SIMPLIFIED METHOD FOR DESIGN OF STIFFENED AND UNSTIFFENED STRUCTURAL TEE HANGERS

by

Michael A. Otegui

Thesis submitted to the faculty of the
Virginia Polytechnic and State University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

CIVIL ENGINEERING

APPROVED:

Thomas M. Murray, Chairman

W. Samuel Easterling

Richard M. Barker

April 1996
Blacksburg, VA

2

SIMPLIFIED METHOD FOR DESIGN OF STIFFENED AND UNSTIFFENED STRUCTURAL TEE HANGERS

by

Michael A. Otegui

Thomas M. Murray, Chairman

Civil Engineering

(ABSTRACT)

Structural tee hangers are a popular tension connection, yet current design procedures are cumbersome, and excessively conservative. Also, no provisions are made for the additional strength gain due to stiffeners. Past research has shown that the design of these connections is controlled by either plate yielding, or bolt rupture.

This study presents a simplified method for determining the ultimate strength of structural tee hanger connections. Yield line analysis is used for the determination of the yield capacity of the connection based on plate strength. A simplified version of the Kennedy method is used for calculating connection capacity based on bolt strength with prying action. The simplified design procedure is verified with comparison to results of 15 connection tests. Design recommendations are made, and examples presented.

ACKNOWLEDGMENTS

The author wishes to express his sincere gratitude to his advisor Dr. Thomas M. Murray, without whom attending graduate school would have been impossible. Thanks also to Dr. W. Samuel Easterling, and Dr. Richard M. Barker for their help and influence as committee members.

Thanks to Dennis Huffman and Brett Farmer for their tremendous help in the fabrication of the test specimens, as well as their entertainment during long frustrating days. Also, thanks to Ron Meng and Brad Davis for all their insight and assistance in helping me learn how to use testing equipment.

The following people also had a great influence on the thesis, either directly or indirectly: Clinton O. Rex, Michelle D. Rambo-Roddenberry, Martin S. Kigudde, Barry Band Jr., Christopher Bathgate, Michelle Floyd, Joseph N. Howard, Cheryl Rottman, Budi R. Widjaja, and Cynthia A. Kraus.

Finally, the author wishes to thank his family. Through good days and bad days, they have always been there. My gratitude for their encouragement and love can never be fully expressed.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER I

INTRODUCTION

1.1 BACKGROUND

Tee hangers are used to transfer tensile force to a support. Usually a plate, pair of angles, or structural tee is used. Figure 1.1 shows an example of a structural tee hanger connected to a beam flange.

Two limit states control the design of hanger connections: yielding of the flange or plate, and bolt rupture. Extensive testing has been done on these and similar types of connections. Tests were conducted to determine the load at which the plate yields, and bolt forces, including prying action (Packer and Morris 1977; Kennedy *et al.* 1981.)

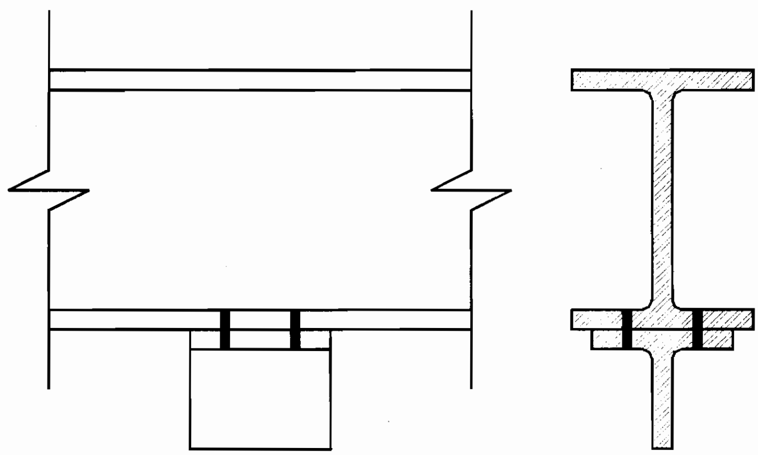


Figure 1.1 Tee Hanger Connection

From these studies came many design procedures to determine required plate thickness as well as bolt force predictions (Kennedy *et al.*1981; Thornton 1985; Astaneh 1985). There is a great deal of difference between these design procedures. The current design procedure (Manual 1994) has an open solution space, that is, the equations can give more than one plate thickness for a given applied load. The current procedure is also overly conservative. Also, none of the above methods treat stiffened tee hanger connections.

The purpose of this study is to introduce a simplified design method for tee hangers, stiffened or unstiffened. Current literature on tee hangers is first reviewed, followed by the development of yield line and simplified bolt force design procedures. Comparisons between prediction and experimental results are given, followed by conclusions and design recommendations.

1.2 Literature Review

In the study of bolted end plate design, the use of the "split tee" has been used extensively. The assumption being that the tension flange connection of the end plate design is the critical point. It is here that failure due to steel yielding or bolt rupture is to occur.

Many studies of this "split tee" have been done, with a variety of design methods developed. Some of these studies were intended for use in end plate design, and some for designing tee hangers. The following review will briefly explore different methods of structural tee hanger design, with a focus on work done by Kennedy *et al.*(1981).

A comparison of results obtained from different design methods was compiled by Kennedy *et al.* (1981) for the specimens in Figure 1.2. The two connections were studied to find the load at first yielding in some portion of the connection. Douty and McGuire (1965) used a plastic as well as a working stress design in which the prying force was calculated as a fraction of the flange force. Kato and McGuire (1973) also proposed elastic and plastic methods for determining connection performance, but evaluation of prying forces was not investigated. Agerskov (1977) used an elastic method which included design charts to find prying forces which depend on plate dimensions, and bolt placement. Grundy *et al.* (1977) accounted for prying forces by increasing the bolt load by 20%.

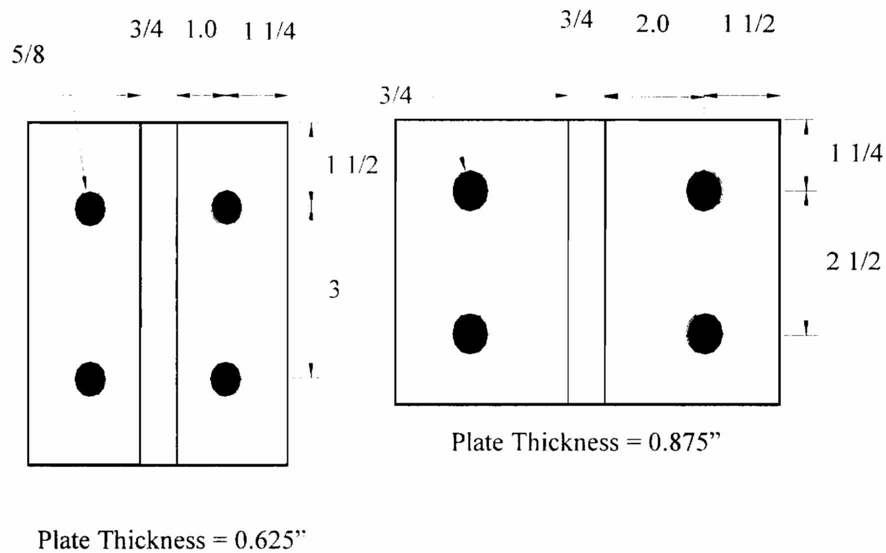


Figure 1.2 Kennedy Test Specimens

Packer and Morris (1977) used the yield line theory. The end plate thickness was found by equating plastic moment capacity to applied moment. Bolt load was increased by 33% to account for prying action. Mann and Morris (1978) also used yield line theory, but did not remove the bolt holes from the calculation of capacity. They claimed that the bolts contribute to the bending strength of the plates.

Kennedy *et al.*(1981) proposed a design method which predicted the bolt force due to prying action in split tees. The model (Figure 1.3) consists of a tee section bolted to a rigid support, where Q is the prying force, and 2F is the applied load. This method describes three modes which the tee goes through before failure. Initially, when loading on the connection is low, the system is assumed to have no deflection, and no yielding. At this point there is no prying action in the connection and the plate is said to be "thick" (Figure 1.4a).

As loading continues, a plastic hinge will form on both sides of the flange. This in turn will cause deformation of the connection at the plate flange interface, thus introducing initial prying forces. The prying force is now between zero and its maximum value. Kennedy *et al.* (1981) describe the plate as being in an "intermediate" phase (Figure 1.4b).

Another set of plastic hinges will form at the bolt lines if load is still added. Here the connection will see its greatest separation, and its maximum and constant, prying force. This was called "thin" plate behavior (Figure 1.4c).

Kennedy *et al.* determine the state of the plate behavior by comparing the plate thickness, t_p, to the thick plate limit, t_1, and the thin plate limit, t_{11}. The thick plate limit is found by iteration using the following equation:

$$t_1 = \sqrt{\frac{2p_f(2F)}{b_f\sqrt{F_{py}^2 - 3\left(\frac{F}{b_f t_1}\right)^2}}} \tag{1.1}$$

The thin plate limit is also found by iteration using:

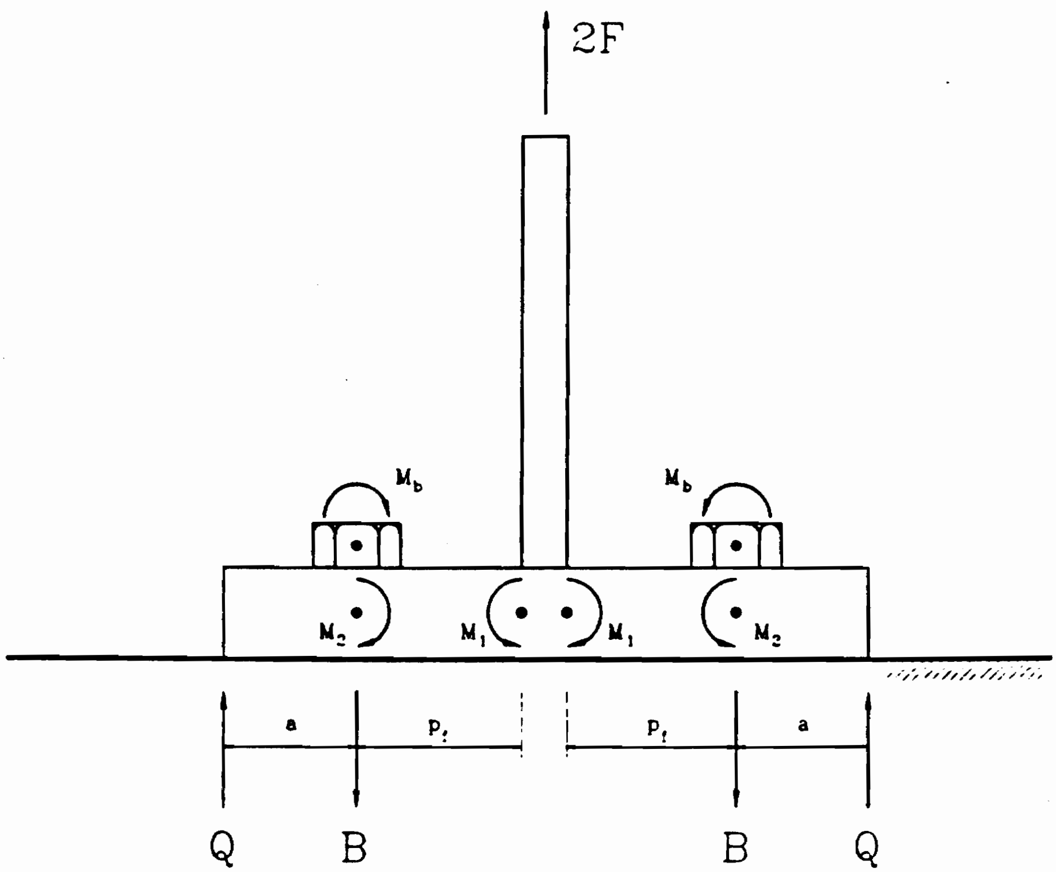
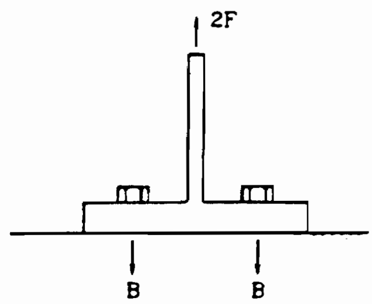
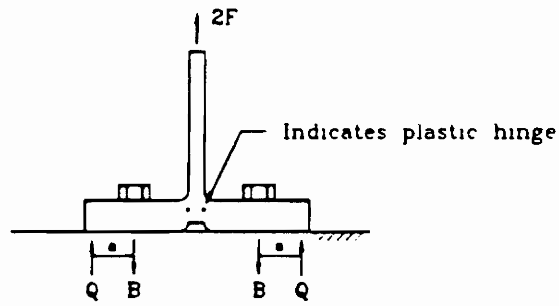


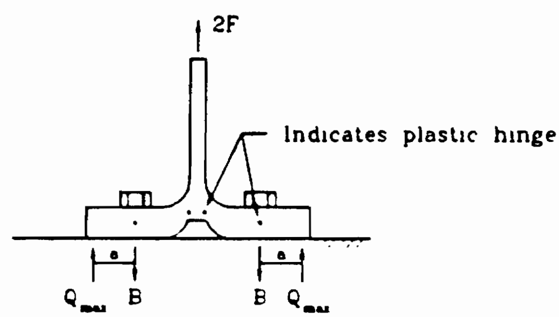
Figure 1.3 Kennedy Split-Tee Analogy
(after Kennedy *et al.*, 1981)



(a) First Stage -- Thick Plate Behavior



(b) Second Stage -- Intermediate Plate Behavior



(c) Third Stage -- Thin Plate Behavior

Figure 1.4 Kennedy Split-Tee Behavior
(after Kennedy *et al.*, 1981)

$$t_{11} = \sqrt{\frac{p_f(2F) - \pi d_b^3 F_{yb}/8}{\frac{b_f}{2}\sqrt{F_{py}^2 - 3\left(\frac{F}{b_f t_{11}}\right)^2} + w'\sqrt{F_{py}^2 - 3\left(\frac{F}{2w' t_{11}}\right)^2}}} \tag{1.2}$$

where F = applied flange force per bolt, b_f = plate flange width, Fpy = plate yield stress, d_b = bolt diameter, F_{yb} = nominal strength of the bolts as defined in table J3.2 of the AISC manual (AISC Load 1993), w' = width of plate per bolt at bolt line minus the hole diameter, and p_f = pitch distance from flange face to center of bolt line.

If $t_p > t_1$, the plate is considered to be thick, and no prying forces are present. The forces in the bolts are assumed to be the applied load divided by the number of bolts. If $t_1 > t_p > t_{11}$, the plate is said to be in the intermediate state. The magnitude of the prying force is given by:

$$Q = \frac{p_f(F)}{a} - \frac{\pi d_b^3 F_{yb}}{32a} - \frac{b_f t_p^2}{8a}\sqrt{F_{py}^2 - 3\left(\frac{2F}{b_f t_p}\right)^2} \tag{1.3}$$

In this equation, a = distance from the centerline of the bolts to the edge of the plate, and is suggested to be between two and three times the bolt diameter. The bolt force at intermediate behavior is then:

$$B = F + Q; \qquad \text{for } t_1 > t_p > t_{11} \tag{1.4}$$

If the plate thickness is considered "thin", $t_{11} > t_p$, then maximum prying forces have developed and is estimated from:

$$Q_{max} = \frac{w' t_p^2}{4a} \sqrt{F_{py}^2 - 3\left(\frac{F'}{w' t_p}\right)^2} \tag{1.5}$$

where

$$F' = \frac{t_p^2 F_{py}\left(0.85 b_f / 2 + 0.80 w'\right) + \pi d_b^3 F_{yb} / 8}{4 p_f} \tag{1.6}$$

The prying force is constant once the thin plate limit has been reached. so the bolt force becomes:

$$B = F + Q_{max} \tag{1.7}$$

Kennedy *et al.* (1981) note that the quantity under the radical in Equations (1.3) and (1.5) may become negative. This would be an indication of local yielding in shear prior to development of bolt prying forces, which would make the connection inadequate for the load.

The use of the Kennedy method was modified by Srouji *et al.* (1983) to determine the bolt forces in moment end plate. The study used the yield line method to determine strength of various end plate bolt configurations. From this and other studies (Srouji *et al.* 1984; Borgsmiller 1995) it was concluded that the yield line, and the modified Kennedy methods were accurate predictors of moment end-plate connection strength.

Hendrick *et al.* (1984) continued the work by Srouji. One of the modifications Hendrick did was the distance “a” from the bolt centerline to the line of action of the prying force. For each moment end-plate test Hendrick *et al.* back calculated the length “a” needed to give the bolt forces that were obtained in testing. The equation that best fit the experimental values, using plate thickness and bolt diameter as the variables is:

$$a = 3.682\left(\frac{t_p}{d_b}\right)^3 - 0.085 \tag{1.8}$$

Borgsmiller (1995) introduced a simplified method for design of moment end plate connections. The method involved designing the end plate for one of two failure modes. The yield line method was used to determine connection strength through plate yielding, and the modified Kennedy method was used to determine connection strength through bolt failure. The bolt calculations were reduced greatly from that of the original Kennedy method. Only the computation for the maximum prying force Q_{max} is involved. The assumption for this method is: substantial yielding in the connection plate, causing maximum prying forces in the bolts. If the plate is thick enough, there will be no yielding and therefore no prying forces.

It can bee seen from the variety of the above design methods for structural tee hanger and moment end-plate designs, that there is a need for an accurate simplified design method. None of the above methods can accurately predict both bolt forces and plate strength in a straight forward manner.

1.3 SCOPE OF RESEARCH

The purpose of this study was to develop a simplified method for design of stiffened and unstiffened tee hangers. The objectives of this study were reached by developing yield line equations that could accurately predict failure load for a wide variety of geometry. Experiments were done to confirm these equations, as well as predict bolt forces. The resulting design procedure provides:

- Required tee plate thickness for a given geometry, bolt configuration, and material strength, as determined by yield line theory.
- Bolt forces, including those forces added due to prying action. Forces are determined using the modified Kennedy method.

CHAPTER II

CONNECTION STRENGTH USING YIELD LINE THEORY

2.1 GENERAL

A yield line is the formation of a continuous plastic hinge, along a straight or curved line, in a plate or slab. Failure mechanisms exist when yield lines form kinetically valid collapse mechanisms. The mechanism can then be separated into rigid plates connected by, and rotating about, the defined yield lines. Originally the yield line theory was developed for concrete slabs, but it is equally applicable to steel plates.

Common guidelines in the determination of yield lines in a steel plate are set forth as follows:

- Axes of rotation generally lie along lines of support
- Yield lines pass through the intersection of the axes of rotation of adjacent plate segments
- Along a yield line, the bending moment is assumed to be constant and equal to the plastic moment of the plate.

Yield line mechanisms may be analyzed using two methods: equilibrium, or virtual work. In this study, virtual work was used due to its simplicity and suitability in tee stub analysis. In this method, the work done by the applied load, from a unit virtual displacement on the tee hanger, is set equal to the internal work done by the tee stub. Yield line theory assumes that elastic deformations are negligible in comparison to plastic deformations, so the infinitesimally small added unit deflection (virtual work) can be used. The resistance to deformation is the plastic moment capacity in the yield lines as it rotates to accommodate the yield line pattern. The yield line method is an upper bound one, therefore many yield lines must be drawn to find the mechanism which gives the lowest failure load, or the highest required plastic moment capacity.

The location of yield lines in this study were found experimentally, rather than theoretically. Specimens were tested, failure loads measured, and yield lines were observed from deformations and mill scale chipping off plates.

The internal energy stored in a yield line mechanism is the sum of the internal energy stored within each yield line. Internal energy per unit length of yield line is the multiplication of the normal moment on the yield line, with the normal rotation of the yield line. So, the energy stored in the n^{th} yield line of length L_n is:

$$w_n = \int_{L_n} m_p \theta_n ds = m_p \theta_n L_n \tag{2.1}$$

where m_p is the plastic moment capacity per linear inch of the steel plate, θ_n is the relative normal rotation the yield line n, and ds is the elemental length of line Ln. The internal energy stored by a yield line mechanism can be written as :

$$W_i = \sum_{n=1}^{N} m_p \theta_n L_n \tag{2.2}$$

where N is the number of yield lines in a mechanism.

For more complicated yield lines it is more convenient to express the internal work in terms of x- and y- components. The result is given as:

$$W_i = \sum m_{px} \theta_{nx} L_{nx} + \sum m_{py} \theta_{ny} L_{ny} \tag{2.3}$$

where m_{px} and m_{py} is the moment capacity per unit length in the x- and y- direction, θ_{nx} and θ_{ny} are the x- and y- components of the relative normal rotation of the plate segments, and L_{nx} and L_{ny} are the x- and y- components of the n^{th} yield line length. For steel plates, the moment capacity per unit length is assumed to be the same in both the x- and y- directions, therefore:

$$m_{px} = m_{py} = m_p = \frac{F_{py} t_p^2}{4} \tag{2.4}$$

where F_{py} is the yield stress of the steel plate and t_p is the plate thickness.

A point load on a plate or slab will give a "fan" circular yield line pattern, which is not dependent upon its radius. The equation for the point load is (See Appendix A):

$$P = 2\pi(m_p + m_p') \tag{2.5}$$

where P is the applied point load, m_p is the positive moment capacity of the plate or slab, and m_p' is the negative moment capacity of the slab (See Appendix A). In steel it is assumed that these capacities are the same, so the equation becomes:

$$P = 4\pi m_p \tag{2.6}$$

Since the equation does not depend on the radius of the yield line circle, it is assumed that if only a portion of the circular yield line is formed, then only that portion of the total load given by the equation of the circular yield line will be realized. The equation then becomes:

$$P = 4\pi\left(\frac{\alpha}{360}\right)m_p \tag{2.7}$$

where $\alpha/360$ is the fraction of the total yield line of the circular point load equation. Derivation of the point load equation can be found in Appendix A.

The expression for the external work done on the tee hanger by the applied load was the same for all configurations and is given by:

$$W_e = P(1) \tag{2.8}$$

where P is the applied load to the connection, and 1 is the unit displacement the connection undergoes due to the applied load.

The work done by the tee hanger to resist the applied load is dependent upon the geometry of the tee and the presence or absence of stiffeners.

2.2 UNSTIFFENED TEE HANGERS

The unstiffened tee hanger was determined to have a yield line pattern such as those shown in Figures 2.1 and 2.2. The pattern depends on the distance between bolts. If the bolts are relatively far apart, a parabolic yield line will form from the web to the edges of the tee flange (Figure 2.1). If the bolts are spaced closer together, then the parabolic yield lines from adjacent bolts will intersect prior to reaching the edges of the flange (Figure 2.2). The internal energy in the unstiffened tee yield line mechanism is given by:

$$W_i = m_p\left[4\pi\left(\frac{\alpha}{360}\right) + (b)\left(\frac{1}{e}\right)\right] \tag{2.9}$$

Where m_p is given by equation 2.4.

Figure 2.3 shows the dimension which define the parabolic yield line. The focal point of the parabola is directly above the bolt, and the vertex is at the edge of the web fillet, directly below the bolt. The length p is the distance from the focal point to the

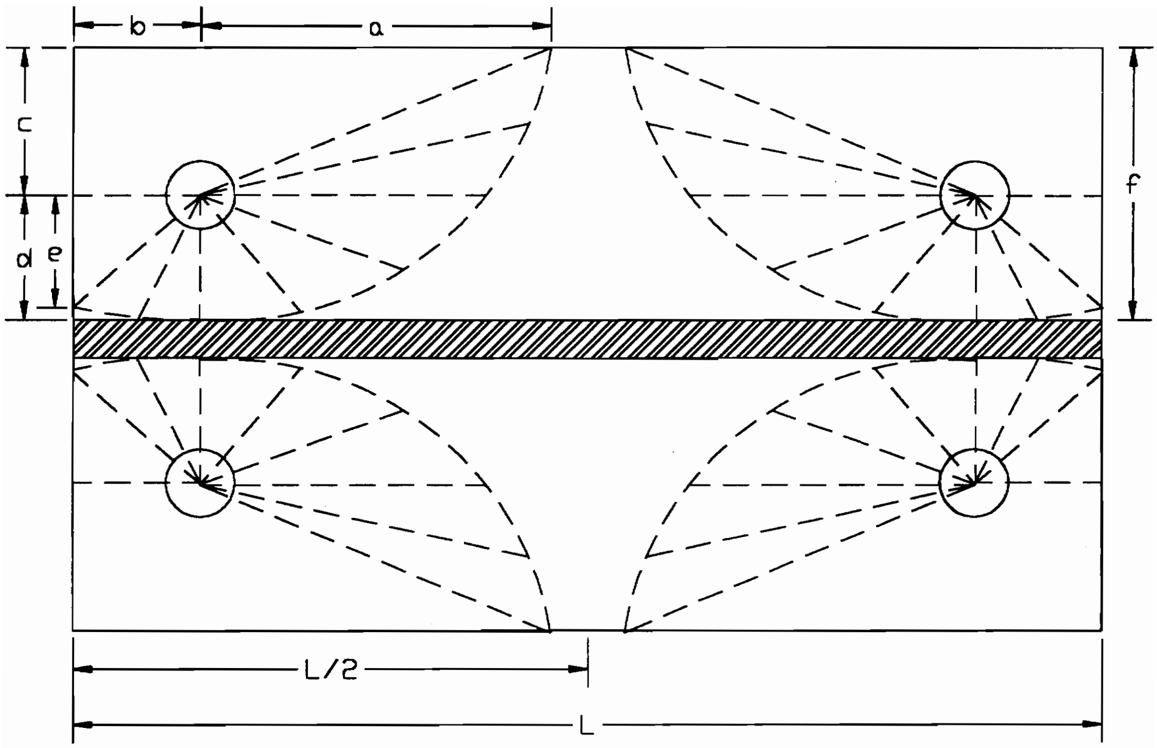


Figure 2.1 Unstiffened Tee Yield Line Pattern
Wide Spacing

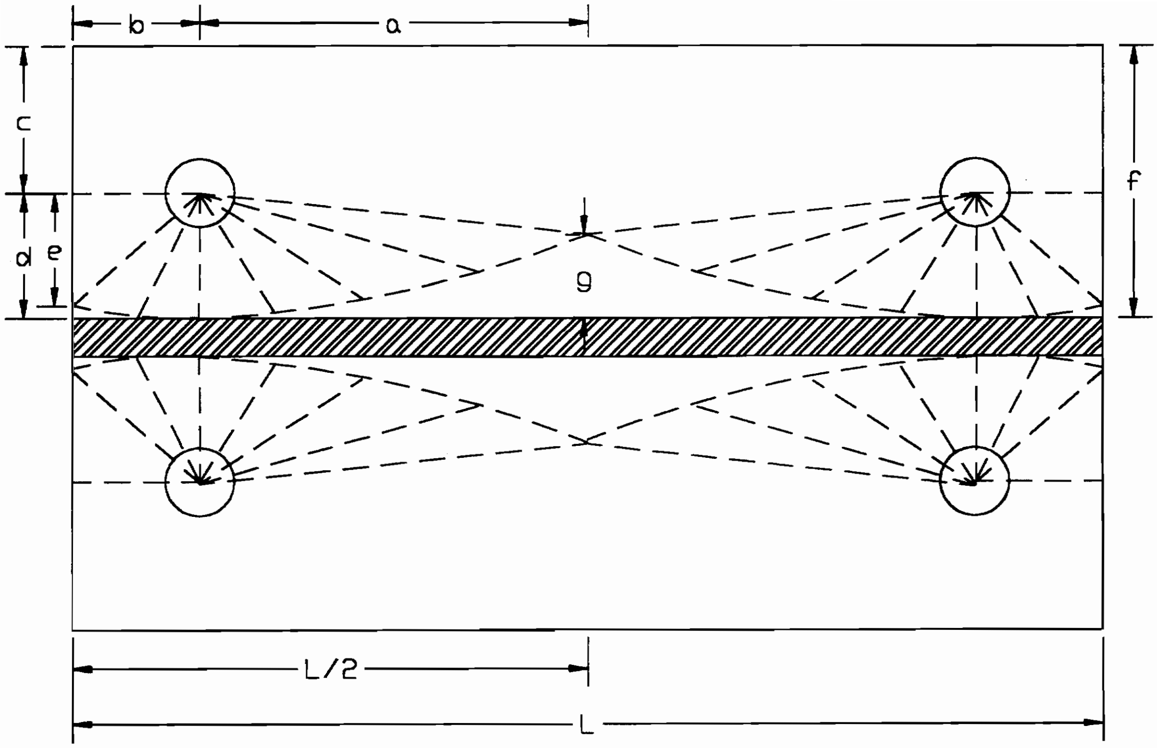


Figure 2.2 Unstiffened Tee Yield Line Pattern
Narrow Spacing

vertex of the parabola, which was found experimentally to be:

$$p = 2.17d \tag{2.10}$$

where d is the distance from the bolt hole center to the edge of the web fillet. The distance e is found through the parabolic equation as:

$$e = d - \left(\frac{b^2}{4p} \right) \tag{2.11}$$

The angle α is subtended by the parabola from the center of the bolt hole (not the focal point of the parabola). The angle α is the sum of θ_1 and θ_2, where θ_1 is the angle subtended from the bolt hole center to the free edge of the tee perpendicular to the web, and is given as:

$$\theta_1 = \tan^{-1}\left(\frac{b}{e} \right) \tag{2.12}$$

and θ_2 is the angle subtended from the bolt hole center to either the free edge of the tee hanger parallel to the web, or the intersection of the yield line with the yield line of the adjacent bolt. The dimension, a, is defined as the minimum of the distance from the center line of the bolt hole to the centerline of the tee hanger (perpendicular to the web), or the intersection of the yield line to the edge of the flange (parallel to the web).

The angle θ_2 is given by:

$$\theta_2 = \tan^{-1}\left(\frac{a}{d - g} \right) \tag{2.13}$$

or, if $g \geq d$

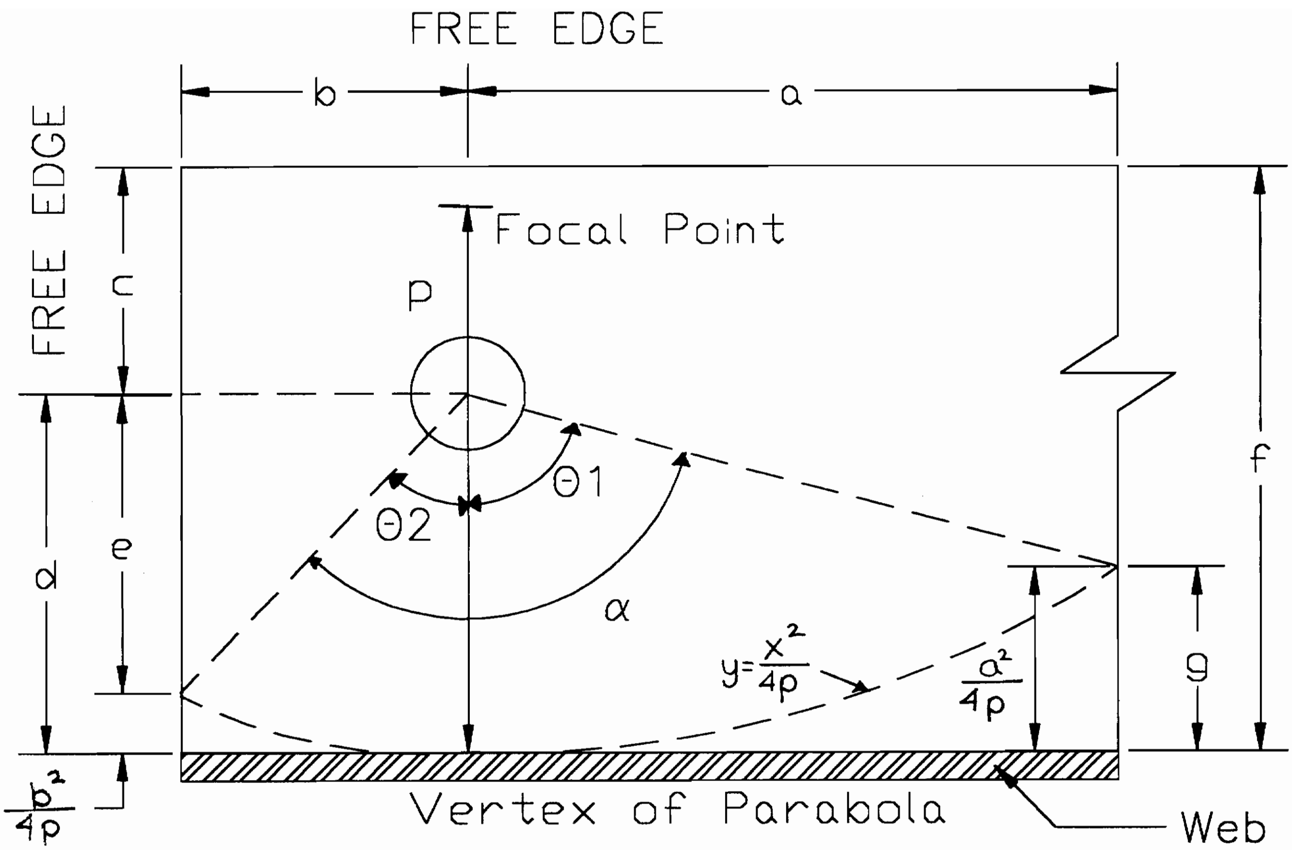


Figure 2.3 Variables Defining Yield Line for One Bolt

$$\theta_2 = \tan^{-1}\left(\frac{g-d}{a}\right) + 90 \tag{2.14}$$

where d is the distance from the bolt hole center to the fillet of the tee web, and a is defined as:

$$a = \min\left|\begin{array}{l} L/2 - b \\ \sqrt{4pf} \end{array}\right. \tag{2.15}$$

and

$$g = \frac{a^2}{4p} \tag{2.16}$$

where L is the total length of the tee. To assure that the length of the parabolic yield line does not extend out further than the tee flange, the following conditions must be satisfied:

$$g \leq f \text{ and } a \leq \sqrt{4pf}$$

If $g \geq f$ then:

$$\theta_2 = \tan^{-1}\left(\frac{f-d}{a}\right) + 90 \tag{2.17}$$

Equating the internal and external work expressions gives the load capacity for the four bolt tee hanger:

$$P_{us} = t_p^2 F_y \left[4\pi\left(\frac{\alpha}{360}\right) + (b)\left(\frac{1}{e}\right)\right] \tag{2.18}$$

A complete mechanism may not form at failure in plates that are relatively thick. A plate may fail at loads lower than those predicted by yielding due to shear effects. A term to reduce the effective capacity due to this effect must be added to determine load capacity. From Kennedy *et al.* (1981), the modified moment capacity per linear inch for steel becomes:

$$m_p = \frac{t_p^2}{4}\sqrt{F_y^2 - 3\left(\frac{V}{wt_p}\right)^2} \tag{2.19}$$

where V is the factored shear force per bolt, or $P_u/4$ for a four bolt configuration, and w is the flange width per bolt. Solving Equation 2.18 for the required tee design thickness in terms of the ultimate load P_u:

$$t_p = \sqrt{\frac{P_u}{\left(4\pi\left(\frac{\alpha}{360}\right) + (b)\left(\frac{1}{e}\right)\right)\sqrt{F_y^2 - 3\left(\frac{P_u}{4wt_p}\right)^2}}} \tag{2.20}$$

The solution requires some iteration due to the term for the plate thickness being on both sides of the equation.

