

A STUDY OF THE COMPLETE DESIGN FOR  
CIRCULAR PRESTRESSED CONCRETE TANK AND ITS DYNAMIC ANALYSIS

by

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IV. LIST OF SYMBOLS

- A area in general,  $A_s$ , and  $A_c$ , the cross section area of steel and concrete,  $A_t$ , the transformed area of concrete.
- a acceleration.
- C creep of concrete  $C_c$  is the creep coefficient,  $c'$  = coefficient.
- D the diameter, or the flexural rigidity.
- E modulus of elasticity,  $E_s$  for steel and  $E_c$  for concrete.
- F force in general,  $F_e$ , the effective prestressing force  
 $F_i$ , the initial prestressing force  
 $F_o$ , the prestressing force at the time of transfer.
- f unit stress, or the subscript to indicate final or flexural.  
 $f_s$  and  $f_c$ , the unit stresses for steel in tension and concrete in compression.  
 $f_t$ , the unit tensile stress of concrete.
- f the natural frequency.
- H height of wall, or the hoop force.
- I moment of inertia, or impulse.
- i the subscript to indicate initial.
- K the curvature =  $1/R$ , or a constant.
- k the spring constant.
- L spacing, or length of tie bar, and high tension steel wire.
- $\Delta L$  elongation of wires.
- $M_z$  the vertical moment.

|            |   |
|------------|---|
| M          | bending moment, or mass.  |
| m          | the Poisson ratio.  |
| $N_z$      | vertical load.  |
| N          | number of wires.  |
| n          | the ratio between moduli of elasticity of steel and concrete.   |
| $P_z$      | vertical loads.   |
| $P_y$      | the horizontal or radial loads.   |
| p          | circular frequency.   |
| Q          | the resistant force of the structure.   |
| R          | radius.   |
| r          | the ratio of $f_s$ and $f_c$ .  |
| S          | spacing in general.   |
| T          | temperature, thrust, or period.   |
| t          | the thickness, $t'$ , the variable thickness.   |
| $T_\phi$   | ring tension or tangential force.   |
| $\Delta T$ | temperature range.  |
| u          | the vertical displacement of tank wall, or the coefficient of friction, or the coefficient of conductivity. |
| v          | the tangential displacement of tank wall.   |
| V          | shear, and $V_y$ is the radial shear of tank wall.  |
| W          | work, or radial displacement of tank wall.  |
| x          | displacement. $x_m$ = maximum displacement.   |
| y          | deflection. ( $y_s$ , shear deformation; $y_f$ , flexural deformation).                                     |
| z          | distance from the base to a point on the tank wall; $z'$ , distance from the top of the tank.               |

- $\alpha$  a constant, or coefficient of expansion and contraction.
- $\sigma$  unit stress in general.
- $\phi$  angle.
- $\beta$  constant.
- $\gamma$  unit weight.
- $\epsilon_c$  creep strain of concrete.
- $\epsilon_e$  elastic shortening strain of concrete.
- $\epsilon_{sh}$  shrinkage strain of concrete.
- $\eta$  % of loss of prestressed forces.
- $\epsilon$  unit strain in general.
- $\epsilon$  ratio,  $z/H$  .

## V. INTRODUCTION

The prestressed concrete type of circular tank is recognized as one of the most economical designs today, (Ref. 15). In the United States, they are built mostly for water reservoirs and sewage treatment tanks; while in Europe, they are also used for the storage of fuel oil, and some granular materials such as, cement, wheat and sugar. Circular prestressing solves the deficiencies of leakage and cracks which occur frequently in conventional reinforced concrete tanks. It provides less maintenance, good fire proofing, sound explosive resistance and is a preferable substitute where steel is expensive or in short supply.

The basic principle of circular prestressing was applied to the construction of wooden barrels, long ago. Metal bands were wound around wooden staves to press them together. When the bands were tightened, they were under tensile prestress which in turn created compressive prestress in the staves and thus enabled them to resist hoop tension produced by internal liquid pressure. In other words, the bands and the staves were both prestressed before they were subjected to any service loads, (Ref. 8). This is the same principle applied to circular prestressed concrete tank structure. Although this principle is easily stated, the structural analysis of tank using the approach of the cylindrical thin shell theory is rather complicated.

The economy of material in prestressing can only be achieved through the proper combination of forces in the distribution of

different stages of stresses. For many years, techniques, systems and competitive patterns regarding tank design and circular prestressing methods have been continuously developed for different conditions in different countries. Indeed, there is still a need for further study of methods of design and construction of this important structural form.

This thesis consists of a study of the complete design of a circular prestressed concrete tank, and a proposal of a method of circular prestressing which is not commonly used in the United States. In addition, an investigation of dynamic analysis is included.

The first two sections review the theory of structural analysis of tanks, and the general principles of prestressed concrete design in their application to circular tanks. They are followed by an explanation of the proposed method of prestressing, and an illustrative example of the complete design procedure. In the final discussion, a comparative study of different prevailing methods of circular prestressing has also been included.

Prestressed concrete circular tanks might be built in those areas subjected to earthquake disturbance. In present design practice, an equivalent static live load is used to simulate the earthquake effect. This is adequate for slowly applied dynamic load, or first mode of vibration of structures due to earthquake shock. For the higher modes, a study of the dynamic behavior of the tank becomes necessary. An analysis of the response of this type of structure to the dynamic load and the influence of stresses caused by the vibra-



tion of the tank structure are investigated. A simplified method is employed in this investigation.

## VI. REVIEW OF LITERATURE

In conventional practice, the structural analysis of the circular tank is based on the theory of long cylindrical thin shells modified with certain simplifying assumptions. S. Timoshenko, in "Theory of Plates and Shells" (1), presents equations for tank analysis in detail. The Portland Cement Association has tabulated, for different dimensions and end conditions, forces and moments produced on a tank wall (11). These are very useful for practical design work. European designers have utilized Reissner's method which was organized by Carpenter and presented in reference (4a). It was also introduced by W. S. Gray in his text (3). It is interesting to note that S. Timoshenko (1) related the solution of the radial deformation of the tank wall to a similar equation of the deflection of a beam on an elastic foundation. In some design methods (14), the term of the coefficient of subgrade reaction even appears in the equations. In Denmark, Helge Lundgren in "Cylindrical Shells" (20), developed a chapter especially for tank shell analysis. Komendant (6) also has a discussion of tank design by the method of superposition similar to that used in solving the redundant reactions of statically indeterminate structures.

As to the application of prestressing principles to circular concrete tanks, Evan Bennett (7), T. Y. Lin (8), and K. W. Mautner (22) have all discussed this problem quite thoroughly. Chr. Ostenfeld, consulting engineer, Denmark (14), and Preload Company, New York, (13)

have compiled the results of their experiences on the designs of prestressed concrete tanks. The basic design principles used are the same but each has his own specification and prestressing practice.

Loss of prestressing is a bothersome problem in tank construction. Methods used for circular prestressing are usually of the post tensioning type. Losses of prestressing forces with time (creep, shrinkage) should be studied very carefully. In many early prestressed concrete tanks the effects of creep of concrete and steel were ignored and concrete was found to be cracked after a limited time of service. J. M. Crom, (15), emphasized this point and reported the results of tests performed at M.I.T. on the creep and shrinkage of concrete. The percentage of losses on these was recorded to be almost 25-30% of the total prestressing force.

There is challenging competition between different patterns and construction methods for prestressing circular concrete tanks. Evan Benett (7), and Leonard R. Creasy (23), show several methods with very complete descriptions. Creasy also has compiled a set of sketches to explain the practical constructions in many field examples.

In 1920, Hewett, U.S.A. (8) first built prestressed concrete tanks by wrapping ordinary steel bars around the wall and stressing them with turnbuckles. The low prestressing force induced on mild steel could not even offset the creep loss of concrete. After high tension steel wire was introduced in prestressed concrete work, the construction of circular prestressed concrete tanks was gradually improved.

At the present time, prestressed concrete tanks can be classified as bonded or unbonded. Tendons are placed in the concrete wall and bonded with grouting after post tensioning, or wound around the wall and protected by a coating of pneumatic mortar after tensioning, without bonding to the concrete. Tanks are also grouped by the method of construction: cable, continuous winding, precast, and hyperboloidal.

High tension steel wire or strand with special anchorage and splicing is also utilized for circular prestressed tanks. Some well-known patterns are the Gifford Udall system in England (7 and 23), Chalos system in France (7), and the steel band method (18) employed by several prestressed concrete water tanks in this country. The Freyssinet cable and cone (12), and the Magnel and Blaton reciprocal sandwich wedge (32) have been modified and applied to circular prestressing with successful results in England and other European countries.

Preload company in the United States is the leading constructor of prestressed concrete tanks using a continuous winding machine (13). The Crom Corporation in Florida, U.S.A. also uses the spiral winding process (33). French and Russian engineers have a similar method with winding machine for tank construction.

Mautner of England (23) has employed high strength wire wrapped around precast concrete units and jacked to stretch the wire. Upon completing the prestressing, the openings are finally filled with concrete. The hyperboloidal method is a relatively new development in France. The vertical section of the inside surface of the tank

is of hyperbolic form. Wire placed on the form is prestressed first. Then the tank wall is finished by shotcrete or gunite on the inside.

References regarding the general vibrating problems on engineering and the structural design for dynamic loads are S. Timoshenko's "Vibration Problems on Engineering" (25), and Proceedings of M.I.T. Conferences, 1952, 1956, (28, 29). At M.I.T., two conferences have been held for the dynamic design and building construction for blast resistance. Further examples of dynamic analysis on frame structures and free vibration of cantilever beams can be found from Rogers' "Dynamics of Framed Structures" (27). Two papers written by Moren and Blum on the earthquake design by the spectrum method were published in A.S.C.E. Proceedings (30 and 31). No literature directly related to the dynamic analysis of the low large diameter tank appears to be available.

VII. EQUATIONS FOR STRUCTURAL ANALYSIS

It is assumed that the thickness of the tank wall is very small compared to its height and that the principle of thin cylindrical shell analysis applies. The tank structure is a closed ring type. On any horizontal ring section (see figures 1-4), the radial loading is the same throughout and  $T_\phi$  is constant on both sides of the infinitely small element "abcd" cut out from the wall. Moments and shears exist only in the zy plane and on the horizontal edges. The material is assumed to be elastic and isotropic.

Geometry of deformations:

vertical displacement = u

tangential displacement = v

radial displacement = w

vertical unit strain  $e_z = \frac{du}{dz}$  . . . . . (1)

tangential unit strain in hori. plane

$$e_\phi = -\frac{2\pi(R+w) - 2\pi R}{2\pi R} = -\frac{w}{R}$$
 . . . . . (2)

Equilibrium of forces:

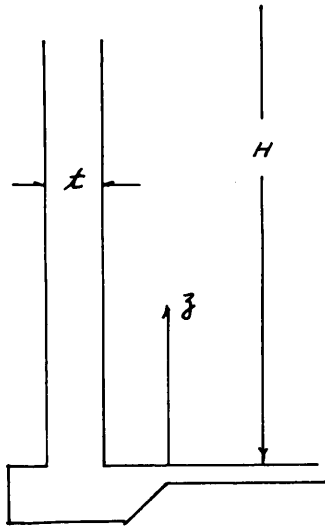
$T_\phi$  is constant . . . . . (3)

$\sum F_z = 0$

$$\frac{dN_z}{dz} + p_z = 0$$

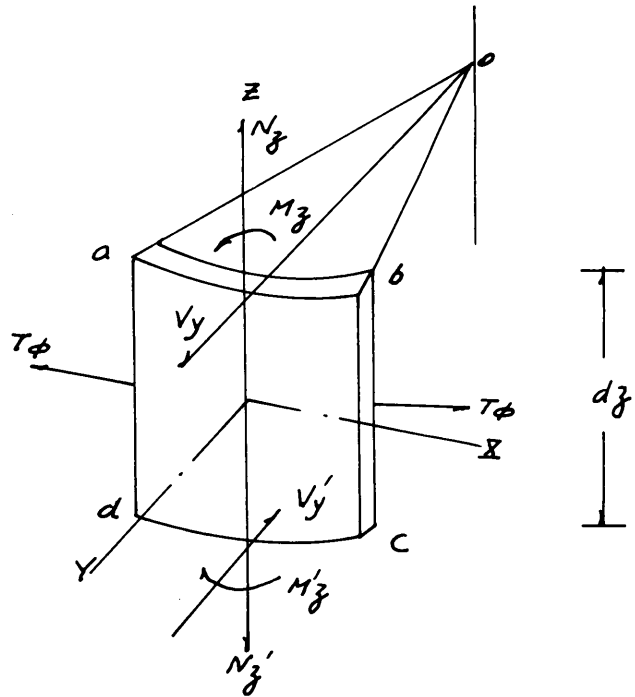
$$N_z = - \int p_z dz + C_1$$
 . . . . . (4)

Internal forces and displacements on the tank wall



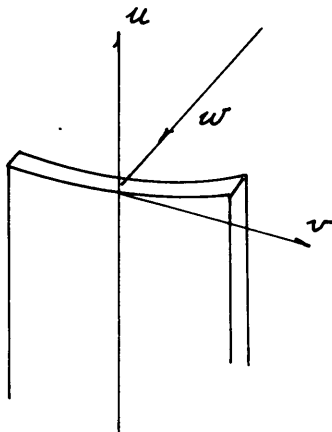
Dimensions

Fig. 1



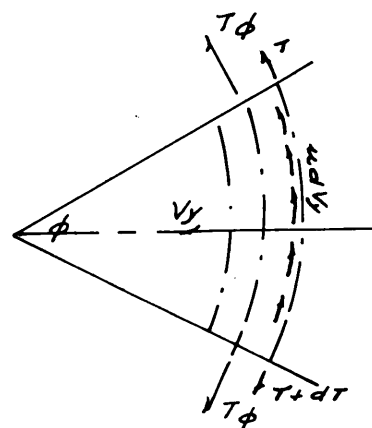
Internal forces

Fig. 2



Displacements

Fig. 3



Radial shear, Ring tension,  
Prestressing and friction forces.

