Sampling Spatial Sediment Variation in Gravel-bed Streams

by

David W. Crowder

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APPROVED:

P. Diplas, Chairman

J. B. Campbell

W. E. Cox

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P. Diplas, Chairman

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(ABSTRACT)

A gravel-bed stream's grain size distribution plays an important role in determining a river's depth, sediment transport rates, and stream bed stability as well as the survival rates of mussels and salmonids. Unfortunately, the material found in gravel-bed rivers exhibits vertical stratification as well as spatial variation in the horizontal direction and is difficult to sample. Previous research has largely dealt with the ability of grid, areal, and bulk sampling techhniques to sample a single spot within a river. Little has been done in characterizing an entire river reach. Of the methods suggested, none is adequate because they are either inherently biased or are incapable of describing the spatial variations within a sampled region. The present research proposes a method that overcomes these problems. It shows that a single large grid sample, or composite grid sample, can be used to obtain an unbiased estimate of an area's overall grain size distribution at a known accuracy level. It then suggests that the arithmetic mean is a suitable parameter to characterize the coarseness of individual sediment deposits within a sampled area. Thus, by recording the size and location of each stone taken in the

composite grid sample one can use statistical hypothesis testing to systematically analyze local means throughout the sampled area and locate sediment boundaries. Once the boundaries are located, stones from the composite grid sample falling within the boundaries of a particular deposit can be analyzed as separate grid samples representative of the individual deposits present and describe the local variability.

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D_{xx} = the grain size in which xx percent of the population's material is finer than.

D_{50} = the median grain size.

$\dot{D}_{max}$ = the maximum grain size present.

E = the percent error.

p = the proportion passing a given grains size.

n = the number of stones in a grid sample.

p(V-W) = the volumetric percent retained by weight on sieve size i obtained in a volumetric sample.

C = a constant.

$p(S)_i$ = the percent retained by weight on sieve size i obtained by a surface sample.

D_i = the geometric mean grain size for grain sizes between sieves i and i+1.

x = an exponent used to convert an areal sample into a volumetric equivalent.

V_i = the volume of material contained in a population distribution.

p_i = the percent by volume a particle of size D_i contains in a given population.

p_{iT} = the total percent by volume a particle of size D_i contains in an area having multiple populations.

z = the test statistic coming from the standard normal distribution.

$\bar{x}$ = the sample mean.

s = the sample standard deviation.

α = the significance level.

μ = the population mean.

θ = the difference in means of two populations.

$OC(\theta)$ = the probability of accepting the null hypothesis for a given (θ).

Φ = the cumulative percentage under the normal distribution curve.

D_g = the geometric mean grain size

σ_g = the geometric standard deviation

f_i = the probability of occurence of D_i

SAMPLING SPATIAL SEDIMENT VARIATION IN GRAVEL-BED STREAMS

CHAPTER 1: INTRODUCTION

1.1 Overview

A gravel-bed stream's grain size distribution plays a key role in the hydraulic and ecological characteristics displayed by a river. Hydraulically, the size distribution helps determine a stream's flood levels, sediment transport rates, and stream bed stability. Ecologically, the amount of fines present in a river bed influences the survival rates of mussels and fish species, some of which are endangered. Unfortunately, obtaining a representative grain size distribution for a gravel-bed stream or river is difficult due to a vertical stratification of the bed material as well as sediment variations in the horizontal direction. Past research in sediment sampling has mainly dealt with obtaining an unbiased, volummetric sample at a single spot within a river; however, little has been done to obtain a distribution which represents an entire river reach or depositional bar. The few methods that have been suggested, as will be shown later, are either biased, require extremely large amounts of material, or are incapable of sampling the armour layer of the stream bed. The next few chapters address this issue and propose a sampling method that is built on the principles of grid sampling, and is capable of obtaining an overall grain size distribution for an area. It also puts forth a statistically based method to locate and describe the spatial variation occurring within the sampled region. Chapter one describes the need for such work, the nature of the problem, and the scope of the work done here. Chapter two describes some common sediment sampling techniques which form an important base for the new procedure along with their limitations, and applicability to sampling river reaches. Chapter three then looks at the mechanics of combining samples coming from different sediment deposits such that the distribution resulting from physically combining the individual samples is an unbiased estimate of the areas overall grain size distribution. After this, chapters four and five, through the use of computer simulations, develop and test a technique designed to obtain a grain size curve for a region and describe any spatial variations within it. Chapters six and seven then turn to testing a previously suggested sampling method, along with the one proposed here, on materials having known distributions. Chapter 8 concludes with a summary of the results and discusses how the results may be used in the field.

1.2 The Needs for Sediment Sampling

Many of the problems that are associated with rivers and streams such as flooding, erosion, and meandering, are related to the frictional characteristics of the materials found on the stream bed. A common procedure to quantify the grain roughness is in terms of a particular grain size such as D_{50}, or D_{90}, where D_{50} and D_{90} are grain sizes which are larger than 50 and 90 percent of the grain size distribution, respectively. However, in order to obtain these values one must sample the relevant area in an unbiased and accurate manner. Once this is done, values from the size distribution are used in equations to predict flow velocity and depth, stream bed stability, sediment transport rates, and flood levels. For example, in calculating flow velocity with the Manning equation, the roughness coefficient is related to D_{50} (Vanoni, 1977). Likewise, D_{50} is used in Shield's diagram to determine the stability of a river bed and when particle motion will begin (Vanoni, 1977). Similarly, common bed load transport equations such as the Einstein, Meyer-Peter & Muller, and DuBoy's equations use D_{65}, D_{50}, and D_{35} as grain size parameters (Vanoni, 1977). Since particle size parameters play a role in all of these equations, it is clear that the more accurately the parameter is determined the better the calculated value is. In turn, hydraulic structures, based upon the more accurate values become more efficient and reliable than those based on less accurate values.

In 1991 only 191 adult salmon returned to the Sacramento River while at one time over 100,000 spawned there each year (Ziemba, 1992). One reason for this decline in the salmon population is the introduction of fine particles into the stream by logging, construction, and agricultural practices (Diplas and Parker, 1992). With the introduction of the fine material to the stream, the geometric mean grain size drops, and as Figure 1.1 shows, so does the salmonid embryo survival rate (Shirazi and Seim, 1981). With a more accurate sampling technique a more precise correlation between embryo survival rates and the amount of fines present can be found . Thus it may be possible to create basin wide erosion management programs that would provide quality spawning grounds to help repopulate the species. Similarly, one may be able to monitor the habitats of certain endangered mussel species that are destroyed by excessive amounts of fine materials (Helfrich et. al, 1986).

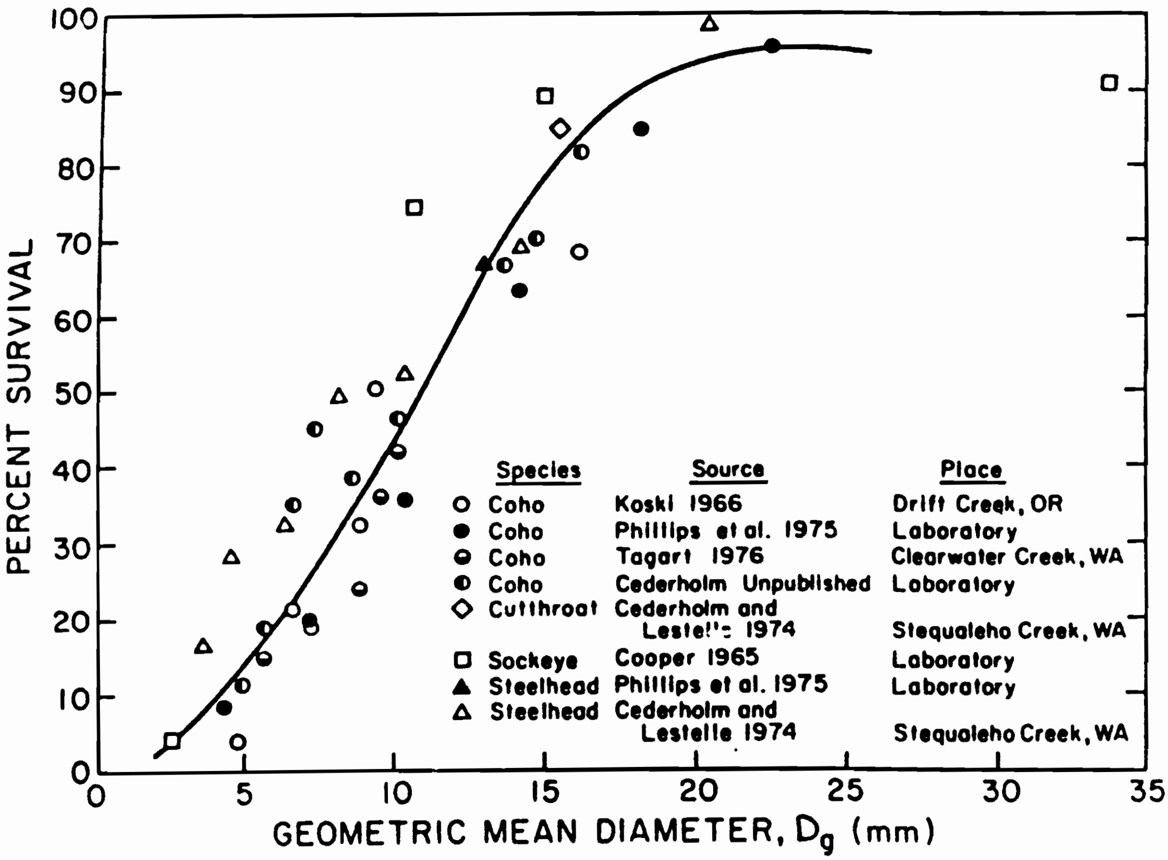


Figure 1.1: Relationship Between the Geometric Mean Grain Size and Embryo Survival Rates of Various Species of Fish. From Shirazi and Seim (1981)

1.3 Coarse-Grained Stream Beds

While more accurate sediment sampling methods would benefit hydraulic calculations and enable the monitoring of spawning grounds, the very nature of gravel-bed streams complicates sampling dramatically. The following few paragraphs describe the physical makeup of a gravel-bed stream. Once this is done it becomes possible to describe common sampling techniques and show their applicability to sampling a river reach or depositional bar.

The composition of gravel-bed sediments typically consists of gravels larger than 2 mm. A vertical cross section typically reveals three distinct sediment populations or strata. The top layer, or pavement, is typically the coarsest and impacts the stream's flow depth, velocity, bed load transport rate, and probably it's suspended transport rate. The second layer, or subpavement, contains substantially finer material and plays a crucial role in determining salmonid embryo survival rates (Diplas and Fripp, 1992). Both the pavement and subpavement, which represent different sediment populations, are only about as thick as the maximum grain size present (Kellerhals and Bray, 1971; Diplas and Sutherland, 1988). The third layer, however, has no predetermined thickness (Diplas and Fripp, 1992). Figure 1.2 shows a sketch of a typical vertical gravel-bed cross section. The next chapter will show that these three different layers play a crucial role in determining how a sediment sample must be gathered. However, in addition to having vertical stratification, coarse-bed rivers often exhibit a spatial variation in the horizontal direction. These changes can be sudden or gradual. Sudden changes can be seen in gravel bars where sediment sizes may change several times over the width of the river. Figure 1.3 demonstrates this for a depositional bar in the Quesnel River in Canada (Wolcott and Church, 1991). In contrast, a gradual decrease in grain size occurring over long distances, known as downstream fining, is also common for gravel rivers.

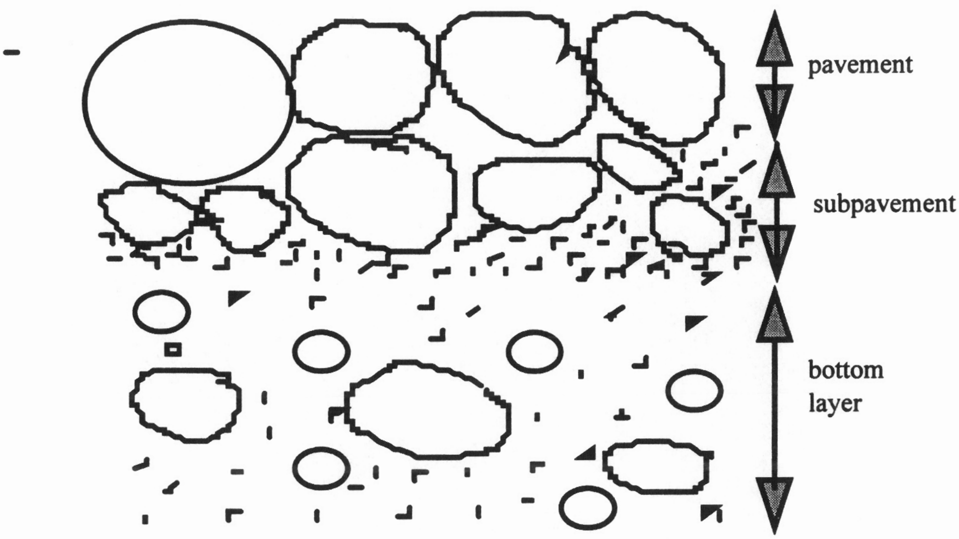


Figure 1.2: Typical Cross Section of Gravel-Bed Stream

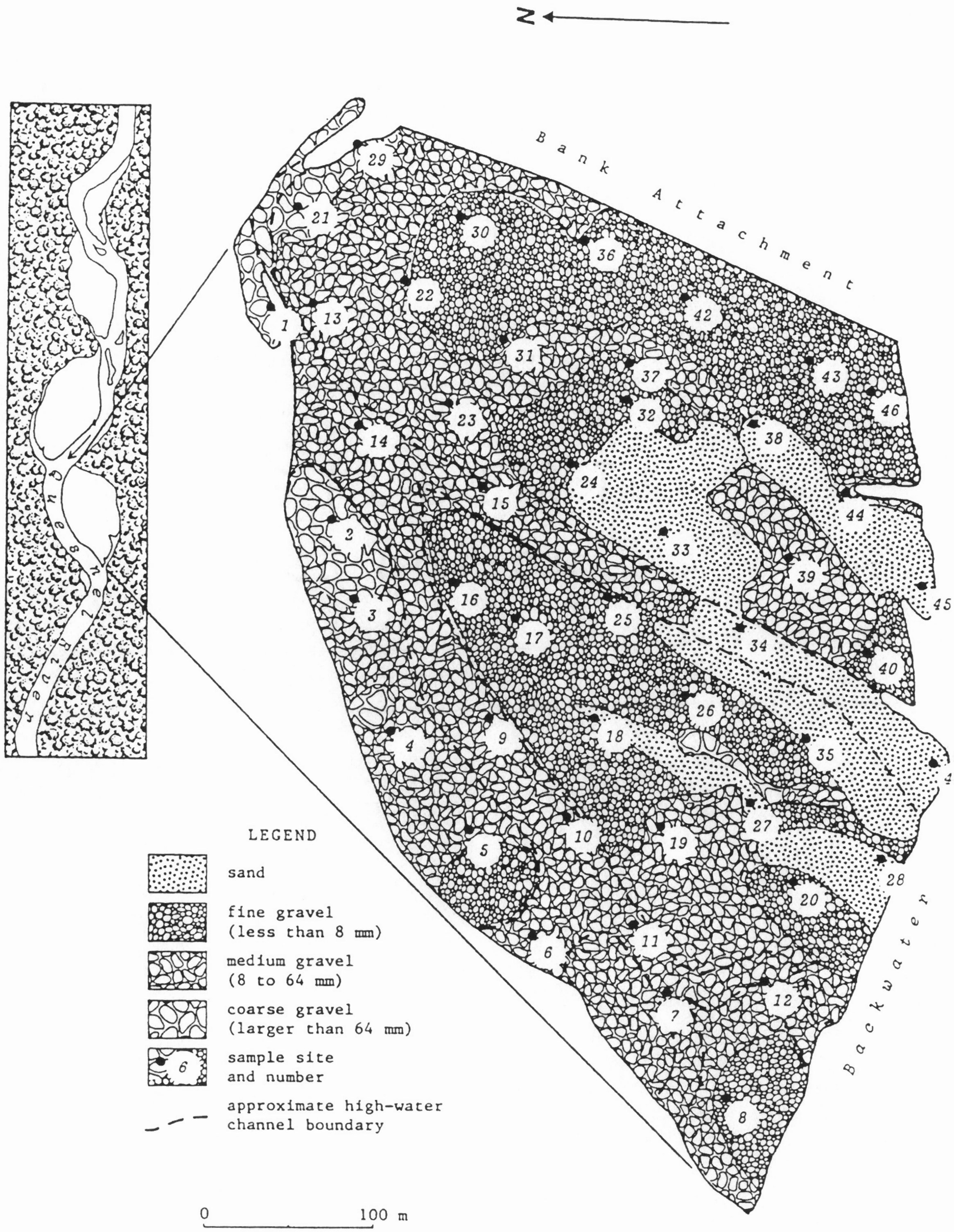


Figure 1.3: Depositional Bar on the Quesnel River. From Wolcott and Church (1991)

CHAPTER 2: SAMPLING BASICS

2.1 Sampling at a Point

The procedure used for sampling a grain size distribution within a gravel-bed stream depends largely on the type of material present and which grain size population is of interest. Bulk sampling, grid sampling, and areal sampling are the main methods of obtaining a sample. However, these techniques are not directly comparable. Each sample must be analyzed in accordance to the method by which it was obtained. Furthermore, each of these methods can provide biased results if used improperly. A brief review of these techniques, their limitations, and their accuracy levels, is provided in the next few paragraphs.

2.1.1 Volumetric or Bulk Sampling

Volumetric or bulk sampling is the most widely used method of sediment sampling. It consists of taking a predetermined volume of material such that its dimensions are independent of the grain sizes present (Kellerhals and Bray, 1971). Once this is done a sieve analysis is performed. The percent by weight retained on each sieve size is calculated and a plot of grain size versus percent passing that sieve size is made. Bulk sampling works well for sand streams where the pavement and subpavement layers do not exist. In this case, the sediment is simply shoveled into a bucket, dried and analyzed according to percent by weight. On gravel-bed streams, bulk sampling cannot be used to sample the pavement and subpavement because they are only one grain diameter thick (Kellerhals and Bray, 1971; Diplas and Sutherland, 1988). If a bulk sample were taken, it would contain grain sizes from all three populations and be representative of none of them. It should be noted that, while the bottom layer can be bulk sampled after removing the pavement and subpavement, the pavement and subpavement must be sampled by a different method. Despite not being able to sample the pavement or subpavement, volummetric samples, because of their unbiased nature, are the basis of all comparisons and as will be shown later, all samples regardless of sampling technique, must be the equivalent to a volummetric sample.

The accuracy of a bulk sample depends on the grain sizes of the material being sampled and the volume of the sample taken. As with all sampling techniques, the larger the sample, the more accurately the size parameters can be determined. Unfortunately, large samples are often physically and financially impractical. In an attempt to address this problem, considerable research has been done to determine what the smallest sample size for a given accuracy level is. In bulk sampling, the samples volume is determined by weight. Both de Vries (1970) and Church, et al. (1987) provide guidelines for determining minimum volumetric sample sizes at different accuracy levels. De Vries provides three accuracy levels and uses D_{84} to determine the sample size. Church, et al. provide similar guidelines, however, they recommend that the maximum grain size present should be used instead of D_{84}. Both methods are based on the results of a single gravel sample and are expected to apply to all gravel-bed materials. Furthermore, they require the operator to assume the D_{84} or D_{max} before the material is sampled. This is a problem because neither of them can be known in advance. Two more drawbacks are associated with these criteria. First, these accuracy estimates do not provide confidence limits for different individual size parameters such as D_{95}, and D_{35} which are measured at different accuracy levels for a given weight. Second, very large amounts of material are often needed. For example, a D_{84} of 35 mm requires over a 90 kg of material to satisfy de Vries' low precision criterion, while over 10,000 kg is necessary to meet the high precision requirements.

2.1.2 Grid Sampling

Grid sampling is an appropriate means of sampling the pavement, subpavement, and bottom layers of gravel-bed streams. Grid sampling is performed by laying out a grid over the area to be sampled and collecting the stones immediately beneath each grid point (Kellerhals and Bray, 1970). For small areas a wire mesh can be used, while for larger areas a pacing procedure can be used. The pacing procedure, known as Wolman's walk is an important basis for the sampling procedure that developed later and should be described (Wolman, 1954). In Wolman's walk, the operator steps off regular intervals such that the distance of the intervals is larger than the largest grain size present. After pacing off an interval the operator, with his eyes averted reaches down and picks up the stone immediately beneath his big toe. This helps reduce any operator bias in selecting grain sizes. The size of the particle is then measured and recorded. A template, known as a gravelometer, having square holes consistent with sieve sizes can be carried with the operator to simplify measuring the stone sizes (Hey and Thorne, 1983). The stone's size is measured as the smallest sieve size opening that the particle just passes. A template works well for particles having a diameter of up to about 215 mm or, 14 kg, which is about as large a stone as a person can handle with one hand (Fripp and Diplas 1993). Stones larger than 215 mm may have to be measured with a tape. Fripp and Diplas (1993) recommend bringing a tape recorder along so that the operator can discard a stone after its size and location have been recorded, making it possible for a single person to perform the sampling procedure. Grid sampling is probably the easiest sampling method to perform in the field and can be used on dry ground as well as underwater. Furthermore, grid sampling, unlike bulk and areal sampling, does not require the material to be weighed, or taken to a lab for analysis. Even if the material were collected, Crowder and Diplas (1994) have shown that a grid sample with the same accuracy level as de Vries' (1970) low accuracy criteria requires about a third of the material a bulk sample needs. Since a grid sample is equivalent to a volumetric sample (Kellerhals and Bray, 1971), the grain size distribution is found by plotting the percentage of stones that pass each sieve size. However, a major limitation of grid sampling is that one can not sample material smaller than about 15 mm in an unbiased manner (Fripp and Diplas, 1993) and thus, has a tendency to truncate the smaller particles from the grain size distribution.

Like bulk samples, the accuracy of grid samples depends on the size of the sample taken. Hey and Thorne (1983) suggested that if a standard deviation of the grain sizes is assumed for the sediment to be sampled and a confidence interval was specified, then the number of stones to obtain the given confidence limit could be calculated based on a log normal distribution. Unfortunately, like the D_{84} or D_{max} in bulk sampling, the standard deviation is not known in advance and must be assumed in order to calculate the number of stones necessary to obtain a given accuracy level. Furthermore, Church, et al. (1987), amoung others, point out that the grain sizes found in gravel-bed streams rarely follow a log normal distribution. Thus, statistical results based on a log-normal distribution such as Hey and Thorne's (1983) may become biased if the sediment is not log normal.

Fripp and Diplas (1993) proposed that the accuracy levels of a grid sample can be determined through the use of the binomial distribution. The test used is whether or not a stone passes through a given sieve size. It does not assume an underlying particle size distribution and provides a means of determining confidence limits for any given grain size parameter of interest. The confidence limits are given in terms of percentage of the distribution and calculated by the following equation,

$$E = \sqrt{\frac{z^2_{(a/2)}\, p(1-p)}{n}} \qquad (1)$$

where E is the error, $z_{(a/2)}$ is the z value obtained from the normal distribution for a $100\times(1-\alpha)$ confidence limit, p is the percentile of interest and n is the number of stones in the sample. However, this equation is valid only when $n \times p$ and $n\times(1-p)$ are greater than 20. Otherwise, the binomial criteria can no longer be approximated with the normal distribution (Ott 1988).

Since the binomial method is used extensively throughout this paper, a description of its use is given below. A common problem is to estimate the error of a specific grain size parameter for a given sample size. For example, given a 200 stones sample (n = 200) and that the particle size of interest is D_{50} (p=.50) one can calculate a 95% confidence interval ($Z_{(a/2)}$= 1.96) using equation (1), which yields an error of 6.9%. Thus, we conclude the true percentage of material finer than the grain size corresponding to the sample D_{50} is between 43.1% and 56.9%. This, however, does not indicate the expected grain size error (in terms of mm) of D_{50}. In order to approximate the grain size error one must plot the confidence limits for the entire distribution and determine the minimum and maximum values for the grain size of interest. An example of this is shown in Figure 2.1 which depicts a sample of 200 stones and has a D_{50} equal to 14.2 mm. By following the

50% passing level across the plot, one finds that it intersects the 95% confidence limits at 11.59 mm and 16.59 mm which represent the 95% confidence limits in terms of grain size. The distribution in Figure 2.1 is truncated at the ends for material larger than D_{90} and below D_{10}. The reason for this is that confidence limits can not be predicted beyond this as n×p and n×(1-p) are less than 20 for these areas. If the sample size were increased to 400 stones, as in Figure 2.2, confidence limits can be calculated for values between D_{95} and D_5 and the confidence limits move toward the sample distribution reflecting the increased accuracy. Similarly, a 100 stone sample's confidence interval would be wider, and be valid only between D_{80} and D_{20}. Another important aspect of the confidence intervals is that paired parameters such as D_{90} and D_{10}, and D_{75} and D_{25} have the same percentile error but do not necessarily have the same grain size error. Figure 2.1 demonstrates this for the values of D_{85} and D_{15}. The D_{85} confidence limits are 24.09 mm and 31.97 mm a difference of 7.88 mm while D_{15} has confidence limits of .60 mm and 4.02 mm a difference of 3.42 mm. These differences will depend on the shape of the distribution and it's standard deviation.

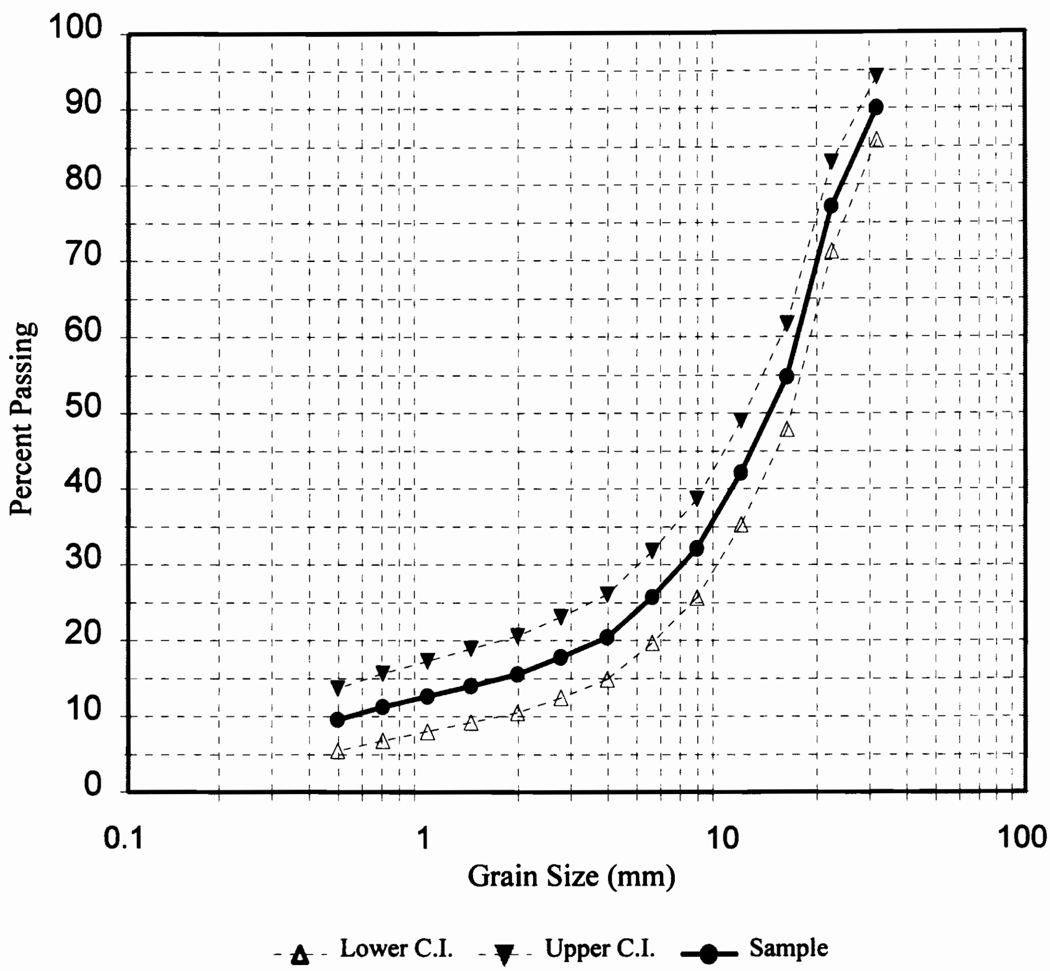


Figure 2.1: Sample of 200 Stones with 95% Confidence Limits

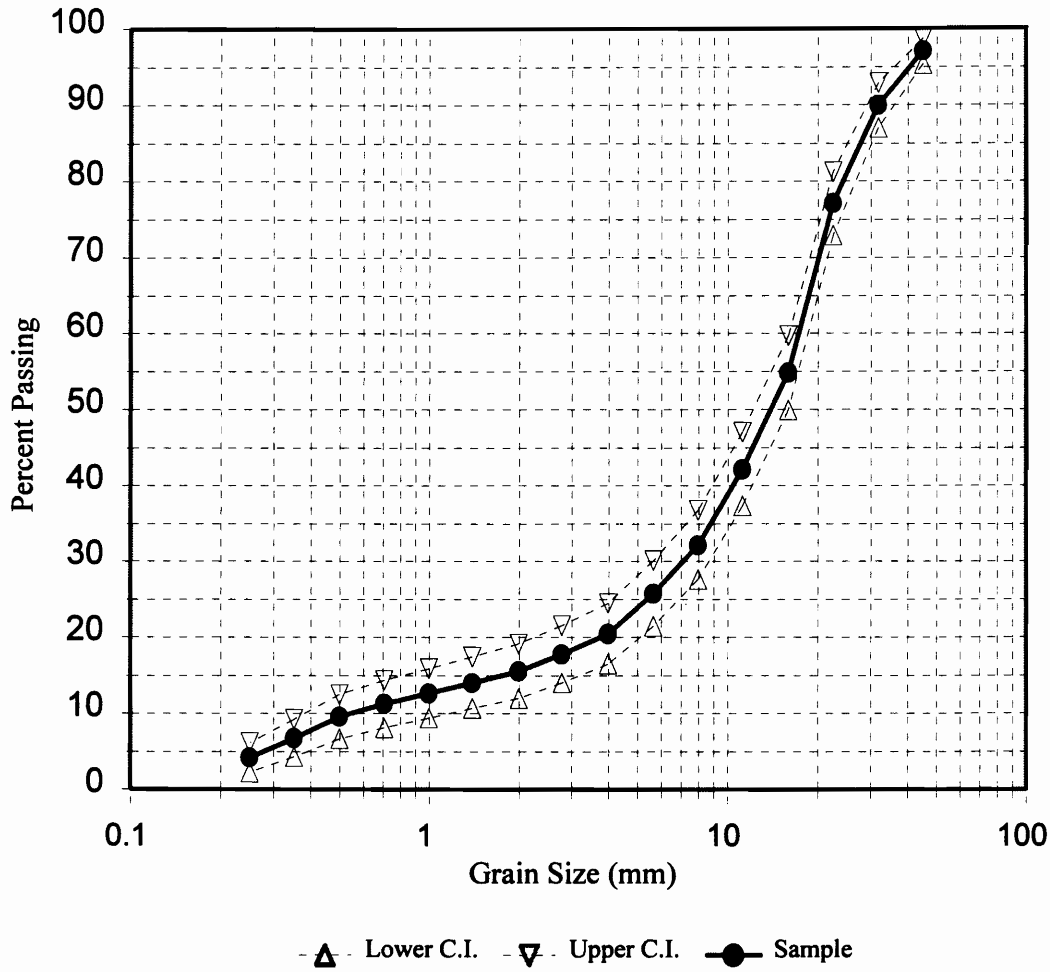


Figure 2.2: Sample of 400 Stones with 95% Confidence Limits

2.1.3 Areal Sampling

Areal sampling, like grid sampling, can be used to sample the pavement, subpavement, and bottom layers of a gravel-bed stream. It also provides a means of collecting material that is smaller than 15 mm, which grid sampling can not obtain in an unbiased manner. The sample is collected by removing all of the particles that are on the surface of a specified area. Some common methods of doing this use clay, wax, or paint to collect the sample (Diplas and Sutherland, 1988; Kellerhals and Bray, 1971). Unfortunately, techniques using wax and paint can not be used underwater and are of limited value in the field. However, clay sampling can be performed underwater as well as on dry surfaces (Diplas and Fripp, 1992). Fripp and Diplas (1993) suggest using a piston-like device containing a round flat base surrounded by a thin plastic shield to sample materials that are finer than about 40 mm. Common moist pottery clay is placed at the bottom of the device and placed inside the shield to protect the sample from the current. The piston head containing the clay is then pushed against the gravel-bed and obtains the sample. The sample is "placed into a sieve with openings smaller than the smallest particle of interest and wet sieved to remove the clay" (Fripp and Diplas, 1993).

This sample, like all areal samples, however, is not equivalent to a volumetric sample. It, therefore, must be converted to a volumetric sample. Substantial research has addressed the question of how this should be done (e.g. Kellerhals and Bray, 1971; Diplas and Sutherland, 1988; Diplas and Fripp, 1992). Kellerhals and Bray (1971) proposed an equation which was adopted with modifications in the exponent x. The equation follows:

$$p(V\text{-}W)_i = C\ p(S)_i\ D_i^x \tag{2}$$

where $p(V\text{-}W)_i$ is the volumetric percentage by weight of material retained on sieve size i, while $p(S)_i$ is the percentage by weight retained on the sieve size i from the surface sample. D_i is the geometric mean of the sieve sizes i and i + 1 or $D_i = \sqrt{D_i \times D_{i+1}}$. The exponent x depends on the type of sampling used. For clay samples x = -1.0, and for wax samples x ≈ -0.47 (Diplas and Sutherland, 1988). The exponent for a grid by number sample (which is equivalent to a volummetric sample) is zero. The constant C is unique to each sample and can be calculated by integrating equation (2) over the entire range of particle sizes D_i which results in the following formula:

$$C = \frac{1}{\sum p(S)_i\ D_i^x} \tag{3}$$

Without making this conversion to a volumetric sample, it is impossible to compare two samples even if they were taken by the same areal technique as each sample has a unique constant and is nonlinearly biased (Fripp and Diplas, 1993).

While no accuracy levels have been established in terms of confidence limits for areal samples, Diplas and Sutherland (1988) found the following two properties to be true of areal samples:

1. "On a sediment surface, each grain, on average, projects an area proportional to the square of its sieve diameter."

2. "A size fraction occupying p% of the volume of solids will occupy p% of the sample surface."

Expressing these statements in a simple mathematical proof Diplas and Fripp (1992) found that the minimum area for a surface sample is equal to $n \times D^2_{max}$ where D_{max} is the maximum grain size present and n corresponds to an adequate number of stones for a grid sample which is recommended to be 100.

Another limitation is that the clay is only reliable in picking up particles that are under about 40 mm in diameter (Diplas and Fripp, 1992). Thus, materials larger than that would be truncated from a sample, resulting in a biased sample.

2.1.4 <u>Sampling the Entire Particle Size Range</u>

Since gravel-bed streams typically contain a wide range of particle sizes, neither grid sampling nor areal sampling is sufficient to sample the surface layer. The reason, as described earlier, is that grid samples tend to truncate material below 15 mm, while clay areal samples, truncate materials above 40 mm. Thus, a major question is how can an entire surface layer be sampled in an unbiased manner?

Fripp and Diplas (1993) addressed this problem and recommended a method that combines the results of a clay sample and a grid sample into one distribution representative of the entire range of grain sizes. The basis of this method is that both the grid and clay methods sample material between 15 mm and 40 mm in an unbiased fashion. Therefore, the results can be adjusted such that both curves match within this overlap region. It is assumed in this process that, the accuracy of an areal sample having an area equal to $n \times D_{max}^2$ is equal to that of a grid sample containing n stones.

2.2 Sampling River Reaches and Gravel-bars

While some research has dealt with obtaining the minimum sample sizes necessary for a specified accuracy level, as well as means of sampling surfacial material for a single deposit, little has been done to develop an unbiased way of sampling a river reach containing several sediment deposits. Some questions that are important to obtaining an unbiased sample follow: How should a sample be taken such that it accounts for the area of each sediment deposit present? Can bulk or volume samples of equal size be combined into a large composite sample representative of the entire area or is it more appropriate to average the individual distributions? Do the samples taken from each deposit have to have the same accuracy level and if so how can we identify the boundaries of deposits that are underwater? Can a measure of local variability be obtained and how can we reduce the amount of labor involved in sampling a river reach? The following sections describe the results of some proposed methods and how they address these issues.

2.2.1 Composite Bulk Sampling

Mosley and Tindale (1985), in an attempt to quantify the grain size distribution of a river reach containing several different deposits, established 7 cross sections throughout the area to be sampled and delineated the different deposit boundaries along them. Eighty six bulk samples of about 30 kg each were then taken from randomly chosen points along these cross sections in a manner considered to be consistent with a stratified random sample as described in statistical books such as Krumbein and Graybill, (1965). When combined, these samples would represent 2.5 tonnes of material. Based on the statistics of these samples Mosley and Tindale (1985) found that 228 samples would be necessary to estimate the mean particle size to within ±10% at a 95% confidence level. Realizing that sampling intensive methods such as this are impractical, Mosley and Tindale (1985) proposed that an alternative to calculating the distributions of each of the eighty six samples independently and computing confidence levels for a parameter such as the mean would be to collect samples and combine them into a single sample and use it's mean as the representative value. While no confidence intervals can be calculated using this approach, it was found that by randomly selecting some of the 86 samples and aggregating them that the resulting mean grain size fell within the previously predicted 95% confidence when 45 samples had been combined.

Wolcott and Church (1991) extended this idea and suggested that if small subsamples of equal volume are taken in a manner such that the number of subsamples coming from the different deposits is proportional to the area of each deposit, that they can be combined into a single sample characteristic of the entire area sampled.

In order to test this hypothesis Wolcott and Church (1991) began by mapping a gravel-bar according to its different sediment deposits and then digitized the results so that computer simulations could be used to test the abilities of random, systematic, and systematic unaligned sampling methods, as described by Krumbein and Graybill (1965), to sample the facies in proportion to their areas. Wolcott and Church (1991) concluded that systematic and systematic unaligned methods worked best and were on average unbiased provided that 100 to 300 sampling sites were used.

Two samples were then collected for comparison using the systematic unaligned method to locate sample sites. The first sample was obtained by "pooling" the results of 46 bulk samples. The bulk samples are of different sizes but presumably of similar accuracy levels (personal communication with Church). The second sample was a composite of 46 subsamples of equal volume. The only requirements on the subsamples were that they have a volume at least that of the largest grain size present and that the cumulative volume be large enough such that the presence or absence of a single large stone would not bias the results. A 95% confidence interval was then plotted around the pooled sample by "basing the standard deviation of the cumulated proportion of the sample at each 1/2 phi sieve break" (Wolcott and Church, 1991). It was found that the composite fell within the confidence limits except for the finer particle sizes which it overestimated. This, as Wolcott and Church (1991) suggest, may be a result of an inadequate number of subsamples being taken, however, as will be shown in Chapter 3 and later in Chapter 6, the overestimation of fine materials may be a result of a bias inherent to combining nonvolumetric samples.

While this approach vastly reduces the amount of labor needed to sample an area and appears to accurately weigh the individual sediments in proportion to their areas within the river reach, it has several drawbacks. First, the only basis for combining nonvolumetric samples of equal volume into a single distribution is the single empirical example provided and thus, assumes that such a method will work in areas having material compositions different than the one sampled. Second, it is limited to the bottom layer of a gravel-bed stream, and even then the pavement and subpavement must be scraped off before it is sampled. If the pavement, or subpavement layer, is to be sampled, this method cannot be used. The reason for this, as mentioned earlier, is that only areal and grid samples are capable of sampling the pavement and subpavement layers without including material from the other populations. Another drawback to this approach is that, since the subsamples are not volummetric, the subsamples can not be analyzed separately to provide information on the local variation of the material. Thus, grains size parameters or distributions for a specific point within the area are lost. Finally, the very nature of combining nonvolumetric subsamples of equal volume promotes biased results and samples the different sediments present at different accuracy levels. A discussion of how nonvolumetric samples are automatically biased and how a composite composed of such subsamples is likewise biased is given in chapter 3. Results of testing the procedure on a material having a known distribution as described in Chapter 6 further confirm this as well.

2.2.2 Composite Grid Sampling

Mosley and Tindale (1985), in addition to describing a river reach's bottom layer through bulk sampling as described in the last section, attempted to characterize the surface material of the same region with grid sampling. Specifically two grid samples were taken. The first consisted of 141 samples of 70 stones each, coming from 141 distinct sediment deposits visually identified along the cross sections. The mean grain size was 22.3 mm. The second surface sample, consisting of 1687 stones, was obtained by collecting a stone every two paces along 12 cross sections. The resulting mean was 13.1 mm. One major reason for this discrepancy as Wolcott and Church (1991) point out is that the first sample taken by Mosley and Tindale (1985) is biased because the areas of the 141 individual deposits were not considered. That is to say the number of stones taken from each deposit was not in proportion to its area. As shown in chapter 4, it is, indeed, crucial to sample each deposit in proportion to its area or else the result becomes biased. The accuracy of the second sample will, likewise, depend on the success that transect sampling (grid sampling in a straight line) has at sampling the different deposits in proportion to their area, which as of now has not been investigated. However, it can be assumed that if the transects are equally spaced and form a sort of grid such that deposits present are sampled in proportion to their areas it should provide an unbiased estimate of the overall grain size.

2.3 Summary of Current Sampling Techniques

The development of methods to sample coarse grained material has been primarily focused on sampling a single distribution and making certain that it samples only the population of interest i.e. the pavement, subpavement or bottom layer in the streambed. While bulk sampling is inappropriate for sampling surface materials, grid and areal sampling techniques have limitations which can lead to truncated results. The hybrid method, however, resolves the problem of truncation.

Sampling an area containing several different sediment deposits is difficult because an unbiased sample must reflect the size of each deposit present. Taking bulk samples in a manner to satisfy this condition requires enormous amounts of material and is often physically or economically impractical. The alternative of combining nonvolumetric subsamples of equal volumes into a single composite sample reduces the amount of material needed dramatically, however, all measure of local variability is lost in this procedure in addition to providing biased results as described in chapter 3 and confirmed in chapter 6.

Current sampling techniques designed to sample the surface material of an entire region exhibiting spatial size variation are therefore inadequate for one or more of the reasons which follow:

1) They do not sample the different sediment deposits in proportion to their areas.
2) They truncate material smaller than 15 mm.
3) Combine nonvolumetric subsamples that are by definition already biased.
4) Are incapable of describing sediment variation within the sampled area
5) Have no guidelines as to the accuracy level of the sample.

Thus, there is a need to establish a method which provides an unbiased grain size distribution representative of a river reach, has a known accuracy level, and describes the spatial variation of the surface material.

CHAPTER 3: MECHANICS OF COMPOSITE SAMPLING

3.1 Combining Samples

As mentioned in Chapter 2, the idea behind composite sampling is to combine samples from different sediments (subsamples) into a single sample representative of the entire area. One approach of doing this is to obtain the distributions of each subsample weight them in proportion to the area they represent and then average the cumulative percentage retained on each sieve size. The resulting averaged values are then plotted to obtain the overall distribution. The second approach is to physically combine the subsamples without having analyzed the distributions of the individual samples separately, but simply computing the overall distribution with a single sieve analysis. However, it will be shown that the latter approach is valid only if the subsamples represent the different deposits in proportion to their areas, and the subsamples are either volumetric and of equal size or grid samples containing an equal number of stones. The next few paragraphs will demonstrate the ideas behind these methods, show how using subsamples of different sizes or nonvolumetric subsamples provide biased results, and discuss the accuracy levels of composite samples.

If one were to combine subsamples into a single composite sample representative of two or more sediment populations, it becomes important to understand the principles of how two known populations can be combined into a single overall distribution in an unbiased fashion. Therefore, consider two known but different sediment populations having volumes V_1 and V_2. Furthermore lets assume that the depth of material in each deposit is the same. The volumes in each deposit occupied by grain particles of size D_i are $p_{i1}xV_1$ and $p_{i2}xV_2$ respectively, where p_i is the percent by volume of grain size D_i , and D_i is the geometric mean of sieve sizes i and i+1. If the materials in the deposits are physically combined, p_{iT}, the percent by volume of grain size D_i for the entire volume, becomes

$$p_{iT} = \frac{p_{i1} \times V_1 + p_{i2} \times V_2}{V_1 + V_2} \quad \textbf{(4)}$$

which can be reduced to $(p_{i1} + p_{i2})/2$ when the volumes are equal ($V_1 = V_2$). For a case having three or more deposits of material this equation becomes

$$p_{iT} = \frac{p_{i1} \times V_1 + p_{i2} \times V_2 + \ldots + p_{ik} \times V_k}{V_1 + V_2 + \ldots + V_k} \quad \textbf{(5)}$$

where k is the number of distributions present. Again, if the volumes of each deposit are equal, this equation reduces to a simple average of the individual percentages retained on each sieve size.

$$p_{iT} = \frac{p_{i1} + p_{i2} + \ldots + p_{ik}}{k} \quad \textbf{(6)}$$

Obviously, in the field neither the volumes of the individual deposits nor their distributions are known. Furthermore, it is impossible to sieve all the material present to obtain the overall distribution. Therefore it is necessary to estimate the overall distribution with samples of considerably less material. However, as equation 5 demonstrates, the volume sampled from each deposit must be in proportion to the volumes of the sediment populations if an unbiased distribution is to be obtained. One approach of guaranteeing that this happens is to take samples with equal volumes in proportion to the area each sediment population covers (assuming depths are constant). For example, if the second deposit has twice the volume and hence covers twice the area as the first deposit, taking one sample from the first deposit and two samples from the second deposit and combining them together using equation 5 or equation 6 will result in the same distribution as combining both of the entire populations together provided the samples are accurate. Wolcott and Church (1991) used this principle in developing their composite sampling procedure. Specifically, Wolcott and Church (1991) demonstrated that samples taken systematically over an area represent each deposit in proportion to its area. Therefore, each subsample represents the same amount (or volume assuming depth is constant) of material and each sediment's distribution is automatically weighted by the number of subsamples taken from it.

It also becomes clear from equations 5 and 6 that, if the number of samples taken from each population are in proportion to the population volumes, but the samples are of unequal volumes the proportions between the sample volumes and the population volumes are destroyed and the resulting composite sample is biased. This bias is demonstrated in the following example. Suppose that the area to be sampled has two sediment deposits of known distributions covering equal areas. Furthermore, to avoid sampling error it is assumed that the samples taken obtain the exact distributions of each deposit. Finally since it is common practice in the field to assume that density is constant in the field, we can assume that samples of the same weight have the same volume and those of different weight have different volumes. Table 3.1 shows the amounts of material retained on each sieve size for three samples from the first area and a single sample taken from area two. Of the three samples taken from area 1, one is less than the weight of sample 2, one is equal to the weight of sample 2, and the third one is larger than sample 2. The cumulative distributions of each area and the area's overall distribution are also tabulated. Finally, the last three columns in Table 3.1 show the distributions resulting from physically combining each of the three samples from area 1 with the sample from area 2. Plotting these distributions in Figure 3.1 demonstrates that the physical combination of samples results in an unbiased distribution only when the samples are of equal volume.

