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Predicting Height to Live Crown Increment for Thinned and Unthinned Loblolly Pine Plantations

by

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Thesis submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Forestry

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May, 1991

Blacksburg, Virginia

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(ABSTRACT)

Several nonlinear, individual tree crown height increment equations were tested for their ability to predict annual crown height increment in loblolly pine plantations. The selected model contained tree height (HT), tree crown ratio (CR) raised to the one-half power, age (A), and a competition index (CI) for the distance-dependent model and the ratio of quadratic mean diameter to tree dbh (DR) for the distance-independent model. The distance-dependent and the distance-independent models were the same form, except for the expression for competition.

Hypothesis tests revealed that thinning, both its intensity and the elapsed time since its occurrence, had a significant effect on crown height increment. A thinning variable, THIN1, which accounted for thinning intensity and the interval since thinning, was developed and incorporated into the final individual tree increment models. Predictions of crown height increment were improved using models with the THIN1 variable as compared to those with no thinning allowance.

In another approach, existing crown height equation was modified to account for the effect of thinning on crown recession. Another thinning variable, THIN2, similar to THIN1, was added to the crown height model. This model yielded better results than its counterpart with no thinning variable; however, the improvement was not as great as for the increment models.

The individual tree increment models were also used to form a stand level crown height increment model. The independent variables were collapsed to stand-level statistics; the final

model contained average height of dominants and codominants (HD), average crown ratio (R), age (A), and the THIN1 variable. Unlike the individual tree models, raising the average crown ratio to .5 did not improve the fit; however, including THIN1 did improve the results.

From this study it was concluded that better data, a standard definition of height to the live crown, and other crown variables, such as crown diameter, will be required to produce more refined individual tree crown height increment models.

Acknowledgements

I sincerely wish to thank my advisor, Dr. Harold Burkhart, for his constant support, expertise and encouragement throughout this study. Also I want to thank Dr. Richard Oderwald and Dr. David Smith who provided many valuable thoughts and suggestions. My work was also made much easier by Ralph Amateis, to whom I owe many thanks as well.

Additionally, I wish to thank the Loblolly Pine Growth and Yield Research Cooperators for providing the data and the financial support for my study.

My deepest love and gratitude to my parents without whose love and support my college achievements would have never been possible.

A hearty thanks and congratulations to my roommates Damon Houghton and Easton Loving who not only tolerated me for two years, but who also kept me smiling during times of frustration.

Finally, I wish to thank everyone in the biometrics section at Virginia Tech - faculty, staff (Gerry), and my fellow graduate students for a great time. I will miss you.

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CHAPTER 1

INTRODUCTION

The importance of growth and yield prediction to forest planning and management is well documented (Clutter *et al.* 1983; Avery and Burkhart 1983). The significance of growth and yield models continues to grow with the demand for more accurate, long term forest resource planning. As Davis and Johnson (1987, 40) stated, "Predicting future growth and yield of managed...stands is absolutely essential to credible forest management planning", and this prediction relies on the accuracy of all elements of the growth and yield model.

Crown size is an important component of many growth and yield models. The significance of crown size to a tree's growth and survival has been well documented (Hamilton 1969; Spurr and Barnes 1980); furthermore, research has shown that crown size is a prime indicator of a tree's potential response to thinning (Chapman 1953; Smith 1986). Also, a tree's height and crown dimension determines the length of its clear bole, which is significant in the merchandizing of the tree into various wood products (Smith 1986; Kershaw *et al.* 1990). Consequently crown size is increasingly used in individual growth models to aid in the modelling of stand and stem dynamics through time (Hatch 1971; Daniels and Burkhart 1975; Wykoff, *et al.* 1982). However, the prediction of crown dimension is relatively new, and it is believed that more thorough knowledge of crown size could lead to further improvement in the prediction of growth and mortality (Sprinz and Burkhart 1987), including the southeastern United States commercial species, loblolly pine

(*Pinus taeda* L.).

Several variables can be used to model crown dimension such as: crown ratio, height to live crown, and crown length. Crown dimension modelling depends upon accurate total height and height to crown equations. Much work exists on total height equations for loblolly pine, such as Amateis and Burkhart (1985), among others. However, relatively little research has focused on modelling the height to live crown for loblolly pine (Dyer and Burkhart 1987). Two reasons for this lack of crown height modelling are lack of data and the difficulty often found in defining the beginning of the live crown for many tree species. This difficulty arises from asymmetric crowns due to noncontiguous live whorls. (Maguire and Hann 1990b). However, once one determines the live crown base, these crown dimension variables can be easily computed by simply knowing the tree's height to live crown and total height.

With the recent increase in crown size data, these crown variables can be computed in most growth and yield models, and given the importance of crown size's in a tree's survival, growth, and wood quality, it follows that a tree's crown dimensions would be an important element in growth and yield models. Nonetheless, most crown dimension models simply predict each tree's new crown size after each growth period based on stand and site characteristics and updated tree dimensions, such as diameter, height, and age. It is believed that incrementing crown height through time, much like diameter and height are now in most models, would improve crown size estimates and would subsequently improve tree growth and mortality prediction. Thus, one of this study's objectives was to develop crown height increment equations for cutover, site-prepared loblolly pine plantations in the southern Piedmont and Coastal Plain for the three basic types of growth and yield models: stand-level, individual-tree distance-dependent and individual-tree distance-independent models.

Stand level models predict growth and yield solely on a stand basis, no information is required about individual trees. The input variables for these models are stand statistics, such as basal area, trees per acre, site index, and age.

Individual tree, distance-independent models require both overall site and stand characteristics and individual tree information such as diameter, height, and crown size to predict tree growth. Stand growth and yield is subsequently determined from the summed growth and yield of each tree. A tree's location in the stand is unknown, thus intertree competition is usually based on the tree's size relative to the stand average.

Distance-dependent models are quite similar to distance-independent models except for their requirement of each tree's location. The theoretical advantage of these models compared to distance-independent models is that the growth and survival of each tree should be better predicted given the knowledge of its competitors' sizes and locations, as well as the subject tree's dimensional characteristics.

In this study it was decided to constrain the the two individual tree equations to be identical except for a competition variable. The distance-dependent model would use a competition index based on a subject tree's competitors and their respective distances from the subject tree, while the distance-independent model would employ a competition variable based on the subject tree's relative size compared to the stand average. The stand level model would be derived by collapsing the basic individual-tree model to one with only site and stand level variables.

Furthermore, most studies have not examined the effect of thinning on the subsequent development of the crown height. It is believed that thinning has an effect on the progression of height to live crown in loblolly pine plantations (Ginn 1989). Thus, if thinning does slow the increment of crown height, it would produce a larger crown compared to unthinned stands which would affect a tree's growth and survival. Therefore, this study will also examine if thinning, and its degree, significantly affects loblolly pine crown recession, and if so, develop an appropriate variable to account for this thinning effect. Also, a loblolly pine crown height prediction equation will be studied to determine if a thinning variable will improve its prediction accuracy as well.

Objectives

1. Develop individual tree distance-dependent, individual tree distance-independent, and stand level crown height increment equations for cutover, site-prepared loblolly pine plantations.

2. Determine if thinning, and its degree, significantly effects loblolly pine crown height increment, and if so, develop an appropriate variable to account for this effect.

3. Determine if a thinning variable will improve the performance of a loblolly pine crown height prediction equation.

CHAPTER II

LITERATURE REVIEW

Crown dimension has received much attention recently, and a number of models have been developed to predict height to live crown or crown ratio in forest stands; however, many have either focused on unthinned stands or made no distinction between thinned and unthinned stands. Also, much of this research has produced stand-level models leaving little research which has examined methods for predicting the height to live crown increment for individual trees in thinned or unthinned stands. Crown height increment models will then be evaluated, followed by individual tree and stand-level crown height models. First, a brief review of crown dimension behavior will be given.

Crown Height Behavior

The behavior of crown height and other crown dimensions through time and their effect on tree growth and survival have been widely studied. Crown height has a strong positive correlation with age and total height, and thus site index (Beekhuis 1965). Also, the larger, dominant trees have lower crown heights relative to smaller trees, and thus longer crown lengths. Most studies have also found that the height to live crown increases with increasing stand density, or closer spacings (Beekhuis 1965; Curtis and Reukema 1970; Keister and Walker 1971). Furthermore, upon reaching crown closure, crown height has been found to increase at the approximate rate of total height for many conifer species such as radiata pine (*Pinus radiata* D. Don) (Beekhuis 1965), Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco) (Curtis and Reukema 1970) and the southern pines (Keister and Walker 1971), producing a constant crown length and decreasing crown ratio at a decreasing rate.

Kershaw *et al.* (1990) studied the longetivity and duration of radial growth in Douglas-fir branches. They found that the greatest branch longetivity was found on the larger, more vigorous trees. This further supports the view that longer live crowns, hence lower crown heights and smaller crown height increments, are found on the larger trees. Conversely, branch life is shorter for the weaker, more suppressed trees producing shorter crown lengths.

Silvicultural studies have also shown that thinning has a significant effect on crown dimensions, yet this knowledge has not been utilized in crown height equations. Beekhuis (1965) and Siemon *et al.* (1976) both found crown height increment to slow after thinning in radiata pine producing greater crown length. Likewise, crown length was found to increase through decreased crown height recession in thinned loblolly pine plantations (Ginn 1989). Kramer (1966) found that various types and levels of thinning resulted in different crown percent responses (crown ratios) for Norway spruce (*Picea abies* (L.) Karst.), Sitka spruce (*Picea sitchensis* (Bong.) Carr.), and Douglas-fir plots in Scotland. Kramer found that heavier thinnings produced larger crown ratios for a longer duration than lighter, low thinnings. Furthermore, Beekhuis (1965) found that varying levels of thinning did produce differing crown height increments after thinning until crown closure was regained.

Clearly, crown height and its increment are highly correlated with age, height, and stand density. Also, thinning does appear to have a significant effect on crown height as well. Thus, any crown height or crown height increment models should account for these correlations.

Crown Height Increment Equations

Perhaps the most thorough research to date concerning crown height increment models was performed by Maguire and Hann (1990a). Three basic model forms - weighted and unweighted nonlinear equations, logarithmic equations, and generalized linear models assuming gamma-distributed observations, were tested and compared for predicting the five-year crown height recession for Douglas-fir stands in southwestern Oregon.

A nonlinear, unweighted equation was developed from a generalization of the existing models that directly predict crown increment (ΔHLC) (Hatch 1971; Krumland and Wensel 1981; Krumland 1982) using crown length (CL), total height (HT), crown ratio (CR), age (A), and crown competition factor (CCF) (Krajicek *et al.*, 1961):

$$\Delta HLC = \frac{(CL + \Delta HT)}{[1 + EXP(XA)]} \tag{1}$$

where :

$$XA = a_0 + a_1 \ln(CR) + a_2 CR + a_3 A + a_4 \ln(CCF) \tag{2}$$

A second nonlinear model was developed using the same variables:

$$\Delta HLC = b_0 HT^{b_1} CCF^{b_2} \exp[b_3 CR + b_4 A] \tag{3}$$

The authors also tested the logarithmic equivalent of this nonlinear equation.

A strong peaking behavior of crown increment over crown ratio was evident in the data

(Maguire and Hann 1990a), thus equation (3) was modified into equation (4):

$$\Delta HLC = c_0 HT^{c_1} CCF^{c_2} \exp[c_3 (1 - CR/38]^{c_4} + c_5 A \quad \textbf{(4)}$$

Logarithmic crown height increment equations were also studied as mentioned. One equation predicted the change in crown height based on diameter, height, crown ratio, age, stand basal area, stand basal area for trees larger than the subject tree, elevation, and change in height. This model was reduced to yield the equation:

$$\ln(\Delta HLC) = e_0 + e_1 CR + e_2 A + e_3 \ln(HT) + e_4 \ln(CR) + e_5 \ln(CCF) + e_6 \ln(\Delta HT) \quad \textbf{(5)}$$

This model was further reduced to the logarithmic equivalent of equation (3).

Two generalized linear models were developed assuming gamma-distributed observations. One was developed using a logarithmic link function which assumes a nonconstant but normal error distribution, the other a reciprocal link function which assumes a constant coefficient of variation and gamma-distributed errors. Both models used the logarithmic equivalent of equation (3). However, neither produced a significant increase in fit, and more importantly neither model's skewness and kurtosis coefficients conformed well with coefficients expected from a gamma distribution.

Maguire and Hann found that generally a low proportion of crown change is explained by these equations with the highest R^2 being under .3, and that the normality and constant variance assumptions of ordinary least squares were violated. This suggests that weighting the equation would be more appropriate. However, no weighted five-variable nonlinear models converged. The best four-variable model was equation (3), and again weighting did not improve the fit. The corresponding logarithmic equation was the best model for the four-variable logarithmic models.

Equation (4) was developed due to the peaking of crown change over crown ratio, peaking at 38 percent, and it produced a slightly better fit than equation (3), however, weighting did not improve its fit either.

The logarithmic equations produced the better fit statistics (R^2, residual mean square, Furnival's index, skewness, and kurtosis), while the nonlinear and generalized linear models produced approximately the same results. Due to the nonlinear models' positive skewness and extreme kurtosis, and thus normality violations combined with the inability for weighted nonlinear least squares to improve the fit, Maguire and Hann concluded that lognormality of residuals was the primary problem rather than nonconstant variance. However, the nonlinear models produced higher coefficients of determination for actual versus predicted 5-year crown change than the logarithmic equations. A final consideration is that logarithmic transformations of crown recession could be used since its five-year change was predicted, if an annual change was predicted, the possible occurence of no annual crown increment would not allow the log transformation.

Individual-Tree Crown Height Prediction Equations

Many recent studies of crown height and crown ratio prediction have produced nonlinear equations. Therefore, these type models will be reviewed first, followed by a brief overview of some linear equations.

Nonlinear Crown Height Equations

Biging and Wensel (1987) estimated the crown height (CH) for individual trees in the California Conifer Timber Output Simulator, CACTOS, using tree height (HT), dbh (DBH), and the stand basal area in trees greater than 5.5 inches at dbh (BA_6):

$$CH = HT(1-\exp-(b_0+b_1\ln BA_6+b_2(DBH/HT))^2) \qquad (6)$$

The standard errors for this equation ranged between 9 and 11 feet. Notice that as diameter increases, the crown height's recession slows.

For cutover, site prepared loblolly pine plantations Dyer and Burkhart (1987) developed a nonlinear crown height (CH) model using stand age (A), tree dbh (DBH), and total tree height (HT), where b_2 and b_3 are positive coefficients:

$$CH = HT\exp[-(b_2 + b_3A^{-1})DBH/HT] \qquad (7)$$

This equation ensures that crown height will always be between zero and total height, and as in Biging and Wensel's model (1987), will decrease with increasing taper due to the D/H term. Also, as age increases, crown height will increase and eventually remain fairly constant.

Dyer and Burkhart's equation was used to estimate the height to live crown for unthinned, lightly thinned, and heavily thinned loblolly pine plantations. Standard errors were slightly over 3 feet, with a slight bias toward overestimation of crown height, especially in the heavily thinned stands.

Zumrawi and Hann (1989) expanded on previous work by Walters and Hann (1986) to predict bole ratio (BR) which is defined as the ratio of the height to live crown (CH) over total height (H) for two conifer and four hardwood species in Oregon. Walters and Hann predicted the height to crown base using a logistic function:

$$CH = \frac{H}{1.0 + EXP[\sum_{i=1}^{K} b_i X_i]} \tag{8}$$

Zumrawi and Hann used this same function to predict bole ratio based on tree height (H), stand basal area (BA), tree taper (D/H), and the crown competition factor from trees larger than the subject tree (CCFL).

$$BR = \frac{1.0}{1.0 + EXP(b_0 + b_1 H + b_2 CCFL + b_3 \ln(BA) + b_4 (D/H))} \tag{9}$$

The equation produced positive values for b_0 and b_4, and negative values for the remaining parameters. Height to crown base could then be calculated by multiplying the predicted bole ratio by tree height. This logistic transformation was used to homogenize the height to crown base's variance which increases with tree height.

