THE OUTPUT VOLTAGE OF CONTROLLED POLYPHASE RECTIFIERS

by

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III. INTRODUCTION

The conversion of electrical energy from alternating to direct current is of primary importance in the field of electricity. Electrical energy is generated and transmitted almost exclusively as alternating current in the United States today. However, this energy is not all used as such for there are many applications for which direct current is either required or preferred. Large amounts of direct current are used in the electrochemical, mining, transportation, and other industries.

Although some of this direct current is generated at the site, a much greater part is purchased from power companies and converted. The conversion is ordinarily accomplished with either rotating machinery or electronic rectifiers, that is, rotary converters or mercury-arc rectifiers. The choice between these two is made on the basis of initial cost, efficiency of operation, maneuverability, cost of maintenance, reliability, resultant efficiency of load devices, resultant cost of maintenance of load devices, and personal prejudice.

The mercury-arc rectifiers are of two types--the multianode, steel tank rectifiers and the separate-unit rectifier tubes. It is possible for the separate-unit rectifier tubes to have a lower arc-voltage drop and they are consequently more efficient than the multianode type. This advantage has produced a trend for replacement of the multianode types with separate-unit rectifier tubes. One of the first of these separate-unit mercury-arc rectifier tubes was the ignitron. The initial development of the ignitron was done by Mr. J. Slepian and Mr. L. R. Ludwig for the Westinghouse Electric Corporation in 1933[1], and the ignitron has gained in popularity for heavy duty rectifier service steadily. At present, there are large numbers of ignitron rectifiers in use. The electrochemical, mining, and transportation companies are particularly large users and have hundreds of ignitron recifier units in service.[2]

In order to aid in the determination of the merits of mercury-arc rectifiers, it is necessary that the quality of the rectified output be determined. When an alternating current is rectified, the output is a periodically-varying, unidirectional current. An analysis of the quality of the output includes a determination of the average value and the number and magnitudes of the harmonics of the power frequency that are present. It is because of the presence of these harmonic frequencies that there has been some dissatisfaction with polyphase rectifiers. The presence of large harmonic currents in a machine rectifier load would increase hysteresis and eddy current losses and consequently reduce the efficiency of the machine. This reduction in efficiency should, of course, be charged to the rectifier.

IV. THE REVIEW OF LITERATURE

Slepian and Ludwig[3] developed the ignitor in 1933 and from their work stemmed the separate-unit rectifier tube industry. The ignitron with its ignitor electrode to initiate the arc is used to supply direct current power to large capacity loads.

A. VOLTAGE CONTROL METHODS:

The ignitrons require a special ignitor circuit to initiate the cathode of an arc. Several ignitor circuits have been developed to fire the tube at the proper time and control the output voltage of the ignitron rectifier.

During the ignitron development stages, a capacitor-thyratron circuit excited from a transformer whose primary was connected across the ignitron was used for arc-initiation.[4] Thyratron firing was determined through the use of a phase shifter. A simplification of this procedure placed the thyratron alone directly across the ignitron: this is referred to as main-anode firing.[5] The thyratron firing is used to control the ignition time according to the thyratron bias, which is determined externally.

In 1939, Klemperer[6] developed an ignitron firing circuit which eliminated the necessity for a thyratron. His circuit was the original one to use a saturating reactor for an ignitor circuit. In 1941 when Myers and Cox[7] surveyed the various ignitron firing circuits, they listed four main types. They included the main-anode thyratron circuit, a capacitor-thyratron circuit, the saturating reactor type firing circuit, and a rotating impulse generator as the primary methods used to fire ignitrons.

Subsequently, Mittag and Schmidt[8] show in a paper of 1942 several circuits which are also of the saturating reactor type.

The voltage control methods of the previous paragraphs were all designed to control the output voltage by controlling the firing time of the ignitrons. To obtain a wide range of voltage control, a circuit has been devised for use in the aluminum industry which uses autotransformers with no load taps, load-ratio control, and firing angle control all together.[9] The losses are higher because of the larger core needed for load-ratio control.[10] The voltage was determined approximately by the choice of the no load tap on the autotransformer. Load-ratio control is used to obtain voltages between the available no load tap voltages. Firing angle control serves only to balance the tube loads in this rectifier.

B. RECTIFIER WAVEFORMS

The principle of rectification and the analysis of rectified voltages is not unique to the controlled rectifier field. The tremendous amounts of power handled and the heavy-duty loads supplied did give rise to some new problems. Two of the chief problems are telephone interference and motor heating due to harmonics.[11]

Telephone interference is of importance where the rectifier load is the principal load on the power lines.[12] The interference may be reduced by using a larger number of phases or interconnecting six phase groups with phase shifting transformers and combining delta to star and star to delta connections.[13] Marti and Taylor[14] give tables of line harmonics measured under various conditions for rectifier groups and their conclusions indicate a preference for a large number of phases and interconnection of phase-shifted units. This problem is confined to the cases in which telephone lines are exposed to power lines whose principal load is rectifiers.

The quality of rectified voltage is of interest in each rectifier installation. The direct voltage and current relations have been determined by Puchlowski[15] for controlled rectifiers with inductive loads. These quantities are readily determined for continuous rectifier current but are more involved when the firing delay becomes so large that the rectifiers do not conduct continuously. The back emf developed by the inductive load then becomes a part of the load voltage determinations. Puchlowski includes calculations for continuous and discontinuous current and solves the problem of locating the angle of extinction for inductive loads. The determination of the harmonics present in the load voltage has been limited to the case in which there is no firing delay. The values of these harmonics may be found in any text-book. The effect of firing delay on harmonics has been neglected.

V. THE INVESTIGATION

A. OBJECT OF INVESTIGATION

The object of this investigation was to determine the variation of direct voltage and harmonic voltages for several polyphase rectifier circuits as a function of the amount of voltage control. This information was desired to establish a mathematical basis for comparison of controlled mercury-arc rectifiers with other methods of conversion.

B. METHOD OF PROCEDURE

The Plan. The investigation of the output of various polyphase rectifier circuits was pursued along analytical lines. The output voltage waveform was determined as a function of the firing angle. This waveform was analysed by a Fourier series analysis to determine the direct voltage and the principal harmonics. The values of these harmonics were calculated for successive firing angles and the ratio of the peak value of the harmonic to the direct voltage determined. Tables and curves were then prepared to show these ripple factors against a wide range of direct output voltage. These calculations were done for three phase, six phase, and twelve phase rectifiers in each case determining harmonics up to and including the twenty-fourth harmonic of the supply frequency. These voltage calculations would apply for continuous rectifier conduction or discontinuous conduction with a pure resistance load. Inductive effects of transformer leakage reactance were assumed to have a negligible effect and the calculated rectified direct voltage included the voltage drop which would appear across the tube. A breadboard circuit of thyratron was prepared to obtain some degree of justification of the assumed conditions and calculated results.

Calculations. The calculations distinquish between continuous and discontinuous current but in both categories the results of the Fourier analysis may be applied to three, six, or twelve phase. In the application, though, attention must be paid to the different limits of firing angle for each number of phases.

A three phase rectifier will fire at the 30 degree point of each of the sine waves if there is no firing delay. Without firing delay each tube will begin to conduct as soon as its anode becomes more positive than the other two and will stop conduction as soon as another anode becomes the most positive anode. Each tube, then, conducts for one-third of a period of the power frequency. The lowest harmonic frequency in the rectified voltage is three times the supply frequency. When firing delay is introduced, the third harmonic is still the lowest frequency present in the rectified voltage but its value changes with the angle of delay. The firing may be delayed to the 60 degree point of each sine wave before the current becomes discontinuous. With greater delay, there will be regular periods of zero current when two anodes are negative and the third positive but not yet fired.