2.3 STIFFENED TEE HANGER

The yield lines for the stiffened tee hanger are shown in Figures 2.4 and 2.5. As with the unstiffened tee hanger, the yield lines are dependent on the position of the bolts.

If the bolts are too far from the stiffener, then the parabolic yield line will form uninterrupted, and the tee will perform as an unstiffened tee hanger (Figure 2.4). If the following relation is true: $g \geq f$, where g is defined by Equation 2.16, then the tee hanger acts as an unstiffened tee, and the equation for the internal work of the unstiffened tee holds.

If the bolts are close to the stiffener, (Figure 2.5) then the parabolic yield line will

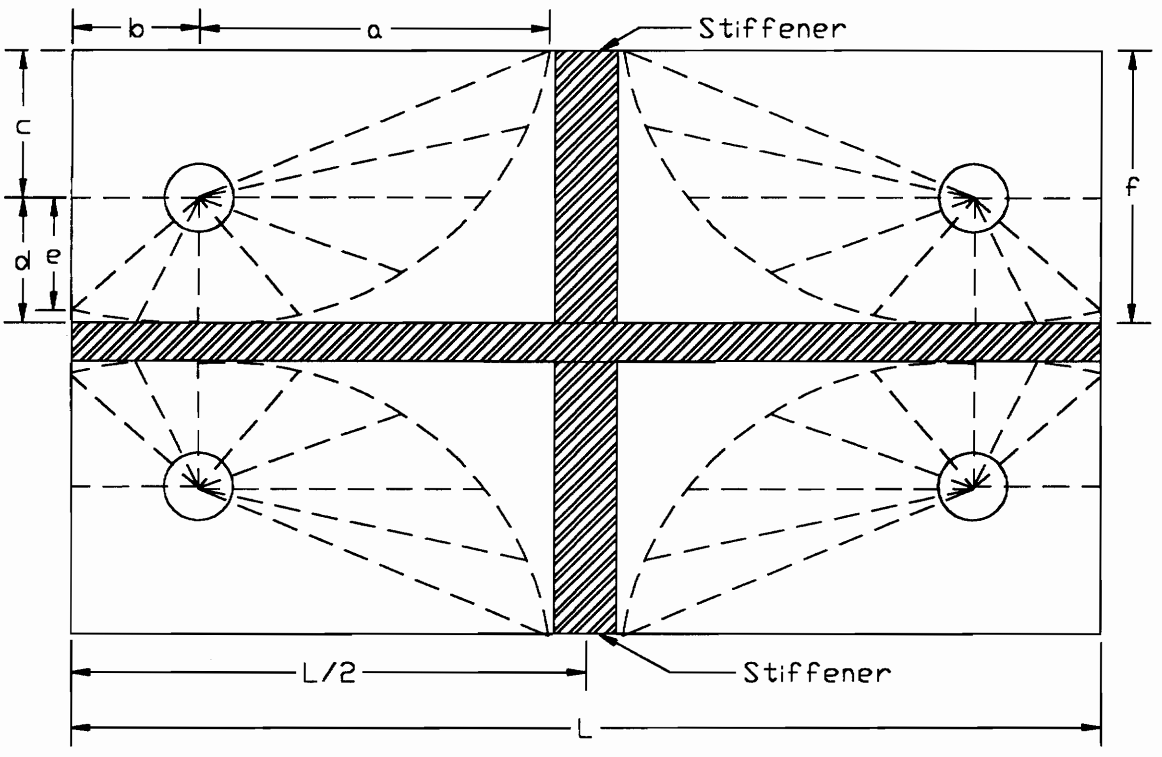


Figure 2.4 Stiffened Tee Yield Line Pattern
Wide Spacing

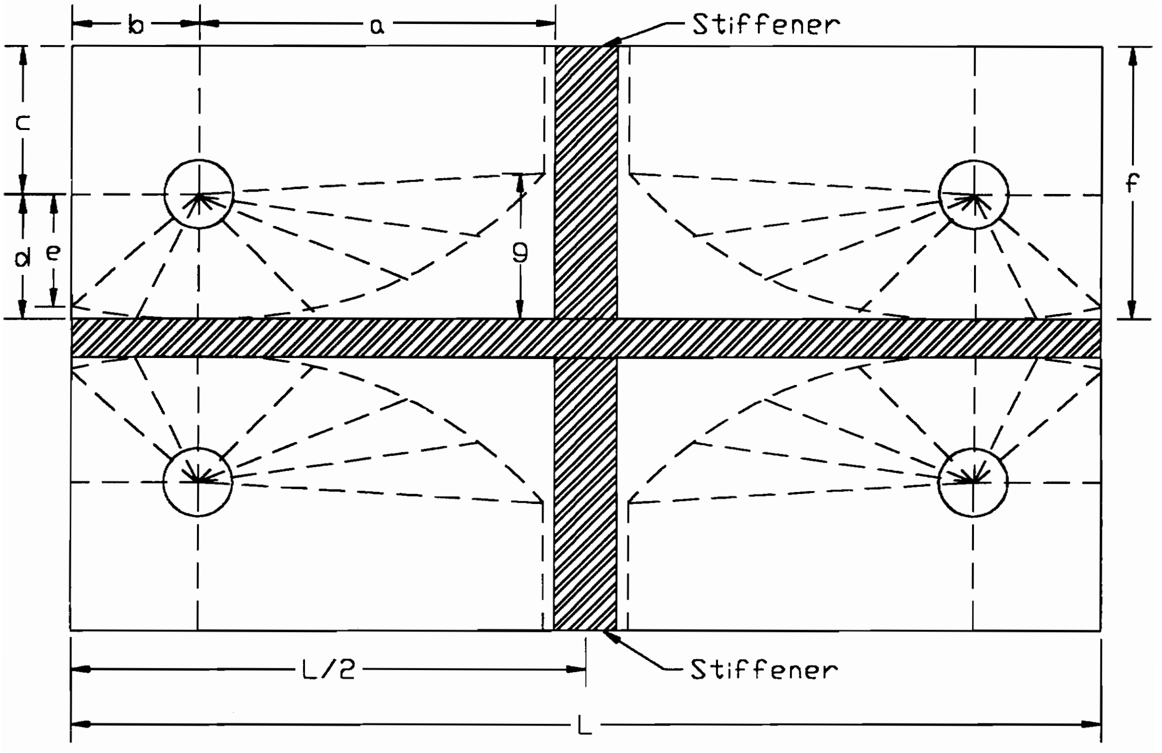


Figure 2.5 Stiffened Tee Yield Line Pattern
Narrow Spacing

form until it reaches the stiffener fillet, then a straight yield line will follow the stiffener fillet until it reaches the free edge.

For $g < f$, where g is defined by Equation 2.16, the internal energy for the stiffened tee yield line mechanism is given by:

$$W_i = m_p\left(4\pi\left[\frac{\alpha}{360}\right] + \left(c + (f - g)\right)\left(\frac{1}{a}\right) + (b)\left(\frac{1}{e}\right)\right) \tag{2.21}$$

Equating the internal and external work expressions gives the yielding load capacity for the stiffened tee hanger:

$$P_u = t_p^2 F_y\left(4\pi\left[\frac{\alpha}{360}\right] + \left(c + (f - g)\right)\left(\frac{1}{a}\right) + (b)\left(\frac{1}{e}\right)\right) \tag{2.22}$$

and solving for the required tee design thickness in terms of the ultimate load P_u with the shear effect incorporated:

$$t_p = \sqrt{\frac{P_u}{\left(4\pi\left[\frac{\alpha}{360}\right] + \left(c + (f - g)\right)\left(\frac{1}{a}\right) + (b)\left(\frac{1}{e}\right)\right)\sqrt{F_y^2 - 3\left(\frac{P_u}{4wt_p}\right)^2}}} \tag{2.23}$$

Again, thickness is found through iteration.

The process for finding θ_1 and θ_2, which are needed to determine the parabolic yield line, are the same as those for the unstiffened tee case, and 'a' is the minimum of the distance from the centerline of the bolt hole to: the distance from the edge of the stiffener fillet, or the intersection of the yield line to the flange (parallel to the web) as given by the parabolic yield line equation. The distance 'a' is given by the following equation:

$$a = \min \begin{cases} \dfrac{L}{2} - b - \dfrac{t_s}{2} - t_w \\ \sqrt{4pf} \end{cases} \qquad (2.24)$$

CHAPTER III

CONNECTION STRENGTH USING BOLT ANALYSIS

3.1 GENERAL

The yield line analysis described in Chapter II predicts the failure of the structural tee hanger due to the yielding of the tee flange. The connection can also fail in bolt rupture, therefore calculation of the bolt forces that arise during ultimate loading conditions are of importance. As the flange of the tee deforms in loading, contact points at the free edges (Figure 1.3) bring rise to prying actions. This prying action increases the load in the bolt, thus decreasing the ultimate allowable applied load. Kennedy *et al.* (1981) developed a method to calculate this added prying force, which was discussed in Chapter I. This prying force has been proven experimentally for moment end plate connections(Srouji *et al.*, 1984; Hendrick *et al.*, 1984; SEI. 1984; Bond and Murray, 1989; Abel and Murray, 1992; Borgsmiller *et al.* 1995) and has been dealt with in a variety of ways. Borgsmiller simplified the method developed by Kennedy *et al.* which is used in this study to predict the limit state for bolt rupture.

The Kennedy method assumes that a plate goes through three stages of behavior during loading: described as thick, intermediate, and thin. The stage is determined by the amount of deformation of the plate and the magnitude of the loading.. Each stage has a corresponding equation for calculating prying forces, and thus the total bolt force. The Kennedy method assumes no pretensioning of the bolts. When bolts are pretensioned, the initial two stages of the Kennedy method are bypassed. Bolt failure is said to occur when the applied load causes one of the bolts to reach its proof load. The proof load, P_t, is determined from:

$$P_t = A_b F_{yb} \tag{3.1}$$

where A_b is the nominal cross sectional area of the bolt, and F_{yb} is the tensile strength of the bolt as defined by Table J3.2, of the AISC LRFD specification(Load 1993).

The Kennedy method requires an iterative solution to determine the design thickness of the plate due to shear effects at the ultimate load. But assumptions can be made to greatly reduce the calculations involved in determining the ultimate applied load a connection can resist. Because of the ductile nature of steel, one bolt may reach its proof load, continue to yield, but carry load without rupture until the rest of the bolts have reached their proof loads. This has been proven by Abel and Murray (1992). The second assumption states that at this proof load, the plate acts as a thin plate as defined by Kennedy. This means that there is maximum additional bolt force due to prying action. This assumption only applies when calculating the ultimate load capacity.

3.2 APPLICATION TO STIFFENED AND UNSTIFFENED TEES

To calculate the connection capacity using the simplified approach, the sum of the bolt forces, P_b, is calculated. First, the maximum prying force from plate deformation, Q_{max}, is calculated. From the free body diagram in Figure 1.3, the prying force is subtracted from the proof load. Giving the following equation for a four bolt tee hanger connection:

$$P_b = 4P_t - 4Q_{max} \tag{3.2}$$

where P_t was defined in Equation 3.1. This equation is independent of whether a stiffener is added to the connection.

In the determination of the capacity of the connection, one must make sure that the quantity in the equation above is not less than the pretension of the bolt, T_b. The determination of the ultimate load then becomes:

$$P_b = \max \begin{vmatrix} 4(P_t - Q_{max}) \\ 4T_b \end{vmatrix} \tag{3.3}$$

The maximum prying force is calculated using :

$$Q_{max} = \frac{w' t_p^2}{4a} \sqrt{F_{py}^2 - 3\left(\frac{F'}{w' t_p}\right)^2} \tag{1.5}$$

with F' as:

$$F' = \frac{t_p^2 F_{py}\left(0.85 b_f / 2 + 0.80 w'\right) + \pi d_b^3 F_{yb} / 8}{4 p_f} \tag{1.6}$$

The value of 'a' in the above equation was found experimentally, and should not be confused with the variable 'a' defined in Chapter II. After completing tests described in Chapter IV, the applied load at which the bolt force became equal to its proof load was noted. An equation for a was developed by using the variables of plate thickness, yield stress of steel, and the diameter of the bolt. The equation was found by plotting all the variables together and finding the best curve to fit the experimental data. The resulting equation is:

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 \qquad (3.4)$$

Kennedy *et al.* (1981) warned that if the quantity under the radical for the maximum prying force equation (Equation 1.5) above was negative, then the plate would fail locally in shear before prying forces have developed, thus making the connection inadequate for the applied load.

The above equations were developed for unstiffened tees. Adding a stiffener to a tee hanger may increase its capacity, in effect, adding plate thickness to an unstiffened tee. To modify the thickness of the stiffened tee, the capacity of the stiffened tee is compared with that of an unstiffened tee of the same geometry.

$$t_{mod} = \sqrt{\frac{P^s}{F_y\left(P_{us} / m_p\right)}} \qquad (3.5)$$

Where P_s is the desired capacity of the stiffened tee, and P_{us} is the yield line capacity of the same tee, but unstiffened. It is assumed that shearing reduction is not needed in this case. Shear reduction effects the value of the yield stress of the material. In this equation there are two F_y terms, one is obvious, the other is nested inside the m_p term. Since there is an F_y term inside the m_p term, and if both F_y terms are reduced by the approximately the same amount, the reduction will cancel itself out.

CHAPTER IV

EXPERIMENTAL RESULTS AND COMPARISON WITH PREDICTIONS

4.1 EXPERIMENTAL PROCEDURE

The testing program consisted of twelve tests designed to have the plate yielding limit state and three to have the bolt rupture limit state. These specimens were fabricated from plates, as well as, cut from H-sections.

Eleven of the specimens designed for the plate yielding limit state were cut into tees from a W8x18 beam. All specimens were cut, with a steel saw, out of the same beam. The average yield stress for this beam, as found by coupons cut from both the flange and web, is 45.5 ksi.

The tee sections were cut to two different lengths, 10-1/2 in. and 6-1/2 in. Bolt holes of 13/16 in. diameter were then drilled. The bolts on all specimens were drilled at approximately the same distance from their respective free edges. Since the width of the flange was held constant, the only difference in bolt placement between the two different length specimens was the pitch between bolts on the same side of the web.

Each of the different length tees had three different stiffener placements: no stiffener, a 1/4 in. stiffener centered between the two bolt rows with a 5/16 in. E70xx weld all the way around the stiffener/flange interface, and the stiffener/web interface, and a 5/8 in. stiffener centered between the two bolt rows with a 5/16 in. E70xx weld all the way around the stiffener/flange interface and the stiffener/web interface. The yield stress of the stiffeners used is unknown, but is not considered to be of importance to the results.

The remaining tee section that was designed for the plate yielding limit state was built up from plates. The flange plate was cut 6-1/2 in. x7 in. x1/2 in. and had 1-1/16 in. diameter bolt holes drilled approximately 1-3/4 in. from the free edges. The specimen had no stiffener. The average yield stress of the flange plate was 68.4 ksi, as found from standard coupon tests. Specimens designed for the plate yielding limit state are shown in Figures 4.1 and 4.2, with dimensions in Table 4.1.

Two of the tees for the bolt rupture tests were cut from a single W21x101. The lengths were again 10-1/2 in. and 6-1/2 in.. The width of the flanges was 6 in., and the bolt hole diameters were 13/16 in.. The bolts were placed in the same configuration as in the plate yielding limit state tests. As mentioned before, there were three tests performed, two unstiffened and one stiffened. One of the unstiffened tees was built up. A 7/8 in. plate with yield stress of 43.85 ksi was used for the flange. The two unstiffened test specimens were 10-1/2 in. long and 6-1/2 in. long, and the stiffened specimen was 6-1/2 in. long and used a 1/2 in. thick stiffener welded as the stiffeners for the plate yielding limit state tests were. The yield stress for the W21x101 beam is 42.2 ksi, and was found by standard coupon tests. Specimens designed for bolt rupture limit state are shown in Figure 4.1, with dimensions listed in Table 4.2.

A SATEC universal testing machine with a 300 kip capacity was used for the tests. The tees were gripped in the SATEC in one of two ways: plates that were bolted to the web of the tee or plates that were welded to the web of the tee. The tee to be tested was then bolted to a larger tee which had a flange thickness of 1-1/2 in., ensuring it would not yield prior to specimen yielding.

Plate Thickness	a (in.)	b (in.)	c (in.)	d (in.)	f (in.)	ts (in.)	tw (in.)	tweld (in.)	d_b (in.)	d_{bh} (in.)	L (in.)	bf (in.)	# Tests
0.355	3.35	1.5	1.25	1.25	2.5	none	0.25	0.2	0.75	0.8125	6.5	5.4	2
0.355	1.482	1.5	1.25	1.25	2.5	0.25	0.25	0.2	0.75	0.8125	6.5	5.4	1
0.355	1.31	1.5	1.25	1.25	2.5	0.625	0.25	0.2	0.75	0.8125	6.5	5.4	2
0.355	5.25	1.5	1.25	1.25	2.5	none	0.25	0.2	0.75	0.8125	10.5	5.4	2
0.355	3.318	1.5	1.25	1.25	2.5	0.25	0.25	0.2	0.75	0.8125	10.5	5.4	2
0.52	3.25	1.75	1.75	1.25	3.0	none	0.75	0.125	1.0	1.0625	6.5	7.0	1
0.355	3.169	1.5	1.25	1.25	2.5	0.625	0.25	0.2	0.75	0.8125	10.5	5.4	2

Table 4.1 Plate Yielding Test Specimens

Plate Thickness	a (in.)	b (in.)	c (in.)	d (in.)	f (in.)	ts (in.)	tw (in.)	tweld (in.)	d_b (in.)	d_{bh} (in.)	L (in.)	bf (in.)	# Tests
0.785	5.25	1.5	1.25	1.25	2.5	none	0.5	0.375	0.75	0.8125	10.5	6.0	1
0.785	3.25	1.5	1.25	1.25	2.5	none	0.5	0.375	0.75	0.8125	6.5	6.0	1
0.875	1.31	1.5	1.25	1.25	2.5	none	0.5	0.375	0.75	0.8125	6.5	6.0	1

Table 4.2 Bolt Rupture Specimens

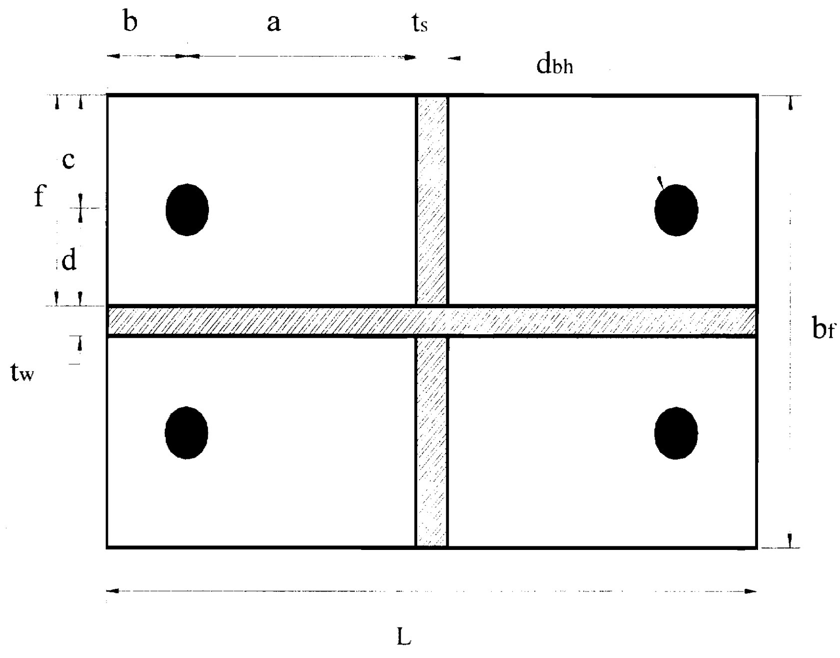


Figure 4.1 Stiffened Tee Hanger Test Specimens

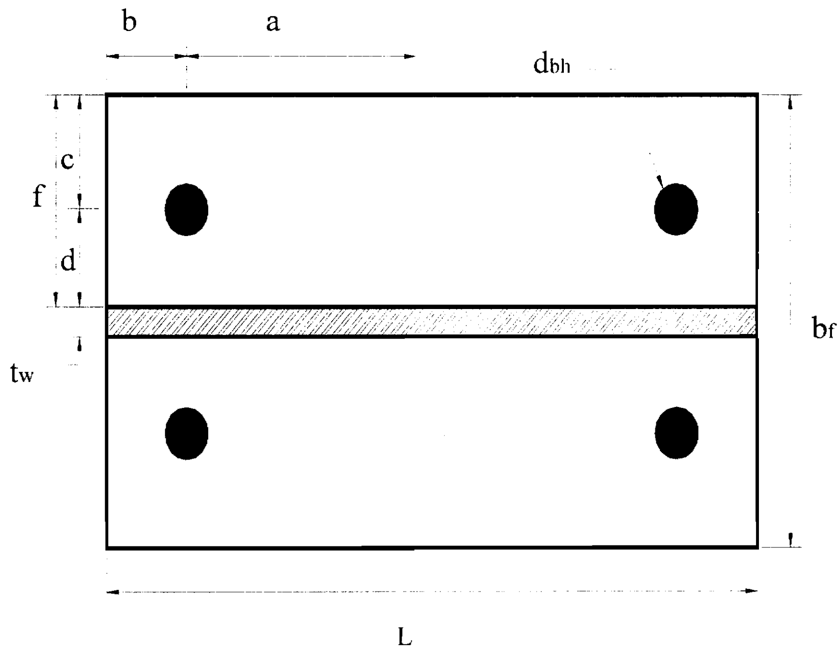


Figure 4.2 Unstiffened Tee Test Specimens

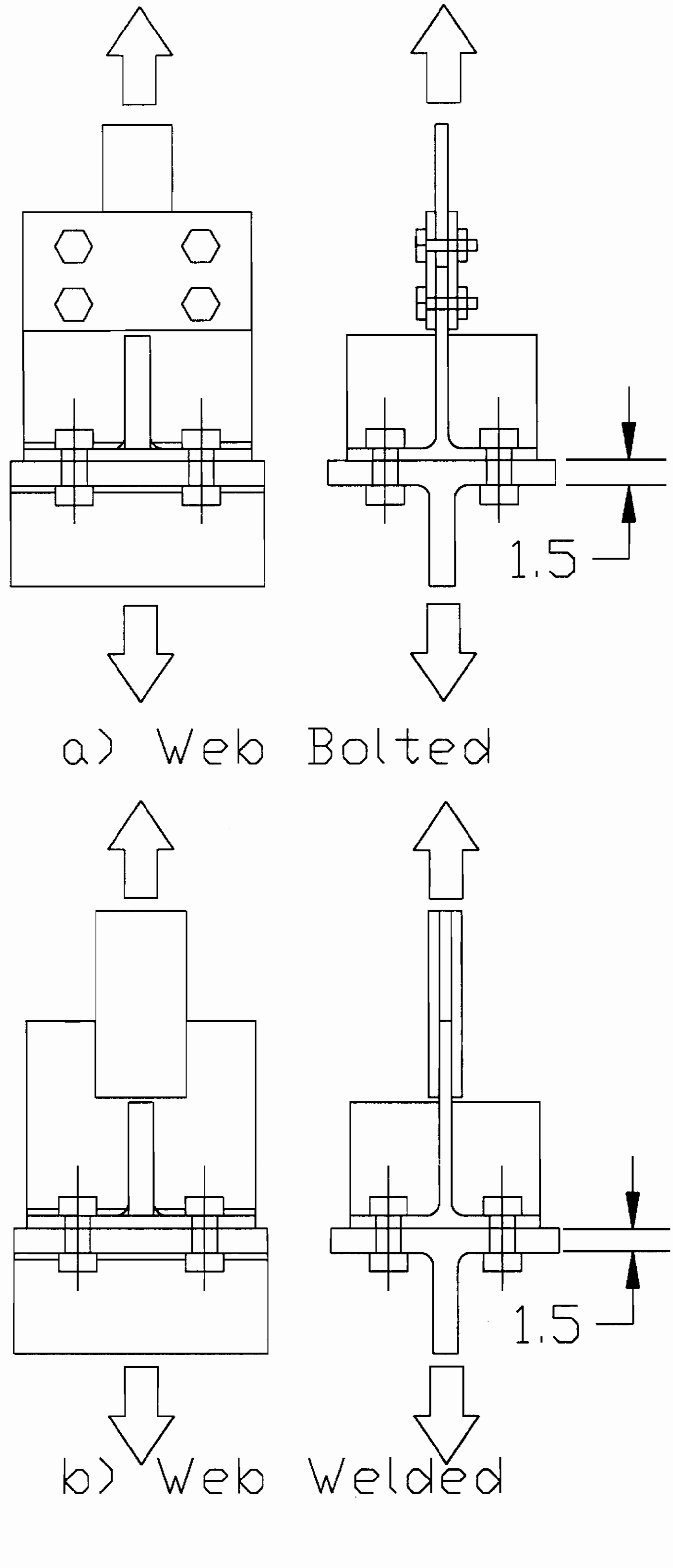


Figure 4.3 Tee Hanger Test Configuration

Both configurations are shown in Figure 4.3. Deflections between plates were measured by dial gages. These gages were attached to the web and stiffener (when applicable) of the tee and measured the plate separation of the specimen with respect to the top of the flange of the heavy tee. Again, the heavy tee, which the specimen was attached to, did not yield.

Bolt forces were found using bolt strain gauges. One or two strain gauged bolts were used on every test. The gauges were type BTM-6C manufactured by Tokyo Sokki Kenyujo Co. and are intended to determine bolt strain in the elastic region. Bolts are prepared as follows: A 2mm hole is drilled from the head of the bolt into the unthreaded portion of the shank. The hole is then cleaned out with acetone and allowed to dry. An epoxy provided by the manufacturer is then mixed and placed in the hole, followed by inserting the strain gauge. The bolt, with strain gauge, is then baked in an oven at 325 F for twelve hours for the epoxy to harden.

The strain gauged bolts were then calibrated. They were wired to a strain indicator and the SATEC testing machine was used to apply load in increments. The corresponding strain at each load increment is noted. The values of load and strain are then linearized using a regression analysis, and a slope of the load/ strain relationship is determined. During testing this slope is then used to calculate the force in the bolt.

The specimens were placed in the SATEC, and the bolts were tightened to either the minimum bolt tension load designated by AISC Table J3.1 (AISC, Load 1993), or as much as possible. In some instances, two people turning a ratchet with an extra long handle, were unable to tighten the bolt up to the minimum pre-tension. Tests in which the minimum bolt tension was not attained can be identified in the Bolt Force versus Applied Load plots in Appendices B, C, and D. Applicable plots indicate an initial pretension less than that required by AISC.

The pretensioning force was determined directly when bolts were strain gauged. Bolts that were not strain gauged were tightened to "feel" the same as those that were.

Load was then applied at 5 kips increments for the plate yielding limit state specimens, and 10 kips increments for the bolt rupture limit state specimens. Plate deflection, applied load, and bolt force quantities were all recorded. Tests were stopped when either plate yielding was excessive as determined from the measured separation or bolt forces became excessive.

4.2 DETERMINATION OF EXPERIMENTAL CONNECTION STRENGTH

Determination of the experimental strength of the tee hanger specimens is essential in showing the validity of the predicted strength. Failure load was determined for both plate yielding and bolt rupture for each test.

Each test has a graph of applied load versus plate separation associated with it. Each graph contains two data series, one for each dial gauge that was used. Figure 4.4 shows a typical load versus plate separation plot. Initially the data points make a straight line where the plates are deforming elastically. As load continues to be applied, the connection softens, and the flange of the tee hanger is now yielding. A straight line is

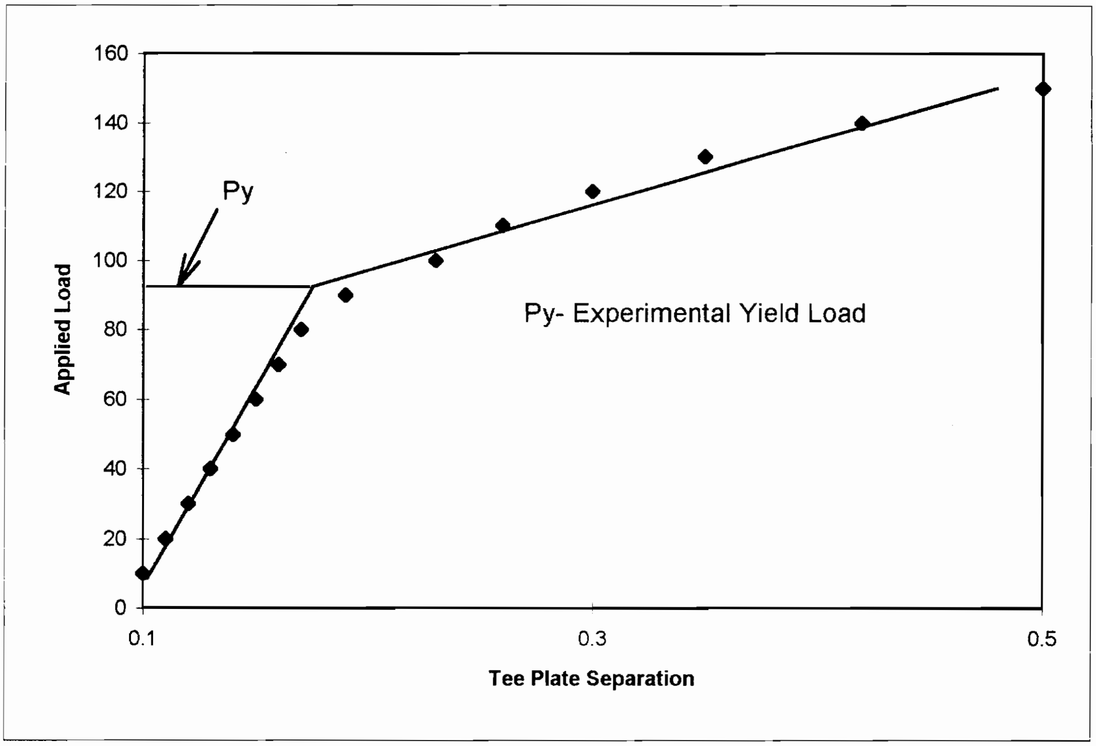


Figure 4.4 Sample Applied Load vs. Plate Separation

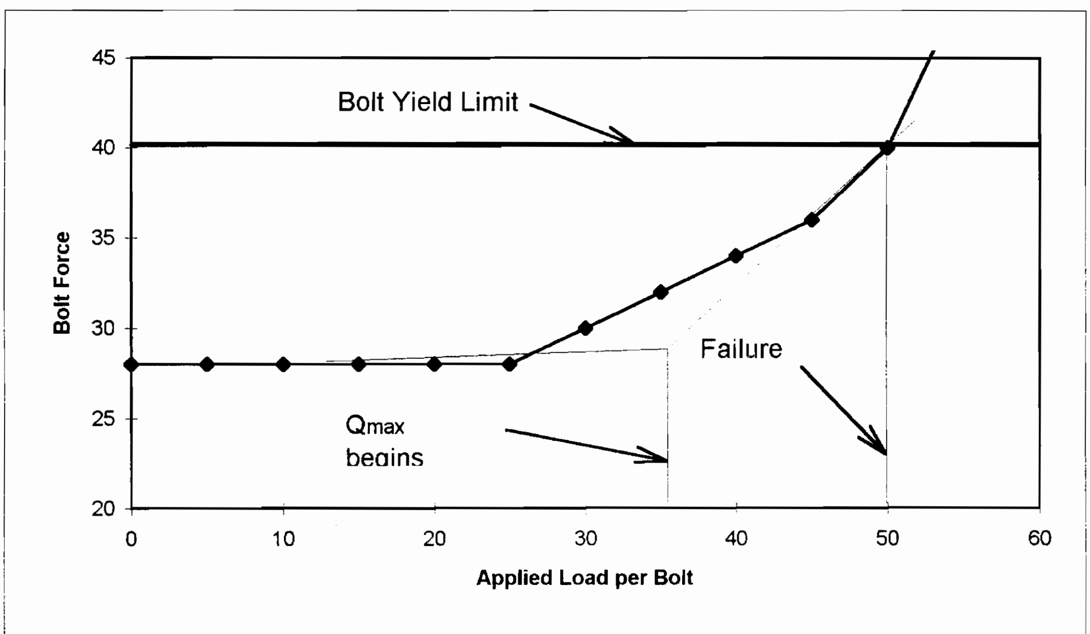


Figure 4.5 Sample Bolt Force vs. Applied Load Plot

drawn connecting the elastic and plastic regions, this intersection is assumed to be the load at which a mechanism has just formed, and thus the failure load.

