FINITE ELEMENT ANALYSIS OF STRESSES IN A UNIAXIALLY LOADED ELASTIC SHEET CONTAINING AN INTERFERENCE-FIT FASTENER

by

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(ABSTRACT)

A finite element model is developed to study the stresses in a uniaxially loaded infinite sheet containing an interference-fit fastener. The sheet-fastener interface is modeled using one-dimensional gap elements. The geometry is chosen so that the performance of the gap element can be compared with known theoretical solutions. The fastener is modeled as a disk with thickness equal to that of the sheet. The effect of the fastener exiting the sheet, referred to as edge-stiffening, is neglected in the current study. Plane stress conditions are assumed for the sheet and fastener. Material response is assumed to be elastic after fastener insertion and during subsequent loading. Frictionless and no-slip conditions for the sheet-fastener interface are investigated. These two idealized conditions are expected to bracket the real behavior of the sheet-fastener interface. The ability of the gap element to predict the sheet-fastener separation stress for frictionless and no-slip interface conditions is investigated. Results obtained from the finite element models compare favorably with theoretical solutions.
LIST OF SYMBOLS

\( E_s \)  
Young's modulus for sheet, psi

\( E_f \)  
Young's modulus for fastener, psi

\( R \)  
radius of hole in sheet, inch

\( D_f \)  
diameter of fastener, inch

\( D_s \)  
diameter of hole in sheet, inch

\( I \)  
diameter based interference level, inch

\( r, \theta \)  
polar coordinates, inch and degree

\( \nu \)  
Poisson's ratio for sheet, dimensionless

\( \nu_f \)  
Poisson's ratio for fastener, dimensionless

\( \alpha_f \)  
coefficient of thermal expansion for fastener, \(1/\circ\text{F}\)

\( \sigma_{rr} \)  
radial stress, psi

\( \sigma_{\theta\theta} \)  
tangential stress, psi

\( \tau_{r\theta} \)  
shear stress, psi

\( \sigma_g \)  
gross stress, psi

\( \sigma_s \)  
separation stress, psi

\( \sigma_e \)  
von Mises stress, psi

\( \sigma_{me} \)  
equivalent mean von Mises stress, psi

\( \sigma_{ae} \)  
equivalent alternating von Mises stress, psi
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</tbody>
</table>
INTRODUCTION

The use of interference-fit fasteners to increase the fatigue life in new and reworked aerospace structures is commonplace. Interference-fit fasteners offer several advantages, including straightforward installation techniques and significant increase in fatigue life at minimal cost [1,2]. A typical interference-fit fastener installation is shown in Fig. 1. For interference-fit applications, the fastener diameter is larger than the diameter of the hole in a structure in which the fastener is to be installed. The difference between the fastener diameter and the sheet-hole diameter is referred to as the diameter-based interference level, which varies from 0.001 inch to 0.0045 inch in most aerospace applications. During insertion of the fastener, the hole in the structure expands radially, resulting in tangential tension stresses and radial compressive stresses while the fastener contracts radially. In most cases, the subsequent applied loading in the structure with the interference-fit fastener installed is a combination of axial tension and compression.

A literature review revealed an elastic solution for the stress field in a uniaxially loaded infinite sheet containing an interference-fit fastener was developed by Muskhelishvili [3] and Savin [4]. Crews [5] applied the solutions presented in [3] and [4] to aerospace structures and explained why interference-fit fasteners improve fatigue life. Crews studied two sheet-fastener interface conditions which are expected to bracket the real behavior of the sheet-fastener interface. The solutions are valid as long as material response is elastic and the fastener remains in contact with the sheet. The first condition assumed a frictionless interface between the sheet and fastener. The second condition assumed a no-slip interface between the sheet and fastener. Crews also investigated tension loading conditions that caused incipient separation between the sheet and fastener.