The six phase rectifier with no firing delay will fire at the 60 degree point of each sine wave and conduct for one-sixth of the period of the supply frequency. The lowest harmonic frequency present in a six phase rectifier is, then, the sixth harmonic of the supply frequency. The firing in a six phase rectifier may be delayed to the 120 degree point of the sine waves before the current becomes discontinuous. The twelve phase rectifier with no firing delay has each anode beginning conduction on the 75 degree point of its sine wave. The firing may be delayed to the 150 degree point before conduction becomes discontinuous. The lowest frequency present in the output of the twelve phase rectifier is the twelfth harmonic of the supply frequency.

To determine the direct component of the output for continuous current, the output of one tube was integrated over its period of conduction and divided by the period of conduction. The period of conduction for a rectifier is 2π divided by the number of phases, since each phase conducts once during a period of the supply frequency. The direct component is, then, found as follows:

$$E_{dc} = \frac{1}{2\pi/p} \int_{\theta}^{\theta + 2\pi/p} e \, d(\omega t)$$

where: p = number of phases
e = $E_m \sin wt$ = rectified voltage for this interval
θ = angle of firing of sine wave
E_m = maximum of impressed alternating voltage

This becomes:

$$E_{dc} = \frac{pE_M}{2\pi}\left[\cos\theta - \cos\left(\theta + \frac{2\pi}{p}\right)\right]$$

When the current becomes discontinuous, each tube ceases to conduct when its voltage becomes zero which occurs at π radians. The direct voltage for discontinuous conduction is then found by changing the upper limit of integration from $\theta + \frac{2\pi}{p}$ to π. The direct voltage for discontinuous conduction is:

$$E'_{dc} = \frac{pE_M}{2\pi}\left[1 + \cos\theta\right]$$

The harmonics present will be integral multiples of the number of phases. In the Fourier series there will be in general both sine and cosine terms for each harmonic frequency. The coefficients of the sine terms are determined as follows:

$$A_N = \frac{1}{\pi}\int_{-\pi}^{\pi} e \sin(N\omega t)\, d(\omega t)$$

and the coefficients of the cosine terms:

$$B_N = \frac{1}{\pi}\int_{-\pi}^{\pi} e \cos(N\omega t)\, d(\omega t)$$

where: e = the rectified voltage

N = number of the harmonic

Since N will be a multiple of the number of phases, sin Nwt and cosNwt will be repeated identically during each anode conduction period. It is, therefore, necessary to integrate only over one anode conduction period and multiply the result by the number of phases. The coefficients may be determined for the continuous current range as follows:

$$A_N = \frac{pE_M}{\pi}\int_{\theta}^{\theta+\frac{2\pi}{p}} \sin(\omega t) \sin(N\omega t)\, d(\omega t)$$

and

$$B_N = \frac{pE_M}{\pi}\int_{\theta}^{\theta+2\pi/p} \sin(\omega t) \cos(N\omega t)\, d(\omega t)$$

The results of this integration give the following expression for the coefficients in the continuous current range of firing angles:

$$A_N = \frac{pE_M}{\pi(N^2-1)}\Big[\sin(N\theta)\Big\{\cos\Big(\theta+\frac{2\pi}{p}\Big)-\cos(\theta)\Big\}+N\cos(N\theta)\Big\{\sin(\theta)-\sin\Big(\theta+\frac{2\pi}{p}\Big)\Big\}\Big]$$

and

$$B_N = \frac{pE_M}{\pi(N^2-1)}\Big[\cos(N\theta)\Big\{\cos\Big(\theta+\frac{2\pi}{p}\Big)-\cos(\theta)\Big\}+N\sin(N\theta)\Big\{\sin\Big(\theta+\frac{2\pi}{p}\Big)-\sin(\theta)\Big\}\Big]$$

The coefficient of the N'th harmonic is then the square root of the sum of the squares of the sine and cosine coefficients. This coefficient is:

$$C_N = \sqrt{A_N^2 + B_N^2}$$

which, upon substitution, becomes

$$C_N = \frac{pE_M}{\pi(N^2-1)}\sqrt{\Big\{\cos\Big(\theta+\frac{2\pi}{p}\Big)-\cos(\theta)\Big\}^2+N^2\Big\{\sin\Big(\theta+\frac{2\pi}{p}\Big)-\sin(\theta)\Big\}^2}$$

To find the harmonic voltages in the discontinuous current range there is again the change of the upper limit of integration.

$$A_N' = \frac{pE_M}{\pi}\int_{\theta}^{\pi}\sin(\omega t)\sin(N\omega t)\,d(\omega t)$$

and

$$B_N' = \frac{pE_M}{\pi}\int_{\theta}^{\pi} \sin(\omega t)\cos(N\omega t)\, d(\omega t)$$

give the coefficients of the N'th harmonic sine and cosine terms for discontinuous current as:

$$A_N' = \frac{pE_M}{\pi(N^2-1)}\left[N\cos(N\theta)\sin(\theta) - \sin(N\theta)\cos(\theta)\right]$$

and

$$B_N' = \frac{pE_M}{\pi(N^2-1)}\left[\cos([N+1]\pi) - N\sin(N\theta)\sin(\theta) - \cos(N\theta)\cos(\theta)\right]$$

The magnitude of the N'th harmonic term is then:

$$C_N' = \frac{pE_M}{\pi(N^2-1)}\sqrt{N^2+1-(N^2-1)\cos^2\theta - 2\cos([N+1]\pi)\left\{N\sin(N\theta)\sin(\theta) + \cos(N\theta)\cos(\theta)\right\}}$$

These expressions for E_{dc}, E_{dc}', C_n and C_n' were used to calculate the direct voltage and the harmonic content of polyphase rectifiers for various firing angles. The calculations were made for as many firing angles as were necessary to determine the character of the curves.

<u>Verification</u>. In order to show that the large harmonics calculated could actually be present a bread board circuit of FG-81A thyratrons was used in the laboratory. A bank of one Kva transformers was connected

delta to zigzag to supply anode voltage and a fourth winding of each transformer was used across a resistance-capacitance circuit to obtain firing control. The results of this test are shown in tabular and curve form with the analytical results. A General Radio wave analyzer was used to determine the magnitudes of the harmonics present.

C. RESULTS

The results of calculations of the harmonic contents as a function of the firing delay are displayed in both tabular and curve form. The curves are plotted as ripple factors versus percent of maximum direct current load voltage. Ripple factor is calculated as the ratio of the peak value of the harmonic voltage to the direct rectified voltage. Table I and Curve I show the effect of small direct load voltage changes brought about by small delays of the firing angle. One-hundred percent direct load voltage is the load voltage expected with no firing delay. The ripple factors increase rapidly with the delay of the firing angle. The sixth harmonic, which is the lowest frequency and largest magnitude harmonic in a six phase rectifier, increases from 5.7 percent for no firing delay to 21.75 percent for enough delay to give a fifteen percent decrease in load voltage. It has doubled its ripple factor in a delay sufficient to give about three percent reduction in load voltage.

The variation of ripple factors for larger delays of firing is shown in Curves 2, 3, 4 and Tables II, III, and IV. The irregular increase of ripple factors beyond the continuous current delay is obvious in all three curves. These variations are due to the harmonic sine and cosine terms of the last term for the harmonic component for the discontinuous current case.