Table 3.1 Bulk Samples of Different Weights from Two Areas and their Composites

Sieve Size (mm)	Di (mm)	Area 1 25.15 kg	Area 1 107.81 kg	Area 1 201.2 kg	Area 2 107.81 kg	Cumulative % Distribution 1	Cumulative % Distribution 2	Overall Distribution	Cumulative % Composite 1	Cumulative % Composite 2	Cumulative % Composite 3
0.06	0.08	0.03	0.11	0.20	0.10781	0.10	0.10	0.10	0.10	0.10	0.10
0.09	0.11	0.08	0.32	0.60	0.10781	0.40	0.20	0.30	0.24	0.30	0.33
0.13	0.15	0.20	0.86	1.61	0.21562	1.20	0.40	0.80	0.55	0.80	0.92
0.18	0.21	0.40	1.72	3.22	0.32343	2.80	0.70	1.75	1.10	1.75	2.07
0.25	0.30	1.23	5.28	9.86	0.75467	7.70	1.40	4.55	2.59	4.55	5.50
0.35	0.42	1.46	6.25	11.67	1.40153	13.50	2.70	8.10	4.74	8.10	9.73
0.50	0.60	1.21	5.17	9.66	2.80306	18.30	5.30	11.80	7.76	11.80	13.76
0.71	0.84	0.68	2.91	5.43	2.91087	21.00	8.00	14.50	10.46	14.50	16.46
1.00	1.18	0.70	3.02	5.63	3.12649	23.80	10.90	17.35	13.34	17.35	19.30
1.40	1.67	0.96	4.10	7.65	3.66554	27.60	14.30	20.95	16.82	20.95	22.96
2.00	2.37	1.51	6.47	12.07	4.42021	33.60	18.40	26.00	21.28	26.00	28.30
2.80	3.35	2.24	9.60	17.91	5.28269	42.50	23.30	32.90	26.93	32.90	35.80
4.00	4.76	2.36	10.13	18.91	5.28269	51.90	28.20	40.05	32.68	40.05	43.63
5.66	6.73	2.92	12.51	23.34	5.60612	63.50	33.40	48.45	39.09	48.45	53.00
8.00	9.47	3.52	15.09	28.17	6.03736	77.50	39.00	58.25	46.28	58.25	64.07
11.20	13.39	3.04	13.05	24.35	6.25298	89.60	44.80	67.20	53.27	67.20	73.97
16.00	18.97	1.51	6.47	12.07	5.60612	95.60	50.00	72.80	58.63	72.80	79.69
22.50	26.83	0.58	2.48	4.63	4.74364	97.90	54.40	76.15	62.63	76.15	82.72
32.00	37.95	0.53	2.26	4.23	9.81071	100.00	63.50	81.75	70.40	81.75	87.27
45.00	53.67	0.00	0.00	0.00	14.76997	100.00	77.20	88.60	81.51	88.60	92.05
64.00	75.89	0.00	0.00	0.00	14.33873	100.00	90.50	95.25	92.30	95.25	96.69
90.00	107.33	0.00	0.00	0.00	6.68422	100.00	96.70	98.35	97.32	98.35	98.85
128.00	151.79	0.00	0.00	0.00	3.55773	100.00	100.00	100.00	100.00	100.00	100.00
180.00	214.66	0.00	0.00	0.00	0	100.00	100.00	100.00	100.00	100.00	100.00
256.00	303.58	0.00	0.00	0.00	0	100.00	100.00	100.00	100.00	100.00	100.00
360.00	0.00	0.00	0.00	0.00	0	100.00	100.00	100.00	100.00	100.00	100.00
Weight (kg)		25.15	107.81	201.2	107.81						

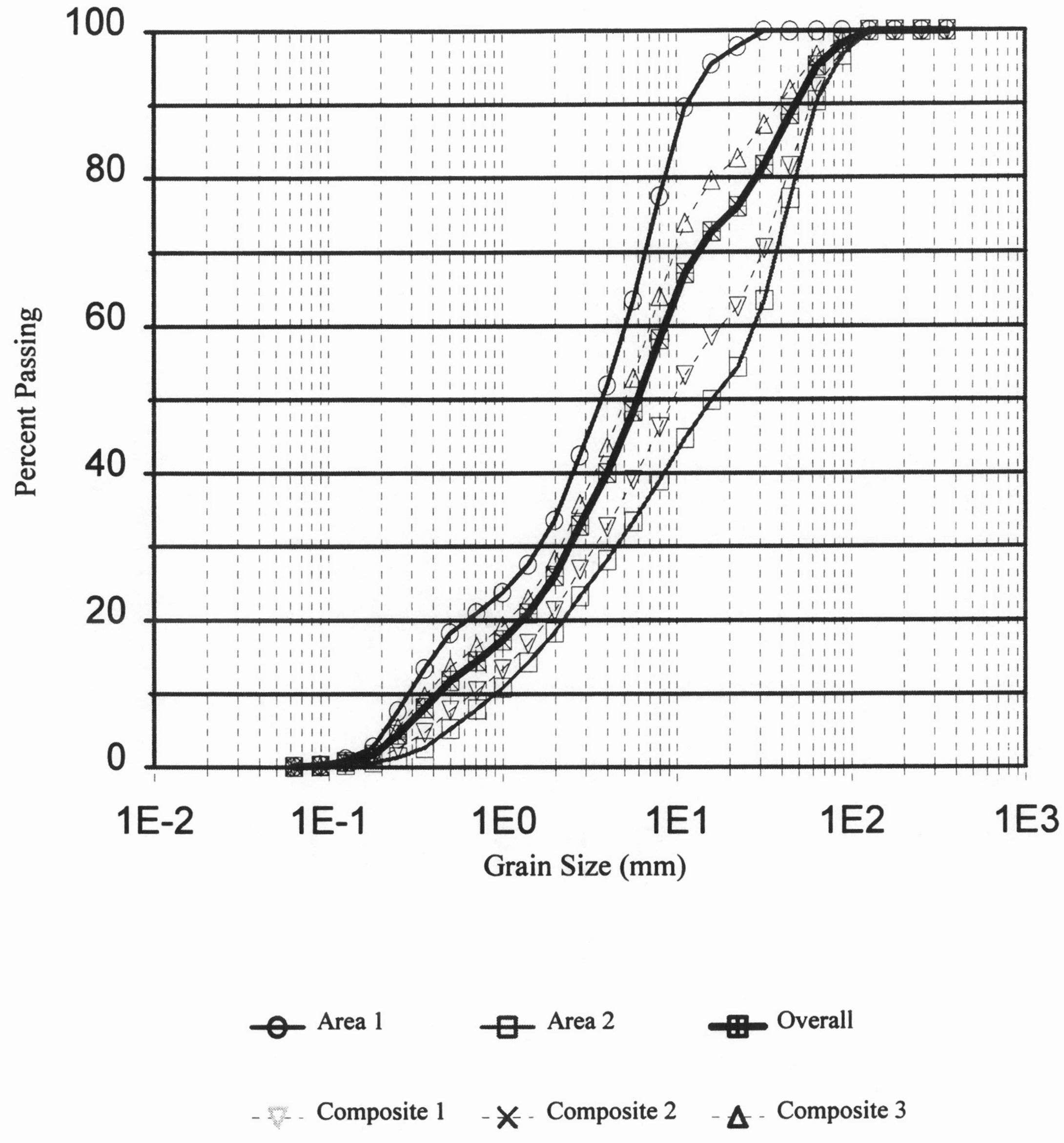


Figure 3.1: Bulk Composites of Various Sizes

These principles also apply to grid by number samples. The reason for this is that grid samples are volumetrically equivalent and equation 5 can be rewritten as follows:

$$p_{iT} = \frac{p_{i1} \times n_1 + p_{i2} \times n_2 + \ldots + p_{ik} \times n_k}{n_1 + n_2 + \ldots + n_k} \tag{7}$$

Where n is the number of stones in each sample. Similar, to bulk sampling, the sample sizes must be proportional to the volume that each sediment population has or a biased result is obtained. Fortunately, as with bulk sampling, collecting the stones systematically over the area to be sampled will automatically weight each sediment present according to its surface area. Table 3.2 and Figure 3.2, similar to Table 3.1 and Figure 3.2, show the results of combining grid subsamples of various sizes together. Again, each of the two deposits are assumed to cover the same amount of area and have the same depth. As one can see only when grid samples sizes are proportional to the sediment population volumes (areas if depths are assumed the same) can they be physically combined into a grain size distribution representative of the entire area. In this case, since both sediment populations cover the same area the samples sizes must be the same size.

Throughout the above discussion, the depth of each of the deposits was assumed to be constant. Unfortunately, this can not be guaranteed in the field and thus, if the two aforementioned deposits covered the same areas, but had different depths, both the bulk and grid composites would become biased as the proportionalities necessary in equation

(5) are destroyed. Thus, if one is unable to accept this assumption, the depths of the individual deposits must be accounted for, especially if the bottom layer of a gravel-bed stream is being sampled. However, if one is mainly concerned with either the pavement or subpavement layer, the depth is of no importance as it is only as thick as the maximum grain size. Hence, taking a grid sample in proportion to the individual deposit's areas is sufficient and will result in an unbiased sample.

Table 3.2 Grid Samples of Different Sizes from Two Areas and their Composites

Sieve Size (mm)	Di (mm)	Area 1 1000 Stones	Area 1 2000 Stones	Area 1 3000 Stones	Area 2 2000 Stones	Cumulative % Distribution 1	Cumulative % Distribution 2	Overall Distribution	Cumulative % Composite 1	Cumulative % Composite 2	Cumulative % Composite 3
0.06	0.08	1.00	2.00	3.00	2.00	0.10	0.10	0.10	0.10	0.10	0.10
0.09	0.11	3.00	6.00	9.00	2.00	0.40	0.20	0.30	0.27	0.30	0.32
0.13	0.15	8.00	16.00	24.00	4.00	1.20	0.40	0.80	0.67	0.80	0.88
0.18	0.21	16.00	32.00	48.00	6.00	2.80	0.70	1.75	1.40	1.75	1.96
0.25	0.30	49.00	98.00	147.00	14.00	7.70	1.40	4.55	3.50	4.55	5.18
0.35	0.42	58.00	116.00	174.00	26.00	13.50	2.70	8.10	6.30	8.10	9.18
0.50	0.60	48.00	96.00	144.00	52.00	18.30	5.30	11.80	9.63	11.80	13.10
0.71	0.84	27.00	54.00	81.00	54.00	21.00	8.00	14.50	12.33	14.50	15.80
1.00	1.18	28.00	56.00	84.00	58.00	23.80	10.90	17.35	15.20	17.35	18.64
1.40	1.67	38.00	76.00	114.00	68.00	27.60	14.30	20.95	18.73	20.95	22.28
2.00	2.37	60.00	120.00	180.00	82.00	33.60	18.40	26.00	23.47	26.00	27.52
2.80	3.35	89.00	178.00	267.00	98.00	42.50	23.30	32.90	29.70	32.90	34.82
4.00	4.76	94.00	188.00	282.00	98.00	51.90	28.20	40.05	36.10	40.05	42.42
5.66	6.73	116.00	232.00	348.00	104.00	63.50	33.40	48.45	43.43	48.45	51.46
8.00	9.47	140.00	280.00	420.00	112.00	77.50	39.00	58.25	51.83	58.25	62.10
11.20	13.39	121.00	242.00	363.00	116.00	89.60	44.80	67.20	59.73	67.20	71.68
16.00	18.97	60.00	120.00	180.00	104.00	95.60	50.00	72.80	65.20	72.80	77.36
22.50	26.83	23.00	46.00	69.00	88.00	97.90	54.40	76.15	68.90	76.15	80.50
32.00	37.95	21.00	42.00	63.00	182.00	100.00	63.50	81.75	75.67	81.75	85.40
45.00	53.67	0.00	0.00	0.00	274.00	100.00	77.20	88.60	84.80	88.60	90.88
64.00	75.89	0.00	0.00	0.00	266.00	100.00	90.50	95.25	93.67	95.25	96.20
90.00	107.33	0.00	0.00	0.00	124.00	100.00	96.70	98.35	97.80	98.35	98.68
128.00	151.79	0.00	0.00	0.00	66.00	100.00	100.00	100.00	100.00	100.00	100.00
180.00	214.66	0.00	0.00	0.00	0.00	100.00	100.00	100.00	100.00	100.00	100.00
256.00	303.58	0.00	0.00	0.00	0.00	100.00	100.00	100.00	100.00	100.00	100.00
360.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00	100.00	100.00	100.00	100.00
# Stones		1000	2000	3000	2000						

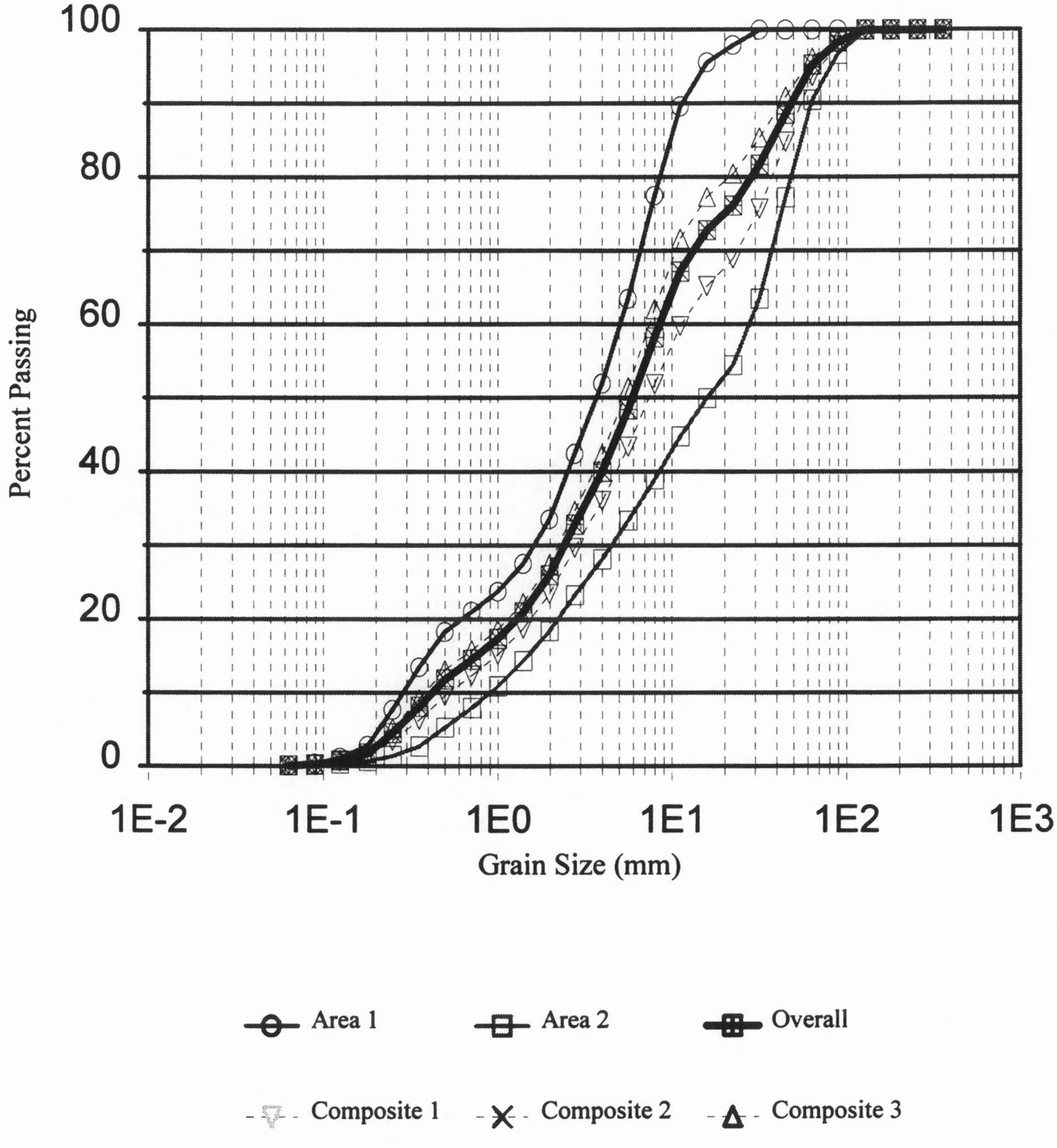


Figure 3.2: Grid Composites of Various Sizes

It should be noted that, unlike physically combining subsamples, averaging the percentages retained on each sieve size results in an unbiased distribution regardless of size or sampling method. However, unbiased results are obtained only when the subsamples are unbiased and represent deposits of the same amount of material. Since these conditions are met when samples are taken systematically as Wolcott and Church (1991) described, and the subsamples are volumetric the distribution 46 volumetric distributions obtained by Wolcott and Church if averaged should provide an unbiased estimate of the areas overall distribution (assuming the depths of the deposits are constant throughout the area). However, the accuracy level of such a sample as well as those of composite bulk, and composite grid samples needs to be addressed.

3.2 Sample Size and Accuracy Levels

While it has been demonstrated that volumetric bulk samples of equal volume, if taken in proportion to the volume of the sediment populations present, can be physically combined into a single sample representative of the entire area, the impact of sample size and the resulting accuracy level has not been evaluated. Likewise, the accuracy of a composite grid needs to be addressed as well.

First, it is important to evaluate the effects of combining nonvolumetric subsamples of equal volume together. Wolcott and Church (1991) combined 46 of these subsamples into a single distribution in order to estimate the overall grain size distribution of a gravel-bar. The only criteria were that the subsamples be at least as large as the largest grain size present, the subsamples represent the sediments in proportion to their areas and that all of the subsamples when combined must weigh at least 100 times the maximum grain size present. This approach implies that if a homogenous deposit was sampled with both a single volumetric sample and a composite bulk sample having the same volume but was made by collecting small nonvolumetric subsamples throughout the area and cumulating them, both samples would provide the same distribution. However, chapter 6 shows that this is not the case and that the composite sample is biased towards the larger particles sizes.

The reason that cumulating nonvolumetric subsamples results in a biased estimate is due to the fact that any sample to small to be volumetric is already biased. The definition of a volumetric sample is a volume such that the dimensions of it are independent of the grain sizes (e.g. much larger than the dimensions of the individual particles sizes present) (Kellerhals and Bray, 1971). Thus, any sample taken smaller than this is biased. Obviously, when the dimensions of a sample is equal to that of the largest grain size, the volume is completely dependent on the grain size and is in direct contradiction to an unbiased volume for a sample. In fact, by driving this technique to the limit, one could take 100 samples equal to the volume of the largest grain size side by side. This, in essence would result in taking an areal sample with depth D_{max} which results in a bias toward the larger particle sizes and must be corrected for by converting the results to a volumetric equivalent.

A more in depth discussion of the degree of bias introduced by taking cumulating nonvolumetric subsamples is given in chapter 6. Sommer (1986), however, provides a way to visualize how this bias occurs by pointing out that realistically sampling with constant mass or volume should only be done when the materials being sampled differ in some way other than grain size, such as color. The reason for this as Sommer (1986) explains is that when a sample volume is filled with materials of different sizes certain particles must be exchanged with a number of particles of different sizes to obtain the exact volume required. This in turn biases the sample because these particles are no longer independent of the particles already in the sample. An example of this can be illustrated by picturing two urns of the same size. The first containing equal volumes of cubes having sizes of 1 and 2 mm. The second urn contains cubes of sizes 4 and 6 mm each occupying the same amount of volume. For sake of illustration let's assume that each urn has a total volume of 3456 mm^3. Thus each cube size occupies 1728 mm^3 or 25% of the total volume. Following Wolcott and Church's (1991) approach allows one to collect volumes as small as that contained by the largest particle size present and combine them into an overall composite. If the largest grain size's volume is used in this case,the true distribution of the two urns cannot be obtained because if one fills a volume of 216 mm cubed (6^3) from the first urn the closest one can get to collecting half of each sized cube is either 13 two mm cubes and 112 one mm cubes or 14 two mm cubes and 104 one mm cubes. Likewise, taking the same volume from the second urn would always result in obtaining a 6 mm cube because it would require 3.375 four mm cubes to fill that exact volume and there is no combination of particle sizes to fill that exact volume. Hence in this simple example we see that an entire grain size would be left out of the composite distribution and the volummetric percentages of the two smallest grain sizes is biased as well. Thus regardless of how many subsamples are taken the result would always be biased. While this is an extreme case it is evident that the accuracy of a composite taken in this fashion will depend on the properties of the material and the canceling out of sampling errors. Furthermore, it is not known if these biases can be corrected by converting the sample into a volumetric equivalent as areal samples can be.

A secondary problem associated with combining non-volumetric samples is that of obtaining subsamples of exactly the same weight or volume. If subsamples are collected according to weight, changes in moisture and density must be accounted for. On the other hand, if samples are collected by putting them in a box of constant volume, changes in porosity must be accounted for. Komura, and Colby (1962) found that the porosity for sediment deposits having a median grain size of .06 mm was 40% while for sediments with a median grain size of 10 mm was 30%. The exact role porosity and density plays in combining volumetric samples is not known and should be addressed in the future. .

Finally, the accuracy level of a composite sample should be addressed. In bulk samples the accuracy level depends on the particle sizes present as well as the volume of the material collected. Thus, two different deposits sampled at the same volume will have different accuracy levels (de Vries, 1970; Church et al, 1987). This is true even if the standard deviation of the material doesn't play a role in the accuracy level. For example, if two materials have the same standard deviation, but one is a sand while the other is a gravel, the gravel is sampled less accurately than the sand simply because more material is needed to describe the gravel at the same level as the sand due to its larger particle sizes (de Vries, 1970; Church et al, 1987). Thus, a composite sample made up of equal volume samples has an unknown accuracy level. Wolcott and Church (1991) recommend that accuracy levels can be found by taking several composite samples and then using the student t-distribution to calculate confidence limits. However, if the largest particle size is about 215 mm and weighs about 15 kg the total mass collected for 5 composite samples would be in excess of 7500 kg.

Unlike bulk samples, grid samples of the same size (having the same number of stones) have the same accuracy level in terms of percent passing a certain size. Therefore, two different materials having the same standard deviations and sampled with the same number of stones would be sampled at the same accuracy level. However, as with bulk samples the accuracy level in terms of the grain size is also dependent on the standard deviation of the sediment being sampled and hence deposits having different standard deviations but sampled at the same intensity would still have different accuracy levels. Fortunately, despite this problem, a composite sample's accuracy level is still given by the binomial method suggested by Fripp and Diplas (1993). This is shown in Chapter 4, along with the fact that the accuracy level in terms of grain size is solely dependent on the standard deviation of the overall distribution and not of the individual deposits present.

From this discussion, it becomes clear that, when samples are collected such that the number of samples taken from each sediment deposit present is in proportion to the individual deposit's area, the subsamples must be either volumetric samples of equal volumes or grid samples having the same number of stones in each subsample in order for a composite sample to be unbiased. This discussion also shows that a composite sample taken by cumulating nonvolumetric subsamples of equal volume results in a biased estimate of the overall grain size distribution which may be difficult if not impossible to correct for.

CHAPTER 4: OBTAINING A DEPOSIT'S OVERALL GRAIN SIZE DISTRIBUTION

4.1 Introduction

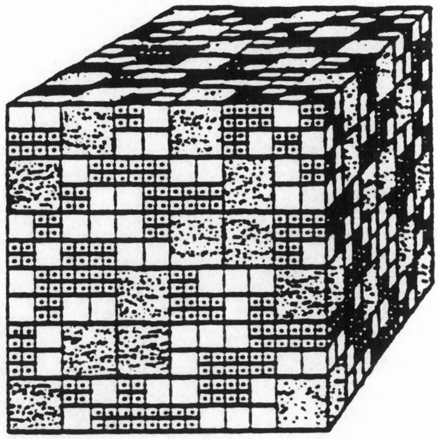
It is clear from Chapter 2 that current grid sampling techniques are inadequate to sufficiently characterize a river reach's overall grain size distribution or its spatial variability. Little attention has been given to appropriately weighing each sediment's distribution in accordance to the area it contains within a region or the effects of combining grid samples taken from different areas together. Furthermore, no accuracy guidelines have been established for samples containing stones from several sediment deposits. Chapter 3 began addressing this issue by examining appropriate ways to physically combine grid samples together in order to obtain a region's overall particle size distribution. The results suggest that, if grid samples are taken such that each sediment is sampled in proportion to it's area, (e.g. maintains a consistent grid spacing throughout the area) the resulting distribution is an unbiased measure of the entire region's size distribution. Unfortunately, the examples used in Chapter 3 assume that the subsamples obtained the exact distributions of the underlying sediments. These examples are, therefore, unrealistic and provide little insight on what role sampling error plays. Thus, an important question is the accuracy of such a sample in relationship to its size. This chapter describes how a simple computer program was used to test the validity of physically combining subsamples in proportion to their areas and to determine the accuracy of the resulting composite grid sample. As will be shown later, tests indicate that if a grid sample is taken such that each deposit is sampled in proportion to it's area, then the overall accuracy can be adequately described by the binomial method which Fripp and Diplas (1993) use to describe a single grid sample's accuracy.

4.2 Simulating Surface Sampling with a Computer

In order to test the validity of sampling each sediment in proportion to its area and the accuracy of the resulting composite sample, a simple basic program was developed. It is designed to simulate grid sampling in an area consisting of up to four different sediments deposits. The programs is written in QBASIC and is briefly described here. Appendix A contains the actual code.

The first step in developing such a program is to represent a deposit's surface in a meaningful and accurate way. In following with Kellerhals and Bray (1971) and Diplas and Sutherland (1988), a cubical surface model seems appropriate. There are two main assumptions in using a cubical model: (1) On a sediment surface, each grain, on average, projects an area proportional to the square of its sieve diameter and (2) A size fraction occupying p% of the volume of solids will occupy p% of the sample surface area occupied by grains. Experimental data has shown these assumptions to be valid (Diplas and Sutherland 1988). Since the results of grid by number sampling are independent of voids, also shown by Diplas and Sutherland (1988), one can assume either the area has a substantial amount of voids or the surface has no voids. It is further assumed that the particle sizes are randomly distributed over the surface area.

Kellerhals and Bray's (1971) Figure 4.1 depicts such a surface. Specifically, Kellerhals and Bray's model cube is made up of three cubes having sides of 1, 2, and 4 respectively, each representing 33.3% of the entire cube's volume by weight. Furthermore, one can see that despite each cube size having a different number of stones exposed to the surface, the amount of surface area covered by each cube size is the same. Thus, the probability of a certain particle size being at a grid point within a particular deposit is given by the proportion of surface area that each particle has, which equals the volumetric size distribution. Hence, one can simulate taking a grid sample for any grid that has a grid spacing equal to or larger than the smallest grain diameter simply by choosing random particles in proportion to the known volumetric percentages. For the case described above, a random number between 0 and 100 would be chosen and, if the value is between 0 and 33.333, a particle size diameter of 1 is assigned to that particular stone. If the value is between 33.333 and 66.667 a particle diameter of 2 is given while values greater than 66.667 indicate a particle size of 4. Expanding upon this idea, one can define a series of sieve sizes, and the percentage passing each sieve size, and have the computer randomly select a number as before and assign a stone size equal to the geometric mean of the two sieve sizes between which it falls. The process can then be repeated until the specified number of stones in the sample is generated. Furthermore, by providing several distributions and specifying the amount of area each distribution covers the computer can calculate the total number of stones of each stone size and the overall distribution of the area. Once this is done, the computer can select a specified number of stones from each distribution and combine them into a single sample representative of the entire area, whereupon, it can be compared to the known overall distribution.



Particle	Linear Size D	Weight W	Total No. In Sample Volume	Total No. In Sample Surface
	1	1	4608	192
	2	8	576	48
	4	64	72	12

Figure 4.1: Kellerhals and Bray's (1971) Cubic Model

The method just described was accepted as a suitable approach to simulate taking a sample from a sediment deposit with a known particle size distribution. This method follows the properties of a sediment surface as put forth by Diplas and Sutherland (1988); is relatively easy to program; and has the following advantages. A fictitious area having different sediments deposits can easily be defined. Furthermore, since the overall size distribution is known as well as the individual deposit distributions, a composite of stones randomly selected from each area can be compared with the overall size distribution. Taking several such samples, therefore, allows one to determine the accuracy of the sample statistically. In turn this can be used to confirm the binomial accuracy criteria suggested by Fripp and Diplas (1993) as well as the ability to obtain an overall distribution through composite grid samples.

4.2.1 Testing the Program

The most important aspect of the program is to randomly select stones from the distributions defining each area. In order to make sure the program was selecting stones in a reliable manner, a chi-squared goodness-of-fit test was used (e.g. Weiss and Hasset, 1991). Specifically, the computer selected an 800 stone sample from an area having a particle size distribution based on one of Wolcott and Church's (1991) volumetric samples. Performing the test at a .05 significance level and having 21 degrees of freedom provides a critical value of 32.67. Since the test statistic χ^2 equals 30.04 and is less than

32.67 one can accept the null hypothesis and conclude that the program is selecting stone sizes according to the provided distribution. Table 4.1 shows the volumetric percentage of each geometric grain size for the sample, the expected number of stones in an 800 stone sample, and the observed number of stones selected by the program. The results of squaring the differences of the observed and expected number of stones divided by the expected number are tabulated and summed as well. While this procedure was not performed on other distributions used in the program, there appears to be no reason for the program to select samples inconsistent with any other defined distribution.

Table 4.1 Goodness of Fits Test Results

D_i	Volumetric %	Expected Number	Observed Number	Observed Number %	$(O - E)^2/E$
107	1.3	10.4	12	1.5	0.25
75.9	0.7	5.6	8	1.0	1.03
53.7	6.5	52	46	5.8	0.692
38	15.7	125.6	150	18.8	4.74
26.8	11.8	94.4	89	11.1	0.31
19	14.3	114.4	119	14.9	0.18
13.4	10.7	85.6	63	7.9	5.97
9.47	6.4	51.2	41	5.1	2.03
6.73	6	48	41	5.1	1.02
4.67	4.7	37.6	34	4.3	0.34
3.35	3.5	28	36	4.5	2.29
2.37	2.4	19.2	28	3.5	4.03
1.67	2	16	12	1.5	1
1.18	1.8	14.4	10	1.3	1.34
0.843	2.3	18.4	26	3.3	3.14
0.596	3.5	28	34	4.3	1.29
0.421	2.9	23.2	24	3.0	0.028
0.297	1.9	15.2	14	1.8	0.095
0.212	0.7	5.6	5	0.6	0.064
0.15	0.5	4	4	0.5	0
0.106	0.2	1.6	2	0.3	0.1
0.075	0.2	1.6	2	0.3	0.1
Total	100	800	800	100	30.04

4.3 Accuracies of Grid Samples According to the Binomial Method

The binomial method proposed by Fripp and Diplas (1993) assumes that a percentile estimate will be normally distributed about its true mean. Thus, 95% of the percentile estimates for a given grain size will be within 1.96 standard deviations of the true mean (Ott 1977). Using this fact, one can test to see if the binomial distribution holds by plotting the 95% confidence intervals around the known percentiles for each sieve size and then plotting a number of sample distributions. If 95% of these samples fall within the confidence limits the binomial distribution is valid. Crowder and Diplas (1994) used this fact and the computer program just described to qualitatively confirm the ability of the binomial method to describe the accuracy of a grid sample. Crowder and Diplas (1994), however, did not provide results indicating how often a sample fell within the 95% confidence region. In addition to this, the particle size distribution used to simulate a deposit was not based on an actual sediment grain size distribution. It is therefore important to validate the binomial method quantitatively and for actual sediment distributions Furthermore, the accuracies and effects of combining stones from different distributions have yet to be explored. Finally, it has been suggested that a more traditional and appropriate means of testing confidence limits is to calculate the confidence intervals around the sample value and determine if it captures the true sample percentage 95% of the time (Statistical Consulting Center VA Tech). Since this is the procedure that would be used for a grid sample taken in the field, this approach is employed in the next two sections to quantitatively test the validity of the binomial method to sample various conditions.

4.3.1 The Accuracy of a Grid Sample from a Single Deposit

To confirm whether or not the binomial method proposed by Fripp and Diplas (1993) describes the accuracy of a grid sample containing a specified number of stones and coming from a single deposit, three computer simulations were performed. The simulated area in each case consisted of a single deposit having the same distribution as that used in the chi squared goodness of fit test. Each simulation collected 20 samples, however, the number of stones making up a sample changed in each simulation. Each sample in the first simulation contained 100 stones, while the samples in the second and third simulations were made up of 236 and 50 stones, respectively. The 95% confidence limits of the percentages retained on each sieve size and for each sample were then computed. Thus, the accuracy level of the sampling technique corresponding to any given sieve size can be determined by counting the number of sample confidence intervals that contain the true percentage passing the given sieve size. Figure 4.2 shows the 95% confidence limits for a 100 stone sample from the first simulation, along with the true volumetric distribution. In this case, all of the true volumetric percentiles are contained within the confidence limits for each sieve size. This was fairly typical of the other 19 samples as well. For example, the confidence limits of samples retained on five of the sieves captured the true percentage at least 95% of the time. Of the other two sieve sizes, 15 percent of the sample confidence limits failed to enclose the volumetric percentage retained on the 32 mm sieve, while 10% failed to contain the true value passing the 11.2 mm sieve. However, two of the three percentages retained on the 32 mm sieve were above 80% and calculating confidence intervals around them is not appropriate as is less than 20. Realizing this and the fact that one of the percentages retained on the 11.2 mm sieve excluded the true percentage passing by only .2% shows that the binomial approach is a good indicator of the accuracy of a grid sample.

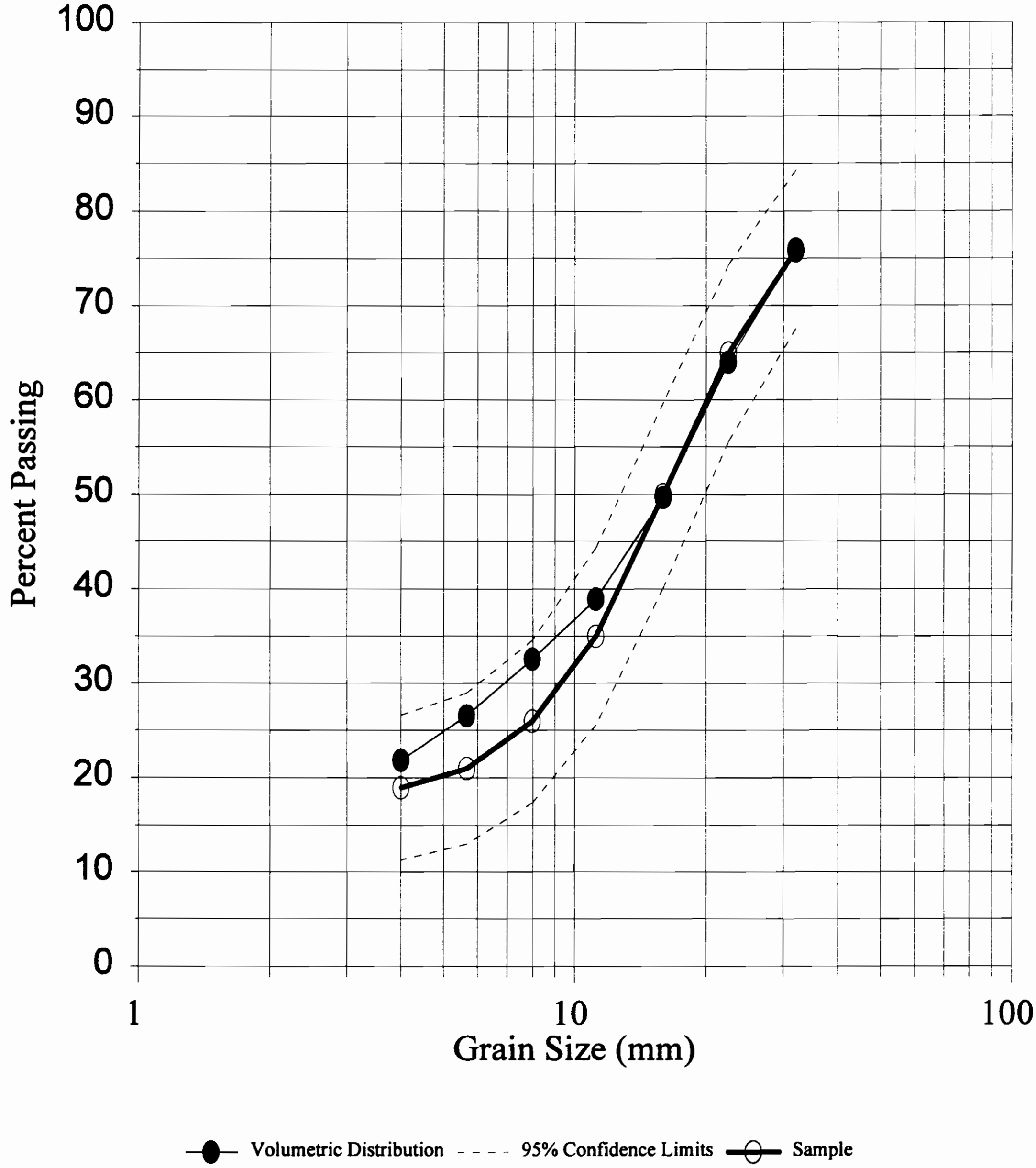


Figure 4.2: 95% Confidence Limits of a 100 Stone Sample from a Distribution Consistent with a Medium Gravel

Similarly, Figure 4.3 shows the 95% confidence limits for a sample of 236 stones obtained in the second simulation. The reason 236 stone samples were taken is the binomial criteria states that this number is sufficient to describe the size parameters of $D_{91.5}$ and $D_{8.5}$ which correspond to the true volumetric amounts passing the 45 mm and .71 mm sieves in this example. As before, all of the sample percentage confidence limits enclosed the true volumetric percentage of the distribution. However, in this simulation, all of the sample percentage's confidence limits, regardless of sieve size, enclosed the true percentage at least 95% of the time. The third simulation collected 20 samples of 50 stones. Here, as with the last simulation, all of the sample percentage's confidence limits, regardless of sieve size, enclosed the true volumetric percentage at least 95% of the time. Figure 4.4 shows one of the samples obtained in the third simulation which does not include the true percentile of the percent passing the 16 mm sieve in its confidence limits.

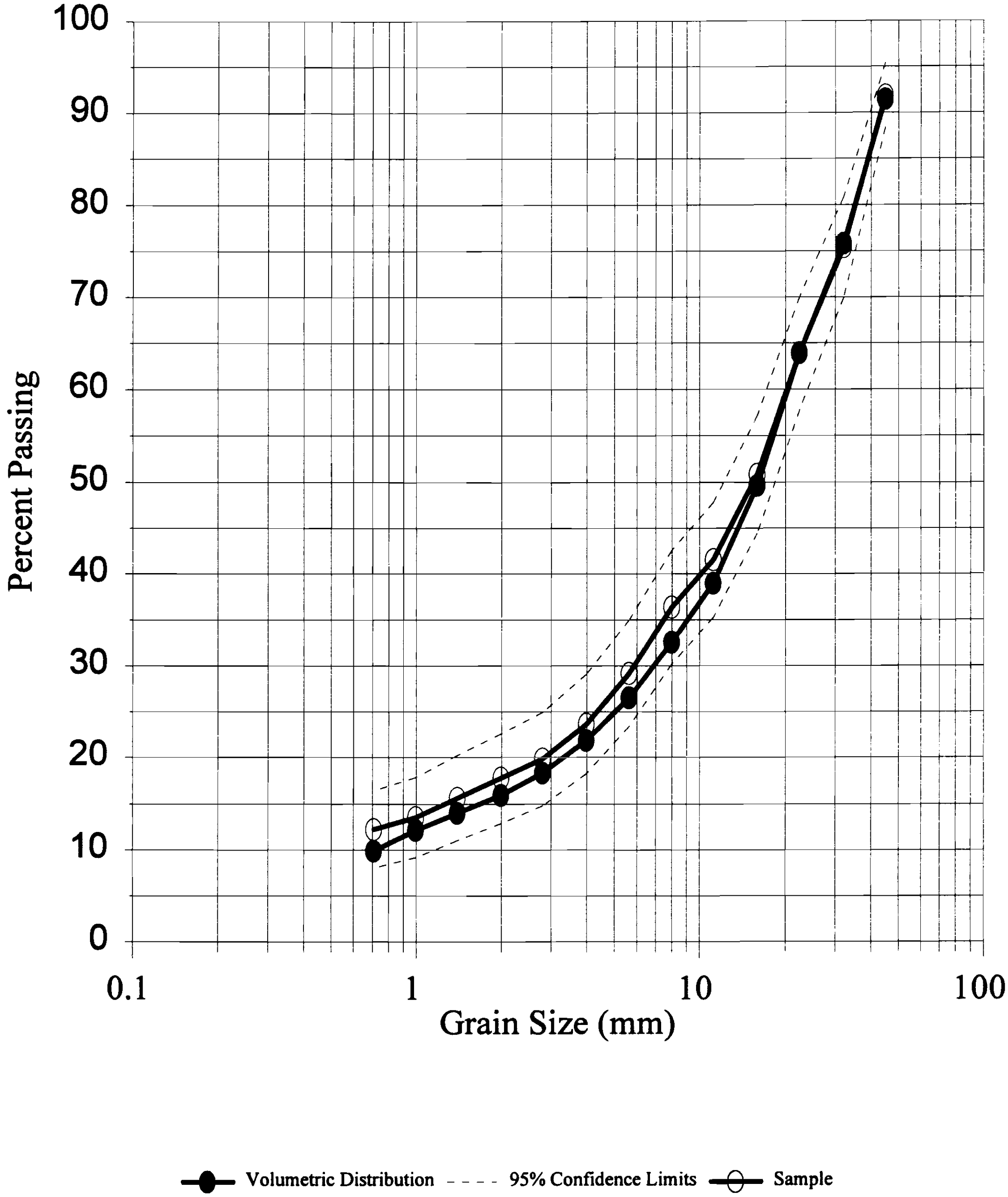


Figure 4.3: 95% Confidence Limits of a 236 Stone Sample from a Distribution Consistent with a Medium Gravel

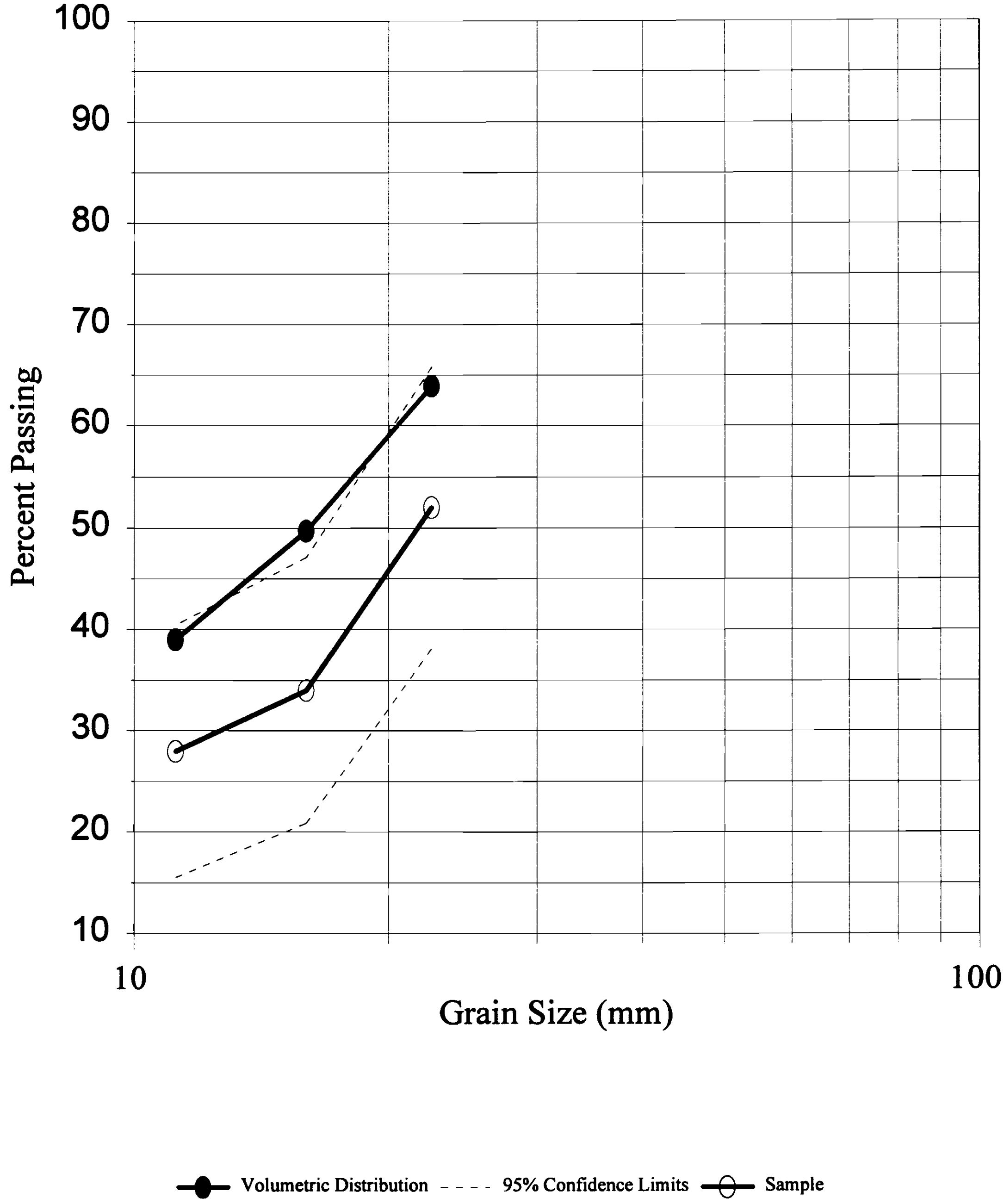


Figure 4.4: 95% Confidence Limits of a 50 Stone Sample from a Distribution Consistent with a Medium Gravel

Two further simulations were run to see if the method worked equally well for distributions having different shapes. The distributions where of actual sediments sampled by Wolcott and Church (1991). One of the distributions represents a coarse gravel and is slightly bimodal, while the other represents a sand. Figure 4.5 shows these two distributions along with the distribution of the medium gravel previously examined. By comparing the accuracy resulting from samples taken from these distributions to those previously taken it can be established how grain size and distribution shape affect the accuracy of the binomial method. It should be noted that while materials smaller than 15 mm are typically truncated in a grid sample this does not happen here as the computer does not have a physical limitation in selecting materials smaller than this as a human operator does. Thus, sandy material as well as the smaller materials in previous sample distributions are collected in an unbiased manner.

As with the previous simulations, twenty samples were taken from each of these distributions. The samples in both cases were made up of 100 stones. Results from the coarse gravel found only one sieve out of 11 did not meet the 95% confidence limits. However, as before, only two of the sample's confidence limits (10%) failed to capture the true percentage. One of which failed to capture the true value by less than .3% of the true percentage for this sieve size. Of the 20 samples taken from the sandy distribution 10% failed to capture the true percentage for the smallest sieve size. The confidence limits resulting from sample percentages retained on the other sieves, however, included the true percentage at least 95% of the time as well.

