Economic Comparisons of Thinning from Above and Below in Loblolly Pine Plantations using Dynamic Programming.

by

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Thesis submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

Master of Science

in

Forestry

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June, 1986

Blacksburg, Virginia

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(ABSTRACT)

Thinning from above and below were compared using an economic optimizing dynamic program, FORTE (Arthaud 1986). Economically optimal (net present value maximizing) thinning regime and rotation age were determined for benchmark economic and model inputs. Sensitivity of net present value and optimal management regime were tested for varying interest rates (6 or 8%), site indexes (50, 60 and 70, base 25 years), fixed and variable thinning costs, planting density (440, 680 and 910 trees per acre), stumpage prices and thinning type. Given the same assumptions, thinning from below consistantly provided the higher net present value for the optimal regime than thinning from above. For the benchmark assumptions, both thinning types had two thinnings in their optimal regimes. Optimal rotation age and thinning timings occur later when thinning from above. Both thinning types provided higher net present values than not thinning under all conditions except pulpwood management.

ACKNOWLEDGEMENTS

I wish to thank my committee members, Harold Burkhart and John Scrivani for their advice and counsel on the biometric aspects of this study. Special thanks go to my committee chairman Dave Klemperer, who prepared the original research proposal, encouraged me when I needed encouragement and trusted me to take my own direction in the research. Thanks also go to the U.S.F.S. Southeastern Forest Experiment Station for providing financial support.

I also wish to thank Brian Greber, an original committee member, for introducing me to dynamic programming.

and deserve recognition for developing the new version of the stand model used (PTAEDA2) and for providing biometric and computer advice.

Finally, I wish to thank my room 200 desk-mates for their friendship and support over the past two years. You made living in "the South" fun.

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CHAPTER 1. INTRODUCTION

Loblolly pine (Pinus taeda) is the most important commercial southern timber type. The South contains about two-fifths of the commercial timberlands in the United States (U.S.D.A. 1982). The area is the most important supplier of pulpwood and nearly equal to the Pacific Coast in supplying softwood lumber, veneer and plywood.

Loblolly pine will grow in a wide variety of soils throughout the Coastal Plains and Piedmont, from Delaware to eastern Texas, with the Appalachian Mountains acting as a northern and western boundary to its range (U.S.D.A. 1965).

More efficient approaches to managing this resource are needed. Specifically, what is the best economic rotation age, planting density, number of thinnings, timing of thinnings, level of thinnings, and type of thinning? This study deals primarily with thinning type, and more specifically thinning from above (high thinning) versus thinning from below (low thinning) in loblolly pine plantations.

Current shifts in timber-holding purposes away from strategic to financial has placed an increasing importance on maintaining the optimal economic management regime for the given physical and economic conditions. Some of the earlier loblolly pine management research has assumed conditions

current at the time, and has ignored changing economic conditions.

Once determined, economic values for thinning from above and thinning from below can be compared. Economic comparison between the two types of thinning implies weighing the relative benefits of receiving a higher early income when thinning from above versus higher income from the final harvest when thinning from below.

OBJECTIVES

The primary goal of this study was to compare the economics of thinning from above and thinning from below in loblolly pine plantations on cutover sites under a wide variety of economic and biological conditions. Specifically, the objectives were to:

1. Estimate and contrast optimal (for thousands of possibilities) planting densities for various thinning strategies.

2. Estimate and contrast optimal thinning timings and intensities and final harvest ages.

3. Test sensitivity of optimal management to economic and end product price assumptions.

4. Compare results from high thinning with the results from low thinning and no thinning.

5. Present a method for efficiently determining optimal management strategy as conditions change.

CHAPTER 2. LITERATURE REVIEW

Literature for this study was reviewed in five major areas; 1)Planting density, 2)Thinning from below, 3)Thinning from above, 4)Optimization techniques, 5)Growth models.

PLANTING DENSITY

Spacing studies have shown that number of trees planted per acre influences future stand state. Shepard (1974) tested loblolly pine plantation spacings of 4x4 feet, 6x6 feet, 6x8 feet, 8x8 feet, and 10x10 feet. When checked at age 23, the 10'x10' stands had eleven times more sawtimber than the 4'x4' stands and three times more sawtimber than the the next closest spacing, 8'x8'. The 10'x10' stands also had the largest cordwood volume. Other studies have shown similar results, that wider spacings provide more merchantable volume, less total volume, more sawtimber, less mortality loss, less basal area, and larger diameter trees (Hansbrough et al. 1964, Wakely 1970, Harms and Lloyd 1982).

By providing larger diameter trees and more sawtimber, studies have shown higher values for wider spaced plantations (Bennett 1963, Wakely 1968, Broderick 1978, Broderick et al. 1982). Shepard (1974) found 10'x10' spacing to provide the highest valued unthinned stand at age 23. The 10'x10' spac-

ing had more sawtimber at age 23 than stands thinned to 300 trees/acre at age 11. As pulp prices rose relative to sawtimber prices, the value of the stands with narrower spacing approached that of wider spacings. Harms and Lloyd (1982) found at age 20, 12x12 feet spacings had 14% larger diameters than 10x10 feet and 58% larger diameters than 6x6 feet spacings.

THINNING FROM BELOW

The recommended number of thinnings in a rotation has varied. Most of the biological based studies have dealt with more than one thinning. Ackerman (1928) tested thinning in loblolly pine every 7 years starting at 17 years. Andrulot et al. (1972) tested 1,2,4, and 5 thinnings in a 53 year period. Biologically, it seems preferable to remove trees often, concentrating on trees which would die before the next thinning treatment (Feduccia and Mosier 1977). In this way, volume of the stand over the rotation can be maximized.

Economically, single thinnings have typically been recommended (Shepard 1974, Broderick 1978, Thurmes 1980, Broderick et al. 1982). Single thinning usually occurs between the ages of 15 and 25 years. Shepard (1974) recommended single thinning normally, but in areas of high glaze damage potentials, two thinnings of lighter intensity were recommended.

Timing of thinnings is related to either crown closure or movement of diameters into product classes. Thinning in the first 8 years is inefficient, producing limby trees with large crowns and more taper (Shepard 1973). By the time the stand reached 18 years, diameter growth and spacing were not correlated. Chapman (1953) recommended removing 50% of crown cover when competition for soil moisture and root space became large. Hardie (1977) recommended thinning at ages 30 and 35 using net present value (NPV) maximization as the objective. Other recent thinning economics studies have recommended thinning at 20 or 25 years, depending on site index (Broderick 1978, Thurmes 1980, Broderick et al. 1982).

Level of thinning depends on stocking before thinning, ability of the stand to occupy the site after thinning, and product desirability. Chapman (1953) found that stagnation and shortening of crowns occurred in loblolly pine stands with basal areas greater than 120 sq. ft. and that the highest level of sustainable basal area (saturated) was 155 sq. ft. Thinning to 75 or 80 square feet per acre was recommended. Hardie (1977) found thinning 1/3 of the stand to be economically preferable to not thinning. Smith and Hafley (1986) reported that increasing the thinning intensity reduced stand yield at harvest while increasing mean annual increment (cu.ft.). Foil et al. (1964) applied thinning at 14 years and found that more intense thinning decreased mean annual increment (cords). Broderick et al. (1982) showed

that NPV was not very sensitive to level of thinning between 20 and 50 percent volume removal.

Different rotation ages have been recommended for loblolly pine plantations. Broderick et al. (1982), based on NPV, have recommended the shortest rotation ages, 20 years for pulpwood and 25 or 30 for multiple products. Higher site indexes had shorter optimal rotation lengths. In other studies, Shepard (1974) recommended 35 year rotation lengths, while Hardie (1977) and Alig et al. (1981) recommended 40 years.

Nelson (1969) reported that volumes from thinned and unthinned stands differ little, so costs, prices and discount rates are more important than biological considerations. Flick (1985) showed that a site index (SI) 60 stand of loblolly pine harvested at age 30 would have returned an internal rate of return (IRR) of 4.4% and an NPV of $43 (4% interest rate), assuming harvest values of June, 1980. If the harvest value were based on price 8 months earlier, the same stand would have provided an IRR of 6.0% and an NPV of $238. Net present value with fixed prices is very stable over differing rotation lengths in thinned stands, so market rates for timber should play a dominant role in harvesting decisions (Flick et al. 1980). This downplaying of biology can also be found in an earlier paper in which Bull (1941) recommended that four areas be considered in thinning decisions: financial status of owner, market value of the

thinning removals, future values of the end product and stand location and accessibility.

Diameter growth-increases on residual trees help to justify thinning as a tool to increase the economic value of the stand. Typically, mean diameters for thinned stands are 20 to 30% greater than unthinned stands (Ackerman 1928, Foil et al. 1964, Andrulot 1972, Goebel et al. 1974, Balmer et al. 1975). On old-field stands of loblolly pine, diameter growth rate decreased significantly at about age 23-25 in fully stocked stands (Chapman 1953). Zahner (1960) found that wider spaced trees left after a thinning had diameter growth into late fall while unthinned plots ceased diameter growth by mid-summer.

There is no consensus on stand density and stand height relationships. Some studies (Shepard 1974, Balmer et al. 1975, Harms and Lloyd 1985) showed that wider planting spacings (10'x10' and 12'x12') yielded trees with codominant and dominant heights from 3-10% taller and average heights 9-25% taller than narrow spacings (4'x4' and 6'x6'). Thinning to lower stand density did not seem to significantly affect dominant stand height and increased average stand height slightly (Foil et al. 1964, Andrulot et al. 1972, Goebel et al. 1974).

Studies have found that thinning increases stand sawtimber volume (Andrulot 1972, Burton and Shoulders 1974). Cordwood volume doesn't seem to be affected significantly by

thinning (Wakely 1968, Goebel et al. 1974). Chapman (1953) reported higher cord volumes on thinned stands, while Somberg and Haas (1971) reported lower volumes. Thinning decreases total cubic foot production according to Burton and Shoulders (1974). Goebel et al. (1974) reported equal cubic foot volumes for thinned and unthinned stands. Andrulot (1972) and Feduccia and Mosier (1977) reported higher volumes from thinned stands, mostly due to their multiple thinning regimes that capture some of the mortality.

THINNING FROM ABOVE

Gingrich (1983) suggested that high thinning in southern forests might sometimes be economically preferable to low thinning. Unfortunately, limited data are available for high thinning in loblolly pine. Economic analysis comparing thinning from above and thinning from below in loblolly pine does not exist.

On even-aged second-growth loblolly-shortleaf pine stands in Arkansas and Louisiana, little difference was found in cubic foot wood production between stands thinned from below and stands thinned from above (Bassett 1966, Burton 1968, Burton 1979). However, sawtimber production at 35 years was much higher for stands thinned from below. Burton (1979) showed the diameter distributions for various high and low thinning regimes. By age 45, the percent of trees in the

sawtimber class was less for the stand thinned from above. Burton (1977) compared pole production in loblolly pine stands at age 45 and found thinning from above produced more poles while thinning from below produced longer poles with higher individual pole values and higher total value.

Mann (1952) found that old-field stands thinned from above had slower annual growth rates in cords than stands thinned from below. In planted stands on abandoned fields in Georgia's Piedmont, Belanger and Brender (1968) found higher cubic foot annual growth rates in stands thinned from below than in stands thinned from above. Their results may reflect the fact that glaze storms had more severely damaged the stands thinned from above.

In a study of multiple thinning, Kennedy (1961) applied both low and high thinning to stands at age 20. Trees in both thinning types were selected as crop trees to be held to rotation age. At age 30, the stands were thinned again. In high thinning, only 29% of the trees chosen as crop trees in the first thinning were again chosen as crop trees in the second thinning. Crop trees from the first thinning were chosen as 98% of the crop trees in the second low thinning. This suggests that selection of other than dominant trees for final crop trees may be inadvisable. Because loblolly pine is classified as an intolerant species, trees held back considerably may lack the vigor to respond well to release.

OPTIMIZATION METHODS

Most thinning research in the past has used a limited set of field trials to help provide data for aiding in forest management decisions. Economic analysis was simply applied to each of the tested regimes to provide management direction. The advent of computer simulation of forest growth and yield has greatly expanded the analytical possibilities.

Chappelle and Nelson (1964) used a marginal analysis technique to obtain optimal management practices in loblolly pine. They simultaneously determined optimal thinning regime and rotation age. The technique used is not applicable to problems where either growth models or value functions produce difficult if not impossible partial differentiation.

Enumerative simulation has been used in loblolly pine to provide comparison between alternative management practices (Broderick 1978, Thurmes 1980, Broderick et al. 1982). All regimes of interest are simulated and then compared to determine the "best " solution among the choices.

Bullard et al. (1985) discussed the use of a random search algorithm to estimate optimal thinning regime and rotation age in mixed-species timber stands. Simple random search selects possible regimes randomly, in an attempt to find one within a desired upper region of the NPV probability density function. Multistage random search uses past trials

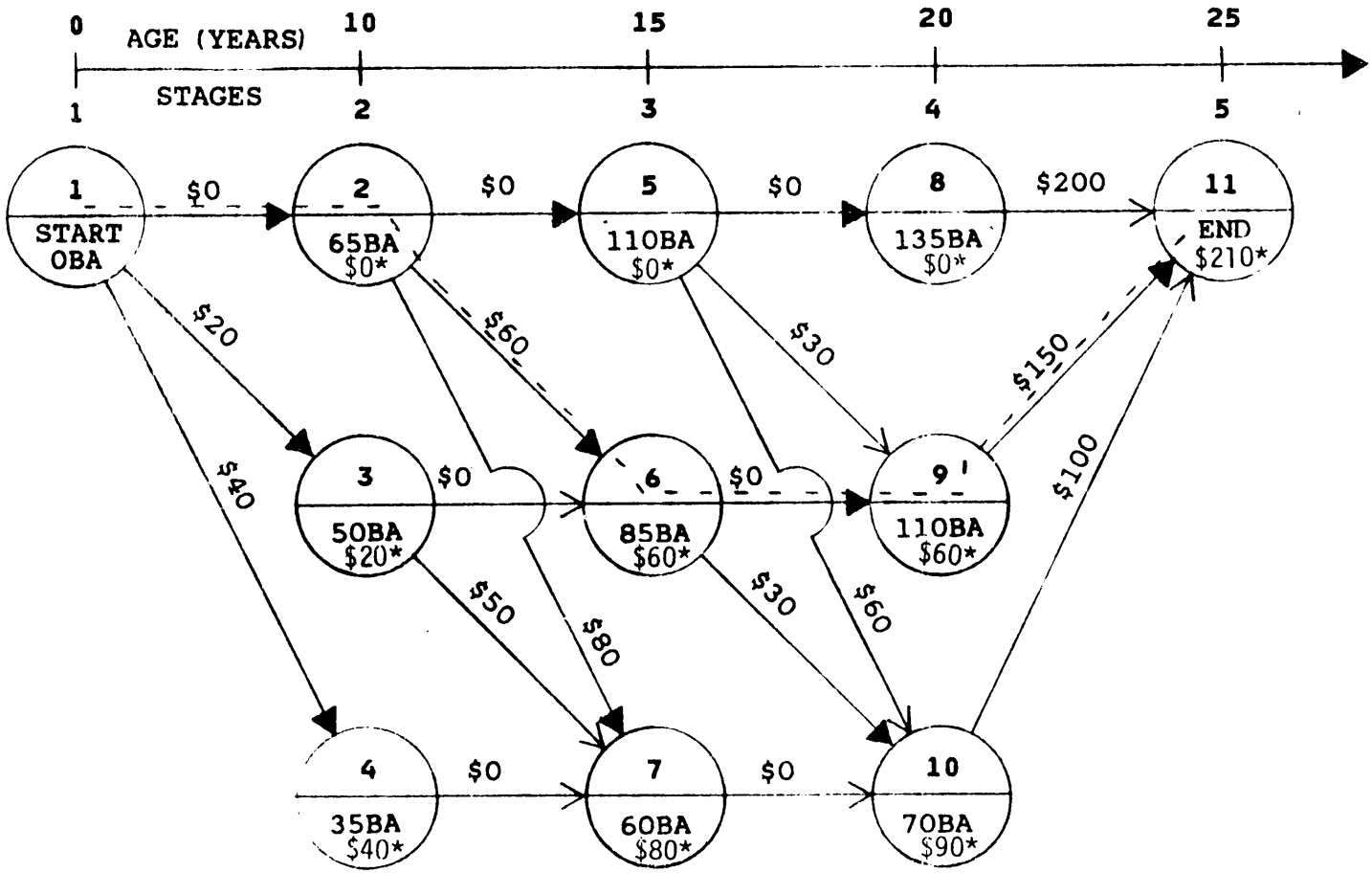
to help narrow the region in which the optimum may exist. Future trials are then selected within the region.

Dynamic programming (DP) is a mathematical programming technique which attempts to find the optimal path in a defined network. Most dynamic programming problems look at a discrete set of possibilities. In such cases, dynamic programming estimates the optimum rather than finding the true optimum. The network is structured using stand states to describe the attributes of the stand, and stages to provide a time frame or horizontal element to the network.

Figure 1 illustrates a 2 state descriptor dynamic programming problem. Each of the nodes (circles) represents a unique combination of stand age and basal are, the two state descriptors. The top half of the node shows an identification number for reference purposes. The lower half shows the stand basal area in square feet per acre. Stand age is represented by the time line on top of the network.

The objective of this problem is to maximize the rotation start net present value for one rotation of the stand. Clearcutting will be at age 25, represented by the terminal node (11). Node 1 is the starting node of the network, when the stand was planted (year 0).

Paths connect the nodes. In this example, the paths move in a left to right direction representing forward recursion. The dollar value alongside each path represents the rotation start NPV associated with taking that path.



* = Maximum accumulated present value for node.

\- - - = Optimal path through network.

Figure 1. A two state descriptor dynamic programming problem.

Positive values signify that a thinning occurred at the age of the ending node of the path. A zero value indicates there was no thinning at the ending node.

The basal area removed in a thinning can be determined by subtracting the residual basal area at the ending node from the basal area that would have existed without thinning. For example, the path 1-3 represents a thinning with an NPV of $20. The stand would have had 65 sq. ft. of basal area if unthinned (node 2). Node 3 has 50 sq. ft. of basal area, so 15 sq. ft. of basal area was removed. In a similar fashion, the path 1-4 represents a thinning of 30 sq. ft. of basal area at age 10 (65 - 35 sq. ft. of basal area).

Dynamic programming involves disection of the network into single stage problems. Stage 1 contains only one node, so it requires no optimization. Stage 2 (nodes 2, 3, and 4) involves finding the highest NPV path entering each node. Since there is only one path entering each of these nodes, the best paths are the only paths and are indicated by the solid arrows. The accumulated values (NPV) are simply the thinning values, since node 1 (beginning node for each of the paths leading into stage 2) has no value. The value of the nodes in stage 2 are $0 for node 2, $20 for node 3, and $40 for node 4.

At stage 3 we see that node 2 would grow from 65 sq. ft. of basal area to 110 sq. ft. of basal area if unthinned (path 2-5). Alternately, it could be thinned at age 15 to

either 85 sq. ft. of basal area (path 2-6) or 60 sq. ft. of basal area (path 2-7). Finding the best path to node 5 is easy since there is only one path entering it (2-5) with a $0 value ($0 + $0).

Node 6 has two paths entering it, 2-6 and 3-6. Path 2-6 has a value of $60 ($0 + $60) and path 3-6 has a value of $20 ($20 from node 3 + $0). The best path coming into node 6 is 2-6 and that path is then marked as best with a solid arrow. An accumulated value of $60 is stored at node 6.

Node 7 is reached through 3 paths. Path 2-7 has a higher value ($0 + $80) than path 3-7 ($20 + $50) or path 4-7 ($40 + $0).

Stage 4 is solved in a manner similar to stage 3. The highest accumulated value for node 8 is $0 from node 5. The highest value for node 9 is $60 from node 6 ($60 + $0). The path 6-10 provides the highest value, $90 ($60 + $30) for node 10.

The clearcut age of 25 years is reached in stage 5. Clearcut values are shown on the paths leading into node 11. The best path is 9-11 with an accumulated NPV of $210 ($60 + $150). The other paths had values of $200 for 8-11 ($0 + $200) and $190 for 10-11 ($90 + $100).

The management regime that yielded the $210 NPV can then be found by reading back through the network following the solid arrows. The best path was 1-2-6-9-11. The regime represented by that path is plant at year 0 (node 1), no removal

at age 10 (node 2), removal of 25 basal area at 15 years (node 6), no removal at age 20 (node 9) and a clearcut at 25 years.

Applications of dynamic programming to forest management decisions first began appearing in the late 1950's and the 1960's. (Arimizu 1958, Hool 1965, Schreuder 1968, Amidon and Akin 1968). One of the first examples of formulating a dynamic program applied to thinning can be found in Johnson (1961). He wasn't sure if the type of problem structure was valid and didn't try to solve the problem.

Hool (1965) discussed the use of stand condition as a probabilistic element to aid in dynamic programming for woodland management. Others have used dynamic programming to determine optimal forest acreage cutting practices (Schreuder 1968), volume stocking (Amidon and Akin 1968), most economical thinning practices (Brodie and Kao 1979), stand volume maximization (Martin and Ek 1981), to compare volume and value maximization practices (Riitters et al 1982), control mountain beetle attack (Haight et al. 1985a) and determine value of hardwood removal (Valsta and Brodie 1985).

States used in dynamic programming contain information which describes the stand and allow for predicting future development of the stand. State descriptors are used to solve for the best paths leading to each possible state in a stage. State descriptors usually use "neighborhood storage locations" (Brodie and Kao 1979) in comparing stands. For

example, assume a dynamic programming algorithm uses 2 state descriptors: stand age and trees per acre. If the trees per acre state descriptor had an interval width of 10 trees per acre centered on every 10 trees per acre, a stand 20 years old with 108 trees/acre would be considered the same as a stand 20 years old with 112 trees/acre. If the dynamic programming algorithm was designed to maximize NPV, the stand with the highest accumulated NPV at 20 years would be stored at the node described by the state descriptors 20 years and 110 trees/acre.

Amidon and Akin (1968) using stand age and volume and Schreuder (1968) using stand age and acreage are two examples of early studies using 2 state descriptor algorithms. Brodie and Kao (1979) used a 3 state descriptor algorithm (stand age, basal area, and number of trees/acre) to optimize thinning in Douglas-fir. Others have worked with 4 state descriptor dynamic programs, adding stand type (Valsta and Brodie 1985), thinning type (Haight et al. 1985b) or time since last thinning (Riitters et al. 1982) to the basic 3 descriptor dynamic program.

To use dynamic programming, the problem must conform to the "principle of optimality" (Bellman and Dreyfuss 1962) or "memoryless" condition. Basically, this requires that a multistage network formulation can be decomposed into a series of single stage problems such that decisions made at earlier stages remain optimal at later stages. In the exam-

ple used earlier, the stand chosen optimal at age 20 (108 or 112 trees/acre) must remain the more desirable of the two thinning regimes when clearcut at a later date.

The choice of forward or backward recursions to solve a dynamic programming problem depends on the individual problem. Recursion deals with the direction of flow through the network of the optimization procedure. The network in Figure 1 was solved using the forward recursion technique. Starting the optimization procedure from node 11 and moving to the left would be backward recursion and would yield the same optimal path.

Forward recursion builds the network to a stage using information from the previous stage. Forward recursion follows the direction of stand growth and has been found preferable when changing rotation age is desired. The optimal stand fitting a node description can then be chosen, providing the highest valued stand for future stages. The next stage is then formed from the states of the current stage. The process continues until the final stage is reached or a set objective is achieved.

Backward recursion was used in earlier studies (Amidon and Akin 1968) and will provide the same answers as forward recursion and define the new optimal path should deviation from the optimal path occur (Hann et al. 1983). Backward recursion moves from right to left in a network. The final stage of the network is the starting point of the backward

recursive process. The states of the final stage are determined through building the entire network first. To evaluate different rotation ages in a backward formulation requires a separate run of the dynamic program starting at each final stage (rotation age). For thinning problems, forward recursion is more efficient because the joint optimization of thinning and rotation can be solved by one run which considers the option of clearcutting (100% thin) at each node. Dynamic programming is more fully described in Bellman and Dreyfuss (1962), Nemhauser (1966), Schreuder (1968), Chen et al. (1980), and Dykstra (1984).

GROWTH MODELS

Three major types of growth and yield models have been used in dynamic programming: whole stand, diameter distribution and individual tree simulation. All three types are capable of predicting growth before and after thinning.

Whole stand models, fit by regression methods, are computationally quick and easy to use. Coile and Schumacher (1964) and Burkhart et al. (1972) are two of the most widely used loblolly pine regression equation growth and yield models. Equations can be used to predict average height of dominant and codominant trees, surviving trees/acre, mean tree diameter, stand volume, basal area, and other stand-

level attributes. These types of models have limited application for thinning and product differentiation.

Amidon and Akin (1968) used a whole stand growth model with dynamic programming to maximize soil rent on a loblolly pine volume control problem. Their optimal path present value differed by less than .02% from the optimal path found by Chapelle and Nelson (1964) using marginal analysis.