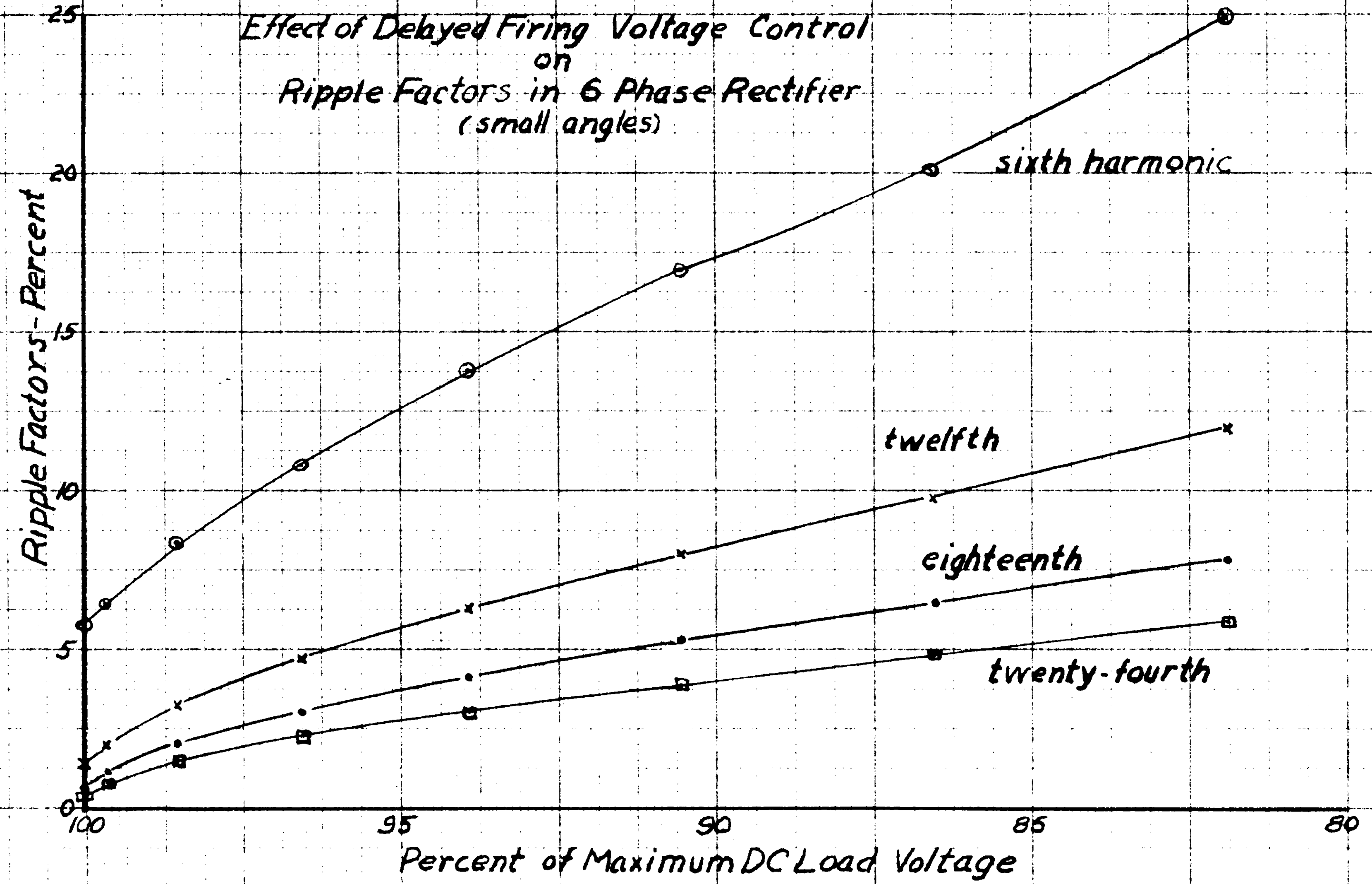
These ripple factors approach two hundred percent for very low load voltages, that is, large delay angles.

Curve 6 shows the relationship between the abscissa of the other curves and firing delay angle. This firing delay angle is not the equivalent of θ the firing angle, but is the firing angle minus the lowest firing angle possible for the number of phases. The firing delay angle for a three phase rectifier is the firing angle minus 30 degrees; the firing delay angle for a six phase rectifier is the firing angle minus 60 degrees; and the firing delay angle for a twelve phase rectifier is the firing angle minus 75 degrees. For all three rectifier circuits, the firing angle must be delayed about 31 degrees to get a fifteen percent reduction in output voltage.

The results of the experimental observations are recorded in Table V and plotted in part in Curve 5. These results substantially confirm the calculated values for the same type rectifier in Table 4 and Curve 4. In the measured values, the tube drop is not included in the output voltage. Because of differences in the control characteristics of the tubes, it is possible that the rectified output does not consist of balanced tube outputs. In spite of these shortcomings, the calculated results are checked quite closely by the experimental data.

TABLE I. Effect of Delayed Firing Voltage Control on Ripple Factors in Six Phase Rectifier (small angles).

θ°	DC LOAD VOLTAGE		SIXTH HARMONIC		TWELFTH HARMONIC		EIGHTEENTH HARMONIC		TWENTY-FOURTH HARMONIC	
FIRING ANGLE	E_{dc}	% of max.	E_{6M}	$\frac{E_{6M}}{E_{dc}}$	E_{12M}	$\frac{E_{12M}}{E_{dc}}$	E_{18M}	$\frac{E_{18M}}{E_{dc}}$	E_{24M}	$\frac{E_{24M}}{E_{dc}}$
	$\frac{3E_m}{\pi}$ volts	%	$\frac{6E_m}{35\pi}$ volts	%	$\frac{6E_m}{143\pi}$ volts	%	$\frac{6E_m}{323\pi}$ volts	%	$\frac{6E_m}{575\pi}$ volts	%
60	1.000	100.	1.00	5.7	1.00	1.4	1.00	.62	1.00	.35
65	.997	99.7	1.13	6.5	1.44	2.0	1.86	1.16	2.32	.81
70	.986	98.6	1.44	8.4	2.31	3.3	3.30	2.1	4.30	1.5
75	.966	96.6	1.83	10.8	3.25	4.7	4.76	3.1	6.29	2.3
80	.939	93.9	2.26	13.8	4.21	6.3	6.22	4.1	8.28	3.1
85	.906	90.6	2.69	17.0	5.15	8.0	7.66	5.2	10.17	3.9
90	.866	86.6	3.12	20.6	6.06	9.8	9.04	6.5	12.0	4.8
95	.819	81.9	3.56	24.9	6.95	11.9	10.3	7.8	13.8	5.9