These results suggest that the binomial approach does indeed provide approximate accuracy levels for a grid sample of a given size, regardless of the distribution shape or gain sizes present. In most cases confidence intervals based on the percentage passing a certain grain size contains the true percentage about 95% of the time.

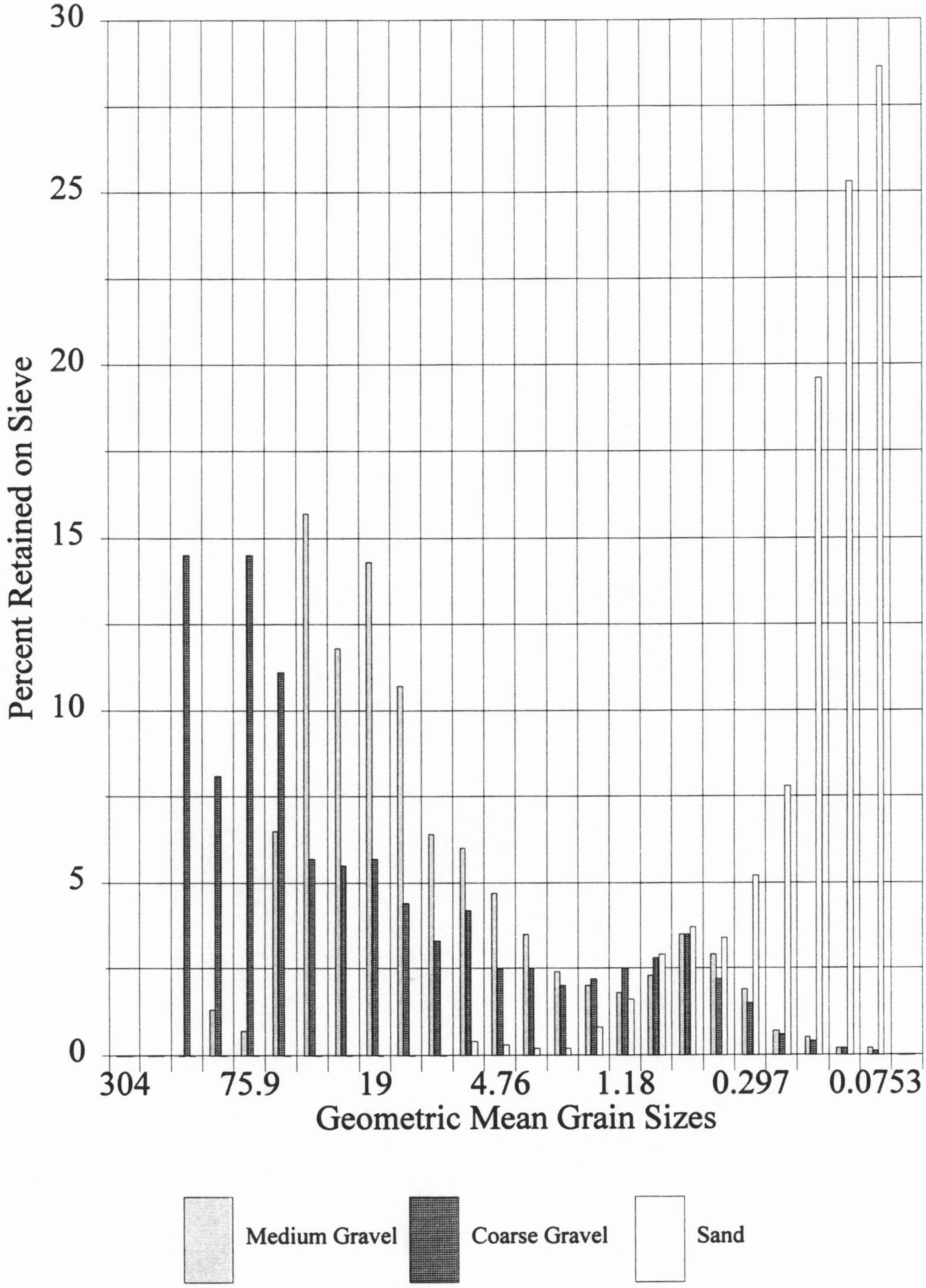


Figure 4.5: The Shapes of Three Grain Size Distributions for which Sampling Accuracy Levels using Equation (1) were Determined

4.3.2 Accuracy of a Composite Grid Sample

The last section shows that a grid sample taken from a single deposit, regardless of its distribution, has an accuracy level approximately equal to that predicted by the binomial method. However, the accuracy level of a grid sample containing stones from several different deposits needs to be addressed as well. This section shows that, if each deposit is sampled in proportion to its area, the composite sample is not only unbiased, as shown earlier, but has an accuracy level approximately equal to that predicted by the binomial method as in equation (1) where n is based on the total number of stones in the composite sample. The bias of not sampling sediment deposits in proportion to their area will also be shown.

To test the accuracy of such a situation, a computer simulation was used. In this case four different areas were defined each having a different distribution. The four different grain size distributions were obtained from volumetric samples from Wolcott and Church's (1991) depositional bar. The distributions describe a sand, a fine gravel, a medium gravel, and a coarse gravel. The first two distributions, covered the same amount of area, while the third one covered twice the area of the first. The fourth area covered four times that of the first area. Twenty 400 stone composite samples were then selected. Fifty stones were obtained from the first and second areas, while 100 and 200 stones were obtained from the third and fourth areas, respectively. Figure 4.6 shows the results of one such sample and its 95% confidence intervals for each sieve size. In this case the confidence limits for each sieve size contain the true volumetric percentage. The same is true of the other samples taken and in this case exceed the 95% confidence limits. To further test the appropriateness of a composite sample and its accuracy levels another simulation was run. It was identical to the previous simulation except that each sample consisted of 80 stones. In this case, 10 stones were taken from areas 1 and 2, while 20 and 40 stones were taken from areas 3 and 4, respectively. As before, the 95% confidence limits for the percentage retained at each sieve size were calculated for all 20 samples. The results show at least 95% of the time these confidence limits contain the true volumetric percentage. Figure 4.7 shows an example of one of these samples and its 95% confidence limits along with the overall volumetric distribution for the area.

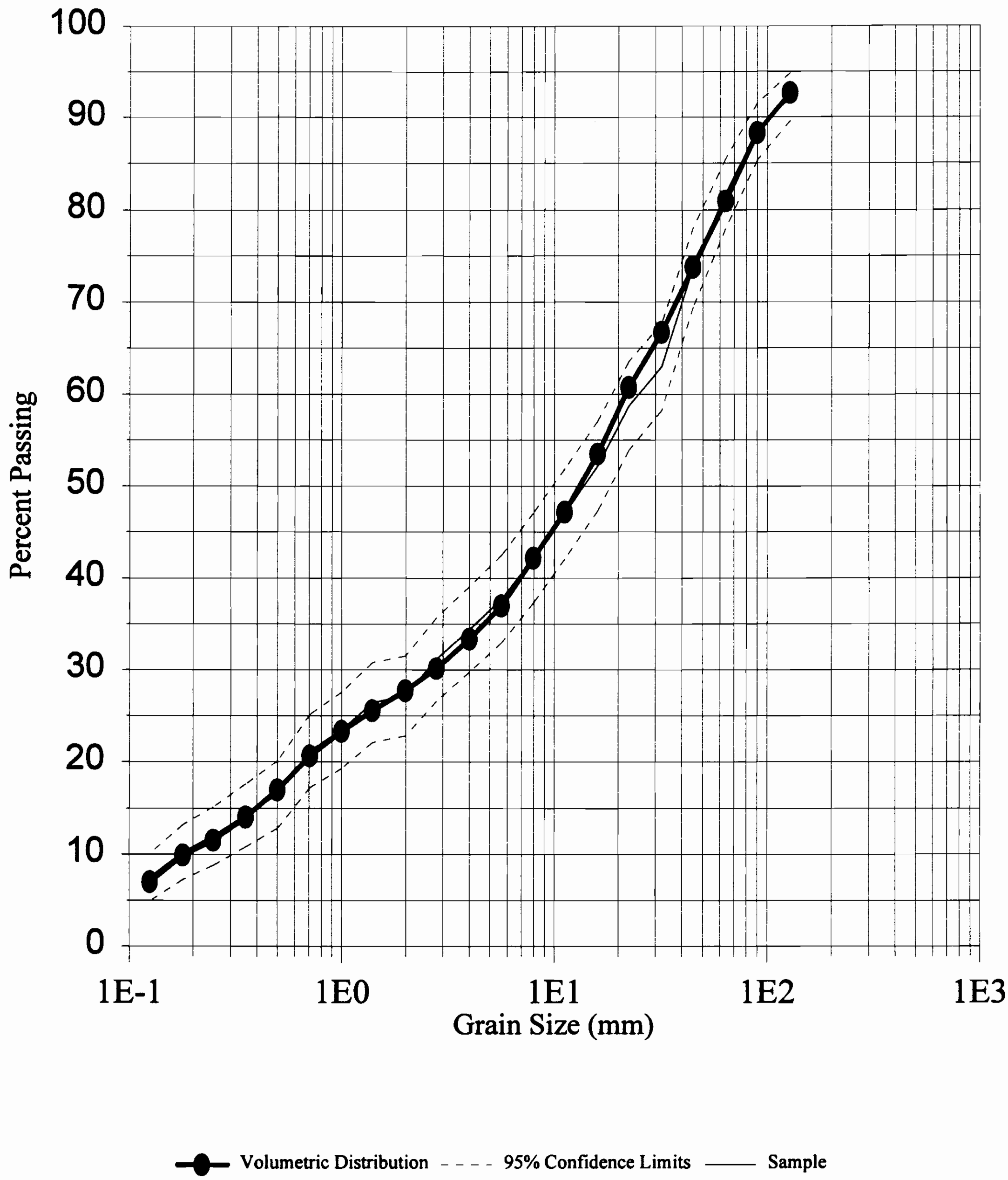


Figure 4.6: 95% Confidence Limits of a 400 Stone Composite Sample Coming from an Area having Four Distinct Sediment Deposits of Various Sizes, each of which was sampled in proportion to its area

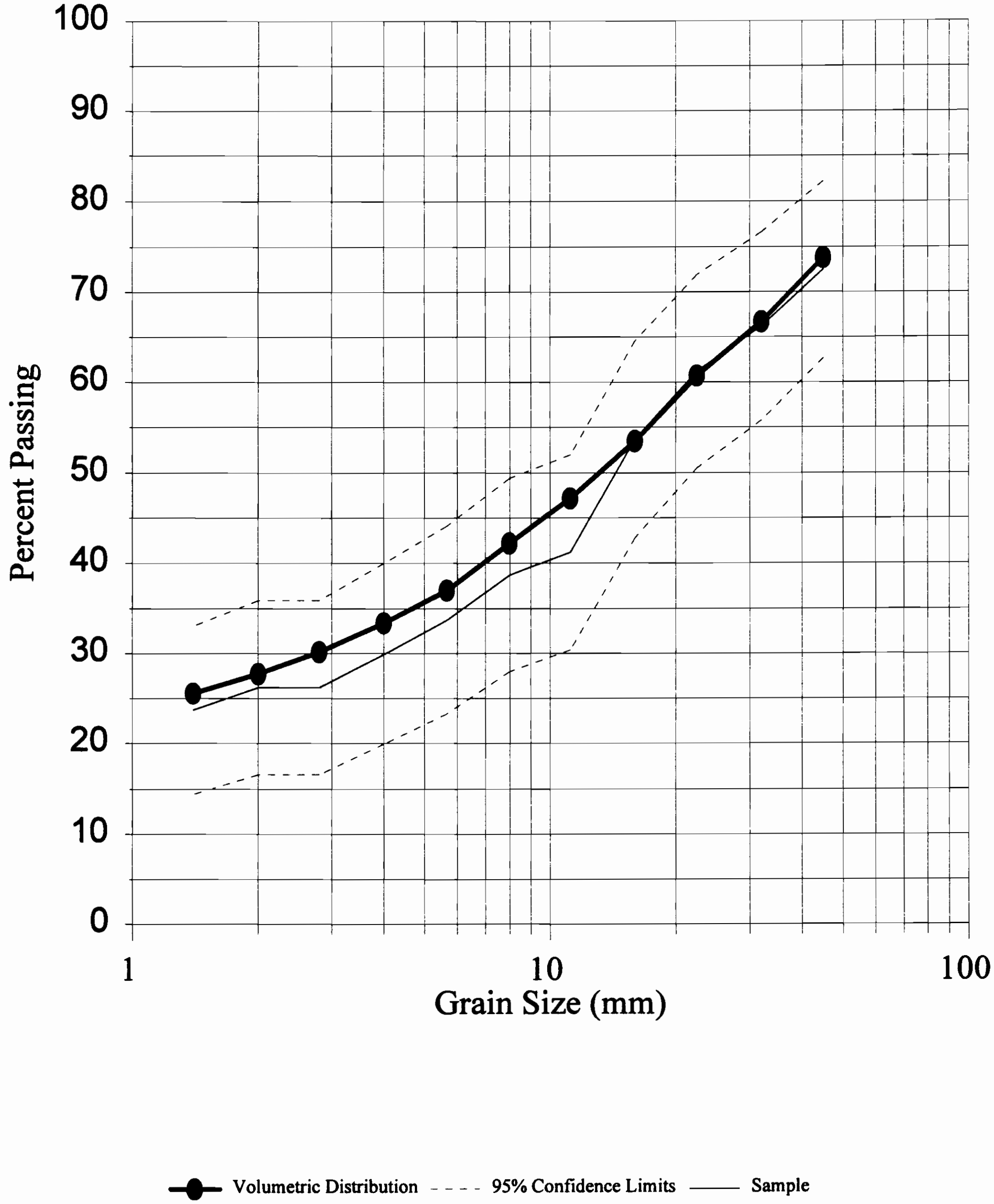


Figure 4.7: 95% Confidence Limits of an 80 Stone Composite Sample Coming from an Area having Four Distinct Sediment Deposits of Various Sizes, each of which was sampled in proportion to its area

To emphasize the bias in sampling each distribution unproportionally, the same areas and distributions as in the previous two simulations were used. In this case, however, the same amount of stones (100) were collected from each area. Of the twenty samples taken none of the confidence intervals calculated contained the true volumetric percentile of the material over the range valid for the binomial approximation. An example of one such sample is shown in Figure 4.8.

The computer simulations just described show that if a region consists of several subareas, each having a different particle size distribution, is sampled so that each subarea is sampled in proportion to the area it covers the resulting sample represents the overall size distribution with an accuracy similar to that predicted by the binomial method outlined in Fripp and Diplas (1993). This makes sense intuitively since as a whole the composite sample is unaware of boundaries within the sampled area and would obtain the same results as if the materials in each separate deposit were thoroughly mixed together and spread uniformly over the same area and then sampled. Likewise, since the accuracy of a thoroughly mixed deposit is solely dependent on the number of stones taken so is that of a composite sample. Recognizing this and the fact that Wolcott and Church (1991) have shown that taking samples systematically or at grid points does in fact sample each strata in proportion to its area, allows one to obtain an estimate of the overall sediment size distribution simply by laying out a grid over a region (despite having several strata) and then collecting a stone at each grid point as done in the Wolman walk.

The resulting sample will sample each deposit in proportion to its area and have an accuracy level set forth by Fripp and Diplas' (1993) criteria.

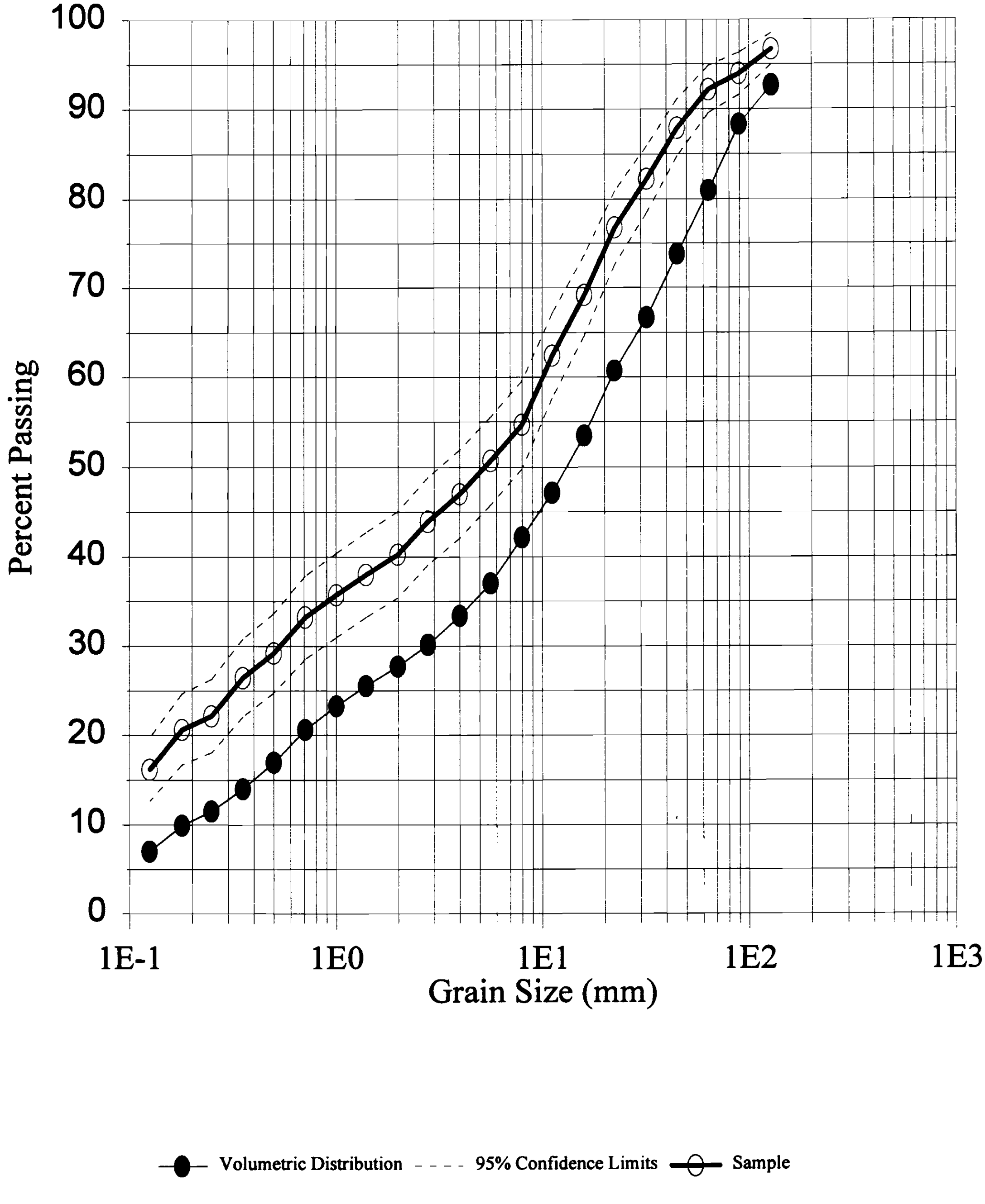


Figure 4.8: 95% Confidence Limits of a 400 Stone Sample Obtained from collecting 100 Stones from each of Four Sediment Deposits Without Regard to the Individual Areas

CHAPTER 5: MAPPING LOCAL VARIATION WITHIN A SAMPLED AREA

5.1 Introduction

Obtaining a grain size distribution characteristic of an entire area with a composite sample as described in chapter 4 is useful in itself however, the resulting distribution provides no information about the spatial variation within the area which is also important to describe. A definite improvement would be to locate distinct boundaries between individual sediments and describe the variation of grain sizes within the area sampled as a single number may mask the actual condition of the river bed. For example, in salmon spawning grounds it may be important to describe how the mean grain size changes from place to place within a river reach to determine what portions, if any, of the area are suitable for spawning. Similarly, a study of sediment transport rates might benefit from a description of how a specific grain size parameter such as D_{10}, D_{50}, or D_{90} changes throughout a region. As it has been shown by Seal, Parker and Paola (1995), patchiness or spatial variation influences both the quantity and the character of bedload transport. The reason for this is that different sediment deposits tend to become active at different flow stages. If a stream bed is exposed, sediment boundaries can be visually located as done by Wolcott and Church (1991), and Mosley and Tindale (1985); whereupon samples can be taken within each deposit to record these changes. However, this becomes difficult, if not impossible, to do when the material to be sampled is underwater. The next few sections propose a method capable of locating sediment deposit boundaries and describing changes in a particular grain size parameter without resorting to a visual analysis of the area. In fact, the only information required is that which is obtained in a composite grid sample. Therefore, this technique can be used on submerged deposits as well as exposed surfaces. More specifically, the composite sample is broken up into small subsamples which are used to estimate a specified parameter at a point within the sampled area. Based on these results statistical hypothesis tests can then be used to determine if the parameters of two subsamples come from the same population distribution. If the two subsamples of interest are statistically different, a moving window procedure, which will be described later, can be used to estimate the exact location of the sediment boundary between the two subsamples. By systematically doing this throughout the area one may locate all of the sediment boundaries, whereupon one can analyze the stones falling within a given sediment deposit as a grid sample characteristic of that particular deposit and determine any given grain size parameter (D_{10}, D_{50}, D_{90}, etc) for that deposit. The accuracy of such parameters would of course be described approximately with the binomial method as described in the last chapter. A more detailed description of this process, its limitations, computer simulation results, and a description of how this may be used in the field, are given in this chapter.

5.2 Identifying Distinct Sediment Deposits by Using Subsamples

As previously mentioned, Wolcott and Church (1991) visually identified four distinct sediment deposits on the depositional bar (shown in Figure 1.3) they sampled. As the boundaries were identified visually, it can be assumed that the sediment changes were sharp and represent significantly different sediment deposits, while the material within each deposit was fairly homogenous and did not exhibit noticeable spatial variability. This suggests that samples taken from within the same deposit should have similar distributions and grain size parameters, while samples of different deposits should have noticeably different grain size distributions and grain size parameters. Thus, plotting the value of a specific grain size parameter for each sample should produce a plot where the sample parameters cluster around the true values of that parameter for each deposit. In other words, the sample values coming from a particular deposit should fall close to the true value of the parameter for that deposit, while the values of samples from different deposits should fall close to the true value for their deposit. If each deposit is distinct, these clusters will be distinct as well. Hence, a simple but useful map could be obtained by simply color coding each cluster (representing each sediment deposit present) and coloring the area around each sample point according to its deposit type.

An important concern in this procedure is that of choosing an appropriate parameter to classify the different deposits. Ideally, entire grain size distributions should be considered. Unfortunately, Figure 5.1 shows that the grain size distributions obtained by Wolcott and Church (1991) do not fall into four distinct sediment deposits. However, an important observation of the 46 volumetric samples collected by Wolcott and Church (1991) is that there is a clear correlation between various sample grain size parameters and their standard deviations. These correlations tend to separate sample values into distinct categories representing the different deposits present. Later, it will be shown how these trends can be used to determine sediment boundaries with a statistical hypothesis test, but first a description of these trends as well as an example of how a simple, map of the sampled area can be obtained from this information is provided.

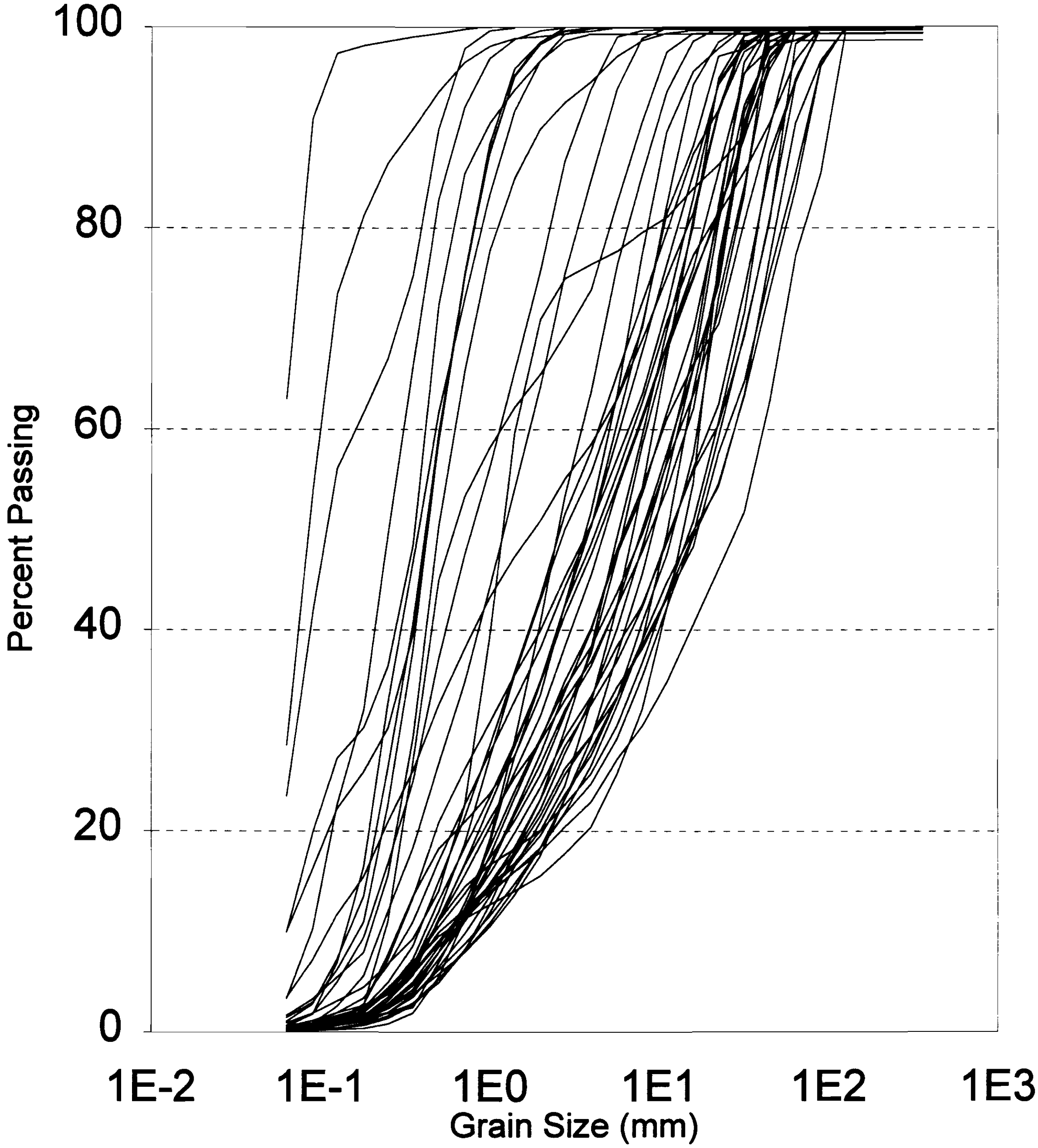


Figure 5.1: The 46 Sample Grain Size Distributions taken from Wolcott and Church's Quesnel River Bar

Figure 5.2 shows a plot of the 46 sample arithmetic means and their standard deviations obtained by Wolcott and Church (1991). As one can see, the standard deviation increases with the mean grain size in a linear fashion. Church and Kellerhals (1978) obtained similar results for 78 grid samples taken from the Peace River in British Columbia. In fact, linear regression analysis shows a correlation coefficient (r^2) of 0.97 for the 46 samples. This trend also holds true for a sample's D_{10}, D_{50}, and D_{90}. Figures 5.3, 5.4, and 5.5 show the relationships between a samples arithmetic standard deviation and its D_{10}, D_{50} and D_{90}, respectively. Figure 5.6 similarly shows the relationship between a sample's geometric mean and geometric standard deviation. Table 5.1 shows the results of linear regression through each of these plots.

A comparison of these trends shows that the correlation coefficient increases with the grain size parameter. That is, D_{10} has a much lower correlation coefficient than does D_{90}, while D_{50} is somewhere in between. This implies that most sediments contain finer particle sizes regardless of their composition and thus have similar D_{10}'s and is clearly seen in Figure 5.1. Hence, D_{10} is probably not a particularly good parameter to use in characterizing a deposit. The plots further confirm this assumption in that D_{10} has the lowest correlation coefficient (0.594) of these plots and that the D_{10}'s do not vary much in size (from 0.07 mm to 1.32 mm). Furthermore, it is clearly difficult to select groups of points that correspond to the individual deposits from this plot. On the other hand, the D_{90} versus standard deviation plot has a correlation coefficient of 0.992, and the D_{90}'s sizes vary dramatically (from 0.12 mm to 144 mm). Hence, we see that the standard deviation is almost solely driven by the largest particle size present and the sample points tend to cluster into more distinct groups. Thus, D_{90} seems to be a more suitable choice to describe a deposit. Likewise, Figure 20 shows that the a plot of D_{50} versus standard deviation is more successful in separating the different deposits than using D_{10}, but not as well as D_{90} or the arithmetic mean.

Thus one can conclude that when selecting a single grain size parameter to describe the overall coarseness of a sediment it should be taken from the upper half of the size distribution preferably D_{90} or higher as it gives the best correlation between grain size and the standard deviation. The reason for this is that the coarsest grain size present drives the standard deviation due to the fact the smaller particle sizes remain about the same. However, it is possible that in a more complicated sediment deposit, the smaller particle sizes may change dramatically while the larger sizes do not. Thus, D_{10} would probably be more appropriate to characterize the sediments present. Similarly, if both the D_{10} and D_{90} were changing from deposit to deposit neither D_{10} or D_{90} would be particularly preferable. Therefore, it is probably best to use a parameter that is influenced by both the larger particles and the smaller particles such as the arithmetic mean or geometric mean. Fortunately, as mentioned earlier, a plot of the arithmetic mean versus standard deviation resulted in a correlation coefficient of 0.97 and separates sample points into fairly distinctive groups. Using a plot of the geometric mean versus geometric standard deviation (shown in Figure 5.6), however, resulted in a much lower correlation coefficient and a plot in which the sediment groups are difficult to locate. It should be noted that the geometric mean and geometric standard deviations were calculated through the following equations:

$$D_g = D_1^{f_1} \times D_2^{f_2} \ldots D_n^{f_n} \tag{8}$$

$$\sigma_g = \exp[\sum(\ln\frac{D_i}{D_g})^2 \times f_i]^{1/2} \tag{9}$$

where D_g and σ_g are the geometric mean and geometric standard deviations, respectively, D_i is the geometric mean grain size between sieve sizes, and f_i is the probability of occurence of D_i.

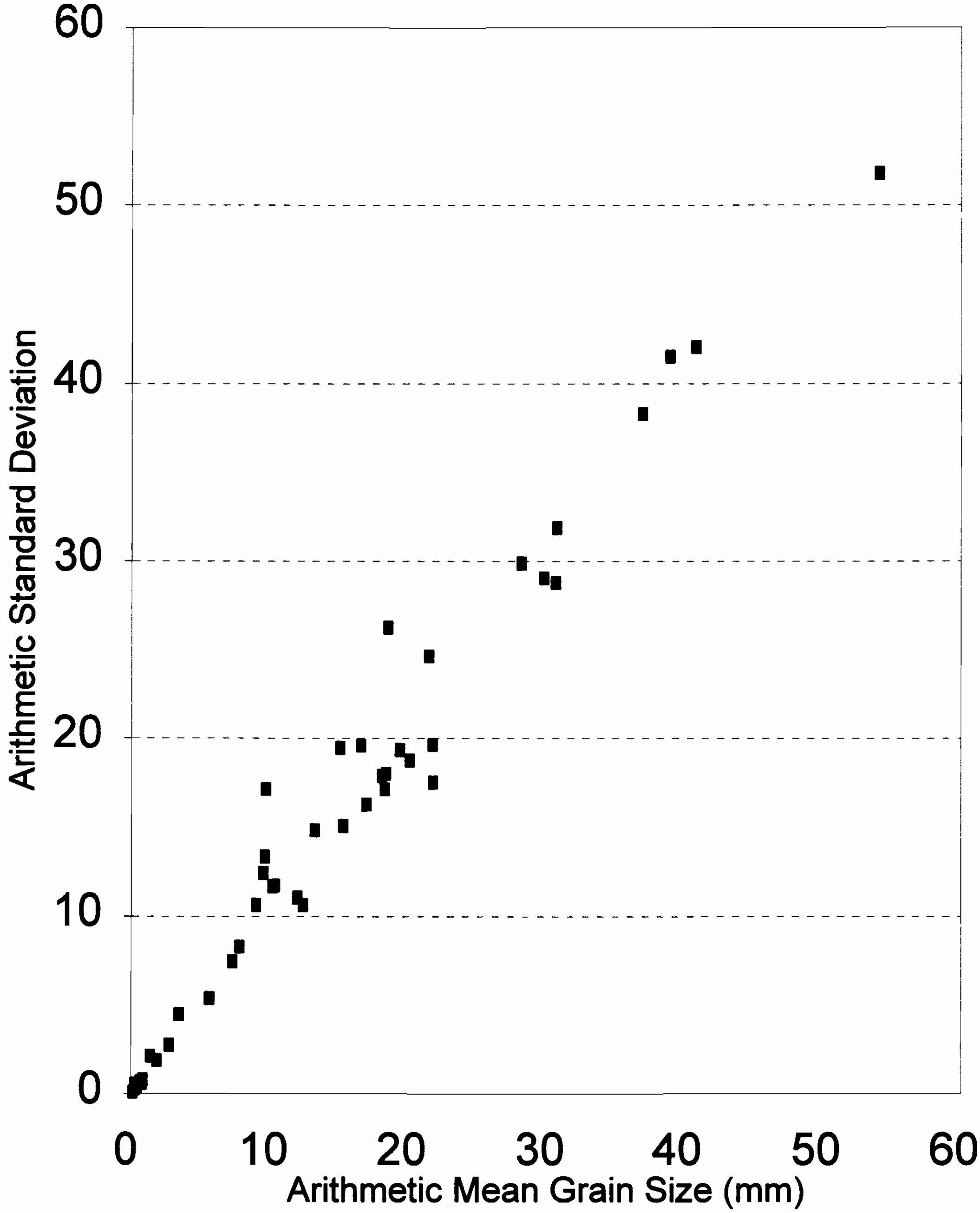


Figure 5.2: The Results of Plotting the Arithmetic Mean Grain Size Against the Arithmetic Standard Deviation of 46 Bulk Samples

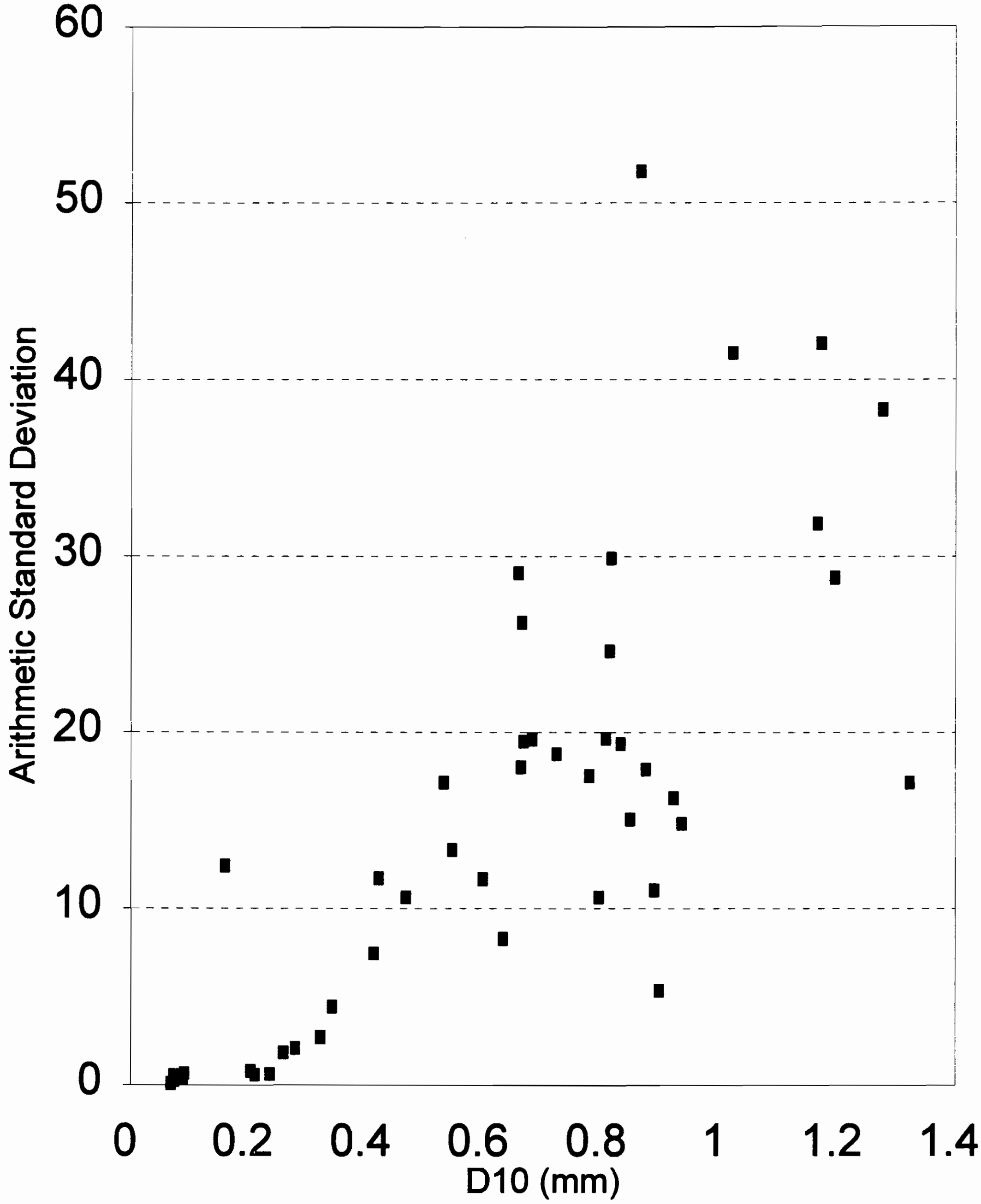


Figure 5.3: The Results of Plotting D_{10} Against the Arithmetic Standard Deviation of the 46 Samples

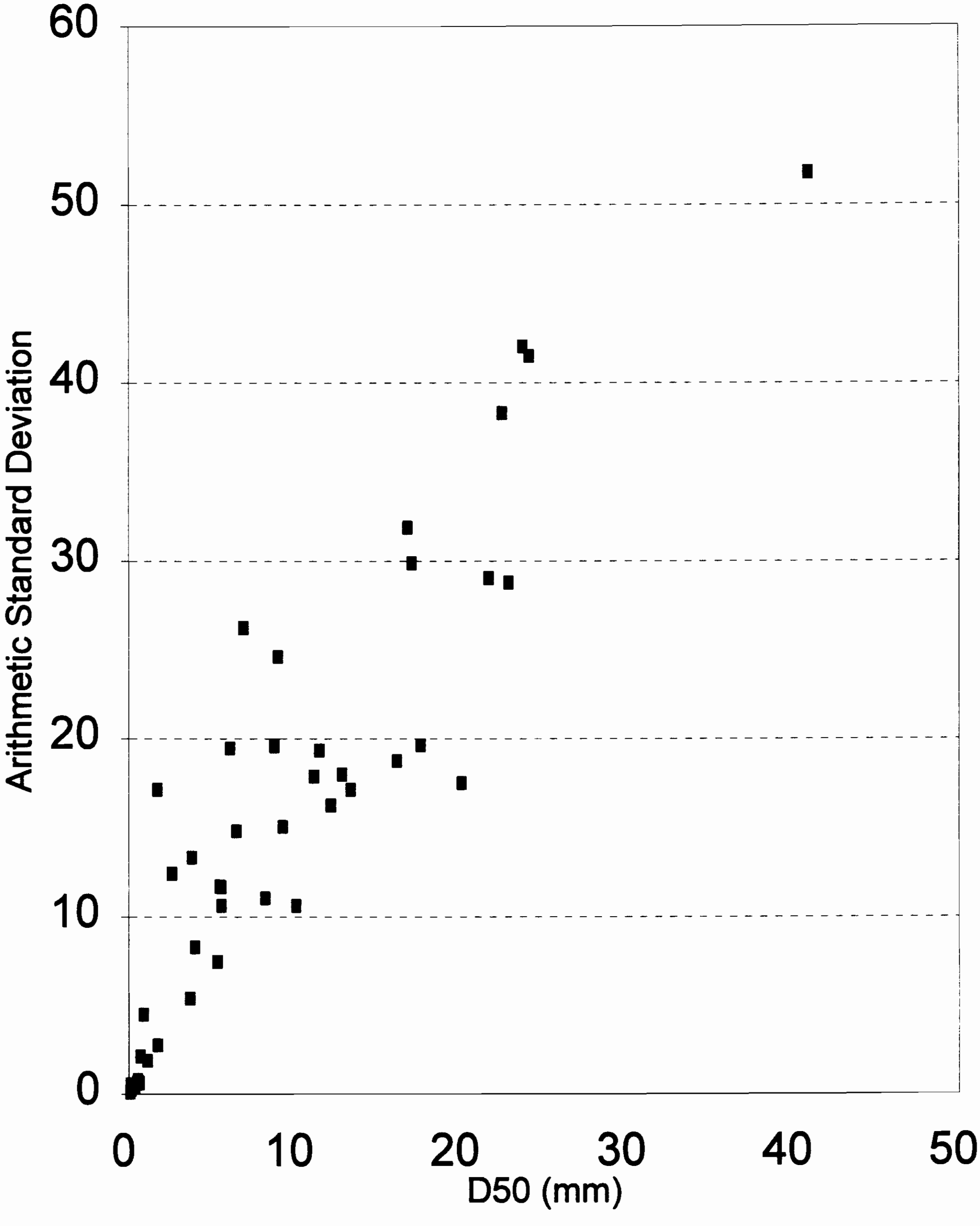


Figure 5.4: The Results of Plotting D_{50} Against the Arithmetic Standard Deviation of the 46 Samples

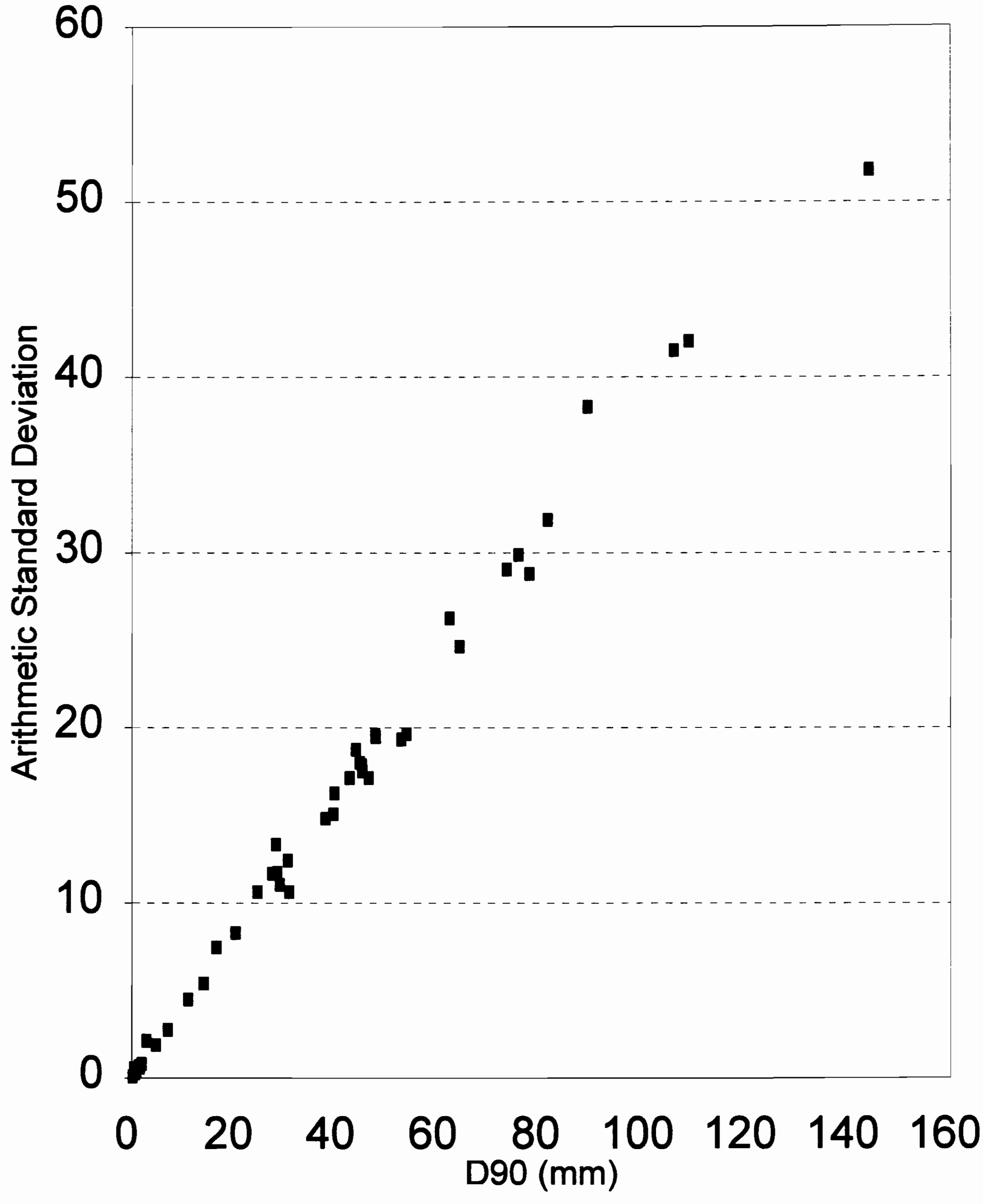


Figure 5.5: The Results of Plotting D_{90} Against the Arithmetic Standard Deviation of the 46 Samples

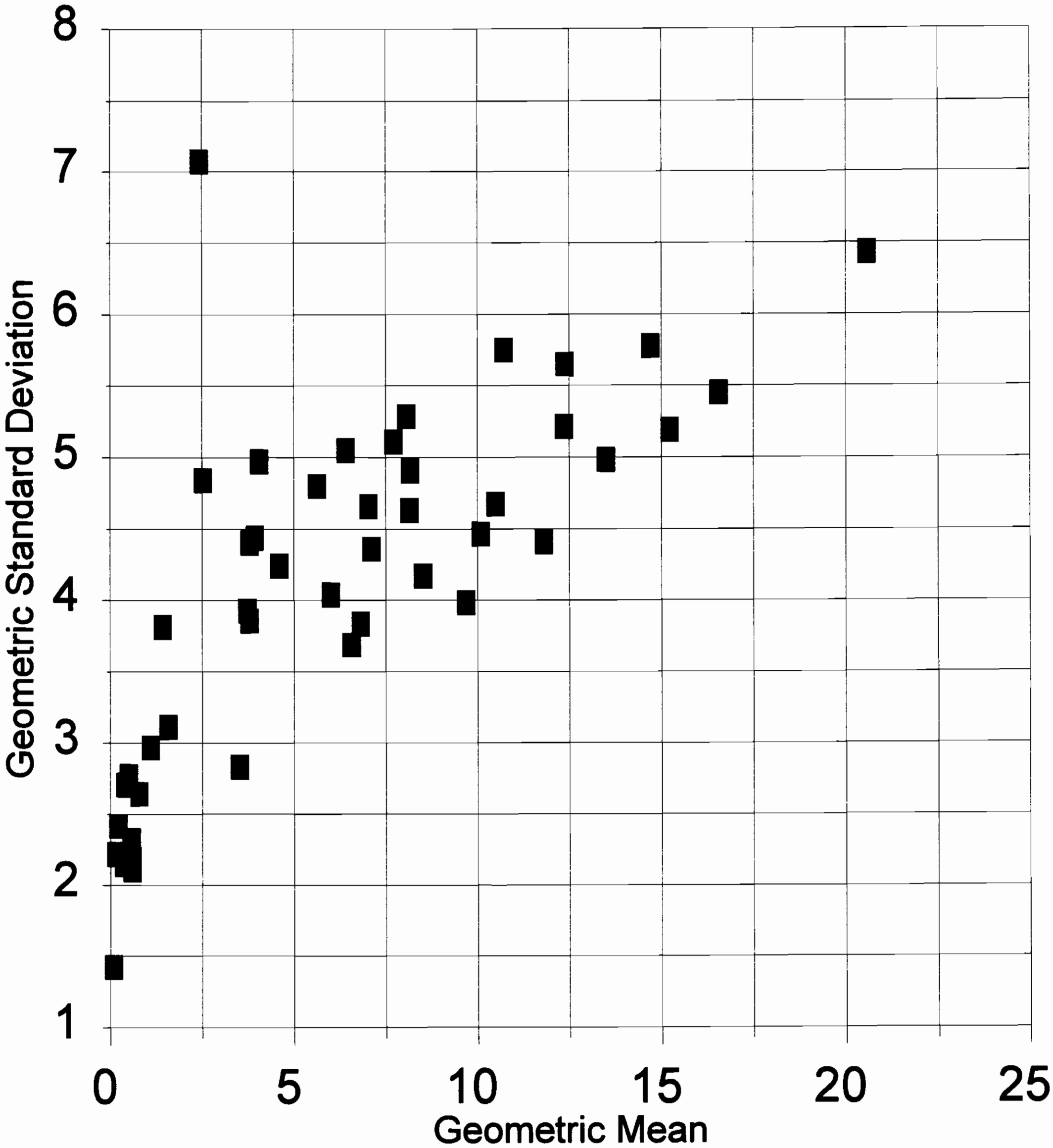


Figure 5.6: The Results of Plotting the Geometric Mean Grain Size Against the Geometric Standard Deviation of 46 Bulk Samples

Table 5.1 Regression Results for Plots in Figures 5.2, 5.3, 5.4, 5.5, and 5.6

D10 Vs. Standard Deviation (Figure 5.3)

Regression Output:		
Constant		-1.88894
Std Err of Y Est		8.226264
R Squared		0.593951
No. of Observations		46
Degrees of Freedom		44
X Coefficient(s)	27.52374	
Std Err of Coef.	3.4308	

D90 Vs. Standard Deviation (Figure 5.5)

Regression Output:		
Constant		0.572969
Std Err of Y Est		1.130074
R Squared		0.992337
No. of Observations		46
Degrees of Freedom		44
X Coefficient(s)	0.381265	
Std Err of Coef.	0.005051	

D50 Vs. Standard Deviation (Figure 5.4)

Regression Output:		
Constant		3.348263
Std Err of Y Est		5.336007
R Squared		0.829153
No. of Observations		46
Degrees of Freedom		44
X Coefficient(s)	1.301989	
Std Err of Coef.	0.089098	

Arithmetic Mean Vs. Standard Deviation (Figure 5.2)

Regression Output:		
Constant		0.817958
Std Err of Y Est		2.23486
R Squared		0.970031
No. of Observations		46
Degrees of Freedom		44
X Coefficient(s)	0.988934	
Std Err of Coef.	0.026205	

Geometric Mean vs. Geometric Standard Deviation (Figure 5.6)

Regression Output:		
Constant		2.994281
Std Err of Y Est		0.864046
R Squared		0.54843
No. of Observations		46
Degrees of Freedom		44
X Coefficient(s)	0.18354	1
Std Err of Coef.	0.025108	

It would therefore seem that if one is attempting to identify changes in sediment types using a single grain size it should be the arithmetic mean as it provides a very high correlation coefficient yet is influenced by both the larger and smaller particle parameters regardless of how they may change. Another advantage in using the arithmetic mean in characterizing a sediment is that it requires less sampling effort to compare the two means of two different sediments than it does their D_{90}'s. The reason for this is that according to equation (1) 200 stones must be collected in order to obtain the D_{90} within a deposit with any reliability, thus a total of 400 stones would be needed to compare just two sediment types. Fortunately, according to the central limit theorem we can quantify the mean of a deposit regardless of distribution with as little as 30 stones (Weiss and Hassett, 1991). Therefore, only 60 stones would be required to sample two different areas before a comparison could be made.

