THE ELASTIC EFFECT OF COLUMNS ON THE MOMENTS IN SLABS DUE TO VERTICAL LOADS

by

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I. INTRODUCTION

The ever-increasing scarcity of many structural materials accentuates the need of continuing investigations into the true interaction of stresses in all the components of a structural frame. These investigations can save materials, both by recognizing the part played by structural elements hitherto disregarded as being too complex to consider, and by the resultant reduction of safety factors employed by the building trades. At the same time it must be realized that, for reasons of economy, designers and engineers are not likely to spend large sums of money for the complex analyses now necessary to determine these stress interactions, when the cost of such analyses cannot be offset by approximately equal savings in the cost of structural materials. For these reasons it is desirable that methods of analysis, which consider the importance of all the structural components of a frame, can be presented to the architect and the engineer in such forms that the analyses can be performed with reasonable rapidity by the regular members of the design staff.

One of the important structural elements which is largely disregarded in present design procedure is the floor slab, both continuous and discontinuous. The floor slab at present is generally designed by considering a typical strip in floors which span in one direction. Slabs which span in both directions have their design rigidly controlled by code specifications, which are based on empirical formulas. Little or no attention is directed to the interaction of stresses between the slabs, the beams, and the columns. It is the purpose of this thesis to continue the investigation, begun by E. L. Miller[1], of a square slab welded to four columns; comparing Miller's experimental results with the results of analytical solutions by N. J. Neilson and H. M. Westergaard. Later formulas for analytical solutions by S. Timoshenko will also be presented. It is hoped that the work done in this thesis will provide part of a usable basis for further experimental work by the Virginia Polytechnic Institute's Department of Architecture.

II. PRECEEDING THESES

Miller's thesis dealt with the loading of a square steel plate supported on round columns, and with the measurement of strains produced by the loading. The columns were welded to the plate and were simply supported in countersunk holes at their lower ends. Results were recorded from readings taken from SR-4 strain rosettes placed at the corners of squares, L/6 on a side, in one quadrant of the plate (Fig. 1). Four SR-4 strain gages, ninety degrees apart, were affixed to the top of the adjacent column. The plate was divided into

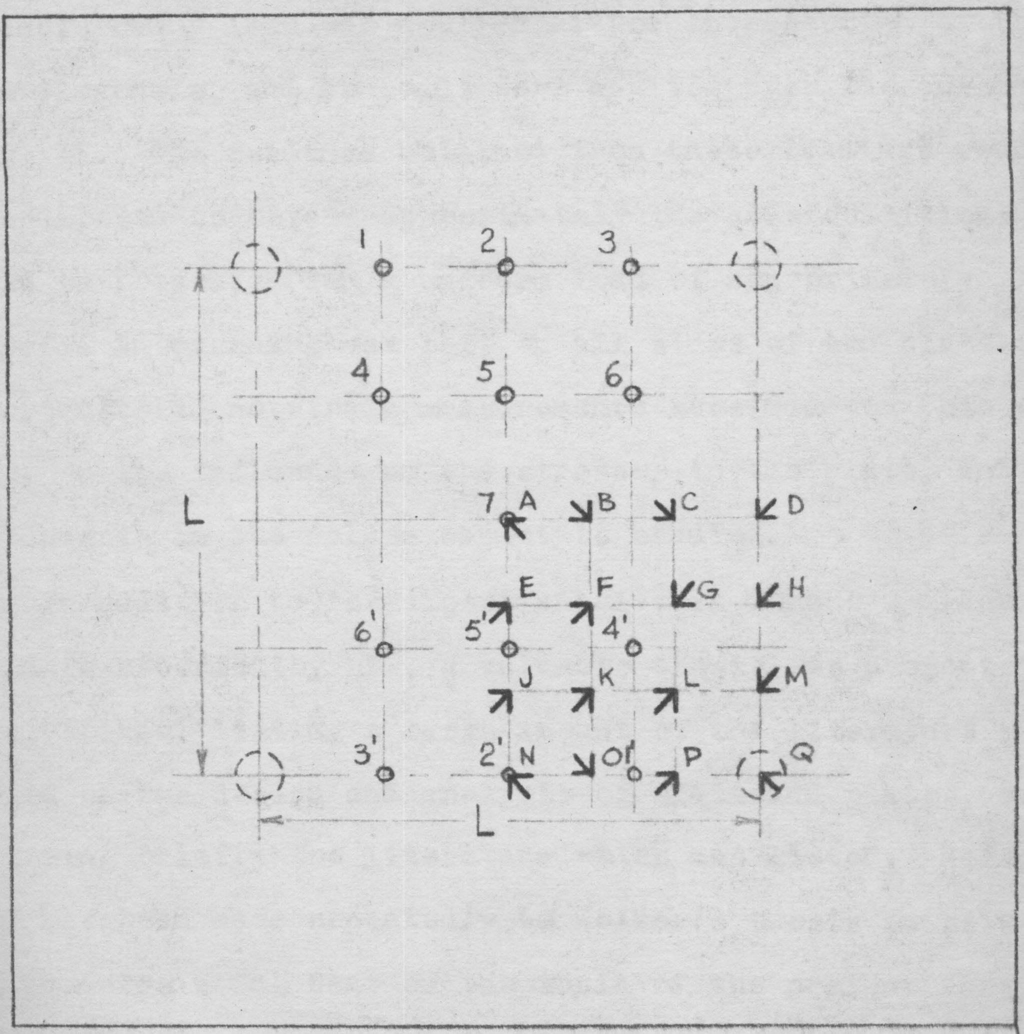


FIG. 1
PLAN OF SLAB, SHOWING LOAD POINTS AND LOCATION OF STRAIN ROSETTES

o = LOAD POINTS
↙ = STRAIN ROSETTES

squares, L/4 on a side, and loads of 5000# were applied in symmetrical pairs at the corners of the squares; except that a single 5000# load was applied at the intersection of the main diagonals, and no loads were applied over the columns (Fig. 1). The readings obtained from these loadings were superimposed to obtain approximately the same conditions as would be obtained from a uniform load of approximately 535psf. An overhang was left on all sides of the plate. Unfortunately, no strain measurements were made on this overhang, so its influence on the stresses in the plate, and on the moments in the column cannot be studied.

In addition to the experimental work done by Miller and the data recorded by him, a valuable thesis was presented by R. J. Kolker,[2] listing a large amount of the literature published on the design and analysis of slabs and plates, and reviewing briefly the literature which was listed. Reference has been made repeatedly to Kolker's thesis in selecting articles which bear on the topic of the present thesis.