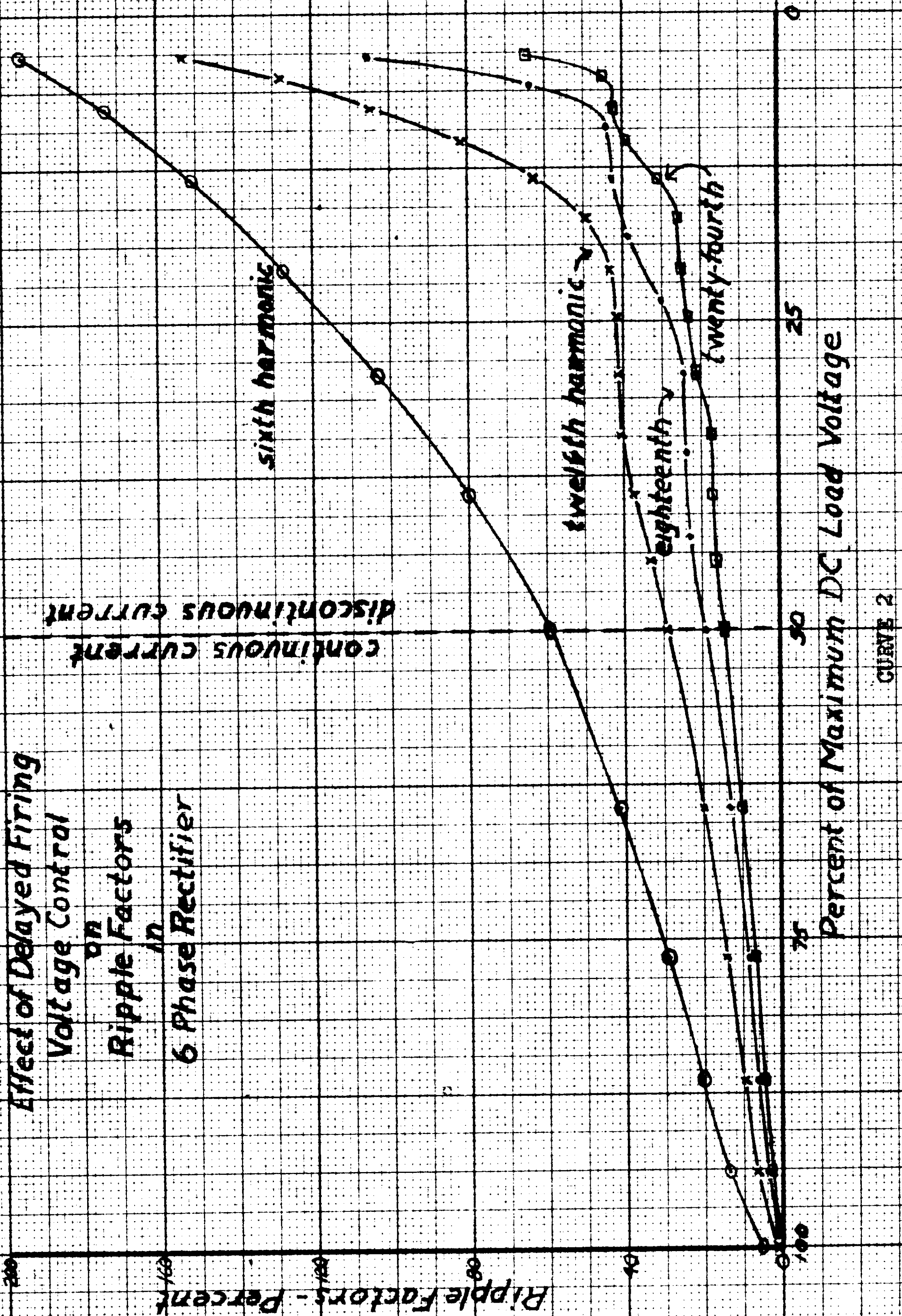


Effect of Delayed Firing Voltage Control
on
Ripple Factors in 6 Phase Rectifier
(small angles)

CURVE 1

TABLE II. Effect of Delayed Firing Voltage Control on Ripple Factors in Six Phase Rectifier

θ°	DC LOAD VOLTAGE		SIXTH HARMONIC		TWELFTH HARMONIC		EIGHTEENTH HARMONIC		TWENTY-FOURTH HARMONIC	
FIRING ANGLE	E_{dc}	% of max.	E_{6M}	$\frac{E_{6M}}{E_{dc}}$	E_{12M}	$\frac{E_{12M}}{E_{dc}}$	E_{18M}	$\frac{E_{18M}}{E_{dc}}$	E_{24M}	$\frac{E_{24M}}{E_{dc}}$
	$\frac{3E_M}{\pi}$ volts	%	$\frac{6E_M}{35\pi}$ volts	%	$\frac{6E_M}{143\pi}$ volts	%	$\frac{6E_M}{323\pi}$ volts	%	$\frac{6E_M}{575\pi}$ volts	%
60	1.0	100	1.0	5.7	1.0	1.4	1.0	.62	1.0	.35
80	.939	93.9	2.26	13.3	4.21	6.3	6.22	4.1	8.28	3.1
90	.866	86.6	3.12	20.6	6.06	9.8	9.04	6.5	12.0	4.8
100	.766	76.6	3.94	29.4	7.75	14.2	11.6	9.4	14.7	6.7
110	.643	64.3	4.65	41.4	9.2	20.0	13.8	13.5	18.4	10.0
120	.500	50.0	5.21	59.5	10.4	29.1	15.6	19.3	20.8	14.5
123.75	.444	44.4			10.[illegible]	33.8			21.0	16.4
125.	.426	42.6					15.7	22.8		
127.5	.391	39.1	5.46	80.0	10.5	37.5			19.1	17.0
130.	.357	35.7					13.7	23.8		
131.25	.341	34.1			9.83	40.4			17.1	17.5
135.	.293	29.3	5.30	103.5	8.53	40.7	11.7	24.7	18.0	21.4
138.75	.248	24.8			7.35	41.5			16.8	23.6
140.	.234	23.4					11.6	30.6		
142.5	.207	20.7	4.61	127.5	6.35	43.0			14.8	24.8
145.	.181	18.1					11.4	38.8		
146.25	.169	16.9			6.00	49.6			12.4	25.6
150.	.134	13.4	3.54	151	6.03	63.0	9.20	42.4	12.0	31.2
153.75	.103	10.3			6.02	81.8			11.7	39.5
155.	.094	9.4					6.70	44.0		
157.5	.076	7.6	2.29	173	5.70	105.			9.24	42.3
160.	.060	6.0					6.20	64.0		
161.25	.053	5.3			4.85	128.			6.75	44.4
165.	.034	3.4	1.16	195	3.73	154.	5.84	106.	6.34	64.8