The appropriateness of separating samples into their deposit types according to their mean and standard deviation can be tested by comparing where each sample point in Figure 5.2 came from in respect to the depositional bar sampled by Wolcott and Church (1991) and shown in Figure 1.3. In this case, one sees that samples coming from the same deposit generally group together and have similar means and standard deviations. More specifically, Figure 5.2 appears to have about 6 groups or clusters of points. This suggests 6 different types of material present in the area sampled, whereas Wolcott and Church (1991) identified only 4. However, two of these groups appear to be outliers and consist of only 4 of the 46 samples. Table 5.2 shows how the 6 groups identified in Figure 5.2 compare to the 4 visually identified sediment types located by Wolcott and Church (1991).

Table 5.2: Comparison of Grouped Samples to Their Actual Deposit Types

Group # (from plot)	Sample Numbers in Group (and range of arithmetic means)	Corresponding Strata (from map)
I	41,38,34,18,33,35,27,28,19, 42, 45,46,20,43 (.10-5.56 mm)	A, A, A, A, A, A, A, A, C, B, A, B, B, B
II	1,8,30,37,36,26,39,32,23,15, 40 (7.00-13.25 mm)	D, B, B, B, B, B, C, B C, C, B
III	5,31,44,6,24,9,7,16, 11,2,13,12,17 (15.00-21.83 mm)	C, B, C, C, B, C, C, B, C, D, C, C, B
IV	3, 25, 21, 22 (28.21-30.76 mm)	D, B/C, D, C/B
V	4, 14, 29 (37.00-40.87 mm)	C/D, C, D
VI	10 (54.00 mm)	C

Table 5.2 designates the 4 different materials identified by Wolcott and Church (1991) as deposits A, B, C, and D, corresponding to sand, fine gravel, medium gravel, and coarse gravel, respectively. Table 5.2 does indeed show that groups I - IV correspond quite closely to the visually identified strata. Groups V and VI have arithmetic means substantially larger than the rest of the material suggesting that they are anomalies or are visually undistinguishable from the other coarse gravels present. Overall, these results of Figure 5.2 predict the 4 different strata very well. In fact, in some cases, the groupings work better than assuming everything within a visually identified strata is uniform. For example, sample 19 has a mean of .82 mm and a D_{90} of 1.32 mm and should be classified as a sand and not a medium gravel. Elsewhere, one sees sample 10, or group VI, having the coarsest material by far classified as a medium gravel. This occurs several times and shows the extreme variability within visually identified strata.

While the samples taken by Wolcott and Church are bulk samples, one can easily extend this procedure to grid subsamples. For example, a composite grid sample, despite being a single sample, can easily be reduced into smaller local grid samples or subsamples. This is done by recording the positions and size of each particle with a tape measure and a gravelometer while collecting the composite sample just as an ordinary grid sample is collected and then subdividing the sampled area into rectangles each containing a specified number of stones or one subsample. Each subsample would have its own particle size distribution and could be compared to the size distributions of other subsamples. Plotting a parameter of interest such as D_{10}, D_{50}, or D_{90} in the center of each subsample would then provide an idea how this parameter changes throughout the area. The subsamples can then be color coded according to their deposit types, which are obtained from plotting the subsamples' arithmetic means and standard deviations as just described. The result would be a simple map of the region capable of roughly defining sediment deposits as well as sediment anomalies in what otherwise might be a fairly uniform strata.

There are drawbacks to this approach. The first is that it is subjective when determining groups. However, use of a cluster analysis such as described by Davis (1986) to reduce this subjectivity may be possible. The other drawback is that boundaries between strata are not precise or distinct. The reason for this is that boundaries will follow the edges of the subsamples creating only horizontal and vertical boundaries. A diagonal boundary would be shown as a zig-zagging line. Another problem is that, if a subsample contains stones from two strata, the resulting subsample may be classified as a third strata halfway between the two strata present. While this may not be acceptable in some cases, it is a quick, simple technique that provides a map of the variability in material found throughout a region without having to resort to pictures and visual classifications of the deposits.

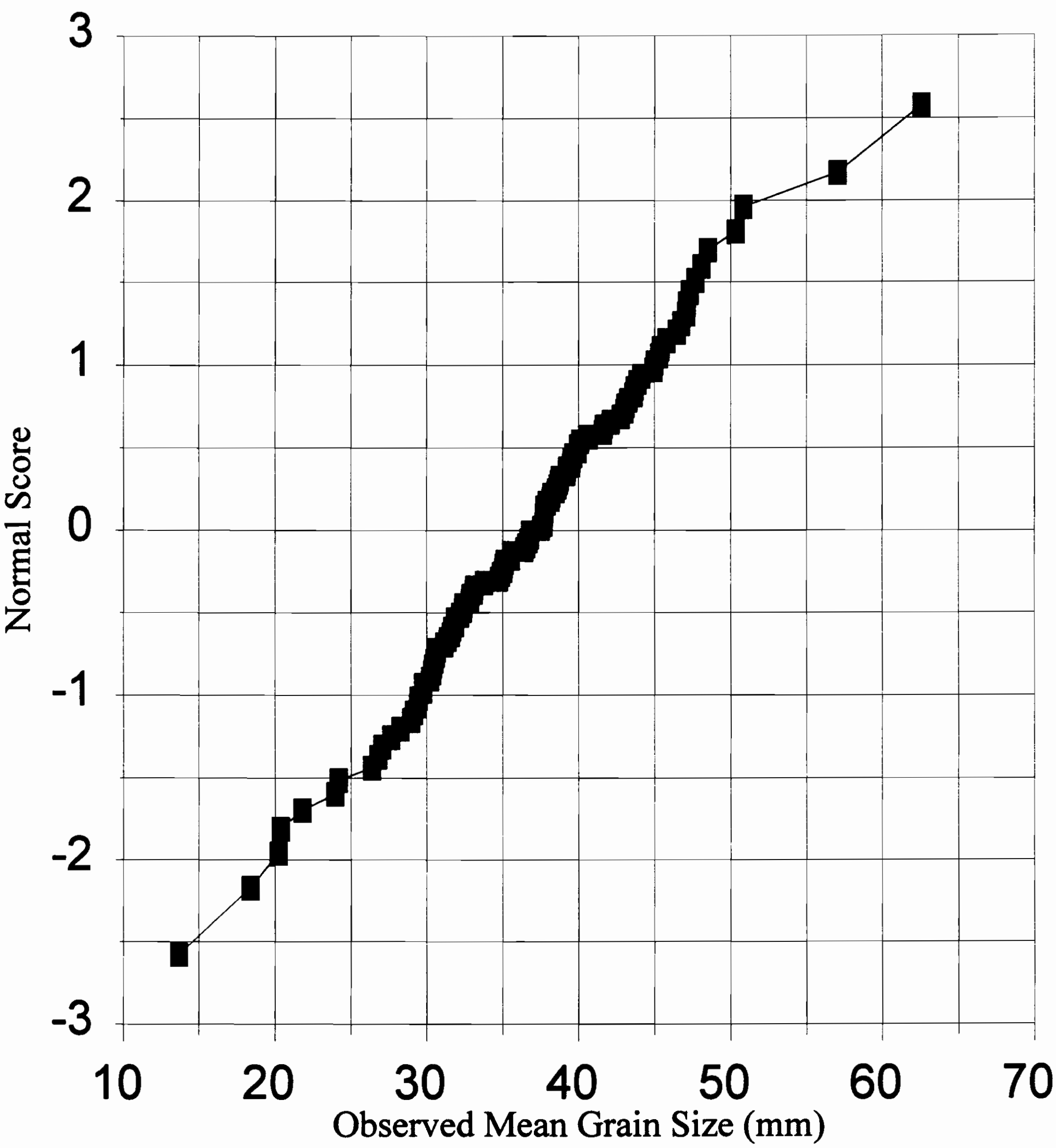
5.2.1 Statistical Comparison of Subsamples

An alternative to identifying groups from a plot of the subsamples' arithmetic means and standard deviations is to compare two subsamples using statistical hypothesis testing. Statistical hypothesis testing overcomes the subjective nature of choosing deposit types based on the clustering of sample points and determines whether or not two sediment subsamples come from the same grain size distribution or deposit. If two subsamples are statistically different, then a change in sediment types occurs and a sediment boundary exists somewhere between the two subsamples compared. As in the last section, this method proposes that different sediments can be adequately differentiated by comparing their arithmetic means and standard deviations. Therefore, this section describes some aspects of hypothesis testing with the arithmetic means of subsamples. Since, hypothesis tests are not limited to comparing the arithmetic means of subsamples, but can be applied to any grain size parameter, a brief description of these alternatives is provided as well.

As previously mentioned, a more precise way of determining whether two sediment subsamples come from the same population is to statistically compare their arithmetic means. The exact comparison between samples is performed just as found in many statistical books such as one written by Weiss and Hassett (1991). The procedure assumes that the samples are independent of each other and that the samples are large (30 or more items according to Weiss and Hassett (1991)). Since the stones selected in one subsample do not affect what stones are selected in another subsample, the subsamples are independent of each other. Therefore, if each of the subsamples contains at least 30 stones, the condition of taking large samples is met, and one can perform a null hypothesis test. The null hypothesis test assumes that the samples have the same mean. If the test statistic falls into the rejection region, we accept the alternative hypothesis which is that the two samples come from different populations. The test statistic is calculated as follows:

$$z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}} \tag{8}$$

where the subscripts refer to the first and second subsamples, $\bar{x}$ is the subsample's arithmetic mean, s is its arithmetic standard deviation and n represents the number of stones in the subsample. For a two-tailed test at a significance level $\alpha = .10$, the test statistic should be larger than 1.645 or less than - 1.645 to conclude that the two subsamples come from different populations. As this equation assumes that the means are normally distributed, this assumption should be tested. By taking a large sample, 30 or more stones according to Weiss and Hasset (1991), one can assume that the means are normally distributed because of the central limit theorem which states: "For a relatively large sample size, the random variable $\bar{x}$ is approximately normally distributed, regardless of how the population is distributed. The approximation becomes better and better with increasing sample size". However, as 30 stones is only a rule of thumb, a normality plot of 100 sample means (obtained from computer simulations) was made to check the appropriateness of this number. Each sample consisted of 20 stones and was obtained from a distribution having a median grain size of 16.1 mm. The resulting plot is shown in Figure 5.7. The points show a linear trend and indicate that the arithmetic means are normally distributed even when samples contain as few as 20 stones. Linear regression shows a correlation coefficient of .98 Likewise, a normality plot was made from 100 composite samples having stones from four different grain size distributions. Each composite sample had 40 stones and had a correlation coefficient of .98 as well. Thus, it appears that a normality assumption is appropriate when each sample consists of at least 30 stones and hypothesis testing can be used to determine if subsamples come from the same deposit or not. Furthermore, if all adjacent subsamples are compared and boundaries are placed between subsamples that have statistically significantly different means, one obtains approximate locations of the boundaries between strata. Unfortunately, this procedure, like the previous method, does not locate a boundary that lies within a subsample and provides only vertical and horizontal boundaries.



Median Grain Size is 16 mm

Figure 5.7: The Normality Plot of the Arithmetic Means resulting from taking 100 Grid Samples, Each of which Consisted of 20 Stones

This procedure, however, is not limited to mean hypothesis testing, but can be used to compare whether the proportion of material passing a particular sieve size for two subsamples are statistically the same or whether they are from different distributions and hence different deposits. An example of this method can be found in statistical books such as Weiss and Hassett (1991). However, as one shall see in a later section, using arithmetic means in hypothesis testing provides a simple means of estimating the intensity in which an area has to be sampled to locate sediment boundaries and is easy to program.

5.2.2 Locating Sediment Boundaries with Moving Windows

The previous sections have described two methods of mapping the sediment changes within a sampled area. The first one used a rather subjective grouping of samples to define the different sediment types present while the second approach used statistical hypothesis tests to ascertain whether two grid subsamples belong to the same deposit type or not. In both cases a sediment boundary could only be placed along one of the edges of the subsamples. Unfortunately, a subsample will more likely sample two deposits at the same time, and the boundary will be somewhere within the subsample. This section proposes a method which builds on the principles of hypothesis testing and locates sediment boundaries more accurately.

As before, the composite grid sample of the sampled area is broken up into subsamples containing equal number of stones. The subsamples sizes should be chosen in such a manner that a subsample containing a sediment boundary does not have adjacent subsamples which contain boundaries. This condition, as will be shown, allows one to use a moving window scheme to locate boundaries throughout a region. An expanded discussion of appropriate window and subsample sizes will be given in a later section, but first the proposed moving window method should be described.

First, the moving window method divides the sampled region into subsamples having dimensions x and y. The subsample's means are then calculated. Next, test statistics are calculated for three adjacent subsamples that are either in a row or a column. The test statistics are referred to as z_1, z_2, and z_3 and result from statistically comparing the mean's of the first and second subsample, second and third subsample, and the first and third subsample as shown in equation (10). If any of these statistics is significant then according to the null hypothesis test a boundary is present. More specifically, if z_1 indicates a strata change, then the boundary lies somewhere in the first or second windows. Likewise, a significant z_2 shows that a boundary exists somewhere in subsample 2 or subsample 3. However, if z_1 and z_2 do not indicate boundaries it does not necessarily mean there is no change in the population between subsamples 1 and 3. The reason for this is that subsample 2 could be a subsample containing materials from two deposits but similar enough to both of them that it is not statistically different from subsamples 1 or 3. In other words, if the 2nd subsample contains stones from two different deposits, the resulting mean may not be statistically different from subsample 1 or subsample 3 while subsamples 1 and 3 are and a boundary exists somewhere in subsample 2. Therefore it is necessary to compute z_3 as well as z_1 and z_2 to locate sharp boundaries.

As previously mentioned, the z-statistics indicate a boundary between two subsamples but do not locate it more precisely than that. However, by placing two moving windows side by side and moving them in small increments over the area containing the boundary allows us to estimate the location of the boundary more precisely. Specifically, the boundary will occur where the arithmetic means of the moving windows have the greatest difference. Figure 5.8 shows a simplified example of this procedure. Figure 5.8a shows two subsamples which represent statistically different strata along with material surrounding them. In this case, we assume that z_1 is significant and know that the boundary is somewhere within subsample 1 or subsample 2 so the moving windows (windows A and B) are placed (as shown in Figure 5.8b) such that window A's right side and window B's left side are located one unit from subsample 1's left border. The difference of the window means is then recorded as shown at the bottom of Figure 5.8c. The windows are then moved in one step increments to the right until window A's edge is one unit from subsamples 2's right edge as depicted in figure 5.8c. The location of the boundary is at the position where largest difference is obtained.

In this case the boundary occurs at position 9 or one stone into subsample 2. If z_2

is significant, the boundary can be found with the same procedure except that the right border of window A is placed one unit right of subsample 2 instead of one unit right of subsample 1. The ending position should be 1 unit left of subsample 4. The third option, as discussed earlier, is that z_3 may be significant while z_1 and z_2 are not. This condition indicates a boundary exists somewhere in subsample 2. As before, this boundary can be located by moving windows A and B in small increments. In this case the moving window A's right border should begin one unit right of subsample 2, however unlike the previous two cases the ending position is one unit left of subsample 3.

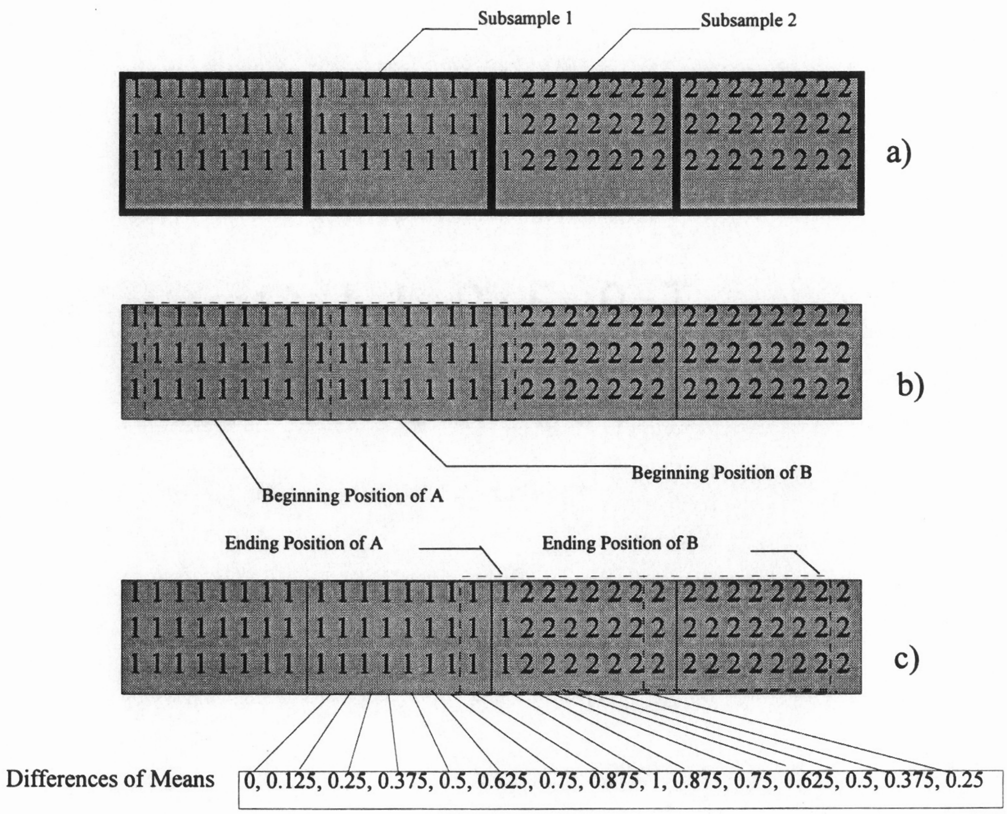


Figure 5.8: a) Depiction of Two Subsamples that are Statistically Different
b) Original Placement of Windows A and B which are used to locate the boundary more precisely
c) Ending Positions of Windows A and B used to locate the Boundary and their differences in means as they incrementally moved across the area

Now that a method to locate a sharp boundary among sediment deposits has been developed, one may systematically analyze an entire area to locate such boundaries. This can be done by analyzing three subsamples at a time beginning at the upper left of the area sampled. For simplicity, call the first of the three subsamples being compared the base subsample. If no boundaries are located, the second subsample becomes the base subsample and is compared with next two subsamples to see if a boundary exists among them. If, however, a boundary exists, it is located and the base window becomes the window immediately to the right of the boundary. This process is then repeated from left to right over the area and for each row of subsamples present. The process is then repeated moving down each column of the area. Thus, both horizontal and vertical boundaries can be located and diagonal boundaries, if correctly located, will appear as alternate horizontal and vertical lines much like steps.

5.2.3 Subsample Sizes and the Ability to Locate Sediment Boundaries

The number of stones making up a subsample plays an important role in the moving window method and should be expanded upon. As it was described in the last section, a minimum of 30 stones must be used if the means of two subsamples are to be compared using statistical hypothesis testing. However, if two sediment deposits form a boundary and have nearly the same arithmetic mean grain size, 30 stone subsamples may not be able to distinguish a difference between these deposits and larger subsamples must be acquired. This section expands upon the issue of subsample size and the reliability with which statistical hypothesis testing can be used to locate sediment boundaries and proposes a method in which a researcher can determine the number of stones that must be in the subsamples to locate a certain amount of change in the arithmetic mean grain size. Ccomputer simulation results testing this idea are discussed as well.

The answer to determining how many stones must be in the subsamples to locate a certain amount of change in the mean diameter lies in finding the probability that the statistical hypothesis test will correctly conclude whether or not two subsamples come from the same deposit. The assumption in our statistical hypothesis test, described earlier, is that if z is greater than 1.645 or less than -1.645 then we have a 90% chance that the two subsamples have different means (these values, however, may be changed for any desired confidence level) . This also means that there is a 10% chance of concluding that the means are different when they are not (a type I error)(Weiss and Hassett 1991). However, this does not tell us the probability of a type II error, or the probability of saying the means are the same when they are actually different (Weis and Hassett 1991). A useful way of dealing with the type II error is through the use of an operating characteristic curve. The operating characteristic curve is simply a plot which shows the type II error for several cases where the means are different (Weiss and Hassett, 1991; Hogg and Ledolter 1992). Hogg and Ledolter (1992) show a specific example of how the type II errors can be calculated for a statistical hypothesis test concerning the means of two independent samples. The means in this example are assumed to be approximately normally distributed. Since normality plots have shown the arithmetic means of gravel deposits to be approximately normally distributed when 30 or more stones are collected, this approach seems useful in determining appropriate subsample sizes. In the example provided by Hogg and Ledolter (1992), the statistical hypothesis tests whether two sample means are the same, $\mu_1 = \mu_2$, versus the alternative hypothesis that first mean is larger than the second, $\mu_1 > \mu_2$ at a significance level of .025. The probability of accepting the null hypothesis for a given θ where $\theta = (\mu_1 - \mu_2)$ is given in the following equation:

$$OC(\theta) = P(\bar{x} - \bar{y} < 1.96\sqrt{s_1^2/n_1 + s_2^2/n_2}) \tag{11}$$

where OC(θ) equals the probability of accepting the null hypothesis for a given θ and is equal to the probability, P, of obtaining a sample mean difference, $(\bar{x} - \bar{y})$, less than $1.96\sqrt{s_1^2/n_1 + s_2^2/n_2}$. However, when comparing two sediment samples it seems more appropriate to compare the null hypothesis against the alternative hypothesis $\mu_1 \neq \mu_2$. Furthermore, since computer simulations, described later, where run at a .10 significance level a $z(\alpha/2)$ value of 1.645 should be assumed rather than a $z(\alpha)$ of 1.96. As the hypothesis test is now two tailed and the significance level has changed, the probability of accepting the null hypothesis can be written as follows:

$$OC(\theta) = P(-1.645\sqrt{s_1^2/n_1+s_2^2/n_2} < \bar{x}-\bar{y} < 1.645\sqrt{s_1^2/n_1+s_2^2/n_2}) \quad (12)$$

$$= \Phi(1.645-\frac{\theta}{\sqrt{s_1^2/n_1+s_2^2/n_2}}) - \Phi(-1.645-\frac{\theta}{\sqrt{s_1^2/n_1+s_2^2/n_2}}) \quad (13)$$

where Φ equals the cumulative percentage under the normal distribution curve and can be found in standard normal distribution tables. This equation can now be used to calculate the type II error for different arithmetic mean values as well as different subsample sizes, thus an appropriate subsample size can be determined based on the amount of change in arithmetic mean grain size ($\theta = \mu_1 - \mu_2$) one is interested in locating. For example, from the 46 bulk samples obtained by Wolcott and Church (1991) one found that a linear regression analysis of the sample means and standard deviations resulted in correlation coefficient of 0.97, a slope of 0.989, and a constant of 0.818. This means that standard deviations for two sediment populations having arithmetic means of μ_1 and μ_2 can be predicted with the following formulas:

$$s_1 = m\times\mu_1 + C \quad (14)$$

$$s_2 = m \times \mu_2 + C \tag{15}$$

where m is the slope and C is the constant. By substituting these into the equation (13) we can calculate the type II error for a given subsample size and hence the reliability of locating a boundary between deposits having a difference of means θ. If this reliability is unacceptable then larger subsamples need to be taken.

This procedure was employed to estimate the type II error for various sediment deposits having differences in means of 5mm based on subsamples of 100 stones. Table 5.3 shows these results and demonstrates that as the particle sizes increase it becomes more and more difficult to identify a boundary between them. The main reason for this is that the standard deviations are increasing with the mean grain size. As one can see, the chance of accepting the null hypothesis in the last column labeled Type II Error becomes extremely large if one is trying to locate a difference in means of 5 mm for sediments having arithmetic means greater than 15 mm. The solution to this problem is to increase subsample size until acceptable probabilities are realized. In our particular case, it would be beneficial to estimate a subsample size that would be able to locate the four boundaries visually identified by Wolcott and Church (1991) at a type II error of 10% or less. Since Table 5.2 demonstrated that the first four groups closely correspond to the four visually identified deposit types, an estimate of each deposit's mean grain size can be made by averaging the arithmetic means of each sample in that group. The type II error can then be calculated based on these values for various subsample sizes as shown in Table 5.4 As one can see a subsample size of 36 stones is only capable of locating boundaries between groups I and II. Subsamples of 100 stones, on the other hand, seem adequate to determine the boundaries between the four visually identified by Wolcott and Church (1991) as the largest type II error is less than 7%. If, however, one wants to locate the boundaries between groups IV and V and V and VI depicted in Table 5.4 subsamples of 200 or more stones should be used. It should be noted that their are other possible boundary combinations than the ones listed in Table 5.4 such as I&III or I&VI. However, all of these will be easier to identify than the ones listed in Table 5.4.

Table 5.3 Relationship of the Type II Error and How It Increases for Deposits Having the Same Difference in Means but Variance Increasing with the Mean. Subsample size is 100 stones.

Mean of Deposit #1 (mm)	Mean of Deposit #2 (mm)	Differences of Means (mm)	Variance of Deposit #1	Variance of Deposit #2	Sum of Variances	Type II Error (%)
0.1	5	4.9	0.84	33.21	34.05	0.00
5	10	5.0	33.21	114.66	147.87	0.68
10	15	5.0	114.66	245.02	359.68	16.07
15	20	5.0	245.02	424.28	669.29	38.66
20	25	5.0	424.28	652.44	1076.72	54.75
25	30	5.0	652.44	929.52	1581.96	64.91
30	35	5.0	929.52	1255.50	2185.02	71.42
35	40	5.0	1255.50	1630.38	2885.88	75.75
40	45	5.0	1630.38	2054.17	3684.56	78.75
45	50	5.0	2054.17	2526.87	4581.05	80.90
50	55	5.0	2526.87	3048.48	5575.35	82.50

Table 5.4 The Type II Errors Expected when Locating Boundaries Between Various Sediment Groups Identified in Table 5.2. The means for each group are based on the average mean value of each sample belonging to that group while the standard deviations were computed by squaring the results obtained in equations (12) and (13).

Groups	Subsample Size	Mean of Deposit #1 (mm)	Mean of Deposit #2 (mm)	Differences of Means (mm)	Variance of Deposit #1	Variance of Deposit #2	Sum of Variances	Type II Error (%)
I & II	36	1.43	10.11	8.69	4.96	117.02	121.99	0.11
II & III	36	10.11	18.62	8.51	117.02	369.80	486.83	25.20
III & IV	36	18.62	29.88	11.26	369.80	922.30	1292.10	40.69
IV & V	36	29.88	38.95	9.07	922.30	1547.60	2469.90	70.58
V & VI	36	38.95	54.00	15.05	1547.60	2940.24	4487.84	61.54
I & II	100	1.43	10.11	8.69	4.96	117.02	121.99	0.00
II & III	100	10.11	18.62	8.51	117.02	369.80	486.83	1.35
III & IV	100	18.62	29.88	11.26	369.80	922.30	1292.10	6.83
IV & V	100	29.88	38.95	9.07	922.30	1547.60	2469.90	42.83
V & VI	100	38.95	54.00	15.05	1547.60	2940.24	4487.84	27.37
I & II	200	1.43	10.11	8.69	4.96	117.02	121.99	0.00
II & III	200	10.11	18.62	8.51	117.02	369.80	486.83	0.01
III & IV	200	18.62	29.88	11.26	369.80	922.30	1292.10	0.27
IV & V	200	29.88	38.95	9.07	922.30	1547.60	2469.90	17.46
V & VI	200	38.95	54.00	15.05	1547.60	2940.24	4487.84	6.27

By modifying the computer program used to simulate taking samples of a single deposit or several deposits, the proposed method of locating sediment boundaries can be tested. The next few paragraphs briefly describe how this was done and the results of several simulations.

As with the earlier program, the individual grain size distributions of different deposits were input as well as the number of stones to be taken from each deposit; however, this time deposit boundaries were created by defining an array consisting of X by Y stones and placing the randomly chosen stone sizes from a certain grain size distribution at specified coordinates within the array. Once this is done, a subsample size can be chosen and the computer can calculate the subsamples arithmetic means, and locate the sediment boundaries within the array by using statistical hypothesis testing and the moving windows as described in previous sections. Appendix B contains a copy of the portion of the program that locates boundaries, but, unlike the original, reads the stone sizes from data lines so that it can be employed on actual data and not just data generated through the computer simulations.

Several computer simulations were run in an attempt to see how sensitive this procedure is in finding boundaries between sediment populations. The simulations evaluated what percentage of the time (out of ten trials) a vertical boundary placed between two different sediment populations could be identified. The results can then be compared to the frequency with which we would expect to find based on the type II error calculations. For example, if a type II error of 10% is expected for a given circumstance, one would expect 90% of the boundaries to be found by the computer simulation. Table 5.5 shows the results of the computer simulations and compares them to the number of boundaries we would expect to locate based on the type II errors. In most cases these results correspond closely to that predicted by the type II error calculations. Most of the larger discrepancies occur when the arithmetic mean of one of the deposits is less than 1 mm. The reason for this is that the standard deviations calculated by equations (14) and (15) significantly overestimate the standard deviations of the finer materials. Thus, the predictions of type II error are higher than they actually are. Table 5.6 shows the same results as Table 5.4, but uses the underlying deposit's actual standard deviation to predict the type II errors. As one can see, the results improve substantially and the number of boundaries located and agree with the predicted values quite closely.

Table 5.5 Computer Simulation Showing the Percentage of Times Subsamples of 100 and 36 Stones Could Locate 10 Boundaries along with the Percentage of Times One Would Expect to Locate a Boundary. Variances were computed by the correlation between arithmetic mean and standard deviation as given in equations (12) and (13).

Simulation Number	Subsample Size	Mean of Deposit #1 (mm)	Mean of Deposit #2 (mm)	Difference of Means (mm)	Variance of Deposit #1	Variance of Deposit #2	Sum of Variances	Type II Error (%)	Expected Boundaries (%)	Boundaries Found (%)
1	100	0.30	0.67	0.37	1.24	2.19	3.43	36.82	63.18	100.00
2	100	10.53	21.44	10.91	126.16	484.98	611.14	0.28	99.72	100.00
3	100	19.44	38.65	19.21	401.77	1524.34	1926.11	0.31	99.69	100.00
4	100	0.30	0.47	0.17	1.24	1.65	2.89	73.64	26.36	90.00
5	100	7.78	11.94	4.16	72.46	159.43	231.89	13.86	86.14	90.00
6	100	20.26	30.87	10.61	434.94	982.72	1417.66	12.04	87.96	90.00
7	100	30.14	37.01	6.87	937.98	1400.32	2338.30	58.76	41.24	40.00
8	100	7.38	9.10	1.72	65.88	96.39	162.27	61.45	38.55	60.00
9	36	0.74	1.90	1.16	2.41	7.27	9.69	27.98	72.02	100.00
10	36	21.44	53.96	32.52	484.98	2935.95	3420.93	4.54	95.46	100.00
11	36	7.38	18.33	10.95	65.88	358.96	424.85	6.15	93.85	90.00
12	36	2.75	5.66	2.91	12.52	41.16	53.68	23.02	76.98	90.00
13	36	10.53	21.43	10.90	126.16	484.54	610.70	15.83	84.17	90.00
14	36	19.43	38.66	19.23	401.37	1525.12	1926.49	16.26	83.74	80.00
15	36	9.58	18.56	8.98	105.94	367.64	473.57	20.30	79.70	80.00
16	36	9.58	18.56	8.98	105.94	367.64	473.57	20.30	79.70	40.00
17	36	30.65	53.96	23.31	969.13	2935.95	3905.08	27.65	72.35	60.00
18	36	0.47	0.82	0.35	1.64	2.66	4.30	72.78	27.22	60.00
19	36	10.53	18.13	7.60	126.16	351.51	477.67	32.94	67.06	80.00
20	36	11.95	18.33	6.38	159.68	358.96	518.65	48.52	51.48	70.00
21	36	11.95	18.33	6.38	159.68	358.96	518.65	48.52	51.48	60.00

Table 5.6 Computer Simulation Showing the Percentage of Times Subsamples of 100 and 36 Stones Could Locate 10 Boundaries along with the Percentage of Times One Would Expect to Locate a Boundary. Variances are those of the underlying deposits.

Simulation Number	Subsample Size	Mean of Deposit #1 (mm)	Mean of Deposit #2 (mm)	Difference of Means (mm)	Variance of Deposit #1	Variance of Deposit #2	Sum of Variances	Type II Error (%)	Expected Boundaries (%)	Boundaries Found (%)
1	100	0.30	0.67	0.37	0.08	0.61	0.68	0.26	99.74	100.00
2	100	10.53	21.44	10.91	146.47	604.31	750.78	0.97	99.03	100.00
3	100	19.44	38.65	19.21	375.76	1695.27	2071.03	0.50	99.50	100.00
4	100	0.30	0.47	0.17	0.08	0.16	0.24	3.56	96.44	90.00
5	100	7.78	11.94	4.16	68.64	120.63	189.28	8.40	91.60	90.00
6	100	20.26	30.87	10.61	370.12	842.18	1212.30	8.04	91.96	90.00
7	100	30.14	37.01	6.87	870.83	1464.98	2335.81	58.74	41.26	40.00
8	100	7.38	9.10	1.72	58.71	117.51	176.22	63.49	36.51	60.00
9	36	0.74	1.90	1.16	0.63	3.86	4.49	8.01	91.99	100.00
10	36	21.44	53.96	32.52	604.31	2677.63	3281.94	3.91	96.09	100.00
11	36	7.38	18.33	10.95	58.71	295.19	353.89	3.23	96.77	90.00
12	36	2.75	5.66	2.91	7.66	30.24	37.91	11.68	88.32	90.00
13	36	10.53	21.43	10.90	146.46	604.31	750.77	22.91	77.09	90.00
14	36	19.43	38.66	19.23	375.76	1695.29	2071.05	18.66	81.34	80.00
15	36	9.58	18.56	8.98	158.93	687.10	846.03	41.76	58.24	80.00
16	36	9.58	18.56	8.98	158.93	687.10	846.03	41.76	58.24	40.00
17	36	30.65	53.96	23.31	1008.13	2677.61	3685.74	25.50	74.50	60.00
18	36	0.47	0.82	0.35	0.16	0.63	0.79	22.78	77.22	60.00
19	36	10.53	18.13	7.60	146.46	321.38	467.84	32.15	67.85	80.00
20	36	11.95	18.33	6.38	120.63	295.19	415.82	40.80	59.20	70.00
21	36	11.95	18.33	6.38	120.63	295.19	415.82	40.80	59.20	60.00

These results demonstrate two things. First, if given some knowledge of the mean grain sizes and standard deviations of deposits within an area, one can reasonably predict the ability of a given subsample size to locate a boundary using the correlation between mean grain size and standard deviation and equation (13). Second, the proposed method used by the computer to locate sediment boundaries locates boundaries with about the same frequency as that given by equation (13) based on the underlying deposit's true mean and standard deviation. Together these facts allow us to develop a method to sample any given sediment deposit and locate changes in sediment types with known accuracy levels as described in the next section. However, before doing this, it is important to develop a method of finding the necessary subsample size to detect a given change in arithmetic means with high reliability without having to plug subsample sizes into equation (13) until a suitable type II error is found. If one assumes that the subsamples are of equal size and decides that the reliability given by a type II error of ten percent (90% chance of locating a boundary) is adequate, then equation (13) can be rewritten as follows:

$$0.10 = \Phi\left(1.645 - \frac{\theta}{\frac{1}{\sqrt{n}}\sqrt{s_1^2 + s_2^2}}\right) - \Phi\left(-1.645 - \frac{\theta}{\frac{1}{\sqrt{n}}\sqrt{s_1^2 + s_2^2}}\right) \tag{16}$$

As one can see from this equation, if one were to solve for n, the subsample size, only two terms would determine its outcome; the difference in means θ and the sum of variances, $(s_1^2 + s_2^2)$. By holding the sum of variances constant and solving for n at several different values of θ a line can be generated that shows the necessary subsample size to locate a boundary 90% of the time regardless of the difference in means as long as the sum of variances fall on that line. By repeating this procedure, using different values for the sum of variances, one can make a plot capable of determining the subsample size for almost any circumstance. All that is needed is the difference in mean grain sizes one is interested in detecting and the sum of variances that the deposits are likely to have. Figure 5.9 and Figure 5.10 show the results of plotting such values for various sums of variance at different θ's. Specifically, Figure 5.9 shows 6 such lines for sums of variances that are between 1 and 100, while Figure 5.10 shows 6 lines that have sums of variances between 100 and 4000. As one can see, as the sum of variances increase so does the subsample size necessary to locate a specified difference in means. For example, in Figure 5.10 one sees that, deposits having a sum of variances equal to 100 and a difference in means of 6 mm, a 30 stone subsample is required, while for two deposits having a sum of variance equal to 3000 a subsample of over 700 stones is necessary to detect the same 6 mm change in means. Thus, an appropriate means of determining the necessary subsample size to sample in area can be given by finding the subsample size based on the minimum difference of means one wants to locate based near the line representing the highest sum of variances one expects to find in the field. Such graphs can be made based on any type II error and for any chosen sums of variances.

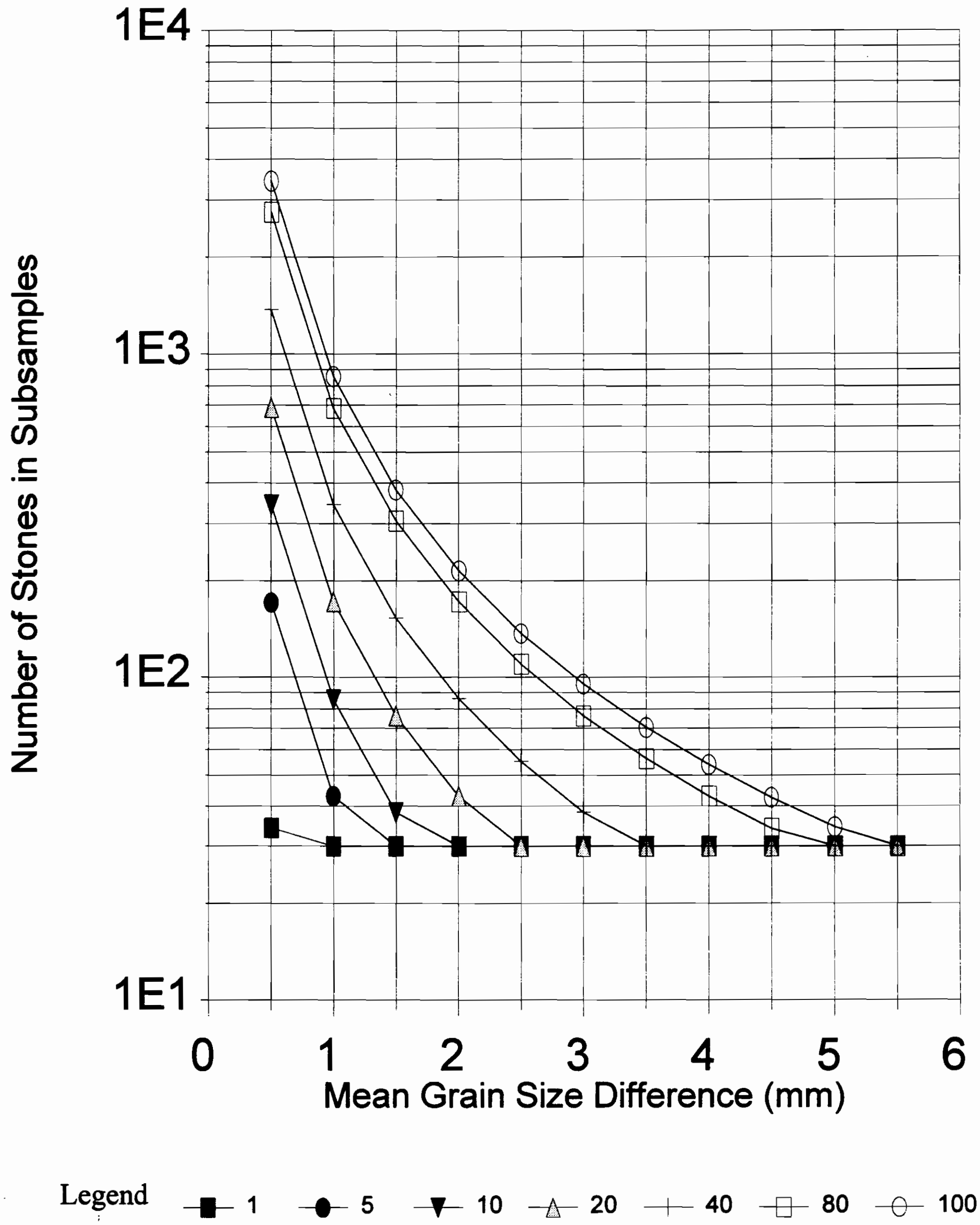


Figure 5.9: Graph of Minimum Subsample Sizes Needed to Locate a Specified Change in Mean Grain Size for Various Sums of Variance between 1 and 100. Legend refers to the sum of variance.