Residuals from the equation showed no bias when plotted over the independent variables or predicted bole ratio. Also, the signs of the parameter estimates conformed to expectations. Crown ratio generally increases with taper (D/H), thus bole ratio would decrease, and this was supported by b_4's positive logistic value. Likewise b_2 and b_3 are negative, indicating that bole ratio increases, i.e. crown ratio decreases, as the tree's relative position in the stand worsens (higher CCFL), and stand density (BA) increases respectively. Furthermore as the tree's height increases its bole ratio should increase to some extent since height growth slows before crown height increment, this is evident through the small negative value of b_1.

Holdaway (1986) developed an individual crown ratio model for 23 hardwood and softwood species in the Lake States to use in the STEMS tree growth projection system (Belcher *et al.* 1982 and USDA For. Serv. 1983). The individual tree model was created from a stand level crown ratio (R) model which uses stand basal area (BA) as the competition measure:

$$R = \frac{b_0}{(1 + b_1 BA)} \tag{10}$$

Both coefficients are positive; b_0 estimates the maximum crown ratio without competition, and b_1 is the rate of crown ratio decrease as competition (BA) increases.

A second term containing tree diameter (DBH) was added to produce the tree crown ratio (CR) model:

$$CR = \frac{b_0}{(1 + b_1 BA)} + b_2(1 - \exp(-b_3 DBH)) \tag{11}$$

The additional term was chosen since Holdaway hypothesized that a tree's individual crown ratio is mainly due to its ability to compete for light, and its degree of dominance compared to the surrounding trees; graphical analyses indicated that tree crown ratio increases with diameter until maximum tree height is attained (Holdaway 1986).

The individual tree crown ratio model was tested on second measurement data from 83,283 trees where crown ratio was measured in 10 percent intervals (i.e. 0-10% is 1, 11-20% is 2, etc.) and produced an average overestimation of .19 units.

The Stand Prognosis Model is a growth and yield model developed for the Northern Rocky Mountain coniferous forests (Wykoff *et al.*, 1982). An individual crown ratio model was developed by Hatch (1980) and is a regression function based on various transformations of the individual species, habitat type, stand basal area (BA), crown competition factor (CCF), tree DBH, tree height, and the tree's ranking percentile (PCT) in the stand's basal area distribution. The equation can contain up to fifteen terms; however, with most coefficients set to zero, most species' equations contain five or six terms.

Wykoff *et al.* found Hatch's model to work quite well. The estimated change in crown ratio is the difference between the predicted crown ratio for the beginning and end of a growth period. The change is then produced by adding the difference to the actual crown ratio.

For stands exhibiting little stand density effects, CCF values are under 100, the same basal area and CCF values are used at both the beginning and end of the growth cycle to predict crown ratio changes. Furthermore, thinning is assumed to encourage crown development; however, thinning from below reduces the remaining trees' percentiles in the stand basal area distribution (PCT) producing smaller crowns. Thus, thinned stands use the same PCT values at the beginning and end of the growth cycle.

Feduccia, *et al.* (1979) and Dell *et al.* (1979) developed an individual tree crown ratio model for unthinned loblolly and unthinned slash pine plantations, respectively, on cutover sites in the west Gulf Coast region using the following assumptions:

(1) crown ratio predictions must be between 0 and 100

(2) when an individual tree's diameter (D_i) is equal to the average stand diameter (D), then its individual crown ratio must equal the stand's average crown ratio

(3) crown ratio increases monotonically with diameter

Thus, individual tree crown ratio (CR) model was:

$$CR = 100[1 - \exp((\frac{D_i}{\bar{D}})^{\lambda} \ln(1-\frac{R}{100}))] \qquad \mathbf{(12)}$$

where A is the number of growing seasons since planting, H is the average dominant and codominant height at age A, T is the number of surviving trees at age A, and λ is a regression equation containing the log transformations of H, A, and T.

The average crown ratio (R) is defined as:

$$R = b_0 - b_1 \log(H) - b_2 \log(A) - b_3 \log(T) \qquad \mathbf{(13)}$$

where the variables are defined as above. The average diameter is found using a gamma function and the Weibull distribution. The authors stated that results were reasonably accurate; however, more thorough validation of the model was needed.

Curtis and Reukema (1970) used log transformations of dbh (D) in a multiple regression equation to predict the height to live crown (CH) in Douglas-fir spacing trials:

$$CH = b_0 + b_1 \log D + b_2 \log^2 D \qquad \mathbf{(14)}$$

producing a standard error of 4.8 feet. Obviously, as diameter increases, so does the height to live crown. Curtis and Reukema also found that height to live crown was inversely related to spacing until crown closure occurred, then live crowns in wider spacing would recede rapidly as well.

Linear Crown Height Equations

Hegyi (1974) developed a distance-dependent, tree-growth simulation model for jack pine (*Pinus banksiana* Lamb.). A linear equation was used to predict individual tree crown lengths. Crown lengths (CL) is believed to initially increase with tree height (H), but to eventually decrease with age (A)

$$CL = b_0 + b_1 * H - b_2 * A \qquad \textbf{(15)}$$

The equation accounted for 72.5 percent of the variation in the 55 estimated crown lengths.

A linear equation was developed by Siemon *et al.* (1976) to simulate the effect of thinning on crown lengths of radiata pine on New Zealand and Australian sites. The equation predicts crown length (CL) using tree diameter (DBH),

$$CL = b_0 + b_1 * DBH \qquad \textbf{(16)}$$

The data consisted of four different thinning regimes, plus a control plot, where each regime was thinned from below at ages 15, 17, and 20 to different basal area limits. The control plot remained near 180 square feet of basal area per acre while the thinning treatments were kept at basal area levels of 60, 80, 100 and 120 square feet per acre. The authors found that one equation could predict all crown lengths except for the lightest thinned stands (approximately 120 square feet of basal area per acre) and the unthinned plots. However, when one equation was used to predict all crown lengths, sixty percent of the crown length variation was explained.

Stand-level Crown Height Equations

Several stand level crown models have been developed, with the majority being nonlinear equations. Cole and Jensen (1982) developed two crown height equations for lodgepole pine (*Pinus contorta* var *latifolia* Dougl. ex Loud.) - one uses stand basal area (BA) as the competition index while the other uses crown competiton factor (CCF). The following equation estimates the average height to the base of the crown for dominants (HBCD) using stand basal area (BA) and average dominant height (HD):

$$HBCD = b_0 HD^{b_1} \left[b_2 \exp - \left| \frac{\frac{BA}{250} - 1}{b_3} \right|^{5.5} - b_4 \right] \tag{17}$$

Cole and Jensen found that the relationship for HBCD over BA could be characterized as a single sigmoidal form over the entire range of average dominant heights (HD), and the unscaled curve is expressed in the brackets above. The curve's maximum height occurred at a basal area of 250 square feet and defined the upper limit of the interaction model (Cole and Jensen 1982). The basal area model explained 82 percent of the variation in the average height to live crown and produced a standard error of 5.2 feet.

A similar model to equation (17) was fitted only using CCF as the competition measure where BA/250 was replaced by CCF/100 and the power of 5.5 on the exponent term was replaced by ten. The equation produced a standard error of 4.7 feet, and explained 85 percent of the variation in the average height to live crown.

Cole and Jensen (1982) recommend using only the model involving basal area (BA) as the stand density measure if predictions for thinned stands are desired, since CCF values produce biased results under repeated thinnings (especially low thinnings).

An additional equation was necessary to predict crown height for the suppressed crown class (HBCS), and is a linear function of dominant height (HD) and the dominant and codominant crown height (HBCD):

$$HBCS = b_0 + b_1 HBCD + b_2 HD \tag{18}$$

The equation explained 84 percent of the variation in the crown height for the suppressed crown class.

Cole and Jensen (1982) found that at the stand level, thinning increases crown lengths; yet, they also found that it raises the height to live crown compared to no thinning since suppressed trees are removed.

Liu *et al.* (1989) developed a timber production function consisting of a four equation system to predict growth and yield in unthinned loblolly pine plantations established on cutover, site-prepared sites in the southeastern United States. One equation was a stand-level crown ratio model based on Holdaway's (1986) individual tree crown model. They hypothesized that stand crown ratio (R) is inversely related to stand basal area, (B) and directly related to dominant height (HD),

$$R = \frac{d_1}{1 + d_2 B} + d_3 \left(1 - e^{d_4 (HD - 4.5)}\right) \tag{19}$$

using 4.5 feet as breast height. A stand crown length model can be derived by multiplying the above equation by the dominant stand height. This equation implies that crown length approaches the dominant height when dominant height equals breast height, and thus basal area approaches zero (Liu *et al.* 1989). This crown length equation produced a standard error of 2.54 feet.

Holdaway *et al.* (1979) developed separate stand level crown ratio models for conifers and hardwoods in the Lake States region. The authors created the model using four basic assumptions:

(1) solving the mathematical equation will yield a value between 0 and 10

(2) crown ratio (CR) cannot increase with increasing basal area (BA)

(3) crown ratio (CR) will eventually decrease as age (A) or average diameter (AD) increases, and

(4) the rate of change of crown ratio (CR) with basal area (BA) will decrease as age (A) or average diameter (AD) increases.

Also, thinning from below was assumed to result in a larger average crown ratio since larger trees usually have the higher crown ratios. Conversely, a thinning from above would produce the opposite result.

Holdaway, *et al.* (1979) developed the following generalized mean crown ratio (R) model based on the above assumptions, where b_1 is zero for conifers, b_2 is zero for hardwoods, and otherwise, all coefficients are species dependent:

$$R = b_1 A + \frac{b_2 e^{b_3 A} + b_4}{1 + \frac{b_5}{AD} BA} \tag{20}$$

Holdaway *et al.* (1979) believe that poor crown ratio estimates will result from point estimates of stand basal area if the stand's history is unknown; however, this problem lessens as the stand ages, or diameter increases. They explain that thinnings in older, larger diameter stands produce less of a crown ratio response than in a similarly stocked stand of younger, smaller trees.

Beekhuis (1965) developed a linear model to predict crown depth (length) for radiata pine based on stand density. He found that over all stocking levels, thinned and unthinned, crown length increased 1.6 feet per foot increase in average spacing. There was also evidence that crown length varies with stand height, particulary in younger stands. Also, site quality affected crown depth, but there were little differences in the rate of crown depth increase across spacings for a given site. Beekhuis developed stand-level crown depth (CD) equations for five different sites, all equations having the form:

$$CD = \frac{b_0}{\sqrt{N}} + b_1 \tag{21}$$

where b_0 and b_1 are positive coefficients. Thus, as stand density (N) decreases, crown depth will increase. An average standard error of approximately 4.5 feet was obtained for all five equations. Beekhuis believedthat once the necessary data became available a crown depth model using stand height, site, and stand density should be developed to provide more accurate results for different thinning regimes.

From past research it is apparent that height to live crown is correlated with age, total height, and inter-tree competition. These variables should be considered in the development of the individual tree models. Likewise, research has shown that the plantation's age, dominant height, and stand density need to be studied during the development of stand level height to live crown models.

CHAPTER III

DATA

The data used for this study comes from the Loblolly Pine Growth and Yield Research Cooperative's regionwide thinning study for the southeastern United States (Figure 1). This study was established during the dormant seasons of 1980-1981 and 1981-1982 on permanent plots throughout the natural range of loblolly pine. Plots were only established in plantations on cutover, site-prepared areas. These areas received the site preparations normal to their site conditions, such as bedding on wet sites. Also, the stand history for each plot location was recorded and included the stand type prior to planting, when it was clearcut, the type of site preparation, the time of planting, and any necessary hardwood release. For a plantation to be included in the study, it had to meet the following guidelines:

- At least eight years old.
- Unthinned.
- Show no evidence of heavy disease or insect attack.
- Free of heavy storm damage.
- Free of interplanting.
- Unpruned.
- Not fertilized within the last four years.

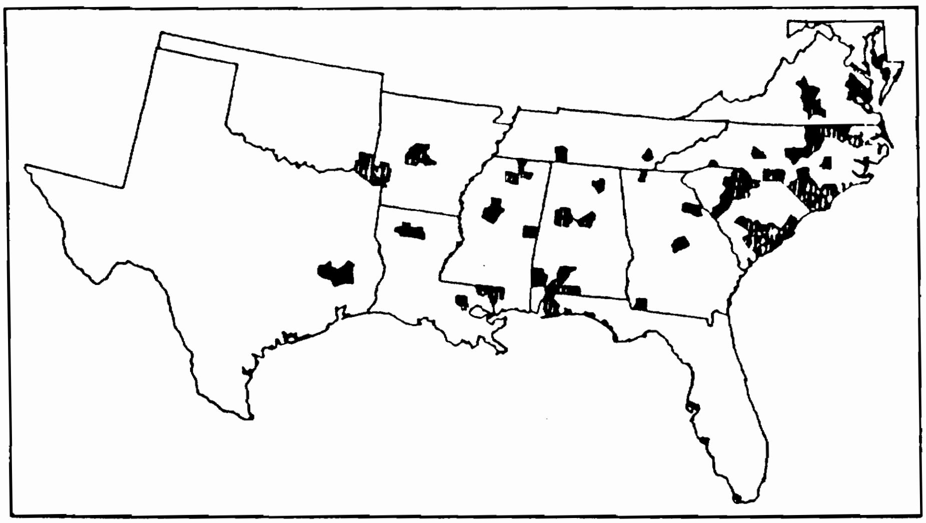

Figure 1. Location of the 186 thinning study plot installations by county (Burkhart et al., 1985). Each shaded county contains one or more plot locations.

- Not planted with genetically improved stock.
- Have a minimum of 200 to 300 planted pines per acre which appear free to grow.
- Have no more than 25 percent of the main canopy occupied by volunteer pines.

Subsequently, for each of the 186 locations, the number of trees per acre and stand age were determined. Site index was determined using the equation from Burkhart *et al.* (1987). For each pine, the following data were measured: dbh to the nearest tenth of an inch, total height to the nearest foot, height to the base of live crown to the nearest foot, crown class, and stem quality (Burkhart *et al.* 1985; Amateis and Burkhart, 1985). At each location, three plots were established - (1) an unthinned control plot, (2) a lightly thinned plot, and (3) a heavily thinned plot. All thinnings were from below, thus average tree characteristics were generally increased by the thinnings. The light thinnings removed approximately one third of the basal area while the heavy thinnings removed about one half of the basal area.

Since plot establishment, the plots have been remeasured every three years. Two remeasurements were available for all plots, and the third remeasurement was available for 55 plots - those established in 1980-1981.

Since the plots were measured at three year intervals, the crown height increment was assumed linear throughout the period. Therefore, the annual increment was simply the three year increment divided by three. All crown height and crown height increment equations were fit to only the years immediately after each measurement since tree attributes were only available for the beginning of the measurement period.

Tables 1 through 4 give summary statistics for the unthinned plots for each measurement, Tables 5 to 8 show the same for lightly thinned plots, and Tables 9 through 12 are for the heavily thinned plots. Note that the third remeasurement's statistics will vary somewhat from the others since only one third of the plots are represented at this time. Also, Tables 5 and 9 contain the minimum, mean, and maximum basal area, trees per acre, and quadratic mean diameter values for both immediately before and after thinning at plot establishment.