III. DERIVATION OF PLATE FORMULAS

The derivations of the plate formulas vary somewhat among different authors, but the results are in general accordance with one another. The derivation presented here follows approximately that of H. M. Westergaard.[3]

Preliminary considerations used in the derivation are:

(1) the material is homogeneous and isotropic throughout,

(2) stress varies linearly with strain,

(3) the neutral axis is a plane (a warped plane during deflection) at all times half the thickness of the plate beneath the upper surface,

(4) the plate is medium thick, i.e. not so thick that an appreciable part of the energy of deformation is contributed by the vertical stresses, and not so thin that an appreciable part of the energy is contributed by the plate's stretching.

The following notations will be used:

x, y = horizontal rectangular coordinates.

w = vertical deflection, positive downward.

V_x = vertical shear per unit length in section perpendicular to x at point (x,y).

V_y = same in section perpendicular to y.

M_x = bending moment per unit length in section perpendicular to x at point (x,y); positive when causing compression at top and tension at bottom.

M_y = same in section perpendicular to y.

M_{xy} = torsional moment per unit length in sections perpendicular to xy plane at point (x,y); positive when it causes shortenings at top along the diagonal through

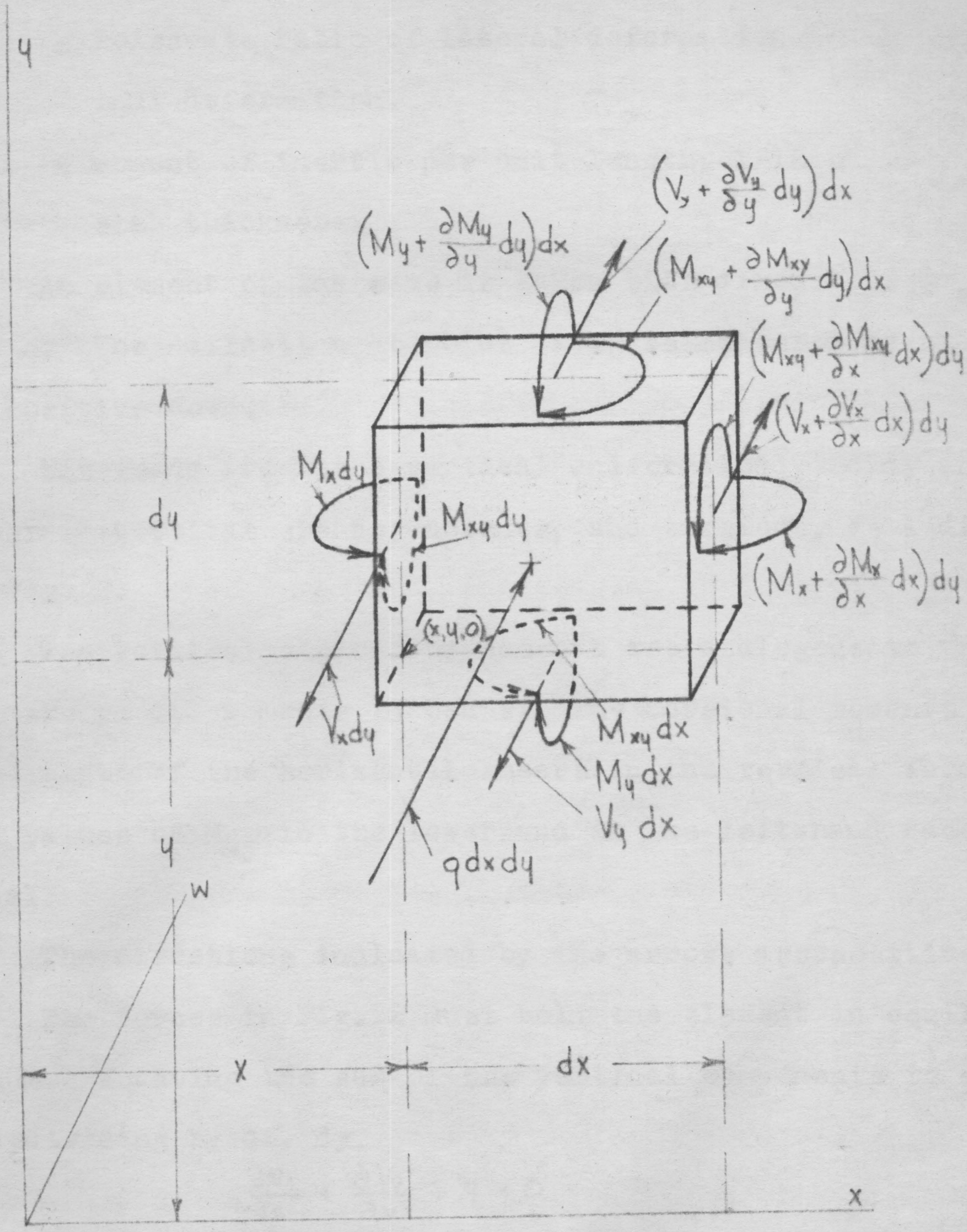


FIG. 2
RECTANGULAR ELEMENT OF SLAB
W-AXIS IS CONSIDERED VERTICAL

the corner (x,y) of the element.

E = modulus of elasticity.

ν = Poisson's ratio of lateral deformation to longitudinal deformation.

I = moment of inertia per unit length; $1/12\ h^3$ when h is slab thickness.

An element of the slab is taken with dimensions dx, dy, and h. The deflection at point (x,y) is measured by w, which is positive downward.

The loads are: the vertical uniform load, qdxdy and the internal vertical shears, moments, and torsions, as indicated by Fig. 2.

The vertical shears and moments are analogous to vertical shears and moments in beams. The torsional moments are resultants of the horizontal shears in the vertical faces. The values of M_{xy} in the lower and in the left-hand faces are equal.

The directions indicated by the arrows are positive.

The forces in Fig. 2 must hold the element in equilibrium. Equating the sum of the vertical components to zero and dividing by dx, dy

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + q = 0 \quad . \qquad (1)$$

By equating to zero the sum of the moments about a line parallel to y and through the center of the element, and dividing by dx, dy

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} = V_x \tag{2}$$

similarly,

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} = V_y \quad . \tag{3}$$

Differentiating (2) and (3) with respect to x and y, respectively, and substituting in (1)

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q \quad . \tag{4}$$

Since equations (1) to (4) are equations of equilibrium only, they do not consider the elastic properties of the plate. If the elastic deformations are considered it is seen that any type of deformation can be resolved into (a) bending in the x direction, (b) bending in the y direction, and/or (c) torsion in the xy-directions. The amounts of deformation in (a) and (b) are measured by the curvatures $-\frac{\partial^2 w}{\partial x^2}$ and $-\frac{\partial^2 w}{\partial y^2}$ respectively, just as in beams, and in (c) by the rate of change of the slope $-\frac{\partial\left(\frac{\partial w}{\partial x}\right)}{\partial y} = -\frac{\partial\left(\frac{\partial w}{\partial y}\right)}{\partial x} = \frac{\partial^2 w}{\partial x \partial y}$.