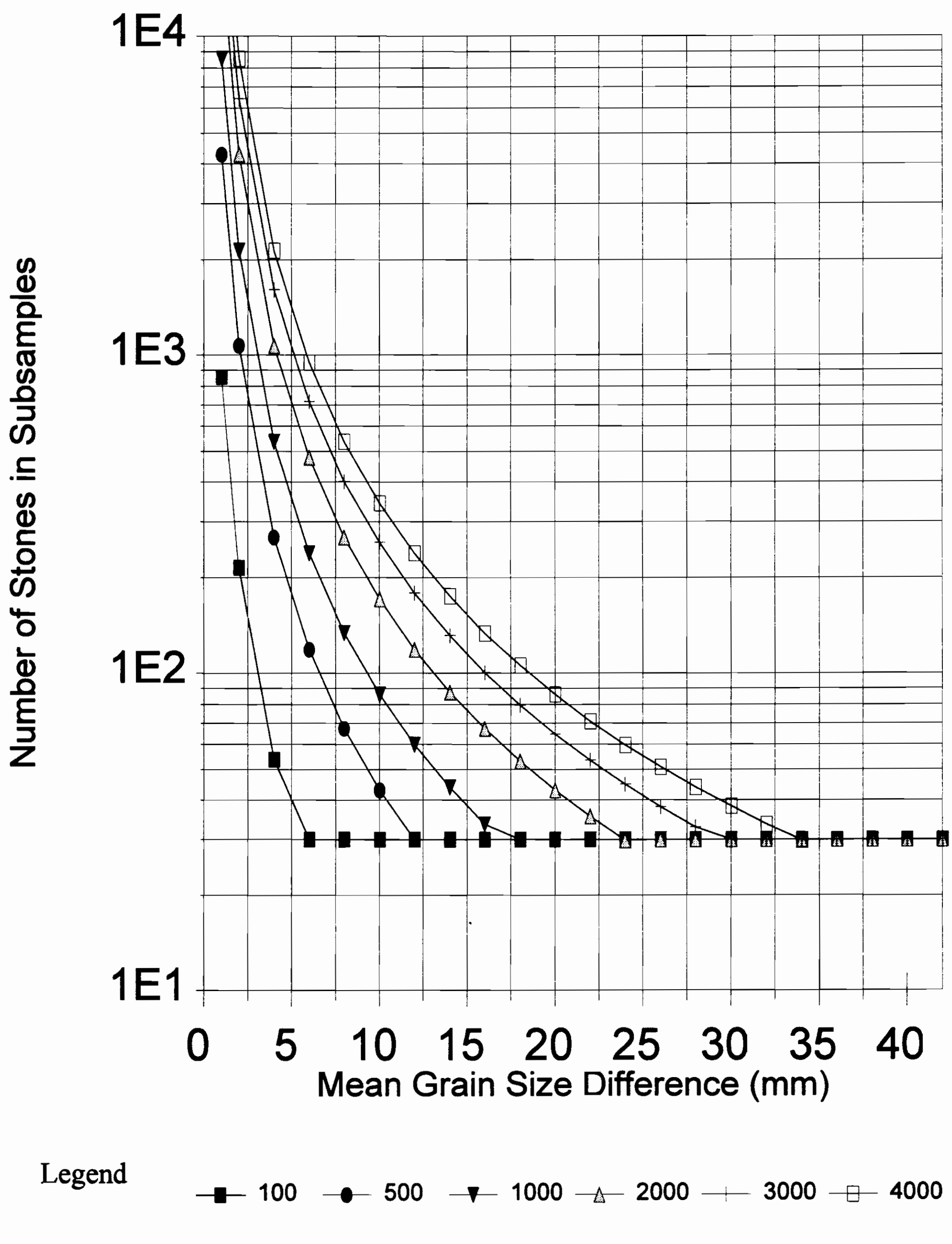


Figure 5.10: Graph of Minimum Subsample Sizes Needed to Locate a Specified Change in Mean Grain Size for Various Sums of Variance between 100 and 4000. Legend refers to sums of variance.

5.3 Sampling Different Sediment Deposits

In the previous section it was suggested that 100 stone subsamples would be sufficient to locate the various boundaries within the Quesnel Bar sampled by Wolcott and Church (1991). Unfortunately, this assumption was based on information collected after the boundaries were located, and average mean values could be calculated for the individual deposits. Thus, a major question becomes how does one sample an area that nothing is known about and determine the accuracy in which boundaries can be located. It is proposed here that this can be done in four steps which are described below.

First, the physical dimensions of the subsample should be selected. These dimensions will obviously be subjective and depend on the degree to which one expects the sediment to vary so that a boundary does not occur more than every other subsample. Thus, the higher the spatial variability is, the smaller the dimensions of each subsample should be. An evaluation of exposed surfaces near the area to be sampled may provide the best idea in determining these dimensions.

Second, grid samples should be taken throughout the area such that at least 30 stones fall within a subsample. This ensures that the means are normally distributed and can be statistically compared. The means and variances of the subsamples should then be sketched in the center of each subsample so that one can obtain an idea of the magnitude of the mean grain sizes present as well as their standard deviations.

Third, select the minimum difference in mean grain size (θ)to be located and find the sum of the two highest variances obtained from the 30 stone subsamples. Using the selected θ and the sum of variances Figures 5.9 and 5.10 or equation (16) can then be used to calculate the minimum subsample size necessary to locate the specified difference θ. If the number of stones is larger than 30, one should increase the sampling intensity throughout the sampled area to meet this criteria.

Finally, a computer program such as that given in Appendix B can be used to locate the boundaries throughout the area. The stones falling within each sediment deposit can then be analyzed as a grid sample that solely characterizes that particular deposit.

An idea of the amount of time and labor involved in performing such an analysis is given by Rice and Church (1995) who state that a 3000 stone sample can be collected in 1 or 2 work days depending on whether the stones are measured or sorted through templates. Employing a sampling scheme of 100 subsamples each containing 100 stones would thus require between 3 and 7 days to complete. As this method is designed to be used underwater (shallow), it may require a day or two more because of problems involved with sampling in water.

CHAPTER 6: TESTING BULK COMPOSITE SAMPLING

6.0 Overview

As chapter three indicated, there is sufficient reason to believe that a composite sample made up of nonvolumetric subsamples of equal volumes will result in a biased estimate of the overall grain size distribution. This chapter confirms this suspicion by showing the results of testing the composite sampling technique proposed by Wolcott and Church (1991) on a material having a known distribution. More specifically, the results show that a composite bulk sample composed of nonvolumetric subsamples is biased towards the larger particle sizes and remains biased regardless of the total amount of material collected. Furthermore, as it will be discussed later, it is difficult if not impossible to correct for this bias as is done for areal samples with equation (2). A brief description of the experiment used to test this method and its results are given in the next few sections.

6.1 Procedure

Material: It was decided that a combination of round glass beads could be used to simulate a gravel deposit. Using these materials has a couple of advantages. First, thes materials allows one to select and use a variety of distributions without extensive sieving. Second, the size of the materials can be chosen such that there is a wide range in diameters yet avoids the problems of sampling extremely large or very small particles. Avoiding materials having extremely large or very fine particle sizes is important because distributions consisting of large particle sizes require enormous amounts of material to be sampled in an unbiased way. Also, if grid sampling is to be performed, as was done to verify the uniformity of the test material, the smallest particle present should be large enough that grid sampling is capable of sampling it in an unbiased manner. Choosing particle sizes ranging from 4 mm to 25.4 mm and spaced in a manner similar to a sieve analysis (4, 5, 7, 10, 14.28, 19.05, and 25.4 mm) seems adequate to provide for these conditions and simulate the diversity and size distribution of a gravel deposit. Figure 6.1 shows the 7 different sphere sizes used. The volumetric distribution used in the test is shown in Figure 6.2. It has a median grain size of 15.77 mm and an arithmetic mean of 14.65 mm. As typical of gravel deposits, the larger particles represent more of the material than the smaller particles. It should be noted that since the specific weights of the different sized spheres used in the test material vary somewhat, the % by weight is slightly different than that of the % by volume and is therefore shown as well in Figure 26.

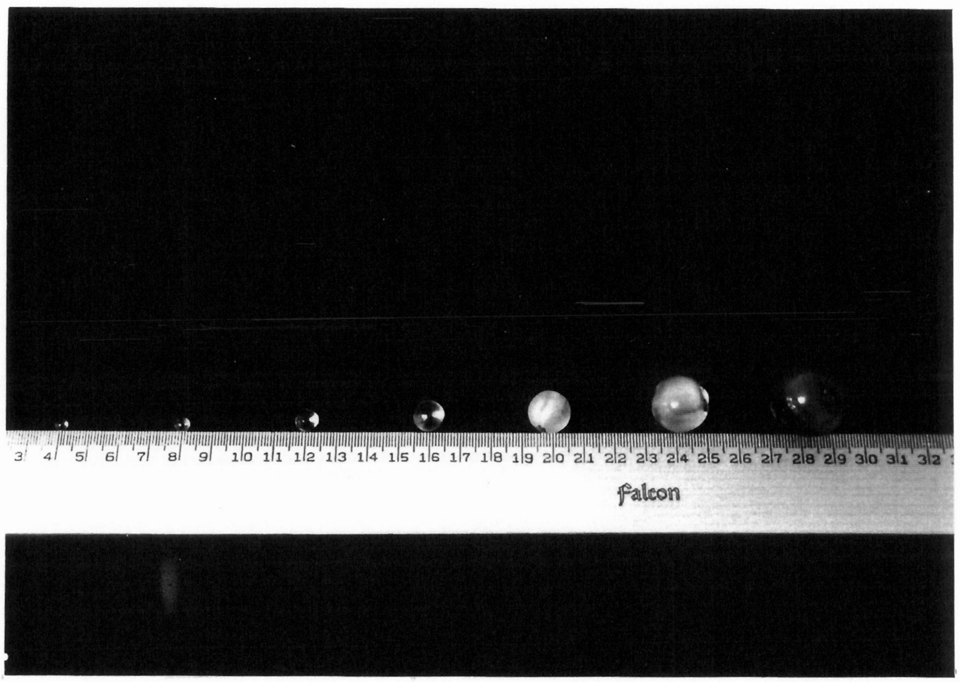


Figure 6.1: The Seven Glass Beads Used in the Test Materials

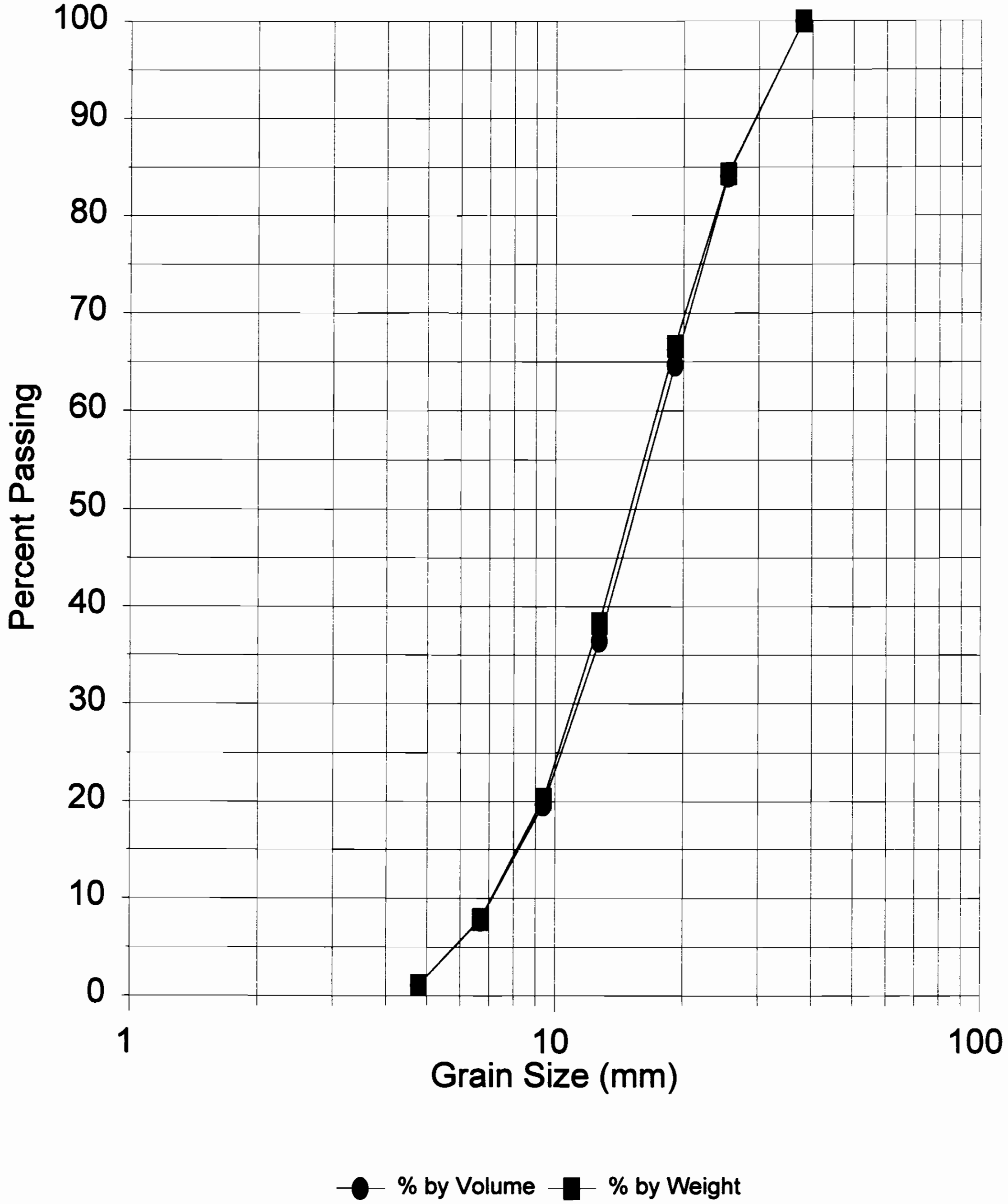


Figure 6.2: The Volumetric Distribution of the Test Material as well as the Percent by Weight Distribution. Differences in specific weights among the particles are responsible for the difference

Placement of the Material: The material was placed in a clear plexiglass box having dimensions of 533.4 mm x 533.4 mm x 254 mm for sampling. The clear sides allow one to make sure that the material is uniform throughout and segregation does not occur while the dimensions of the box were determined so that the material would be several times the minimum size of a volumetric sample. Thus, the material remaining after taking a bulk composite sample would still be volumetric. More specifically, Diplas and Fripp (1992) determined that the minimum volumetric depth of a material occurs at a depth equal to twice the maximum grain size present and that the minimum volumetric area and volume are nD_{max}^2 and $2nD_{max}^3$, respectively where n corresponds to the number of stones in a grid sample having an error as in equation (1). Diplas and Fripp (1992) recommend that n equal 100 or more. Therefore, with a maximum grain size of 25.4 mm the minimum volumetric depth, area, and volume, are 50.8 mm, 0.0645 m^2 and 0.00328 m^3, respectively. Thus, when the box is filled to a depth of 101.6 mm, or four times the maximum grain size, the volume of material present is eight times that of the minimum volumetric sample size and four times the minimum volumetric area providing ample material for sampling.

Special care was taken in the actual placement of the material into the box so segregation would not occur. Specifically, small amounts of each of the seven grain sizes present were measured out in proportion to their volumetric weights and placed in small paper bags. The contents of each bag were then sprinkled uniformly over the entire area of the box one at a time. The amount of material from the seven bags was sufficient to cover only about half of the surface area of the box thus it was impossible for any one particle size to bury another. After the contents of the seventh bag were sprinkled over the area, the box was the rotated 90 degrees and the procedure repeated until the material reached a depth of about 101 mm. Figure 6.3 shows a typical side of the box after placing the material in it. As one can see all of the grain sizes are present throughout the material and no obvious segregation occurs. To further confirm that the material was placed uniformly within the box and for purposes discussed later, several grid samples were taken of the material. Typically, two grid samples of 49 stones and 100 stones were taken when the material reached a depth of about 51 mm while grid samples of 49, 100, 196 and 400 stones were taken at the top layer. The grid samples were taken such that the material was not disturbed or physically moved. This was done by measuring off grid intervals on the top left side of the box and then placing a T-square across the top of the box at a grid interval. By placing a measure on top of the T-square and using a home-made plumb bob each row of the grid can be sampled by dropping the plumb-bob at each grid interval indicated by the measure on top of the T-square and recording the size of the glass bead or marble directly below it. Again, since the spheres can be visually classified according to their sizes, a sphere need not be removed and the material remains untouched. Figure 6.4 shows the top of the box and the T-square as well as the plumb-bob hanging from the T-square. Overall, the grid samples predicted the volumetric distribution quite well and indicate that the material is uniformly mixed. Figures 6.6 and 6.7, discussed later, show the results of the 49 stone and 400 stone grid samples.

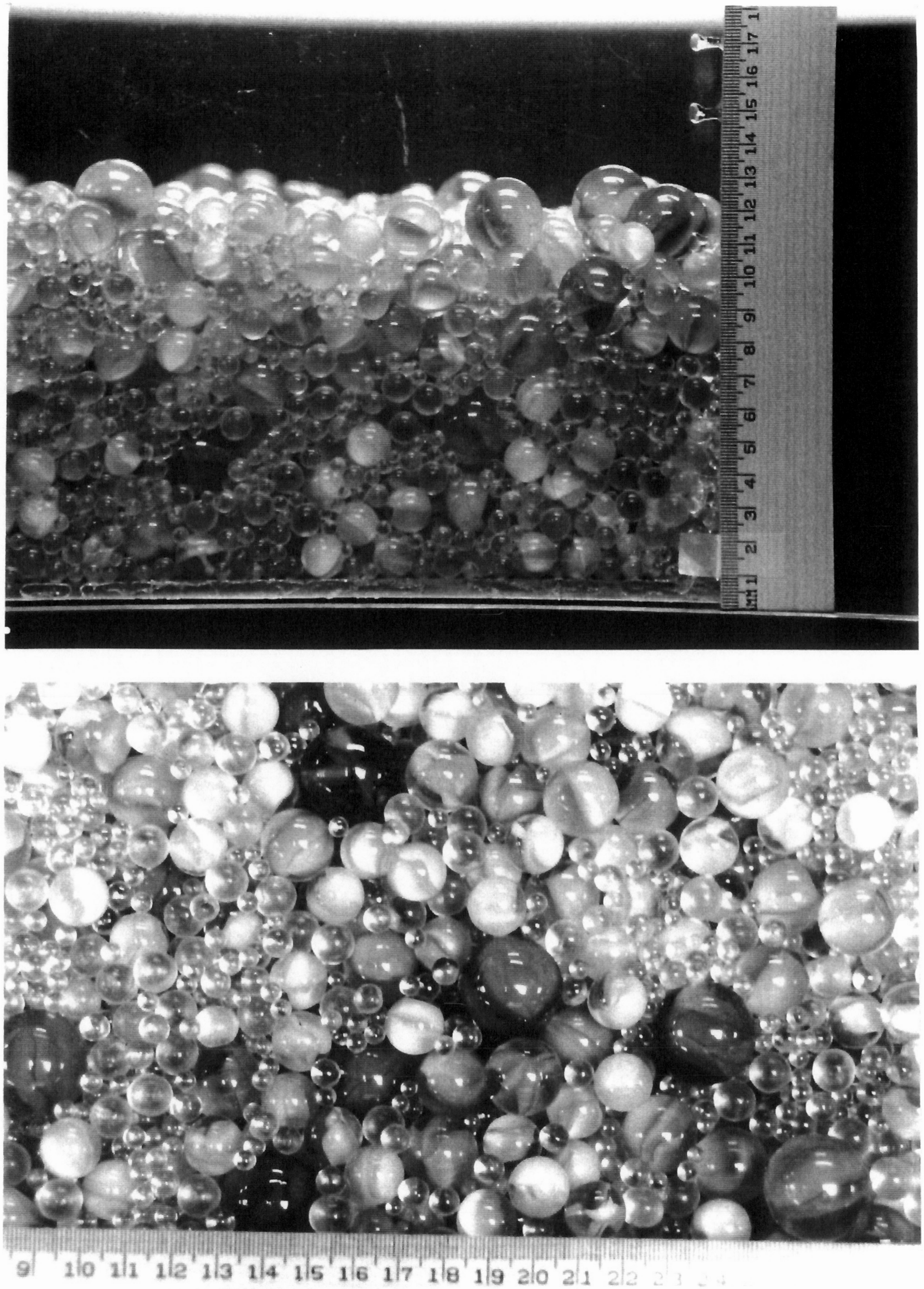


Figure 6.3: a) (top) A Representative Picture of the Material After it was Placed in the Plexiglass Box
b) (bottom) A Closeup View of the Material Along the Side of the Box

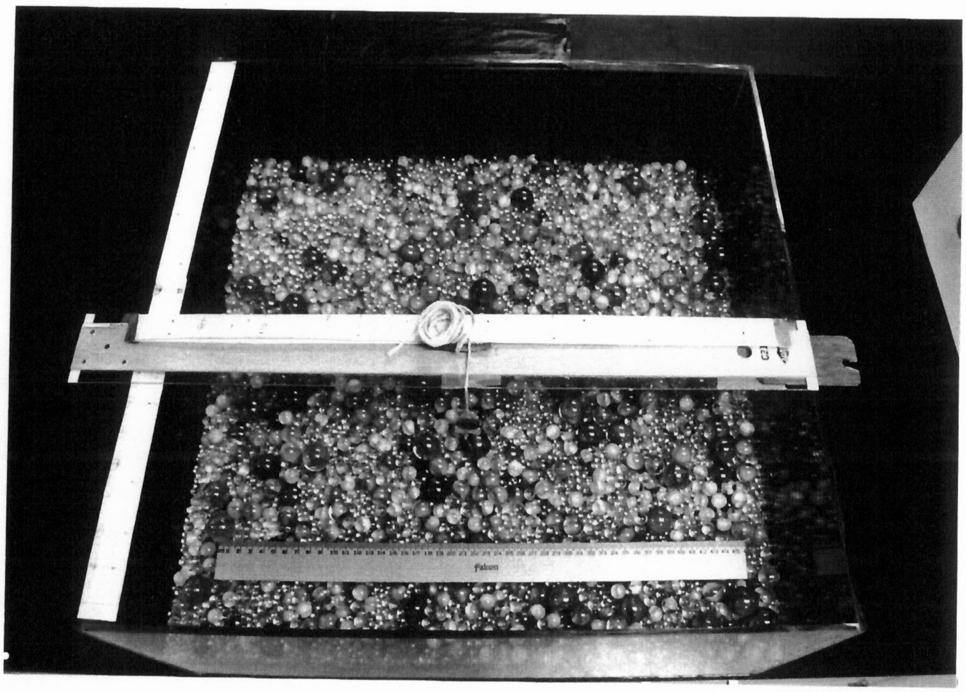


Figure 6.4: Picture of the Surface Material after its Placement as well as the T-Square and Plumb-bob Used to Take Grid Sample

Bulk Composite Sampling: After the material was placed in the box and grid samples were taken as previously described, a composite sample was taken in a manner meeting the criteria put forth by Wolcott and Church (1991). The criteria that each sediment be sampled in proportion to its area is automatically satisfied since only one distribution is being sampled. This also means that it is not necessary to sample 100 to 300 sites to satisfy this criteria as Wolcott and Church (1991) recommend. Hence, the only other requirement is to take subsamples of equal volumes that are at least as large as the largest grain size present, and that when combined have a weight such that the largest particle weighs no more than 1% of the combined sample. As the largest marble 25.4 mm on average weighs about 21 grams this will occur when a total weight of at least 2100 grams has been obtained.

In all, four composite samples and two volumetric, or bulk samples, were taken. Each of the composite, however, was distinct in that the subsample volumes used to make up the composites were different. Figure 6.5 shows different devices used to collect the subsamples as well as the bulk samples. The smallest container was a contact lens cleaning container which held on average about 21 g of material or 1 times the maximum grain size weight. The second container is used to store camera film and holds on average about 42 g of material or about 2 times the maximum grain size. The third and fourth containers were a cardboard box and a laboratory beaker which on average contained about 132 g and 985 g or 6 and 47 times the maximum grain size's weight. The fifth container is an 203.2 mm diameter sieve pan having a depth of 50.8 mm and a volume of about 1648 cm^3. It held about 103 of the largest particles. Since the volume of this pan is at least 100 times the maximum grain size it can also be considered a volumetric sample according to Church et al. (1987). The first composite, taken with the contact lens container, was obtained by systematically taking 100 subsamples from a predetermined grid over the area having a grid intersection every 50.8 mm. Likewise, the second, third, and fourth composites consisted of 64 subsamples taken with the film case, 25 subsamples taken with the cardboard box, and two subsamples taken with the laboratory beaker, respectively. The distances between the subsamples taken with the film case and the cardboard box, like the subsamples taken with the contact lens case, were taken at predetermined grid locations, however, the spacing between grid points were 63.5 mm and 101.6 mm apart. In retrieving subsamples for the first three composites, the container was placed over the spot to be sampled and pushed downward until the bottom of the container was near flush with the surface of the material, whereupon a finger(s) was then placed under the opening of the container to prevent the smaller particles from rolling out and then removed. For the fourth composite and the bulk samples, the two subsamples and bulk samples were collected by simply scooping the material out as it was impossible to drive them flush and put something directly under them to prevent material from rolling out. If a particle protruded more than 50% above the rim of the subsample/sample containers, it was not included in the composite sample.

Adjacent subsamples points were placed far enough away that the material sampled later on remained visually undisturbed from the removal of previous subsamples. After taking a composite sample, all of the material in the box as well as the sample taken was removed, sieved and placed back in the box as described earlier, except between the fourth composite and bulk samples which were taken from the same box as there was sufficient undisturbed material to obtain all of them..

It is important to note that, before combining the subsamples making up each composite together, the number and size of the particles in each subsample was recorded and then placed into one of several bags so the distributions resulting from the combining of smaller numbers of subsamples could be compared with the overall distribution. For example, in obtaining the first composite the subsamples were placed into five separate bags each containing the particles of twenty consecutive subsamples whereupon their individual distributions were recorded. Likewise, the second composite was broken up into 7 bags, six of which contained ten subsamples while the seventh contained four subsamples. The third composite was broken up into 5 bags each containing the particles of five consecutive subsamples. Similarly, both of the subsamples in the fourth composite as well as the bulk samples were put in two separate bags for analysis.

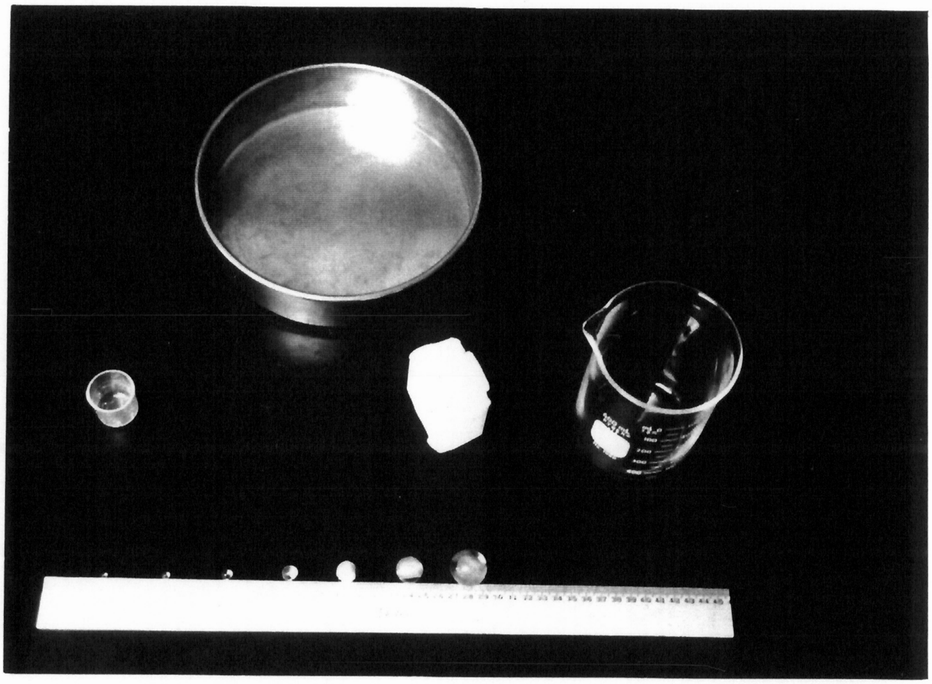


Figure 6.5: The Four Containers Used to Collect the Composite Bulk Samples as well as the Sieve Pan used to Collect the Bulk Samples

6.2 Results

As mentioned earlier, the results of the grid samples used to confirm the uniformity of the material throughout the box turned out quite well. Figures 6.6 and 6.7 show the results of the 49 stone samples taken along with the 400 stone samples. As expected, the 49 stone samples are not as accurate as the 400 stone samples, however, both approximate the volumetric distribution quite closely and confirm that the test material is uniformly mixed and segregation is not occurring. It is also important to note that the individual distributions obtained with the grid samples as a whole neither overestimate or underestimate the true grain size distribution. In fact, most of them tend to slightly overestimate the true size distribution in some parts while slightly underestimating it in others. This feature indicates that there is no obvious bias in either the uniformity of the material or the grid sampling technique. Grid samples consisting of 100 and 200 stones exhibited similar properties and had accuracy levels that appeared to be between those of the 49 stone samples and the 400 stone samples.

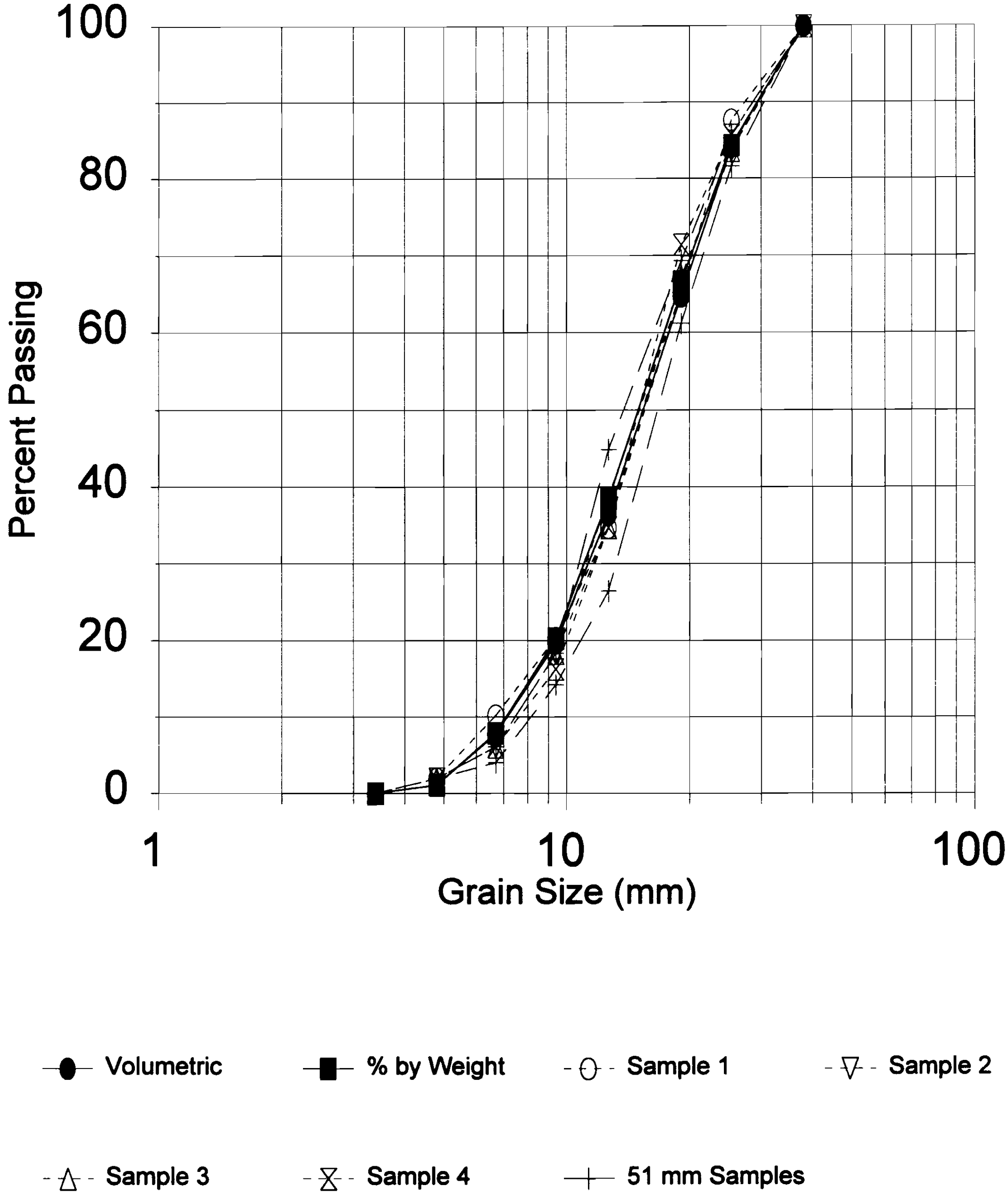


Figure 6.6: The Results of Various 49 Stone Grid Samples Taken from the Test Material throughout the experiment.

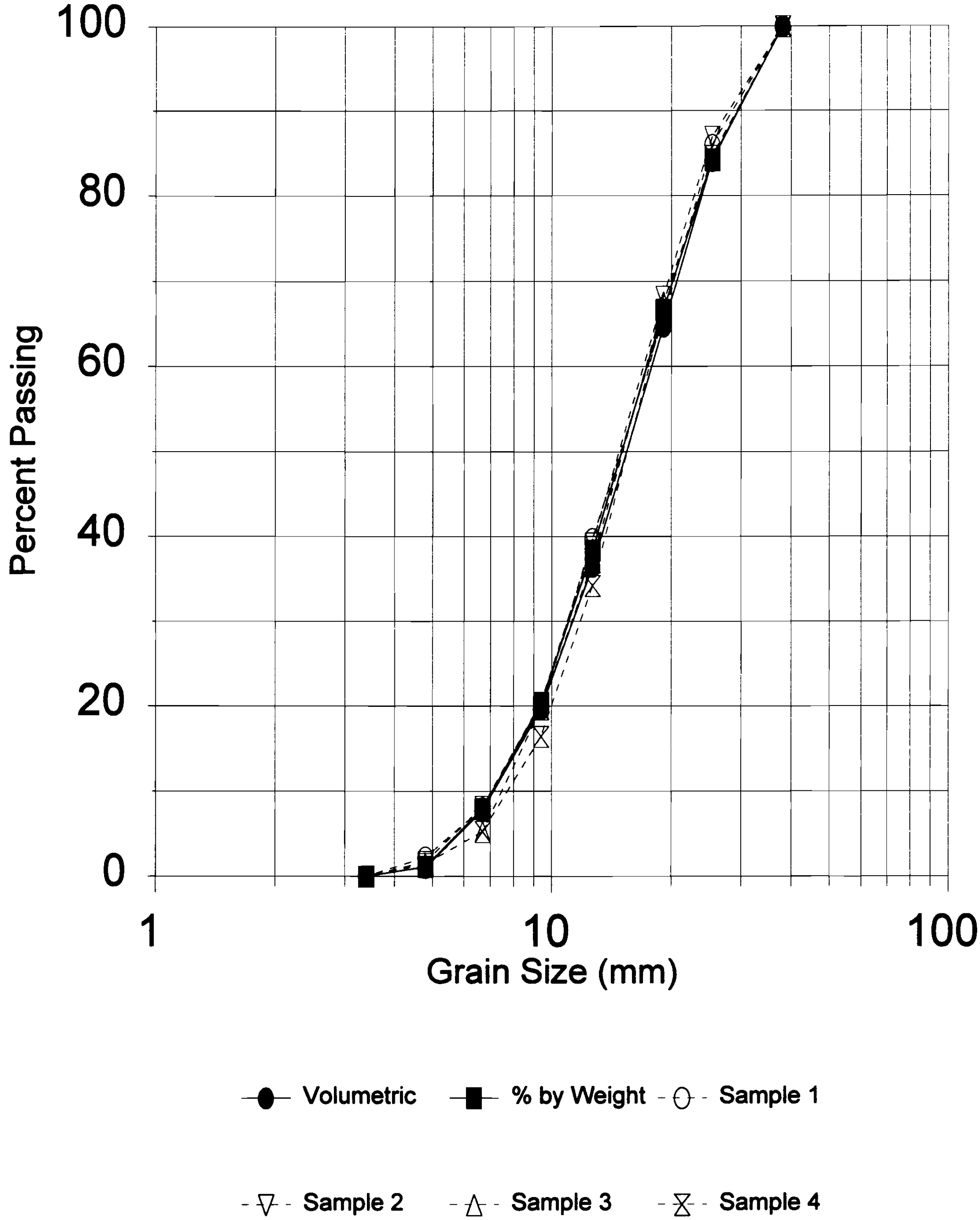


Figure 6.7: Distributions Obtained from 400 Stone Grid Samples taken throughout the experiment

In contrast to these grid samples, Figure 6.8 shows that all four of the composite distributions obtained by cumulating nonvolumetric subsamples as described earlier overestimate the particle grain sizes. Specifically, Composite #1 represents the composite retained by taking 100 subsamples with the contact lens container and weighs 2117.7 grams. Composite #2 shows the results of combining the 64 subsamples taken with the film case and weighs 2710.2 grams while composite #3 is composed of the 25 subsamples taken with the cardboard box and weighs 3328.6 grams. The fourth composite shows the results of combing two samples taken with the laboratory beaker and weighs 1969.9 grams. Likewise, bulk samples #1 and #2 taken with the 203.2 mm diameter sieve pan weighing 2495.4 and 2161.7 grams, respectively are also shown. As one can see, while the bulk samples are close to the true volumetric distribution, composites samples composed of subsamples become more and more biased towards the larger grain sizes as the subsample volume decreases. In fact, collecting subsamples as small as the largest grain size present as suggested by Wolcott and Church (1991) overestimates the D_{50} to be 19.64 mm when the actual value is 15.77 or nearly a 25% overestimation. In fact, the composite #1 has about the same bias as an areal sample that is analyzed by weight and not converted to a volumetric equivalent. The distribution marked "'Areal Equivalent" shows the distribution that would result from an areal sample of the test material and was calculated using the sieve by weight to area by weight conversion provided by Kellerhals and Bray (1971). As one can see both composite #1 and the areal equivalent have very similar distributions. The reason for this is that the contact lens container collects material only to a depth of about that of the maximum grain size present. Thus, the material collected in the 100 subsamples is very similar to that we would obtain if 100 subsamples were taken side by side, which, in essence, is an areal sample that must be converted to a volumetric equivalent.

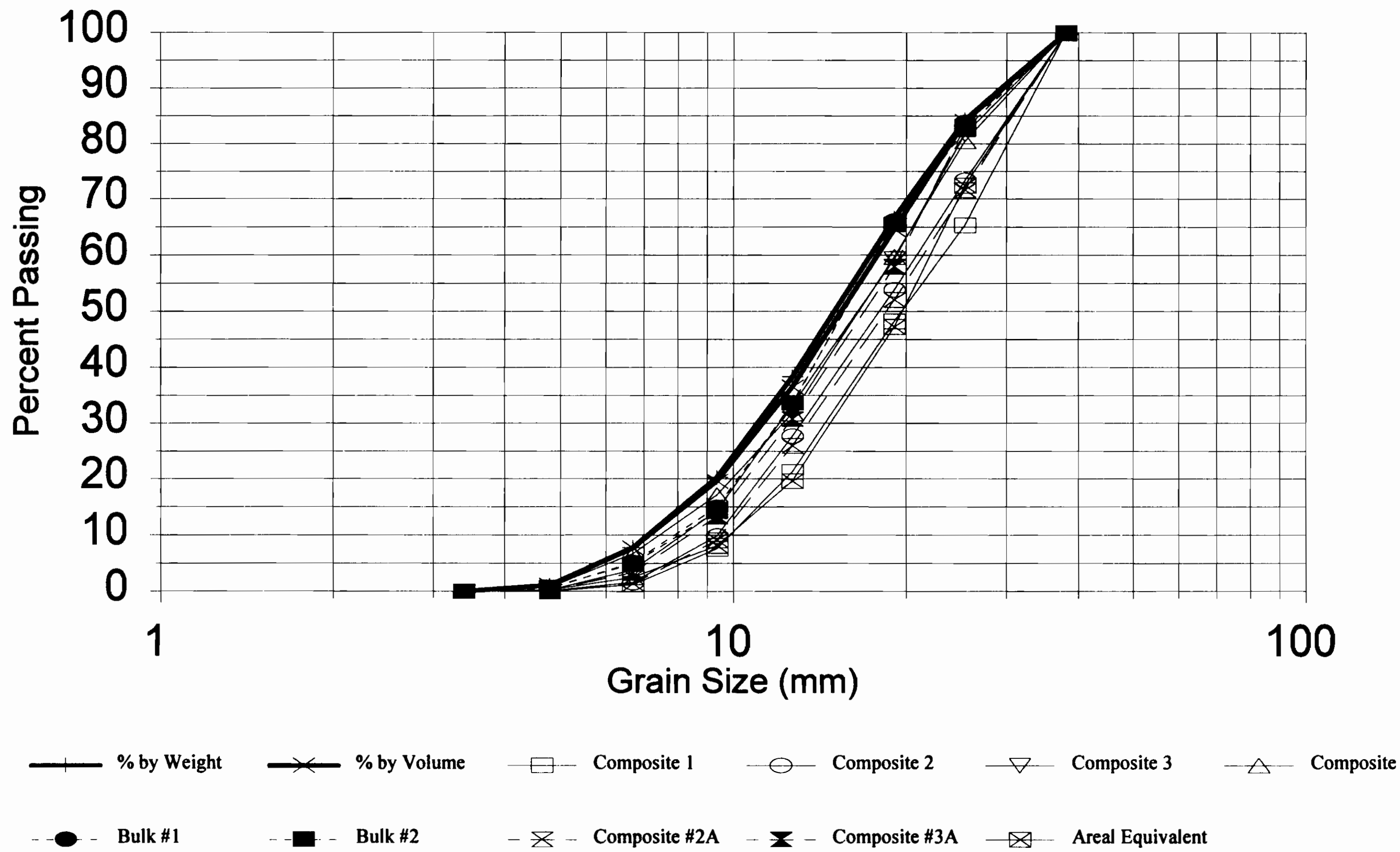


Figure 6.8: The Sample Distributions Obtained in Various Bulk Composite Samples. All distibutions were analyzed by weight.

It could be argued that the differences in weight might account for these differences therefore, it is important to compare the different composites when they have nearly the same weight. However, by using the first 50 subsamples taken with the film case (Composite #2A) and the first 16 subsamples taken with the cardboard box, (Composite #3A) one can obtain composites weighing nearly that of Composite #1 or 2107.7 and 2111.1 grams, respectively, and see that the resulting distributions are very similar to that obtained by combining all of the subsamples. Thus, even when the composite samples have similar amounts of material (about 2100 grams), the composites become more biased toward the larger particle sizes as the subsample sizes decrease in volume. Furthermore, the fact that there was little change between these distributions and those representing larger amounts of material taken in the same fashion suggests that this bias does not decrease as the total material sampled increases. If the bias did decrease with an increase in sample size, composite distributions should move closer and closer to the true volumetric distribution as the sample size increased. However, as can be seen in Figure 6.9, the composite distributions do not move towards the true volumetric distribution but remain biased as the number of subsamples increases. Specifically, Figure 6.9 shows the cumulative distribution of composites #1 and #3 after 20%, 40%, 60%, 80% and 100% of the material has been collected for composites #1 and #3. Composite #2, not shown here, had a similar pattern. This indicates that no matter how many subsamples are taken and how large the total weight of the composite is the result will always be biased. In fact, for this specific cases, it seems that a composite distribution does not change significantly after about 20% of the material has been collected. Instead, it approaches a constant size distribution with a built-in bias that reflects the sampling procedure and its inaccuracies. Furthermore, the resulting distributions are always biased toward the larger particle sizes.

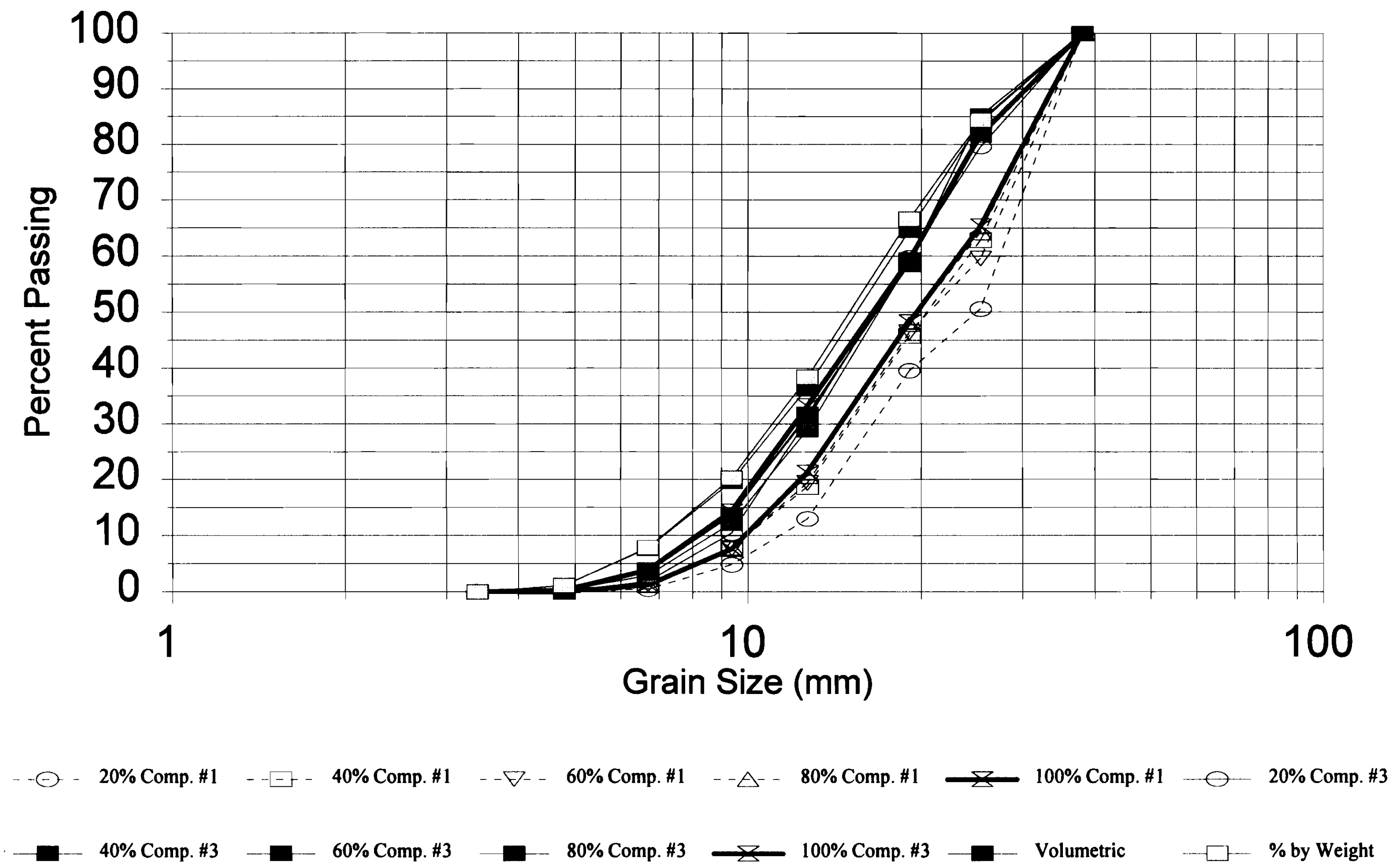


Figure 6.9: The Effect of Cumulating Subsamples

6.3 Conclusions

The results indicating that combining nonvolumetric subsamples together will result in a biased distribution is not surprising given the discussion in Chapter 3. However, it now becomes clear that the results obtained are further corroborated with earlier work as well. For example, Kellerals and Bray (1971) showed that if a grid sample, which is volumetrically equivalent is analyzed on a percentage by weight basis, the results overestimate the true percentages by D^3. In order to correct for this bias, the sample should be converted to its volumetric equivalent by using equation (2) and an exponent of $x = -3$. Therefore, if a subsample the size of the largest grain size present collects the largest grain size present that subsample is automatically weighted by D^3 because the overall composite is analyzed as percent by weight. On the other extreme, if several deposits are present the subsample may be large enough to render a subsample taken from a fine sand volumetric whereupon an exponent of zero is appropriate for equation (2). Unfortunately, most subsamples will have an exponent in between $x = 0$ and $x = -3$. Therefore, different subsamples will require different conversion exponents to render it volumetric. In the case where a single material is sampled, the smaller the subsample is, the closer the subsample volume becomes to representing a grid sample (or representing a single stone) and the more biased the composite becomes toward the larger particle sizes assuming it is analyzed on a percent by weight basis. In contrast, the larger the subsample becomes the closer it becomes to being volumetric and the less biased it becomes toward the larger particle sizes. This is exactly what occurs in the experiments and shows the bias inherent in combining nonvolumetric subsamples into a single sample. Correcting for this bias, however, may be difficult if not impossible as the proper conversion for each subsample will depend on the dimensions of the subsample taken as well as the material sampled. Furthermore, correcting this bias, would require that each subsample be analyzed individually or an average conversion exponent for the entire sample be used. How the later can be done is unknown.

