THEORETICAL CONSIDERATIONS OF THE MAGNETOHYDRODYNAMIC GENERATOR

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IV. SYMBOLS

a	dimensionless constant, $\frac{m_n u_{n,o}^2}{kT}$
A	cross-sectional area of generating channel
$\vec{B}$	magnetic flux density, webers/m^2
C_p	specific heat at constant pressure, $\frac{\text{joules}}{\text{kgm}\ {}^{\circ}K}$
e	electronic charge, coulombs
$\vec{E}$	electric field strength, volts/m
$\vec{j}$	current density, amps/m^2
k	Boltzmann's constant, $\frac{\text{joules}}{\text{particle}\ {}^{\circ}K}$
K	dimensionless constant, $(\lambda + \mu + \mu\lambda)\left(\frac{\varepsilon_{in}\ n_{n,o}}{eB}\right)^2$
L	dimensionless constant, $\frac{\vec{E} \cdot \vec{j}}{\vec{j} \times \vec{B} \cdot \vec{V}_n}$
m	particle mass, kgm
n	particle number density, particles/m^3
$\vec{p}$	pressure, newtons/m^2
$\vec{P}$	momentum exchange variable, newtons/m^3
Q	collision cross section, m^2
t	time, sec
T	temperature, ${}^{\circ}K$
$\vec{u}$	axial velocity, m/sec
$\vec{v}$	transverse velocity, m/sec
$\vec{V}$	resultant velocity, m/sec
$\vec{W}$	random velocity, m/sec
x	axial coordinate, m

α initial mole fraction of seeding material, $\frac{n_e}{n_n}$

β dimensionless variable, $e^{-a\left(\frac{1-L}{2}\right)(1-\theta^2)}$

Γ collision rate per unit volume

δ reduced mass, kgm

ε constant, joule-m-sec

θ dimensionless variable, $\frac{u_n}{u_{n,o}}$

λ dimensionless constant, $\frac{\varepsilon_{en}}{\varepsilon_{in}}$

μ dimensionless constant, $\alpha \frac{\varepsilon_{ei}}{\varepsilon_{in}}$

ρ mass density, kgm/m^3

σ scalar electrical conductivity, mho/m

ξ dimensionless variable, $\frac{\sigma B^2}{\rho_{n,o}\, u_{n,o}}\, x$

τ mean free time, sec

ψ stress tensor, $\frac{m \sum_r (\vec{W})(\vec{W})}{\text{volume}}$, newtons/m^2

ω cyclotron frequency, radians/sec

Subscripts

e electron

i ion

n neutral particle

o initial condition at $x = 0$

x axial component

y transverse component

V. INTRODUCTION

The distinction between the magnetohydrodynamic generator and the conventional wire wound generator is that the role of the armature in the latter is played by an electrically conducting fluid in the former. In the conventional generator currents are induced in highly conducting wires; in the magnetohydrodynamic generator currents are induced in a relatively low conducting fluid. This fluid is passed through a transverse magnetic field and between two parallel plate electrodes. (See fig. 1.) The induced electric field in the fluid, which is proportional to the fluid velocity and magnetic flux density, maintains a voltage drop across the electrodes and a current is generated when the electrodes are connected to an external load, closing the circuit. Thus energy is extracted from the conducting fluid and delivered to a load as electrical power.

Prior investigators (refs. 1, 2, and 3) have formulated this basic concept of a magnetohydrodynamic generator and each has contributed significantly. However, a literature survey of this work reveals two conspicuous points. First is the lack of quantitative information currently available which pertains to such devices as the magnetohydrodynamic generator and the other is the omission of ion slip considerations.

The results of this analysis indicate the heat associated with ion slip performs a major function in the constant temperature generating flow. When the ions move on the average through an angle of 10^{-2} radians or more between collisions with neutral particles, that is, $\omega_i \tau_i \geq 10^{-2}$, the energy transformed to heat from ion "friction" is of the same order of magnitude as the conventionally referred to joule heat, j^2/σ_o. As the number density diminishes, $\omega_i\tau_i$ increases, and the role of ion slip becomes more prominent.

Expressions are obtained for the ion, electron, and neutral particle velocities and densities, and for the axial and transverse components of the electric field as a function of the channel distance x.

Results for a cesium seeded nitrogen gas as the working fluid are presented in graphical form.

VI. BASIC ASSUMPTIONS

Magnetic Field

The present method of determining flow behavior will involve a basic assumption about the net magnetic field; namely, the magnetic flux density due to currents in the fluid is negligible over a fairly large region compared with the externally applied transverse B. Since the current is due largely to the externally applied B, we have from Ampere's law

$$B(\text{currents}) \propto \frac{1}{2}\, r\mu\sigma V B(\text{applied})$$

In the m.k.s. system of units

$$\mu(\text{permeability}) \sim 10^{-7}$$

$$\sigma(\text{conductivity}) \sim 10^{2}\ \frac{\text{mho}}{\text{m}}$$

$$V(\text{velocity}) \sim 10^{3}\ \frac{\text{m}}{\text{sec}}$$

Thus

$$\frac{B(\text{currents})}{B(\text{applied})} \sim 10^{-2}\, r$$

where r is a length large compared with the width of the channel. This analysis considers a channel width of 2.5 centimeters such that

$$r \sim 1 \gg 0.025$$

and

$$\frac{B(\text{currents})}{B(\text{applied})} \sim 10^{-2} \tag{3}$$

Current Density

In the generator the ion and electron cyclotron frequencies differ greatly due to their difference in mass. The ions on the average travel only a fraction of a cyclotron path between collisions and the electrons necessarily travel over many cycloidal paths between collisions. To prevent a net charge from building up high potentials, by exhausting a net electrical charge, we assume the axial components of the electron and ion velocities be equal, that is

$$u_e = u_i \tag{1}$$

This requires the net current be normal to the channel axis since

$$\vec{j} = n_e \, e\left(\vec{V}_i - \vec{V}_e\right) \tag{2}$$

The transverse electron velocity will be much greater than the transverse ion velocity due to the difference in inertia of the electrons and ions - hence, the electric field will be slanted. This difference in transverse velocities will contribute an axial component to the electric field of the form

$$\vec{E}_x = -\left(C_1 \vec{V}_e \times \vec{B} + C_2 \vec{V}_i \times \vec{B}\right)$$

Constant Temperature

The requirement that flow energy be transformed into electrical energy implies the only suitable working fluids are gases. This requirement, however, introduces a basic difficulty, that is, gases are relatively poor conductors. A consideration of three physical properties of gases emphasizes certain criteria which may be complied with in order to achieve and maintain optimum conductivity characteristics.

(a) As a gas becomes ionized the number of positive ions increases thereby decreasing the mean free path of electrons.

(b) Consequently, the gas conductivity is not a linear function of electron density.

(c) Gases which have lower ionization potentials generally have higher collision cross sections with charges. (More loosely bound electrons → higher polarizability → greater charge-induced dipole forces → larger cross section.)

An easily ionizable gas is obviously desirable because of the small energy required to obtain the charged particles but property (c) limits this desirability since the conductivity decreases as the collision cross section increases. Probably the best physical means available for producing a plasma consists of an easily ionizable gas mixed with a gas of small electron scattering cross section. The low partial pressure of the seeding gas permits a greater percentage of it to be ionized than if it alone were used and the seeded gas permits the electron mean free path to be longer.

A convenient choice of material for a seeding gas is the alkali metal cesium, and for an inert seeded gas, nitrogen. At the temperature (4000° K) and pressure (1 atmosphere) of interest, dissociation and ionization of the diatomic nitrogen molecule are negligible. Furthermore, the ionization of the seeding gas (0.01 mole fraction) cesium is more than 95 percent complete and electron concentrations on the order of 10^{16} electrons per cc can be obtained (ref. 9). However, even under optimum conditions, conductivities of the order of 10^3 mhos per meter probably are the best which can be achieved. As the fluid is expanded through the variable area channel the charged particle number density will decrease due to the falling temperature, thereby diminishing the conductivity. To maintain the temperature at a constant value, we require the channel cross-sectional area to vary in such a manner that

$$\nabla T = 0 \tag{4}$$

Degree of Ionization

In the plasma all electrons are initially obtained from thermal ionization of the seeding gas. For a typically seeded plasma (ref. 9) it has been shown that in the temperature range of interest only single ionization need be considered. Consequently, for each ion in the plasma there is one corresponding electron. The densities are thus

$$n_e = n_i \tag{5}$$

provided no charge separation occurs spatially. If it is further assumed that the ionization of the cesium is practically complete, then

$$n_e = n_i = \alpha n_n$$

where α is the fraction of seeding material (0.01).