From the theory of beams, a bending moment M_x, acting alone, produces the curvatures $-\frac{\partial^2 w}{\partial x^2} = \frac{M_x}{EI}$ in the x-direction, $-\frac{\partial^2 w}{\partial y^2} = \nu\frac{M_y}{EI}$ in the y-direction, and no twist. The torsional couples produce a twist $-\frac{\partial^2 w}{\partial x \partial y}$ which may be determined by introducing temporarily another system of coordinates, x', y', making angles of 45° with the system of x, y. The couples, M_{xy}, are replaced by an equivalent combination of couples consisting of the bending moments $M_x' = M_{xy}$ in the x'-direction and $M_y' = -M_{xy}$ in the y'-direction. These bending moments produce the curvatures

$$-\frac{\partial^2 w}{\partial x'^2} = \frac{M_{x'} - \nu M_{y'}}{EI} = \frac{M_{xy}(1+\nu)}{EI} \quad , \quad -\frac{\partial^2 w}{\partial y'^2} = -\frac{M_{xy}(1+\nu)}{EI}$$

in terms of which the twist may be expressed by the transformation formula $-\frac{\partial^2 w}{\partial x \partial y} = \frac{1}{2}\left(-\frac{\partial^2 w}{\partial x'^2} + \frac{\partial^2 w}{\partial y'^2}\right) = \frac{M_{xy}(1+\nu)}{EI}$.

The derivation of the transformation formula follows from the relationships

$$x' = x\cos\alpha + y\sin\alpha$$

$$y' = -x\sin\alpha + y\cos\alpha$$

where α = the angle between the two sets of coordinates. In this case $\cos\alpha = \sin\alpha = 1/\sqrt{2}$. Then

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial w}{\partial y'}\frac{\partial y'}{\partial x} = \frac{\partial w}{\partial x'}\frac{1}{\sqrt{2}} - \frac{\partial w}{\partial y'}\frac{1}{\sqrt{2}}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}\right) = \frac{1}{\sqrt{2}}\left[\frac{\partial}{\partial x'}\left(\frac{\partial w}{\partial x'}\right)\frac{\partial x'}{\partial y} + \frac{\partial}{\partial y'}\left(\frac{\partial w}{\partial x'}\right)\frac{\partial y'}{\partial y} - \frac{\partial}{\partial x'}\left(\frac{\partial w}{\partial y'}\right)\frac{\partial x'}{\partial y} - \frac{\partial}{\partial y'}\left(\frac{\partial w}{\partial y'}\right)\frac{\partial y'}{\partial y}\right]$$

but $\frac{\partial x'}{\partial y} = \frac{1}{\sqrt{2}}$, $\frac{\partial y'}{\partial y} = \frac{1}{\sqrt{2}}$.

$$\therefore \frac{\partial^2 w}{\partial x \partial y} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left[\frac{\partial^2 w}{\partial x'^2} + \frac{\partial^2 w}{\partial x'\partial y'} - \frac{\partial^2 w}{\partial x'\partial y'} - \frac{\partial^2 w}{\partial y'^2}\right] = \frac{1}{2}\left[\frac{\partial^2 w}{\partial x'^2} - \frac{\partial^2 w}{\partial y'^2}\right]$$

which equals the transformation formula above when both sides of the equation are multiplied by minus one.

Taking the general case where M_x, M_y, and M_{xy} are all present, the resultant deformations are

$$-\frac{\partial^2 w}{\partial x^2} = \frac{M_x - \nu M_y}{EI} \tag{5}$$

$$-\frac{\partial^2 w}{\partial y^2} = \frac{M_y - \nu M_x}{EI} \tag{6}$$

$$-\frac{\partial^2 w}{\partial x \partial y} = \frac{M_{xy}(1+\nu)}{EI} \tag{7}$$

Solving (5), (6), and (7) for the moments give

$$M_x = \frac{EI}{(1-\nu^2)}\left(-\frac{\partial^2 w}{\partial x^2} - \nu\frac{\partial^2 w}{\partial y^2}\right) \tag{8}$$

$$M_y = \frac{EI}{(1-\nu^2)}\left(-\nu\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2}\right) \tag{9}$$

$$M_{xy} = \frac{EI}{(1+\nu)}\left(-\frac{\partial^2 w}{\partial x \partial y}\right) . \tag{10}$$

If the values of (8), (9) and (10) are substituted in equation (4) the Lagrange equation for the flexure of plates results. This is the general "plate equation" used by the majority of authors on the subject.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1-\nu^2}{EI} q . \tag{11}$$

In the solution of the Lagrange equation, Poisson's ratio is generally taken as zero and the solution is converted by

$$\begin{aligned} M'_x &= M_x + \nu M_y & V'_x &= V_x \\ M'_y &= M_y + \nu M_x & V'_y &= V_y \\ M'_{xy} &= (1-\nu) M_{xy} & w' &= (1-\nu^2) w \end{aligned} \tag{12}$$

There is as yet no general solution of equation (11). Marcus[4] gave a reasonably accurate solution for several cases, which involved, however, the solution of a number of simultaneous equations. For some cases Sir Richard Southwell's Relaxation method[5] is applicable. In all cases a minimum of two boundary conditions is required for a solution.

IV. PRELIMINARY CONSIDERATIONS IN ANALYTICAL SOLUTION

In attempting an analytical solution of the slab under discussion, the following conditions must be recognized: (1) the plate is rigidly perpendicular to the column at the points of connection, i.e. at the welds, (2) the plate is free to deflect vertically at all points, except at the columns, (3) the columns are hinged at the base and rigidly supported at the plate.

The difficulty with the slab under consideration is the selection of two satisfactory boundary conditions which can be applied in a workable fashion to the Lagrange equation. A reasonable approach to the problem may be found, however, in a solution by H. M. Westergaard, which will be discussed later.

V. THE REDISTRIBUTION OF STRESSES

One of the principal reasons a purely elastic approach to plate problems fails to give solutions exactly corresponding to experimental results is the phenomena of stress redistribution. Under very light loads stresses tend to occur as the theory of elasticity predicts, but as the load increases regions of high stress fail to increase proportionally. Instead, other areas of somewhat lower stress patterns tend to absorb more than their proportional share of stresses. A discussion by Westergaard on this subject considers the example of a rectangular interior panel supported on beams. A uniform load was considered applied over the whole slab. With small loads the panel behaved as predicted by elastic theory and the stresses at the center of the long sides were the greatest in the panel. As the load was increased the stresses at the center of the longest sides failed to increase proportionally as the deformations, while the stresses at the corners increased more than proportionally. In other words, the corner had gained in stiffness while the center of the long side had lost. Secondary and tertiary redistributions occurred toward the center of the panel and finally from the longer span to the shorter. This redistribution is probably the result of local stresses exceeding the yield point. In effect, it allows critical design coefficients to be reduced below theoretical coefficients. An example of the magnitude of this redistribution is mentioned by Newmark: "Calculations reported by Westergaard[3] for tests by Bach and Graf[6] on slabs subjected to uniform loads, indicated computed stresses in the reinforcement at failure from 1.05 to 1.57 of the yield point of the reinforcing steel..."[7]

The plate discussed in this thesis would have large negative moments at the columns, if the columns were rigid, and would have a smaller positive moment near the center. Even though the columns are elastic, they are stiff and would theoretically behave nearly the same. Redistribution will produce greater moments along the edges of the plate than predicted; greater moments at the center of the plate than predicted; and lesser negative moments at the corners of the plate than predicted.

