REYNOLDS STRESS MEASUREMENTS

DOWNSTREAM OF A TURBINE CASCADE

by

Damon M. Shaffer

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MASTER OF SCIENCE in Mechanical Engineering

APPROVED:

J. Moore, Chairman

W. F. Ng

W. F. O'Brien

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An experimental investigation was performed to measure Reynolds stresses in the turbulent flow downstream of a large-scale linear turbine cascade. A rotatable X-wire hot-wire probe that allows redundant data to be taken with solution for mean velocities and turbulence quantities by least-squares fitting procedures was developed. This measurement technique was verified in a fully-developed turbulent pipe flow; the results show the accuracy of the probe when used in an end-flow orientation at various incidence angles and with a multiple number of angular settings.

Traverses with a single hot-wire at mid-span near the blade row exit show very high levels of turbulence locally in the blade wake near the trailing edge which quickly lessen in magnitude downstream. The rotatable X-wire was used to obtain the Reynolds stresses on a measurement plane located 10% of an axial chord downstream of the trailing edge. Here the turbulence kinetic energy exhibits a distribution resembling the contours of total pressure loss obtained previously, but is highest in the blade wake where losses are relatively low. The turbulent shear stresses obtained are consistent in sign and magnitude with the gradients of mean velocity. The mass-averaged turbulence kinetic energy accounts for 21% of the total pressure loss at this measurement plane.
II. ACKNOWLEDGMENTS

I am most indebted to my major professor, Dr. John Moore, for all the guidance he provided throughout my research. He served not only as an advisor, but also as a good friend who was always present when help was needed. Without him, this thesis simply would not have been possible.

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a  wire linearity exponent, Eq. (10)
pipe radius, Fig. 29

(AK,N1,N2)  wire coordinate directions, Fig. C1

b  blade axial chord, Fig. 2

C_{pt}  total pressure loss coefficient, Eq. (6)

d  wire yaw constant, Eq. (9)

D/Dt  substantial derivative (\( \frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \))

E,e  mean and fluctuating wire voltage

H_{12}  shape factor (\( \delta^*/\delta \)), Table 4

(i,j,k)  flow direction unit vectors, Figs. B1, C1

k^2  wire yaw constant, Eq. (9)

P,p  mean and fluctuating static pressure

Q,q  mean and fluctuating effective cooling velocity, Eqs. (9,10)

Q_o  velocity magnitude, Eq. (9)

Q_{ref}  velocity magnitude at reference position used to set wire voltage

(r,\( \theta \),z)  pipe coordinate directions, Fig. E1

q^2/2  turbulence kinetic energy, Eq. (3)

r'  measurement coordinate in pipe (\( \equiv a-r \)), Fig. 29

T  rms turbulence intensity, Eq. (2)

U,u  mean and fluctuating primary velocity components, Fig. 3

U_{fs}  local freestream velocity, Fig. 36

U_{max}  velocity at pipe center

U_o  upstream freestream velocity

\( \frac{U}{U_o} \)  shear velocity (\( \equiv \sqrt{\tau_o/\rho} \)) in pipe, Eq. (E3)

\( \overline{u^2,v^2,w^2} \)  Reynolds normal stresses
NOMENCLATURE (continued)

\( \overline{uv}, \overline{uw}, \overline{vw} \) Reynolds shear stresses

\( V, v \) mean and fluctuating lateral velocity components, Fig. 3

\( V_n \) velocity component normal to measuring plane, axial velocity, Figs. 3, 4

\( W, w \) mean and fluctuating spanwise velocity components, Fig. 3

\( (x, y, z) \) cascade coordinates, Fig. 2

\( x_i \) coordinate directions, Cartesian tensor notation

\( y' \) single-wire measurement location, Fig. 5

\( \Delta y \) blade pitch

\( \Delta z \) blade span

\( \alpha_o, \alpha_w \) wire angles, vertical-stemmed probe, Fig. 27

\( \alpha'_o, \alpha'_w \) wire angles, horizontal-stemmed probe, Fig. 27

\( \alpha_r, \alpha_p \) flow angles relative to fixed coordinates, Fig. B1

\( \beta_1, \beta_2 \) flow angles relative to blade, Fig. 2

\( \delta_{0.99} \) boundary layer thickness, Table 4

\( \delta^* \) displacement thickness of boundary layer, Table 4

\( \theta \) momentum thickness of boundary layer, Table 4

\( \mu \) viscosity

\( \rho \) density

\( \tau_o \) wall shear stress in pipe, Fig. E1

\( \phi \) wire yaw angle, Eq. (9), Fig. B1

\( \phi_{cone} \) cone angle, three-dimensional incidence angle, Eq. (14)

Subscripts

\( A, B \) wire designation

\( o \) upstream of cascade in freestream
NOMENCLATURE (continued)

\( t \)  
\( \text{total, stagnation} \)

\( i,j \)  
\( \text{coordinate directions, Cartesian tensor notation} \)
\( \text{wire, position, Appendix B} \)

**Overbars**

\( \overline{\cdot} \)  
\( \text{time-averaged} (\overline{1} = \frac{1}{\Delta T} \int_{0}^{\Delta T} dt) \)

\( \overline{=} \)  
\( \text{mass-averaged, Eq. (7)} \)

**Abbreviations**

\( \text{rms} \)  
\( \text{root mean square} \)
VII. INTRODUCTION

Aircraft gas turbine engines have been continuously improved by increasing component efficiencies. Traditionally, the state of the aerodynamic design of the turbine section of the engine has been considered adequate, and it is only recently that attention has been focused toward improving turbine efficiencies.

Better blade designs resulting from understanding the flow in blade rows is one way that efficiencies have been increased. Blade cascades are often used to model the flow in turbine blade rows and in conducting flow studies, and an important part of studying cascade flows is to locate the regions of the flow where losses are generated. Studies in turbine cascade flows have shown that the region downstream of the blade row exit is responsible for a significant portion of the losses generated through the blade row. Questions still remain regarding the mechanisms that create the losses and in what form, if any, the losses appear before being dissipated.

One possible explanation is turbulence, since flows in turbine blade rows are invariably turbulent. The fact that a flow is turbulent implies that there is energy contained in turbulent motion of the fluid. This 'turbulence kinetic energy' itself may be a form that the losses take before dissipation. The turbulent stresses may also be a contributing factor in the loss production through shear work and the decay of mean secondary velocities. However, the levels of turbulence existing in turbine cascades are not in general known from the loss studies that have been conducted.
This investigation has set out to measure the turbulence kinetic energy and turbulent stresses downstream of a turbine blade cascade whose losses have already been measured. A new rotatable hot-wire (X) probe has been designed to measure the turbulence in the highly three-dimensional flow known to exist in this flow. It is hoped that by measuring the turbulence quantities a better understanding of the loss mechanisms taking place in this cascade can be obtained.
A. Reynolds Stresses and Their Importance

Here the Reynolds stresses and other turbulence quantities are introduced and their importance to the loss-producing energy transfer that takes place in a turbulent three-dimensional flow is discussed. The reader is referred to a book on turbulent flow such as Hinze [1] or White [2] for a complete discussion and derivations of the equations that follow.

In turbulence analysis, it is customary to divide an instantaneous quantity into a time-averaged (mean) and a fluctuating component. In this thesis, upper-case symbols are used for time-average quantities, and lower-case for fluctuating quantities. For example, the instantaneous velocity, $Q(t)$, can be written as $Q(t) = Q + q$. A single overbar is used to denote the process of time-averaging: $\overline{Q(t)} = Q$.

When the incompressible equations of motion are expanded with instantaneous quantities and then time-averaged, the common time-averaged turbulent equations of motion result. In Cartesian tensor notation:

$$\rho \left( \frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} \right) = - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial U_i}{\partial x_j} - \rho \overline{u_i u_j} \right) \overline{F_i} \quad (1)$$

Where $U_i + u_i$ are the instantaneous velocities in the three coordinate directions $x_i$, $P$ is the static pressure, and $F_i$ are the body forces in
The quantities $- \rho \overline{u_i u_j}$ are called the Reynolds stresses*. They are not actually stresses, but terms that result from time-averaging the expanded convection terms in the equations of motion. They are often combined with the viscous stress terms as in (1), resulting in the name stresses. For $i = j$, there are three normal stresses and for $i \neq j$, three shear stresses.

Two quantities are often used to quantify turbulence levels. They are combinations of the Reynolds normal stresses and are defined as (in orthogonal coordinate notation):

the rms turbulence intensity

$$T = \sqrt{\frac{1}{3} (u^2 + v^2 + w^2)}$$  \hspace{1cm} (2)

and turbulence kinetic energy

$$\frac{q^2}{2} = \frac{1}{2} (u^2 + v^2 + w^2) = \frac{3}{2} T^2.$$  \hspace{1cm} (3)

Multiplying equations (1) by $U_1$ and adding the results gives the transport equation for mean kinetic energy $\frac{1}{2} U_1 U_1$:

---

*For simplicity, the quantities $\overline{u_i u_j}$ are referred to as the Reynolds stresses in this thesis.
where constant viscosity $\mu$ is assumed. The first term on the right is the turbulence production term. As Hinze points out ([1], p. 72), when $-\rho \overline{u_i u_j}$ is a shear stress ($i \neq j$), it usually (but not always) has the same sign as the associated mean velocity gradient, $\partial u_i / \partial x_j$. Hence, the turbulent shear stresses contribute to a decrease in mean kinetic energy. To relate this to the level of turbulence, consider the equation for the transport of turbulence kinetic energy:

$$
\frac{\partial}{\partial t} \left( \frac{1}{2} \overline{u_i u_j} \right) + \frac{\partial}{\partial x_k} \left( p + \frac{1}{2} \rho \overline{u_i u_j} \right) =
- \left( -\rho \overline{u_i u_j} \right) \frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j} \left( -\rho \overline{u_i u_j} u_i \right) +
\mu \frac{\partial}{\partial x_k} \left[ u_j \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] - \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j}.
$$

(4)

The same turbulence production term appears as in (4), but with opposite sign. Hence there is an equal increase in turbulence kinetic energy as there is a decrease in mean kinetic energy.

To relate the above energy transfer to loss production in a turbine flow, consider the following simplified description. The useful
mechanical energy in a flow can be quantified by the total pressure \( P_t = P + \frac{1}{2} \rho U_i U_i \). In a turbine, the flow is accelerated and a large portion of the total pressure appears as mean kinetic energy which is vulnerable to the parasitic action of turbulent stresses. The turbulent shear stresses combined with the large mean velocity gradients typically found in turbine flows result in a conversion of mean kinetic energy into turbulence kinetic energy. The turbulence kinetic energy is then converted into internal energy of the fluid (heat) by viscous dissipation and not recovered as total pressure. The result is a loss in total pressure.

B. Flow Development in Linear Turbine Cascades

1) Overview of Mechanisms

A three-dimensional flow can be described as a primary flow (usually defined as the inviscid potential flow), and secondary flow defined as the deviation from the primary flow due to viscous effects. The secondary flow that develops in turbine blade cascades is dominated by two general effects:

1) A passage vortex forms from the generation of streamwise vorticity brought about by turning a flow initially containing normal vorticity, such as the endwall boundary layer. This effect is enhanced by the migration of the endwall boundary layer to the suction surface: the free-stream pressure, decreasing toward the suction side is imposed on the slower-moving boundary layer fluid, forcing it to the suction side of the passage.
2) A horseshoe vortex forms from the interaction of the endwall boundary layer with the blade leading edge. Two legs are formed, one each on the pressure and suction sides of the leading edge. The pressure-side leg rotates in the same direction as the passage vortex, while the suction-side leg rotates in the opposite sense.