Diameter distribution models partition the total number of trees in the stand into diameter classes. The diameter distribution is commonly described by a probability density function, either the beta probability function or the Weibull function. Burkhart and Strub (1974) and Feduccia et al. (1979) are two examples of diameter distribution models. PCWTHIN (Burk et al. 1984) is a diameter distribution growth and yield model for old-field loblolly pine plantations which is available for use on a personal computer. It is based on the mainframe program WTHIN (Cao et al. 1982) and contains options for low thinning, row thinning, "average" thinning, and a combination low/row thinning. Thinning data for this model is from low thinned plots.

Valsta and Brodie (1985) combined the PCWTHIN model with a hardwood model (Burkhart and Sprinz 1984) to analyze the economics of hardwood removal. The Weibull function was used to describe the diameter distribution and provided for differentiation of product classes based on tree diameters. The dynamic programming algorithm used forward recursion and 4

state descriptors: age, basal area, number of trees and type of stand (pure pine or pine/hardwood mix). State intervals were 5 years, 10 sq.ft./acre basal area, and 10 trees/acre.

Daniels and Burkhart (1975) developed the individual tree simulation program PTAEDA for old-field loblolly pine plantations. It is a stochastic model, requiring multiple runs to give a stable estimate of mean volume and other stand attributes. Low thinning, row thinning, or a combination thinning are available in PTAEDA. Because each tree is grown individually as a function of size characteristics and competition from neighbors, individual tree models have the greatest potential for simulating different thinning types (including high thinning).

Types of individual tree simulators are distance dependent and distance independent. Individual tree-distance dependent models like PTAEDA have not been used in dynamic programming. This is due to the stochastic nature of the model and the amount of calculations and data storage needed to use the model with dynamic programming. Stone and Ek (1985) have developed a dynamic programming algorithm that uses a multi-species distance independent model. Haight et al. (1985a) used the LPSIM (Dahms 1983) simulator to optimize thinning treatment and rotation age in lodgepole pine. Currently, a distance independent individual tree model for loblolly pine does not exist.

CHAPTER 3. PROCEDURES

Growth and yield equations are from data generated by a revised version of PTAEDA2 (unpublished), a cutover-site edition of the individual-tree loblolly pine plantation growth simulator PTAEDA (Daniels and Burkhart 1975). Once formed, the equation parameters are entered into FORTE (Arthaud 1986), an interactive program for determining optimal management regimes in even-aged stands, using dynamic programming. Changes in economic assumptions can quickly be simulated and sensitivity to the changes determined.

The optimal management regime is that which maximizes rotation-start present value. For one rotation, net present value is (NPV):

$$NPV = \sum_{t=0}^{n} \frac{R_t}{(1+i)^t} - \sum_{t=0}^{n} \frac{C_t}{(1+i)^t} \qquad (1)$$

Where: NPV = Single rotation net present value.
t = Year of cost or revenue.
R_t = Revenue occurring in year t
C_t = Cost occurring in year t.
n = Rotation length.
i = Interest rate.

Single rotation present values (equation 1) are used to compare stands with the same rotation age and are converted to infinite series present values (equation 2) to compare the

highest value stands with differing rotation lengths. Infinite series present value (NPV or SEV) is calculated from the single rotation NPV as:

$$NPV_{\infty} = SEV = NPV \left\{ \frac{1}{(1+i)^n - 1} + 1 \right\} \qquad (2)$$

Infinite series present values are used because comparisons made with single rotation present values would not maximize present value and would yield longer-than-optimal rotation lengths.

REVISED PTAEDA2 GROWTH SIMULATOR

Since field data on thinning from above in loblolly pine is inadequate to formulate optimization, it was necessary to use a growth and yield model which was capable of simulating the impacts of such thinnings. PTAEDA2 can simulate individual tree growth given any tree removal criterion. A competition index is used to indicate relative competition for each tree. This index is based on diameter of the tree of interest, diameter of competing trees, distance to competing trees, and number of competing trees. Through incorporating individual tree dynamics, the response of the model can approximate that in the forest.

Table 1. Variables used in the thinning routine equations.

V_r = Volume per acre removed (cubic feet).

V_b = Volume per acre before thinning (cubic feet).

B_r = Basal area per acre removed (sq. ft.)

B_b = Basal area per acre before thinning (sq. ft.).

N_r = Number of stems per acre removed.

N_b = Number of stems per acre before thinning.

SC_i = The standardized competition value for the ith tree.

C_i = Individual tree competition index for the ith tree.

C = Mean stand competition index.

C_s = The standard deviation of competition indexes.

SD_i = The standardized deiameter for the ith tree.

D_i = Individual tree diameter for the ith tree.

D = Arithmetic mean stand diameter.

D_s = The standard deviation of diameters.

B_i = Individual tree basal area (ft^2/ac.) for the ith tree.

n = number of trees per acre in stand.

D_q = Stand quadratic mean diameter.

D_{qr} = Quadratic mean diameter of trees removed.

SD_r = The standardized diameter for the trees removed.

DV_i = The deviation of individual tree standardized diameter of ith tree from the standardized diameter of the trees removed.

X = The value for choosing trees to be removed.

It was necessary to develop a routine which would allow high thinning in the PTAEDA2 model. The routine had to provide a consistent method of removal. Table 1 shows the variables which will be used in defining the thinning routine. A function that relates percent of trees removed to basal area removed was derived from functions developed by Field et al (1978). They provided equations to determine volume removed from typical thinnings in slash pine, basing the ratio of volume removed on either the ratio of basal area removed to a power or on the ratio of trees removed to a power:

$$\frac{V_r}{V_b} = \left\{\frac{B_r}{B_b}\right\}^{P_1} \tag{3}$$

$$\frac{V_r}{V_b} = \left\{\frac{N_r}{N_b}\right\}^{P_2} \tag{4}$$

Where P_1 and P_2 are power terms. Working beyond Field et al. (1978), we can set equation 3 equal to equation 4 and solve for a combined power term of $P = P_1 / P_2$:

$$\frac{N_r}{N_b} = \left\{\frac{B_r}{B_b}\right\}^{P} \tag{5}$$

or,

$$P = \frac{\log (N_r / N_b)}{\log (B_r / B_b)} \tag{6}$$

Equation 5 can be solved for number of trees removed, given the beginning number of trees, basal area removed and original basal area. The power term (P) is used to represent the type of thinning. A power greater than one indicates a high thinning. Power terms less than one are low thinnings and a power term equal to one represents a mechanical thinning.

A thinning criterion was developed using a combination of tree competition and diameter of trees. First, in PTAEDA2 the competition indexes of the trees were converted to standardized numbers with equation 7.

$$SC_i = \frac{C_i - \bar{C}}{C_s} \tag{7}$$

The standardized form provides an equal weighting to competition during tree removal regardless of the absolute levels of competition in the stand. The standardized competition index will provide for distribution of trees removed in a thinning over diameter classes. The diameters were also converted to standardized form with equation 8.

$$SD_i = \frac{D_i - \bar{D}}{D_s} \tag{8}$$

The quadratic mean diameter of the removals can be determined (equation 13), and can also be converted to a standardized number (equation 14). Because FORTE will need a measure of diameter, and individual tree diameters are not available, quadratic mean diameter is used because it can be formed from the basal area and number of trees variables used in FORTE.

For a single tree:

$$B_i = 0.005454\ D_i^{\ 2} \tag{9}$$

and,

$$D_i = \sqrt{\frac{B_i}{0.005454}} \tag{10}$$

Average stand diameter:

$$\bar{D} = \frac{\sum D_i}{n} \tag{11}$$

Quadratic mean diameter: of stand before removal

$$\bar{D}_q = \sqrt{\frac{B_b\ /\ N_b}{0.005454}} \tag{12}$$

Quadratic mean diameter: of trees removed

$$\bar{D}_{qr} = \sqrt{\frac{B_r\ /\ N_r}{0.005454}} \tag{13}$$

Standardized diameter: of trees removed

$$SD_r = \frac{\bar{D}_{qr} - \bar{D}_q}{D_s} \tag{14}$$

Standardized deviation:
of trees from the quad.
mean diameter of removal

$$DV_i = \left| SD_r - SD_i \right| \qquad (15)$$

The standardized diameters are then converted to deviations from the standardized mean diameter of the removals by taking the absolute value of the difference of the two (equation 15).

The revised PTAEDA2 program requires entry of both the desired power term for thinning and the desired basal area removal. Choice of tree to be thinned is based on a value, X, which is the standardized competition index minus the standardized deviation of the individual tree from the mean diameter of removal (equation 16). Trees with the highest X values are chosen for thinning. Using this type of removal will center the removals on the mean diameter of trees to be removed by penalizing trees as their diameters deviate from the mean removal diameter while favoring those trees with higher competition indexes.

$$X = SC_i - DV_i \qquad (16)$$

The program first removes the tree with the highest X value above the mean diameter of trees to be removed. Trees are chosen based on current value for the power term based on the trees removed so far. If the current power term is greater than the user-specified desired power term, the next

tree will be chosen with a diameter less than the mean diameter to decrease the power term. Since the first tree was above the mean diameter of trees to be removed, the next tree will be taken below the mean diameter. Removal of trees proceeds in this manner until the desired basal area has been removed. While trees are being removed, the current power term should approach the desired power term. As can be seen by the parameter estimates of the power term in Appendix B, the actual power terms came close to approaching the desired power terms with standared errors in the range of .001 to .005. After a tree is removed, the competition indexes are recalculated.

Four different types of thinning were modelled with PTAEDA2. They will be known as strong low thinning (power = .70), weak low thinning (power = .85), weak high thinning (power = 1.20), and strong high thinning (power = 1.30). Sampling the different types of thinning will provide insight into what diameters in a thinning provide the highest NPV. Figures 2 through 5 show typical thinnings of the 4 types for 20 year old stands (site index 60, 440 trees/acre planted).

The thinning routine removed some small trees in the high thinning due to the very high competition indexes associated with the small diameter trees. It is not unreasonable to assume that some small trees would be lost in high thinning, especially those closest to the large trees (those trees with high competition indexes). A more complex

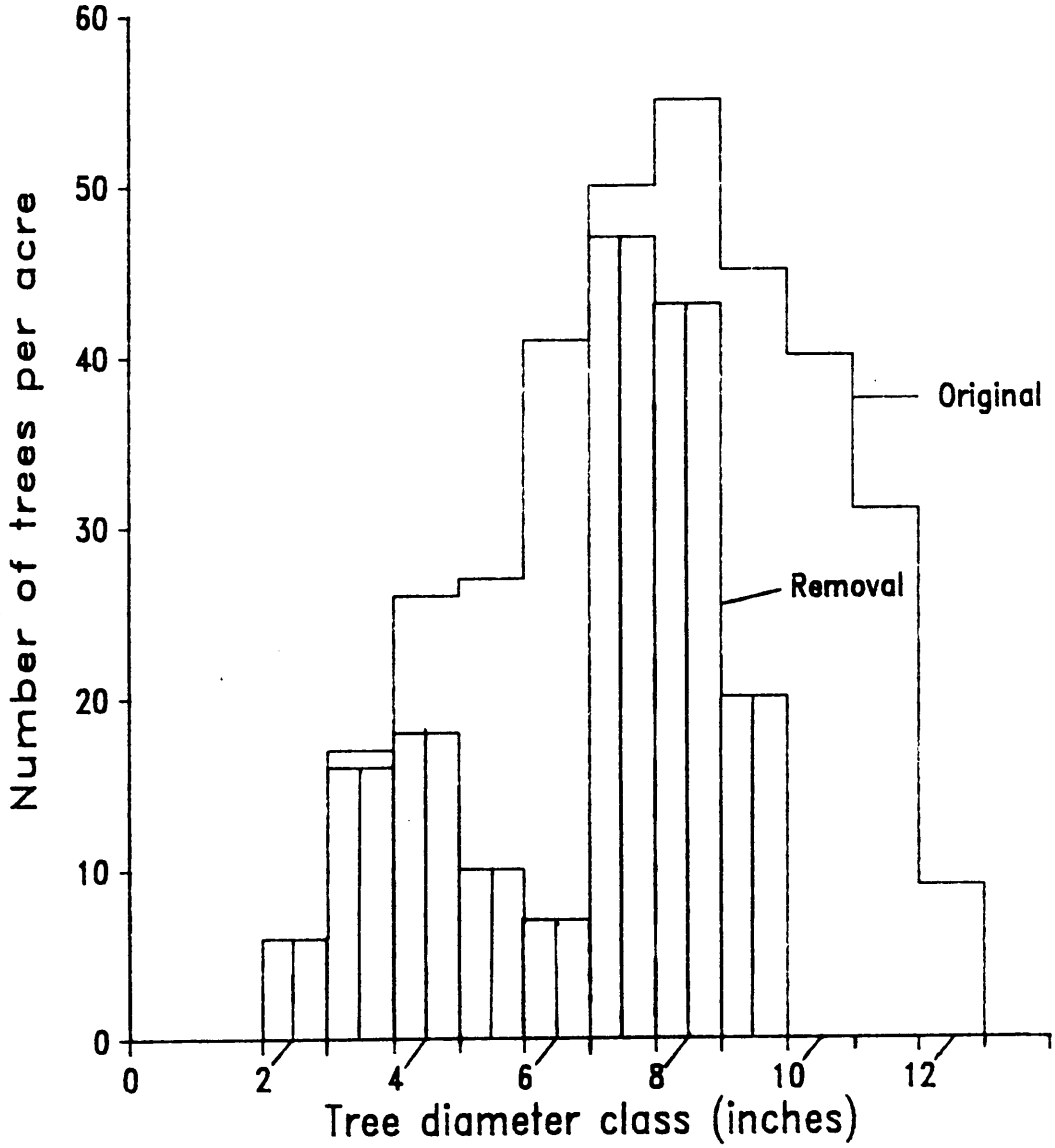


Figure 2. Typical strong low thinning on a 20 year old stand.

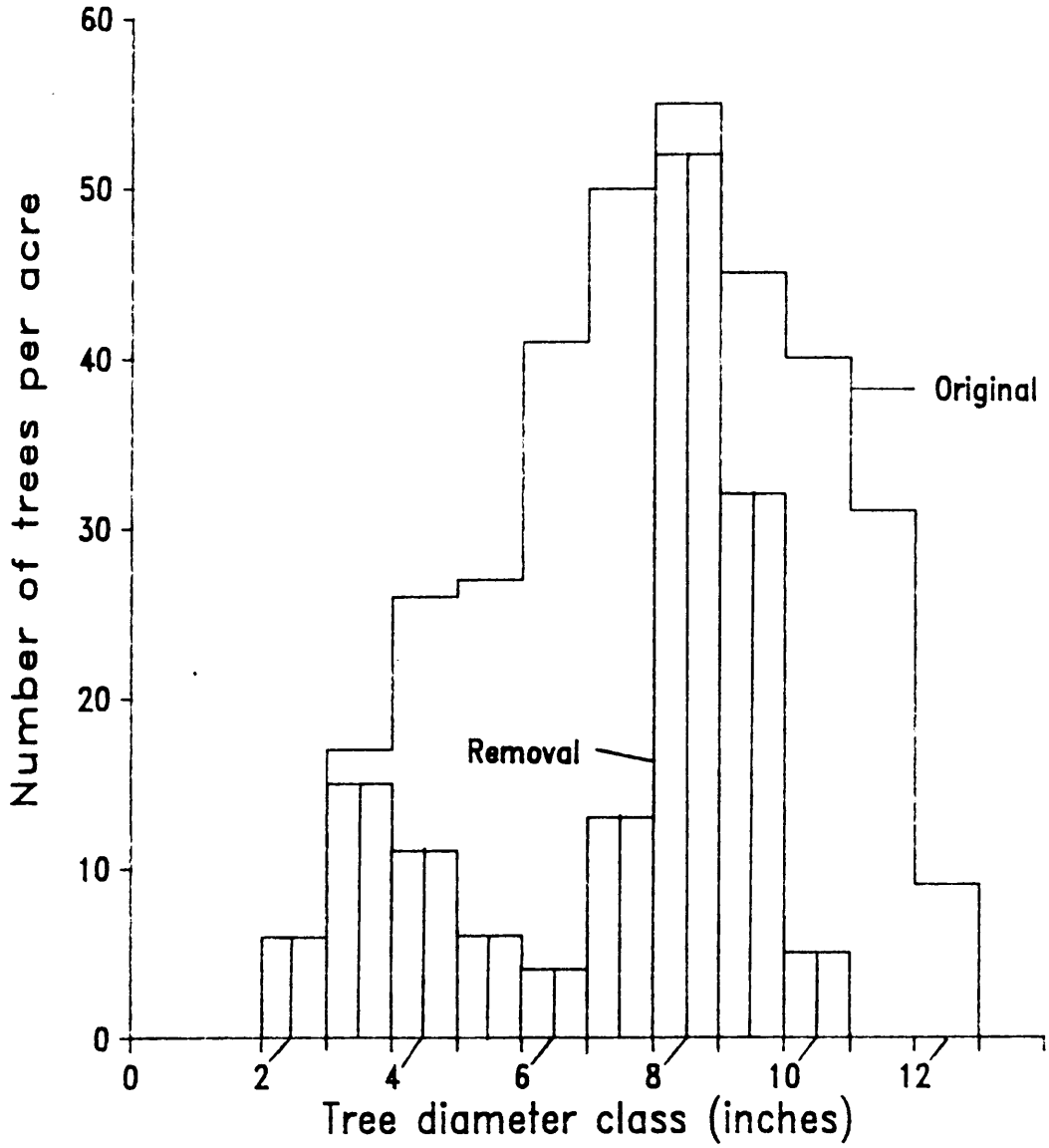


Figure 3. Typical weak low thinning on a 20 year old stand.

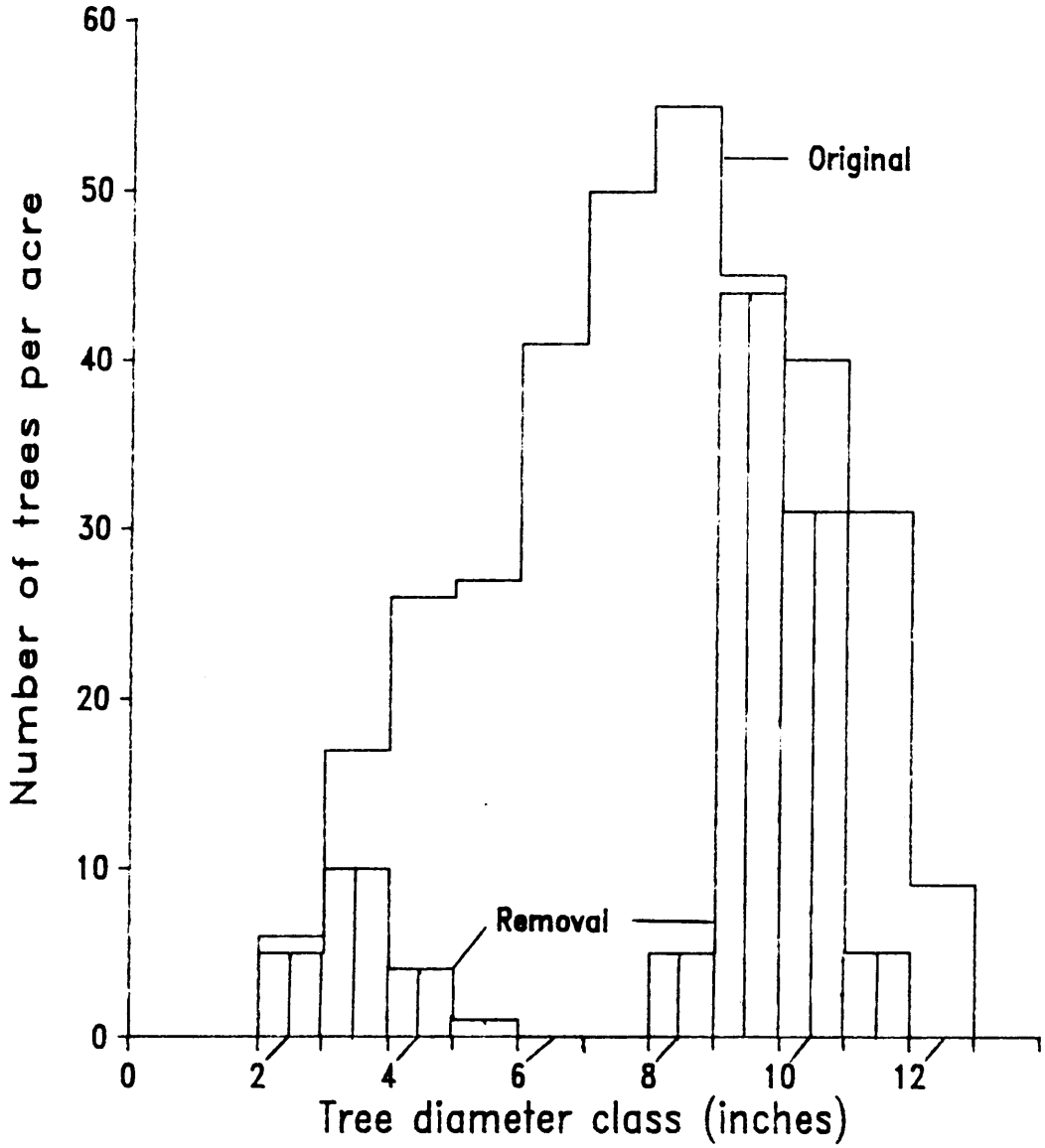


Figure 4. Typical weak high thinning on a 20 year old stand.

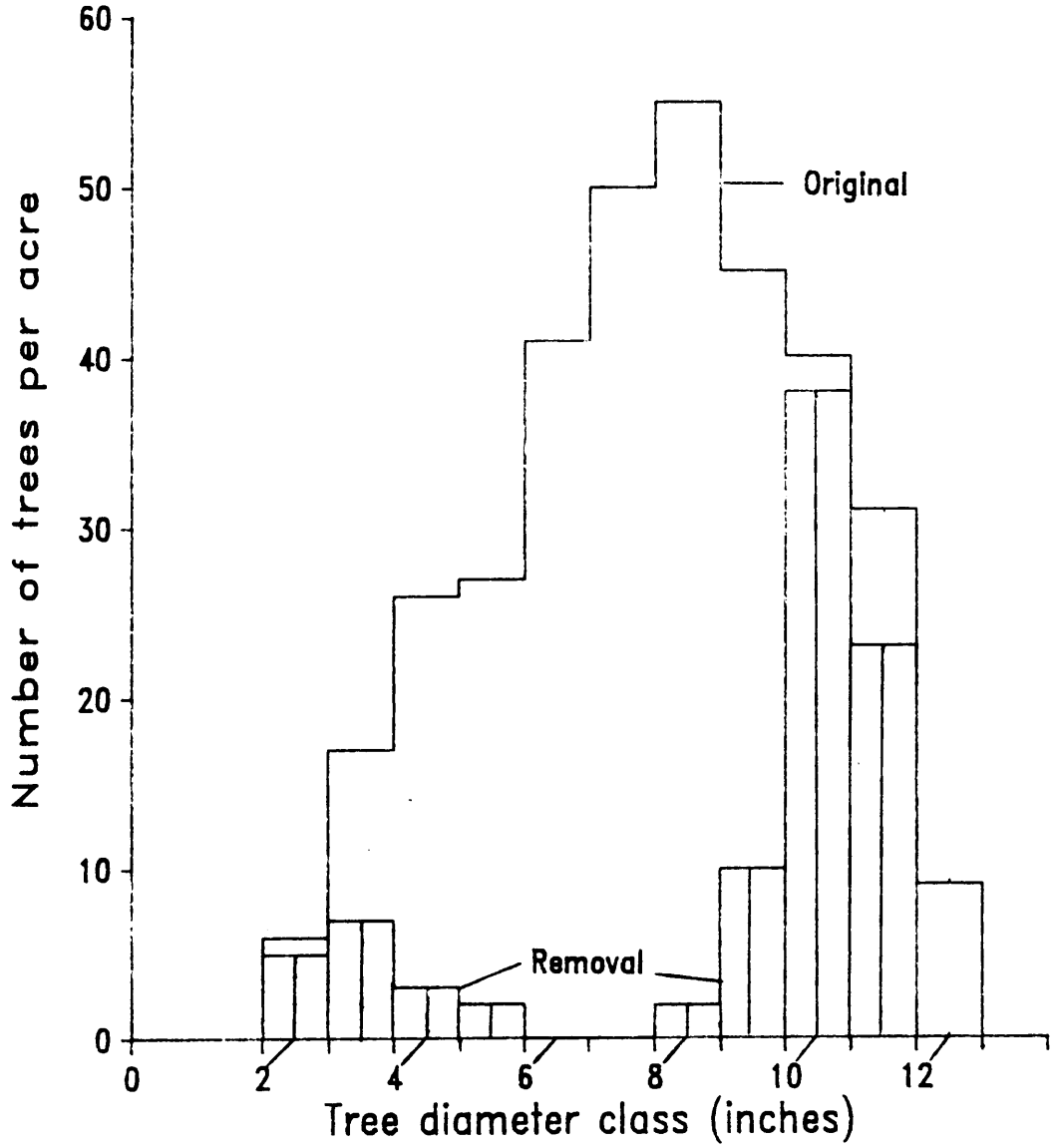


Figure 5. Typical strong high thinning on a 20 year old stand.

thinning removal routine could be developed to suppress removal of small trees in a high thinning. By changing the weighting of diameter deviations and competition index deviations, this bi-modal trait of removal could be removed. The weights used in this analysis were 1 for both, as seen by the lack of coefficients in equation 16.