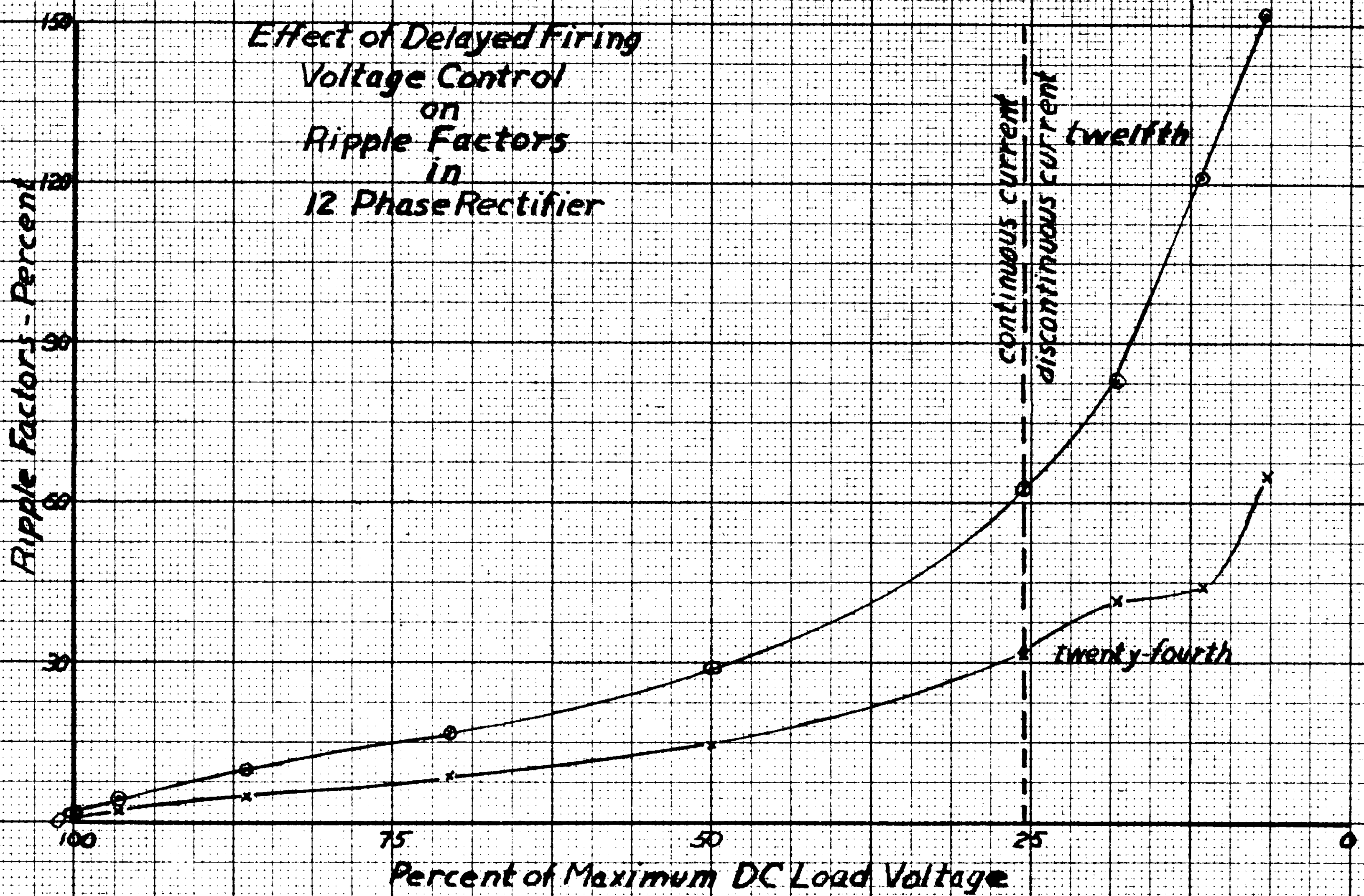


Effect of Delayed Firing Voltage Control on Ripple Factors in 6 Phase Rectifier

CURVE 2

TABLE III. Effect of Delayed Firing Voltage Control on Ripple Factors in Twelve Phase Rectifier

θ°	DC LOAD VOLTAGE		TWELFTH HARMONIC		TWENTY-FOURTH HARMONIC	
FIRING ANGLE	E_{dc}	% of max.	E_{12M}	$\frac{E_{12M}}{E_{dc}}$	E_{24M}	$\frac{E_{24M}}{E_{dc}}$
	$\frac{3E_m}{\pi}$ volts	%	$\frac{6E_m}{143\pi}$ volts	%	$\frac{6E_m}{575\pi}$ volts	%
75	1.036	100	1.036	1.4	1.036	.35
90	1.000	96.6	3.37	4.6	6.81	2.4
105	.896	86.5	6.28	9.8	12.5	4.8
120	.732	70.6	8.81	16.8	17.6	8.4
135	.518	50.0	10.7	29.0	21.4	14.4
150	.268	25.9	12.0	62.6	24.0	31.2
155	.188	18.2	11.3	84.0	22.4	41.5
160	.120	11.6	10.4	121.2	15.2	44.1
165	.068	6.6	7.4	152.2	12.7	64.9

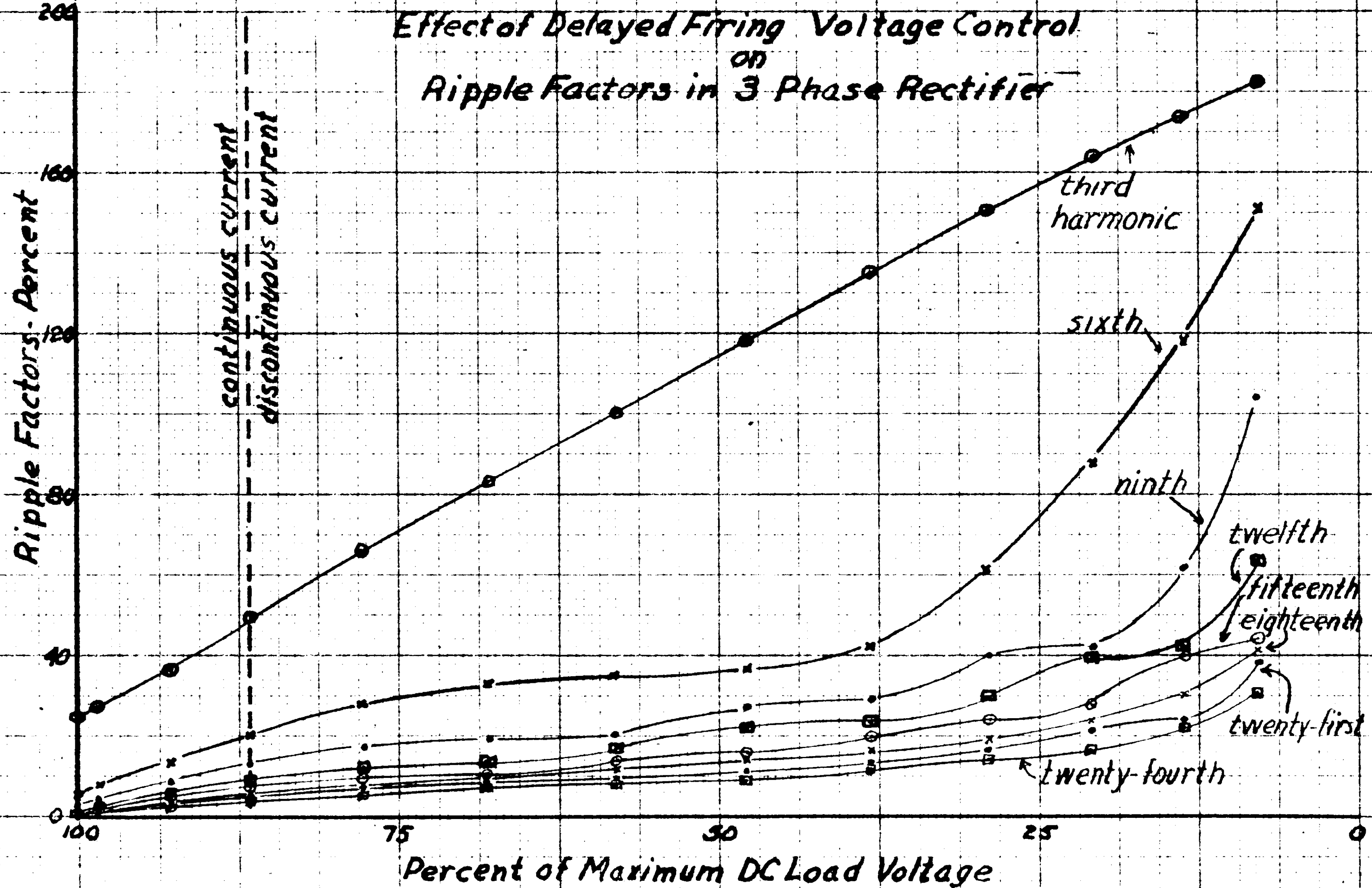

Effect of Delayed Firing
Voltage Control
on
Ripple Factors
in
12 Phase Rectifier
Ripple Factors - Percent
150
120
90
60
30
continuous current
discontinuous current
twelfth
twenty-fourth
100
75
50
25
0
Percent of Maximum DC Load Voltage