Table 1. **Plot establishment statistics for the unthinned plots (N=186).**

Variable	Minimum	Mean	Maximum
Age (years)	8.0	15.2	25.0
Trees per acre	270.0	569.4	1020.0
Basal area per acre (square feet)	24.3	111.9	234.9
Dominant height (feet)	14.4	41.2	74.4
Site index (feet, base age 25 years)	32.5	58.3	86.1
Quadratic mean dbh (inches)	2.9	6.0	10.1
Height (feet)	5.0	36.2	78.0
Crown height (feet)	1.0	20.4	53.0
Annual crown height inc. (feet)	0.0	1.8	9.0
Crown ratio	0.03	0.45	0.94

Table 2. **First remeasurement statistics for the unthinned plots (N=184).**

Variable	Minimum	Mean	Maximum
Age (years)	11.0	18.2	28.0
Trees per acre	270.0	544.0	1020.0
Basal area per acre (square feet)	50.9	130.1	236.0
Dominant height (feet)	21.7	47.3	80.1
Site index (feet, base age 25 years)	34.2	58.4	84.3
Quadratic mean dbh (inches)	4.3	6.7	10.8
Height (feet)	9.0	42.6	80.0
Crown height (feet)	2.0	25.6	58.0
Annual crown height inc. (feet)	0.0	1.7	11.7
Crown ratio	0.03	0.41	0.93

Table 3. Second remeasurement statistics for the unthinned plots (N=176).

Variable	Minimum	Mean	Maximum
Age (years)	14.0	21.2	31.0
Trees per acre	90.0	510.6	935.0
Basal area per acre (square feet)	34.3	140.5	209.4
Dominant height (feet)	26.8	52.2	80.2
Site index (feet, base age 25 years)	38.1	58.3	80.1
Quadratic mean dbh (inches)	4.9	7.3	10.8
Height (feet)	14.0	48.4	89.0
Crown height (feet)	7.0	30.7	71.0
Annual crown height inc. (feet)	0.0	1.6	8.7
Crown ratio	0.04	0.37	0.78

Table 4. Third remeasurement statistics for the unthinned plots (N=50).

Variable	Minimum	Mean	Maximum
Age (years)	17.0	24.2	34.0
Trees per acre	240.0	467.0	759.0
Basal area per acre (square feet)	93.1	154.7	207.3
Dominant height (feet)	43.8	58.3	85.4
Site index (feet, base age 25 years)	46.7	59.6	77.7
Quadratic mean dbh (inches)	5.7	8.0	11.1
Height (feet)	21.0	54.6	92.0
Crown height (feet)	15.0	36.5	74.0
Annual crown height inc. (feet)	*	*	*
Crown ratio	0.06	0.33	0.65

(*) Crown height increment could not be calculated for the third remeasurement since the fourth remeasurement has not been made.

Table 5. Plot establishment statistics for the lightly thinned plots (N=186).

Variable	Minimum		Mean		Maximum	
	Before	After	Before	After	Before	After
Age (years)	8.0		15.3		25.0	
Trees per acre	280.0	191.0	556.2	341.4	938.0	633.0
Basal area per acre (square feet)	27.4	19.6	110.1	78.4	239.0	155.5
Dominant height (feet)	14.8		40.9		72.6	
Site index (feet, base age 25 years)	31.5		57.8		85.7	
Quadratic mean dbh (inches)	3.0	3.2	6.0	6.5	9.7	10.2
Height (feet)	10.0		38.3		79.0	
Crown height (feet)	1.0		20.7		53.0	
Annual crown height inc. (feet)	0.0		1.5		8.3	
Crown ratio	0.04		0.48		0.97	

Table 6. First remeasurement statistics for the lightly thinned plots (N=186).

Variable	Minimum	Mean	Maximum
Age (years)	11.0	18.2	28.0
Trees per acre	191.0	335.3	633.0
Basal area per acre (square feet)	33.2	96.0	174.4
Dominant height (feet)	14.8	47.2	72.6
Site index (feet, base age 25 years)	33.9	58.2	82.6
Quadratic mean dbh (inches)	4.3	7.3	10.9
Height (feet)	13.0	44.2	83.0
Crown height (feet)	1.0	25.1	61.0
Annual crown height inc. (feet)	0.0	1.5	7.7
Crown ratio	0.03	0.45	0.97

Table 7. Second remeasurement statistics for the lightly thinned plots (N=180).

Variable	Minimum	Mean	Maximum
Age (years)	14.0	21.2	31.0
Trees per acre	89.0	328.6	633.0
Basal area per acre (square feet)	30.7	109.9	193.5
Dominant height (feet)	29.0	52.4	82.0
Site index (feet, base age 25 years)	37.9	58.4	81.5
Quadratic mean dbh (inches)	5.1	7.9	11.2
Height (feet)	15.0	49.8	90.0
Crown height (feet)	6.0	29.7	67.0
Annual crown height inc. (feet)	0.0	1.6	7.0
Crown ratio	0.03	0.41	0.83

Table 8. Third remeasurement statistics for the lightly thinned plots (N=51).

Variable	Minimum	Mean	Maximum
Age (years)	17.0	24.2	34.0
Trees per acre	173.0	308.3	450.0
Basal area per acre (square feet)	79.3	126.2	158.3
Dominant height (feet)	40.8	58.7	82.2
Site index (feet, base age 25 years)	44.9	60.0	78.6
Quadratic mean dbh (inches)	6.3	8.8	12.0
Height (feet)	22.0	56.2	88.0
Crown height (feet)	14.0	35.7	64.0
Annual crown height inc. (feet)	*	*	*
Crown ratio	0.05	0.37	0.72

(*) Crown height increment could not be calculated for the third remeasurement since the fourth remeasurement has not been made.

Table 9. Plot establishment statistics for the heavily thinned plots (N=186).

Variable	Minimum		Mean		Maximum	
	Before	After	Before	After	Before	After
Age (years)	8.0		15.2		25.0	
Trees per acre	271.0	130.0	556.0	259.7	957.0	479.0
Basal area per acre (square feet)	24.7	15.0	110.1	63.3	221.5	117.5
Dominant height (feet)	14.8		40.6		74.1	
Site index (feet, base age 25 years)	29.8		57.4		87.4	
Quadratic mean dbh (inches)	2.8	2.9	6.0	6.7	9.3	11.2
Height (feet)	9.0		38.7		82.0	
Crown height (feet)	1.0		20.6		53.0	
Annual crown height inc. (feet)	0.0		1.3		8.0	
Crown ratio	0.11		0.49		0.95	

Table 10. First remeasurement statistics for the heavily thinned plots (N=186).

Variable	Minimum	Mean	Maximum
Age (years)	11.0	18.2	28.0
Trees per acre	130.0	254.9	479.0
Basal area per acre (square feet)	26.9	79.1	132.5
Dominant height (feet)	22.9	47.0	78.1
Site index (feet, base age 25 years)	32.0	57.9	84.1
Quadratic mean dbh (inches)	4.5	7.6	12.2
Height (feet)	14.0	44.5	84.0
Crown height (feet)	3.0	24.6	59.0
Annual crown height inc. (feet)	0.0	1.4	8.0
Crown ratio	0.03	0.46	0.89

Table 11. Second remeasurement statistics for the heavily thinned plots (N=180).

Variable	Minimum	Mean	Maximum
Age (years)	14.0	21.2	31.0
Trees per acre	44.0	250.5	479.0
Basal area per acre (square feet)	18.9	92.7	145.2
Dominant height (feet)	29.5	52.2	80.4
Site index (feet, base age 25 years)	37.4	58.2	82.8
Quadratic mean dbh (inches)	5.4	8.4	12.1
Height (feet)	18.0	50.3	93.0
Crown height (feet)	4.0	28.9	65.0
Annual crown height inc. (feet)	0.0	1.4	7.7
Crown ratio	0.08	0.44	0.89

Table 12. Third remeasurement statistics for the heavily thinned plots (N=52).

Variable	Minimum	Mean	Maximum
Age (years)	17.0	24.2	34.0
Trees per acre	129.0	227.9	410.0
Basal area per acre (square feet)	74.6	105.1	152.1
Dominant height (feet)	44.3	58.3	84.8
Site index (feet, base age 25 years)	47.2	59.6	78.7
Quadratic mean dbh (inches)	6.9	9.3	12.9
Height (feet)	30.0	56.8	94.0
Crown height (feet)	10.0	34.0	64.0
Annual crown height inc. (feet)	*	*	*
Crown ratio	0.05	0.41	0.77

(*) Crown height increment could not be calculated for the third remeasurement since the fourth remeasurement has not been made.

CHAPTER IV

METHODS AND PROCEDURES

Data Splitting for Model Evaluation

The data were split into two parts - the plots with three remeasurements and the plots with only two remeasurements. The three remeasurement data were used to evaluate the candidate models; the two remeasurement data were then used to validate any potential models. Originally, it was hoped that the plots with two remeasurements could be used to evaluate the models, and then the data set with three remeasurements could be used as the validation data. If so, the models could be evaluated to see how well they would predict crown height increment for the third measurement period without being fit to any data from that period; however, it became evident that the third remeasurement period was necessary to better understand the crown height behavior. Consequently, the candidate models were fit to all three remeasurements of the evaluation data, thus every tree was represented three times. This does violate the independence assumption of least squares theory in that (1) one tree's crown height increment is affected by others on the plot, and (2) a tree's crown height at one remeasurement period is not independent of a past period's increment; however, with such a large sample size, over 10,000 trees for the evaluation data, it was believed that this problem was minimial.

All candidate models were first evaluated using the unthinned plots. Only the models that performed well with the unthinned plots were then fit to the data from the thinned plots.

Hypothesis Testing of the Influence of Thinning on Crown Height

An objective of this study was to determine if thinning had an effect on crown height and its increment, and if so, did the two different thinning levels have differing effects. Likewise, if thinning did have a significant effect, would it it continue through time or disappear after one or two remeasurements. To test for a thinning effect on crown height, a proven crown height model with no thinning allowance was fit to the data with three remeasurements. It was fit to all the data for each measurement and also separately to the unthinned, lightly thinned, and heavily thinned evaluation data for each measurement. Tests for significant differences between the thinning treatments were done using F-tests as shown by Swindel (1970). The regression fit to all the data was the reduced model and the three equations individually fit to the thinning regimes combined was the complete model. Dyer and Burkhart's (1987) crown height equation, see equation (7), was chosen for the testing.

Prior to the study it was hypothesized that thinning would have a significant effect on crown height and its increment; however, the hypothesis tests were needed to give support to the idea. Furthermore, these tests would show if light and heavy thinnings produced significantly different crown height responses. If so, then a categorical variable for thinned versus unthinned stands would not be sufficient, a continuous variable expressing the intensity of thinning would be required. Finally, assuming a thinning effect, the hypothesis tests were necessary to determine if the thinning variable was required throughout the remainder of a rotation or if just for a certain time period, possibly until stand reclosure.

Potential Independent Variables for the Increment Equations

Several stand and individual tree measures were seen as possible independent variables for the crown height increment equations. A competition index variable was desired. Two variable types could be used to estimate competition - (1) individual tree competition measures and (2) stand-level competition measures. Certainly an individual tree competition index would be favorable, if not required, for the two individual tree models, and perhaps a stand-level measure would also be necessary. Daniel's (1976) modification of Hegyi's (1974) competition index was chosen as the individual tree, distance-dependent competition measure. This is a distance-weighted size ratio index defined as:

$$CI_i = \sum_{j=1}^{n} (D_j/D_i)/L_{ij} \qquad (22)$$

where the competition exerted upon tree i is the sum of the ratios of competitors' diameters (D_j) to its diameter (D_i) inversely weighted by the distance between the subject tree and each competitor (L_{ij}). The number of competitors, n, is defined as those trees included in a ten basal area factor point sample centered at the subject tree. This is an attractive index because both the size of competing trees relative to the subject tree and the intertree distance are considered. Furthermore, the index is relatively easy to compute compared to other distance-dependent competition indexes, and it has proven to work well in the loblolly pine growth and yield model

PTAEAD2 (Burkhart *et al.* 1987).

The competition variable for the distance-independent tree model is one used in TRULOB (Amateis and Burkhart 1986), a distance-independent loblolly pine growth and yield simulator for cutover, site-prepared plantations. It is the ratio of the stand's quadratic mean diameter ($\bar{D}_q$) to the subject tree's dbh (D_i). This competition measure is computationally simple, yet it has proven effective.

Besides an individual tree competition index, other variables were also considered. It was believed that crown height increment is also dependent upon a tree's vigor, age, and perhaps overall stand density, in addition to a possible thinning effect. Variables which have often been used to express a tree's vigor are:

- dbh
- total height
- crown ratio
- crown length
- taper (dbh/height)

Common average stand density measures are:

- basal area
- trees per acre
- relative spacing
- crown competition factor

Crown competition factor, though commonly used in species modelling for the western United States, was not considered since empirical evaluations have shown that it is generally not more effective for predicting growth and yield than simpler measures of stand density.

The potential independent variables for the stand-level crown height increment model would be those of the individual tree models' which could be collapsed to a stand level statistic.

Potential Variables for Expressing the Influence of Thinning

Assuming a thinning effect on crown height, two variable types were possible. If thinning had an effect but crown height increment could not be related to the intensity of thinning, then a simple zero or one categorical variable should be sufficient. That is, the variable would be used only when thinning occurred, and would not be included, or would have no effect, on the increment equation for unthinned stands. Additionally, if the thinning effect disappeared after a certain time, then this variable would also be eliminated from the equation once the appropriate time interval had elapsed since thinning.

Nonetheless, if different thinning regimes produced different crown height responses, as was hypothesized, then a continuous variable would be necessary. Current basal area (BA), basal area before thinning (BAB), and basal area immediately after thinning (BAA) or some combination of these statistics were considered as possible candidates. Also, if the effect of thinning was manifested through time, then time must also be considered, perhaps a combination of the stand's current age (AGE) along with its age at thinning (THINAGE) or the time interval (THININT) since thinning. Assuming thinning produced a multiplicative effect on crown height and not an additive effect, then ideally the variable should equal one in unthinned stands, thus having no effect on increment except for different model parameter estimates due to including a thinning variable. For thinned stands, the thinning statistic would be less than one. Assuming the age at time of thinning, the interval since thinning, and the thinning degree all affect crown height increment, two thinning variables were proposed:

$$THIN1 = \left(\frac{BAA}{BAB}\right)^{\left(\frac{THINAGE}{AGE}\right)} \tag{23}$$

and

$$THIN2 = \left(\frac{BAA}{BAB}\right)^{\left(\frac{THININT}{AGE}\right)} \tag{24}$$

where the variables are defined above. Both variables equal one for unthinned stands, i.e. the basal area after thinning equals the basal area before thinning since there is no thinning. Likewise, the heavier the thinning the smaller the ratio, producing a smaller crown height recession. The only difference between the variables is the use of THINAGE or THININT. Variable THIN1, by using THINAGE, approaches one as the thinning interval increases, while THIN2 does not. Also, as thinning age increases, THIN1 becomes smaller, predicting a larger relative impact on crown height increment for older stands, while THIN2 approaches one. Each variable was tested with the final increment model to determine which better emulated the response of trees to thinning. These two variables were also incorporated into Dyer and Burkhart's (1987) crown height equation to model response to thinning.

Candidate Increment Models

Maguire and Hann's (1990a) research showed that a nonlinear equation or a logarithmic transformation of the dependent variable was necessary to predict 5-year crown height increment for Douglas-fir; however, since annual increment was the dependent variable in this study, logarithmic equations were inappropriate since a crown height increment of zero was possible for a given year. Thus, only nonlinear increment equations could be considered.