The basic flow mechanisms are illustrated in Fig. 1. As the flow proceeds through the blade passage, the pressure-side leg of the horseshoe vortex crosses to the suction side of the passage and joins with the suction-side leg in the formation of the passage vortex. The endwall boundary layer fluid is swept down the suction side of the blade and into the passage vortex, feeding the passage vortex with high total-pressure-loss fluid and causing the growth of a new, thin endwall boundary layer. At the blade exit, vorticity shed from the blade trailing edge contributes to the mixing of fluid from the two sides of the blade in the blade wake.

The above mechanisms are discussed in greater detail in the following section on the flow in the VPI&SU cascade.

ii) VPI&SU Cascade

The cascade used in the present investigation is the same one used by previous investigators Moore and Ransmayr [3], Moore and Smith [4], and Moore and Adhye [5]. The cascade geometry and nomenclature are introduced first, followed by a summary of their results.
Fig. 1. Secondary Flow Mechanisms in a Turbine Cascade
a) **Description of the Cascade**

The VPI&SU cascade contains five large-scale linear airfoils in the shape of reaction turbine blades. The middle three blades along with dimensions are shown in Fig. 2. The cascade and the blade profiles are geometrically similar to those used by Langston, et al. [6]; the cascade geometry is summarized in Table 1. This cascade turns the flow through $110^\circ$ while accelerating it with a velocity ratio of about 1.6. The Reynolds number based on an exit velocity of 37.5 m/s and axial chord is $5.2 \times 10^5$. The blade boundary layers are tripped turbulent near the leading edges of the suction and pressure surfaces.

<table>
<thead>
<tr>
<th>Blade Geometry</th>
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<tbody>
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<tr>
<td>Span</td>
<td>234.4 mm</td>
</tr>
<tr>
<td>Pitch</td>
<td>224.8 mm</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>0.997 (span/axial chord)</td>
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<th>Airfoil Mean-Camber-Line Angles</th>
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<tr>
<td>$\beta_2$</td>
<td>25.98°</td>
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<th>Inlet Flow Velocity</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$U_0$</td>
<td>23.5 m/s</td>
</tr>
</tbody>
</table>
Fig. 2. Geometry and Nomenclature of the VPI&SU Cascade
The measuring planes referred to hereafter are also shown in Fig. 2 with their location as a fraction of axial chord, x/c. The cascade coordinate system used to identify measurement location is shown with the (x,y,z) directions corresponding to the axial, pitchwise, and spanwise directions, respectively. Spanwise location, z, is measured from the bottom endwall.

The velocity components used do not coincide with the cascade coordinates. They are shown in Fig. 3. The primary flow direction (U,u) is in the direction of the camberline at the blade trailing edges, 26° to the pitchwise direction. The secondary velocities are contained in a cross-sectional plane. The lateral component (V,v) is defined orthogonal to the primary (U,u) and spanwise directions (W,w). The secondary flow angle is then defined as the angle between the total flow direction and the primary flow direction and is the same as the cone angle defined in Eq. (14), Section X-D-ii. The geometry is seen more clearly in Fig. 4 which shows the normal velocity component, V_n, coinciding with the axial direction. The normal velocity is used primarily in evaluating mass fluxes for mass-averaged flow properties.

The blade row losses are quantified by considering the loss in total pressure. The (local) total pressure loss coefficient, C_{pt}, is defined by:

\[
C_{pt} = \frac{P_{to} - P_t}{\frac{1}{2} \rho U_o^2}
\]

where \(P_t\) is the local total pressure, \(P_{to}\) is the total pressure in the upstream freestream flow, and \(U_o\) is the upstream freestream velocity.
Fig. 3. Velocity Components at Cascade Exit
Fig. 4. Velocity Components Relative to Measuring Plane
At each plane, the mass-averaged total pressure loss coefficient, $\overline{C_{pt}}$, is defined by:

$$
\overline{C_{pt}} = \frac{\int_{\Delta y} \int_{\Delta z/2} \rho V_n C_{pt} \, dz \, dy}{\int_{\Delta y} \int_{\Delta z/2} \rho V_n \, dz \, dy}
$$

where $\Delta y$ is the blade pitch and $\Delta z$ is the blade span. Mass-averaged values of other properties are similarly defined. Measurements are taken in the upper half-passage, assuming a symmetric flow in the lower half.

b) Previous Findings

Preliminary measurements in this cascade were performed by Moore and Ransmayr. They measured a rather thick upstream boundary layer with displacement thickness 2.1 to 2.4 per cent of span. They collected data on downstream plane 4 ($x/c = 1.40$) using a five-hole probe and found that for the mass-averaged total pressure loss coefficient ($\overline{C_{pt}}$) of 0.38, 58.6% was convected in a high-loss core near mid-span, as shown in Fig. 5. This loss distribution was seen not to change significantly when the basic round leading edges were replaced by wedge-shaped ones; the basic profile was used in subsequent investigations.

Moore and Smith then used ethylene gas injection with flame-ionization detection to study flow development. Of the ethylene gas injected in the horseshoe vortex near the blade inlet, over 65% was found in the high-loss core near mid-span at the downstream plane 4. At
Fig. 5 Reproduction of Moore [7] Fig. 6 Obtained by Moore and Ransmayr Defining Loss Regions at Plane 4 (x/c = 1.40).
the blade exit, fluid from the pressure-side leg was found throughout
the passage vortex; whereas, suction-leg fluid was observed to be
convected around the passage vortex. This observation led to the
conclusion that the suction-side leg wraps itself around the pressure-
side leg after the latter crosses to the suction side of the passage.
The paths of the two legs of the horseshoe vortex are shown in Fig. 6;
these observations are consistent with the qualitative flow picture in
Fig. 1.

Moore and Adhye conducted an extensive study of flow development
and losses on plane 1 just upstream of the blade exit, and on planes 2,
3, and 4 downstream of the blade row exit. Vector plots of their
measured secondary velocities projected in the cross-sectional plane
(Fig. 3) are shown for planes 1, 2, and 4 in Figs. 7, 8, and 9, respec-
tively. The flow is dominated by a single, large passage vortex which
is seen to migrate towards mid-span and away from the suction surface
while it decays in strength from \( x/c = 0.96 \) to 1.4. Initially, the
vortex is asymmetric with very large negative spanwise velocities near
the suction surface, but it becomes more symmetric as it weakens
downstream. Maximum secondary flow angles decrease from about 40° at
plane 1 to about 35° and 16° at planes 2 and 4, respectively.

Plots of the corresponding total pressure loss coefficient \( (C_{pt}) \)
distribution for these planes are shown in Figs. 10, 11, and 12. They
show an endwall boundary layer that grows downstream but remains
relatively thin. There is a high-loss core nearly coincident with the
center of the passage vortex in which the peak local \( C_{pt} \) grows from 1.1
to 1.3. At plane 1, a high-loss region exists at mid-span near the
Fig. 6. Reproduction of Moore and Smith Fig. 3: Likely Path of Horseshoe Vortex Obtained with Ethylene Injection
Fig. 7. Reproduction of Moore and Adhye Fig. 4: Secondary Velocity Vectors at Plane 1 (x/c = 0.96)
Fig. 8. Reproduction of Moore and Adhye Fig. 9: Secondary Flow Vectors at Plane 2 (x/c =1.10)
Fig. 9. Reproduction of Moore and Adhye Fig. 10: Secondary Flow Vectors at Plane 4 (x/c = 1.40) Showing Migration of Passage Vortex Core from Planes 1 to 4.
Fig. 10. Reproduction of Moore and Adhaye Fig. 3: Contours of Cpt at Plane L (x/c = 0.96)
Fig. 11. Reproduction of Moore and Adhye Fig. 7: Contours of $C_{pt}$ at Plane 2 ($x/c = 1.10$)
Fig. 12. Reproduction of Moore and Adhye, Fig. 8. Contours of $C_p$ at Plane 4 ($x/c = 1.40$)
suction surface, but this becomes mixed out as part of the passage vortex by plane 4. The blade wake between the passage vortex and the endwall is seen to exhibit relatively low losses. There is a low-loss region between the passage vortex and the wake of the adjacent blade where $C_{pt}$ is typically less than 0.1. The low-loss region and blade wake are seen to become skewed by plane 4.

The variation of the mass-averaged flow properties for the planes downstream of the trailing edge is shown in Fig. 13. The mass-averaged total pressure loss grows downstream accompanied by a corresponding decrease in secondary kinetic energy coefficient $(\overline{v^2 + w^2}/U_0^2)$. The sum of $C_{pt}$ and the secondary kinetic energy coefficient remains nearly constant, suggesting that the increase in total pressure loss can be nearly totally accounted for as an irrecoverable decay of secondary kinetic energy. A downstream mixing analysis gave results which suggested a reversible primary flow and an irreversible secondary flow.

Thus, the general features of the flow in this cascade were known prior to the present investigation. The basic flow patterns as well as the magnitude and location of losses had already been measured. This knowledge proved to be a valuable asset in the experimental work that followed.

c) Role of the Present Investigation

The present work set out to determine the role of turbulence in the flow downstream of the VPI&SU cascade. In particular, the answers to two questions were sought:

1) What portion of the total pressure losses generated in the
Fig. 13. Reproduction of Moore and Adhye Fig. 12: Variation of Mass-Averaged Flow Properties Downstream of Trailing Edge
cascade appears as an increase in turbulence kinetic energy, to be dissipated by viscous action?

2) Could the rapid decay of secondary velocities, especially the spanwise component, downstream of the blades be due to the action of turbulent shear stresses?

Obviously, before these questions could be addressed properly, it was first necessary to measure the turbulent stresses and turbulence kinetic energy downstream of the cascade. Hot-wire anemometry was chosen as the technique to be used.

C. Turbulence Measurements in Turbine Cascades

Although no comprehensive study of turbulence levels in turbine cascades has been made, several authors have included turbulence measurements as part of more general loss studies.

Bailey [8] measured turbulent stresses in a single turbine vane passage including headboards, bleeds along the blade leading edges, and tailboards. He used hot-wire anemometry and laser-Doppler velocimetry. His passage had a low core-flow turbulence kinetic energy of about 0.015% of the free-stream velocity head (0.7% rms turbulence intensity). Just upstream of the blade exit, he found a maximum turbulence kinetic energy (based on inlet velocity head) of about 3.5%, as estimated from his Fig. 13. The maximum occurred within the end wall boundary layer 5 mm from the end wall (1% of span). At the exit, his flow had a passage vortex centered midway between the blades and 12-15 mm from the end wall (2.4-3% of span), extending to 30-40 mm from the end wall (5.9-7.9% of span). The turbulence kinetic energy within the
passage vortex was of a comparable magnitude (about 1.2% of the inlet free-stream velocity head) to that found in the boundary layer at the blade inlet (0.1-3%). The turbulence decreased to core-flow levels within 20 mm of the endwall, leading Bailey to conclude that the turbulent stresses were insignificant in large regions of the passage vortex. However, leading-edge vortices and trailing-edge mixing were absent from his single-vane flow, which may be the reason for the small changes in turbulence kinetic energy through the passage.