With the evolution of high-speed computers, finite element models of interference-fit fasteners in sheets developed along with methods to model the sheet-fastener interface. Finite element modeling of an interference-fit fastener in an unloaded sheet is straightforward. Since insertion of the interference-fit fastener in the sheet produces only radial forces at the interface, the nodes at the interface can be common. One method to
develop the required interference is to thermally expand the fastener in the sheet. Another method is to place a small hole in the center of the fastener and apply a radial pressure in the hole that will expand the fastener. The small hole is assumed to have a negligible effect on fastener stiffness. In each case the temperature or pressure required for correct expansion of the fastener is calculated using basic elasticity theory.

Special consideration must be given to sheet-fastener interface conditions when tension or compression loading is applied to the sheet. As presented in [5], two interface conditions are expected to bracket the actual behavior. The first condition is the frictionless interface. In this case, only radial transfer of load across the sheet-fastener interface is allowed. Shear stresses are not allowed to develop at the interface. Load transfer due to in-plane shearing action between sheet and fastener does not occur. The frictionless interface assumption is considered conservative, since in reality, shear transfer occurs at the interface. Also, as reported in [5], the frictionless interface tends to under-predict the sheet-fastener separation stress due to lack of shear transfer. Various methods have been developed to model this condition. Carey [6] made use of multi-point constraints to ensure radial-only load transfer. Crews [7] used orthotropic elements with zero tangential stiffness on the outer layer of fastener elements to ensure radial-only load transfer. Crews, Hong, and Raju [8] connected nodes on the sheet-fastener interface with stiff radial spring elements which had zero transverse stiffness to model the frictionless interface condition.

The second condition is the no-slip interface where shear stresses are assumed to fully transfer across the interface. In reality, slip at the interface can occur, especially when high tension loads are applied to the sheet. The no-slip condition serves as an upper-bound on the amount of shear transfer that can occur at the interface for a given sheet tension load. As reported in [5], the no-slip interface over-predicts the sheet-fastener separation stress, since full shear transfer does not occur in the real case. The nodes at the sheet-fastener interface are common when the no-slip condition is imposed.
With advances in non-linear finite element analysis, the sheet-fastener interface can now be easily modeled using contact elements. However, a recent search revealed no published literature exists on this topic. Along with modeling the frictionless and no-slip interface conditions, contact elements also permit friction to be modeled at the interface thus allowing a more accurate prediction of separation stresses. Output from the contact element allows for easy visualization of normal and tangential displacements as well as radial and shear load transfer at the interface.

The purpose of the current work is to study the sheet-fastener interface using one-dimensional gap elements. The geometry under consideration is an infinite sheet containing an interference-fit fastener and is shown in Fig. 2. This geometry is chosen so that the performance of the gap element can be compared with solutions presented in [5]. The fastener is modeled as a disk with thickness equal to that of the sheet. The effect of the fastener exiting the sheet, referred to as edge-stiffening, causes an increase in stresses at the edge of the hole in the sheet. This effect is neglected in the current study. Refer to [9] and [10] for more details on this effect. The materials for the sheet and fastener, aluminum and steel respectively, are chosen to be typical of what would be used in aerospace structures. Material response is assumed to be elastic during fastener insertion and subsequent axial loading. Frictionless and no-slip interface conditions will be investigated for a loaded and unloaded sheet. The ability of the element to predict the sheet-fastener separation stress for frictionless and no-slip interface conditions will also be investigated.

The PATRAN P3 pre/post processor release 1.3-2 developed by PDA Engineering will be used to develop the finite element model and review results. The ADVANCED FEA finite element code release 1.3-1 will be used to solve the model developed in PATRAN. ADVANCED FEA is a limited version of the finite element code ABAQUS that has been totally integrated into the PATRAN P3 pre/post processor environment.
THEORETICAL SOLUTIONS

The theoretical solution for stresses in a uniaxially loaded elastic sheet containing an interference-fit fastener is derived in [3] and [4]. Crews [5] presented the results in a more straightforward manner and they are repeated here for convenience. The solutions are valid as long as material response is elastic and the fastener remains in contact with the sheet. Plane stress conditions are assumed for the sheet and fastener. The fastener is modeled as a disk with thickness equal to that of the sheet. Fig. 2 shows the geometry under consideration. Since failures usually occur in the sheet, stresses \( \sigma_r, \sigma_{\theta\theta} \) and \( \tau_{r\theta} \) for the sheet only are presented for the frictionless and no-slip interface conditions.