Bolt force versus applied load plots, such as the one in Figure 4.5, were also made for each test. The bolt force is initially the pre-tension force in the bolt. The force in the bolt remains constant until it reaches an applied load sufficient to cause plate separation. At this load, the force in the bolt increases, and prying action (which was discussed in Chapter I) is developed. The strain gauges in the bolts are unable to accurately read strains once the bolt has yielded, therefore readings are taken only to the proof load of the bolt.

The plots contain one or three sets of data series. If only one data series is shown, then either only one bolt was strain gauged, or the second strain gauged bolt was giving faulty readings. If three data series are shown, two are bolt force readings, and one is their average. Failure of the bolt is assumed to occur at the applied load which causes the bolt force to reach the bolt yield load. Yield stress of the bolt is taken as 90 ksi (from AISC Load (1994), Table J3.2) and the yield load is the cross-sectional area of the unthreaded portion of the bolt, multiplied by the yield stress. Plots for all tests can be found in Appendices B, C, and D.

The bolt force remains constant until the applied load causes the plate to deform enough to allow prying action to occur. It is assumed that when the bolt force first deviates from the initial pre-tension load, it is increasing at the same rate as the applied load, shortly thereafter, the maximum prying force is realized. The applied load at which prying action begins was determined to be the intersection of lines drawn along the portion of the plot of initial deviation from pre-tensioning and the bolt force increase due to prying action. An example of the bolt force versus applied load chart is shown in Figure 4.5.

4.3 DETERMINATION OF PREDICTED CONNECTION STRENGTH

Equations to predict the strength of tee hanger connections were given for the limit states of plate yielding and bolt rupture. Plate yielding, P_y, is determined by using the yield line method as discussed in Chapter II, where P_y is the yield load for either the stiffened or unstiffened case. Bolt rupture, P_b, using the modified Kennedy method was discussed in Chapter III. Calculations for P_y and P_b for all tests are presented in Appendices B through J.

Once the connection strengths for the two limit states have been calculated, a controlling connection strength, P_{pred}, is chosen. Here an assumption must be made. If prying action is to occur, the plate must sufficiently deform. If the plate has not sufficiently deformed, or begun to form plastic hinges, there can be no points of application for the prying forces to act (see Figure 1.3). This concept was introduced by Kennedy *et al.* (1981) as different stages of plate behavior in tee stubs. With this assumption in mind, a "prying action threshold" can be determined. This threshold is the applied load at which the maximum prying force, Q_{max}, has just begun. Prior to this applied load, the plate acts as a thick plate, or one that is un-deformed. After this load, the plate has begun forming plastic hinges to cause maximum prying force.

If the ratio of the applied load which causes yielding in the plate to the applied load which first causes prying forces to be realized is less than one, then prying forces are present prior to the formation of a mechanism in the plate. In Table 4.3 one can see that the applied load at which Q_{max} begins, is after the formation of a full mechanism in tees with thin flanges ($t_p = 0.355$ in.), and prior to the formation of full mechanism in tees with thicker flanges ($t_p \geq 0.5$ in.). Test 13a saw Q_{max} begin at an applied load slightly larger than the load producing a full mechanism. The ratio of applied yield load to applied load at which Q_{max} begins, in the case of test 13a, is close enough to unity (as compared to tests having a plate thickness of 0.355 in.) that the above relationship holds. Following previous research on the modified Kennedy method (Borgsmiller 1995) and to be conservative, it will be assumed that the applied load at which Q_{max} begins is 90% of the yield strength of the tee, or $0.9P_y$.

Table 4.3 Prying Action Threshold of Test Specimens

Test	Flange Thickness (in)	Plate Yielding Load (PY) (kips)	Applied Load Q_{max} Begins (AL) (kips)	AL/PY
4a&10a	0.355	44	64	1.45
1a&11a	0.355	42	64	1.52
2a	0.355	50	74	1.48
8a&3a	0.355	54	80	1.48
9a&5a	0.355	47	66	1.40
13a	0.899	140	156	1.11
12a	0.785	128	120	0.94
1	0.520	100	80	0.80

The predicted strength of the connection can be determined by the following guidelines:

If applied load < $0.9P_y$ Thick plate behavior

If applied load > $0.9P_y$ Thin Plate Behavior

If the plate behaves as a thick plate, then no prying action takes place. To calculate connection strength with no prying action, P_{np}, one follows the guidelines set forth in Chapter III, except Q_{max} is set equal to zero.

Once the predicted forces P_y, P_r, and P_{np} are known, the prediction of the connection strength, P_{pred}, can be determined as follows:

$$P_{pred} = P_{np} \qquad \text{if } P_{np} < 0.9P_y \tag{4.1}$$

$$P_{pred} = P_r \qquad \text{if } 0.9P_y \leq P_{np} \text{ and } P_r \leq P_y \tag{4.2}$$

$$P_{pred} = P_y \qquad \text{if } P_y < P_r \tag{4.3}$$

If the strength for the limit state of bolt rupture with no prying action, P_{np}, is less than the prying action threshold, $0.9P_y$, then yielding will not occur in the plate and the connection will fail in bolt rupture with no prying action in the bolts. If the limit state for bolt rupture with no prying forces is greater than the prying action threshold, $0.9P_y$, prying action is taking place due to plate yielding prior to bolt rupture. If the strength of the limit state for bolt rupture with prying forces, P_r, is less than the yield line strength of the tee stub flange plate, P_y, the connection will fail in bolt rupture, with prying action prior to complete plate yielding. If P_r is greater than P_y, then the connection will fail in plate yielding.

Table 4.4 Test Results-Unstiffened Tee Hangers
Flange Yield-Line Failure Design

Test	P_b (kips)	$P_{b,pred}$ (kips)	P_y (kips)	P_r (kips)	P_{np} (kips)	P_{pred} (kips)	P_{exp} (kips)	P_{pred}/P_{exp}	$P_{b,pred}/P_b$
4a/10a	74.00	72.00	38.90	112.00	160.00	38.90	44.00	0.88	0.97
1a/11a	68.00	72.00	34.04	112.00	160.00	34.04	42.00	0.81	1.06
1	110.00	106.00	105.80	208.00	280.00	105.80	100.00	1.06	0.96

Table 4.5 Test Results-Stiffened Tee Hangers
Flange Yield-Line Failure Design

Test	P_b (kips)	$P_{b,pred}$ (kips)	P_y (kips)	P_r (kips)	P_{np} (kips)	P_{pred} (kips)	P_{exp} (kips)	P_{pred}/P_{exp}	$P_{b,pred}/P_b$
9a/5a	77.00	75.00	40.56	112.00	160.00	40.56	47.00	0.86	0.97
7a/6a	N/A	76.00	40.53	112.00	160.00	40.53	47.25	0.86	N/A
2a	77.00	82.00	40.40	112.00	160.00	40.40	50.50	0.80	1.06
8a/3a	85.00	84.00	41.27	112.00	160.00	41.27	54.00	0.76	0.99

Table 4.6 Test Results - Bolt Rupture Design

Test	P_b (kips)	$P_{b,pred}$ (kips)	P_y (kips)	P_r (kips)	P_{np} (kips)	P_{pred} (kips)	P_{exp} (kips)	P_{pred}/P_{exp}	$P_{b,pred}/P_b$
12a	134.00	120.00	133.00	119.60	160.00	119.60	128.00	1.04	0.90
13a	166.00	126.00	153.00	127.00	160.00	127.00	140.00	1.09	0.76
14a	N/A	N/A	123.00	127.00	280.00	105.80	160.00	0.77	N/A

Notes for above Tables: N/A for tests giving faulty readings

P_b- Experimental applied load at which the bolt force is equal to the bolt proof load.

$P_{b,pred}$-Predicted applied load at which bolt force is equal to the bolt proof load.

P_y-Predicted applied load at which mechanism in flange plate commences.

P_r-Predicted applied load at which bolt rupture with prying action is the predicted mode of failure.

P_{np}-Predicted applied load at which bolt rupture with no prying action is the predicted mode of failure.

P_{pred}-Predicted failure load for the connection.

P_{exp}-Experimental failure load.

4.4 UNSTIFFENED TEE HANGER COMPARISON-YIELD DESIGN

The results for the unstiffened tee hangers which were designed to fail by plate yielding are found in Table 4.4. The value P_{pred}/P_{exp} is the ratio of the predicted failure load to the experimental failure load. If this ratio is less than one ($P_{pred}/P_{exp} < 1.0$) then the design is conservative, if the ratio is greater than one ($P_{pred}/P_{exp} > 1.0$) then the design was unconservative. The tee hangers with thin flanges (t_p = 0.355 in., d_b = 3/4 in.) showed conservative ratios at about 0.85. The thicker tee (t_p = 0.52 in., d_b = 1.0 in.) was slightly unconservative with a ratio of 1.06.

In all tests, the specimens failed in yielding as predicted. In Table 4.4 it can be seen that bolt forces, P_b, reach their proof loads at an applied load much larger than the applied load that caused failure in yielding, P_y. Comparison between experimental and predicted applied load at which the bolt force reached its proof load showed very good results. The average of the predicted to experimental applied loads that caused bolt force to equal proof load, $P_{b,pred}/P_b$, was 1.0. Calculations for the predicted loads can be found in Appendix B.

4.5 STIFFENED TEE HANGER COMPARISON-YIELD DESIGN

The results for the stiffened tee hangers, which were designed to fail by plate yielding are found in Table 4.5. In all tests, the specimens failed in yielding as predicted. The ratio of the predicted to experimental yield loads, P_{pred}/P_{exp}, averaged 0.82, which is conservative.

In Table 4.5 it can be seen that bolt forces, P_b. reach their proof loads at an applied load much larger than the applied load that caused failure in yielding, P_y. The average for the predicted to experimental bolt forces $P_{b,pred}/P_b$, was 1.01, which is slightly unconservative, but still very good.

The use of a stiffener increased the capacity of the connection as predicted. In the tests using shorter tees (2a,8a/3a), where the distance between bolts on the same side of the web was small, the use of thicker stiffeners increased the strength significantly. The experimental strength rose from 42 kips for the unstiffened case, to 50 kips and 54 kips for the stiffened samples. This is due to the fact that the parabolic yield line intersected the stiffener yield line very close to the stiffener/web intersection (see Figure 2.4). In the case of the tests which contained longer tees (9a/5a, 7a/6a) the increase in strength due to increased stiffener thickness was negligible because the parabolic yield line intersected the stiffener yield line very far from the stiffener/web intersection (see Figure 2.3). Calculations for the predicted loads can be found in Appendix C.

4.6 BOLT RUPTURE DESIGN COMPARISON

The results for the tee hangers designed for bolt rupture can be found in Table 4.6. All tests, with the exception of Test 14a, had the predicted failure load, P_{pred}, to be less than the predicted yield failure load, P_y , thus the predicted failure mode was bolt rupture. Test 14a had a predicted bolt rupture failure load, P_r, slightly greater than the predicted plate yielding load, P_y, thus giving a predicted failure mode of plate yielding. The average ratio of predicted to experimental bolt forces, $P_{b,pred}/P_b$. was 0.83 which is conservative, but a reasonable result. For maximum prying forces to be realized, plate yielding must begin, but a full mechanism need not occur. In all the tests, plate yielding had begun, but the formation of a complete mechanism did not form until after the bolts reached their proof loads. The ratio of the predicted to experimental yield load, P_{pred}/P_{exp}, was 1.06 which is slightly unconservative. The predicted to experimental yield load ratio in test 14a was 0.76, this is considered to be an out liar, and is not considered.

4.7 COMPARISON WITH OTHER RESULTS

Table 4.7 shows a comparison between the results from the method developed in this study, with results from design methods developed by others. The tee examined was one with the dimensions found in Figure 1.2, with a plate thickness of 0.625 in. The failure load is assumed to be the applied load that causes the first plastic hinge to form. This plastic hinge is located at the web to flange interface (See Figure 1.3). This is not the load at which the connection has formed a complete mechanism. The design method developed in this study determines the applied load at which a full mechanism forms. For comparison, the failure load will be assumed to be the load at which yielding begins or $0.9P_{us}$, or 90% of the yield strength of the unstiffened connection.

Table 4.7 Comparison of Failure Loads for Different Design Methods on Single Tee Section

Researcher	Failure Load per bolt (kips)
Douty & McGuir	18.9
Grundy *et al.*	12.9
Packer & Morris	22.8
Mann & Morris	25.8
Kennedy *et al.*	12.3
Murray & Otegui	24.3

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

5.1 SUMMARY

This study introduced a simplified yet accurate method for designing structural tee hangers with or without stiffeners. The method covers two limit states: failure of the connection through plate yielding, and failure of the connection through bolt rupture.

When a tee hanger is loaded in tension, it will deform elastically until it reaches an applied load which will cause it to start deforming inelasically. At this load, plastic hinges begin to form in the tee flanges, until a complete mechanism forms. The equations controlling the limit state of plate yielding were derived using the yield line method. In these equations, the maximum applied load is that which causes the connection to form a complete mechanism. The equation controlling bolt rupture, with or without prying action, was derived from the modified Kennedy method. This simplification of the Kennedy method is based on the assumption that once the connection has begun to yield substantially, but has not formed a complete mechanism, the prying forces are at their maximum, and therefore only the maximum prying force, Q_{max}, is used in the calculation of the bolt force. If the plate is strong enough, no yielding will occur, and the bolts will have no added forces other than those applied by the load.

The two limit state equations were used to calculate the failure load, and mode, for 15 tests of tee hangers with various lengths, plate thickness, bolt diameters, and stiffener thickness. The proposed procedure for design of tee hangers is summarized in

Section 5.3. The design strength, ϕP_n, is calculated for the limit states of flange yield, P_y, and bolt rupture with and without prying action, P_r, and P_{np}, respectively. The resistance factors ϕ_r, and ϕ_y, have been incorporated into the equations to make them applicable to Load Factor and Resistance Design. The resistance factor for bolt rupture ϕ_r, is 0.75, and the resistance factor for plate yielding ϕ_y, is 0.9 (Load 1993).

5.2 CONCLUSIONS

The major conclusions drawn from this study are:

1) The yield-line mechanisms described in Chapter II adequately predict the strength of stiffened and unstiffened tee hanger connections. Out of the 15 tests, 14 gave usable predicted versus experimental values. One test had a ratio of 0.76, and was considered an outlier and thrown out. The mean value of the predicted yield load to the experimental yield load is 0.94. The standard deviation is 0.119, with a variance of 0.014.

2) The simplified Kennedy method described in Chapter III adequately predicted the bolt forces in all tests. Out of the 15 tests, 11 gave usable data for bolt force calculations. One test had a predicted - to - experimental ratio of 0.76 and was considered an outlier and thrown out. The remainder of the bolt readings that were thrown out were due to faulty bolt strain gage readings. The mean value of the predicted applied load at bolt proof load was 0.99 with a standard deviation of 0.057 and a variance of 0.003.

3) The threshold at which prying action becomes a maximum is assumed to be 90% of the yield capacity of the plate or $0.9P_y$. If the applied load is less than this threshold, then the end plate behaves as a thick plate, and prying action is negligible.

Once the load has crossed this threshold, the plate is considered to be thin and the maximum prying force is incorporated into the bolt analysis.

5.3 DESIGN RECOMMENDATIONS

Two LRFD design procedures have been devised depending on the limiting provisions of the design. If it is necessary to limit bolt diameter, Design Procedure 1 is recommended. If it is necessary to limit the thickness of the plate, Design Procedure 2 is recommended. Design Procedure 1 is preferred, because it ensures that no prying action takes place at the ultimate load, thus reducing the number of calculations needed. Plate thicknesses for this procedure are only approximatley 10% thicker than for the design procedure includung prying action.

Design Procedure 1: The following procedure results in a relatively thick plate for the tee hanger and smaller diameter bolts. The design is governed by bolt rupture with no prying action included. The design steps are:

1) Compute the ultimate factored load, P_u, using factors specified in Chapter A4-1 of the 1995 AISC specification (Load 1993). Set the connection design strength, ϕP_n, equal to the desired ultimate load, which is set equal to the proof load of the bolts.

$$P_u = \phi_r P_r = \phi_r(4P_t) \tag{5.1}$$

Solve for the required bolt proof load P_t:

$$P_t = \frac{P_u}{4\phi_r} \tag{5.2}$$

Where ϕ_r is the reduction factor for bolt rupture and is equal to 0.75.

Solve for the required bolt diameter, d_b, from the expression:

$$d_b = \sqrt{\frac{4_{pt}}{\pi F_{yb}}} \tag{5.3}$$

(where F_{yb} for an A325 bolt is 90ksi.) and select the appropriate bolt diameter. With the selected bolt diameter, re-calculate the bolt proof load P_t:

$$P_t = \left(\frac{\pi d_b^2}{4}\right) F_{by} \tag{5.4}$$

and check that it is sufficient to resist the applied load:

$$P_u \leq \phi_r (4P_t) = \phi_r P_r \tag{5.1}$$

2) Choose a stiffened or unstiffened tee configuration and the geometry required as shown in Figure 5.1. Also choose type of plate material.

3) To ensure that the flanges of the plate will be strong enough to resist yielding and cause the connection to fail by bolt rupture with no prying action, divide P_u by 0.9 and set the quantity equal to the yield strength of the flanges of the tee hanger:

$$P_u/0.9 = \phi_y P_y \tag{5.5}$$

Where ϕ_y is the reduction factor for plate yielding and is equal to 0.9.

4) Solve for the required flange thickness for either the stiffened or unstiffened case, depending on which configuration is needed, using one of the following equations:

For the unstiffened tee

$$t_p = \sqrt{\frac{P_u / 0.9}{\left(4\pi\left(\frac{\alpha}{360}\right) + (b)\left(\frac{1}{e}\right)\right)(\phi_y)\sqrt{F_y^2 - 3\left(\frac{P_u / 0.9}{4wt_p}\right)^2}}} \tag{5.6}$$

For the stiffened tee:

$$t_p = \sqrt{\frac{P_u / 0.9}{\left(4\pi\left[\frac{\alpha}{360}\right] + \left(c + (f-g)\right)\left(\frac{1}{a}\right) + (b)\left(\frac{1}{e}\right)\right)\left(\phi_y\right)\sqrt{F_y^2 - 3\left(\frac{P_u / 0.9}{4wt_p}\right)^2}}} \qquad (5.7)$$

Choose an appropriate plate thickness, then recalculate the yield strength of the tee flange, $\phi_y P_y$, using the appropriate equation, e.g. P_{us} or P_s, in the equation summary table, Table 5.1.

5)Check that $\phi_r P_r < 0.9\phi_y P_y$ for the chosen values of t_p and d_b. If the inequality is true, the design is complete. Otherwise, increase plate thickness, or add stiffeners until the inequality stands.

<u>Design Procedure 2</u>: The following procedure results in a design with a relatively thin end plate and larger diameter bolts. The design is governed by either yielding in the plate connection or bolt rupture when prying action is included.

1) Compute the ultimate factored load, P_u, using factors specified in Chapter A4-1 of the 1995 AISC specifications (Load 1993). Set the connection yield design strength, $\phi_y P_{us}$, or $\phi_y P_s$ equal to the desired ultimate load.

$$P_u = \phi_y P_{us} \text{ or } \quad P_u = \phi_y P_s \qquad (5.8)$$

2) Choose a stiffened or unstiffened tee configuration and required tee hanger geometry as shown in Figure 5.1. Also choose type of plate material. Solve for the plate thickness by iteration using equations in Table 5.1. A good first guess for plate thickness is 1in. per 100 kips of applied load.

3) Choose a plate thickness, and recalculate the stiffened or unstiffened yield strength of the connection using equations in Table 5.1. Check that it is greater than the factored design load:

$$\phi_y P_{us} \text{ or } \phi_y P_s \geq P_u \tag{5.9}$$

If using an unstiffened tee, and the yield strength is below the factored design load, addition of a stiffener may increase the yield strength sufficiently to make the inequality true.

4) Select a trial bolt diameter and calculate the connection resistance for the limit state of bolt rupture with prying action, $\phi_r P_r$, using the equations in Table 5.1 for bolt rupture. If a stiffener is being used, the modified plate thickness, t_{mod}, must be calculated to find the maximum prying force, Q_{max}. To calculate t_{mod}, the yield strength of the same connection, unstiffened, ($\phi_y P_{us}$) mustalso be calculated. The equations can be found in Table 5.1

5) Check that $\phi_r P_r > P_u$, this will ensure that the bolts will not yield prior to the factored design load. If necessary, adjust the bolt diameter until $\phi_r P_r$ is slightly larger than P_u .

5.4 DESIGN EXAMPLES

Example 1

Determine the required thickness and bolt diameter for a Structural Tee Hanger, with the geometry shown in Figure 5.1, that ensures the mode of failure to be bolt rupture. The plate material is A572 Gr 50 and the bolts are A325. The factored applied load is 65 kips.

1) P_u was given as 65 kips, therefore:

Table 5.1 Tee Hanger Equation Summary

Variables Defining Yield Line Mechanism for Stiffened and Unstiffened Tees

$$d = \frac{b_f}{2} - \frac{t_w}{2} - t_{weld} \qquad p = 2.17d \qquad e = d - \left(\frac{b^2}{4p}\right)$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) \qquad \theta_2 = \tan^{-1}\left(\frac{a}{d-g}\right)$$

$$\text{if } g > d \text{ then } \quad \theta_2 = \tan^{-1}\left(\frac{g-d}{a}\right) + 90 \qquad \text{if } g > f \text{ then } \quad \theta_2 = \tan^{-1}\left(\frac{f-d}{a}\right) + 90$$

$$a =_{min} \left| \begin{array}{l} \text{measured} \\ L/2 - b \\ \sqrt{4pf} \end{array} \right. \qquad g = \frac{a^2}{4p}$$

Equations for Determining Tee Flange Thickness

Unstiffened Tee

$$t_p = \sqrt{\frac{P_u}{\left(4\pi\left(\frac{\alpha}{360}\right) + (b)\left(\frac{1}{e}\right)\right)\sqrt{F_y^2 - 3\left(\frac{P_u}{4wt_p}\right)^2}}}$$

Stiffened Tee

$$t_p = \sqrt{\frac{P_u}{\left(4\pi\left[\frac{\alpha}{360}\right] + (c + (f-g))\left(\frac{1}{a}\right) + (b)\left(\frac{1}{e}\right)\right)\sqrt{F_y^2 - 3\left(\frac{P_u}{4wt_p}\right)^2}}}$$

Table 5.1 Tee Hanger Equation Summary (cont.)

Equations for Determining Tee Yield Load Strength

Unstiffened Tee

$$P_{us} = \left(4\pi\left[\frac{\alpha}{360}\right] + b\left(\frac{1}{e}\right)\right)\sqrt{F_y^2 - 3\left(\frac{P_u}{4wt_p}\right)^2}$$

Stiffened Tee Hanger

$$P_s = \left(4\pi\left[\frac{\alpha}{360}\right] + \left(c + (f - g)\right)\left(\frac{1}{a}\right) + (b)\left(\frac{1}{e}\right)\right)\sqrt{F_y^2 - 3\left(\frac{P_u}{4wt_p}\right)^2}$$

Equations to Calculate Bolt Forces with Prying Action

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 \qquad w' = \frac{b_f}{2} - d_{bh} \qquad t_{mod} = \sqrt{\frac{P_s}{F_y\left(P_{us} / m_p\right)}}$$

$$F' = \frac{t_p^2 F_{py}\left(0.85b_f / 2 + 0.80w'\right) + \pi d_b^3 F_{yb} / 8}{4p_f}$$

$$Q_{max} = \frac{w' t_p^2}{4a}\sqrt{F_{py}^2 - 3\left(\frac{F'}{w' t_p}\right)^2}$$

$$P_r = \max\left|\begin{matrix} 4(P_t - Q_{max}) \\ 4T_b \end{matrix}\right.$$

$$65\text{ k} = \phi_r P_r = \phi_r(4P_t) \quad \text{with } \phi_r = 0.75$$

The required bolt capacity is

$$P_t = \frac{P_u}{4\phi_r} = \frac{65}{4(0.75)} = 21.67\text{ kips}$$

The required bolt diameter is:

$$d_b = \sqrt{4\frac{P_t}{\pi F_{yb}}} = \sqrt{\frac{4(21.67)}{\pi(90)}} = 0.55\text{ in.}$$

Try a 5/8" diameter bolt

$$P_t = \frac{\pi(5/8)^2}{4}(90) = 27.59$$

$$\phi_r(4P_t) = 0.75(4)(27.59) = 82\text{k} > 65\text{k} - \text{ok}$$

2) To ensure that the plate does not yield before bolt yielding, divide P_u by 0.9

$$\frac{P_u}{0.9} = \frac{65}{0.9} = 72.2\text{k}$$

3) Solve for the required thickness that ensures no yielding at bolt proof load. The following variables must now be defined to find the capacity of the plate:

$$d = \frac{b_f}{2} - c - \frac{t_w}{2} - t_{weld} = \frac{8}{2} - 2 - \frac{0.5}{2} - \frac{5}{16} = 1.4375\text{ in.}$$

$$p = 2.17d = 2.17(1.4375) = 3.119\text{ in.}$$

$$e = d - \frac{b^2}{4p} = 1.4375 - \frac{2^2}{4(3.119)} = 1.1168\text{ in}$$

$$a = \min \begin{vmatrix} \sqrt{4pf} \\ \frac{L}{2} - b \end{vmatrix} = 4 \text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{4^2}{4(3.119)} = 1.2822 \text{ in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{2}{1.1168}\right) = 60.82^o$$

$$\theta_2 = \tan^{-1}\left(\frac{a}{d-g}\right) = \tan^{-1}\left(\frac{4}{1.4375 - 1.282}\right) = 87.78^o$$

$$\alpha = \theta_1 + \theta_2 = 60.82^o + 87.78^o = 148.6^o$$

$$\beta = \frac{P_{us}}{m_p^{'}}$$

$$\beta = \left(4p\left(\frac{a}{360}\right) + b\left(\frac{1}{e}\right)\right) = \left(4p\left(\frac{148.6}{360}\right) + 2\left(\frac{1}{1.1168}\right)\right) = 6.97$$

Where $m_p^{'}$ is the moment resistance per linear inch, modified for shear. The required plate thickness is deteremined by iteration. Assuming a plate thickness of 0.7 in.

$$t_p = \sqrt{\frac{P_u / 0.9}{\beta(\phi)\sqrt{(F_y)^2 - 3\left(\frac{\phi P_y}{\phi(4)(wt_p)}\right)^2}}}$$

$$t_p = \sqrt{\frac{72.2}{6.97(0.9)\sqrt{(50)^2 - 3\left(\frac{72.2}{(4)(3.433)(0.7)}\right)^2}}} = 0.488 \text{ in}$$

A second iteration using $t_p = 0.488$ in. gives $t_p = 0.498$ in.

<u>Try 1/2 in. plate</u>

4) Check that yield load of the plate is greater than proof load of bolts

$$P_{us} = \left(4\pi\left(\frac{\alpha}{360}\right) + b\left(\frac{1}{e}\right)\right)(t_p)^2 \sqrt{(F_y)^2 - 3\left(\frac{P_u / 0.9}{(4)(wt_p)}\right)^2}$$

$$= \left(4\pi\left(\frac{148.6}{360}\right) + 2\left(\frac{1}{1.1168}\right)\right)(0.5)^2 \sqrt{(50)^2 - 3\left(\frac{72.2}{(4)(3.433)(0.5)}\right)^2} = 81.2\text{k}$$

check $\phi P_y = (0.9)\text{x}81.2 = 73.08\text{k} > 72.2\text{k} = P_u/0.9$ ok

Summary: For the given loading, geometry, and materials, use 1/2 in. thick A572 Gr50plate, with 5/8 in. diameter A325 bolts.

Example 2

Determine the required thickness and bolt diameter for a Structural Tee Hanger with the geometry listed in Figure 5.1, that ensures the mode of failure to be bolt rupture. The plate material is A572 Gr 50 and the bolts are A325. The factored applied load is 250 kips. Also, the plate thickness is not to exceed 1in.

1) P_u was given as 250 kips, therefore:

$$250\text{ k} = \phi P_n = \phi_r P_r = \phi_r(4P_t) \quad \text{with } \phi_r = 0.75$$

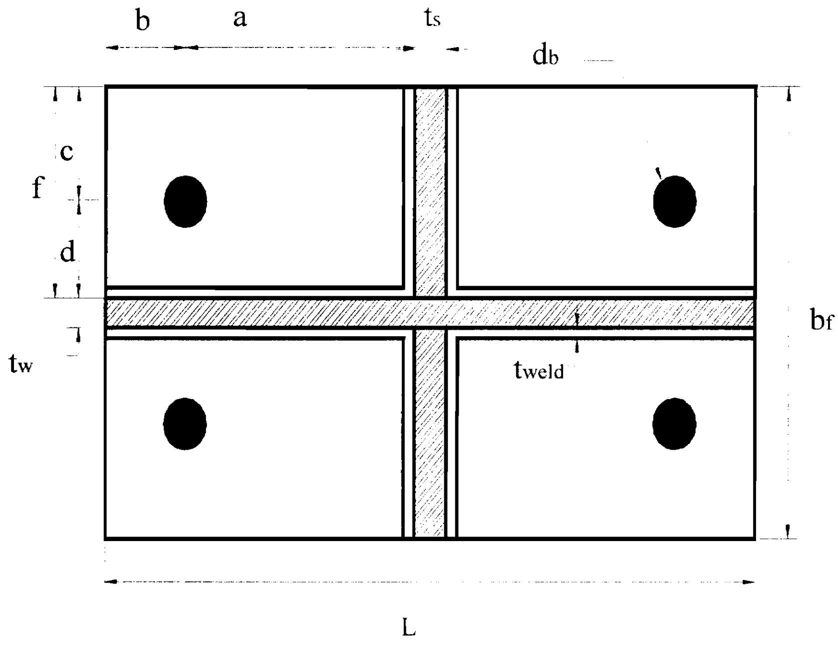
The required bolt capacity is

$$P_t = \frac{P_u}{4\phi_r} = \frac{250}{4(0.75)} = 83.33 \text{ kips}$$

The required bolt diameter is:

$$d_b = \sqrt{\frac{4P_t}{\pi F_{yb}}} = \sqrt{\frac{4(83.33)}{\pi(90)}} = 1.08\,\text{in.}$$

Try a 1-1/8 in. diameter bolt



Example	b_f (in.)	b (in.)	c (in.)	d (in.)	f (in.)	t_s (in.)	t_w (in.)	d_b (in.)	d_{bh} (in.)	L (in.)	t_{weld} (in.)
1	8.0	2.0	2.0	1.435	3.435	none	0.5	0.625	0.6875	12.0	5/16
2	10.0	2.0	1.5	2.0	3.5	0.5	1.0	1.0	1.0625	10.0	1/2
3	10.0	2.0	1.5	2.0	3.5	0.5	1.0	1.0	1.0625	10.0	5/16
4	8.0	2.0	2.0	1.435	3.435	none	0.5	0.75	0.8125	12.0	1/2

Figure 5.1 Dimensions for Example Problems

$$P_t = \frac{\pi(1.125)^2}{4}(90) = 89.41$$

$$\phi_r(4P_t) = 0.75(4)(89.41) = 268.2\text{k} > 250 \text{ ok}$$

2) To ensure that the plate does not yield before bolts do, divide P_u by 0.9

$$\frac{P_u}{0.9} = \frac{250}{0.9} = 277.7 \text{ k}$$

3) Solve for the required thickness that ensures no yielding at the bolt proof load.