Some of the larger trees were left in high thinnings. Since thinning was based on the power term, it was necessary to leave a stand that could be thinned again at a later age using the same power term. If the power term was about 1.5, it may have removed only the largest trees. The residual stand would not be able to produce a thinning with a power term of 1.5. The strong high and strong low thinning both left stands that were capable of supporting additional thinning with the same power. The power relationship is also used in the dynamic programming algorithm whole stand equations. One of the requirements of the whole stand equations is that the power term is a fixed parameter, providing consistency in removals of number of trees per acre for a given basal area removal.

A second routine was added to PTAEDA2 to give stand volume in terms of cords, MBF sawtimber and MBF peeler logs. Conversion of cubic feet to cords used the equations from Burkhart et al. (1972) (see Appendix A). Percent of wood falling into the different product classes was determined

using the diameter-product schedule from Broderick (1978) (see Appendix A).

Since the PTAEDA2 model is stochastic, it was necessary to create deterministic equations from it. These whole stand equations give quick predictions of growth and yield while providing the consistency necessary in a dynamic programming optimization procedure.

A sampling scheme was developed to equally sample site index 50, 60 and 70 (base age 25 years) stands. Thinnings were simulated at ages 10, 16, 22, 28 and 34. Stands were thinned twice such that thinnings at earlier ages were mostly first thinnings and later aged thinnings were primarily second thinnings. Three levels of thinning were applied; light (10-15% basal area removal), medium (25-35% basal area removal) and heavy (45-55% basal area removal). Initial planting densities of 400, 700, and 1000 trees/acre were equally sampled in each site index class. These densities were chosen to represent the acceptable sampling range of the PTAEDA2 data. Growth of the stands was checked randomly at from 1 to 5 years after the thinning.

For each thinning type, 180 thinnings were simulated. All thinning types used the same exact sampling scheme, including random number seeds.

Unthinned stands were sampled so that 147 unthinned stands were generated. Site indexes 50, 60 and 70 were equally tested. Planting densities were equally tested be-

tween 400 and 1000 trees, using 100 tree increments. These data provided 126 points for determining growth between ages, and 147 points for determining volume of unthinned stands. Combining a thinned dataset with the unthinned dataset provided 78 points to predict unthinned stand growth to age 10, when the first thinning was allowed. After age 10, basal area equations use past basal areas as independent variables in predicting stand growth. No hardwood competition was assumed in the stands other than ambient levels in data used when fitting equations.

Variables stored during the PTAEDA2 program runs were: age, thinning number, original planting density, basal areas before removal, basal area after removal, basal area growth, trees before removal, trees after removal, mortality, stand volumes (cunits, cords, MBF sawtimber, and MBF peelers), and average stand diameter. Equations in Table 2 were fitted to these data using linear and non-linear regression. These functions were developed for, and have been applied to, loblolly pine stands. The survival of thinned stands equation was originally used in slash pine by Clutter and Jones (1980) but was applied to loblolly pine by Lemin and Burkhart (1983). Parameters for the equations in Table 2 were determined from the PTAEDA2 generated data.

Table 2. Whole stand equations used in the FORTE program.

Survival of unthinned stands:

$$\log(N_2)=\log(N_0)+(a_1+a_2(\log(N_0))\frac{A_2}{100}$$

Source: Coile and Schumacher (1964)

Basal area of stand before initial thinning:

$$\log(B_2)=b_1(\log(SI))+\frac{b_2}{A_2}+b_3(\log(N_2))+b_4$$

Source: Coile and Schumacher (1964)

Basal area of unthinned stand after initial thinning:

$$\ln(B_2)=c_1+c_2SI-\left[\frac{A_1}{A_2}\right]\left[c_1+c_2SI-\ln(B_1)\right]$$

Source: Cao et al. (1982)

Volume (cubic feet) of unthinned stands:

$$\ln(Y)=d_1+\frac{d_2}{A_2}+d_3\frac{H}{A_2}+d_4A_2\ln(N_2)+d_5\ln(B_2)$$

Source: Amateis et al. (1986)

Basal area of thinned stands:

$$\ln(B_2)=f_1+f_2SI-\left[\frac{A_1}{A_2}\right]\left[f_1+f_2SI-\ln(B_1)\right]$$

Source: Cao et al. (1982)

Table 2. Whole stand equations used in the FORTE program. continued

Volume (cubic feet) of thinned stand:

$$\ln(Y)=g_1+\frac{g_2}{A_2}+g_3\frac{H}{A_2}+g_4A_2\ln(N_2)+g_5\ln(B_2)$$

Source: Amateis et al. (1986)

Trees remaining following thinning of given basal area:

$$N_3=N_2\left[1-\left\{\frac{B_2-B_3}{B_2}\right\}^{POWER}\right]$$

Derived from: Field et al. (1978)

Height of dominant / codominant trees:

$$\ln(H)=\ln(SI)\left\{\frac{j_1}{A_2}\right\}\left\{j_2e^{\left\{j_3\left(\frac{1}{A_2}-\frac{1}{25}\right)\right\}}\right\}$$

Source: Amateis et al. (1986)

Survival of thinned stands:

$$N_2=\left\{N_1^{k_1}+k_2(A_2^{k_3}-A_1^{k_3})\right\}^{(1/k_1)}$$

Source: Cao et al. (1982)

Table 2. Whole stand equations used in the FORTE program.

where: $a_1, a_2, \ldots, k_1, k_2, k_3^*$ = estimated parameters for equations.

A_1 = age of stand at beginning (before growth)

A_2 = age of stand at end (after growth)

N_0 = number of trees/acre planted

N_1 = number of trees/acre at beginning

N_2 = number of trees/acre at end

N_3 = number of trees/acre left after thinning

B_1 = basal area (ft^2) at beginning

B_2 = basal area at end (after growth)

B_3 = basal area left after thinning

SI = site index (base age set by j_1)

Y = volume of stands in cubic feet

H = height of dominant / codominant stand

POWER = term that specifies the type of thinning

---

* e, i and h excluded.

THE FORTE PROGRAM

The general structure of the FORTE dynamic programing algorithm is the same as that shown in Figure 1. However, rather than having two state descriptors, FORTE has four: age, basal area, number of trees and type of stand (thinned or unthinned). The whole stand equations in Table 1 are used to predict growth and yield.

The FORTE program was designed to be user friendly. The users guide for the program can be found in Appendix D. The program is basically divided into 2 parts: input/output and the dynamic programming algorithm. The input/output segment allows for menu driven entry of equation parameters, economic variables, model constraints and printing of output.

Figure 6 shows a part of the FORTE algorithm. For simplicity, assume the nodes shown are for thinned stands. Unthinned stands are stored in the first node of a stage. At the top of the node is listed a node identification number, and underneath are the basal area (square feet per acre) and number of trees per acre. Across the top of the figure are listed stand ages.

Represented at node 1 is a stand that is thinned, 18 years old, 108 basal area (sq. ft.), and has 439 trees/acre. The node is identified in Figure 6 as 110 basal area (BA) and 440 trees/acre (t/a), but the true stand descriptors are stored (as shown in Figure 6 by the numbers just before each

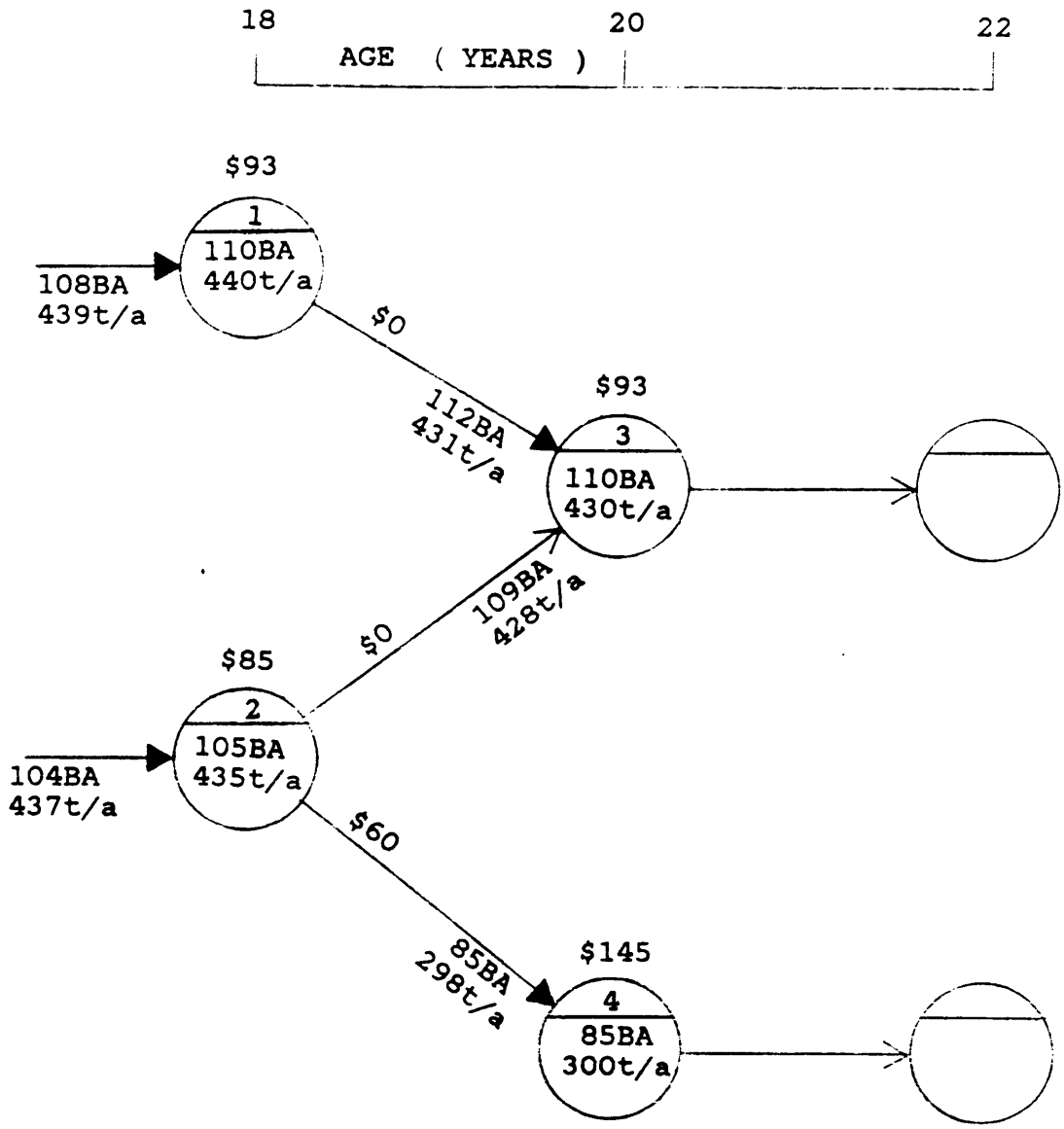


Figure 6. A section of a typical FORTE network.

node). Node 2 is similar in that a thinned 18 year old stand with 104 BA and 437 t/a is stored in it. Because node identification is in steps of 5 BA and 5 t/a, node 1 stores stands with basal areas between 102.5 and 107.5 and number of trees between 437.5 and 442.5. This is the "neighborhood storage location" described by Brodie and Kao (1979). The program could be restructured to use different step intervals. Associated with the nodes are accumulated net present values (NPV) of previous thinnings, $93 for node 1 and $85 for node 2.

Moving to age 20, we see that the stand at node 1 has grown to a stand with 112 BA and 431 t/a while node 2 grew to 109 BA and 428 t/a. Both of these stands have the same node identifiers: 110 BA and 430 t/a. The stand grown from node 1 will occupy node 3 due to its higher value. The stand represented by the path leading from node 2 to node 3 will be dropped. The darkened arrows represent optimal paths entering a node.

The path from node 2 to node 4 represents the growth of the stand in node 2 to node 3, with a thinning at age 20 reducing basal area to that indicated at node 4. The value of $60 along the path represents the NPV of the thinning. Thinning in FORTE uses basal area as the decision variable in thinning intensity. The paths leading from node 2 to 3 and 4 show that thinning removed 24 BA (109 BA - 85 BA) and 130 trees/acre (428 t/a - 298 t/a). This thinning would

yield a power term of about .8, representing low thinning. The value of the stand at node 4 would be $145 ($85 + $60). Should no other paths lead to node 4, the 85 BA - 298 t/a stand would be stored there.

When all paths have been formed and nodes determined, the algorithm will move on to the next stage, 22 years. Three possible fates exist for paths. They can displace a lower value path as the path 2-3 would have done if it had been worth more than $93. They can be discarded if a node exists with the same state descriptors and higher value, as happened to the 2-3 path. Or, a new node can be formed if there are no current nodes with the same state descriptors, as happened to the 2-4 path.

Stand values are also checked at each stage to find the path with the highest value if clearcut at the age associated with that stage. Only those stands not thinned in the stage are checked. In Figure 6, node 3 wasn't thinned at 20 years so its NPV would be determined by finding the present value of a clearcut 20 year old thinned stand with 112 BA and 431 t/a. To that would be added the $93 of accumulated NPV before age 20. After checking all nodes, the stand with the highest NPV at age 20 would then be printed and the random access file storing the paths read to give the complete history of the optimal thinning regime for that rotation. Other rotations must be tested to determine the rotation with maximum present value.

The FORTE algorithm uses forward recursion as seen by the left to right movement in figure 6. Backward recursion was initially used, but proved to cause network storage problems and was also slower due to the need to make complete passes through the network each time rotation age was changed. Forward recursion is especially efficient because once optimal paths to a node are determined, they remain optimal for any rotation age.

To restrict problem size, the following variables can be set: minimum basal area and number of trees left after thinning, minimum thinning removal, time between thinnings, thinning removal increments, and minimum thinning age. For this analysis the model was constrained to leave a minimum of 50 sq. ft./acre basal area and 100 trees/acre after thinning. Thinning increment was varied, with time between possible thinnings ranging from 1 to 13 years. Thinning was not permitted in stands less than 10 years old. Thinning was performed in increments of 10 sq. ft./acre basal area with a minimum removal of 15% of volume or 300 cubic feet.

The planting density of 440 trees per acre was the benchmark density. Past research has shown that wider spaced plantations produce higher valued stands (Bennett 1963, Wakely 1968, Broderick et al. 1982)

PRICE-DIAMETER RELATIONSHIP

The quadratic mean diameter was used for determining stand values. It is also the diameter displayed in the output of the FORTE program. One of the requirements of dynamic programming is that any required stand attribute must be obtainable through manipulation of available stand variables (state descriptors). In the case of FORTE these are basal area, number of trees, age and type of stand. Quadratic mean diameter meets the requirement.

The program uses a linear value function which bases cubic foot value on quadratic mean diameter ($\bar{D}_q$) of the stand. This type of value relationship was used by Haight et al. (1985b) in optimizing management of lodgepole pine. They used a function which passed through $0/cunit (cunit=100 cubic feet) at average stand diameter of 6", and $40/cunit at 15". This is equivalent to the function:

$$\$/\text{cunit} = -26.667 + 4.444 \; (\bar{D}_q) \qquad (17)$$

For this problem the base conditions of $11/cord for pulpwood, $100/MBF sawtimber and $135/MBF peeler logs converts to:

$$\$/\text{cunit} = -18.929 + 5.454 \; (\bar{D}_q) \qquad (18)$$

This equation was determined by applying the product values to the data generated by PTAEDA2 and then solving the equation through simple linear regression.

The base value equation was adjusted to determine the effects of changing slope and intercept terms. The base equation intersects the Y-axis at -19.77711 and intersects the X-axis at 3.4". By increasing the slope and pivoting on the 3.4" diameter intersection, extra premium for larger diameters is simulated. Likewise, decreasing the slope will flatten the function and simulate less premium for larger diameters.

Increasing the Y-axis intersection value simulates increasing the number of trees considered merchantable by shifting the X-axis intercept to the left. By setting the Y-axis intercept to a positive number and the slope term to 0, a fixed value per cunit of wood is represented.

ECONOMIC DATA

Benchmark economic variables were obtained from a review of literature. Virginia costs and revenues were used whenever available. The benchmark inputs are shown in Table 3.

Product values of $11/cord for pulpwood, $100/MBF sawtimber and $135/MBF for peelers (veneer logs) were obtained from the December 1985 edition of Timber Mart South (1985) for Virginia. These values show the reduction in

Table 3. Benchmark inputs.

Type	Amount	Comment
COSTS		
Planting	$.06/seedling	Includes seedling and labor costs.
Site preparation	$75/acre	Cutover conditions.
Fixed thinning cost	$20/acre	Entry/administration costs.
REVENUES		
Stumpage prices		
pulpwood	$11/cord	Converted to a linear price-diameter relationship for clearcuts.
sawtimber	$100/MBF	
peelers	$137/MBF	
thinnings		Thinning value was 80% of the clearcut prices. .
INTEREST RATE	6%	Real, (no inflation), before tax.
SITE INDEX	60 (base age 25)	Unless otherwise specified.

premiums for peelers when compared to the $75/MBF for sawtimber and $150/MBF for peelers used by Broderick (1978) in his analysis.

Site preparation and planting costs were set at $75/acre. This amount is comparable to values reported by Broderick et al. (1982), Straka and Hotvedt (1985) and Valsta and Brodie (1985). Added to these costs were a $.06 cost per seedling planted as used in Broderick et al. (1982). The difference in planting costs could affect the NPV considerably. For the spacings tested the total costs of site preparation and planting were $101/acre for 440 trees/acre, $116/acre for 680 trees/acre and $130/acre for 910 trees/acre.

Annual management costs were excluded from this analysis. Since comparisons made are for an infinite series of rotations, discounted annual management costs would be the same for any rotation length. The discounted annual management cost could be subtracted from the NPV's reported here to arrive at a lower NPV, but wouldn't affect the optimal management regime.

Taxes were not included in this analysis. As constructed, the FORTE algorithm cannot store the basis needed for capitalization of costs under the capital gains tax. The residual basis after thinning would need to be included as an additional state descriptor to the dynamic programming algorithm. This would expand the problem too much.

It is possible to apply taxes to the optimal regimes given by the program, however, application of taxes to these regimes no longer guarantees that these stands will remain optimal. The FORTE program can provide several stands that are very close in value to the optimal management regimes. Taxes such as property taxes, yield taxes and income taxes can be applied to optimal and sub-optimal stands to check the sensitivity of results to taxation.

Currently, uncertainty about future federal income tax rates exists. One proposal would tax timber income at rates up to to 27% for individuals and at 33% for corporations. Taxes of this type could be approximated by simply reducing the product values by the appropriate tax.

Thinning costs and revenues are very important to this analysis. Ideally, an equation would predict the cost and revenues of thinning based on available stand parameters such as basal area removed, percent of stand removed, type of thinning, average diameter of stand, and so on. Broderick et al. (1982) assumed thinning product values to be 80% of clearcut product values in loblolly pine. Valsta and Brodie (1985) added a $20 fixed cost to thinning. Stone and Ek (1985) used an elaborate set of functions that based costs and revenues of thinning on volume removed and average diameter removed.

Thinning product values in this analysis are 80% of clearcut values. A fixed (entry) cost of $20 was also used.

The fixed cost is especially important because this analysis uses 1 year increments for stages and allows multiple thinnings in a rotation. Without the entry cost, no penalty would exist for lightly thinning in successive years instead of medium thinning in one of the years.

A 6% alternative rate-of-return (real, before tax) was used in this analysis. Other interest rates can easily be used in the FORTE program.

CHAPTER 4. RESULTS

OPTIMA FOR BENCHMARK INPUTS

Given the Table 3 benchmark inputs, Table 4 shows the management regimes yielding the maximum net present values generated by FORTE given no thinning, strong low thinning and strong high thinning. When minimally constrained, FORTE always generated higher present values for strong thinning than for weak thinning, given the benchmark inputs. Strong low thinning yielded the maximum present value of $223/acre. In the course of a rotation, low thinning removes more volume from thinning than does high thinning, 18.6 cunits in a 34 year rotation for low versus 13.3 cunits in a 40 year rotation for high.

At its optimal rotation age, low thinning has fewer trees and larger diameters than does high thinning with a 6 year longer rotation length. These fewer trees have a higher value per cunit. The low thinnings at 22 and 30 years remove a considerable number of small trees to favor crop trees. The two low thinnings remove a total of 228 trees per acre while the high thinnings only remove 52 trees per acre. It would seem for the benchmark assumptions that increases in growth on larger trees from low thinning increase NPV more than does removing larger value trees in high thinning.

Table 4. Optimal regimes using the benchmark inputs.

Thinning type	Unthinned	Strong Low Thinned	Strong High Thinned
Planting density	440 trees/acre for all options		
Number of thinnings.	0	2	2
Age of thinnings (years).	-	22/30	28/35
Residual basal area after thinning (sq. ft./acre).	-	80/80	120/120
Percent of cubic foot volume removed.	-	33/28	17/15
Thinning yields (cunits).	-	9.6/9.0	6.6/6.7
Rotation age (years).	31	34	40
Average diameter of clearcut stand.	9.5	14.0	11.4
Number of trees at rotation age.	310	94	138
Basal area at rotation age.	153	101	138
Final yield (cunits/acre).	45.7	31.5	33.1
Value/cunit of clearcut.	$33	$58	$43
Present value/acre.	$176	$223	$198

SENSITIVITY ANALYSIS

Weak versus strong thinning

The relationship between net present value (NPV) and rotation age for the four types of thinnings tested is shown in Figure 7. Note that for different rotation ages, the optimal frequency, intensity and timing of thinnings determined by FORTE can change. Over the range of ages tested, the strong low thinning has the highest present value. It becomes superior to not thinning when rotation ages are 19 years or greater. The difference in NPV between the strong thinning and the other types of thinning is largest for rotation ages from 25 to 30 years. The NPV maximizing rotation age for the strong low thinning is 34 years.

The weak low thinning has the second highest NPV, followed by the strong high thinning. By the rotation age of 44 years, strong high thinning surpasses weak low thinning. That age is beyond the recommended rotation lengths of 36 years for weak low thinning and 40 years for strong high thinning.

Weak high thinning gives higher net present values than not thinning beyond 30 years but has the lowest value of the thinning types tested. Apparently, if the removal of trees is in the larger diameters it should be in the largest diameters available. If high thinning was to be a recommended

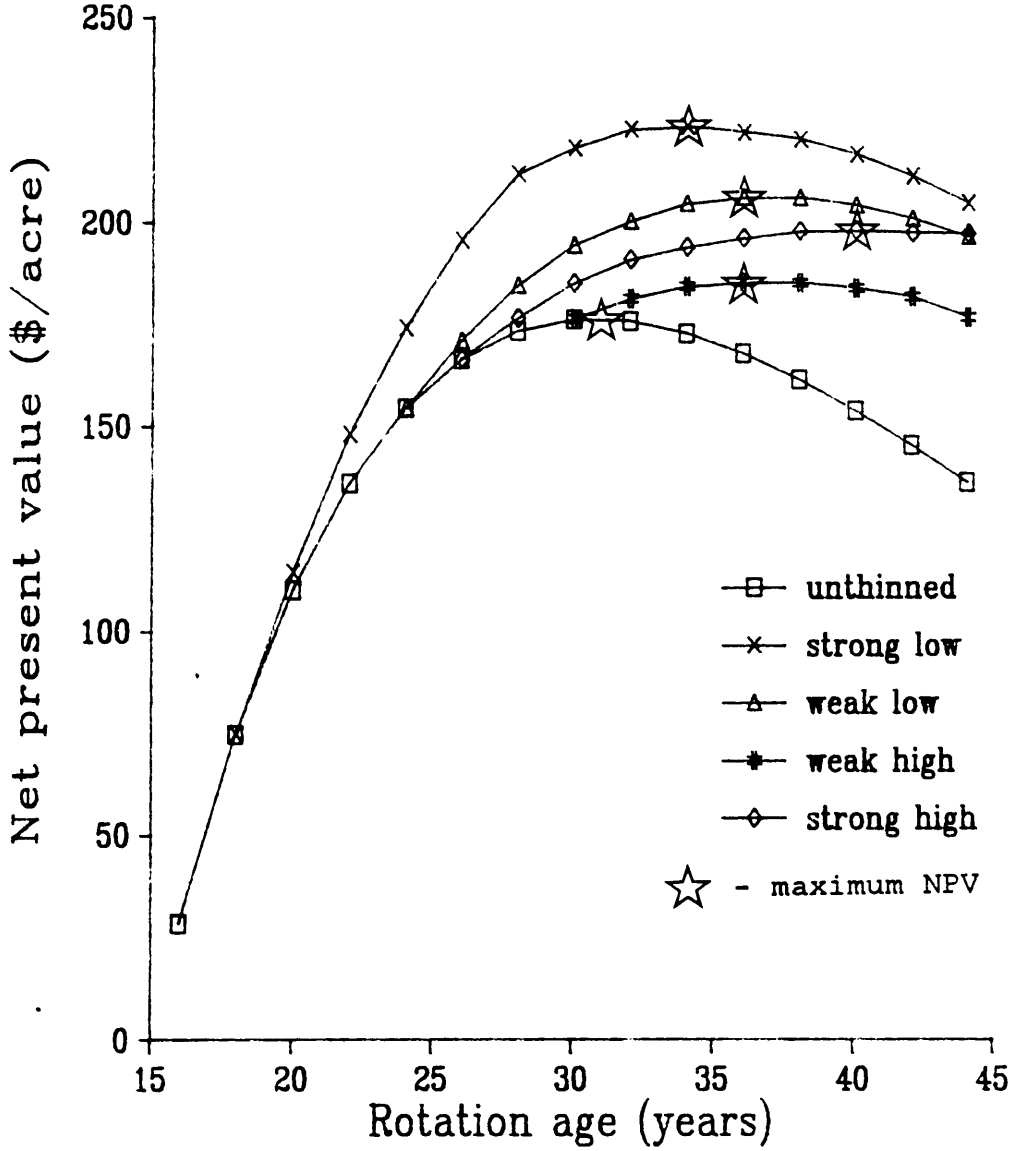


Figure 7. Net present values for the four thinning types: benchmark case assumptions.

practice it would be because high value trees were removed before rotation age. Weak high thinning doesn't adequately capture this early income.

