TWO-GROUP COMPARISONS IN THE RANK ANALYSIS OF INCOMPLETE BLOCK DESIGNS

by

Richard Peyton Bartlett, Jr.

Thesis submitted to the Graduate Faculty of the

Virginia Polytechnic Institute

in candidacy for the degree of

MASTER OF SCIENCE

in

STATISTICS

APPROVED:

Director of Graduate Studies

Dean of the School of Applied Science and Business Administration

APPROVED:

Head of Department

Major Professor

June, 1952

Blacksburg, Virginia

TABLE OF CONTENTS

I. INTRODUCTION

1.1 Problems in Sensory Testing:

Several of the standard statistical techniques now in use are not satisfactorily adaptable to many of the current problems arising in sensory testing, where qualitative measures alone are available as reliable data. Hence, there is a need for more elaborate ranking methods.

The best experimental testing of data which require subjective measures has involved the comparison of two items only. Experiments involving more than two items or treatments have had varying success. Problems or difficulties occur in the experiments that require more than one sitting or in those experiments where it is difficult to select a suitable scoring scale.

The analysis of variance has been used to a great extent in other types of experimentation when more than two treatments are compared. It is then possible to make comparisons of any two of a number of treatments, as well as other comparisons, by the single degree of freedom method. A similar test is needed for rank methods.

The following work is designed to increase the flexibility of a ranking method of analysis by introducing a rank analogue to the single degree of freedom technique of analysis of variance. The principle mathematical formulation of the method is given in section 2.1.

1.2 Rank Order and Non-Parametric Procedures:

The problem of testing whether two samples (groups of observations) come from the same population has been treated in various ways in statistical methodology. Several statisticians have proposed tests of this type, both for the case where there are observed measurements with no assumptions about the forms of the populations sampled and for the case where ranks alone have been recorded.

An early test based on qualitative data was proposed by R. A. Fisher [8]* and has become known as the "sign" test. Fisher considers the problem, related to the two sample situation described above, of testing the null hypothesis that pairs of observations in each of k pairs come from a common population. He considers differences in such pairs and records the numbers of positive and negative differences. The test procedure is simply related to the well-known binomial distribution. The test may easily be adjusted to consider ranking within the pairs of samples instead of the differences used.

Later, E. J. G. Pitman [16] introduced the "spread test", which involves no assumptions about the nature of

*Numbers in square brackets refer to the Bibliography.

the populations sampled. The Pitman test may be modified for ranked data and is then known as the "rank spread test". Closely related to that test are others by F. Wilcoxon [25] and by H. B. Mann and D. R. Whitney [11]. All three test procedures may be shown to be exactly equivalent and each has prepared some tables to facilitate the use of their tests. These tables supplement each other to some degree.

W. J. Dixon [7], using one additional assumption to the effect that the cumulative distribution function (c.d.f.) common to the two populations in question be continuous, developed a C^2-test with tables of critical values for sample sizes not greater than 20.

A short time later, A. Wald and J. Wolfowitz [24], using a statistic based on the theory of runs, derived a consistent test, subject to some slight restrictions on the distribution functions. The Wald and Wolfowitz test differs from those discussed above in that it is also consistent against differences in the distribution functions of the populations other than in location. F. S. Swed and C. Eisehhart [19] have computed tables of significant values of the Wald and Wolfowitz statistic for sample sizes of twenty or less.

Taking a different approach, M. E. Terry [20] has derived most powerful rank order tests against specific parametric alternatives. His $C_1(R)$-test, in general, is more sensitive than the Mann and Whitney U-test. In particular, he has shown that his method leads to the most powerful rank order test in those situations where, had observations been available, a two-sample t test would have been correctly used. Tables are included, giving the complete array of values of $C_1(R)$ for all possible subsample sizes which do not exceed ten, together with the corresponding value of the Mann and Whitney U. A second table gives critical values of $C_1(R)$ for samples up to size 10 and significance levels equal to or lower in probability than .10. Terry shows how k repetitions of his rank procedure may be combined for an over-all test of significance based on a statistic designated by $C_1(R^k)$.

In the present study we are interested in experiments involving more than two populations or groups of observations. We shall, however, restrict our interest to that experimental design known as the method of paired comparisons and to two-group comparisons within the framework of such a design.

One of the early considerations of paired comparisons was developed by L. L. Thurstone [22]. His procedure, developed for psychological scaling, postulated the existence of a subjective continuum whereon items or treatments had certain location points. Assumptions involved in the analysis included one of inherent normality and one that postulated zero correlations between preferences for pairs of items to be scaled. F. Mosteller [13, 14, 15] following the developments of Thurstone, shows that Thurstone's assumption of zero correlations is too restrictive and that the customary solution to paired comparisons is the Least Squares Solution.

M. G. Kendall and B. Babington Smith [10] developed a method of paired comparisons that disregards any scalar relationships among the items. They form a coefficient of consistency of ranking for a single set of ranks based on the theory of "circular triads" and a coefficient of agreement for several sets of ranks.

R. A. Bradley and M. E. Terry [4] have opened the way to a consideration of ranking in incomplete block designs. As a first step, they also considered the method of paired comparisons as a special case. This involves a test of a hypothesis of no differences among treatments, in a ranking experiment, with only two treatments in a block of their balanced incomplete block design. They have computed tables for this test for three and four treatments up to and including ten repetitions of all possible treatment pairs.

As an addendum to the work of Bradley and Terry, we shall consider the second special test (ii) [4] which they formulated. That test postulates the existence, for t treatments, or true treatment ratings which fall into one or another or two groups within which no differences in ratings exist. The existence of differences between these two groups is under test. We review this test procedure and provide tables for the easy conduct of the test. If s be the number of treatments in the first group, and consequently (t-s) the number in the second group, and, if n be the number of times the complete set of paired comparisons is repeated, in this notation tables are included for t=3, s=1, n up to and including 10 and for t=4, s=1 and 2, n up to and including 5.

An approximate test closely related to the special test (ii) is formulated and discussed.

II. MATHEMATICAL DEVELOPMENT

2.1 General Formulation and Notation for the Method of Paired Comparisons:

A review of the mathematical model and test procedure given by Bradley and Terry will be instructive. In so far as possible we will here use notation consistent with their notation and this work may be properly regarded as a continuation of their research.

Let t denote the total number of treatments which may themselves be referred to as $T_1, \ldots, T_t$. We postulate that these treatments have true ratings, $\pi_1, \ldots, \pi_t$ with the restrictions that $\pi_i \geq 0$, $i = 1, \ldots, t$ and $\sum_{i=1}^{t} \pi_i = 1$. This latter restriction is for convenience only. The treatment parameters are further defined with regard to their role in pairwise comparisons. π_i and π_j are taken to be such that in a comparison of T_i with T_j, the probability that T_i obtains top rating (or perhaps preference) is $\pi_i/(\pi_i+\pi_j)$ with the complementary probability that T_j obtains top rating of $\pi_j/(\pi_i+\pi_j)$.

Let r_{ijk} denote the rank of T_i in the k^{th} repetition of the block in which T_i appears with T_j. Since the ranks 1 and 2 are assigned to each pair of treatments, it follows that $r_{jik} = 3-r_{ijk}$. We retain n to denote the total number of repetitions of the experiment where a repetition by definition consists of a complete set of comparisons of all possible pairs of treatments.

The method of maximum likelihood is used throughout this work for the estimation of treatment ratings. All test procedures depend on the corresponding likelihood ratio statistics. The general properties of such estimation and test procedures are well known and a discussion may be found, for example, in A. M. Mood [12]. We shall denote the maximum likelihood estimates of $\pi_1, \ldots, \pi_t$ by $p_1, \ldots, p_t$ respectively.

Bradley and Terry formulate a general class of tests which may be developed under the framework of this mathematical model. They indicate that two special cases are of particular interest and develop the theory and tables for the first of these. Thus, they test the null hypothesis

H_0: $\pi_i = 1/t$, $i = 1, \ldots, t$ against the alternative

H_1: π_i may have any value subject to the restrictions in their definition. The test depends on the statistic*

$$B_1 = n \sum_{i<j} \log(p_i+p_j) - \sum_i [2n(t-1)-\Sigma r_i] \log p_i \tag{2.1}$$

where Σr_i is the total sum of ranks for T_i.

*We use logarithms to base 10 unless otherwise specified.

We turn now to a consideration of the second special test.

2.2 The Theory of the Two-Group Comparison:

A minimum goal in the development of rank analysis for qualitative measures is the provision of test procedures which permit the flexibility of analysis of variance techniques. This goal has by no means been reached at the present time.

The two-group comparison in the Rank Analysis of Incomplete Block Designs is a step towards achieving the purpose. In the analysis of variance pairwise comparisons may be made using the single degree of freedom techniques. We shall now formulate a rank test procedure which is practically somewhat analogous to that situation. As in analysis of variance it is understood that the comparisons made are chosen on the basis of a priori knowledge of treatment behaviour.

We are concerned with testing the null hypothesis

$$H_0: \ \pi_i = 1/t, \ i = 1, \ldots, t \text{ against the}$$

alternative

$$H_2: \ \pi_i = \pi, \ i = 1, \ldots, s \quad , \ s < t$$

$$\pi_i = \frac{1-s\pi}{t-s}, \ i = s+1, \ldots, t.$$

That is, we are considering that situation wherein treatments in an experiment belong to one or another of two groups which may or may not differ. Under the null hypothesis it is postulated that the two groups themselves do not differ in the attribute in question. Thus true treatment ratings are assumed to be identical. The two groups may differ in size; one group may consist of only a single treatment as is the case in one situation which we consider. It is necessary that all treatments be included in one or the other of the two possibly different groups of treatments.

In the analysis of variance it is often desired to compare group means for two groups of treatments. There it is not required that means within groups differ only due to sampling variation, but this assumption is often inherent in the thinking of the experimenter. For such situations the hypotheses which we have here formulated constitute the complete analogue of the analysis of variance test.

Information bearing on the correctness of the null hypothesis can only be obtained from those paired comparisons which involve rankings of one treatment from each group. The nature of the test definition eliminates information from pairings of two treatments within a group. We do, however, retain the rankings from within group comparisons to facilitate application of this method.

In the general test formulation, for within group rankings of T_i and T_j, the probability that T_i obtains rank 1 is $\frac{1}{2}$ and similarly for T_j; for between group rankings, if T_i be in group 1 and T_j in group 2, the probability of rank 1 for T_i is $(t-s)\pi/[(t-2s)\pi+1]$ and for T_j is $(1-s\pi)/[(t-2s)\pi+1]$. Upon assuming probability independence between blocks or pairs of treatments, we may write the general likelihood function as

$$(2.2)\quad L = (\tfrac{1}{2})^{n[\binom{s}{2}+\binom{t-s}{2}]}\ \pi^{ns[2(t-s)+\frac{3}{2}(s-1)]-\sum_{i=1}^{s}\sum_{j\neq i}\sum_{k=1}^{n} r_{ijk}}$$

$$\left(\frac{1-s\pi}{t-s}\right)^{n(t-s)[2s+\frac{3(t-s-1)}{2}]-\sum_{i=s+1}^{t}\sum_{j\neq i}\sum_{k=1}^{n} r_{ijk}}$$

$$\left(\frac{(t-2s)\pi+1}{t-s}\right)^{-ns(t-s)}.$$

$\sum_{i=1}^{s}\sum_{j\neq i}\sum_{k=1}^{n} r_{ijk}$ is the total sum of ranks for all treatments in the first group of s treatments summed over the k repetitions.

Following the method of maximum likelihood, we then obtain p, the maximum likelihood estimate of π, which may be written:

$$(2.3)\quad p = \frac{ns(4t-s-3)-2\sum_{i=1}^{s}\sum_{j\neq i}\sum_{k=1}^{n} r_{ijk}}{ns(5st-2t^2-6s+3t)-2(2s-t)\sum_{i=1}^{s}\sum_{j\neq i}\sum_{k=1}^{n} r_{ijk}}.$$

Strictly the likelihood ratio statistic is computed as a ratio of maximum values of L (2.2) obtained under the restrictions of H_0 and H_2 on the parameters. In practice any function which is monotonically related to the likelihood ratio will suffice and may be simpler to compute. It has been suggested [4] that we use the statistic

(2.4)

$$B_2 = \left[\sum_{i=1}^{s} \sum_{j\neq i} \sum_{k=1}^{n} r_{ijk} + \frac{n}{2}s(s-1) - 2n(t-1)s\right]\log\left[\frac{(t-s)p}{(t-2s)p+1}\right]$$

$$+\left[\sum_{i=s+1}^{t} \sum_{j\neq i} \sum_{k=1}^{n} r_{ijk} + \frac{n}{2}(t-s)(t-s-1) - 2n(t-1)(t-s)\right]\log\left[\frac{1-sp}{(t-2s)p+1}\right]$$

which bears the required relationship to the likelihood ratio.

We shall now turn to a consideration of the distribution of this statistic.

III. THE DISTRIBUTIONAL PROBLEM

3.1 The Computation of Tables:

For the easy application of the B_2-test, it is desirable that tables be available in such a form that the experimenter may complete his numerical analysis with the calculations of treatment total sums of ranks. Tables for the test procedure are calculated on the assumption H_o. Now this null hypothesis is identical with that for the B_1-test already developed. Tables for the distribution of B_2 may be computed with the aid of Table I prepared by Bradley and Terry.

In computing tables for B_2, the first step was to list all sets of sums of ranks possible in two-group comparison experiments excluding all but one permutation of the within group sums of ranks. Then values of p and B_2 corresponding to each recorded set of sums of ranks were computed by substituting in (2.3) and (2.4) respectively. The probabilities for individual listed sets of sums of ranks were obtained from the probabilities of Table 1 [4] from which a sample section is shown below for three repetitions of the paired comparison design with three treatments.