Matched Impedence

From simple generator theory the ratio of the external impedence to the internal impedence is equal to 1 for optimum conditions. In the magnetohydrodynamic generator we consider that element of the external load which represents the power extracted from an increment of length along the generator channel. (To satisfy the conditions for a slanted electric field, segmented electrodes are required. This increment of length may be thought of as representing the length of one of these electrodes.) If the temperature remains constant, the energy extracted from the flow per unit volume and per unit time $(-\vec{E} \cdot \vec{j})$ is equal to the decrease in kinetic energy of that volume, whereas the total work done on the charged particles is $(-\vec{j} \times \vec{B} \cdot \vec{V}_n)$. (The difference is the total heat generated in the gas and serves to keep T constant.) If the ratio of external to internal impedence is denoted by L, then

$$L = \frac{\vec{E} \cdot \vec{j}}{\vec{j} \times \vec{B} \cdot \vec{V}_n} = \text{constant} < 1 \tag{6}$$

since E_y varies with u_n. For L greater than 1 the device ceases to generate and operates as an accelerator with energy being fed into the gas through the electric field.

VII. FORMULATION AND SOLUTION OF MACROSCOPIC EQUATIONS

Prior investigators have done considerable work concerning the behavior of individual particles in such devices as the magnetohydrodynamic generator. However, a study of individual particles is not the most convenient method for obtaining quantitative information of such flow problems. This is partly because the current density, a macroscopic quantity, plays an important part in the process giving rise to both electric and magnetic fields. Moreover, for any accurate computations, a distribution of particle velocities of single particles requires the consideration of a discouragingly large number of particles. For rigorous results, the magnetohydrodynamic generator can most conveniently be analyzed in terms of the individual macroscopic equations of motion for electrons, ions, and neutral particles. The macroscopic quantities, j, (current density), and V (velocity) are determined by the so-called transfer equations of kinetic theory. Solutions obtained assume an nonviscous, compressible, constant temperature power extraction model.

The equation of motion for a particular species (ions, electrons, or neutral particles) is derived from the Liouville equation. This derivation requires that Hamilton's equations of motion be satisfied, i.e., the external force fields be conservative. An unpublished analysis (based on the equation of continuity in six-dimensional phase space and Newton's second law) by Dr. Willard E. Meador, Jr. shows that this restriction is not necessary. As far as the use of the momentum equation in this analysis is concerned, restrictions will come only in the interpretation of P, the net momentum gained by the species in collisions with other species. Neglecting the momentum gained due to collisions between like particles in this interpretation implies neglect of long-range cumulative effects (Fokker-Planck form of the Boltzmann equation). In the sense, binary collisions will be assumed in deriving an expression for P.

For ions of charge Ze, mass m_i, and number density n_i, the equation of momentum transfer is

$$n_i \, m_i \left(\frac{\partial \vec{V}_i}{\partial t} + (\vec{V}_i \cdot \vec{\nabla}) \vec{V}_i \right) = n_i \, Ze \left(\vec{E} + \vec{V}_i \times \vec{B} \right)$$

$$- \vec{\nabla} \cdot \bar{\psi}_i - n_i \, m_i \, \vec{\nabla} \varphi + \vec{P}_{ie} + \vec{P}_{in} \qquad (7)$$

where:

φ = gravitational potential

V_i = mean ion velocity in an element of volume

$\bar{\psi}_i$ = stress tensor, or dyadic defined by

$$\bar{\psi}_i = \frac{m_i \, \Sigma(\vec{W}_i)(\vec{W}_i)}{\text{unit volume}}$$

$(\vec{W}_i$ = random velocity)

the particle summation extending over the volume element

P_{ie} = net momentum gained by the ions per unit volume per unit time by collisions with the electrons.

P_{in} = net momentum gained by the ions per unit volume per unit time by collisions with the neutral particles.

Similarly the momentum balance for electrons is

$$n_e\, m_e\left(\frac{\partial \vec{V}_e}{\partial t} + (\vec{V}_e \cdot \vec{\nabla})\vec{V}_e\right) = -n_e\, e\left(\vec{E} + \vec{V}_e \times \vec{B}\right) - \vec{\nabla} \cdot \bar{\bar{\psi}}_e$$

$$-n_e\, m_e\, \vec{\nabla}\varphi + \vec{P}_{ei} + \vec{P}_{en} \tag{8}$$

and the neutral particle equation of motion

$$n_n\, m_n\left(\frac{\partial \vec{V}_n}{\partial t} + (\vec{V}_n \cdot \vec{\nabla})\vec{V}_n\right) = -\vec{\nabla} \cdot \bar{\bar{\psi}}_n - n_n\, m_n\, \vec{\nabla}\varphi + \vec{P}_{ni} + \vec{P}_{ne} \tag{9}$$

The form of these equations is exact with the previously mentioned restrictions for a nonrelativistic gas: they are useful, however, only when the distribution of random velocities is sufficiently well behaved so that the stress tensor $\bar{\bar{\psi}}$ may be approximated in a relatively simple way. The stress tensor has nine components, ψ_{lm} where l and m represent directions along each of the three coordinate axes. Since ψ_{lm} equals ψ_{ml} there are only six independent components. We assume the distribution of random velocities is isotropic such that ψ_{lm} vanishes unless l equals m and the three diagonal components ψ_{xx}, ψ_{yy}, and ψ_{zz} are all equal to each other and to the scaler pressure p. In this situation

$$\nabla \cdot \bar{\bar{\psi}} = \nabla p \tag{10}$$

Consider a Maxwellian set of particles of density n_a per colliding with another Maxwellian set of density n_b. Assuming that collisions are due primarily to thermal motion the mean relative speed is, from kinetic theory (ref. 7)

$$\bar{V} = \left(\bar{V}_a{}^2 + \bar{V}_b{}^2\right)^{1/2} = \left(\frac{8kT}{\pi\ \delta}\right)^{1/2}$$

where δ is the reduced mass. The number of collisions in the +x direction is the same as in the -x direction such that the net collision rate of particles moving in the x direction depends only on the motion of particles in the y-z plane. Thus the collision rate of particles a with particles b per unit volume is

$$\Gamma_{ab} = \frac{2}{3}\, n_a\ n_b\ Q_{ab}\ \bar{V} = \frac{2}{3}\, n_a\ n_b\ Q_{ab}\left(\frac{8kT}{\pi\ \delta}\right)^{1/2} \tag{11}$$

and is equal to the rate of momentum exchange per unit volume, P_{ab}, divided by the momentum exchange per collision. The latter is easily obtained from Newton's second and third laws of motion as follows. When particle a approaches particle b the forces exerted on a due to the presence of b is equal and opposite to those exerted on b due to the presence of a, that is

$$\frac{d}{dt}(m_a\ \vec{V}_a) = f(\vec{r}) = -\frac{d}{dt}(m_b\ \vec{V}_b)$$

and by definition

$$\vec{V}_{relative} = \vec{V}_b - \vec{V}_a$$

Thus the exchange of momentum during one collision is $2\delta(\vec{V}_b - \vec{V}_a)$ and the net momentum gained by particles a per unit time per unit volume

$$\vec{P}_{ab} = 2\Gamma_{ab}\,\delta\left(\vec{V}_b - \vec{V}_a\right) = n_a\, n_b\, Q_{ab}\,\frac{8}{3}\left(\frac{2kT}{\pi}\,\delta\right)^{1/2}\left(\vec{V}_b - \vec{V}_a\right)$$

$$= n_a\, n_b\, \epsilon_{ab}\left(\vec{V}_b - \vec{V}_a\right) \tag{12}$$

where

$$\epsilon \equiv \frac{8}{3}\left(\frac{2kT}{\pi}\,\delta\right)^{1/2} Q_{ab}$$

and is hereafter referred to as a drag or friction parameter (ref. 5).

If either particle a or b is an electron, and the temperature is constant, the drag or friction term is directly proportional to and dependent only on the collision cross section.

To give some insight into the phenomena permitted by these basic equations in the magnetohydrodynamic generator, steady-state solutions are obtained where conditions are relatively simple and results are more readily understood.