**Frictionless Interface**

\[
\sigma_r = \frac{\sigma_s}{2} \left[ 1 - \frac{\frac{E_f}{E_s} \left( 2 - \frac{2IE_s}{\sigma_g R} \right)}{1 - \nu_f + (1 + \nu_s) \frac{E_f}{E_s}} \left( \frac{R}{r} \right)^2 \right]
\]

\[
\sigma_{\theta\theta} = \frac{\sigma_s}{2} \left[ 1 + \frac{\frac{E_f}{E_s} \left( 2 - \frac{2IE_s}{\sigma_g R} \right)}{1 - \nu_f + (1 + \nu_s) \frac{E_f}{E_s}} \left( \frac{R}{r} \right)^2 \right]
\]

\[
\tau_{r\theta} = \frac{\sigma_s}{2} \left[ 1 + 2 \frac{\frac{E_f}{E_s} \left( \frac{6}{3 + \nu_f + (1 + \nu_s) \frac{E_f}{E_s}} \right)}{3 + \nu_f + (5 + \nu_s) \frac{E_f}{E_s}} \left( \frac{R}{r} \right)^2 - 3 \frac{\frac{E_f}{E_s} \left( \frac{4}{3 + \nu_f + (5 - \nu_s) \frac{E_f}{E_s}} \right)}{3 + \nu_f + (5 - \nu_s) \frac{E_f}{E_s}} \left( \frac{R}{r} \right)^4 \right] \sin(2\theta)
\]

The stress \( \sigma_s \) for incipient sheet-fastener separation is obtained by setting \( \sigma_r = 0, r = R \), and \( \theta = 90^\circ \) in Eq. 1 and solving for \( \sigma_s \). The separation stress is then given as:

\[
\sigma_s = \frac{IE_s}{2R} \left[ \frac{1 + \nu_f + (5 - \nu_s) \frac{E_f}{E_s}}{9 - 5\nu_f + (11 + 5\nu_s) \frac{E_f}{E_s}} \right]
\]
No-Slip Interface

\[
\sigma_r = \frac{\sigma_s}{2} \left[ 1 - \frac{\frac{E_f}{E_s} \left( 2 - 3 \frac{E_f}{E_s} \right)}{1 - \nu_f + (1 + \nu_s) \frac{E_f}{E_s}} \left( \frac{R}{r} \right)^2 \right] - 1 \left[ \frac{\frac{4E_f}{E_s}}{1 + 3 \nu_f + (3 - 3 \nu_s) \frac{E_f}{E_s}} \left( \frac{R}{r} \right)^2 - 3 \left( \frac{R}{r} \right)^4 \right] \cos(2\theta) \right)
\]

(5)

\[
\sigma_{\theta\theta} = \frac{\sigma_s}{2} \left[ 1 + \frac{\frac{E_f}{E_s} \left( 2 - 3 \frac{E_f}{E_s} \right)}{1 - \nu_f + (1 + \nu_s) \frac{E_f}{E_s}} \left( \frac{R}{r} \right)^2 \right] + 1 \left[ \frac{\frac{4E_f}{E_s}}{1 + 3 \nu_f + (3 - 3 \nu_s) \frac{E_f}{E_s}} \left( \frac{R}{r} \right)^4 \right] \cos(2\theta) \right)
\]

(6)

\[
\tau_{\theta} = \frac{\sigma_s}{2} \left[ 1 + \frac{\frac{4E_f}{E_s}}{1 + 3 \nu_f + (3 - 3 \nu_s) \frac{E_f}{E_s}} \left( \frac{R}{r} \right)^2 - 3 \left( \frac{R}{r} \right)^4 \right] \sin(2\theta) \right)
\]

(7)

The stress \( \sigma_s \) for incipient sheet-fastener separation is obtained by setting \( \sigma_r = 0, \ r = R \), and \( \theta = 90^\circ \) in Eq. 5 and solving for \( \sigma_s \). The separation stress is then given as:

\[
\sigma_s = \frac{IE_s}{2R} \left[ \frac{1 + 3 \nu_f + (3 - 3 \nu_s) \frac{E_f}{E_s}}{3 - 3 \nu_f + (3 + 3 \nu_s) \frac{E_f}{E_s}} \right]
\]