Fig. 4

$$\sum F_r = 0$$

$$\frac{d V_y}{dz} + T_\phi \sin d\phi + p_y = 0$$

$$\frac{d V_y}{dz} + \frac{T_\phi}{R} + p_y = 0 \dots\dots\dots (5)$$

Compatibility relations:

stress-strain relations:

$\sigma_z$  = vertical unit stress

$\sigma_\phi$  = tangential unit stress

$\sigma_y$  = radial unit stress

$$e_z = \frac{1}{E} (\sigma_z - m \sigma_\phi)$$

$$e_\phi = \frac{1}{E} (\sigma_\phi - m \sigma_z) = - \frac{W}{R}$$

$$\sigma_z = \frac{E}{1 - m^2} (e_z + m e_\phi)$$

$$\sigma_\phi = \frac{E}{1 - m^2} (e_\phi + m e_z)$$

Area = l x t

$$N_z = \sigma_z \times t = \frac{Et}{1 - m^2} (e_z + m e_\phi)$$

$$N_z = \frac{Et}{1 - m^2} \left( \frac{du}{dz} - m \frac{W}{R} \right) \dots\dots\dots (6)$$



$$\text{If } N_z = - \int p_z dz + C_1 = 0$$

$$\text{then } \frac{du}{dz} - m \frac{w}{R} = 0 \quad (7)$$

Similarly

$$T_\phi = \frac{Et}{1 - m^2} (e_\phi + m e_z)$$

$$T_\phi = \frac{Et}{1 - m^2} \left( -\frac{w}{R} + m \frac{du}{dz} \right) \quad (8)$$

If  $N_z = 0$ , vertical load and weight of structure both are neglected.

$$\text{Then } T_\phi = \frac{Et}{1 - m^2} \left( -\frac{w}{R} + m^2 \frac{w}{R} \right)$$

$$T_\phi = -\frac{Et w}{R} \quad (9)$$

Combining equations (5) and (9)

$$\frac{dV_y}{dz} - \frac{Et w}{R^2} + p_y = 0 \quad (10)$$

Moment-curvature relation:

$$\text{Curvature of a surface parallel to } zy \text{ plane} = K_z = \frac{1}{R_z}$$

$$\text{Curvature of a surface parallel to } xy \text{ plane} = K_x = \frac{1}{R_x} = 0$$

$$e_z = t \times K_z = \frac{t}{R_z}$$

$$e_x = t_x K_x = \frac{t}{R_x} = 0$$

$$\sigma_z = \frac{E}{1 - m^2} (e_z + m e_x) = \frac{E}{1 - m^2} \left( \frac{t}{R_z} + \frac{t}{R_x} m \right)$$

$$\sigma_x = \frac{E}{1 - m^2} (e_x + m e_z) = \frac{E}{1 - m^2} \left( \frac{t}{R_x} + \frac{t}{R_z} m \right)$$

$$K_x = 0$$

$$\sigma_z = \frac{E}{1 - m^2} \left( \frac{t}{R_z} \right)$$

$$K_z = - \frac{d\theta}{dz} \quad \theta = \frac{dw}{dz} \quad K_z = - \frac{d^2w}{dz^2}$$

$$\sigma_z = \frac{Et}{1 - m^2} \left( - \frac{d^2w}{dz^2} \right)$$

$$M_z = \sigma_z \frac{I}{C} \quad I = \frac{1}{12} t^3$$

$$M_z = \frac{Et^3}{12(1 - m^2)} \left( - \frac{d^2w}{dz^2} \right)$$

$$\text{let } D = \text{flexural rigidity} = \frac{Et^3}{12(1 - m^2)}$$

$$\text{then } M_z = - D \frac{d^2w}{dz^2}$$

$$V_y = - D \frac{d^3w}{dz^3}$$

(11)

Expressing unknown forces and Moments in terms of w

$$\text{From (10): } \frac{d}{dz} \left( - D \frac{d^3w}{dz^3} \right) - \frac{Et w}{R^2} + p_y = 0$$

$$\frac{d^4 w}{dz^4} + \frac{Et w}{DR^2} - \frac{P_y}{D} = 0 \quad (12)$$

in case  $N_z \neq 0$

then from (8)

$$T_\phi = \frac{Et}{1 - m^2} \left( -\frac{w}{R} + m \frac{du}{dz} \right)$$

$$\frac{du}{dz} = \frac{1 - m^2}{Et} (N_z) + m \frac{w}{R}$$

$$T_\phi = \frac{Et}{1 - m^2} \left( -\frac{w}{R} + m^2 \frac{w}{R} \right) + m N_z$$

$$T_\phi = -\frac{Et w}{R^2} + m N_z \quad (13)$$

$$\text{or } \frac{d^4 w}{dz^4} + \frac{Et w}{DR^2} = \frac{P_y}{D} + \frac{m P_z}{D} \quad (14)$$

$$\text{let } \beta = \sqrt[4]{\frac{12(1 - m^2)}{(2R)^2 t^2}}$$

$$D = \frac{Et}{(2R)^2 \beta^4}$$

Substituting into (12) and (14) they become

$$\frac{d^4 w}{dz^4} + 4\beta^4 w = \frac{P_y}{D} \quad (15)$$

$$\text{or } \frac{d^4 w}{dz^4} + 4\beta^4 w = \frac{P_y}{D} + \frac{m P_z}{D}$$

This equation is analogous to the equation of a beam on an elastic foundation (Ref. 22).

The general solution of equation (15):

$$w = w_h + w_p$$

$w_{(h)}$  is the homogeneous solution due to restraint reactions at the ends.

$w_{(p)}$  is the particular solution due to loads with end conditions ignored.

It is assumed that  $p_z$ , the load from the roof and the dead load weight of the wall itself, acts concentrically to the wall. Usually it is assumed that  $p_z$  has no effect on the internal forces and moments except on the direct vertical compression on the wall.  $P_y$  varies as the height of wall, if  $\gamma$  is the unit weight of the material stored in the tank.

$$\text{then, } p_y = C' \gamma (H-z)$$

Equation (15) can be written:

$$\frac{d^4 w}{dz^4} + 4 \beta^4 w = \frac{C' \gamma (H-z)}{D} \quad (16)$$

S. Timoshenko (1) has introduced a solution:

$$w = e^{\beta z} (C_1 \cos \beta z + C_2 \sin \beta z) + e^{-\beta z} (C_3 \cos \beta z + C_4 \sin \beta z) + f(z) \quad (17)$$

where  $f(z) = w_p$

Substituting:  $w_h'''$  and  $w_h$  into the homogeneous equation:

$$\frac{d^4 w}{dz^4} + 4 \beta^4 w = 0$$

with the end conditions satisfied the constants,  $C_1, C_2, C_3$

and  $C_4$  will be evaluated.

From this solution, it is seen that  $e^{-\beta z}$  will approach zero as a limit as  $z$  becomes greater. For shell action we also know that the bending or loading effect near an edge is a localized action which will die out a short distance from the edge. Saint Venant's principle states that "If the forces acting on a small portion of the surface of an elastic body are replaced by another statically equivalent system of forces acting on the same portion of the surface, this redistribution of loads produces substantial change in the stress locally but has a negligible effect on the stress at distances which are large in comparison with the linear dimension of the surface on which the forces act." Since  $t$  is much smaller than  $H$ , one end restraint has negligible effect to the other. Therefore, when we analyze the fixed end effect at the base, it is reasonable to ignore the top end condition and let  $C_1$  and  $C_2$  both equal to zero.

$$\text{then } w_h = e^{-\beta z} (C_3 \cos \beta z + C_4 \sin \beta z) \quad (18)$$

This reason also is true for the top of the wall. When we consider top restraint we change  $z$  to  $z'$ , where  $z'$  is measured from the top. Preload Company (13) has set a limit on wall height. If the tank is considered by this analysis, then the height of the wall should not be less than  $2.3 \sqrt{Rt}$ .

$$w_p = - \frac{C_3 \gamma (H-z)}{4 \beta^4 D} \quad (19)$$

$$w = w_h + w_p = e^{-\beta z} (C_3 \cos \beta z + C_4 \sin \beta z) - \frac{C' \gamma (H-z)}{4\beta^4 D} \quad (20)$$

When the bottom is assumed to be a fixed end condition:

$$(w)_{(z=0)} = 0 = C_3 - \frac{C' \gamma (H-z)}{4\beta^4 D}$$

$$C_3 = \frac{C' \gamma H R^2}{Et}$$

$$\frac{dw}{dz} (z=0) = 0 = \left[ -\beta C_3 e^{-\beta z} (\cos \beta z + \sin \beta z) + \beta C_4 e^{-\beta z} (\cos \beta z - \sin \beta z) + \frac{C' \gamma R^2}{Et} \right]_z = 0$$

$$\frac{dw}{dz} (z=0) = \beta (C_4 - C_3) + \frac{C' \gamma R^2}{Et} = 0$$

$$C_4 = \frac{C' \gamma R^2}{Et} \left( H - \frac{1}{\beta} \right)$$

$$w = \frac{C' \gamma R^2}{Et} \left\{ H - z - e^{-\beta z} \left[ H \cos \beta z + \left( H - \frac{1}{\beta} \right) \sin \beta z \right] \right\} \quad (21)$$

$$T_\theta = - \frac{Et w}{R} = C' \gamma R \left[ H - z - e^{-\beta z} \left( H \cos \beta z + \left( H - \frac{1}{\beta} \right) \sin \beta z \right) \right] \quad (22)$$

$$M_z = -D \frac{d^2 w}{dz^2} = \frac{2\gamma R^2 D \beta^2}{Et} C' e^{-\beta z} \left[ -H \sin \beta z + \left( H - \frac{1}{\beta} \right) \cos \beta z \right] \quad (23)$$

$$V_y = -D \frac{d^3 w}{dz^3} = + \frac{2\gamma R^2 D \beta^3}{Et} C' e^{-\beta z} \left[ 2H \cos \beta z - \sin \beta z - \cos \beta z \right] \quad (24)$$

So, when  $z = 0$  at the fixed end base connection

$$M_o = \frac{\gamma H R t}{\sqrt{12(1 - m^2)}} \left(1 - \frac{1}{\beta H}\right) \quad (25)$$

$$V(o) = - \frac{\gamma H R t}{\sqrt{12(1 - m^2)}} \left(2 \beta - \frac{1}{H}\right) \quad (26)$$

$$T_\phi = 0$$

At the top, a doweled joint is assumed in this case. Then no expansion is allowed, and the shear is resisted. At first, we can calculate as if the top is free. Shear and displacement computed is the amount taken by the dowel.

$$w' = e^{-\beta z'} (C_3 \cos \beta z' + C_4 \sin \beta z') - \frac{C \gamma z'}{4 \beta^4 D} \quad (27)$$

where  $z'$  is measured from the top of tank wall.

$$\text{At } z' = 0, \text{ (at top), } \frac{d^2 w}{dz'^2} = M = 0$$

$$\text{then } 2 \beta^2 (C_3 \sin \beta z' e^{-\beta z'} - C_4 \cos \beta z' e^{-\beta z'}) = 0$$

$$C_4 = 0$$

$$\text{at } z' = 0 \quad T_\phi' = - \frac{Et}{R} (w') = - \frac{Et}{R} \times C_3$$

$T_\phi$  (from fixed end condition) at top, computed from the above case, when  $z = H$  equation (22) gives:

$$T_\phi = C' \gamma R \left[ -e^{-\beta H} \left( H \cos \beta H + \left( H - \frac{1}{\beta} \right) \sin \beta H \right) \right]$$

$$T_\phi = T_\phi' \quad (\text{it is the same, } w = w')$$

$$C_3 = \frac{C' \delta R^2}{Et} \left[ e^{-\beta H} \left( H \cos \beta H + \left( H - \frac{1}{\beta} \right) \sin \beta H \right) \right]$$

$$w' = +e^{-\beta z'} \left[ \frac{C' \delta R^2}{Et} \left\{ e^{-\beta H} \left( H \cos \beta H + \left( H - \frac{1}{\beta} \right) \sin \beta H \right) \right\} \cos \beta z' \right. \\ \left. - \frac{C' \delta z'}{4 \beta^4 D} \right] \quad (28)$$

$$T_{\phi}' = - \frac{Et w'}{R} = - \frac{Et}{R} \left[ C' e^{-\beta z'} \frac{\delta R^2}{Et} \left\{ e^{-\beta H} \left( H \cos \beta H + \left( H - \frac{1}{\beta} \right) \sin \beta H \right) \right\} \right. \\ \left. \cos \beta z' - \frac{C' \delta z'}{4 \beta^4 D} \right] \quad (29)$$

$$M_z' = - D \frac{d^2 w'}{dz'^2} = - D \left[ 2 \beta^2 \left\{ e^{-\beta H} \left( H \cos \beta H + \left( H - \frac{1}{\beta} \right) \sin \beta H \right) \right\} \frac{C' \delta R^2}{Et} \right. \\ \left. \times \sin \beta z' e^{-\beta z'} \right] \quad (30)$$

$$V_y' = D \frac{d^3 w'}{dz'^3} = - D \left[ 2 \beta^3 \left\{ \frac{C' \delta R^2}{Et} \left( e^{-\beta H} \left( H \cos \beta H + \left( H - \frac{1}{\beta} \right) \sin \beta H \right) \right) \right. \right. \\ \left. \left. \times \left( - \sin \beta z' e^{-\beta z'} + \cos \beta z' e^{-\beta z'} \right) \right\} \right] \quad (31)$$

for  $z' = 0$

$$w' = \frac{\delta R^2}{Et} e^{-\beta H} \left( H \cos \beta H + \left( H - \frac{1}{\beta} \right) \sin \beta H \right)$$

$$T_{\phi}' = - \frac{Et}{R} \times w' \quad (32)$$

$$M_z' = 0$$

$$V_y' = - D \left[ 2 \beta^3 \left\{ \frac{\delta R^2}{Et} \left( e^{-\beta H} \left( H \cos \beta H + \left( H - \frac{1}{\beta} \right) \sin \beta H \right) \right) \right\} \right]$$

Equations (32) should be added to equations (25) and (26) to get the total results of  $T_{\phi}$ ,  $M_z$ , and  $V_y$  at any level on the

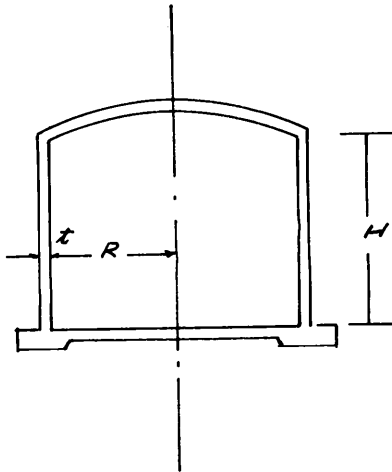


wall due to both top and end restraints and the loading effect, (see figures 5 - 8).