Gregory-Smith and Graves [9] performed hot-wire turbulence measurements just downstream of a six-blade turbine cascade. Their cascade, shown in Fig. 14, is seen to be very similar to the present one (see Section B-iia), providing an opportunity for comparison with present findings. Both blade rows turn the flow through 110° and have similar inlet boundary layer displacement thickness relative to span (δ*/span = 2.7% vs. 2.3%). Gregory-Smith and Graves' slot 8 (x/c = 1.03) and plane 2 (x/c = 1.10) of the VPI&SU cascade were the respective measurement planes.

Gregory-Smith and Graves observed considerably higher levels of turbulence than did Bailey. Their inlet free-stream turbulence kinetic energy was significantly higher at 0.27% of velocity head (3% rms turbulence intensity). Peak values as high as 90% were observed in the passage vortex at slot 8, not an insignificant level as found by Bailey. Overall, the turbulence kinetic energy at slot 8 could account for as much as 75% of the blade row losses.

Gregory-Smith and Graves found the turbulence distribution at slot 8 to conform closely to their measured total pressure losses. Fig. 15
Fig. 14. Geometry and Nomenclature of Gregory-Smith and Graves' Cascade
Fig. 15. Contours of $\frac{q^2}{U_o^2}$ at Slot 8 with $C_{pt}$ Contours Superimposed and Flow Regions Identified: Combination of Gregory-Smith and Graves Figs. 26 and 28
is a reproduction of their turbulence kinetic energy contours with total pressure loss contours superimposed for comparison. Both quantities are high in the blade wake regions and in the passage vortex. The peak value of turbulence kinetic energy is nearly coincident with the vortex core. Gregory-Smith and Graves observed the turbulence to be "fairly isotropic" at slot 8. However, the upstream turbulence was anisotropic, with the lateral component about twice the spanwise or streamwise components.

Clearly the findings of Bailey and of Gregory-Smith and Graves do not present a unified picture of the turbulence levels in turbine blade cascades. The present work aims to help resolve this dilemma.

D. Hot-Wire Techniques for Turbulence

i) General Considerations

For an orthogonal three-dimensional coordinate system, the Reynolds stress tensor has six components: \( \overline{u^2}, \overline{v^2}, \overline{w^2}, \overline{uv}, \overline{uw}, \) and \( \overline{vw} \) where \( U, V, W \) and \( u, v, w \) represent the mean and fluctuating velocity components in the three coordinate directions, respectively. The Reynolds stress quantities can be related to wire voltages by geometry and the method used to model wire response (e.g. the cosine law). The analysis method adopted here is that of Gorton and Lakshminarayana [10], presented in detail in Appendix C and summarized below.

The method is developed for two wires (A and B), but is equally valid for single-wire systems. For the first wire (A), the instantaneous output voltage \( (E + e)_{A} \) is related to the instantaneous cooling velocity \( (Q + q)_{A} \) by a response law, giving \( (Q + q)_{A} = f(E + e)_{A} \).
Expanding and time-averaging allows the mean-square of the wire voltage, $e_A^2$, obtained from the measured wire rms signal, to be transformed into a corresponding fluctuating cooling velocity, $q_A^2$. Geometry then relates the cooling velocity to the fixed-coordinate velocity components, which after linearization and time-averaging results in a linear expression:

\[
q_A^2 = A_1 \overline{u^2} + A_2 \overline{v^2} + A_3 \overline{w^2} + A_4 \overline{uv} + A_5 \overline{uw} + A_6 \overline{vw}
\]  

(8)

where the coefficients, $A_1$, ..., $A_6$, are functions of the mean velocities, $U, V, W$. The mean velocities can be found from the wire mean voltages (see Appendix B and Forlini [11]) or obtained from another sensing technique such as a pressure probe. If a second wire (B) is available, its mean-square signal, $e_B^2$, and the mean-product $e_A e_B$ of the two wire signals (obtained from combining the rms sum and difference signals) produce a second and third equation in the Reynolds stresses for $q_B^2$ and $q_A q_B$.

It is important to note that an inherent assumption made in the above linearized analysis is that fluctuations are small compared to mean quantities. A further assumption that secondary velocities ($V$ and $W$) are small compared to the primary velocity ($U$) is made. Violation of either of the above assumptions may reduce the accuracy of the procedure or even render it invalid.

It is up to the experimenter to devise a procedure to provide six independent equations which can be solved for the Reynolds stresses. Traditionally, single- and multiple-wire probes are used in various configurations and combinations. Following is a review of the more
common techniques that have been used to measure the Reynolds stresses. Only stationary wire methods are considered, since it was preferred to avoid the complexity of continuously-rotating-wire techniques.

ii) Methods of Measurement

a) Single-Wire Method

This method employs a single slantwire probe which is usually rotated about its axis to produce different wire positions relative to the flow. Each position gives one equation from the wire rms signal, requiring a minimum of six angular settings to solve for all the Reynolds stresses. The method often fails due to the inability to resolve six sufficiently independent equations from closely-spaced wire positions.

Kool [12] employed this method to measure the Reynolds stresses behind a low-speed compressor and reported success. However, he performed a careful sensitivity analysis and found it desirable to take redundant data, solving with a least-squares method. Other investigators have been less successful using the single-wire method.

Wang [13] measured Reynolds stresses in a curved rectangular duct using an X-wire probe. He found that using only wire rms signals produced an inverse matrix with large coefficients, resulting in unacceptable errors in the Reynolds stresses. He found better success by including the mean-product signal of the two wires. His flow was less three-dimensional than the present one, with secondary flow angles never exceeding 20°, compared with the maximum values of about 35°.
expected downstream of the VPI&SU cascade.

Yowakim [14] recently attempted to use a single slantwire to measure Reynolds stresses in an annular flow and obtained unrealistic results. He believes that a maximum of five independent angular positions is the limit for a single slantwire probe. A second probe is then required to obtain the sixth equation.

In short, this method was rejected due to the difficulties reported by past researchers.

b) Stationary Two-Wire (X-Array) Probes

A crossed-wire probe with its sensors in a coordinate plane can be used to determine the two normal stresses and one shear stress associated with that plane, as long as there is no flow normal to the plane. This technique is often used in boundary layer flows where the mean flow direction is well-defined, but in a three-dimensional flow it can be difficult to align an X-array so there is no flow normal to the wire plane.

Menna [15] attempted to use three X-arrays to measure the shear stresses in a boundary layer and could not do so with accuracy. Gregory-Smith and Graves [9], mentioned previously, used two X-arrays in their work, but could not determine the third shear stress. They argued that this stress was not important in their flow and thereby justified not solving for it. In the present investigation, it was hoped to measure all six Reynolds stresses.

A serious practical problem with using multiple X-arrays is the difficulty of reproducibly setting the probes in the same location. It
is also time-consuming to have to switch probes or take data on separate occasions. It was felt that a single probe technique would be best if one could be found.

c) Three-Wire Probes

Typically probes of this type contain three mutually-orthogonal wire sensors (A,B,C). In theory, they are the most general because all six Reynolds stresses and the three mean velocity components can be obtained at once. The three wire mean voltages \(E_A, E_B, E_C\) are used to solve for the mean velocities; the Reynolds stresses are solved from the three wire \(rms\) signals and the three mean-product signals \(\overline{e_A^2}, \overline{e_B^2}, \overline{e_C^2}, \overline{e_A e_B}, \overline{e_A e_C}, \overline{e_B e_C}\). Since the probe does not have to be rotated or repositioned, relatively quick measurements are possible. Gorton and Lakshminarayana [10] used this method to measure turbulent stresses in an axial flow inducer.

There are, however, practical problems associated with three-wire systems, as pointed out by Yowakim. More channels of hot-wire equipment and a more complex and difficult to fabricate probe imply greater cost. Calibration procedures become more complicated and time-consuming. The possibility of one wire failing increases, often requiring a complete rewiring and recalibration, and subsequent delays.

The three wires are in a fixed orientation and of necessity not in an optimum arrangement for the various three-dimensional flow situations to be encountered, and such probes are usually not designed to be rotated or repositioned to account for this. The greater number of wire support prongs increases the likelihood of prong interference effects
being significant, a consideration later found to be important in the present experimental work.

The present investigation had only two channels of hot-wire equipment immediately available and it was decided not to introduce the cost of another channel. A rotatable two-wire probe was designed and used.

d) Rotatable* Two-Wire (X) Probe

The probe used in the present investigation combines the advantages of single- and multiple-wire systems and is illustrated in Fig. 16. It consists of two slantwires separated by a fixed angular distance about the probe axis, and is also designed to be rotated in its support. This is an extension of the rotatable single-wire idea. Probe complexity is reduced over the three-wire probe, an important consideration when wires are to be replaced by hand as in the present work.

For each angular setting of the probe, the two rms signals and the mean-product produce three equations for the Reynolds stresses. Angular settings can be spaced more widely, avoiding the singularity problem encountered in single-wire solution procedures. Since the probe can be rotated to as many angular settings as desired, as much redundant data can be taken as deemed applicable. A least-squares fitting method can then be used to solve the redundant set of equations and increase accuracy. Those positions which exhibit prong interference tend to be of diminished significance by the strengthening effect of the least-

*This probe can be rotated into different stationary angular positions, not to be confused with continuously-rotating probes.
Fig. 16. Simple Figure of Rotatable X-Wire Probe
squares fitting. This probe was used as an end-flow probe, i.e., with the primary flow in the direction of the probe axis. It was anticipated that the effects of prong interference would be minimized by using the probe in this orientation.
IX. EXPERIMENTAL

A. Plan of Investigation

Turbulence measurement were taken on the first downstream plane of the previous investigations. This was plane 2 located at x/c = 1.10 (see Fig. 2). This plane was chosen from the four planes studied by Moore and Adhye since it was anticipated that the turbulent quantities would be highest there due to trailing edge effects.

The upper half of plane 2 was traversed using the two-wire rotatable probe designed for this investigation. Single-wire traverses were done near the endwall where probe blockage effects were expected if the larger two-wire probe were used.

The two-wire probe allowed measurement of the complete set of Reynolds stresses, including turbulence kinetic energy, and all three mean velocity components. The single wire was used only to gain an approximation of the turbulence intensity. One of the VPI SU mainframe IBM computers was used for data analysis and for producing selected graphical plots.

The experimental apparatus and measurement technique are discussed in the following sections.

B. Experimental Equipment

1) Wind Tunnel

The experimental measurements were made in an open-cycle forced-draft wind tunnel shown in Fig. 17. The tunnel rig consisted of the following components:
1. A 20.4 kW blower.

2. Diffuser section.

3. Plenum chamber containing screens, filters, and honeycomb flow straighteners.

4. Contraction to 0.91 m width and 0.3 m height.

5. Inlet duct.

   An adjustable 6.35 mm thick plexiglas roof was used to reduce the height to 234.4 mm, the airfoil span. The remaining walls were made of 19.0 mm plywood with 1.6 mm white formica on the inside surface. One sidewall contained a transparent plexiglas window for observation during testing.

6. Cascade test section.

   ii) Cascade Test Section

   The blade geometry is as previously described in the discussion of the VPI&SU cascade findings. The blade surfaces were made of 3.2 mm (1/8 in.) plexiglas with a 0.5 mm (0.02 in.) diameter trip wire near the leading edge of both surfaces. The leading edge was constructed from a 25.4 mm (1.0 in.) diameter plexiglas tube, and the trailing edge from a 14.3 mm (9/16 in.) diameter aluminum rod. The details of blade fabrication are given by Moore [16].

