Fatigue Optimization of an Induction Hardened Shaft Under Combined Loading

by

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(ABSTRACT)

An integrated procedure, combining finite element modeling and fatigue analysis methods, is developed and applied to the fatigue optimization of a notched, induction hardened, steel shaft subjected to combined bending and torsional loading. Finite element analysis is used first to develop unit-load factors for generating stress-time histories, and then, employing thermo-elastic techniques, to determine the residual stresses resulting from induction hardening. These stress fields are combined using elastic superposition, and incorporated in a fatigue analysis procedure to predict failure location and lifetime. Through systematic variation of geometry, processing, and loading parameters, performance surfaces are generated from which optimum case depths for maximizing shaft fatigue performance are determined. General implications of such procedures to the product development process are discussed.
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1 Introduction

Fatigue failure of components and structures has commanded the attention of engineers for more than a century. While considerable progress has been made in understanding the fatigue process and in predicting structural performance, it has only been since the 1970’s that fatigue methods have begun to appear as an integral part of the product development process [1, 2].

In 1962, the Society of Automotive Engineers (SAE) formed the Fatigue Design and Evaluation (FD&E) Committee with the mission of investigating the practical aspects of fatigue technology. During the 1980’s, this committee began an extensive investigation of a "component like" test specimen under multi-axial loading [3]. The first phase of the program involved testing and analysis of a notched shaft in an unheat treated condition. In the second phase, the shaft was surface processed by induction hardening and tested under combined bending and torsional loading [4]. Under these circumstances, fatigue life is influenced by:

- the hardness gradient, which results in a variation of the material properties and the development of residual stresses,
- the notch acuity,
- and the nature of the combined loading.

The interplay of these three issues, component processing, geometry, and loading, makes fatigue life estimates for the shaft a complex problem and creates the need for an integrated design approach. It is the development of a comprehensive fatigue analysis procedure that provides the focus for this study. First, methods are developed for predicting shaft fatigue life as a function of:

- the notch factor,
- the depth of hardening,
- and the applied loading.
An approach combining finite element modeling with modern fatigue life analysis techniques is employed. Then, a fatigue optimization scheme for induction hardened components is developed. Finally, general guidelines are established for the implementation of these analytical tools in product development and evaluation.
2 Background

In arriving at an integrated analysis scheme for this component, it is necessary to combine the essential features of the induction hardening process with finite element modeling and fatigue analysis. This section will provide an overview of key issues in each of these areas in preparation for the detailed development of analysis procedures in later sections.

2.1 Surface Processing

Surface processing of steel can take many forms (shot peening, selective hardening, coatings, chemical conversions, etc.) to serve a variety of purposes (changes in material properties, development of residual stress, protection against external environment, etc.) [5]. Induction, the focus of this study, is a common technique for providing the fatigue resistance of steel components under bending and torsional loading through material strengthening and the generation of favorable residual stresses.

2.1.1 Hardening of Steels

Hardening of steel is a three-step process involving an initial heating (austenitization), a rapid cooling (quenching) followed by a re-heating (tempering).

Step one: **Austenitization**. In this initial step carbon goes into solid solution with iron in the gamma, face-centered cubic phase, called **austenite**. Austenite is formed by heating the steel above its critical temperature and holding for a period of time; it will transform as soon as the temperature falls below the critical temperature (generally around 700 °C).

Step two: **Rapid cooling**. A fast cooling, or quenching, of the austenite obtained
during the heating process will produce martensite, a body centered tetragonal phase. The amount of martensite produced is dependent on the amount of austenite at the beginning of the fast cooling process and the rate of cooling. The more austenite present at the beginning of the quenching and the higher the cooling rate, the more martensite will be present at the end of the process.

Step Three: Re-heating. Most steels are not serviceable ”as quenched”; they are usually too brittle. Therefore, it is necessary that the steel be re-heated after the quenching process. The heating temperature, this time, stays below the critical temperature. This step, called tempering, is actually a softening process; it gives the material back some ductility.

This martensitic transformation induces two major changes in the material:

- an increase in strength properties of the steel, which will lead to a better endurance at long lives, and
- a volume expansion due to carbon atoms trapped in the iron lattice during the quenching.

2.1.2 Induction Hardening

Because steel is electrically conductive, it can be heated locally by means of magnetic induction powered by AC current [6]. A typical induction hardening assembly is presented in Figure 1. Induction heating is fast, clean and repeatable. It is not necessary for the coil to be in contact with the component. In addition, it produces a very precise heating, allowing localized or surface zone heating. Unfortunately, part of this preciseness in heating is lost during the quenching process, where thermal conduction and geometry play an important role.

The volume expansion created during the hardening process can generate internal
residual stresses in the component. In fact, when the component is subjected to this partial hardening, only the hardened area is affected by the volume expansion (here, the surface of the shaft) leaving the core unaffected. Due to this volume mismatch, residual stresses are developed: compressive at the surface, tensile in the core. Such stresses can influence significantly fatigue life of the shaft and, therefore, have to be taken into account during the stress analysis.
2.2 Finite Element Analysis

A complete mechanics analysis of the shaft requires determination of the stresses induced both by external loading and by the surface processing. The finite element method provides a powerful tool for conducting such calculations. The technique is based on the division of a geometric structure into a large number of elements. The physical behavior of these elements is assumed to follow simple mathematical equations. Applying, then, physical equilibrium on each element leads to a system of equations in which the unknowns are the physical quantities (strain, temperature, etc.) of the elements at each node (or Gaussian integration point) of the model. This numerical method is particularly well adapted to computer application.

In order to handle time-varying load histories, individual unit loads (for bending and torsion) are applied to finite element model. The results of these computations provide stress coefficients for critical elements \( S_{\text{UnitBending}} \) and \( S_{\text{UnitTorque}} \), that through elastic superposition allows determination of the time-varying stresses from the time-varying loading:

\[
S_{\text{Bending}}(t) = S_{\text{UnitBending}} \times F_{\text{Bending}}(t) \tag{1}
\]

\[
S_{\text{Torque}}(t) = S_{\text{UnitTorque}} \times F_{\text{Torque}}(t) \tag{2}
\]

If plastic deformations are present, they can be taken into account during the fatigue life analysis.

While surface processing affects the strength properties of the metal, \( E \) (Young modulus), \( G \) (Shear modulus) and \( \nu \) (Poisson ratio) remain unchanged. Therefore, the residual stresses due to surface processing in elastic analysis are only dependent on the volume expansion and can be calculated using a thermo-elastic finite element model. The volume expansion due to hardening is then simulated by thermal expansion: temperatures are applied at each node of the model as a function of the assumed hardness number at this location thus providing a method to compute elastic residual
stresses due to hardening.

2.3 Fatigue Methods

Modern approaches to fatigue life prediction, based on "local stress-strain" concepts, involve modeling of the material deformation response at critical regions in a structure under complex loading sequences [7]. From this local response, individual events are identified, their stress-strain characteristics documented, and damage assessed using material fatigue curves. These methods have been successfully applied to a range of structural problems over the past thirty years and are used increasingly in product design.

For our purpose, the cyclic loading will be analyzed as follow:

- multichannel stress-time histories are combined through a superposition process (unit stresses are multiplied by their respective load-time histories and the residual stresses are added to their sum),
- the orientation of planes at critical locations experiencing the highest stress excursions are identified,
- a cycle counting algorithm is applied that identifies events by strain range and mean stress (see [8] p707 - p715),
- damage is assessed using appropriate mean stress correction and a life time is estimated.

Next, Chapter 3 details the finite element analysis, while Chapter 4 presents the fatigue life computations.
3 Finite Element Analysis

The finite element analysis serves two purposes in this study:

- the computation of the stresses due to unit loading, and
- the prediction of the residual stresses generated by the induction hardening.

All the following computations were done using the ALGOR finite element code [9], supported on a Silicon Graphics Indigo 2 workstation.

3.1 Loading Analysis

The finite element analysis of the stresses due to loading involves first, the creation of a mesh according to the geometry of the shaft, then boundary conditions are applied. Finally, the prediction of loading stresses are presented.

3.1.1 Geometry

A first model of the shaft was developed using the dimensions presented in Figure 2. The finite element model is axisymmetric. It is built of 3952 three-dimensional brick and 944 hybrid brick elements (Figure 3) for a total number of 4896 elements and 4780 nodes.

Element density increases in the vicinity of the notch, in order to precisely analyze the stress gradient in this area. A half cut of the finite element model is presented in Figure 4.

This model was subsequently modified to obtain the stresses for notch radii of 4, 5, 6 and 7 mm. Zoomed cuts of the notch area for these 4 different models are presented in Figure 5.
The relevant material properties for the SAE 1045 steel are:

\[ E = 200 \, MPa \quad , \quad \nu = 0.3 \quad , \quad G = \frac{E}{2 \times (1 + \nu)} = 76.9 \, MPa \quad (3) \]
3.1.2 Boundary Conditions

The experimental set-up used by the SAE committee for shaft fatigue testing is presented in Figure 6. The shaft is clamped at its largest extremity (area A of Figure 2) with the loads applied at the other extremity (area B of Figure 2) through hydraulic pistons mounted perpendicular to the shaft axis. When the two pistons are moving in phase a pure bending is applied; when moving in opposite phase, a
pure torque is applied to the shaft. In this way any combination of bending moment and torque can be applied to the shaft. To reproduce the clamping on the finite element models, the three degrees-of-freedom in translation of the nodes located on the external layer have been set to zero (note that the nodes of the brick and hybrid brick elements do not allow degrees of freedom in rotation).

To reproduce the bending load, a force was applied in the Z-direction on the node positioned at $X = 310$ mm, $Y = 0$ mm and $Z = 21.5$ mm. To reproduce the torsional loading, two opposite forces were applied in the Y-direction: the first at the same point as the bending force; the second was applied to the point symmetric of the first one, with respect to the axis of symmetry.

The applied forces were chosen to obtain a unit bending moment and a unit torque at the notch as follows:
- unit bending moment: 6.897 N applied at 145 mm from the notch
- unit torque: 23.257 N applied at 21.5 mm from the neutral axis

In this way, the stresses created by every combination of bending moment and torque can easily be obtained using the principle of superposition.

### 3.1.3 Loading Stress Results

Longitudinal stresses due to bending of the 5 mm notched shaft are presented in Figure 7(a) while the circumferential stresses due to torque of the shaft are presented in Figure 7(b). These two figures present typical stress contour plots for axisymmetric notched components with the notch concentration readily apparent. Note that the notch effect is greater in bending than in torsion.

![Tensor](image)

(a) Longitudinal stresses due to bending moment

![Tensor](image)

(b) Shear stresses due to torsional moment

Figure 7: Loading stress
In order to obtain the notch factors in bending ($K_b$) and in torque ($K_t$), stresses due to bending and torque at the surface of unnotched shaft were computed by linear interpolation of the stresses obtain from the finite element model at the core, and compared to the stresses at the notch. The notch factors are presented in Table 1. In addition, this table shows notch factors for the same radii obtained from photo-elastic tests reported by R.E. Peterson [10]. Good correlation is noted between the finite element computations and the measured values of the notch factors.