(8)

For future equivalent stress discussions, the von Mises definition will be used and is defined as:

\[
\sigma_e = \sqrt{\sigma_{\nu}^2 + \sigma_{\nu\nu}^2 + \sigma_{\theta\theta}^2 + 3 \tau_{\theta}^2}
\]

(9)

**FINITE ELEMENT MODEL DESCRIPTION**

For development of the finite element model, the PATRAN P3 pre/post processor was used. To approximate the infinite sheet condition shown in Fig. 2, the width of the sheet was chosen to be 20 times the diameter of the fastener. The thickness of the sheet was 0.1 inch and the hole diameter was 0.250 inch. The fastener was modeled as a disk with a diameter of 0.251 inch and thickness equal to that of sheet. The diameter-based interference level was 0.001 inch. The materials for the sheet and fastener were chosen to be typical of what would be used in aerospace structures. The sheet was selected to be
aluminum with \( E_s = 10.5 \times 10^6 \) psi and \( \nu_s = 0.3 \). The fastener was selected to be steel with \( E_f = 30.0 \times 10^6 \) psi, \( \nu_f = 0.3 \), and \( \alpha_f = 9 \times 10^{-6} \) 1/°F.

Finite element meshes with 1°, 2°, and 3° nodal spacing at the sheet-fastener interface were used. A finite element mesh with 2° nodal spacing is shown in Fig. 3a. A detail of the mesh at the sheet-fastener interface is shown in Fig. 3b. Due to symmetry, only a quarter of the sheet is modeled. The sheet and fastener were modeled using first-order plane-stress elements. First order elements were chosen since their shape functions produce work-equivalent forces which are equal to the statically-equivalent forces generated by the gap element forces.

To model the interface between the sheet and fastener, a one-dimensional gap element from the ADVANCED FEA element library was chosen. Initially, the fastener diameter was equal to the sheet hole diameter resulting in common nodes with different identification numbers at the interface. The gap elements connected these initially common nodes with different identification numbers. For example, there are a total of 46 gap elements at the interface shown in Fig 3b. The fastener was then thermally expanded to create the required interference between the sheet and fastener using the relationship:

\[
\Delta T = \frac{I}{\alpha_f D_f} \tag{10}
\]

The Langrange multiplier method was chosen for contact and friction formulations. Contact between the sheet and fastener was modeled using a hard contact model. The hard contact model allows any force to be transmitted when the gap is closed. When the clearance between the gap is positive, there is no contact and zero force is transmitted. When modeling the frictionless interface condition, the coefficient of friction is set equal to zero. By setting the coefficient of friction equal to zero, radial only load transfer is assured. The NO-SLIDING CONTACT option is turned on to model the no-slip condition. This option allows no sliding at the interface boundary by setting the
coefficient of friction equal to essentially infinity. The nodes at the sheet-fastener interface behave as if they were common when the NO-SLIDING CONTACT option is turned on.

FINITE ELEMENT MODEL RESULTS AND DISCUSSION

Interference-fit fastener in an unloaded infinite sheet

The frictionless interface condition was used when analyzing stresses resulting from insertion of the interference-fit fastener in the sheet. The no-slip condition was not considered since fastener insertion results in only radial forces at the sheet-fastener interface. The coefficient of friction was set equal to zero to model the frictionless interface condition. Using Eq. 10, a temperature difference of 444.4°F created a diameter based interference level of 0.001 inch. A convergence study was performed to investigate the effect of boundary nodal spacing on stress and displacement values. Radial displacement at \( r=R, \theta=90^\circ \) and maximum differences in unaveraged values of \( \sigma_{rr}, \sigma_{\theta\theta}, \) and \( \sigma_z \) at common nodes were monitored to determine spacing that would yield acceptable engineering results. Results of the convergence study are presented in Table 1.