   The top and bottom endwalls were made of plywood with formica inner surfaces. The top endwall contained four access slots at the measurement planes shown in Fig. 18, which is a plan view of the top endwall. Only the slot for plane 4 (measuring 325 x 9 mm) was left open during testing and was used to insert a probe support. The others were covered
Fig. 18. Plan View of the Top Endwall of the Cascade Test Section
with cellophane tape to reduce flow disturbances. Two 20 mm (25/32 in.)
diameter instrumentation ports were located upstream of the leading
edges and were filled with plexiglas plugs when not in use.

iii) Calibration Pipe

Hot-wire calibration and fully-developed turbulent pipe flow
measurements were done in a calibration pipe adjacent to the wind
tunnel. The pipe is illustrated in Fig. 19 and is described below.

The flow was forced by an open-cycle axial-inflow, radial-outflow
centrifugal blower. The flow area was then contracted from the blower
exit diameter of 130 mm to the pipe inside diameter of 80.0 mm. A
butterfly valve located after the contraction allowed the peak exit
dynamic head to be varied from a maximum 50.8 mm (2.0 in.) H$_2$O to 12.5
mm (0.5 in.) H$_2$O. Screens and filters were placed at the blower exit
and just downstream of the butterfly valve. There was a honeycomb flow
straightener and screen 610 mm downstream of the valve. This was
followed by an uninterrupted straight length of pipe 19.5 m (240
diameters) in length to the pipe exit. A platform to support the probe
holder was built at the pipe exit.

Using the maximum exit dynamic head of 50.8 mm H$_2$O, a velocity of
30.0 m/s at the pipe center is obtained. Assuming a 1/7-power law
velocity profile, the average velocity is 0.82 of this or 24.6 m/s. The
Reynolds number based on diameter and maximum average velocity is then
about 120,000, indicating a flow which is clearly turbulent. The
length-to-diameter ratio of 240 for the straight section assures a
fully-developed turbulent flow (L/D of 40-100 is typical for turbulent
internal flows to reach fully-developed conditions).

Static pressure tappings were located near the pipe exit and 5.79 m (19.0 ft.) upstream of the exit, allowing the pressure drop along the fully-developed section of pipe to be measured.

iv) Hot-wire Probe

The basic components of the X-wire probe are shown in Fig. 20. They include:

1. Probe stem made from 6.35-mm (0.250-in.) outside diameter steel tubing.
2. Plastic insulation to hold the prongs in place and insulate them from the metal stem.
3. Four prongs made from steel rod with base diameter of 1.02 mm (0.040 in.) tapered to a point. The prongs were given a heavy copper coating over the end 8-12 mm which was smoothed and rounded on the end.
4. Wire sensors.

Two hot-wire sensors made of 0.0038-mm (0.00015-in.) diameter tungsten wire. The wires were given a copper coating except for about 1.3-1.6 mm exposed length at the middle. The copper coating shortened the effective length of the wire and helped facilitate soldering to the prongs.

An end view of the prongs showing their location on the probe and relation to each other is given in Fig. 21. Each prong is labeled A or B to designate which wire it supports, and L or S for long or short. Note that the prongs place the two wires so as to be separated by
Fig. 20. Components of X-Wire Probe

not to scale
Fig. 21. End View of X-Wire Probe with Dimensions

Dimensions in mm
nominally 120° about the axis of the probe. Figure 22 is a side view in the plane of one wire showing only that wire and its support. The wire angle, $\alpha$, is labeled and the dimensions along the probe axis are given.

For a nominal wire angle of 35.3°, three wire positions separated by 120° about the probe axis form a mutually-orthogonal set of axes; this probe is designed so that its two wires form two out of three axes of an orthogonal coordinate set. It can be rotated to produce the complete set. In this way it is an extension of a single slantwire system commonly used to measure mean velocities, and it produces all the wire positions of three-wire systems used to measure Reynolds stresses.

The probe was designed and built, and the wires were copper coated and soldered to the probe prongs in the Internal Flow Research Laboratory at VPI&SU. The probe angles and dimensions needed in the experimentation and data analysis to follow were then measured with an optical microscope with rotatable crosshair. The measurements included:

- the wire angle, $\alpha$, for both wires
- locating the probe rotational positions where each wire was vertical and horizontal
- the exposed length of each wire.

The exposed length allowed a wire resistance based on 4900 ohm/m (1500 ohm/ft) to be predicted. If a wire's expected resistance greatly exceeded this, a faulty solder connection with a prong was known to exist and the wire was replaced.
Fig. 22. View in Plane of One Wire Sensor Showing Prongs and Axial Dimensions
v) **Probe Holder and Traversing Apparatus**

The probe was held in a traversing apparatus designed for pitchwise (constant percent of span) traverses with the probe facing into the flow in the direction of the blade trailing edges. This apparatus is illustrated in Fig. 23. The orientation of the probe as it was used in the cascade and its relation to the velocity components is shown in Fig. 24. Spacer blocks were used to position the probe in the spanwise direction; with all the spacers removed, the probe was located at mid-span. The probe was manually positioned pitchwise using a scale drawn on the outer surface of the test section top endwall. A brass probe support was inserted through the slot cut previously for plane 4, helping limit vibrations and further securing the probe. The probe could be rotated about its axis and the angular position measured on a dial graduated to the nearest 0.1°. Fore/aft positioning was accomplished with a lockable sliding adjustment.

This traversing mechanism was also used in the calibration pipe as shown in Fig. 25. In this configuration, the angle between the probe axis and the flow direction could be varied, by swinging the entire support about an anchor bolt, and measured on a scale on the support platform.

vi) **Measuring Equipment**

Figure 26 is a schematic diagram of the instrumentation used in the hot-wire experimentation. Each hot-wire signal was fed into a constant temperature anemometer (TSI model 1010A) set to operate with an overheat ratio of 1.5. Next, each wire signal was processed by a linearizer (TSI
Fig. 23. Probe Holder and Traversing Apparatus in Cascade
Fig. 24. Location of Probe in Cascade
Fig. 26. Schematic Diagram of Measuring Equipment
model 1005B) which squared the signal twice. The linearizer zero suppression adjustment was used to subtract the additive constant in King's Law.

Both signals were then fed into a correlator (TSI model 1015C) which allowed either signal or the sum or difference of the two to be selected. The correlator's variable gain feature was used to set the wire voltages to the proper value at the reference position, with the linearizers' gain adjustments defeated. The correlator output was directed through a true rms meter (Fluke model 8920A) or directly to a switch, allowing selection of either an rms or mean voltage signal.

All signals were processed with an R-C averaging circuit (a low-pass filter) to reduce signal fluctuations. A resistance of 15.9 k-ohm and capacitance of 120 μf combined to give a time constant of nominally 1.9s. Signals were read manually on a digital voltmeter (Keithley model 179 TRMS Digital Multimeter).

The upstream mean velocity was measured at position B (Fig. 18) with a total-pressure probe, a wall static pressure tapping, and an inclined manometer (Meriam Instrument Co. model A-434).

vii) Test Conditions

All experimentation was conducted in the Internal Flow Research Laboratory at VPI&SU. Atmospheric air was used as the test medium, and atmospheric temperature and pressure were monitored. The atmospheric conditions were used to calculate the air density (assumed constant at the low speeds used) for determining velocities with the pitot and pitot-static probes used.
Standard test conditions were 298 K and 95.6 kPa, resulting in a density of 1.12 kg/m$^3$ and viscosity of $1.88 \times 10^{-5}$ kg/m·s.

C. Experimental Procedure

i) Wire Calibration

The modified cosine law was used to model wire response:

$$Q = Q_o (\cos^2 \phi + k^2 \sin^2 \phi)^{1/2}$$

(9)

where $Q$ is the effective cooling velocity, $Q_o$ is the total flow velocity (i.e., the magnitude), $\phi$ is the wire yaw angle (angle between the wire normal and flow directions), and $k^2$ is an experimentally-determined constant to include the effect of flow along the wire axis. Although the wire signals were linearized, the effective cooling velocity was found to follow a power law:

$$Q = E^a$$

(10)

where $E$ is the linearized wire voltage and $a$ is another experimental constant. Calibration for $a$ and $k^2$ was done in the calibration pipe.

The exponent, $a$, was found by placing the wire normal to the flow whose velocity was varied with the pipe's butterfly valve. The yaw constant, $k^2$, was found by fixing the flow at maximum velocity and varying the wire yaw angle. As noted above, the platform that supported the probe allowed the angle that the probe axis made with the flow to be varied; thus, by placing a wire horizontal, its yaw angle could be
varied. Details of the calibration analysis are presented in Appendix A.

ii) Setting Reference Voltage

In all phases of the experimentation, the wire voltage was set to one volt when the wire was normal to some reference flow. Results were then obtained normalized with respect to the reference flow velocity, $Q_{\text{ref}}$. For work done in the calibration pipe, this was simply accomplished by placing the wire normal to the flow at the pipe center and setting the voltage directly to one volt. The reference velocity was measured with a pitot-static probe and the inclined manometer.

In the wind tunnel, the reference voltage had to be set indirectly. Interpolating Adhye's five-hole pressure probe data [17], a location at the same spanwise height in the relatively undisturbed part of the flow with a negligible spanwise velocity was found. The probe was placed facing into the flow at this location (it was necessary to place the probe at an angle to the blade trailing-edge direction to accommodate the V-component of velocity). The probe was rotated until each wire was horizontal and the wire voltage set to the calibrated voltage at the wire angle, $\alpha_0$, which was now the wire yaw angle. In practice, the wire voltage changed slightly when the probe was rotated $180^\circ$ between the two wire horizontal positions; the wire voltage was set to average to the correct value. $Q_{\text{ref}}$ was then the total velocity at this location as measured by Adhye.
iii) Measurement

For each wire angular setting at a measurement point, the two wire mean voltages were measured. Also, the rms value of each wire voltage as well as the rms of the sum and difference of the two wire signals were taken. This is summarized in Table 2.

Table 2. Measured Quantities

Mean Voltages: $E_A, E_B$

Fluctuating Voltages: $\sqrt{\frac{e_A^2}{2}}, \sqrt{\frac{e_B^2}{2}}, \sqrt{(e_A + e_B)^2}, \sqrt{(e_A - e_B)^2}$

After being steadied by the R-C averaging circuit, the mean wire voltages could be read directly as one value. However, the fluctuating voltages often varied rapidly in a small range about an average value. The high and low values were recorded and the average of the two used in the data analysis.

D. Solution Procedures

1) Specifying Wire Orientation

The solution procedures used to solve for mean velocities and Reynolds stresses were developed for a vertical-stemmed probe, facing in
the - W direction. With the probe in such a vertical orientation, the wire geometry could produce wire yaw angles \( \phi \) as high as 70° with secondary flow angles of only 15°. By placing the probe horizontal, facing into the primary flow (in the -U direction), yaw angles were kept below 70° with flow angles as high as 35°, which was the maximum expected in the cascade at plane 2. It was decided to adopt the horizontal orientation for this reason. Rather than rederiving all the applicable equations, a transformation of axes was performed.