VI. REVIEW OF WESTERGAARD'S[3] ARTICLE

The most common occurence of the flat plate rigidly connected to columns is in the so-called "flat slab" construction, where a whole floor system is supported on a series of columns with no connecting girders. This system is generally constructed in reinforced concrete and is rigidly limited by the American Concrete Institute's code requirements in its ratio of distances between columns. H. M. Westergaard uses this system for his analysis, and chooses the theoretical example of an interior panel of an infinitely large flat slab. The panel selected is square, homogeneous, and has Poisson's ratio of zero. The columns on which it is supported are assumed to be infinitely stiff and the slab is assumed to be horizontal at all points along the edge of the column capitals, and horizontal at all points along the panel edges. There is no limitation placed on the deflection between columns, except that it be small as compared to the overall panel size, i.e. not a skin-type construction.

Westergaard based his investigation on certain results reported by N. J. Neilson[8] in his analysis of plates by the method of difference equations. Neilson analyzed a square interior slab, point-supported, and uniformly loaded. He divided the panel into square sections, 0.1L on a side, and used the deflections at the corners of each square as his variables. Westergaard improved on Nielson's results by drawing a smooth curve through the values obtained at the corners of the squares.

Westergaard used the point supported slab "as a substitute structure which temporarily replaces the slab supported on columns capitals, and which is made to act like the original slab, that is, have the same deflections and moments at all points outside the circles marked by the edges of the column capitals..."[9] He accomplished this by using "ring loads" over the column capitals, i.e. uniformly distributed upward loads along the capital's circumference and an equal downward load at the center of the capital. In addition he added a "uniformly distributed bending moment applied at the edge of the whole slab,"[10] and combined the whole with the uniform load and the point reactions.

In order to determine the intensity of the ring loads, and to make the proper addition to the point-supported moments, Westergaard solved the Lagrange equation, (1) for the case of the ring load by using a double-infinite series.

Westergaard presents his results for ratios of column capitals to length of side ranging from 0.15 to 0.30. He presents also approximate formulas for moment coefficients at strategic points based on ratios of 0.15 to 0.30. To find the results for a ratio of 0.10 it was necessary to interpolate, using the approximate formulas and the curves for reference.

Curves of σ_x based on Westergaard's interpretation of Neilson's results for capital to length ratio of zero are presented later, along with curves of σ_x for capital to length ratio of 0.10. These curves are compared with those obtained for σ_x from Miller's experimental results. It was necessary to assume that the diagonals were stress trajectories in order to obtain some of the values of σ_x. This assumption seemed logical in view of the symmetry of the square plate.

VII. REVIEW OF TIMOSHENKO'S[11] METHOD

S. Timoshenko approaches the same problem of the interior panel of an infinitely large slab through the use of an infinite series. However, it has been found in the department of Applied Mechanics at V.P.I. that reasonable accuracy could be obtained in the case of the homogeneous plate by the use of only the first term in the series.

Timoshenko's equations will be stated here, although no effort will be made to follow his derivations. The assumptions he makes are in accord with those of Westergaard. The equations are somewhat laborious, but their solution can be obtained with the use of any good mathematical handbook and a rudimentary knowledge of partial derivatives. No mathematics beyond calculus should be necessary.

The first case considered is that of the panels supported on columns of zero cross-sectional area. Axes are taken as shown in Fig. 3.

$$w = \frac{qb^4}{384D}\left(1-\frac{4y^2}{b^2}\right)^2 - \frac{qa^3b}{2\pi^3D}\sum_{m=2,4,6\cdots}^{\infty}\frac{1}{m^3}\left(\alpha_m - \frac{\alpha_m+\tanh\alpha_m}{\tanh^2\alpha_m}\right) + \frac{qa^3b}{2\pi^3D}\sum_{m=2,4,6\cdots}^{\infty}\frac{(-1)^{m/2}\cos\frac{m\pi x}{a}}{m^3\sinh\alpha_m\tanh\alpha_m}\left[\tanh\alpha_m\,\frac{m\pi y}{a}\sinh\frac{m\pi y}{a} - (\alpha_m+\tanh\alpha_m)\cosh\frac{m\pi y}{a}\right].$$

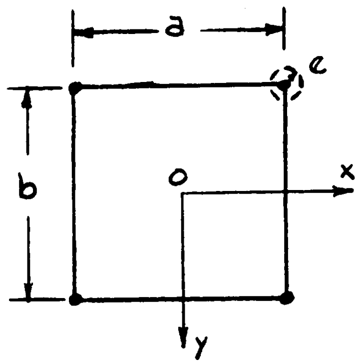


Fig. 3
Single Panel of Slab

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right),\quad M_y = -D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right),$$
$$M_{xy} = -M_{yx} = D(1-\nu)\frac{\partial^2 w}{\partial x\partial y},$$

where

w = deflection in vertical direction

q = intensity of loading

ν = Poisson's ratio

h = thickness of plate

D = $Eh^3/12(1-\nu^2)$

α_m = $\frac{m\pi b}{2a}$.

As the columns take on a finite cross-sectional area, the moments at the supports cease to be infinitely large. Timoshenko states that in a square panel the moments in a radial direction become approximately zero along a

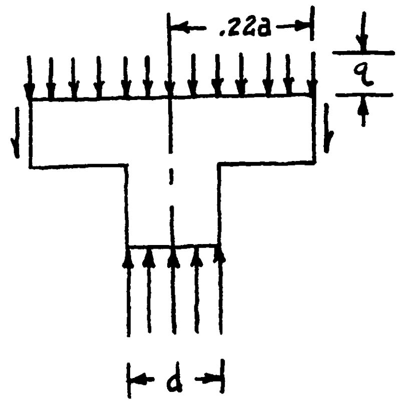


Fig. 4
Conditions of Loading around Panel

circle of radius $e = 0.22a$ (Fig. 3), so that the plate around the column and inside the circle is virtually a simply supported plate. The conditions of loading are as shown in Fig. 4.

The maximum stresses in the section are obtained from the formulas

$$\sigma_{max} = k\frac{qa^2}{h^2} \quad \text{or} \quad \sigma_{max} = \frac{kP}{h^2}$$

where k can be determined accurately enough by considering a radial strip as a beam with end conditions and loading the same as in the actual plate.