Table 1: Notch factors

| Notch factor (mm) | Finite element results | Measurements |
|:---|:---|:---|:---|:---|
| | $K_b$ | $K_t$ | $K_b$ | $K_t$ |
| 4 | 1.68 | 1.34 | 1.67 | 1.35 |
| 5 | 1.59 | 1.30 | 1.57 | 1.29 |
| 6 | 1.52 | 1.27 | 1.50 | 1.25 |
| 7 | 1.47 | 1.25 | 1.45 | 1.22 |

### 3.2 Residual Stress Analysis

As explained earlier, residual stresses develop in the shaft during surface processing as a result of the volume expansion of the hardened area of the component. It is important to be able to predict these residual stresses because they can significantly affect the fatigue life of the shaft.

### 3.2.1 Volume Expansion Mechanism

The volume expansion accompanying the surface processing is due to the formation of martensite. In order to relate the volume expansion to the hardness level, we must, first find a relationship between the martensite content and the hardness of the steel.
As shown in Figure 8, the percentage of martensite is a linear function of the hardness [11]:

\[ \%\text{Martensite} = 0.28023 \times BHN - 66.127 \]  

(4)

where \( BHN \) is the Brinell hardness number.

![Graph showing the relationship between percentage of martensite and Brinell Hardening Number (BHN).](image)

**Figure 8: Percent martensite as a function of hardness number**

In addition, the linear strain due to martensite volume expansion is about 0.1% for the SAE 1045 steel. Oschner [11] has successfully calculated residual stresses created by the hardening process by means of a thermal expansion model. The idea is to use the thermal expansion capability of finite element analyzes to simulate the
volume expansion due to hardening, using the simple formula:

\[ \Delta \epsilon = \alpha \ast \Delta T \]  \hspace{1cm} (5)

where \( \Delta T \) is the temperature range, \( \alpha \) the coefficient of thermal expansion and \( \Delta \epsilon \) is the linear strain range.

Instead of using a fixed \( \Delta T \) and varying \( \alpha \), as Oschner did, in this model the thermal expansion coefficient \( \alpha \) is kept fixed while the temperature range \( \Delta T \) is allowed to vary. The volume expansion due to hardening is proportional to the percentage of martensite which, in turn, is a linear function of hardness. Therefore, the temperature applied to simulate the stresses due to volume expansion created during surface processing can be assumed a linear function of the hardness.

A thermal expansion coefficient of 3.81 \(10^{-5} \, ^\circ C^{-1} \) was chosen, while the range of temperature to be used to represent the volume expansion induced by surface processing was chosen to range from 0 to 100 \( ^\circ C \), with 0 \( ^\circ C \) representing a hardness of 225 BHN and 100 \( ^\circ C \) representing a hardness of 595 BHN. The linear relationship between hardness and temperature can then be written:

\[ \Delta T = 0.28023 \ast BHN - 66.127 \]  \hspace{1cm} (6)

where \( \Delta T \) is the temperature range and \( BHN \) is the Brinell hardness number.

The fictive temperature to be applied at each node of the model can now be determined as a function of the hardness at the location of the shaft represented by this node, using Equation 6. In order to simulate the volume expansion due to the induction hardening, we need to know the hardness profile of the shaft through the thickness.

Oschner used hardness profiles at different locations of the shaft, including the notch, to design a precise finite element model in which the positions of the nodes follow the iso-hardness contours based on experimental measurements. This finite element model is therefore specially adapted to the case depth that he was looking at.
But, for our purpose, we want to be able to include the volume expansion due to induction hardening in order to predict the residual stresses, using the previously developed finite element models. So, we can obtain the residual stresses at the same location where stresses due to loading were computed, without using any interpolation. Plus, we want to be able to predict the residual stresses due to several case depths of hardening. It is thus necessary to find a way to predict the hardness number at the location of a node, instead of positioning the nodes with respect to the hardness number they are supposed to represent.

In order to predict the hardness number of a node in our model, we must obtain the hardness number as a continuous function of the depth. As shown in Figure 9, using a curve fitting of the form of an arctangent of a polynomial of the seventh degree gives us the following approximation of the hardness number as a function of depth:

\[
BHN = -\frac{380}{\pi} \arctan \left[ 3.762610^{-7}X^5 - 0.67245X^4 + 4.78426623484166X^3 \\
- 17.0638X^2 + 33.0582X - 29.9391 \right] + 415
\]  \hspace{1cm} (7)

where \( X \) represents the distance from the surface, and \( BHN \) the Brinell hardness number.

This curve fitting is based on eight measurements, and uses eight constants of interpolation, thus giving exact values at each measurement depth. The measurements presented by Oszchner correspond to a case depth of 16\%, which means that the distance corresponding to the mean value of hardness (between hardness at the surface and at the neutral axis), is 16\% of the distance between the surface and the neutral axis.

A simple approximation to obtain the hardness number as a function of the depth is to assume that hardness profiles are proportional to the depth of hardening. This means, for example, that if we want to obtain this hardness profile for a depth of hardening of 8\%, instead of 16\%, we just have to multiply by two the labels of the horizontal axis in Figure 9. The curve fitting of Equation 7 now becomes a function
of the distance from the surface, and of the case depth:

\[
BHN = -\frac{380}{\pi} \arctan \left[ 3.762610^{-7} \left( X \times \frac{D}{16\%} \right)^5 - 0.67245 \left( X \times \frac{D}{16\%} \right)^4 \\
+ 4.7843 \left( X \times \frac{D}{16\%} \right)^3 - 17.0638 \left( X \times \frac{D}{16\%} \right)^2 \\
+ 33.0582 \left( X \times \frac{D}{16\%} \right) - 29.9391 \right] + 415
\] (8)

where \( D \) represents the case depth, \( X \) the distance from the surface, and \( BHN \) the Brinell hardness number.

![Graph showing the relationship between Brinell Hardness Number (BHN) and distance from surface with a case depth of 16%.](image)

Figure 9: Hardness number as a function of the depth from the surface
We are now able to approximate the value of the hardness number at the notch, between the surface and the neutral axis, for every node, and for every case depth. But we need the hardness number, not only for the nodes positioned at the notch, but for each node of the finite element model.

It is important to notice that the resulting hardness profile through the shaft can quite sensitive to processing details. The induction method of heating the metal offers a precise control on the area to be hardened and, even if the quenching removes a large part of this preciseness, it is quite easy to vary the depth of hardening along the neutral axis of the shaft. Therefore, the hardness profiles may vary from one coil to another.

However, our concern is centered around the notch and along the area of the shaft with a constant diameter of 40 mm because this is where all the failures are expected to occur (all the other areas having much larger diameters).

According to Oschner's hardness measurements, it appeared that the depth of hardening is deeper at 20 mm away from the notch than at the notch. However, the hardness profile at 60 mm from the notch is similar to the hardness profile at the notch. Although the shape of the hardness profile can have a major effect on the residual stresses due to induction hardening, the hardness profile is highly dependent on the induction hardening mounting. In addition, notch geometry that has an important effect on the stresses due to loading, are erased from the hardness profile, due to thermal conduction occurring during the surface processing.

Therefore, in order to present a general and simple prediction of the residual stresses due to induction hardening, we can assume that the hardness profile is only dependent on the distance from a fictive surface presented in Figure 10. This surface corresponds to the surface of the shaft everywhere except in the area before the notch and at the notch, where it corresponds to the tangent between the notch and the extremity of the 63.5 mm diameter area (tangent between Circle A and Point B in Figure 10).
Although somewhat less precise than Oschner’s, this approach has the advantage of being able to handle many different depths of hardening with only one measurement of the hardness number through the thickness at the notch. For a more precise analysis, this approach can be refined, based on experimental results for different case depths and various induction hardening mountings.

3.2.2 Finite Element Model

The finite element models used previously for the prediction of the stresses due to loading are taken as bases for these new finite element models. Using the symmetry around the neutral axis, the loading finite element models can be cut through any plane going through the neutral axis. We chose to cut them in the plane x-z (x: neutral axis, z: vertical direction) and the plane x-y. Each one of these thermal finite element models (4 models corresponding to the 4 notch diameters mentioned earlier) includes a total of 1224 elements, partitioned as:
- 988 brick elements
- 236 hybrid brick elements

Each has a total of 1513 nodes.

To conserve the neutral axis symmetry during the finite element analysis, the freedom of translation in the y-direction was canceled for all the nodes located on the plane x-z, and the freedom of translation in the z-direction was canceled for all the nodes located on the plane x-y.

To fix the thermal finite element models in the three-dimensional space, all the freedoms in translation of one of the nodes on the neutral axis were removed (this is the equivalent of the boundary conditions due to the clamping in the loading finite element models, but here, all the stresses are due to internal deformations and we do not need to clamp the finite element models in several nodes because the external forces applied are non-existent).

Finally, a FORTRAN code was developed to read the finite element analysis input file of these models and automatically rewrite them, applying the appropriate temperature to each node of the models. This code computes, for each node, its distance from the fictive surface described earlier, and using Equation 8 combined with Equation 4, computes the temperature to be applied at this node. In order to analyze the effect of the hardening depth on the fatigue life of the shaft, seven case depths were utilized: from 5% of the depth hardened to 35%, by increments of 5%. In addition to the material properties used for the loading computations, a coefficient of thermal expansion, $\alpha$, of 3.81 °C$^{-1}$ was included.

A plot of the results obtained for the longitudinal residual stresses due to hardening for a case depth of 20% is presented in Figure 11(a), and, for the same case depth, a plot of the circumferential residual stress is presented in Figure 11(b). On these plots, we can see that induction hardening induces compressive residual stresses at the surface of the shaft (represented by the dark shades of gray), while tensile residual
stresses appear at the core. Notice the stress relaxation around the upper part of the notch for longitudinal residual stress. This stress relaxation is due to the boundary condition around this area. Stress relaxation also occurs for the circumferential residual stress, but is weaker. In addition, the compressive circumferential residual stress goes deeper into the shaft than the compressive longitudinal residual stress.

In order to give a more precise representation of these residual stresses, plots of the longitudinal and residual stresses as a function of the distance from the surface at the notch is presented in Figure 12. Here again differences are noted between the longitudinal and circumferential residual stresses: at the core, the longitudinal component displays a much higher tensile value than the circumferential component.

(a) Longitudinal residual stresses

(b) Circumferential residual stresses

Figure 11: Residual stresses due to induction hardening for a case depth of 20%
Figure 12: Residual stresses due to hardening versus distance from the neutral axis

However, they both present similar values of compressive stress at the surface. In addition, the longitudinal component increases in compression at the surface, whereas the circumferential stress diminishes slightly. Although these results are very comparable to Oschner’s predictions, his measurements of the longitudinal residual stress do not show an increase in compressive stress at the surface.

Table 2 presents the values of maximum tensile residual stresses in the core of the shaft, the values of maximum compressive residual stresses at the surface, and the location of zero residual stresses. We can see that as the depth of hardening increases,
the compressive stresses at the surface decrease, while the tensile stresses in the core of the shaft increase, and the locations of free state of stress go deeper into the shaft. Notice that between 5% and 10% of hardening, for the circumferential residual stress, the location of free state of stress presents the opposite tendency. This is due to linear interpolation approximation.