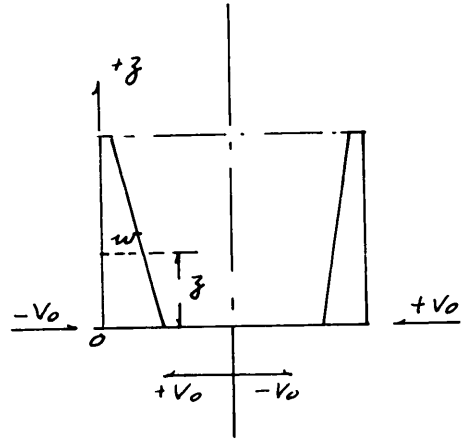
For a wall of variable thickness, these equations can be applied to get approximate values. S. Timoshenko (1) gives a comparison between wall of constant and variable "t". When H equal 14' the diameter equal 60' and t equal 14". In the first case the moment and shear at the base are 13960 ft. lbs. and 564 lbs. In the case of variable thickness, t varies from 14" at the base to 3.5" at top, H = 14' D = 60', the moment is 13900 ft. lbs. and shear is 527 lbs. These differences are so small that it is not absolutely necessary to work out a very complicated solution for the variable thickness case. The equations derived for the case of constant thickness can be used with reasonable accuracy.

The equations listed in Table 1 can be used with different values of parameters to compile tables for design.

Reactions at the base and the top of the tank wall

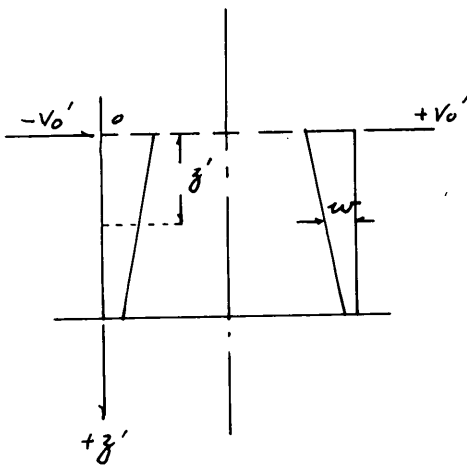


Fixed end  
Doweled top  
Fig. 5



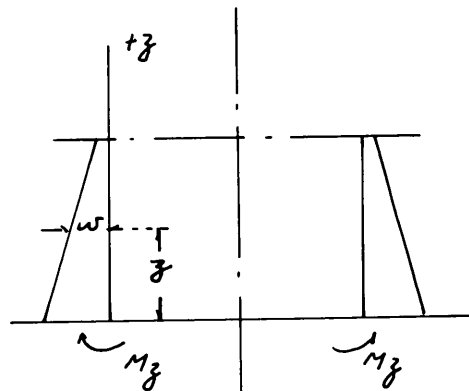
Shear at base

Fig. 6



Shear at top

Fig. 7



Moment at base

Fig. 8

Table I

Table of equations and parameters  
for structural analysis

| Items    | Conditions                   | Equations  | Parameters                               |
|----------|------------------------------|--|--|
| w        | Loading                      | $-\frac{\gamma(H-z)}{4\beta^4 D}$  | $= \sqrt[4]{\frac{12(1-m^2)}{(2R)2t^2}}$ |
| $T_\phi$ | "                            | $\frac{Et}{R} \left( \frac{\gamma(H-z)}{4\beta^4 D} \right)$   |  |
| $M_z$    | "                            | 0  | $D = \frac{Et^3}{12(1-m^2)}$             |
| $V_y$    | "                            | 0  |  |
| w        | Fixed end<br>(and loading)   | $-\frac{\gamma R^2}{Et} \left[ H - z - (H\phi + (H - \frac{1}{\beta})\psi) \right]$  | $\phi = e^{-\beta z} (\cos \beta z)$     |
| $T_\phi$ | "                            | $+\gamma R \left\{ H - z - (H\phi + (H - \frac{1}{\beta})\psi) \right\}$   | $\psi = e^{-\beta z} (\sin \beta z)$     |
| $M_z$    | "                            | $+\frac{2\gamma R^2 D \beta^2}{Et} \left[ -H\psi + (H - \frac{1}{\beta})\phi \right]$  |  |
| $V_y$    | "                            | $+\frac{2\gamma R^2 D \beta^3}{Et} \left[ 2H\phi - \psi - \phi \right]$  |  |
| w        | doweled top<br>(and loading) | $\left[ \frac{\gamma R^2}{Et} (H\phi_H + (H - \frac{1}{\beta})\psi_H) \right] - \frac{\gamma z'}{4\beta^4 D}$                                | $\phi' = e^{-\beta z'} (\cos \beta z')$  |
| $T_\phi$ | "                            | $-\frac{Et}{R} \left\{ \left[ \frac{\gamma R^2}{Et} (H\phi_H + (H - \frac{1}{\beta})\psi_H) \right] - \frac{\gamma z'}{4\beta^4 D} \right\}$ | $\psi' = e^{-\beta z'} (\sin \beta z')$  |
| $M_z$    | "                            | $-D \left[ 2\beta^2 \frac{\gamma R^2}{Et} (H\phi_H + (H - \frac{1}{\beta})\psi_H) \right]$   | $\phi_H = e^{-\beta H} (\cos \beta H)$   |
| $V_y$    | "                            | $-D \left[ 2\beta^3 \frac{\gamma R^2}{Et} (H\phi_H + (H - \frac{1}{\beta})\psi_H) \right] (-\psi' + \phi')$                                  | $\psi_H = e^{-\beta H} (\sin \beta H)$   |

### VIII. PRESTRESSED CONCRETE DESIGN

Prestressed concrete design employs the application of prestressing principles in addition to the ordinary design methods of concrete structures. For a successful design, the prestressing force, the load, and stress distribution on the structure should be understood.

In a circular tank, the load from the contained material is resisted by a combination of two visualized elastic elements. The horizontal element takes the ring tension, while the vertical strip, without shear on the vertical edges, receives a bending effect. At any point, the computed deflections of these two elements must be equal, and the load is distributed between them. W. S. Gray has a very clear explanation (3) of this analysis in which he introduces Reissner's method for the calculation of the load distribution factor for tank structures. He presents some diagrams for load distribution used in reinforced concrete tank design. Figures 9 and 10 are taken from Gray's text, showing load distribution and bending diagrams for fixed base and doweled top end conditions. From the equations derived in this thesis discussing the structural analysis,  $T_{\phi}$ , the ring tension, is equal to  $C \gamma HR$ . The coefficient  $C$  is determined by the load received by the horizontal element in sharing the total loading effect with the vertical elastic strip.

Under this loading condition, both circumferential and vertical prestressing is necessary. The circumferential wire provides hoop compression to eliminate the ring tension in the concrete, and the vertical steel obviates the tensile stress caused by the bending

effect. In a circular tank with a fixed end condition at the base, the maximum positive bending moment due to live load occurs at nearly one third of the height above the base, and negative bending moment occurs at the base. Thus maximum tensile stresses are induced at the outer fiber at one third the height and at the inner fiber at the base connection. However, in a prestressed concrete tank, the moment caused by live load when the tank is full can be balanced in whole or in part by the effect of the horizontal prestressing force. This is the principal characteristic of prestressing in a concrete tank structure. When the tank is empty, the bending effect of the circumferential prestressing wire will be acting alone. The empty tank usually presents the critical condition in design. The tensile stresses caused by prestressing forces must be investigated. Some engineers prefer to add ordinary mild steel reinforced bars to take care of the tensile stresses; this results in partial prestressing. Others use a free or sliding base design to reduce the bending moment (Ref. 8 and 13).

For prestressed concrete circular tank design in the United States, it is usually specified that: (a) when the tank is full, there should be residual compression in the concrete wall in order to assure tightness and a safety margin for superimposed overloading; (b) when the tank is empty, no tension is allowed at any section of the wall. The British code for liquid containers of concrete permits 150 p.s.i. vertical flexural tensile stress in the concrete when the tank is empty, and requires 100 p.s.i. compression stress

in the concrete when the tank is full (23). Figures 11 and 12 show the bending effects due to the loading and the circumferential prestressing.

The general prestressing principles for linear prestressed concrete are also true for circular prestressing. In a circular prestressed concrete tank, the losses of prestress, the special techniques of prestressing on the curved surface and the anchorage design have to be studied more carefully. The following losses in prestressing force must be considered:

a, Anchorage slip:

This loss depends upon the kind of anchorage used. In Sweden, from tests made on Freyssinet cones, it is recommended that an average of 6mm. loss in elongation for each cone should be used (14).

b, Friction loss:

For post tensioning of the circular loop wire or cable,  
(See figure 4)

$u$  is the coefficient of friction between the steel wire and the concrete wall.

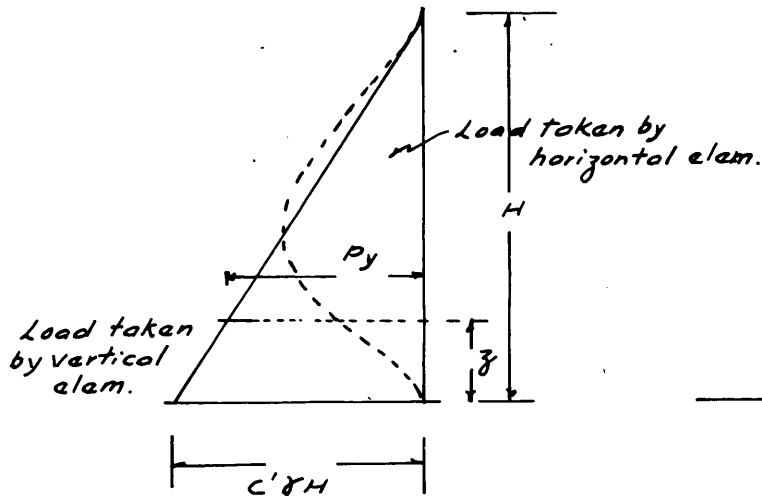
$T$  is the circular prestressing force.  $dT$  = increment of  $T$ .

$ds$  is the change in length of steel wire.

$d\phi$  is the change of central subtended angle  $\phi$  by length  $ds$

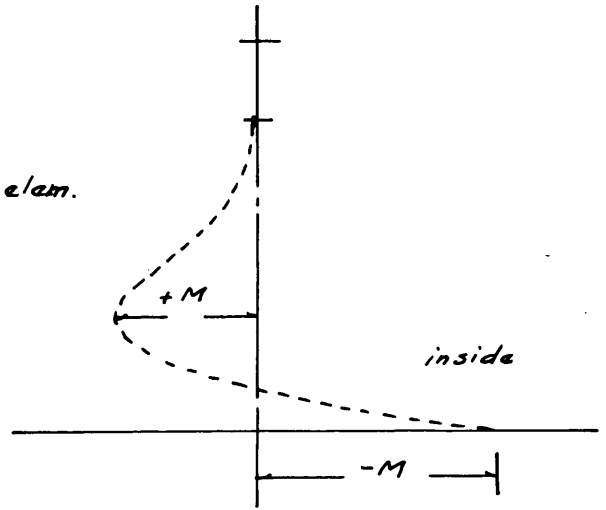
$dV_y$  is the change of radial shearing force in the concrete normal to  $ds$ .

Load distribution and bending moment diagrams for circular tank



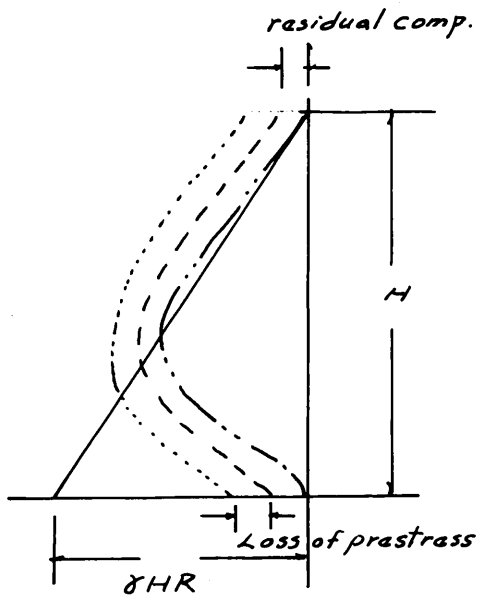
Load distribution curve

Fig. 9



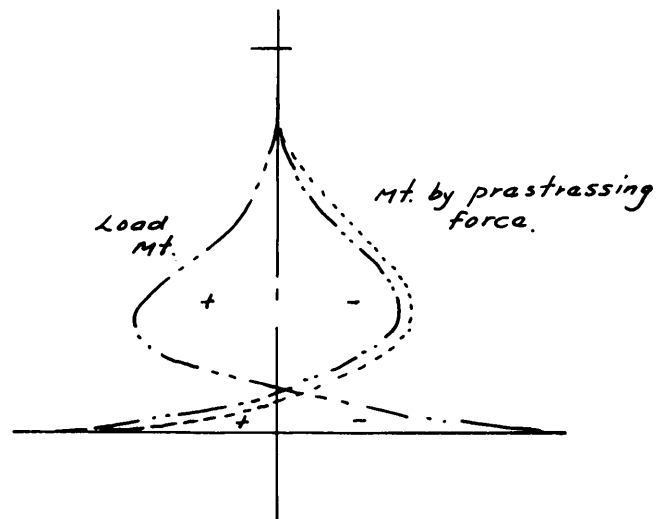
Bending moment diagram

Fig. 10



Ring tension and ring compression

Fig. 11



Bending moment due to prestressing force

Fig. 12

$$\sum F_n = 0 \text{ (normal forces)}$$

$$2T \sin \frac{d\phi}{2} = dV_y, \quad dV_y = T d\phi$$

$$\sum F_t = 0 \text{ (tangential force)}$$

$$dT = u dV_y = u T d\phi$$

when  $d\phi$  increases from 0 to  $\phi$        $T$  increases from  $T_1$  to  $T_2$

$$\text{therefore } \int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\phi} u d\phi \quad T_2 = T_1 e^{u\phi}$$

where  $\phi$  is measured in radians.

$$T_2 - T_1 \text{ is the friction loss}$$

Mautner (22) has calculated the friction loss of the Freyssinet cable stretched 90° away around the tank using a coefficient of friction of 0.15. It was found that the effective  $T$  is only 83 % of the initial jacking prestressing force. The loss is 17%, a relatively large amount.

c, Creep of steel:

Creep of steel, verified by test, results in about 4 % loss of prestress (8).

d, Creep and shrinkage of concrete:

1, Elastic shortening due to elastic strain of the concrete is:

$$\epsilon_e = \frac{f_c}{E_c} \quad (\text{where } \epsilon_e \text{ and } f_c \text{ are unit strain and unit stress due to load or prestressing force.})$$



loss of prestress  $\Delta f_s = n f_c = \frac{f_c}{E_c} \times E_s$ ; this is normally about 1 to 2 %.