Table 1
Results of convergence study for an interference-fit fastener in an unloaded infinite sheet

| spacing (degree) | radial displacement, \( r=R, \theta=90^\circ \) | maximum difference in unaveraged stresses | | | |
|-----------------|---------------------------------------------|------------------------------------------|-----------------|-----------------|-----------------|-----------------|
|                 | FEA (inch)                                 | Theory (inch)                            | \% difference\(^1\) | \( \sigma_{rr} \) (psi) | \( \sigma_{\theta\theta} \) (psi) | \( \sigma_z \) (psi) |
| 3               | 0.004215819                                | 0.004228156                              | 0.2918          | 3.559           | 1.070           | 2.147           |
| 2               | 0.004216128                                | 0.004228156                              | 0.2845          | 2.500           | 751             | 1.507           |
| 1               | 0.004216326                                | 0.004228156                              | 0.2797          | 1.307           | 393             | 788             |

At 3° spacing, displacement results are more than adequate for most engineering applications. However, the maximum difference in the unaveraged stress for \( \sigma_{rr} \) is more than 14% of the maximum finite element value of -24,963 psi. As the mesh is refined from 3° to 1° spacing, the maximum difference in the unaveraged stress for \( \sigma_{rr} \) drops to 5% of the maximum finite element value -26,400 psi, which is adequate for most engineering

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\( ^1\) % difference is calculated as \[ \frac{\sigma_{\text{Theory}} - \sigma_{\text{FEA}}}{\sigma_{\text{Theory}}} \times 100 \]
applications. Considering results of the convergence study, a mesh with 1° spacing is
determined to yield acceptable engineering results.

The theoretical solutions for stresses in the sheet, uniform on the hole boundary,
were calculated from Eqs. 1 and 2 by setting $\sigma_x$ to a small number. Equivalent stresses
were calculated from Eq. 9. Finite element results were obtained from a mesh with 1°
spacing. A comparison of theoretical and finite element results is presented in Table 2.

**Table 2**
Comparison of theoretical and finite element solutions for $\sigma_{rr}$, $\sigma_{\theta\theta}$ and $\sigma_\theta$ on the case of
an interference-fit fastener in an unloaded infinite sheet

<table>
<thead>
<tr>
<th></th>
<th>psi (Theory)</th>
<th>psi (FEA)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{rr}$</td>
<td>-27,184</td>
<td>-26,400</td>
<td>2.88</td>
</tr>
<tr>
<td>$\sigma_{\theta\theta}$</td>
<td>27,184</td>
<td>26,891</td>
<td>1.08</td>
</tr>
<tr>
<td>$\sigma_\theta$</td>
<td>47,084</td>
<td>46,159</td>
<td>2.00</td>
</tr>
</tbody>
</table>

As can be seen in Table 2, finite element and theoretical results compare favorably.
Finite element solutions for $\sigma_{rr}$, $\sigma_{\theta\theta}$, and $\sigma_\theta$ along the transverse axis are presented in Fig.
4. As expected, solutions approach zero away from the hole. Finite element solutions for
$\sigma_{rr}$, $\sigma_{\theta\theta}$, and $\sigma_\theta$ were compared with theoretical values at each node on the transverse axis.
The maximum percent difference was 2.88% and occurred at the hole boundary for $\sigma_{rr}$.

**Interference-fit fastener in a loaded infinite sheet / frictionless interface**

To investigate stresses in a loaded infinite sheet containing an interference fit
fastener with a frictionless interface condition, a 10,000 psi load was applied. The
frictionless interface condition was modeled by setting the coefficient of friction equal to
zero. Using Eq. 10, a temperature difference of 444.4°F created a diameter based
interference level of 0.001 inch. As was done for the previous case, a convergence study
was performed to investigate the effect of boundary nodal spacing on stress and
displacement values. Results of the convergence study are presented in Table 3.
Table 3

Results of convergence study for an interference-fit fastener in a loaded infinite sheet with a frictionless interface