Table 1. The Distribution of the Likelihood Ratio for General Alternatives*

(Design: t=3, λ=1, b=3, r=2, k=2)

Σr_1	Σr_2	Σr_3	p_1	p_2	p_3	S.S.	B_1	Cumulative Probability
				n=3				
6	9	12	1	0	0	18	0	.0117
6	10	11	1	0	0			
7	8	12	.6667	.3333	0	14	0.8293	.0820
7	9	11	.7045	.2241	.0713	8	1.8402	.2226
7	10	10	.7143	.1429	.1429			
8	8	11	.4545	.4545	.0909	6	2.0771	.4336
8	9	10	.4955	.3102	.1943	2	2.5112	.8906
9	9	9	.3333	.3333	.3333	0	2.7093	1.0000

The probabilities of individual entries in Table 1 were obtained by reduction of the cumulative probabilities shown in that table. Under the null hypothesis, each permutation of the sets of sums of ranks may occur with equal probability. Thus, if we consider the entry 7 8 12 in the sample table above, the extreme right hand column gives, by reduction .0352 as the probability for this entry. Each of the six permutations of this set of sums of ranks then has probability of occurrence .0352/6. Under the formulation of the B_2-test we are interested in the sets 8 : 7, 12; 12: 7, 8 and 7: 8, 12. We are not interested in the

* This table is a section taken from Table I of Bradley and Terry [4].

permutations of the rank sums in the second group, and thus do not show 7 : 12, 8 as distinct from 7: 8, 12. Under the null hypothesis, the probability associated with each of the recorded sets of rank sums is in this case .0352/3 = .0117.

If we take for a second example the entry 8 8 11, we will now be concerned only with the sets 8 : 8 11 and 11 : 8 8 and these carry respectively 2/3 and 1/3 of the probability of this entry in the sample table.

We have now indicated how table entries are obtained together with the appropriate values of p and B_2 and the individual probabilities of each entry. In the tables computed, since small values of B_2 are significant under H_o, we have accumulated probabilities with ascending values of B_2. These cumulative probabilities give significance levels under H_o.

Although the B_2-test is now complete, we have added an additional column in the tables, which lists the sum of squares and may be found under the heading S.S. Here the appropriate sums of squares for rank sums has been computed following the well-known method for single degrees of freedom* with the exception that the sums of squares have not been reduced to a per item basis or for orthogonality.

* The reader is referred to G. W. Snedecor [18] for one discussion of this procedure.

This saves division by a constant throughout this column within each section of the tables. This column is added in the tables for comparison of the ordering by the B_2 statistic and the order that could be obtained on the basis of the sum of squares.

Tables for B_2 have been computed for $t=3$, $s=1$, $n=1,\ldots,10$ and for $t=4$, $s=1,2$, $n=1,\ldots,5$. These tables appear in the appendix of this thesis.

3.2 <u>The Approximate Distribution</u>:

The large sample approximation to this test, as noted by Bradley and Terry [4], may be made by recognizing that

$$(3.1) \qquad -2 \log_e \lambda_2 = 2ns(t-s)\log_e 2 - 2B_2 \log_e 10$$

has approximately the chi square distribution with one degree of freedom for large sample sizes. This follows from the properties of the likelihood ratio tests.

An approximate distribution and test is only useful when we can be confident that the approximation is sufficiently good. Accordingly, a comparison of the means and variances of $-2 \log_e \lambda_2$, obtained where the exact distribution has been tabled, with the mean and the variance of the appropriate chi square distribution has been undertaken. Such a comparison does not give irrefutable evidence on the goodness of the approximation, but a satisfactory comparison is comforting. Numerical values of the mean and variance of $-2 \log_e \lambda_2$ have been computed for t=3, s=1, through n=10 and t=4, s=1,2 through n=5 and appear in Table 2 below.

Table 2. Means and Variances of $-2 \log_e \lambda_2$

n	t=3, s=1		t=4, s=1		t=4, s=2	
	Mean	Variance	Mean	Variance	Mean	Variance
1	1.39	1.92	1.29	2.73	1.22	2.91
2	1.22	2.91	1.12	2.70	1.08	2.46
3	1.12	2.70	1.07	2.38	1.05	2.24
4	1.08	2.46	1.05	2.24	1.03	2.15
5	1.06	2.33	1.04	2.19	1.03	2.11
6	1.05	2.24				
7	1.04	2.17				
8	1.03	2.15				
9	1.03	2.12				
10	1.03	2.11				
∞	1.00	2.00	1.00	2.00	1.00	2.00

From the table, it is seen that the means and variances are converging very rapidly to the values of the approximate distribution. It may also be noted that when t=4 and s=2, the convergence is twice as fast as that for t=3 and s=1. Thus it would appear that the approximation may be used with some confidence for t=4, s=2 when $n \geq 8$ and for t=3, s=1 and t=4, s=1 for $n \geq 15$. If the approximation method were used for smaller values of n, the statistic, $-2 \log_e \lambda_2$, would have too large a value if taken to be distributed as χ_1^2, thus leading to the announcement of too many significant results.

3.3 A Related Test and Distribution

We may sometimes be interested in testing the conditions set forth under H_2, namely, that

$$H_0: \pi_i = \pi, \; i = 1, \ldots, s$$

$$\pi_i = \frac{1-s\pi}{t-s}, \; i = s+1, \ldots, t$$ against the

alternative $H_a: \pi_i, \; i = 1, \ldots, t.$

The exact test is not free of nuisance parameters, but the large sample test is possible. Let λ_3 be the likelihood ratio statistic for this new test. It follows that

$$(3.2) \qquad -2 \log_e \lambda_3 = -2 \log_e \lambda_1 + 2 \log_e \lambda_2$$

where

$$(3.3) \qquad -2 \log_e \lambda_1 = nt(t-1)\log_e 2 - 2B_1 \log_e 10$$

and $-2 \log_e \lambda_3$ has, for large samples, approximately a chi square distribution with $(t-2)$ degrees of freedom. A method of checking the goodness of this approximation has not been found.

The approximate test procedures in the analysis of the paired comparisons experiment may be conveniently summarized in the following table.

Table 3. Large Sample Analysis

(Note that $\log_e 2 = 0.69315$ and $2 \log_e 10 = 4.60518$)

Statistic	Hypotheses	Limiting Distribution
$2ns(t-s)\log_e 2 - 2B_2 \log_e 10$	H_0: $\pi_i = 1/t$, $i=1,\ldots,t$ H_1: $\pi_i = \pi$, $i=1,\ldots,s$ $\pi_i = \frac{1-s\pi}{t-s}$, $i=s+1,\ldots,t$	χ^2_1
$-2 \log_e \lambda_1 + 2 \log_e \lambda_2$	H_0: $\pi_i = \pi$, $i=1,\ldots,s$ $\pi_i = \frac{1-s\pi}{t-s}$, $i=s+1,\ldots,t$ H_1: π_i, $i=1,\ldots,t$	χ^2_{t-2}
$nt(t-1)\log_e 2 - 2B_1 \log_e 10$	H_0: $\pi_i = 1/t$, $i=1,\ldots,t$ H_1: π_i	χ^2_{t-1}

IV. THE EXPERIMENTAL PROCEDURE AND ANALYSIS ILLUSTRATED

4.1 Experimental Procedure:

For the easy application of new statistical techniques, it is useful to divide the procedure into simple elementary steps.

The steps listed below outline the general procedure for applying the B_2-test. For simplicity, three treatments are used to describe the procedure, but the generalisation to more than three treatments will be obvious. Two cases are described, (A) where tables are available and (B) where approximate solutions must be used.

Procedure: (steps 1 through 6 are similar to those set forth by Terry, Bradley and Davis [21]).

Step 1. (Preliminary) Select a panel. The panel should be trained for judgements on material similar to that of the proposed experiment.

Step 2. For each judge and for each repetition take six small containers and number or code them. Place two samples of each of the three treatments in separate, but exactly similar containers and record the identity of the containers. Pair the containers as follows:

Pair	I	II	III
Treatment	$(T_1)(T_2)$	$(T_1)(T_3)$	$(T_2)(T_3)$

Record each pair by the code numbers on a suitable score card in a random order. Now place these pairs before the judge in a random order together with the score card.

Step 3. A judge will test each pair, recording on the score card the value one (1) to that member of the pair containing more of the attribute and the value two (2) to that member containing less of the attribute.

Step 4. Repeat the steps 2 and 3 for the desired number of repetitions.

Step 5. The experimenter will collect and decode the score cards, recording the ranks as follows:

		T_1	T_2	T_3
Score Card	Pair 1 $(T_1)(T_2)$	--	--	x
No. __	2 $(T_1)(T_3)$	--	x	--
Judge ____	3 $(T_2)(T_3)$	x	--	--
	Total	--	--	--

Step 6. Add the ranks on each score card for each treatment. Then for each treatment add the treatment totals on all the score cards. These three numbers (for three treatments) constitute the test data for this experiment.

Step 7. Rearrange the data of step 6 so that the treatment sums of ranks will be listed in ascending order from left to right within groups for the two treatment groups to be tested.

A. The Method Using Tables.

Step 8a. Enter the two-group tables, in the appendix of this thesis, under the total number, n, of repetitions made and for the appropriate values of t and of s. Find the entry corresponding to the rearranged rank sums. These rank sums appear in the left hand columns of the table and the two groups are separated by a colon. The extreme right hand column shows the significance level of the experimental data. If that entry is less than the predetermined rejection level of significance (usually .05 or .01), the experimenter may conclude that significant group differences exist.

Since we do not differentiate between repetitions of the paired comparisons design performed by different judges, cases will often occur where the number of repetitions in an experiment exceed those for which tables are available. The following approximate test may be used in this case.

B. The Approximate Method.

Step 8b. Obtain the estimate, p, by substituting the first group total sums of ranks and the values of n, t, and s in equation (2.3).

Step 9b. When the estimate, p, is obtained, compute B_2 by substituting the values of p, n, t, s and both group total sums of ranks in equation (2.4), where common logarithms are used.

Step 10b. Compute $\chi_1^2 = 2ns(t-s)\log_e 2 - 2B_2\log_e 10$, (the subscript (1) denotes the degrees of freedom). The approximate test is made by comparing this computed value of χ_1^2 with tables of chi square. Large values of χ^2 are significant.

4.2 Illustrative Examples:

Let us consider an experiment* involving pork roasts, which were compared on their flavor characteristics by ranking in pairs. The roasts were obtained from three groups of hogs which had been fattened on three different rations: corn, corn plus a peanut supplement, and corn plus a large peanut supplement. The object was to determine whether there was a significant difference between roasts taken from the hogs that were corn fed as compared to those taken from the hogs which were fed corn plus peanuts (moderate and large supplements). Hence, we have the design t=3, s=1, where the corn-fed samples form the first

* The illustrative example is taken from preliminary experimental results of L. L. Davis, C. M. Kincaid and H. R. Thomas at the Virginia Agricultural Experiment Station.

group and the two corn-plus-peanut-fed sets of samples form the second group.

The analysis for this test is shown below with reference to the separate results of two of the several judges used in the experiment described above. Method (A) with the tables was used for the results of each judge separately and Method (B) was applied for an over-all approximate test. Each judge performed ten repetitions of the paired comparison design ($t=3$, $s=1$).

Procedure:

Steps 1-4. The experimenter followed steps 1-4 with each of two judges for 10 repetitions of the paired comparison design.

Step 5. (Analysis) The experimenter collected and decoded the score cards, and recorded the ranks as shown in Table 4. C denotes the corn ration, Cp, the corn plus peanut supplement ration, and CP, the corn plus large peanut supplement ration.

Step 6. The treatment sums of ranks for C, Cp, and CP are respectively 28, 31, 31 and 33, 28, 29 for the two judges. Hence the total treatment sums of ranks are 61, 59, 60 respectively. The treatment sums of ranks for each judge may be tested following method (A) and the approximate method (B) will be used to test the total data.

Table 4. Rankings for Two Judges in the Pork Experiment.

Repetition	1			2			3			4			5			6			7			8			9			10		
	C	Cp	CP	C	Cp	CP	C	Cp	CP	C	Cp	CP	C	Cp	CP	C	Cp	CP	C	Cp	CP	C	Cp	CP	C	Cp	CP	C	Cp	CP
Pair	Judge 1																													
C, Cp	1	2	x	1	2	x	2	1	x	1	2	x	2	1	x	1	2	x	1	2	x	2	1	x	1	2	x	1	2	x
C, CP	2	x	1	1	x	2	1	x	2	1	x	2	2	x	1	2	x	1	1	x	2	2	x	1	1	x	2	2	x	1
Cp,CP	x	1	2	x	1	2	x	1	2	x	2	1	x	1	2	x	2	1	x	1	2	x	1	2	x	2	1	x	2	1
	Judge 2																													
C, Cp	2	1	x	2	1	x	2	1	x	2	1	x	2	1	x	2	1	x	1	2	x	1	2	x	1	2	x	2	1	x
C, CP	2	x	1	1	x	2	2	x	1	2	x	1	2	x	1	1	x	2	1	x	2	1	x	2	2	x	1	2	x	1
Cp,CP	x	2	1	x	1	2	x	2	1	x	1	2	x	2	1	x	2	1	x	1	2	x	1	2	x	2	1	x	1	2

Step 7. The data is arranged in the proper order as we are testing C against the group Cp and CP.

A. The Method Using Tables

Step 8a. For Judge 1 we enter the tables, Appendix of this thesis, under n=10 and for t=3, s=1. Here we find the entry 28: 31 31 which has a significance level of .5034. It may therefore be concluded that the results of Judge 1 did not present a significant difference (at the .05 or .01 level) between the two groups.

By the same method, we find that the entry 33: 28 29, for Judge 2, has the significance level of .2632. Thus, we may conclude the same for Judge 2 as we did for Judge 1.