The following variables must now be calculated to find the strength of the plate

$$d = \frac{b_f}{2} - c - \frac{t_w}{2} - t_{weld} = \frac{10}{2} - 2 - \frac{1}{2} - \frac{1}{2} = 2 \text{ in.}$$

$$p = 2.17d = 2.17(2) = 4.39 \text{ in.}$$

$$e = d - \frac{b^2}{4p} = 2 - \frac{2^2}{4(4.39)} = 1.76 \text{ in.}$$

$$a = \min\left|\begin{matrix}\sqrt{4pf} \\ \frac{L}{2} - b\end{matrix}\right. = 3 \text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{3^2}{4(4.39)} = 0.518 \text{ in}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{2}{1.76}\right) = 48.65^o$$

$$\phi_2 = \tan^{-1}\left(\frac{a}{d-g}\right) = \tan^{-1}\left(\frac{3}{2-0.518}\right) = 63.7^o$$

$$\alpha = \theta_1 + \theta_2 = 48.65^o + 63.7^o = 112.36^o$$

$$\beta = \frac{P_{us}}{m_p'}$$

$$\beta = \left(4\pi\left(\frac{\alpha}{360}\right) + b\left(\frac{1}{e}\right)\right) = \left(4\pi\left(\frac{112.36}{360}\right) + 2\left(\frac{1}{1.76}\right)\right) = 5.057$$

the required plate thickness can then be solved by iteration, and assuming a plate thickness of 1.0 in. by using:

$$t_p = \sqrt{\frac{P_u / 0.9}{\beta(\phi)\sqrt{(F_y)^2 - 3\left(\frac{P^u / 0.9}{(4)(wt_p)}\right)^2}}}$$

$$= \sqrt{\frac{277.7}{5.057(0.9)\sqrt{(50)^2 - 3\left(\frac{277.7}{(4)(4)(1)}\right)^2}}} = 1.172 \text{ in.}$$

a second iteration using t_p = 1.172 in. gives t_p = 1.192 in.

The plate is thicker than the permissible thickness of 1 in. in the problem statement.

Try a 1 in. plate and add a 1/2 in. stiffener with a 5/16 in. weld.

4) Determine the plate capacity using the stiffened plate equations

$$a = \frac{L}{2} - b - \frac{t_s}{2} - t_{weld} = 5 - 2 - \frac{0.5}{2} - \frac{5}{16} = 2.43\text{in.}$$

$$g = \frac{a^2}{4p} = \frac{2.43^2}{4(4.34)} = 0.3422\text{in.}$$

$$\theta_2 = \tan^{-1}\left(\frac{a}{d-g}\right) = \tan^{-1}\left(\frac{2.43}{2 - 0.3422}\right) = 55.7^o$$

θ_1 remains the same, all other quantities remain the same

$$\alpha = \theta_1 + \theta_2 = 55.7 + 48.65 = 104.35$$

5) Check that yield load of the plate is greater than the proof load of the bolts

$$P_s = \left(4\pi\left(\frac{\alpha}{360}\right) + b\left(\frac{1}{e}\right) + (c + (f - g))\left(\frac{1}{a}\right)\right)(t_p)^2 \sqrt{(F_y)^2 - 3\left(\frac{P_u / 0.9}{(4)(wt_p)}\right)^2}$$

$$P_s = \left(4\pi\left(\frac{148.6}{360}\right) + 2\left(\frac{1}{1.1168}\right) + (1.5 + (3.5 - 0.3422))\left(\frac{1}{2.43}\right)\right)(1.0)^2 \sqrt{(50)^2 - 3\left(\frac{277.7}{(4)(4)(1.0)}\right)^2} = 355\text{ k}$$

check $\phi P_y = 0.9\text{x } 355 = 320\text{k} > 277.7\text{k} = P_u/0.9$ ok

Note: The value is somewhat conservative, but any standard thickness plate below 1 in. for A572 Gr 50 steel, gives insufficiant strength.

<u>Summary</u>: For the given loading, geometry, and materials, use 1.0 in. thick A572 Gr50 plate, with 1.0 in. diameter A325 bolts, and 1/2 in. stiffeners with a 5/16 in. weld.

Example 3

Determine the required thickness and bolt diameter for a Structural Tee Hanger with the geometry listed in Figure 5.1, that ensures the mode of failure to be plate yielding. The plate material is A572 Gr 50, and the bolts are A325. The factored applied load is 100 kips. Also, plate thickness cannot exceed 5/8 in.

1) P_u was given as 100 kips, therefore:

$$100\ k = P_u$$

2) Find the strength of the plate.

The following variables must now be defined to find the capacity of the plate:

$$d = \frac{b_f}{2} - \frac{t_w}{2} - t_{weld} = \frac{6}{2} - \frac{1}{2} - \frac{1}{2} = 2\text{in.}$$

$$p = 2.17d = 2.17(2) = 4.39\text{in.}$$

$$e = d - \frac{b^2}{4p} = 2 - \frac{2^2}{4(4.39)} = 1.76\text{in.}$$

$$a = \left|\begin{matrix} \sqrt{4pf} \\ \dfrac{L}{2} - b \end{matrix}\right|_{min} = 3\text{in.}$$

$$g = \frac{a^2}{4p} = \frac{3^2}{4(4.39)} = 0.518\text{in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{2}{1.76}\right) = 48.65^o$$

$$\theta_2 = \tan^{-1}\left(\frac{a}{d-g}\right) = \tan^{-1}\left(\frac{3}{2-0.518}\right) = 63.7^o$$

$$\alpha = \theta_1 + \theta_2 = 48.65^o + 63.7^o = 112.36^o$$

$$\beta = \frac{P_{us}}{m_p'}$$

$$\beta = \left(4\pi\left(\frac{\alpha}{360}\right) + b\left(\frac{1}{e}\right)\right) = \left(4\pi\left(\frac{112.36}{360}\right) + 2\left(\frac{1}{1.76}\right)\right)m_p = 5.057$$

3) Solve for plate thickness by iteration, using an assumed thickness of 1.0”

$$t_p = \sqrt{\frac{P_u}{\beta(\phi)\sqrt{(F_y)^2 - 3\left(\frac{P_u}{(4)(wt_p)}\right)^2}}}$$

$$t_p = \sqrt{\frac{100}{5.057(0.9)\sqrt{(50)^2 - 3\left(\frac{100}{(4)(4)(1)}\right)^2}}} = 0.67\text{in.}$$

A second iteration using $t_p = 0.67$ in. gives $t_p = 0.68$ in.

The plate is thicker than the permissible thickness of 5/8 in. in the problem statement.

<u>Try using a 5/8 in. plate and adding a 1/2 in. stiffener with a 5/16 in. weld.</u>

4) Determine plate capacity using the stiffened plate equation

$$a = \frac{L}{2} - b - \frac{t_s}{2} - t_{weld} = 5 - 2 - \frac{0.5}{2} - \frac{5}{16} = 2.43$$

$$g = \frac{a^2}{4p} = \frac{2.43^2}{4(4.34)} = 0.3422$$

$$\theta_2 = \tan^{-1}\left(\frac{a}{d-g}\right) = \tan^{-1}\left(\frac{2.43}{2-0.3422}\right) = 55.7^o$$

θ_1 remains the same, all other quantities remain the same

$$\alpha = \theta_1 + \theta_2 = 55.7 + 48.65 = 104.35$$

A trial plate thickness of 1/2 in. gave ϕP_s = 98 k < 100 k = P_u. Try 5/8 in. plate

$$P_s = \left(4\pi\left(\frac{\alpha}{360}\right) + b\left(\frac{1}{e}\right) + \left(c + (f - g)\right)\left(\frac{1}{a}\right)\right)\left(t_p\right)^2 \sqrt{\left(F_y\right)^2 - 3\left(\frac{P_u}{(4)\left(wt_p\right)}\right)^2}$$

$$P_s = \left(4\pi\left(\frac{148.6}{360}\right) + 2\left(\frac{1}{1.1168}\right) + \left(1.5 + (3.5 - 0.3422)\right)\left(\frac{1}{2.43}\right)\right)(0.625)^2 \sqrt{(50)^2 - 3\left(\frac{100}{(4)(4)(0.625)}\right)^2} = 170.9\text{k}$$

check $\phi P_s = 0.9\text{x}170.9 = 153\text{k} > 100\text{k} = P_u$ ok

5) Bolt force must now be checked and include prying action, since this is a stiffened tee, the quantity t_{mod} must be calculated. The yield strength of the connection, unstiffened, was calculated earlier in the example.

Try 1 in. diameter bolts

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 = 0.02317(50)\left(\frac{0.821}{1}\right)^3 = 0.641 \text{ in.}$$

$$t_{mod} = \sqrt{\frac{P^s}{F_y\left(P_{us} / m_p\right)}} = \sqrt{\frac{170.9}{50(5.07)}} = 0.821 \text{ in.}$$

$$w' = \frac{b^f}{2} - d_{bh} = 5 - 1.0625 = 3.9375$$

$$F' = \frac{t_p^2 F_{yp}\left[0.85\left(\frac{b^f}{2}\right) + 0.8(w')\right] + \pi d_b^3 F_{yb} / 8}{4(p_f)}$$

$$F' = \frac{(0.821)^2 (50)\left[0.85\left(\frac{10}{2}\right) + 0.8(3.9375')\right] + \pi(1)^3 (90)/8}{4(2)} = 32.87$$

$$Q_{max} = \frac{w' t_p^2}{4a}\sqrt{(F_y)^2 - 3\left(\frac{F'}{w' t_p}\right)^2}$$

$$= \frac{(3.9375)(0.821)^2}{4(0.641)}\sqrt{(50)^2 - 3\left(\frac{32.87}{(3.9375)(0.821)}\right)^2} = 55\text{ k}$$

$$\phi_r P_r = \max\left|\begin{matrix}\phi 4(P_t - Q_{max}) \\ \phi 4T_b\end{matrix}\right. = \left|\begin{matrix}0.75(4)(70-55) \\ 0.75(4)(51)\end{matrix}\right. = 153\text{ k}$$

6) Check that bolt yield is greater than plate yield load.

$$\phi_r P_r = 153 \geq \phi P_s = 153\text{k}$$

<u>Summary</u>: For the given loading, geometry, and materials, use 5/8 in. thick A572 Gr50 plate, with 1.0 in. diameter A325 bolts, and 1/2 in. stiffeners with a 5/16 in. weld.

Example 4

Determine the required thickness and bolt diameter for a Structural Tee Hanger with the geometry listed Figure 5.1, that ensures the mode of failure to be plate yielding. The plate material is A572 Gr 50, and the bolts are A325. The factored applied load is 65 kips.

1) P_u was given as 65 kips, therefore:

$$65\text{ k} = P_{us}$$

2)Find strength of the plate.

The following variables must now be defined to find the capacity of the plate.

$$d = \frac{b_f}{2} - \frac{t_w}{2} - t_{weld} = \frac{4}{2} - \frac{1}{4} - \frac{5}{16} = 1.4375\text{in.}$$

$$p = 2.17d = 2.17(1.4375) = 3.119\text{in.}$$

$$e = d - \frac{b^2}{4p} = 1.4375 - \frac{2^2}{4(3.119)} = 1.1168\text{in.}$$

$$a = \left|\begin{array}{l} \sqrt{4pf} \\ \frac{L}{2} - b \end{array}\right._{min} = 4\text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{4^2}{4(3.119)} = 1.282\text{ in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{2}{1.116}\right) = 60.82^o$$

$$\theta_2 = \tan^{-1}\left(\frac{a}{d-g}\right) = \tan^{-1}\left(\frac{4}{1.4375 - 1.282}\right) = 87.87^o$$

$$\alpha = \theta_1 + \theta_2 = 60.82^o + 87.87^o = 148.6^o$$

$$\beta = \frac{P_{us}}{m_p'}$$

$$\beta = \left(4\pi\left(\frac{\alpha}{360}\right) + b\left(\frac{1}{e}\right)\right) = \left(4\pi\left(\frac{148.6}{360}\right) + 2\left(\frac{1}{1.1168}\right)\right) = 6.97$$

3) Solve for plate thickness by iteration using assumed thickness of 1/2 in.

$$t_p = \sqrt{\frac{Pu}{\beta(\phi)\sqrt{(F_y)^2 - 3\left(\frac{Pu}{(4)(wt_p)}\right)^2}}}$$

$$t_p = \sqrt{\frac{65}{6.97(0.9)\sqrt{(50)^2 - 3\left(\frac{65}{(4)(3.4375)(0.5)}\right)^2}}} = 0.468\text{ in.}$$

a second iteration using $t_p = 0.468$ in. gives $t_p =$ 0.47 in. <u>Try a 1/2 in. thick plate</u>

4) Determine plate capacity

$$P_{us}=\left(4\pi\left(\frac{\alpha}{360}\right)+b\left(\frac{1}{e}\right)\right)\left(t_p\right)^2\sqrt{\left(F_y\right)^2-3\left(\frac{P_u}{\phi(4)\left(wt_p\right)}\right)^2}$$

$$P_{us}=\left(4\pi\left(\frac{148.6}{360}\right)+2\left(\frac{1}{1.1168}\right)\right)(0.5)^2\sqrt{(50)^2-3\left(\frac{65}{(4)(3.4375)(0.5)}\right)^2}=82.3\text{k}$$

check $\phi P_y = 0.9x82.3 = 74k > 65k = P_u$ ok

5) Bolt force must now be checked and prying action <u>Try 3/4 in. diameter bolts</u>

$$a=0.02317F_y\left(\frac{t_p}{d_b}\right)^3=0.02317(50)\left(\frac{0.5}{0.75}\right)^3=0.343\text{ in.}$$

$$w'=\frac{b_f}{2}-d_{bh}=4-0.8125=3.1875$$

$$F'=\frac{t_p^2F_{yp}\left[0.85\left(\frac{b_f}{2}\right)+0.8(w')\right]+\pi d_b^3F_{yb}/8}{4(p_f)}$$

$$F'=\frac{(0.5)^2(50)\left[0.85\left(\frac{8}{2}\right)+0.8(3.1875)\right]+\pi(0.75)^3(90)/8}{4(2)}=11.1$$

$$Q_{max}=\frac{w't_p^2}{4a}\sqrt{\left(F_y\right)^2-3\left(\frac{F'}{w't_p}\right)^2}$$

$$Q_{max}=\frac{(3.1875)(0.625)^2}{4(0.343)}\sqrt{(50)^2-3\left(\frac{11.1}{(3.1875)(0.5)}\right)^2}=28.17\text{ k}$$

$$\phi_rP_r=\max\begin{vmatrix}\phi4\left(P_t-Q_{max}\right)\\ \phi4T_b\end{vmatrix}=\max\begin{vmatrix}0.75(4)(40-28.17)\\ 0.75(4)(28)\end{vmatrix}=84\text{ k}$$

6) Check that bolt yield is greater than plate yield

$\phi_rP_r = 84k > \phi P_y = 74$ k ok

Summary: For the given loading, geometry, and materials, use 1/2 in. thick A572 Gr50 plate, with 3/4 in. diameter A325 bolts.

REFERENCES

Abel, M.S. and T.M. Murray (1992) "Analytical and Experimental Investigation of the Extended Unstiffened Moment End-Plate Connections with Four Bolts at the Beam Tension Flange," Research Report CE/VPI-ST-93/08, Department of Civil Engineering, Virginia Polytechnic and State University, Blacksburg, VA, October 1994.

Agerskov, H. (1977) " Analysis of Bolted Connections Subject to Prying," Journal of the Structural Division, ASCE, Volume 103, Proc. Paper 4298.

Astaneh, Abolhassan (1985) "Procedure for Design and Analysis of Hanger Type Connections" Engineering Journal AISC, March 1985

Barker, Richard M. (1995) "Advanced Reinforced Concrete Design Lecture Notes" Virginia Polytechnic and State Institute, September 1995.

Bond, D.E. and T.M. Murray (1989) "Analytical and Experimental Investigation of Flush Moment End-Plate Connections with Six Bolts at the Tension Flange," Research Report CE/VPI-ST-89/10, Department of Civil Engineering, Virginia Polytechnic and State University, Blacksburg, VA, December 1989.

Borgsmiller, J. T., (1995) "Simplified Method for Design of Moment End-Plate Connections"Masters Thesis, Department of Civil Engineering, Virginia Polytechnic and State University, Blacksburg, VA, March 1995.

Douty, R.T. and McGuire, W. (1965) "High Strength Bolted Moment Connections," Journal of the Structural Division, ASCE, Volume 91, Proc. Paper 4298.

Grundy, P., Thomas, I., and Bennetts, I. (1977) "The Design of Beam -to- Column Moment Connections using End Plates and High Strength Bolts," AISC Second Conference on Steel Developments. Monash University, Melborne, Australia.

Hendrick, D., A.R. Kukreti and T.M. Murray (1984) "Analytical and Experimental Investigation of Stiffened Flush End-Plate Connections with Four Bolts at the Tension Flange," Research Report FSEL/MBMA 84-02, Fears Structural Engineering Laboratory, University of Oklahoma, Norman, OK, September 1984.

Kato, B. and McGuire, W. (1973) " Analysis of T-Stub Flange to Column Connections," Journal of the Structural Division, ASCE, Volume 99, Proc. Paper 9725.

Kennedy, N.A., Vinnakota, S., and A.N. Sherbourne (1981) "The Split-Tee Analogy in Bolted Splices and Beam-Column Connections," *Joints in Structural Steelwork*, John Wiley and Sons, London-Toronto, 1981, pp. 2.138-2.157.

"Load and Resistance Factor Design Specification for Steel Buildings" (1993) American Institute of Steel Construction, Inc. Chicago, Illinois

Mann, A.P., and Morris, L.J. (1979) "Limit Design of Extended End-Plate Connections," Journal of the Structural Division, ASCE

"Manual of Steel Construction Load and Resistance Factor Design, Second Edition," (1994) American Institute of Steel Construction, Inc. Chicago, Illinois

Packer, J. and Morris, L. (1977) " A Limit State Design Method for the Tension Region of Bolted Beam-Column Connections," The Structural Engineer, Number 10, Volume 51.

SEI (1984), "Multiple Row, Extended End-Plate Connections Tests," Research Report, Structural Engineers, Inc., Norman, OK, July 1983.

Srouji, R., A.R. Kukreti and T.M. Murray (1983) "Yield Line Analysis of End-Plate Connections with Bolt Force Predictions," Research Report FSEL/MBMA 83-05, Fears Structural Engineering Laboratory, University of Oklahoma, Norman, OK, December 1983.

Srouji, R., A.R. Kukreti and T.M. Murray (1984) "Yield Line Analysis of End-Plate Connections with Bolt Force Predictions -- Addendum," Research Report FSEL/MBMA 83-05, Fears Structural Engineering Laboratory, University of Oklahoma, Norman, OK, August 1984.

Thornton, William (1985) "Prying Action, a General Treatment" Engineering Journal, AISC, March 1985

APPENDIX A

PARABOLIC YIELD LINE DERIVATION

PARABOLIC YIELD LINE DERIVATION

The purpose of this Appendix is to show the derivation and the assumptions made to determine the internal work for a parabolic yield line. The "Resisting Moment along a Skewed Line" and "Fan Patterns at Concentrated Loads" derivations were taken from Barker (1995).

Using the principle of virtual work, the steel plate is given a small arbitrary virtual displacement, and the corresponding rotations at the various yield lines are determined. By equating the internal and external work, the relation between the applied loads and the resisting moments of the plates are obtained. In the development of a parabolic yield line equation, it was first necessary to understand how to predict the internal work done by a yield line when it forms at an angle to its support.

Resiting Moment Along a Skewed Line

Consider the segment ABCD (Figure A.1) with the vector moment m per unit length acting along the yield line AD. The plane ABCD rotates about the support line BC through a small angle θ. The deflection of point A is $L_2\theta$, and the rotation of AE (perpendicular to AD) is β. This rotation is equal to:

$$\beta = \frac{L_2\theta}{L_2 / \cos\alpha} = \theta\cos\alpha \tag{A.1}$$

which is the rotation through which the resisting moment m will act. The internal work done by m is:

$$W = mL\beta = mL\theta\cos\alpha \tag{A.2}$$

Now, $\cos\alpha = L_1/L$, so that

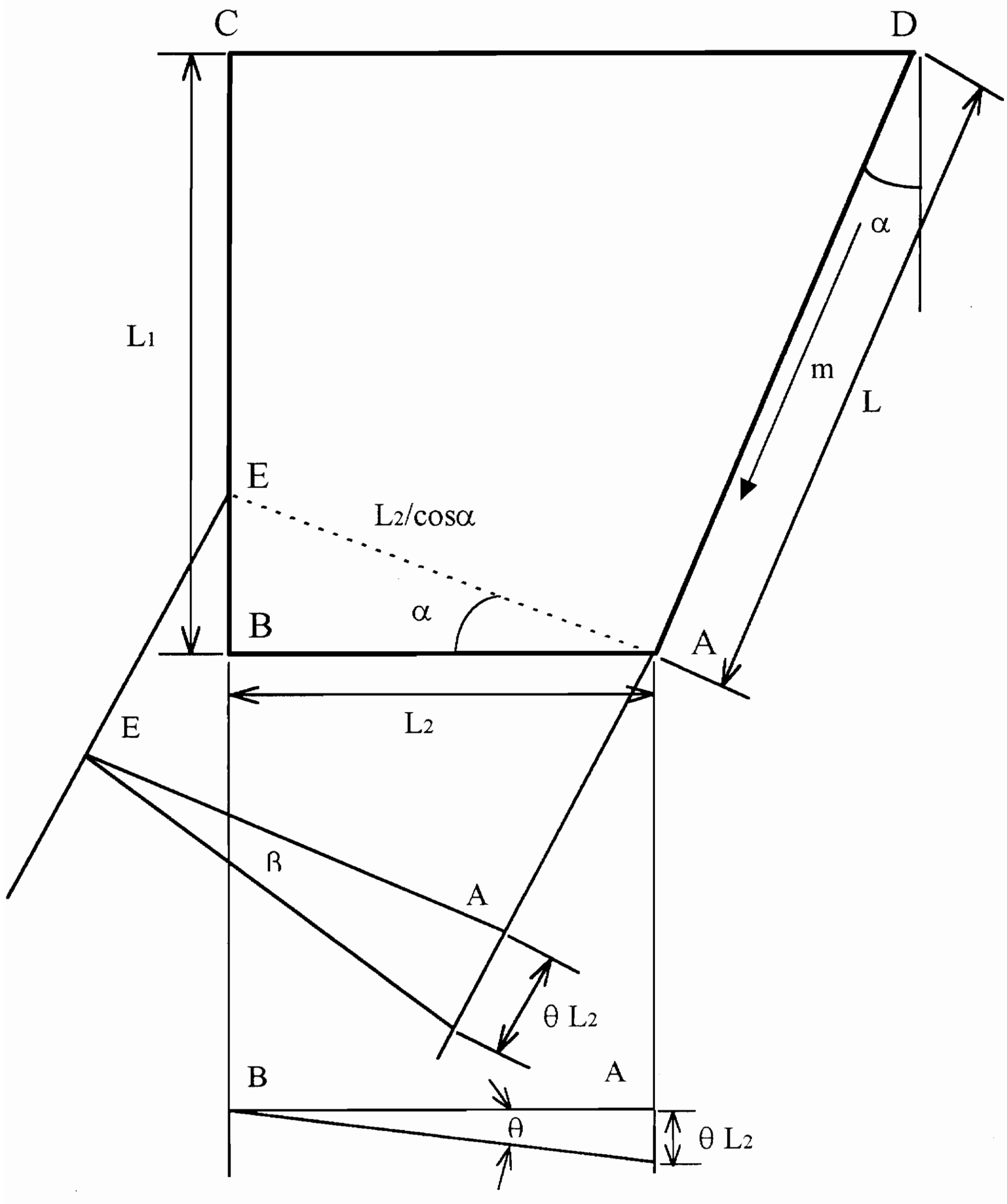


Figure A.1 Yield Line Along a Skewed Line

$$W = mL\theta \frac{L_1}{L} = mL_1\theta \qquad \text{(A.3)}$$

Thus the work done by m acting on AD is the same as if m had been acting on the support line BC.

The concept that the work done by a moment acting on a yield line at a skew, is equal in magnitude to to the work done by the moment acting on a length of support equal to the projection of the yield line, is very important in the derivation of the yield line pattern formed by a point load.

Fan Patterns at Concentrated Loads

If a concentrated load (such as a bolt) acts on a steel plate at an interior location, away from edges, a negative yield line will form in a circular pattern, with positive yield lines radiating outwards from the load point. A negative yield line is one whose resisting moment will try to bend the plate concave up, conversley, a positive yield line will try to bend the plate concave down. Figure A.2 shows the fan pattern which is produced due to the point load. Figure A.3 shows a slice of the yield line pattern, in which the angle α, which is the angle subtended between positive yield lines is unknown. The length of the positive yield lines (which is also the radius of the circle) is also unknown. The length of the negative yield line is equal to the radius of the circle multiplied by the angle α (in radians).

The external work done on this pie section by the applied load, is equal to the fraction of

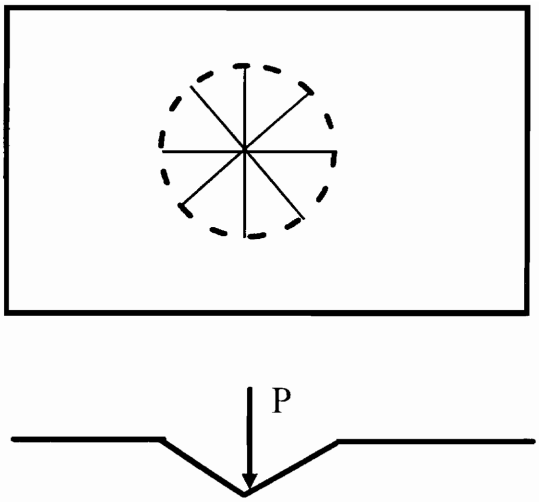


Figure A.2 Fan Yield Line Pattern

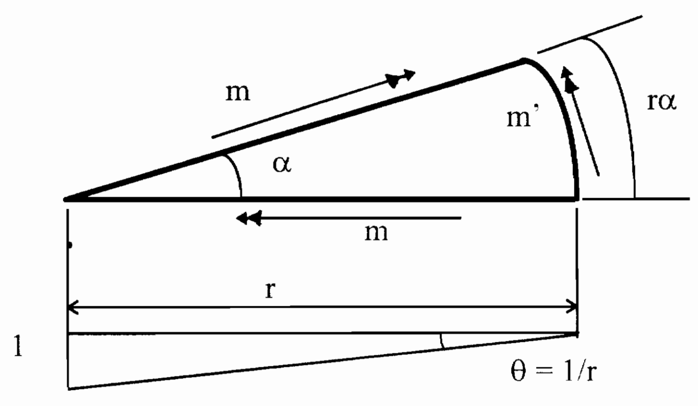


Figure A.3 Pie Section of Circular Yield Line Pattern the load which is resisted by the section, multiplied by the unit displacement:

$$W_e = P\left(\frac{\alpha}{2\pi}\right)1 \tag{A.4}$$

It was shown earlier that the work done by a resisting moment acting along a line at a skew is equal to the work done by the same moment acting on a length of support equal to the projection of the yield line. By this we can assume the positive yield lines (which are at a skew relative to the negative yield line) is equal to the work done as if the positive moment acted on the negative yield line. We can then show the internal work as:

$$W_i = (m + m')r\alpha \frac{1}{r} \tag{A.5}$$

Equating the internal and external work:

$$\begin{aligned} P\left(\frac{\alpha}{2\pi}\right) &= (m + m')\alpha \\ \frac{P}{2\pi} &= (m + m') \end{aligned} \tag{A.6}$$

Since the positive and negative resisting moments in steel are the same, the equation for a point load on a steel plate is:

$$P = 4\pi(m) \tag{A.8}$$

Figure A.4 shows a typical parabolic yield line pattern. As in the case of the circular yield line pattern, positive moment yield lines radiate outward to a negative yield line which is parabolic in shape. A slice is taken from the yield line pattern (Figure A.5), and analyzed in the same manner as the circular pattern. Since the shape is parabolic, the distance from the focal point to the parabola is changing from point to point along the parabola. For simplicity the change in the radius is considered to be small, for small

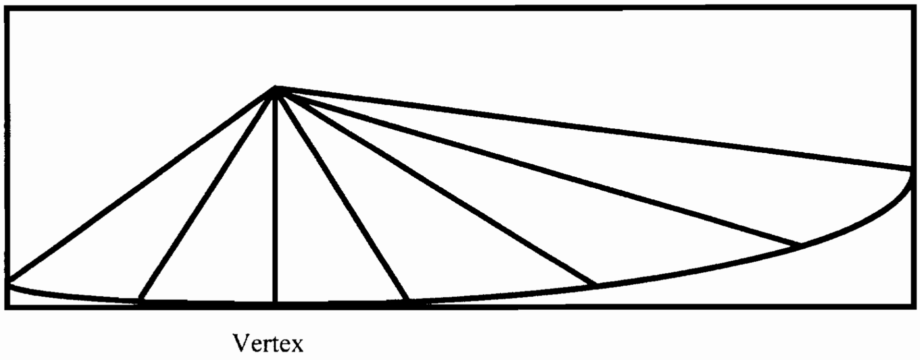


Figure A.4 Parabolic Yield Line Fan Pattern

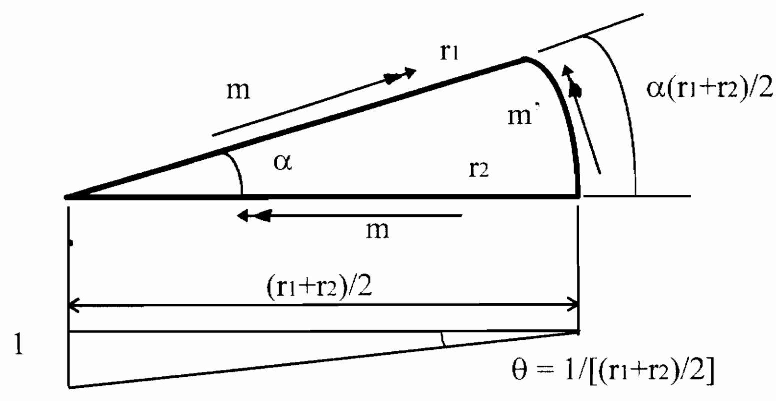


Figure A.5 Pie Section of Parabolic Yield Line Pattern angles, so the length of the parabola subtended by an angle α is:

$$s = \frac{r_1 + r_2}{2}\alpha \tag{A.9}$$

The internal work done by the plate is:

$$W_i = (m + m')\left(\frac{r_1 + r_2}{2}\right)\alpha\left(\frac{2}{r_1 + r_2}\right) \tag{A.0}$$

Equating internal and external work gives the same result as that of the circular yield line pattern in Equation A.8. In this study, the parabolic yield lines had some restrictions. The yield lines would end when the parabola intersected with a stiffener, or the edge of the plate. So, a complete 360 degree arc is not possible. The assumption made is that the strength of the yield line is proportional to the angle it has subtended. So, the equation for the parabolic yield line is:

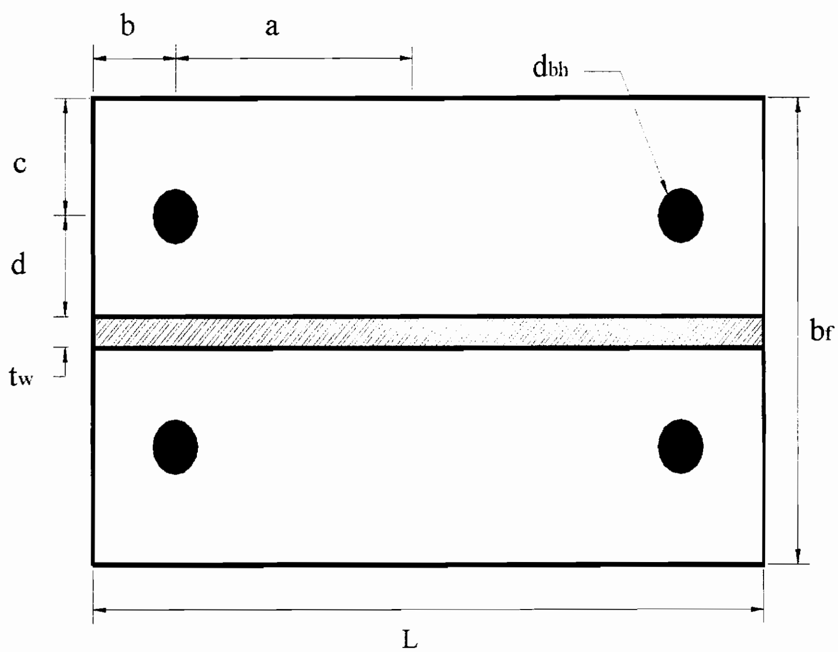
$$P = 4\pi\left(\frac{\alpha}{360}\right)m \tag{A.11}$$

where α is redefined as the angle subtended by the parabola, in degrees.