<table>
<thead>
<tr>
<th>spacing (degree)</th>
<th>radial displacement, ( r=R ), ( \theta=90^\circ )</th>
<th>maximum difference in unaveraged stresses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FEA (inch)</td>
<td>Theory (inch)</td>
</tr>
<tr>
<td>3</td>
<td>.0004873667</td>
<td>.0004889913</td>
</tr>
<tr>
<td>2</td>
<td>.0004873695</td>
<td>.0004889913</td>
</tr>
<tr>
<td>1</td>
<td>.0004873817</td>
<td>.0004889913</td>
</tr>
</tbody>
</table>

Trends for results displayed in Table 3 reflect those presented in Table 1. At 1° spacing, displacement results are within 1% of the theoretical value. As the mesh is refined from 3° to 1° spacing, the maximum difference in the unaveraged stress for \( \sigma_{rr} \) drops to 5.55% of the maximum finite element value of -29,778 psi, which is adequate for most engineering applications. Based on results of convergence study, 1° spacing is determined to yield acceptable engineering results.

The theoretical solutions for stresses in the loaded sheet were calculated from Eqs. 1 and 2. Equivalent stresses were calculated from Eq. 9. Finite element results were obtained from a mesh with 1° spacing. A comparison of theoretical and finite element results are presented in Table 4.

Table 4

Comparison of theoretical and finite element solutions for \( \sigma_{rr} \), \( \sigma_{\theta \theta} \), and \( \sigma_c \) on the case of an interference-fit fastener in a loaded sheet with a frictionless interface

<table>
<thead>
<tr>
<th></th>
<th>psi (Theory)</th>
<th>psi (FEA)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{rr} @ \theta = 0^\circ, r=R )</td>
<td>-30,960</td>
<td>-29,778</td>
<td>3.82</td>
</tr>
<tr>
<td>( \sigma_{rr} @ \theta = 90^\circ, r=R )</td>
<td>-10,464</td>
<td>-9,780</td>
<td>6.54</td>
</tr>
<tr>
<td>( \sigma_{\theta \theta} @ \theta = 0^\circ, r=R )</td>
<td>40,464</td>
<td>40,864</td>
<td>&lt; 1.00</td>
</tr>
<tr>
<td>( \sigma_{\theta \theta} @ \theta = 90^\circ, r=R )</td>
<td>20,960</td>
<td>21,246</td>
<td>1.36</td>
</tr>
<tr>
<td>( \sigma_c @ \theta = 0^\circ, r=R )</td>
<td>62,037</td>
<td>61,431</td>
<td>&lt; 1.00</td>
</tr>
<tr>
<td>( \sigma_c @ \theta = 90^\circ, r=R )</td>
<td>27,715</td>
<td>27,475</td>
<td>&lt; 1.00</td>
</tr>
</tbody>
</table>

As can be seen in Table 4, finite element and theoretical results compare favorably. Finite element solutions for \( \sigma_{rr} \), \( \sigma_{\theta \theta} \), and \( \sigma_c \) along the transverse axis are presented in Fig. 5. As expected, results approach the uniform gross stress away from the hole. Finite element solutions for \( \sigma_{rr} \), \( \sigma_{\theta \theta} \), and \( \sigma_c \) along the hole boundary are presented in Fig. 6.
Finite element solutions for $\sigma_{rr}$, $\sigma_{\theta\theta}$, and $\sigma_e$ were compared with theoretical values at each node on the transverse axis and hole boundary. The maximum percent difference was 6.54% and occurred on the hole boundary at $\theta = 90^\circ$ for $\sigma_{rr}$.

The gross stress required to cause incipient sheet-fastener separation at $\theta = 90^\circ$ is referred to as the separation stress. The theoretical solution was calculated from Eq. 4 as 16,259 psi. The gross stress which caused the gap element at $\theta = 90^\circ$ to open defined the separation stress for the finite element model. Using an incremental search method, a gross stress of 16,050 psi caused the gap element at $\theta = 90^\circ$ to open. The percent difference in solutions was less than 1%.