Two of Maguire and Hann's (1990a) crown height increment equations, equations (3) and (4), were considered as possible candidates. Also, the total height increment equation (HT_{inc}) used by the loblolly pine growth and yield model PTAEDA2 (Burkhart *et al.* 1987) was also considered. The equation consists of a potential increment and a modifier function. The potential height increment, PHIN, is the annual height increment of the dominants, while the remainder of the equation is a modifier function depending upon the individual tree's crown ratio (CR) and Daniel's (1976) distance-dependent competition index (CI):

$$HT_{inc} = PHIN(b_1 + b_2 CR^{b_3} \exp(-b_4 CI - b_5 CR)) \qquad (25)$$

The competition index term (CI) was replaced by quadratic mean diameter over tree dbh (DR) for the distance-independent equation.

A final candidate model form was:

$$\Delta HLC = b_0 HT^{b_1} \exp(b_2 CR + b_3 CI + b_4 A) \qquad (26)$$

where CI was replaced by DR for the distance-independent model. This model was considered since increment was based on a tree's vigor (CR), its size (HT), its competition (CI or DR), and its age (A). The exponential function would serve as a modifier-type function for the total height variable. Hopefully the parameter for age, (b_4), would be negative, allowing crown height increment to approach zero as age increased.

All candidate models were tested in their original form, and then by substitution of similar variables, such as taper for crown ratio, into the models.

Crown Height Model

Since this study's primary goal was to develop a crown height increment model, only one crown height model was studied. Dyer and Burkhart's (1987) model:

$$CH = HT \exp[-(b_2 + b_3 A^{-1}) DBH/HT] \tag{7}$$

was chosen since it is currently used in the PTAEDA2 growth model and has worked well. The two proposed variables for expressing thinning effects were each added to the model to test if either significantly improved the model's overall fit, or if they improved the model's predictions for thinned stands since the authors noted that the model slightly overpredicted crown height, especially in heavily thinned stands.

Stand Level Increment Model

The candidate stand models were versions of the individual tree models using only stand level variables. Thus, there were four candidate stand level increment models, each a collapsed version of the individual tree increment models:

$$\Delta HLC = b_0 HD^{b_1} CCF^{b_2} \exp[b_3 R + b_4 A] \tag{27}$$

$$\Delta HLC = c_0 HD^{c_1} CCF^{c_2} \exp[c_3 (1 - R/38]^{c_4} + c_5 A \tag{28}$$

$$\Delta HLC = PHIN(b_1 + b_2 R^{b_3} \exp(-b_4 SC - b_5 R)) \tag{29}$$

$$\Delta HLC = b_0 HD^{b_1} \exp(b_2 R + b_3 SC + b_4 A) \tag{30}$$

where SC is a stand level competition index such as basal area, and the other variables were defined previously.

Model and Data Evaluation

All model and data evaluations were done using the SAS computer package (1982). All nonlinear equations were fit using the package's PROC NLIN procedure using either the Gauss-Newton or the Marquardt algorithms. Other statistical calculations were also done using the SAS package.

Model Validation and Final Model Selection

Candidate increment model selection was based on the following criteria:

- mean square error (MSE).
- proportion of variation in the dependent variable explained (R^2)
- mean residuals by thinning regime and interval since thinning.
- residual plots.
- predicted versus observed increment plots

After the best candidate model was selected based on its performance criteria for the evaluation data, it was then used to predict the crown height increment for the validation data. As with the evaluation data, the assumption of independence among observations was violated - each tree in the validation data was represented twice, once for each remeasurement. Again, with such a large data set, over 25000 trees for the validation data, the independence problem was negligible. The selected candidate model performance with the validation data was studied for any potential biases. These biases not only included any general tendency to over or underpredict increment, but also any potential biases through time or across thinning treatments.

Once the final model was chosen its final parameters were estimated by fitting the model to all of the data. This was done in an effort to try to get the best parameter estimates. Again residual plots and mean residuals were used to check for any biases in the final model. This procedure was originally done for the two individual tree equations, and then performed again for the stand-level version.

The crown height model with and without a thinning variable was similarly tested using the validation data. The final form was then also fit using all the data to estimate its final parameters.

CHAPTER V

RESULTS AND DISCUSSION

Hypothesis Testing of the Influence of Thinning on Crown Height

The hypothesis tests revealed that thinning and its degree both have a significant effect on crown height.

Hypothesis tests for the influence of thinning on crown height used Dyer and Burkhart's (1987) crown height equation. The hypothesis tests showed that the thinning itself had an effect on crown height (Table 13). The tests revealed that immediately after thinning there was a significant difference in crown height among the three regimes, this was expected since the thinning was from below. Further tests showed that there was a signifant difference between the crown heights of the light and heavy thinned plots but not between the unthinned and lightly thinned plots. Thus, it was concluded that the crown heights for the heavily thinned plots were significantly different from those of the unthinned and lightly thinned plots immediately after thinning. Hypothesis tests show that the thinning levels have significantly different effects on crown height; in fact, all three regimes' crown heights are significantly different for all three remeasurements (Tables 14 through 16). Furthermore, the test statitics show that the difference between the three treatments becomes greater with time. Thus, it was concluded that a continuous variable was needed to incorporate

Table 13. Results of hypothesis tests for significance of the effect of thinning on crown height at plot establishment.[*]

Hypothesis test	F test statitic	df_{num}	df_{den}	p-value
No significant difference among all three regimes	7.60	4	9938	< 0.0001
No significant difference between light and heavy thinnings	10.80	2	6936	< 0.0001
No significant difference between unthinned and light thinnings	0.42	2	7035	0.6569
No significant difference between unthinned and heavy thinnings	12.53	2	5905	< 0.0001

[*] Using Dyer and Burkhart's (1987) crown height model.

Table 14. Results of hypothesis tests for significance of the effect of thinning on crown height at the first remeasurement.[*]

Hypothesis test	F test statitic	df_{num}	df_{den}	p-value
No significant difference among all three regimes	27.27	4	9447	< 0.0001
No significant difference between light and heavy thinnings	25.12	2	6705	< 0.0001
No significant difference between unthinned and light thinnings	8.28	2	6630	0.0003
No significant difference between unthinned and heavy thinnings	52.44	2	5559	< 0.0001

[*] Using Dyer and Burkhart's (1987) crown height model

Table 15. Results of hypothesis tests for significance of the effect of thinning on crown height at the second remeasurement.*

Hypothesis test	F test statitic	df_{num}	df_{den}	p-value
No significant difference among all three regimes	29.15	4	8441	< 0.0001
No significant difference between light and heavy thinnings	12.06	2	6064	< 0.0001
No significant difference between unthinned and light thinnings	22.89	2	5840	< 0.0001
No significant difference between unthinned and heavy thinnings	60.37	2	4978	< 0.0001

* Using Dyer and Burkhart's (1987) crown height model.

Table 16. Results of hypothesis tests for significance for the effect of thinning on crown height at the third remeasurement.*

Hypothesis test	F test statitic	df_{num}	df_{den}	p-value
No significant difference among all three regimes	87.28	4	8441	< 0.0001
No significant difference between light and heavy thinnings	60.38	2	6064	< 0.0001
No significant difference between unthinned and light thinnings	42.59	2	5840	< 0.0001
No significant difference between unthinned and heavy thinnings	175.46	2	4978	< 0.0001

* Using Dyer and Burkhart's (1987) crown height model.

the effects of thinning in Dyer and Burkhart's crown height model since differing thinning intensities produced different crown height levels. Furthermore, since these tests showed that the difference between the crown height levels became greater with time, it was suspected that the thinning intensities were producing different crown height increments. These hypotheses were supported by graphs of the data which indicated that mean crown height and mean crown height increment were different through time for the three thinning treatments (Figures 2 and 3). Furthermore, different thinning intensities produced different mean crown heights and mean crown height increments across different age classes; these results are presented in Appendix A.

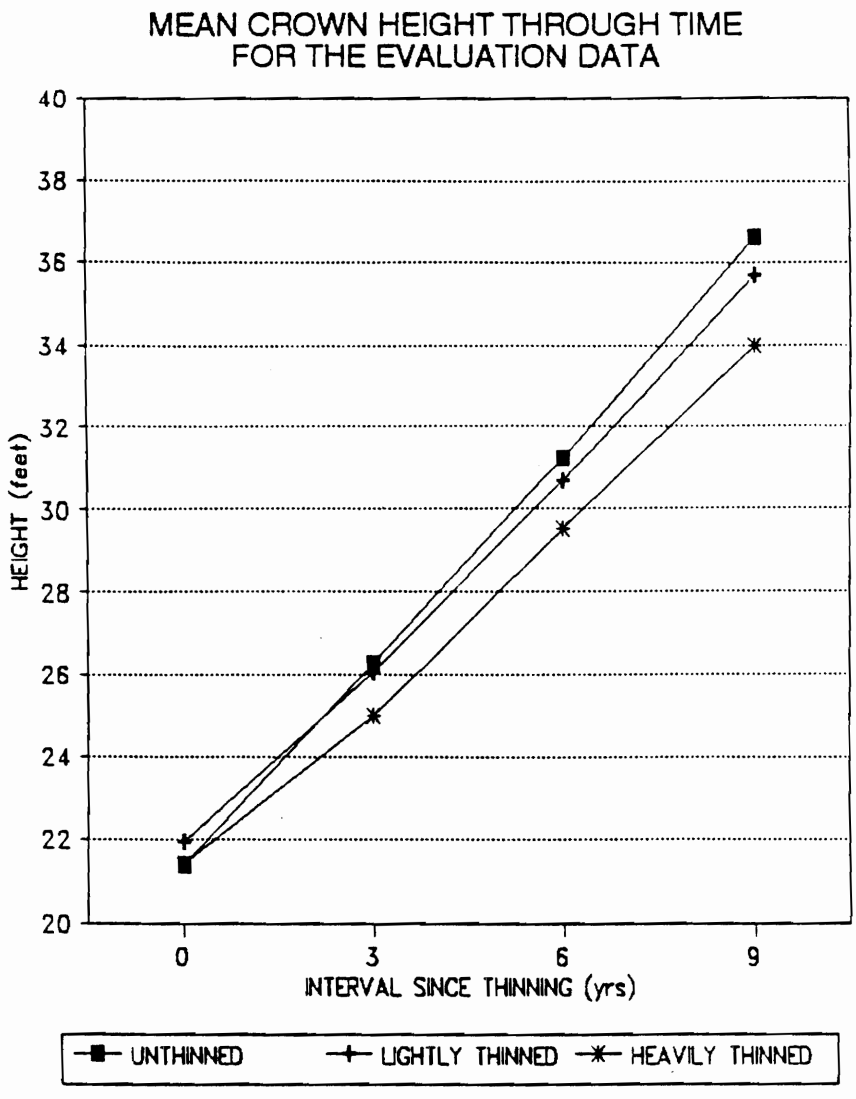

Figure 2. Mean loblolly pine crown height, by thinning treatment, through time for the evaluation data.

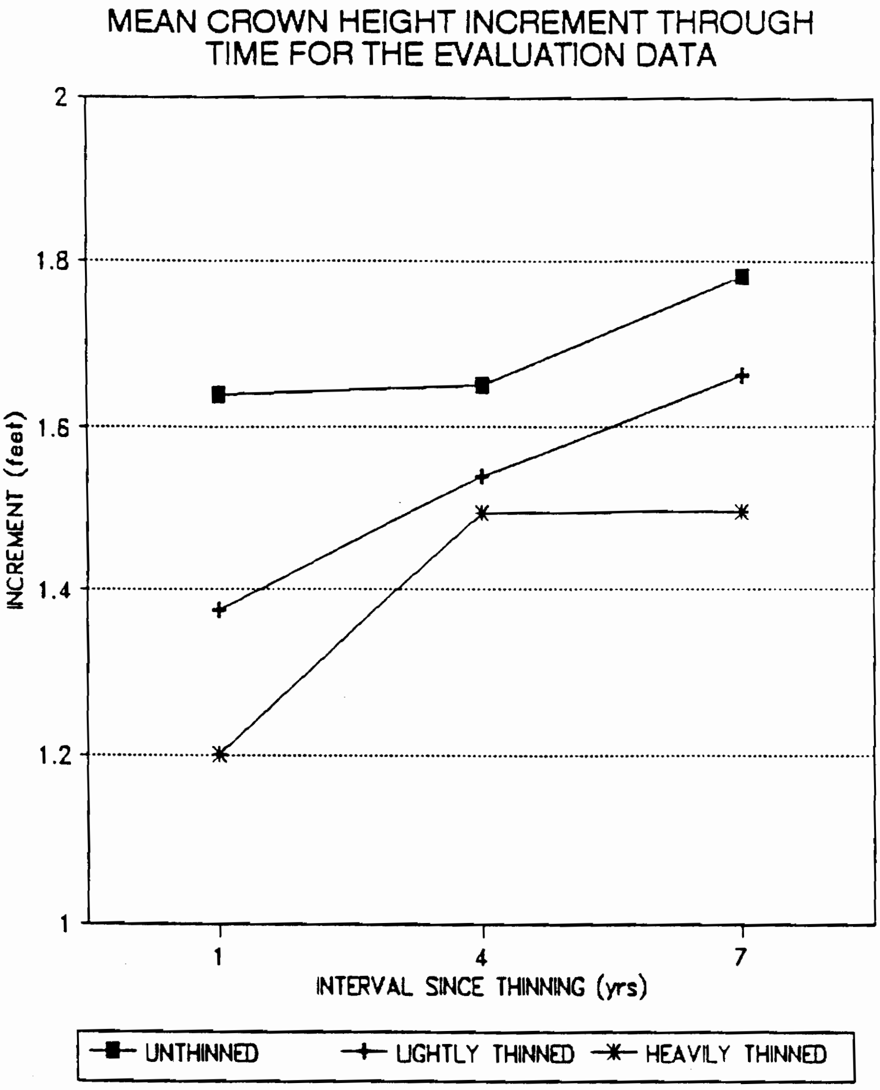

Figure 3. Mean loblolly pine crown height increment, by thinning treatment, through time for the evaluation data.

Individual Tree Crown Height Increment Candidate Models

The four candidate increment models were tested first on the unthinned plots with three remeasurements. Additionally, different tree and stand variables were substituted into the models to test for better models. To review, the original candidate models were:

$$\Delta HLC = b_0 HT^{b_1} CCF^{b_2} \exp[b_3 CR + b_4 A] \quad (3)$$

$$\Delta HLC = c_0 HT^{c_1} CCF^{c_2} \exp[c_3 (1 - CR/38]^{c_4} + c_5 A \quad (4)$$

$$\Delta HLC = PHIN(b_1 + b_2 CR^{b_3} \exp(-b_4 CI - b_5 CR)) \quad (25)$$

$$\Delta HLC = b_0 HT^{b_1} \exp(b_2 CR + b_3 CI + b_4 A) \quad (26)$$

The models' performances are presented below.

The PTAEDA2 height increment model (25) performed the worst. The original equation would not even converge when fit to crown height increment. Only when the exponent for crown ratio, b_3, was set equal to its estimated value in the height increment equation did the equation converge; however the resulting R^2 was .06. Since the model has two parts, PHIN operating as a potential and the remainder of the equation as a modifier, it was hypothesized that perhaps one or both of these functions were incorrect for crown height increment.

It was hypothesized that the height increment of the dominant and codominants (PHIN) was inappropriate for the potential crown height increment, therefore a different potential was developed. An equation based on total height (HT) and age (A) was fit to data from open-grown loblolly pine trees to predict height to live crown (CH):

$$CH = b_0 + b_1 HT + b_2 AGE \tag{31}$$

The first difference of the equation with respect to age produced a potential crown height increment (PHCIN), the final fitted model was:

$$PHCIN = 0.1416\ (HT_{inc}) + 0.09439 \tag{32}$$

However, when PHCIN was substituted for PHIN the equation still did not converge. Thus, the modifier function was changed and different variables were used. Yet, this did not substantially improve the model's performance either. Furthermore, age was not significant when placed in the equation. Since a significant age term with a negative parameter was desirable, so that increment would go approach zero as age increased, the PTAEDA2 height increment model was eliminated as a candidate model.