2, Shrinkage of concrete (21):

For post tensioning, the shrinkage loss is essentially zero and can be neglected.

Normally the shrinkage unit strain  $\epsilon_{sh} = 0.0002$  in per in.

In pretensioning this represents a loss of prestress approximately 5 to 6 %.

3, Creep of concrete or plastic flow of concrete (8) (9):

Usually the coefficient of creep  $C_c$  is assumed to be 3

The loss of prestress is  $(C_c - 1) n f_c = \Delta f_s$

It is estimated to be approximately 20 %

Prestress losses for items 1, 2 and 3 can also be calculated as follows:

The elastic change in the length of the steel is the total change in the length of concrete due to elastic shortening and creep together.

If we assume  $\epsilon_c$  is the unit creep strain per unit of stress of concrete,

$$\text{then } - \frac{df_s}{E_s} = \frac{df_c}{E_c} + f_c d \epsilon_c$$

$$n = \frac{E_s}{E_c} \quad r = \frac{f_s}{f_c}$$

Substituting into the above equation:

$$- \frac{d f_c r}{n} = df_c + E_c f_c d \epsilon_c$$

While  $\delta_c$  changes from 0 to  $\delta_c$ ,

$f_c$  changes from  $f_{ci}$  to  $f_{cf}$

$$\int_{f_{ci}}^{f_{cf}} \frac{df_c}{f_c} = - \frac{E_c n}{n+r} \int_0^{\delta_c} d\delta_c$$

$$\text{Let } \alpha = \frac{n+r}{E_s} = \frac{n+r}{E_c n}$$

$$\text{then } f_{cf} = f_{ci} e^{-\frac{\delta_c}{\alpha}}$$

$$\text{and } f_{ci} - f_{cf} = \text{loss of } f_c$$

J. M. Crom (15) reported on tests performed at M.I.T. on creep and shrinkage of concrete. The following are the results of losses found in terms of steel stress:

|  |               |
|--|---------------|
| loss by shrinkage and plastic flow       | 24,000 p.s.i. |
| allowable for test                       | 4,000 "       |
| permissible variation of rod stress used |               |
| in test                                  | 10,000 "      |
| Total                                    | 38,000 p.s.i. |

This represents about 25 % loss of prestress.

Other details of prestressing principles and specifications concerning the circular prestressed concrete tank are included in the two sections following.

## IX. PROPOSED METHOD OF PRESTRESSING

In the prestressed concrete tank high stress is induced in the steel by stretching the wires wound around the tank or the cables bonded into the tank wall.

Different methods have been used for the circular prestressing of the tanks. In the continuous winding method, a machine is used to wrap the wires around the tank. The wire is prestressed by cold drawing it through a die attached to the machine. The cable method for tank prestressing is the same as that used for linear prestressing. Special jacks and anchorages are adopted to stretch the wires against the concrete. Many prestressed concrete companies have their own patented methods of prestressing.

A problem that has been in the author's mind for a long time is: can a simple method be devised for the circular prestressing of the tank without using elaborate machines and special jacking devices? A zig-zag extensioning method of post tensioning with a continuous winding process offers one solution. This procedure has been used in making prestressed concrete railway ties in Taiwan, China. It was not used extensively because the ties can be easily made by the pretensioning method in a central plant without difficulty.

In the zig-zag extensioning method, wire is wrapped around the tank, hand tight, to form a series of parallel wire loops. Each wire is then prestressed by displacing one point on it laterally up or down, while holding adjacent points in place at certain spacing intervals, (see figure 14). The extensioning process requires only

an ordinary hydraulic jack. Steel spacing bars are employed to fix the wires in position. Prestretching of the wires is not required during the wrapping procedure. No special anchorage is needed. The end of the wires can be embedded in the wall. After prestressing, a coating of gunite is applied for the protection.

The details of this proposed method are best shown by the following example with numerical data and explanations:

The assumed data for calculations are:

Given a circular prestressed concrete tank with the wall base fixed to the tank below and the top dome roof doweled to the wall.

H, the height of the wall of the tank is 30'

D, diameter of the tank is 84'

t, the thickness of the tank wall is 8" , the minimum thickness required to facilitate the pouring of concrete and the installation of vertical cables.

r, the unit weight of the contained material is 50 p.c.f.

C', the coefficient of the horizontal pressure of the contained material is 1

The hoop tension  $T_{\phi}$  due to load  $p_y$  at the various heights of the wall is shown on table 2.

The maximum ring tension is 43,000 p.p.f. at 0.7 H from the top of the wall. This is the critical section for design.

It is supposed that high tension wire with an ultimate strength  $f_s = 240,000$  p.s.i. is used for the tank prestressing.

Table 2

Ring tension and number of wires for circumferential prestressing

$$T_{\phi} = C \times \gamma \times H \times R = C \times 50 \times 30 \times 42 = C \times 63,000 \text{ p.p.f.}$$

$$\frac{H^2}{Dt} = 16 \quad C\text{'s values are taken from Table 1 Ref. (11)}$$

| Point              | 0.0H  | 0.1H   | 0.2H   | 0.3H   | 0.4H   | 0.5H   | 0.6H   | 0.7H   | 0.8H   | 0.9H   | 1.0H |
|--------------------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|------|
| C = Coefficient    | 0     | +0.099 | +0.199 | +0.304 | +0.412 | +0.531 | +0.641 | +0.687 | +0.582 | +0.265 | 0    |
| $T_{\phi}$         | 0     | +6300  | +12600 | +19200 | +26000 | +33500 | +40500 | +43000 | +36600 | +16700 |      |
| average $T_{\phi}$ | +3150 | +9450  | +15900 | +22600 | +29750 | +37000 | +41750 | +39800 | +26650 | +8350  |      |
| N (calculated)     | 2.3   | 6.8    | 11.4   | 16.3   | 21.5   | 26.6   | 30     | 28.5   | 19.2   | 6      |      |
| N (used)           | 3     | 7      | 12     | 17     | 22     | 27     | 36 *   | 29     | 20     | 6      |      |

Total N = 179      Total length = 48500'

\* Add 6 wires for the convenience of arrangements at the section of maximum  $T_{\phi}$

Table 1 of Ref. (11) is for the fixed end and free top case, since doweled top has no effect on ring tension of the wall, the influence of top connection need not be considered here.

The design stress of the wire is limited to 60 % of the ultimate strength or  $0.60 \times 240,000 = 144,000$  p.s.i.

Some designers reduce the design stress by 5,000 p.s.i. to provide for residual compression stress in the concrete when the tank is full and 1,000 p.s.i. for construction tolerance.

This represents an average deduction of nearly 5 %. The design stress used here will be  $144,000 - 6,000 = 138,000$  p.s.i.

If 5 mm. ( 0.196 " ) wire is used, the design prestressing force =  $0.0302$  sq. in. times  $138,000$  p.s.i. =  $4170$  pounds per wire.

The maximum number of wires required per foot is  $43,000/4170 = 10.4$ .

Using 11 wires, then area furnished is  $11 \times 0.0302 = 0.334$  square inch per foot of the wall.

$F_e$ , the effective prestressing force of wires is  $43,000$  p.p.f.

$F_i$ , the initial prestress of the steel =  $F_e + \text{losses} = F_0$  (the prestress force at the time of transfer) + anchorage and splice losses.

Losses of the prestressing force are assumed as follows:

|                                 |      |
|---------------------------------|------|
| shrinkage and creep of concrete | 25 % |
| creep of steel                  | 4 %  |
| elastic shortening of concrete  | 2 %  |
| anchorage and splice losses     | 2 %  |
| Total losses                    | 33 % |

(The wires are zig-zag extensioned to a certain

elongation to induce the stress; therefore no friction loss is considered.)

$$F_1 = \frac{43,000}{1-0.33} = \frac{43,000}{0.67} = 64,000 \text{ lbs. per foot of the wall height.}$$

$$\text{The } f_{si} \text{ of the wires} = \frac{F}{A_s} = \frac{64,000}{0.334} = 192,000 \text{ p.s.i.}$$

The allowable maximum initial temporary prestressing stress of the high tension wire can reach 80 % of  $f_s' = 80 \% \times 240,000 = 192,000 \text{ p.s.i. (8)}$ .

The tank is a closed ring type of structure and is statically indeterminate. The theory of linear transformation can be applied. Any wire through the center of gravity of the concrete (c.g.c.) section is called a concordant wire, any other wire parallel to it, is simply that line transformed, whose line of pressure will still remain through the c.g.c. line. That is to say, the compressive force of concrete  $= C = F_1$  of the steel wire and the C-line coincides with c.g.c. line of the tank wall (8).

The twenty eight day compressive ultimate stress of the concrete ( $f_c'$ ) should be at least 3500 p.s.i. The design stress of the concrete is  $0.4 \times 3500 = 1400 \text{ p.s.i.} = f_c$

$$A_c = \frac{C}{f_c} = \frac{F_1}{f_c} = \frac{64,000}{1400} \times \frac{1}{12} = 1 \times t$$

$$t = 3.8 \text{ inch}$$

t assumed is 8" which is ample. For the sake of conven-

ience of construction, the minimum thickness of 8" will be maintained.

Check the concrete stress:

At the time of initial prestressing (temporary stress):

$$f_c = \frac{F_i}{A_c} = \frac{64,000}{12 \times 8} = 670 \text{ p.s.i. (comp.)}$$

After loss of prestressing

empty tank

$$f_c = \frac{F_e}{A_c} = \frac{43,000}{12 \times 8} = 450 \text{ p.s.i. (comp.)}$$

full tank

$$f_c = -\frac{43,000}{12 \times 8} + \frac{T\phi}{A_t}$$

$$A_t = \text{transformed section of concrete} = (n-1)A_s + A_c$$

$$n = \frac{E_s}{E_c} \quad E_s = 29,000,000 \text{ p.s.i.} \quad E_c = 3,500,000 \text{ p.s.i.}$$

$$n = 8.3$$

$$f_c = -450 + \frac{43000}{(12 \times 8) + (8.3 - 1) \times 0.334} = 15 \text{ p.s.i. (comp.)}$$

Residual compression = 3.4 % which is less than the assumed value of 5 %, because of the large value of t used.

This is a check of the direct stress only due to ring tension and the circumferential prestressing compressive force. The combination of bending effect due to the fixed end restraint at the base will be investigated in the section on the design of the vertical prestressing steel.



The above procedure follows the general process of preliminary design using the critical section of maximum ring tension for a check. The design steps of the proposed method of prestressing are explained as follows in detail.

- 1, Divide the wall into ten equal parts:

The whole height of the tank wall is divided into ten intervals. Each of which is three feet, for the convenience of arranging the circumferential prestressing wires.

- 2, Calculate the average ring tension in each interval:

The average ring tension due to the pressure of the contained material for each three foot interval is shown in table 2.

- 3, Calculate the number of wires needed, (N):

$N = \frac{\text{the average ring tension}}{\text{the design stress of each wire}} = \frac{\text{average } T_{\phi}}{4170}$

The results are listed in table 2.

The distribution of wires into rows and layers for each three foot interval is shown in figure 13 b, and c.

The spacing of the slots of the spacing bar is assumed 6". Each slot contains a maximum of three wires per layer and a maximum of two layers.

The spacing of the slots in the spacing bar can be larger or smaller as needed. This design is quite flexible. An even number of slots in each interval is preferred so that half of the wire can be extended up and half down.

- 4, The elongation of the wire needed to acquire the necessary pre-

stressing force:

Wires are wound around the tank wall hand tight only. The slack of the wire assumed is one inch per turn of the circular loop.

The unit strain of the wire corresponded to the prestressing stress is:

$$\begin{aligned}\epsilon_e &= \frac{f_{si}}{E_s} + \text{slack} = \frac{192000}{29 \times 10^6} + \frac{1''}{\pi(84 \times 12 + 16)} \\ &= 0.0066 + 0.0003 = 0.0069\end{aligned}$$

Total strain =  $0.0069 \times \pi(84 \times 12 + 16) = 22.2''$  per turn.

S = spacing of slots in the spacing bar = 6" c.c.

C = 6" - 1 x 0.196 = 5.804" used in calculation

See figure 13 c.

5. Design the zig-zag extensioning pattern:

(See figure 13 d)

L = spacing of the vertical steel spacing bars along the circumferential direction.