To summarize the finite element results, we first obtained the elastic stresses due to unit bending moment with respect to the notch and due to unit torque with respect to the neutral axis. Then we were able to predict the elastic residual stresses due to induction hardening for a range of case depths. All these results were obtained for 4 different notch values (4, 5, 6 and 7 mm), at every node of the finite element model. The utilization of this information in fatigue life analysis is detailed in the next section.
Table 2: Residual stresses for the different case depths

<table>
<thead>
<tr>
<th>depth of hardening (%)</th>
<th>longitudinal stress</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>maximum in tension (MPa)</td>
<td>maximum in compression (MPa)</td>
<td>location of zero* (mm)</td>
</tr>
<tr>
<td>5</td>
<td>178.1</td>
<td>-1297</td>
<td>18.2</td>
</tr>
<tr>
<td>10</td>
<td>283.2</td>
<td>-1191</td>
<td>17.5</td>
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<tr>
<td>15</td>
<td>381.3</td>
<td>-1079</td>
<td>16.6</td>
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<td>20</td>
<td>469.5</td>
<td>-963.5</td>
<td>15.8</td>
</tr>
<tr>
<td>25</td>
<td>551.9</td>
<td>-854.5</td>
<td>15.0</td>
</tr>
<tr>
<td>30</td>
<td>621.6</td>
<td>-751.9</td>
<td>14.2</td>
</tr>
<tr>
<td>35</td>
<td>671.7</td>
<td>-655.2</td>
<td>13.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>depth of hardening (%)</th>
<th>circumferential stress</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>maximum in tension (MPa)</td>
<td>maximum in compression (MPa)</td>
<td>location of zero* (mm)</td>
</tr>
<tr>
<td>5</td>
<td>61.67</td>
<td>-1111</td>
<td>15.9</td>
</tr>
<tr>
<td>10</td>
<td>115.3</td>
<td>-1016</td>
<td>16.2</td>
</tr>
<tr>
<td>15</td>
<td>160.5</td>
<td>-925.9</td>
<td>15.5</td>
</tr>
<tr>
<td>20</td>
<td>208.3</td>
<td>-851.9</td>
<td>14.7</td>
</tr>
<tr>
<td>25</td>
<td>251.8</td>
<td>-769.0</td>
<td>14.0</td>
</tr>
<tr>
<td>30</td>
<td>285.4</td>
<td>-691.8</td>
<td>13.3</td>
</tr>
<tr>
<td>35</td>
<td>304.0</td>
<td>-616.4</td>
<td>12.4</td>
</tr>
</tbody>
</table>

* Distance from the neutral axis
4 Fatigue Life Analysis

In order to perform a fatigue life analysis of the shaft, in addition to the elastic stresses obtained from the finite element analysis, it is necessary to know:

- the cyclic material properties of the SAE 1045 steel as a function of hardness, and
- the load-time histories in bending moment and torque.

4.1 Cyclic Material Properties

The hardness number computed by the FORTRAN code for each node depends on the position of the node with respect to the fictive surface described in Figure 10 and the case depth of hardening. However, the position of the node with respect to the fictive surface is not really controllable. Therefore, the hardness numbers applied at the nodes of the finite element models can vary anywhere between the maximum hardness number reached at the surface (595 BHN) and the minimum on the neutral axis of the shaft (225 BHN). So, in order to compute the fatigue life of each node, we need a continuous set of values for the cyclic material properties \( \sigma'_f, \epsilon'_f, b, c, n', \) and \( K' \) for the SAE1045 steel hardened from 225 to 595 BHN.

Such continuous sets of values for the cyclic material properties does not exist in current literature, but, using two discrete sets of data presented in Table 3, it is possible to interpolate continuous functions of the hardness number representative of the cyclic material properties using curve fitting methods.

Curve fitting of c: It is assumed that the value of c is not sensitive to the level of hardening of the metal. Therefore, as shown in Figure 13(a), the constant chosen as value for c is the the average of the values of the data sets:

\[
c = 0.68
\]
Table 3: Cyclic material properties data sets

<table>
<thead>
<tr>
<th>Set of Data</th>
<th>BHN</th>
<th>$\sigma_f'$</th>
<th>b</th>
<th>$\epsilon_f'$</th>
<th>c</th>
<th>K'</th>
<th>n'</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>225</td>
<td>180</td>
<td>-.095</td>
<td>1.000</td>
<td>-.66</td>
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<td>-.090</td>
<td>1.000</td>
<td>-.65</td>
<td>207</td>
<td>.180</td>
</tr>
<tr>
<td></td>
<td>280</td>
<td>183</td>
<td>-.077</td>
<td>1.000</td>
<td>-.66</td>
<td>199</td>
<td>.150</td>
</tr>
<tr>
<td></td>
<td>330</td>
<td>200</td>
<td>-.070</td>
<td>1.000</td>
<td>-.69</td>
<td>239</td>
<td>.140</td>
</tr>
<tr>
<td>#2**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>390</td>
<td>260</td>
<td>-.079</td>
<td>.450</td>
<td>-.68</td>
<td>238</td>
<td>.110</td>
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<tr>
<td></td>
<td>410</td>
<td>270</td>
<td>-.073</td>
<td>.600</td>
<td>-.70</td>
<td>284</td>
<td>.100</td>
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<td>278</td>
<td>-.070</td>
<td>.270</td>
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<td>.110</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>384</td>
<td>-.096</td>
<td>.210</td>
<td>-.66</td>
<td>361</td>
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<td>595</td>
<td>455</td>
<td>-.084</td>
<td>.140</td>
<td>-.76</td>
<td>561</td>
<td>.120</td>
</tr>
<tr>
<td></td>
<td>670</td>
<td>340</td>
<td>-.071</td>
<td>.004</td>
<td>-.57</td>
<td>682</td>
<td>.125</td>
</tr>
</tbody>
</table>

* Provided by Dr. R. W. Landgraf, Virginia Polytechnic Institute.

** Provided by the Material Properties Subcommittee, Society of Automotive Engineers.

** Curve fitting of n': By looking at the plot of n' versus the hardness number presented in Figure 13(b), we can reasonably apply a curve fitting using a second order polynomial, resulting in the following equation is obtained:

\[
n' = 9.366110^{-7}(\text{Hardness})^2 - 8.7295110^{-4}\text{Hardness} + 0.3111\quad (10)
\]

where \text{Hardness} is the Brinell hardness number.
Figure 13: Cyclic material properties

Curve fitting of $\sigma'_f$ and $\epsilon'_f$: By plotting the sets of data corresponding to these two material properties, using a semi-logarithmic scale as shown in Figures 13(c) & (d), we can assume that $\sigma'_f$ is an exponential function of the hardness number, and $\epsilon'_f$ a piecewise exponential function of the hardness number. Applying a least-squares method using exponential and piecewise exponential functions, the following functions are obtained for these 2 material properties:

\[
\sigma'_f = e^{(2.946310^{-3} \text{ Hardness} + 4.37711)}
\]

\[
\epsilon'_f = \begin{cases} 
1 & \text{for Hardness < 350BHN} \\
 e^{(-8.729110^{-3} \text{ Hardness} + 2.97)} & \text{for Hardness > 350BHN}
\end{cases}
\]
Finally, instead of using curve fitting methods to obtain continuous data sets for \( b \) and \( K' \), we can use the following equations, combined with the values obtained for \( n' \), c, \( \sigma_f' \) and \( \epsilon_f' \) [7]:

\[
\begin{align*}
  b &= n' * c \\
  K' &= \frac{\sigma_f'}{(\epsilon_f')^{n'}}
\end{align*}
\]  

We now have continuous data sets for the cyclic material properties of the SAE1045 steel, hardened between 225 and 595 BHN.

4.2 Load-Time Histories

The last inputs necessary for the fatigue life analysis are the load-time histories. Two types of histories were selected for the purpose of our study:

- Automotive load-time histories *
- Log skidder load-time histories **

In order to test the specimen, these load-time histories are repeatedly applied to the shaft until failure occurs.

4.2.1 Automotive Load-Time Histories

These load-time histories represents measurements made on a vehicle during durability testing at Ford Motor Company. These histories were originally composed of 49039 time indices, but have been filtered to 5816 time indices in order to reduce the computation time. The filter removes the time indices whose corresponding moment value differs from the moment value of the previous time indices by less than 5% of the maximum moment amplitude. Note that this filter is based on consecutive peaks

*provided by Dr. S. Thangjitham at Virginia Tech.

**obtained from the SAE data base.
or valleys. Some cycles of range value less than 5% of the maximum range can still occur in a load-time history submitted to this type of filtering.

In addition, the histories were scaled such that the maximum amplitude is equal to 1000 Nm in order to simplify their manipulation. The history corresponding to the bending moment is presented in Figure 14(a) while the torque history is presented in Figure 14(b).

![Graphs showing bending moment and torque versus time]

Figure 14: Automotive load-time histories

### 4.2.2 Log Skidder Load-Time Histories

These load-time histories are composed of 109420 time increments for the bending, and 73349 time increments for the torque; they also have been scaled to a maximum
amplitude of 1000 Nm. They have not been filtered, but time indices that represent intermediate values of bending and torque have been removed (time indices that do not represent a peak or a valley either in bending or in torque, and therefore, will never appear in the rainflow cycle counting). The number of time indices in these load-time histories is then reduced to 68624. The history corresponding to the bending moment is presented in Figure 15(a), while the torque history is presented in Figure 15(b). In these histories, the number of high amplitude events is small compared to the number of small amplitude events. Therefore, the percentage of damage due to small range cycles will have a tendency for be higher than for the previous automotive histories.

![Bending moment versus time](image)

![Torque versus time](image)

Figure 15: Skidder load-time histories

We now have all the inputs necessary to proceed to the fatigue life analysis. A
detailed explanation of this analysis is presented in the following.

4.3 Fatigue Life Code

As described in section 2.3, several steps are involved in performing fatigue life analysis. For this study, a code developed by F.A. Conle of Ford Motor Company was used [12]. The main features of this code are shown in the flow-chart of Figure 16; a detail description of the analysis steps follows.

![Fatigue life analysis procedure flow-chart](image)

**Figure 16: Fatigue life analysis procedure flow-chart**

4.3.1 Superposition Process

The stresses obtained from the finite element analysis represent an equivalent multi-channel multi-axial state of stress. They must to be recombined into a single-channel, multi-axial state of stress before further analysis. Using the superposition principal, the elastic stresses due to unit loads in bending and torque are multiplied by their respective load-time histories thus creating multi-axial stress-time histories in bending
and torsion. These stress-time histories are then combined up to produce a single-channel multi-axial stress-time history. Finally, the residual stresses due to induction hardening are added to this history as follows:

\[
\sigma_{zz}(t) = k_{zz_{\text{Bending}}} C_{\text{Bending}} F_{\text{Bending}}(t) + k_{zz_{\text{Torque}}} C_{\text{Torque}} F_{\text{Torque}}(t) + \sigma_{zz_{\text{Residual}}} \tag{15}
\]

\[
\sigma_{yy}(t) = k_{yy_{\text{Bending}}} C_{\text{Bending}} F_{\text{Bending}}(t) + k_{yy_{\text{Torque}}} C_{\text{Torque}} F_{\text{Torque}}(t) + \sigma_{yy_{\text{Residual}}} \tag{16}
\]

\[
\tau_{xy}(t) = k_{xy_{\text{Bending}}} C_{\text{Bending}} F_{\text{Bending}}(t) + k_{xy_{\text{Torque}}} C_{\text{Torque}} F_{\text{Torque}}(t) + \tau_{xy_{\text{Residual}}} \tag{17}
\]

where \( k_{zz_{\text{Bending}}}, k_{yy_{\text{Bending}}}, k_{xy_{\text{Bending}}}, k_{zz_{\text{Torque}}}, k_{yy_{\text{Torque}}} \) and \( k_{xy_{\text{Torque}}} \) are the unit bending moment and unit torque coefficients (stresses obtained from the finite element analysis under unit bending moment and unit torque), \( F_{\text{Bending}}(t) \) and \( F_{\text{Torque}}(t) \) are the load-time histories, \( t \), the time indices, \( C_{\text{Bending}} \) and \( C_{\text{Torque}} \), the magnification factors applied to the load-time histories, \( \sigma_{zz_{\text{Residual}}}, \sigma_{yy_{\text{Residual}}} \) and \( \tau_{xy_{\text{Residual}}} \), the residual stresses due to induction hardening, and \( \sigma_{zz}(t), \sigma_{yy}(t) \) and \( \tau_{xy}(t) \), the single-channel multi-axial stress-time history.