The remaining three candidate models yielded much better results based on R^2 and MSE values when fit to the unthinned plots in the evaluation data (Table 17). Residuals from these models were all quite good, with each model's average residual under a tenth of a foot and standard deviations under a foot. The first four models are patterned after equation (4). The next two models are derived from equation (3), while the last two are forms of equation (26) (Table 17). The competition term for the distance-independent model (DR) is the quadratic mean diameter ($\bar{D}_q$) divided by tree DBH, while CI is Daniels' distance-dependent competition index.

Table 17. Best individual tree increment models based on fit criteria for the unthinned evaluation plots.

Candidate Model	R^2	MSE
$c_0 HT^{c1} CR^{c2} \exp(c_3(1\text{-}DBH/HT)) + c_4 RS$	.2102	0.821
$c_0 HT^{c1} CR^{c2} \exp(c_3(1\text{-}DBH/HT)) + c_4 D_q/DBH$	.2070	0.824
$c_0 HT^{c1} CR^{c2} \exp(c_3(1\text{-}DBH/HT))$	.2061	0.825
$c_0 HT^{c1} CR^{c2} \exp(c_3(1\text{-}DBH/HT)) + c_4 A$	.2058	0.826
$b_0 HT^{b1} CI^{b2} \exp(b_3 CR + b_4 A)$	.2019	0.612
$b_0 HT^{b1} DR^{b2} \exp(b_3 CR + b_4 A)$	.1860	0.846
$b_0 HT^{b1} \exp(b_2 CR^{1/2} + b_3 CI + b_4 A)$	.2294	0.592
$b_0 HT^{b1} \exp(b_2 CR^{1/2} + b_3 DR + b_4 A)$	.1883	0.844

where: RS = relative spacing
HT = total tree height
CR = tree crown ratio
DR = quadratic mean diameter/tree dbh
CI = Daniels' competition index
DBH = tree dbh
A = tree age

Models with Daniels' CI term were fit to a subset of the unthinned evaluation data since only those trees whose competitors, defined by a 10 BAF sweep, were not outside the measured plot could be used.

Models similar to equation (4) yielded the best fit statistics. Unlike Maguire and Hann's findings (1990a), crown height increment did not exhibit a strong peaking behavior when plotted against crown ratio. In fact the best results were found by replacing the competition term with CR and then replacing the crown ratio variable in the exponent with a taper variable (DBH/HT). Replacing crown ratio with taper was not surprising since others, such as Dyer and Burkhart (1987), have found taper to be well correlated with crown characteristics. Unlike Maguire and Hann's work, raising the exponential function to a power did not improve the fit. Also, adding the final term (c_4) with AGE actually worsened the fit; furthermore the confidence intervals about (c_4) included zero, so age was judged insignificant in this equation. This was a disadvantage since it was hoped that the final equation would include age with a negative coefficient. Also, adding the distance-independent competition term DR to the model did not significantly increase the fit either, nor did basal area or trees per acre, only relative spacing (RS) showed any measureable contribution. Thus, since the equation did not contain an age variable or a competition index, this equation was not further considered.

Models like equation (6) produced adequate results. This model was attractive since it contained age and a competition term. The distance-dependent equation produced a better fit than the distance-independent model; this was appealing since knowing the location of a tree's competitors should improve one's predictions. The distance-independent equation produced a negative parameter with age as desired; however, the distance-dependent model produced a positive parameter for age, and the confidence interval included zero. Substituting other variables into the model, such as taper for crown ratio, only worsened the fit and age still remained nonsignificant. Also, having the competition term and total height outside the expontential function did not make much intuitive sense. Having only height outside the exponent as a potential-like function seemed more appropriate when combined with a competition term, age and a tree vigor variable such as crown ratio in the exponent. These considerations made equation (26) the most attractive model.

Equation (26) produced slightly better results than equation (3), and again the distance-dependent model yielded a better fit than the distance-independent model. Also, taking the square root of crown ratio improved the fit. One possible reason for this improvement is that the square root narrows the range of the crown ratio term from approximately .30 to .95 before to .55 to .98 after taking the square root. This improvement was discovered when exponents were estimated for CR, DR or CI, and AGE; estimating an exponent for crown ratio improved the fit, whereas no improvement was realized when estimating an exponent for the other variables. The exponent of CR was estimated as .41, thus .5 was chosen as an approximation and an extra parameter was eliminated. Age's parameter, b_4, was negative for both equations; however, for the CI model its confidence interval included zero. This lack of significance was disturbing; however since the CI equation was fit to a relatively small subset of the unthinned data, 372 of the total 8126 observations, it was hoped that b_4 would become negative when fit to all the evaluation data. Consequently, equation (28) was chosen as the best model based on its fit to the evaluation data from unthinned plots and its biological soundness. It was then fit to the data from the thinned plots.

Fitting the two versions to the thinned data produced similar results. The age parameter for the CI model remained negative but its confidence interval still included zero. Yet, overall the models still performed well. It was then necessary to decide if a variable expressing thinning was needed for the increment model. The crown height hypothesis tests had shown that a thinning variable was likely needed for crown height increment as well, but a formal test was considered appropriate. However, due to this evidence, the tests were done simply on the combined data, i.e. not by individual measurements. The distance-independent version was used since it was based on a much larger data set, and it seemed reasonable that results from the distance-independent model could be assumed to hold for the distance-dependent model as well. If this was not the case, then biases in the residuals should appear once the final distance-dependent model was fit with or without a thinning allowance.

The hypothesis tests for the influence of thinning on crown height increment using the distance-independent model demonstrated that thinning and its intensity has a very significant impact (Table 18). Due to this overwhelming evidence, hypothesis tests for each measurement period were considered unnecessary.

Table 18. Results of hypothesis tests for significance of the effect of thinning on crown height increment.[a]

Hypothesis test	F test statitic	df_{num}	df_{den}	p-value
No significant difference among all three regimes	108.50	10	27829	< 0.0001
No significant difference between light and heavy thinnings	53.86	5	19707	< 0.0001
No significant difference between unthinned and light thinnings	80.90	5	19507	< 0.0001
No significant difference between unthinned and heavy thinnings	202.10	5	16444	< 0.0001

[a] Using the model:

$$\Delta HLC = b_0 HT^{b_1} \exp(b_2 CR^{1/2} + b_3 DR + b_4 A)$$

where: HT = total tree height
CR = tree crown ratio
DR = quadratic mean diameter/tree dbh
A = tree age

Thinning Variable Selection

Two variables were proposed to describe the effect of thinning on crown height increment. To review, they were:

$$THIN1 = \left(\frac{BAA}{BAB}\right)^{\left(\frac{THINAGE}{AGE}\right)} \qquad (23)$$

or

$$THIN2 = \left(\frac{BAA}{BAB}\right)^{\left(\frac{THININT}{AGE}\right)} \qquad (24)$$

Each variable was added to the distance-dependent and the distance-independent versions of equation 26, with THIN1 producing the best results, THIN2 did not significantly improve the models' fit and was eliminated. Variable THIN1, by using THINAGE, approaches one as the thinning interval increases, apparently this better captured the trees' thinning response than THIN2 which does not include the thinning age. Adding a parameter b_5 as a power to THIN1 significantly improved its performance; also THIN1 showed significant improvement of fit over simply using a basal area after/ basal area before variable to express the effects of thinning.

Three distance-independent crown increment models were fit to the evaluation data - THIN1 with the parameter b_5 (model 1), THIN1 without the parameter (model 2), and without THIN1 (model 3). Since the remeasurements were every three years, the equations were fit to only the years immediately after each measurement because the tree attributes were only available for the beginning of the measurement periods. Thus, for all tables, time 0 is the year directly after thinning, time 4 is the fourth year after thinning and time 7 is the seventh year after thinning.

Model 1 which included THIN1 with an estimated parameter produced the best fit (Table 19). Likewise it produced the smallest mean residuals by thinning level or the interval since thinning for these same models (Table 20). Similar results were achieved with the distance-dependent model although adding THIN1 did not improve the results as significantly as with the distance-independent model. Also, adding the THIN1 variable caused age's parameter to become positive. This was disturbing, but it was decided to leave the variable in the equation and see how the model performed against the validation data before trying another variable.

Additionally, the model's fit improved slightly by using trees per acre instead of basal area in the THIN1 variable. However, it was thought that basal area was a better measure of thinning intensity, and thus basal area remained in the THIN1 variable.

From the results one can see that adding THIN1 raised to a power significantly improves the fit and improves the mean residuals. With no thinning allowance (model 3) the model overpredicts increment for the thinned plots producing negative residuals (Table 20). Also, model 2, with no exponent on the thinning variable, overpredicts increment in the unthinned stands. Furthermore, when these residuals are broken down into combinations of thinning intensity and the interval since thinning, model one's superiority is more apparent. These residuals are listed in Appendix B. Thus, model 1, using THIN1 as the thinning variable raised to a power, was chosen as the most appropriate model.

Table 19. Fit statistics for three candidate distance-independent crown height increment models fitted to the data from thinned and unthinned plots.

Model	R^2	MSE
1. $b_0 THIN1^{b5} HT^{b1} \exp(b_2 CR^{1/2} + b_3 DR + b_4 A)$	.1847	0.889
2. $b_0 THIN1 \bullet HT^{b1} \exp(b_2 CR^{1/2} + b_3 DR + b_4 A)$	.1740	0.901
3. $b_0 HT^{b1} \exp(b_2 CR^{1/2} + b_3 DR + b_4 A)$	.1504	0.926

where: HT = total tree height
CR = tree crown ratio
DR = quadratic mean diameter/tree dbh
A = tree age
THIN1 = thinning variable defined in equation (25)

Table 20. Mean residuals by thinning intensity or elapsed time since thinning for the three candidate crown height increment models.

Thinning intensity or the elapsed time since thinning	Mean Residuals		
	Model 1 *	Model 2 *	Model 3 *
Unthinned plots, all measurements	-0.006	-0.118	0.221
Lightly thinned plots, all measurements	0.014	0.055	-0.009
Heavily thinned plots, all measurements	-0.026	0.094	-0.213
Time = 1, all thinning intensities	-0.039	-0.003	-0.070
Time = 4, all thinning intensities	0.007	0.026	0.009
Time = 7, all thinning intensities	0.025	0.027	0.064

* Models 1, 2, and 3 defined in Table 19.

Individual Tree Crown Height Increment Model Validation

The distance-dependent and distance-independent tree models were then tested against the validation data. Only two remeasurements have been made on the validation data, so only two intervals since thinning were available for validation purposes. Both individual tree models performed well with an overall average residual of approximately one tenth of a foot which is approximately seven percent of the average increment. Yet, residuals showed that the models slightly underpredicted the increment especially for the first measurement period (Table 21). The models then slightly overpredicted increment for the second measurement (Table 21); whether this overprediction would grow with time is unknown. Hopefully the models are simply performing better for the second measurement and would continue to do so at a third remeasurement. This initial underprediction bias was present across all thinning levels as can be seen in Appendix B where the mean residuals are further classified by thinning intensity and the interval since thinning.

Even though both models performed adequately, the problem of a positive parameter estimate for the age variable in the distance-dependent model remained. To evaluate the extent of this problem, the CI model with THIN1 was fit to the validation data, and then parameter for age became negative and its entire confidence interval was also negative. Thus, it was decided to keep the models' form and fit both models to the entire data set and examine their performance. If no large biases resulted, then these models would be chosen as the final individual tree models.

Table 21. Mean residuals for the validation data by thinning intensity or elapsed time since thinning for the the individual tree increment models.

Thinning intensity or the elapsed time since thinning	Sample size and mean residuals			
	Model DR *		Model CI *	
	N	Residual	N	Residual
Unthinned plots, both measurements	13664	0.094	847	0.050
Lightly thinned plots, both measurements	18993	0.089	4029	0.102
Heavily thinned plots, both measurements	14707	0.092	3057	0.131
Time = 1, all thinning intensities	24493	0.204	4631	0.193
Time = 4, all thinning intensities	22871	-0.030	3302	-0.012

* Model DR is defined as:

$$\Delta HLC = b_0\, THIN^{b_5}\, HT^{b_1} \exp\left(b_2\, CR^{1/2} + b_3\, DR + b_4\, A\right)$$

* Model CI is defined as:

$$\Delta HLC = b_0\, THIN^{b_5}\, HT^{b_1} \exp\left(b_2\, CR^{1/2} + b_3\, CI + b_4\, A\right)$$

Final Individual Tree Increment Models

The two individual tree models were fit to all the data; again mean residuals were calculated and also residual plots were made. Adding the thinning variable substantially improves the fit of the DR model, and also slightly improves the fit of the distance-dependent model (Table 22). Furthermore, the predicted crown height increments from the models which include THIN1 are more correlated with the observed increments than the versions without THIN1 (Table 23). Mean residuals indicated that the DR model with no thinning variable overpredicts crown increment for the thinned stands, but this bias is removed with the addition of the THIN1 variable (Table 23); appendix B gives the mean residuals by thinning intensity and the interval since thinning. The smaller improvement of fit resulting from the addition of THIN1 for the CI model compared to the distance-independent DR model is expected since the location and size of a tree's competitors are known, and consequently the loss of competitors due to thinning is better modelled with the distance-dependent model. However, since the THIN1 variable also improved the fit of the model containing the CI term, it was left in the model to keep the two models consistent. The final fitted individual tree models were:

$$\Delta HLC = b_0 \, THIN1^{\,b_5} HT^{\,b_1} \exp\left(b_2 CR^{1/2} + b_3 DR + b_4 A\right) \tag{33}$$

where:

$b_0 = 0.001552$
$b_1 = 1.096939$
$b_2 = 4.113819$
$b_3 = 0.480997$
$b_4 = -0.019342$
$b_5 = 0.693263$

Table 22. Fit statistics for the individual tree increment models when fit to the entire data set both with and without the THIN1 variable.

Model	R^2	MSE
1. $b_0 THIN1^{b5} HT^{b1} \exp(b_2 CR^{1/2} + b_3 DR + b_4 AGE)$	.2003	0.895
2. $b_0 HT^{b1} \exp(b_2 CR^{1/2} + b_3 DR + b_4 AGE)$	.1601	0.940
3. $b_0 THIN1^{b5} HT^{b1} \exp(b_2 CR^{1/2} + b_3 CI + b_4 AGE)$	.2148	0.776
4. $b_0 HT^{b1} \exp(b_2 CR^{1/2} + b_3 CI + b_4 AGE)$	.1988	0.792

where: HT = total tree height
CR = tree crown ratio
DR = quadratic mean diameter/tree dbh
CI = Daniels' competition index
A = tree age
THIN1 = thinning variable defined in equation (25)

Table 23. Mean residuals by thinning intensity or elapsed time since thinning for the individual tree increment models with and without the THIN1 variable.

Thinning intensity or the elapsed time since thinning	Mean Residuals			
	Model 1*	Model 2*	Model 3*	Model 4*
Unthinned plots, all measurements	-0.026	0.230	-0.125	0.088
Lightly thinned plots, all measurements	0.004	-0.021	-0.009	0.016
Heavily thinned plots, all measurements	0.009	-0.196	0.031	-0.057
Time = 1, all thinning intensities	0.051	0.025	0.040	0.029
Time = 4, all thinning intensities	-0.065	-0.047	-0.080	-0.063
Time = 7, all thinning intensities	0.013	0.060	0.038	0.078
Coeff. of determination, all data	0.4476	0.4002	0.4635	0.4461

* Models 1, 2, 3, and 4 are defined in Table 22.

$$\Delta HLC = b_0 THIN1^{b_5} HT^{b_1} \exp(b_2 CR^{1/2} + b_3 CI + b_4 A) \tag{34}$$

where:

b_0 = 0.003880
b_1 = 0.789696
b_2 = 4.415248
b_3 = 0.381313
b_4 = -0.005950
b_5 = 0.528954

Individual Tree Crown Height Increment Model Problems

Both individual tree models do well for most of the trees, those whose increments are from one half a foot to two feet. However, for the extreme increments, those near zero and those over two feet, the model's predictions are poorer. Plots of predicted crown height increment versus observed crown height increment demonstrate this problem well, and a good example is a plot of the distance-dependent model's predicted crown height increments versus the observed increments for all of the heavily thinned plots (Figure 4). This problem is mainly due to the exponential function's moderation of the prediction increment, it is very difficult for the model to predict an increment of zero or over three. Thus, the model overpredicts many small increments and underpredicts the larger increments. Perhaps a weighted regression would make an improvement; however, Maguire and Hann's study (1990a) found that weighting did not improve the fit of their crown height increment equations. Also, any logarithmic transformation of the increment is not possible because annual increment is often zero. In general, crown height increment has proven difficult to model.