The strong low thinning and strong high thinning are the best of the low and high thinning types and are used for the rest of this analysis. They will be referred to from this point on as simply low thinning and high thinning.

Low thinning tends to produce more thinnings in a rotation than does high thinning. There are many 1, 2 and 3 thinning regimes that have net present values within $3 of the $223 optimal regime for low thinning. Most of the high thinning regimes that approach the optimal high thinning regime are single or double thinnings.

Part of the reason for this difference in thinning frequency may be attributed to the thinning cost assumptions. Values of the low thinnings are not as great as those from high thinning, so variable costs are less for low thinning when stumpage is worth a percentage of clearcut stumpage. Growth response of the residual stand to low thinning may also be greater, requiring more frequent thinning to keep the stand optimally stocked.

Site index

Lower site indexes provide the best opportunities for high thinning. Figures 8, 9 and 10 show comparisons of high

and low thinning for site indexes 50, 60 and 70. The highest NPV for low thinning ($128) in SI 50 stands with a rotation age of 36 years is approached by high thinning with a rotation age of 42 years and an NPV of $126. The gap between high and low thinning expands from $2 for SI 50 to $26 for SI 60 and $92 for SI 70. In site index 60 stands, high thinning approaches low thinning in value at the longer rotation ages tested (44 years). The gap between the value of the two thinning types remains larger throughout the rotation ages for site index 70 stands.

Site index affects optimal rotation length for both thinning types. Optimal rotation length is 36, 34 and 32 for site index 50, 60 and 70 low thinned stands and 43, 40 and 37 for site index 50, 60 and 70 high thinned stands.

Number of thinnings providing the highest NPV is not affected by site index. One or two thinnings are favored for all site indexes. Effectiveness of thinning depends more on stand stocking than on stand age. Thinning occurs sooner on site index 70 than on site index 50 due to the earlier competition for growing space on better sites.

Interest rate

Increasing the interest rate decreases the net present values and rotation ages of both high and low thinning as can be seen by comparing Figures 9 and 11. At the 8% interest

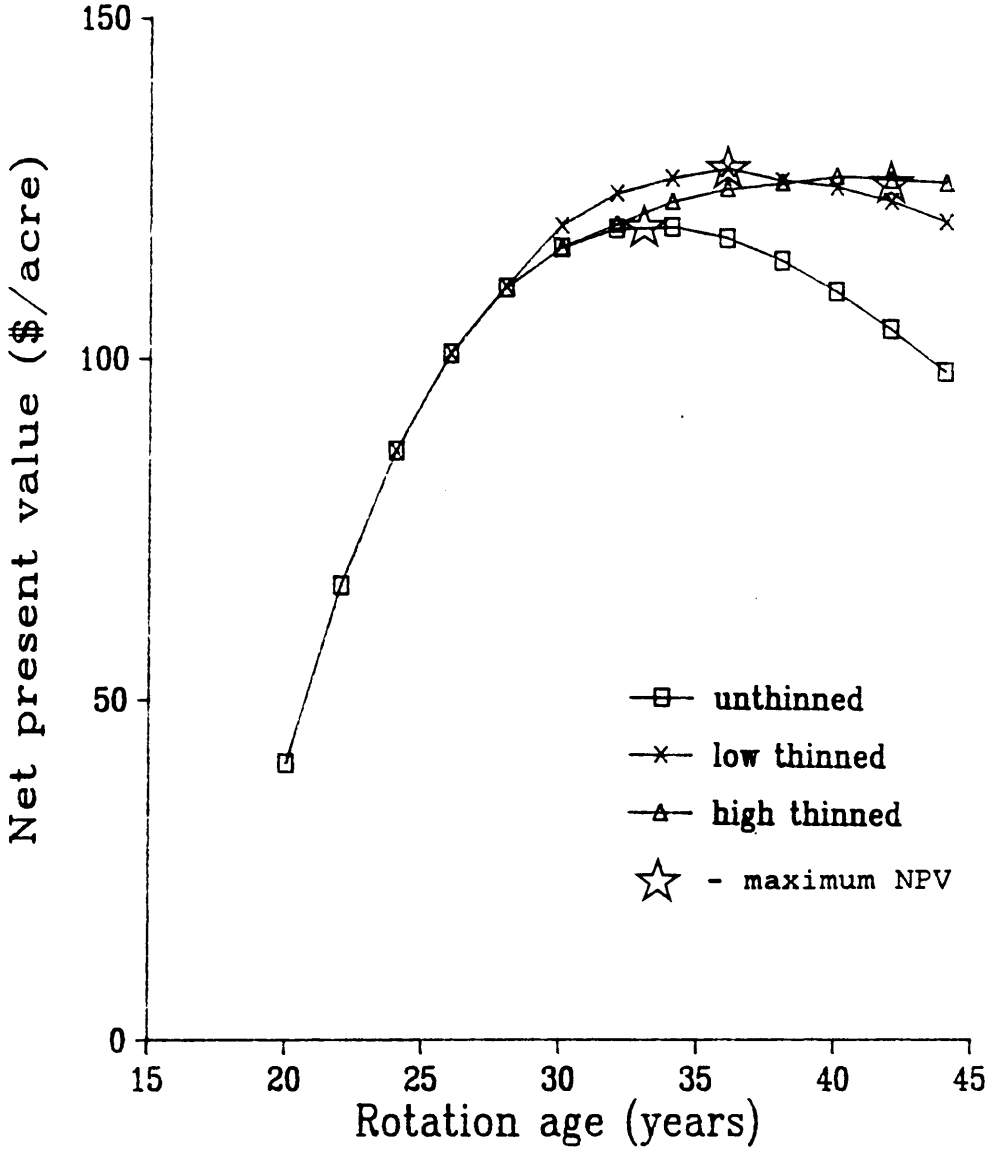


Figure 8. Net present values for site index 50 stands: all other benchmark assumptions.

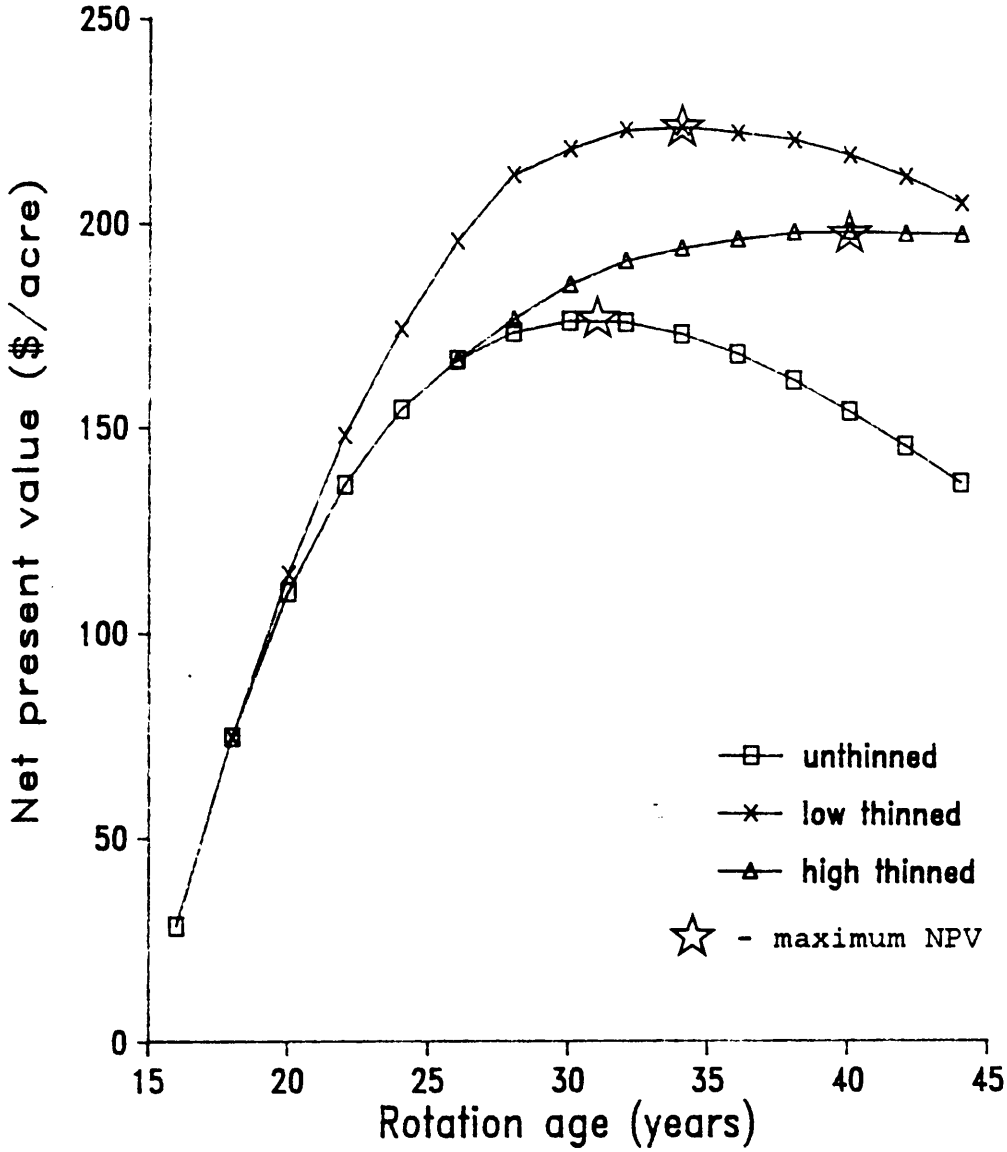


Figure 9. Net present values for site index 60 stands: benchmark assumptions.

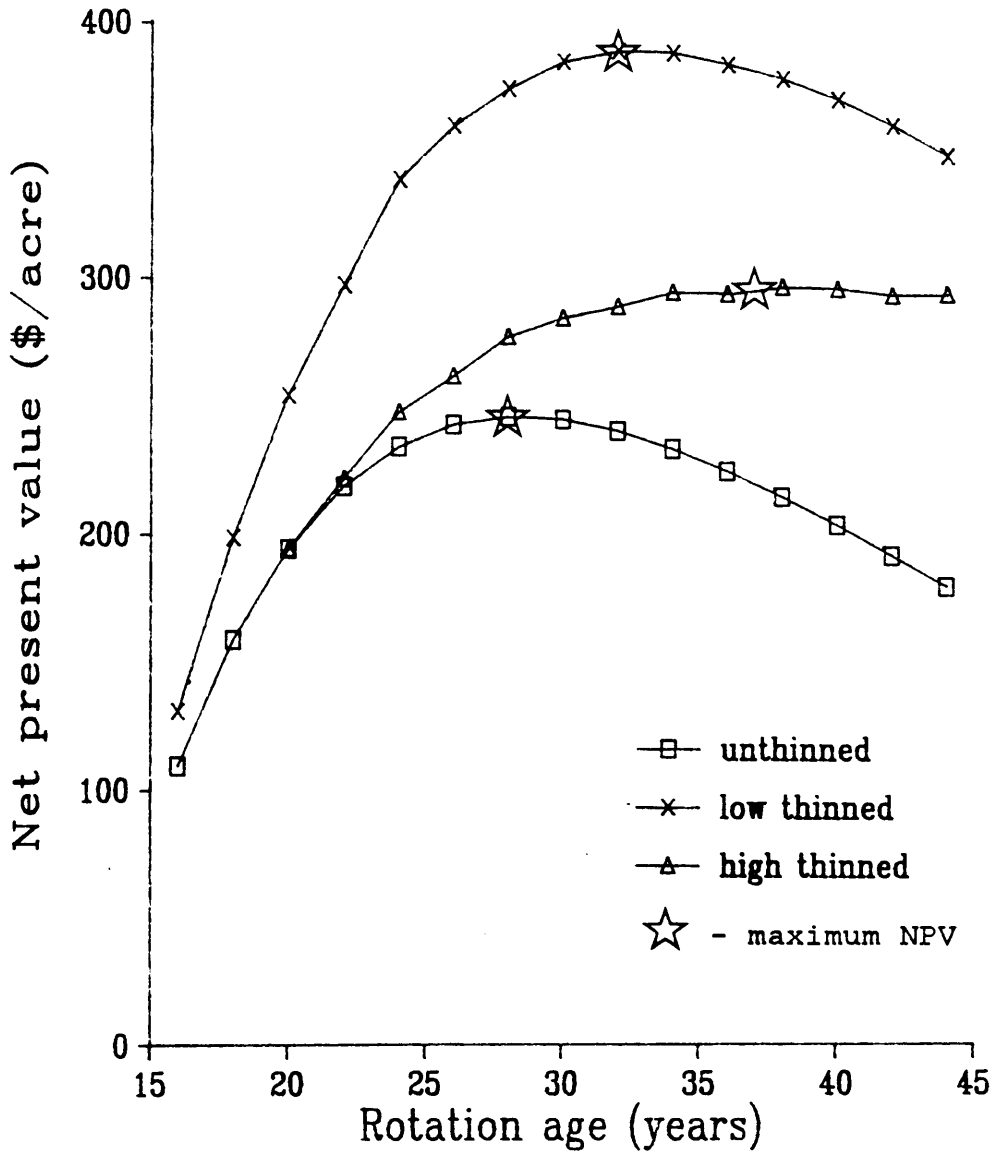


Figure 10. Net present values for site index 70 stands: all other benchmark assumptions.

rate, the high thinning present value is fairly insensitive to rotation age. It peaks at 32 years with a value of $53.50 and remains above $50 past rotation ages of 44 years. Low thinning peaks at a higher NPV but declines more rapidly thereafter.

The optimal thinning regimes change with 8% interest. The higher interest rate shortened both rotation age and thinning timings for both thinning types. The highest NPV for low thinning is still with two thinnings. The optimal regime at 8% interest removes 22 sq. ft. basal area at 15 years and 37 sq. ft. at 20 years with a final harvest at 30 years. The highest NPV for high thinning at 8% interest is derived from thinning lightly when the stand is about 22 years old and again when 25 years old with a final harvest at 32 years.

In addition to the 6% and 8% interest rates, 4% and 10% were tested. The 4% rate caused rotation length to go beyond 40 years (the limit of the data). With 10% interest, net present values were negative using the other benchmark assumptions.

Fixed thinning costs

Increasing the fixed cost of thinning reduces the highest NPV and the economic benefit of thinning. Compared to Figure 9, figure 12 shows that thinning provides less advan-

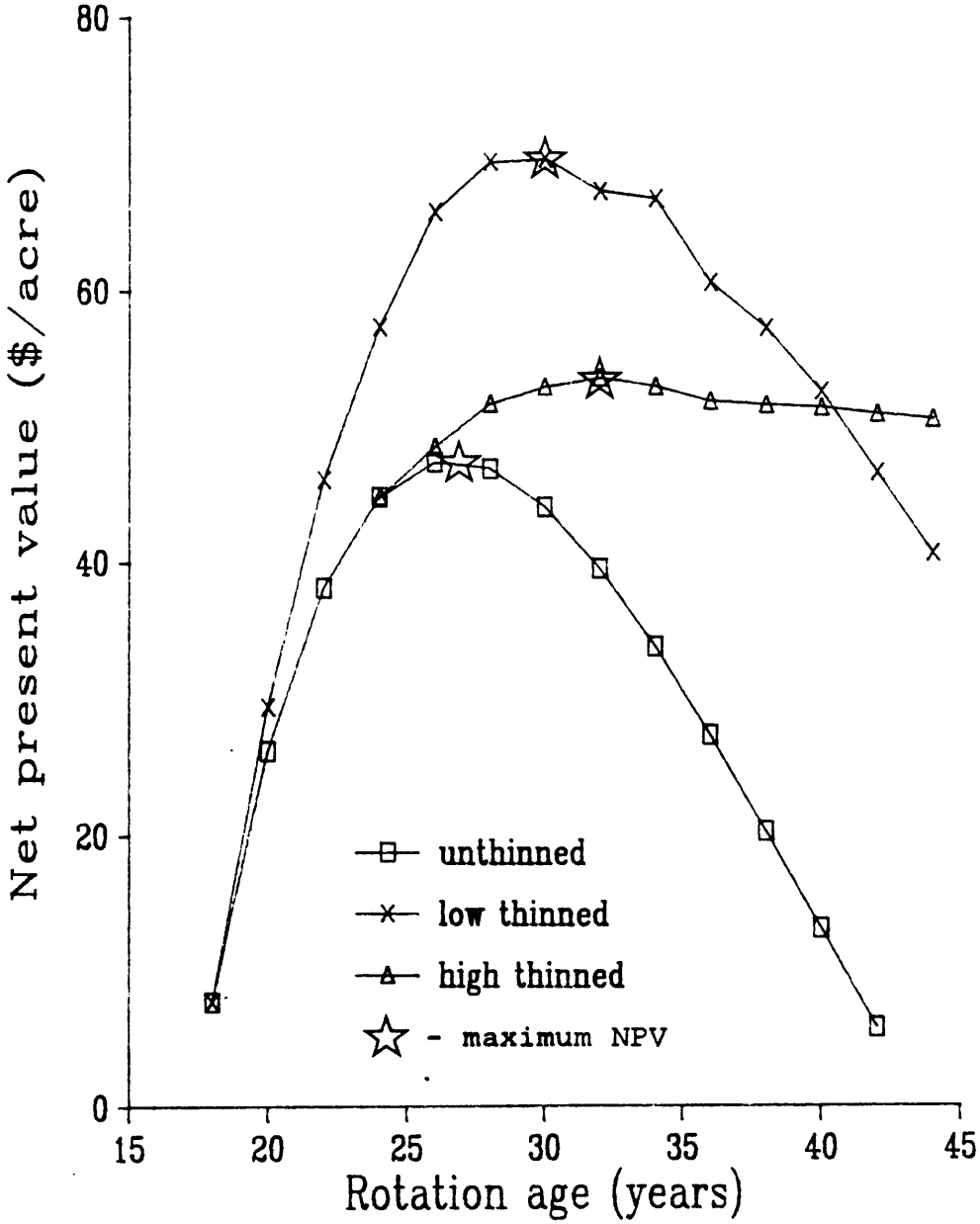


Figure 11. Net present values with 8% interest: all other benchmark assumptions.

tage over not thinning, when the fixed cost per thinning per acre increases from $20 to $50 per acre. The difference between high thinning and low thinning NPV also increases.

Single thinnings are optimal when using the $50 fixed thinning cost. The low thinning optimal regime with the $20 fixed thinning cost was 2 medium thinnings at 22 and 30 years. With the higher fixed cost ($50), a heavy thinning of over half the stand at 28 years is optimal. Likewise, the two thinnings in high thinning are replaced with a single moderate thinning with the increased fixed thinning costs.

Variable thinning costs

Thinning costs and revenues affect timing, intensity and frequency of thinning greatly. Better equations for predicting thinning costs and revenues based on average diameter of trees removed, basal area removed, number of trees removed and other stand attributes are needed. The base assumptions for this study were that fixed thinning cost was $20 per acre and thinned stumpage value was 80% of clearcut stumpage value. This implies that low valued thinnings from below would be cheaper to apply than would high thinnings removing the same basal area but with higher stumpage values.

In practice, it may be cheaper to remove the fewer larger trees in high thinning than the many small trees of low thinning. To model this, thinned stumpage prices were

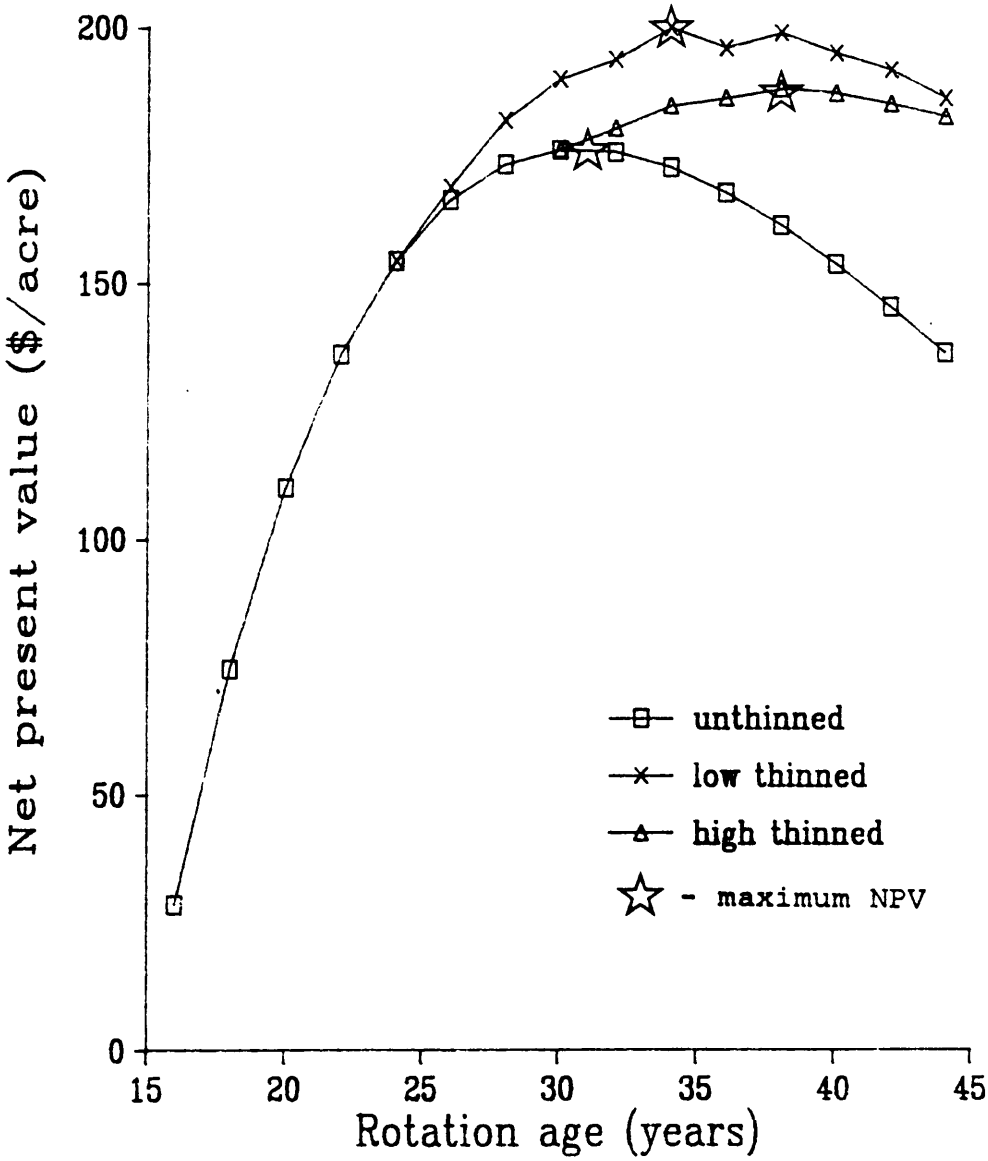


Figure 12. Net present values for $50/acre fixed thinning cost: all other benchmark assumptions.

set at 50% of clearcut prices for low thinning and 85% of clearcut price for high thinning. Fixed thinning costs remained at $20 per acre. Figure 13 shows that high thinning net present value would exceed that of low thinning at optimal rotation lengths by $3.

Low thinning rotation age didn't change with the new thinning values, but a single thinning at 5-10 years before clearcutting replaced the two thinnings found optimal under the benchmark conditions. High thinning with the lower thinning costs seemed to be approaching the maximum net present value by 44 years. Less confidence can be placed in results beyond at 40, since the data used to form the equations in FORTE only extend to 40 years. However, the longer rotation age for high thinning would seem reasonable in combination with 2 or 3 thinnings when thinning costs are lowered.