### 4.3.2 Principal Angle Determination

In order to determine the principal angle, a two dimensional zero matrix is created. The first dimension of this matrix represents the angle categories (from \(-90^\circ\) to \(90^\circ\)), the second dimension represents the stress categories. For each time increment of the multi-axial stress-time history, the principal stresses and their corresponding angles with respect to the reference axis are computed using the following formulas:

\[
\tan(2\theta(t)) = \frac{2\tau_{xy}(t)}{\sigma_{xx}(t) - \sigma_{yy}(t)} \tag{18}
\]

\[
\sigma_{\text{Principal}} = \begin{pmatrix}
\sigma_{xx}(t) & \sigma_{xy}(t) \\
\sigma_{xy}(t) & \sigma_{yy}(t)
\end{pmatrix}
\begin{pmatrix}
\cos^2(\theta(t)) & \sin(\theta(t))
\end{pmatrix}
\begin{pmatrix}
\cos(\theta(t)) & \sin(\theta(t))
\end{pmatrix}

= \sigma_{xx}(t)\cos^2(\theta(t)) + \sigma_{yy}(t)\sin^2(\theta(t))

\pm 2\tau_{xy}(t)\sin(\theta(t))\cos(\theta(t)) \tag{19}
\]
where $\sigma_{xx}(t)$, $\sigma_{yy}(t)$ and $\tau_{xy}(t)$ are the multi-axial stress-time history, $t$ the time indices, $\theta(t)$ the principal angle computed for each time indices, and $\sigma_{principal}$ the principal stresses corresponding to this angle.

The matrix cells corresponding to these angles and principal stresses are then incremented by 1 (note that when the minus sign is used for the computation of the principal stress, the corresponding angle for this stress is $90^\circ$ away from the actual $\theta(t)$ computed in Equation 18). Two cells of the matrix are incremented in this way for each time increment throughout the history.

Once the matrix is totally filled, the most active fatigue angle is determined using the following rules:

1. Select the angle for which the column presents the maximum stress range.
2. If two or more columns present the same maximum stress range, select the angle corresponding to the column with the maximum moment of inertia.

Finally, the multi-axial stress-time history is transformed into a principal stress-time history for this principal angle using the following equation:

$$S(t) = \sigma_{xx}(t)\cos^2(\alpha) + \sigma_{yy}(t)\sin^2(\alpha) + 2\tau_{xy}(t)\sin(\alpha)\cos(\alpha)$$

where $\sigma_{xx}(t)$, $\sigma_{yy}(t)$ and $\tau_{xy}(t)$ are the multi-axial stress-time histories and $t$ the time indices. The principal angle $\alpha$ is computed following the previous rules and $S(t)$ is the principal stress-time history.

4.3.3 Rainflow Cycle Counting

In order to proceed to the damage calculation, the cycles of the principal stress-time history must be identified. This is accomplished using a rainflow cycle counting procedure [8], in which stresses reversals are identified going from a peak to a valley.
or vice-a-versa. The actual algorithm used for the rainflow cycle counting in Conle’s code, was developed by S. Downing [13]. As shown in figure 17(a), the peak B and the valley C form a cycle according to the rainflow cycle counting procedure. To form a cycle, these peaks and valleys do not have to be consecutive in the time history, as shown in Figure 17(b), where the valley A and the peak D form another cycle. Note, that each cycle is formed by one and only one peak and one and only one valley. In addition, each peak and valley of the time history is associated with a particular cycle.

![Figure 17: Example of cycles](image)

Once identified, the cycles are classified according to their stress range and mean stress (or maximum and minimum of stress) into a rainflow matrix. The maximum stress of a cycle corresponds to the stress at the peak of this cycle, and the minimum of stress corresponds to the valley. The range and mean of the cycle are then obtained using:

\[
S_{\text{range}} = \frac{S_{\text{maximum}} - S_{\text{minimum}}}{2} \tag{21}
\]

\[
S_{\text{mean}} = \frac{S_{\text{maximum}} + S_{\text{minimum}}}{2} \tag{22}
\]

where, \( S_{\text{maximum}} \) and \( S_{\text{minimum}} \) are the maximum and minimum of stress for the cycle,
and $S_{\text{range}}$ and $S_{\text{mean}}$ are the stress range and mean stress.

Each cell of the rainflow matrix contains the number of cycles for which the stress range and mean stress are the closest to the value of the stress range and mean stress of this cell. It is an approximation, but it is currently assumed to represent well the stress-time history.

Note that the time of occurrence of each peak and valley of the cycles is lost during the rainflow cycle counting procedure. The consequences of this loss of information is explained later in part 4.3.4. The cycle formed by the overall maximum and minimum of stress in the history is called the principal cycle while the others are called secondary cycles.

**4.3.4 Damage Calculation**

The damage calculation is the key procedure of the fatigue life analysis. It involves several steps outlined in Figure 18 and explained in detail in the following.

![Flowchart](image)

Figure 18: Damage calculation flow-chart
**Plasticity Transformation:** Until now, all the stresses that have been presented are elastic stresses. However, plastic deformations may occur and are known to play an important role in fatigue damage accumulation. Therefore, the elastic stresses obtained in the rainflow matrix have to be transformed into plastic stresses and strains. This can be done by means of a Neuber notch analysis and a material stress-strain relation based on the Ramberg-Osgood model [7].

For the maximum absolute value of stress of the principal cycle, the Neuber and Ramberg-Osgood equations are:

\[
\frac{S_a^2}{E} = \sigma_a \epsilon_a \quad (\text{Neuber}) \quad (23)
\]

\[
\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{\frac{1}{n'}} \quad (\text{Ramberg - Osgood}) \quad (24)
\]

where \(S_a\) is the elastic stress value corresponding to the maximum absolute value of stress for the principal cycle, \(\sigma_a\) and \(\epsilon_a\) are the plastic stress and strain amplitudes for the same extremity of this cycle, and \(E, n'\) and \(K'\), the material properties.

**Observation 1:** Because Equation 24 is only defined for positive plastic stress amplitude \(\sigma_a\), if \(S_a\) is negative (i.e., the maximum absolute value of elastic stress corresponds to the bottom of the principal loop), \(S_a\) is taken as positive value before the elasto-plastic transformation, and the plastic stress and strain are transformed back to negative numbers after this transformation.

**Observation 2:** The notch factor, \(K_t\), usually present in the parentheses next to \(S_a\) in the Neuber equation, is not present here. This is due to the fact that the geometry effects have already been taking care of during the finite element analysis and therefore, the notch factor can be set equal to 1.

Figure 19 shows more precisely how equations 23 and 24 are used in order to transform the elastic stress amplitude \(S_a\) (defined as \(S_{\text{max,princ}}\) in this figure) to plastic stress and strain, \(\sigma_a\) and \(\epsilon_a\) (defined as \(\sigma_{\text{max,princ}}\) and \(\epsilon_{\text{max,princ}}\) in this figure). All the other elasto-plastic transformations involve the Neuber and Ramberg-Osgood
Figure 19: Plasticity transformation for the principal loop

equations expressed in terms of stress and strain ranges:

\[
\frac{(\Delta S)^2}{E} = (\Delta \sigma)(\Delta \epsilon) \quad \text{(Neuber)} \quad (25)
\]

\[
\Delta \epsilon = \frac{\Delta \sigma}{E} + 2 \left( \frac{\Delta \sigma}{2K'} \right)^{\frac{1}{n}} \quad \text{(Ramberg - Osgood)} \quad (26)
\]

where $\Delta S$ is the elastic stress range, $\Delta \sigma$ and $\Delta \epsilon$ are the plastic stress and strain ranges.
Note that the equations 25 and 26 are obtained from equations 23 and 24 by replacing \( S_a, \sigma_a \) and \( \epsilon_a \) by \( \Delta S/2, \Delta \sigma/2 \) and \( \Delta \epsilon/2 \). This is to reflect the change of regime occurring at the maximum absolute value of stress for the principal cycle. To go from the origin to this point, the mechanical behavior is monotonic (i.e. no unloading is considered), while after this point, the mechanical behavior is cyclic and therefore, hysteresis loops formed by fully reversed cycles have to be symmetrical about the origin. The hysteresis loop paths are obtained by scaling the initial loading curve by a factor-of-two.

Figure 19 presents the construction of the elasto-plastic transformation for the second extremity of the principal cycle. In this figure, \( S_{\min_{\text{princ}}} \) represents the elastic stress amplitude, \( \sigma_{\min_{\text{princ}}} \) and \( \epsilon_{\min_{\text{princ}}} \) the plastic stress and strain amplitude for the second extremity of the principal cycle. Therefore, we have:

\[
\Delta S = S_{\max_{\text{princ}}} - S_{\min_{\text{princ}}} \quad (27)
\]
\[
\Delta \sigma = \sigma_{\max_{\text{princ}}} - \sigma_{\min_{\text{princ}}} \quad (28)
\]
\[
\Delta \epsilon = \epsilon_{\max_{\text{princ}}} - \epsilon_{\min_{\text{princ}}} \quad (29)
\]

Details for determining the ranges \( \Delta S, \Delta \sigma \) and \( \Delta \epsilon \) for the elasto-plastic transformation of the secondary loops are given below.

**Hanging Loop Process:** During the rainflow cycle counting, the time indices of the stresses are lost. Therefore, it is not possible to position the cycles at their exact location with respect to the principal cycle. For computing the damage due to each cycle, using a simple strain-life approach, this loss of information is not important. The only data required is the plastic strain amplitude, which can be obtained from Equations 25 and 26. But, this approach does not include the mean stress effect.

Two popular variations of the simple strain-life approach that include mean stress effects are: the Smith/Watson/Topper and the Morrow methods [7]. But these approaches need complementary information such as the plastic mean stress, for Morrow,
or the maximum plastic stress, for Smith/Watson/Topper. These plastic stresses are not available after the loss of time indices due to the rainflow cycle counting procedure.

However, we know that the secondary cycles have to be positioned within the boundaries of the principal loop. Therefore, we can obtain the limits of the life by shifting the secondary loop to the lobes of the principal cycle. This is the hanging loop process. The minimum in life corresponds to the case where all the secondary loops are attached to the tensile lobe of the principal cycle, while the maximum in life is obtained when all the secondary loops are attached to the compressive lobe.