CHAPTER 7: GRID COMPOSITE SAMPLES: TESTING THEIR ACCURACY AND ABILITY TO LOCATE SEDIMENT BOUNDARIES

7.0 Overview

Chapter 6 described how a mixture of glass marbles was used to simulate a gravel deposit with a known distribution and demonstrated that a composite bulk sample composed of nonvolumetric subsamples of equal volume is biased towards the larger grain sizes within a homogeneous deposit. Obviously, if such a composite was taken of an area containing more than one sediment deposit, the result would likewise be biased. Composite grid sampling, however, as developed and tested in computer simulations in Chapter 4, seems to be an appropriate method to sample an area containing more than one sediment deposit. Furthermore, chapter 5 suggests that by using the information contained in a grid sample in connection with statistical hypothesis testing, allows one to locate sediment deposit boundaries when they cannot be visually located due to being underwater. The results obtained in chapters 4 and 5, however, are based on computer simulations and not on an actual deposit. Therefore, a procedure, similar to that used in chapter 6, was developed to test the ability of a grid composite sample to sample an area having more than one sediment deposit as well as the ability to use the information obtained in that grid sample to locate sediment boundaries. The results indicate that a composite grid sample has an accuracy approximately equal to that of the binomial theory as suggested by Fripp and Diplas (1993). Furthermore, the results show that using statistical hypothesis testing in connection with a moving window scheme as proposed in chapter 5 detects sediment boundaries having a specified change in the arithmetic mean grain size with reasonable accuracy. The next few sections describe these results and how they were obtained.

7.1 Procedure

Using the glass marbles and plexiglass box described in chapter 6, two sediment deposits, each containing two different, but known, grain size distributions, were made and sampled to test the results of composite grid samples. Each deposit represented 160.02 cm x 106.68 cm of material or six times the surface area of the plexiglass box. Due to the limited amount of glass marbles, each deposit had to be made and sampled in six increments beginning with the upper left corner of the deposit and moving across, and then down and across. The plexiglass box was filled with a desired distribution as described in chapter 6 to a depth of 5.08 cm after which a 400 stone grid sample of the material was taken along with a record of the position within the box each particle came from. The process was then repeated until the desired number of boxes containing specific grain size distributions had been made and sampled. Recording the position of each marble and its size then allows one to place the recorded data at the positions they would have been sampled if six different boxes had been filled and placed three abreast and two deep and then grid sampled all at one time, which in turn allows one to use the method proposed in chapter 5 to locate the boundaries of an actual deposit.

The first depositional area created in this manner is depicted in Figure 7.1 and consists of two different mixtures having arithmetic mean grain sizes and variances of 17.55 mm and 38.07, and 14.65 mm and 40.20, respectively. The four corner boxes (boxes 1, 3, 4 and 6) contain the coarser material while the material in the middle is finer, thus creating two distinct boundaries. Since each box contains 20 x 20, or 400, sample points, the grid sample representing the entire area would have 60 sample points in each row and 40 rows for a total of 2400 particles. The boundaries, as depicted in Figure 7.1, would therefore occur after the 20th and 40th grid sample points in each row. Likewise, the second deposit, which was made simply by replacing boxes 2 and 5 with boxes containing a grain size distribution having an arithmetic mean grain size of 12.51 mm and variance 31.58, would also have boundaries after the 20th and 40th sample points in each row.

Box #1 Mean = 17.55 mm Variance = 38.07	Box #2 Mean = 14.65 mm Variance = 40.20	Box #3 Mean = 17.55 mm Variance = 38.07
Box #4 Mean = 17.55 mm Variance = 38.07	Box #5 Mean = 14.65 mm Variance = 40.20	Box #6 Mean = 17.55 mm Variance = 38.07

Figure 7.1: The Layout of the First Deposit Made with the Test Material

7.2 <u>Results</u>

The first aspect of the composite sampling approach to be investigated is how well the composite grid samples described the actual volumetric distribution of the areas. In chapter 4, this was done by calculating the 95% confidence limits around the percent passing each sieve size of 20 samples and determining how often these limits included the true volumetric percent passing each sieve size. The same approach is used here; unfortunately, since only one sample is available from each deposit, the percentage of times this approach successfully captures the volumetric percent passing each sieve size is impossible to estimate. However, such a plot will show any obvious bias inherent to the method. Figure 7.2 shows the volumetric distribution of the first deposit, along with the 2400 stone composite and its 95% confidence limits. In addition to this, the results of taking a grid sample of 600 stones (every other stone of the 2400 stone composite) and its 95% confidence limits are shown as well. As one can see, both composites reflect the actual distribution quite well. However, the true volumetric percentages passing the 19.1 mm and the 12.7 mm sieve fall outside the confidence limits for the 2400 stone sample by 1.4% and 0.3 %, respectively. Likewise, the true percentage passing the 12.7 mm sieve for the 600 stone sample falls outside the 95% confidence limits by 0.8%. Figure 7.3, likewise, shows the volumetric distribution of the second deposit along with its 2400 and 600 stone composite samples. Again, one finds that for the 2400 stone sample the 95% confidence limits obtained for the 19.1 mm and 12.7 mm sieves exclude the true percentage passing these sieves by 1.7% and 1.3%, respectively. On the other hand, the 600 stone 95% confidence limits enclose the true percent passing each sieve size in all cases.

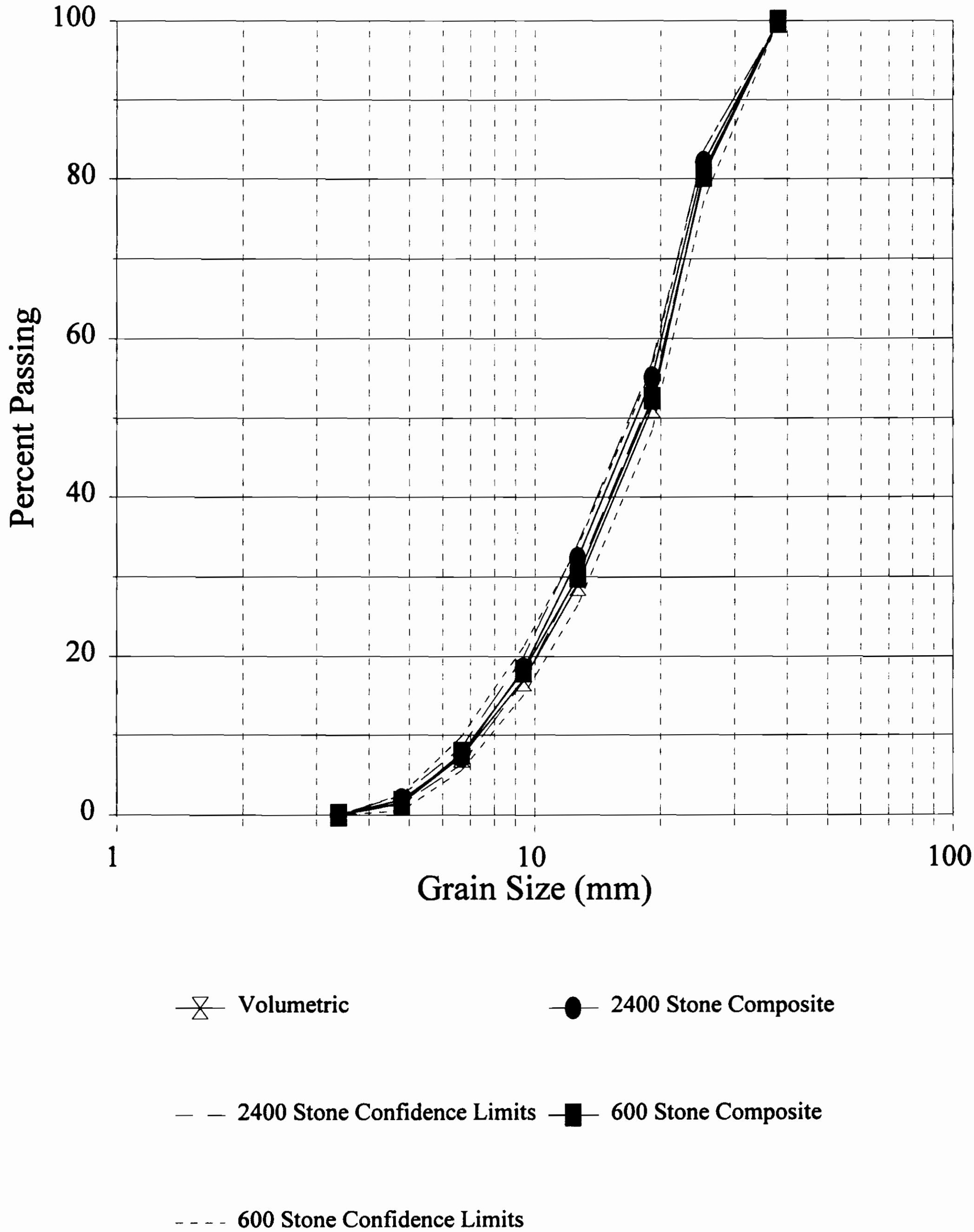


Figure 7.2: Composite Samples of 2400 and 600 Stones for Area 1

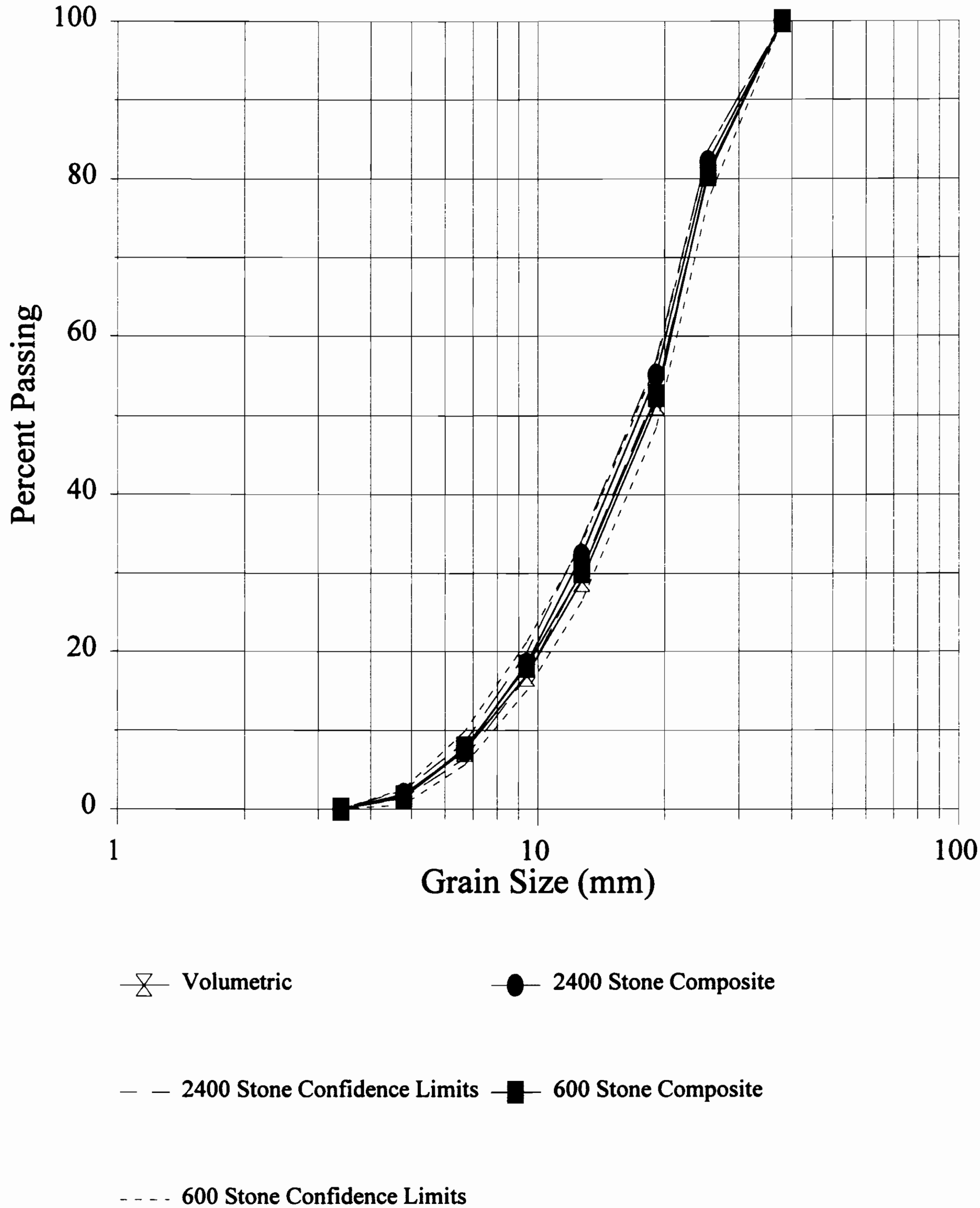


Figure 7.3: Composite Samples of 2400 and 600 Stones for Area 2

It should be noted that all of the sample distributions are slightly finer than those of the volumetric distribution. The most likely reason for the distributions being finer than the volumetric distribution is that the boxes were filled by repeatedly placing small amounts of each of the seven grain sizes present in small paper bags, whereupon, the contents of each bag were then sprinkled uniformly over the entire area of the box, one at a time, starting with the largest, and ending with the smallest size, making it possible for the smaller particles to cover the larger particles more than they should. This unintentional increase of smaller particles on the surface layer biases the grid sample towards the finer materials as grid samples are highly dependent on the surface material present. A bias introduced in this fashion would become more evident for finer materials. Figure 7.4, which shows the results of the 400 stone grid samples taken from each box that combined make up the 2400 stone composite samples, suggests that this is indeed occurring. The samples coming from the distribution having an arithmetic mean of 12.51 mm are slightly more biased towards the finer material than samples from coarser material. However, Figure 7.5 shows a plot of 100 stone grid samples which came from the individual boxes making up the two test deposits and suggests that such a bias is minor and is masked in the wider confidence intervals (not shown here) present in smaller grid samples and is the reason for the 100 stone samples to both underestimate and overestimate the actual grain size distributions yet always include the true percentage passing the sieve sizes in their confidence limits. This would also explain why fewer points fall outside the confidence intervals for the 600 stone composites than those of the 2400 stone composites. Therefore, it is suggested that the reason for some of the points falling slightly outside the predicted confidence limits is due to the experimental error involved in placing the material, as well as the fact the materials' depth was only 50.8 mm instead of 101.6 mm as in the previous chapter.

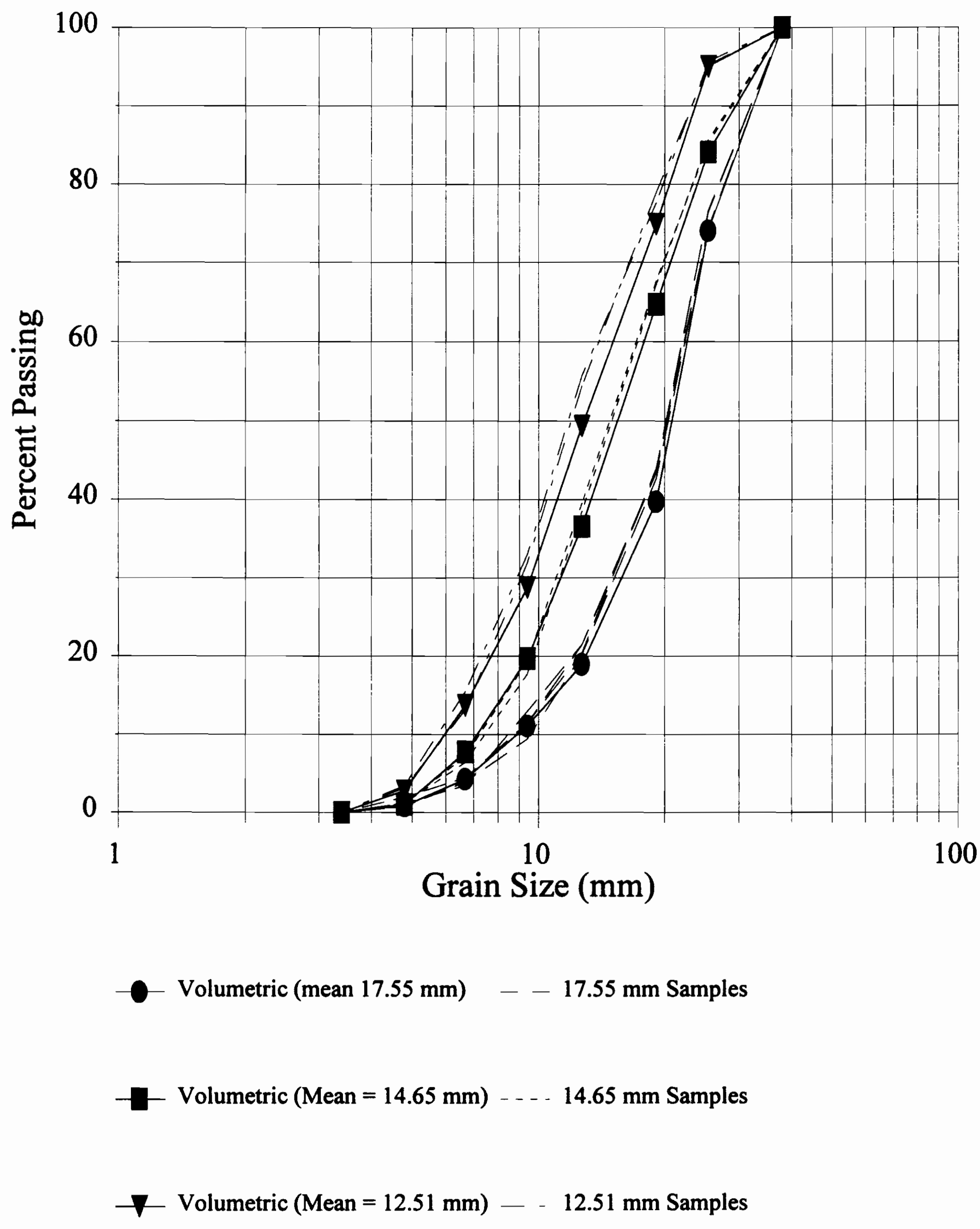


Figure 7.4: 400 Stone Samples from the Individual Boxes

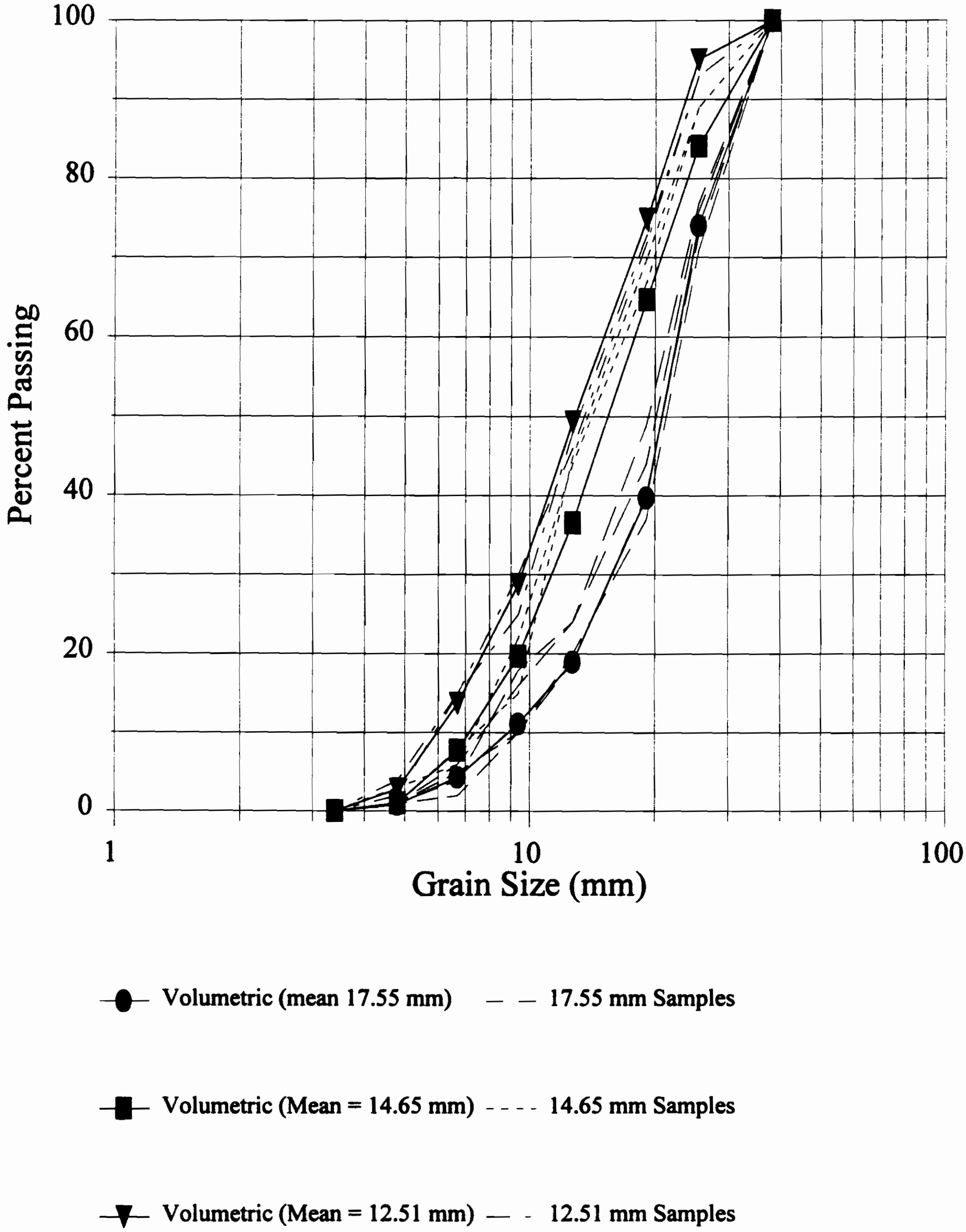


Figure 7.5: 100 Stone Samples from the Individual Boxes

number in the last row shows the mean for the subsample located in the lower right corner of the sampled area. Similarly, the second block of data shows the locations of "vertical" boundaries that were located throughout the area. For this discussion, a vertical boundary implies a boundary running from the top of the page toward the bottom of the page while a horizontal boundary runs from left to right across the page or sampled area. If no boundary was located within a subsample or between two subsamples, "none" is printed in that location. If, however, a boundary was located within or between two subsamples, the stone number after which the boundary is located is printed out. As one can see from the plots of vertical boundaries, the computer consistently located all of the boundaries around the 20th and 40th stones from the left side of the sampled area which corresponds exactly with the actual sediment boundaries and confirms the ability of the proposed method to locate sediment boundaries. It should, however, be noted that along with locating the actual boundaries "false boundaries" were also located. The reason for this is not surprising, considering that if one uses a 90% confidence interval to test whether two means are statistically the same or not there is a 10% chance of concluding that the two means are different when they are actually the same. Specifically, this occurs when subsamples coming from the same distribution by chance have a particularly high or low arithmetic mean. For example, consider the second and third subsample means in the last row. The means are 19.50 and 15.83 mm, respectively. Despite, both subsamples coming from the same distribution the difference in the means is large enough for statistical analysis to conclude that the two samples come from different distributions, hence the computer locates a boundary between them. We see this phenomena further in the last block of data which displays the horizontal boundaries present in the area. As there are no horizontal boundaries in the deposits made in these experiments, all of these boundaries are "false boundaries".

Table 7.1 The Results of Locating Sediment Boundaries through the Use of the Computer Program on the Second Deposit Using a Subsample Size of 6x5. The first block of data shows the means of the subsamples while the second two depict where horizontal and vertical boundaries were located within them.

PLOT OF THE MEANS

17.10	15.62	15.96	13.47	12.64	11.48	12.92	17.54	17.54	18.19
17.80	19.87	19.18	13.69	11.49	12.01	11.72	16.67	17.87	16.61
16.35	16.76	16.70	14.20	12.46	11.91	14.12	16.18	18.63	16.09
18.18	16.97	16.93	15.58	10.76	11.40	13.81	18.36	17.76	18.12
17.11	16.66	16.01	16.02	10.93	12.63	13.27	18.88	17.23	18.56
15.83	17.91	17.61	11.85	11.34	11.56	14.05	16.97	18.26	16.48
17.16	17.96	17.67	15.00	9.45	12.11	14.30	17.22	14.87	16.16
16.73	19.50	15.83	13.04	11.80	10.92	14.00	18.15	16.53	17.17

VERTICAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	20	NONE	NONE	38	NONE	NONE	NONE
NONE	NONE	NONE	20	NONE	NONE	NONE	42	NONE	NONE
NONE	NONE	NONE	19	NONE	NONE	NONE	44	NONE	NONE
NONE	NONE	NONE	22	NONE	34	40	NONE	NONE	NONE
NONE	NONE	NONE	23	NONE	NONE	40	NONE	NONE	NONE
NONE	NONE	NONE	18	NONE	NONE	41	NONE	NONE	NONE
NONE	NONE	NONE	23	25	NONE	40	NONE	NONE	NONE
NONE	11	NONE	20	NONE	NONE	40	NONE	NONE	NONE

HORIZONTAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	10	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	17	NONE	17
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	25	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	31	NONE	NONE	NONE	NONE	30	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE

Table 7.2 shows the same results except for the first deposit sampled. In this case, as in the last case, a subsample size of 6 x 5 was used. However, as the differences in the means of the two deposits is 2.9 mm instead of 5.04 mm, our ability to locate the boundaries decreases. In this example, the computer failed to locate the boundaries that exist after the 20th stone in the subsamples found in the 1st, 5th, 6th, and 8th rows along with the boundary existing after the 40th stone in the 4th row of subsamples. Thus, only 68.75 percent of the boundaries were found in this scenario where previously all of them were found. This inability to locate the boundaries, however, is predicted by equation (13) which shows that for subsamples of 30 stones we expect to find about 56% of the boundaries and further confirms the use of statistical hypothesis testing with the use of moving windows to locate sediment deposit boundaries. Table 7.3 summarizes the results of using the different subsample sizes on the two data sets, while Appendix C shows the individual results. The first column shows which deposit was analyzed while the second column gives the window size's dimensions in terms of stones. The next two columns show the difference in arithmetic means between the underlying distributions in which the boundaries were present as well as the sum of the variances of the two distributions. With the information provided in columns 2-4 , one can calculate the type II error according to equation (13) and hence the percentage of boundaries one expects to find under these conditions as shown in columns 5 and 6. Column (7) shows the percentage of boundaries that were correctly located within 3 stones of the true boundaries, hence, showing how accurately the moving windows are able to locate sharp boundaries. The final column (8), shows the percent of boundaries correctly located and correspond quite well with the predicted values. Specifically, one sees that the larger the subsample size is the more likely it is that the method will locate a boundary. It also shows that the difference in the arithmetic means of the mixtures being compared affects the ability to detect a boundary. That is, for the same subsample size, a difference in arithmetic means of 5 mm can be located more easily than a difference of 3 mm. These observations are what one would expect and support the method proposed in chapter 5 to locate sediment boundaries.

Table 7.2 The Results of Locating Sediment Boundaries through the Use of the Computer Program on the First Deposit Using a Subsample Size of 6x5. The first block of data shows the means of the subsamples while the second two depict where horizontal and vertical boundaries were located within them.

PLOT OF THE MEANS

17.10	15.62	15.96	17.11	14.84	13.44	12.77	17.54	17.54	18.19
17.80	19.87	19.18	15.09	14.32	14.13	14.42	16.67	17.87	16.61
16.35	16.76	16.70	14.90	13.85	14.82	14.83	16.18	18.63	16.09
18.18	16.97	16.93	16.14	14.04	15.54	16.53	18.36	17.76	18.12
17.25	16.52	15.85	15.69	16.28	13.56	13.64	17.97	18.14	17.80
16.56	17.70	17.04	16.15	15.00	16.15	13.62	16.77	17.67	16.89
17.88	17.05	18.09	16.18	14.47	13.19	14.36	17.72	13.83	15.55
17.40	19.74	15.60	15.13	14.22	15.20	14.33	17.68	16.90	16.50

VERTICAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	NONE	NONE	NONE	41	NONE	NONE	NONE
NONE	NONE	NONE	20	NONE	NONE	NONE	46	NONE	NONE
NONE	NONE	NONE	19	NONE	NONE	NONE	44	NONE	NONE
NONE	NONE	NONE	20	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	42	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	42	NONE	NONE
NONE	NONE	NONE	22	NONE	NONE	41	NONE	48	NONE
NONE	NONE	12	NONE	NONE	NONE	40	NONE	NONE	NONE

HORIZONTAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	10	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	20	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	29	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	30	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE

Table 7.3 Probability of Locating Existing Boundaries Based on the Mixtures Underlying Properties

(1) Deposit	(2) Subsample Size # Stones	(3) Difference in Means (mm)	(4) Sum of Variances	(5) Type II Error (%)	(6) Expected Boundaries (%)	(7) Boundaries Within 3 Stones	(8) Boundaries Found (%)
1	6 x 8 (48)	2.90	78.26	26.56	73.44	70.00	70.00
1	6 x 5 (30)	2.90	78.26	43.99	56.01	56.25	68.75
1	6 x 4 (24)	2.90	78.26	51.50	48.50	55.00	65.00
2	6 x 8 (48)	5.04	69.65	0.56	99.44	100.00	100.00
2	6 x 5 (30)	5.04	69.65	4.82	95.18	93.75	100.00
2	6 x 4 (24)	5.04	69.65	9.45	90.55	90.00	95.00

As noted earlier, the grid samples' distributions are slightly finer than expected due to the placement of the material and hence the arithmetic means associated with the grid samples are also smaller than expected. Therefore, it is useful to calculate the probability of detecting the boundaries given this tendency. This can be done by calculating the type II error exactly as it was done in Table 7.3. In this case, however, the difference in means and sum or variances are based on the values given by grid samples of the different deposits and not the true grain size distribution. Table 7.4 shows the results of such an analysis. As one can see, the difference in the arithmetic means of the two mixtures based on grid samples in the first area changes from 2.9 mm to 2.81 mm while the sum of variances changes from 78.26 to 76.63. As the change in these values is small, so are those of the type II errors and hence the percentage of boundaries one expects to find is virtually the same as before. The changes in the second deposit are likewise small and result in similar type II errors. Thus, one can be confident that the apparent underestimation of the actual size distributions reflected in the grid samples, whether it be by chance or a slight bias introduced in the placement of the material, does not significantly increase or decrease the chances of locating the sediment boundaries in the two areas which were sampled.

Table 7.4 Probability of Locating Existing Boundaries Based on Grid Sample Values

(1) Deposit	(2) Subsample Size # Stones	(3) Difference in Means (mm)	(4) Sum of Variances	(5) Type II Error (%)	(6) Expected Boundaries (%)	(7) Boundaries Within 3 Stones	(8) Boundaries Found (%)
1	6 x 8 (48)	2.81	76.63	28.13	71.87	70.00	70.00
1	6 x 5 (30)	2.81	76.63	45.46	54.54	56.25	68.75
1	6 x 4 (24)	2.81	76.63	52.82	47.18	55.00	65.00
2	6 x 8 (48)	5.25	69.24	0.32	99.68	100.00	100.00
2	6 x 5 (30)	5.25	69.24	3.51	96.49	93.75	100.00
2	6 x 4 (24)	5.25	69.24	7.41	92.59	90.00	95.00

7.3 Conclusions

Distributions obtained through composite grid samples for the most part coincided quite well with the actual volumetric distributions of the two deposits tested. Some of the confidence limits of these grid samples failed to capture the true percent passing a given sieve size. However, the maximum error was 1.7%, some or all of, which could be accounted for in experimental error due to the placement of the material or by chance. When considering this and the results of the computer simulations provided in chapter 4, it appears that a composite grid sample is capable of describing an area's overall grain size distribution with an accuracy level approximately equal to that given by the binomial method described by Fripp and Diplas (1993). Furthermore, using statistical hypothesis testing in conjuction with the moving windows method proposed in chapter 5, located the boundaries present in the two deposits with approximately the same accuracy levels as predicted. Thus, this experiment, along with previous computer simulations, suggests that the proposed composite grid sampling approach is capable of both obtaining an unbiased estimate of the overall grain size distribution within a sampled area as well as providing the information necessary to locate sediment boundaries.

CHAPTER 8: CONCLUSIONS AND APPLICATIONS OF THE CURRENT WORK

An accurate description of the grain size distributions found in gravel-bed streams is important since the bed material influences both the hydraulic and ecological characteristics of a gravel-bed river. While several sampling techniques have been developed to characterize the material found within gravel-bed streams, none of them is capable of describing both the overall grain size distribution of a river reach and the spatial variation within it at a known accuracy level. The current work addressed this problem by first looking at how samples taken from different grain size distributions can be combined into a single sample, or composite sample, capable of describing the overall grain size distribution of the area sampled. Next, the accuracy levels of the composite samples were analyzed, whereupon, a method for locating sediment boundaries in order to describe the spatial variability in a sampled area was developed. A brief synopsis of these results follows as well as a brief description of how these results can be applied in the field and what needs to be done in the future.

First, it was confirmed that in order to obtain an unbiased composite sample, the number of subsamples taken from the individual deposits present must be proportional to the areas of each deposit. Wolcott and Church (1991) previously showed this and that if samples are taken systematically throughout the area under consideration the deposits are automatically sampled in proportion to their areas. Since a grid sample takes samples systematically over an area, a grid sample automatically weighs each deposit in proportion to its area and, in essence, becomes a composite sample. This principle holds true for volumetric bulk samples as well, provided that all of the subsamples are volumetric and are of equal volumes. However, this approach requires an enormous amount of material to be collected and has an unknown accuracy level because, in bulk sampling, the volume of material necessary to obtain a desired accuracy level increases with the grain sizes. Hence, when two materials are sampled with the same volume, the material having the smallest grain sizes will always be sampled more accurately than the coarser material. Of equal or more importance in combining bulk samples together is the fact that combining nonvolumetric subsamples together into a composite sample is inherently biased towards the larger grain sizes. The reason for this is that nonvolumetric samples are analogous to taking samples whose nature is somewhere between a grid sample and a volumetric sample much like an areal sample which must be converted into a volumetric equivalent. Unfortunately, while an areal sample can be converted into a volumetric equivalent, the nonvolumetric subsample cannot be, due to the fact the conversion exponent will change with the dimensions of the volume collected as well as the grain sizes present.

Second, accuracy levels of composite grid samples were evaluated. Specifically, the binomial approach proposed by Fripp and Diplas (1993) was tested first on deposits containing only a single grain size distribution and then on areas having more than one grain size distribution with the aid of computer simulations. Simulations that sampled a single homogenous deposit showed that, on average, when 95% confidence intervals are calculated around the sample percentages retained on each of the sieve sizes according to the binomial method, approximately 95% of the time, they enclosed the true volumetric percent passing the given sieve size. As this was found to be true for a variety of grain size distributions, including distributions from a sand, a medium gravel, and a coarse gravel, it appears this approach is independent of the grain size distribution being sampled and can be used in any circumstance regardless of the materials shape (i.e. skewness or bimodality). Likewise, computer simulations showed that composite samples made up of stones from several different distributions had overall accuracy levels approximately equal to those predicted by the binomial method. The accuracy levels of the composites are not dependent on the number of stones collected from each deposit present but simply on the total number of stones collected throughout the area.

Third, having proposed a method to sample a river reach having several different deposits present at a known accuracy level, the question of how sediment boundaries can be located in cases where they cannot be visually identified was addressed so that the variability within the sampled area could be better described. A valuable characteristic of sediments that can be used to help us do this is in the observation that the standard deviation of a sediment increases with the arithmetic mean grain size of the deposit. Thus, if the arithmetic means and standard deviations of samples taken from different grain size distributions are plotted against each other the result is that samples taken from the same deposits cluster together while those coming from distinctly different deposit form different clusters. While this trend holds true for other grain size parameters such as D_{10}, D_{50}, and D_{90}, it appears that the arithmetic mean distinguishes between sediment deposits better than most other grain size parameters and can be easily used in statistical hypothesis tests to distinguish whether two samples are statistically different without having to resort to the process of subjectively identifying groups from a plot of the arithmetic means and their standard deviations.

By using a single parameter such as the arithmetic mean, one can adequately characterize a distinct sediment deposit and distinguish it from other deposits and hence locate sediment boundaries through the use of statistical hypothesis testing. More specifically, the grid sample which is taken throughout the area can be broken down into small samples, or subsamples, which are small enough that one would only expect to have a boundary to occur no more frequently than every other subsample. This allows adjacent subsamples to be statistically compared to one another, whereupon, if it is concluded the two subsamples come from different deposits, a boundary is located through the use of moving windows. Specifically, the boundary is located by calculating the difference in the arithmetic means of two windows that are placed side by side and move across the length of the subsamples in which the boundary is presumed to exist. It is then assumed that the location at which the difference in the arithmetic means is the greatest is where the boundary is located. The accuracy of this approach will of course depend on the sharpness in the change in sediment types as well as the amount the arithmetic mean changes. The sharper the boundary is, and the greater the differences in the arithmetic means is between the two deposits, the more precisely the boundary will be located. By systematically analyzing adjacent subsamples both across and down the sampled area, horizontal and vertical sediment boundaries can be located. The size of a subsample necessary to locate a specified change in grain size is a function of the type II error and can be calculated accordingly. Computer simulations show that this approach does locate boundaries within a sampled area with approximately the accuracy level predicted by the type II error calculations (or power curves).

Finally, as the proposed method was developed largely through computer simulations, it was tested on materials having known boundaries and grain size distributions. Results showed that the composite grid samples, while close to that of the overall grain size distributions, slightly underestimated the actual grain size distribution and thus 95% confidence limits failed to incorporate some of the true percentages passing certain sieve sizes. However, at the worst point, the true percentage passing fell outside the confidence limits by only 1.7% and does not seem unreasonable considering the difficulties in placing a mixture in a completely unbiased manner. The results of locating boundaries with the proposed method corresponded with that predicted by the relationships between subsample size and the difference in arithmetic means one wants to detect.

Together, the results described in the preceding paragraphs provide a basis for sampling the pavement layer of a river reach or depositional bar containing several different deposits within it and acquire a grain size distribution representative of the entire area with a known accuracy level. In addition to this sediment boundaries that are underwater and cannot be visually located can be located through the use of statistical hypothesis testing and moving windows as proposed here. With the boundaries located within the sampled area, one can analyze the stones falling within each sediment deposit as a grid sample solely representative of that deposit and determine the D_{10}, D_{50}, D_{90} or any other grain size parameter of interest for that particular deposit. Thus, one can analyze the changes of a specific parameter throughout the sampled area to determine the suitability of the area for spawning grounds or the bedload transport rates for the area.

While this sampling technique is designed primarily to sample the pavement layer of a gravel-bed stream, it is capable of sampling the subpavement and bottom layer of the stream as well, provided that the pavement, or pavement and subpavement layers, are removed before sampling. One drawback to this method is that grid sampling only samples material larger than approximately 15 mm. Thus, it should be combined with clay areal samples as proposed by Fripp and Diplas (1993). This can probably be done simply be taking an appropriately sized areal sample within each of the grid subsamples and combining them into a single distribution as proposed by Fripp and Diplas (1993), whereupon the subsamples can then be compared and analyzed as before using their means so that boundaries can be located. The exact details of doing this are important and should be fully addressed in the future.

REFERENCES

American Society of Civil Engineers, (1977). "Sedimentation Engineering", Manual No. 54', edited by V.A. Vanoni, New York, N.Y.

Church, M.A. and Kellerhal, R. (1978). "On the Statistics of Grain Size Variation Along a Gravel River", Can. J. Earth Sci., 15, 1151-1160.

Church, M.A., McLean, D.G., and Wolcott J.F., (1987). "River Bed Gravels: Sampling and Analysis", 'Sediment Transport in Gravel Bed Rivers', edited by C.R. Thorne, J.C. Bathurst, and R.D. Hey. John Wiley and Sons Ltd., London, U.K., 43-88.

Crowder, D.W., and Diplas P. (1994). "Some Benefits of Grid by Number Sampling", 'Hydraulic Engineering 94', edited by George V. Cotroneo, and Ralph R. Rumer, ASCE, New York, N.Y., 795-799.

Davis, J.C. (1986). *Statistics and Data Analysis in Geology*. 2nd Ed., John Wiley & Sons, New York.

De Vries, M. (1970). "On Accuracy of Bed-Material Sampling", J. of Hydr. Research, 8(4), 523-533.

Diplas, P. and Fripp, J.B. (1992). "Properties of Various Sediment Sampling Procedures", J. Hydr. Eng. ASCE, 118(7), 955-970.

Diplas, P. and Parker, G. (1992). "Deposition and Removal of Fines in Gravel-bed Streams", 'Dynamics of Gravel-bed Rivers', Edited by P. Billi, R.D. Hey, C.R. Thorne and P. Tacconi. John Wiley & Sons Ltd., Chichester, 313-329.

Diplas, P. and Sutherland, A.J. (1988). "Sampling Techniques of Gravel Sized Sediments", J. Hydr. Eng. ASCE, 114(5), 484-501.

Fripp, J.B. and Diplas, P. (1993). "Surface Sampling in Gravel Steams", J. Hydr. Eng. ASCE, 119(4), 473-490.

Helfrich, L.A. Weigmann, D.L. Neves, R.J., Bromley, P.T. (1986). "The Clinch, Powell, and Holston Rivers of Virginia and Tennessee: Wildlife and Water Quality", Virginia Cooperative Extension Service Publication 460-110, Virginia Polytechnic Institute and

State University, Blacksburg, Va.

Hey, R.D., and Thorne, C.R. (1983). "Accuracy of Surface Samples from Gravel Bed Material", J. Hydr. Eng. ASCE, 109(6), 842-851.

Hogg, R.V., and Ledolter, J. (1992). *Applied Statistics for Engineers and Physical Scientists*. 2nd Ed., Macmillan Publishing Company, New York.

Holtzman, G., Neff, A., Price, D., and Yan, J. (1994 -1995), Statistical Consulting Center Va Tech, Blacksburg, VA.

Kellerhals, R. and Bray, D.I. (1971). "Sampling Procedures for Coarse Fluvial Sediments", J. Hydr. Eng. Div. ASCE, 103(HY8), 1165-1180.

Komura, S., and Colby, B.R. (1962). "Sediment Transportation Mechanics: Introduction and Properties of Sediment", by Task Committee on Preparation of Sedimentation Manual, Committee on Sedimentation (Proc. Paper 3194)

Krumbein, W.C., and Graybill, F.A. (1965). An introduction to statistical methods in geology: New York: McGraw Hill p.475.

Mosley, M.P., and Tindale, D.S. (1985). "Sediment Variability and Bed Material Sampling in Gravel-Bed Rivers", Earth Surface Processes (10), 465-482

Ott, L.(1988). An introduction to statistical methods and data analysis. 3rd Ed., PWS-Kent Publishing Co., Boston, Mass.

Rice, S., and Church, M. (1995). "Sampling Surficial Fluvial Gravels: The Precision of Size Distribution Percentile Estimates". Submitted for Publication to Journal of Sedimentary Research.

Seal, R., Parker, G., and Paola, C. (1993). "The Effect of Local Patchiness of Gravel Grain Size Distributions on Bed Load Transport in Braided Rivers", 'Advances in Hydro-Science and - Engineering, Volume 1, edited by S. Y. Wang , Center for Computational Hydorscience and Engineering, School of Engineering, University of Mississippi.

Shirazi, M.A. and Seim, W.K. (1981). "Stream System Evaluation with Emphasis on Spawning Habitat for Salmonids", Water Resources Res., 17(3), 592-594.

Sommer, K. (1986) Sampling of Powders and Bulk Materials: Berlin, Heidelberg:

Springer-Verlag p. 142.

Weiss, N.A., and Hasset, M.J., (1991). *Introductory Statistics*. 3rd Ed., Addison-Wesley Publishing Company.

Wolcott, J. and Church, M., (1991). "Strategies for Sampling Spatially Heterogeneous Phenomena: The Example of River Gravels", J. of Sedimentary Petrology, 61(4), 534-543.

Wolman, M.G. (1954). "A Method of Sampling Coarse River-Bed Material", American Geophysical Union, 35(6), 951-956.

Ziemba, V. (1992). "In California, the Issue is People and Fish, not People vs. Fish", U.S. Water News, June, pp. 13.