APPENDIX B

UNSTIFFENED TEE HANGER TEST RESULTS

UNSTIFFENED TEE HANGER TEST



Test Designation..1a,11a

F_y.............45.5 ksi	t_p.........................0.355 in.
t_w..............0.25 in.	t_{weld}......................0.325 in.
a...............1.743 in.	b..........................1.501 in.
c...............1.276 in.	d..........................1.004 in.
f................2.28 in.	L..........................6.5 in.
b_f..............5.4 in.	d_b.........................0.75 in.
d_{bh}.............0.8125 in.	

V...10.5 kips

Average Experimental Yield Load...42 kips

Average Experimental Ultimate Applied Bolt Load..............................17 kips

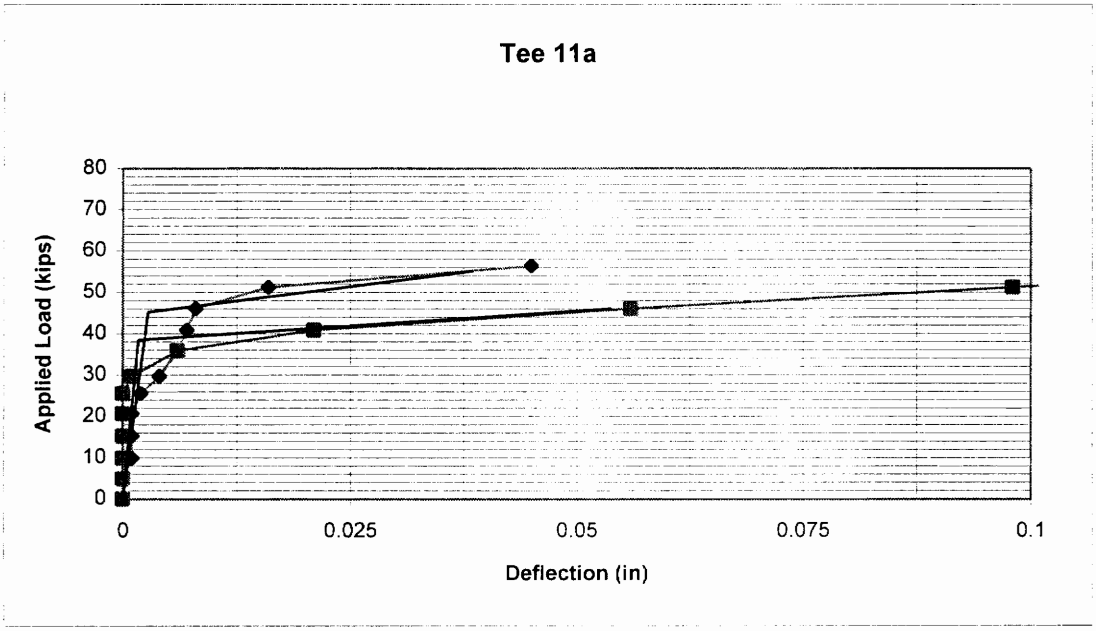
P_y..34.04 kips

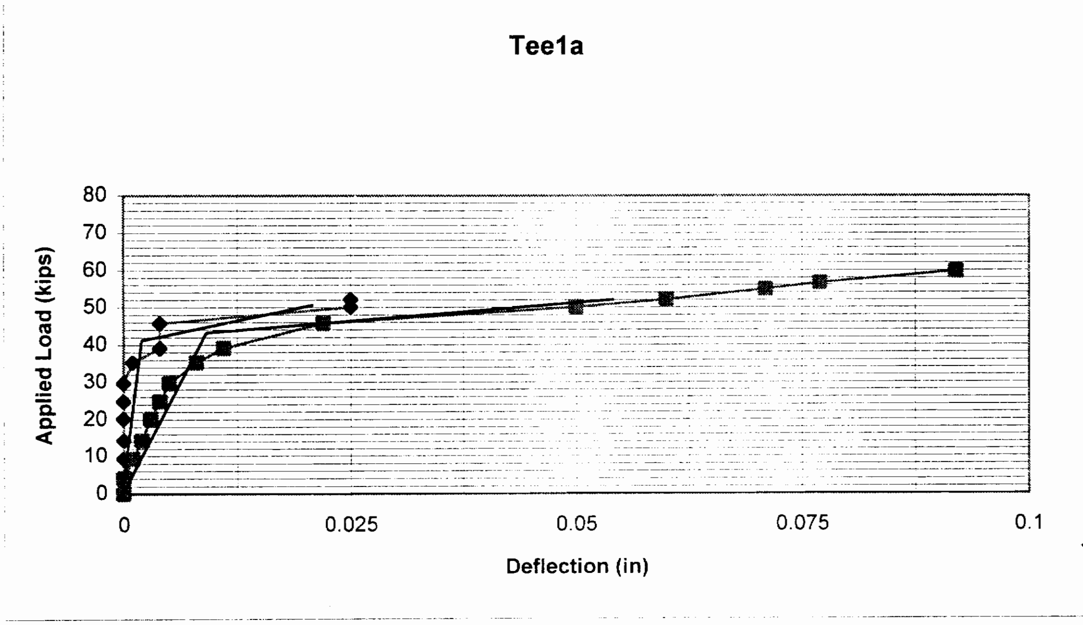
P_r..112 kips

P_{np}..160 kips

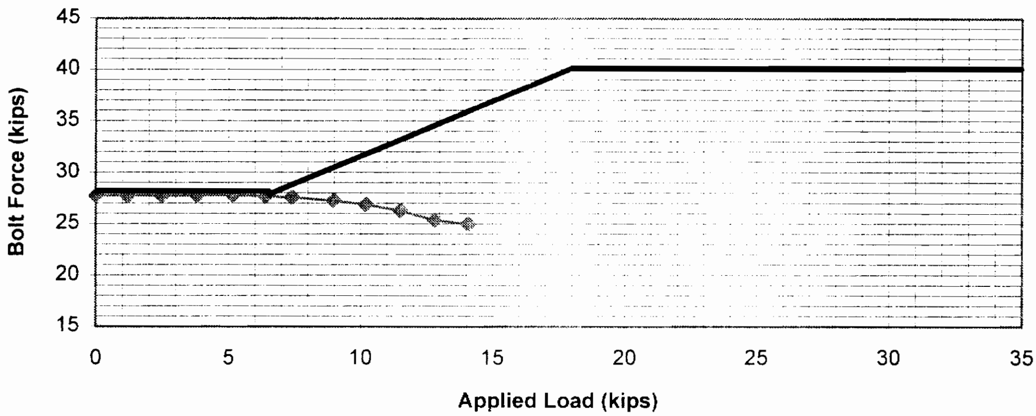
P_{pred}..34.04 kips

Notes: Bolt load for test 11a was not used in bolt force calculations

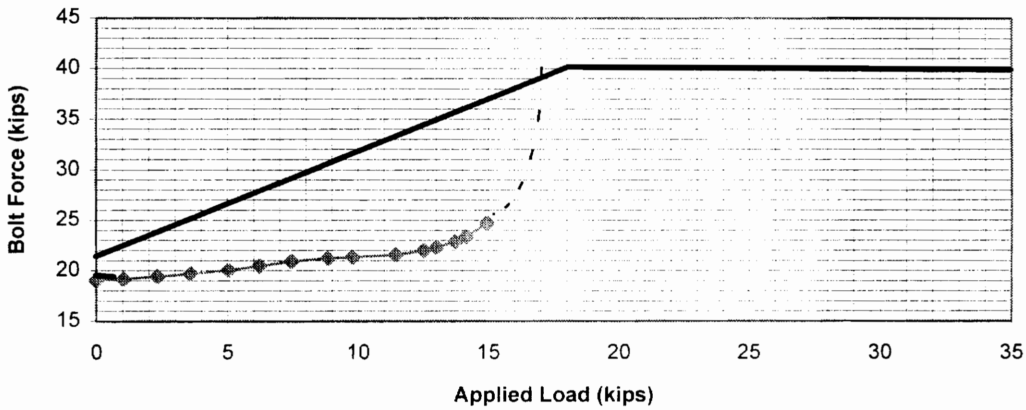

Tee 11a
80
70
60
50
40
30
20
10
0
Applied Load (kips)
0
0.025
0.05
0.075
0.1
Deflection (in)


Tee1a
80
70
60
50
40
30
20
10
0
Applied Load (kips)
0
0.025
0.05
0.075
0.1
Deflection (in)

Tee 11a



Tee1a



CALCULATION OF PREDICTED VALUES

Test:...1a,11a
Limit State:...Plate Yielding

$$p = 2.17d = 2.17(1.004) = 2.179 \text{ in.}$$

$$e = d - \frac{b^2}{4p} = 1.004 - \frac{(1.501)^2}{4(2.179)} = 0.745 \text{ in.}$$

$$b \le 3.53e = 2.63 - \text{ok!}$$

$$a = \min\begin{vmatrix} l/2 - b \\ \sqrt{4pf} \end{vmatrix} = \begin{vmatrix} 1.732" \\ 4.457" \end{vmatrix} = 1.732 \text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{(1.734)^2}{4(2.179)} = 0.344" \le f = 2.28 \text{ in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{1.501}{0.745}\right) = 63.6^{o}$$

$$g \le d \Rightarrow 0.344 \le 1.004$$

$$\theta_2 = \tan^{-1}\left(\frac{a}{d-g}\right) = \tan^{-1}\left(\frac{1.732}{1.004 - 0.344}\right) = 69.19^{o}$$

$$\alpha = \theta_1 + \theta_2 = 63.6^{o} + 69.19^{o} = 132.79^{o}$$

$$m_p = \frac{t_p^2}{4}\sqrt{(F_y)^2 - 3\left(\frac{V}{wt_p}\right)^2} = \frac{(0.355)^2}{4}\sqrt{(45.5)^2 - 3\left(\frac{10.5}{2.5(0.355)}\right)^2} = 1.303\text{kip} - \text{in}/\text{in}$$

$$P_a = 4\pi\left(\frac{\alpha}{360}\right)m_p = 4\pi\left(\frac{132.79^{o}}{360^{o}}\right)m_p = 4.632m_p$$

$$P_c = b\left(\frac{1}{e}\right)m_p = 1.501\left(\frac{1}{0.745}\right)m_p = 2.014m_p$$

$$P_y = 4(P_a + P_c) = 4(4.632m_p + 2.014m_p) = 4(6.65m_p) = 4(6.65(1.303)) = 34.04 \text{ kips}$$

CALCULATION OF PREDICTED VALUES

Test:..1a,11a
Limit State:...Bolt Rupture with Prying Action

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 = 0.02317(45.5)\left(\frac{0.355}{0.75}\right)^3 = 0.112 \text{ in.}$$

$$w' = \frac{b_f}{2} - d_{bh} = 2.7 - 0.8125 = 1.8875 \text{ in.}$$

$$F' = \frac{t_p^2 F_{py}\left(0.85b_f / 2 + 0.80w'\right) + \pi d_b^3 F_{yb} / 8}{4p_f}$$

$$F' = \frac{(0.355)^2(45.5)\left[0.85(5.4)/2 + 0.80(1.8875)\right] + \pi(0.75)^3(90)/8}{4(1.25)} = 7.345 \text{ kips}$$

$$Q_{max} = \frac{w't_p^2}{4a}\sqrt{F_{py}^2 - 3\left(\frac{F'}{w't_p}\right)^2} = \frac{1.8875(0.355)^2}{4(0.112)}\sqrt{(45.5)^2 - 3\left(\frac{7.345}{1.8875(0.355)}\right)^2}$$

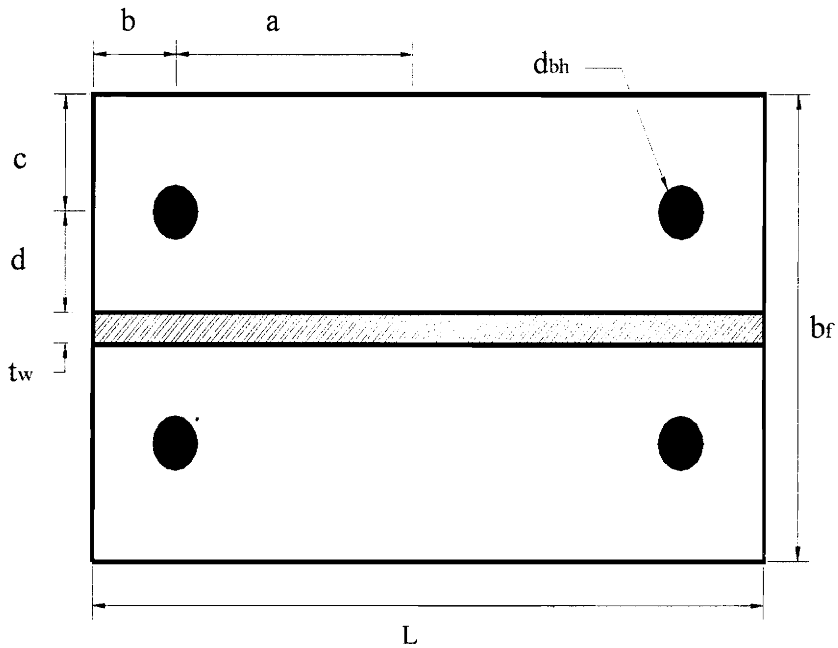
$$Q_{max} = 21.96 \text{ kips}$$

$$P_r = \max\left|\begin{matrix} 4(P_t - Q_{max}) \\ 4T_b \end{matrix}\right. = \left|\begin{matrix} 4(40 - 21.96) \\ 4(28) \end{matrix}\right. = 112 \text{ kips}$$

Test:..1a,11a
Limit State:...Bolt Rupture with no Prying Action

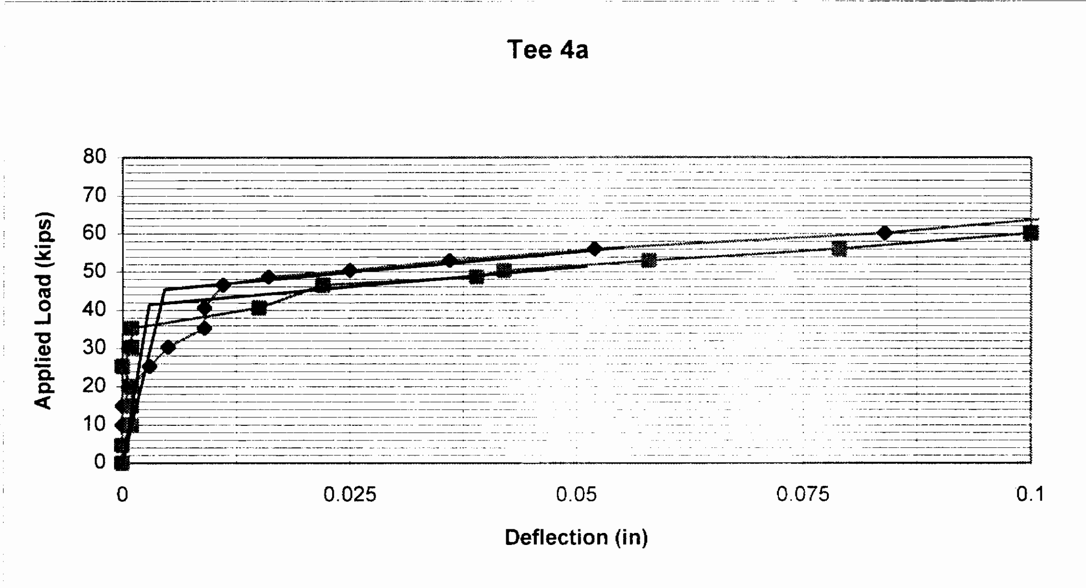
$$P_{np} = \max\left|\begin{matrix} 4Pt \\ 4T_b \end{matrix}\right. = \left|\begin{matrix} 4(40) \\ 4(28) \end{matrix}\right. = 160\text{kips}$$

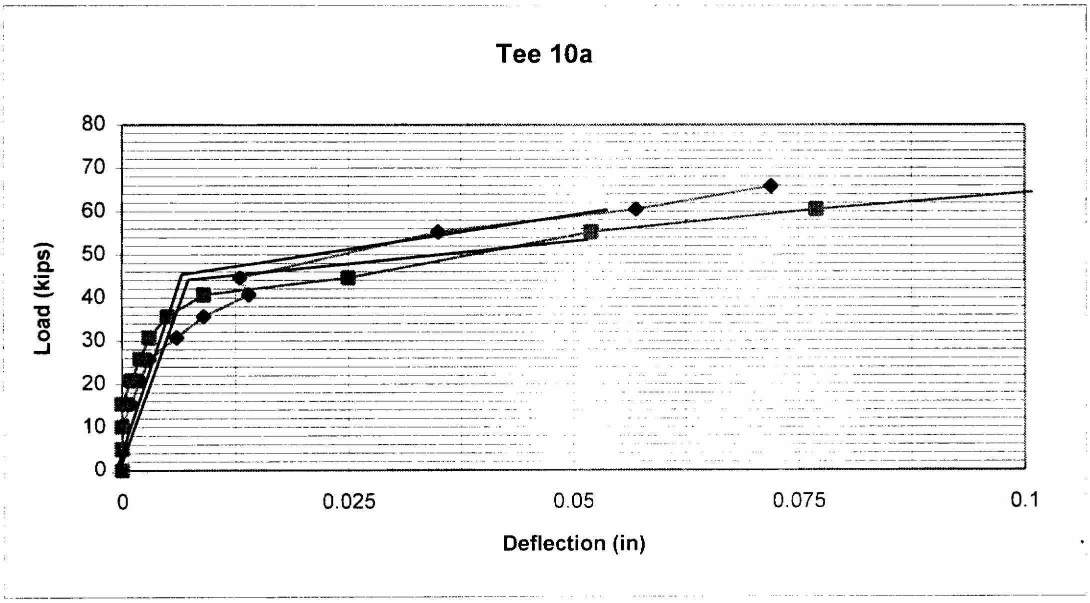
UNSTIFFENED TEE HANGER TEST

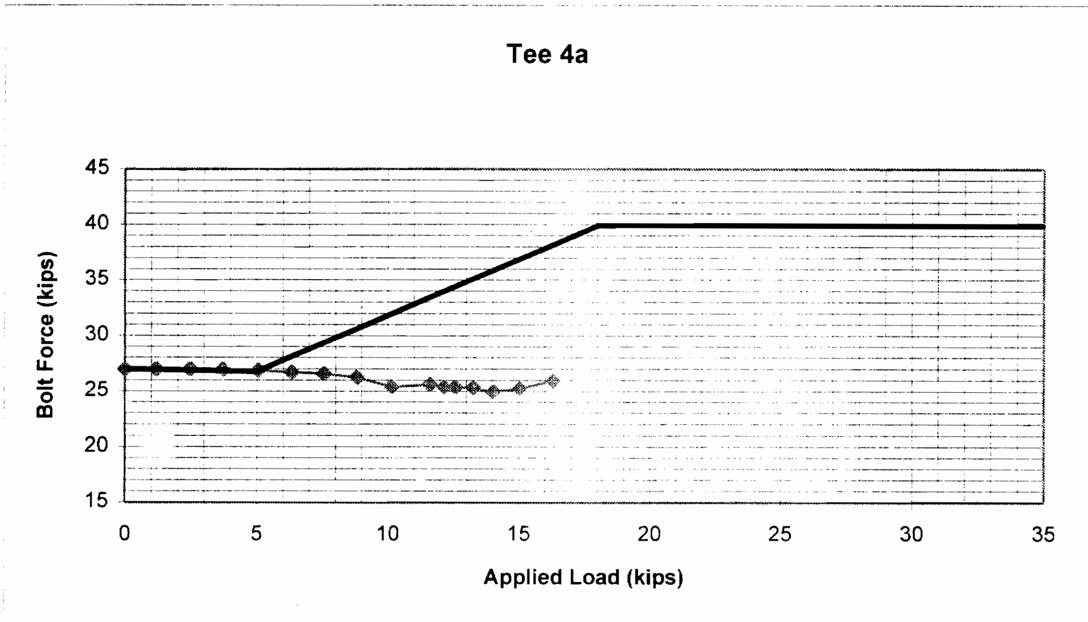


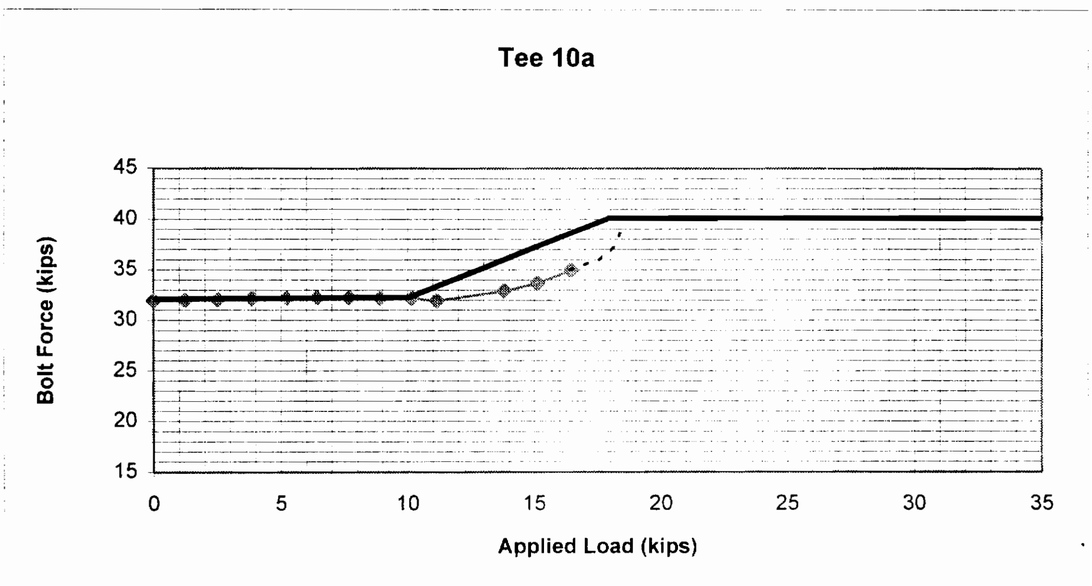
Test Designation	4a,10a
F_y ... 45.5 ksi	t_p ... 0.355 in.
t_w ... 0.25 in.	t_{weld} ... 0.325 in.
a ... 3.749 in.	b ... 1.501 in.
c ... 1.276 in.	d ... 1.004 in.
f ... 2.23 in.	L ... 10.5 in.
bf ... 5.4 in.	d_b ... 0.75 in.
d_{bh} ... 0.8125 in.	
V	11 kips
Average Experimental Yield Load	44 kips
Experimental Ultimate Applied Bolt Load	18.5 kips
P_y	38.9 kips
P_r	112 kips
P_{np}	160 kips
P_{pred}	38.9 kips

Notes: Bolt load for test 4a was not used


Tee 4a
80
70
60
50
40
30
20
10
0
Applied Load (kips)
0
0.025
0.05
0.075
0.1
Deflection (in)


Tee 10a
80
70
60
50
40
30
20
10
0
Load (kips)
0
0.025
0.05
0.075
0.1
Deflection (in)


Tee 4a
45
40
35
30
25
20
15
Bolt Force (kips)
0
5
10
15
20
25
30
35
Applied Load (kips)


Tee 10a
45
40
35
30
25
20
15
Bolt Force (kips)
0
5
10
15
20
25
30
35
Applied Load (kips)

CALCULATION OF PREDICTED VALUES

Test:..4a,10a
Limit State:..Plate Yielding

$$p = 2.17d = 2.17(1.004) = 2.179 \text{ in.}$$

$$e = d - \frac{b^2}{4p} = 1.004 - \frac{(1.501)^2}{4(2.179)} = 0.745 \text{ in.}$$

$$b \le 3.53e = 2.63 - \text{ok!}$$

$$a = \min \begin{vmatrix} l/2 - b \\ \sqrt{4pf} \end{vmatrix} = \begin{vmatrix} 3.749" \\ 4.457" \end{vmatrix} = 3.749 \text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{(3.749)^2}{4(2.179)} = 1.613" \le f = 2.28 \text{ in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{1.501}{0.745}\right) = 63.6^o$$

$$g \ge d \Rightarrow 1.613 \ge 1.004$$

$$\theta_2 = \tan^{-1}\left(\frac{g-d}{a}\right) + 90^o = \tan^{-1}\left(\frac{1.613 - 1.004}{3.749}\right) + 90^0 = 99.22^o$$

$$\alpha = \theta_1 + \theta_2 = 63.6^o + 99.22^o = 162^o$$

$$m_p = \frac{t_p^2}{4}\sqrt{(F_y)^2 - 3\left(\frac{V}{wt_p}\right)^2} = \frac{(0.355)^2}{4}\sqrt{(45.5)^2 - 3\left(\frac{11}{2.5(0.355)}\right)^2} = 1.263\text{kip} - \text{in}/\text{in}$$

$$P_a = 4\pi\left(\frac{\alpha}{360}\right)m_p = 4\pi\left(\frac{162^o}{360^o}\right)m_p = 5.68m_p$$

$$P_c = b\left(\frac{1}{e}\right)m_p = 1.501\left(\frac{1}{0.745}\right)m_p = 2.014m_p$$

$$P_y = 4(P_a + P_c = 5.68m_p + 2.014m_p) = 4(7.695m_p) = 4(7.695(1.263)) = 38.9 \text{ kips}$$

CALCULATION OF PREDICTED VALUES

Test:..4a, 10a
Limit State:..Bolt Rupture with Prying Action

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 = 0.02317(45.5)\left(\frac{0.355}{0.75}\right)^3 = 0.112 \text{ in.}$$

$$w' = \frac{b_f}{2} - d_{bh} = 5.4 - 0.8125 = 1.8875 \text{ in.}$$

$$F' = \frac{t_p^2 F_{py}\left(0.85b_f / 2 + 0.80w'\right) + \pi d_b^3 F_{yb} / 8}{4p_f}$$

$$F' = \frac{(0.355)^2(45.5)\left[0.85(5.4)/2 + 0.80(1.8875)\right] + \pi(0.75)^3(90)/8}{4(1.25)} = 7.345 \text{ kips}$$

$$Q_{max} = \frac{w't_p^2}{4a}\sqrt{F_{py}^2 - 3\left(\frac{F'}{w't_p}\right)^2} = \frac{1.8875(0.355)^2}{4(0.112)}\sqrt{(45.5)^2 - 3\left(\frac{7.345}{1.8875(0.355)}\right)^2}$$

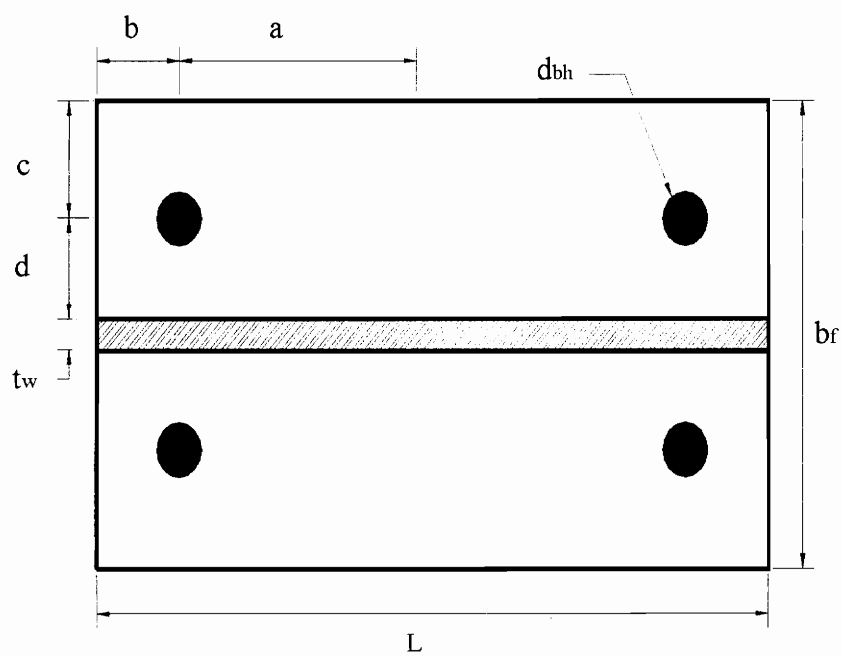
$$Q_{max} = 21.96 \text{ kips}$$

$$P_r = \max\begin{vmatrix}4(P_t - Q_{max}) \\ 4T_b\end{vmatrix} = \begin{vmatrix}4(40 - 21.96) \\ 4(28)\end{vmatrix} = 112 \text{ kips}$$

Test:..4a, 10a
Limit State:..Bolt Rupture with no Prying Action

$$P_{np} = \max\begin{vmatrix}4Pt \\ 4T_b\end{vmatrix} = \begin{vmatrix}4(40) \\ 4(28)\end{vmatrix} = 160 \text{ kips}$$

UNSTIFFENED TEE HANGER TEST

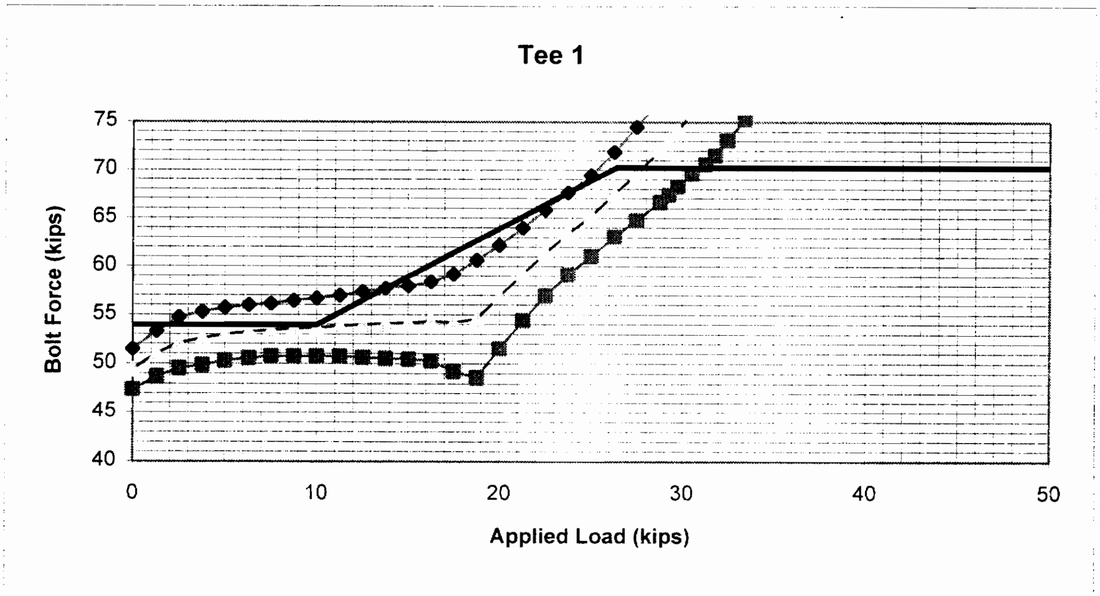


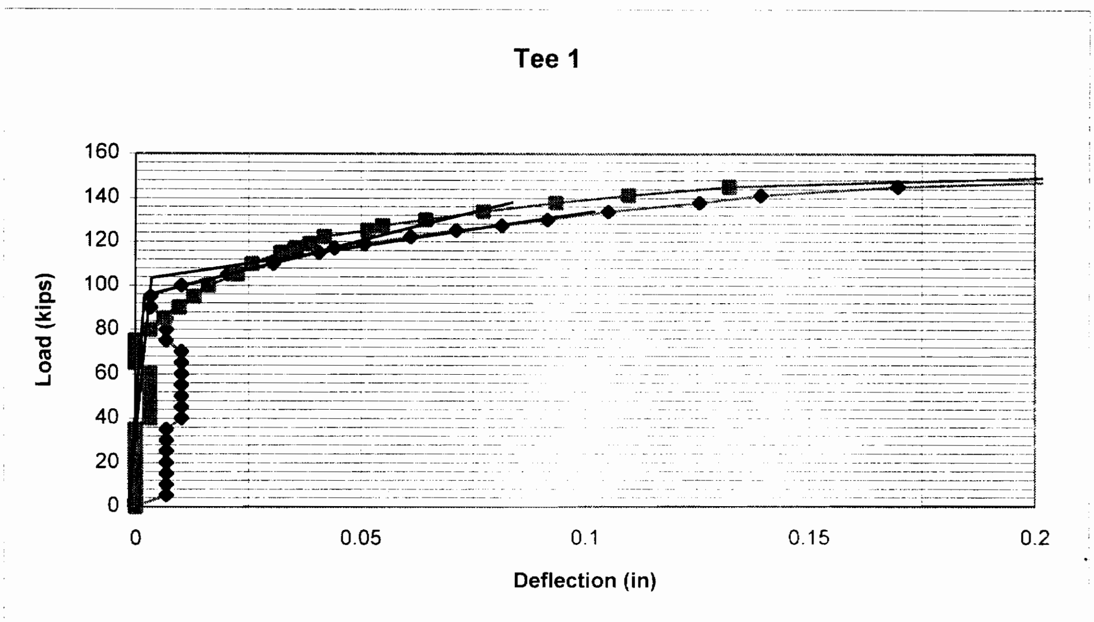
Test Designation...1

F_y.............68.5 ksi	t_p.........................0.52 in.
t_w..............0.75 in.	t_{weld}.......................0.19 in.
a...............1.48 in.	b..........................1.77 in.
c...............1.77 in.	d..........................1.165 in.
f................2.935 in.	L..........................6.5 in.
b_f..............7.0 in.	d_b........................1.0 in.
d_{bh}.............1.0625 in.	