Planting density

Changing the planting spacing doesn't affect the optimal management regime significantly. Rotation ages remain similar for both thinning types. One or two thinnings are optimal with the 440 trees/acre assumption. Comparing Figure 9 (440 trees/acre) with Figures 14 and 15 showing 680 and 910 trees/acre planted shows similar patterns for NPV comparisons of the thinning types. However, the wider spacing provides

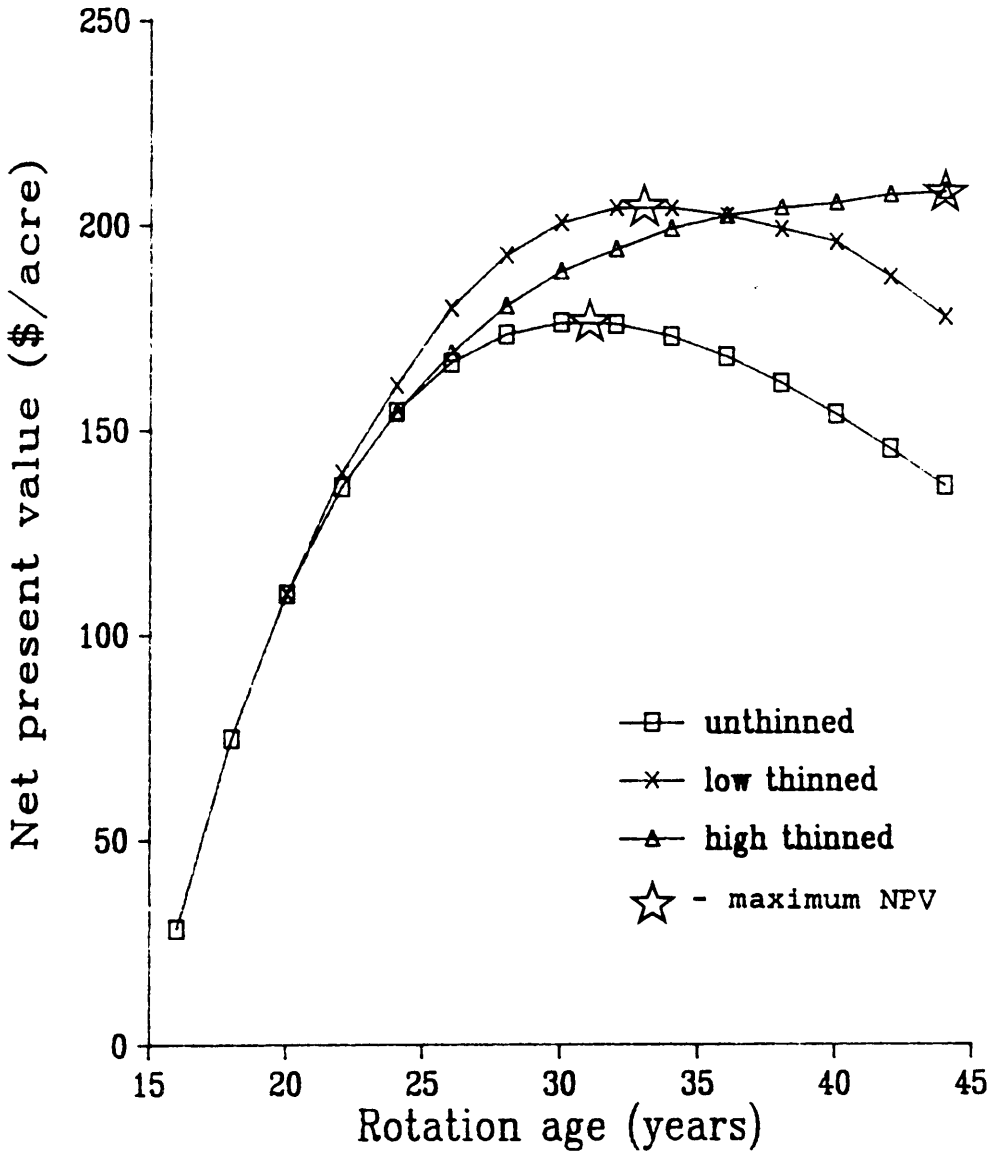


Figure 13. Net present values for different stumpage prices: low thinning prices at 50% of clearcut prices and high thinning prices at 85% with all other benchmark assumptions.

the highest net present values as seen in Figure 16. Wider spacings were not tested because of both lack of data in the PTAEDA2 growth simulator for wider spacings, and because the data used to form the whole stand growth equations only included planting densities down to 400 trees/acre.

Variation of stumpage prices

To simulate a greater premium on larger trees, a linear price-diameter function was fitted to $11/cord pulpwood, $150/MBF sawtimber and $300/MBF peelers. The equation was determined to be: $/cunit=-45.692+11.05(D). The slope is about twice that of the benchmark equation slope. Part of the problem of the function form is that the X-axis intercept (diameter) is 4.1" for the new equation and 3.5" for the benchmark value equation ($11/cord pulpwood, $100/MBF sawtimber and $137/MBF peeler values). Increasing board foot values shouldn't increase the average stand diameter necessary to produce a positive value from cutting. A third equation assumes a constant $20 per cord for all trees. Figure 17 shows how the three price-diameter equations value the wood removed.

Using the higher premium price-diameter function, rotation age for low thinning was shortened to 32 years and increased to at least 44 years (last age of analysis) for high thinning with the increased premiums. Thinning regime wasn't

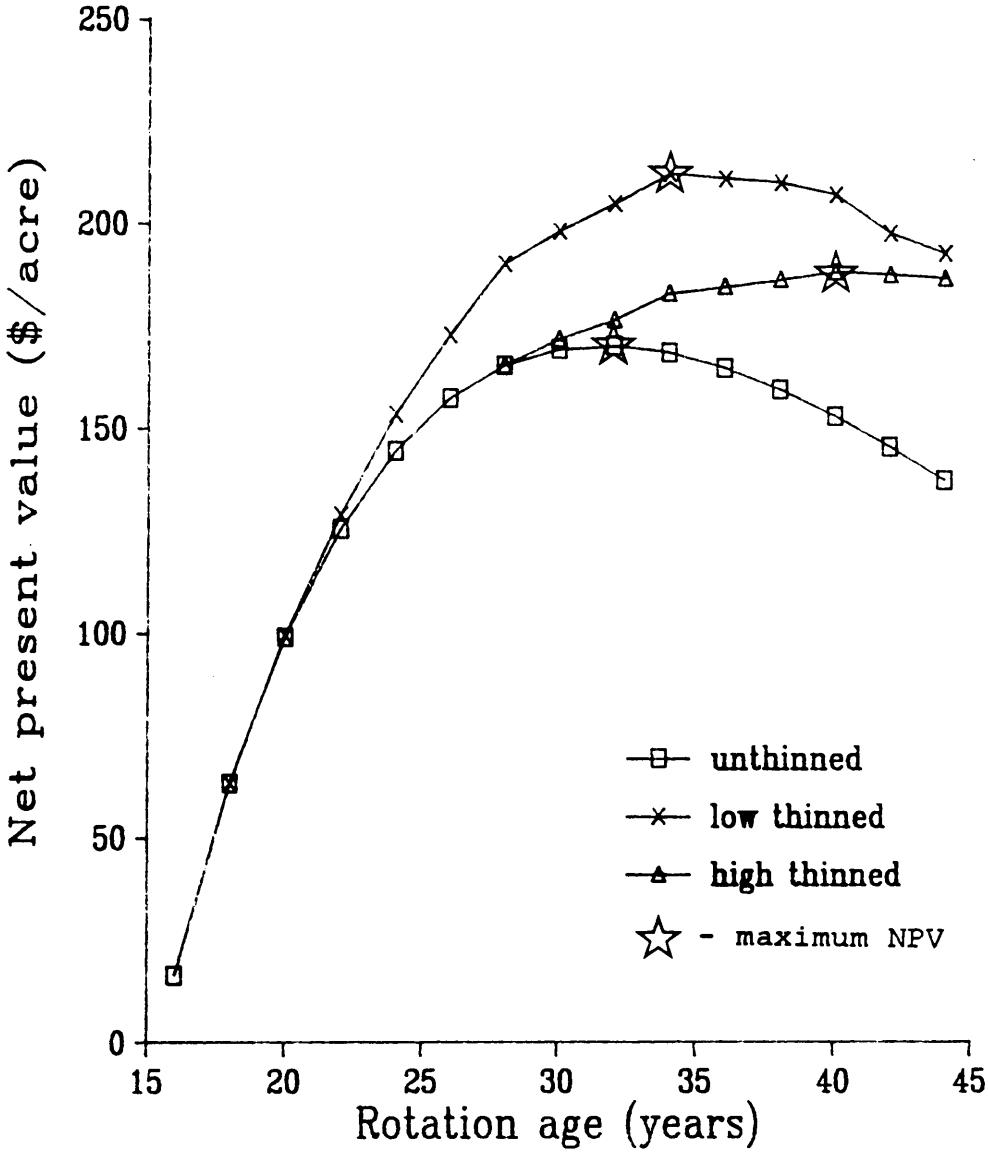


Figure 14. Net present values for planting 680 trees/acre: all other benchmark assumptions.

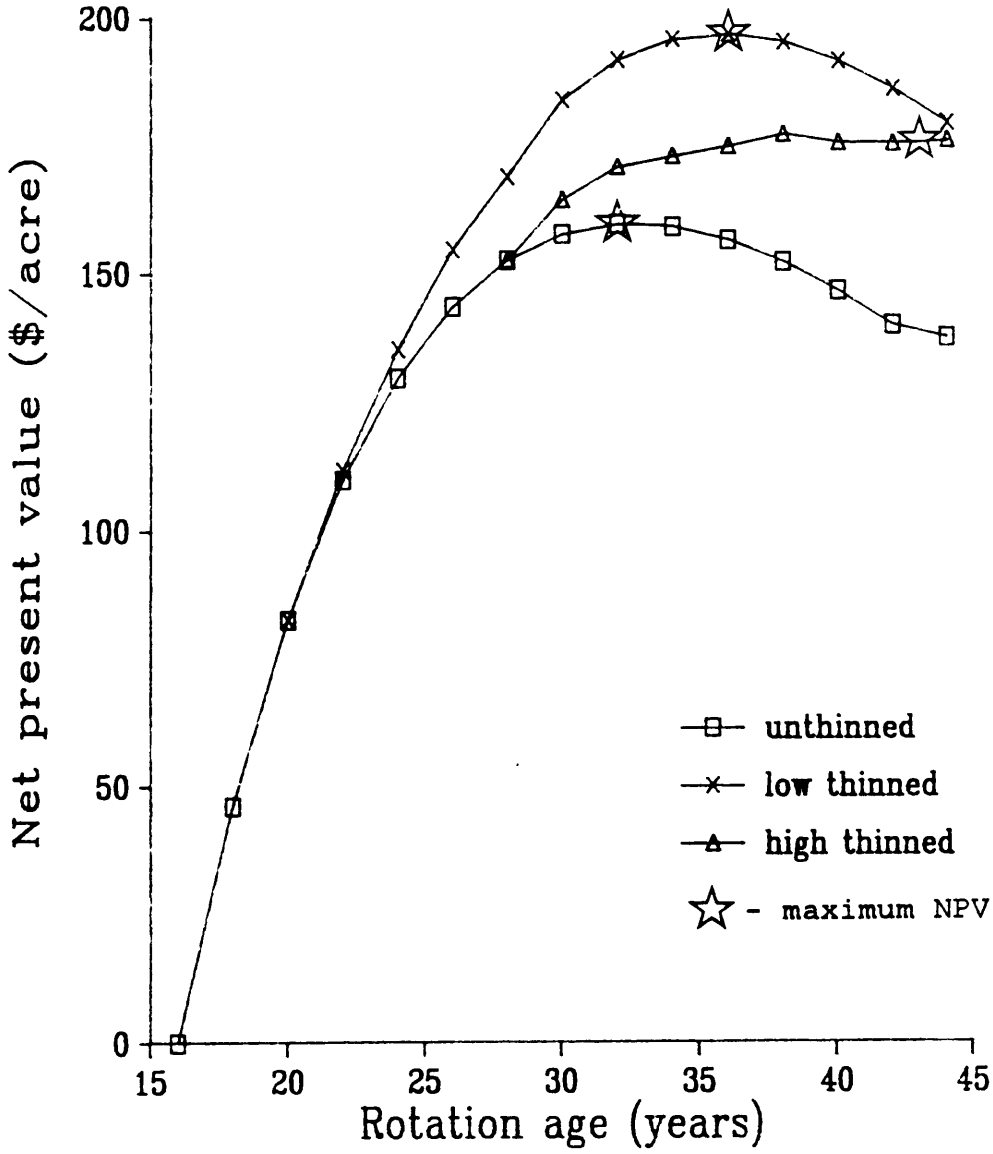


Figure 15. Net present values for planting 910 trees/acre: all other benchmark assumptions.

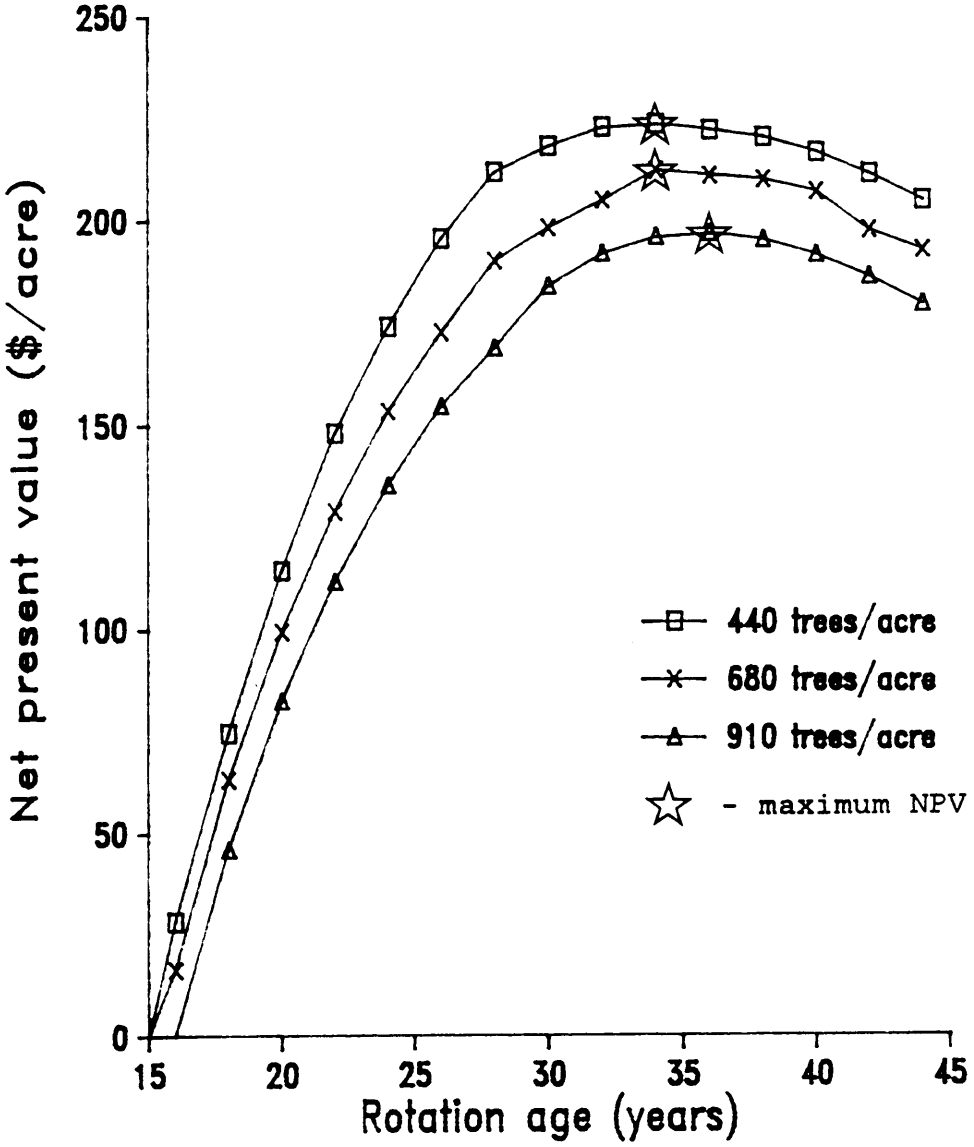


Figure 16. Net present values with changing planting densities: all other benchmark inputs.

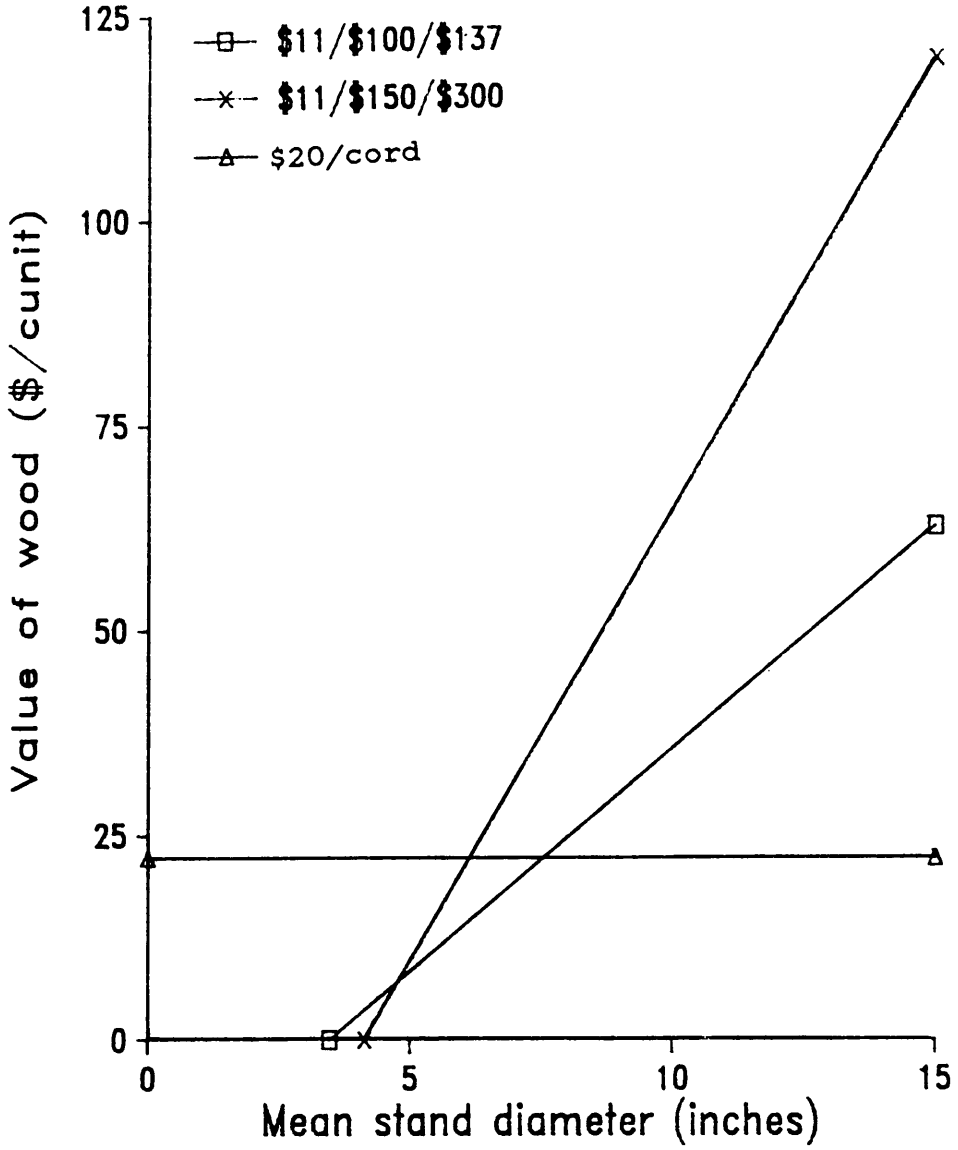


Figure 17. Price/cunit of wood for different price assumptions: price for different diameters using $11/$100/ $137 versus $11/$150/$300 for $/cord pulpwood, $/MBF sawtimber, $/MBF peeler and $20/cord linear price-diameter equations.

significantly affected. Figure 18 shows the base case present values and the new premium present values.

The general relationship between NPV's for high and low thinning remains the same. Present values are higher when product values are increased, as expected. However, low thinning shows roughly the same percentage NPV increase over high thinning for both price assumptions

Setting the price-diameter function slope to zero represents a fixed value per volume of wood. In Figure 19, the value of wood is assumed to be $20/cord. Thinning didn't become advantageous in the fixed value case until age 32, well beyond the net present value peak. The highest present value of ($106/acre) occurred with no thinning and a rotation of 22 years. Low thinning had a slight advantage over high thinning, but both were suboptimal.

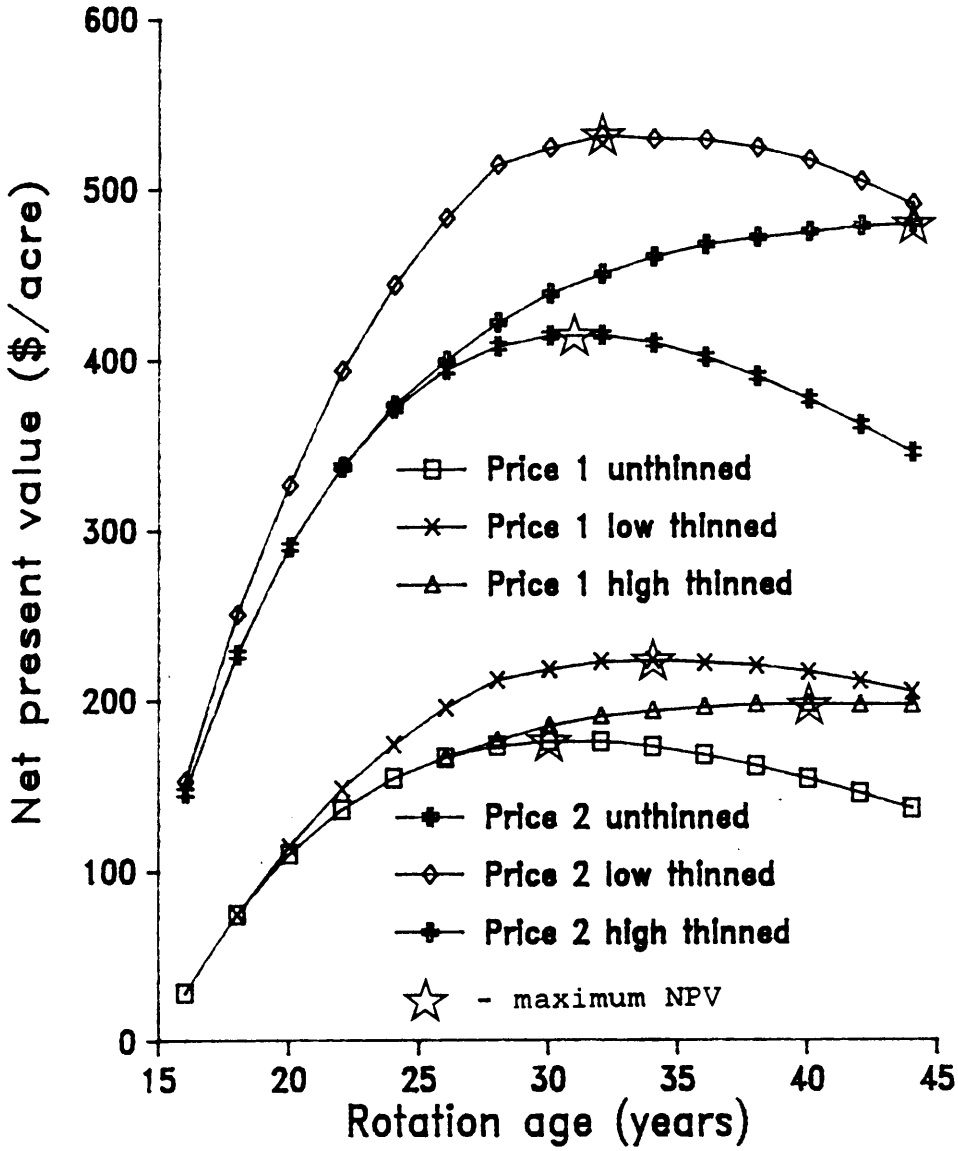


Figure 18. Net present values with changing stumpage value assumptions: (1) $11/$100/$137 and (2) $11/$150/$300 for $/cord pulpwood, $/MBF sawtimber and $/MBF peelers with all other benchmark assumptions.