Take one section for investigation

Elongation,  $\Delta$ , of wire of length L is  $0.0069 L$

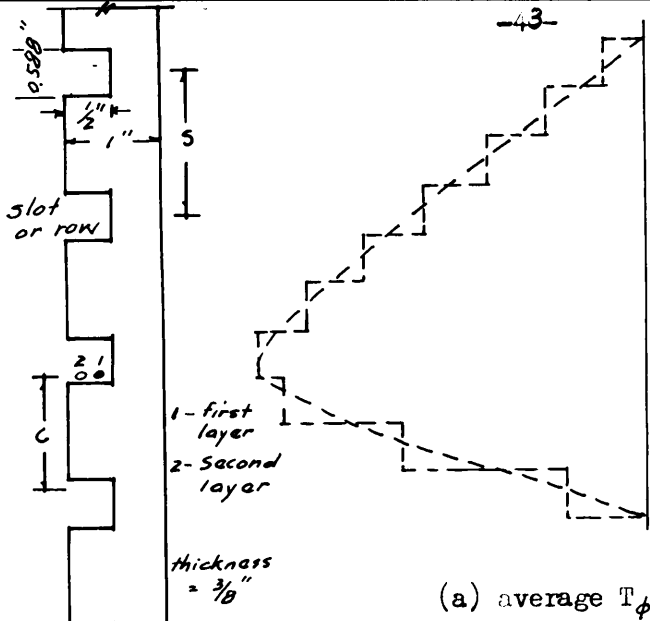
C = 5.804 and L = 62"

The number of spacing bars =  $\frac{(D + 2t)}{62} = 52$  pcs

6. Design the steel spacing bars.

See figures 13 c and d)

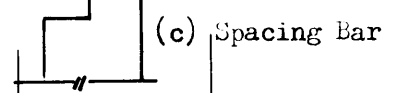
The steel spacing bars are cast into the concrete wall. The projected portion should be at least one-half inch. The cast in



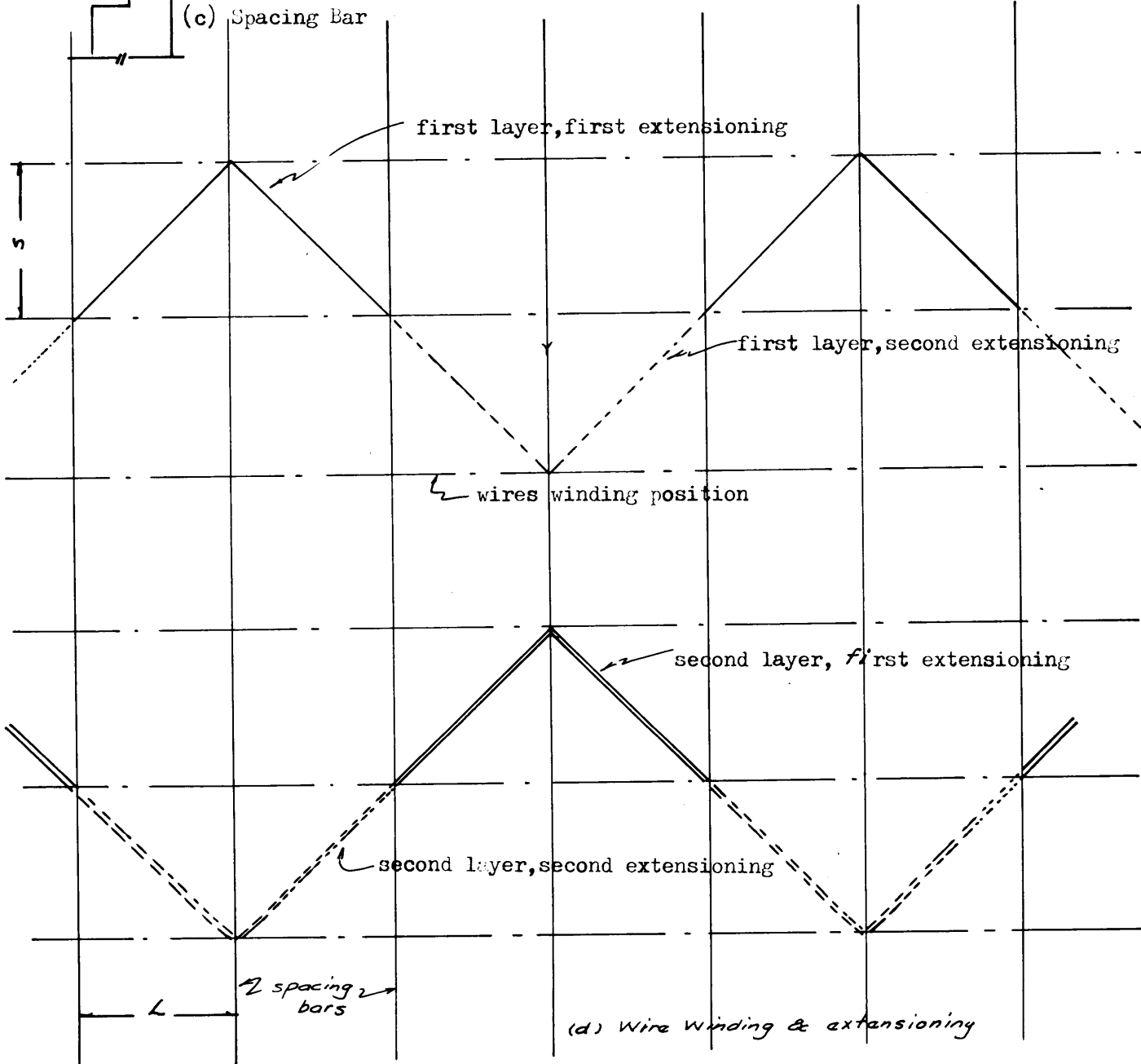
| 0.1H | 3  | 3   | 1     | 1,1,1         |
|------|----|-----|-------|---------------|
| 0.2H | 7  | 6   | 1     | 1,1,1,1,1,2   |
| 0.3H | 12 | 6   | 2     | 2,2,2,2,2,2   |
| 0.4H | 17 | 6   | 2     | 2,3,3,3,3,3   |
| 0.5H | 22 | 6   | 2     | 3,3,4,4,4,4   |
| 0.6H | 27 | 6   | 2     | 4,4,4,4,5,6   |
| 0.7H | 36 | 6   | 2     | 6,6,6,6,6,6   |
| 0.8H | 29 | 6   | 2     | 6,6,5,4,4,4   |
| 0.9H | 20 | 6   | 2     | 4,4,3,3,3,3   |
| 1.0H | 6  | 6   | 1     | 1,1,1,1,1,1   |
| H    | N  | Row | Layer | Wires per row |

(a) average  $T_\phi$

(b) Distribution of wires



(c) Spacing Bar



(d) Wire Winding & extensioning

Figure 13 Proposed zig-zag extensioning Method.

part should be two inches. Width of each slot is 0.588". Its depth should be at least one-half inch. The spacing of the slots are 6" center to center.

Let  $t$  = thickness of the spacing bar. High tension wire  $f_s = 138,000$  p.s.i.

$$A_s = 0.0302 \text{ sq. in.}$$

For 3 wires in each slot, the maximum case

Check for compression: allowable stress = 18,000 p.s.i. for steel bar

$$18000 \times t \times \frac{1''}{2} = 3 (A_s f_s \sin i) \times 2 \quad t = 0.265'' \text{ use } t = \frac{3''}{8}$$

Check for shear: allowable stress = 11,000 p.s.i. for steel bar

$$11000 \times t \times 6'' = 3 (A_s f_s \sin i) \times 2 \quad t = 0.0364'' \text{ use } t = \frac{3''}{8}$$

#### 7. Design of the anchorages:

Anchorages consisting of coils of wire in spiral form are embedded into the wall. Position of anchorages should be designed according to the length of wire per each wire coil used. Anchorages at the adjacent intervals are placed some distance apart with an overlap of wire between them.

Wires are connected by the Torpedo splice to the anchorages. The length of wire in the spiral anchorage coil is checked as follows:

let  $l$  be the required length of the spiral coil anchorage

$$l \times \text{perimeter} \times \text{allowable unit bond stress} = A_s f_s$$

$$l \times 0.59 \times 375 = 0.0302 \times 192000$$

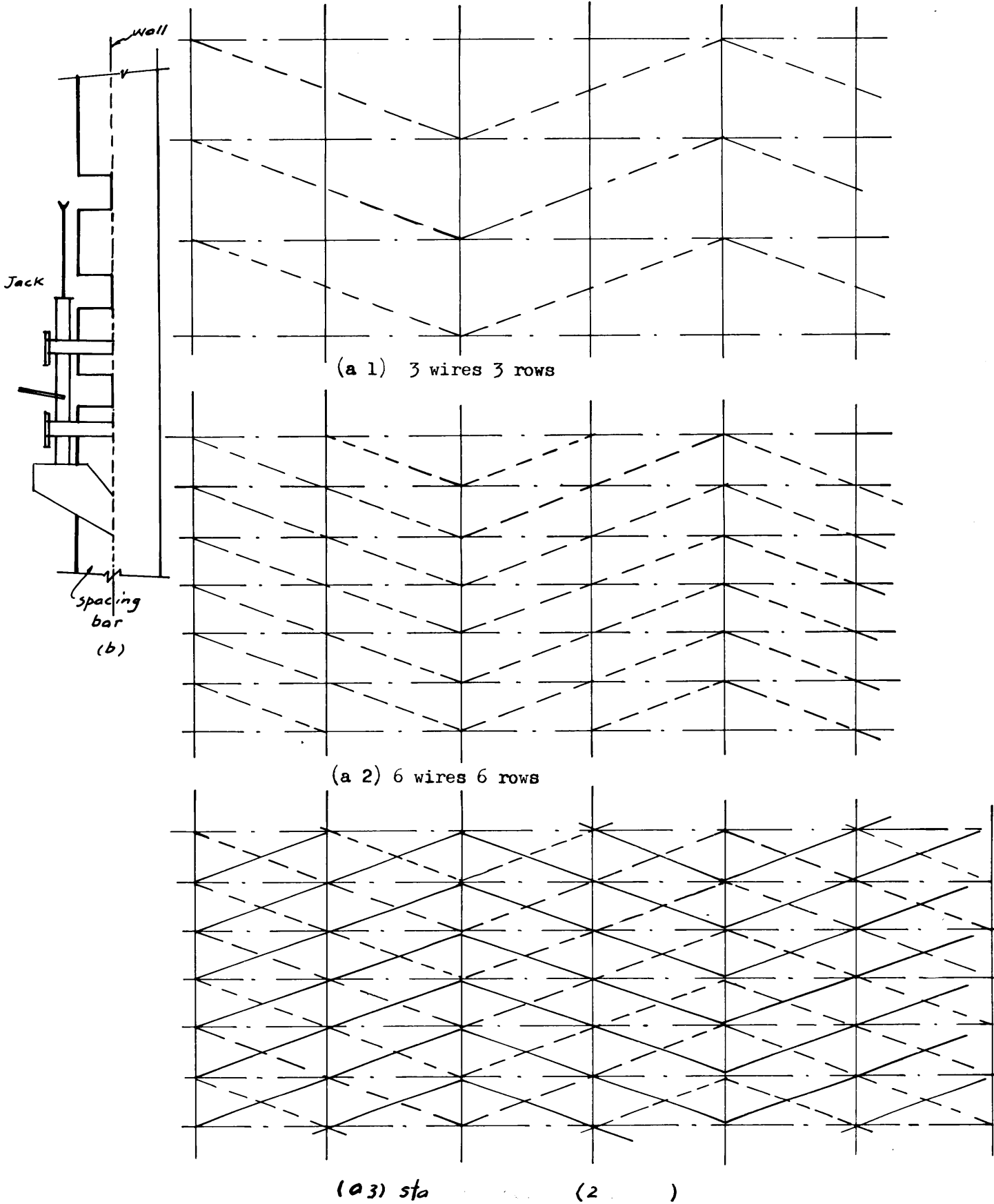
$$l = 26''$$

If diameter of the coil is 4", then 2 coils are required. The pitch of the spiral is 2".

8, Process of zig-zag extensioning:

- a, Winding of the wires: Wires are wound around the tank wall spaced according to the distribution table. They should be hand tight and firmly connected to the splice. Continuous winding may be facilitated by using a revolving wire drum placed on the platform at the top of the tank on the dome moulding.
- b, Distribution of the wires: The distribution of the wires should be carefully arranged before the construction. The following three items can be used as references. (1) It is better to use an even number of rows in each three foot interval. (2) At least two wires in one row is preferred. (3) If we use less number of rows in one interval, then three rows, with one wire in each, are supposed to be the minimum. These three typical forms are shown in figure 14.
- c, Sequence of extensioning: The first layer is extended first. After it is completed the second layer is extended and placed on the top of the first layer (see figure 13 d and figure 14).
- d, Jacking of the wires: The zig-zag extensioning is done by ordinary jacks clamped to the spacing bar and supported on the wall. They can work up or down. Two jacks or more can work at the same time. Wires are pulled in one direction at every fourth bar all around the tank. Then they are pulled in the opposite direction at the intermediate intervals. This makes the prestressing stress more uniform and also balance

Figure 14 Typical sections for Zig-zag extensioning



the slips of the wires which occur during the first pulling. Auxillary tools are used to pull the wires out of the slots. The pulling force of the jack required for each wire is equal to the sum of the prestressing forces of two adjacent wire intervals plus the friction force between the wire and the wall.

$$\text{Pulling force} = F$$

For each wire

$$\begin{aligned} F &= 2 \times A_s \times f_s \sin i + u \times (2 \times A_s f_s) \sin \frac{360^\circ}{52} \\ &= 2 \times 0.0302 \times 192000 \times \frac{6}{62.4} + 0.15 \times 2 \times 0.0302 \\ &\quad \times 192000 \times \sin 7.0^\circ \\ &= 1120 + 212 = 1332 \text{ lbs.} \end{aligned}$$

For three wires

$$F = 3 \times 1332 = 3996 \text{ lbs.}$$

- 9, Typical winding and extensioning diagrams are shown in figure 14.
- 10, After extensioning the wire, one to one and half inch of pneumatic motar is sprayed for outside protection and 4" x 4" x No. 12 wire mesh is used for the temperature reinforcement of the pneumatic motar layer.

X. DESIGN EXAMPLE

In this section, only three parts of the complete design procedure (vertical prestressing, dome roof, and the analysis for earthquake force), are discussed:

a, vertical prestressing

Vertical steel is designed to compensate for the tensile stress produced by the bending effect either due to the loading or due to the circumferential prestressing force.

From table 7 (Ref. 11)  $M_z = \text{coefficient} \times H^3 = C \times 50 \times 30^3 = C \times 1,350,000$   
in ft. lbs per ft.  $H^2 / Dt = 16$

| pt.   | 0.0H | 0.1H | 0.2H | 0.3H    | 0.4H    | 0.5H    | 0.6H    | 0.7H    | 0.8H    | 0.9H    | 1.0H    |
|-------|------|------|------|---------|---------|---------|---------|---------|---------|---------|---------|
| C     | 0    | 0    | 0    | -0.0001 | -0.0002 | -0.0001 | +0.0004 | +0.0013 | +0.0019 | +0.0001 | -0.0079 |
| $M_z$ | 0    | 0    | 0    | -135    | -270    | -135    | +540    | +1755   | +2567   | +135    | -10665  |

for the top joint  $M_z = VH$  since  $V = 0$   $M = 0$

By the flexural formula,  $f_c = MC/I$ , we can calculate the fiber stresses at points of maximum positive and negative moments for the different conditions of loadings. Critical sections at the base and 0.8H section are investigated.

$$f_c = MC/I$$

$$f_c = M \times 12 \times 4 \div (1/12 \times 12 \times 8 \times 8 \times 8) = 3/32 M$$



|  | Base         |             | 0.08H      |       |
|--|--------------|-------------|------------|-------|
|  | inner fiber  | outer fiber | inner      | outer |
| full tank<br>(loading moment)  |              |             |            |       |
| -Max. M = 10,665   | +1,000p.s.i. | -1,000      |            |       |
| +Max. M = 2,567  |              |             | -240p.s.i. | +240  |
| empty tank<br>(prest. force)<br>1.05 of case 1   | -1050        | +1050       | +252       | -252  |
| Initial prestressing<br>1.25 times case 2<br>(losses 25 % for<br>vertical prestress-<br>ing) | -1320        | +1320       | +315       | -315  |
| Dead load (assumed)<br>(roof and<br>dome plus wall)  | -40p.s.i.    | -40         | -40        | -40   |
| Temperature<br>difference<br>f. inwall (Ref.<br>13) Full:                                    | -169         | -169        | -169       | -169  |
| Empty:   | -70          | -70         | -70        | -70   |

Partial prestressing: this is omitted because the vertical steel will be prestressed before the horizontal wire.