Figure 20 presents the hanging loop process on the tensile lobe of the principal cycle. Because this results in the minimum life time (conservative), this configuration is used in all subsequent calculations. The location of the attachment point of the sec-

![Diagram](image)

Figure 20: Hanging loop process in tension
ondary loop on the tensile lobe of the principal cycle is determined using Equations 25 and 26 with the following elastic stress range:

\[
\Delta S = S_{\text{max sec}} - S_{\text{min prin}}
\]  

(30)

where \( \Delta S \) is the elastic stress range, \( S_{\text{min prin}} \), the elastic stress value of the bottom of the principal loop, and \( S_{\text{max sec}} \), the elastic stress value top of the secondary loop. The location of the second extremity of the secondary loop is determined using the procedure introduced for the second extremity of the principal cycle. Note that the ranges associated with the second step of construction of the secondary loop are denoted by a superscript prime (').

For the case of a hanging loop process on the compressive lobe of the principal cycle, the equation becomes:

\[
\Delta S = S_{\text{max prin}} - S_{\text{min sec}}
\]  

(31)

where \( \Delta S \) is the elastic stress range, \( S_{\text{max prin}} \), the elastic stress value of the top of the principal loop, and \( S_{\text{min sec}} \), the elastic stress value bottom of the secondary loop.

We now have the plastic stress ranges and the plastic stress position for every cycle so we can proceed to the life calculation.

**Life Calculation:** In order to compute the total life at a node of the shaft, the damages due to each cell of the rainflow cycle matrix are calculated and, then, summed to obtain the total life. To obtain the damage due to the cycles present in one cell of the rainflow matrix, the strain-life approach is used. This approach assumes that the strain amplitude can be divided into an elastic and a plastic part:

\[
\epsilon_a = \epsilon_{ea} + \epsilon_{pa} = \frac{\sigma_f}{E} (2N_f)^b + \epsilon'_f (2N_f)^c
\]  

(32)

where \( \epsilon_a \) is the total strain amplitude, \( \epsilon_{ea} \) the elastic part of this strain, \( \epsilon_{pa} \) the plastic part, \( E \), \( \sigma_f \), \( \epsilon'_f \), \( b \) and \( c \) are the material properties and, \( N_f \) is the life.
Equation 32 can be formulated in terms of damage:

\[ \epsilon_a = \frac{\sigma_f}{E} (\text{Dam})^{-b} + \epsilon_f (\text{Dam})^{-c} \]  

(33)

where \( \text{Dam} \) represents the damage created by one reversal of strain amplitude \( \epsilon_a \). This equation can be solved numerically for each cycle of the rainflow cycle matrix. The total damage created by the all cycles present in rainflow matrix is obtained using Miner’s rule:

\[ \text{TotalDamage} = \sum_{i=1}^{N} \text{Nb.Rev}_i \times \text{Dam}_i \]  

(34)

where \( N \) is the total number of cells of the rainflow cycle matrix, \( \text{Dam}_i \) the damage due to one reversal of the \( i^{th} \) cell, and \( \text{Nb.Rev}_i \) the number of reversals counted for the \( i^{th} \) cell.

Finally, the total life is defined as the inverse of the total damage:

\[ \text{TotalLife} = \frac{1}{\text{TotalDamage}} \]  

(35)

This total life corresponds to the number of sequences of the load-time history that can be applied before failure occurs.

Figure 21 presents the strain amplitude \( \epsilon_a \) as a function of the life: the \( \epsilon-N \) curve. The plots present the life response for the SAE 1045 steel at a hardness level of 595 BHN and 250 BHN thus encompassing the range of observed hardnesses. Notice that this graphs correspond to constant amplitude loading. Therefore, the life corresponds to the inverse of the damage created by two reversals (or one cycle). The two X-axis show the equivalence between fatigue life and damage for constant amplitude loading.

The \( \epsilon-N \) curves presenting the two hardness levels intersect at a life of approximately \( 10^3 \) cycles and a strain amplitude of \( 10^{-2} \). This behavior is typical of a large number of engineering steels and is indicative of the interplay of material strength and ductility in determining overall fatigue resistance; short-life resistance is governed by ductility, while long-life resistance is governed by strength.
Figure 21: $\epsilon$-N curves

In order to account for mean stresses, the strain-life approach can be modified using either the Smith/Watson/Topper or the Morrow method [7].

Smith/Watson/Topper assumes that the life depends on the product $\sigma_{\text{max}}\epsilon_a$, where:

$$\sigma_{\text{max}} = \sigma_m + \sigma_a$$  \hspace{1cm} (36)

Therefore, if we set the mean stress value equal to zero, for a complete reversed cycle,
for example, we have:

$$\sigma_{\text{max}} \varepsilon_a = \sigma_{\text{ar}} \varepsilon_{\text{ar}}$$  \hspace{1cm} (37)

where $\sigma_{\text{ar}}$ and $\varepsilon_{\text{ar}}$ are the stress and strain amplitude value for a cycle of mean stress equal to zero. This stress and strain can be obtained using the stress-life relationship [7] and Equation 33. The Smith/Watson/Topper version of the strain-life approach is, then, obtained by substituting these equations in Equation 37:

$$\sigma_{\text{max}} \varepsilon_a = \sigma'_f (D_{\text{am}})^{-b} \left[ \frac{\sigma'_f}{E} (D_{\text{am}})^{-b} + \varepsilon'_f (D_{\text{am}})^{-c} \right]$$

$$= \frac{(\sigma'_f)^2}{E} (D_{\text{am}})^{-2b} + \sigma'_f \varepsilon'_f (D_{\text{am}})^{-(b+c)}$$  \hspace{1cm} (38)

On another hand, the Morrow method accounts for mean stress effects by modifying the stress-strain curve as follows:

$$\varepsilon_a = \frac{(\sigma'_f - \sigma_m)}{E} (D_{\text{am}})^{-b} + \varepsilon'_f (D_{\text{am}})^{-c}$$  \hspace{1cm} (39)

Here the elastic strain curve intercept is shifted by the magnitude of the mean stress.

### 4.4 Mean Stress Relaxation

It has been observed that the residual stresses resulting from surface processing may relax, or decay, during fatigue cycling. This mean stress relaxation is due to the local reversed plastic deformation occurring in the material. The plastic deformations tend to balance the maximum and minimum stresses during cyclic loading and therefore, tend to reduce the residual stress value to zero. A precise description of the residual stress relaxation and its effects can be found in [14].

Mean stress relaxation can be quantified as follows:

$$\sigma_{mN} = \sigma_{m1} (N)^r$$  \hspace{1cm} (40)

where $\sigma_{m1}$ is the mean stress at the beginning of the loading, $\sigma_{mN}$ the mean stress after $N^{th}$ cycles, and $r$ the relaxation exponent. The relaxation exponent depends mainly
on the strain range of the cycles and the hardness number of the material. Relaxation increases with decreasing material hardness and/or increasing strain range.

For an induction hardened member, relaxation is most likely to occur in the core of the shaft (softer material), thus reducing the tensile residual stress. This in turn, by internal stress equilibrium, causes an accompanying reduction in surface compressive stress. Residual stress relaxation was not taken into account in the fatigue life analysis procedure. However, parallel analysis was performed without including the residual stress in the superposition process in order to gage the possible importance of this phenomenon.

To summarize, a procedure has been developed to combine the predicted stresses due to loading and induction hardening with cyclic material properties obtained (section 4.1) and load-time histories (section 4.2) in order to arrive at a prediction of the fatigue life at every node of the shaft. Next, the results of these computations are presented.
5 Results

Using the tools described in previous chapters, the fatigue life of the induction hard-ened shaft was computed at the nodes located around the notch in the X-Z plan going through the neutral axis. These nodes were selected according to the following rules on their position; this selection is somewhat arbitrary, but includes all the nodes where failure should occur:

\[
185 \text{ mm} \geq X \geq 158 \text{ mm} \\
Y = 0 \text{ mm} \\
Z \geq 7 \text{ mm}
\]

(41) (42) (43)

In this way, 106 nodes were selected for the 4 and 5 mm notch radius finite element models, while 102 and 97 nodes are selected for the 6 and 7 mm notch radius models, respectively. In all, computations were performed for 411 nodes.

The fatigue life at these nodes were computed for seven different depths of hardening (from 5% to 35% of the depth of the shaft, by increments of 5%), five different load magnification factors (35, 40, 50, 70 and 100), and six different bending/torque ratios (from 100% of bending and 0% of torque, to 0% of bending and 100% of torque, by increments of 20%). Note that the amplitude of the load-time history in torque has been scaled by a factor of three, in order to make the lives under pure torque of the same order as the lives under pure bending. Therefore, the maximum amplitude in bending is still 1000 Nm while the maximum torque amplitude becomes 3000 Nm. This gives us a total of 210 computation cases for each node, or an overall total of 86310 fatigue life computations.

For the automotive histories, the average computation time is 1.5 second per fatigue life calculation. Therefore, the total computation time for these load-time histories is approximately 36 hours. Using the skidder histories, the average computation time is 20 seconds per fatigue life calculation giving a total of 480 hours or 20
days. The presentation and interpretation of such large data sets can be efficiently accomplished using three-dimensional graphic portrayals.

In the following, the effect of the depth of hardening on life will be discussed assuming that the notch, the bending/torque ratio, and the load magnification factor are kept constant. Then the notion of optimum case depth is presented. A discussion on the variation of this optimum with respect to the load magnification factor follows with a heavy reliance on three-dimensional visualization methods. The notch effect is, then, presented in a similar manner. Finally, using an optimization code, the bending/torque ratio effect is explored.

5.1 Life Versus Depth of Hardening: Mechanism

The effect of induction hardening on fatigue life is portrayed in Figure 22. Here the top plot represents the life as a function of the distance from the neutral axis, the middle plot represents the Brinell hardness number as a function of the same distance, while the bottom plot represents the residual stress due to hardening with respect to the same X-axis. The scale of the X-axis is the same for the three plots, so that the life can be compared to the hardness number and the residual stress. These data come from the 5mm radius notch finite element model, with a case depth of 20%. The loading applied to this model is a pure bending using the automotive histories with a magnification factor of 50.

Considering first the top plot, we can see that the curve presents three regions. Starting from the surface, as the distance from the surface of the shaft increases, the life increases until it reaches a local maximum, then the life decreases to a local minimum. Finally, as the location comes closer to the the neutral axis, the life increases again until it reaches a infinite value at the the neutral axis.

The first region, between the surface and the local maximum, is located in the hardened area of the shaft (surface area), as can be seen in the middle plot. Therefore,
Figure 22: Life, hardness and residual stress through the thickness
the material properties of the steel is this area stay constant. In addition, the bottom plot shows that the residual stress is nearly constant for the entire phase. In this region, as the distance from the surface (and from the notch) increases, the stress due to loading decreases and, therefore, the life increases.

The second region, between the local maximum and the minimum of life, corresponds to the transition zone. The plot of residual stresses as a function of the distance from the neutral axis shows that the residual stresses, in this region, go from compressive to tensile. This has the effect of decreasing the life. In addition, as the plot of hardness as a function of the distance from the neutral axis shows, the material goes from hard to soft, which has the effect of decreasing the life at long lives. These two effects combined are more important on life than the decrease of the stress due to loading and, therefore, in this region, the life decreases.

The third region, between the minimum of life and the neutral axis, is the equivalent of the first region, but for the soft material. As the location gets closer to the neutral axis, the residual stresses, hardness level, and the material properties are constant and the life increases due to the decrease of the loading stress.