When performing fatigue calculations for the sheet, the alternating stress and mean stress on the hole boundary at $\theta = 0^\circ$ are of interest. Since the stress state on the hole boundary at $\theta = 0^\circ$ is bi-axial, an equivalent stress definition is needed. For comparison purposes only, the equivalent mean von Mises stress, $\sigma_{me}$, and the alternating von Mises stress, $\sigma_{ae}$, defined in [11] are used. Using stress values provided in Tables 2 and 4, a comparison of theoretical and finite element solutions for $\sigma_{me}$ and $\sigma_{ae}$ is presented in Table 5.

<table>
<thead>
<tr>
<th></th>
<th>psi (Theory)</th>
<th>psi (FEA)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{me}$</td>
<td>54,521</td>
<td>53,743</td>
<td>1.43</td>
</tr>
<tr>
<td>$\sigma_{ae}$</td>
<td>5,926</td>
<td>6,314</td>
<td>6.55</td>
</tr>
</tbody>
</table>

Interference-fit fastener in a loaded infinite sheet / no-slip interface

To investigate stresses in a loaded infinite sheet containing an interference fit fastener with a no-slip interface condition, a 10,000 psi load was applied. The NO-SLIP option in the ADVANCED FEA finite element code was activated to model the no-slip interface condition. Based on convergence studies presented in Tables 1 and 3, $1^\circ$ spacing
will be used to investigate the stress field. Using Eq. 10, a temperature difference of 444.4°F created a diameter based interference level of 0.001 inch.

The theoretical solutions for stresses in the loaded sheet were calculated from Eqs. 5, 6, and 7. Equivalent stresses were calculated from Eq. 9. Finite element results were obtained from a mesh with 1° spacing. A comparison of theoretical and finite element solutions are presented in Table 6.

<table>
<thead>
<tr>
<th></th>
<th>psi (Theory)</th>
<th>psi (FEA)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{rr} \at \theta = 0^\circ, r=R)</td>
<td>-27,051</td>
<td>-26,147</td>
<td>3.34</td>
</tr>
<tr>
<td>(\sigma_{rr} \at \theta = 90^\circ, r=R)</td>
<td>-14,373</td>
<td>-13,412</td>
<td>6.69</td>
</tr>
<tr>
<td>(\sigma_{\theta \theta} \at \theta = 0^\circ, r=R)</td>
<td>31,695</td>
<td>32,007</td>
<td>&lt; 1.00</td>
</tr>
<tr>
<td>(\sigma_{\phi \phi} \at \theta = 90^\circ, r=R)</td>
<td>29,729</td>
<td>30,112</td>
<td>1.29</td>
</tr>
<tr>
<td>(\sigma_e \at \theta = 0^\circ, r=R)</td>
<td>50,928</td>
<td>50,449</td>
<td>&lt; 1.00</td>
</tr>
<tr>
<td>(\sigma_e \at \theta = 90^\circ, r=R)</td>
<td>38,958</td>
<td>38,609</td>
<td>&lt; 1.00</td>
</tr>
<tr>
<td>(\tau_{r \theta} \at \theta = 0^\circ, r=R)</td>
<td>0</td>
<td>-234</td>
<td>N/A</td>
</tr>
<tr>
<td>(\tau_{r \theta} \at \theta = 45^\circ, r=R)</td>
<td>6,339</td>
<td>6,197</td>
<td>2.26</td>
</tr>
<tr>
<td>(\tau_{r \theta} \at \theta = 90^\circ, r=R)</td>
<td>0</td>
<td>209</td>
<td>N/A</td>
</tr>
</tbody>
</table>

As can be seen in Table 6, finite element and theoretical results compare favorably. Finite element solutions for \(\sigma_{rr}, \sigma_{\theta \theta}, \) and \(\sigma_e\) along the transverse axis are presented in Fig. 7. As expected, results approach the uniform gross stress away from the hole. Finite element solutions for \(\sigma_{rr}, \sigma_{\theta \theta}, \tau_{r \theta},\) and \(\sigma_e\) along the hole boundary are presented in Fig. 8. Finite element solutions for \(\sigma_{rr}, \sigma_{\theta \theta}, \tau_{r \theta},\) and \(\sigma_e\) were compared with theoretical values at each node on the transverse axis and hole boundary. The maximum percent difference was 6.69% and occurred on the hole boundary at \(\theta = 90^\circ\) for \(\sigma_{rr}\).