Another problem encountered when developing the crown height and crown increment

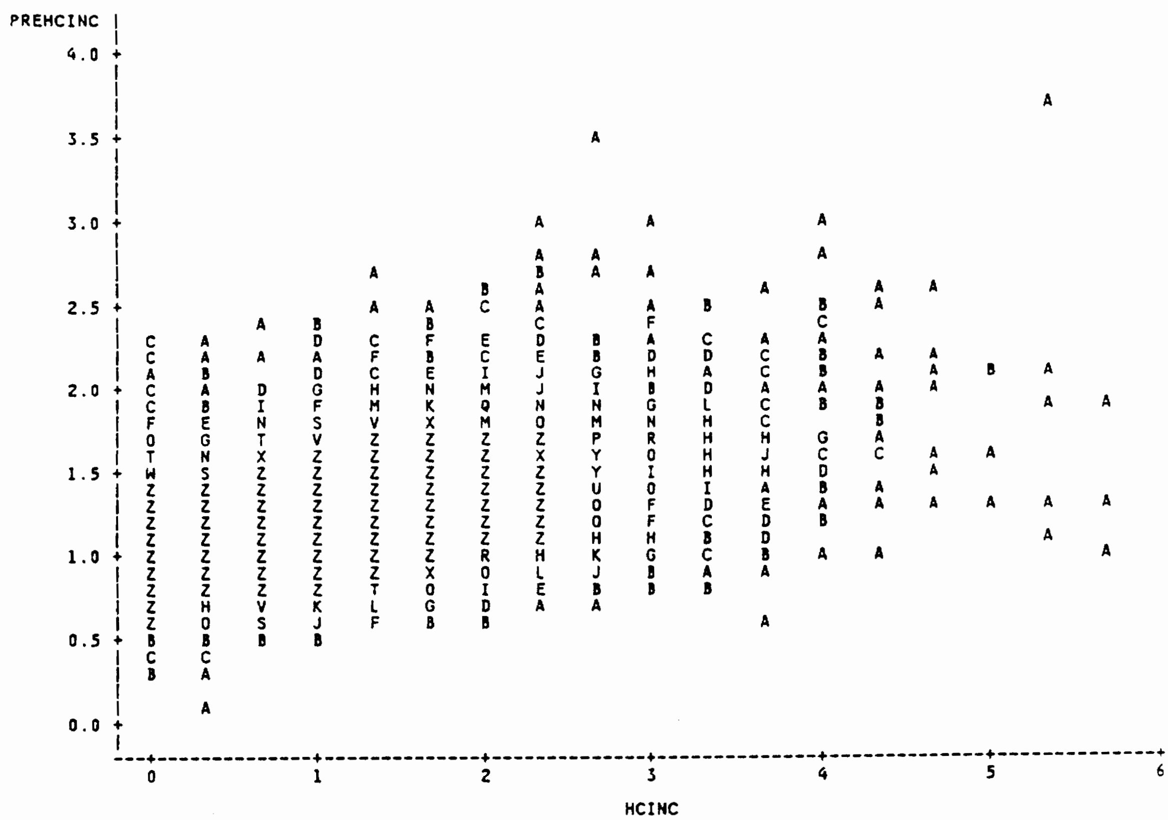

Figure 4. Predicted versus observed loblolly pine crown height increment for the distance-dependent tree model for the heavily thinned plots.

models was due to the subjectivity in the judgement of exactly where the live crown begins. Some define the live crown base as the first green limb, others define it as the first contiguous green whorl. These data showed that for this study, some companies were using the first definition while others the latter. A standard delineation is necessary if crown increment equations are to work well.

Additionally the data for the model development was taken every three years, and the resulting problems are obvious. The assumption that the increment over the three year period was linear is shaky at best, especially in the thinned stands. It is suspected, particularly in the thinned stands, that where some crown height recession was observed over the three-year increment, there may have been no increment in the initial year followed by a substantial increment over the last one or two years when crown closure reoccurred. However, this was impossible to determine with these data. Data gathered on an annual basis are needed in order to develop a more refined crown increment model. Improvements are needed in the ability to predict very small increments, especially in the thinned stands.

Adding a Thinning Variable to the Crown Height Model

The two thinning variables THIN1 and THIN2, equations (23) and (24) respectively, were tested with Dyer and Burkhart's (1987) crown height model using the evaluation data. Unlike the increment models, THIN2 performed better with the crown height model. This is likely due to the fact that over the range of these data, THIN2, by using the thinning interval in the numerator of the exponent, produces a number closer to one for the majority of the plots than does THIN1 which uses the thinning age. This larger number, therefore closer to one, produces a smaller thinning effect on crown height. It is logical that thinning will produce a smaller percentage effect on a tree's overall crown height than its crown's annual recession. Including THIN2 in the model (model 1) produced an R^2 of .9078 and a mean square error of 11.05 opposed to an R^2 of .9047 and a mean square error of 11.42 without THIN2 (model 2). Using THIN2 in the equation also improved the model's mean residuals (Table 24); Appendix C contains the further classification of the residuals by thinning intensity and the interval since thinning. With this prediction equation, it is possible to predict the crown height for each measurement, thus there are residuals for plot establishment (time 0) and the three remeasurements.

Without the thinning variable, the model significantly overpredicts crown height compared to including THIN2; also, the original model has an overprediction bias for the longer thinning intervals so the THIN2 model was tested against the validation data.

Table 24. Mean residuals by thinning intensity or elapsed time since thinning for the crown height models with and without the THIN2 variable.

Thinning intensity or the elapsed time since thinning	Mean Residuals	
	With THIN2 *	Without THIN2 *
Unthinned plots, all measurements	0.009	0.521
Lightly thinned plots, all measurements	0.118	0.066
Heavily thinned plots, all measurements	-0.081	-0.498
Time = 0, all thinning intensities	0.383	0.699
Time = 3, all thinning intensities	-0.166	-0.053
Time = 6, all thinning intensities	-0.267	-0.381
Time = 9, all thinning intensities	0.148	-0.211

* where the model with THIN2 is:

$$CH = HT \cdot THIN2^{\,b_4} \exp[-(b_2 + b_3 A^{-1})\, DBH/HT]$$

and the model without THIN2 is:

$$CH = HT \exp[-(b_2 + b_3 A^{-1})\, DBH/HT]$$

Crown Height Model Validation

Models 1 and 2 were tested against the validation data. Both models underpredict crown height for the unthinned stands and overpredict for the thinned stands; however, the model without the thinning variable performs worse for the unthinned and heavily thinned stands (Table 25). This is further demonstrated in Appendix C, where the residuals are grouped by thinning intensity and thinning interval. Thus, the crown height model with THIN2 would be the final model if it performed equally as well when fit to all the data.

Table 25. Mean validation data residuals by thinning intensity or elapsed time since thinning for the two versions of the crown height model.

Thinning intensity or the elapsed time since thinning	Mean residuals	
	with THIN2 *	without THIN2 *
Unthinned plots, all three measurements	0.157	0.553
Lightly thinned plots, all three measurements	-0.119	-0.062
Heavily thinned plots, all three measurements	-0.078	-0.239
Time = 0, all thinning intensities	0.278	0.545
Time = 3, all thinning intensities	0.061	0.153
Time = 6, all thinning intensities	-0.454	-0.565

* where the model with THIN2 is:

$$CH = HT \cdot THIN2^{b_4} \exp[-(b_2 + b_3 A^{-1}) DBH/HT]$$

and the model without THIN2 is:

$$CH = HT \exp[-(b_2 + b_3 A^{-1}) DBH/HT]$$

Final Crown Height Model

The crown height model was fit to all the data both with and without the THIN2 variable and the mean residuals were calculated. Including THIN2 produced an R^2 of .9051 and a mean square error of 10.782, while not including it resulted in an R^2 of .9015 and a mean square error of 11.187. The thinning variable produced an increase in accuracy of the model's predictions. Without THIN2 the model overpredicts crown height in thinned stands, especially heavily thinned plantations, and underpredicts for the unthinned stands (Table 26). This increase in accuracy by including THIN2 is also apparent in Appendix C which contains a table which categorizes the mean residuals by thinning level and the interval since thinning. Therefore, the final fitted stand level model was:

$$CH = HT \cdot THIN2^{\,b_4} \exp[-(b_2 + b_3 A^{-1}) DBH/HT] \qquad (35)$$

where THIN2 is:

$$THIN2 = \left(\frac{BAA}{BAB}\right)^{\left(\frac{THININT}{AGE}\right)} \qquad (24)$$

where THININT is the number of years since thinning. The final parameters for the crown height model were:

b_2 = 1.161411
b_3 = 41.328126
b_4 = 0.362285

Table 26. Mean residuals by thinning intensity or elapsed time since thinning for the crown height models with and without the THIN2 variable.

Thinning intensity or the elapsed time since thinning	Mean Residuals	
	with THIN2 *	without THIN2 *
Unthinned plots, all measurements	0.032	0.534
Lightly thinned plots, all measurements	0.020	-0.027
Heavily thinned plots, all measurements	0.058	-0.335
Time = 0, all thinning intensities	0.309	0.647
Time = 3, all thinning intensities	0.032	0.095
Time = 6, all thinning intensities	-0.324	-0.572
Time = 9, all thinning intensities	0.269	-0.345
Coeff. of determination, all data	0.9515	0.9496

* where the model with THIN2 is:

$$CH = HT \cdot THIN2^{b_4} \exp[-(b_2 + b_3 A^{-1}) DBH/HT]$$

and the model without THIN2 is:

$$CH = HT \exp[-(b_2 + b_3 A^{-1}) DBH/HT]$$

Stand Level Crown Height Increment Model

The individual tree increment model was collapsed to a model containing only stand level statistics; it was hoped that this specification would be sufficient for the stand level model since this would bring a consistency to the increment models. The resultant model was:

$$\Delta HLC = b_0 HD^{b_1} THIN1^{b_3} \exp(b_2 R + b_4 A) \quad \textbf{(36)}$$

where HD is the dominant and codominant height, R is the average crown ratio for the stand and the other variables were defined earlier.

The model was fit to the evaluation data using the plots' mean crown ratio as R. For situations in which an initial stand level crown ratio is not available, then the model developed by Liu *et al.* (1989) based on stand basal area (B) and dominant stand height (H) can be used:

$$R = \frac{d_1}{1 + d_2 B} + d_3 (1 - e^{d_4 (H - 4.5)}) \quad \textbf{(19)}$$

and then the beginning crown ratio can be updated annually using the increment model.

Model (32) produced a similar fit as the individual tree model with a R^2 of .2127 and a mean square error of 0.344 feet, this was opposed to an R^2 of .1459 and a mean square error of 0.373 feet without the THIN1 variable. Consequently, the thinning variable was included in the model. Furthermore, unlike the individual tree models, raising crown ratio to the one half power did not improve the model's fit, so the power was set at one. Also, no stand competition term such as basal area, relative spacing, or trees per acre significantly improved the fit of the model.

Before developing the model it was thought that a separate increment model may be necessary for the suppressed and intermediate portion of the stand as was concluded by Cole and Jensen (1982) for lodgepole pine. Accordingly, the model with the THIN1 variable was fit to all the data and separately to the dominant and codominant classes and the intermediate and suppressed classes. The hypothesis tests firmly concluded that separate models were unnecessary. This was advantageous because crown class determination is subjective and some stand growth and yield models do not maintain this measure. Thus, model (36) fitted to all the data combined, was tested against the validation data.

Stand Level Crown Height Increment Model Validation

This version of the stand level model (model A) consistently overpredicted the average plot increment for the validation data for all thinning intensities and the intervals since thinning (Table 27). Appendix D shows the mean residuals for each combination of thinning intensity and the interval since thinning. Since this overprediction bias was present across all thinning levels, it was questioned if fitting the model to the third remeasurement was causing the model's poor performance with the two remeasurement data; the stand model was refit to only the first two remeasurements of the evaluation data. Predictions from the model fitted to the first two time periods, called model B, for the validation data were much improved (Table 27). Evidently some characteristic of the third remeasurement when included in the stand model greatly altered its ability to predict for data with only two remeasurements. These residuals are characteristic of the individual tree models' residuals whereby there is an underprediction bias for the first remeasurement period followed by a slight overprediction in the second period. This

Table 27. Mean validation data residuals by thinning intensity or elapsed time since thinning for the two versions of the stand level increment model.

Thinning intensity or the elapsed time since thinning	Sample size and mean residuals			
	Model A *		Model B *	
	N	Residual	N	Residual
Unthinned plots, both measurements	498	-0.964	498	0.047
Lightly thinned plots, both measurements	491	-0.831	491	0.077
Heavily thinned plots, both measurements	445	-0.764	445	0.075
Time = 1, all thinning intensities	725	-0.726	725	0.163
Time = 4, all thinning intensities	709	-0.990	725	-0.033

* Model A is equation (32) fitted to all four measurements of the evaluation data.
Model B is equation (32) fitted to only the first three measurements of the evaluation

improved performance combined with its consistency with the individual tree models, was sufficient to choose it as the stand-level model.

Final Stand Level Crown Height Increment Model

The stand level model was fit to all the data both with and without the THIN1 variable and the mean residuals were calculated. Including THIN1 produced an R^2 of .2858 and a mean square error of 0.337, while not including it resulted in an R^2 of .2101 and a mean square error of 0.372. Again, including the thinning variable produced a substantial increase in accuracy for the model's predictions. Without the THIN1 variable the model overpredicts increment for the thinned stands, particularly the heavily thinned data, and underpredicts for the unthinned plots (Table 28). Appendix D contains a table which categorizes the mean residuals by thinning level and the interval since thinning as well. Therefore, the final fitted stand level model was:

$$\Delta HLC = b_0 HD^{b_1} THIN1^{b_5} \exp(b_2 R + b_4 A) \qquad (37)$$

where:

b_0 = 0.009681
b_1 = 1.230687
b_2 = 2.586149
b_4 = -0.037665
b_5 = 0.622877

Table 28. Mean residuals by thinning intensity or elapsed time since thinning for the stand level increment models with and without the THIN1 variable.

Thinning intensity or the elapsed time since thinning	Mean Residuals	
	With THIN1 *	Without THIN1 *
Unthinned plots, all measurements	-0.029	0.178
Lightly thinned plots, all measurements	0.016	-0.021
Heavily thinned plots, all measurements	0.009	-0.176
Time = 1, all thinning intensities	0.029	0.002
Time = 4, all thinning intensities	-0.058	-0.042
Time = 7, all thinning intensities	0.086	0.130
Coeff. of determination, all data	0.5347	0.4584

* where the model with THIN1 is:

$$\Delta HLC = b_0 HD^{b_1} THIN1^{b_3} \exp(b_2 R + b_4 A)$$

and the model without THIN1 is:

$$\Delta HLC = b_0 HD^{b_1} \exp(b_2 R + b_4 A)$$

Chapter VI

Summary and Conclusions

Several individual tree, candidate crown height increment models were tested to obtain a model that predicts the annual recession of crown height. The best model based on its performance and its biological soundness contained the tree's total height, the square root of crown ratio, age, and a competition measure. Competition index (CI) was used in the distance-dependent form while the distance-indepedent equation was identical except DR (quadratic mean diameter/dbh) replaced CI.