VIII. COMPARISON OF RESULTS

Figures 5 - 9 show the curves for moment coefficients given by Westergaard for values of $C/L = 0$, and for Poisson's ratio of zero. They also show the interpolated curves for $C/L = 0.10$ and Poisson's ratio equal zero. These curves were used to obtain the contours for the theoretical values of .
All of the moment coefficients are to be multiplied by qL^2 to obtain values of unit moments.

Figures 10 - 11 are stress contours for σ_x derived from the values shown in Fig. 5 - 9. Final values were obtained by use of formula 12. The values of $\sigma_x = 0$ on the upper and lower boundaries occur at points approximately .22L from the corners. This confirms, for one point at least, Timoshenko's statement that the radial moments practically vanish in a

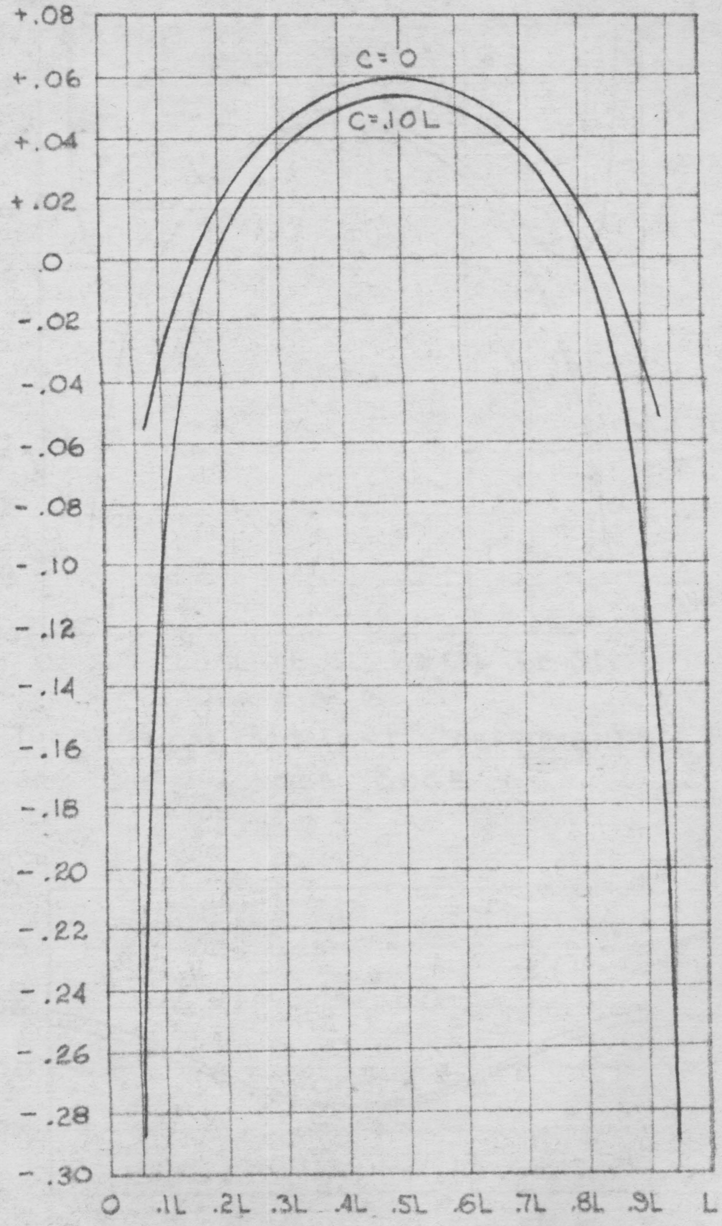


FIG. 5
THEORETICAL MOMENT COEFFICIENTS
ALONG EDGE

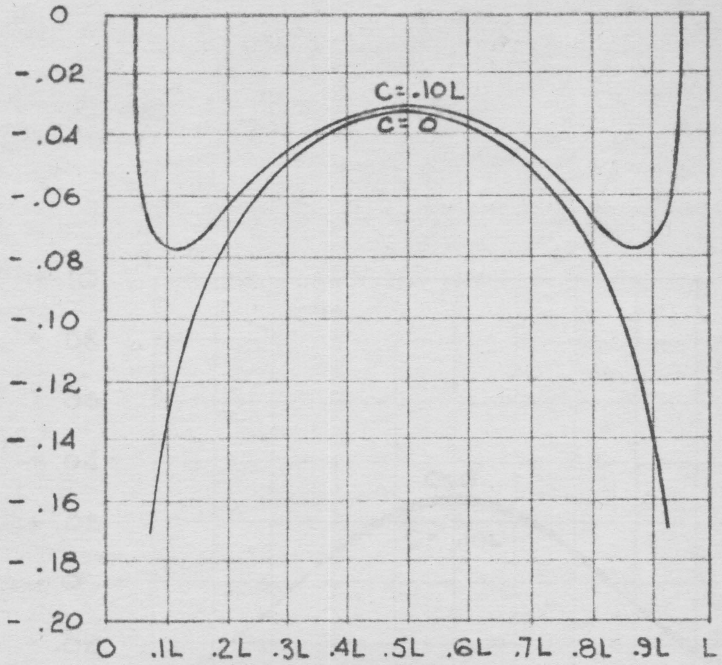


FIG. 6
THEORETICAL MOMENT COEFFICIENTS
ACROSS EDGE

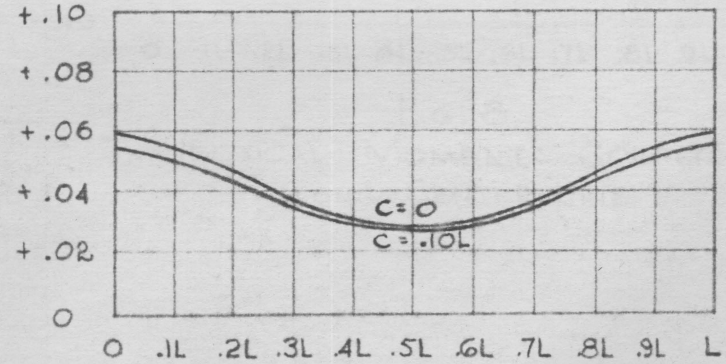


FIG. 7
THEORTICAL MOMENT COEFFICIENTS
ACROSS CENTER LINE

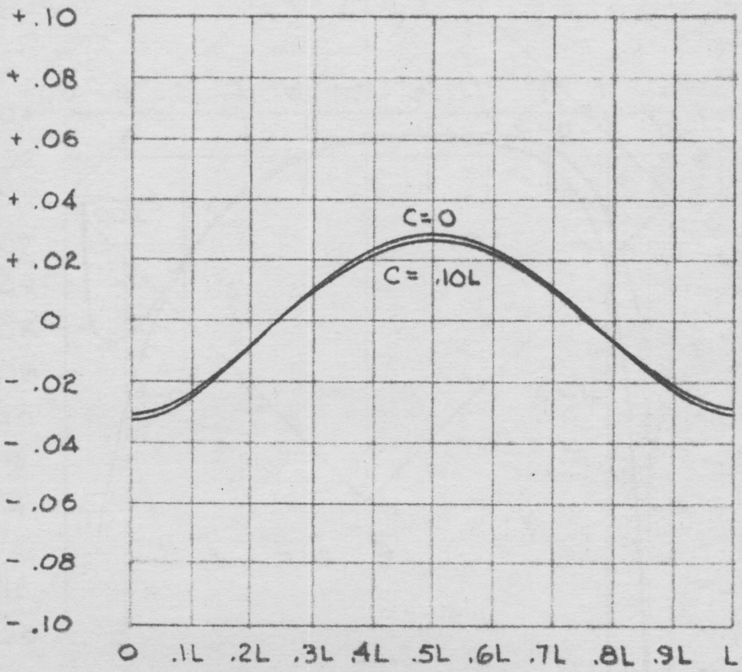
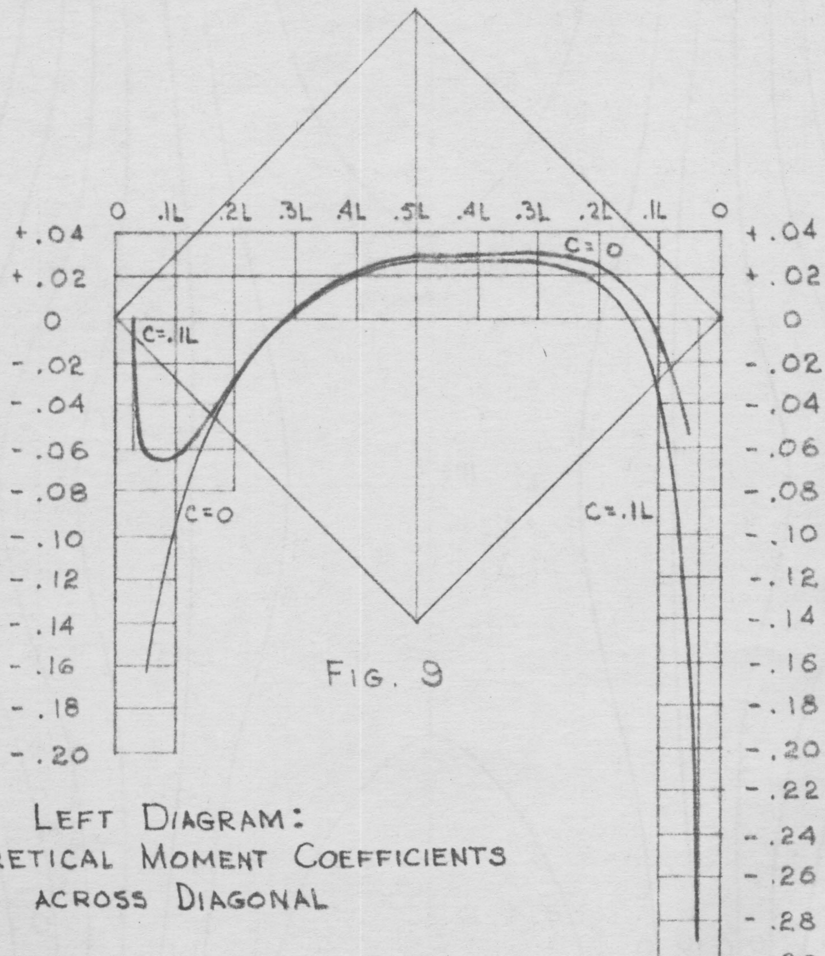