B. The Approximate Method

Step 8b. For the total data, n=20, t=3, s=1 and

$\sum_{i=1}^{s} \sum_{j \neq i} \sum_{k=1}^{n} r_{ijk} = 61$. Upon substituting these values in equation (2.3), we obtain the estimate p = .3115.

Step 9b. By substitution in equation (2.4), we have $B_2 = 12.0195$.

Step 10b. Computing χ_1^2, we obtain $\chi_1^2 = .10004$. Consulting the chi square tables, we find that this value of χ_1^2 is not significant at the .05 or .01 levels. Thus, on the basis of the data, we cannot conclude that there is a difference between the two groups tested.

4.3 Illustrative Analysis of the Related Test:

We may be interested in checking our a priori assumption that the experimental items fall into the two specified groups. This may be accomplished by applying the approximate test procedure set forth in section 3.3. The reader will recall that the null and alternative hypotheses were specified mathematically by

$$H_o : \pi_i = \pi, \ i = 1, \ldots, s$$
$$\pi_i = \frac{1-s\pi}{t-s}, \ i = s+1, \ldots, t$$

and

$$H_a : \pi_i, \ i = 1, \ldots, t.$$

This test depends on the statistic $-2 \log_e \lambda_3$ as shown in the summary table 3.

We return to the experimental data of the preceding sections. Total treatment sums of ranks were 61, 59 and 60 for treatment C, Cp and CP respectively. Cp and CP were taken to be equivalent in the second group.

To compute $-2 \log_e \lambda_3$ in (3.2), we require values of B_1 and λ_1, which were obtained following the prescribed methods for the approximate test [21]. For our data $B_1 = 18.0334$ and $-2 \log_e \lambda_1 = .1310$. From the preceding section $-2 \log_e \lambda_2$ had the value .1000 and it follows that

$$-2 \log_e \lambda_3 = .1310 - .1000 = .0310$$

This quantity in the approximate test is taken as an observation in a chi square variate with t-2 = 1 degree of freedom for this example. The result obtained is clearly nonsignificant at any reasonable level of significance. This is as we would expect here. The data has exhibited no effect of treatment on either a B_1 or a B_2 test and any suggested grouping of the sort here tested should be permissible.

This illustration was included to indicate the computational procedure.

V. THE VERIFICATION OF THRESHOLD VALUES

5.1 Introductory Discussion:

M. E. Terry and R. A. Bradley in their article "The Adaptation of the Rank Analysis to Threshold Values" [5] have presented a new technique concerning the verification of threshold values. It is the purpose of this writer to show the applications of the two-group comparisons here developed as they were anticipated in that paper.

Three types of threshold are distinguished: (i) the absolute (or stimulus) threshold, defined as the lower limit of stimulus magnitude that arouses sensation; (ii) the difference threshold, defined as the smallest difference between two stimuli that gives a difference of response; (iii) the terminal threshold, defined as the upper limit of the stimulus beyond which no further increase in the specific response can be obtained. We shall discuss these types separately.

5.2 Special Applications of the Two-Group Comparison:

(i) The Absolute Threshold:

It is assumed that there is a true absolute threshold stimulus, T_ε. We then compare zero stimulus, T_0, and a third stimulus, T_δ, where T_δ represents a possible stimulus of smaller magnitude than T_ε. (i.e. if T_ε represents a dilution and T_ϵ represents a dilution, $\delta = k\epsilon$, $0 < k < 1$.) If T_ϵ is the true threshold value, it should be distinguishable from T_o and T_δ although they themselves are not distinguishably different. To check this situation we hypothesize that T_ϵ is too low. Then we are interested in comparing

H_o: $\pi_\epsilon = \pi_o = \pi_\delta$ against the alternative

H_a: $\pi_\epsilon : \pi_o = \pi_\delta$,

a situation covered in the two-group comparison. Now, the π's denote the true stimulus ratings.

(ii) The Difference Threshold:

In this case, it is assumed that there is a "just noticeable difference," $\epsilon(T_o)$, associated with a stimulus level T_o, and $\delta(T_o)$ is a small increment of stimulus. We then have three stimulus levels, T_o, $T_o-\delta(T_o)$, and $T_o+\epsilon(T_o)$. If $\epsilon(T_o)$ is such that it gives a difference of response, $T_o+\epsilon(T_o)$ will not be equal to T_o and $T_o-\delta(T_o)$. The extent of this inequality may be determined by testing the above hypotheses using the ϵ, o, and δ as defined here and π_o, π_δ, π_ϵ associated with T_o, $T_o-\delta(T_o)$ and $T_o+\epsilon(T_o)$ respectively.

(iii) The Terminal Threshold:

The following discussion follows that of the reference [5] very closely.

If T_f is the terminal threshold value and δ is the smallest positive increment of stimulus, such that $T_{f-\delta}$ and T_f give differential responses, then we postulate that $\pi_f = \pi_{f+k\delta}$, and $\pi_f \neq \pi_{f-k\delta}$ for all $k \geq 1$.

In this case several designs can be used profitably as follows:

1. Using three treatments, $T_{f-\delta}$, T_f, and $T_{f+\delta}$, we have as our null hypothesis

$$H_o: \ \pi_{f-\delta} = \pi_f = \pi_{f+\delta} \text{ with our alternative}$$

$$H_a: \ \pi_{f-\delta} : \pi_f = \pi_{f+\delta}.$$

This model leads to the same methods of testing and analysis as previously outlined.

2. Using four treatments $T_{f-\delta}$, T_f, $T_{f+\delta}$, $T_{f+2\delta}$, the null hypothesis becomes

$$\pi_{f-\delta} = \pi_f = \pi_{f+\delta} = \pi_{f+2\delta}$$

and the alternative hypothesis is now

$$\pi_{f-\delta} : \pi_f = \pi_{f+\delta} = \pi_{f+2\delta}.$$

3. Using the three treatments $T_{f-\delta}$, T_f, $T_{f+\delta}$ and a fourth treatment $T_{f-k\delta}$ so chosen that the increment of stimulus $k\delta$ yields a treatment stimulus which gives no differential response when compared with treatment $T_{f-\delta}$, we have the following situation. Our null hypothesis is

$$H_o : \pi_{f-k\delta} = \pi_{f-\delta} = \pi_f = \pi_{f+\delta} \text{ and the alternative}$$

$$H_a : \pi_{f-k\delta} = \pi_{f-\delta} : \pi_f = \pi_{f+\delta}.$$

Here the method of analysis is straight forward, using the table in the appendix of this thesis for $t=4$, $s=2$.

The decision as to which group of four treatments is to be used will depend to a large extent on the accuracy of location of the terminal threshold. When the value is suspected of being high the more advantageous design would appear to be $f-\delta$, f, $f+\delta$, $f+2\delta$; and when the terminal value may be low, it would seem desirable to use the stimulus $f-k\delta$, $f-\delta$, f, $f+\delta$.

VI. SUMMARY AND CONCLUSION

The purpose of this work was to investigate certain two-group comparisons within a paired comparison experimental design. Tests of the null hypothesis that true treatment ratings are equal against the alternative hypothesis that there are only two groups of treatments which may have different ratings, the treatments within groups having equal ratings, are developed. In fulfilling this purpose, tables were derived for use in the analysis. The method of maximum likelihood is used in developing the mathematical procedure and the tests depend on the likelihood ratio statistics.

The procedure for using the test is outlined and examples are given to illustrate this use.

The problem of threshold values is discussed and an indication of the possible application of these rank analysis methods to this problem is shown.

The procedures given are applicable in most problems where qualitative measurements alone are reliable and are particularly useful in problems involving subjective ranking by a small panel of judges for the detection of differences in food products, color processes and test items of psychology.

VII. ACKNOWLEDGMENTS

The author would like to express his sincere appreciation to Dr. R. A. Bradley, whose guidance has made this paper possible. The author also wishes to acknowledge the assistance of Dr. Boyd Harshbarger, Dr. D. B. Duncan, and Dr. M. E. Terry during his graduate study.

The assistance of for the final typing of this thesis is appreciated.

VIII. BIBLIOGRAPHY

1. Bradley, R. A. and Duncan, D. B. (1950). Statistical Methods for Sensory Difference Tests of Food Quality. Bi-Annual Report to the Bureau of Agricultural Economics, No. 1, (Mimeo).

2. Bradley, R. A. and Terry, M. E. (1951). Statistical Methods for Sensory Difference Tests of Food Quality. Bi-Annual Report to the Bureau of Agricultural Economics, No. 2, (Mimeo).

3. ______________________ (1951). Statistical Methods for Sensory Difference Tests of Food Quality. Bi-Annual Report to the Bureau of Agricultural Economics, No. 3, (Mimeo).

4. ______________________ (1952). Rank Analysis of Incomplete Block Designs I - The Method of Paired Comparisons. Biometrika, ?, (To be published in December).

5. ______________________ (1952). Statistical Methods for Sensory Difference Tests of Food Quality. Bi-Annual Report to the Bureau of Agricultural Economics, No. 4, (Mimeo).

6. Davis, L. L. and Terry, M. E. (1950). A Better, Cheaper Taste Test Method for Smaller Packers, Food Packer, 31, 28.

7. Dixon, W. J. (1940). A Criterion for Testing the Hypothesis that Two Samples are From the Same Population. Annals of Mathematical Statistics, 11, 199.

8. Fisher, R. A. (1925). Statistical Methods for Research Workers, 1st ed., Edinburgh: Oliver and Boyd.

9. ______________ (1947). The Design of Experiments, Edinburgh: Oliver and Boyd.

10. Kendall, M. G. and Babington Smith, B. (1939-40). On the Method of Paired Comparisons. Biometrika, 31, 324.

11. Mann, H. B. and Whitney, D. R. (1947). On a Test of Whether One of Two Random Variables is Stochastically Larger than the Other. Annals of Mathematical Statistics, 18, 50.

12. Mood, A. M. (1950). Introduction to the Theory of Statistics, 1st ed. New York: McGraw-Hill.

13. Mosteller, F. (1951). Remarks on the Method of Paired Comparisons: I. The Least Squares Solutions Assuming Equal Standard Deviations and Equal Correlations. Psychometrika, 16, 3.

14. Mosteller, F. (1951). Remarks on the Method of Paired Comparisons: II. The Effect of an Aberrant Standard Deviation When Equal Standard Deviations and Equal Correlations are Assumed. Psychometrika, 16, 203.

15. ______ (1951). Remarks on the Method of Paired Comparisons: III. A Test of Significance for Paired Comparisons When Equal Standard Deviations and Equal Correlations are Assumed. Psychometrika, 16, 207.

16. Pitman, E. J. G. (1937). Significance Tests Which may be applied to Samples from any Population. Journal of the Royal Statistical Society, Supplement, 4, 119.

17. Scheffé, H. (1943). Statistical Inference in the Non-Parametric Case. Annals of Mathematical Statistics, 14, 305.

18. Snedecor, G. W. (1946). Statistical Methods, 4th ed. Ames: The Iowa State College Press.

19. Swed, F. S. and Eisenhart, C. (1943). Tables for Testing Randomness of Grouping in a Sequence of Alternatives. Annals of Mathematical Statistics, 14, 66.

20. Terry, M. E. (1952). Some Rank Order Tests Which are Most Powerful Against Specific Parametric Alternatives. Annals of Mathematical Statistics, ?, (to be published).

21. Terry, M. E., Bradley, R. A. and Davis, L. L. (1952). New Designs and Techniques for Organoleptic Testing. Food Technology, 6 (7), 250.

22. Thurstone, L. L. (1927). Psychophysical Analysis. American Journal of Psychology, 38, 368.

23. ______________ (1945). The Prediction of Choice. Psychometrika, 10, 237.

24. Wald, A. and Wolfowitz, J. (1940). On a Test Whether Two Samples are from the Same Population. Annals of Mathematical Statistics, 11, 147.

25. Wilcoxon, F. (1945). Individual Comparisons By Ranking Methods. Biometrics Bulletin, 1, 80.

The vita has been removed from the scanned document

X. APPENDIX OF TABLES

TABLES FOR THE TWO-GROUP COMPARISONS IN THE RANK ANALYSIS OF INCOMPLETE BLOCK DESIGNS

The following tables give the values of the likelihood ratio statistic, B_2, and the probabilities that these values of B_2 will not be exceeded. The likelihood estimates of the true treatment ratings are given under the columns $p_1, \ldots, p_t$; Σr_i is the sum of ranks for treatment i, and S.S. is the sum of squares of deviations for treatment rank sums. t denotes the number of treatments and s, the number of treatments in the first group of the two-group comparisons.

Sets of sums of ranks grouped together have identical values of S.S. and B_2, and these values are shown opposite the last entry in each group. In some cases subgroups of each such group have identical estimates for the p's, and these estimates are shown opposite the last entry of the subgroup. Square brackets have been used to indicate the grouping and subgrouping.