V..25 kips
Average Experimental Yield Load..100 kips
Average Experimental Ultimate Applied Bolt Load...............................27.5 kips
P_y...105.8 kips
P_r...208 kips
P_{np}...280 kips
P_{pred}...105.8 kips

Notes:


Tee 1
75
70
65
60
55
50
45
40
Bolt Force (kips)
0
10
20
30
40
50
Applied Load (kips)


Tee 1
160
140
120
100
80
60
40
20
0
Load (kips)
0
0.05
0.1
0.15
0.2
Deflection (in)

CALCULATION OF PREDICTED VALUES

Test:..1
Limit State:..Plate Yielding

$$p = 2.17d = 2.17(1.165) = 2.528 \text{ in.}$$

$$e = d - \frac{b^2}{4p} = 1.165 - \frac{(1.77)^2}{4(2.528)} = 0.855 \text{ in.}$$

$$b \le 3.53e = 2.63 - \text{ok!}$$

$$a = \min\begin{vmatrix} l/2 - b \\ \sqrt{4pf} \end{vmatrix} = \begin{vmatrix} 1.48'' \\ 5.44'' \end{vmatrix} = 1.48 \text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{(1.48)^2}{4(2.528)} = 0.217'' \le f = 2.935 \text{ in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{1.77}{0.855}\right) = 64.22^o$$

$$g \le d \Rightarrow 0.217 \le 1.165$$

$$\theta_2 = \tan^{-1}\left(\frac{a}{d-g}\right) = \tan^{-1}\left(\frac{1.48}{1.165 - 0.217}\right) = 57.35^o$$

$$\alpha = \theta_1 + \theta_2 = 64.22^o + 57.35^o = 121.58^o$$

$$m_p = \frac{t_p^2}{4}\sqrt{(F_y)^2 - 3\left(\frac{V}{wt_p}\right)^2} = \frac{(0.52)^2}{4}\sqrt{(68.4)^2 - 3\left(\frac{25}{2.9(0.52)}\right)^2} = 4.19 \text{kip} - \text{in}/\text{in}$$

$$P_a = 4\pi\left(\frac{\alpha}{360}\right)m_p = 4\pi\left(\frac{121.58^o}{360^o}\right)m_p = 4.24 m_p$$

$$P_c = b\left(\frac{1}{e}\right)m_p = 1.77\left(\frac{1}{0.855}\right)m_p = 2.07 m_p$$

$$P_y = 4(P_a + P_c) = 4(4.24m_p + 2.07m_p) = (4)6.312m_p = 4(6.312)(4.19) = 105.78 \text{ kips}$$

CALCULATION OF PREDICTED VALUES

Test:..1
Limit State:...Bolt Rupture with Prying Action

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 = 0.02317(68.5)\left(\frac{0.52}{1.0}\right)^3 = 0.223 \text{ in.}$$

$$w' = \frac{b_f}{2} - d_{bh} = 3 - 1.0625 = 1.9375 \text{ in.}$$

$$F' = \frac{t_p^2 F_{py}\left(0.85b_f / 2 + 0.80w'\right) + \pi d_b^3 F_{yb} / 8}{4p_f}$$

$$F' = \frac{(0.52)^2(68.4)\left[0.85(6)/2 + 0.80(1.9375)\right] + \pi(1.0)^3(90)/8}{4(1.25)} = 22.23 \text{ kips}$$

$$Q_{max} = \frac{w' t_p^2}{4a}\sqrt{F_{py}^2 - 3\left(\frac{F'}{w' t_p}\right)^2} = \frac{1.9375(0.52)^2}{4(0.223)}\sqrt{(68.4)^2 - 3\left(\frac{22.23}{1.9375(0.52)}\right)^2}$$

$$Q_{max} = 39.88 \text{ kips}$$

$$P_r = \max\left|\begin{matrix}4(P_t - Q_{max}) \\ 4T_b\end{matrix}\right. = \left|\begin{matrix}4(70 - 39.88) \\ 4(52)\end{matrix}\right. = 208 \text{ kips}$$

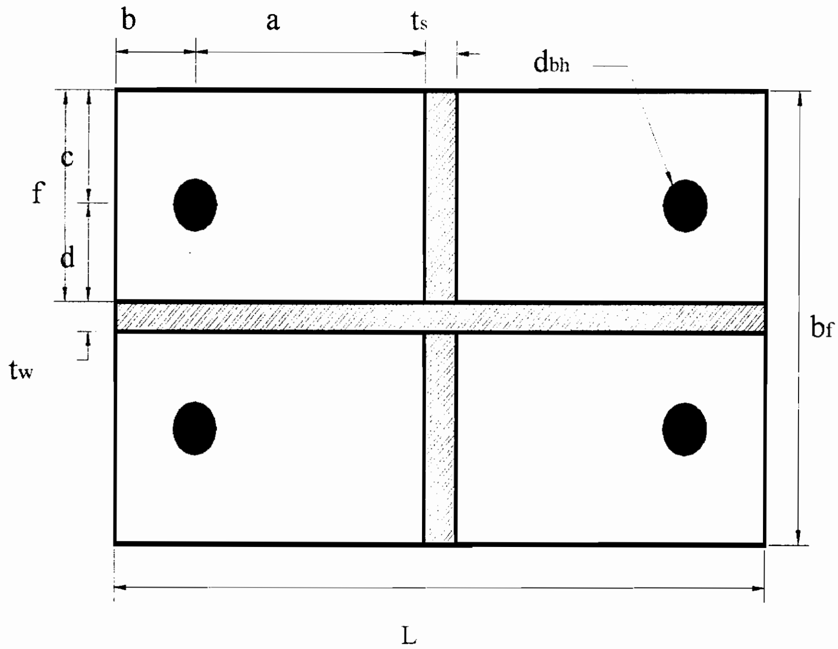
Test:..1
Limit State:...Bolt Rupture with no Prying Action

$$P_{np} = \max\left|\begin{matrix}4Pt \\ 4T_b\end{matrix}\right. = \left|\begin{matrix}4(70) \\ 4(52)\end{matrix}\right. = 280 \text{ kips}$$

APPENDIX C

STIFFENED TEE HANGER TEST RESULTS

STIFFENED TEE HANGER TEST



Test Designation...8a, 3a

F_y.............45.5 ksi

t_p...................0.355 in.

t_s...............0.25 in.

t_w....................0.25 in.

t_{weld}...........0.325 in.

a.....................1.31 in.

b...............1.5 in.

c.....................1.25 in.

d...............1.0 in.

f......................2.25 in.

L................6.5in.

b_f....................5.4 in.

d_b.............0.75 in.

d_{bh}..................0.8125 in.

V...13.5 kips

Average Experimental Yield Load ....54 kips

Average Experimental Ultimate Applied Bolt Force............................21.25 kips

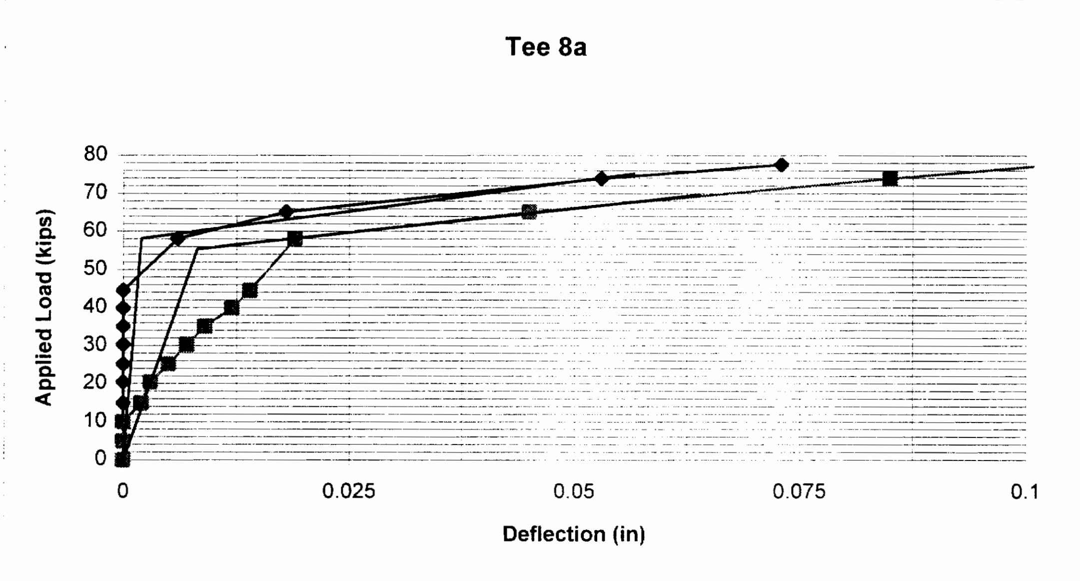
P_y...41.27 kips

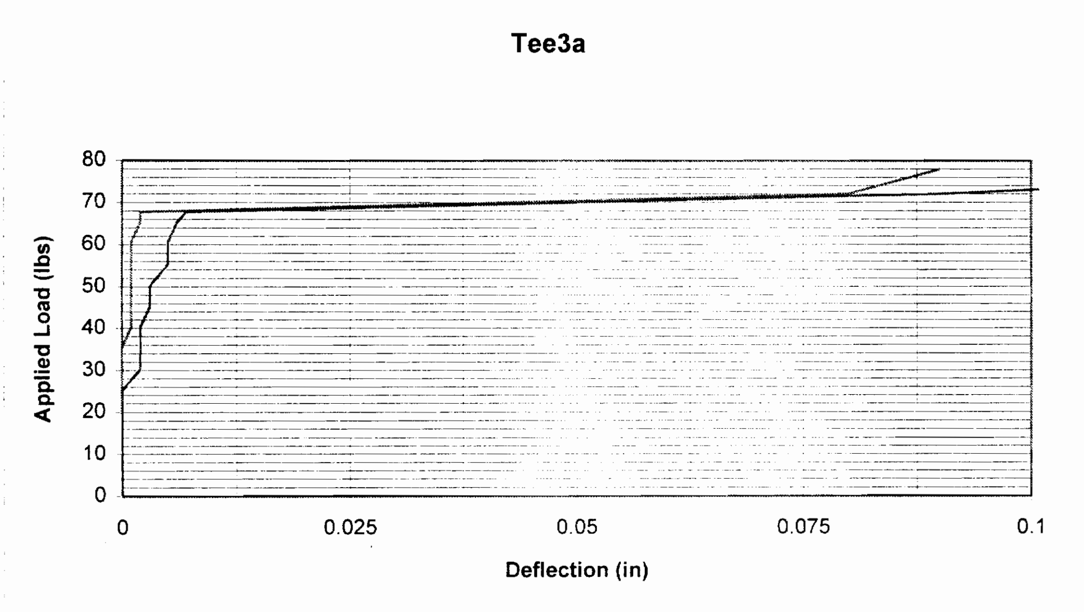
P_r...112 kips

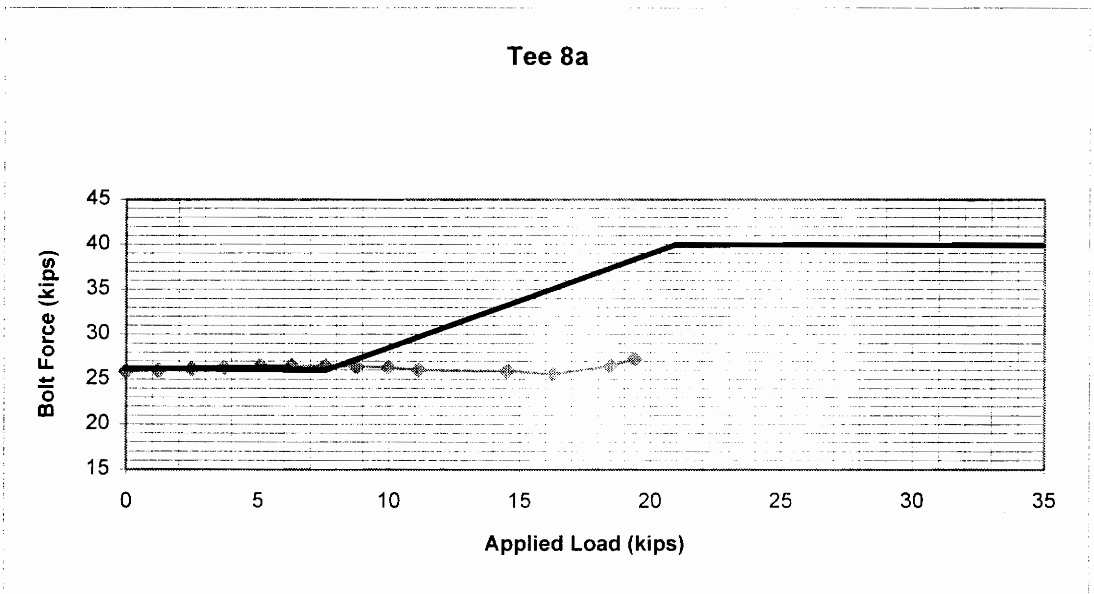
P_{np}...160 kips

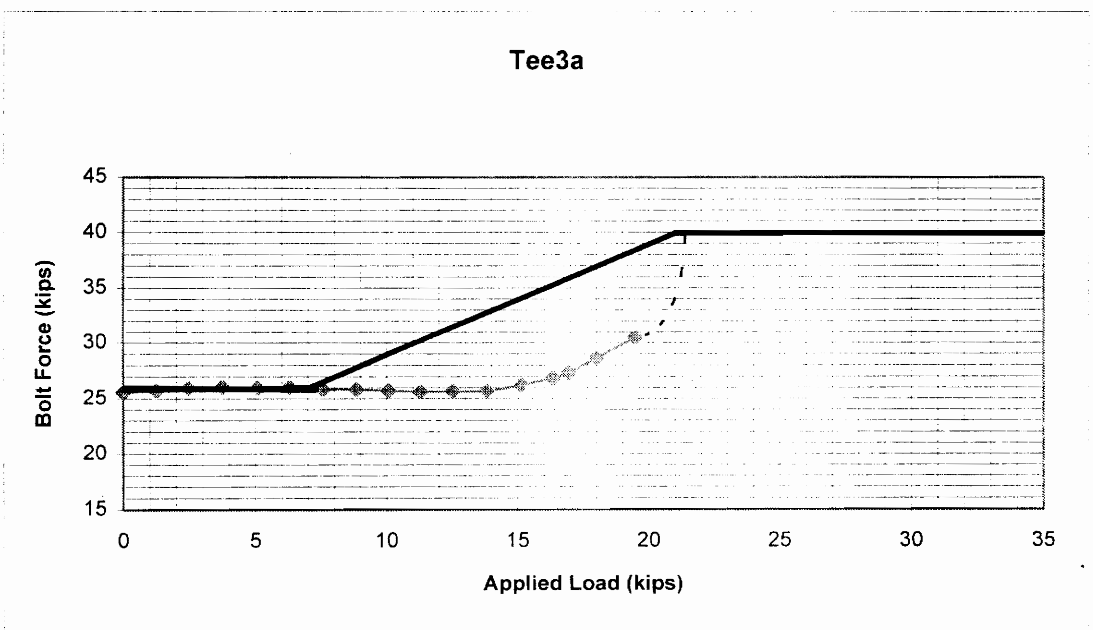
P_{pred}..41.27 kips

Notes: Bolt in test 8a was not used for bolt force calculation. Test 3a was not used in determining yield load due to faulty gage readings.


Tee 8a
80
70
60
50
40
30
20
10
0
Applied Load (kips)
0
0.025
0.05
0.075
0.1
Deflection (in)


Tee3a
80
70
60
50
40
30
20
10
0
Applied Load (lbs)
0
0.025
0.05
0.075
0.1
Deflection (in)


Tee 8a
Bolt Force (kips)
45
40
35
30
25
20
15
0
5
10
15
20
25
30
35
Applied Load (kips)


Tee3a
Bolt Force (kips)
45
40
35
30
25
20
15
0
5
10
15
20
25
30
35
Applied Load (kips)

CALCULATION OF PREDICTED VALUES

Test:..8a,3a
Limit State:...Plate Yielding

$$p = 2.17d = 2.17(1.004) = 2.179 \text{ in.}$$

$$e = d - \frac{b^2}{4p} = 1.004 - \frac{(1.501)^2}{4(2.179)} = 0.745 \text{ in.}$$

$$b \le 3.53e = 2.63 - \text{ok!}$$

$$a = \min\begin{vmatrix} \text{measured} \\ \sqrt{4pf} \end{vmatrix} = \begin{vmatrix} 1.31'' \\ 4.457'' \end{vmatrix} = 1.31 \text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{(1.31)^2}{4(2.179)} = 0.196'' \le f = 2.28 \text{ in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{1.501}{0.745}\right) = 63.6^o$$

$$g \le d \Rightarrow 0.196 \le 1.004$$

$$\theta_2 = \tan^{-1}\left(\frac{a}{d-g}\right) = \tan^{-1}\left(\frac{1.31}{1.004 - 0.196}\right) = 58.36^o$$

$$\alpha = \theta_1 + \theta_2 = 63.6^o + 58.36^o = 121.9^o$$

$$m_p = \frac{t_p^2}{4}\sqrt{(F_y)^2 - 3\left(\frac{V}{wt_p}\right)^2} = \frac{(0.355)^2}{4}\sqrt{(45.5)^2 - 3\left(\frac{13.5}{2.5(0.355)}\right)^2} = 1.168\text{kip} - \text{in} / \text{in}$$

$$P_a = 4\pi\left(\frac{\alpha}{360}\right)m_p = 4\pi\left(\frac{121.9^o}{360^o}\right)m_p = 4.25m_p$$

$$P_b = (c + (f - g))\left(\frac{1}{a}\right)m_p = (1.276 + (2.28 - 0.196))\left(\frac{1}{1.31}\right)m_p = 2.564m_p$$

$$P_c = b\left(\frac{1}{e}\right)m_p = 1.501\left(\frac{1}{0.745}\right)m_p = 2.014m_p$$

$$P_y = 4(P_a + P_b + P_c) = 4(4.25m_p + 2.564m_p + 2.014m_p) = 4(8.829m_p) = 4(8.829(1.168)) = 41.27 \text{ ki}$$

CALCULATION OF PREDICTED VALUES

Test:..8a,3a
Limit State:..Bolt Rupture with Prying Action

$$t_{mod} = \sqrt{\frac{P_s}{F_y\left(P_{us}/m_p\right)}} = \sqrt{\frac{50.6}{45.5(6.65)}} = 0.409 \text{ in.}$$

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 = 0.02317(45.5)\left(\frac{0.409}{0.75}\right)^3 = 0.170 \text{ in.}$$

$$w' = \frac{b_f}{2} - d_{bh} = 2.7 - 0.8125 = 1.8875 \text{ in.}$$

$$F' = \frac{t_p^2 F_{py}\left(0.85b_f/2 + 0.80w'\right) + \pi d_b^3 F_{yb}/8}{4p_f}$$

$$F' = \frac{(.409)^2(45.5)\left[0.85(5.4)/2 + 0.80(1.8875)\right] + \pi(0.75)^3(90)/8}{4(1.25)} = 9.75 \text{ kips}$$

$$Q_{max} = \frac{w't_p^2}{4a}\sqrt{F_{py}^2 - 3\left(\frac{F'}{w't_p}\right)^2} = \frac{1.8875(0.409)^2}{4(0.170)}\sqrt{(45.5)^2 - 3\left(\frac{9.75}{1.8875(0.409)}\right)^2}$$

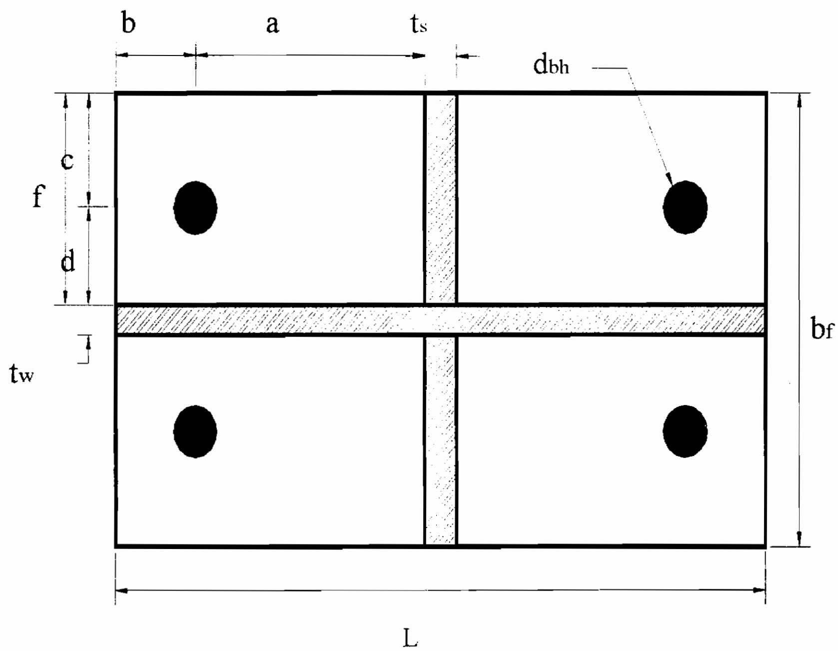
$$Q_{max} = 18.53 \text{ kips}$$

$$P_r = \max\begin{vmatrix} 4(P_t - Q_{max}) \\ 4T_b \end{vmatrix} = \begin{vmatrix} 4(40 - 18.53) \\ 4(28) \end{vmatrix} = 112 \text{ kips}$$

Test:..8a,3a
Limit State:..Bolt Rupture with no Prying Action

$$P_{np} = \max\begin{vmatrix} 4Pt \\ 4T_b \end{vmatrix} = \begin{vmatrix} 4(40) \\ 4(28) \end{vmatrix} = 160 \text{ kips}$$

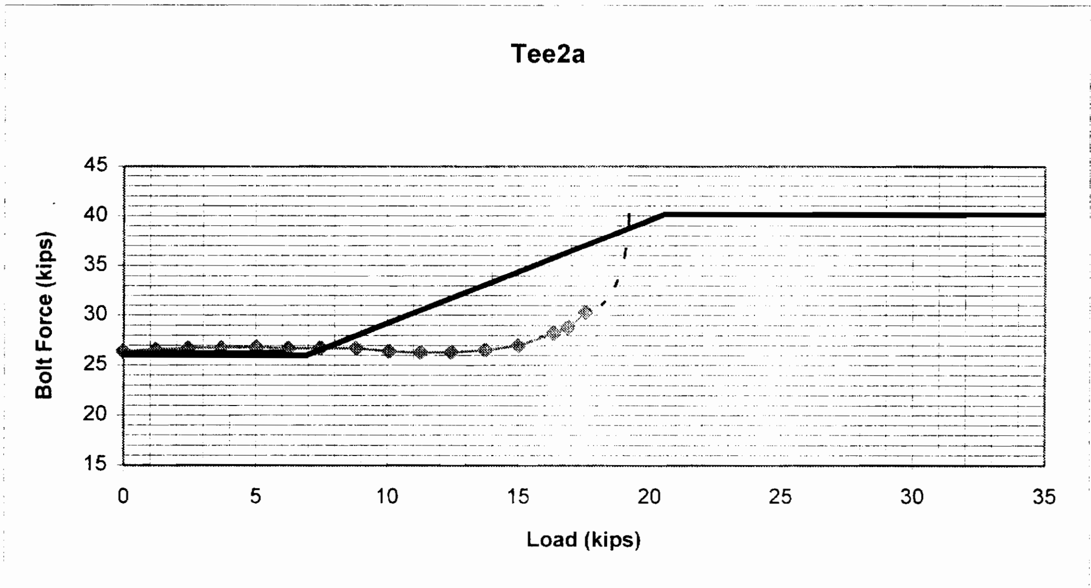
STIFFENED TEE HANGER TEST

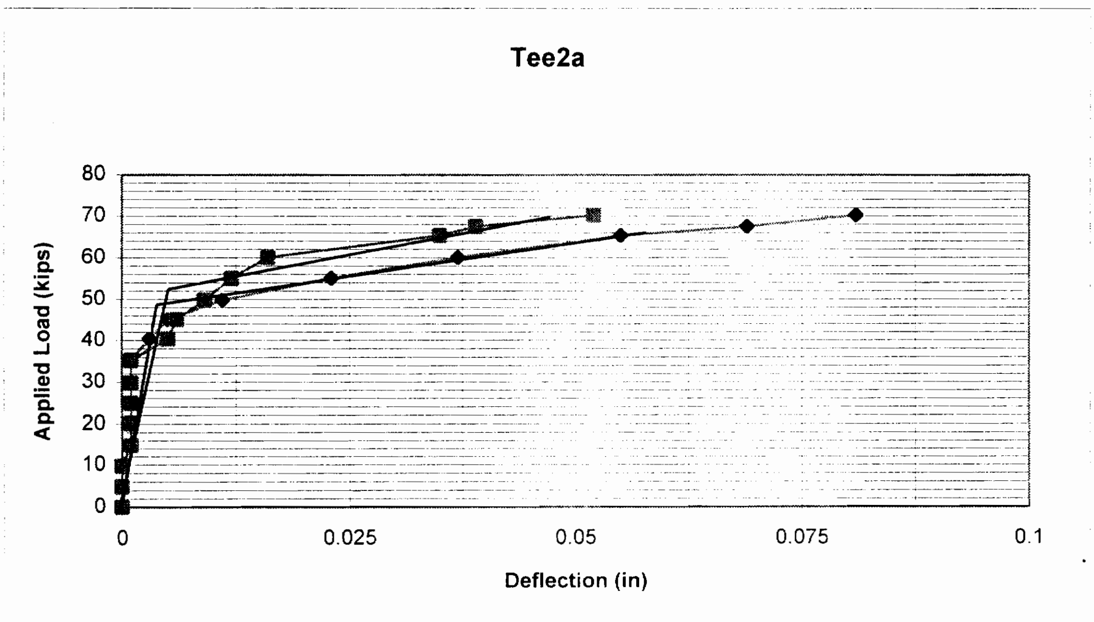


Test Designation..2a

F_y.............45.5 ksi	t_p...................0.355 in.
t_s...............0.25 in.	t_w...................0.25 in.
t_{weld}...........0.325 in.	a.....................1.428 in.
b...............1.5 in.	c.....................1.25 in.
d...............1.0 in.	f.....................2.25 in.
L................6.5in.	b_f...................5.4 in.
d_b.............0.75 in.	d_{bh}..................0.8125 in.

V..12.625 kips
Average Experimental Yield Load ....50.5 kips
Average Experimental Ultimate Applied Bolt Force...........................19 kips
P_y..40.4 kips
P_r..112 kips
P_{np}..160 kips
P_{pred}..40.4 kips
Notes:


Tee2a
45
40
35
30
25
20
15
Bolt Force (kips)
0
5
10
15
20
25
30
35
Load (kips)


Tee2a
80
70
60
50
40
30
20
10
0
Applied Load (kips)
0
0.025
0.05
0.075
0.1
Deflection (in)

CALCULATION OF PREDICTED VALUES

Test:..2a
Limit State:..Plate Yielding

$$p = 2.17d = 2.17(1.004) = 2.179 \text{ in.}$$

$$e = d - \frac{b^2}{4p} = 1.004 - \frac{(1.501)^2}{4(2.179)} = 0.745 \text{ in.}$$

$$b < 3.53e = 2.66 - \text{ok!}$$

$$a = \min\begin{vmatrix} \text{measured} \\ \sqrt{4pf} \end{vmatrix} = \begin{vmatrix} 1.482" \\ 4.457" \end{vmatrix} = 1.482 \text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{(1.482)^2}{4(2.179)} = 0.253" \leq f = 2.28 \text{ in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{1.501}{0.745}\right) = 63.6^o$$

$$g \leq d \Rightarrow 0.253 \leq 1.004$$

$$\theta_2 = \tan^{-1}\left(\frac{a}{d-g}\right) = \tan^{-1}\left(\frac{1.482}{1.004 - 0.253}\right) = 63.12^o$$

$$\alpha = \theta_1 + \theta_2 = 63.6^o + 63.12^o = 126^o$$

$$m_p = \frac{t_p^2}{4}\sqrt{(F_y)^2 - 3\left(\frac{V}{wt_p}\right)^2} = \frac{(0.355)^2}{4}\sqrt{(45.5)^2 - 3\left(\frac{12.625}{2.5(0.355)}\right)^2} = 1.16\text{kip} - \text{in}/\text{in}$$

$$P_a = 4\pi\left(\frac{\alpha}{360}\right)m_p = 4\pi\left(\frac{126^o}{360^o}\right)m_p = 4.42m_p$$

$$P_b = (c + (f - g))\left(\frac{1}{a}\right)m_p = (1.276 + (2.28 - 0.253))\left(\frac{1}{1.482}\right)m_p = 2.23m_p$$

$$P_c = b\left(\frac{1}{e}\right)m_p = 1.501\left(\frac{1}{0.745}\right)m_p = 2.014m_p$$

$$P_y = 4(P_a + P_b + P_c) = 4(4.632m_p + 2.23m_p + 2.014m_p) = 4(8.66m_p) = 4(8.66(1.16)) = 40.4 \text{ kips}$$

CALCULATION OF PREDICTED VALUES

Test:..2a
Limit State:..Bolt Rupture with Prying Action

$$t_{mod} = \sqrt{\frac{P_s}{F_y\left(P_{us}/m_p\right)}} = \sqrt{\frac{49}{45.5(6.65)}} = 0.402 \text{ in.}$$

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 = 0.02317(45.5)\left(\frac{0.402}{0.75}\right)^3 = 0.162 \text{ in.}$$

$$w' = \frac{b_f}{2} - d_{bh} = 2.7 - 0.8125 = 1.8875 \text{ in.}$$

$$F' = \frac{t_p^2 F_{py}\left(0.85b_f/2 + 0.80w'\right) + \pi d_b^3 F_{yb}/8}{4p_f}$$

$$F' = \frac{(0.402)^2(45.5)\left[0.85(5.4)/2 + 0.80(1.8875)\right] + \pi(0.75)^3(90)/8}{4(1.25)} = 8.57\text{kips}$$

$$Q_{max} = \frac{w't_p^2}{4a}\sqrt{F_{py}^2 - 3\left(\frac{F'}{w't_p}\right)^2} = \frac{1.8875(0.402)^2}{4(0.162)}\sqrt{(45.5)^2 - 3\left(\frac{8.57}{1.8875(0.402)}\right)^2}$$

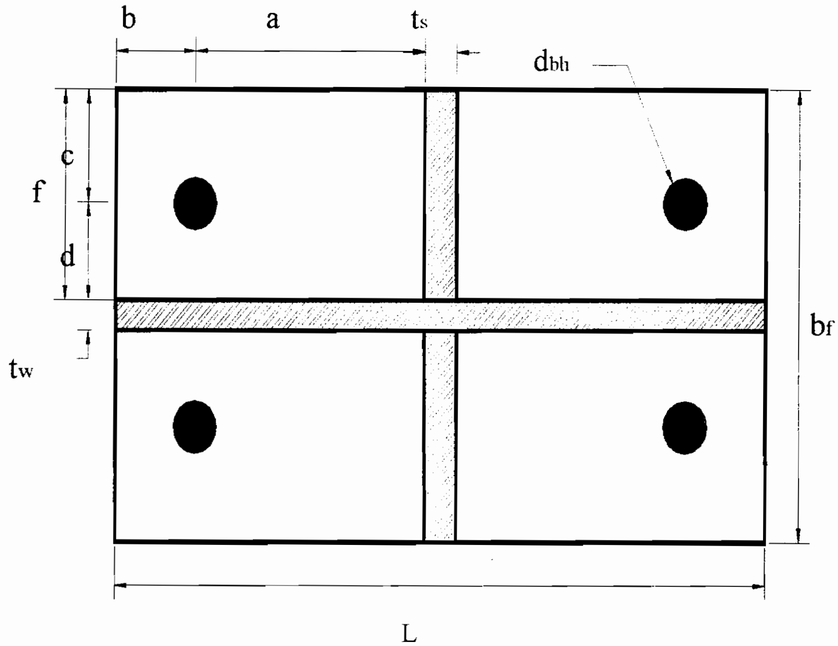
$$Q_{max} = 19.33 \text{ kips}$$

$$P_r = \max\left|\begin{matrix} 4(P_t - Q_{max}) \\ 4T_b \end{matrix}\right. = \left|\begin{matrix} 4(40 - 19.33) \\ 4(28) \end{matrix}\right. = 112 \text{ kips}$$

Test:..2a
Limit State:..Bolt Rupture with no Prying Action

$$P_{np} = \max\left|\begin{matrix} 4Pt \\ 4T_b \end{matrix}\right. = \left|\begin{matrix} 4(40) \\ 4(28) \end{matrix}\right. = 160 \text{ kips}$$

STIFFENED TEE HANGER TEST



Test Designation...9a, 5a

F_y.............45.5 ksi	t_p....................0.355 in.
t_s...............0.25 in.	t_w....................0.25 in.
t_{weld}...........0.325 in.	a.....................3.318 in.
b...............1.5 in.	c.....................1.25 in.
d...............1.0 in.	f.....................2.25 in.
L................10.5in.	b_f....................5.4 in.
d_b.............0.75 in.	d_{bh}..................0.8125 in.