FIG. 8
THEORETICAL MOMENTS COEFFICIENTS
ALONG CENTER LINE



LEFT DIAGRAM:
THEORETICAL MOMENT COEFFICIENTS
ACROSS DIAGONAL

RIGHT DIAGRAM:
THEORETICAL MOMENT COEFFICIENTS
ALONG DIAGONAL

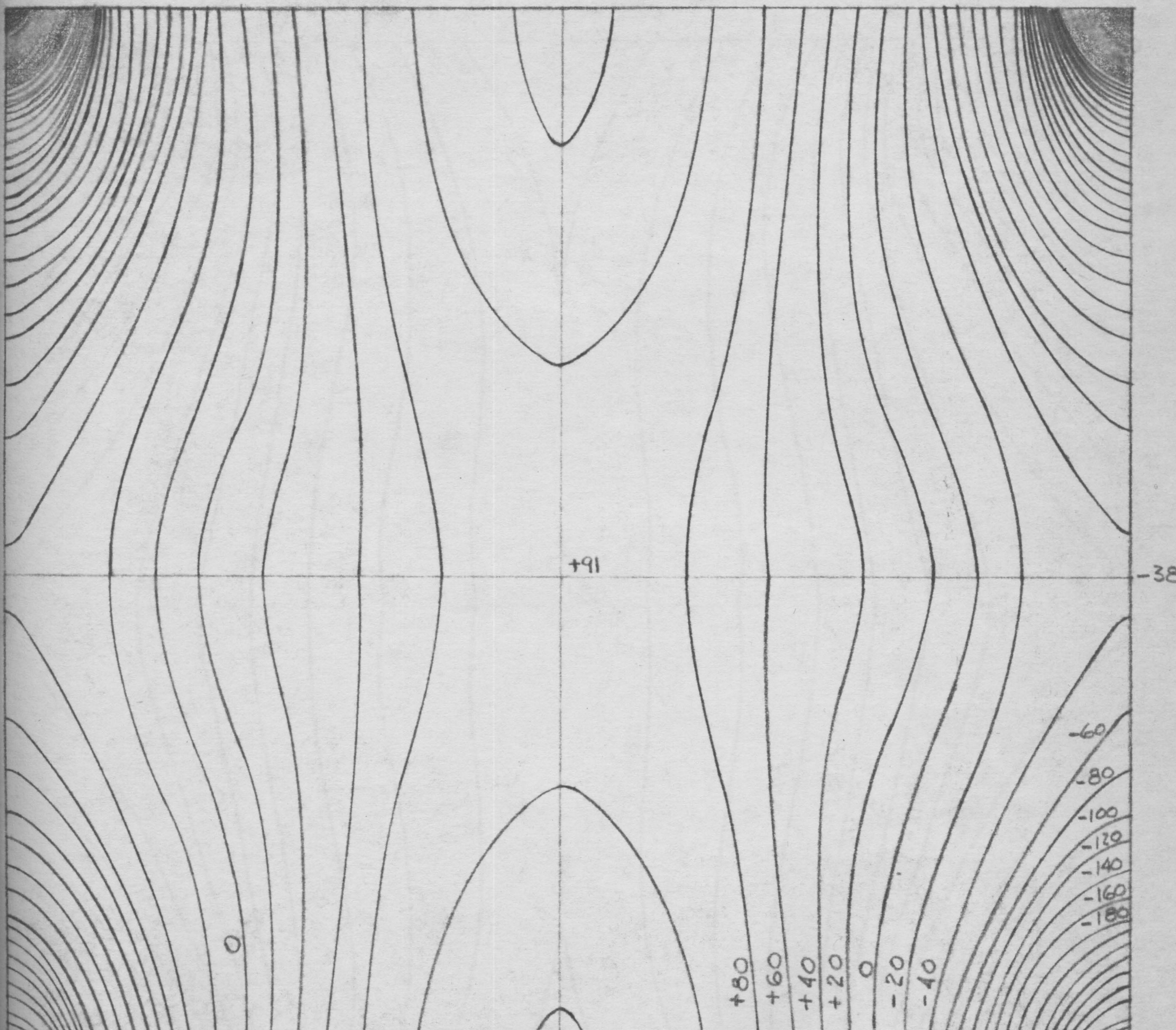


FIG. 10
σ_x FOR PANEL ; C/L = 0

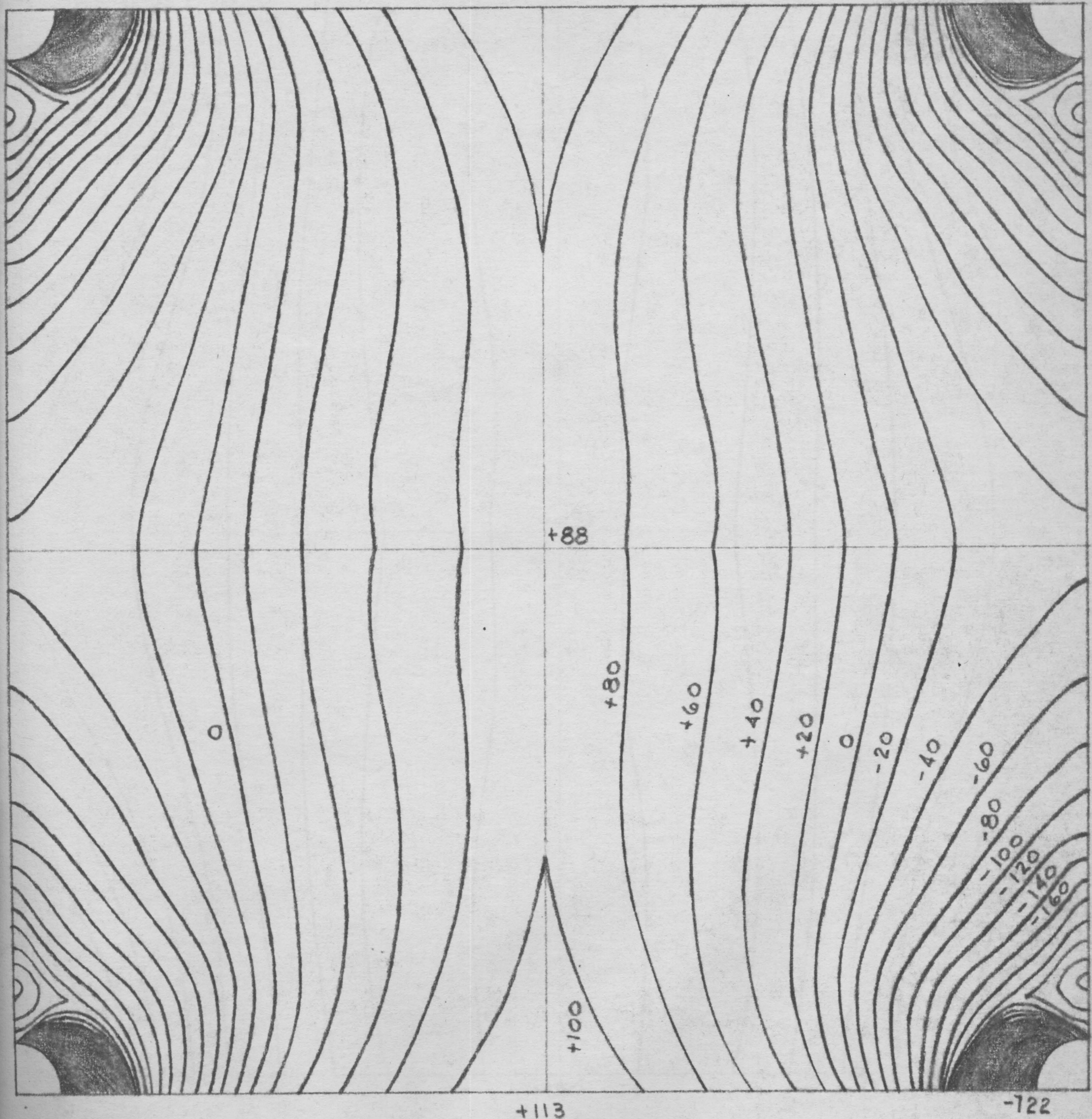


FIG. 11
σ_x FOR PANEL ; C/L = 0.10

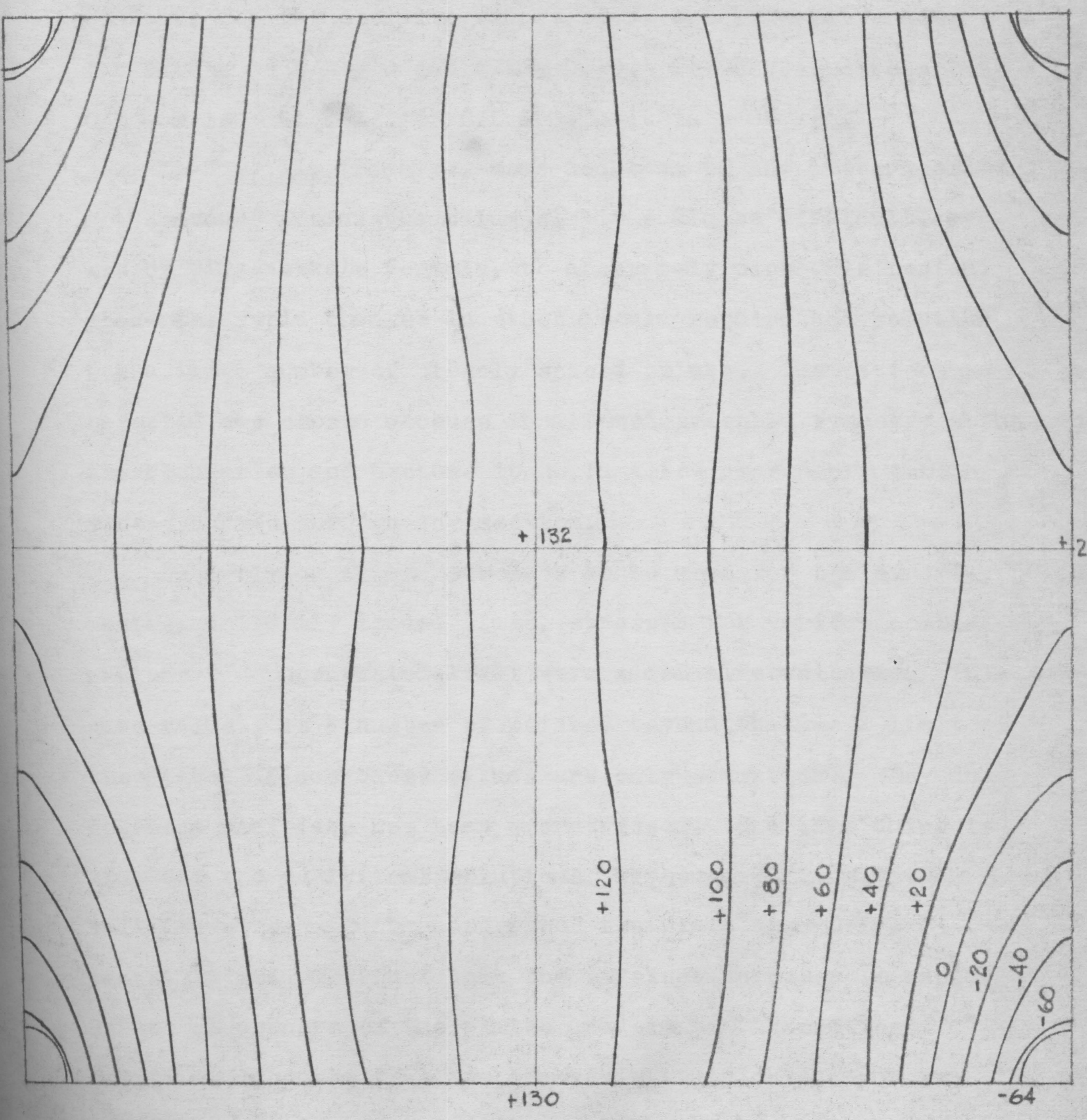


FIG. 12
σ_x FOR PANEL ; EXPERIMENTAL RESULTS

circle of radius .22L from the corners in a square slab. The contours for the positive values of σ_x are somewhat similar for values of C/L = 0 and C/L = 0.10. The center stress for C/L = 0 is + 91 ksi; for C/L = 0.10 it is + 88 ksi.

In Fig. 11, there was some doubt as to the pattern of the contours around the columns. It would be difficult, by use of Timoshenko's formula, to accurately plot this region, since the rapid changes in stress would require the solution for a large number of closely spaced points. The pattern selected was chosen because it allowed probable symmetry about the boundaries and because it would allow reasonably smooth stress curves through any section.

In arriving at the stresses to be used for the experimental, uniformly loaded plate, stresses for various combinations of concentrated loads were added algebraically. This gave values, at a number of points, beyond the yield limit of the plate. Since these values are only summations, they do not mean the plate has been overstressed. The same thing is true for the analytical solutions, although obviously the infinite stress at the corners of the plate with C/L = 0 cannot exist. The fact that the stresses increase so rapidly at the corners of the plates used in the theoretical solution, suggests that the corners will quickly reach the yield point and redistribution of the stresses will begin.

In the experimental plate, the shifting of the zero moment values (and consequently the zero stress curve) toward the columns is analogous to the shifting of the points of contraflexure toward the supports in a fixed-ended beam whose supports have been relaxed. This analogy is intensified by the increase in positive moment (stress) at the center of the slab, and the decrease in negative moment toward the columns.

IX. CONCLUSIONS

In trying to approach, analytically, the results of the tests on the experimental plate, the case of an interior panel of an infinitely large slab was used. This case produced solutions of greater negative stresses at the columns and lesser positive stresses at the center, than the experimental results. If an analogy is drawn between the slab supported on its welded columns and a single span bent, the relationship between the experimental results and the analytical results is similar to the relationship between the beam of the elastic bent and the beam of one with infinitely stiff columns. If this thesis is to be used as part of a basis for further exploration into the interaction of slabs and columns, it is suggested that the plate-bent analogy be carried further. Just as the moment values in the beam of the elastic bent lie between the values found by considering infinitely stiff columns and columns with no stiffness, so will the moment values in the slab lie between the cases of the interior panel of an infinitely large slab and a single panel hinged at the four corners. It is therefore suggested that an analytical solution based on a single panel hinged at the four corners would produce a limit for the positive moments at the center of the plate.