The gross stress required to cause incipient sheet-fastener separation at \(\theta = 90^\circ\) is referred to as the separation stress. The theoretical solution was calculated from Eq. 8 as 22,749 psi. The gross stress which caused the gap element at \(\theta = 90^\circ\) to open defined the separation stress for the finite element model. Using an incremental search method, a gross
stress of 20,850 psi caused the gap element at $\theta = 90^\circ$ to open. The percent difference in solutions was 8.45%.

When performing fatigue calculations for the sheet, the alternating stress and mean stress on the hole boundary at $\theta = 0^\circ$ are of interest. For comparison purposes only, the equivalent mean von Mises stress, $\sigma_{me}$, and the alternating von Mises stress, $\sigma_{ae}$, defined in [11] are used. Using stress values provided in Tables 2 and 6, a comparison of theoretical and finite element solutions for $\sigma_{me}$ and $\sigma_{ae}$ is presented in Table 7.

<table>
<thead>
<tr>
<th></th>
<th>psi (Theory)</th>
<th>psi (FEA)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{me}$</td>
<td>48,993</td>
<td>48,283</td>
<td>1.45</td>
</tr>
<tr>
<td>$\sigma_{ae}$</td>
<td>2,223</td>
<td>2,497</td>
<td>12.4</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

A finite element analysis to determine the stresses in a uniaxially loaded elastic sheet containing an interference-fit fastener has been performed. The interface between the sheet and fastener was modeled using one-dimensional gap elements. Frictionless and no-slip interface conditions were investigated. Overall, the gap element did an excellent job of modeling the sheet-fastener interface. All finite element models with $1^\circ$ nodal boundary spacing yielded results which compared favorably with theory.

The frictionless interface condition was investigated for a loaded and unloaded sheet. A convergence study revealed one degree nodal spacing at the hole boundary would produce acceptable engineering results. All stresses obtained from the finite element model were within 6.54% of theoretical values. The finite element value for the stress which caused incipient separation between the sheet and fastener was found to be within 1% of the theoretical value. Trends for stresses along the transverse axis and hole boundary compared favorably with theoretical results with no oscillations noted.

The no-slip condition was investigated for a loaded sheet. All stresses obtained from the finite element model were within 6.69% of theoretical values. The finite element
value for the stress that caused incipient separation between the sheet and fastener was found to be within 8.45% of the theoretical value. Trends for stresses along the transverse axis and hole boundary compared favorably with theoretical results with no oscillations noted.

Considering a review of results for the current analysis, several areas warrant future investigation. The effect of the fastener exiting the sheet, referred to as edge-stiffening, was neglected in the current study. The increase in edge stresses due to edge-stiffening can effect the fatigue life of the sheet and is an area for further research. Inspection of results for $\sigma_e$ in Tables 2, 4, and 6 indicate that for most structural aluminum, an elastoplastic analysis will be necessary at higher interference and/or load levels in order to accurately predict the stress field in the sheet. Crews [7] discusses plasticity effects during fastener installation and subsequent tension axial loading. The effect of friction at the interface with various combinations of interference level and axial load is an area of additional study. Slippage at the interface due to frictional effects can cause fretting and reduction in fatigue life. Finally, the post-separation behavior of the sheet and the effect on stress distributions is an area for further research.
REFERENCES


REFERENCES


Figure 1, View of a typical interference-fit fastener installation
Figure 2, Infinite sheet under tension with an interference-fit fastener installed
Figure 3a, Finite element model of an infinite sheet under tension with an interference-fit fastener installed, two degree nodal spacing
Figure 3b, Detail of sheet-fastener interface, finite element model
Figure 4. Stress distribution along transverse axis, fastener insertion only.
Figure 5. Stress distribution along transverse axis, 10,000 psi load, frictionless interface
Figure 6, Stress distribution along hole boundary, 10,000 psi load, frictionless interface
Figure 7, Stress distribution along transverse axis, 10,000 psi load, no-slip interface
Figure 8, Stress distribution along hole boundary, 10,000 psi load, no-slip interface