Preliminary calculations reveal the ion and electron pressure gradients to be several orders of magnitudes smaller than the electric and magnetic force terms on the right-hand side of equations (7) and (8), respectively. The spatial acceleration terms are also much smaller than the terms of the right-hand side. Further justification for this assumption is given by the resulting form of Ohm's law and also by the agreement between the numerical results of this analysis and those in reference 1.

In fact, the assumption

$$\vec{\nabla} p_i = \vec{\nabla} p_e = 0 = (\vec{V}_e \cdot \vec{\nabla})\vec{V}_e = (\vec{V}_i \cdot \vec{\nabla})\vec{V}_i \tag{13}$$

introduces a negligible error. Summarizing, we have

$$\vec{\nabla} \cdot \bar{\Psi}_n = \vec{\nabla} p_n$$

$$\vec{\nabla} \cdot \bar{\Psi}_i = \vec{\nabla} p_i = 0 = \vec{\nabla} \cdot \bar{\Psi}_e = \vec{\nabla} p_e$$

$$\vec{P}_{ab} = n_a\, n_b\, \epsilon_{ab}\left(\vec{V}_b - \vec{V}_a\right)$$

$$\vec{\nabla}\varphi = 0 \qquad \text{(neglect gravitational effects)}$$

$$(\vec{V}_i \cdot \vec{\nabla})\vec{V}_i = 0 = (\vec{V}_e \cdot \vec{\nabla})\vec{V}_e$$

$$\frac{\partial \vec{V}_e}{\partial t} = \frac{\partial \vec{V}_i}{\partial t} = \frac{\partial \vec{V}_n}{\partial t} = 0 \qquad \text{(steady state)} \tag{14}$$

These conditions applied to the macroscopic equations of motion gives

$$n_e\, n_i\, \epsilon_{ei}\left(\vec{V}_e - \vec{V}_i\right) + n_e\, n_n\, \epsilon_{en}\left(\vec{V}_e - \vec{V}_n\right) = -n_e\, e\left(\vec{E} + \vec{V}_e \times \vec{B}\right) \tag{15}$$

$$n_i\, n_e\, \epsilon_{ie}\left(\vec{V}_i - \vec{V}_e\right) + n_i\, n_n\, \epsilon_{in}\left(\vec{V}_i - \vec{V}_n\right) = n_e\, e\left(\vec{E} + \vec{V}_i \times \vec{B}\right) \tag{16}$$

$$n_e\, n_n\, \epsilon_{ne}\left(\vec{V}_n - \vec{V}_e\right) + n_n\, n_i\, \epsilon_{ni}\left(\vec{V}_n - \vec{V}_i\right) + n_n\, m_n\, (\vec{V}_n \cdot \vec{\nabla})\vec{V}_n + \vec{\nabla} p = 0 \tag{17}$$

Assuming the energy of the ions and electrons to be negligible compared with that of the neutrals, the energy equation is derived from the following considerations. Let n_n be the number of neutral particles in unit volume and $n'_n m_n = 1$. The work done on charges by unit mass of gas while moving a distance dx in time dt, is

$$-\frac{(\vec{j} \times \vec{B} \cdot \vec{V}_n)n'_n}{n_n} dt = -\frac{\vec{j} \times \vec{B} \cdot \vec{V}_n}{n_n m_n u_n} dx$$

and is equal to the work done in maintaining the current

$$-\frac{\vec{E} \cdot \vec{j}}{n_n m_n u_n} dx$$

plus the total heat generated. The external work done on unit mass of gas in time dt is

$$\Big(p - (p + dp)\Big)\frac{V}{dx} dx = -V\, dp$$

where V is the volume occupied by unit mass. The increase in energy of unit mass of gas is the increase in internal energy $\left(d \int_0^T C_v \, dT\right)$ plus the decrease in internal energy (pdV) plus the decrease in kinetic energy. Thus,

$$-\frac{\vec{E} \cdot \vec{j}}{n_n m_n u_n} dx = -Vdp - d\int_0^T C_v \, dT - pdV - \frac{1}{2} d\, u_n^2$$

or

$$\vec{E} \cdot \vec{j} = n_n \, m_n \, \vec{V}_n \cdot \left(C_v \, \vec{\nabla} T + \vec{\nabla}(pV) + \vec{V}_n \cdot \vec{\nabla} \vec{V}_n \right)$$

But,

$$C_v \, \vec{\nabla} T + \vec{\nabla}(pV) = C_v \, \vec{\nabla} T + \vec{\nabla}(n_n' \, kT) = (C_v + n_n' \, k) \, \vec{\nabla} T = C_p \, \vec{\nabla} T$$

where C_p is the constant pressure heat capacity of unit mass. Finally

$$\vec{E} \cdot \vec{j} = n_n \, m_n \, \vec{V}_n \cdot \left(C_p \, \vec{\nabla} T + \vec{V}_n \cdot \vec{\nabla} \vec{V}_n \right)$$

Since $C_p \nabla T$ includes the heat generated (or an increase in T) as well as heat lost (or decrease in T), the assumption $\nabla T = 0$ leads to a solution which specifies the increase in A along the channel. Assuming no heat lost through the walls the increasing area tends to drop the temperature by the same amount as the generated heat tends to raise the temperature. Thus, for constant temperature the energy equation reduces to

$$n_n \, m_n \, \vec{V}_n \cdot (\vec{V}_n \cdot \vec{\nabla}) \vec{V}_n = \vec{E} \cdot \vec{j} \tag{18}$$

Other Relations

$$n_e = n_i = \alpha n_n \qquad \text{(space neutralization)} \tag{19}$$

$$u_e = u_i \qquad \text{(no axial current)} \tag{20}$$

$$\frac{\vec{E} \cdot \vec{j}}{\vec{j} \times \vec{B} \cdot \vec{V}_n} = L \qquad \text{(Constant impedence ratio)} \tag{21}$$

$$\vec{\nabla} \cdot (n_n \vec{V}_n) = 0 \qquad \text{(Neutral particle continuity)} \qquad (22)$$

$$p_n = n_n kT \qquad \text{(Perfect gas)} \qquad (23)$$

We further assume that the neutral particles move on the average in the axial direction such that

$$\frac{v_n}{v_e} \ll \frac{v_n}{v_i} \ll 1 \qquad (24)$$

and

$$\frac{v_n}{u_n} \ll 1 \qquad (25)$$

Although this reduces the problem to a quasi one-dimensional analysis, similar to that commonly used in aerodynamics, valid for small channel openings, it does not contradict the assumption of constant temperature. The latter determines the area increase whereas the former determines a minimum length for the area increase.

Equations (15) through (25) constitute a set of seventeen equations, including the vector component equations, containing seventeen dependent variables. These equations are solved simultaneously using elementary algebraic techniques to obtain a first-order nonlinear differential equation expressing the channel length as a function of neutral particle velocity.

Imposing the condition $u_e = u_i$, the quotient of the axial components of the electron and ion momentum equations is

$$\lambda v_i + v_e + (1 + \lambda)\frac{Ex}{B} = 0 \tag{26}$$

Where

$$\lambda = \frac{\epsilon_{en}}{\epsilon_{in}}$$

Similarly the sum of the y-components of equations (15) and (16) is (with eq. (24))

$$v_i = -\lambda \; v_e \tag{27}$$

Solving equations (26) and (27) for v_e we have

$$v_e = -\frac{1}{1 - \lambda}\frac{E_x}{B} \tag{28}$$

Using the definition of current density, that is,

$$\vec{j} = n_e \; e\left(\vec{V}_i - \vec{V}_e\right)$$

in conjunction with equations (18), (25), (27), and (28) yields

$$\alpha \; e \; \frac{1 + \lambda}{1 - \lambda} E_x = \frac{m_n \; u_n}{L}\frac{du_n}{dx} \tag{29}$$

The neutral particle momentum equation may then be written

$$\frac{1 - L}{L} u_n \; du_n = kT \frac{dn_n}{n_n}$$

which integrates to

$$n_n = n_{n,o} \; e^{\frac{a}{2}\frac{1-L}{L}(\theta^2-1)} \tag{30}$$

where

$$\theta = \frac{u_n}{u_{n,o}}$$

$$a = \frac{m_n\ u_{n,o}^2}{kT}$$

Equations (27), (28), and (30) in equations (15) and (16) result in an expression for the axial electric field, E_x, as a function of the neutral particle velocity.