(Design: t = 3, s = 1)

Σr_1	:	Σr_2	Σr_3	p_1	$p_2 = p_3$	S.S.	B_2	Cumulative Probability
				n=1				
2	:	3	4	1	0			
4	:	2	3	0	.5000	9	0	.5000
3	:	2	4					
3	:	3	3	.3333	.3333	0	.6021	1.0000
				n=2				
4	:	6	8					
4	:	7	7	1	0			
8	:	4	6					
8	:	5	5	0	.5000	36	0	.1250
5	:	5	8					
5	:	6	7	.6000	.2000			
7	:	4	7					
7	:	5	6	.1429	.4285	9	.9768	.6250
6	:	4	8					
6	:	5	7					
6	:	6	6	.3333	.3333	0	1.2041	1.0000
				n=3				
6	:	9	12					
6	:	10	11	1	0			
12	:	6	9					
12	:	7	8	0	.5000	81	0	.0312
7	:	8	12					
7	:	9	11					
7	:	10	10	.7143	.1428			
11	:	6	10					
11	:	7	9					
11	:	8	8	.0909	.4545	36	1.1740	.2187

Σr_1 : Σr_2 Σr_3	p_1	$p_2=p_3$	S.S.	B_2	Cumulative Probability
n=3 (continued)					
8 : 7 12					
8 : 8 11					
8 : 9 10	.5000	.2500			
10 : 6 11					
10 : 7 10					
10 : 8 9	.2000	.4000	9	1.6586	.6875
9 : 6 12					
9 : 7 11					
9 : 8 10					
9 : 9 9	.3333	.3333	0	1.8062	1.0000
n=4					
8 : 12 16					
8 : 13 15					
8 : 14 14	1	0			
16 : 8 12					
16 : 9 11					
16 : 10 10	0	.5000	144	0	.0078
9 : 11 16					
9 : 12 15					
9 : 13 14	.7778	.1111			
15 : 8 13					
15 : 9 12					
15 : 10 11	.0667	.4666	81	1.3090	.0703
10 : 10 16					
10 : 11 15					
10 : 12 14					
10 : 13 13	.6000	.2000			
14 : 8 14					
14 : 9 13					
14 : 10 12					
14 : 11 11	.1429	.4285	36	1.9538	.2891

Σr_1	:	Σr_2	Σr_3	p_1	$p_2=p_3$	S.S.	B_2	Cumulative Probability
				n=4 (continued)				
11	:	9	16					
11	:	10	15					
11	:	11	14					
11	:	12	13	.4545	.2728			
13	:	8	15					
13	:	9	14					
13	:	10	13					
13	:	11	12	.2308	.3846	9	2.2985	.7266
12	:	8	16					
12	:	9	15					
12	:	10	14					
12	:	11	13					
12	:	12	12	.3333	.3333	0	2.4082	1.0000
				n=5				
10	:	15	20					
10	:	16	19					
10	:	17	18	1	0			
20	:	10	15					
20	:	11	14					
20	:	12	13	0	.5000	225	0	.0020
11	:	14	20					
11	:	15	19					
11	:	16	18					
11	:	17	17	.8182	.0909			
19	:	10	16					
19	:	11	15					
19	:	12	14					
19	:	13	13	.0526	.4737	144	1.4118	.0215
12	:	13	20					
12	:	14	19					
12	:	15	18					
12	:	16	17	.6667	.1667			
18	:	10	17					
18	:	11	16					
18	:	12	15					
18	:	13	14	.1111	.4444	81	2.1732	.1094

Σr_1	:	Σr_2	Σr_3	p_1	$p_2=p_3$	S.S.	B_2	Cumulative Probability
n=5 (continued)								
13	:	12	20					
13	:	13	19					
13	:	14	18					
13	:	15	17					
13	:	16	16	.5385	.2308			
17	:	10	18					
17	:	11	17					
17	:	12	16					
17	:	13	15					
17	:	14	14	.1765	.4118	36	2.6529	.3437
14	:	11	20					
14	:	12	19					
14	:	13	18					
14	:	14	17					
14	:	15	16	.4286	.2857			
16	:	10	19					
16	:	11	18					
16	:	12	17					
16	:	13	16					
16	:	14	15	.2500	.3750	9	2.9229	.7539
15	:	10	20					
15	:	11	19					
15	:	12	18					
15	:	13	17					
15	:	14	16					
15	:	15	15	.3333	.3333	0	3.0103	1.0000
n=6								
12	:	18	24					
12	:	19	23					
12	:	20	22					
12	:	21	21	1	0			
24	:	12	18					
24	:	13	17					
24	:	14	16					
24	:	15	15	0	.5000	324	0	.0005

Σr_1 : Σr_2 Σr_3	p_1	$p_2=p_3$	S.S.	B_2	Cumulative Probability
n=6 (continued)					
13 : 17 24					
13 : 18 23					
13 : 19 22					
13 : 20 21	.8462	.0769			
23 : 12 19					
23 : 13 18					
23 : 14 17					
23 : 15 16	.0435	.4782	225	1.4949	.0063
14 : 16 24					
14 : 17 23					
14 : 18 22					
14 : 19 21					
14 : 20 20	.7143	.1428			
22 : 12 20					
22 : 13 19					
22 : 14 18					
22 : 15 17					
22 : 16 16	.0909	.4546	144	2.3481	.0386
15 : 15 24					
15 : 16 23					
15 : 17 22					
15 : 18 21					
15 : 19 20	.6000	.2000			
21 : 12 21					
21 : 13 20					
21 : 14 19					
21 : 15 18					
21 : 16 17	.1429	.4286	81	2.9306	.1460
16 : 14 24					
16 : 15 23					
16 : 16 22					
16 : 17 21					
16 : 18 20					
16 : 19 19	.5000	.2500			
20 : 12 22					
20 : 13 21					
20 : 14 20					
20 : 15 19					
20 : 16 18					
20 : 17 17	.2000	.4000	36	3.3172	.3877

Σr_1 : Σr_2 Σr_3	p_1	$p_2 = p_3$	S.S.	B_2	Cumulative Probability
n=6 (continued)					
17 : 13 24					
17 : 14 23					
17 : 15 22					
17 : 16 21					
17 : 17 20					
17 : 18 19	.4118	.2941			
19 : 12 23					
19 : 13 22					
19 : 14 21					
19 : 15 20					
19 : 16 19					
19 : 17 18	.2632	.3684	9	3.5396	.7744
18 : 12 24					
18 : 13 23					
18 : 14 22					
18 : 15 21					
18 : 16 20					
18 : 17 19					
18 : 18 18	.3333	.3333	0	3.6124	1.0000
n=7					
14 : 21 28					
14 : 22 27					
14 : 23 26					
14 : 24 25	1	0			
28 : 14 21					
28 : 15 20					
28 : 16 19					
28 : 17 18	0	.5000	441	0	.0001
15 : 20 28					
15 : 21 27					
15 : 22 26					
15 : 23 25					
15 : 24 24	.8667	.0666			
27 : 14 22					
27 : 15 21					
27 : 16 20					
27 : 17 19					
27 : 18 18	.0370	.4815	324	1.5646	.0018

Σr_1 : Σr_2 Σr_3	p_1	$p_2=p_3$	S.S.	B_2	Cumulative Probability
n=7 (continued)					
16 : 19 28					
16 : 20 27					
16 : 21 26					
16 : 22 25					
16 : 23 24	.7500	.1250			
26 : 14 23					
26 : 15 22					
26 : 16 21					
26 : 17 20					
26 : 18 19	.0769	.4616	225	2.4936	.0129
17 : 18 28					
17 : 19 27					
17 : 20 26					
17 : 21 25					
17 : 22 24					
17 : 23 23	.6471	.1764			
25 : 14 24					
25 : 15 23					
25 : 16 22					
25 : 17 21					
25 : 18 20					
25 : 19 19	.1200	.4400	144	3.1592	.0574
18 : 17 28					
18 : 18 27					
18 : 19 26					
18 : 20 25					
18 : 21 24					
18 : 22 23	.5556	.2222			
24 : 14 25					
24 : 15 24					
24 : 16 23					
24 : 17 22					
24 : 18 21					
24 : 19 20	.1667	.4166	81	3.6376	.1796

Σr_1	:	Σr_2	Σr_3	p_1	$p_2=p_3$	S.S.	B_2	Cumulative Probability
				n=7 (continued)				
19	:	16	28					
19	:	17	27					
19	:	18	26					
19	:	19	25					
19	:	20	24					
19	:	21	23					
19	:	22	22	.4737	.2632			
23	:	14	26					
23	:	15	25					
23	:	16	24					
23	:	17	23					
23	:	18	22					
23	:	19	21					
23	:	20	20	.2174	.3913	36	3.9628	.4239
20	:	15	28					
20	:	16	27					
20	:	17	26					
20	:	18	25					
20	:	19	24					
20	:	20	23					
20	:	21	22	.4000	.3000			
22	:	14	27					
22	:	15	26					
22	:	16	25					
22	:	17	24					
22	:	18	23					
22	:	19	22					
22	:	20	21	.2727	.3636	9	4.1522	.7905
21	:	14	28					
21	:	15	27					
21	:	16	26					
21	:	17	25					
21	:	18	24					
21	:	19	23					
21	:	20	22					
21	:	21	21	.3333	.3333	0	4.2144	1.0000

Σr_1 : Σr_2	Σr_3	p_1	p_2=p_3	S.S.	B_2	Cumulative Probability
						n=8
16 : 24	32					
16 : 25	31					
16 : 26	30					
16 : 27	29					
16 : 28	28	1	0			
32 : 16	24					
32 : 17	23					
32 : 18	22					
32 : 19	21					
32 : 20	20	0	.5000	576	0	.0000
17 : 23	32					
17 : 24	31					
17 : 25	30					
17 : 26	29					
17 : 27	28	.8824	.0588			
31 : 16	25					
31 : 17	24					
31 : 18	23					
31 : 19	22					
31 : 20	21	.0323	.4838	441	1.6246	.0005
18 : 22	32					
18 : 23	31					
18 : 24	30					
18 : 25	29					
18 : 26	28					
18 : 27	27	.7778	.1111			
30 : 16	26					
30 : 17	25					
30 : 18	24					
30 : 19	23					
30 : 20	22					
30 : 21	21	.0667	.4666	324	2.6180	.0042

Σr_1	:	Σr_2	Σr_3	p_1	$p_2=p_3$	S.S.	B_2	Cumulative Probability
			n=8 (continued)					
19	:	21	32					
19	:	22	31					
19	:	23	30					
19	:	24	29					
19	:	25	28					
19	:	26	27	.6842	.1579			
29	:	16	27					
29	:	17	26					
29	:	18	25					
29	:	19	24					
29	:	20	23					
29	:	21	22	.1034	.4483	225	3.3533	.0213
20	:	20	32					
20	:	21	31					
20	:	22	30					
20	:	23	29					
20	:	24	28					
20	:	25	27					
20	:	26	26	.6000	.2000			
28	:	16	28					
28	:	17	27					
28	:	18	26					
28	:	19	25					
28	:	20	24					
28	:	21	23					
28	:	22	22	.1429	.4286	144	3.9075	.0768
21	:	19	32					
21	:	20	31					
21	:	21	30					
21	:	22	29					
21	:	23	28					
21	:	24	27					
21	:	25	26	.5238	.2381			
27	:	16	29					
27	:	17	28					
27	:	18	27					
27	:	19	26					
27	:	20	25					
27	:	21	24					
27	:	22	23	.1852	.4074	81	4.3158	.2101

Σr_1	:	Σr_2	Σr_3	p_1	$p_2=p_3$	S.S.	B_2	Cumulative Probability
				n=8 (continued)				
22	:	18	32					
22	:	19	31					
22	:	20	30					
22	:	21	29					
22	:	22	28					
22	:	23	27					
22	:	24	26					
22	:	25	25	.4545	.2728			
26	:	16	30					
26	:	17	29					
26	:	18	28					
26	:	19	27					
26	:	20	26					
26	:	21	25					
26	:	22	24					
26	:	23	23	.2308	.3846	36	4.5970	.4545
23	:	17	32					
23	:	18	31					
23	:	19	30					
23	:	20	29					
23	:	21	28					
23	:	22	27					
23	:	23	26					
23	:	24	25	.3913	.3044			
25	:	16	31					
25	:	17	30					
25	:	18	29					
25	:	19	28					
25	:	20	27					
25	:	21	26					
25	:	22	25					
25	:	23	24	.2800	.3600	9	4.7621	.8036
24	:	16	32					
24	:	17	31					
24	:	18	30					
24	:	19	29					
24	:	20	28					
24	:	21	27					
24	:	22	26					
24	:	23	25					
24	:	24	24	.3333	.3333	0	4.8165	1.000

Σr_1	:	Σr_2	Σr_3	P_1	$P_2=P_3$	S.S.	B_2	Cumulative Probability
				n=9				
18	:	27	36					
18	:	28	35					
18	:	29	34					
18	:	30	33					
18	:	31	32	1	0			
36	:	18	27					
36	:	19	26					
36	:	20	25					
36	:	21	24					
36	:	22	23	0	.5000	729	0	.0000
19	:	26	36					
19	:	27	35					
19	:	28	34					
19	:	29	33					
19	:	30	32					
19	:	31	31	.8947	.0526			
35	:	18	28					
35	:	19	27					
35	:	20	26					
35	:	21	25					
35	:	22	24					
35	:	23	23	.0286	.4857	576	1.6772	.0001
20	:	25	36					
20	:	26	35					
20	:	27	34					
20	:	28	33					
20	:	29	32					
20	:	30	31	.8000	.1000			
34	:	18	29					
34	:	19	28					
34	:	20	27					
34	:	21	26					
34	:	22	25					
34	:	23	24	.0588	.4706	441	2.7269	.0013

Σr_1	:	Σr_2	Σr_3	p_1	$p_2 = p_3$	S.S.	B_2	Cumulative Probability
n=9 (continued)								
21	:	24	36					
21	:	25	35					
21	:	26	34					
21	:	27	33					
21	:	28	32					
21	:	29	31					
21	:	30	30	.7143	.1428			
33	:	18	30					
33	:	19	29					
33	:	20	28					
33	:	21	27					
33	:	22	26					
33	:	23	25					
33	:	24	24	.0909	.4546	324	3.5222	.0075
22	:	23	36					
22	:	24	35					
22	:	25	34					
22	:	26	33					
22	:	27	32					
22	:	28	31					
22	:	29	30	.6364	.1818			
32	:	18	31					
32	:	19	30					
32	:	20	29					
32	:	21	28					
32	:	22	27					
32	:	23	26					
32	:	24	25	.1250	.4375	225	4.1408	.0309
23	:	22	36					
23	:	23	35					
23	:	24	34					
23	:	25	33					
23	:	26	32					
23	:	27	31					
23	:	28	30					
23	:	29	29	.5652	.2174	144	4.6188	.0962