CURVE 3

TABLE IV. Effect of Delayed Firing Voltage Control on

Ripple Factors in Three Phase Rectifier

$\theta°$	DC LOAD VOLTAGE		THIRD HARMONIC		SIXTH HARMONIC		NINTH HARMONIC		TWELFTH HARMONIC		FIFTEENTH HARMONIC		EIGHTEENTH HARMONIC		TWENTY FIRST HARMONIC		TWENTY FOURTH HARMONIC	
FIRING ANGLE	E_{dc}	% of max.	E_{3M}	$\frac{E_{3M}}{E_{dc}}$	E_{6M}	$\frac{E_{6M}}{E_{dc}}$	E_{9M}	$\frac{E_{9M}}{E_{dc}}$	E_{12M}	$\frac{E_{12M}}{E_{dc}}$	E_{15M}	$\frac{E_{15M}}{E_{dc}}$	E_{18M}	$\frac{E_{18M}}{E_{dc}}$	E_{21M}	$\frac{E_{21M}}{E_{dc}}$	E_{24M}	$\frac{E_{24M}}{E_{dc}}$
	$\frac{3E_m}{\pi}$ volts	%	$\frac{6E_m}{8\pi}$ volts	%	$\frac{6E_m}{35\pi}$ volts	%	$\frac{6E_m}{80\pi}$ volts	%	$\frac{6E_m}{143\pi}$ volts	%	$\frac{6E_m}{224\pi}$ volts	%	$\frac{6E_m}{323\pi}$ volts	%	$\frac{6E_m}{440\pi}$ volts	%	$\frac{6E_m}{575\pi}$ volts	%
30	.866	100	.866	25	.866	5.7	.866	2.5	.866	1.4	.866	.84	.866	.62	.866	.46	.866	.35
40	.853	98.6	.930	27	1.24	8.3	1.55	4.6	1.94	3.2	2.40	2.5	2.74	2.0	3.30	1.7	3.70	1.5
50	.804	92.9	1.20	37	1.90	13.5	2.78	8.7	3.64	6.3	4.51	5.0	5.39	4.2	6.20	3.5	7.14	3.1
60	.750	86.7	1.50	50	2.70	20.6	3.96	13.2	5.25	9.8	6.53	7.8	7.83	6.5	7.13	5.6	10.4	4.8
70	.671	77.6	1.77	66	3.28	27.9	4.73	17.6	6.06	12.6	7.31	9.7	8.46	7.8	7.63	6.5	10.8	5.6
80	.587	67.9	1.94	83	3.39	33.0	4.45	19.0	5.49	13.1	6.96	10.6	8.9	9.4	10.8	8.4	12.3	7.9
90	.500	57.8	2.00	100	3.04	34.8	4.00	20.0	6.19	17.3	8.00	14.3	9.02	11.2	10.0	9.1	12.0	8.4
100	.413	47.6	1.94	118	2.55	36.3	4.45	27.0	6.35	21.6	6.96	15.1	8.90	13.3	10.8	11.9	11.4	9.6
110	.329	38.0	1.17	135	2.39	41.5	3.13	28.3	5.21	22.2	7.31	19.9	8.50	16.0	9.62	13.3	11.[illegible]	12.4
120	.250	28.9	1.50	150	2.66	60.9	3.96	39.6	5.21	29.2	6.55	23.4	7.81	19.3	9.14	16.6	10.4	14.5
130	.179	20.7	1.17	164	2.73	87.1	2.96	41.3	5.06	39.5	5.50	21.4	6.94	24.0	8.29	21.1	8.76	17.0
140	.117	13.5	.816	174	2.42	118.	2.85	61.0	3.42	40.9	5.21	39.8	5.69	30.1	6.24	24.3	8.05	24.0
150	.067	7.7	.490	183	1.77	151	2.78	104.	3.00	62.6	3.28	43.7	4.60	42.5	5.76	39.2	6.00	31.2



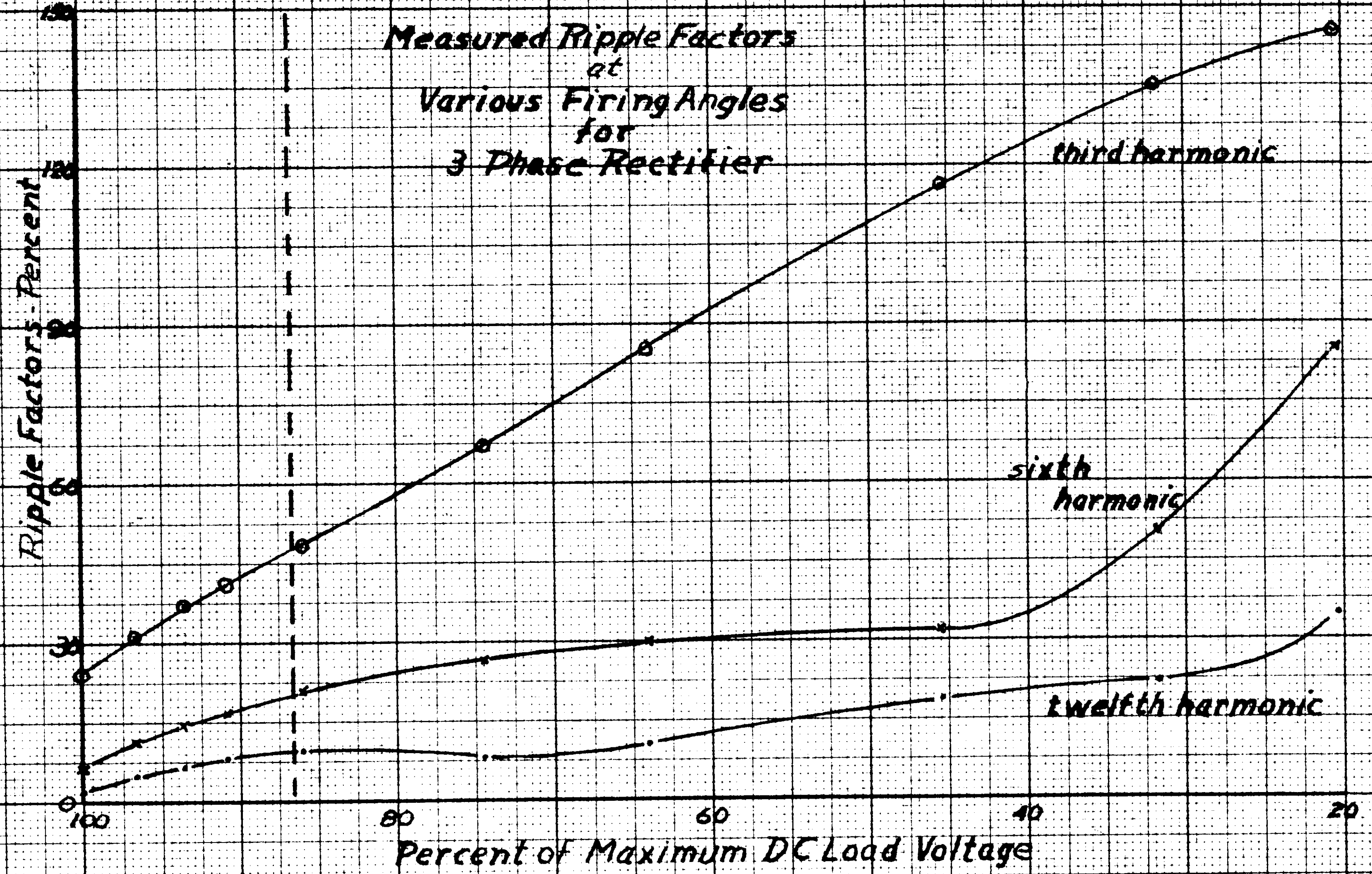
Effect of Delayed Firing Voltage Control on Ripple Factors in 3 Phase Rectifier