$$E_x = -\frac{1-\lambda}{2} B \frac{1}{K\beta + 1}\left(\frac{K\beta}{\lambda + \mu + \mu\lambda}\right)^{1/2} u_n \tag{31}$$

where

$$K = (\lambda + \mu + \mu\lambda)\left(\frac{\epsilon_{in}\ n_{n,o}}{eB}\right)^2$$

$$\beta = e^{-a \frac{1-L}{L}(1-\theta^2)}$$

$$\mu = \alpha \frac{\epsilon_{ie}}{\epsilon_{in}}$$

E_x may now be eliminated from equation (29) and the variables separated to yield

$$d\xi = \frac{-d\theta}{K\beta^{1/2}}\left(1 + \frac{K\beta}{1 + \frac{\lambda}{K\beta}}\right) \tag{32}$$

where

$$d\xi = \frac{\alpha e^2(1 + \lambda)}{\epsilon_{in} (\lambda + \mu + \mu\lambda)} \frac{B^2}{\rho_{n,o} u_{n,o}} dx = \frac{\sigma_o B^2}{\rho_{n,o} u_{n,o}} dx$$

One of the more recent and complete analyses of the magnetohydrodynamic generator (ref. 1) which is currently available, results in a relation similar to but is in fact a limiting case of equation (32). For $K\beta \gg 1$, which corresponds to $1 > \theta > 0.9$, equation (32) becomes

$$d\xi = -d\theta \ \beta^{1/2} \tag{33}$$

or

$$\frac{1 - L}{2}\left(\frac{\alpha e^2(1 + \lambda)}{\epsilon \quad (\lambda + \mu + \mu\lambda)}\right)\frac{B^2}{\rho_{n,o} u_{n,o}} x = \int_\theta^1 d\theta \ e^{\frac{m_n u_{n,o}^2}{2kT}(\theta^2-1)} \tag{34}$$

this compares with the results of the reference analysis

$$\frac{1}{4} \frac{\sigma B^2}{\rho_{n,o} u_{n,o}} x = \int_\theta^1 d\theta \ e^{\frac{m_n u_{n,o}^2}{2kT}(\theta^2-1)} \tag{35}$$

where L has been set equal to one-half.

The bracketed term on the left side of equation (34) has the units of conductivity, and as we shall see later, is the scalar conductivity independent of the magnetic field.

$$\sigma_o = \frac{\alpha e \ (1 + \lambda)}{\epsilon_{in} \ (\lambda + \mu + \mu\lambda)} \tag{36}$$

Substituting this expression into equation (34)

$$\frac{1}{4} \frac{\sigma_o B^2}{\rho_{n,o} u_{n,o}} x = \int_\theta^1 d\theta \, e^{\frac{m_n u_{n,o}^2}{2kT}(\theta^2 - 1)}$$

which is identical to equation (35). Thus the results of the two independent analyses should, and in fact do check when one considers the extraction of a relatively small quantity of energy, on the order of 10 percent or less. As previously assumed, the only manner in which power may be extracted is by a decrease in the kinetic energy of the flow. As expansion occurs, a greater amount of heat energy must be supplied to hold the temperature constant.

Thus, to understand equation (34), it is necessary to understand this internal fluid heating mechanism. The rate of useful work done per unit volume in maintaining the current $\vec{j}$ is given by the sum of equation (15) dotted into $\vec{V}_e$ and equation (16) dotted into $\vec{V}_i$, that is

$$\begin{aligned} -\vec{E} \cdot \vec{j} = {} & -n_e \, n_n \, \epsilon_{en} \left[\vec{V}_e - \vec{V}_n\right] \cdot \vec{V}_e - n_i \, n_n \, \epsilon_{in} \left[\vec{V}_i - \vec{V}_n\right] \cdot \vec{V}_i \\ & -n_e \, n_i \, \epsilon_{ie} \left[\vec{V}_i - \vec{V}_e\right] \cdot \left[\vec{V}_i - \vec{V}_e\right] \end{aligned} \tag{37}$$

and is equal to the power per unit volume extracted from the flow.

The rate at which work is done by the neutral particles on the charges, from the sum of equations (15) and (16) dotted into $\vec{V}_n$, is

$$-\vec{j} \times \vec{B} \cdot \vec{V}_n = -n_e \, n_n \, \epsilon_{en} \left[\vec{V}_e - \vec{V}_n\right] \cdot \vec{V}_n - n_i \, n_n \, \epsilon_{in} \left[\vec{V}_i - \vec{V}_n\right] \cdot \vec{V}_n \tag{38}$$

The total heat generated in the flow per unit volume per unit time is the amount of work done by the neutral particles on the charges minus the energy extracted from the flow and is after some manipulation equal to

$$-\vec{j} \times \vec{B} \cdot \vec{V}_n + \vec{E} \cdot \vec{j} = \frac{\vec{j} \cdot \vec{j}}{\sigma_o}\left[1 + \frac{1}{\lambda + \mu}\left(\frac{eB}{\epsilon_{in}\, n_n}\right)^2\right] \tag{39}$$

The ion cyclotron frequency is given by

$$\omega_i = \frac{eB}{m_i}$$

and the number of collisions per ion per unit time is given by

$$\frac{\Gamma_{i,n}}{n_i} = \frac{1}{\tau_i} = \frac{1}{2}\,\frac{m_i + m_n}{m_i\, m_n}\,\epsilon_{in}\, n_n \tag{40}$$

Thus, the angle subtending the mean free path is

$$\omega_i\, \tau_i = \frac{2m_n}{m_i + m_n}\,\frac{eB}{\epsilon_{in}\, n_n} \tag{41}$$

and its effect on fluid heating may be seen from equations (40) and (39)

$$-\vec{j} \times \vec{B} \cdot \vec{V}_n + \vec{E} \cdot \vec{j} = \frac{\vec{j} \cdot \vec{j}}{\sigma_o}\left[1 + \frac{1}{\lambda + \mu}\left(\frac{m_i + m_n}{2m_n}\right)^2 (\omega_i\, \tau_i)^2\right] \tag{42}$$

Specifically for a nitrogen gas λ and μ are each of the order of 10^{-3} (see appendix), so that the ion slip becomes appreciable when $\omega_i\, \tau_i \approx 10^{-2}$ and increases with the square of $\omega_i\, \tau_i$.

The requirement $\nabla T = 0$ imposed the condition that the generated heat balance the heat loss due to expansion, i.e.,

$$\frac{PdV}{\rho_n \; u_n(\vec{j} \times \vec{B} \cdot \vec{V}_n - \vec{E} \cdot \vec{j})} = \text{constant} \tag{43}$$

Equation (42) demonstrates how the generated heat increases with increasing A, since (from eqs. (41) and (22))

$$u_n \; A \propto \omega_i \; \tau_i$$

The heat loss due to expansion must be the same, otherwise, a solution with $\nabla T = 0$ would not exist. Consequently, the heat created due to ion slip is an important factor in realizing a constant temperature condition in the MHD generator.

To obtain a generalized expression of Ohm's law equation (15) is divided by ϵ_{en}, equation (16) by ϵ_{in}, and the two resulting equations are subtracted. Simplification of this expression yields

$$\frac{\vec{j}}{\sigma_o} = \vec{E} + \frac{1}{1 + \lambda}\left[\vec{V}_e + \lambda\vec{V}_i\right] \times \vec{B} \tag{44}$$

However, the electron velocity is greater than the ion velocity due to the difference in inertia of the two particles, and λ, the ratio of "friction" parameters $\dfrac{\epsilon_{en}}{\epsilon_{in}}$, is less than 1, that is,

$$\lambda = \frac{\epsilon_{en}}{\epsilon_{in}} = \left[\frac{m_e(m_i + m_n)}{m_i m_n}\right]^{1/2} \frac{Q_{en}}{Q_{in}} \ll 1$$

Thus, to a first approximation, equation (44) becomes

$$\frac{\vec{J}}{\sigma_o} = \vec{E} + \vec{V}_e \times \vec{B}$$

which is the well known and commonly used form of Ohm's law. Prior analyses (refs. 1, 2, and 3) have assumed this equation with $\vec{V}_e$ replaced with the velocity of center of mass.