For the probe axes in a horizontal orientation, the wire orientation could be specified completely by two angles, \( \alpha'_o \) and \( \alpha'_w \) (see Fig. 27):

- \( \alpha'_o \) - the angle between the wire normal and the probe axis
- \( \alpha'_w \) - the angle the wire-prong plane has been rotated relative to the vertical (W) axis.

These angles were known from measurements with an optical microscope and from the dial setting on the probe holder.

These angles are transformed into the equivalent angles for a vertical-stemmed probe, \( \alpha_o \) and \( \alpha_w \). From Fig. 27, it can be seen that the appropriate transformation equations are:

\[
\begin{align*}
\alpha_w &= \tan^{-1}(\cos \alpha'_o \sin \alpha'_w / \sin \alpha'_o) \\
\alpha_o &= \sin^{-1}(\cos \alpha'_o \cos \alpha'_w). \quad (11)
\end{align*}
\]

The angles \( \alpha_o \) and \( \alpha_w \) are used as geometric parameters in the solution procedures for the mean velocities and Reynolds stresses.
Fig. 27. Geometry for Transformation of Probe Axes
ii) **Solution for Mean Velocities**

Theoretically, the three mean velocity components (U, V, W) could be determined from any three non-duplicating wire orientations. The method used here was a least-squares technique which produces a solution most consistent with all the data. The procedure resembles that of Hirsch and Kool [18] and Forlini [11], but is extended to use all of the available data. Details are given in Appendix B. It is not necessary to selectively choose positions to solve, and the effect of wire orientations influenced by prong interference tends to be minimized.

iii) **Solution for Reynolds Stresses**

The measured rms wire voltages can be reduced into effective cooling velocity fluctuating components \(q_A^2\), \(q_B^2\), and \(q_{AB}^2\) for each probe angular setting. With known wire geometry and mean velocities these can be expressed as three linear equations in the six unknown Reynolds stresses. The techniques for reducing the measured values and setting up the equations are developed in Appendix C.

Thus, for each probe angular setting, three equations for the six Reynolds stresses could be generated. The experimental procedure always called for at least three such angular settings, resulting in an over-specified set of linear equations.

It should be noted that the geometry of the X-wire probe is such that the two wires could duplicate orientations when the probe was rotated. This occurred when three angular settings equally-spaced by 120° were used; the two wires assumed nearly the same three mutually-orthogonal positions as the probe was rotated. Nine equations were
obtained, but only six (three from rms and three from mean product signals) were independent. By increasing the number of settings beyond three, redundant independent information was obtained as opposed to just redundant information as with three settings. This distinction between redundant independent and just redundant information is important. However, having just redundant information does allow the least-squares fitting method to take advantage of the self-consistency of the extra data.

The solution procedure was to least-squares fit the system of equations into a set of six equations in six unknowns as discussed in Appendix D. This resulted in a more strongly diagonal coefficient matrix, increasing confidence in the results obtained. Row reduction was then used to solve the least-squares-fit system for the Reynolds stresses. The turbulence kinetic energy was then calculated from the three normal stresses.

The results thus obtained were normalized with respect to the reference velocity, $Q_{\text{ref}}$. For the purpose of comparison, all values were then uniformly renormalized with respect to the mean upstream velocity, $U_o$. For example:

$$\frac{\overline{u^2}}{U_o^2} = \frac{\overline{u^2}}{Q_{\text{ref}}^2} \times \left( \frac{Q_{\text{ref}}}{U_o} \right)^2$$

(12)

where $\frac{\overline{u^2}}{Q_{\text{ref}}^2}$ is the quantity obtained from the solution procedure. The same was done for the other Reynolds stresses. The turbulence kinetic energy was normalized with respect to the inlet velocity head as
\[
\frac{q^2/2}{U_0^2/2} = \frac{q^2}{U_0^2} = \frac{u^2 + v^2 + w^2}{\frac{Q_{\text{ref}}}{U_0}} \times \left(\frac{Q_{\text{ref}}}{U_0}\right)^2
\]  

which allows direct comparison with the total pressure loss coefficient, \(C_{pt}\), defined in Section VIII-B-ii-a.

iv) Number of Angular Settings

In most regions of the flow three angular settings of the probe produced satisfactory results. However, in parts of the passage vortex and blade wake exhibiting either high levels of turbulence or large secondary velocities (usually spanwise) the solution procedure failed with very small diagonal entries appearing in the coefficient matrix. It is believed that this was due to prong interference effects arising from large probe incidence angles.

This difficulty was overcome by increasing the number of probe angular settings and taking advantage of the ability of the least-squares fitting method to stabilize the system of equations for the Reynolds stresses. Figure 28 shows how the number of angular settings affected the turbulence kinetic energy in one position where the solution procedure failed \((y = 2.54 \text{ cm}, z = 9.8 \text{ cm})\). At this point, the secondary flow angle was 26° as measured by the X-wire probe and 35° as measured by a five-hole probe [17]. Twelve angular settings equally spaced at 30° increments were taken. The solution procedure was then implemented for the possible combinations of six, four, and three equally-spaced settings.
Fig. 28. Effect on $\overline{q^2}/U_0^2$ of Increasing Number of Angular Setting at a Point with Large Incidence Angle in the Cascade ($y = 2.54$ cm, $z = 9.8$ cm at Plane 2)
The value obtained for $\frac{q^2}{U_0^2}$ was seen to change by as much as a factor of two. Much greater variation (by as much as an order of magnitude) was observed if inconsistent mean velocities were used; this difficulty was avoided by the least-squares method of solving for mean velocities. The method appears to have stabilized by four angular settings, but it was decided to use six angular settings in regions of the flow with large secondary velocities. By increasing the number of angular settings and solving with redundant data, a consistent solution has been obtained when the basic method of using three angular settings has failed. The effect of flow angle on the solution procedure is illustrated further in Section X-A-ii on flow angle variation tests in the calibration pipe.
X. RESULTS AND DISCUSSION

A. Verification in Fully-Developed Pipe Flow

1) Simple Horizontal Traverse

The calibration pipe was traversed at mid-height in the exit plane with the rotatable X-wire probe using 3 x 120° angular settings at the five measurement locations shown in Fig. 29. The object was to test the probe's ability to reproduce the turbulent stresses in a relatively simple, known flow.

The results are plotted against those of Laufer [19] in Figs. 30, 31, 32, and 33. All the turbulence quantities are normalized by the shear velocity, \( U_\tau = \sqrt{\tau_w/\rho} \). The wall shear stress, \( \tau_w \), was calculated from the static pressure drop along the last 5.79 m of pipe (see Appendix E, Section 1). The present Reynolds number of about 120,000 was between Laufer's 50,000 and 500,000.

The mean axial velocity, Fig. 30, reproduces Laufer's distribution very well. The difference in magnitude varies from -0.7% (point 1) to +2.2% (5) and averages 0.9%. The flow is nominally axial with zero angle of incidence. The measured flow angles varied from 0.54° (3) to 1.34° (5), averaging 0.83°.

The normal stresses are shown in Fig. 31. The axial normal stress, \( \overline{u'^2}/U_\tau^2 \), is from 25.5% (4) to 36.7% (5) higher than Laufer's, averaging 31.0%. The radial normal stress, \( \overline{v'^2}/U_\tau^2 \), was also measured high, from 10.4% (1) to 27.7% (2), with an average of 21.2%. The azimuthal normal stress, \( \overline{w'^2}/U_\tau^2 \), varied about Laufer's results, from -10.8% (5) to +23.4% (1), with an average magnitude difference of 12.5%.
Fig. 29. Location of Measurement Positions for Simple Horizontal Traverse in Calibration Pipe
Fig. 30. Comparison of Axial Velocity, $U/U_{\text{max}}$, with Results of Laufer -- X-Wire Probe with Three Angular Settings
Fig. 31. Comparison of Normal Stresses, $\frac{\overline{u^2}}{U_T^2}$, $\frac{\overline{v^2}}{U_T^2}$ and $\frac{\overline{w^2}}{U_T^2}$ with Results of Laufer -- X-Wire Probe with Three Angular Settings.
Fig. 32. Comparison of Turbulence Kinetic Energy, $\frac{q^2}{U^2_r}$, with Values Obtained from Normal Stresses of Laufer — X-Wire Probe with Three Angular Settings
Fig. 33. Comparison of Primary Shear Stress, $\frac{\overline{uv}}{U^2}$, with Results of Laufer -- X-Wire Probe with Three Angular Settings
The turbulence kinetic energy, Fig. 32, shows the measured values within about 25% of those expected from Laufer's normal stress results. All the values were from 15.0% (4) to 24.0% (2) high. Since the turbulence kinetic energy was consistently higher (average of 19.6%) than Laufer, there is reason to believe that the present pipe flow may have a higher level of turbulence than did Laufer's.

The radial-axial (primary) shear stress, $\overline{uv}/U^2_\tau$, Fig. 33, shows the linear variation expected across a fully-developed pipe flow (see Appendix E, Section ii). The center value (1) was smaller (-0.015) than the other points by an order of magnitude as expected. The points on the left (4,5) have negative values since the r- and v-directions oppose on this side of the pipe (see Fig. 29). All the values were lower in magnitude than Laufer's, from -8.1% (3) to -19.8% (2), with an average of -13.9%.

**ii) Tests with Variable Flow Angle**

The calibration pipe was also used to test the ability of the rotatable X-wire probe to measure turbulence quantities with variable angles of incidence (angle between flow and probe axis). Measurements were taken at mid-height at $r'/a = 0.50$ with incidence angles of 0, 10, 20, and 35° (maximum expected in the cascade at plane 2). Data was taken for 6 x 60° angular settings and solutions for the mean velocity components and turbulence quantities were obtained using all six settings and the two included 3 x 120° data set. Ideally all the results would be the same.
The results using 6 x 60° are given in Table 3a. The flow angle should ideally be zero (V = W = 0). The largest flow angle obtained was 3.6° at 35°, and the velocity magnitude was measured to within 0.8% of Laufer's results for all the 6 x 60° data. For incidence angles of 0, 10, and 20° all the normal stresses and turbulence kinetic energy agree with Laufer to within 29% and the $\overline{uv}/\overline{U^2}$ shear stress agrees to within 21%. By 35°, the turbulent stresses have increased, suggesting that the limit of the probe's usefulness is being reached. At 35°, the maximum normal stress disagreement is +46% (in $\overline{v^2}/\overline{U^2}$) and the turbulence kinetic energy is 28% high. The $\overline{uv}/\overline{U^2}$ shear stress is still accurate to within 24%, but the $\overline{vw}/\overline{U^2}$ shear stress is no longer negligible (25% of $\overline{uv}/\overline{U^2}$).

Table 3a. Results of Flow Angle Variation Tests
Using 6 x 60° Angular Settings, r'/a = 0.50.