Because of the probable redistribution of stresses, it would appear to be overcautious to assume that the large negative moments at the columns, resulting from the analytical solution, actually occur. Certainly the results of the experimental solution indicate they do not. If the plate-bent analogy could be carried farther it would seem that the maximum negative moment, for the interior panel of the infinite slab, should be two times the value of the maximum positive moment at the center of the panel. It is doubtful, however, that the plate-bent analogy can be carried this far with any degree of accuracy. A more satisfactory limit, until further experimental data can be gathered, would seem to be a negative moment at the column equal in magnitude to the positive moment at the center of a single panel supported on hinges at the corners. In a single span beam the moment curve retains the same shape regardless of the degree of fixity of the supports. The total difference between the maximum negative moment and the maximum positive moment is $wL^2/8$ is also the value for the moment at the center of a simply supported beam. Therefore a beam designed for this moment value, would be designed for the maximum moment value possible under a uniform load only. The idea of the moment curve having the same shape at all times in a plate would not be correct, of course, in theory. The case of the pin-supported center panel considered in this thesis goes from a finite moment value at the center to an infinitely large negative moment value at the supports. The case of a hinge-supported single panel would go from a positive moment value at the center to zero at the supports. Nevertheless, in practice, it is probable the moment curves along the diagonals will be similar for the various kinds of supports, and that the maximum positive moment in the hinge-supported single panel will be the maximum condition of moment existing.

In view of the large negative moments found at the columns it would be highly desirable to secure a more accurate strain measurement than that recorded by the use of a single SR-4 strain rosette placed over the center of a column. Admittedly the size of the rosette itself makes this difficult. However, the placing of two rosettes along the perimeter of the column; one with the diagonal gage tangent to the perimeter, and one with all three gages crossing the perimeter at about their mid-points; would appear to be a more accurate method of obtaining the critical strain values than the method previously used. An alternative method would be to obtain readings in a fashion similar to those obtained in the plate discussed in this thesis, and then to apply a stress coat to the whole plate. This would give critical quantitative results at the points where the gages were located, and a good idea of the qualitative stress patterns at points between the gages.

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