VIII. FLOW AND FIELD CHARACTERISTICS

The following equations were derived in the previous section:

$$v_e = -\frac{1}{1-\lambda}\frac{E_x}{B}$$

$$E_x = -\frac{1-\lambda}{2} B \frac{1}{K\beta + 1}\left[\frac{K\beta}{\lambda + \mu + \mu\lambda}\right]^{1/2} u_n$$

$$v_i = -\lambda v_e$$

$$u_e = u_i = \frac{e}{n_n \epsilon_{in}}\left[v_i B + E_x\right] + u_n \quad \text{(from eq. (16))}$$

$$E_y = LB\, u_n$$

(L is a load characteristic subject to external determination through the current drawn. This implies a minimum current such that $n_e = n_i$ remains valid.)

$$n_n = n_{n,o}\, \beta^{1/2}$$

$$\xi = \int_\theta^1 \frac{d\theta}{K\beta^{1/2}}\left[1 + \frac{K\beta}{1 + \frac{\lambda}{K\beta}}\right] \tag{45}$$

From the neutral particle continuity equation we have

$$A = A_o \frac{n_{n,o}}{n_n}\frac{1}{\theta} \tag{46}$$

To demonstrate the behavior of these quantities inside the generating channel, a typical calculation was performed for a nitrogen gas partially seeded with cesium. A numerical integration of equation (45) was accomplished on a desk calculator using the basic trapezoidal rule. This method is quite applicable and yields good results if the curve $x = f(u_n)$ does not vary abruptly.

The variations in the flow and field characteristics are plotted in figures 2 - 7. One notes that for $L = 1/2$ the area opens very rapidly and suggests that it would be better to prevent the possibility of the flow breaking away from the walls by increasing L, that is, decreasing the rate at which the area opens. The area is plotted for $L = 3/8$, $1/2$, and $5/8$, and the effect in area change is apparent.

The amount of energy extracted per unit time per unit volume (from eq. (18)) is

$$\vec{E} \cdot \vec{j} = \rho_n \vec{V}_n \cdot (\vec{V}_n \cdot \vec{\nabla})\vec{V}_n$$

or the total energy extracted per unit time in distance dx (using eqs. (22) and (25)) is

$$- \int_o^x A(\vec{E} \cdot \vec{j})dx = \int_\theta^1 \rho_{n,o}\, u_{n,o}^3 \frac{n_n}{n_{n,o}} A\, \theta^2\, d\theta$$

$$= \frac{1}{2} \rho_{n,o}\, u_{n,o}^3\, A_o\left[1 - \theta^2\right]$$

As previously mentioned, the power extracted depends only on the change in the kinetic energy of the neutral particles. Figure 7 represents the power extracted as a function of neutral particle velocity.

A basic difficulty one is confronted with in evaluating these relations for a specific gas is the evaluation of the momentum interaction parameters ϵ_{ie}, ϵ_{in}, ϵ_{en}. Appendix I outlines these difficulties and suggests a reasonable solution.

IX. ACKNOWLEDGMENTS

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XII. APPENDIX

Evaluation of Friction Parameters, ϵ_{ei}, ϵ_{en}, ϵ_{in}

The interaction parameter as defined previously is

$$\epsilon_{jk} = \frac{8}{3}\left[\frac{2}{\pi} kT \frac{m_j\, m_k}{m_j + m_k}\right]^{1/2} Q_{jk} \qquad \text{(A-1)}$$

The difficulty in evaluation of this expression for electron-neutral interactions is due to the Ramsauer effect (ref. 7) which causes a larger variation in Q_{en} near the lower energies. This difficulty of rapid and large variation of Q_{en} with electron velocity is overcome by using experimental data on mobilities of swarms (monoenergetic) of electrons. Consider the electron momentum equation

$$n_e\, n_i\, \epsilon_{ie}\,(\vec{V}_e - \vec{V}_i) + n_e\, n_n\, \epsilon_{en}\,(\vec{V}_e - \vec{V}_n) = -n_e\, e\left[\vec{E} + \vec{V}_e \times \vec{B}\right] \qquad \text{(A-2)}$$

ϵ_{en} is independent of the magnetic field, B, and also the number density of ions. As these terms approach zero, equation (A-2) becomes

$$n_n\, \epsilon_{en}\,(\vec{V}_e - \vec{V}_n) = -e\vec{E}$$

or

$$n_n\, \epsilon_{en} = -e \left|\frac{\vec{E}}{\vec{V}_{n,e}}\right| \qquad \text{(A-3)}$$

where $\vec{V}_e - \vec{V}_n = \vec{V}_{n,e}$ = relative electron-neutral velocity and

$\left|\frac{\vec{E}}{\vec{V}_{n,e}}\right| = \frac{1}{\text{electron mobility}}$ (definition). Thus, ϵ_{en} may be evaluated from experimental data (ref. 7).

The Ramsauer effect is not significant in an ion-neutral interaction. However, for convenience, ϵ_{in} may be evaluated in a manner analogous to that used in the evaluation of ϵ_{en}. From the ion momentum equation

$$n_n \epsilon_{in} = e \left|\frac{\vec{E}}{\vec{V}_{n,i}}\right| \tag{A-4}$$

The data used in the calculations (figs. 2 - 6) was taken at NTP (ref. 6) and was adjusted accordingly by use of equation (A-1). For a first approximation the collision cross sections, Q_{en} and Q_{in}, were held constant in this adjustment.

The basic difficulty in evaluating ϵ_{ie} is due to the long-range forces between charged particles. There is no single acceptable theory by which Q_{ie} can be evaluated.

For a singly ionized gas (ref. 5)

$$Q_{ie} = 8.1 \times 10^{19} \frac{e^4}{(kT)^2} \ln \Lambda \tag{A-5}$$

where Λ contains the limit at which the coulomb forces can be considered significant.

Finkelnburg and Maecker (ref. 5) suggest

$$\Lambda = \frac{10^{-9} kT}{9 n_i^{1/3} \; e^2} \tag{A-6}$$

based on experimental data obtained in an arc.

Spitzer (ref. 4), however, based his calculations on the Debye shielding distance and obtains

$$\Lambda = 7 \times 10^{-16} \frac{kT}{n_i^{1/3} \; e^2} \tag{A-7}$$

In the evaluations of the interaction terms for the cesium seeded nitrogen gas the results obtained from equation (A-6) and from equation (A-7) were compared and the difference proved to be negligible. Since Q_{ie} varies as $\ln \Lambda$, the dependency is not critical and considerable error in Λ can be tolerated without significantly altering the results.

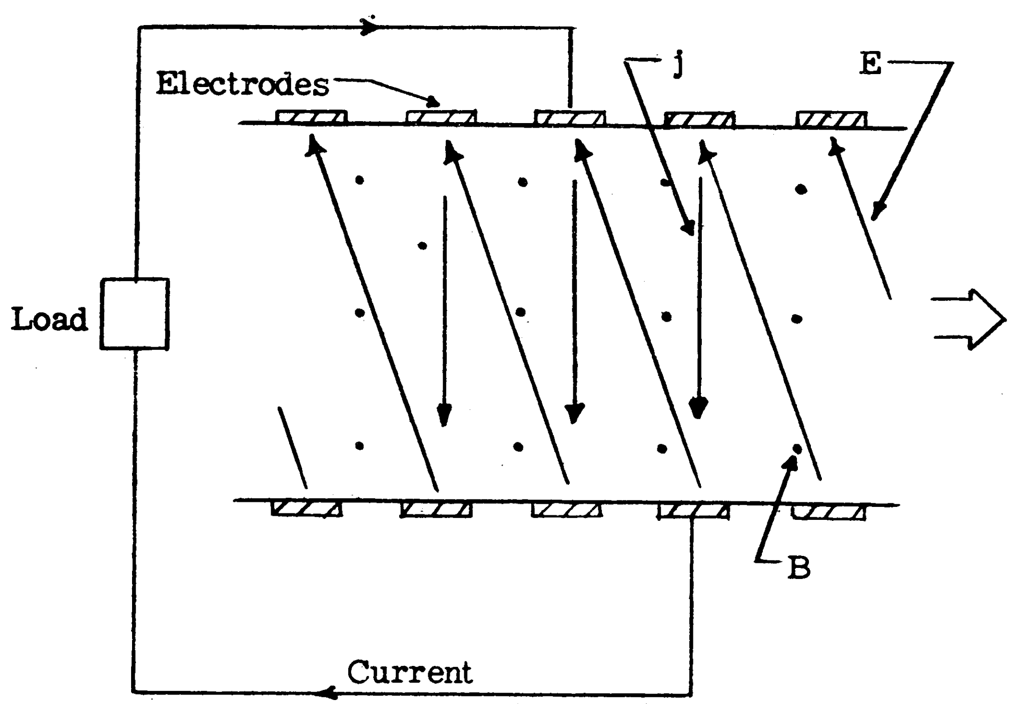

Figure 1.- Schematic diagram of crossed field generator.