Σr_1 : Σr_2 Σr_3	p_1	$p_2=p_3$	S.S.	B_2	Cumulative Probability
n=9 (continued)					
31 : 18 32					
31 : 19 31					
31 : 20 30					
31 : 21 29					
31 : 22 28					
31 : 23 27					
31 : 24 26					
31 : 25 25	.1613	.4194	144	4.6188	.0962
24 : 21 36					
24 : 22 35					
24 : 23 34					
24 : 24 33					
24 : 25 32					
24 : 26 31					
24 : 27 30					
24 : 28 29	.5000	.2500			
30 : 18 33					
30 : 19 32					
30 : 20 31					
30 : 21 30					
30 : 22 29					
30 : 23 28					
30 : 24 27					
30 : 25 26	.2000	.4000	81	4.9758	.2379
25 : 20 36					
25 : 21 35					
25 : 22 34					
25 : 23 33					
25 : 24 32					
25 : 25 31					
25 : 26 30					
25 : 27 29					
25 : 28 28	.4400	.2800	36	5.2239	.4807

Σr_1 : Σr_2	Σr_3	p_1	p_2=p_3	S.S.	B_2	Cumulative Probability
n=9 (continued)						
29 : 18	34					
29 : 19	33					
29 : 20	32					
29 : 21	31					
29 : 22	30					
29 : 23	29					
29 : 24	28					
29 : 25	27					
29 : 26	26	.2414	.3793	36	5.2239	.4807
26 : 19	36					
26 : 20	35					
26 : 21	34					
26 : 22	33					
26 : 23	32					
26 : 24	31					
26 : 25	30					
26 : 26	29					
26 : 27	28	.3846	.3077			
28 : 18	35					
28 : 19	34					
28 : 20	33					
28 : 21	32					
28 : 22	31					
28 : 23	30					
28 : 24	29					
28 : 25	28					
28 : 26	27	.2857	.3572	9	5.3701	.8145
27 : 18	36					
27 : 19	35					
27 : 20	34					
27 : 21	33					
27 : 22	32					
27 : 23	31					
27 : 24	30					
27 : 25	29					
27 : 26	28					
27 : 27	27	.3333	.3333	0	5.4185	1.0000

Σr_1	:	Σr_2	Σr_3	p_1	$p_2 = p_3$	S.S.	B_2	Cumulative Probability
				n=10				
20	:	30	40					
20	:	31	39					
20	:	32	38					
20	:	33	37					
20	:	34	36					
20	:	35	35	1	0			
40	:	20	30					
40	:	21	29					
40	:	22	28					
40	:	23	27					
40	:	24	26					
40	:	25	25	0	.5000	900	0	.0000
21	:	29	40					
21	:	30	39					
21	:	31	38					
21	:	32	37					
21	:	33	36					
21	:	34	35	.9048	.0476			
39	:	20	31					
39	:	21	30					
39	:	22	29					
39	:	23	28					
39	:	24	27					
39	:	25	26	.0256	.4872	729	1.7244	.0000
22	:	28	40					
22	:	29	39					
22	:	30	38					
22	:	31	37					
22	:	32	36					
22	:	33	35					
22	:	34	34	.8182	.0909			
38	:	20	32					
38	:	21	31					
38	:	22	30					
38	:	23	29					
38	:	24	28					
38	:	25	27					
38	:	26	26	.0526	.4737	576	2.8237	.0004

Σr_1	:	Σr_2	Σr_3	P_1	P_2=P_3	S.S.	B_2	Cumulative Probability
n=10 (continued)								
23	:	27	40					
23	:	28	39					
23	:	29	38					
23	:	30	37					
23	:	31	36					
23	:	32	35					
23	:	33	34	.7391	.1304			
37	:	20	33					
37	:	21	32					
37	:	22	31					
37	:	23	30					
37	:	24	29					
37	:	25	28					
37	:	26	27	.0811	.4594	441	3.6716	.0026
24	:	26	40					
24	:	27	39					
24	:	28	38					
24	:	29	37					
24	:	30	36					
24	:	31	35					
24	:	32	34					
24	:	33	33	.6667	.1666			
36	:	20	34					
36	:	21	33					
36	:	22	32					
36	:	23	31					
36	:	24	30					
36	:	25	29					
36	:	26	28					
36	:	27	27	.1111	.4444	324	4.3464	.0118
25	:	25	40					
25	:	26	39					
25	:	27	38					
25	:	28	37					
25	:	29	36					
25	:	30	35					
25	:	31	34					
25	:	32	33	.6000	.2000	225	4.8844	.0414

Σr_1	:	Σr_2	Σr_3	p_1	$p_2 = p_3$	S.S.	B_2	Cumulative Probability
				n=10 (continued)				
35	:	20	35					
35	:	21	34					
35	:	22	33					
35	:	23	32					
35	:	24	31					
35	:	25	30					
35	:	26	29					
35	:	27	28	.1429	.4286	225	4.8844	.0414
26	:	24	40					
26	:	25	39					
26	:	26	38					
26	:	27	37					
26	:	28	36					
26	:	29	35					
26	:	30	34					
26	:	31	33					
26	:	32	32	.5385	.2308			
34	:	20	36					
34	:	21	35					
34	:	22	34					
34	:	23	33					
34	:	24	32					
34	:	25	31					
34	:	26	30					
34	:	27	29					
34	:	28	28	.1765	.4118	144	5.3059	.1153
27	:	23	40					
27	:	24	39					
27	:	25	38					
27	:	26	37					
27	:	27	36					
27	:	28	35					
27	:	29	34					
27	:	30	33					
27	:	31	32	.4815	.2592	81	5.6237	.2632

Σr_1 : Σr_2 Σr_3	p_1	$p_2=p_3$	S.S.	B_2	Cumulative Probability
n=10 (continued)					
33 : 20 37					
33 : 21 36					
33 : 22 35					
33 : 23 34					
33 : 24 33					
33 : 25 32					
33 : 26 31					
33 : 27 30					
33 : 28 29	.2121	.3940	81	5.6237	.2632
28 : 22 40					
28 : 23 39					
28 : 24 38					
28 : 25 37					
28 : 26 36					
28 : 27 35					
28 : 28 34					
28 : 29 33					
28 : 30 32					
28 : 31 31	.4286	.2857			
32 : 20 38					
32 : 21 37					
32 : 22 36					
32 : 23 35					
32 : 24 34					
32 : 25 33					
32 : 26 32					
32 : 27 31					
32 : 28 30					
32 : 29 29	.2500	.3750	36	5.8457	.5034
29 : 21 40					
29 : 22 39					
29 : 23 38					
29 : 24 37					
29 : 25 36					
29 : 26 35					
29 : 27 34					
29 : 28 33					
29 : 29 32					
29 : 30 31	.3793	.3104	9	5.9772	.8238

Σr_1 : Σr_2 Σr_3	p_1	$p_2=p_3$	S.S.	B_2	Cumulative Probability
n=10 (continued)					
31 : 20 39					
31 : 21 38					
31 : 22 37					
31 : 23 36					
31 : 24 35					
31 : 25 34					
31 : 26 33					
31 : 27 32					
31 : 28 31					
31 : 29 30	.2903	.3548	9	5.9772	.8238
30 : 20 40					
30 : 21 39					
30 : 22 38					
30 : 23 37					
30 : 24 36					
30 : 25 35					
30 : 26 34					
30 : 27 33					
30 : 28 32					
30 : 29 31					
30 : 30 30	.3333	.3333	0	6.0206	1.0000

(Design: $t=4$, $s=1$)

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
					$n=1$				
3	:	4	5	6					
3	:	5	5	5	1	0			
6	:	3	4	5					
6	:	4	4	4	0	.3333	36	0	.2500
4	:	3	5	6					
4	:	4	4	6					
4	:	4	5	5	.4000	.2000			
5	:	3	4	6					
5	:	3	5	5					
5	:	4	4	5	.1429	.2857	4	.8293	1.0000
					$n=2$				
6	:	8	10	12					
6	:	8	11	11					
6	:	9	9	12					
6	:	9	10	11					
6	:	10	10	10	1	0			
12	:	6	8	10					
12	:	6	9	9					
12	:	7	7	10					
12	:	7	8	9					
12	:	8	8	8	0	.3333	144	0	.0312
7	:	7	10	12					
7	:	7	11	11					
7	:	8	9	12					
7	:	8	10	11					
7	:	9	9	11					
7	:	9	10	10	.6250	.1250			
11	:	6	8	11					
11	:	6	9	10					
11	:	7	7	11					
11	:	7	8	10					
11	:	7	9	9					
11	:	8	8	9	.0625	.3125	64	1.1740	.2187

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=2 (continued)					
8	:	6	10	12					
8	:	6	11	11					
8	:	7	9	12					
8	:	7	10	11					
8	:	8	8	12					
8	:	8	9	11					
8	:	8	10	10					
8	:	9	9	10	.4000	.2000			
10	:	6	8	12					
10	:	6	9	11					
10	:	6	10	10					
10	:	7	7	12					
10	:	7	8	11					
10	:	7	9	10					
10	:	8	8	10					
10	:	8	9	9	.1429	.2857	16	1.6586	.6875
9	:	6	9	12					
9	:	6	10	11					
9	:	7	8	12					
9	:	7	9	11					
9	:	7	10	10					
9	:	8	8	11					
9	:	8	9	10					
9	:	9	9	9	.2500	.2500	0	1.8062	1.0000
				n=3					
9	:	12	15	18					
9	:	12	16	17					
9	:	13	14	18					
9	:	13	15	17					
9	:	13	16	16					
9	:	14	14	17					
9	:	14	15	16					
9	:	15	15	15	1	0			
18	:	9	12	15					
18	:	9	13	14					
18	:	10	11	15					
18	:	10	12	14					
18	:	10	13	13					
18	:	11	11	14					
18	:	11	12	13					
18	:	12	12	12	0	.3333	324	0	.0039

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=3 (continued)									
10	:	11	15	18					
10	:	11	16	17					
10	:	12	14	18					
10	:	12	15	17					
10	:	12	16	16					
10	:	13	13	18					
10	:	13	14	17					
10	:	13	15	16					
10	:	14	14	16					
10	:	14	15	15	.7273	.0909			
17	:	9	12	16					
17	:	9	13	15					
17	:	9	14	14					
17	:	10	11	16					
17	:	10	12	15					
17	:	10	13	14					
17	:	11	11	15					
17	:	11	12	14					
17	:	11	13	13					
17	:	12	12	13	.0400	.3200	196	1.3634	.0391
11	:	10	15	18					
11	:	10	16	17					
11	:	11	14	18					
11	:	11	15	17					
11	:	11	16	16					
11	:	12	13	18					
11	:	12	14	17					
11	:	12	15	16					
11	:	13	13	17					
11	:	13	14	16					
11	:	13	15	15					
11	:	14	14	15	.5385	.1538	100	2.0704	.1797

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=3 (continued)					
16	:	9	12	17					
16	:	9	13	16					
16	:	9	14	15					
16	:	10	11	17					
16	:	10	12	16					
16	:	10	13	15					
16	:	10	14	14					
16	:	11	11	16					
16	:	11	12	15					
16	:	11	13	14					
16	:	12	12	14					
16	:	12	13	13	.0870	.3043	100	2.0704	.1797
12	:	9	15	18					
12	:	9	16	17					
12	:	10	14	18					
12	:	10	15	17					
12	:	10	16	16					
12	:	11	13	18					
12	:	11	14	17					
12	:	11	15	16					
12	:	12	12	18					
12	:	12	13	17					
12	:	12	14	16					
12	:	12	15	15					
12	:	13	13	16					
12	:	13	14	15					
12	:	14	14	14	.4000	.2000			
15	:	9	12	18					
15	:	9	13	17					
15	:	9	14	16					
15	:	9	15	15					
15	:	10	11	18					
15	:	10	12	17					
15	:	10	13	16					
15	:	10	14	15					
15	:	11	11	17					
15	:	11	12	16					
15	:	11	13	15					
15	:	11	14	14					
15	:	12	12	15					
15	:	12	13	14					
15	:	13	13	13	.1429	.2857	36	2.4879	.5078

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
					n=3 (continued)				
13	:	9	14	18					
13	:	9	15	17					
13	:	9	16	16					
13	:	10	13	18					
13	:	10	14	17					
13	:	10	15	16					
13	:	11	12	18					
13	:	11	13	17					
13	:	11	14	16					
13	:	11	15	15					
13	:	12	12	17					
13	:	12	13	16					
13	:	12	14	15					
13	:	13	13	15					
13	:	13	14	14	.2941	.2353			
14	:	9	13	18					
14	:	9	14	17					
14	:	9	15	16					
14	:	10	12	18					
14	:	10	13	17					
14	:	10	14	16					
14	:	10	15	15					
14	:	11	11	18					
14	:	11	12	17					
14	:	11	13	16					
14	:	11	14	15					
14	:	12	12	16					
14	:	12	13	15					
14	:	12	14	14					
14	:	13	13	14	.2104	.2632	4	2.6851	1.0000
					n=4				
12	:	16	20	24					
12	:	16	21	23					
12	:	16	22	22					
12	:	17	19	24					
12	:	17	20	23					
12	:	17	21	22					
12	:	18	18	24					
12	:	18	19	23	1	0	576	0	.0005