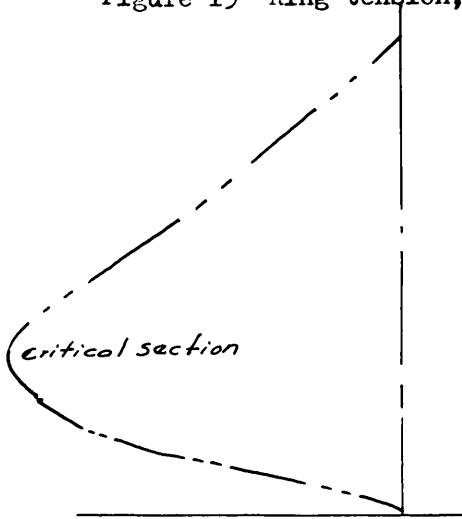
Anchorage: same as the partial prestressing.

The critical case is the case II: (see figure 15) in this design the eccentric vertical prestressing cable will be used. Let "F" be the force required and "e" be the eccentric distance from the center of the wall to the cable in the outer side.

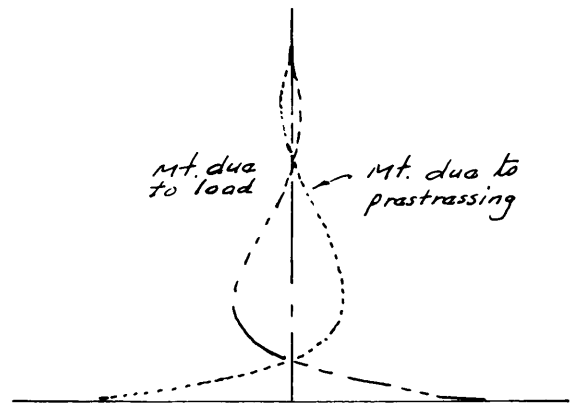
$$\text{outer fiber: } -940 = -\frac{F}{A_c} - \frac{F_e C}{I}$$

$$\text{inner fiber: } -240 = -\frac{F}{A_c} + \frac{F_e C}{I}$$

Figure 15 Ring tension, bending moment and wall stress distribution  
(Design example)

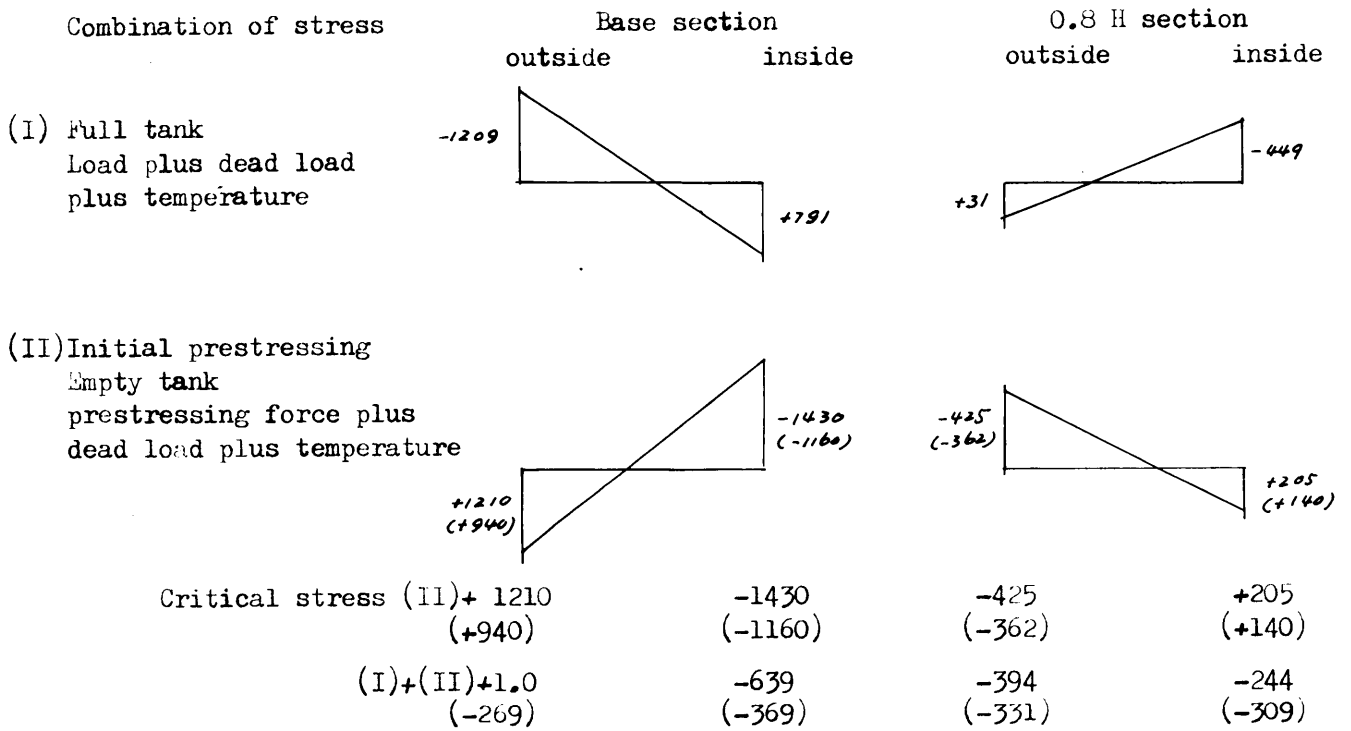


Ring tension



Bending moment

Combination of stress



+ tension, - compression

( ) affectiva prest. force.

Solving for F and e,  $F = 70,000 \text{ lbs}$   $e = 1''$  (computed this way)

Then wire needed =  $F \div (144,000 \times 0.0302) = 70,000 \div 4350 = 16$

Use an 18 wire group, the spacing =  $\frac{18}{16} = 1.12' \text{ c.c.}$

The anchorages can be a spiral of wire cores at the bottom and Freyssinet cones at the top. Cables are placed eccentrically 1" outside from the center of the wall.

b, Dome roof:

Assume a trial section as shown in the general drawing. A simple method for checking the stress is (Ref. 19): Assume an uniform thickness: 5", Rise: 10.5', Radius of the dome: 89', Rise span ratio: one-eighth, Central angle of the arch roofing:  $28^\circ 06'$ , tan. of angle: 0.535

L.L. = 40 lbs per sq. ft., D.L.: 62.5 p.s.f.  $W_T = 102.5 \text{ p.s.f.}$

Allowable max. stress = 200 p.s.i. Meridian thrust is T, hoop force is H. At the top  $T = H = \frac{1}{2} \times 102.5 \times 89 = 4500 \text{ p.p.f.}$

stress =  $4500 \div 5 \times 12 = 75 \text{ p.s.i.}$

at the point  $14^\circ$  from the top sin, of angle is 0.242 cos is 0.970

Total load above this point is  $2\pi 89^2 \times 102.5(1 - 0.970) = 152000 \text{ lbs.}$

$T = 152000 \div 2\pi 89 \times \sin^2 \phi = 4700 \text{ p.p.f. (comp.)}$

stress =  $4700 \div 5 \times 12 = 79 \text{ p.s.i. (comp.)}$

$H = -T + 89 \times 0.970 \times 102.4 = 4200 \text{ p.p.f. (comp.)}$

stress =  $4200 \div 60 = 70 \text{ p.s.i.}$

at the edge sin  $28^\circ 06'$  is 0.4715 cos is 0.8818

Total load is  $2\pi 89^2 \times 102.4(1 - 0.882) = 600,000 \text{ p. (above edge)}$

$$T = 600,000 \div 2 \pi 89 \times \sin^2 28^\circ 08' = 4800 \text{ p.p.f. stress} = 80 \text{ p.s.i. (comp.)}$$

$$H = -T + 104.5 \times 89 \times \cos 28^\circ 08' = 4000 \text{ lbs/ft. stress} = 67 \text{ p.s.i. (comp.)}$$

From the above, no reinforcement is needed, for temp.  $4'' \times 4'' \times 4''$  wire mesh is used.

Ring tension on the edge member  $600,000 \times \cos \phi \div 2 \pi \sin \phi = 178,000 \text{ p.p.f.}$

Use  $5^{\text{mm}}$  high tension wires. Number of wire needed  $= \frac{178,000}{4350} = 41 \text{ pcs/ft.}$

The edge beam assumed is  $14'' \times 8''$ , the prestressing method is the same as that used on the tank wall.

c, Floor slab

The floor slab is designed as an ordinary slab footing. Its thickness =  $8''$ . Use number 4 bars  $10''$  c.c. in both directions.

d, Check for earthquake force:

Lateral force of 15 % of the weight of contained materials is applied to the tank as equivalent to the earthquake effect.

Uniformed lateral load =  $0.15 \times \gamma \times D = 630 \text{ p.p.f.}$

$M_z = \text{coefficient} \times F H^2$ . The values of coefficient is taken from table 9 Ref. (11). :  $M_z$  in ft. lbs per ft.

Negative maximum moment at base =  $- 0.0091 \times 630 \times 30^2 = - 5160$ .

Positive maximum moment at  $0.8H = + 0.0020 \times 630 \times 30^2 = +1130$ .

$$f_c = \frac{MC}{I} = \frac{3}{32} M \text{ p.s.i.}$$

|                 |            |       |            |       |
|-----------------|------------|-------|------------|-------|
| $f_c$ at base   | inner side | + 480 | outer side | - 480 |
| $f_c$ at $0.8H$ |            | - 106 |            | + 106 |

Check shear at base:

$$\text{weight of dome} = w_T \times \pi r^2 = 102.5 \times \pi \times 42^2 = 565,000 \text{ lbs.}$$

$$\text{weight of wall} = 150 \times H \times \pi \left\{ \left( 42 + \frac{8}{12} \right)^2 - 42^2 \right\} = 846,000 \text{ lbs.}$$

$$\text{weight of contained materials} = 50 \times H \times \pi \times 42^2 = 8,265,000 \text{ lbs.}$$

$$\text{Total} \quad \underline{\quad \quad \quad} \quad 9,676,000 \text{ lbs.}$$

$$\text{Total shear } V = 0.15 \times 9,676,000 = 1,451,400 \text{ lbs.}$$

$$I = \frac{\pi (d_o^4 - d_i^4)}{64} = 1.68 \times 10^5 \text{ ft}^4$$

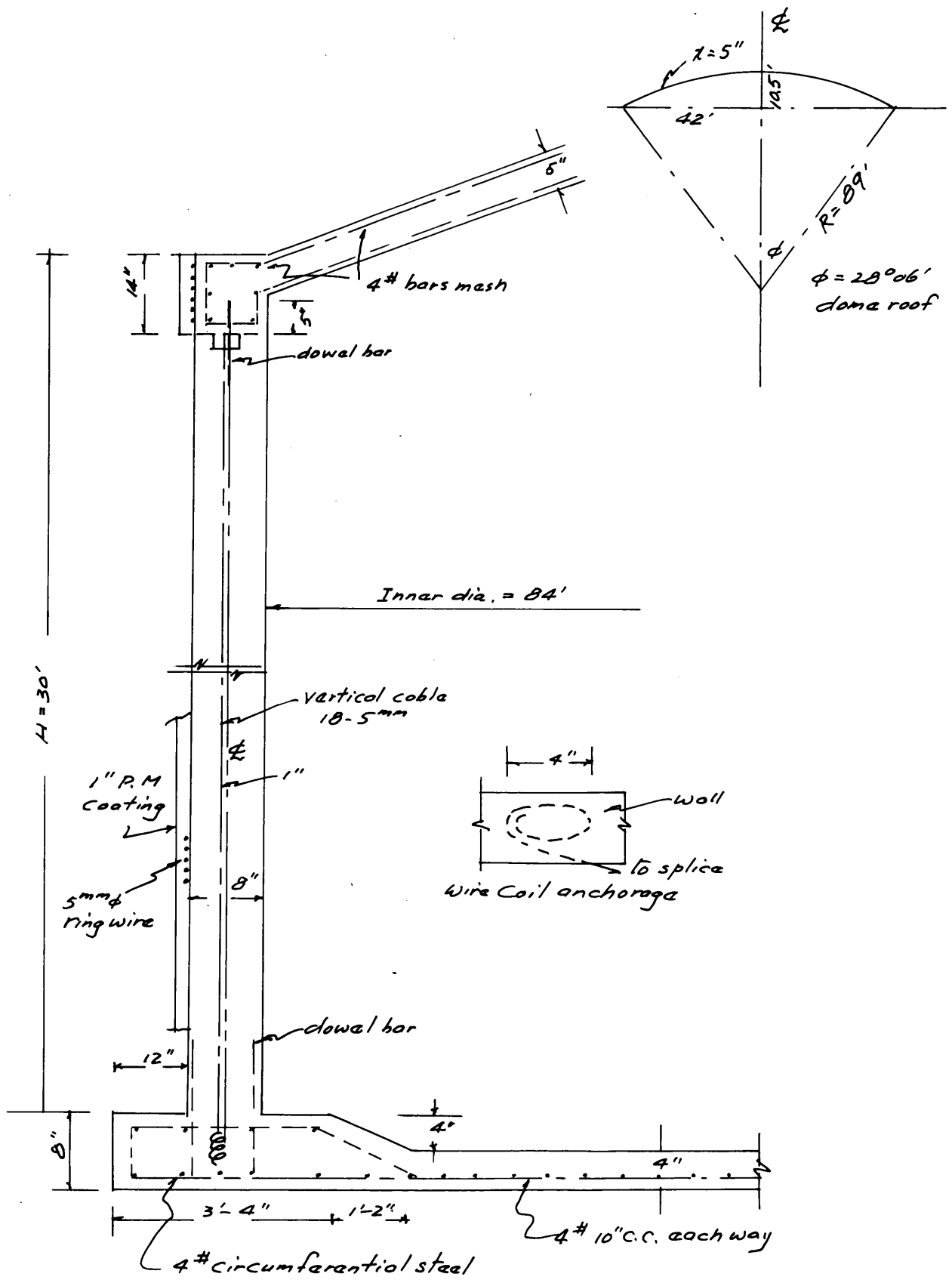
$$Q = \frac{1}{2} \pi r_o^2 (1 - 0.5756 r_o) - \frac{1}{2} \pi r_i^2 (1 - 0.5756 r_i)$$

$$= \frac{1}{2} \pi (42.7^2)(0.4244 \times 42.7) - \frac{1}{2} \pi (42)^2(0.4244 \times 42) = 2200 \text{ ft}^3$$

$$b = 2 \times 8 = 16''$$

$$v = \frac{VQ}{Ib} = \frac{145/100 \times 2200 \times 1728}{1.68 \times 10^5 \times 144 \times 144 \times 16} = 99 \text{ p.s.i.}$$

Figure 16 Complete sketch of circular prestressed concrete Tank



## XI. DYNAMIC ANALYSIS

This analysis is limited to the effect of earthquake forces. In tank design practice, we usually select the predominant lateral forces D.L. + L.L. + either wind or earthquake force. It is usually found that the earthquake force is larger than wind force.