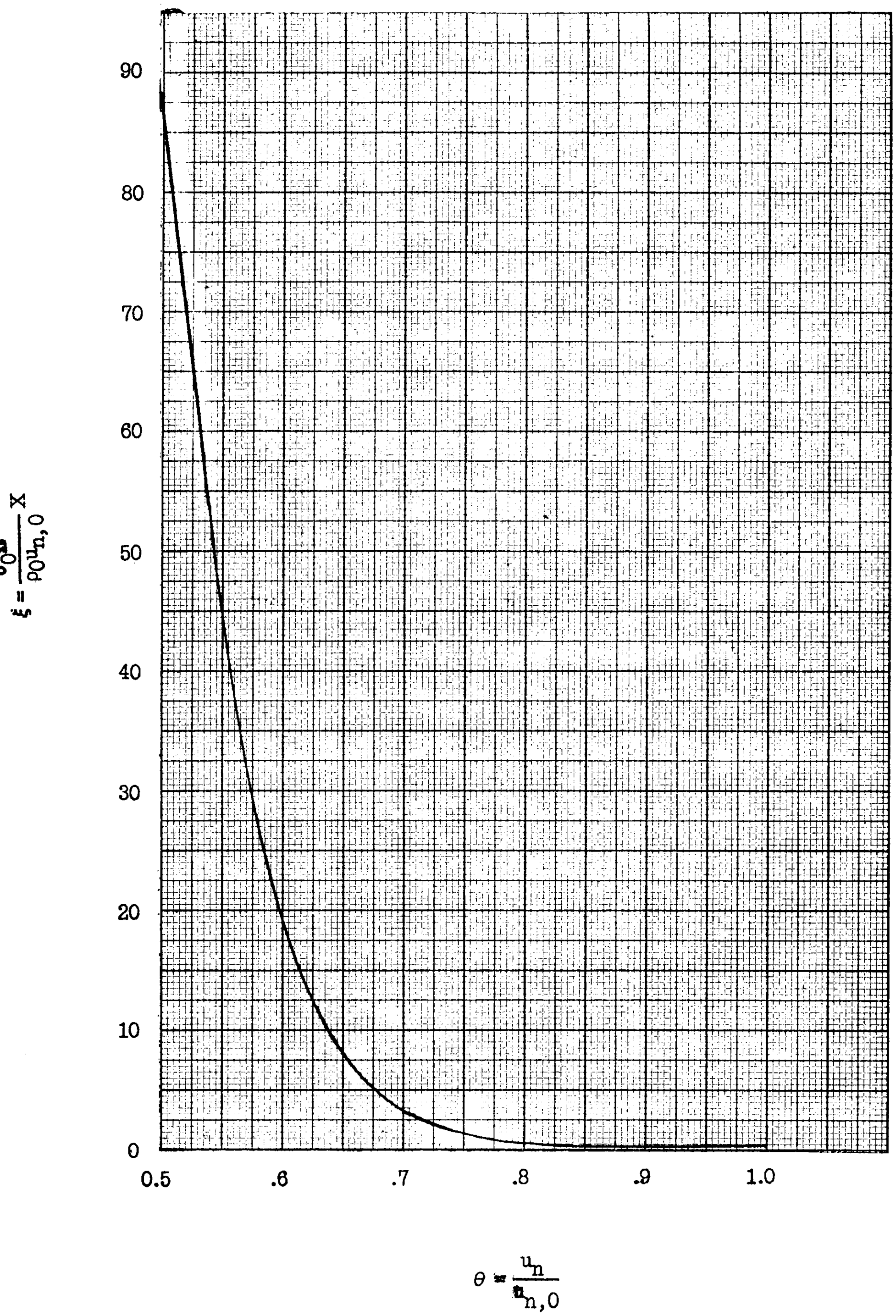

Figure 2.- Variation of neutral particle velocity with channel length.

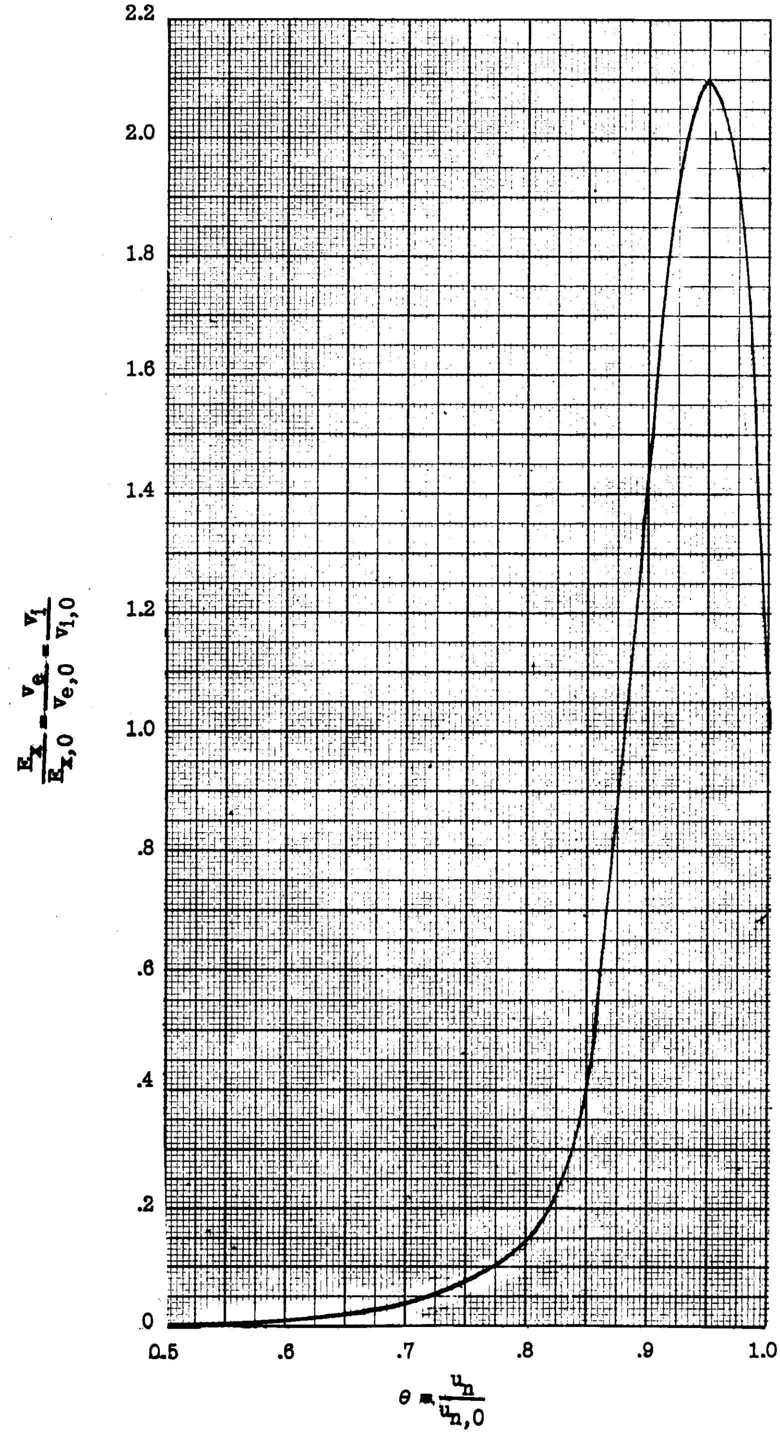

Figure 3.- Variation of axial electric field, transverse electron and ion velocities with netural particle velocity.