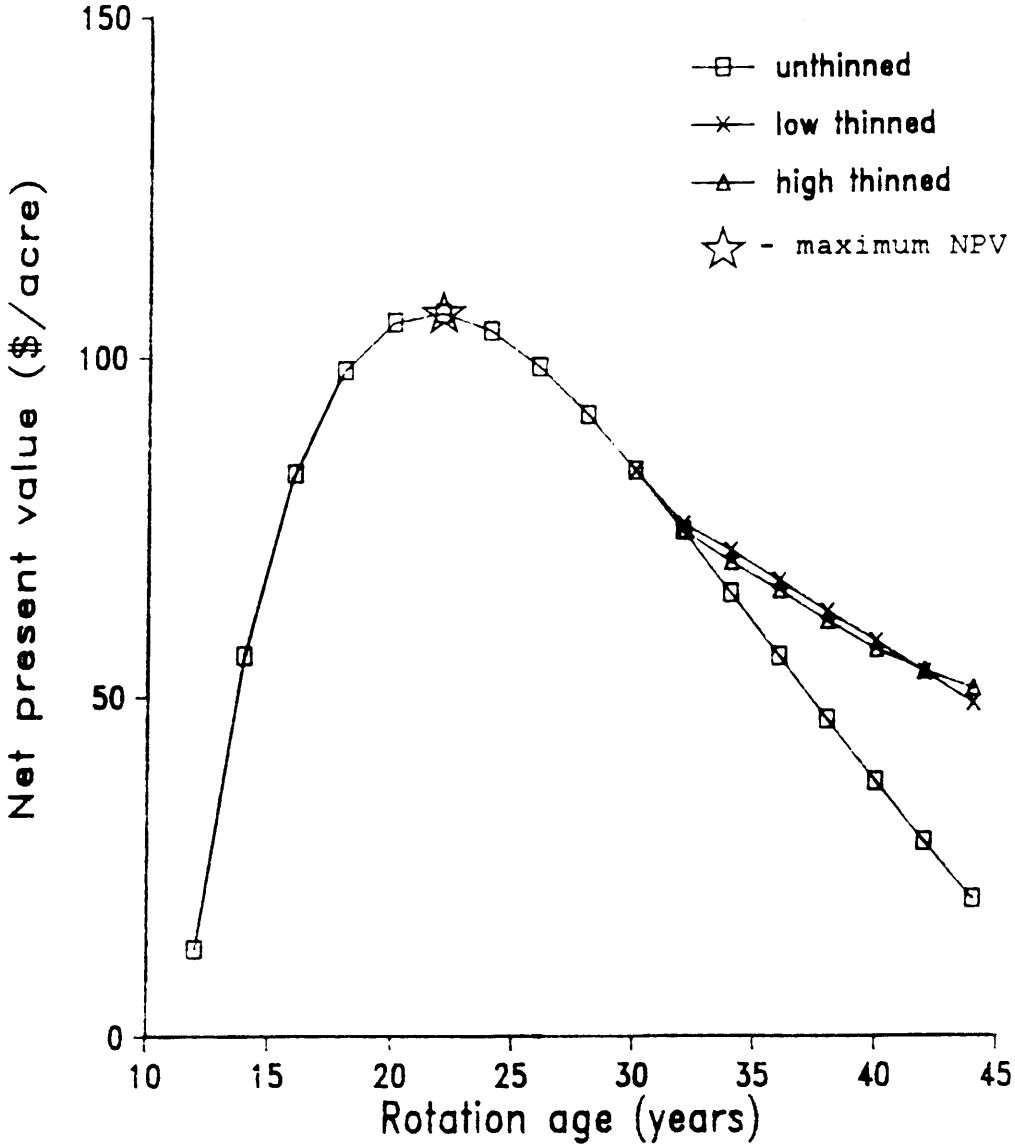


Figure 19. Net present values of thinning type with $20/cord value: all other benchmark assumptions.

CHAPTER 5. DISCUSSION

Using the FORTE program with the specified stand equations created several problems in both the equations and in the dynamic programming algorithm. Sometimes FORTE produced the highest present value by thinning the year before clearcutting. When this occured, only a minimal amount of basal area was thinned, which suggests that the regime is chosen to move the stand into the more profitable thinned volume equation. This implies there may be a problem in using different volume equations for the thinned and unthinned stands, or there may be a problem in not specifying a minimum amount of time between thinning and clearcutting.

Situations also occured where low thinning was in adjacent years. This may be due to problems with the diameter-price function and the determination of quadratic mean diameter. By removing trees in the first thinning, the residual stand has a larger diameter and subsequently a higher value per cunit. In the second thinning, the value increase of the stand must not only offset the costs of the thinning but also the additional fixed cost incurred from not removing all the wood in just a single thinning in one year. The problem of back-to-back thinning could be alleviated by increasing fixed thinning costs or minimum thinning removals.

Dynamic programming, as applied here, is subject to suboptimal solutions. This became apparent in the many iterations of FORTE used in this analysis. When a stand is chosen as the highest valued stand of a node description, no allowance is given for future stand growth and value potential. If two stands are vying for the same node, one stand is chosen. The discarded stand has a smaller accumulated value that, if grown, may have more than offset the difference in accumulated value with a higher clearcut value. For example, assume that a node was described as thinned, 120 sq. ft./acre basal area (BA), 300 trees/acre (t/a) and 20 years old. Two thinned stands at age 20 with attributes of 122 BA, 302 t/a and 118 BA, 298 t/a would compete for the node. Suppose the 122 BA, 302 t/a stand has an accumulated net present value of $32 and is stored in favor of the 118 BA, 298 t/a stand with a value of $31. Assume that by age 35 the 118 BA, 298 t/a stand would have produced a maximum net present value of $200 from clearcutting while the 122 BA, 298 t/a stand would have provided $195 from clearcutting. The accumulated net present value of the 118 BA, 298 t/a stand at clearcutting would be higher at $231 ($31+$200) than the $227 ($32+$195) of the 122 BA, 302 t/a stand. Choosing the 122 BA, 302 t/a stand at 20 years would produce a less than optimal solution.

The "neighborhood storage" was needed to keep the stand from sticking at one level of number of trees. If stands were

grown based on the node description, not the actual stand description, mortality could be inadequate to reduce the number of trees descriptor. For example, if a stand had 118 trees/acre it would be stored at 120 trees/acre. If mortality was 2 trees in the next period, the stand would have 116 trees/acre and be stored in the 115 trees/acre node. If "neighborhood storage" wasn't used, the 120 trees/acre stand descriptor would be used and the stand in the following period would be 118 trees/acre (120-2) and stored again in the 120 trees/acre node.

The "principle of optimality" may be violated as a result of this loss of a better stand through using "neighborhood storage". However, "neighborhood storage" is needed to eliminate rounding errors in stand state descriptors which could result in "sticking" in a state descriptor. To provide more information, the FORTE model was run using 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13 year increments. Many times, the 2, 3, 4, and 5 year increment runs provided higher net present values at some rotation ages. Loss of these "better" stands occured through "neighborhood storage" using the 1 year increment. This loss generally accounted for less than $1 decrease in optimal net present value.

Using 1 year increments provides more opportunity for thinning and gives more exact thinning timing information. The disadvantage of using the narrow time increments is that the number of stages is large, and at each stage the true

maximum net present value stand among those tested may be dominated by a stand with a higher accumulated NPV up to that stage.

Choosing correct age-increments implies weighing the level of detail needed versus the problems associated with the finer detail: thinning too close to clearcut, thinning in adjacent years and many stages requiring long execution times.

The results of this study differ from earlier thinning economic studies in loblolly pine. Hardie (1977) recommended thinning at 30 and 35 years, removing 1/3 of the stand (row thinning) with 7% interest and increasing annual costs and land rent values. Optimal rotation age was 40 years. These thinnings were later than the low thinned stands and more intense than the high thinned stands of this study. Results from Hardie are limited to five year increments and the equations were derived from stands 35 years or younger. Planting density was not varied in Hardie's model, but would have been about 1300 trees/acre based on his reported 686 trees/acre at age 20. Wood product values were less than this studies and the sawtimber prices were substantially less than peeler prices.

Broderick (1978), using the PTAEDA growth model (old-field), recommended 10'x10' planting spacing with a thinning of 20-30% of stand volume at age 20 and a final harvest at 30 years using 6% interest and after tax values. The old

version of PTAEDA showed a tendency to generate declining total stand volumes beyond age 30 which would tend to limit longer rotation lengths. Again, 5 year increments were used. Lower site preparation and planting costs were used. Pulpwood price was the same, sawtimber price lower and peeler price higher than this studies.

Thurmes (1980) also used a modified version of PTAEDA (PTAEDAJ) to determine optimal thinning regimes and rotation ages with hardwood competition or fertilization. Heavier hardwood competition favored 6'x10' spacing with a 40% volume thinning at 20 years and a final harvest at 30-35 years. Lighter hardwood competition optimal regime was 10'x10' spacing with a 30% thinning at 20 or 25 years and a final harvest at 30-35 years. The optimal fertilization regime was 10'x10' spacing, thinning at 15 years to remove 32-42% volume and final harvesting at 25 or 30 years. Thurmes used higher pulpwood prices, lower sawtimber and peeler prices, and lower site preperation costs than this study. Age increment of 5 years, 6% interest and after tax values were used.

Broderick et al. (1982) used the same assumptions as Thurmes (1980) except for excluding hardwood competition and fertilization. They found 10'x10' spacing, thinning 22% volume at 20 years and a final harvest at 25 years to provide the highest net present value. Their approach was again constrained by 5 year increments and used the old PTAEDAJ

growth simulator which has many of the same problems as the original PTAEDA model.

The difference between the results of this study and past studies can be attributed to differences in the growth and yield projections, costs and price assumptions, age increments, number of stands tested and thinning removal type.

Because of the inability to validate the high thinning results and the amount of assumptions made overall, results of this study should be considered tentative until further validated in field trials.

CHAPTER 6. CONCLUSIONS AND SUMMARY

FORTE, a stand level economic optimization dynamic programming model, was developed and applied to compare thinning from above and below in loblolly pine plantations. The program uses whole stand growth and yield equations and user specified constraints and economic assumptions to determine optimum management regimes which will maximize rotation-start net present value for selected thinning types.

Growth and yield equations were estimated from a revised version of the individual tree growth simulator PTAEDA2. Two types of low and two types of high thinning were simulated. Decision variables for each thinning type were planting spacing, number and timing of thinnings, basal area removed at each harvest, and rotation age

For the benchmark inputs (Table 4), the optimum low thinning had a maximum net present value (NPV) of $223, high thinning had a maximum NPV of $198 and not thinning produced an maximum NPV of $176. Low thinning left residual basal areas of 80 sq. ft. at ages 22 and 30. High thinning left residual basal areas of 120 sq. ft. at ages 28 and 35. Rotation ages were 31 years for no thinning, 34 years for low thinning and 40 years for high thinning. Low thinning produced the largest diameter trees, fewest remaining trees and highest valued trees at the final harvest age. Of the

three planting spacings tested, optimal planting spacing for all cases was 440 trees per acre.

Low thinning gave higher net present values for all situations tested when the same assumptions were applied to all thinning types. When assigned lower thinning costs, present value of the optimal high thinning regime slightly exceeded that of the optimal low thinning regime.

The optimal rotation ages for high thinning were longer than for low thinning. Rotation age declines by 3 years for each 10 unit decrease of site index. The advantage of low thinning over high thinning net present value is greatest at higher site indexes.

For the benchmark assumptions, the high thinning optimal regime removes less volume and basal area than low thinning. Both types of thinning had optimal regimes which included two thinnings. High thinnings occured at later ages than low thinnings.

Dynamic programming was a quick method for analyzing changing economic assumptions. Because the FORTE program was developed for the IBM personal computer, it can be used by forest managers who don't have access to a mainframe computer.

Problems sometimes occurred using the 4 state-descriptor dynamic program. Among these were: optimal regimes that had thinning shortly before clearcutting, thinning in adjacent periods, and slightly higher net present value regimes when

one increased the increments of age tested. These problems did not occur always, but often enough to suggest that the "principle of optimality" which is assumed in dynamic programming may not hold for this problem. On the other hand, numerous trials revealed that possible increases in present value above the FORTE solution were insignificant (within a couple of dollars).

Because dynamic programming is a network search technique which tests a discrete set of possibilities, true optimization is not guaranteed. However, one can generate many alternate regimes which are not significantly different from the highest net present value management regime. Dynamic programming is a powerful tool which will see continuing use in forestry applications.

The conclusions of this analysis can only be considered preliminary until adequate field data is obtained to support the results.

CHAPTER 7. RECOMMENDATIONS FOR FUTURE RESEARCH

There is a lack of field data available for comparing thinning from above and below. Results of this study are based on whole stand equations that are generated from a probabilistic model which wasn't designed for high thinning. Creating an adequate growth model for high thinning would require much more data than is available. Since high thinning is not commonly applied in loblolly pine, it is unlikely that data will become available in the quantity necessary to produce a good growth and yield prediction model.

Ideally, diameter distribution should be directly considered in comparisons of thinning type. Valsta and Brodie (1985) used a diameter distribution model in their loblolly pine dynamic program. Whether the model could be adapted to simulate high thinning utilizing the Weibull distribution needs to be researched.

This study also raised questions about the "principle of optimality". Past forestry research has not directly addressed whether the principle actually holds for dynamic programming applied to thinning. If violated, research needs to address the implications.

The best type of growth and yield model for analyzing different cultural treatments is the individual tree growth simulator. PTAEDA2 is a such a model, but doesn't adapt

easily to use in dynamic programming because of its stochastic elements. If it could be converted to a deterministic model, it would be the most flexible loblolly pine growth and yield model available for dynamic programming use.

The PTAEDA2 model might also be directly used in a dynamic program and run enough times to find which management regimes were most often selected. Probabilistic dynamic programming has not yet been used for economic optimization of thinning.

The FORTE program could be revised in many ways. Rather than only printing the highest NPV regime, a number of the highest regimes could be printed. It would also be easy to change the growth and yield functions in the program if better ones were available.

Two more state descriptors could be used in FORTE. Including stand height as a state descriptor might eliminate the need to utilize separate yield functions for high thinned and unthinned stands. Number of years since last thinning is a state descriptor that would allow the user to constrain the program to eliminate some extraordinary thinning timings.

The program could be altered to only allow a single thinning in the length of one rotation. This would allow comparison between the optimal solution of the unconstrained FORTE which may recommend multiple thinnings with single thinning. Preliminary indications are that a correctly timed

single thinning could replace two thinnings without an appreciable difference in net present value

Dynamic programming holds great promise for analyzing other cultural treatments besides thinning, such as fertilizing, pruning, hardwood removal and site preparation techniques. Damage appraisal is also a good application of dynamic programming. Damage can be simulated by adjusting the mortality or growth functions.

Development of an interactive program which would allow forest managers to test his/her prescriptions against the optimum given by the FORTE program could be written. Combining high and low thinning into a single program run would also be desirable. This was not done due to the complexity of defining growth and yield relationships when both types of thinning have occured in a stand.

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APPENDIX A. PRODUCT CLASSIFICATIONS

Table 5. Product merchantability limits.

1. Pulpwood	- Trees 5 inches d.b.h.(o.b.) and larger which are not of sawtimber quality to a 4 inch top (o.b.), and tops of sawtimber and peeler trees to a 4 inch top (o.b.). Measured in cords.
2.Sawtimber	- Trees 8 inches d.b.h.(o.b.) and larger which are not of peeler quality to a 6 inch tip (i.b.) Measure in thousand of board feet: Internation 1/4 inch rule.
3. Peelers	- Trees 11 inches d.b.h.(o.b.) and larger of peeler quality to a 6 inch top (i.b.). Measured in thousands of board feet: International 1/4 inch rule.
4. Pulpwood only	- All trees 5 inches d.b.h. (o.b.) and larger to a 4 inch top (o.b.). Measured in cords.

Table 6. Percentage of stems by diameter class falling into each product category.

D.b.h. (inches)	Percentages pulpwood*	sawtimber**	peelers***
5-7	100	--	--
8	68	32	--
9	47	53	--
10	7	93	--
11	2	49	49
12	--	50	50
13	--	42.5	57.5
14	--	35	65
15	--	30	70
16	--	25	75
>16	--	25	75

* trees 5 inches D.b.h. (o.b.) and larger to a 4 inch (o.b.) top, and tops of sawtimber and peeler trees to a 4 inch top (o.b.) - standard cords.

** trees 8 inches D.b.h. (o.b.) and larger to a 6 inch top (o.b.) - International 1/4 inch rule.

*** trees 11 inches D.b.h. (o.b.) and larger to a 6 inch top (o.b.) - International 1/4 inch rule.

Source: Broderick (1978)

Table 7. Cubic-foot to standard cord conversion.

Diameter (inches)	Conversion* factor
5	84
6	85
7	87
8	90
9	91
10	92
11	93
12	94
>12	95

* Standard cords (o.b.) to 4 inch top diameter (o.b).
Cords = cu. ft.(o.b.) / appropriate conversion factor.

Source: Burkhart et al. (1972)

APPENDIX B. GROWTH AND YIELD PARAMETER VALUES FOR COMPONENT EQUATIONS FITTED TO THE OUTPUT FROM RUNS OF PTAEDA2

Survival of unthinned stands

$$\log(N_2)=\log(N_0)+(a_1+a_2(\log(N_0))\frac{A_2}{100}$$

Equation form from: Coile and Schumacher (1964)

Stand	Parameter	Estimate	Se	t-value	r^2
unthinned	a_1	2.111513	.219525	9.619	.9963
	a_2	-1.120269	.077598	-14.437	

Basal area of stands before initial thinning

$$\log(B_2)=b_1(\log(SI))+\frac{b_2}{A_2}+b_3(\log(N_2))+b_4$$

Equation form from: Coile and Schumacher (1964)

Stand	Parameter	Estimate	Se	t-value	r^2
unthinned	b_1	1.070547	.069975	15.274	.9234
	b_2	0 (not used)			
	b_3	.830926	.025829	32.274	
	b_4	-2.406679	.138638	-17.485	

Basal area of unthinned stand after initial thinning age

$$\ln(B_2)=c_1+c_2SI-\left[\frac{A_1}{A_2}\right]\left[c_1+c_2SI-\ln(B_1)\right]$$

Equation form from: Cao et al. (1982)

Stand	Parameter	Estimate	Se	t-value	r^2
unthinned	c_1	5.578588	.094204	59.218	.9986
	c_2	.000656195	.00159235	.412	

Appendix B. Growth and yield parameter values for component equations fitted to the output from runs of PTAEDA2

Volume (cubic feet) of unthinned stands

$$\ln(Y) = d_1 + \frac{d_2}{A_2} + d_3 \frac{H}{A_2} + d_4 A_2 \ln(N_2) + d_5 \ln(B_2)$$

Equation form from: Amateis et al. (1986)

Stand	Parameter	Estimate	Se	t-value	r^2
unthinned	d_1	3.073178	.034964	87.897	.9922
	d_2	-16.034681	.256349	-62.550	
	d_3	.280939	.005152	54.530	
	d_4	.003135831	.000104	30.211	
	d_5	.937307	.007205	130.096	

Height of dominant / codominant trees

$$\ln(H) = \ln(SI) \left\{ \frac{j_1}{A_2} \right\} \left\{ j_2 \, e^{\left\{ j_3 \left[\frac{1}{A_2} - \frac{1}{25} \right] \right\}} \right\}$$

Source: Amateis and Burkhart (1985)

Stand	Parameter	Estimate	
all	j_1	25	(used directly from
	j_2	-.10283	Amateis and Burkhart 1985)
	j_3	-2.1676	

Appendix B. Growth and yield parameter values for component equations fitted to the output from runs of PTAEDA2

Trees remaining following thinning of given basal area

$$N_3=N_2\left\{1-\left\{\frac{B_2-B_3}{B_2}\right\}^{P}\right\}$$

Derived from equation forms by: Field et al. (1978)

Stand	Parameter	Estimate	Se	t-value	r^2
strong low	POWER	.704596	.001114	632.675	.9996
weak low	POWER	.849059	.005235	162.188	.9932
weak high	POWER	1.156089	.003090	374.100	.9987
strong high	POWER	1.298639	.003131	414.770	.9991

Basal area of thinned stands

$$\ln(B_2)=f_1+f_2SI-\left[\frac{A_1}{A_2}\right]\left[f_1+f_2SI-\ln(B_1)\right]$$

Equation form from: Cao et al. (1982)

Stand	Parameter	Estimate	Se	t-value	r^2
strong low	f_1	4.569874	.370690	33.340	.9980
	f_2	.016688	.002280	7.318	
weak low	f_1	4.839991	.110763	43.697	.9987
	f_2	.012342	.001843	6.698	
weak high	f_1	4.780369	.115397	41.425	.9986
	f_2	.013518	.001919	7.043	
strong high	f_1	4.800633	.198273	40.063	.9987
	f_2	.013276	.001994	6.658	

Appendix B. Growth and yield parameter values for component equations fitted to the output from runs of PTAEDA2

Volume (cubic feet) of thinned stands

$$\ln(Y) = g_1 + \frac{g_2}{A_2} + g_3\left(\frac{H}{A_2}\right) + g_4 A_2 \ln(N_2) + g_5 \ln(B_2)$$

Equation form from: Burkhart et al. (1972)

Stand	Parameter	Estimate	Se	t-value	r^2
strong low	g_1	3.530895	.054252	65.084	.9913
	g_2	-20.901036	.434947	-48.054	
	g_3	.344410	.009454	36.428	
	g_4	.003109782	.000226	13.787	
	g_5	.858602	.013029	65.899	
weak low	g_1	3.189246	.074172	42.998	.9930
	g_2	-18.173826	.513404	-35.399	
	g_3	.321147	.009378	34.244	
	g_4	.003384216	.000191	17.692	
	g_5	.907650	.015749	57.631	
weak high	g_1	3.045928	.052912	57.566	.9918
	g_2	-16.746035	.436346	-38.378	
	g_3	.304988	.008603	35.451	
	g_4	.003302152	.000196	16.853	
	g_5	.929702	.010586	87.820	
strong high	g_1	2.944411	.058186	50.603	.9919
	g_2	-15.132416	.464963	-32.545	
	g_3	.277489	.009048	30.669	
	g_4	.003294184	.000209	15.740	
	g_5	.948115	.011638	81.470	

Appendix B. Growth and yield parameter values for component equations fitted to the output from runs of PTAEDA2

Survival of thinned stands

$$N_2 = \left\{ N_1^{k_1} + k_2 (A_2^{k_3} - A_1^{k_3}) \right\}^{(1/k_1)}$$

Equation form from: Clutter and Jones (1980)

Stand	Parameter	Estimate	Se
strong low	k_1	-.916609	.208899
	k_2*	3.1933E-7	
	k_3	2.408472	.228293
weak low	k_1	-1.095724	.210387
	k_2*	3.2139E-8	
	k_3	2.881649	.235910
weak high	k_1	-1.037742	.208501
	k_2*	3.2889E-8	
	k_3	2.904052	.240071
strong high	k_1	-.828945	.201907
	k_2*	2.1054E-7	
	k_3	2.688604	.215044

* k_2 was estimated from a combination of k_1, k_3 and another variable.

Appendix B. Growth and yield parameter values for component equations fitted to the output from runs of PTAEDA2

APPENDIX C. MODEL VALIDATION

Table 8. Comparison of yields of FORTE and revised PTAEDA with yields from Coile and Schumacher (1964) on low thinned stands.

Stand predictor	Thinning removal(cds) at age 17	22	--Residual stand age 30--- Basal area sq.ft.	Trees per acre	Average D.b.h. inches*	Volume cds/ acre	Total production cds/ac
plant 440 trees/ac.** thinnings at 17 and 22 years removing basal areas of 32 and 26 sq.ft.							
Coile/Schu.	4	6	83	89	13.1	21.2	31.2
FORTE	4.5	7.5	85	87	13.3	25.0	37.0
PTAEDA***	4.4	7.2	101	109	12.9	31.8	43.4
plant 680 trees/ac.** thinnings at 17 and 22 years removing basal of areas 32 and 26 sq.ft.							
Coile/Schu.	5	5	101	165	10.6	26.6	36.6
FORTE	5.3	6.1	107	144	11.7	32.3	43.7
PTAEDA***	5.3	5.9	121	166	11.4	38.4	49.6
plant 910 trees/ac.** thinnings at 17 and 22 years removing basal areas of 32 and 26 sq.ft.							
Coile/Schu.	6	4	126	241	9.8	30.6	40.6
FORTE	5.9	4.4	138	212	10.9	42.2	52.5
PTAEDA***	6.1	4.4	130	215	10.3	40.6	51.1

Site index 60 loblolly pine.

* FORTE D.b.h. is quadratic mean diameter, others are arithmetic mean diameters.

** Coile and Schumacher used 400, 600 and 800 trees/ac. at age 5. Planting 440, 680 and 910 trees/acre approximately produces that number of trees at age 5. They used non-old field sites which would tend not to produce as well as site prepared land.

*** PTAEDA data is from 5 runs averaged.

APPENDIX D. FORTE USERS GUIDE VERSION 1.0

FORTE Users Guide version 1.0

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FORTE is a personal computer program that uses dynamic programming (a network analysis technique) to seek the maximum economic return from an acre of forest land. Economic return is determined using infinite series net present value (NPV) of forest investment, also known as soil expectation value (SEV) or discounted cash flow. As will be seen in Section 4 of the program main menu, there are many economic assumptions underlying the choice of the highest valued (optimal) management regime. The economic assumptions are combined with whole stand growth and yield functions in the FORTE algorithm to obtain the optimal management regime (of the discrete set of possibilities simulated).

Maximization of economic return from a forest stand is a complex task. Linear programming cannot adequately deal with stand level non-linear growth and yield relationships. Analytical methods (calculus) for interpretting production functions cannot adequately handle intermediate stand removal and shifting production functions. Dynamic programming is used because it can handle the complex relationships involved.

FORTE uses forward recursion which builds the network and solves it at the same time. It is especially efficient because once optimal "paths" to a "node" are determined, they remain optimal for any rotation age, so one program run will solve for many rotation ages.