The position of the local maximum and minimum is highly dependent on the depth of hardening. The effect of variation of depth of hardening on life is demonstrated in Figure 23. This figure shows the plot of life through thickness for seven different depths of hardening. Except for the variation of depth of hardening, the data presented in this figure corresponds to the same situation as that in Figure 22: a pure bending load with a magnification factor of 50 using the automotive histories applied to the 5mm radius notch model.

The seven curves presented in this figure show the same three-region characteristic discussed previously. However, we can notice that the second region (transition region) is located deeper into the shaft as the depth of hardening increases. In addition, if we look at the curve representing the life for the case depth of 5%, we can see that the minimum of life for the transition region is actually an absolute minimum.
Figure 23: Life through the thickness for multiple case depth

Therefore, the failure should occur at the sub-surface (location of the minimum life of the transition region).

If we now look at the curve representing the life for a depth of hardening of 10%, we can see that, as the transition region gets deeper into the shaft, the minimum of life at the transition region gets higher. This is due to the fact that, as the location of the minimum life of the transition region gets deeper into the shaft, the stress due to loading decreases and, therefore, the life increases.

Notice that the variation of residual stress at this location has the opposite effect
on life: as the depth of hardening increases, the tensile residual stress at the core of the shaft increases and the compressive residual stress at the surface decreases as shown in section 3.2.2. This has the effect of decreasing the life but, in this case, it has a minor effect compared to the decrease of the loading stress. However, this can become more important as the amplitude of the loading decreases and the life increases. This behavior is discussed in section 5.3.

As the depth of hardening is increased to 15% of thickness, the increase of life at the minimum of the transition region is once again true, and the same pattern is shown as the depth of hardening is increased until 35%. However, as the depth of hardening reaches 30%, the minimum of life at the transition region reaches a superior value than the minimum of life at the surface. Therefore, the failure does not occur at the sub-surface, but at the surface.

As the depth of hardening is increased to 35%, the minimum of life at the sub-surface still increases, but the failure occurs at the surface. And, as the depth of hardening was increased from 30% to 35%, the life at the surface decreased due to the decrease in compressive stress at this location. Therefore, in order to reach the maximum life, there is an optimum depth of hardening to be applied to the shaft. This optimum is reached when the surface and the sub-surface failure occur at the same life. For this case of loading and geometry, the optimum of case corresponds to a value between 25% and 30% of the depth. However, this optimum depth varies with the loading and the geometry.

Next the effect of the load magnification factor on this optimum case depth is considered.
5.2 Optimization of the Case Depth with Respect to the Load Magnification Factor

In the previous section, it was shown that the induction hardened shaft presents two possible critical locations for fatigue life: at the surface and sub-surface (the location where the soft steel begins to appear). In addition, we defined the optimum depth of hardening as the depth at which surface and the sub-surface failure are equally likely.

In order to optimize the case depth with respect to the magnification factor, we need to be able to visualize three variables simultaneously: the depth of hardening, the load magnification factor and the critical life at the surface and the sub-surface. Three-dimensional plots are developed for this purpose.

Figure 24 presents the surface and sub-surface failure as functions of the depth of hardening and the magnification factor applied to the load. The two graphs in this figure represent the same three dimensional plot viewed from different perspectives. The X-axis is the load magnification factor, the Y-axis, the depth of hardening in percent of the depth of the shaft from the surface, and the Z-axis, the life in blocks to failure. The surface with the steepest slope in the X-direction corresponds to the surface failure, while the other surface corresponds to the sub-surface failure.

The data presented in this figure correspond to the 5mm radius notch finite element model under pure bending loading using the automotive histories. The results for other bending/torque ratios are presented in Appendix A, while the results of the computations using the log skidder histories are shown in Appendix B.

Upon examination of the data associated with the surface failure, we can see that they present the typical characteristic of the ε-N curves. As the magnification factor increases, the life decreases. When the magnification factor gets very low or very high, the surface tends to converge to asymptotes. The asymptote for low magnification factors has a steeper slope than the asymptote for high magnification factors. These asymptotes represent the elastic and plastic part of the damage as
Figure 24: Response surfaces for surface & subsurface failure
explained in section 4.3.4.

Notice that the $\epsilon$-$N$ curves are usually presented as load amplitude as a function of life and not, life as a function of the load. Therefore, the slopes are inverted, and the steepest slope in our three-dimensional plot corresponds to the flattest slope in the typical $\epsilon$-$N$ curve diagram. In addition, we note that life decreases slightly as the depth of hardening increases. This is due to the decrease of compressive stress at the surface that follows an increase of the depth of hardening.

Upon examination of the data associated with the sub-surface failure, we can see that they present the same general characteristics as the data associated with the surface failure. We can observe an decrease in life as the load magnification factor increases. In addition, this surface presents asymptotes for the high and low magnification factors.

However, the slopes of these asymptotes are different. The logical explanation for this change in slope can be the change in material properties. The slopes of the asymptotes are governed by the stress and strain exponents $b$ and $c$, in the $\epsilon$-$N$ curve diagram, which corresponds to $1/b$, for the low magnification factors, and $1/c$, for the high magnification factors in our three-dimensional plot. The surface failure occurs in a 595 BHN steel while the sub-surface failure occurs for a steel of hardness around 250 BHN. So, according to section 4.1 the stress exponent $b$ is higher for the surface failure than for the sub-surface failure. Therefore, the slope $1/b$ is lower for the surface failure than for the sub-surface failure.

This explains the change of slope between surface and sub-surface failure at low magnification factors, but, according to section 4.1, the strain exponent, $c$, does not vary with respect to the hardness level of the steel. Therefore, the asymptotes for high magnification factors should have the same slope for the surface and the sub-surface failure. But, remember that the $\epsilon$-$N$ curves presented in section 4.3.4 correspond to the life as a function of stress amplitude for a constant amplitude loading. In our three-dimensional plot, the loading amplitude is not constant. So while while some
of the cycles applied to the shaft are plasticity dominant, some others are elastic dominant. That leads the asymptotes for the high magnification factors to be reached more slowly in our figure than in the $\epsilon$-$N$ curve diagram.

In addition, this behavior is coupled by the fact that for high levels of hardening, the elastic part of the damage, even for short lives, is more important than the elastic part of the damage for low levels of hardening. In fact, for lives under $10^3$ cycles (in constant amplitude loading), the damage for the low level of hardening SAE 1045 steel can be assumed totally plastic while this assumption can be made for the high level of hardening only when the lives are under 10 cycles. Therefore, for high magnification factors (corresponding to the short lives of the $\epsilon$-$N$ curve diagram), the surface representing the sub-surface failure reaches its asymptote faster than the surface representing the surface failure. And, while we can assume that all the damages for the first surface are plastic, the second one presents still, even at very high magnification factors, a non-negligible part of elastic damages. Notice that this behavior depends on the loading applied to the specimen tested: If the loading applied was a constant amplitude loading, our three-dimensional plot would have the same characteristics than the $\epsilon$-$N$ curve diagram.

In order to find the optimum depth of hardening with respect to the magnification factor, remember that we defined this optimum as the depth of hardening for which surface and sub-surface failure occur for the same fatigue life. These lives are presented on the Z-axis of our three-dimensional plot. So the optimum design is achieved when the two surfaces of our plot have the same Z-value for the same depth of hardening and magnification factor. Therefore, this optimum design corresponds to the three-dimensional curve defined by the intersection of the two surfaces. This is the design for which the surface and the sub-surface failure occur simultaneously. Using this curve, for a given magnification factor, we are able to define which depth of hardening to be applied to the shaft in order to reach to longest life.

From this optimum curve, we can see that as the magnification factor increases,
the optimum life very logically decreases and, the optimum depth of hardening decreases. This means that, in order to achieve an optimum design, the higher the applied loads are, the lower the depth of hardening should be. We can also noticed that, if the magnification factor or the depth of hardening are increased from the optimum design, the failure will occur at the sub-surface, while, if they are increased, the failure will occur at the surface.

Although, we can theoretically define precisely the depth of hardening to be applied to the shaft in order to achieve the optimum design, the reality of engineering design involves many approximations. Therefore, the depth of hardening to be applied to the shaft in order to achieve the optimum design can be approached, but never precisely reached. So, it is important to know the behavior of the parameters studied in a neighborhood of the design optimum.

The effect of variation of depth of hardening around the design point is demonstrated in Figure 25. This figure presents the fatigue life as a function of the depth of hardening for a constant magnification factor of 60 (cut of the three-dimensional plot at this magnification factor). On this plot, we can observe that, starting from 5% case depth, as the depth of hardening increases, the life increases, until a maximum is reached at 23% of hardening, then the life decreases. The maximum of life corresponds to the optimum design. The first part of the curve (before 23% of hardening) corresponds to a sub-surface failure, while the second part (after 23% of hardening) corresponds to a surface failure.

We can notice that the slope of the first part of the curve (corresponding to a sub-surface failure) is much steeper than the slope corresponding to the surface failure. Therefore, fatigue life drops down much faster as the depth of hardening gets lower than the optimum design, than when the depth of hardening gets higher than the optimum design. This means that it would be better for the design to have a more depth of hardening than the optimum design than less depth of hardening, assuming that the load magnification factor is kept constant. However, the loading is usually
the less precise input of a design problem.

The effect on life of a variation of loading under constant depth of hardening is presented in Figure 26 for the case depths of 10% and 30%. The two curves presented in this figure show the same general shape: the life decreases smoothly as the magnification factor increases, except for a slope discontinuity corresponding to the optimum design and marking the transition between sub-surface and surface failure.

According to this plot, when the failure occurs at the surface, the depth of hardening does not really matter: at the surface, the depth of hardening influences the residual stress only, and according to our computation, this has a minor effect on life.

![Graph showing life as a function of depth of hardening](image)

Figure 25: Fatigue life as a function of the case depth for constant magnification factor
Figure 26: Fatigue life as a function of the magnification factor for constant case depth.

On-the-other-hand, the depth of hardening has a major effect on life when the failure is located at the sub-surface. And, the deeper the depth of hardening, the higher is the life at the sub-surface.

Therefore, we could assume that it is better for the designer of harden somewhat more than the optimum depth: if the load is higher than expected, surface failure will occur, and the depth of hardening has a minor effect on the life at the surface; but if the load is lower than expected, a higher depth of hardening can result in major gain.
in life when the failure occurs at the sub-surface.

However, during the life of the shaft, the surface will be exposed to corrosion and other external parameters which will have for effect to decrease the life. But the sub-surface will not be affected by these effects. Therefore, usually, designers prefer to choose the sub-surface as the failure location. In addition, the slope of life for the sub-surface is flatter than the slope of life at the surface. Therefore, there will be less variation of life for sub-surface failure design than for surface failure design.

5.3 Mean Stress Effect

As mentioned previously, mean stresses can significantly affect fatigue life. In this study, mean stresses are introduced either as residual stresses from the hardening process, or by the applied loading as reflected in the hanging loop process.

5.3.1 Residual Stress Effect

In order to explore residual stress effects on life, fatigue predictions for the 5mm radius finite element model were made with and without residual stresses. By comparing these results, we can deduce the importance of residual stress on the life of the shaft.