<table>
<thead>
<tr>
<th></th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>35°</th>
<th>Laufer</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>0.917</td>
<td>0.919</td>
<td>0.908</td>
<td>0.912</td>
<td>0.915</td>
</tr>
<tr>
<td>V</td>
<td>-0.004</td>
<td>-0.003</td>
<td>0.001</td>
<td>-0.005</td>
<td>(0)</td>
</tr>
<tr>
<td>W</td>
<td>0.001</td>
<td>-0.013</td>
<td>-0.030</td>
<td>-0.057</td>
<td>(0)</td>
</tr>
<tr>
<td>$\overline{u^2}/\overline{U^2}$</td>
<td>1.97</td>
<td>1.96</td>
<td>1.98</td>
<td>2.21</td>
<td>1.54</td>
</tr>
<tr>
<td>$\overline{v^2}/\overline{U^2}$</td>
<td>0.87</td>
<td>0.77</td>
<td>0.87</td>
<td>1.13</td>
<td>0.77</td>
</tr>
<tr>
<td>$\overline{w^2}/\overline{U^2}$</td>
<td>1.09</td>
<td>1.00</td>
<td>1.00</td>
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<td>$\overline{uv}/\overline{U^2}$</td>
<td>0.42</td>
<td>0.40</td>
<td>0.44</td>
<td>0.62</td>
<td>0.50</td>
</tr>
<tr>
<td>$\overline{uw}/\overline{U^2}$</td>
<td>0.02</td>
<td>0.05</td>
<td>-0.01</td>
<td>0.04</td>
<td>0*</td>
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<tr>
<td>$\overline{vw}/\overline{U^2}$</td>
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<td>-0.01</td>
<td>0.09</td>
<td>0.12</td>
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<tr>
<td>$\overline{q^2}/\overline{U^2}$</td>
<td>3.93</td>
<td>3.73</td>
<td>3.85</td>
<td>4.38</td>
<td>3.43</td>
</tr>
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</table>

*Expected small, not reported by Laufer.
The results using 3 x 120° are shown in Table 3b. For each incidence angle, the results for the two 3 x 120° data sets obtained from the 6 x 60° data set are shown. The measured quantities at 0° incidence angle agree with Laufer to within 30% (28% maximum disagreement in $\overline{u^2}/U_t^2$) which is consistent with part 1 of this section. By comparing the zero-incidence results in Tables 3a and 3b, it can be seen that for flow straight onto the probe three and six angular settings produce results with similar accuracy. At incidence angles as low as 10° using 3 x 120° angular settings, the $\overline{ww}$ and $\overline{uw}$ shear stress are being overpredicted on the same order of magnitude as the other turbulence quantities (60% and 35% of the expected $\overline{uw}/U_t^2$, respectively). Much the same occurs at 20° (68% and 8%), and by 35° the measurement technique clearly breaks down using 3 x 120° angular settings.

Overall with 3 x 120° angular settings, through 10° incidence, the maximum measured flow angle is 1.0° and the velocity magnitude is within 0.6%, the normal stresses are within 28%, the turbulence kinetic energy is within 15%, and the $\overline{uw}/U_t^2$ shear stress is within 27% of Laufer's results. Since 3 x 120° angular settings were used in the cascade flow where incidence angles were less than 12°, the normal stresses and turbulence kinetic energy are expected to be accurate to within 30%, the $\overline{uw}$ shear stress to within 30%, and the $\overline{uw}$ and $\overline{ww}$ shear stresses only to within about 50%. In the turbine cascade, six angular settings were used in regions of high turbulence and high shear stress, so that maximum values were obtained with greater accuracy.
Table 3b. Results of Flow Angle Variation Tests
Using 3 x 120° Angular Settings, r'/a = 0.50

<table>
<thead>
<tr>
<th></th>
<th>0°</th>
<th>10°</th>
<th>20°</th>
<th>35°</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>0.918</td>
<td>0.920</td>
<td>0.911</td>
<td>0.904</td>
</tr>
<tr>
<td>V</td>
<td>-0.002</td>
<td>-0.002</td>
<td>-0.013</td>
<td>-0.029</td>
</tr>
<tr>
<td>W</td>
<td>0.001</td>
<td>-0.005</td>
<td>-0.015</td>
<td>-0.047</td>
</tr>
<tr>
<td>u^2/u^2</td>
<td>1.97</td>
<td>1.94</td>
<td>2.01</td>
<td>2.30</td>
</tr>
<tr>
<td>v^2/u^2</td>
<td>0.79</td>
<td>0.79</td>
<td>1.32</td>
<td>5.88</td>
</tr>
<tr>
<td>w^2/u^2</td>
<td>1.17</td>
<td>1.03</td>
<td>1.01</td>
<td>1.05</td>
</tr>
<tr>
<td>u v /u^2</td>
<td>0.40</td>
<td>0.37</td>
<td>0.39</td>
<td>-0.80</td>
</tr>
<tr>
<td>u w /u^2</td>
<td>0.02</td>
<td>0.04</td>
<td>0.04</td>
<td>0.26</td>
</tr>
<tr>
<td>v w /u^2</td>
<td>0.04</td>
<td>-0.11</td>
<td>0.20</td>
<td>1.01</td>
</tr>
<tr>
<td>q^2/u^2</td>
<td>3.94</td>
<td>3.77</td>
<td>4.34</td>
<td>9.23</td>
</tr>
</tbody>
</table>
B. **Upstream Flow**

The upstream boundary layer was most recently measured by Adhye [17]. He did traverses of upstream locations A and B, Fig. 18, through the top endwall boundary layer using a total pressure probe. The results of his measurements are given in Table 4.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ₀.99</td>
<td>36.0 mm</td>
<td>41.0 mm</td>
</tr>
<tr>
<td>δ*</td>
<td>5.0 mm</td>
<td>5.9 mm</td>
</tr>
<tr>
<td>θ</td>
<td>3.9 mm</td>
<td>4.5 mm</td>
</tr>
<tr>
<td>H₁₂</td>
<td>1.28</td>
<td>1.31</td>
</tr>
</tbody>
</table>

The shape factor, H₁₂, is close to the turbulent, flat-plate, zero-pressure-gradient value of 1.3-1.4. The inlet free-stream velocity U₀ was 23.5 ± 0.4 m/s.

The upstream boundary layer was presently traversed at locations A and B with a horizontal single-wire probe, allowing measurement of the rms streamwise fluctuation, $\sqrt{u'^2}/U₀$. The results are plotted as fraction of span in Fig. 34. The turbulence extends further toward mid-span at B, as expected by the thicker boundary layer. By 25% of span, the turbulence has nearly reached the free-stream rms value of 0.3% observed at both A and B.

The streamwise fluctuation was mass-averaged through the boundary layer to mid-span, resulting in values for $u'^2/U₀^2$ of .00074 and .00088 at A and B. To estimate the inlet turbulence kinetic energy, the present u-fluctuations were combined with Klebanoff's [20] v- and w-fluctuations
Fig. 34. Streamwise Velocity Fluctuation, $\sqrt{u'^2}/U_0$, Measured in Upstream Boundary Layer with Single-Wire Probe
appropriately scaled by the u-fluctuation ratio to obtain mass-averaged values. This resulted in values for $q^2/U_o^2$ of .0015 and .0019 at A and B. Assuming isotropic turbulence in the boundary layer and the measured distribution of $\sqrt{u'^2/U_o}$, these values for $q^2/U_o^2$ become 0.0023 and 0.0026.

C. Measurements at Mid-Span

1) Single-Wire Traverses

Horizontal traverses with a vertical single-wire probe were done at mid-span downstream of the blade row throat. Traverses were done in the lateral ($V$) direction on the planes shown in Fig. 35, which shows their location measured in mm from the trailing edge. Five planes (A, AB, B, BC, and C) were upstream of the trailing edge, originating on the suction surface of the middle blade. Three planes (P1D, P2D, and P3D) were downstream of the trailing edge and extended through the blade wake. Plane 2, used for X-wire measurements, is also shown. Measurement location, $y'$, is specified as the distance from the suction surface in the single-wire measurement plane.

These single-wire traverses only give approximate information about the streamwise fluctuation, $u$. They were intended to be interpreted in a qualitative sense to gain a feel for the turbulence levels to be expected at the cascade exit.

Figure 36 shows the mean primary velocity distribution, $U/U_{fs}$, measured on the upstream planes, A–C. The boundary layer thickens rapidly, increasing in thickness ($\delta_{0.99}$) from 6.6 mm to 38.4 mm from planes A to C, which can be compared with the blade-row throat width of
Fig. 35. Measurement Planes for Single-Wire Traverses at Mid-Span (Distance from Trailing Edge in mm)
Fig. 36. Variation of Primary Velocity, $U/U_{fs}$, in Suction Surface Boundary Layer Upstream of Trailing Edge at Mid-Span
about 94 mm. Between planes A and BC, the shape factor is quite high, 1.7 ± 0.1, reflecting the adverse pressure gradient near the suction surface due to blade unloading (see Moore and Ransmayr [3], Fig. 6). The shape factor then falls to 1.41 at plane C just upstream of the trailing edge. Figure 37 shows the turbulence measurements in this region. The turbulence intensity builds rapidly and is seen to extend further from the blade surface as the boundary layer thickens on the suction surface. Peak values at plane C are about three times those at plane A.

It is interesting to note that the level and distribution of u-fluctuations in the boundary layer at plane A are similar to those of a flat plate turbulent boundary layer. In terms of the local free stream velocity, the peak value of the rms turbulence intensity, \( \sqrt{u'^2/U} \), is about 0.07. By plane C, the turbulence level has increased and the peak value of the turbulence intensity is then 0.14.

The mean velocity distribution on the planes downstream of the trailing edge is shown in Fig. 38. There is a well-defined wake at P1D which quickly mixes out downstream. There exists a steep velocity gradient on the pressure side of the wake and a somewhat weaker one on the suction side. This is because the boundary layer is thin on the pressure surface [5], causing mixing with high-velocity fluid on this side of the wake; the thick suction-surface boundary layer allows more gradual mixing. These velocity gradients also progressively decrease in severity from P1D to P3D.

The turbulence results for planes P1D, P2D, and P3D are shown in Fig. 39. Very high values of \( \frac{u'^2}{U^2} \) appear in the wake at P1D, reaching
Fig. 37. Variation of Primary Velocity Fluctuation, $\frac{u'^2}{U_0^2} \times 10^3$, in Suction Surface Boundary Layer Upstream of Trailing Edge at Mid-Span
Fig. 38. Variation of Primary Velocity, U/U₀, in Blade Wake at Mid-Span
Fig. 39. Variation of Primary Velocity Fluctuation, $\frac{\overline{u^2}}{U_0^2} \times 10^3$, in Blade Wake at Mid-Span
a maximum of 0.093 in the pressure-side steep-gradient region and 0.080 on the suction side. These peak values decay quickly to 0.041 and 0.044 by P2D, after which the decay is not so rapid. This indicates that the highest turbulence levels exist for only about three trailing-edge diameters downstream. By P3D, the peak turbulence has been convected toward mid-passage by action of the passage vortex. Upstream planes A, B, and C are included in Fig. 39 for comparison.