According to the explanation of a Japanese expert on earthquakes, Professor John K. Minami, at the M.I.T. conference on dynamic design problems; the earthquake force is a shock or exciting force to the structures; it is measured by the amount of kinetic energy released by the shock. (28).

Professor Newmark of the University of Illinois (29) concludes that the earthquake impulse exists only a very short moment on the structure and transfers an amount of kinetic energy to it; after the impulse is over, the structure comes to a stage of free vibration. The energy is then absorbed by the internal strain and the vibration is damped out. In all dynamic analysis Newton's second law and the law of conservation of energy are used as the basic principles.

Most dynamic analyses are very complicated in mathematical derivation and solution. In practical work, however, simplification is necessary. Professor Newmark has a very good observation on this (29). He states that due to the uncertainties of loading, duration of the disturbance, and resistance of the structure, a very accurate calculation really does not give an exact solution. A rapid checking method is considered desirable. In this thesis an approximate method will be considered.

From a paper written by John E. Rinne at the conference on the discussion of the earthquake and blast effect on structures, held at the University of California in 1956, (33), the following is quoted: "In the vast majority of structures, vibration in the first or the fundamental mode accounts for most earthquake forces. Only in the tall building does the effect of the second mode become appreciable and the third and higher mode can be neglected for almost all practical purposes." In the tank problem, only the fundamental mode is considered.

As to the damping effect, it is stated in chapter 7, p. 135 of Ref. (28): "..... The effect of damping is neglected. This is possible because in most problems of structural dynamics we are only interested in the first peak value of deflection and not the continuous state of vibration. For this reason, the damping is of little importance." The viscous damping character of the concrete, therefore, is omitted here.

These statements are important to a study on the earthquake dynamic analysis for the tank problem.

The lateral force code prepared by the Joint Committee of San Francisco, California, A.S.C.E. and the Structural Engineer Association of Northern California and published in Ref. 34, represents a work of outstanding significance and value in pointing toward a rational approach to the dynamic problem of earthquake resistant design.



The code specifies that:

$$T \text{ (the fundamental period of vibration in sec.)} = 0.06 \frac{H}{\sqrt{b}}$$

H is the height of the building in feet and b is the width at the base in feet, in the direction of movement.

$$\text{The seismic coefficient } C = \frac{0.015}{T}$$

The lateral force  $F = C W$  where W is the weight of the structure.

In some places subjected to the severe earthquake disturbances, the assumed C value may be as high as 15 %.

For an approximate method of structural dynamic analysis, the structure should be idealized first in such a way that the total mass of the structure is distributed to several concentrated points connected by the rest of the idealized structural elements which are weightless but have the same rigidity as the original structure.

There are several ways to study this problem. If we consider the tank as a vertical hollow structure clamped at the base subjected to the lateral load, the whole structure will probably deflect as shown in figure 17b. The shearing forces at the sides of point A should be taken into consideration. Or we may consider circular ring sections placed horizontally and acted upon by the lateral force. Then the deflections of points A and Points B will be different. However in this approximate method, we assume the tank is subjected to the ground motion from any direction due to the earthquake and

point A is at the top of a vertical strip of the idealized tank wall. This strip is the critical section to be investigated. In our calculation we further assume that the mass of the tank is concentrated at two parts, one half at the top and one half at the base. The lumped mass at the top will be used to estimate the maximum deflection resulting from the earthquake disturbance. Its spring constant is the force required to produce a unit deflection of the top section of the tank wall. After the maximum deflection is calculated, then the equivalent lateral force corresponding to this dynamic effect can be figured.

Basic equation for dynamic analysis  $F = ma$

$F$  = external force,  $Q$  is the resistance of the structure

$M$  = mass,  $a$  is the acceleration

$F = f(t)$ ,  $Q = f(x)$  and  $x$  the displacement,  $k$  the spring constant

$$M \frac{d^2x}{dt^2} = F(t) - Q(x) \quad (1)$$

From equation (1):

$$\int_0^x F(t) dx = \frac{1}{2} M \left( \frac{dx}{dt} \right)^2 + \int_0^x Q(x) dx \quad (2)$$

Equation (2) derived from equation (1) is a statement of energy conservation, in which the first term is the external energy, the second term is the internal kinetic energy while the last is the potential energy or the internal strain energy.

The external energy due to the dynamic disturbance from an

earthquake is considered to be transferred to potential energy and vibrating kinetic energy in the structure. When the displacement  $x$  is the maximum, the velocity is zero and all the external energy will convert to the internal strain energy. At that time:

$$\int_0^x F(t) dx = \int_0^x Q(x) dx \quad (3)$$

External load and its duration curve can be obtained from the actual earthquake record. For the illustrated problem, we assume that the impulse diagram is triangular due to the impulse load as shown in Figure 17 (d).

$$W_e \text{ ( external work )} = \int_0^x F(t) dx = \int_0^t F(t) \frac{dx}{dt} dt \quad (4)$$

From equation (1)

$$v = \frac{dx}{dt} = \frac{1}{M} \int_0^t [F(t) - Q(x)] dt \quad (5)$$

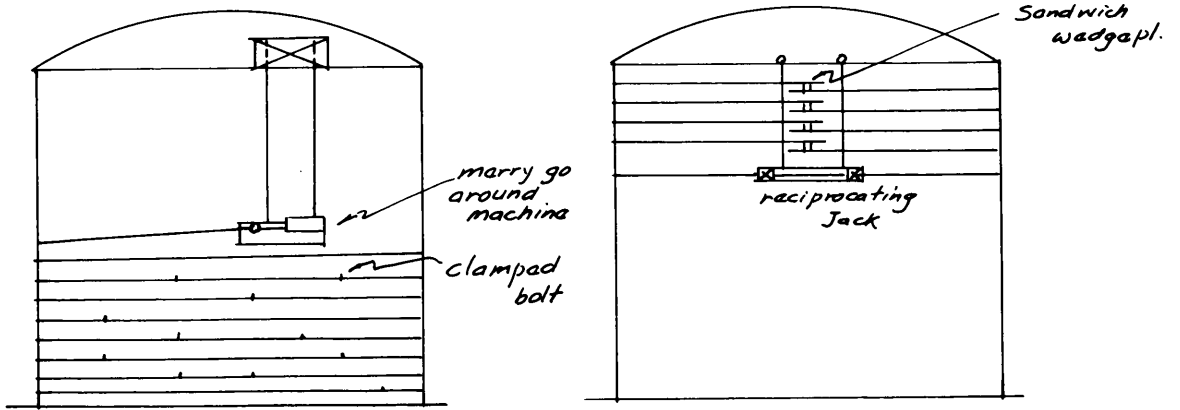
At the very beginning  $Q(x)$  is not fully mobilized and it may be considered zero.

$$\text{Then } v = \frac{1}{M} \int_0^t F(t) dt \quad (6)$$

$$\text{and } W_e = \int_0^t F(t) \left[ \frac{1}{M} \int_0^t F(t) dt \right] dt \quad (7)$$

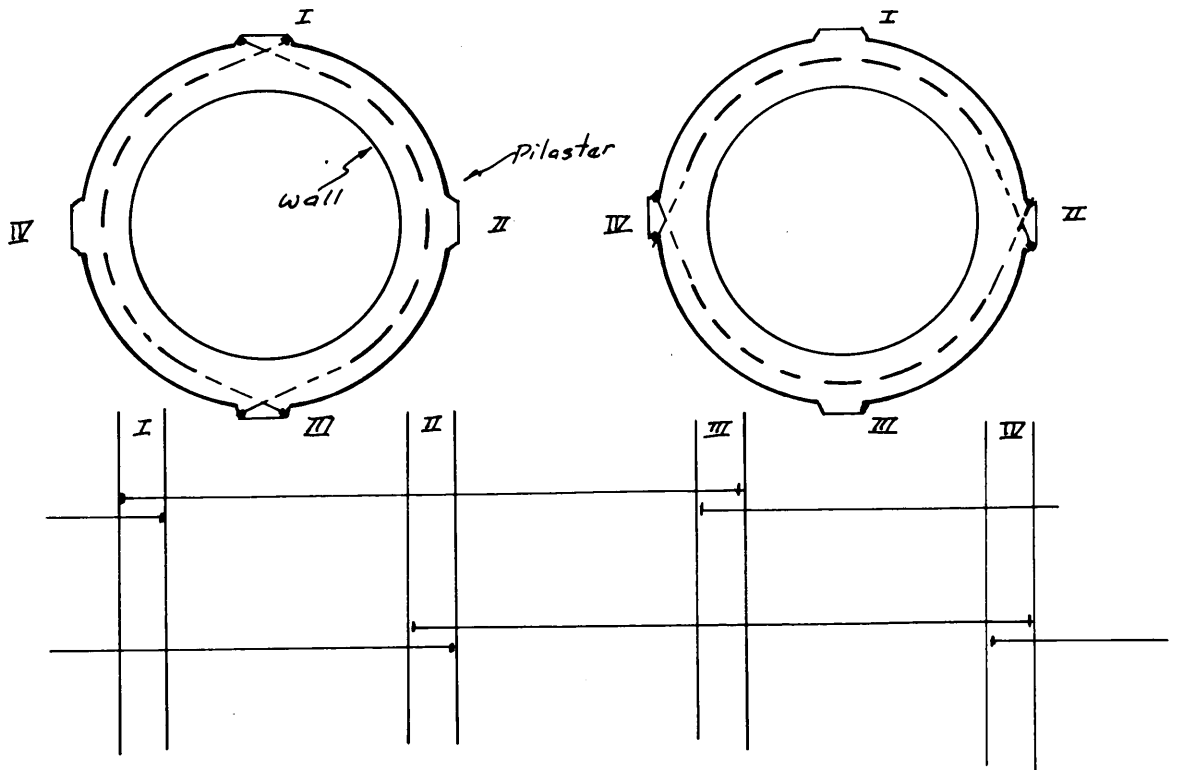
If  $I$  = total impulse = area of triangular load  
duration diagram

Figure 18 Different prestressing systems for circular Tank



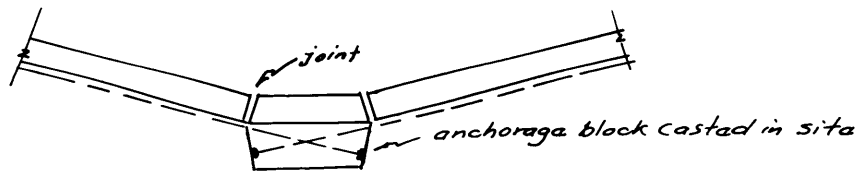
(a) Preload method

(b) Magnel-Blaton method



(c) Freyssinet cable method

(d) Precast method



$$I = \int_0^t F(t) dt$$

$$W_e = \frac{1}{M} \int_0^t F(t) \left[ \int_0^t F(t) dt \right] dt = \frac{I^2}{2M} \quad (8)$$

$\frac{I^2}{2M}$  is the initial energy given by the earthquake impulse.

At the maximum deflection  $x_m$ ,  $v = 0$

$$\frac{I^2}{2M} = \int_0^x Q(x) dx = \frac{1}{2} k x_m^2 \quad (9)$$

This equation is used to estimate  $x_m$  and thus calculate the equivalent lateral load.

Assume:  $H = 30'$        $D = 84'$        $t = 8''$

$W_T$  = total weight of tank including contained material

$W_T$  = wt. of dome ( $565^k$ ) + wt. of wall ( $846^k$ )  
+ wt. of contained material ( $8265^k$ )

Half of the mass is assumed to concentrate at the top.

$$M = (565^k + \frac{1}{2} \times 846^k + \frac{1}{2} \times 8265^k) \times \frac{1}{32.2}$$

$$= 158 \text{ k} - \text{sec}^2 \text{ per ft.}$$

$$\text{For a one foot strip} \quad M = 158 \div 84 = 1880 \frac{\text{lbs-sec}^2}{\text{ft.}}$$

$$F = \frac{0.15 \times 5121}{84} = \frac{770}{84} = 9170 \text{ lbs.}$$

$$t = \text{duration} = \frac{1}{2} \text{ sec}$$

$$I = \frac{1}{2} \times 9170 \times \frac{1}{2} = 2290 \text{ lbs-sec}$$

From table 5 Ref. (11), ring tension  $T_{\phi}$  at top due to a shear  $V$  at the top acted horizontally:

$$T_{\phi} = 14.74 \times \frac{VR}{H}$$

$$\left(\text{for } \frac{H^2}{Dt} = 16\right)$$

$$\text{Deflection at top} = w = T_{\phi} R \div Et = 14.74 \frac{VR^2}{H} \times \frac{1}{Et}$$

if  $w = 1''$ , then

$$V = \frac{1'' \times H \times E \times t}{14.74 \times R^2} = 32,200 \text{ lbs. per ft. per inch of deflection}$$

For one half concentrated mass and two sides of the wall:

$$\begin{aligned} k &= \left(V \times \frac{H}{2}\right) \times 2 = 32,200 \times 15 \times 2 \\ &= 966,000 \text{ lbs/in of deflection (approximately)} \end{aligned}$$

By equation (9)

$$\frac{1}{2} \times \frac{(2290)^2 \times 12''}{1880} = \frac{1}{2} \times 966,000 \times x_m^2$$

$$x_m = 0.19 \text{ inch}$$

From this, the equivalent load, reactions and stresses induced can be easily checked in the same way as the numerical example before.

The above is a rapid check for dynamic analysis. It is only an approximate method.

The vertical unit strip of wall has been analyzed as a free standing section at A' or A'' (Figure 15 (b) ).

Actually the rest of the wall, particularly at B' and B'' is fairly rigid and will offer considerable radial shearing resistance to the vertical unit strip. The resulting deflection  $x_m = 0.19$  inch is therefore considerably larger than what may be expected.