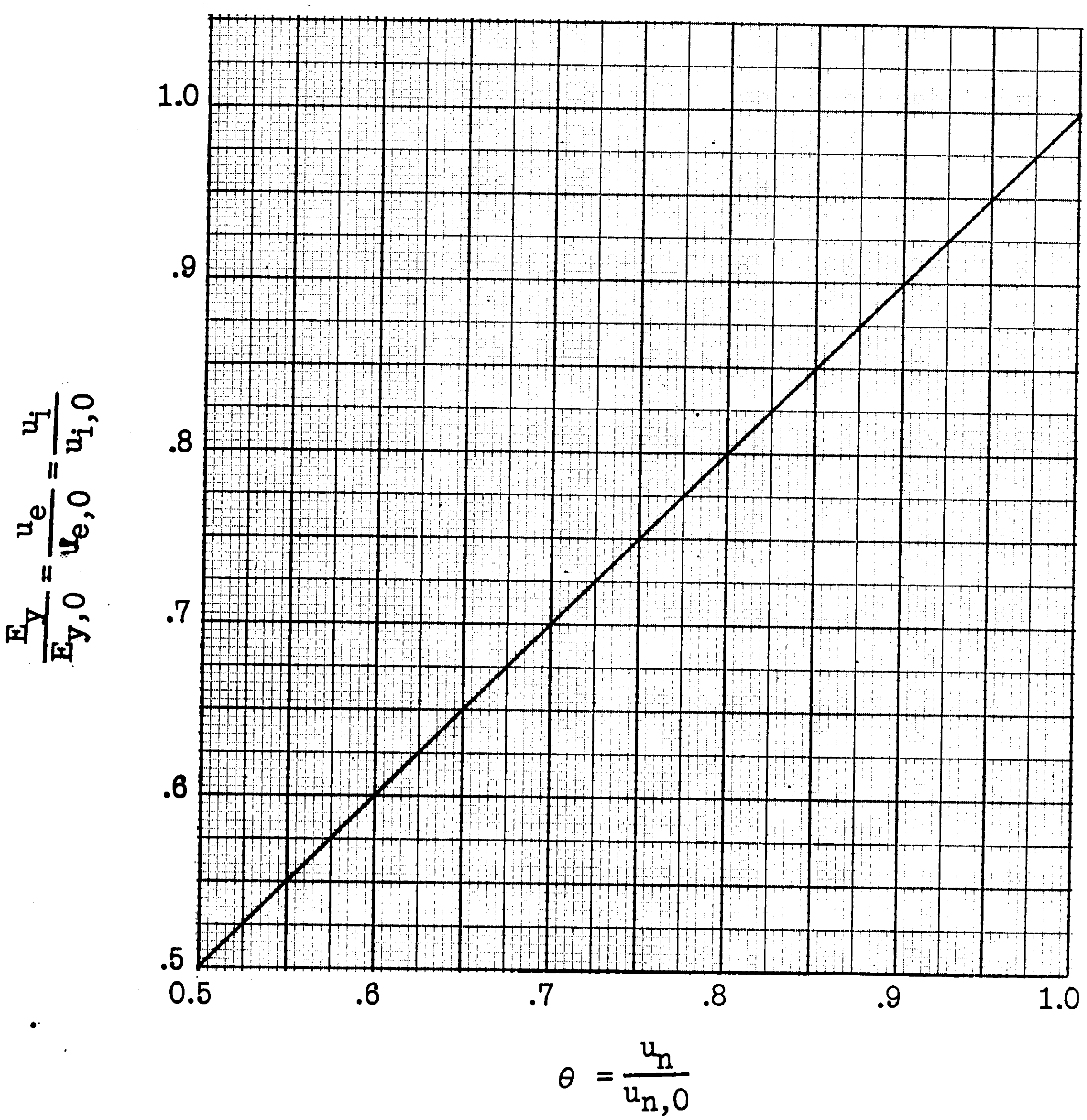

Figure 4.- Variation of transverse electric field, axial electron and ion velocities with neutral particle velocity.

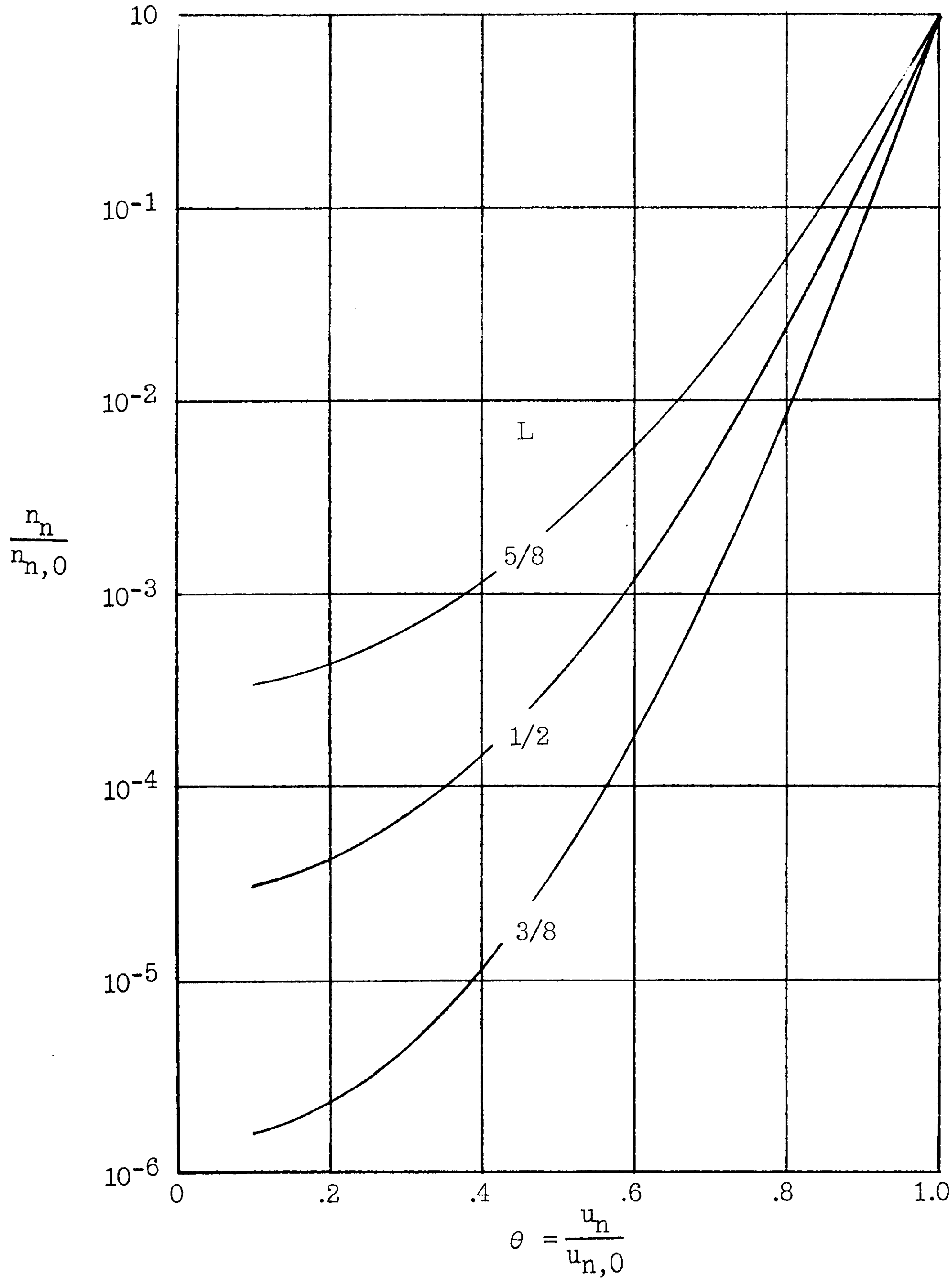

Figure 5.- Variation of neutral particle density with neutral particle velocity.