Hypothesis tests indicated that thinning intensity significantly affected crown height and its recession, thus a thinning variable (THIN1) was developed and added to the models. THIN1 is the ratio of basal area immediately after thinning to the basal area before thinning, with this quantity raised to the ratio of the stand age at time of thinning to current age. The variable equals one in unthinned stands and becomes increasingly less than one as thinning intensity increases, yet as the interval since thinning lengthens it reapproaches one. The fit of increment models improved with the addition of the THIN1 variable, especially the distance-independent model. The two tree models, with THIN1 included, were tested with validation data, and the results indicated only a slight underprediction bias. The two increment equations were refit to all the data to obtain their final parameter estimates .

The selected crown height prediction model was tested to determine if it required a variable expressing thinning intensity and timing, and its performance was improved with the variable, THIN2. THIN2 is identical to THIN1 except it uses the interval since thinning instead of the stand age at time of thinning. This expression produces a number closer to one over the range of this data than THIN1, and thus performs better for crown height prediction since thinning has less of a percentage effect on overall crown height than on annual crown increment. The model with the thinning variable was chosen as the crown height model after validation tests also indicated that this version improved crown height prediction over that of the comparable model without a thinning variable included.

A stand level crown height increment model was developed by collapsing the individual tree model to only stand level statistics. Tree height was replaced by dominant and codominant height, tree crown ratio was replaced with the stand's average crown ratio, THIN1 and age remained, and the competition term was eliminated since neither basal area, trees per acre or relative spacing improved predictions for the model. Raising the stand level crown ratio to the one-half power was eliminated since it did not improve the fit of the model. Hypothesis tests indicated that the intermediate and suppressed crown classes were not significantly different from the combined dominant and codominant crown classes. Validation tests indicated that fitting the model to all three plot remeasurements produced a significant overprediction bias when it was tested against data with only two remeasurements. The model was refit to only the first two remeasurements of the evaluation data, which significantly improved the model's performance with the validation data. Consequently this reduced version of the individual tree models, with the thinning allowance, was selected as the stand level increment model.

This study has shown much promise, and it is believed that this method can be applied to most other conifers as well; however, more work is needed to produce a fully acceptable crown height increment model for loblolly pine. Equations developed here do not predict well for the extreme observed ranges of crown height increment. Furthermore, annual increment data is needed, not just three year data which must be linearily interpolated to derive an observed annual increment. Also, a standard definition of the base of the live crown will be necessary to eliminate any biases due to data measurement error. Lastly, other tree measures, such as crown diameter, may be necessary to produce reliable crown height increment equations. Knowing the rate of lateral expansion of crowns in thinned stands would enable one to estimate points of achieving full crown closure and times when the base of the live crown would be expected to again recede at a more rapid rate.

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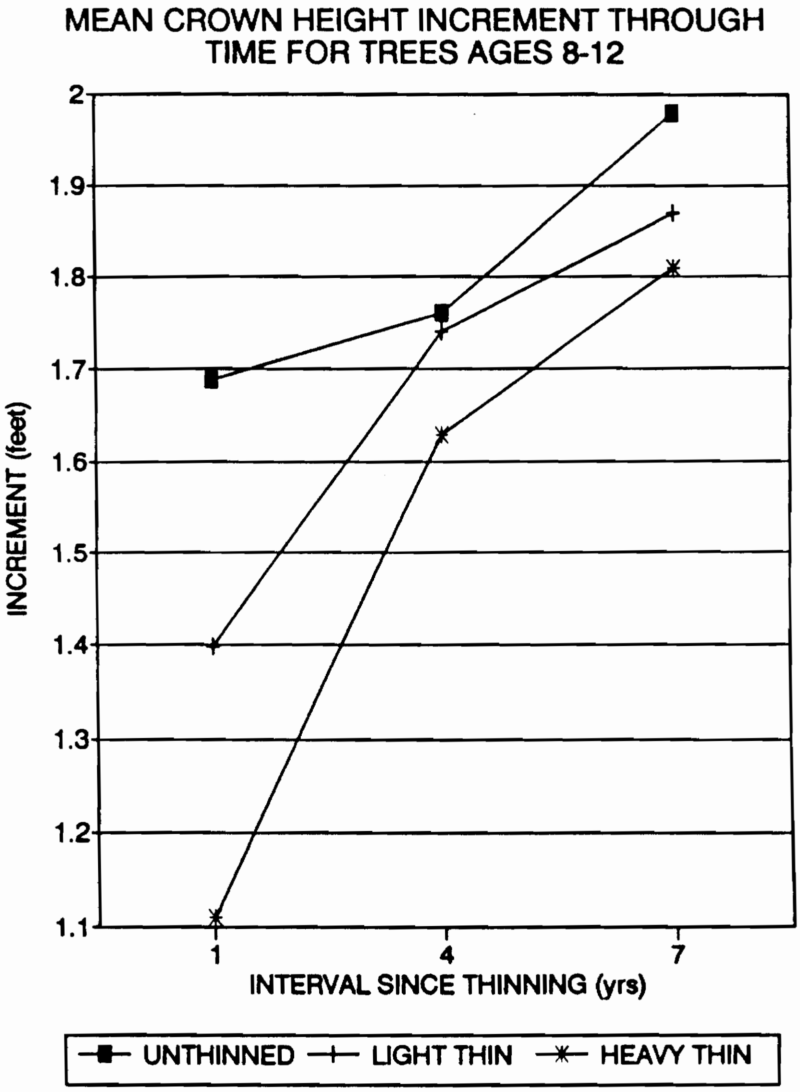
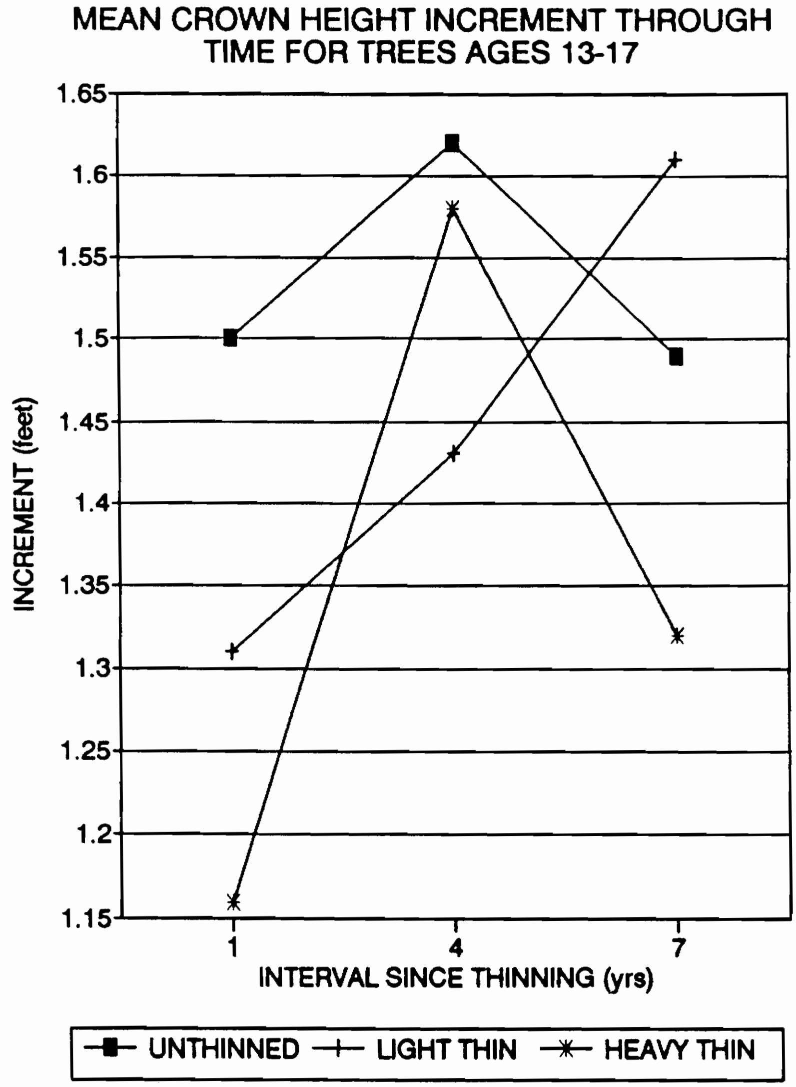
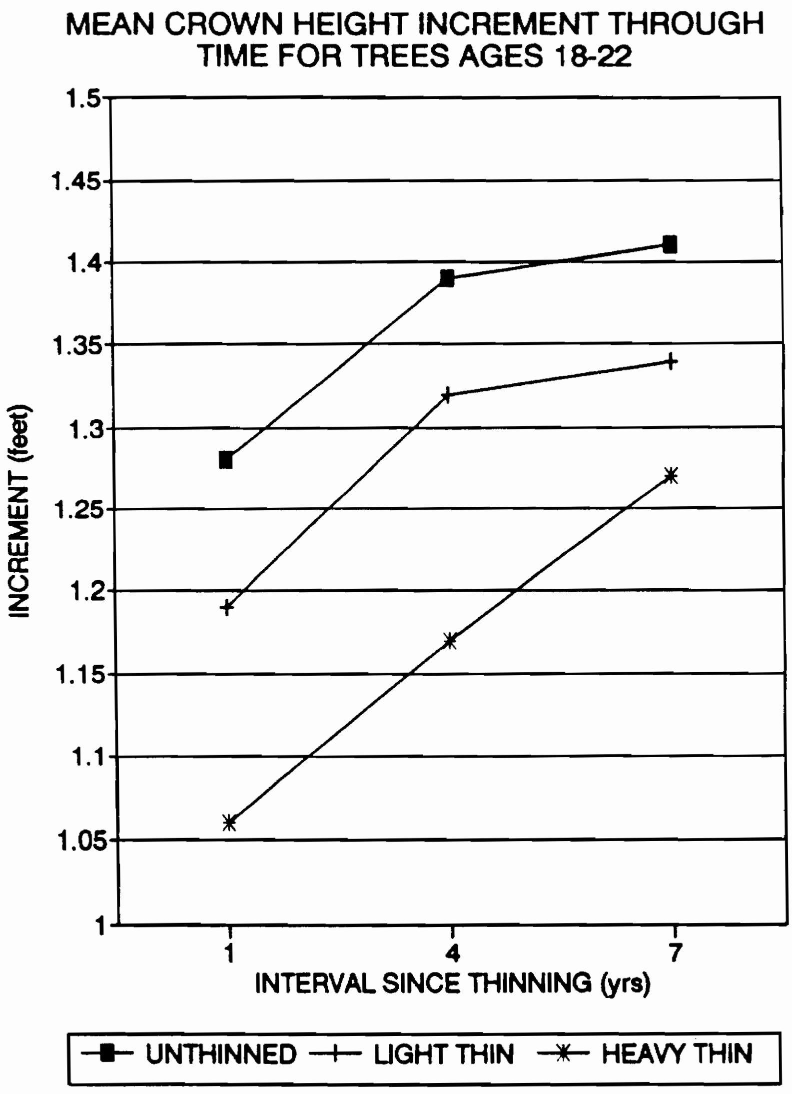
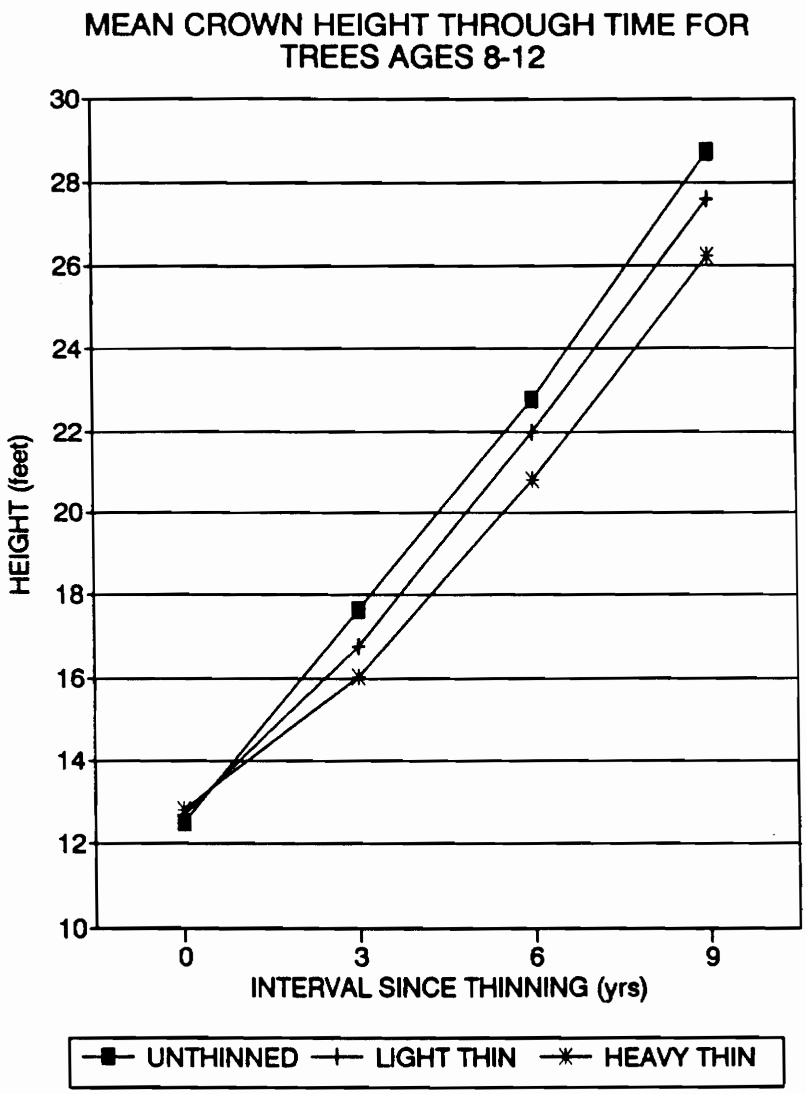
Appendix A

Mean Crown Height and Crown Height Increment by Age Classes for the Evaluation Data