## XII. DISCUSSION

A comparative study of the prevailing circular prestressing systems and construction methods is included in this section for reference.

The Preload continuous winding method of circular prestressing is probably the best at the present time. In the United States, nearly all circular prestressed concrete tanks are built by the Preload method. They are designed with sliding base and free top construction. Internal pressure is taken by the circumferential wires alone. The winding machine is a self propelled merry-go-around machine which is suspended from the top of the wall and travels at about 3 to 7 m. p. h. The wires are pulled through a die to control the stress. The base friction coefficient used in design is 0.5. Rubber pads and rubber water stops are provided at the base and at construction joints. The only problem is the congestion of crowded wires. This sometimes causes trouble in construction work. (see figure 18 a).

The Magnel-Blaton method is a modification of the linear cable prestressing method applied to the construction of circular tanks. Cables are jacked around the curved wall of the tank and anchored by a set sandwich wedge plates. Jacks are hung from the trestle on the top of the wall and placed horizontally during the prestressing process. Friction loss is the chief problem in this system. The loss of friction may be up to 24 % of the prestressing force. (see



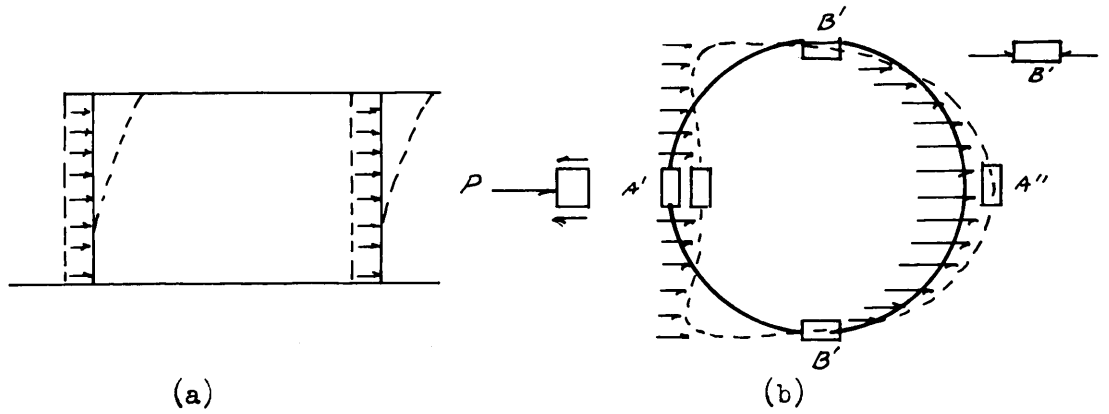
figure 18 b).

The Freyssinet method is also a modified linear prestressing method applied to circular tank construction. Freyssinet cones are used for the anchorages. Cables are tensioned at a distance of one-quarter of the circumference apart. Friction loss is the chief problem also. However, this method is preferred when it is necessary to put the cables inside the wall section. In some European countries where the climate is severely cold, this method is more suitable. Freyssinet cables are put inside metal conduit tubes. After tensioning, they are grouted to bond with the wall, (see figure 18 c).

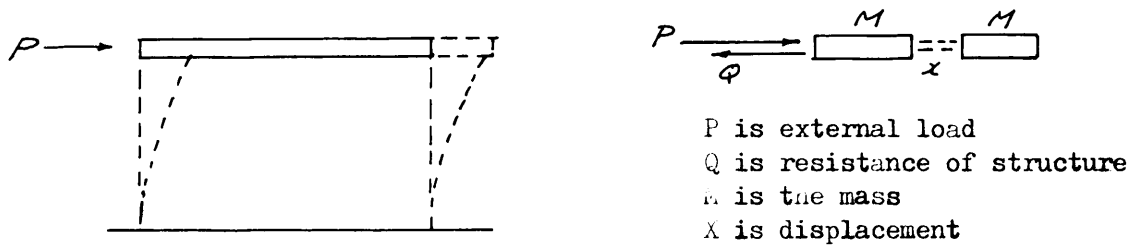
In the precast method, precast vertical concrete staves and horizontal steel bands are used. This is the same as the water barrel. In England and Africa, this method had been practiced. Cables or wires may be used. After tensioning, the spaces between the staves are filled with concrete. Water leakage is the main defect, (see figure 18 d).

The proposed zig-zag extensioning method is a continuous winding post-tensioning method. The wall is designed to be fixed at the base connection. The introduction of end restraint moments at the base will decrease the number of wires required in circumferential prestressing. No elaborate machine, and no special anchorages are needed. A simple jacking device is sufficient to overcome the friction force during the zig-zag extensioning process. No grouting is necessary. The only problem will be the large moment at the base and it might be necessary to have additional dowels at the base

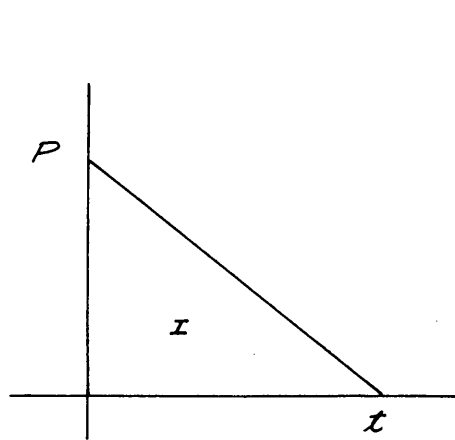
Figure 17 Sketches on Dynamic analysis for Tank



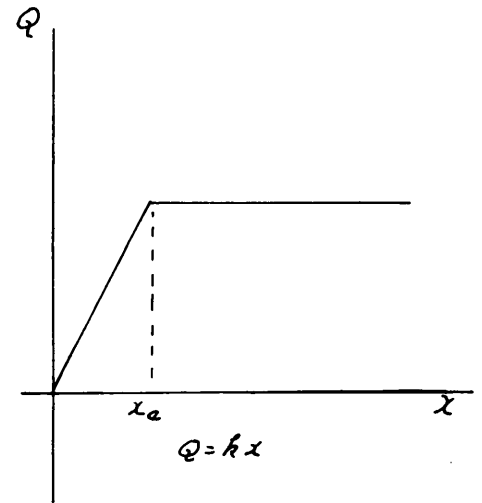
Lateral Earthquake Impulse



(c) Idealized Structure



(d) Load duration diagram



(e) Resistance displacement Diagram

connection to take care of a part of the tensile stress produced.

The zig-zag extensioning method is more simple and flexible both in design and construction. Future applications will verify these facts, the author believes.

Some difficulties may be anticipated in the prestressing of the wires in the proposed zig-zag extensioning method such as the sequence of extensioning the wires and the stretching of wire to produce uniform stress in the wire. This may be determined from a full size model test in the field.

The continuous winding of the wires around the wall and connecting of the wires to the splices also presents field problems. All of these should be well-arranged before construction.

Several practical construction problems require special mention:

1, The concrete work of the tank can be divided into several steps

(This method is generally used in Europe):

- a, The lower part of the wall nearly one-third of the height of the wall can be poured first. Due to the large moment at the base some engineers prefer to use stub wall construction. This means that part of the vertical prestressing wires can be cut off at the height of the stub wall and post tensioned. Not all the vertical wires are needed through the whole height of the wall.
- b, The upper part of the wall and the mid-portion of the base slab are the next to be poured. The effect of shrinkage of the base away from the wall is therefore minimized.

- c, The roof and that part of the slab near the wall complete the concrete work, (see figure 19 a).
- 2, The concrete work can also be accomplished by a sliding form to facilitate the continuous pouring and eliminate the horizontal construction joints. This is the method generally used in the United States.
  - 3, For the fixed end connection, the end anchorages of the vertical cables should be arranged very carefully. For the sliding base, water tightness must be given special attention, (see figure 19 b).
  - 4, Vertical prestressing is required to be done before the circumferential prestressing work in order to avoid the tensile stress caused by the horizontal wires.
  - 5, Measurement of the deflections during and after construction are needed to have a check on the calculations in the design.
  - 6, Problems concerning the stress in that portion of the wire over the slot opening of the spacing bar are investigated as follows:

a, Bending stress:

Bending strains induced in the extreme fibers of the wires where they pass over the spacing bar are of the order of magnitude of 5 %. Whether or not these additional strains will induce premature failure of the wire will depend upon the stress-strain characteristics of the wire. Preliminary laboratory tests are necessary to verify the limits of curvature permitted. The curvature can be decreased by using wider spacing bars.

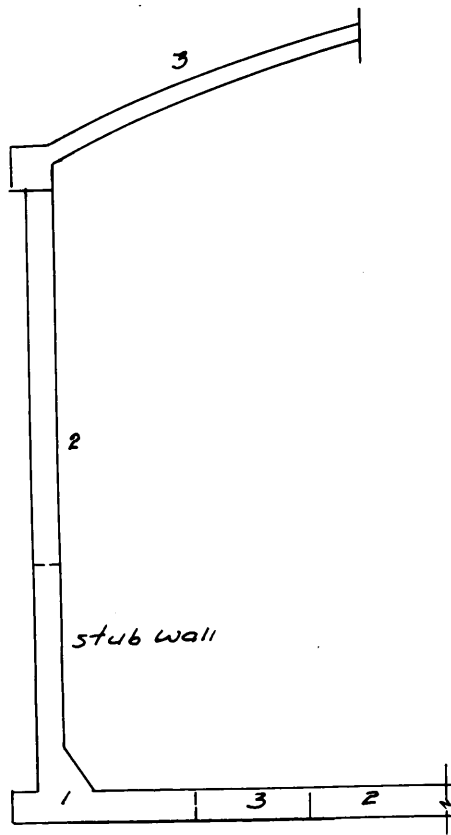
b, The concentration of stress due to bearing pressure:

The wire is tensioned in two directions. The tensile stress in the wire is assumed to be uniform throughout the whole length. Over the slot, the contact stress on the wire due to the bearing pressure can be assumed to effect the whole diametral section of the wire, because the size of the wire is very small. It is approximately equal to

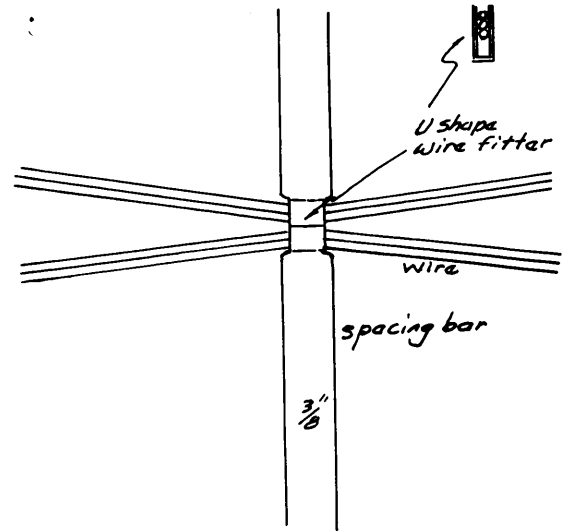
$2 A_s \times f_s \times \sin i \div (3/8'' \times d) = 16,000 \text{ p.s.i.}$  According to Timoshenko, the material at the center of the surface of contact can sustain high pressure without failure, since it is under compression from all sides. (p. 341, Strength of Material, part 2 by S. Timoshenko).

c, The opening of the slot should be smoothed as shown in figure 19 c. The sharp edges of the slot are eliminated to avoid the cutting of wire. It is better to use a U-shape steel wire fitter to fit the wire in position and to furnish larger bearing area.

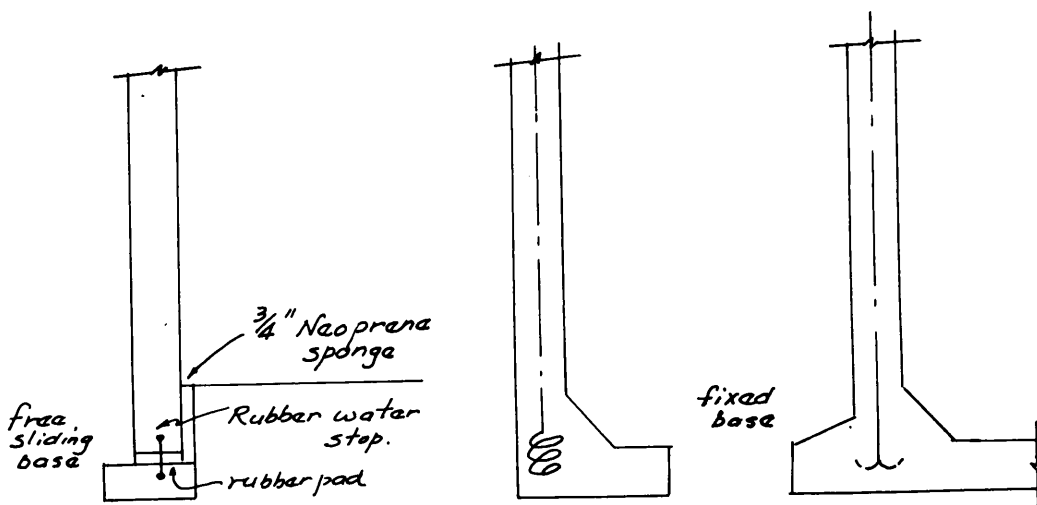
Figure 19 Construction Methods of Prestressed concrete Tank



(a) Orders of concrete work



(c) Wire connection in slot of spacing bar



(b) Base connection of Tank wall and footing

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These all will never be forgotten for my whole life.

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A STUDY OF THE COMPLETE DESIGN FOR CIRCULAR  
PRESTRESSED CONCRETE TANK AND ITS DYNAMIC ANALYSIS

Abstract

The purpose of this thesis is to study the complete design for a circular prestressed concrete tank and the application of the principle of dynamic analysis to the design of the tank structure. A proposed method of circular prestressing is presented and the comparative study of the prevailing circular prestressing system is also included.

The thesis is divided into four parts. The first part is a study of the structural analysis using the theory of the thin shell as an approach and the prestressed concrete design. The second part presents the proposed zig-zag extensioning method of circular prestressing. Numerical examples for the complete design procedure follow as illustrations. The third part makes a brief review on the dynamic analysis for the tank structure due to the earthquake disturbance. In this section, only the application of an approximate energy method is discussed. The fourth part of this thesis is the comparison between different methods of circular prestressing including the Preload method, Magnel-Blaton method, Freyssinet cable method, Precast method, and the Zig-zag extensioning method. Several construction problems are discussed.