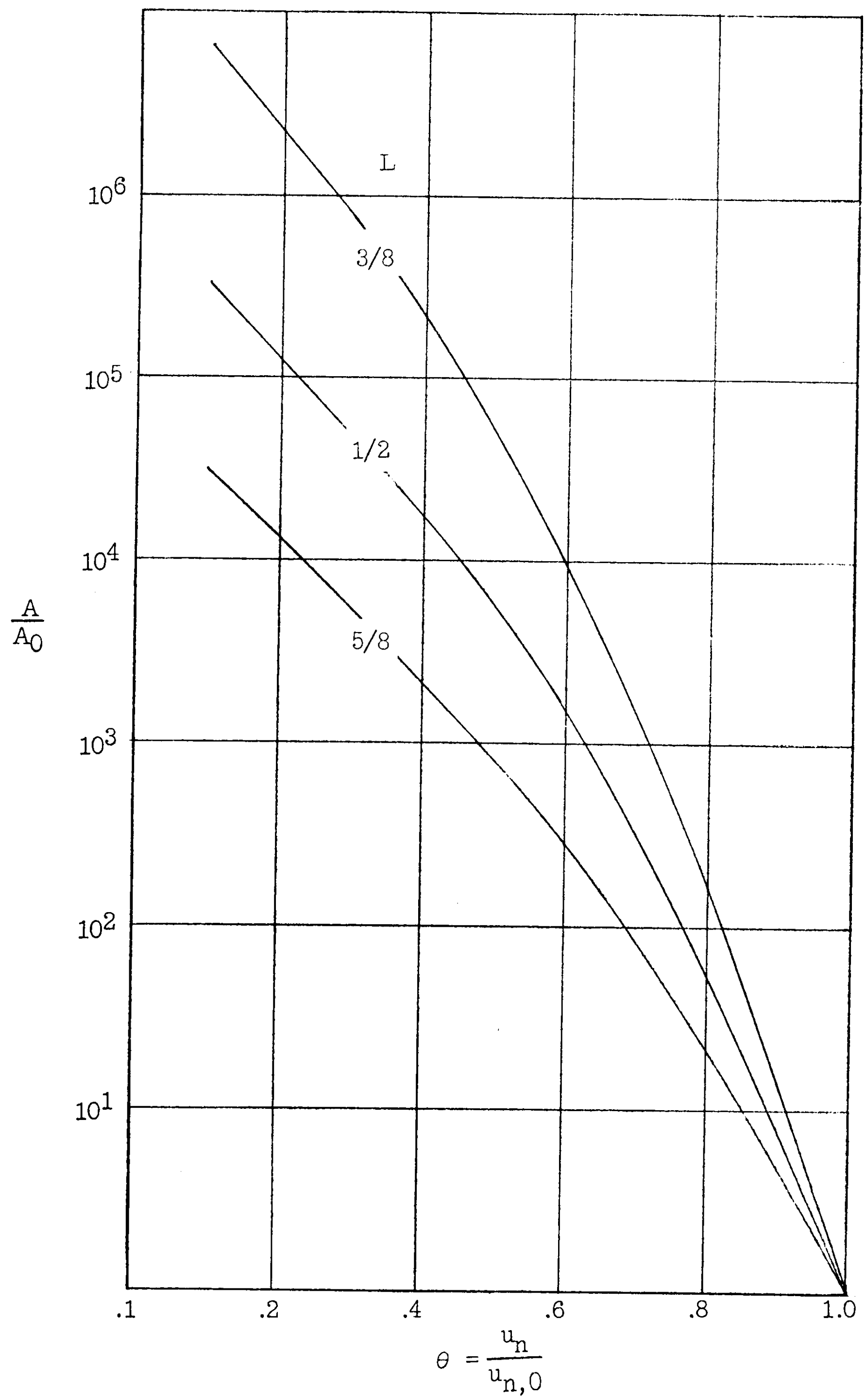

Figure 6.- Variation of cross-section area with neutral particle velocity.

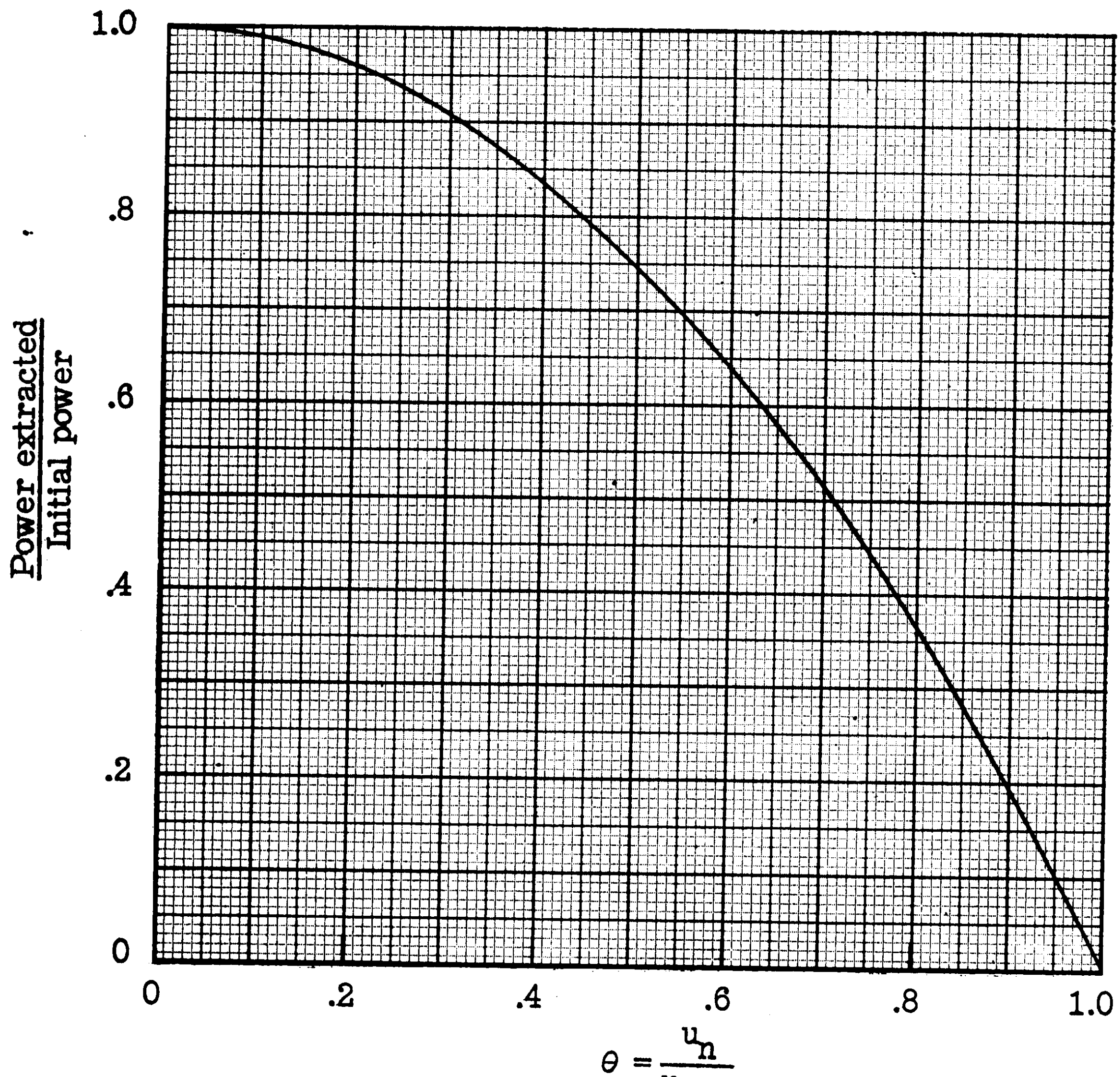

Figure 7.- Variation of power extracted with neutral particle velocity.

THEORETICAL CONSIDERATIONS OF THE

MAGNETOHYDRODYNAMIC GENERATOR

by

J. Byron Pennington

ABSTRACT

The distinction between the magnetohydrodynamic generator and the conventional wire-wound generator is that the role of the armature in the latter is played by an electrically conducting fluid in the former. This fluid is passed through a transverse electric and magnetic field and between two parallel plate electrodes. The induced electric field in the fluid, which is proportional to the fluid velocity and magnetic flux density, maintains a voltage drop across the electrodes and a current is generated when the electrodes are connected to an external load, closing the circuit. Thus energy is extracted from the fluid and delivered to an external load as electrical power.

This analysis considers a quasi- one-dimensional, inviscid, compressible, expanding area, channel flow for the steady-state case of a constant temperature magnetohydrodynamic generator. The theory is developed from the individual equation of motion for ions, electrons, and neutral particles.

Results of this analysis indicate the heat associated with ion slip performs a major function in realizing the constant temperature flow. When the ions move on the average through an angle of 10^{-2} radians or more between collisions with neutral particles, that is, $\omega_i \tau_i \geq 10^{-2}$, the energy transformed to heat from ion "friction" is of the same order

of magnitude as the joule heat, j^2/σ_0. As the number density diminishes, $\omega_i \tau_i$ increases, and the role of ion slip becomes more prominent.

Expressions are obtained for the ion, electron, and neutral particle macroscopic velocities and densities, and for the axial and transverse components of the electric field as a function of the channel distance, X.

Results, including the power extracted as a function of channel length, for a cesium seeded nitrogen gas are presented in graphical form.