Stands are merged when they share the same four state variables: age, basal are, trees/acre and stand type (thinned or unthinned). Age increments can be varied by the user down to single year increments. Basal area is rounded to 5 sq. ft./acre and number of trees is rounded to the nearest 5 trees/acre for node storage. Actual stand values are stored at a node for use in valuation and projection.

A "memoryless" nature of stands is assumed in FORTE. This condition lets the stands with equal attributes be merged by assuming that the method by which the stand reaches that state is of little importance and that future growth of stands with the same state would be similar. Applying a single type of thinning to stands of equal planting density strengthens the assumption.

Only one type of thinning per run is allowed in FORTE. Thinning type is defined by a power term P (referred to as L24 in the removal equation found in Appendix B). The value of the power term is that which equates the ratio of basal area removed with the ratio of trees removed. In row thinning P=1.00, in a low thinning P<1.00 and in high thinning P>1.00. Growth reactions through applying several types of thinning in a single rotation involves complex relationships which only can be modelled through more complex growth and yield simulation not possible with the current form of FORTE.

Sensitivity analysis is possible through changing asssumptions such as planting density, interest rate and product values. Varying the age increments will provide several alternate management regimes with only slight differences in NPV.

The program will run relatively quickly in its compiled form. Arrays contain enough room to store 1400 nodes per stage, enough room for all but the largest problems.

Operating instructions

Starting the program

Boot up DOS in drive A. Input through the date and time queries to the A> prompt. Replace the DOS disk with the FORTE disk. Enter FORTE and hit RETURN. The main menu of the program should now appear.

FORTE Figure 1. FORTE main menu listing

```
Main menu selections are:
-------------------------

 1 -- READ function parameter file
 2 -- CREATE new function parameter file
 3 -- EDIT function parameter file
 4 -- READ economic and model variables file
 5 -- CREATE new economic and model variables
 6 -- EDIT economic and model variables
 7 -- HELP
 8 -- PRINT utilities
 9 -- RUN model
10 -- END model

Note: Before running the model a parameter file must
      be READ or CREATEd and the economic and model
      parameters must also be formed.

Select a number and RETURN
```

All segments of the program are accessed through this menu, and return to it. Descriptions of the routines can be found in the following sections.

Sections of the main menu

1 -- READ function parameter file

Parameters for growth and yield functions can be stored in disk files. Once computed for an individual use,they will form the basis for predictions of stand attributes and will be used repeatedly. BAS.DAT and HI.DAT are sets of parameters available on this disk. They were estimated using linear and non-linear regression techniques on data obtained from repeated runs of the program PTAEDA2 (unpublished) a cutover version of the old-field loblolly pine growth simulator PTAEDA (Daniels and Burkhart 1972). They should provide adequate predictions for site indexes between 50 and 70 (base age 25) and planting densities between 400 and 1000 trees/acre. BAS.DAT is a low thinning data set and HI.DAT is a high thinning data set. Care must be excercised in not pushing these data sets beyond their acceptable range of values. Also, because these equations are indirectly formed from field data, the use of the predictions from them should be used with great caution.

2 -- CREATE function parameter file

Once parameters have been estimated for the whole stand equations used in FORTE (see Appendix B), this routine allows for easy input of the parameter values. The parameters can be used for current and, if saved to a disk file, future runs

(as BAS.DAT and HI.DAT were). Opportunity is given for storage of the data in a user named file. Once the parameters are entered, the program shifts to the EDIT routine (section 3).

3 -- EDIT function parameter file

Individual parameter values can be changed for the function parameter file. The program will prompt for the number of the parameter to be change. Enter the number (not including letter assigned) of the parameter (1 to 27) and then give the new value when prompted. It is possible to form a whole file by editing, but it is quickest to read a disk file or to utilize the prompt characteristics of the CREATE function parameter file section (2).

FORTE Figure 2. Example function parameters file screen

```
GROWTH AND YIELD EQUATION PARAMETERS
SEE USER GUIDE FOR HELP

A 1 = 4.396929        D 9 =  1.54058          G 19 = 1.917294
A 2 =-1.849307        D 10 =-4.591066         G 20 =-6.56134
                      D 11 = .316724          G 21 = .36871
B 3 = 1.19469         D 12 = 5.820874E-03     G 22 = 5.99632E-03
B 4 =-7.221833        D 13 = .988886          G 23 = .918996
B 5 = .532329
B 6 =-1.127654        E 14 =-.65808           H 24 = .720278
                      E 15 = 7.5795E-06
C 7 = 5.166031        E 16 = 1.78019          J 25 =-.10283
C 8 = 5.83403E-03                             J 26 =-2.1676
                      F 17 = 5.477131         J 27 = 25
                      F 18 = 2.34655E-03

 CHANGE ON OR MORE VARIABLES (Y/N)?
```

4 -- READ economic and model variable file

Variables can be stored to file and recalled (READ) at a later time. This is most useful when a set of economic and model assumptions is used repeatedly in analysis. A sample variable file is VAR.DAT. Table 1 gives definitions of the variables that are needed to run FORTE. For complete new entry of the variables, section 5 (CREATE) prompts the user for easy entry. Section 6 (EDIT) can then be used to edit the variables should changes be required.

5 -- CREATE economic and model variable file

As can be seen in Figure 5, there are many economic and model variable options. This section allows for initial input of the variable. Once entered, the program shifts to the EDIT (section 6) routine.

6 -- EDIT economic and model variables

This section allows for easily changing one or more variables. The variables are stored for use in the run portion of the program. This allows for quick changes of variable values for sensitivity analysis.

FORTE Figure 3. Economic and model variables sceen

```
                    ECONOMIC AND MODEL VARIABLES

 1 --- MINIMUM THINNING AGE = 10                10 -- MINIMUM BASAL AREA LEFT = 50
        (10 SUGGESTED)                                    AFTER THIN
 2 --- MAXIMUM ROTATION AGE = 45                11 - INCREMENTS OF BASAL AREA = 10
 3 - HOW MANY YEARS BETWEEN = 5                          TO THIN AT
       DECISION PERIODS                        12 -- VALUE OF INTERCEPT TERM =-18.929
 4 - STOP PROGRAM AFTER NPV = 1                13 ------ VALUE OF SLOPE TERM = 5.454
    DECLINES (0=YES, 1=NO)                     14 ------ THINNING VALUE AS % = 80
 5- # OF TREES/ACRE PLANTED = 440                        OF HARVEST VALUE
 6 --- SITE INDEX (25 YRS.) = 60               15- COSTS OCCURING EVERY THIN = 20
 7 - MINIMUM THIN REMOVAL % = 15               16 - SITE PREP/PLANTING COSTS = 101
 8 -- MINIMUM CUNIT REMOVAL = 3                17 ------ ALTERNATIVE RATE OF = 6
       FOR FEASIBLE THIN                              RETURN (INTEREST RATE)
 9- MINIMUM TREES/ACRE LEFT = 100
          AFTER THIN
```

FORTE Table 1. Economic and model variable definitions

---

1 -- Age of first thinning, also the first year that the program will produce output.

2 -- Latest age that program will grow stands to, and produce output for.

3 -- Number of years before next thinning is allowed following a thinning. This is also the frequency at which harvest present values are generated and output.

4 -- This is a flag to let the program know whether to keep running and printing after NPV starts to decline.

5 -- Number of trees per acre planted initially.

6 -- Site index of stand (base age set by variable J27).

7 -- Minimum percent removal of stand volume needed from a thinning.

8 -- Minimum volume (cunits) removed from a thinning.

9 -- Minimum trees/acre that must remain after thinning.

10 -- Minimum basal area (sq. ft./acre) that must remain following thinning.

11 -- Increments of basal area used in thinning. If = 10: a stand of 100 sq.ft/ac. will be thinned to 90,80,...

12 -- Intercept term in linear price-diameter function Value/cunit = intercept + slope (mean stand diameter)

13 -- Value of the slope term in the linear price-diameter function (rate of value increase).

14 -- Decreases thinning value to percent of harvest value.

15 -- Fixed (entry) costs per acre occuring every thinning.

16 -- Costs associated with site preperation and planting. (per acre).

17 -- Alternative rate of return (interest rate). The going rate for an investment with similar risk.

7 -- HELP files

The help files are accessed through the HELP menu. These files contain information similar to that in this Users Guide.

FORTE Figure 4. HELP menu

```
---------------------------------------------------------------

       HELP menu selections are:
       -------------------------

       1 -- General model description
       2 -- Function parameter file
       3 -- Economic and model variables
       4 -- Return to main menu

---------------------------------------------------------------
```

8 -- PRINT utilities

The PRINT utilities section has several functions. Sub-sections 1, 2 and 3 allow for user selection of program output type. The output is always displayed to the screen and can also be either printed or written to an output file specified by the user.

Sub-section 4 lets the user input a title for the program run. This can be useful for identifying different program runs during analysis of output.

Sub-section 5 displays a user specified file to screen. It allows scrolling through the file or exiting from display.

Sub-section 6 allows printing of disk files. This is best used by writing program output to a disk file and then printing it later using this routine.

FORTE Figure 5. Print utilities menu

```
---------------------------------------------------------------

       PRINT utilities menu selections are:

           OUTPUT settings for program runs
           --------------------------------

       1 -- DISPLAY (only) program output on the screen
       2 -- PRINT program output to the printer
       3 -- WRITE program output to a disk file

           Other utilities
           ---------------
       4 -- INPUT title name for output
       5 -- DISPLAY program output from a disk file
       6 -- PRINT program output from a disk file
       7 -- Return to main menu

---------------------------------------------------------------
```

9 -- RUN model

The run portion of the program will first display a "status" screen showing information of interest before the run begins. It checks several function parameters and model variables to see if the program can run. If the function

parameters or model variables have not been entered, the program will not run correctly. Before running, opportunity is allowed to return to the main menu to update the run set-up. Be sure that the function parameter file and economic and model variable file have been completely entered. Figure 6 shows a sample screen.

During the run, output is generated for each stage of the program, starting at the first thinning year and continuing for each decision period until NPV declines or maximum rotation age is reached. The output from the program contains information on the stand at the time of harvest (clearcut) as well as information on any thinnings that may have occured.

FORTE Figure 6. RUN status screen

```
---------------------------------------------------------------

        Program run status for FORTE
        --------------------------------------------

        File to be printed: NO (not required)

        File to be written to disk file: test.dat

        Status of function parameter file: has been entered

        Status of economic/model variables: has been entered

        Title of report files:
Test run of model using var.dat and bas.dat files    3/14/86

CONTINUE this run (C and return)
or RETURN to menu (R and return)

---------------------------------------------------------------
```

All values, unless otherwise specified, are in terms of amount per acre. The values listed can be thought of as expected values, since they are derived form deterministic equations. Of most importance in deciding the "best" management regime is infinite series present value. This value is derived from applying the management regime an infinite number of times to the land. Utilization of an infinite time horizon allows comparison of values for different rotation lengths.

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FORTE Appendix A. Example output

```
page 1       Test run of FORTE   6/24/86.

Economic and model variables used for this program run
------------------------------------------------------
Minimum thinning age= 10     Maximum rotation age= 40
Trees/acre planted = 440     Site Index (base age 25 yrs)= 60
Minimum thinning removal= 15 % of stand volume or 3 cunits
Stand thinned in increments of 20 basal area down to 50 basal area
Stand value ($/cunit)=  -18.929  +  5.454  x (mean stand diameter)
Thinning value= 80 % of harvest value
Thinning cost (fixed)=$ 20     Site prep/planting costs=$ 101
Alternative rate of return= 6
```

Age Yrs	Activity	Type of Stand	Basal Area	Trees /Acre	Mean Dia.	Cunits	Value /Cunit	Cash Value	Present Value
10	CLEARCUT		45	393	4.58	6.3	$6.05	$37.93	$21.18

Minus planting / site preparation costs= -$101.00

Single rotation net present value= -$79.82
Infinite series net present value= -$180.75

Age Yrs	Activity	Type of Stand	Basal Area	Trees /Acre	Mean Dia.	Cunits	Value /Cunit	Cash Value	Present Value
15	CLEARCUT		82	371	6.37	15.4	$15.82	$243.18	$101.47

Minus planting / site preparation costs= -$101.00

Single rotation net present value= $0.47
Infinite series net present value= $0.81

Age Yrs	Activity	Type of Stand	Basal Area	Trees /Acre	Mean Dia.	Cunits	Value /Cunit	Cash Value	Present Value
20	CLEARCUT		111	351	7.62	25.0	$22.65	$566.71	$176.70

Minus planting / site preparation costs= -$101.00

Single rotation net present value= $75.70
Infinite series net present value= $110.00

page 2 Test run of FORTE 6/24/86.

Age Yrs	Activity	Type of Stand	Basal Area	Trees /Acre	Mean Dia.	Cunits	Value /Cunit	Cash Value	Present Value
25	CLEARCUT		97	168	10.29	25.6	$37.20	$953.01	$222.05
15		ORIGINAL	82	371	6.37	15.4			
	THINNING	REMOVAL	32	192	5.65	5.8	$9.49	$34.59	$14.43
		RESIDUAL	50	179	7.15	9.6			

Minus planting / site preparation costs= -$101.00

Single rotation net present value= $135.49
Infinite series net present value= $176.64

Age Yrs	Activity	Type of Stand	Basal Area	Trees /Acre	Mean Dia.	Cunits	Value /Cunit	Cash Value	Present Value
30	CLEARCUT		87	97	12.86	25.5	$51.21	$1304.61	$227.15
25		ORIGINAL	97	168	10.29	25.6			
	THINNING	REMOVAL	27	68	8.75	7.0	$23.05	$142.37	$33.17
		RESIDUAL	70	100	11.35	18.6			
15		ORIGINAL	82	371	6.37	15.4			
	THINNING	REMOVAL	32	192	5.65	5.8	$9.49	$34.59	$14.43
		RESIDUAL	50	179	7.15	9.6			

Minus planting / site preparation costs= -$101.00

Single rotation net present value= $173.75
Infinite series net present value= $210.38

page 3 Test run of FORTE 6/24/86.

Age Yrs	Activity	Type of Stand	Basal Area	Trees /Acre	Mean Dia.	Cunits	Value /Cunit	Cash Value	Present Value
35	CLEARCUT		102	93	14.17	32.4	$58.36	$1892.40	$246.21
25		ORIGINAL	97	168	10.29	25.6			
	THINNING	REMOVAL	27	68	8.75	7.0	$23.05	$142.37	$33.17
		RESIDUAL	70	100	11.35	18.6			
15		ORIGINAL	82	371	6.37	15.4			
	THINNING	REMOVAL	32	192	5.65	5.8	$9.49	$34.59	$14.43
		RESIDUAL	50	179	7.15	9.6			

Minus planting / site preparation costs= -$101.00

Single rotation net present value= $192.82
Infinite series net present value= $221.66

Age Yrs	Activity	Type of Stand	Basal Area	Trees /Acre	Mean Dia.	Cunits	Value /Cunit	Cash Value	Present Value
40	CLEARCUT		115	89	15.36	39.4	$64.87	$2552.98	$248.21
25		ORIGINAL	97	168	10.29	25.6			
	THINNING	REMOVAL	27	68	8.75	7.0	$23.05	$142.37	$33.17
		RESIDUAL	70	100	11.35	18.6			
15		ORIGINAL	82	371	6.37	15.4			
	THINNING	REMOVAL	32	192	5.65	5.8	$9.49	$34.59	$14.43
		RESIDUAL	50	179	7.15	9.6			

Minus planting / site preparation costs= -$101.00

Single rotation net present value= $194.81
Infinite series net present value= $215.79

FORTE Appendix B. Growth and yield equations with parameter values fitted to output from runs of PTAEDA2.

Survival of unthinned stands

$$\log(N_2)=\log(N_0)+(a_1+a_2(\log(N_0))\frac{A_2}{100}$$

Equation form from: Coile and Schumacher (1964)

Stand	Parameter	Estimate	Se	t-value	r^2
unthinned	a_1	2.111513	.219525	9.619	.9963
	a_2	-1.120269	.077598	-14.437	

(In FORTE $A1=a_1$ and $A2=a_2$)

Basal area of stands before initial thinning

$$\log(B_2)=b_1(\log(SI))+\frac{b_2}{A_2}+b_3(\log(N_2))+b_4$$

Equation form from: Coile and Schumacher (1964)

Stand	Parameter	Estimate	Se	t-value	r^2
unthinned	b_1	1.070547	.069975	15.274	.9234
	b_2	0 (not used)			
	b_3	.830926	.025829	32.274	
	b_4	-2.406679	.138638	-17.485	

(In FORTE $B3=b_1$, $B4=b_2$, $B5=b_3$ and $B6=b_4$)

Basal area of unthinned stand after initial thinning age

$$\ln(B_2)=c_1+c_2SI-\left[\frac{A_1}{A_2}\right]\left[c_1+c_2SI-\ln(B_1)\right]$$

Equation form from: Cao et al. (1982)

Stand	Parameter	Estimate	Se	t-value	r^2
unthinned	c_1	5.578588	.094204	59.218	.9986
	c_2	.000656195	.00159235	.412	

(In FORTE C7=c_1 and C8=c_2)

Volume (cubic feet) of unthinned stands

$$\ln(Y)=d_1+\frac{d_2}{A_2}+d_3\frac{H}{A_2}+d_4A_2\ln(N_2)+d_5\ln(B_2)$$

Equation form from: Amateis et al. (1986)

Stand	Parameter	Estimate	Se	t-value	r^2
unthinned	d_1	3.073178	.034964	87.897	.9922
	d_2	-16.034681	.256349	-62.550	
	d_3	.280939	.005152	54.530	
	d_4	.003135831	.000104	30.211	
	d_5	.937307	.007205	130.096	

(In FORTE D9=d_1, D10=d_2, D11=d_3, D12=d_4, D13=d_5)

Height of dominant / codominant trees

$$\ln(H) = \ln(SI) \left\{ \frac{j_1}{A_2} \right\} \left\{ j_2 \; e^{\left\{ j_3 \left(\frac{1}{A_2} - \frac{1}{25} \right) \right\}} \right\}$$

Source: Amateis and Burkhart (1985)

Stand	Parameter	Estimate	
all	j_1	25	(used directly from
	j_2	-.10283	Amateis and Burkhart 1985)
	j_3	-2.1676	

(In FORTE J25=j_1, J26=j_2 and J27=j_3)

Trees remaining following thinning of given basal area

$$N_3 = N_2 \left\{ 1 - \left\{ \frac{B_2 - B_3}{B_2} \right\}^P \right\}$$

Derived from equation forms by: Field et al. (1978)

Stand	Parameter	Estimate	Se	t-value	r^2
BAS.DAT	P	.704596	.001114	632.675	.9996
HI.DAT	P	1.298639	.003131	414.770	.9991

(In FORTE H24=P)

Basal area of thinned stands

$$\ln(B_2)=f_1+f_2SI-\left[\frac{A_1}{A_2}\right]\left[f_1+f_2SI-\ln(B_1)\right]$$

Equation form from: Cao et al. (1982)

Stand	Parameter	Estimate	Se	t-value	r^2
BAS.DAT	f_1	4.569874	.370690	33.340	.9980
	f_2	.016688	.002280	7.318	
HI.DAT	f_1	4.800633	.198273	40.063	.9987
	f_2	.013276	.001994	6.658	

(In FORTE F17=f_1 and F18=f_2)

Survival of thinned stands

$$N_2=\left\{N_1^{k_1}+k_2(A_2^{k_3}-A_1^{k_3})\right\}^{(1/k_1)}$$

Equation form from: Clutter and Jones (1980)

Stand	Parameter	Estimate	Se
BAS.DAT	k_1	-.916609	.208899
	k_2*	3.1933E-7	
	k_3	2.408472	.228293
HI.DAT	k_1	-.828945	.201907
	k_2*	2.1054E-7	
	k_3	2.688604	.215044

* k_2 was estimated from a combination of k_1, k_3 and another variable.

(In FORTE E14=k_1, E15=k_2 and E16=k_3)

Volume (cubic feet) of thinned stands

$$\ln(Y)=g_1+\frac{g_2}{A_2}+g_3\left(\frac{H}{A_2}\right)+g_4A_2\ln(N_2)+g_5\ln(B_2)$$

Equation form from: Burkhart et al. (1972)

Stand	Parameter	Estimate	Se	t-value	r^2
BAS.DAT	g_1	3.530895	.054252	65.084	.9913
	g_2	-20.901036	.434947	-48.054	
	g_3	.344410	.009454	36.428	
	g_4	.003109782	.000226	13.787	
	g_5	.858602	.013029	65.899	
HI.DAT	g_1	2.944411	.058186	50.603	.9919
	g_2	-15.132416	.464963	-32.545	
	g_3	.277489	.009048	30.669	
	g_4	.003294184	.000209	15.740	
	g_5	.948115	.011638	81.470	

(In FORTE G19=g_1, G20=g_2, G21=g_3, G22=g_4 and G23=g_5)

FORTE Appendix C. Variables used in the FORTE program

A$ - file storage of past node index
B$ - file storage of before thin index
C$ - file storage of after thin index
D$ - file storage of basal area after thin
E$ - file storage of trees/acre before thin
F$ - file storage of trees/acre after thin
G$ - file storage of value of optimal thinning regime to node
IPASTSIZE(J) - number of nodes in year number J
IFASTSIZE(J) - number of nodes in year number J
IIFIRST(J) - location of first node in year J
IPAST(K) - past node index
IGROW(K) - before thin node index
ITHIN(K) - after thin node index
SURV(K) - before thin trees/acre
SURVB(K) - after thin trees/acre
BA(K) - basal area after thin
OPTVAL(K) - value of optimal thins to node
KPAST(K) - storage of ITHIN used for next years IPAST
SURVPAST(K) - storage of SURVB for use in next year
BAPAST(K) - storage of BA for use in next year
PASTVAL(K) - storage of OPTVAL for use in next year
XX - inverse log of 10
REG - trees/acre planted
SI - site index (base age 25 years)
SI - site index (base age 25 years)
MINCUNIT - minimum cunit volume removal for allowable thin
MINSURV - minimum surviving trees following a thin
MINBA - minimum residual basal area following a thin
THINPERC - percent value of thin as percent of harvest value
THINFIXED - cost occuring every thin
YR - year
INDEX1 - past node index
INDEX2 - before thin index
INDEX3 - after thin index
SURVEXACT - single precision storage of trees acre before thin
SURVEXACT2 - trees/acre following a thin
BAEXACT - single precision storage of basal area per acre after thin
HARVAL - NPV of harvest
MAXVAL - stores highest NPV for regime
MAXPAST - past node index for highest NPV node
MAXGROW - current node index for highest NPV node
MAXCUNIT - volume (cunits) for highest NPV node
MAXVALUE - value ($/cd) for highest NPV node
MAXBA - basal area for highest NPV node
MAXSURV trees/acre for highest NPV node
MAXSURV - trees/acre for highest NPV node
CUNIT - total cunits/acre
VALUE - value of wood ($/cd)
BA1 - basal area (rounded to 5) before thinning at current node
BA2 - basal area (rounded to 5) after thinning

```
  SURV1 - surviving trees/acre (rounded to 5) before th
  SURV2 - surviving trees/acre (rounded to 5) after thin
  SURV3 - same as SURV1
  SURV4 - same as SURV2
  CUNITREMOVE - cunits removed in a thin
  VALTHIN - value ($/cd) of thin as contribution to NPV
  ILOCATION - storage location for output
  START - first node location for year
  ENDD - last node location for year
  MAXNPV - highest NPV value
  OPTPATH - past node for highest NPV node
  KLOCATION - storage location for input
  OPTVALUE - value of optimal thing for node
  IPASTB - same as IPAST
  IGROWB - same as IGROW
  ITHINB - same as ITHIN
ARR - Alternative rate of return
LOWESTAGE - lowest age at which thinning is allowed
ROTATION - maximum rotation age allowed
INTWIDTH - years between thins
NPVDECLINE - when=1 then continue to ROTATION, + so fill NPV array
MINREMOVE - minimum percent of volume removed
BAINCREM - basal area thinning increment
INTERCEPT - intercept term for a linear value equation ($=intercept
SLOPE - slope term for linear value equation
INTERVALS - number of stages in dynamic program
DIA - diameter of stand at harvest age
DIA1 - diameter of stand before thin
DIA2 - diameter of stand after thin
DIAREMOVE - diameter of removals from thinning
MAXINDEX - node location of highest NPV node
MAXDIA - mean diameter of highest NPV node
VALTHIN2 - ADJUSTED VALUE OF THIN (ADJUSTED FOR % VALUE OF HARVEST)
VALTHIN3 - total value of thin above fixed costs
HEIGHT - Height of dominant/codominant trees
  NPVINF - NPV for infinite series of rotations
  NPVINFLAST - NPV (infinite series) for rotation age minus one
  VALTHIN1 - value ($/cunit) of thin at time of thin
 Z(VAR) - growth and yield function parameters
 XLOC(VAR) - X axis location of Z(VAR)
 YLOC(VAR) - Y axis location of Z(VAR)
 ZZ(VAR) - economic and model variables
 XXLOC(VAR) - X axis location of ZZ(VAR)
 YYLOC(VAR) - Y axis location of ZZ(VAR)
 NUMB - menu selection variable
 A$ - yes/no variable
 FILE$ - temporary file storage
 VAR - variable/parameter counter
 STOREFILE$ - temporary function parameter file storage name
 WRITOUT - output flag
 OUTFILE$ - temporary output file storage name
 TITLE$ - title of program run
 PRINTFILE$ - temporary printing file storage name
```

FORTE Appendix D. Files used in FORTE

BAS.DAT ----- Contains growth and yield parameters for loblolly pine plantations. Derived from multiple runs of a revised version (PTAEDA2) of the individual tree growth simulator PTAEDA (Daniels and Burkhart 1972). Low thinning.