V...11.75 kips

Average Experimental Yield Load ....47.0 kips

Average Experimental Ultimate Applied Bolt Force...........................19.25 kips

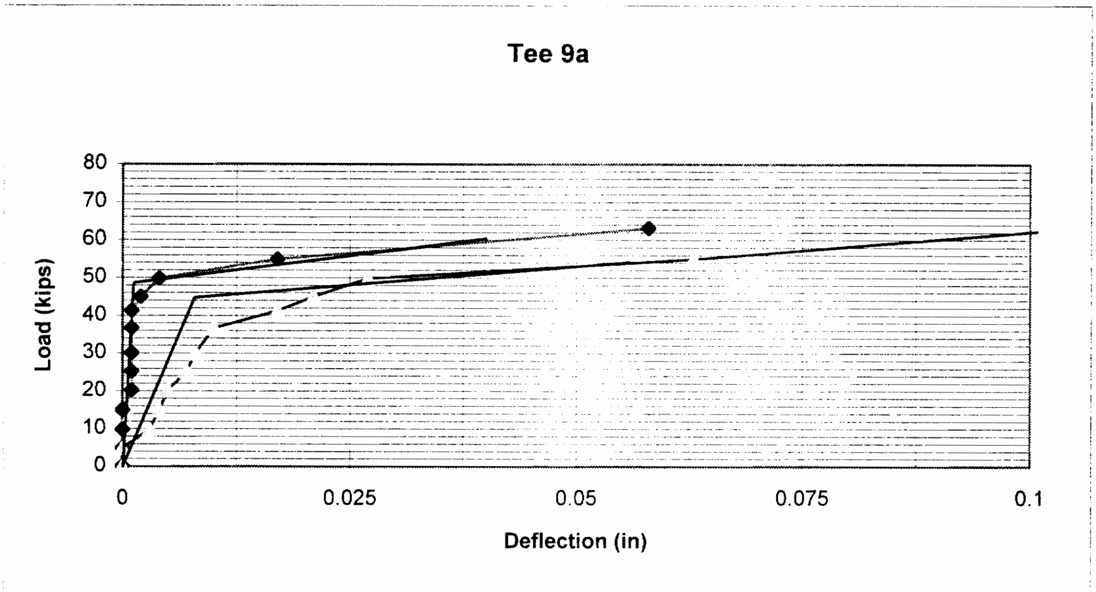
P_y..40.56 kips

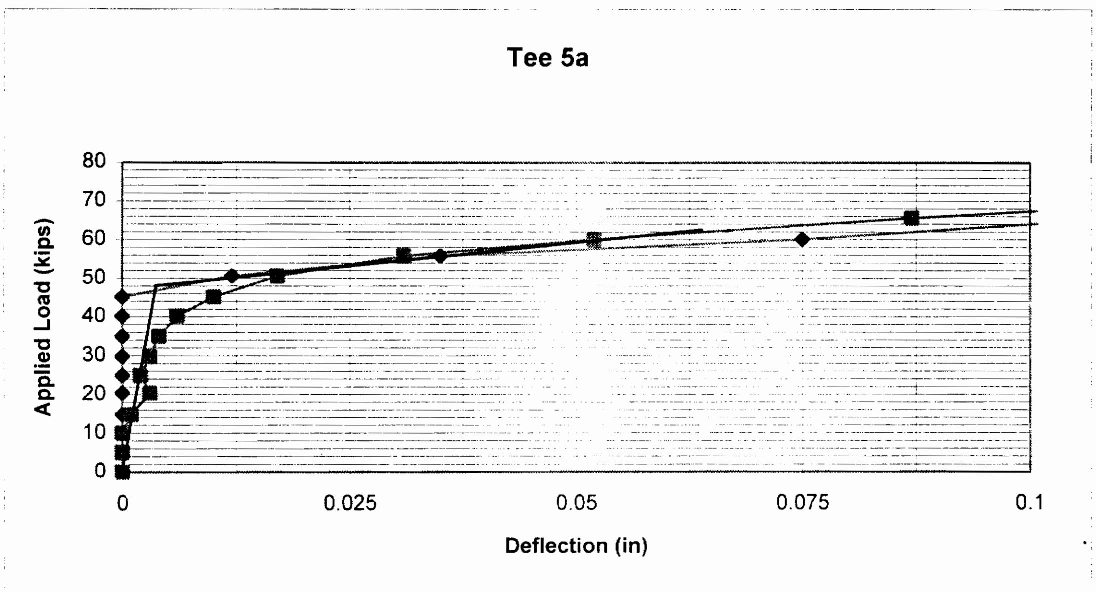
P_r..112 kips

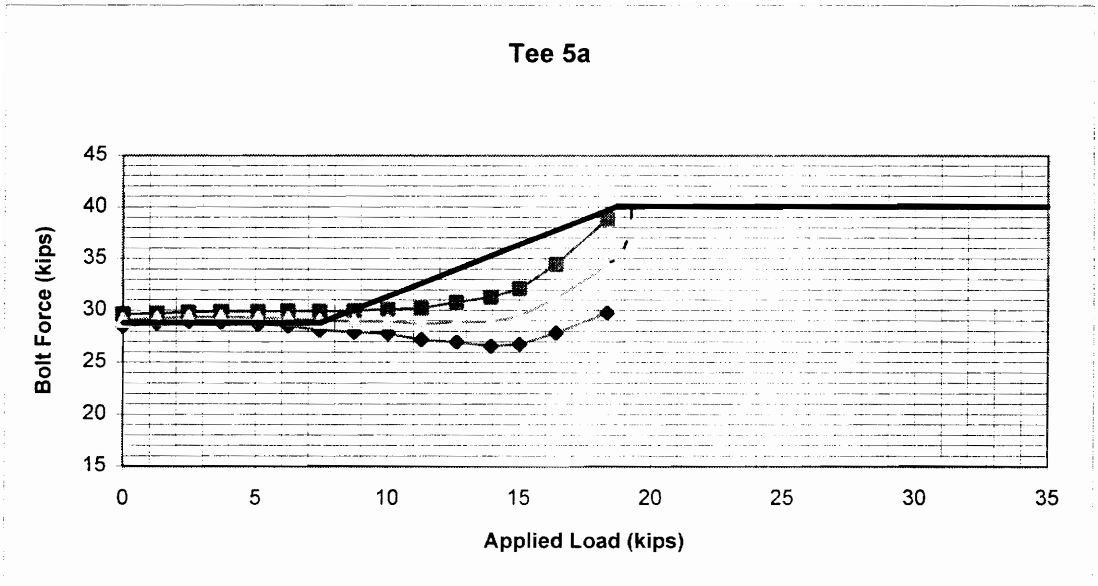
P_{np}..160 kips

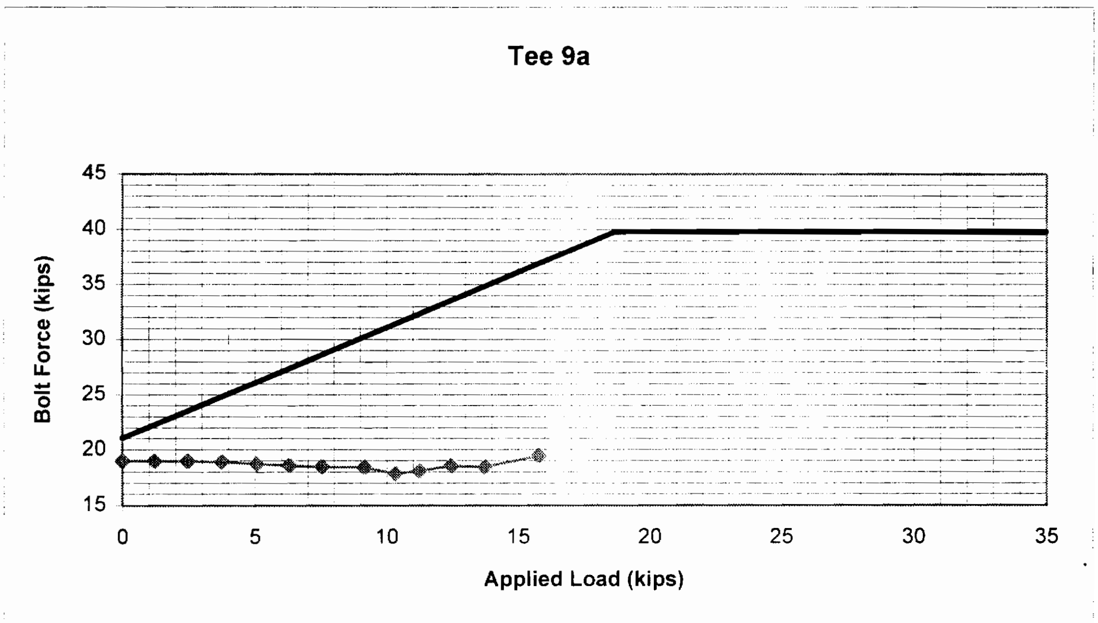
P_{pred}..40.56 kips

Notes: Test 9a could not be used in bolt force predictions.


Tee 9a
Load (kips)
80
70
60
50
40
30
20
10
0
0
0.025
0.05
0.075
0.1
Deflection (in)


Tee 5a
Applied Load (kips)
80
70
60
50
40
30
20
10
0
0
0.025
0.05
0.075
0.1
Deflection (in)


Tee 5a
45
40
35
30
25
20
15
Bolt Force (kips)
0
5
10
15
20
25
30
35
Applied Load (kips)


Tee 9a
45
40
35
30
25
20
15
Bolt Force (kips)
0
5
10
15
20
25
30
35
Applied Load (kips)

CALCULATION OF PREDICTED VALUES

Test:..9a,5a
Limit State:..Plate Yielding

$$p = 2.17d = 2.17(1.004) = 2.179 \text{ in.}$$

$$e = d - \frac{b^2}{4p} = 1.004 - \frac{(1.501)^2}{4(2.179)} = 0.745 \text{ in.}$$

$$b \le 3.53e = 2.63 - \text{ok!}$$

$$a = \min\begin{vmatrix}\text{measured} \\ \sqrt{4pf}\end{vmatrix} = \begin{vmatrix}3.318" \\ 4.457"\end{vmatrix} = 3.318 \text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{(3.318)^2}{4(2.179)} = 1.26" \le f = 2.28 \text{ in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{1.501}{0.745}\right) = 63.6^o$$

$$g \ge d \Rightarrow 1.216 \ge 1.004$$

$$\theta_2 = \tan^{-1}\left(\frac{g-d}{a}\right) + 90^o = \tan^{-1}\left(\frac{1.216 - 1.004}{3.318}\right) + 90^0 = 93.84^o$$

$$\alpha = \theta_1 + \theta_2 = 63.6^o + 93.84^o = 157.4^o$$

$$m_p = \frac{t_p^2}{4}\sqrt{(F_y)^2 - 3\left(\frac{V}{wt_p}\right)^2} = \frac{(0.355)^2}{4}\sqrt{(45.5)^2 - 3\left(\frac{11.75}{2.5(0.355)}\right)^2} = 1.238 \text{kip} - \text{in} / \text{in}$$

$$P_a = 4\pi\left(\frac{\alpha}{360}\right)m_p = 4\pi\left(\frac{157.4^o}{360^o}\right)m_p = 5.49m_p$$

$$P_b = (c + (f - g))\left(\frac{1}{a}\right)m_p = (1.276 + (2.28 - 1.26))\left(\frac{1}{3.318}\right)m_p = 0.692m_p$$

$$P_c = b\left(\frac{1}{e}\right)m_p = 1.501\left(\frac{1}{0.745}\right)m_p = 2.014m_p$$

$$P_y = 4(P_a + P_b + P_c) = 4(5.49m_p + 0.692m_p + 2.014m_p) = 4(8.19m_p) = 4(8.19(1.238)) = 40.56 \text{ kips}$$

CALCULATION OF PREDICTED VALUES

Test:..9a,5a
Limit State:..Bolt Rupture with Prying Action

$$t_{mod} = \sqrt{\frac{P_s}{F_y\left(P_{us}/m_p\right)}} = \sqrt{\frac{47}{45.5(7.695)}} = 0.366 \text{ in.}$$

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 = 0.02317(45.5)\left(\frac{0.366}{0.75}\right)^3 = 0.122 \text{ in.}$$

$$w' = \frac{b_f}{2} - d_{bh} = 2.7 - 0.8125 = 1.8875 \text{ in.}$$

$$F' = \frac{t_p^2 F_{py}\left(0.85b_f/2 + 0.80w'\right) + \pi d_b^3 F_{yb}/8}{4p_f}$$

$$F' = \frac{(0.366)^2(45.5)\left[0.85(5.4)/2 + 0.80(1.8875)\right] + \pi(0.75)^3(90)/8}{4(1.25)} = 7.81 \text{ kips}$$

$$Q_{max} = \frac{w't_p^2}{4a}\sqrt{F_{py}^2 - 3\left(\frac{F'}{w't_p}\right)^2} = \frac{1.8875(0.366)^2}{4(0.122)}\sqrt{(45.5)^2 - 3\left(\frac{7.81}{1.8875(0.366)}\right)^2}$$

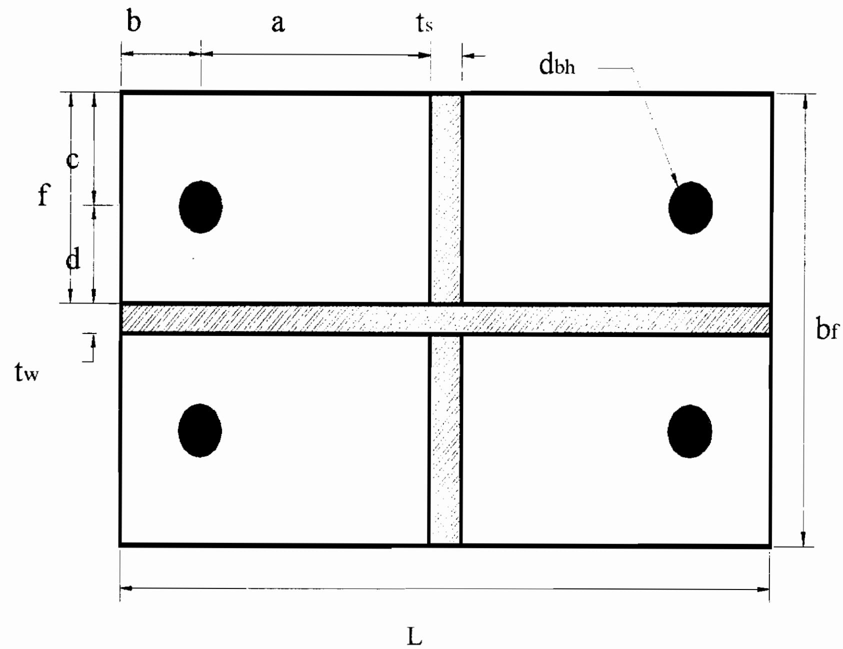
$$Q_{max} = 21.27 \text{ kips}$$

$$P_r = \max\left|\begin{matrix}4(P_t - Q_{max}) \\ 4T_b\end{matrix}\right. = \left|\begin{matrix}4(40 - 21.27) \\ 4(28)\end{matrix}\right. = 112 \text{ kips}$$

Test:..9a,5a
Limit State:..Bolt Rupture with no Prying Action

$$P_{np} = \max\left|\begin{matrix}4Pt \\ 4T_b\end{matrix}\right. = \left|\begin{matrix}4(40) \\ 4(28)\end{matrix}\right. = 160 \text{ kips}$$

STIFFENED TEE HANGER TEST



Test Designation....................7a, 6a

F_y.............45.5 ksi	t_p....................0.355 in.
t_s...............0.25 in.	t_w....................0.25 in.
t_{weld}...........0.325 in.	a.....................3.169 in.
b...............1.5 in.	c.....................1.25 in.
d...............1.0 in.	f.....................2.25 in.
L...............10.5in.	b_f....................5.4 in.
d_b.............0.75 in.	d_{bh}...................0.8125 in.

V....................11.8 kips

Average Experimental Yield Load ....47.25 kips

Average Experimental Ultimate Applied Bolt Force....................N/A

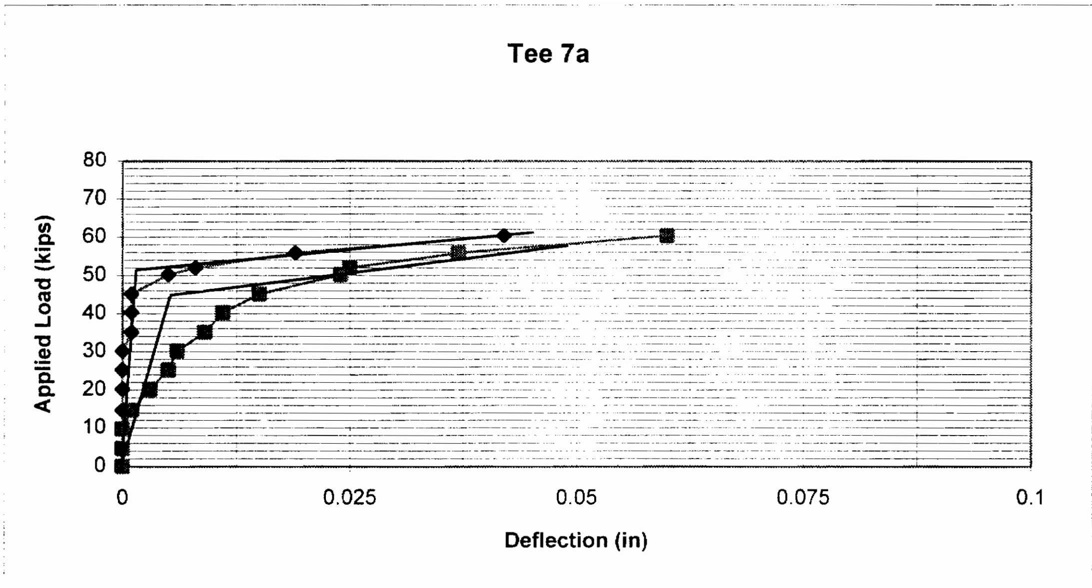
P_y....................40.53 kips

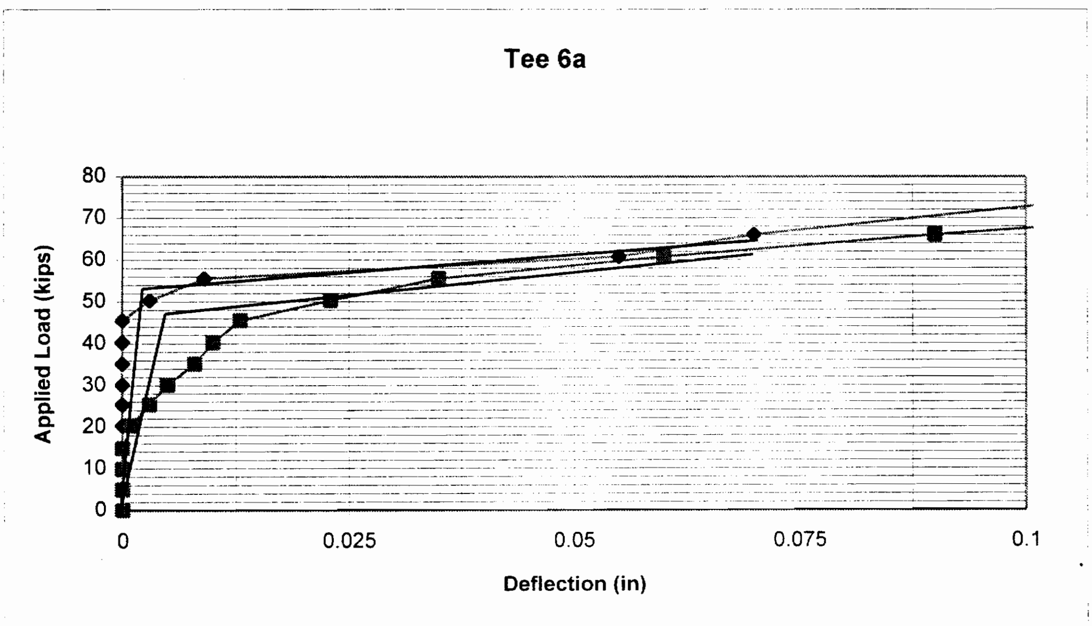
P_r....................112 kips

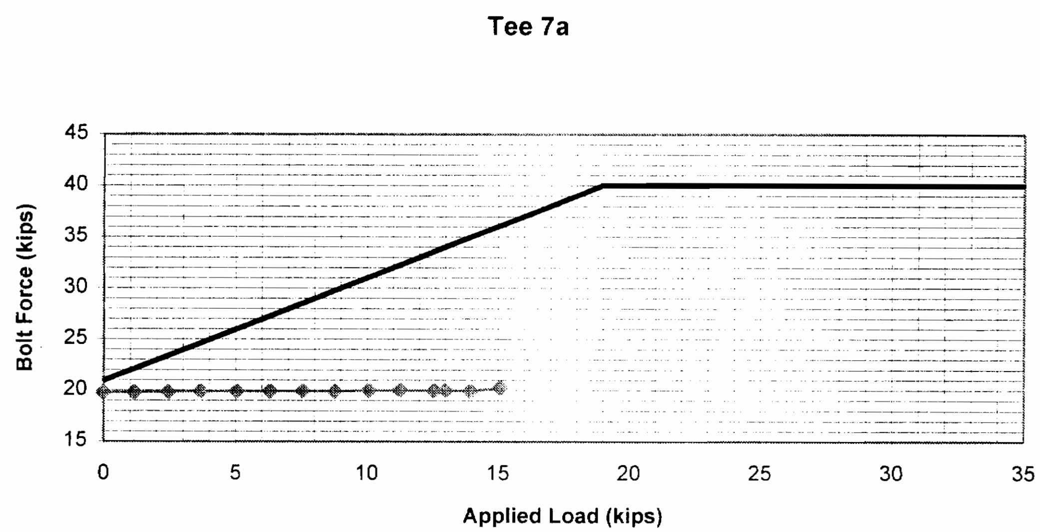
P_{np}....................160 kips

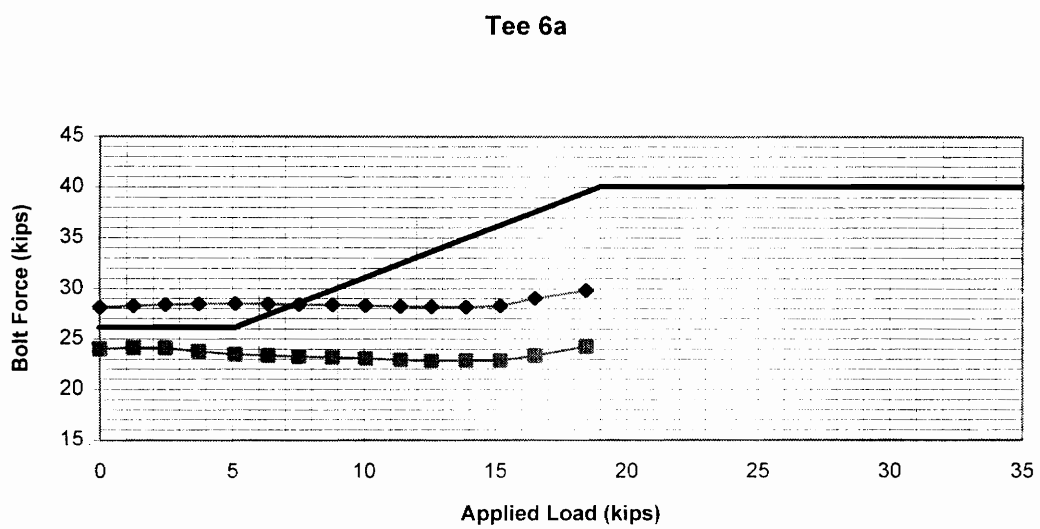
P_{pred}....................40.53 kips

Notes: Neither test gave usable bolt force test data


Tee 7a
Applied Load (kips)
80
70
60
50
40
30
20
10
0
0
0.025
0.05
0.075
0.1
Deflection (in)


Tee 6a
Applied Load (kips)
80
70
60
50
40
30
20
10
0
0
0.025
0.05
0.075
0.1
Deflection (in)


Tee 7a
Bolt Force (kips)
45
40
35
30
25
20
15
0
5
10
15
20
25
30
35
Applied Load (kips)


Tee 6a
Bolt Force (kips)
45
40
35
30
25
20
15
0
5
10
15
20
25
30
35
Applied Load (kips)

CALCULATION OF PREDICTED VALUES

Test:..6a,7a
Limit State:..Plate Yielding

$$p = 2.17d = 2.17(1.004) = 2.179 \text{ in.}$$

$$e = d - \frac{b^2}{4p} = 1.004 - \frac{(1.501)^2}{4(2.179)} = 0.745 \text{ in.}$$

$$b \le 3.53e = 2.63 - \text{ok!}$$

$$a = \min\begin{vmatrix}\text{measured} \\ \sqrt{4pf}\end{vmatrix} = \begin{vmatrix}3.169" \\ 4.457"\end{vmatrix} = 3.169 \text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{(3.169)^2}{4(2.179)} = 1.152" \le f = 2.28 \text{ in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{1.501}{0.745}\right) = 63.6^o$$

$$g \ge d \Rightarrow 1.152 \ge 1.004$$

$$\theta_2 = \tan^{-1}\left(\frac{g-d}{a}\right) + 90^o = \tan^{-1}\left(\frac{1.152 - 1.004}{3.169}\right) + 90^0 = 92.67^o$$

$$\alpha = \theta_1 + \theta_2 = 63.6^o + 92.67^o = 156.27^o$$

$$m_p = \frac{t_p^2}{4}\sqrt{(F_y)^2 - 3\left(\frac{V}{wt_p}\right)^2} = \frac{(0.355)^2}{4}\sqrt{(45.5)^2 - 3\left(\frac{11.8}{2.5(0.355)}\right)^2} = 1.236\text{kip} - \text{in}/\text{in}$$

$$P_a = 4\pi\left(\frac{\alpha}{360}\right)m_p = 4\pi\left(\frac{156.27^o}{360^o}\right)m_p = 5.45m_p$$

$$P_b = (c + (f-g))\left(\frac{1}{a}\right)m_p = (1.276 + (2.28 - 1.152))\left(\frac{1}{3.169}\right)m_p = 0.758m_p$$

$$P_c = b\left(\frac{1}{e}\right)m_p = 1.501\left(\frac{1}{0.745}\right)m_p = 2.014m_p$$

$$P_y = 4(P_a + P_b + P_c) = 4(5.45m_p + 0.758m_p + 2.014m_p = 4(8.22m_p) = 4(8.22(1.236)) = 40.53 \text{ kips}$$

CALCULATION OF PREDICTED VALUES

Test:..6a,7a
Limit State:...Bolt Rupture with Prying Action

$$t_{mod} = \sqrt{\frac{P_s}{F_y\left(\frac{P_{us}}{m_p}\right)}} = \sqrt{\frac{47.13}{45.5(7.695)}} = 0.367 \text{ in.}$$

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 = 0.02317(45.5)\left(\frac{0.367}{0.75}\right)^3 = 0.122 \text{ in.}$$

$$w' = \frac{b_f}{2} - d_{bh} = 2.7 - 0.8125 = 1.8875 \text{ in.}$$

$$F' = \frac{t_p^2F_{py}\left(0.85b_f/2 + 0.80w'\right) + \pi d_b^3F_{yb}/8}{4p_f}$$

$$F' = \frac{(0.367)^2(45.5)\left[0.85(5.4)/2 + 0.80(1.8875)\right] + \pi(0.75)^3(90)/8}{4(1.25)} = 7.85 \text{ kips}$$

$$Q_{max} = \frac{w't_p^2}{4a}\sqrt{F_{py}^2 - 3\left(\frac{F'}{w't_p}\right)^2} = \frac{1.8875(0.367)^2}{4(0.122)}\sqrt{(45.5)^2 - 3\left(\frac{7.85}{1.8875(0.367)}\right)^2}$$

$$Q_{max} = 21.38 \text{ kips}$$

$$P_r = \max\left|\begin{matrix}4(P_t - Q_{max})\\4T_b\end{matrix}\right. = \left|\begin{matrix}4(40-21.38)\\4(28)\end{matrix}\right. = 112 \text{ kips}$$

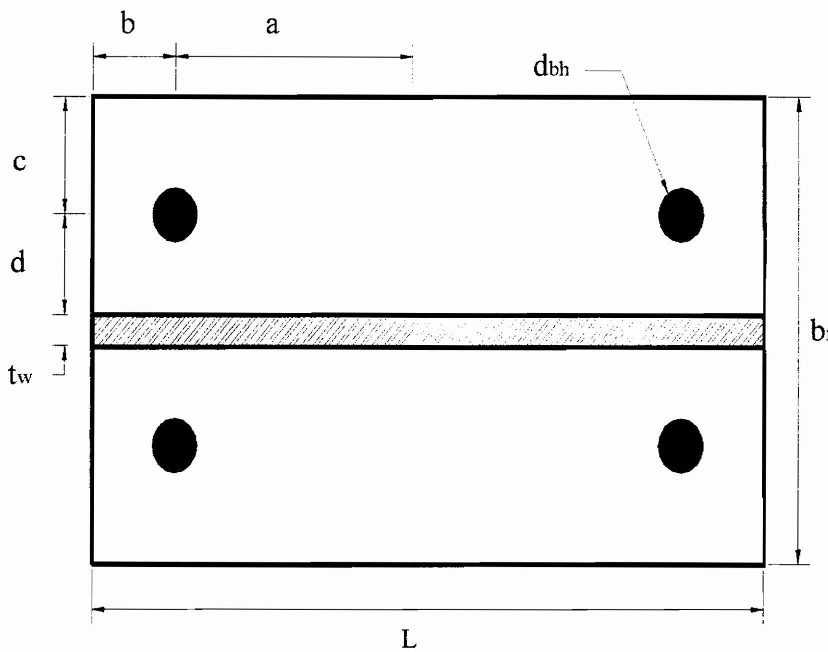
Test:..6a,7a
Limit State:...Bolt Rupture with no Prying Action

$$P_{np} = \max\left|\begin{matrix}4Pt\\4T_b\end{matrix}\right. = \left|\begin{matrix}4(40)\\4(28)\end{matrix}\right. = 160 \text{ kips}$$

APPENDIX D

TEE HANGER BOLT RUPTURE TEST RESULTS

UNSTIFFENED TEE HANGER TEST



Test Designation..12a

F_y.............42.2 ksi	t_p.........................0.785 in.
t_w..............0.5 in.	t_{weld}......................0.25 in.
a...............3.75 in.	b..........................1.5 in.
c...............1.25 in.	d..........................1.03 in.
f...............2.27 in.	L..........................10.5 in.
bf..............6.0 in.	d_b.........................0.75 in.
d_{bh}.............0.8125 in.	