The values of \( \frac{u^2}{U_0^2} \) observed at P1D were the highest encountered in the experimentation. The peak rms values relative to local velocity occurred between the turbulence peaks at P1D (where the local velocity is lower) and reached about 50%. This is significant, because the solution method used with the X-wire probe is valid to only about 20%. By plane 2, between P2D and P3D, the highest rms-to-mean has been reduced to about 20%, allowing the linearized solution procedure to be used.

ii) **Comparison with X-Wire Results**

The primary mean velocity (U) measured with the X-wire and a five-hole probe at plane 2 mid-span are compared in Fig. 40. The values are in excellent agreement, and show a wake region centered on the suction side of the trailing edge. The single-wire turbulence results were interpolated onto plane 2 and are compared with the mid-span X-wire measurements of \( \frac{u^2}{U_0^2} \) in Fig. 41. The general shape of the turbulence distribution agrees quite well, but the X-wire predicts peak values about 20% higher than the single wire. This is better agreement than expected from the results of the verification tests in fully-developed pipe flow (see Section A).
Fig. 40. Comparison of Primary Velocity, $U$, Obtained with X-Wire and Five-Hole Pressure Probes at Mid-Span Plane 2
Fig. 41. Comparison of Primary Velocity Fluctuation, $\frac{u''^2}{U_0^2} \times 10^3$, Obtained with X-Wire and Single-Wire Probes at Mid-Span, Plane 2
D. Turbulence Measurements at Plane 2

1) Measurement Locations

Plane 2 was traversed with the X-wire probe. Horizontal (pitch-wise, y) traverses were made at the following distances from the top endwall: 2.2, 4.1, 6.0, 7.9, 9.8, and 11.7 cm (mid-span). At each spanwise height, measurements were made from y = -2.54 cm to 17.8 cm every 2.54 cm. This defined a coarse grid as shown in Fig. 42, which also indicates the number of angular settings used. Three settings were used in regions of low turbulence and regions of mild secondary velocities (generally cone angles less than 15°, see below). The number of settings was increased to six or twelve in those regions of distorted flow exhibiting large secondary velocities and turbulence levels. The vertical lines on Fig. 42 show the trailing edges of the middle and adjacent blades projected in the primary flow direction. The results at y = -2.54, 0, and 2.54 cm were duplicated over the wake region of the adjacent blade to fill in the region to the right.

The region between z = 2.2 cm and the top endwall was traversed spanwise with a horizontal single-wire probe at selected pitchwise locations (not shown).

ii) Mean Velocities

The mean velocities were solved for particularly to complete the coefficient matrix in the Reynolds stress solution (Appendix C). The results are compared with the mean velocities obtained by Adhye [17] at plane 2 with a five-hole pressure probe.
number of angular settings: three ○ six □ twelve △

end wall

mid-span

Fig. 42. Grid for X-Wire Measurement Locations at Plane 2
The primary velocity, $U$, was obtained within 4 m/s of Adhye's results. The X-wire secondary velocities are vector plotted in Fig. 43. The same large passage vortex was measured as observed by Adhye, Fig. 44, and the overall secondary flow structure appears similar. The X-wire probe, however, gave smaller maximum secondary velocities. Maximum spanwise ($W$) velocities near the suction side were about $-16$ m/s, whereas Adhye observed values of about $-20$ m/s (interpolated) at the same locations. Overall, $V$ agreed to within 5 m/s and $W$ to within 7 m/s of Adhye's measurements.

These secondary velocity discrepancies can be explained in part by considering the flow angles encountered. The cone angle is defined as the angle between the total flow direction and the primary flow direction (see Fig. 3) in a three-dimensional sense:

$$
\phi_{cone} = \tan^{-1} \left( \frac{\sqrt{V^2 + W^2}}{U} \right)
$$

and is simply the incidence angle to the probe. It was anticipated that the X-wire probe would have greatest difficulty with high incidence angles, due to prong interference. In Fig. 45, the difference in cone angle is plotted against cone angle where the five-hole probe results are used as the reference values. Most large disagreement occurred in regions where the cone angle was large and secondary velocities were strong. With cone angles less than 15°, the X-wire probe agreed within 3°, but differences as large as 9° were observed in regions of highly three-dimensional flow. The effect of the number of angular settings is
Fig. 43. Secondary Velocity Vectors Obtained with X-Wire Probe at Plane 2
Fig. 44. Secondary Velocity Vectors Obtained with Five-Hole Probe at Plane 2
Fig. 45. Difference in Cone Angle vs. Cone Angle for X-Wire and Five-Hole Probe Results at Plane 2
not seen clearly since three settings were typically used up to a cone angle of about 15°, where agreement was good.

The per cent difference in velocity magnitude versus cone angle is shown in Fig. 46. There seems to be a more random relationship than with difference in cone angle. The X-wire probe generally gave the velocity magnitude to within 4% (1.7 m/s for a maximum of 42 m/s) with five points (out of 54) exhibiting greater disagreement. The number of angular settings does not seem to have had a dramatic effect on the determination of velocity magnitude.

These comparisons of mean velocity results must be interpreted with some reservation. The mean velocity components of Adhye were interpolated linearly to the X-wire measurement locations (Adhye used the same y-locations, but different z-locations except at mid-span) over regions of the flow where large velocity gradients existed. Also, small changes in location could dictate large differences in local velocity, and the traversing procedure used with the X-wire probe was less accurate than that used by Adhye. Also some uncertainty is bound to exist in Adhye's results, although he claims to have measured mean velocities to within ± 0.5 m/s (see Adhye [17], Appendix C).

iii) Turbulence Kinetic Energy

a) Distribution

Contours of $\overline{u'^2}/U'_0^2 \times 10^3$ measured by the X-wire probe are plotted in Fig. 47. The end-wall region was traversed with a single-wire, giving information only for $\overline{u'^2}$; the turbulence in this region was assumed to be isotropic ($\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$) for the purposes of contour plotting. The
Fig. 46. Per Cent Difference in Velocity Magnitude vs. Cone Angle for X-Wire and Five-Hole Probe Results at Plane 2
Fig. 47. Contours of $\frac{\bar{q}^2}{u_o^2} \times 10^3$ at Plane 2
computer routine that produced the contours used linear interpolation between grid points, and this should be considered when interpreting the following contour plots.

The contours of $\frac{q^2}{U_0^2}$ show a peak of 0.20 in the blade wake downstream of the trailing edge, at about 20% of span. Two peaks of 0.16 occur near the center of the passage vortex and in the mid-span/suction-side corner region. The latter two peaks correspond to regions of high-total-pressure-loss fluid, as can be seen in Fig. 11. However, the blade-wake peak is in a region of relatively low-loss fluid ($C_{pt} \approx 0.8$ versus a maximum of 1.8). The turbulence here is probably due to a different mechanism, possibly residual trailing-edge vorticity causing violent mixing of different velocity fluids. The turbulence kinetic energy contours also show the end-wall boundary layer to be quite thin, about 2 mm thick at mid-passage.

A very low turbulence region occupies much of the passage, from the passage vortex region to the wake of the adjacent blade. Values of $\frac{q^2}{U_0^2}$ here are typically 0.001 to 0.002, with a minimum of 0.0008 (corresponding to an rms turbulence intensity of 1.7% of inlet flow velocity) occurring at mid-span and 68% pitch. This minimum value of turbulence intensity at plane 2 is of the same order as at the edge of the inlet endwall boundary layer (about 1.5%, see Fig. 34).
b) **Mass-Averaged Turbulence Kinetic Energy**

The mass-averaged turbulence kinetic energy is defined as:

\[
\frac{q^2}{U_o^2} = \frac{\int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} \left( \frac{1}{2} (u^2 + v^2 + w^2) \rho V_n \right) dz dy}{\int_{-\Delta y/2}^{\Delta y/2} \int_{-\Delta z/2}^{\Delta z/2} \frac{1}{2} \rho U_o^2 \rho V_n dz dy}
\]

(15)

which is consistent with the definition of $C_{pt}$ used in previous investigations, [3-5,17] (see Section VIII-B-ii-a). The integrals were approximated with a numerical integration procedure using Adhye's interpolated values of $V_n$, and assuming constant density. A four-point average over each grid cell was used for property values, and summation carried out over the entire flow region (see [17], Appendix D, for details). The isotropic approximation made near the end wall did not have a large effect on the answer since overturning there results in a small $V_n$.

The resultant value for $q^2/U_o^2$ is 0.062, which can be compared to a $C_{pt}$ of 0.294 at plane 2 [5,17]. The turbulence kinetic energy then accounts directly for about 21% of the total pressure loss at plane 2.

Gregory-Smith and Graves [9] believe turbulence may account for 75% of the total pressure losses in their cascade. However, their measurement plane (slot 8, Fig. 14) was closer to the trailing edge ($x/c = 1.03$) than plane 2 ($x/c = 1.10$). As noted in Section B, the peak turbulence intensities decay abruptly downstream of the trailing edge, but the decreases seen in Fig. 39 would not explain a factor of 3.5 in mass-averaged turbulence kinetic energy.
iv) Normal Stresses

Contour plots of $\overline{u^2}$, $\overline{v^2}$, and $\overline{w^2}$ normalized by $U_o^2$ are shown in Figs. 48, 49, and 50, respectively. When interpreting these plots, it is helpful to keep in mind the relationship between the turbulence kinetic energy and the three normal stresses:

$$\frac{\overline{q^2}/2}{U_o^2/2} = \frac{\overline{q^2}}{U_o^2} = \frac{\overline{u^2}}{U_o^2} + \frac{\overline{v^2}}{U_o^2} + \frac{\overline{w^2}}{U_o^2}$$  \hspace{1cm} (16)

where $\overline{q^2}/2$ is the turbulence kinetic energy.

The secondary normal stresses, $\overline{v^2}$ and $\overline{w^2}$, show a distribution similar to $\overline{q^2}/U_o^2$. Local peak values of $\overline{v^2}/U_o^2$ reach 0.100, accounting for half the corresponding local peak turbulence kinetic energies. The $\overline{w^2}/U_o^2$ values are next largest, with peak values of 0.070 occurring in the blade wake.

The primary normal stress distribution is quite different. Contours of $\overline{u^2}/U_o^2$ show a peak of only about 0.050 occurring at mid-span, and are more symmetric than the secondary normal stress and turbulence kinetic energy contours. The larger values of $\overline{v^2}$ and $\overline{w^2}$ may result from the rapid mixing-out of the mean secondary velocities, $V$ and $W$, downstream of the cascade, while the primary flow, $U$, behaves as a reversible flow (see Mixing Analysis, [5,17]).
Fig. 48. Contours of $\frac{\overline{u^2}}{u_0^2} \times 10^3$ at Plane 2
Fig. 50. Contours of $\frac{w^2}{V_{10}^2} \times 10^3$ at Plane 2
v) **Shear Stresses**

The turbulent shear stresses, $\overline{uv}$, $\overline{uw}$, and $\overline{vw}$, normalized by $U_0^2$, are contour plotted in Figs. 51, 52, and 53. The contours of $\overline{uv}/U_0^2$ show a peak value of $+0.020$ in the blade wake near mid-span. This is a region of a high negative $\partial U/\partial y^{\star}$ velocity gradient (see Fig. 40) and one would expect a large positive $\overline{uv}$ shear stress (see Section VIII-A or [1], p. 72). There is a corresponding region of not so large positive $\partial U/\partial y$ on the other side of the wake which is accompanied by a region of negative $\overline{uv}$.

Contours of $\overline{uw}/U_0^2$, Fig. 52, have a large negative ($-0.020$ peak) region at about 40% of span and 20% of pitch. This is in a region of large positive $\partial U/\partial z$ as might be expected.

The $\overline{vw}/U_0^2$ shear stress, Fig. 53, has a peak of $+0.040$ in the blade wake closely corresponding to the location of highest $q^2$, $v^2$, and $w^2$. It is more difficult to associate this with a mean velocity gradient since both $\partial W/\partial y$ and $\partial V/\partial z$ may be important and the y-direction does not lie in a measurement plane. From the vector plot of secondary velocities (Figs. 43, 44) $W$ is decreasing (becoming more negative) in the y-direction near peak $\overline{vw}$, which is consistent with a positive $\overline{vw}$.