Figure 27 presents the variation of life as a percentage of the life computed including the residual stress, as a function of the depth of hardening and the load magnification factor. These results are for the 5 mm notch under pure bending using the automotive histories. The variation of life can be expressed as follows:

\[
\%\text{Life Variation} = \frac{\text{Life}_{w/\text{t_residual}} - \text{Life}_{\text{with_residual}}}{\text{Life}_{\text{with_residual}}} \times 100
\]  

(44)

where \(\text{Life}_{w/\text{t_residual}}\) is the life computed without residual stresses, and \(\text{Life}_{\text{with_residual}}\) is the life computed with the residual stresses. The top plot corresponds to the variation of life for a surface failure, while the bottom plot corresponds to the variation of life for a sub-surface failure.
(a) Surface failure

(b) Sub-surface failure

Figure 27: Residual stress effect
Considering first the variation of life for the surface failure, we see that at low magnification factors, the residual stress plays a relative important role, while, as the load magnification factor increases, the difference between the life computed with and without residual stresses approaches zero as the magnification factor reaches 90. Note that the values presented in this graph are negative, which means that the life computed without residual stress is lower than the life computed with residual stress; at the surface the residual stress is compressive, therefore, it increases the fatigue life in this area.

In addition, for low magnification factors, as the depth of hardening increases, the variation of life decreases; as the depth of hardening increases, the compressive residual stress at the surface decreases, therefore, the difference between the life computed with and without residual stress decreases. For high magnification factors, the residual stress effect is negligible even for shallow depths of hardening (high compressive residual stress).

If residual stress relaxation occurs, its effect will be relatively important at long lives only, especially for shallow depth of hardening. However, residual stress relaxation will not start at the surface, because at this location, the steel is hard and allows little plastic deformation compared to the soft steel of the sub-surface: if residual stress relaxation occurs, it will most probably start at the sub-surface and induce a residual stress relaxation at the surface.

The bottom plot, representing the sub-surface failure, is displayed from a different perspective than the surface failure plot. On this graph, we see that the residual stress effect is negligible everywhere except for deep depths of hardening under low load magnification factors. For the sub-surface failure, the difference between the life computed with and without residual stress is positive, which means that the residual stress at the sub-surface has for effect to decrease the life. In addition, the variation of life for the sub-surface failure is lower than the variation of life for the surface failure.
Further, while the automotive histories contain 800 to 1000 cycles after the rainflow cycle counting, most of the damage is due to a few large cycles. Therefore, for this type of loading, the residual stress relaxation is not likely to have any effect on the optimum depth of hardening. If residual stress relaxation occurs, it occurs at a high magnification factor, where the plastic deformation would be expected. But the graphs of Figure 27 show that this will have a negligible effect on life. Therefore, the optimum depth of hardening defined earlier will not be affected by the residual stress relaxation under this type of loading.

Next, similar comparison are made for the log skidder histories. Figure 28 presents these differences for a pure bending load applied on the 5mm finite element model. The plot of difference in life between the computations with and without residual stresses are essentially identical to the ones involving the automotive histories.

There is a simple explanation for this behavior: the residual stress affects only the mean stress of the principal loop. The mean stress of the secondary loops is defined by the hanging loop process. In addition, the principal cycle for the automotive histories presents the same amplitude as the principal cycle for the log skidder histories because these histories were scaled to a maximum amplitude of 1000 Nm, were multiplied by the same unit load coefficient from the finite element model and the same load magnification factors. Therefore, it is normal and expected that the residual stress effect is the same for both time-load histories.

Note that the log skidder histories contain about 20000 cycles after the rainflow cycle counting, however, the principal loop does most of the damage (at least 50% for short lives and more for long lives) and the secondary damaging cycles are smaller than for the automotive histories.

From these results, we can assume that residual stresses will have little effect on the optimum depth of hardening. However, it was shown that the results presented in this section were affected the principal loop only. But the secondary loops might play an important role in the life of the shaft (50 to 60% of the damage at short lives).
(a) Surface failure

(b) Sub-surface failure

Figure 28: Residual stress effect (log skidder)
Therefore, it is important to look at the effect of the hanging loop process.

5.3.2 Hanging Loop Effect

As explained earlier, during the rainflow cycle counting the order of occurrence of the cycles is lost, thus necessitating use of the hanging loop process to compute the fatigue life. In this procedure, the mean stress of each secondary cycle is shifted either to its maximum or minimum value, according to the principal cycle. Industry usually prefers a shift to the maximum, because this value gives the most conservative life. This value of mean stress has also been used to compute all the results presented until now. However, if the values of life computed using the maximum and the minimum value of mean stress are not close one to another, the optimum depth of hardening can change drastically. Therefore, it is important to look at the results of life computed with the minimum value of mean stress.

Figure 29 presents the variation of life between lives computed using the maximum and minimum values of mean stress:

\[
\text{%Life Variation} = \frac{\text{Life}_{\text{min, mean}} - \text{Life}_{\text{max, mean}}}{\text{Life}_{\text{max, mean}}} \times 100
\]  

(45)

where \( \text{Life}_{\text{min, mean}} \) is the life computed using the minimum value of mean stress, and \( \text{Life}_{\text{max, mean}} \) is the life computed using the maximum value of mean stress. The top plot presents the variation of life at the surface, while the bottom plot presents the variation of life at the sub-surface. These data correspond to a 5mm radius finite element model loaded using the automotive histories in pure bending.

From the top plot, we see that the variation of life is not sensitive to the depth of hardening, but is sensitive to the magnification factor. The variation in depth of hardening creates a variation of residual stress at the surface which changes the position of the principal loop used to compute damage. But it does not change its shape. In addition, the hanging loop process defines the maximum and minimum of mean stress based on the shape of the principal cycle. Therefore, it is normal that
Figure 29: Hanging loop effect
the depth of hardening does not influence the difference in life between computation using maximum and minimum of mean stress.

On the other hand, the change of magnification factor influences the shape of the principal loop and, therefore, influences the difference in life between computation using maximum and minimum of mean stress. We can see that the higher the magnification factor, the bigger the difference in life. However even for magnification factor of 110, the difference is less then 6%, and thus of little importance.

The bottom plot presents a different shape: starting from low magnification factor, and high depth of hardening, as the case depth decreases and the magnification factor increases, the difference in life between computations involving maximum and minimum of mean stress increases until it reaches a plateau. The irregular shape on this plateau around high magnification factor, and small depth of hardening, is due to the rounding of life to the first decimal. Therefore, for short lives (around ten cycles) the error on the difference can be up to 1%.

For the sub-surface, the difference in life is sensitive to the depth of hardening (at least around small magnification factors) because a change of depth of hardening induced a change in the location of the sub-surface, and therefore a change in loading stress equivalent to a change in magnification factor.

The plateau corresponds to a level of stress where the hanging loop process attaches the damaging secondary cycles on the plastic part of the stress-strain curve of the principal cycle. At this point, the plastic stress range between the peek and valley of the principle cycle increases more slowly as the magnification factor increases. This limits the maximum and minimum of mean stress of the secondary cycles. In addition, secondary cycles still increase their stress range therefore the difference should decreases. However, for this type of loading, the damaging secondary cycles present a stress range close to the stress range of the principal cycle. Thus, when these secondary cycles are attached to the plastic part of the stress-strain curve of the principal cycle, they present some plastic deformation so their stress range does not increases
more than the range of mean stress.

As for the surface, at the sub-surface the difference in life between computations using the maximum and minimum of mean stress is small (around 6% at most). Therefore we can assume that the optimum depth of hardening defined using the maximum value of mean stress is not influenced by the hanging loop process as far as the automotive histories are concerned.

Next we consider differences in life using the log skidder histories. Figure 30 presents these differences the same way as before except, we are now using log skidder load time histories. We can first notice that the general shapes of the graphs presenting the differences in life for the log skidder histories are the same than the graphs presenting the automotive histories however in both cases (surface and subsurface failure for the log skidder histories) the maximum value of the difference in life can reach up to 18% this is can be explained by the fact that the damaging secondary cycles for the log skidder histories are smaller and more damaging than for the automotive histories. Therefore the difference between the maximum and minimum value of mean stress is larger for the log skidder histories which creates a larger difference in life.

If we now look at the graph representing the sub-surface failure we can see that the plateau like shape has now a slight negative slope as the depth of hardening decreases and the magnification factor increases. Here, the damaging secondary cycles are significantly smaller than the principal cycle. Therefore, once these secondary cycles are attached to the plastic part of the stress-strain loop they are still in the elastic part of their stress-strain curve. Therefore, as the level of stress increases, the difference between the maximum and minimum of mean stress decreases.

Although the difference in life between the computation using maximum and minimum values of mean stress can be up to 20% for a real loading, this is still very reasonable for fatigue life computations, especially if we consider that these changes in life will occur in the same direction for the surface and the sub-surface; this will
(a) Surface failure

(b) Sub-surface failure

Figure 30: Hanging loop effect (log skidder)
affect the life defined for the optimum of hardening, but not really the optimum itself.

5.4 Radius effect

The geometry of a part is an important factor because of its influence on the local stress field. The critical geometry of the shaft is the notch area which will affect primarily the surface stress field with only a minor effect on the sub-surface life.

Figure 31 presents the location of the sub-surface failure with respect to the neutral axis, as a function of the depth of hardening, and the stress magnitude (due to unit load in bending as obtained from the finite element analysis) as a function of the distance from the neutral axis. The data correspond to a 5mm radius. The location of the sub-surface failure is computed using the automotive histories under pure bending for a magnification factor of 70.

![Graph showing stress and depth of hardening](image)

Figure 31: Effect of notch factor at the sub-surface
From this figure, we can see that the location of the sub-surface failure varies between 11 mm and 17.4 mm from the neutral axis (as the depth of hardening varies from 5% to 35%), while the area where the stress profile is influenced by the notch is located above 14 mm from the neutral axis. Therefore, for sub-surface located below 14 mm from the neutral axis (depths of hardening superior to 20%), the notch will have no effect on the sub-surface failure. In addition, when the depth of hardening is less than 20%, the location of the sub-surface failure is located at the beginning of the area where the notch appears. For these locations, the notch effect is still minor; for 5% of hardening, the subsurface failure is located at 17.4 mm of the neutral axis where the notch factor is 1.18, and at 10% of hardening the notch factor is 1.12. These values are small compare to a notch factor of 1.58 at the surface. Therefore, we can make the reasonable assumption that the sub-surface will not be influenced by the notch radius.

The effect of the notch radius on the optimum depth of hardening is demonstrated in Figure 32. The top graph of this figure presents the surface and sub-surface failure as a function of the depth of hardening and the magnification factor, for the four notch radii and this (4, 5, 6, 7 mm). Because we assumed the notch factor does not affect sub-surface behavior, the sub-surface failure is only plotted for the 5mm radius and corresponds in the graph in the surface with the flattest slope for a constant depth of hardening. The four other surfaces correspond to the surface failures for the four notch radii. The surface with the shortest life corresponds to the 4 mm radius while in increasing value of life the other surfaces correspond to 5, 6 and 7 mm radius.

The bottom plot is similar to the bottom plot but the surfaces representing the surface failures for the different radii were replaced by their intersects and only the subsurface failure is shown. On this graph we see that the optimum depth of hardening is highly sensitive to the geometry of the shaft. It is interesting to note that as the notch radius decreases (and the life at the surface increases), the life corresponding to a constant optimum depth of hardening increases due to the decrease of the
(a) All surfaces

(b) Intersects only

Figure 32: Response surfaces for different notch factors
magnification factor.