CURVE 4

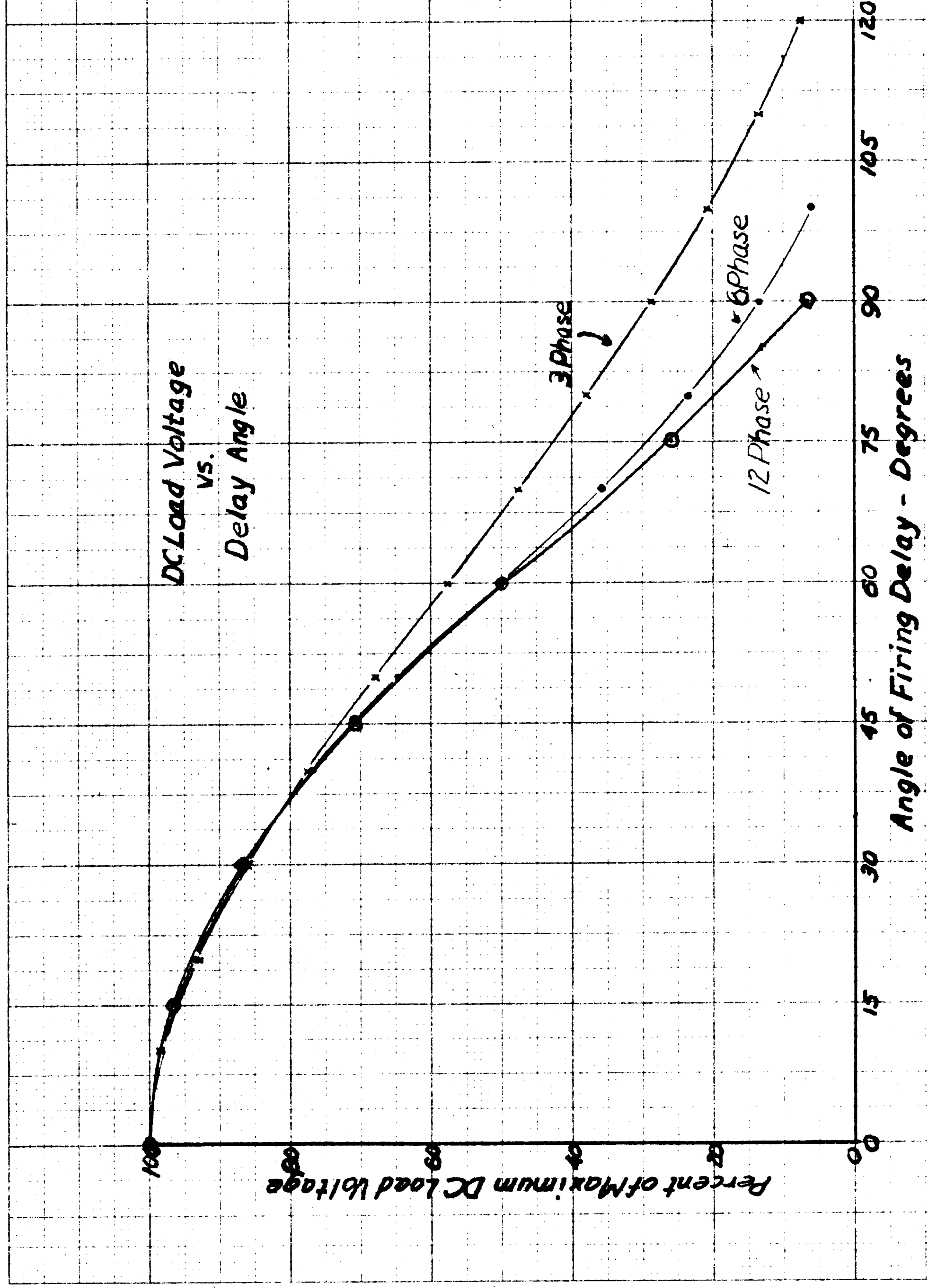
TABLE V. Measured Ripple Factors at Various Firing Angles

for Three Phase Rectifier

DC LOAD VOLTAGE		THIRD HARMONIC		SIXTH HARMONIC		NINTH HARMONIC		TWELFTH HARMONIC		FIFTEENTH HARMONIC		EIGHTEENTH HARMONIC		TWENTY-FIRST HARMONIC		TWENTY-FOURTH HARMONIC	
E_{dc}	% of max.	E_3	$\frac{E_{3M}}{E_{dc}}$	E_6	$\frac{E_{6M}}{E_{dc}}$	E_9	$\frac{E_{9M}}{E_{dc}}$	E_{12}	$\frac{E_{12M}}{E_{dc}}$	E_{15}	$\frac{E_{15M}}{E_{dc}}$	E_{18}	$\frac{E_{18M}}{E_{dc}}$	E_{21}	$\frac{E_{21M}}{E_{dc}}$	E_{24}	$\frac{E_{24M}}{E_{dc}}$
volts	%	volts	%	volts	%	volts	%	volts	%	volts	%	volts	%	volts	%	volts	%
220	100	38	24.4	10.0	6.4	4.7	3.0	2.5	1.6	1.7	1.1	1.25	.80	1.0	.64	.88	.56
213	96.8	47	31.2	17.0	11.3	10.0	6.6	6.9	4.6	5.4	3.6	4.5	3.0	3.8	2.5	3.2	2.1
206	93.7	54	37.1	21.5	14.8	13.5	9.3	9.7	6.7	7.7	5.3	6.3	4.3	5.3	3.6	4.6	3.2
200	90.9	58	41.0	23.5	16.7	14.5	10.3	11.0	7.8	8.6	6.1	7.0	5.0	6.0	4.3	5.2	3.7
189	86.0	65	48.6	28.0	21.0	17.0	12.7	12.5	9.4	10.0	7.5	7.8	5.8	6.4	4.8	5.2	3.9
164	74.5	73	67.2	31.0	26.7	16.0	13.8	9.1	7.9	7.8	6.7	7.1	6.1	6.2	5.4	5.3	4.6
141	64.1	85	85.3	30.0	30.1	16.0	16.1	10.5	10.5	8.5	8.5	6.5	6.5	4.4	4.4	2.8	2.8
100	45.4	82	116	22.5	31.8	20.0	28.3	13.5	19.1	10.0	14.1	9.3	13.2	6.9	9.7	6.3	8.9
70	31.8	67	135	25.0	50.5	16.0	32.3	11.0	22.2	8.5	17.2	6.4	12.9	4.9	9.9	3.7	7.5
45	20.4	46	145	27.0	85.0	12.5	39.3	11.0	34.6	9.0	28.3	6.3	19.8	6.2	19.5	5.2	16.3


Measured Ripple Factors
at
Various Firing Angles
for
3 Phase Rectifier
third harmonic
sixth
harmonic
twelfth harmonic
Ripple Factors - Percent
150
120
90
60
30
0
100
80
60
40
20
Percent of Maximum DC Load Voltage

CURVE 5


DC Load Voltage
vs.
Delay Angle
3 Phase
6 Phase
12 Phase
Angle of Firing Delay - Degrees
0
15
30
45
60
75
90
105
120
Percent of Maximum DC Load Voltage
100
80
60
40
20
0

CURVE 6

VI. DISCUSSION OF RESULTS

The principal control methods now used to control the output of rectifiers control the firing angle of the rectifier tubes. The firing angle may be controlled automatically in order to maintain a constant load voltage or it may be controlled manually for setting the load voltage at different values. This operation varies the firing angle of the rectifier tubes and consequently varies the harmonic content of the rectified output.