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
					n=4 (continued)				
12	:	18	20	22					
12	:	18	21	21					
12	:	19	19	22					
12	:	19	20	21					
12	:	20	20	20	1	0			
24	:	12	16	20					
24	:	12	17	19					
24	:	12	18	18					
24	:	13	15	20					
24	:	13	16	19					
24	:	13	17	18					
24	:	14	14	20					
24	:	14	15	19					
24	:	14	16	18					
24	:	14	17	17					
24	:	15	15	18					
24	:	15	16	17					
24	:	16	16	16	0	.3333	576	0	.0005
13	:	15	20	24					
13	:	15	21	23					
13	:	15	22	22					
13	:	16	19	24					
13	:	16	20	23					
13	:	16	21	22					
13	:	17	18	24					
13	:	17	19	23					
13	:	17	20	22					
13	:	17	21	21					
13	:	18	18	23					
13	:	18	19	22					
13	:	18	20	21					
13	:	19	19	21					
13	:	19	20	20	.7857	.0714	400	1.4949	.0063

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=4 (continued)									
23	:	12	16	21					
23	:	12	17	20					
23	:	12	18	19					
23	:	13	15	21					
23	:	13	16	20					
23	:	13	17	19					
23	:	13	18	18					
23	:	14	14	21					
23	:	14	15	20					
23	:	14	16	19					
23	:	14	17	18					
23	:	15	15	19					
23	:	15	16	18					
23	:	15	17	17					
23	:	16	16	17	.0294	.3235	400	1.4949	.0063
14	:	14	20	24					
14	:	14	21	23					
14	:	14	22	22					
14	:	15	19	24					
14	:	15	20	23					
14	:	15	21	22					
14	:	16	18	24					
14	:	16	19	23					
14	:	16	20	22					
14	:	16	21	21					
14	:	17	17	24					
14	:	17	18	23					
14	:	17	19	22					
14	:	17	20	21					
14	:	18	18	22					
14	:	18	19	21					
14	:	18	20	20					
14	:	19	19	20	.6250	.1250	256	2.3481	.0386

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=4 (continued)									
22	:	12	16	22					
22	:	12	17	21					
22	:	12	18	20					
22	:	12	19	19					
22	:	13	15	22					
22	:	13	16	21					
22	:	13	17	20					
22	:	13	18	19					
22	:	14	14	22					
22	:	14	15	21					
22	:	14	16	20					
22	:	14	17	19					
22	:	14	18	18					
22	:	15	15	20					
22	:	15	16	19					
22	:	15	17	18					
22	:	16	16	18					
22	:	16	17	17	.0625	.3125	256	2.3481	.0386
15	:	13	20	24					
15	:	13	21	23					
15	:	13	22	22					
15	:	14	19	24					
15	:	14	20	23					
15	:	14	21	22					
15	:	15	18	24					
15	:	15	19	23					
15	:	15	20	22					
15	:	15	21	21					
15	:	16	17	24					
15	:	16	18	23					
15	:	16	19	22					
15	:	16	20	21					
15	:	17	17	23					
15	:	17	18	22					
15	:	17	19	21					
15	:	17	20	20					
15	:	18	18	21					
15	:	18	19	20					
15	:	19	19	19	.5000	.1667	144	2.9306	.1460

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=4 (continued)									
21	:	12	16	23					
21	:	12	17	22					
21	:	12	18	21					
21	:	12	19	20					
21	:	13	15	23					
21	:	13	16	22					
21	:	13	17	21					
21	:	13	18	20					
21	:	13	19	19					
21	:	14	14	23					
21	:	14	15	22					
21	:	14	16	21					
21	:	14	17	20					
21	:	14	18	19					
21	:	15	15	21					
21	:	15	16	20					
21	:	15	17	19					
21	:	15	18	18					
21	:	16	16	19					
21	:	16	17	18					
21	:	17	17	17	.1000	.3000	144	2.9306	.1460
16	:	12	20	24					
16	:	12	21	23					
16	:	12	22	22					
16	:	13	19	24					
16	:	13	20	23					
16	:	13	21	22					
16	:	14	18	24					
16	:	14	19	23					
16	:	14	20	22					
16	:	14	21	21					
16	:	15	17	24					
16	:	15	18	23					
16	:	15	19	22					
16	:	15	20	21					
16	:	16	16	24					
16	:	16	17	23					
16	:	16	18	22					
16	:	16	19	21					
16	:	16	20	20	.4000	.2000	64	3.3172	.3877

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=4 (continued)									
16	:	17	17	22					
16	:	17	18	21					
16	:	17	19	20					
16	:	18	18	20					
16	:	18	19	19	.4000	.2000			
20	:	12	16	24					
20	:	12	17	23					
20	:	12	18	22					
20	:	12	19	21					
20	:	12	20	20					
20	:	13	15	24					
20	:	13	16	23					
20	:	13	17	22					
20	:	13	18	21					
20	:	13	19	20					
20	:	14	14	24					
20	:	14	15	23					
20	:	14	16	22					
20	:	14	17	21					
20	:	14	18	20					
20	:	14	19	19					
20	:	15	15	22					
20	:	15	16	21					
20	:	15	17	20					
20	:	15	18	19					
20	:	16	16	20					
20	:	16	17	19					
20	:	16	18	18					
20	:	17	17	18	.1429	.2857	64	3.3172	.3877
17	:	12	19	24					
17	:	12	20	23					
17	:	12	21	22					
17	:	13	18	24					
17	:	13	19	23					
17	:	13	20	22					
17	:	13	21	21					
17	:	14	17	24					
17	:	14	18	23					
17	:	14	19	22					
17	:	14	20	21	.3182	.2273	4	3.5396	.7744

Σr_1 :	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=4 (continued)								
17 :	15	16	24					
17 :	15	17	23					
17 :	15	18	22					
17 :	15	19	21					
17 :	15	20	20					
17 :	16	16	23					
17 :	16	17	22					
17 :	16	18	21					
17 :	16	19	20					
17 :	17	17	21					
17 :	17	18	20					
17 :	17	19	19					
17 :	18	18	19	.3182	.2273			
19 :	12	17	24					
19 :	12	18	23					
19 :	12	19	22					
19 :	12	20	21					
19 :	13	16	24					
19 :	13	17	23					
19 :	13	18	22					
19 :	13	19	21					
19 :	13	20	20					
19 :	14	15	24					
19 :	14	16	23					
19 :	14	17	22					
19 :	14	18	21					
19 :	14	19	20					
19 :	15	15	23					
19 :	15	16	22					
19 :	15	17	21					
19 :	15	18	20					
19 :	15	19	19					
19 :	16	16	21					
19 :	16	17	20					
19 :	16	18	19					
19 :	17	17	19					
19 :	17	18	18	.1923	.2692	4	3.5396	.7744

Σr_1 :	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=4 (continued)								
18 :	12	18	24					
18 :	12	19	23					
18 :	12	20	22					
18 :	12	21	21					
18 :	13	17	24					
18 :	13	18	23					
18 :	13	19	22					
18 :	13	20	21					
18 :	14	16	24					
18 :	14	17	23					
18 :	14	18	22					
18 :	14	19	21					
18 :	14	20	20					
18 :	15	15	24					
18 :	15	16	23					
18 :	15	17	22					
18 :	15	18	21					
18 :	15	19	20					
18 :	16	16	22					
18 :	16	17	21					
18 :	16	18	20					
18 :	16	19	19					
18 :	17	17	20					
18 :	17	18	19					
18 :	18	18	18	.2500	.2500	0	3.6124	1.0000
n=5								
15 :	20	25	30					
15 :	20	26	29					
15 :	20	27	28					
15 :	21	24	30					
15 :	21	25	29					
15 :	21	26	28					
15 :	21	27	27					
15 :	22	23	30					
15 :	22	24	29					
15 :	22	25	28					
15 :	22	26	27	1	0	900	0	.0001

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2 = p_3 = p_4$	S.S.	B_2	Cumulative Probability
n=5 (continued)									
15	:	23	23	29					
15	:	23	24	28					
15	:	23	25	27					
15	:	23	26	26					
15	:	24	24	27					
15	:	24	25	26					
15	:	25	25	25	1	0			
30	:	15	20	25					
30	:	15	21	24					
30	:	15	22	23					
30	:	16	19	25					
30	:	16	20	24					
30	:	16	21	23					
30	:	16	22	22					
30	:	17	18	25					
30	:	17	19	24					
30	:	17	20	23					
30	:	17	21	22					
30	:	18	18	24					
30	:	18	19	23					
30	:	18	20	22					
30	:	18	21	21					
30	:	19	19	22					
30	:	19	20	21					
30	:	20	20	20	0	.3333	900	0	.0001
16	:	19	25	30					
16	:	19	26	29					
16	:	19	27	28					
16	:	20	24	30					
16	:	20	25	29					
16	:	20	26	28					
16	:	20	27	27					
16	:	21	23	30					
16	:	21	24	29					
16	:	21	25	28					
16	:	21	26	27					
16	:	22	22	30					
16	:	22	23	29					
16	:	22	24	28					
16	:	22	25	27					
16	:	22	26	26	.8235	.0588	676	1.5955	.0010

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=5 (continued)									
16	:	23	23	28					
16	:	23	24	27					
16	:	23	25	26					
16	:	24	24	26					
16	:	24	25	25	.8235	.0588			
29	:	15	20	26					
29	:	15	21	25					
29	:	15	22	24					
29	:	15	23	23					
29	:	16	19	26					
29	:	16	20	25					
29	:	16	21	24					
29	:	16	22	23					
29	:	17	18	26					
29	:	17	19	25					
29	:	17	20	24					
29	:	17	21	23					
29	:	17	22	22					
29	:	18	18	25					
29	:	18	19	24					
29	:	18	20	23					
29	:	18	21	22					
29	:	19	19	23					
29	:	19	20	22					
29	:	19	21	21					
29	:	20	20	21	.0233	.3256	676	1.5955	.0010
17	:	18	25	30					
17	:	18	26	29					
17	:	18	27	28					
17	:	19	24	30					
17	:	19	25	29					
17	:	19	26	28					
17	:	19	27	27					
17	:	20	23	30					
17	:	20	24	29					
17	:	20	25	28					
17	:	20	26	27					
17	:	21	22	30					
17	:	21	23	29					
17	:	21	24	28					
17	:	21	25	27	.6842	.1053	484	2.5581	.0074

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
					n=5 (continued)				
17	:	21	26	26					
17	:	22	22	29					
17	:	22	23	28					
17	:	22	24	27					
17	:	22	25	26					
17	:	23	23	27					
17	:	23	24	26					
17	:	23	25	25					
17	:	24	24	25	.6842	.1053			
28	:	15	20	27					
28	:	15	21	26					
28	:	15	22	25					
28	:	15	23	24					
28	:	16	19	27					
28	:	16	20	26					
28	:	16	21	25					
28	:	16	22	24					
28	:	16	23	23					
28	:	17	18	27					
28	:	17	19	26					
28	:	17	20	25					
28	:	17	21	24					
28	:	17	22	23					
28	:	18	18	26					
28	:	18	19	25					
28	:	18	20	24					
28	:	18	21	23					
28	:	18	22	22					
28	:	19	19	24					
28	:	19	20	23					
28	:	19	21	22					
28	:	20	20	22					
28	:	20	21	21	.0488	.3171	484	2.5581	.0074
18	:	17	25	30					
18	:	17	26	29					
18	:	17	27	28					
18	:	18	24	30					
18	:	18	25	29					
18	:	18	26	28					
18	:	18	27	27	.5714	.1429	324	3.2598	.0351

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=5 (continued)					
18	:	19	23	30					
18	:	19	24	29					
18	:	19	25	28					
18	:	19	26	27					
18	:	20	22	30					
18	:	20	23	29					
18	:	20	24	28					
18	:	20	25	27					
18	:	20	26	26					
18	:	21	21	30					
18	:	21	22	29					
18	:	21	23	28					
18	:	21	24	27					
18	:	21	25	26					
18	:	22	22	28					
18	:	22	23	27					
18	:	22	24	26					
18	:	22	25	25					
18	:	23	23	26					
18	:	23	24	25					
18	:	24	24	24	.5714	.1429			
27	:	15	20	28					
27	:	15	21	27					
27	:	15	22	26					
27	:	15	23	25					
27	:	15	24	24					
27	:	16	19	28					
27	:	16	20	27					
27	:	16	21	26					
27	:	16	22	25					
27	:	16	23	24					
27	:	17	18	28					
27	:	17	19	27					
27	:	17	20	26					
27	:	17	21	25					
27	:	17	22	24					
27	:	17	23	23					
27	:	18	18	27					
27	:	18	19	26					
27	:	18	20	25					
27	:	18	21	24					
27	:	18	22	23	.0769	.3077	324	3.2598	.0351

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=5 (continued)									
27	:	19	19	25					
27	:	19	20	24					
27	:	19	21	23					
27	:	19	22	22					
27	:	20	20	23					
27	:	20	21	22					
27	:	21	21	21	.0769	.3077	324	3.2598	.0351
19	:	16	25	30					
19	:	16	26	29					
19	:	16	27	28					
19	:	17	24	30					
19	:	17	25	29					
19	:	17	26	28					
19	:	17	27	27					
19	:	18	23	30					
19	:	18	24	29					
19	:	18	25	28					
19	:	18	26	27					
19	:	19	22	30					
19	:	19	23	29					
19	:	19	24	28					
19	:	19	25	27					
19	:	19	26	26					
19	:	20	21	30					
19	:	20	22	29					
19	:	20	23	28					
19	:	20	24	27					
19	:	20	25	26					
19	:	21	21	29					
19	:	21	22	28					
19	:	21	23	27					
19	:	21	24	26					
19	:	21	25	25					
19	:	22	22	27					
19	:	22	23	26					
19	:	22	24	25					
19	:	23	23	25					
19	:	23	24	24	.4783	.1739	196	3.7778	.1185