HI.DAT ------ Same as BAS.DAT but high thinning.

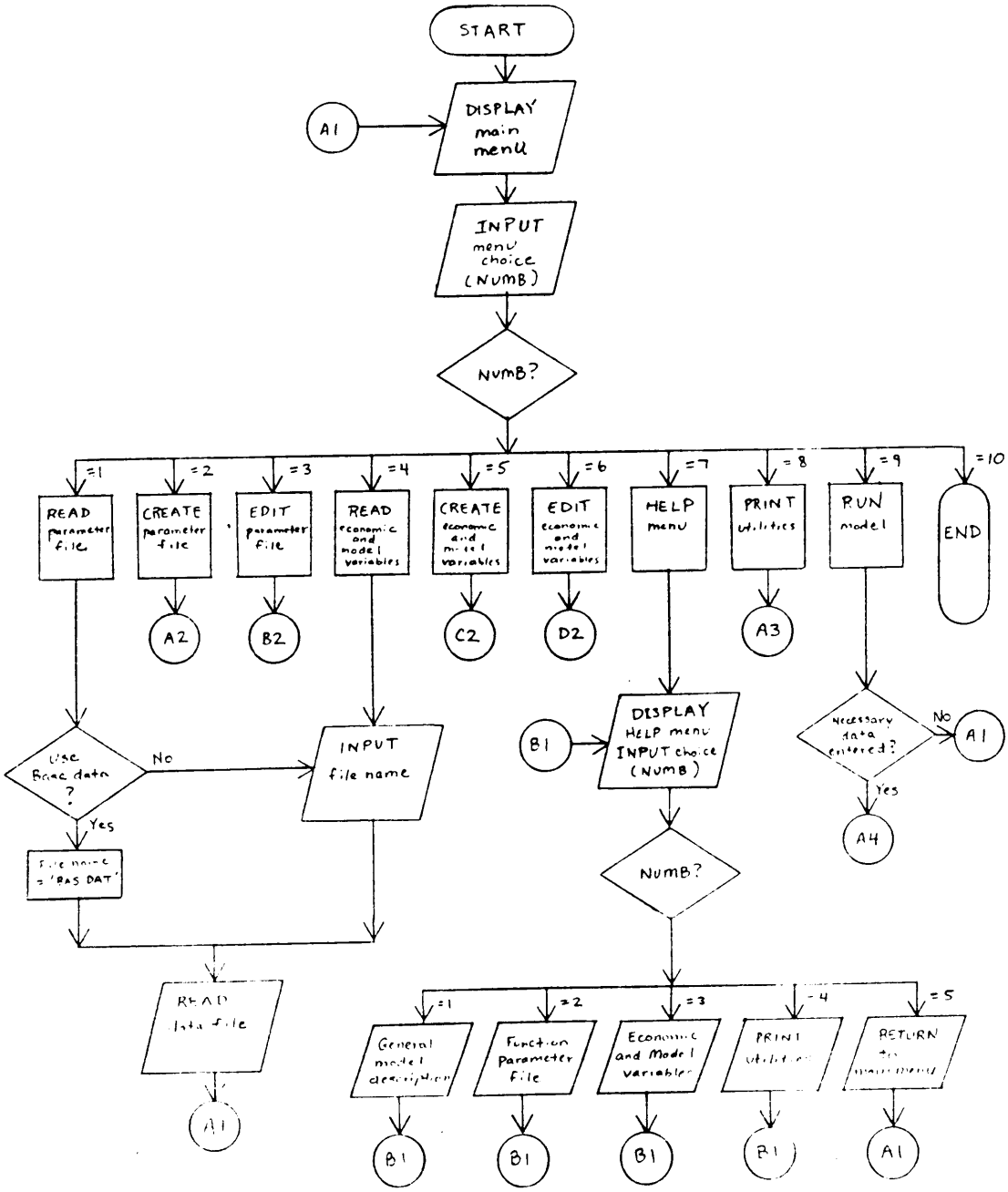
STOREFILE$ -- User defined file of BAS.DAT type.

OUTFILE$ ---- User defined file storing program output sequentially on a line-by-line basis.

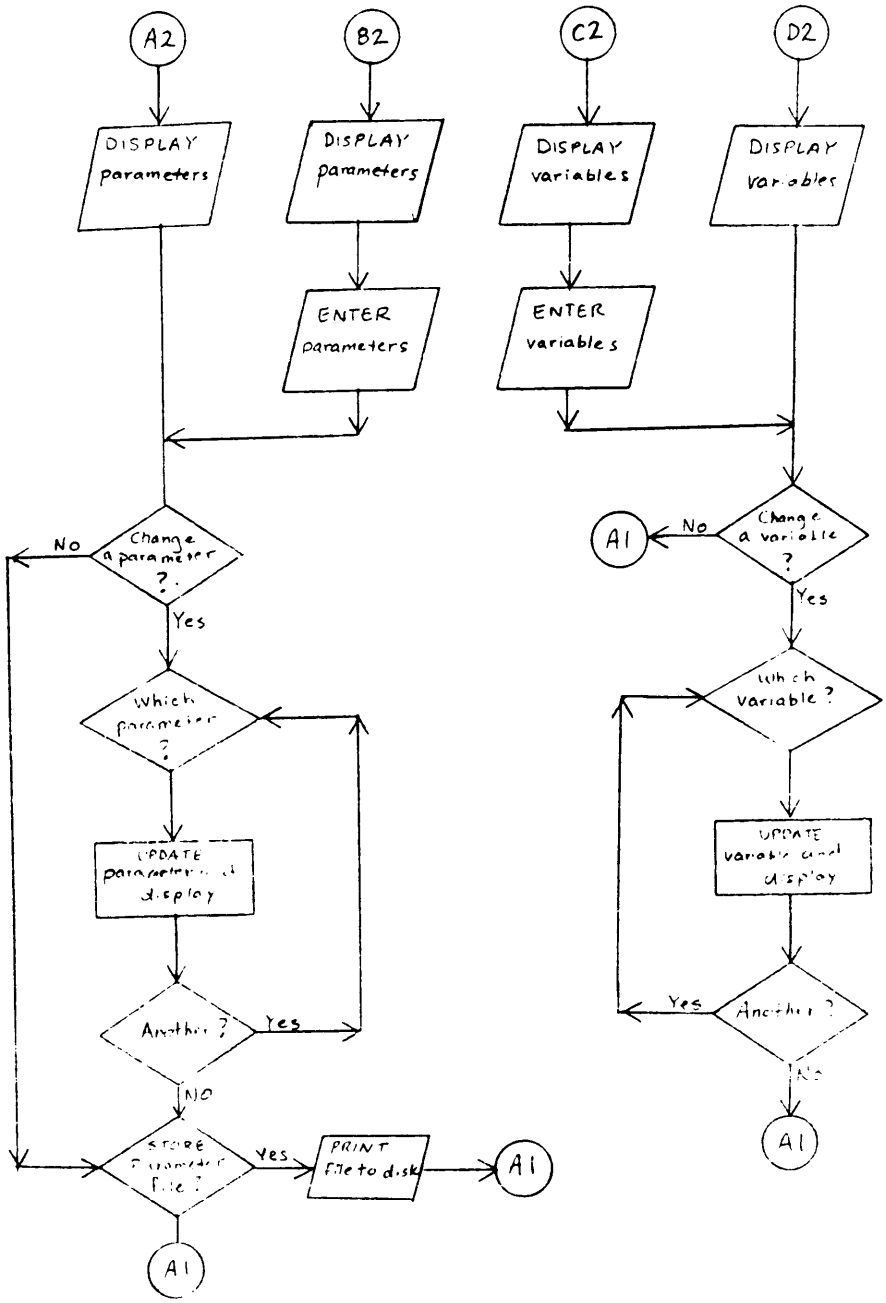
F5.DAT ------ Input/Output random access file used by program to store optimal paths, stand attributes, and values. Field has length of 22 bytes:

3-2 byte var's:	Past node Current node before thinning Current node after thinning
4-4 byte var's:	Basal area before thinning Trees/acre before thinning Trees/acre after thinning Accumulated NPV of thinnings

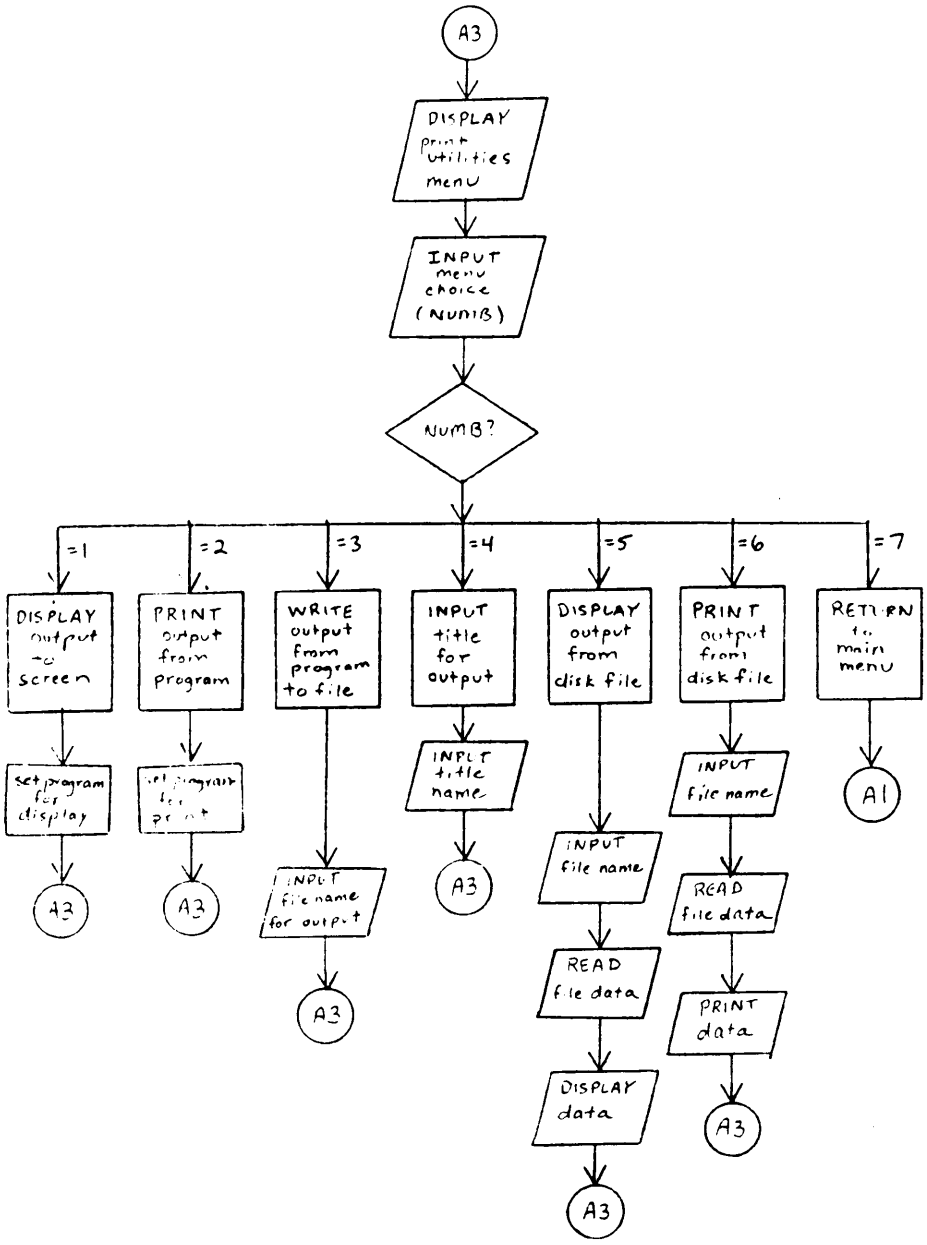
APPENDIX E. FLOWCHART OF THE FORTE PROGRAM

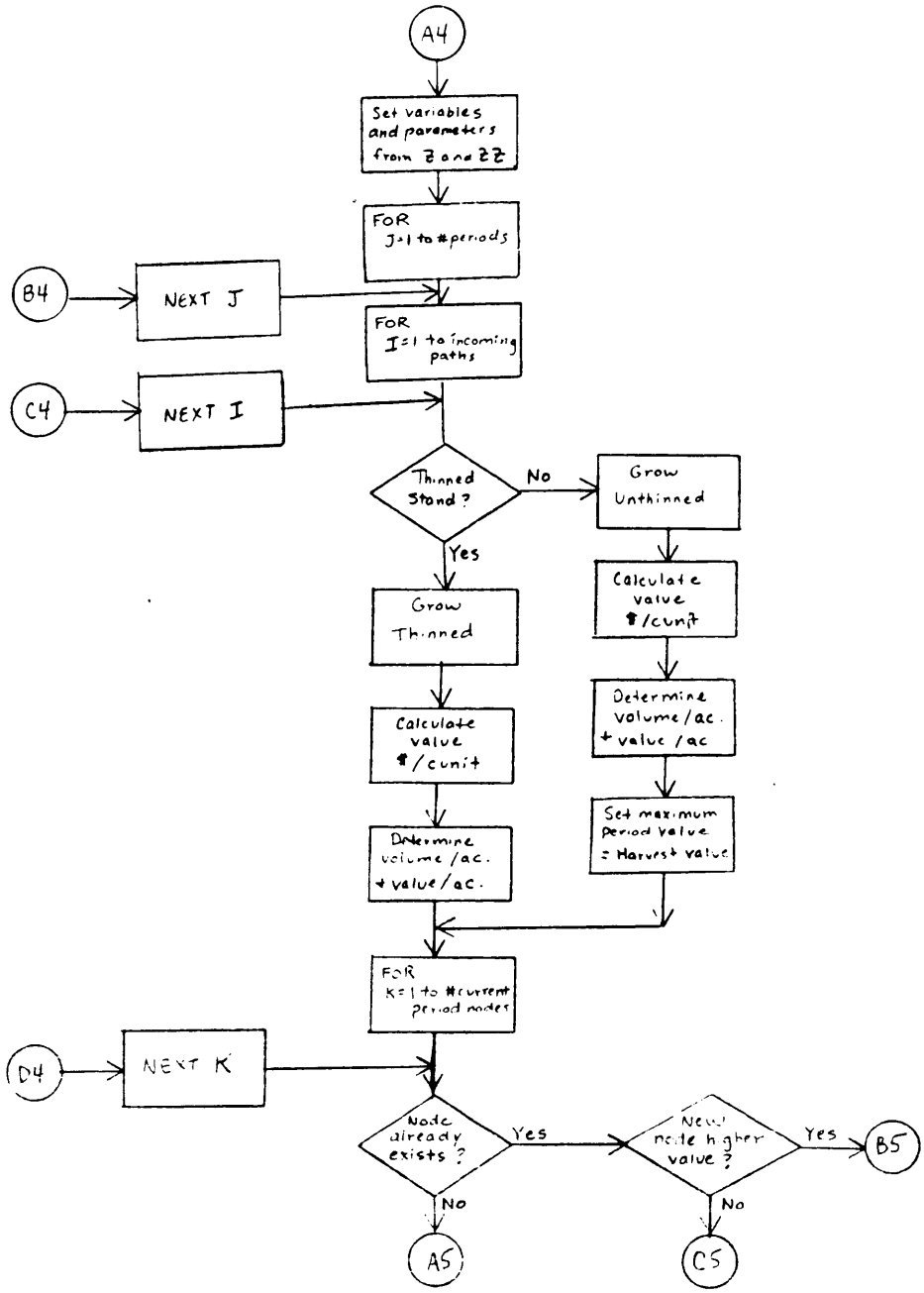


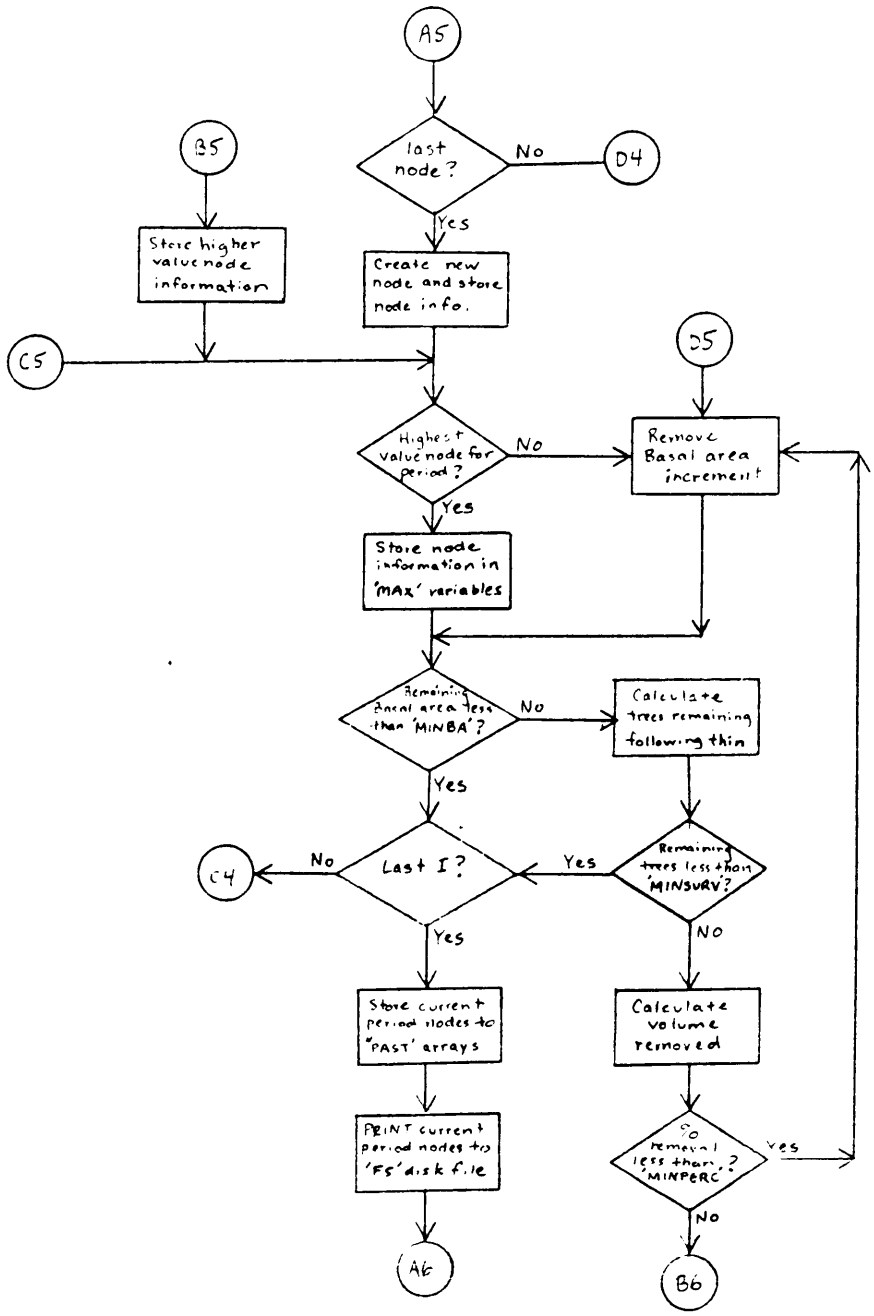
Appendix E. Flowchart of the FORTE program

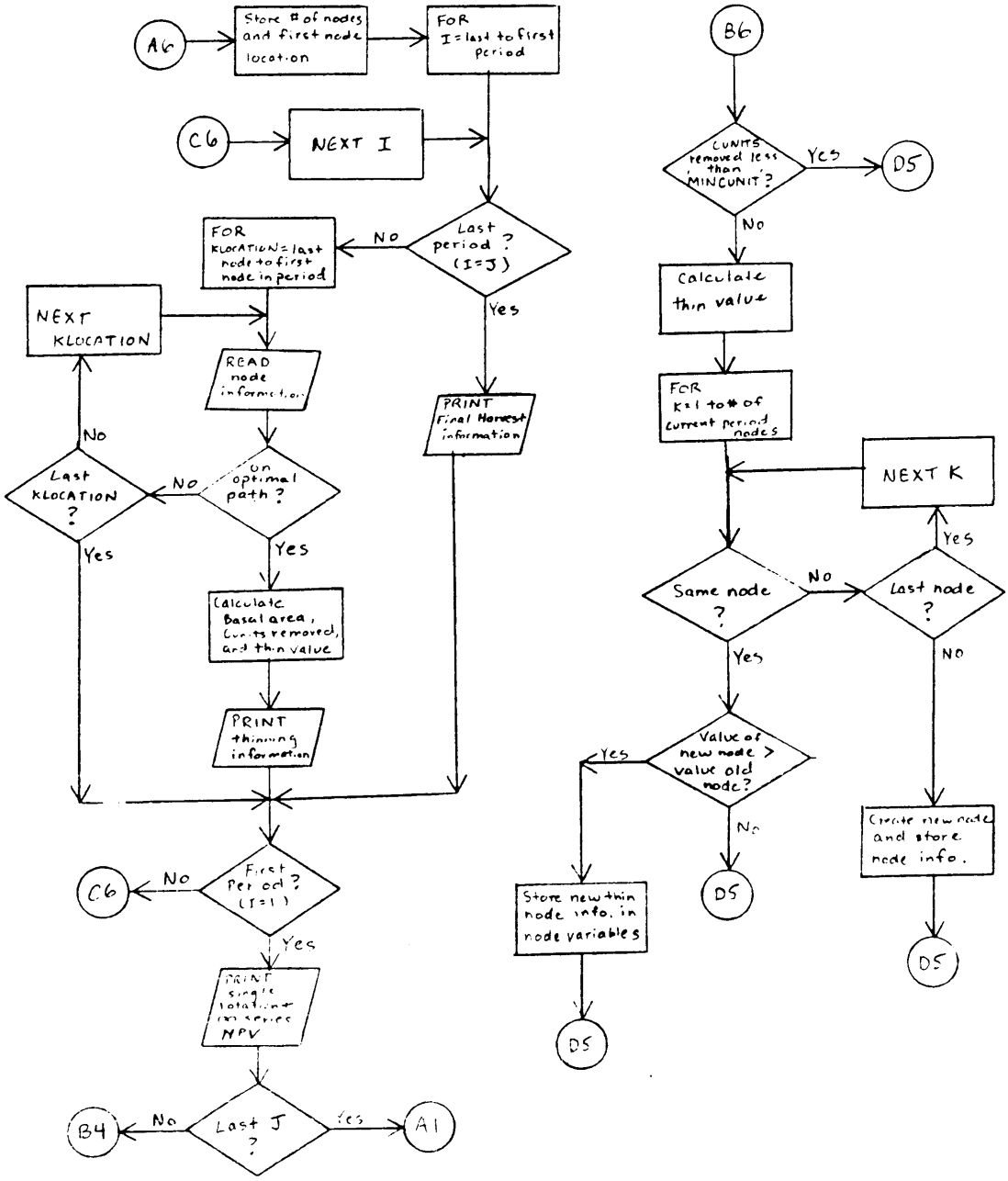


Appendix E. Flowchart of the FORTE program


A3
DISPLAY print utilities menu
INPUT menu choice (NUMB)
NUMB?
=1
=2
=3
=4
=5
=6
=7
DISPLAY output to screen
PRINT output from program
WRITE output from program to file
INPUT title for output
DISPLAY output from disk file
PRINT output from disk file
RETURN to main menu
set program for display
INPUT title name
INPUT file name
INPUT file name for output
INPUT file name
READ file data
READ file data
PRINT data
DISPLAY data
A3
A3
A3
A3
A3
A3
A1


A4
Set variables and parameters from Z and ZZ
FOR J=1 to #periods
B4
NEXT J
FOR I=1 to incoming paths
C4
NEXT I
Thinned Stand ?
No
Grow Unthinned
Yes
Calculate value $/cunit
Grow Thinned
Determine volume/ac. + value/ac
Calculate value $/cunit
Set maximum period value = Harvest value
Determine volume/ac. + value/ac.
FOR K=1 to #current period nodes
D4
NEXT K
Node already exists ?
Yes
New node higher value ?
Yes
B5
No
No
A5
C5


A5
last node?
No
D4
Yes
B5
Store higher value node information
Create new node and store node info.
C5
D5
Highest value node for period?
No
Remove Basal area increment
Yes
Store node information in 'MAX' variables
Remaining Basal area less than 'MINBA'?
No
Calculate trees remaining following thin
Yes
C4
No
Last I?
Yes
Remaining trees less than 'MINSURV'?
Yes
NO
Store current period nodes to 'PAST' arrays
Calculate volume removed
PRINT current period nodes to 'FS' disk file
% removal less than 'MINPERC'?
Yes
No
A6
B6



Appendix E. Flowchart of the FORTE program

The vita has been removed from the scanned document