V..32 kips

Average Experimental Yield Load..128 kips

Average Experimental Ultimate Applied Bolt Load...........................33.5 kips

P_y..133 kips

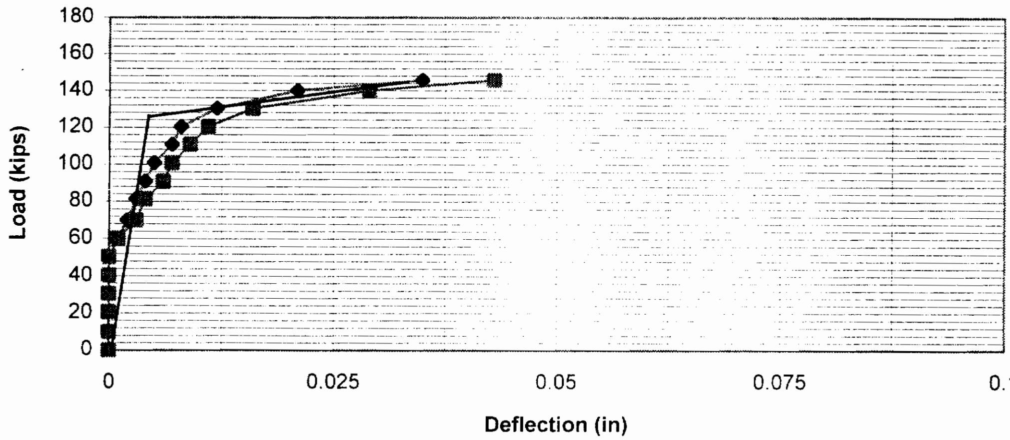
P_r..119.6 kips

P_{np}..160 kips

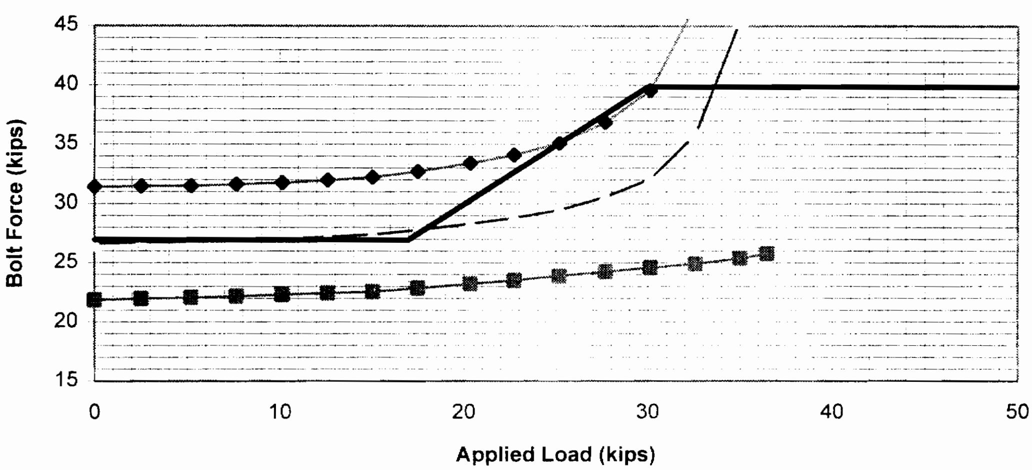
P_{pred}..119.6 kips

Notes:

Tee 12a



Tee 12a



CALCULATION OF PREDICTED VALUES

Test:..12a
Limit State:..Plate Yielding

$$p = 2.17d = 2.17(1.03) = 2.235 \text{ in.}$$

$$e = d - \frac{b^2}{4p} = 1.03 - \frac{(1.5)^2}{4(2.235)} = 0.778 \text{ in.}$$

$$b \le 3.53e = 2.63 - \text{ok!}$$

$$a = \min\begin{vmatrix} l/2 - b \\ \sqrt{4pf} \end{vmatrix} = \begin{vmatrix} 3.75" \\ 4.514" \end{vmatrix} = 3.75 \text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{(3.75)^2}{4(2.235)} = 1.57" \le f = 2.28 \text{ in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{1.5}{0.778}\right) = 62.58^o$$

$$g \ge d \Rightarrow 1.57 \ge 1.03$$

$$\theta_2 = \tan^{-1}\left(\frac{g-d}{a}\right) + 90^o = \tan^{-1}\left(\frac{1.57 - 1.03}{3.75}\right) + 90^o = 98.19^o$$

$$\alpha = \theta_1 + \theta_2 = 62.58^o + 98.19^o = 160.7^o$$

$$m_p = \frac{t_p^2}{4}\sqrt{(F_y)^2 - 3\left(\frac{V}{wt_p}\right)^2} = \frac{(0.785)^2}{4}\sqrt{(42.2)^2 - 3\left(\frac{32}{2.28(0.785)}\right)^2} = 4.41 \text{kip} - \text{in}/\text{in}$$

$$P_a = 4\pi\left(\frac{\alpha}{360}\right)m_p = 4\pi\left(\frac{160.7^o}{360^o}\right)m_p = 5.61m_p$$

$$P_c = b\left(\frac{1}{e}\right)m_p = 1.5\left(\frac{1}{0.778}\right)m_p = 1.93m_p$$

$$P_y = 4(P_a + P_c) = 4(5.61m_p + 1.93m_p) = (4)7.54m_p = 4(7.54)(4.41) = 133 \text{ kips}$$

CALCULATION OF PREDICTED VALUES

Test:..12a
Limit State:..Bolt Rupture with Prying Action

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 = 0.02317(42.2)\left(\frac{0.785}{0.75}\right)^3 = 1.121 \text{ in.}$$

$$w' = \frac{b_f}{2} - d_{bh} = 3 - 0.8125 = 2.1875 \text{ in.}$$

$$F' = \frac{t_p^2 F_{py}\left(0.85b_f/2 + 0.80w'\right) + \pi d_b^3 F_{yb}/8}{4p_f}$$

$$F' = \frac{(0.785)^2(42.2)\left[0.85(6)/2 + 0.80(2.1875)\right] + \pi(0.75)^3(90)/8}{4(1.25)} = 25.34 \text{ kips}$$

$$Q_{max} = \frac{w't_p^2}{4a}\sqrt{F_{py}^2 - 3\left(\frac{F'}{w't_p}\right)^2} = \frac{2.1875(0.785)^2}{4(1.121)}\sqrt{(42.2)^2 - 3\left(\frac{25.34}{2.1875(0.785)}\right)^2}$$

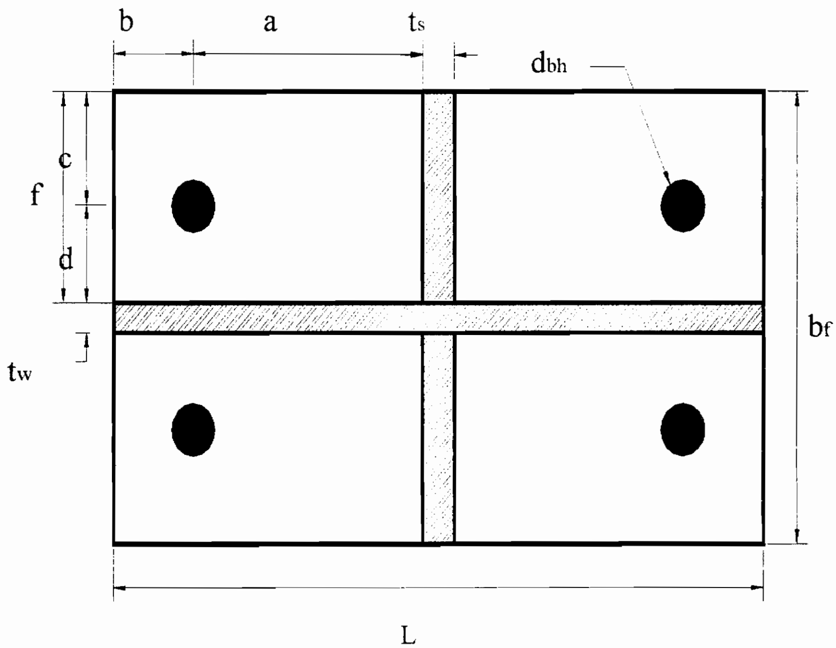
$$Q_{max} = 10.09\text{kips}$$

$$P_r = \max\begin{vmatrix}4(P_t - Q_{max})\\ 4T_b\end{vmatrix} = \begin{vmatrix}4(40 - 10.09)\\ 4(28)\end{vmatrix} = 119.6 \text{ kips}$$

Test:..12a
Limit State:..Bolt Rupture with no Prying Action

$$P_{np} = \max\begin{vmatrix}4Pt\\ 4T_b\end{vmatrix} = \begin{vmatrix}4(40)\\ 4(28)\end{vmatrix} = 160 \text{ kips}$$

STIFFENED TEE HANGER TEST



Test Designation....14a	
F_y.............42.2 ksi	t_p....................0.785 in.
t_s...............0.75 in.	t_w....................0.25 in.
t_{weld}...........0.325 in.	a.....................1.25 in.
b...............1.5 in.	c.....................1.25 in.
d...............1.0 in.	f......................2.25 in.
L...............6.5in.	b_f....................6.0 in.
d_b.............0.75 in.	d_{bh}..................0.8125 in.

V...40.5 kips

Average Experimental Yield Load ....162 kips

Average Experimental Ultimate Applied Bolt Force.........................N/A

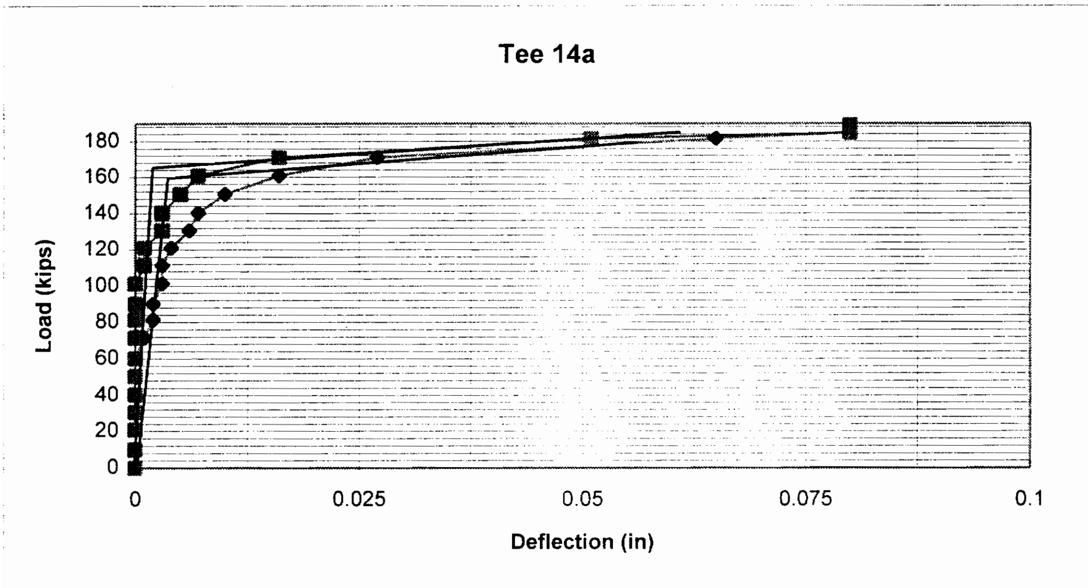
P_y...123 kips

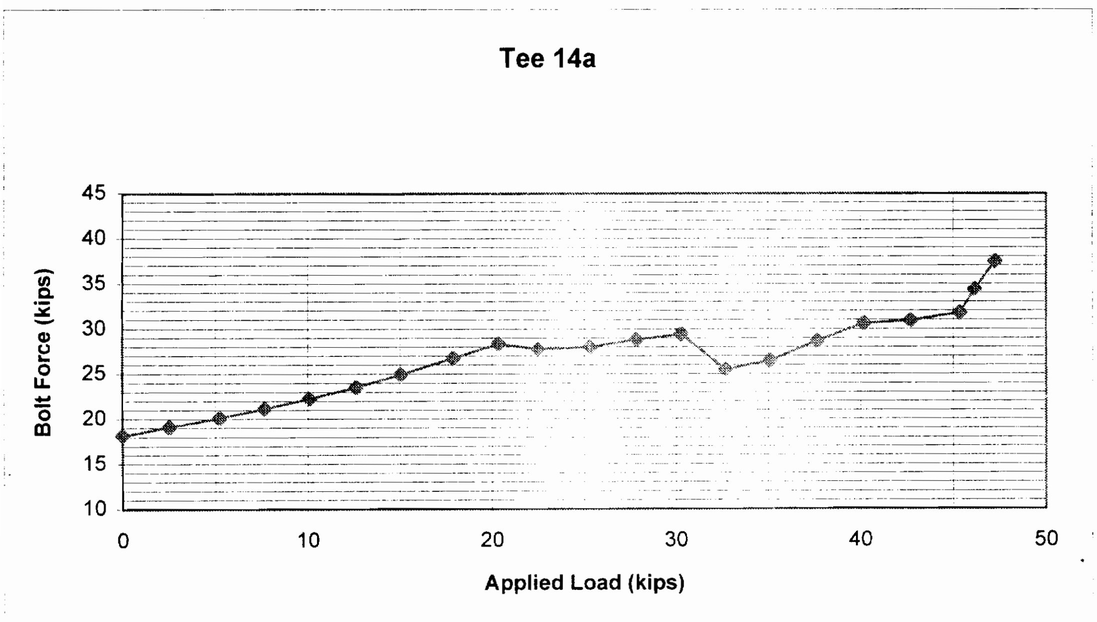
P_r...127 kips

P_{np}...160 kips

P_{pred}...123 kips

Notes: Both strain gaged bolts gave faulty readings.


Tee 14a
180
160
140
120
100
80
60
40
20
0
Load (kips)
0
0.025
0.05
0.075
0.1
Deflection (in)


Tee 14a
45
40
35
30
25
20
15
10
Bolt Force (kips)
0
10
20
30
40
50
Applied Load (kips)

CALCULATION OF PREDICTED VALUES

Test:..14a
Limit State:..Bolt Rupture with Prying Action

$p = 2.17d = 2.17(1.0) = 2.17 \text{ in.}$

$e = d - \dfrac{b^2}{4p} = 1.0 - \dfrac{(1.5)^2}{4(2.17)} = 0.741 \text{ in.}$

$b < 3.53e = 2.61 - \text{ok!}$

$a = \min\left|\begin{matrix}\text{measured} \\ \sqrt{4pf}\end{matrix}\right. = \left|\begin{matrix}1.25" \\ 4.41"\end{matrix}\right. = 1.25 \text{ in.}$

$g = \dfrac{a^2}{4p} = \dfrac{(1.25)^2}{4(2.17)} = 0.18" \le f = 2.25 \text{in.}$

$\theta_1 = \tan^{-1}\left(\dfrac{b}{e}\right) = \tan^{-1}\left(\dfrac{1.5}{0.741}\right) = 63.7^o$

$g \le d \Rightarrow 0.18 \le 1.0$

$\theta_2 = \tan^{-1}\left(\dfrac{a}{d-g}\right) = \tan^{-1}\left(\dfrac{1.25}{1.0-0.18}\right) = 56.73^o$

$\alpha = \theta_1 + \theta_2 = 63.7^o + 56.73^o = 120^o$

$m_p = \dfrac{t_p^2}{4}\sqrt{(F_y)^2 - 3\left(\dfrac{V}{wt_p}\right)^2} = \dfrac{(0.785)^2}{4}\sqrt{(42.2)^2 - 3\left(\dfrac{40.5}{2.25(0.785)}\right)^2} = 3.46 \text{kip} - \text{in/in}$

$P_a = 4\pi\left(\dfrac{\alpha}{360}\right)m_p = 4\pi\left(\dfrac{120^o}{360^o}\right)m_p = 4.2m_p$

$P_b = (c + (f-g))\left(\dfrac{1}{a}\right)m_p = (1.25 + (2.25 - 0.18))\left(\dfrac{1}{1.25}\right)m_p = 2.656m_p$

$P_c = b\left(\dfrac{1}{e}\right)m_p = 1.5\left(\dfrac{1}{0.741}\right)m_p = 2.02m_p$

$P_y = 4(P_a + P_b + P_c) = 4(4.2m_p + 2.656m_p + 2.02m_p) = (4)8.88m_p = 4(8.88)(3.46) = 123 \text{ kips}$

CALCULATION OF PREDICTED VALUES

Test:..14a
Limit State:..Bolt Rupture with Prying Action

$$t_{mod} = \sqrt{\frac{P_s}{F_y\left(P_{us}/m_p\right)}} = \sqrt{\frac{230}{42.2(6.68)}} = 0.90 \text{ in.}$$

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 = 0.02317(42.2)\left(\frac{0.90}{0.75}\right)^3 = 1.69\text{in.}"$$

$$w' = \frac{b_f}{2} - d_{bh} = 3 - 0.8125 = 2.1875 \text{ in.}$$

$$F' = \frac{t_p^2F_{py}\left(0.85b_f/2 + 0.80w'\right) + \pi d_b^3F_{yb}/8}{4p_f}$$

$$F' = \frac{(0.9)^2(42.2)\left[0.85(6)/2 + 0.80(2.1875)\right] + \pi(0.75)^3(90)/8}{4(1.25)} = 32.3 \text{ kips}$$

$$Q_{max} = \frac{w't_p^2}{4a}\sqrt{F_{py}^2 - 3\left(\frac{F'}{w't_p}\right)^2} = \frac{2.1875(0.9)^2}{4(1.69)}\sqrt{(42.2)^2 - 3\left(\frac{32.3}{2.1875(0.9)}\right)^2}$$

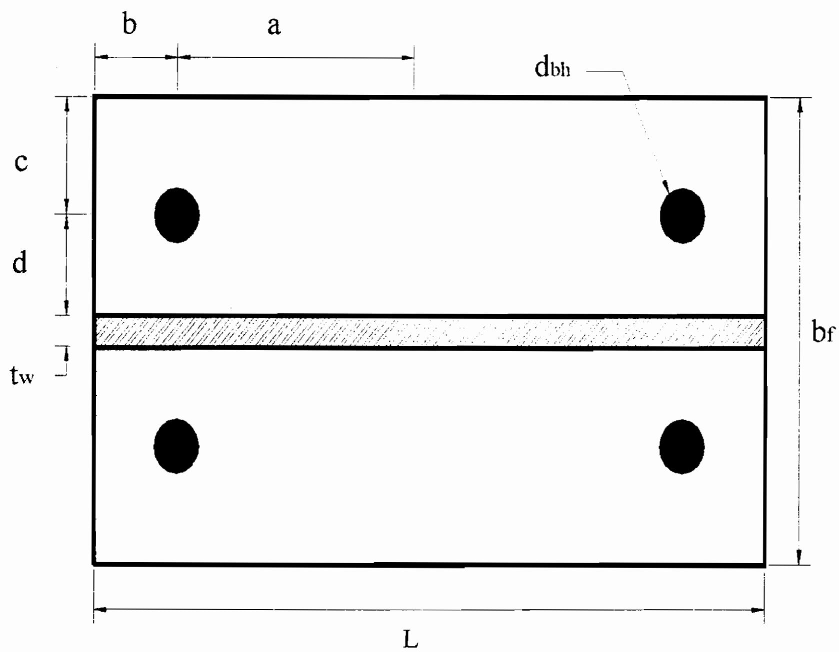
$$Q_{max} = 8.16 \text{ kips}$$

$$P_r = \max\left|\begin{matrix}4(P_t - Q_{max})\\ 4T_b\end{matrix}\right. = \left|\begin{matrix}4(40 - 8.16)\\ 4(28)\end{matrix}\right. = 127 \text{ kips}$$

Test:..14a
Limit State:..Bolt Rupture with no Prying Action

$$P_{np} = \max\left|\begin{matrix}4Pt\\ 4T_b\end{matrix}\right. = \left|\begin{matrix}4(40)\\ 4(28)\end{matrix}\right. = 160 \text{ kips}$$

UNSTIFFENED TEE HANGER TEST



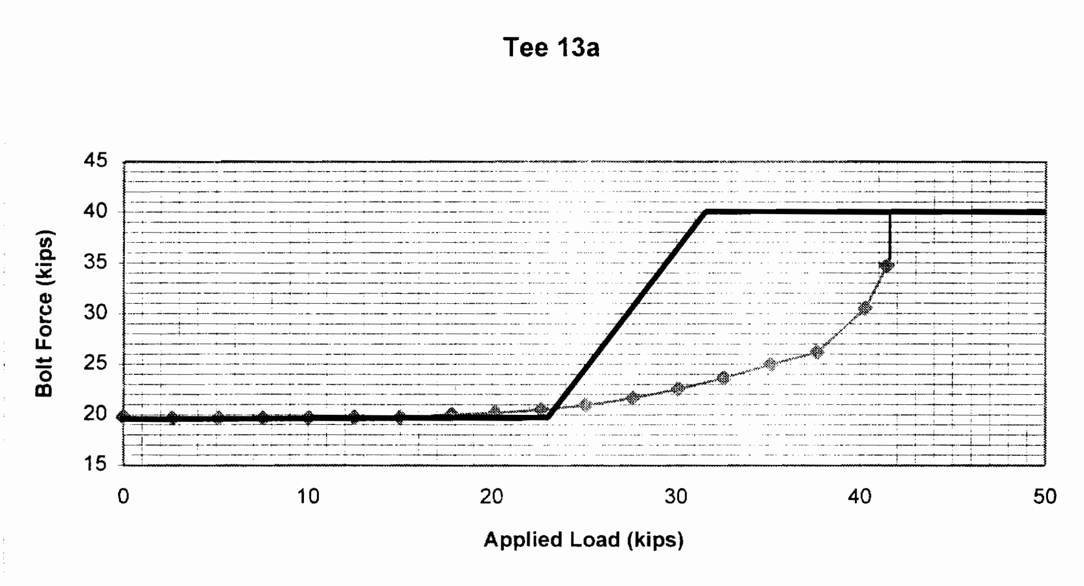
Test Designation 13a

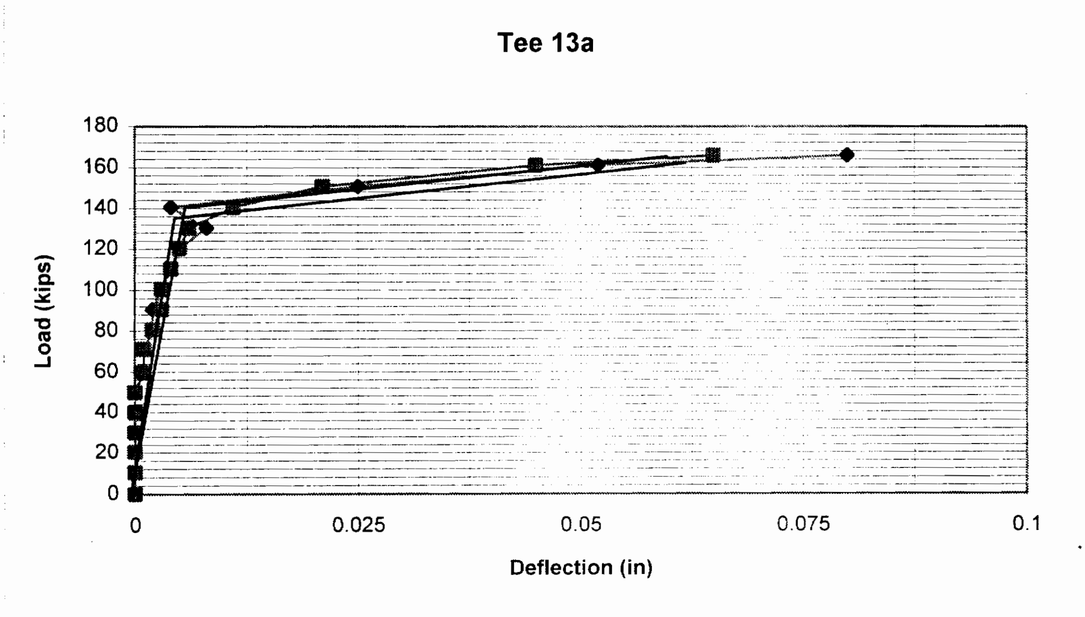
F_y .... 43.85 ksi
t_w .... 0.5 in.
a .... 1.75 in.
c .... 1.25 in.
f .... 2.5 in.
bf .... 6.0 in.
d_{bh} .... 0.8125 in.

t_p .... 0.785 in.
t_{weld} .... 0.25 in.
b .... 1.5 in.
d .... 1.25 in.
L .... 6.5 in.
d_b .... 0.75 in.

V .... 35 kips
Average Experimental Yield Load .... 140 kips
Average Experimental Ultimate Applied Bolt Load .... 42 kips
P_y .... 153 kips
P_r .... 127 kips
P_{np} .... 160 kips
P_{pred} .... 127 kips

Notes: One bolt gave faulty readings and could not be used in bolt force calculations.


Tee 13a
Bolt Force (kips)
45
40
35
30
25
20
15
0
10
20
30
40
50
Applied Load (kips)


Tee 13a
Load (kips)
180
160
140
120
100
80
60
40
20
0
0
0.025
0.05
0.075
0.1
Deflection (in)

CALCULATION OF PREDICTED VALUES

Test:..13a
Limit State:...Bolt Rupture with Prying Action

$$p = 2.17d = 2.17(1.25) = 2.7125 \text{ in.}$$

$$e = d - \frac{b^2}{4p} = 1.25 - \frac{(1.5)^2}{4(2.7125)} = 1.04 \text{ in.}$$

$$b < 3.53e = 3.67 - \text{ok!}$$

$$a = \min \begin{vmatrix} l/2 - b \\ \sqrt{4pf} \end{vmatrix} = \begin{vmatrix} 1.75'' \\ 5.21'' \end{vmatrix} = 1.75 \text{ in.}$$

$$g = \frac{a^2}{4p} = \frac{(1.75)^2}{4(2.7125)} = 0.2822'' \le f = 2.5 \text{ in.}$$

$$\theta_1 = \tan^{-1}\left(\frac{b}{e}\right) = \tan^{-1}\left(\frac{1.5}{1.04}\right) = 55.26^{\circ}$$

$$g \le d \Rightarrow 0.2822 \le 1.25$$

$$\theta_2 = \tan^{-1}\left(\frac{a}{d-g}\right) = \tan^{-1}\left(\frac{1.75}{1.25 - 0.288}\right) = 61.05^{\circ}$$

$$\alpha = \theta_1 + \theta_2 = 55.26^{\circ} + 61.05^{\circ} = 116.31^{\circ}$$

$$m_p = \frac{t_p^2}{4}\sqrt{(F_y)^2 - 3\left(\frac{V}{wt_p}\right)^2} = \frac{(0.899)^2}{4}\sqrt{(43.85)^2 - 3\left(\frac{35}{2.5(0.899)}\right)^2} = 6.98 \text{kip} - \text{in}/\text{in}$$

$$P_a = 4\pi\left(\frac{\alpha}{360}\right)m_p = 4\pi\left(\frac{116.31^{\circ}}{360^{\circ}}\right)m_p = 4.06m_p$$

$$P_c = b\left(\frac{1}{e}\right)m_p = 1.5\left(\frac{1}{1.04}\right)m_p = 1.44m_p$$

$$P_y = 4(P_a + P_c) = 4(4.06m_p + 1.44m_p) = (4)5.5m_p = 4(5.5)(6.98) = 153 \text{ kips}$$

CALCULATION OF PREDICTED VALUES

Test:..13a
Limit State:...Bolt Rupture with Prying Action

$$a = 0.02317F_y\left(\frac{t_p}{d_b}\right)^3 = 0.02317(43.85)\left(\frac{0.899}{0.75}\right)^3 = 1.75\text{ in.}$$

$$w' = \frac{b_f}{2} - d_{bh} = 3 - 0.8125 = 2.1875\text{ in.}$$

$$F' = \frac{t_p^2F_{py}\left(0.85b_f/2 + 0.80w'\right) + \pi d_b^3F_{yb}/8}{4p_f}$$

$$F' = \frac{(0.899)^2(43.85)\left[0.85(6)/2 + 0.80(2.1875)\right] + \pi(0.75)^3(90)/8}{4(1.25)} = 33.4\text{ in.}$$

$$Q_{max} = \frac{w't_p^2}{4a}\sqrt{F_{py}^2 - 3\left(\frac{F'}{w't_p}\right)^2} = \frac{2.1875(0.899)^2}{4(1.75)}\sqrt{(43.85)^2 - 3\left(\frac{33.4}{2.1875(0.899)}\right)^2}$$

$$Q_{max} = 8.2\text{ kips}$$

$$P_r = \max\left|\begin{matrix}4(P_t - Q_{max})\\4T_b\end{matrix}\right. = \left|\begin{matrix}4(40-8.2)\\4(28)\end{matrix}\right. = 127\text{ kips}$$

Test:..13a
Limit State:...Bolt Rupture with no Prying Action

$$P_{np} = \max\left|\begin{matrix}4Pt\\4T_b\end{matrix}\right. = \left|\begin{matrix}4(40)\\4(28)\end{matrix}\right. = 160\text{ kips}$$

NOMENCLATURE

α- Sum of angles subtended by the parabolic yield line

θ_1-Angle subtended by the parabolic yield line, from the center of the bolt hole to the free edge

θ_2- Angle subtended by the parabolic yield line, from the center of the bolt hole to either the stiffener, tee center line, or intersection of free edge of flange towards center of tee

ϕ_r-Resistance factor for plate yielding

ϕ_y-Resistance factor for bolt rupture

a-Distance from bolt center line to application of prying force

a-Distance from center of bolt hole to the minimum of: the measure distance to the center of the tee, the measured distance to the edge of the stiffener fillet, the calculated distance of the intersection of the parabolic yield line with adjacent yield line, or free edge of flange

A_b-Crosectional area of bolt

B-Bolt force

b-Distance from center of bolt hole to free edge of tee, parallel to web

b_f-plate flange width

c-Distance from center of bolt hole to free edge of tee, perpendicular to web

d-distance from center of bolt hole to fillet, or weld, of web

d_b-Bolt diameter

d_{bh}-Bolt hole diameter

e-Calculated distance from center line of bolt hole to intersection of parabolic yield line on free edge of tee

F-1/2 total applied flange force

f-Sum of c and d

F_{py}-Tensile yield strength of steel plate

F_y- Tensile yield strength of steel plate

F_{yb}-Nominal strength of bolts

g- Calculated distance from fillet or weld of tee web to intersection of parabolic yield line to: stiffener weld, or intersection with adjacent parabolic yield line

L-Length of tee

L_n-Length of yield line n

m_p-moment capacity of steel per linear inch

m_{px}-moment capacity of steel per linear inch in x- direction

m_{py}-moment capacity of steel per linear inch in y- direction

p-Length of line from focal point of parabola, to vertex of parabola

P_b-Bolt load

p_f-Pitch distance from flange face to center of bolt line

P_n-Design strength

P_{np}-Maximum applied load which causes no prying action in connection

P_{pred}-Predicted failure load

P_r-Bolt rupture load

P_s-Yield load for stiffened tee hanger

P_t-Proof load of bolts

P_u-Ultimate applied tensile load

P_{us}-Yield load for unstiffened tee hanger

P_y-Experimental yield load

Q-Magnitude of prying force

Q_{max}-Maximum prying force that can be achieved

t_I-Kennedy thick plate limit

t_{II}-Kennedy thin plate limit

t_{mod}- Modifeid plate thickness, when stiffener is added

t_p-Plate thickness

t_s-Thickness of stiffener

t_w-Thickness of the web

t_{weld}-Thickness of weld of fillet

V-Shear force per bolt

w'-width of plate per bolt at bolt line minus the hole diameter

w-width of flange per bolt

W_e-External work done on the connection

W_i-Internal energy stored by a yield line mechanism

VITA

Michael Armando Otegui was born in Miami, Florida on August 1, 1969. He was raised in Fairfax, Virginia where he graduated from James W. Robinson High School. In May of 1992 he received his Bachelor of Science degree in Civil Engineering from Virginia Polytechnic and State University. He entered the structures graduate program at Virginia Polytechnic Institute and State University in the Fall of 1994 to pursue a Master of Science degree in Civil Engineering.