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=5 (continued)					
26	:	15	20	29					
26	:	15	21	28					
26	:	15	22	27					
26	:	15	23	26					
26	:	15	24	25					
26	:	16	19	29					
26	:	16	20	28					
26	:	16	21	27					
26	:	16	22	26					
26	:	16	23	25					
26	:	16	24	24					
26	:	17	18	29					
26	:	17	19	28					
26	:	17	20	27					
26	:	17	21	26					
26	:	17	22	25					
26	:	17	23	24					
26	:	18	18	28					
26	:	18	19	27					
26	:	18	20	26					
26	:	18	21	25					
26	:	18	22	24					
26	:	18	23	23					
26	:	19	19	26					
26	:	19	20	25					
26	:	19	21	24					
26	:	19	22	23					
26	:	20	20	24					
26	:	20	21	23					
26	:	20	22	22					
26	:	21	21	22	.1081	.2973	196	3.7778	.1185
20	:	15	25	30					
20	:	15	26	29					
20	:	15	27	28					
20	:	16	24	30					
20	:	16	25	29					
20	:	16	26	28					
20	:	16	27	27	.4000	.2000	100	4.1465	.3017

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=5 (continued)									
20	:	17	23	30					
20	:	17	24	29					
20	:	17	25	28					
20	:	17	26	27					
20	:	18	22	30					
20	:	18	23	29					
20	:	18	24	28					
20	:	18	25	27					
20	:	18	26	26					
20	:	19	21	30					
20	:	19	22	29					
20	:	19	23	28					
20	:	19	24	27					
20	:	19	25	26					
20	:	20	20	30					
20	:	20	21	29					
20	:	20	22	28					
20	:	20	23	27					
20	:	20	24	26					
20	:	20	25	25					
20	:	21	21	28					
20	:	21	22	27					
20	:	21	23	26					
20	:	21	24	25					
20	:	22	22	26					
20	:	22	23	25					
20	:	22	24	24					
20	:	23	23	24	.4000	.2000			
25	:	15	20	30					
25	:	15	21	29					
25	:	15	22	28					
25	:	15	23	27					
25	:	15	24	26					
25	:	15	25	25					
25	:	16	19	30					
25	:	16	20	29					
25	:	16	21	28					
25	:	16	22	27					
25	:	16	23	26					
25	:	16	24	25	.1429	.2857	100	4.1465	.3017

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=5 (continued)					
25	:	17	18	30					
25	:	17	19	29					
25	:	17	20	28					
25	:	17	21	27					
25	:	17	22	26					
25	:	17	23	25					
25	:	17	24	24					
25	:	18	18	29					
25	:	18	19	28					
25	:	18	20	27					
25	:	18	21	26					
25	:	18	22	25					
25	:	18	23	24					
25	:	19	19	27					
25	:	19	20	26					
25	:	19	21	25					
25	:	19	22	24					
25	:	19	23	23					
25	:	20	20	25					
25	:	20	21	24					
25	:	20	22	23					
25	:	21	21	23					
25	:	21	22	22	.1429	.2857	100	4.1465	.3017
21	:	15	24	30					
21	:	15	25	29					
21	:	15	26	28					
21	:	15	27	27					
21	:	16	23	30					
21	:	16	24	29					
21	:	16	25	28					
21	:	16	26	27					
21	:	17	22	30					
21	:	17	23	29					
21	:	17	24	28					
21	:	17	25	27					
21	:	17	26	26					
21	:	18	21	30					
21	:	18	22	29					
21	:	18	23	28					
21	:	18	24	27					
21	:	18	25	26	.3333	.2222	36	4.3843	.6072

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=5 (continued)									
21	:	19	20	30					
21	:	19	21	29					
21	:	19	22	28					
21	:	19	23	27					
21	:	19	24	26					
21	:	19	25	25					
21	:	20	20	29					
21	:	20	21	28					
21	:	20	22	27					
21	:	20	23	26					
21	:	20	24	25					
21	:	21	21	27					
21	:	21	22	26					
21	:	21	23	25					
21	:	21	24	24					
21	:	22	22	25					
21	:	22	23	24					
21	:	23	23	23	.3333	.2222			
24	:	15	21	30					
24	:	15	22	29					
24	:	15	23	28					
24	:	15	24	27					
24	:	15	25	26					
24	:	16	20	30					
24	:	16	21	29					
24	:	16	22	28					
24	:	16	23	27					
24	:	16	24	26					
24	:	16	25	25					
24	:	17	19	30					
24	:	17	20	29					
24	:	17	21	28					
24	:	17	22	27					
24	:	17	23	26					
24	:	17	24	25					
24	:	18	18	30					
24	:	18	19	29					
24	:	18	20	28					
24	:	18	21	27					
24	:	18	22	26					
24	:	18	23	25	.1818	.2727	36	4.3843	.6072

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=5 (continued)									
24	:	18	24	24					
24	:	19	19	28					
24	:	19	20	27					
24	:	19	21	26					
24	:	19	22	25					
24	:	19	23	24					
24	:	20	20	26					
24	:	20	21	25					
24	:	20	22	24					
24	:	20	23	23					
24	:	21	21	24					
24	:	21	22	23					
24	:	22	22	22	.1818	.2727	36	4.3843	.6072
22	:	15	23	30					
22	:	15	24	29					
22	:	15	25	28					
22	:	15	26	27					
22	:	16	22	30					
22	:	16	23	29					
22	:	16	24	28					
22	:	16	25	27					
22	:	16	26	26					
22	:	17	21	30					
22	:	17	22	29					
22	:	17	23	28					
22	:	17	24	27					
22	:	17	25	26					
22	:	18	20	30					
22	:	18	21	29					
22	:	18	22	28					
22	:	18	23	27					
22	:	18	24	26					
22	:	18	25	25					
22	:	19	19	30					
22	:	19	20	29					
22	:	19	21	28					
22	:	19	22	27					
22	:	19	23	26					
22	:	19	24	25	.2759	.2414	4	4.5009	1.0000

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=5 (continued)					
22	:	20	20	28					
22	:	20	21	27					
22	:	20	22	26					
22	:	20	23	25					
22	:	20	24	24					
22	:	21	21	26					
22	:	21	22	25					
22	:	21	23	24					
22	:	22	22	24					
22	:	22	23	23	.2759	.2414			
23	:	15	22	30					
23	:	15	23	29					
23	:	15	24	28					
23	:	15	25	27					
23	:	15	26	26					
23	:	16	21	30					
23	:	16	22	29					
23	:	16	23	28					
23	:	16	24	27					
23	:	16	25	26					
23	:	17	20	30					
23	:	17	21	29					
23	:	17	22	28					
23	:	17	23	27					
23	:	17	24	26					
23	:	17	25	25					
23	:	18	19	30					
23	:	18	20	29					
23	:	18	21	28					
23	:	18	22	27					
23	:	18	23	26					
23	:	18	24	25					
23	:	19	19	29					
23	:	19	20	28					
23	:	19	21	27					
23	:	19	22	26					
23	:	19	23	25					
23	:	19	24	24					
23	:	20	20	27					
23	:	20	21	26					
23	:	20	22	25					
23	:	20	23	24	.2258	.2581	4	4.5009	1.0000

Σr_1	:	Σr_2	Σr_3	Σr_4	p_1	$p_2=p_3=p_4$	S.S.	B_2	Cumulative Probability
n=5 (continued)									
23	:	21	21	25					
23	:	21	22	24					
23	:	21	23	23					
23	:	22	22	23	.2258	.2581	4	4.5009	1.0000

(Design: t=4, s=2)

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=1					
3	4	:	5	6	.5000	0	16	0	.1250
3	5	:	4	6					
3	5	:	5	5					
4	4	:	4	6					
4	4	:	5	5	.3750	.1250	4	.9769	.6250
3	6	:	4	5					
4	5	:	4	5	.2500	.2500	0	1.2041	1.0000
				n=2					
6	8	:	10	12					
6	8	:	11	11					
7	7	:	10	12					
7	7	:	11	11	.5000	0	64	0	.0078
6	9	:	9	12					
6	9	:	10	11					
7	8	:	9	12					
7	8	:	10	11	.4375	.0625	36	1.3090	.0703
6	10	:	8	12					
6	10	:	9	11					
6	10	:	10	10					
7	9	:	8	12					
7	9	:	9	11					
7	9	:	10	10					
8	8	:	8	12					
8	8	:	9	11					
8	8	:	10	10	.3750	.1250	16	1.9538	.2891
6	11	:	8	11					
6	11	:	9	10					
7	10	:	7	12					
7	10	:	8	11					
7	10	:	9	10					
8	9	:	7	12					
8	9	:	8	11					
8	9	:	9	10	.3125	.1875	4	2.2985	.7266

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=2 (continued)					
6	12	:	8	10					
6	12	:	9	9					
7	11	:	7	11					
7	11	:	8	10					
7	11	:	9	9					
8	10	:	8	10					
8	10	:	9	9					
9	9	:	9	9	.2500	.2500	0	2.4082	1.0000
				n=3					
9	12	:	15	18					
9	12	:	16	17					
10	11	:	15	18					
10	11	:	16	17	.5000	0	144	0	.0005
9	13	:	14	18					
9	13	:	15	17					
9	13	:	16	16					
10	12	:	14	18					
10	12	:	15	17					
10	12	:	16	16					
11	11	:	14	18					
11	11	:	15	17					
11	11	:	16	16	.4583	.0417	100	1.4949	.0063
9	14	:	13	18					
9	14	:	14	17					
9	14	:	15	16					
10	13	:	13	18					
10	13	:	14	17					
10	13	:	15	16					
11	12	:	13	18					
11	12	:	14	17					
11	12	:	15	16	.4167	.0833	64	2.3481	.0386
9	15	:	12	18					
9	15	:	13	17					
9	15	:	14	16					
9	15	:	15	15					
10	14	:	12	18					
10	14	:	13	17					
10	14	:	14	16	.3750	.1250	36	2.9306	.1460

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
n=3 (continued)									
10	14	:	15	15					
11	13	:	12	18					
11	13	:	13	17					
11	13	:	14	16					
11	13	:	15	15					
12	12	:	12	18					
12	12	:	13	17					
12	12	:	14	16					
12	12	:	15	15	.3750	.1250	36	2.9306	.1460
9	16	:	12	17					
9	16	:	13	16					
9	16	:	14	15					
10	15	:	11	18					
10	15	:	12	17					
10	15	:	13	16					
10	15	:	14	15					
11	14	:	11	18					
11	14	:	12	17					
11	14	:	13	16					
11	14	:	14	15					
12	13	:	11	18					
12	13	:	12	17					
12	13	:	13	16					
12	13	:	14	15	.3333	.1667	16	3.3172	.3877
9	17	:	12	16					
9	17	:	13	15					
9	17	:	14	14					
10	16	:	11	17					
10	16	:	12	16					
10	16	:	13	15					
10	16	:	14	14					
11	15	:	10	18					
11	15	:	11	17					
11	15	:	12	16					
11	15	:	13	15					
11	15	:	14	14					
12	14	:	10	18					
12	14	:	11	17					
12	14	:	12	16					
12	14	:	13	15	.2917	.2083	4	3.5396	.7744

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=3 (continued)					
12	14	:	14	14					
13	13	:	10	18					
13	13	:	11	17					
13	13	:	12	16					
13	13	:	13	15					
13	13	:	14	14	.2917	.2083	4	3.5396	.7744
9	18	:	12	15					
9	18	:	13	14					
10	17	:	11	16					
10	17	:	12	15					
10	17	:	13	14					
11	16	:	11	16					
11	16	:	12	15					
11	16	:	13	14					
12	15	:	12	15					
12	15	:	13	14					
13	14	:	13	14	.2500	.2500	0	3.6124	1.0000
				n=4					
12	16	:	20	24					
12	16	:	21	23					
12	16	:	22	22					
13	15	:	20	24					
13	15	:	21	23					
13	15	:	22	22					
14	14	:	20	24					
14	14	:	21	23					
14	14	:	22	22	.5000	0	256	0	.0000
12	17	:	19	24					
12	17	:	20	23					
12	17	:	21	22					
13	16	:	19	24					
13	16	:	20	23					
13	16	:	21	22					
14	15	:	19	24					
14	15	:	20	23					
14	15	:	21	22	.4688	.0312	196	1.6246	.0005

Σr_1	Σr_2	:	Σr_3	Σr_4	$P_1=P_2$	$P_3=P_4$	S.S.	B_2	Cumulative Probability
n=4 (continued)									
12	18	:	18	24					
12	18	:	19	23					
12	18	:	20	22					
12	18	:	21	21					
13	17	:	18	24					
13	17	:	19	23					
13	17	:	20	22					
13	17	:	21	21					
14	16	:	18	24					
14	16	:	19	23					
14	16	:	20	22					
14	16	:	21	21					
15	15	:	18	24					
15	15	:	19	23					
15	15	:	20	22					
15	15	:	21	21	.4375	.0625	144	2.6180	.0042
12	19	:	17	24					
12	19	:	18	23					
12	19	:	19	22					
12	19	:	20	21					
13	18	:	17	24					
13	18	:	18	23					
13	18	:	19	22					
13	18	:	20	21					
14	17	:	17	24					
14	17	:	18	23					
14	17	:	19	22					
14	17	:	20	21					
15	16	:	17	24					
15	16	:	18	23					
15	16	:	19	22					
15	16	:	20	21	.4062	.0938	100	3.3533	.0213
12	20	:	16	24					
12	20	:	17	23					
12	20	:	18	22					
12	20	:	19	21					
12	20	:	20	20	.3750	.1250	64	3.9075	.0768