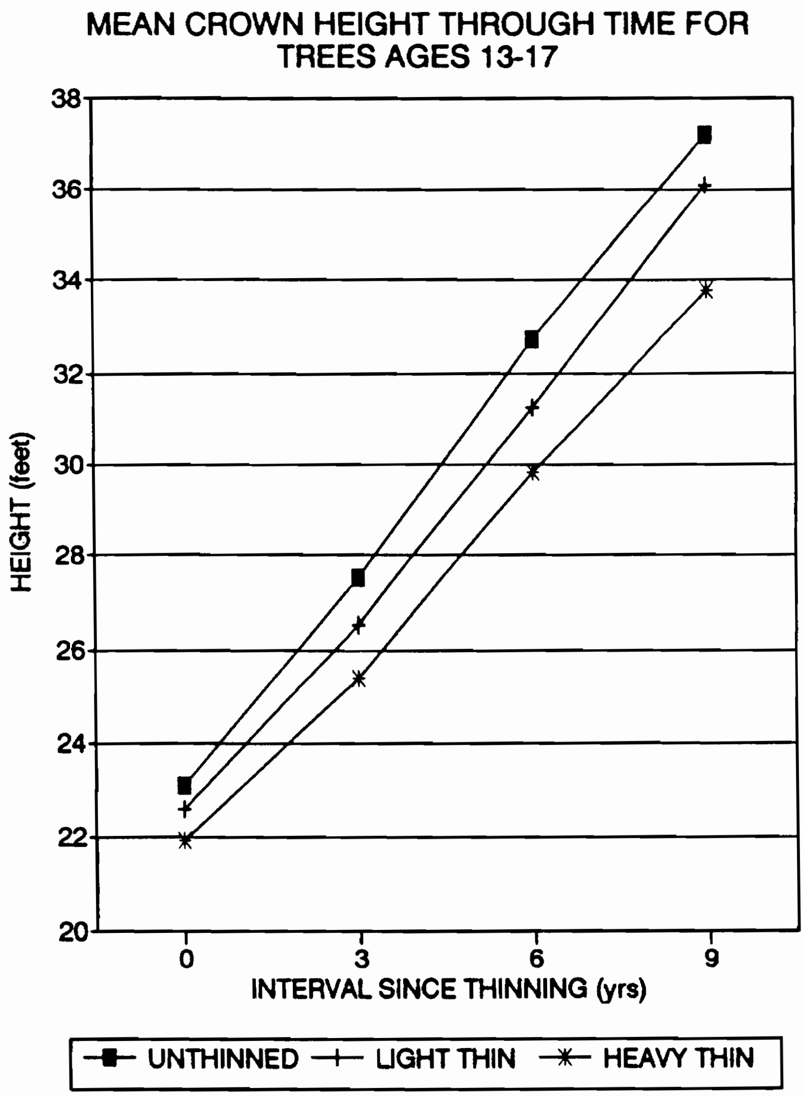

MEAN CROWN HEIGHT INCREMENT THROUGH
TIME FOR TREES AGES 8-12
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
INCREMENT (feet)
1
4
7
INTERVAL SINCE THINNING (yrs)
UNTHINNED
LIGHT THIN
HEAVY THIN

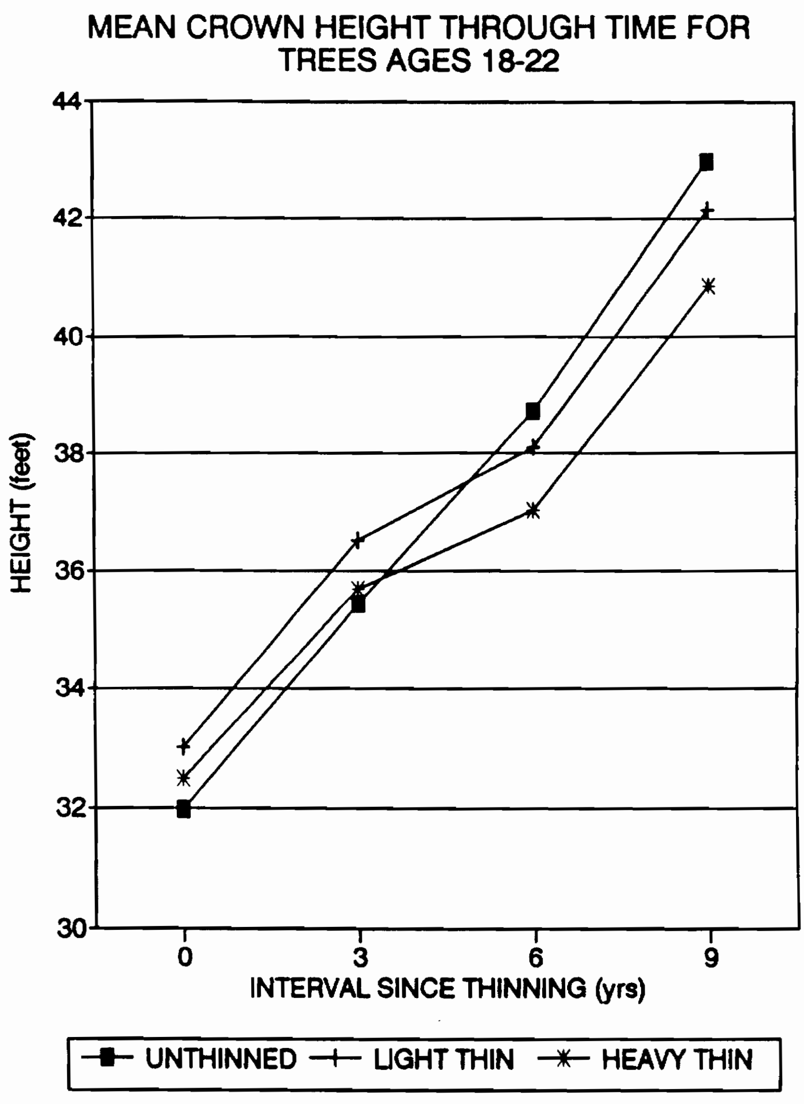

MEAN CROWN HEIGHT INCREMENT THROUGH TIME FOR TREES AGES 13-17
1.65
1.6
1.55
1.5
1.45
1.4
1.35
1.3
1.25
1.2
1.15
INCREMENT (feet)
1
4
7
INTERVAL SINCE THINNING (yrs)
UNTHINNED
LIGHT THIN
HEAVY THIN


MEAN CROWN HEIGHT INCREMENT THROUGH TIME FOR TREES AGES 18-22
INCREMENT (feet)
1.5
1.45
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
1
4
7
INTERVAL SINCE THINNING (yrs)
UNTHINNED
LIGHT THIN
HEAVY THIN



Mean Crown Height and Crown Height Increment by Age Classes for the Evaluation Data


MEAN CROWN HEIGHT THROUGH TIME FOR
TREES AGES 13-17
HEIGHT (feet)
38
36
34
32
30
28
26
24
22
20
0
3
6
9
INTERVAL SINCE THINNING (yrs)
UNTHINNED
LIGHT THIN
HEAVY THIN


MEAN CROWN HEIGHT THROUGH TIME FOR
TREES AGES 18-22
HEIGHT (feet)
44
42
40
38
36
34
32
30
0
3
6
9
INTERVAL SINCE THINNING (yrs)
UNTHINNED
LIGHT THIN
HEAVY THIN

Appendix B

Individual Tree Crown Height Increment Model Residuals

Mean residuals for the evaluation data by thinning intensity and elapsed time since thinning for the three candidate thinning models.

Thinning intensity and the elapsed time since thinning	Mean Residuals		
	Model 1 *	Model 2 *	Model 3 *
Unthinned plots, time = 1	-0.060	-0.168	0.160
Unthinned plots, time = 4	0.012	-0.103	0.243
Unthinned plots, time = 7	0.043	-0.073	0.273
Lightly thinned plots, time = 1	-0.011	0.054	-0.081
Lightly thinned plots, time = 4	-0.005	0.035	-0.023
Lightly thinned plots, time = 7	0.064	0.079	0.092
Heavily thinned plots, time = 1	-0.054	0.086	-0.294
Heavily thinned plots, time = 4	0.017	0.142	-0.174
Heavily thinned plots, time = 7	-0.042	0.051	-0.165

* where model 1 is:

$$\Delta HLC = b_0 \, THIN1^{b_5} HT^{b_1} \exp(b_2 CR^{1/2} + b_3 DR + b_4 A)$$

model 2 is:

$$\Delta HLC = b_0 \, THIN1 \cdot HT^{b_1} \exp(b_2 CR^{1/2} + b_3 DR + b_4 A)$$

and model 3 is:

$$\Delta HLC = b_0 HT^{b_1} \exp(b_2 CR^{1/2} + b_3 DR + b_4 A)$$

Mean validation data residuals by thinning intensity and elapsed time since thinning for the selected individual tree increment models.

Thinning intensity and the elapsed time since thinning	Sample size and mean residuals			
	Model DR *		Model CI *	
	N	Residual	N	Residual
Unthinned plots, time = 1	7219	0.212	568	0.176
Unthinned plots, time = 4	6445	-0.039	279	-0.206
Lightly thinned plots, time = 1	9740	0.202	2332	0.188
Lightly thinned plots, time = 4	9253	-0.031	1697	-0.015
Heavily thinned plots, time = 1	7534	0.200	1731	0.207
Heavily thinned plots, time = 4	7173	-0.021	1326	0.033

* Model DR is defined as:

$$\Delta HLC = b_0\, THIN^{b_5}\, HT^{b_1} \exp(b_2 CR^{1/2} + b_3 DR + b_4 A)$$

and model CI is defined as:

$$\Delta HLC = b_0\, THIN^{b_5}\, HT^{b_1} \exp(b_2 CR^{1/2} + b_3 CI + b_4 A)$$

Mean residuals for all the data by thinning intensity and elapsed time since thinning for the individual tree increment models with and without the THIN1 variable.

Thinning intensity and the elapsed time since thinning	Mean Residuals			
	Model 1 *	Model 2 *	Model 3 *	Model 4 *
Unthinned plots, time = 1	0.030	0.284	-0.045	0.169
Unthinned plots, time = 4	-0.096	0.161	-0.285	-0.070
Unthinned plots, time = 7	0.005	0.265	-0.166	0.012
Lightly thinned plots, time = 1	0.058	-0.006	0.039	0.046
Lightly thinned plots, time = 4	-0.066	-0.068	-0.092	-0.050
Lightly thinned plots, time = 7	0.056	0.090	0.082	0.152
Heavily thinned plots, time = 1	0.063	-0.188	0.070	-0.044
Heavily thinned plots, time = 4	-0.036	-0.212	-0.018	-0.080
Heavily thinned plots, time = 7	-0.038	-0.167	0.008	-0.022

* where models 1, 2, 3, and 4 are defined respectively as:

$$\Delta HLC = b_0\, THIN1^{\,b_5}\, HT^{\,b_1} \exp\left(b_2\, CR^{1/2} + b_3\, DR + b_4\, A\right)$$

$$\Delta HLC = b_0 \cdot HT^{\,b_1} \exp\left(b_2\, CR^{1/2} + b_3\, DR + b_4\, A\right)$$

$$\Delta HLC = b_0\, THIN1^{\,b_5}\, HT^{\,b_1} \exp\left(b_2\, CR^{1/2} + b_3\, CI + b_4\, A\right)$$

$$\Delta HLC = b_0 \cdot HT^{\,b_1} \exp\left(b_2\, CR^{1/2} + b_3\, CI + b_4\, A\right)$$

Appendix C

Crown Height Model Residuals

Mean residuals for the evaluation data by thinning intensity and elapsed time since thinning for the crown height models with and without the THIN2 variable.

Thinning intensity or the elapsed time since thinning	Mean Residuals	
	With THIN2 *	Without THIN2 *
Unthinned plots, time = 0	0.430	0.736
Unthinned plots, time = 3	-0.055	0.400
Unthinned plots, time = 6	-0.361	0.247
Unthinned plots, time = 9	-0.050	0.696
Lightly thinned plots, time = 0	0.443	0.762
Lightly thinned plots, time = 3	-0.098	-0.030
Lightly thinned plots, time = 6	-0.238	-0.437
Lightly thinned plots, time = 9	0.386	-0.082
Heavily thinned plots, time = 0	0.250	0.573
Heavily thinned plots, time = 3	-0.373	-0.553
Heavily thinned plots, time = 6	-0.214	-0.915
Heavily thinned plots, time = 9	0.012	-1.210

* where the model with THIN2 is:

$$CH = HT \cdot THIN2^{b_4} \exp[-(b_2 + b_3 A^{-1}) DBH/HT]$$

and the model without THIN2 is:

$$CH = HT \exp[-(b_2 + b_3 A^{-1}) DBH/HT]$$

Mean validation data residuals by thinning intensity and elapsed time since thinning for the the crown height model with and without the THIN2 variable.

Thinning intensity or the elapsed time since thinning	Mean residuals	
	With THIN2 *	Without THIN2 *
Unthinned plots, time = 0	0.580	0.834
Unthinned plots, time = 3	0.254	0.661
Unthinned plots, time = 6	-0.450	0.100
Lightly thinned plots, time = 0	0.162	0.432
Lightly thinned plots, time = 3	-0.021	0.037
Lightly thinned plots, time = 6	-0.526	-0.699
Heavily thinned plots, time = 0	0.131	0.404
Heavily thinned plots, time = 3	-0.017	-0.181
Heavily thinned plots, time = 6	-0.367	-0.990

* where the model with THIN2 is:

$$CH = HT \cdot THIN2^{b_4} \exp[-(b_2 + b_3A^{-1})\,DBH/HT]$$

and the model without THIN2 is:

$$CH = HT\exp[-(b_2 + b_3A^{-1})\,DBH/HT]$$

Mean residuals for all data by thinning intensity and elapsed time since thinning for the crown height models with and without the THIN2 variable.

Thinning intensity or the elapsed time since thinning	Mean Residuals	
	With THIN2 *	Without THIN2 *
Unthinned plots, time = 0	0.534	0.859
Unthinned plots, time = 3	0.098	0.584
Unthinned plots, time = 6	-0.555	0.088
Unthinned plots, time = 9	-0.263	0.571
Lightly thinned plots, time = 0	0.245	0.587
Lightly thinned plots, time = 3	0.005	0.018
Lightly thinned plots, time = 6	-0.343	-0.683
Lightly thinned plots, time = 9	0.543	-0.218
Heavily thinned plots, time = 0	0.167	0.513
Heavily thinned plots, time = 3	0.003	-0.283
Heavily thinned plots, time = 6	-0.087	-1.031
Heavily thinned plots, time = 9	0.390	-1.350

* where the model with THIN2 is:

$$CH = HT \cdot THIN2^{b_4} \exp[-(b_2 + b_3 A^{-1}) DBH/HT]$$

and the model without THIN2 is:

$$CH = HT \exp[-(b_2 + b_3 A^{-1}) DBH/HT]$$

Appendix D

Stand Level Crown Height Increment Model Residuals

Mean validation data residuals by thinning intensity and elapsed time since thinning for the two versions of the stand level increment model.

Thinning intensity or the elapsed time since thinning	Sample size and mean residuals			
	Model A *		Model B *	
	N	Residual	N	Residual
Unthinned plots, time = 1	255	-0.828	255	0.163
Unthinned plots, time = 4	243	-1.108	243	-0.075
Lightly thinned plots, time = 1	247	-0.695	247	0.173
Lightly thinned plots, time = 4	244	-0.969	244	-0.021
Heavily thinned plots, time = 1	223	-0.643	223	0.151
Heavily thinned plots, time = 4	222	-0.885	222	-0.002

* Model A is equation (32) fitted to all four measurements of the validation data. Model B is equation (32) fitted to only the first three measurements of the validation data.

Mean residuals for all data by thinning intensity and elapsed time since thinning for the stand level increment models with and without the THIN1 variable.

Thinning intensity and the elapsed time since thinning	Mean Residuals	
	With THIN1 *	Without THIN1 *
Unthinned plots, time = 1	0.015	0.221
Unthinned plots, time = 4	-0.096	0.110
Unthinned plots, time = 7	0.046	0.256
Lightly thinned plots, time = 1	0.048	-0.027
Lightly thinned plots, time = 4	-0.053	-0.068
Lightly thinned plots, time = 7	0.142	0.164
Heavily thinned plots, time = 1	0.023	-0.217
Heavily thinned plots, time = 4	-0.023	-0.179
Heavily thinned plots, time = 7	0.069	-0.034

* where the model with THIN1 is:

$$\Delta HLC = b_0 HD^{b_1} THIN1^{b_3} \exp(b_2 R + b_4 A)$$

and the model without THIN1 is:

$$\Delta HLC = b_0 HD^{b_1} \exp(b_2 R + b_4 A)$$

Vita

Eli Austin Short, III was born on October 3, 1967 in Milford, Delaware. He attended Sussex Central High School in Georgetown, Delaware where he graduated in June, 1985. He enrolled at Virginia Polytechnic Institute and State University in September, 1985, and completed his Bachelor of Science degree in Forestry in December 1989. Upon graduation, he entered the graduate school at Virginia Polytechinc Institute and State University where he earned his Master of Science degree in Forest Biometrics in May 1991.