In automatic voltage control, the harmonic content may be increased four or five times over is minimum value. This minimum harmonic content occurs under conditions of no firing delay. When the voltage is adjusted manually, the firing delays may be even greater than those encountered in automatic control. The ripple factors resulting from this variation in output voltage are shown in the results of this investigation.

If the autotransformer, load-ratio control method is used to obtain voltage variation over a wide range, the ripple factors are larger, even then, than the minimum. The delayed firing is used in this method, to maintain tube balance, and five to six percent delayed firing voltage reduction will most likely be necessary to keep a sufficient leeway for balancing the tube loads.

The no delay firing results are, of course, identical with those commonly attributed to the polyphase rectifiers. These ripple factors are very optimistic as the results of this investigation indicate. Actual ripple factors in nearly all cases where controlled rectifiers are used will be much greater than these minimum figures.

The large delay of firing necessary to bring about an appreciable change in the rectified voltage is clearly indicated in Curve 6. This is true for three, six and twelve phase and since the rectified voltage is a cosine function, the results would be very similar, regardless of the number of phases. An increase in the number of phases will not bring a decrease in the amount of delay necessary to reduce the rectified voltage a specified amount.

The advantage of an increased number of phases lies in the fact that only higher harmonics are present. The ripple factor of the twelfth harmonic, for instance, is almost the same for three, six and twelve phase for the same reduction in load voltage. In the twelve phase rectifier, the twelfth harmonic is the lowest frequency and greatest magnitude harmonic, whereas in six and three phase rectifiers, there are lower frequencies with larger magnitudes. Even in a twelve phase rectifier, firing delay will produce ripple factors of annoying magnitude. A twenty-five percent voltage reduction by firing delay in a twelve phase rectifier will produce a fifteen percent ripple factor.

The results of these calculations have been plotted as ripple factors versus percent of maximum load voltage. This should facilitate their use in determining ripple factors since in most rectifiers the firing angle is obscure and the load voltage readily ascertained. For values of ripple factor within the continuous current range, the load currents may be determined from a knowledge of the load impedance.

The fact that these results are indicated as being for three phase, six phase, and twelve phase in generalization of actual circuit arrangement. The application of these results to a specific circuit arrangement would be made according to the nature of the output. Specifically, if three phase circuits are used, but through interconnection on the load side of the transformers, the resultant rectified wave is that of a twelve phase rectifier. The results to be followed are those indicated as for a twelve phase rectifier.

VII. CONCLUSIONS

The values of ripple factor ordinarily specified for the harmonics of a polyphase rectifier are: twenty-five percent for the third, 5.7 percent for the sixth, 2.5 percent for the ninth, 1.4 percent for the twelfth, etc. From the calculated results of this investigation, it is apparent that for controlled rectifiers these figures are no indication of the actual ripple factors existing. The actual ripple factors may be determined for specific firing delays by use of the curves prepared in this investigation.

The efficiency of load equipment and the cost of maintenance of load equipment are two of the bases for comparing methods of conversion. These conditions can only be fairly predicted from a knowledge of the actual quality of the rectified waveform. A prediction of these quantities, then, may be based on the results of this investigation. The final comparison of controlled mercury-arc rectifiers and other methods of conversion will not be made in this investigation. Such a comparison depends upon the type of load, the amount of voltage control necessary, the rectifier circuit employed, and the individual characteristics of the load device. These are all determined only in an actual application.

In order to operate a polyphase rectifier with the smallest harmonic content possible, the firing delay should be as low as possible. For a wide-range of voltage control, it may be advantageous to use control methods such as load-ratio control if the load device is sensitive to the large harmonics produced by extreme delay of firing. It is, however, possible to obtain wide-range voltage control with firing delay alone when the quality of the rectified output is of secondary importance to the expense of the additional voltage control equipment.

VIII. SUMMARY

The controlled polyphase rectifiers in current use employ delayed firing in some degree to obtain voltage control. Delayed firing increases the harmonic content of the rectified voltage to such an extent that the harmonics determined without firing delay are insignificant. The harmonic content which exists is displayed in the results of this investigation. These results are the foundation for analysing the suitability of a particular rectifier circuit for a given application.

IX. ACKNOWLEDGMENTS

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XI. BIBLIOGRAPHY

A. Literature Cited

1. Slepian, J. and Ludwig, L. R., "A New Method for Initiating the Cathode of an Arc", A.I.E.E. Trans. 52:693-700. 1933.
2. Bulletin 3024 of the Westinghouse Electric Corporation, "Ignitron Rectifiers."
3. Slepian, J. and Ludwig, L. R., op. cit.
4. Knowles, D. D., "The Ignitron--A New Controlled Rectifier", Electronics 6:164-166. June, 1933.
5. Jones, G. F., "Ignitrons for the Transportation Industry", Electrical Engineering, Trans. 58:618-622. 1939.
6. Klemperer, H., "A New Ignitron Firing Circuit", Electronics 12:12-15. December, 1939.
7. Myers, H. C. and Cox, J. H., "Excitation Circuits for Ignitron Rectifiers", Electrical Engineering, Trans. 60:943-48. 1941.
8. Mittag, A. H. and Schmidt, A., Jr., "Ignitor Excitation Circuit and Misfire Indication Circuits", Electrical Engineering, Trans. 61:574-577. 1942.
9. Rhea, T. R. "How to Provide for Wide-Range DC Voltage Adjustment in Large Mercury-Arc Rectifiers", General Electric Review. 44:611-613. 1941.
10. Coates, R. E. and Hunter, E. M., "Load-Ratio Control Equipment for Unit Substations", General Electric Review. 49:8-12. November, 1946.

11. Jones, G. R. and Cox, J. H., "Rectifiers and Inverters Have Broad Application Field", *Electrical World*. 123:86-88. December, 1945.

12. Cox, J. H. and Jones, G. R., "Ignitron Rectifiers in Industry", *Electrical Engineering*, Trans. 61:713-718. 1942.

13. *Ibid*.

14. Marti., O. K. and Taylor, T. A., "Wave Shape of 30- and 60-Phase Rectifier Groups", *Electrical Engineering*, Trans. 59:218-226. 1940.

15. Puchlowski, K. P., "Voltage and Current Relations for Controlled Rectifiers with Inductive and Generative Loads", *Electrical Engineering*. 64:255-260. 1945.

B. Additional Literature

Bohn, Ward, Dickinson, and Marshall, "Large Rectifier Station Practice", *Electrical Engineering*. 66:957-963. 1947.

Evans, R. D., "Harmonics and Load Balance of Multiphase Rectifiers", *Electrical Engineering*, Trans. 62:182-187. 1943.

MIT Staff, *Applied Electronics*, John Wiley and Sons, Inc. New York. 1943.

Read, J. C., "New Method for Improving Waveforms of Rectifier Equipments—Phase Doubled Twelve Phase Connection", *Institution of Electrical Engineers*, Journal, part II. 95:756. 1949.

Ryder, J. D., *Electronic Engineering, Principles*, Prentice-Hall, Inc., New York, 1947.

Scharz, W. E., "Ignitron for Direct Current Power", *Power Plant Engineer*. 47:76-79; 110. March, 1943.

Steiner, H. C., "Industrial Applications of Electronic Power Rectifiers", *Industry and Power*. 44:45-48; 102; 104, January, 1943. 44:52-54; 128. February, 1943.

Wagner, C. F., "Applications of Electronics in Electric Power Industry", *Electrical Engineering*. 64:323-327. 1945.

"Ignitron Mercury-Arc Rectifiers—War Machine Extraordinary", *Westinghouse Engineer*. 4:51-55. March, 1944.