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
			n=4 (continued)						
13	19	:	16	24					
13	19	:	17	23					
13	19	:	18	22					
13	19	:	19	21					
13	19	:	20	20					
14	18	:	16	24					
14	18	:	17	23					
14	18	:	18	22					
14	18	:	19	21					
14	18	:	20	20					
15	17	:	16	24					
15	17	:	17	23					
15	17	:	18	22					
15	17	:	19	21					
15	17	:	20	20					
16	16	:	16	24					
16	16	:	17	23					
16	16	:	18	22					
16	16	:	19	21					
16	16	:	20	20	.3750	.1250	64	3.9075	.0768
12	21	:	16	23					
12	21	:	17	22					
12	21	:	18	21					
12	21	:	19	20					
13	20	:	15	24					
13	20	:	16	23					
13	20	:	17	22					
13	20	:	18	21					
13	20	:	19	20					
14	19	:	15	24					
14	19	:	16	23					
14	19	:	17	22					
14	19	:	18	21					
14	19	:	19	20					
15	18	:	15	24					
15	18	:	16	23					
15	18	:	17	22					
15	18	:	18	21					
15	18	:	19	20	.3438	.1562	36	4.3158	.2101

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S. S.	B_2	Cumulative Probability
				n=4 (continued)					
16	17	:	15	24					
16	17	:	16	23					
16	17	:	17	22					
16	17	:	18	21					
16	17	:	19	20	.3438	.1562	36	4.3158	.2101
12	22	:	16	22					
12	22	:	17	21					
12	22	:	18	20					
12	22	:	19	19					
13	21	:	15	23					
13	21	:	16	22					
13	21	:	17	21					
13	21	:	18	20					
13	21	:	19	19					
14	20	:	14	24					
14	20	:	15	23					
14	20	:	16	22					
14	20	:	17	21					
14	20	:	18	20					
14	20	:	19	19					
15	19	:	14	24					
15	19	:	15	23					
15	19	:	16	22					
15	19	:	17	21					
15	19	:	18	20					
15	19	:	19	19					
16	18	:	14	24					
16	18	:	15	23					
16	18	:	16	22					
16	18	:	17	21					
16	18	:	18	20					
16	18	:	19	19					
17	17	:	14	24					
17	17	:	15	23					
17	17	:	16	22					
17	17	:	17	21					
17	17	:	18	20					
17	17	:	19	19	.3125	.1875	16	4.5970	.4545

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=4 (continued)					
12	23	:	16	21					
12	23	:	17	20					
12	23	:	18	19					
13	22	:	15	22					
13	22	:	16	21					
13	22	:	17	20					
13	22	:	18	19					
14	21	:	14	23					
14	21	:	15	22					
14	21	:	16	21					
14	21	:	17	20					
14	21	:	18	19					
15	20	:	13	24					
15	20	:	14	23					
15	20	:	15	22					
15	20	:	16	21					
15	20	:	17	20					
15	20	:	18	19					
16	19	:	13	24					
16	19	:	14	23					
16	19	:	15	22					
16	19	:	16	21					
16	19	:	17	20					
16	19	:	18	19					
17	18	:	13	24					
17	18	:	14	23					
17	18	:	15	22					
17	18	:	16	21					
17	18	:	17	20					
17	18	:	18	19	.2812	.2188	4	4.7621	.8036
12	24	:	16	20					
12	24	:	17	19					
12	24	:	18	18					
13	23	:	15	21					
13	23	:	16	20					
13	23	:	17	19					
13	23	:	18	18	.2500	.2500	0	4.8165	1.0000

Σr_1	Σr_2	:	Σr_3	Σr_4	$P_1=P_2$	$P_3=P_4$	S.S.	B_2	Cumulative Probability
				n=4 (continued)					
14	22	:	14	22					
14	22	:	15	21					
14	22	:	16	20					
14	22	:	17	19					
14	22	:	18	18					
15	21	:	15	21					
15	21	:	16	20					
15	21	:	17	19					
15	21	:	18	18					
16	20	:	16	20					
16	20	:	17	19					
16	20	:	18	18					
17	19	:	17	19					
17	19	:	18	18					
18	18	:	18	18	.2500	.2500	0	4.8165	1.0000
				n=5					
15	20	:	25	30					
15	20	:	26	29					
15	20	:	27	28					
16	19	:	25	30					
16	19	:	26	29					
16	19	:	27	28					
17	18	:	25	30					
17	18	:	26	29					
17	18	:	27	28	.5000	0	400	0	0
15	21	:	24	30					
15	21	:	25	29					
15	21	:	26	28					
15	21	:	27	27					
16	20	:	24	30					
16	20	:	25	29					
16	20	:	26	28					
16	20	:	27	27					
17	19	:	24	30					
17	19	:	25	29					
17	19	:	24	28					
17	19	:	27	27	.4750	.0250	324	1.7244	0

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
n=5 (continued)									
18	18	:	24	30					
18	18	:	25	29					
18	18	:	26	28					
18	18	:	27	27	.4750	.0250	324	1.7244	0
15	22	:	23	30					
15	22	:	24	29					
15	22	:	25	28					
15	22	:	26	27					
16	21	:	23	30					
16	21	:	24	29					
16	21	:	25	28					
16	21	:	26	27					
17	20	:	23	30					
17	20	:	24	29					
17	20	:	25	28					
17	20	:	26	27					
18	19	:	23	30					
18	19	:	24	29					
18	19	:	25	28					
18	19	:	26	27	.4500	.0500	256	2.8237	.0004
15	23	:	22	30					
15	23	:	23	29					
15	23	:	24	28					
15	23	:	25	27					
15	23	:	26	26					
16	22	:	22	30					
16	22	:	23	29					
16	22	:	24	28					
16	22	:	25	27					
16	22	:	26	26					
17	21	:	22	30					
17	21	:	23	29					
17	21	:	24	28					
17	21	:	25	27					
17	21	:	26	26					
18	20	:	22	30					
18	20	:	23	29					
18	20	:	24	28					
18	20	:	25	27					
18	20	:	26	26	.4250	.0750	196	3.6716	.0026

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=5 (continued)					
19	19	:	22	30					
19	19	:	23	29					
19	19	:	24	28					
19	19	:	25	27					
19	19	:	26	26	.4250	.0750	196	3.6716	.0026
15	24	:	21	30					
15	24	:	22	29					
15	24	:	23	28					
15	24	:	24	27					
15	24	:	25	26					
16	23	:	21	30					
16	23	:	22	29					
16	23	:	23	28					
16	23	:	24	27					
16	23	:	25	26					
17	22	:	21	30					
17	22	:	22	29					
17	22	:	23	28					
17	22	:	24	27					
17	22	:	25	26					
18	21	:	21	30					
18	21	:	22	29					
18	21	:	23	28					
18	21	:	24	27					
18	21	:	25	26					
19	20	:	21	30					
19	20	:	22	29					
19	20	:	23	28					
19	20	:	24	27					
19	20	:	25	26	.4000	.1000	144	4.3464	.0118
15	25	:	20	30					
15	25	:	21	29					
15	25	:	22	28					
15	25	:	23	27					
15	25	:	24	26					
15	25	:	25	25	.3750	.1250	100	4.8844	.0414

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=5 (continued)					
16	24	:	20	30					
16	24	:	21	29					
16	24	:	22	28					
16	24	:	23	27					
16	24	:	24	26					
16	24	:	25	25					
17	23	:	20	30					
17	23	:	21	29					
17	23	:	22	28					
17	23	:	23	27					
17	23	:	24	26					
17	23	:	25	25					
18	22	:	20	30					
18	22	:	21	29					
18	22	:	22	28					
18	22	:	23	27					
18	22	:	24	26					
18	22	:	25	25					
19	21	:	20	30					
19	21	:	21	29					
19	21	:	22	28					
19	21	:	23	27					
19	21	:	24	26					
19	21	:	25	25					
20	20	:	20	30					
20	20	:	21	29					
20	20	:	22	28					
20	20	:	23	27					
20	20	:	24	26					
20	20	:	25	25	.3750	.1250	100	4.8844	.0414
15	26	:	20	29					
15	26	:	21	28					
15	26	:	22	27					
15	26	:	23	26					
15	26	:	24	25					
16	25	:	19	30					
16	25	:	20	29					
16	25	:	21	28					
16	25	:	22	27					
16	25	:	23	26					
16	25	:	24	25	.3500	.1500	64	5.3059	.1153

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
			n=5 (continued)						
17	24	:	19	30					
17	24	:	20	29					
17	24	:	21	28					
17	24	:	22	27					
17	24	:	23	26					
17	24	:	24	25					
18	23	:	19	30					
18	23	:	20	29					
18	23	:	21	28					
18	23	:	22	27					
18	23	:	23	26					
18	23	:	24	25					
19	22	:	19	30					
19	22	:	20	29					
19	22	:	21	28					
19	22	:	22	27					
19	22	:	23	26					
19	22	:	24	25					
20	21	:	19	30					
20	21	:	20	29					
20	21	:	21	28					
20	21	:	22	27					
20	21	:	23	26					
20	21	:	24	25	.3500	.1500	64	5.3059	.1153
15	27	:	20	28					
15	27	:	21	27					
15	27	:	22	26					
15	27	:	23	25					
15	27	:	24	24					
16	26	:	19	29					
16	26	:	20	28					
16	26	:	21	27					
16	26	:	22	26					
16	26	:	23	25					
16	26	:	24	24					
17	25	:	18	30					
17	25	:	19	29					
17	26	:	20	28					
17	25	:	21	27					
17	25	:	22	26					
17	25	:	23	25					
17	25	:	24	24	.3250	.1750	36	5.6237	.2632

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
n=5 (continued)									
18	24	:	18	30					
18	24	:	19	29					
18	24	:	20	28					
18	24	:	21	27					
18	24	:	22	26					
18	24	:	23	25					
18	24	:	24	24					
19	23	:	18	30					
19	23	:	19	29					
19	23	:	20	28					
19	23	:	21	27					
19	23	:	22	26					
19	23	:	23	25					
19	23	:	24	24					
20	22	:	18	30					
20	22	:	19	29					
20	22	:	20	28					
20	22	:	21	27					
20	22	:	22	26					
20	22	:	23	25					
20	22	:	24	24					
21	21	:	18	30					
21	21	:	19	29					
21	21	:	20	28					
21	21	:	21	27					
21	21	:	22	26					
21	21	:	23	25					
21	21	:	24	24	.3250	.1750	36	5.6237	.2632
15	28	:	20	27					
15	28	:	21	26					
15	28	:	22	25					
15	28	:	23	24					
16	27	:	19	28					
16	27	:	20	27					
16	27	:	21	26					
16	27	:	22	25					
16	27	:	23	24	.3000	.2000	16	5.8457	.5034

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
				n=5 (continued)					
17	26	:	18	29					
17	26	:	19	28					
17	26	:	20	27					
17	26	:	21	26					
17	26	:	22	25					
17	26	:	23	24					
18	25	:	17	30					
18	25	:	18	29					
18	25	:	19	28					
18	25	:	20	27					
18	25	:	21	26					
18	25	:	22	25					
18	25	:	23	24					
19	24	:	17	30					
19	24	:	18	29					
19	24	:	19	28					
19	24	:	20	27					
19	24	:	21	26					
19	24	:	22	25					
19	24	:	23	24					
20	23	:	17	30					
20	23	:	18	29					
20	23	:	19	28					
20	23	:	20	27					
20	23	:	21	26					
20	23	:	22	25					
20	23	:	23	24					
21	22	:	17	30					
21	22	:	18	29					
21	22	:	19	28					
21	22	:	20	27					
21	22	:	21	26					
21	22	:	22	25					
21	22	:	23	24	.3000	.2000	16	5.8457	.5034
15	29	:	20	26					
15	29	:	21	25					
15	29	:	22	24					
15	29	:	23	23	.2750	.2250	4	5.9772	.8238

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
			n=5 (continued)						
16	28	:	19	27					
16	28	:	20	26					
16	28	:	21	25					
16	28	:	22	24					
16	28	:	23	23					
17	27	:	18	28					
17	27	:	19	27					
17	27	:	20	26					
17	27	:	21	25					
17	27	:	22	24					
17	27	:	23	23					
18	26	:	17	29					
18	26	:	18	28					
18	26	:	19	27					
18	26	:	20	26					
18	26	:	21	25					
18	26	:	22	24					
18	26	:	23	23					
19	25	:	16	30					
19	25	:	17	29					
19	25	:	18	28					
19	25	:	19	27					
19	25	:	20	26					
19	25	:	21	25					
19	25	:	22	24					
19	25	:	23	23					
20	24	:	16	30					
20	24	:	17	29					
20	24	:	18	28					
20	24	:	19	27					
20	24	:	20	26					
20	24	:	21	25					
20	24	:	22	24					
20	24	:	23	23					
21	23	:	16	30					
21	23	:	17	29					
21	23	:	18	28					
21	23	:	19	27					
21	23	:	20	26					
21	23	:	21	25					
21	23	:	22	24					
21	23	:	23	23	.2750	.2250	4	5.9772	.8238

Σr_1	Σr_2	:	Σr_3	Σr_4	$p_1=p_2$	$p_3=p_4$	S.S.	B_2	Cumulative Probability
n=5 (continued)									
22	22	:	16	30					
22	22	:	17	29					
22	22	:	18	28					
22	22	:	19	27					
22	22	:	20	26					
22	22	:	21	25					
22	22	:	22	24					
22	22	:	23	23	.2750	.2250	4	5.9772	.8238
15	30	:	20	25					
15	30	:	21	24					
15	30	:	22	23					
16	29	:	19	26					
16	29	:	20	25					
16	29	:	21	24					
16	29	:	22	23					
17	28	:	18	27					
17	28	:	19	26					
17	28	:	20	25					
17	28	:	21	24					
17	28	:	22	23					
18	27	:	18	27					
18	27	:	19	26					
18	27	:	20	25					
18	27	:	21	24					
18	27	:	22	23					
19	26	:	19	26					
19	26	:	20	25					
19	26	:	21	24					
19	26	:	22	23					
20	25	:	20	25					
20	25	:	21	24					
20	25	:	22	23					
21	24	:	21	24					
21	24	:	22	23					
22	23	:	22	23	.2500	.2500	0	6.0206	1.0000