For a constant magnification factor, an increase of notch radius increases the optimum depth of hardening leading to an increase in the life of the shaft. Notice that once a design is chosen for the depth of hardening and the notch radius, according to the optimum design, an increase of the notch radius will not induce an increase in life: the increase of the notch radius will increase the life at the surface, but the failure will be located at the sub-surface and will not be influenced by this change of radius. On the other hand, a decrease of notch radius from the optimum design point will push the failure to the surface causing a decrease in life for the shaft. Therefore, it is important to match the depth of hardening to the notch radius or vice versa.

In addition, we note the very similar shape of the curve representing the optimum depth of hardening for the four different notch radii. This can be explained simply by the fact that all the surfaces representing the surface failures for different notch radii can be obtained one from another. In fact, the notch factor can be considered a local magnification factor; instead of a load magnification factor it is a stress magnification factor. Therefore once we have the lives corresponding to our sets of depth of hardening and magnification factor for one notch radius, we can obtain the lives at the surface failure corresponding to other notch radii by scaling the load magnification factor according to the new notch factor using the following formula:

\[
Life(R_a, M_a, D) = Life(R_b, M_b, D) \tag{46}
\]

\[
R_a * M_a = R_b * M_b \tag{47}
\]

where \( R_a \) and \( M_a \) are the radius and the magnification factor for the notch radius computed, \( R_b \) and \( M_b \) are the radius and the magnification factor for new notch radius, \( D \) is the depth of hardening.

Notice that in pure torque or pure bending results can be used directly for design analysis. However, in cases of combined loading, according to our finite element analysis, the ratio of the notch factor in bending to the notch factor in torque varies
with the notch radius. This means that as the radius changes, it is not possible to compute a change in load magnification factor that matches the change in notch factors in bending and torque simultaneously. Therefore equations 46 and 47 do not apply in this case. However, the variations of notch factor in bending and torque with respect to the notch radius are not very important, especially compared to the loading tolerances. Therefore equations 46 and 47 can be used as a good first approximation in order to obtain the optimum depth of hardening as a function of the geometry of the shaft.

Next, the effect of the ratio bending/torque on life is presented.

5.5 Bending/Torque Ratio Effect

In order to analyze the effect of the bending/torque ratio on life and the optimum depth of hardening, shaft fatigue lives were computed for the six different bending/torque ratios listed in Table 4.

Table 4: Bending/torque ratios

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Bending</th>
<th>Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>2</td>
<td>80%</td>
<td>20%</td>
</tr>
<tr>
<td>3</td>
<td>60%</td>
<td>40%</td>
</tr>
<tr>
<td>4</td>
<td>40%</td>
<td>60%</td>
</tr>
<tr>
<td>5</td>
<td>20%</td>
<td>80%</td>
</tr>
<tr>
<td>6</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>

The results of these computations are shown in Appendix A for the automotive histories, while the computations using the log skidder histories are presented in Appendix B.

For this purpose, an optimization code was developed that allows one to find the
load magnification to be applied to the shaft in order to obtain a desired life. In this way it is possible to compare different bending/torque ratios leading to the same fatigue life for the shaft. Notice that, for this computation, the torque history is not scaled by a factor of 3: both bending and torque histories are used as defined in section 4.2.1.

Figures 33 to 36 present the load magnification factor to be applied in order to obtain minimum lives of $10^1$, $10^2$, $10^3$ and $10^4$ blocks, respectively, as a function of the depth of hardening and the bending/torque ratio. The data used to obtain these figures correspond to the 5mm radius finite element model loaded using the log skidder histories.

Notice that in these figures, the bending/torque ratio is identified by the percentage of bending applied. The percentage of torque corresponding to the percentage of bending according to Table 4 is not shown on the graphs but is present in the computations and the results presented.

![Figure 33: Ratio bending/torque for minimum life of 10 blocks](image)
Figure 34: Ratio bending/torque for minimum life of $10^2$ blocks

Figure 35: Ratio bending/torque for minimum life of $10^3$ blocks
Figure 36: Ratio bending/torque for minimum life of $10^4$ blocks

From these figures a general pattern emerges: as the depth of hardening increases, for a constant bending/torque ratio, the load magnification factor increases first, then reaches a maximum and, finally, decreases slightly. This is similar to the behavior in Figure 25 except now life is kept constant while we are looking at the variation of the load magnification factor as a function of the depth of hardening. The increase in life for shallow depths of hardening corresponds to a sub-surface failure, while the slight decrease in life at deep depths of hardening corresponds to a surface failure. The maximum of load magnification factor corresponds to the optimum design.

We see that, as the life increases (from Figure 33 to Figure 36), the corresponding optimum depth of hardening increases. Notice that for Figure 36 the maximum of the load magnification factor is not reached: for such long lives, the depth of hardening has to be deeper. Therefore, this graph presents only a sub-surface failure.

In addition, if we look more carefully at Figure 34, we can see that the optimum depth of hardening varies with the bending/torque ratio applied to the shaft. In
particular, we note a sudden jump in the optimum depth of hardening when the percentage of bending goes from 40% to 20%; the optimum depth of hardening jumps from 15% to 25%. This behavior can be explained by the difference of notch factor in bending and torque. During the finite element discussion, we mentioned that the notch effect in bending was more important than the notch effect in torque. So, the life at the surface has a tendency to be lower when the bending is dominant than when the torque is dominant. Therefore, the optimum depth of hardening is deeper for a torque dominant loading than for a bending dominant loading. In addition, the transition between bending dominant and torque dominant loading will result in a jump of the optimum depth of hardening as sudden as number of damaging cycles is small.

If we now look at these graphs for constant depths of hardening, we see that as the percentage of bending decreases, the load magnification factor increases, then reaches a maximum and the decreases. However, the individual behavior of the bending moment and the torque are not readily apparent. For this purpose, the values of bending moment and torque were plotted separately, as function of the depth of hardening and the bending torque ratio in Figure 37, for a minimum of life of $10^2$ cycles. By looking at these graphs, we can again clearly see the bending/torque ratios which bending or torque are dominant for the life. We can also distinguish the area where the surface or the sub-surface failure occurs.

Considering first the graph representing the bending part of the loading, we see that the bending is dominant for ratios corresponding to 40% of bending and higher. The interesting point here is that once the bending is dominant, the load applied in bending does not vary much. The only difference is that the optimum depth of hardening has a tendency to go a little deeper as the percentage of bending increases.

On-the-other-hand, if we now look at the graph representing the torque part of the loading, we see that it is dominant for only small percentages of bending. However, we can still distinguish a sort of plateau for high percentages of torque (less than 20%
Figure 37: Bending moment and torque for minimum life of $10^2$ blocks
of bending) and high depth of hardening (30% and 35%). This corresponds to the plateau in bending dominant loading for percentages of bending more than 40%.

Once again, on this graph we see the effect of the bending/torque ratio on the optimum depth of hardening; a change from 15% for bending dominant loading to 30% for torque dominant loading. Even if the effect of the bending torque ratio on the optimum depth of hardening can be important, when bending and torque are combined, unless the bending/torque ratio corresponds to percentages of bending of 20% to 40%, the effect of a variation of the bending/torque ratio on the optimum depth of hardening will not be major. It is also important to remember that torque dominant loading will result in an optimum depth of hardening considerably deeper than for bending dominant loading.

5.6 Discussion

A major goal of this study was to combine finite element modeling and modern fatigue analysis methods into an integrated scheme for predicting and optimizing the fatigue performance of a structural component. The induction hardened shaft provided a well-documented and suitably challenging problem in that its behavior depends, in a complex way, on a number of geometric, processing, and loading parameters.

It was demonstrated that finite element techniques can be used both to determine unit-load factors for generating stress-time histories and, through thermal analysis, to determine the residual stresses resulting from the surface hardening process. These superposed stress fields were then utilized in a fatigue program to predict failure location and expected lifetime for specified situations.

With this capacity in place, the various design parameters were varied systematically to obtain multiparameter performance surfaces for the shaft to serve as design aids for understanding and controlling component durability. For example, it was shown that there is an optimum depth of hardening that depends on the applied
loading and shaft geometry:

- as the amplitude of the loading decreases, the optimum depth of hardening increases, and,
- as the notch radius increases, the optimum depth of hardening decreases.

Further, it was concluded that residual stress relaxation will not have a major effect on the optimum design point. Also, unless the ratio bending/torque is in the range of 20% to 40% of the load in bending, the bending torque/ratio will not have a major effect on the optimum depth of hardening. However, a bending dominant loading will result in a much shallower optimum depth of hardening than a torsion dominant loading.

On-the-other-hand, it was shown that the effect of the notch acuity will be predominant for the surface failure, while the depth of hardening will mainly affect the fatigue life at the sub-surface. In addition, we mentioned in section 5.2 that a designer will generally choose to design the shaft such that the failure occurs sub-surface.

Therefore, the basic guidelines for an optimum design of the shaft can be presented as follows:

- the depth of hardening must be chosen in order to give the critical life of the shaft, according to the performance requirements;
- the notch acuity must be chosen such that the life at the surface is at least as high as the life at the sub-surface (this gives a minimum value of the notch radius). Usually, a safety margin will be applied in order to account for corrosion and other external parameter.

In a more general view, the methods presented here are representative of a wide range of computer modeling and simulation tools that are being used increasingly by industry to improve the efficiency and rigor of the product design process. When
properly implemented, and validated, they hold great promise for achieving more optimum designs in a shorter period of time.
6 Conclusions

Based on the results of this study, the following conclusions can be drawn:

- finite element modeling provides an efficient technique for performing unit-load analysis, from which stress-time histories can be generated, and, through thermo-elastic analysis, for determining the residual stresses developed during induction hardening;

- these stress fields, when combined by elastic superposition, provide necessary inputs to fatigue analysis routines from which failure location and lifetime can be estimated;

- through systematic variation of design parameters, performance surfaces can be generated that serve as a valuable aid in understanding and improving component durability.

This integrated method provides a powerful problem solving tool for simultaneously assessing the effects of the range of geometry, processing, service parameters on component performance. In the future, this method can be extended to incorporate a rigorous design sensitivity analysis capability that, in turn, can provide the basis for formalized optimization procedures. This will improve significantly the overall efficiency of the product design process.
7 Appendix A

(a) 80% in bending
(b) 60% in bending
(c) 40% in bending
(d) 20% in bending
(e) 0% in bending

Figure 38: Life versus magnification factor and case depth (automotive histories)
8 Appendix B

(a) 100% in bending
(b) 80% in bending

(c) 60% in bending
(d) 40% in bending

(e) 20% in bending
(f) 0% in bending

Figure 39: Life versus of magnification factor and case depth (log skidder)
9 References


10 Vitae

Patrick Le Moal was born July 21, 1970 in Saint Maur des Fossés, Val de Marne (94), France. He was raised in Paris, France and graduated from high-school in July 1988. He did his military service in 1990 and pursued his studies at "Université de Technologie de Compiègne (U.T.C)”, France, majoring in Industrial Systems and Engineering. During his undergraduate studies, Patrick completed a six-month training period as an assistant engineer at SNECMA, an aircraft engine manufacturer. In August 1994, he was offered the opportunity to study at Virginia Polytechnic Institute and State University. Under the supervision of Dr. R.W. Landgraf, he completed his Master of Science in the Engineering Science and Mechanics Department in September 1996. His career plans involve engineering applications in the area of structural analysis, and automotive manufacturing.