APPENDIX A

Computer Program Used to Simulate Grid Sampling

```
          'DEFINE VARIABLES AND ARRAYS

CLS
TIMER ON
RANDOMIZE (TIMER)

DIM SAMP(10, 400): DIM A(10): DIM N(10):
     'SAMP - ARRAY THAT STORES A REGION'S STONE SIZES COLLECTED IN THE SAMPLE.
     'P - PERCENTAGE OF TOTAL AREA EACH REGION COVERS.
     'N - NUMBER OF STONES TO PICK UP IN REGIONS 1-10.

DIM SORT(10, 400): DIM CSORT(400)
     'SORT - PARTICLE SIZES IN INCREASING ORDER USED IN MEDIAN CALCULATIONS
     'CSORT - PARTICLE SIZES IN INCREASING ORDER FOR COMPOSITE SAMPLE

DIM DIST(10, 25): 'DIST - VOLUMETRIC PERCENTAGE EACH SIEVE SIZE RETAINS

DIM RMEAN(10): DIM RSTDV(10): DIM RMEDIAN(10): DIM PMEAN(10): DIM PSTDV(10): DIM PMEDIAN(10)
     ' R INDICATES A REGIONS SAMPLE  MEAN, MEDIAN, OR STD DEV
     ' P INDICATES A REGIONS POPULATION OR ACTUAL MEAN, MEDIAN, OR STD DEV

DIM PTS(10): DIM NDIST(10, 25): DIM GDIST(10, 26)
     ' PTS TOTAL # OF STONES IN A REGIONS POPULATION
     ' PDIST - THE NUMBER OF STONES ON EACH SIEVE MAKING UP A REGION'S POPULATION
     ' GDIST - THE NUMBER OF GRIDS THE STONES IN EACH POPULATION COVER
     ' TA - TOTAL AREA SAMPLED
     ' R - # OF REGIONS
     ' GSPACE - THE MINIMUM GRID SPACING TYPICALLY SMALLEST DIAMETER PARTICLE

DIM SNDIST(25): DIM PNDIST(25): DIM DI(25): : DIM SIEVE(26): DIM PGDIST(25): DIM SGDIST(25)
```

```
      'SNDIST - NUMBER OF STONES BETWEEN SIEVE DIAMETERS FOR THE COMPOSITE SAMPLE
      'PNDIST - NUMBER OF STONES BETWEEN SIEVE DIAMETERS FOR THE COMPOSITE POPULATION
      'DI - THE GEOMETRIC MEANS BETWEEN SIEVE SIZES BEING USED
      'SIEVE - THE SIEVE SIZES
DIM SNUMS(10, 25) 'NUMBER OF STONES IN EACH SIZE CLASS FOR THE SAMPLED SUBAREA
DIM GSUM(10)      'TOTAL NUMBER OF GRIDS IN A SUBAREA
DIM GCDIST(25)    'GRID PERCENTAGES OF THE ENTIRE AREA - VOLUMETRIC PERCENTAGES
DIM SMEDIAN(200): DIM CSNINETY(200): DIM CSTEN(200)
DIM AMEDIAN(10, 200): DIM ANINETY(10, 200): DIM ATEN(10, 200)
DIM AVE(200): DIM CSVOL(200)
DIM AAVE(10, 200): DIM AVOL(10, 200): DIM ASKEW(10, 200)
DIM SVDIST(25): DIM AVDIST(10, 25)
DIM RSKEW(10): DIM SSKEW(10): DIM CSKEW(200): DIM SNINETY(10): DIM STEN(10)
DIM PVAR(10): DIM SVOL(10)
DIM COMPA(10) 'COMPUTED AREA OF POPULATION BASED ON STONES NOT INPUT AREA
DIM AMEDTOT(10): DIM AAVETOT(10): DIM ASKSTOT(10): DIM AVOLTOT(10)
        'SET POPULATION ARRAYS TO ZERO
FOR X - 1 TO NS
 PNDIST(X) - 0
 PGDIST(X) - 0
NEXT X

        'INPUT OF SAMPLING SCENARIO INFORMATION

PRINT "HOW MANY REGIONS DO YOU HAVE IN THE AREA?"; : READ R: PRINT
PRINT "WHAT IS THE MINIMUM GRID SPACING DISTANCE?"; : READ GSPACE: PRINT
FOR X - 1 TO R
 PRINT "HOW MUCH AREA DOES REGION "; X; " COVER?"; : READ A(X): PRINT
 PRINT "WHAT IS ITS MEDIAN PARTICLE SIZE?"; : READ PMEDIAN(X)
 PRINT "HOW MANY STONES DO YOU WISH TO PICK UP IN REGION? "; X; : READ N(X): PRINT
NEXT X
PRINT "HOW MANY GEOMETRIC MEAN SIEVE SIZES ARE THERE BEING USED TO EVALUATE THE OVERALL DISTRIBUTION? ": READ NS: PRINT
PRINT "HOW MANY RUNS ARE TO BE MADE"; : INPUT RUNS
PRINT "HOW MANY SAMPLES DO YOU WISH TO TAKE"; : INPUT REPS
```

```
    COMPA(X) = COMPA(X) + NDIST(X, Y) * (DI(Y)) ^ 2
    GDIST(X, Y) = NDIST(X, Y) * DI(Y) ^ 2 / GSPACE ^ 2
    PTS(X) = PTS(X) + NDIST(X, Y)
  NEXT Y
  TA = TA + COMPA(X)
  CPTS = CPTS + PTS(X)
NEXT X
TGSUM = 0
FOR X = 1 TO NS
  FOR Y = 1 TO R
    PNDIST(X) = PNDIST(X) + NDIST(Y, X)
    PGDIST(X) = PGDIST(X) + GDIST(Y, X)
    GSUM(Y) = GSUM(Y) + GDIST(Y, X)
  NEXT Y
NEXT X
FOR X = 1 TO R
  TGSUM = TGSUM + GSUM(X)
  FOR Y = 1 TO NS
    GDIST(X, Y) = GDIST(X, Y) / GSUM(X) * 100 + GDIST(X, Y - 1)
  NEXT Y
NEXT X
CPMEDIAN = 0
FOR X = 1 TO NS
  PGDIST(X) = (PGDIST(X) / TGSUM * 100 + PGDIST(X - 1))
  IF (PGDIST(X) > 49.999) AND PGDIST(X) < 50.001 THEN CPMEDIAN = (DI(X) + DI(X + 1)) / 2
NEXT X
FOR Y = 1 TO R        'POPULATION MEANS
  FOR X = 1 TO NS
    PMEAN(Y) = PMEAN(Y) + DI(X) * (GDIST(Y, X) / 100 - GDIST(Y, X - 1) / 100)
  NEXT X
NEXT Y
FOR X = 1 TO NS
CPMEAN = CPMEAN + DI(X) * (PGDIST(X) / 100 - PGDIST(X - 1) / 100)
NEXT X
```

```
      NEXT K
      SMEDIAN(I) = SCMED: CSNINETY(I) = CNINETY
      CSKEW(I) = CSSKEW: CSTEN(I) = CTEN
      CSVOL(I) = CVOL
      MEDTOT = MEDTOT + SCMED   'SUM OF THE SAMPLE MEDIANS
      AVETOT = AVETOT + CSMEAN
      SKTOT = SKTOT + CSSKEW
      VOLTOT = VOLTOT + CVOL
      IF DF$ = "Y" THEN GOSUB 3000
    NEXT I
    MEDAVE = MEDTOT / REPS
    XBAR = AVETOT / REPS
    SKAVE = SKTOT / REPS
    VOLAVE = VOLTOT / REPS
    GOSUB 11000    'CALCULATE MEDIAN AND STANDARD DEVIATION OF THE SAMPLE MEDIANS
    GOSUB 12000
    IF DF$ = "Y" THEN GOSUB 12400
  NEXT J
  IF DF$ = "Y" THEN CLOSE #1
  IF DF$ = "Y" THEN CLOSE #2
  END

  1000         'CALCULATE NUMBER OF STONES IN EACH POPULATION AND
               'VOLUMETRIC DISTRIBUTION OF THE OVERALL COMPOSITE

  FOR X = 1 TO NS
    READ DI(X)
  NEXT X
  CPTS = 0      '# OF STONES IN THE COMPOSITE OF THE POPULATIONS
  FOR X = 1 TO R
    PTS(X) = 0   '# OF STONES IN A POPULATION
    FOR Y = 1 TO NS
      READ DIST(X, Y)
      NDIST(X, Y) = FIX((DIST(X, Y) - DIST(X, Y - 1)) / 100 * A(X) / (DI(Y)) ^ 2 + .5)
```

```
PRINT "DO YOU WISH TO PUT RESULTS IN A DATA FILE Y OR N"; : INPUT DF$
IF DF$ = "Y" THEN PRINT "WHAT IS THE FILE NAME?"; : INPUT FILE$
IF DF$ = "Y" THEN OPEN FILE$ FOR OUTPUT AS #1
IF DF$ = "Y" THEN PRINT "WHAT FILE DO YOU WANT THE STONES TO BE SAVED IN ": INPUT SF$
IF DF$ = "Y" THEN OPEN SF$ FOR OUTPUT AS #2
IF DF$ = "Y" THEN WRITE #2, "RUN", "REGION", "SAMPLE"
GOSUB 1000
GOSUB 9000   'PRINT OUT POPULATION AND POPULATION COMPOSITES VOLUMETRIC DISTRIBUTIONS
IF DF$ = "Y" THEN GOSUB 9300
FOR J = 1 TO RUNS
 FOR B = 1 TO R
  AMEDTOT(B) = 0: AAVETOT(B) = 0: ASKTOT(B) = 0: AVOLTOT(B) = 0
 NEXT B
 MEDTOT = 0
 AVETOT = 0
 SKTOT = 0
 VOLAVE = 0
 FOR I = 1 TO REPS
  GOSUB 2000   'GENERATE RANDOM SAMPLES
  GOSUB 4000   'CALCULATE SAMPLE, POPULATION, COMPOSITE SAMPLE AND COMPOSITE POPULATION MEANS
  GOSUB 5000   'COMPUTE SAMPLE, COMPOSITE SAMPLE, POPULATION, AND COMPOSITE POPULATION STD DEVS
  GOSUB 6000   'SORT SAMPLES AND COMPOSITE OF SAMPLE ARRAYS
  IF DF$ = "Y" THEN GOSUB 6500
  GOSUB 8000   'CALCULATE MEDIANS OF SAMPLES, COMPOSITE SAMPLE, POPULATIONS AND COMPOSITE POPULATION
  GOSUB 8500
  IF I = 1 AND J = 1 THEN GOSUB 9500
  CLS : PRINT "RUN: "; J; " SAMPLING SAMPLE NUMBER"; I
  FOR K = 1 TO R
   AMEDIAN(K, I) = RMEDIAN(K): ASKEW(K, I) = SSKEW(K): AAVE(K, I) = RMEAN(K)
   ANINETY(K, I) = SNINETY(K): ATEN(K, I) = STEN(K): AVOL(K, I) = SVOL(K)
   AMEDTOT(K) = AMEDTOT(K) + RMEDIAN(K)
   AAVETOT(K) = AAVETOT(K) + RMEAN(K)
   ASKTOT(K) = ASKTOT(K) + SSKEW(K)
   AVOLTOT(K) = AVOLTOT(K) + SVOL(K)
```

```
            'POPULATION STANDARD DEVIATIONS AND SKEW
FOR X = 1 TO R
 FOR Y = 1 TO NS
  PVAR(X) = PVAR(X) + (DI(Y) - PMEAN(X)) ^ 2 * (GDIST(X, Y) / 100 - GDIST(X, Y - 1) / 100)
  RSKEW(X) = RSKEW(X) + (DI(Y) - PMEAN(X)) ^ 3 * (GDIST(X, Y) / 100 - GDIST(X, Y - 1) / 100)
 NEXT Y
PSTDV(X) = (PVAR(X)) ^ .5
IF PSTDV(X) = 0 THEN RSKEW(X) = 0: GOTO 1100
RSKEW(X) = RSKEW(X) / (PSTDV(X)) ^ 3
1100        'AVOIDS DIVISION BY ZERO ERROR
NEXT X
FOR X = 1 TO NS
 CPVAR = CPVAR + (DI(X) - CPMEAN) ^ 2 * (PGDIST(X) / 100 - PGDIST(X - 1) / 100)
 PSKEW = PSKEW + (DI(X) - CPMEAN) ^ 3 * (PGDIST(X) / 100 - PGDIST(X - 1) / 100)
NEXT X
CPSTDV = (CPVAR) ^ .5
PSKEW = PSKEW / (CPSTDV) ^ 3
FOR X = 1 TO NS + 1: READ SIEVE(X): NEXT X
RETURN

2000   'SET ARRAYS = 0 FOR REPEATED SAMPLING & CHOOSE RANDOM SAMPLES

FOR X = 1 TO 10: FOR Y = 1 TO 100: SAMP(X, Y) = 0: SORT(X, Y) = 0: NEXT Y: NEXT X
FOR X = 1 TO NS: SNDIST(X) = 0: SGDIST(X) = 0: NEXT X
FOR X = 1 TO R: FOR Y = 1 TO NS: SNUMS(X, Y) = 0: NEXT Y: NEXT X
FOR X = 1 TO R: RMEAN(X) = 0: SSTDV(X) = 0: SVOL(X) = 0: NEXT X
FOR X = 1 TO 100: CSORT(X) = 0: NEXT X
CVOL = 0
FOR X = 1 TO R
 IF DF$ = "Y" THEN PRINT #2, J; ","; I; ","; R; ",";
 FOR Y = 1 TO N(X)
  RAND = (RND * 100000)
  RAND = RAND / 1000
  IF RAND <= DIST(X, 1) THEN SAMP(X, Y) = DI(1)
```

```
  IF RAND <= DIST(X, 2) AND RAND > DIST(X, 1) THEN SAMP(X, Y) = DI(2)
  IF RAND <= DIST(X, 3) AND RAND > DIST(X, 2) THEN SAMP(X, Y) = DI(3)
  IF RAND <= DIST(X, 4) AND RAND > DIST(X, 3) THEN SAMP(X, Y) = DI(4)
  IF RAND <= DIST(X, 5) AND RAND > DIST(X, 4) THEN SAMP(X, Y) = DI(5)
  IF RAND <= DIST(X, 6) AND RAND > DIST(X, 5) THEN SAMP(X, Y) = DI(6)
  IF RAND <= DIST(X, 7) AND RAND > DIST(X, 6) THEN SAMP(X, Y) = DI(7)
  IF RAND <= DIST(X, 8) AND RAND > DIST(X, 7) THEN SAMP(X, Y) = DI(8)
  IF RAND <= DIST(X, 9) AND RAND > DIST(X, 8) THEN SAMP(X, Y) = DI(9)
  IF RAND <= DIST(X, 10) AND RAND > DIST(X, 9) THEN SAMP(X, Y) = DI(10)
  IF RAND <= DIST(X, 11) AND RAND > DIST(X, 10) THEN SAMP(X, Y) = DI(11)
  IF RAND <= DIST(X, 12) AND RAND > DIST(X, 11) THEN SAMP(X, Y) = DI(12)
  IF RAND <= DIST(X, 13) AND RAND > DIST(X, 12) THEN SAMP(X, Y) = DI(13)
  IF RAND <= DIST(X, 14) AND RAND > DIST(X, 13) THEN SAMP(X, Y) = DI(14)
  IF RAND <= DIST(X, 15) AND RAND > DIST(X, 14) THEN SAMP(X, Y) = DI(15)
  IF RAND <= DIST(X, 16) AND RAND > DIST(X, 15) THEN SAMP(X, Y) = DI(16)
  IF RAND <= DIST(X, 17) AND RAND > DIST(X, 16) THEN SAMP(X, Y) = DI(17)
  IF RAND <= DIST(X, 18) AND RAND > DIST(X, 17) THEN SAMP(X, Y) = DI(18)
  IF RAND <= DIST(X, 19) AND RAND > DIST(X, 18) THEN SAMP(X, Y) = DI(19)
  IF RAND <= DIST(X, 20) AND RAND > DIST(X, 19) THEN SAMP(X, Y) = DI(20)
  IF RAND <= DIST(X, 21) AND RAND > DIST(X, 20) THEN SAMP(X, Y) = DI(21)
  IF RAND <= DIST(X, 22) AND RAND > DIST(X, 21) THEN SAMP(X, Y) = DI(22)
  IF RAND <= DIST(X, 23) AND RAND > DIST(X, 22) THEN SAMP(X, Y) = DI(23)
  IF RAND <= DIST(X, 24) AND RAND > DIST(X, 23) THEN SAMP(X, Y) = DI(24)
  IF RAND <= DIST(X, 25) AND RAND > DIST(X, 24) THEN SAMP(X, Y) = DI(25)
  IF DF$ = "Y" THEN PRINT #2, SAMP(X, Y); ",";
  FOR Z = 1 TO NS
   IF DI(Z) = SAMP(X, Y) THEN SNDIST(Z) = SNDIST(Z) + 1
   IF DI(Z) = SAMP(X, Y) THEN SNUMS(X, Z) = SNUMS(X, Z) + 1
   IF DI(Z) = SAMP(X, Y) THEN SVOL(X) = SVOL(X) + DI(Z) ^ 3
  NEXT Z
 NEXT Y
 IF DF$ = "Y" THEN PRINT #2,
 CVOL = CVOL + SVOL(X)
NEXT X
```

```
FOR V = 1 TO R: FOR Z = 1 TO NS 'CREATE VOL DIST OF SAMPLE FOR EACH AREA
   AVDIST(V, Z) = SNUMS(V, Z) / N(V) * 100 + AVDIST(V, Z - 1)
NEXT Z: NEXT V
RETURN

3000      'BREAKDOWN OF COMPOSITE SAMPLE INTO EACH AREA SAMPLED
WRITE #1,
WRITE #1, "RUN " + STR$(J): WRITE #1, "SAMPLE " + STR$(I)
WRITE #1, "SIEVE", "DI", "AREA 1", "AREA 2", "AREA 3", "AREA 4", "COMPOSITE", "% PASSING", "% PASSING", "% PASSING", "% PASSING", "% PASSING"
WRITE #1, "SIZE", "SIZE", " ", " ", " ", " ", " ", "AREA 1", "AREA 2", "AREA 3", "AREA 4", "COMPOSITE": WRITE #1,
FOR Y = NS TO 1 STEP -1
 WRITE #1, SIEVE(Y + 1), DI(Y), SNUMS(1, Y), SNUMS(2, Y), SNUMS(3, Y), SNUMS(4, Y), SNDIST(Y), AVDIST(1, Y), AVDIST(2, Y), AVDIST(3, Y), AVDIST(4, Y), SVDIST(Y)
NEXT Y
WRITE #1, SIEVE(1), "-", "-", "-", "-", "-", "-", "-", "-", "-", "-", "-": WRITE #1,
WRITE #1, "MEAN", RMEAN(1), RMEAN(2), RMEAN(3), RMEAN(4), CSMEAN
WRITE #1, "MEDIAN", RMEDIAN(1), RMEDIAN(2), RMEDIAN(3), RMEDIAN(4), SCMED
WRITE #1, "D90", SNINETY(1), SNINETY(2), SNINETY(3), SNINETY(4), CNINETY
WRITE #1, "D10", STEN(1), STEN(2), STEN(3), STEN(4), CTEN
WRITE #1, "STD DEV", SSTDV(1), SSTDV(2), SSTDV(3), SSTDV(4), CSSTDV
WRITE #1, "SKEW", SSKEW(1), SSKEW(2), SSKEW(3), SSKEW(4), CSSKEW
WRITE #1, "VOLUME", SVOL(1), SVOL(2), SVOL(3), SVOL(4), CVOL
WRITE #1, "# OF STONES", N(1), N(2), N(3), N(4), CTS
RETURN

4000        'CALCULATION OF SAMPLE, POPULATION, AND COMPOSITE MEANS

CSUM = 0            'SUM OF STONE  SIZES IN COMPOSITE SAMPLE
CPTS = 0
CTS = 0             'TOTAL # STONES IN THE COMPOSITE SAMPLE
FOR X = 1 TO R
 SSUM = 0           'SUM OF STONE SIZES FOR A REGION'S SAMPLE
 CTS = CTS + N(X)
 PTS = 0            'TOTAL # STONES IN A REGION OR POPULATION
  FOR Y = 1 TO N(X)
```

```
   SSUM = SSUM + SAMP(X, Y)
   CSUM = CSUM + SAMP(X, Y)
  NEXT Y
              'CALCULATING POPULATION MEANS
 FOR Z = 1 TO NS
  CPTS = CPTS + NDIST(X, Z)  '# OF STONES IN THE ENTIRE SURFACE
  PTS = PTS + NDIST(X, Z)    'TOTAL # OF STONES IN A REGIONS POPULATION
  SVDIST(Z) = SNDIST(Z) / CTS * 100 + SVDIST(Z - 1)
 NEXT Z
 RMEAN(X) = SSUM / N(X)
NEXT X
CSMEAN = CSUM / CTS
AVE(I) = CSMEAN
RETURN

4500   'PRINT SAMPLE MEANS
FOR X = 1 TO R: PRINT "SAMPLE MEAN "; RMEAN(X): NEXT X
PRINT "COMPOSITE SAMPLE MEAN "; CSMEAN
RETURN


5000   'ROUTINE TO CALCULATE STANDARD DEVIATIONS
CSSS = 0      'COMPOSITE SAMPLE SUM OF SQUARES
SCS = 0       'STONES IN THE COMPOSITE SAMPLE
FOR X = 1 TO R
 SSS = 0      'SAMPLE SUM OF SQUARES
 FOR Y = 1 TO N(X)
  SSS = SSS + (SAMP(X, Y) - RMEAN(X)) ^ 2
  CSSS = CSSS + (SAMP(X, Y) - CSMEAN) ^ 2
  SCS = SCS + 1
 NEXT Y
 SSTDV(X) = (SSS / (N(X) - 1)) ^ .5      'SAMPLE STD DEV
NEXT X
```

```
CSSTDV = (CSSS / (SCS - 1)) ^ .5       'COMPOSITE SAMPLE STD DEV
RETURN

6000        'SORT SAMPLES AND COMPOSITE SAMPLES IN ARRAYS IN ORDER
            'TO FIND THE MEDIANS
COUNT = 0
FOR X = 1 TO R
 FOR Y = 1 TO N(X)
  SORT(X, Y) = SAMP(X, Y)
  COUNT = COUNT + 1
  CSORT(COUNT) = SAMP(X, Y)
 NEXT Y
NEXT X
FOR X = 1 TO R
 FOR Y = 1 TO N(X)
  FOR Z = N(X) TO Y STEP -1
   IF SORT(X, Z - 1) > SORT(X, Z) THEN SWAP SORT(X, Z - 1), SORT(X, Z)
  NEXT Z
 NEXT Y
NEXT X
FOR Y = 1 TO COUNT
 FOR Z = COUNT TO Y STEP -1
  IF CSORT(Z - 1) > CSORT(Z) THEN SWAP CSORT(Z - 1), CSORT(Z)
 NEXT Z
NEXT Y
RETURN

6500        'Print Sorted Composite to #2
PRINT #2, "SORTED ORDER"; ",";
FOR X = 1 TO COUNT
PRINT #2, CSORT(X); ",";
NEXT X
PRINT #2,
RETURN
```

```
8000            'CALCULATE ALL MEDIANS, D90, D10
FOR X = 1 TO R
 NFIXA = N(X) * .9 + .5: NFIXB = N(X) * .1 + .5
 NFIXC = COUNT * .9 + .5: NFIXD = COUNT * .1 + .5
 IF N(X) / 2 = INT(N(X) / 2) THEN COND$ = "EVEN" ELSE COND$ = "ODD"
 IF COND$ = "ODD" THEN RMEDIAN(X) = SORT(X, INT((N(X) / 2 + .5)))
 IF COND$ = "EVEN" THEN RMEDIAN(X) = (SORT(X, N(X) / 2) + SORT(X, N(X) / 2 + 1)) / 2
 IF ABS(N(X) * .9 + .5 - INT(NFIXA)) < .00001 THEN SNINETY(X) = SORT(X, INT(NFIXA))
 IF ABS(N(X) * .9 + .5 - INT(NFIXA)) < .00001 THEN PRINT "EQUAL"
 IF ABS(N(X) * .9 + .5 - INT(NFIXA)) > .00001 THEN SNINETY(X) = (SORT(X, INT(NFIXA)) + SORT(X, INT(NFIXA + 1))) / 2
 IF ABS(N(X) * .9 + .5 - INT(NFIXA)) > .00001 THEN PRINT "UNEQUAL"
 IF ABS(N(X) * .1 + .5 - INT(NFIXB)) < .00001 THEN STEN(X) = SORT(X, INT(NFIXB))
 IF ABS(N(X) * .1 + .5 - INT(NFIXB)) > .00001 THEN STEN(X) = (SORT(X, INT(NFIXB)) + SORT(X, INT(NFIXB + 1))) / 2
 PRINT INT(NFIXA) - (N(X) * .9 + .5); N(X)
NEXT X
IF COUNT / 2 = INT(COUNT / 2) THEN COND$ = "EVEN" ELSE COND$ = "ODD"
IF COND$ = "ODD" THEN SCMED = CSORT(INT((COUNT / 2 + .5)))
IF COND$ = "EVEN" THEN SCMED = (CSORT(COUNT / 2) + CSORT(COUNT / 2 + 1)) / 2
IF ABS(COUNT * .9 + .5 - INT(NFIXC)) <= .00001 THEN CNINETY = CSORT(INT(NFIXC))
IF ABS(COUNT * .9 + .5 - INT(NFIXC)) > .00001 THEN CNINETY = (CSORT(INT(NFIXC)) + CSORT(INT(NFIXC + 1))) / 2
IF ABS(COUNT * .1 + .5 - INT(NFIXD)) <= .00001 THEN CTEN = CSORT(INT(NFIXD))
IF ABS(COUNT * .1 + .5 - INT(NFIXD)) > .00001 THEN CTEN = (CSORT(INT(NFIXD)) + CSORT(INT(NFIXD + 1))) / 2
IF CPMEDIAN > 0 THEN GOTO 8100
IF 50 <= PGDIST(1) THEN PMEDIAN(X) = DI(1)
IF 50 <= PGDIST(2) AND 50 > PGDIST(1) THEN CPMEDIAN = DI(2)
IF 50 <= PGDIST(3) AND 50 > PGDIST(2) THEN CPMEDIAN = DI(3)
IF 50 <= PGDIST(4) AND 50 > PGDIST(3) THEN CPMEDIAN = DI(4)
IF 50 <= PGDIST(5) AND 50 > PGDIST(4) THEN CPMEDIAN = DI(5)
IF 50 <= PGDIST(6) AND 50 > PGDIST(5) THEN CPMEDIAN = DI(6)
IF 50 <= PGDIST(7) AND 50 > PGDIST(6) THEN CPMEDIAN = DI(7)
IF 50 <= PGDIST(8) AND 50 > PGDIST(7) THEN CPMEDIAN = DI(8)
IF 50 <= PGDIST(9) AND 50 > PGDIST(8) THEN CPMEDIAN = DI(9)
IF 50 <= PGDIST(10) AND 50 > PGDIST(9) THEN CPMEDIAN = DI(10)
```

```
IF 50 <= PGDIST(11) AND 50 > PGDIST(10) THEN CPMEDIAN = DI(11)
IF 50 <= PGDIST(12) AND 50 > PGDIST(11) THEN CPMEDIAN = DI(12)
IF 50 <= PGDIST(13) AND 50 > PGDIST(12) THEN CPMEDIAN = DI(13)
IF 50 <= PGDIST(14) AND 50 > PGDIST(13) THEN CPMEDIAN = DI(14)
IF 50 <= PGDIST(15) AND 50 > PGDIST(14) THEN CPMEDIAN = DI(15)
IF 50 <= PGDIST(16) AND 50 > PGDIST(15) THEN CPMEDIAN = DI(16)
IF 50 <= PGDIST(17) AND 50 > PGDIST(16) THEN CPMEDIAN = DI(17)
IF 50 <= PGDIST(18) AND 50 > PGDIST(17) THEN CPMEDIAN = DI(18)
IF 50 <= PGDIST(19) AND 50 > PGDIST(18) THEN CPMEDIAN = DI(19)
IF 50 <= PGDIST(20) AND 50 > PGDIST(19) THEN CPMEDIAN = DI(20)
IF 50 <= PGDIST(21) AND 50 > PGDIST(20) THEN CPMEDIAN = DI(21)
IF 50 <= PGDIST(22) AND 50 > PGDIST(21) THEN CPMEDIAN = DI(22)
IF 50 <= PGDIST(23) AND 50 > PGDIST(22) THEN CPMEDIAN = DI(23)
IF 50 <= PGDIST(24) AND 50 > PGDIST(23) THEN CPMEDIAN = DI(24)
IF 50 <= PGDIST(25) AND 50 > PGDIST(24) THEN CPMEDIAN = DI(25)
8100 RETURN

8500 'CALCULATIONS OF SKEW
FOR X = 1 TO R
 SSKEW(X) = 0
 FOR Y = 1 TO NS
  SSKEW(X) = SSKEW(X) + (DI(Y) - RMEAN(X)) ^ 3 * SNUMS(X, Y) / N(X)
 NEXT Y
IF SSTDV(X) = 0 THEN GOTO 8600
SSKEW(X) = SSKEW(X) / (SSTDV(X)) ^ 3
8600 'AVOIDS DIVISION BY ZERO ERROR
NEXT X
FOR X = 1 TO NS
 CSSKEW = CSSKEW + (DI(X) - CSMEAN) ^ 3 * (SNDIST(X) / CTS)
NEXT X
CSSKEW = CSSKEW / (CSSTDV) ^ 3
RETURN

9000 'CONDITIONS PRINT
```

```
CLS
PRINT "Conditions:"
PRINT "Number of subareas or (regions) in surface = "; R
PRINT
PRINT TAB(25); "VOLUMETRIC DISTRIBUTION": PRINT
PRINT "REGION"; TAB(15); "1"; TAB(25); "2"; TAB(35); "3"; TAB(45); "4"; TAB(51); "COMPOSITE"
PRINT : PRINT "SIEVE"; TAB(15); "%"; TAB(25); "%"; TAB(35); "%"; TAB(45); "%"; TAB(55); "%"
PRINT "SIZE": PRINT
FOR X = NS + 1 TO 1 STEP -1
PRINT SIEVE(X);
PRINT USING "###.###"; TAB(12); GDIST(1, X - 1); TAB(22); GDIST(2, X - 1); TAB(32); GDIST(3, X - 1); TAB(42); GDIST(4, X - 1); TAB(52); PGDIST(X - 1)
NEXT X
INPUT Q
PRINT
CLS
PRINT TAB(20); "DISTRIBUTION OF STONES FOR THE AREA":
PRINT : PRINT "REGION"; TAB(15); "1"; TAB(25); "2"; TAB(35); "3"; TAB(45); "4"; TAB(51); "COMPOSITE"
PRINT : PRINT "DI"; TAB(15); "#"; TAB(25); "#"; TAB(35); "#"; TAB(45); "#"; TAB(55); "#"
PRINT "SIZE"; TAB(15); "STONES"; TAB(25); "STONES"; TAB(35); "STONES"; TAB(45); "STONES"; TAB(55); "STONES"
FOR X = NS TO 1 STEP -1
PRINT DI(X);
PRINT TAB(14); NDIST(1, X); TAB(24); NDIST(2, X); TAB(34); NDIST(3, X); TAB(44); NDIST(4, X); TAB(54); PNDIST(X)
NEXT X
PRINT "TOTAL"; TAB(14); PTS(1); TAB(24); PTS(2); TAB(34); PTS(3); TAB(44); PTS(4); TAB(54); CPTS
INPUT ; Q
RETURN


9300   'CONDITIONS TO DATA FILE
REG$ = STR$(R)
WRITE #1, FILE$
WRITE #1, "Conditions:"
WRITE #1, "Number of subareas or (regions) in surface: " + REG$
WRITE #1,
```

```
WRITE #1, "VOLUMETRIC DISTRIBUTION": WRITE #1,
WRITE #1, "REGION", "1", "2", "3", "4", "COMPOSITE"
WRITE #1, : WRITE #1, "SIEVE", "%", "%", "%", "%", "%"
WRITE #1, "SIZE", "PASSING", "PASSING", "PASSING", "PASSING", "PASSING"
WRITE #1,
FOR X = NS + 1 TO 1 STEP -1
WRITE #1, SIEVE(X), GDIST(1, X - 1), GDIST(2, X - 1), GDIST(3, X - 1), GDIST(4, X - 1), PGDIST(X - 1)
NEXT X
WRITE #1,
WRITE #1, "DISTRIBUTION OF STONES FOR THE AREA":
WRITE #1, : WRITE #1, "REGION", "1", "2", "3", "4", "COMPOSITE"
WRITE #1, : WRITE #1, "DI", "#", "#", "#", "#", "#"
WRITE #1, "SIZE", "STONES", "STONES", "STONES", "STONES", "STONES"
WRITE #1,
FOR X = NS TO 1 STEP -1
WRITE #1, DI(X), NDIST(1, X), NDIST(2, X), NDIST(3, X), NDIST(4, X), PNDIST(X)
NEXT X
WRITE #1, : WRITE #1, "TOTAL", PTS(1), PTS(2), PTS(3), PTS(4), CPTS
WRITE #1,
RETURN

9500  CLS
PRINT : PRINT TAB(25); "POPULATION STATISTICS": PRINT
PRINT "AREA"; TAB(12); " 1", TAB(22); " 2"; TAB(32); " 3"; TAB(42); " 4"; TAB(52); " COMPOSITE"
PRINT
PRINT "MEAN"; : PRINT USING "###.###"; TAB(12); PMEAN(1); TAB(22); PMEAN(2); TAB(32); PMEAN(3); TAB(42); PMEAN(4); TAB(52); CPMEAN
PRINT
PRINT "MEDIAN"; TAB(12); PMEDIAN(1); TAB(22); PMEDIAN(2); TAB(32); PMEDIAN(3); TAB(42); PMEDIAN(4); TAB(52); CPMEDIAN
PRINT
PRINT "STD DEV"; : PRINT USING "###.###"; TAB(12); PSTDV(1); TAB(22); PSTDV(2); TAB(32); PSTDV(3); TAB(42); PSTDV(4); TAB(52); CPSTDV
PRINT
PRINT "SKEW"; TAB(12); RSKEW(1); TAB(22); RSKEW(2); TAB(32); RSKEW(3); TAB(42); RSKEW(4); TAB(52); PSKEW
PRINT
PRINT "TOTAL AREA"; TAB(12); COMPA(1); TAB(22); COMPA(2); TAB(32); COMPA(3); TAB(42); COMPA(4); TAB(52); TA
```

```
PRINT
PRINT "# STONES "; TAB(12); N(1); TAB(22); N(2); TAB(32); N(3); TAB(42); N(4); TAB(52); COUNT
PRINT "IN SAMPLE": PRINT
PRINT "THE NUMBER OF SAMPLES TO BE TAKEN IS"; REPS
INPUT Q
IF DF$ - "Y" THEN GOSUB 9900
RETURN

9900  ' POPULATION STATISTICS TO DATA FILE
REP$ - STR$(REPS)
WRITE #1, : WRITE #1, "POPULATION STATISTICS": WRITE #1,
WRITE #1, "AREA", " 1", " 2", " 3", " 4", " COMPOSITE"
WRITE #1,
WRITE #1, "MEAN", PMEAN(1), PMEAN(2), PMEAN(3), PMEAN(4), CPMEAN
WRITE #1, "MEDIAN", PMEDIAN(1), PMEDIAN(2), PMEDIAN(3), PMEDIAN(4), CPMEDIAN
WRITE #1, "STD DEV", PSTDV(1), PSTDV(2), PSTDV(3), PSTDV(4), CPSTDV
WRITE #1, "SKEW", RSKEW(1), RSKEW(2), RSKEW(3), RSKEW(4), PSKEW
WRITE #1, "TOTAL AREA", COMPA(1), COMPA(2), COMPA(3), COMPA(4), TA
WRITE #1, "# STONES ", N(1), N(2), N(3), N(4), COUNT
WRITE #1, "IN SAMPLE": WRITE #1,
WRITE #1, "THE NUMBER OF SAMPLES TO BE TAKEN IS: " + REP$
WRITE #1,
RETURN

11000        'SORT MULTIPLE SAMPLES FIND MEDIAN AND STANDARD DEVIATION
SS - 0
SSA - 0
FOR B - 1 TO R: ASS(B) - 0: ASSA(B) - 0: NEXT B
FOR Y - 1 TO REPS
 FOR Z - REPS TO Y STEP -1
  IF SMEDIAN(Z - 1) > SMEDIAN(Z) THEN SWAP SMEDIAN(Z - 1), SMEDIAN(Z)
  IF CSNINETY(Z - 1) > CSNINETY(Z) THEN SWAP CSNINETY(Z - 1), CSNINETY(Z)
  IF CSTEN(Z - 1) > CSTEN(Z) THEN SWAP CSTEN(Z - 1), CSTEN(Z)
  FOR B - 1 TO R
```

```
      IF AMEDIAN(B, Z - 1) > AMEDIAN(B, Z) THEN SWAP AMEDIAN(B, Z - 1), AMEDIAN(B, Z)
      IF ANINETY(B, Z - 1) > ANINETY(B, Z) THEN SWAP ANINETY(B, Z - 1), ANINETY(B, Z)
      IF ATEN(B, Z - 1) > ATEN(B, Z) THEN SWAP ATEN(B, Z - 1), ATEN(B, Z)
    NEXT B
  NEXT Z
NEXT Y
FOR X = 1 TO REPS
SS = SS + (SMEDIAN(X) - MEDAVE) ^ 2
SSA = SSA + (AVE(X) - XBAR) ^ 2
FOR B = 1 TO R
ASS(B) = ASS(B) + (AMEDIAN(B, X) - (AMEDTOT(B) / REPS)) ^ 2
ASSA(B) = ASSA(B) + (AAVE(B, X) - (AAVETOT(B) / REPS)) ^ 2
NEXT B
NEXT X
STDV = (SS / (REPS - 1)) ^ .5
ASTDV = (SSA / (REPS - 1)) ^ .5
RETURN

12000
PRINT : PRINT "RUN" + STR$(J)
PRINT "THE AVERAGE OF THE SAMPLE MEDIANS IS "; (MEDAVE)
PRINT "THE AVERAGE OF THE SAMPLE MEANS IS "; (XBAR)
PRINT "THE AVERAGE OF THE SAMPLE VOLUMES IS "; (VOLAVE)
PRINT "THE STANDARD DEVIATION OF THE SAMPLE MEDIANS IS "; (STDV)
PRINT "THE STANDARD DEVIATION OF THE SAMPLE MEANS IS "; (ASTDV)
PRINT "THE AVERAGE OF THE SAMPLE COEFFICIENTS OF SKEW IS "; (SKAVE)
PRINT "THE SAMPLE MEDIAN FREQUENCIES FOLLOW: "
PRINT "MEDIAN"; TAB(10); "FREQUENCY"
VALUE = SMEDIAN(1): QUANT = 1
FOR X = 2 TO REPS
  IF SMEDIAN(X) = SMEDIAN(X - 1) THEN QUANT = QUANT + 1
  IF SMEDIAN(X) = SMEDIAN(X - 1) AND X = REPS THEN PRINT VALUE; TAB(15); QUANT
  IF SMEDIAN(X) = SMEDIAN(X - 1) THEN GOTO 12100
  PRINT VALUE; TAB(15); QUANT
```

```
 VALUE - SMEDIAN(X)
 QUANT - 1
 IF X - REPS THEN PRINT VALUE; TAB(15); QUANT
12100 NEXT X
PRINT : PRINT "THE MEAN VALUES FOLLOW: "
PRINT
FOR X - 1 TO REPS
 PRINT AVE(X);
 IF X / 10 - FIX(X / 10) THEN PRINT
NEXT X
PRINT
PRINT : PRINT "THE COEFFICIENTS OF SKEW FOLLOW: "
PRINT
FOR X - 1 TO REPS
 PRINT CSKEW(X); ;
 IF X / 10 - FIX(X / 10) THEN PRINT
NEXT X
PRINT
PRINT : PRINT "THE COMPOSITE SAMPLE VOLUMES FOLLOW: "
PRINT
FOR X - 1 TO REPS
 PRINT CSVOL(X);
 IF X / 10 - FIX(X / 10) THEN PRINT
NEXT X
PRINT
PRINT : PRINT "THE COMPOSITE SAMPLE D90 FOLLOW: "
PRINT
FOR X - 1 TO REPS
 PRINT CSNINETY(X);
 IF X / 10 - FIX(X / 10) THEN PRINT
NEXT X
PRINT : PRINT "THE COMPOSITE SAMPLE D10 FOLLOW: "
PRINT
FOR X - 1 TO REPS
```

```
 PRINT CSTEN(X);
 IF X / 10 - FIX(X / 10) THEN PRINT
NEXT X
PRINT
RETURN


12400  'WRITE SAMPLE AVERAGE VALUES
WRITE #1, : WRITE #1, "RUN" + STR$(J)
WRITE #1, "THE AVERAGE OF THE COMPOSITE SAMPLE MEDIANS IS " + STR$(MEDAVE)
WRITE #1, "THE AVERAGE OF THE THE COMPOSITE SAMPLE MEANS IS " + STR$(XBAR)
WRITE #1, "THE AVERAGE OF THE COMPOSITE SAMPLE VOLUMES IS " + STR$(VOLAVE)
WRITE #1, "THE STANDARD DEVIATION OF THE COMPOSITE SAMPLE MEDIANS IS " + STR$(STDV)
WRITE #1, "THE STANDARD DEVIATION OF THE COMPOSITE SAMPLE MEANS IS " + STR$(ASTDV)
WRITE #1, "THE AVERAGE OF THE COMPOSITE SAMPLE COEFFICIENTS OF SKEW IS " + STR$(SKAVE)
WRITE #1,
WRITE #1, "THE COMPOSITE SAMPLE MEDIAN FREQUENCIES FOLLOW: "
WRITE #1, "MEDIAN", "FREQUENCY"
VALUE - SMEDIAN(1): QUANT - 1
FOR X - 2 TO REPS
 IF SMEDIAN(X) - SMEDIAN(X - 1) THEN QUANT - QUANT + 1
 IF SMEDIAN(X) - SMEDIAN(X - 1) AND X - REPS THEN WRITE #1, VALUE, QUANT
 IF SMEDIAN(X) - SMEDIAN(X - 1) THEN GOTO 12500
 WRITE #1, VALUE, QUANT
 VALUE - SMEDIAN(X)
 QUANT - 1
 IF X - REPS THEN WRITE #1, VALUE, QUANT
12500 NEXT X
WRITE #1, : WRITE #1, "THE COMPOSITE SAMPLE D90 FREQUENCIES FOLLOW: "
WRITE #1, "D90", "FREQUENCY"
VALUE - CSNINETY(1): QUANT - 1
FOR X - 2 TO REPS
 IF CSNINETY(X) - CSNINETY(X - 1) THEN QUANT - QUANT + 1
 IF CSNINETY(X) - CSNINETY(X - 1) AND X - REPS THEN WRITE #1, VALUE, QUANT
```

```
 IF CSNINETY(X) = CSNINETY(X - 1) THEN GOTO 12510
 WRITE #1, VALUE, QUANT
 VALUE = CSNINETY(X)
 QUANT = 1
 IF X = REPS THEN WRITE #1, VALUE, QUANT
12510 NEXT X
WRITE #1,
WRITE #1, "THE COMPOSITE SAMPLE D10 FREQUENCIES FOLLOW: "
WRITE #1, "D10", "FREQUENCY"
VALUE = CSTEN(1): QUANT = 1
FOR X = 2 TO REPS
 IF CSTEN(X) = CSTEN(X - 1) THEN QUANT = QUANT + 1
 IF CSTEN(X) = CSTEN(X - 1) AND X = REPS THEN WRITE #1, VALUE, QUANT
 IF CSTEN(X) = CSTEN(X - 1) THEN GOTO 12520
 WRITE #1, VALUE, QUANT
 VALUE = CSTEN(X)
 QUANT = 1
 IF X = REPS THEN WRITE #1, VALUE, QUANT
12520 NEXT X
WRITE #1, : WRITE #1, "THE COMPOSITE SAMPLE MEAN VALUES FOLLOW: "
WRITE #1,
FOR X = 1 TO REPS
 PRINT #1, AVE(X); ",";
 IF X / 10 = FIX(X / 10) THEN PRINT #1,
NEXT X
WRITE #1,
WRITE #1, : WRITE #1, "THE COMPOSITE SAMPLE COEFFICIENTS OF SKEW FOLLOW: "
WRITE #1,
FOR X = 1 TO REPS
 PRINT #1, CSKEW(X); ",";
 IF X / 10 = FIX(X / 10) THEN PRINT #1,
NEXT X
WRITE #1,
WRITE #1, : WRITE #1, "THE COMPOSITE SAMPLE VOLUMES FOLLOW: "
```

```
WRITE #1,
FOR X = 1 TO REPS
 PRINT #1, CSVOL(X); ",";
 IF X / 10 = FIX(X / 10) THEN PRINT #1,
NEXT X
WRITE #1,

FOR P = 1 TO R
WRITE #1, : WRITE #1, "FOR AREA " + STR$(P)
WRITE #1, "THE AVERAGE OF THE SAMPLE MEDIANS IS " + STR$(AMEDTOT(P) / REPS)
WRITE #1, "THE AVERAGE OF THE THE SAMPLE MEANS IS " + STR$(AAVETOT(P) / REPS)
WRITE #1, "THE AVERAGE OF THE SAMPLE VOLUMES IS " + STR$(AVOLTOT(P) / REPS)
WRITE #1, "THE STANDARD DEVIATION OF THE SAMPLE MEDIANS IS " + STR$((ASS(P) / (REPS - 1)) ^ .5)
WRITE #1, "THE STANDARD DEVIATION OF THE SAMPLE MEANS IS " + STR$((ASSA(P) / (REPS - 1)) ^ .5)
WRITE #1, "THE AVERAGE OF THE SAMPLE COEFFICIENTS OF SKEW IS " + STR$(ASKTOT(P) / REPS)
WRITE #1,
WRITE #1, "THE SAMPLE MEDIAN FREQUENCIES FOLLOW: "
WRITE #1, "MEDIAN", "FREQUENCY"
VALUE = AMEDIAN(P, 1): QUANT = 1
FOR X = 2 TO REPS
 IF AMEDIAN(P, X) = AMEDIAN(P, X - 1) THEN QUANT = QUANT + 1
 IF AMEDIAN(P, X) = AMEDIAN(P, X - 1) AND X = REPS THEN WRITE #1, VALUE, QUANT
 IF AMEDIAN(P, X) = AMEDIAN(P, X - 1) THEN GOTO 12600
 WRITE #1, VALUE, QUANT
 VALUE = AMEDIAN(P, X)
 QUANT = 1
 IF X = REPS THEN WRITE #1, VALUE, QUANT
12600 NEXT X
WRITE #1, : WRITE #1, "THE SAMPLE D90 FREQUENCIES FOLLOW: "
WRITE #1, "D90", "FREQUENCY"
VALUE = ANINETY(P, 1): QUANT = 1
FOR X = 2 TO REPS
 IF ANINETY(P, X) = ANINETY(P, X - 1) THEN QUANT = QUANT + 1
 IF ANINETY(P, X) = ANINETY(P, X - 1) AND X = REPS THEN WRITE #1, VALUE, QUANT
```

```
 IF ANINETY(P, X) = ANINETY(P, X - 1) THEN GOTO 12610
 WRITE #1, VALUE, QUANT
 VALUE = ANINETY(P, X)
 QUANT = 1
 IF X = REPS THEN WRITE #1, VALUE, QUANT
12610 NEXT X
WRITE #1,
WRITE #1, "THE SAMPLE D10 FREQUENCIES FOLLOW: "
WRITE #1, "D10", "FREQUENCY"
VALUE = ATEN(P, 1): QUANT = 1
FOR X = 2 TO REPS
 IF ATEN(P, X) = ATEN(P, X - 1) THEN QUANT = QUANT + 1
 IF ATEN(P, X) = ATEN(P, X - 1) AND X = REPS THEN WRITE #1, VALUE, QUANT
 IF ATEN(P, X) = ATEN(P, X - 1) THEN GOTO 12620
 WRITE #1, VALUE, QUANT
 VALUE = ATEN(P, X)
 QUANT = 1
 IF X = REPS THEN WRITE #1, VALUE, QUANT
12620 NEXT X
WRITE #1, : WRITE #1, "THE SAMPLE MEAN VALUES FOLLOW: "
WRITE #1,
FOR X = 1 TO REPS
 PRINT #1, AAVE(P, X); ",";
 IF X / 10 = FIX(X / 10) THEN PRINT #1,
NEXT X
WRITE #1,
WRITE #1, : WRITE #1, "THE SAMPLE COEFFICIENTS OF SKEW FOLLOW: "
WRITE #1,
FOR X = 1 TO REPS
 PRINT #1, ASKEW(P, X); ",";
 IF X / 10 = FIX(X / 10) THEN PRINT #1,
NEXT X
WRITE #1,
WRITE #1, : WRITE #1, "THE SAMPLE VOLUMES FOLLOW: "
```

```
WRITE #1,
FOR X = 1 TO REPS
 PRINT #1, AVOL(P, X); ",";
 IF X / 10 = FIX(X / 10) THEN PRINT #1,
NEXT X
WRITE #1,
NEXT P
RETURN

14000 'TEST RANDOMNESS
CLS
F1 = 0: F2 = 0: F3 = 0: F4 = 0: F5 = 0: F6 = 0: F7 = 0: F8 = 0: F9 = 0: F10 = 0
FOR X = 1 TO 10000
RAND = (RND * 100000)
  RAND = RAND / 1000
  CT = CT + 1
  IF RAND <= 10 THEN F1 = F1 + 1: PRINT "F1"; RAND, CT
  IF RAND <= 20 AND RAND > 10 THEN F2 = F2 + 1: PRINT "F2"; RAND, CT
  IF RAND <= 30 AND RAND > 20 THEN F3 = F3 + 1: PRINT "F3"; RAND, CT
  IF RAND <= 40 AND RAND > 30 THEN F4 = F4 + 1: PRINT "F4"; RAND, CT
  IF RAND <= 50 AND RAND > 40 THEN F5 = F5 + 1: PRINT "F5"; RAND, CT
  IF RAND <= 60 AND RAND > 50 THEN F6 = F6 + 1: PRINT "F6"; RAND, CT
  IF RAND <= 70 AND RAND > 60 THEN F7 = F7 + 1: PRINT "F7"; RAND, CT
  IF RAND <= 80 AND RAND > 70 THEN F8 = F8 + 1: PRINT "F8"; RAND, CT
  IF RAND <= 90 AND RAND > 80 THEN F9 = F9 + 1: PRINT "F9"; RAND, CT
  IF RAND <= 100 AND RAND > 90 THEN F10 = F10 + 1: PRINT "F10"; RAND, CT
NEXT X
PRINT F1, F2, F3, F4, F5, F6, F7, F8, F9, F10, CT
INPUT Q
RETURN

DATA 4,0.0753
DATA 2.5E10,3.76,50
DATA 2.5E10,11.3,50
```

```
DATA 5.0E10,16.0,100
DATA 1.0E11,29.0,200
DATA 25
DATA 0.0753,0.106,0.150,0.212,0.297,0.421,0.596,0.843,1.18,1.67,2.37,3.35,4.76,6.73,9.47,13.4,19.0,26.8,38.0,53.7,75.9,107,152,215,304
DATA 0.100,0.400,1.200,2.800,7.700,13.500,18.300,21.000,23.800,27.600,33.600,42.500,51.900,63.500,77.500,89.600,95.600,97.900,100,100,100,100,100,100,100
DATA 0.200,0.400,0.900,1.600,3.500,6.400,9.900,12.200,14.000,16.000,18.400,21.900,26.600,32.600,39.000,49.700,64.000,75.800,91.500,98.000,98.700,100,100,100,100
DATA 0.100,0.200,0.400,0.700,1.400,2.700,5.300,8.000,10.900,14.300,18.400,23.300,28.200,33.400,39.000,44.800,50.000,54.400,63.500,77.200,90.500,96.700,100,100,100
DATA 0.100,0.300,0.700,1.300,2.800,5.000,8.500,11.300,13.800,16.000,18.000,20.500,23.000,27.200,30.500,34.900,40.600,46.100,51.800,62.900,77.400,85.500,100,100,100
DATA 0.063,0.090,0.125,0.180,0.250,0.354,0.500,0.710,1.00,1.40,2.00,2.80,4.00,5.66,8.00,11.2,16.0,22.5,32.0,45.0,64.0,90.0,128,180,256,360
```

APPENDIX B

Computer Program Used to Locate Sediment Boundaries
(Includes the data taken from test materials)

```
        'DEFINE VARIABLES AND ARRAYS

CLS
DIM MAP(60, 60): 'STONE COORDINATES ON MAP
DIM HB$(20, 20)
DIM VB$(20, 20)
DIM WMEAN(100): DIM WDEV(100): DIM ZSTATX(100): DIM ZSTATY(100) 'THE ADJACENT SUBSAMPLES' MEAN, STANDARD DEVIATION AND CONFIDENCE INTERVALS


        'INPUT OF SAMPLING SCENARIO INFORMATION
PRINT "WHAT ARE THE DIMENSIONS OF THE AREA IN TERMS GRID POINTS"
PRINT "HOW MANY GRIDS IN THE X DIRECTION": INPUT GRIDX
PRINT "HOW MANY GRID POINTS IN THE Y DIRECTION": INPUT GRIDY
PRINT "WHAT ARE THE DIMENSIONS OF THE WINDOW IN TERMS INPUT POINTS"
PRINT "HOW MANY GRIDS IN THE X DIRECTION": INPUT WSIZEX
PRINT "HOW MANY GRID POINTS IN THE Y DIRECTION": INPUT WSIZEY
PRINT "DO YOU WISH TO PUT RESULTS IN A DATA FILE Y OR N"; : INPUT DF$
IF DF$ = "Y" THEN PRINT "WHAT IS THE FILE NAME?"; : INPUT FILE$
IF DF$ = "Y" THEN OPEN FILE$ FOR OUTPUT AS #1

GOSUB 15000 'READ STONE SIZES OF MAP
GOSUB 16000 'CALCULATE WINDOW MEAN, STANDARD DEVIATION AND CONFIDENCE INTERVALS
GOSUB 17000 'FIND VERTICAL BOUNDARIES
GOSUB 18000 'FIND HORIZONTAL BOUNDARIES
GOSUB 19000 'OUTPUT WINDOW MEANS, STD DEV
GOSUB 20000 'OUTPUT POUNDARIES
IF DF$ = "Y" THEN CLOSE #1
END
```

```
15000 'OUTPUT OF THE GRID SAMPLES
IF DF$ = "Y" THEN WRITE #1, : WRITE #1, "THE STONE SIZE OF EACH GRID POINT IS DISPLAYED BELOW.": WRITE #1,
FOR Y = 1 TO GRIDY
  FOR X = 1 TO GRIDX
     READ MAP(X, Y)
     'PRINT "X,Y, MAP(X, Y)", X, Y, MAP(X, Y)
     'INPUT Q
  NEXT X
NEXT Y
RETURN

16000 'CALCULATE SUBSAMPLE MEAN, STANDARD DEVIATION
WCOUNT = 0
WSTONES = WSIZEX * WSIZEY
WNUMX = GRIDX / WSIZEX
WNUMY = GRIDY / WSIZEY
FOR Y = 1 TO WNUMY
  FOR X = 1 TO WNUMX
    WSUM = 0
    WCOUNT = WCOUNT + 1
    FOR V = 1 TO WSIZEY
      FOR W = 1 TO WSIZEX
        XTEMP = ((X - 1) * WSIZEX + W)
        YTEMP = ((Y - 1) * WSIZEY + V)
        WSUM = WSUM + MAP(((X - 1) * WSIZEX + W), ((Y - 1) * WSIZEY + V))
'PRINT XTEMP, YTEMP, MAP(XTEMP, YTEMP): INPUT Q
      NEXT W
    NEXT V
    WMEAN(WCOUNT) = WSUM / (WSIZEX * WSIZEY)
  NEXT X
NEXT Y
WCOUNT = 0
FOR Y = 1 TO WNUMY
```

```
  FOR X = 1 TO WNUMX
    WSUM = 0
    WCOUNT = WCOUNT + 1
    FOR V = 1 TO WSIZEY
      FOR W = 1 TO WSIZEX
        WSS = WSS + (MAP(((X - 1) * WSIZEX + W), ((Y - 1) * WSIZEY + V)) - WMEAN(WCOUNT)) ^ 2
      NEXT W
    NEXT V
    WDEV(WCOUNT) = (WSS / ((WSIZEX * WSIZEY) - 1)) ^ .5
    WSS = 0
  NEXT X
NEXT Y
RETURN

16500 'CALCULATE HORIZONTAL AND VERTICAL ZSTATS FOR PRINTOUT
WC = 0
FOR Y = 1 TO WNUMY 'HORIZONTAL ZSTATS
  FOR X = 1 TO (WNUMX - 1)
    WC = WC + 1
    TOP = (WMEAN((Y - 1) * WNUMX + X) - WMEAN((Y - 1) * WNUMX + X + 1))
    BOTTOM = ((WDEV((Y - 1) * WNUMX + X)) ^ 2 / WSTONES) + ((WDEV((Y - 1) * WNUMX + X + 1)) ^ 2 / WSTONES)
    ZSTATX(WC) = TOP / (BOTTOM) ^ .5
  NEXT X
NEXT Y
WC = 0
FOR Y = 1 TO (WNUMY - 1) ' VERTICAL BOUNDARIES
  FOR X = 1 TO WNUMX
    WC = WC + 1
    TOP = (WMEAN(WC) - WMEAN(Y * WNUMX + X))
    BOTTOM = ((WDEV(WC)) ^ 2 / WSTONES) + ((WDEV(Y * WNUMX + X)) ^ 2 / WSTONES)
    ZSTATY(WC) = TOP / (BOTTOM) ^ .5
  NEXT X
NEXT Y
RETURN
```

```
17000 'LOCATE BOUNDARIES
FOR Y = 1 TO WNUMY
  FOR X = 1 TO WNUMX
    HB$(X, Y) = "NONE "
  NEXT X
NEXT Y
FOR ROW = 1 TO WNUMY 'HORIZONTAL ZSTATS
WBASE = 1
WTWO = 2
WTHREE = 3
17010 IF WBASE >= WNUMX OR WTWO > WNUMX OR WTHREE > WNUMX THEN GOTO 17399

    TOP1 = (WMEAN((ROW - 1) * WNUMX + WBASE) - WMEAN((ROW - 1) * WNUMX + WTWO))
    BOT1 = ((WDEV((ROW - 1) * WNUMX + WBASE)) ^ 2 / WSTONES) + ((WDEV((ROW - 1) * WNUMX + WTWO)) ^ 2 / WSTONES)
    ZSTAT1 = TOP1 / (BOT1) ^ .5
    TOP2 = (WMEAN((ROW - 1) * WNUMX + WTWO) - WMEAN((ROW - 1) * WNUMX + WTHREE))
    BOT2 = ((WDEV((ROW - 1) * WNUMX + WTWO)) ^ 2 / WSTONES) + ((WDEV((ROW - 1) * WNUMX + WTHREE)) ^ 2 / WSTONES)
    ZSTAT2 = TOP2 / (BOT2) ^ .5
    TOP3 = (WMEAN((ROW - 1) * WNUMX + WBASE) - WMEAN((ROW - 1) * WNUMX + WTHREE))
    BOT3 = ((WDEV((ROW - 1) * WNUMX + WBASE)) ^ 2 / WSTONES) + ((WDEV((ROW - 1) * WNUMX + WTHREE)) ^ 2 / WSTONES)
    ZSTAT3 = TOP3 / (BOT3) ^ .5

'PRINT "ZSTAT1,2, AND 3", ZSTAT1, ZSTAT2, ZSTAT3
'PRINT "WBASE, WTWO, WTHREE", WBASE, WTWO, WTHREE
'INPUT Q

IF ZSTAT1 < -1.645 OR ZSTAT1 > 1.645 THEN GOTO 17100 ELSE GOTO 17200
17100 MPOSX = (WBASE * WSIZEX) - (2 * WSIZEX - 2)
    MPOSY = (WSIZEY * (ROW - 1)) + 1
    DIVM = 2
IF WBASE = 1 THEN SIDE$ = "L"
IF WBASE = 1 THEN GOTO 17105
GOSUB 17400
```

```
17105 IF SIDE$ = "L" THEN WBASE = WBASE + 1
IF SIDE$ = "L" THEN WTWO = WTWO + 1
IF SIDE$ = "L" THEN WTHREE = WTHREE + 1
IF SIDE$ = "M" THEN WBASE = WBASE + 1
IF SIDE$ = "M" THEN WTWO = WTWO + 1
IF SIDE$ = "M" THEN WTHREE = WTHREE + 1
IF SIDE$ = "R" THEN WBASE = WBASE + 2
IF SIDE$ = "R" THEN WTWO = WTWO + 2
IF SIDE$ = "R" THEN WTHREE = WTHREE + 2
GOTO 17010

17200 IF ZSTAT2 < -1.645 OR ZSTAT2 > 1.645 THEN GOTO 17210 ELSE GOTO 17300
17210 MPOSX = (WTWO * WSIZEX) - (2 * WSIZEX - 2)
    MPOSY = (WSIZEY * (ROW - 1)) + 1
    DIVM = 2
    IF WTHREE = WNUMX THEN WBASE = WNUMX
    IF WTHREE = WNUMX THEN GOTO 17010
GOSUB 17400
IF SIDE$ = "L" THEN WBASE = WBASE + 2
IF SIDE$ = "L" THEN WTWO = WTWO + 2
IF SIDE$ = "L" THEN WTHREE = WTHREE + 2
IF SIDE$ = "M" THEN WBASE = WBASE + 2
IF SIDE$ = "M" THEN WTWO = WTWO + 2
IF SIDE$ = "M" THEN WTHREE = WTHREE + 2
IF SIDE$ = "R" THEN WBASE = WBASE + 3
IF SIDE$ = "R" THEN WTWO = WTWO + 3
IF SIDE$ = "R" THEN WTHREE = WTHREE + 3
GOTO 17010

17300 IF ZSTAT3 < -1.645 OR ZSTAT3 > 1.645 THEN GOTO 17310 ELSE GOTO 17350
17310 MPOSX = (WTWO * WSIZEX) - (2 * WSIZEX - 2)
    MPOSY = (WSIZEY * (ROW - 1)) + 1
    DIVM = 1
GOSUB 17400
```

```
IF SIDE$ = "L" THEN WBASE = WBASE + 2
IF SIDE$ = "L" THEN WTWO = WTWO + 2
IF SIDE$ = "L" THEN WTHREE = WTHREE + 2
IF SIDE$ = "M" THEN WBASE = WBASE + 2
IF SIDE$ = "M" THEN WTWO = WTWO + 2
IF SIDE$ = "M" THEN WTHREE = WTHREE + 2
IF SIDE$ = "R" THEN WBASE = WBASE + 2
IF SIDE$ = "R" THEN WTWO = WTWO + 2
IF SIDE$ = "R" THEN WTHREE = WTHREE + 2
GOTO 17010

17350 WBASE = WBASE + 2
    WTWO = WTWO + 2
    WTHREE = WTHREE + 2
    GOTO 17010
17399 NEXT ROW
RETURN

17400 'FIND EXACT BOUNDARY LOCATION
MAX = 0
FOR Z = 1 TO ((DIVM * WSIZEX) - 1)
WASUM = 0
WBSUM = 0
PRINT "COODINATES FOR STONES TO BE SUMMED"
  FOR Y = MPOSY TO (MPOSY + (WSIZEY - 1))
    FOR X = MPOSX TO (MPOSX + (WSIZEX - 1))
      WASUM = WASUM + MAP(X, Y)
      WBSUM = WBSUM + MAP((X + WSIZEX), Y)
    'PRINT X; Y; X + WSIZEX; Y,
    NEXT X
  NEXT Y
  'INPUT Q
  WAMEAN = WASUM / WSTONES
  WBMEAN = WBSUM / WSTONES
```

```
  DIF = ABS(WAMEAN - WBMEAN)
  IF DIF > MAX THEN HBPOS = (MPOSX + (WSIZEX - 1))
  IF DIVM = 1 THEN GOTO 17410 ELSE GOTO 17420
17410 SIDE$ = "M"
    GOTO 17430
17420  IF DIF > MAX AND Z <= (WSIZEX - 1) THEN SIDE$ = "L"
      IF DIF > MAX AND Z = WSIZEX THEN SIDE$ = "M"
      IF DIF > MAX AND Z > WSIZEX THEN SIDE$ = "R"
17430  IF DIF > MAX THEN MAX = DIF
MPOSX = MPOSX + 1
'PRINT "X,Y, WAMEAN, X + WSIZEX,Y, WBMEAN, DIF, MAX"
'PRINT X; Y; WAMEAN, X; (Y + WSIZEY); WBMEAN, DIF, MAX
'INPUT Q
NEXT Z
HB$(INT(HBPOS / WSIZEX) + 1, ROW) = STR$(HBPOS)
RETURN

18000 'LOCATE BOUNDARIES
FOR Y = 1 TO WNUMY
  FOR X = 1 TO WNUMX
    VB$(X, Y) = "NONE "
  NEXT X
NEXT Y
FOR COL = 1 TO WNUMX 'VERTICAL ZSTATS
WBASE = 1
WTWO = 2
WTHREE = 3
18010 IF WBASE >= WNUMY OR WTWO > WNUMY OR WTHREE > WNUMY THEN GOTO 18399

    TOP1 = (WMEAN((COL - 1) + (WBASE - 1) * WNUMX + 1) - WMEAN((COL - 1) + (WTWO - 1) * WNUMX + 1))
    BOT1 = ((WDEV((COL - 1) + (WBASE - 1) * WNUMX + 1)) ^ 2 / WSTONES) + ((WDEV((COL - 1) + (WTWO - 1) * WNUMX + 1)) ^ 2 / WSTONES)
    ZSTAT1 = TOP1 / (BOT1) ^ .5
    TOP2 = (WMEAN((COL - 1) + (WTWO - 1) * WNUMX + 1) - WMEAN((COL - 1) + (WTHREE - 1) * WNUMX + 1))
    BOT2 = ((WDEV((COL - 1) + (WTWO - 1) * WNUMX + 1)) ^ 2 / WSTONES) + ((WDEV((COL - 1) + (WTHREE - 1) * WNUMX + 1)) ^ 2 / WSTONES)
```

```
    ZSTAT2 = TOP2 / (BOT2) ^ .5
    TOP3 = (WMEAN((COL - 1) + (WBASE - 1) * WNUMX + 1) - WMEAN((COL - 1) + (WTHREE - 1) * WNUMX + 1))
    BOT3 = ((WDEV((COL - 1) + (WBASE - 1) * WNUMX + 1)) ^ 2 / WSTONES) + ((WDEV((COL - 1) + (WTHREE - 1) * WNUMX + 1)) ^ 2 / WSTONES)
    ZSTAT3 = TOP3 / (BOT3) ^ .5

'PRINT "HORIZONTAL ZSTAT1,2, AND 3", ZSTAT1, ZSTAT2, ZSTAT3
'PRINT "WBASE, WTWO, WTHREE", WBASE, WTWO, WTHREE
'INPUT Q

IF ZSTAT1 < -1.645 OR ZSTAT1 > 1.645 THEN GOTO 18100 ELSE GOTO 18200
18100 MPOSX = (COL * WSIZEX) - (WSIZEX - 1)
    MPOSY = (WBASE * WSIZEY) - (2 * WSIZEY - 2)
    DIVM = 2
IF WBASE = 1 THEN SIDE$ = "U"
IF WBASE = 1 THEN GOTO 18105
GOSUB 18400
18105 IF SIDE$ = "U" THEN WBASE = WBASE + 1
IF SIDE$ = "U" THEN WTWO = WTWO + 1
IF SIDE$ = "U" THEN WTHREE = WTHREE + 1
IF SIDE$ = "M" THEN WBASE = WBASE + 1
IF SIDE$ = "M" THEN WTWO = WTWO + 1
IF SIDE$ = "M" THEN WTHREE = WTHREE + 1
IF SIDE$ = "L" THEN WBASE = WBASE + 2
IF SIDE$ = "L" THEN WTWO = WTWO + 2
IF SIDE$ = "L" THEN WTHREE = WTHREE + 2
GOTO 18010

18200 IF ZSTAT2 < -1.645 OR ZSTAT2 > 1.645 THEN GOTO 18210 ELSE GOTO 18300
18210 MPOSX = (COL * WSIZEX) - (WSIZEX - 1)
    MPOSY = (WTWO * WSIZEY - (2 * WSIZEY - 2))
    DIVM = 2
    IF WTHREE = WNUMY THEN WBASE = WNUMY
    IF WTHREE = WNUMY THEN GOTO 18010
GOSUB 18400
```

```
IF SIDE$ = "U" THEN WBASE = WBASE + 2
IF SIDE$ = "U" THEN WTWO = WTWO + 2
IF SIDE$ = "U" THEN WTHREE = WTHREE + 2
IF SIDE$ = "M" THEN WBASE = WBASE + 2
IF SIDE$ = "M" THEN WTWO = WTWO + 2
IF SIDE$ = "M" THEN WTHREE = WTHREE + 2
IF SIDE$ = "L" THEN WBASE = WBASE + 3
IF SIDE$ = "L" THEN WTWO = WTWO + 3
IF SIDE$ = "L" THEN WTHREE = WTHREE + 3
GOTO 18010

18300 IF ZSTAT3 < -1.645 OR ZSTAT3 > 1.645 THEN GOTO 18310 ELSE GOTO 18350
18310 MPOSX = (COL * WSIZEX) - (WSIZEX - 1)
    MPOSY = ((WTWO * WSIZEY) - ((2 * WSIZEY) - 2))
    DIVM = 1
GOSUB 18400
IF SIDE$ = "U" THEN WBASE = WBASE + 2
IF SIDE$ = "U" THEN WTWO = WTWO + 2
IF SIDE$ = "U" THEN WTHREE = WTHREE + 2
IF SIDE$ = "M" THEN WBASE = WBASE + 2
IF SIDE$ = "M" THEN WTWO = WTWO + 2
IF SIDE$ = "M" THEN WTHREE = WTHREE + 2
IF SIDE$ = "L" THEN WBASE = WBASE + 2
IF SIDE$ = "L" THEN WTWO = WTWO + 2
IF SIDE$ = "L" THEN WTHREE = WTHREE + 2
GOTO 18010

18350 WBASE = WBASE + 2
    WTWO = WTWO + 2
    WTHREE = WTHREE + 2
GOTO 18010
18399 NEXT COL
RETURN
```

```
18400 'FIND EXACT BOUNDARY LOCATION
MAX = 0
FOR Z = 1 TO ((DIVM * WSIZEY) - 1)
WASUM = 0
WBSUM = 0
PRINT "COORDINATES FOR STONES SUMMED"
  FOR Y = MPOSY TO (MPOSY + (WSIZEY - 1))
    FOR X = MPOSX TO (MPOSX + (WSIZEX - 1))
      WASUM = WASUM + MAP(X, Y)
      WBSUM = WBSUM + MAP(X, Y + WSIZEY)
    'PRINT X; Y; X; Y + WSIZEY,
    NEXT X
  NEXT Y
  'INPUT Q
  WAMEAN = WASUM / WSTONES
  WBMEAN = WBSUM / WSTONES
  DIF = ABS(WAMEAN - WBMEAN)
  IF DIF > MAX THEN VBPOS = (MPOSY + (WSIZEY - 1))
  IF DIVM = 1 THEN GOTO 18410 ELSE GOTO 18420
18410 SIDE$ = "M"
    GOTO 18430
18420  IF DIF > MAX AND Z <= (WSIZEY - 1) THEN SIDE$ = "U"
      IF DIF > MAX AND Z = WSIZEY THEN SIDE$ = "M"
      IF DIF > MAX AND Z > WSIZEY THEN SIDE$ = "L"
18430  IF DIF > MAX THEN MAX = DIF
'PRINT "X,Y, WAMEAN, X,Y+WSIZEY, WBMEAN, DIF, MAX"
'PRINT X; Y; WAMEAN, X; (Y + WSIZEY); WBMEAN, DIF, MAX
'INPUT Q
MPOSY = MPOSY + 1
NEXT Z
VB$(COL, (INT(VBPOS / WSIZEY) + 1)) = STR$(VBPOS)
RETURN

19000 'PLOT OF MEANS
```

```
IF DF$ = "Y" THEN WRITE #1, : WRITE #1, "PLOT OF THE MEANS": WRITE #1,
HC = 0
VC = 0
FOR Y = 1 TO WNUMY
  FOR X = 1 TO WNUMX
    HC = HC + 1
    IF X = WNUMX THEN PRINT USING "###.##"; WMEAN(HC)
    IF X = WNUMX AND DF$ = "Y" THEN PRINT #1, USING "###.##"; WMEAN(HC)
    IF X = WNUMX THEN GOTO 19600
    PRINT USING "###.##"; WMEAN(HC);
    IF DF$ < > "Y" THEN GOTO 19600
    PRINT #1, WMEAN(HC);
19600 NEXT X
NEXT Y
IF DF$ = "Y" THEN PRINT #1,
INPUT Q
RETURN

20000 'OUTPUT WINDOW RESULTS
WC = 0
PRINT "VERTICAL BOUNDARIES WERE LOCATED IMMEDIATELY AFTER THE FOLLOWING STONE #'S"
IF DF$ = "Y" THEN WRITE #1,
IF DF$ = "Y" THEN WRITE #1, "VERTICAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS"
FOR Y = 1 TO WNUMY
  PRINT
  IF DF$ = "Y" THEN PRINT #1,
  FOR X = 1 TO WNUMX
    PRINT HB$(X, Y);
    IF DF$ = "Y" THEN PRINT #1, CHR$(34); HB$(X, Y); CHR$(34); ",";
  NEXT X
NEXT Y
PRINT
IF DF$ = "Y" THEN PRINT #1,
PRINT "HORIZONTAL BOUNDARIES WERE LOCATED IMMEDIATELY AFTER THE FOLLOWING STONE #'S"
```

```
IF DF$ = "Y" THEN WRITE #1, "HORIZONTAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS"
FOR Y = 1 TO WNUMY
  PRINT
  IF DF$ = "Y" THEN PRINT #1,
  FOR X = 1 TO WNUMX
    PRINT VB$(X, Y);
    IF DF$ = "Y" THEN PRINT #1, CHR$(34); VB$(X, Y); CHR$(34); ",";
  NEXT X
NEXT Y
PRINT
IF DF$ = "Y" THEN PRINT #1,
IF DF$ = "Y" THEN WRITE #1, "WINDOW", "MEAN", "STD DEV": WRITE #1,
PRINT "WINDOW", "MEAN", "STD DEV": PRINT
FOR X = 1 TO WNUMX * WNUMY
IF DF$ = "Y" THEN WRITE #1, X, WMEAN(X), WDEV(X)
PRINT X, WMEAN(X), WDEV(X)
NEXT X
RETURN

'COMPLETED DATA

'MEAN = 17.55 MM  STONES FROM TOP LEFT TO RIGHT THEN DOWN BY EACH ROW BOX #1
'MEAN = 12.51 MM  STONES FROM TOP LEFT TO RIGHT THEN DOWN BY EACH ROW BOX #1
'MEAN = 14.65 MM  STONES FROM TOP LEFT TO RIGHT THEN DOWN BY EACH ROW BOX #1
'MEAN = 17.55 MM  STONES FROM TOP LEFT TO RIGHT THEN DOWN BY EACH ROW BOX #2


DATA 25.4, 14.29, 19.05, 25.4, 14.29, 25.4, 14.29, 19.05, 7, 19.05, 19.05, 19.05, 14.29, 19.05, 14.29, 14.29, 14.29, 19.05, 25.4, 25.4
'DATA 14.29, 14.29, 14.29, 7, 19.05, 19.05, 19.05, 7, 14.29, 5, 19.05, 19.05, 19.05, 5, 14.29, 7, 10, 5, 19.05, 14.29
DATA 25.4, 14.29, 25.4, 14.29, 14.29, 19.05, 7, 14.29, 19.05, 25.4, 10, 25.4, 7, 19.05, 19.05, 7, 19.05, 10, 10, 25.4
DATA 7, 25.4, 25.4, 19.05, 25.4, 10, 7, 7, 14.29, 7, 25.4, 25.4, 19.05, 7, 14.29, 19.05, 10, 25.4, 5, 19.05


DATA 14.29, 7, 14.29, 7, 19.05, 14.29, 25.4, 19.05, 19.05, 10, 25.4, 14.29, 25.4, 14.29, 7, 25.4, 25.4, 19.05, 5, 19.05
```

```
'DATA 7, 19.05, 4, 4, 7, 14.29, 10, 7, 14.28, 7, 19.05, 7, 10, 10, 7, 14.29, 10, 5, 19.05, 14.29
DATA 19.05, 25.4, 7, 7, 14.29, 10, 19.05, 25.4, 7, 25.4, 14.29, 10, 19.05, 14.29, 25.4, 10, 10, 5, 19.05, 10
DATA 19.05, 14.29, 19.05, 19.05, 25.4, 7, 25.4, 19.05, 25.4, 19.05, 25.4, 14.29, 25.4, 19.05, 14.29, 19.05, 19.05, 25.4, 7, 25.4

DATA 7, 10, 19.05, 25.4, 19.05, 19.05, 19.05, 25.4, 14.29, 25.4, 10, 5, 4, 10, 14.29, 14.29, 19.05, 19.05, 14.29, 19.05
'DATA 14.29, 10, 14.29, 10, 19.05, 7, 10, 25.4, 14.29, 19.05, 10, 4, 5, 19.05, 10, 14.29, 14.29, 7, 25.4, 19.05
DATA 7, 19.05, 10, 10, 10, 7, 19.05, 14.29, 14.29, 25.4, 14.29, 10, 19.05, 5, 14.29, 7, 7, 5, 14.29, 10
DATA 19.05, 10, 19.05, 14.29, 25.4, 19.05, 14.29, 5, 14.29, 10, 25.4, 7, 14.29, 19.05, 19.05, 7, 19.05, 10, 25.4, 25.4

DATA 4, 25.4, 19.05, 19.05, 14.29, 19.05, 10, 10, 5, 14.29, 25.4, 7, 19.05, 19.05, 25.4, 10, 19.05, 14.29, 25.4, 25.4
'DATA 7, 7, 7, 14.29, 10, 7, 5, 19.05, 10, 14.29, 10, 14.29, 25.4, 7, 7, 10, 7, 10, 10, 10
DATA 25.4, 14.29, 14.29, 14.29, 19.05, 14.29, 7, 10, 14.29, 10, 10, 10, 19.05, 14.29, 14.29, 14.29, 7, 14.29, 25.4, 5
DATA 7, 14.29, 25.4, 19.05, 25.4, 19.05, 7, 14.29, 14.29, 25.4, 19.05, 10, 19.05, 5, 19.05, 19.05, 14.29, 25.4, 5, 19.05

DATA 19.05, 19.05, 25.4, 19.05, 5, 25.4, 7, 19.05, 7, 14.29, 25.4, 14.29, 10, 10, 7, 14.29, 19.05, 19.05, 19.05, 14.29
'DATA 10, 19.05, 5, 10, 14.29, 5, 25.4, 10, 14.29, 7, 14.29, 7, 14.29, 7, 5, 10, 4, 19.05, 19.05, 19.05
DATA 19.05, 14.29, 25.4, 10, 14.29, 5, 14.29, 7, 25.4, 14.29, 14.29, 7, 7, 14.29, 14.29, 14.29, 5, 25.4, 10, 19.05
DATA 4, 7, 19.05, 19.05, 7, 14.29, 25.4, 25.4, 25.4, 7, 14.29, 25.4, 19.05, 25.4, 25.4, 25.4, 25.4, 14.29, 25.4, 19.05

DATA 19.05, 19.05, 14.29, 25.4, 14.29, 14.29, 25.4, 19.05, 7, 25.4, 19.05, 25.4, 14.29, 10, 19.05, 19.05, 19.05, 25.4, 14.29, 19.05
'DATA 5, 5, 19.05, 10, 7, 7, 10, 7, 10, 7, 14.29, 4, 10, 7, 19.05, 14.29, 7, 10, 14.29, 7
DATA 14.29, 7, 5, 25.4, 14.29, 10, 19.05, 14.29, 19.05, 14.29, 14.29, 4, 5, 25.4, 14.29, 14.29, 19.05, 14.29, 25.4, 5
DATA 4, 19.05, 14.29, 19.05, 14.29, 25.4, 19.05, 14.29, 19.05, 25.4, 10, 25.4, 25.4, 5, 14.29, 10, 14.29, 19.05, 19.05, 14.29

DATA 25.4, 7, 25.4, 25.4, 14.29, 14.29, 19.05, 19.05, 25.4, 25.4, 19.05, 25.4, 25.4, 14.29, 10, 25.4, 14.29, 19.05, 19.05, 14.29
'DATA 19.05, 14.29, 10, 4, 7, 19.05, 10, 5, 19.05, 14.29, 19.05, 7, 14.29, 7, 7, 14.29, 19.05, 4, 10, 19.05
DATA 14.29, 14.29, 10, 7, 14.29, 25.4, 25.4, 5, 4, 10, 10, 25.4, 19.05, 7, 10, 19.05, 19.05, 7, 4, 14.29
DATA 7, 4, 25.4, 10, 10, 14.29, 14.29, 19.05, 19.05, 14.29, 19.05, 14.29, 7, 25.4, 7, 19.05, 10, 19.05, 25.4, 19.05

DATA 19.05, 19.05, 19.05, 14.29, 14.29, 14.29, 10, 25.4, 25.4, 14.29, 25.4, 19.05, 19.05, 25.4, 7, 25.4, 14.29, 25.4, 19.05, 25.4
'DATA 7, 4, 14.29, 19.05, 14.29, 5, 7, 19.05, 19.05, 10, 7, 10, 7, 14.29, 14.29, 7, 4, 14.29, 14.29, 5
DATA 10, 14.29, 14.29, 10, 14.29, 14.29, 5, 7, 14.29, 25.4, 14.29, 10, 5, 10, 14.29, 10, 25.4, 5, 19.05, 25.4
DATA 25.4, 5, 19.05, 14.29, 19.05, 10, 25.4, 10, 25.4, 19.05, 4, 25.4, 10, 19.05, 7, 25.4, 7, 19.05, 19.05, 25.4
```

```
DATA 19.05, 19.05, 19.05, 19.05, 25.4, 25.4, 14.29, 19.05, 7, 10, 19.05, 19.05, 25.4, 7, 19.05, 25.4, 19.05, 25.4, 5, 19.05
'DATA 5, 14.29, 10, 7, 19.05, 14.29, 7, 10, 19.05, 19.05, 5, 14.29, 5, 25.4, 7, 14.29, 7, 7, 7, 14.29
DATA 14.29, 25.4, 14.29, 14.29, 7, 19.05, 7, 14.29, 19.05, 14.29, 14.29, 14.29, 14.29, 4, 14.29, 19.05, 14.29, 14.29, 14.29, 19.05
DATA 25.4, 14.29, 19.05, 14.29, 14.29, 14.29, 19.05, 19.05, 19.05, 25.4, 14.29, 19.05, 25.4, 19.05, 14.29, 25.4, 14.29, 25.4, 14.29, 10

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'MEAN = 17.55 MM  STONES FROM TOP LEFT TO RIGHT THEN DOWN BY EACH ROW BOX #3
'MEAN = 12.51 MM  STONES FROM TOP LEFT TO RIGHT THEN DOWN BY EACH ROW BOX #2
'MEAN = 14.65 MM  STONES FROM TOP LEFT TO RIGHT THEN DOWN BY EACH ROW BOX #2
'MEAN = 17.55 MM  STONES FROM TOP LEFT TO RIGHT THEN DOWN BY EACH ROW BOX #4

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DATA 4, 14.29, 10, 14.29, 10, 14.29, 14.29, 14.29, 14.29, 10, 10, 14.29, 25.4, 7, 19.05, 14.29, 10, 5, 14.29, 5
DATA 25.4, 7, 14.29, 25.4, 14.29, 19.05, 19.05, 25.4, 7, 10, 25.4, 19.05, 14.29, 14.29, 5, 7, 7, 25.4, 14.29, 25.4

DATA 25.4, 19.05, 14.29, 19.05, 14.29, 14.29, 25.4, 25.4, 25.4, 25.4, 19.05, 14.29, 19.05, 14.29, 19.05, 25.4, 19.05, 14.29, 5, 5
'DATA 10, 5, 10, 19.05, 19.05, 5, 14.29, 10, 19.05, 7, 4, 7, 10, 7, 19.05, 19.05, 5, 25.4, 19.05, 7
DATA 10, 14.29, 14.29, 25.4, 7, 19.05, 10, 7, 19.05, 19.05, 10, 14.29, 10, 7, 10, 7, 10, 14.29, 14.29, 25.4
DATA 25.4, 19.05, 19.05, 19.05, 19.05, 14.29, 7, 14.29, 10, 14.29, 25.4, 19.05, 19.05, 10, 19.05, 19.05, 19.05, 10, 25.4, 19.05

DATA 14.29, 19.05, 14.29, 19.05, 25.4, 19.05, 19.05, 19.05, 7, 19.05, 25.4, 10, 14.29, 25.4, 7, 14.29, 14.29, 25.4, 14.29, 19.05
'DATA 19.05, 14.29, 25.4, 5, 10, 5, 14.29, 14.29, 14.29, 14.29, 7, 5, 14.29, 19.05, 14.29, 14.29, 7, 7, 5, 19.05
DATA 19.05, 10, 25.4, 14.29, 19.05, 10, 10, 14.29, 19.05, 25.4, 5, 14.29, 19.05, 25.4, 19.05, 10, 10, 14.29, 7, 19.05
DATA 7, 25.4, 14.29, 14.29, 19.05, 10, 14.29, 25.4, 19.05, 14.29, 19.05, 5, 7, 14.29, 19.05, 19.05, 19.05, 14.29, 14.29, 25.4
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APPENDIX C

Computer Output Locating Sediment Boundares in the Test Deposits

Deposit #1: Subsample Size = 6 x 4: Difference in the two means is 2.9 mm

PLOT OF THE MEANS

16.67	15.90	16.64	17.13	15.20	13.84	13.02	17.34	17.06	17.11
17.96	19.22	17.13	14.96	13.98	12.60	13.24	16.97	17.87	17.85
17.20	17.22	17.73	16.29	14.24	14.79	15.92	16.86	18.73	15.74
16.48	17.21	17.77	13.79	14.05	15.51	14.14	16.27	18.79	18.53
18.47	16.98	16.69	16.88	13.82	15.67	16.85	18.50	17.29	17.03
17.56	16.00	15.81	15.86	16.58	13.20	13.96	17.81	18.09	17.47
16.62	17.38	16.91	16.16	15.50	15.42	13.84	16.71	17.18	18.13
16.54	18.88	16.88	16.04	14.91	13.36	13.56	17.69	17.23	16.31
17.69	16.28	17.81	16.92	13.38	15.69	13.88	18.07	13.76	15.21
17.95	20.22	15.81	13.95	14.58	14.96	14.69	17.41	16.91	16.31

VERTICAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	NONE	NONE	NONE	41	NONE	NONE	NONE
NONE	NONE	NONE	21	NONE	NONE	NONE	46	NONE	NONE
NONE	NONE	NONE	21	NONE	NONE	NONE	43	NONE	NONE
NONE	NONE	NONE	19	NONE	NONE	NONE	46	NONE	NONE
NONE	NONE	NONE	20	NONE	32	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	30	NONE	42	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	39	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	41	NONE	NONE	NONE
NONE	NONE	NONE	22	NONE	NONE	41	NONE	48	NONE
NONE	NONE	12	NONE	NONE	NONE	NONE	NONE	NONE	NONE

HORIZONTAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	7	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	16	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	21	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	30	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE

Deposit #1: Subsample Size = 6 x 5: Difference in the two means is 2.9 mm

PLOT OF THE MEANS

17.103	15.618	15.957	17.1073	14.8383	13.4413	12.767	17.542	17.5363	18.19
17.797	19.866	19.1783	15.091	14.317	14.127	14.4223	16.6703	17.8693	16.61
16.3527	16.7623	16.7017	14.903	13.8453	14.8187	14.8267	16.1763	18.6257	16.09
18.1773	16.974	16.925	16.1377	14.0407	15.538	16.5273	18.3573	17.76	18.12
17.254	16.5173	15.8527	15.6857	16.2763	13.5627	13.6413	17.9673	18.1417	17.8
16.5567	17.699	17.0427	16.1453	15.001	16.151	13.6157	16.774	17.6697	16.89
17.8753	17.0543	18.089	16.1803	14.466	13.1867	14.362	17.7223	13.827	15.55
17.4007	19.7387	15.5983	15.1263	14.2233	15.2047	14.3347	17.6813	16.8957	16.5

VERTICAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	NONE	NONE	NONE	41	NONE	NONE	NONE
NONE	NONE	NONE	20	NONE	NONE	NONE	46	NONE	NONE
NONE	NONE	NONE	19	NONE	NONE	NONE	44	NONE	NONE
NONE	NONE	NONE	20	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	42	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	42	NONE	NONE
NONE	NONE	NONE	22	NONE	NONE	41	NONE	48	NONE
NONE	NONE	12	NONE	NONE	NONE	40	NONE	NONE	NONE

HORIZONTAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	10	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	20	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	29	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	30	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE

Deposit #1: Subsample Size - 6 x 8: Difference in the two means is 2.9 mm

PLOT OF THE MEANS

17.31791	17.55688	16.88583	16.04583	14.59333	13.22062	13.13313	17.1552	17.465	17.48
16.84167	17.21395	17.74542	15.03792	14.14792	15.14687	15.03187	16.56354	18.76187	17.14
18.01625	16.49021	16.25124	16.36854	15.19937	14.43666	15.40541	18.15208	17.68562	17.25
16.57958	18.13	16.89437	16.10104	15.20812	14.39	13.70187	17.19917	17.20667	17.22
17.8175	18.2525	16.81354	15.43208	13.98125	15.32458	14.28833	17.73687	15.33416	15.76

VERTICAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	22	NONE	NONE	NONE	42	NONE	NONE
NONE	NONE	NONE	19	NONE	NONE	NONE	NONE	48	NONE
NONE	NONE	NONE	NONE	NONE	NONE	40	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	42	NONE	NONE
NONE	NONE	NONE	22	NONE	NONE	41	43	NONE	NONE

HORIZONTAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	9	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	26	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE

Deposit #2: Subsample Size = 6 x 4: Difference in the two means is 5.04 mm

PLOT OF THE MEANS

16.6725	15.89625	16.63833	13.61583	12.63083	11.95042	13.14583	17.33625	17.06458	17.11
17.96333	19.2175	17.13333	13.30208	11.365	10.60083	11.02375	16.97416	17.86541	17.85
17.20167	17.21875	17.72583	14.94375	12.74375	12.23958	14.255	16.85625	18.73	15.74
16.48166	17.20917	17.765	13.32958	11.89917	11.985	13.08	16.27083	18.79375	18.53
18.46833	16.98375	16.69	15.98208	10.53708	11.7225	14.20917	18.495	17.28542	17.03
17.56417	15.99667	15.8125	15.68833	10.86042	13.47417	14.61292	18.31125	17.70875	17.64
15.7125	17.82375	17.37542	12.36667	11.35042	11.72458	12.97917	17.61292	16.48875	19.02
16.73875	18.94583	16.85666	13.54	9.657916	11.57792	13.07958	17.23875	18.64916	15.94
15.77875	17.73083	17.54958	15.49958	10.83125	11.31125	14.45583	17.69167	14.95083	15.24
17.7475	19.54083	16.29542	12.80042	11.70792	10.93375	14.39458	18.17167	15.80083	17.61

VERTICAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	20	NONE	NONE	39	NONE	NONE	NONE
NONE	NONE	NONE	20	NONE	NONE	NONE	42	NONE	NONE
NONE	NONE	NONE	21	NONE	NONE	NONE	43	NONE	NONE
NONE	NONE	NONE	19	NONE	NONE	41	NONE	NONE	NONE
NONE	NONE	NONE	23	NONE	34	40	NONE	NONE	NONE
NONE	NONE	NONE	NONE	24	30	40	NONE	NONE	NONE
NONE	NONE	NONE	20	NONE	NONE	NONE	42	NONE	NONE
NONE	NONE	NONE	23	NONE	35	40	NONE	NONE	NONE
NONE	NONE	NONE	20	NONE	NONE	40	NONE	NONE	NONE
NONE	11	14	NONE	NONE	NONE	40	NONE	NONE	NONE

HORIZONTAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	9	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	25	NONE	NONE	NONE	NONE	NONE	27
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	32	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE

Deposit #2: Subsample Size = 6 x 5: Difference in the two means is 5.04 mm

PLOT OF THE MEANS

17.103	15.618	15.957	13.47233	12.63733	11.47967	12.92166	17.542	17.53633	18.19
17.797	19.866	19.17833	13.68767	11.48533	12.01267	11.71833	16.67033	17.86933	16.61
16.35266	16.76233	16.70166	14.20167	12.45933	11.909	14.119	16.17633	18.62567	16.09
18.17733	16.974	16.925	15.577	10.75867	11.39733	13.812	18.35733	17.76	18.12
17.111	16.66033	16.01133	16.022	10.933	12.632	13.26667	18.88233	17.22666	18.56
15.83166	17.91067	17.609	11.84966	11.34267	11.55967	14.05266	16.97	18.25933	16.48
17.162	17.96367	17.66567	15.001	9.447666	12.10867	14.303	17.217	14.86733	16.16
16.72867	19.49566	15.82567	13.04333	11.803	10.917	13.99533	18.15166	16.52533	17.17

VERTICAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	20	NONE	NONE	38	NONE	NONE	NONE
NONE	NONE	NONE	20	NONE	NONE	NONE	42	NONE	NONE
NONE	NONE	NONE	19	NONE	NONE	NONE	44	NONE	NONE
NONE	NONE	NONE	22	NONE	34	40	NONE	NONE	NONE
NONE	NONE	NONE	23	NONE	NONE	40	NONE	NONE	NONE
NONE	NONE	NONE	18	NONE	NONE	41	NONE	NONE	NONE
NONE	NONE	NONE	23	25	NONE	40	NONE	NONE	NONE
NONE	11	NONE	20	NONE	NONE	40	NONE	NONE	NONE

HORIZONTAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	10	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	17	NONE	17
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	25	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	31	NONE	NONE	NONE	NONE	30	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE

Deposit #2: Subsample Size = 6 x 8: Difference in the two means is 5.04 mm

PLOT OF THE MEANS

17.31791	17.55688	16.88583	13.45896	11.99792	11.27562	12.08479	17.1552	17.465	17.48
16.84167	17.21395	17.74542	14.13667	12.32146	12.11229	13.6675	16.56354	18.76187	17.14
18.01625	16.49021	16.25124	15.83521	10.69875	12.59833	14.41104	18.40312	17.49708	17.34
16.22562	18.38479	17.11604	12.95333	10.50417	11.65125	13.02937	17.42583	17.56896	17.48
16.76312	18.63583	16.9225	14.15	11.26958	11.1225	14.42521	17.93166	15.37583	16.43

VERTICAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	20	NONE	NONE	NONE	42	NONE	NONE
NONE	NONE	NONE	19	NONE	NONE	41	43	NONE	NONE
NONE	NONE	NONE	23	29	NONE	40	NONE	NONE	NONE
NONE	NONE	NONE	20	NONE	35	40	NONE	NONE	NONE
NONE	NONE	NONE	20	NONE	NONE	40	46	NONE	NONE

HORIZONTAL BOUNDARIES ARE LOCATED AFTER THE FOLLOWING STONE NUMBERS

NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	9	NONE	NONE	9	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	25	NONE	NONE	NONE	NONE	NONE	NONE
NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE	NONE

Vita

The author was born December 11, 1968 in Alamosa Colorado. In 1987 he graduated from Alamosa High School. He then enrolled at Virginia Polytechnic Institute and State University and in 1992 received his bachelor 's degree in civil engineering. That same year, he stayed on to begin his studies for his masters degree.