All the shear stresses are small in the undisturbed flow near mid-passage with contours of zero appearing there.

The most significant finding here is that the maximum $\overline{vw}$ shear stress is the largest, about twice the maximum value of $\overline{uv}$ or $\overline{uw}$. The $\overline{vw}$ stress was the one not obtained by Gregory-Smith and Graves [9]. It

---

*Here, the $(x,y,z)$ used to evaluate velocity derivatives correspond to $(U,V,W)$ of Fig. 3 and not the cascade coordinates used previously.*
Fig. 51. Contours of $\overline{uv}/u_0^2 \times 10^3$ at Plane 2
Fig. 53. Contours of $\bar{w}w/u_o^2 \times 10^3$ at Plane 2
is believed that it may contribute to the rapid decay of mean secondary kinetic energy, particularly $W^2/2$, downstream of the cascade.

The shear stresses can be combined with mean velocity gradients to produce a shear work term which may be a contributing factor to the build-up of losses in the downstream region (see Section VIII-A). A quantitative analysis of this was considered beyond the scope of this thesis.
XI. CONCLUSIONS

A. Measurement with the Rotatable X-Wire Probe

The rotatable X-wire probe used in this investigation was designed to allow redundant data to be taken by rotating the probe about its axis into different angular positions. Least-squares solution procedures were then used to obtain the mean velocity components and Reynolds stresses. The purpose of taking redundant data was to obtain sufficient information so that if prong interference existed in some positions, all the data could be included in the solution with the unaffected data dictating the results. In this way, it was hoped to both increase the allowable incidence angles with the probe used in an end-flow orientation, and to improve accuracy at all angles of incidence. The following conclusions are drawn from the measurements taken in this investigation.

1) Fully-Developed Pipe Flow

Verification tests were performed at the exit of a pipe with a Reynolds number of 120,000 based on average velocity and diameter. An uninterrupted length of 240 diameters upstream of the exit insured fully-developed flow and the Reynolds number indicates a clearly turbulent flow. The results are compared with those of Laufer [19], and all errors are based on locally expected values.

a) Zero Incidence

1. Using the basic three angular settings 120° apart, the mean velocity magnitude was measured within 2.2% of Laufer's results,
and the measured flow angle was not in excess of $1.3^\circ$. The Reynolds stresses and turbulence kinetic energy were obtained to within $\pm 40\%$.

2. Increasing the number of angular settings to six did not effectively improve the results for mean velocities or Reynolds stresses.

3. The turbulent normal stresses and turbulence kinetic energy were measured consistently higher than expected from Laufer; the primary shear stress ($\overline{uv}$) was measured lower in magnitude.

b) Non-Zero Incidence

1. Using six angular settings, the velocity magnitude was obtained within $0.8\%$ and the flow angle within $3.6^\circ$ through $35^\circ$ incidence. The Reynolds stresses and turbulence kinetic energy were obtained within $30\%$ through $20^\circ$ incidence, and within $46\%$ up to $35^\circ$ incidence.

2. Using three angular settings through $10^\circ$ incidence, the velocity magnitude and flow angle were obtained to within $0.6\%$ and $1.0^\circ$, respectively. The turbulent normal stresses and turbulence kinetic energy were measured to within $28\%$ and the primary shear stress ($\overline{uv}$) to within $27\%$. However, the secondary shear stresses ($\overline{uw}$, $\overline{vw}$) were measured as large as $60\%$ of the expected primary shear stress when they should have been small in comparison.

The verification tests in fully-developed pipe flow showed that the X-wire probe was capable of measuring the Reynolds stresses and turbulence kinetic energy with the accuracy desired in the cascade flow.
They also indicated that six angular settings were desirable when incidence angles were larger than about 15°.

ii) Three-Dimensional Cascade Flow

1. The velocity magnitude was measured to within ±4% (10:1 odds) of earlier five-hole pressure probe results [17] up to cone (three-dimensional incidence) angles of 35°. The cone angle was obtained within 4° when less than 20°, but only within 9° for cone angles up to 35°.

2. In regions of high incidence (cone angle > 30°) three angular settings gave results for turbulence kinetic energy with large scatter. Increasing the number of angular settings (to six or twelve) reduced the results by as much as a factor of two and produced consistent reproducible results.

B. Turbulence Downstream of the VPI&SU Cascade

Cascade measurements were made in a large-scale five-blade linear turbine cascade. The blades are in the shape of reaction turbine rotor blades and are geometrically similar to those used by Langston, et al. at United Technologies Research Center [6]. For the VPI&SU cascade, the aspect ratio (span/axial chord) is 0.997, and the Reynolds number based on axial chord and exit velocity is $5.2 \times 10^5$. The blades turn a flow of atmospheric air through 110° and accelerate with a velocity ratio of about 1.6. The upstream flow has a freestream velocity of $23.5 \pm 0.4$ m/s and an end wall boundary layer displacement thickness of 2.1 to 2.5 per cent of span.
The downstream flow where turbulence measurements were made is an example in the secondary flow limit. Downstream, the flow is dominated by the decay of passage vortices and secondary kinetic energy; more than one-third of the cascade losses occur downstream.

1) **At Mid-Span**

A single hot-wire was used to obtain estimates of the turbulence intensity at mid-span near the suction surface downstream of the blade row throat and in the blade wake. The following observations were made.

1. The boundary layer grows rapidly on the exposed part of the suction surface. At \( x/c = 0.76 \), the turbulence intensity is similar to that in a flat plate boundary layer, but it doubles by the trailing edge.

2. Very high levels of turbulence (50% rms turbulence intensity relative to local velocity) are found in the blade wake near the trailing edge. The highest levels exist for only about three trailing-edge diameters downstream of the trailing edge (reduced to 20% rms).

11) **At Plane 2 \((x/c = 1.10)\)**

The rotatable X-wire probe was used to measure all the Reynolds stresses and turbulence kinetic energy over a coarse grid at plane 2. A single hot-wire was used to estimate turbulence intensity near the end wall. The following conclusions were reached concerning the turbulence at plane 2.

1. The turbulence kinetic energy is highest in the blade wake, with
peak values of 20% of the inlet velocity head. The distribution resembles that for total-pressure loss.

2. The lowest values of turbulence kinetic energy are found at mid-span and are of the same order as at the edge of the inlet end wall boundary layer.

3. The mass-averaged turbulence kinetic energy is 0.062 times the inlet velocity head. It accounts directly for 21% of the mass-averaged total-pressure loss at plane 2 (\(\frac{C_{pt}}{C_{pt}} = 0.294\)); this is less than the 75% of \(\frac{C_{pt}}{C_{pt}}\) found by Gregory-Smith and Graves [9] and does not appear to have exceeded 30% closer to the trailing edge.

4. The lateral normal stress (\(v^2\)) has the largest maximum value of the three normal stresses and accounts for half of the local turbulence kinetic energy at the location of highest turbulence in the blade wake. The distributions of \(v^2\) and \(w^2\) resemble those for turbulence kinetic energy; whereas, the \(u^2\) distribution is more uniform with a lower peak value that occurs at mid-span.

5. The turbulent shear stresses obtained are consistent in sign and relative magnitude with gradients of mean velocity.

6. The \(vw\) shear stress has the highest peak value of the turbulent shear stresses measured. The peak value of \(vw\) occurs in the blade wake where there is a shear layer between pressure- and suction-side fluids with large gradients of spanwise velocity, \(W\).
XII. REFERENCES


Appendix A. Wire Calibration Procedure

Here the experimental wire calibration method for determining the constants $a$ and $k^2$ is described. The form of the cosine law used here is:

$$E^a = Qo (\cos^2 \phi + k^2 \sin^2 \phi)^{1/2}$$  \hspace{1cm} (A1)

where $E$ is the wire voltage, $\phi$ is the wire yaw angle (angle between velocity and wire normal), $Qo$ is the velocity magnitude, and $a$ and $k^2$ are to be determined.

1) Determination of $a$

The wire is placed normal to the flow ($\phi = 0$) in the calibration pipe and the velocity is varied. Equation (A1) reduces to

$$E^a = Qo$$  \hspace{1cm} (A2)

where $Qo$ is being varied via the flow velocity, which is measured with a pitot-static probe. For a pitot-static probe:

$$Qo \sim \sqrt{\Delta P}$$  \hspace{1cm} (A3)

where $\Delta P$ is the difference in the total and static pressures, $P_t - P$, measured with an inclined manometer. Thus, a set of readings of the form $E^a_1 \sim \sqrt{\Delta P_1}$ is obtained. This is written as:
\[(E_i/E_1)^a = \sqrt{\Delta P_i/\Delta P_1}\]  \hspace{1cm} (A4)

where subscript 1 refers to a reference position. The maximum flow condition was used as a reference, where the voltage \(E_1\) was set to one volt. Taking the logarithm of this gives:

\[a \log(E_i/E_1) = \log \sqrt{\Delta P_i/\Delta P_1} . \hspace{1cm} (A5)\]

The exponent \(a\) is then determined by minimizing the sum of the squares of errors:

\[S = \sum_i [a \log (E_i/E_1) - \log \sqrt{\Delta P_i/\Delta P_1}]^2 . \hspace{1cm} (A6)\]

Taking the derivative with respect to \(a\) and setting equal to zero results in:

\[2 \sum_i [a \log (E_i/E_1) - \log \sqrt{\Delta P_i/\Delta P_1}] \log (E_i/E_1) = 0 \hspace{1cm} (A7)\]

which is rearranged to:

\[a = \frac{\sum_i [\log \sqrt{\Delta P_i/\Delta P_1} \log (E_i/E_1)]}{\sum_i [\log (E_i/E_1)]^2} . \hspace{1cm} (A8)\]
II) Determination of $k^2$

Now the flow velocity is kept constant at its maximum value and the flow angle, $\phi$, is varied. A set of values of $E_1$ versus $\phi$ is obtained which is then solved for $k^2$ using a least-squares method as described above for determining $a$. A set of equations of the form:

$$E_1^a = Q_o (\cos^2 \phi_1 + k^2 \sin \phi_1)^{1/2}$$  \hspace{1cm} (A9)

is obtained where $a$ is a known constant previously determined. The wire voltage was set to one volt when $\phi = 0$ resulting in $Q_o = 1$. Then minimizing the sum of the squares of the errors in $E_1^a$:

$$S = \sum \left( E_1^a - \cos^2 \phi_1 - k^2 \sin^2 \phi_1 \right)^2$$ \hspace{1cm} (A10)

results in:

$$k^2 = \frac{\sum E_1^a \sin \phi_1 - \sum \cos^2 \phi_1 \sin^2 \phi_1}{\Sigma \sin^4 \phi_1}$$ \hspace{1cm} (A11)

iii) Calibration Curve

A calibration curve is obtained by plotting $E_1^a$ versus $(\cos^2 \phi_1 + k^2 \sin^2 \phi_1)^{1/2}$. An example is given in Fig. A1 showing the results of the calibration for both wires of the probe.
Fig. A1. Results of Wire Calibration for X-Wire Probe
Appendix B. **Solution Procedure for Mean Velocities**

For each wire (i) and probe angular setting (j), an effective cooling velocity (Q) is calculated from the wire mean voltage (E):

\[ Q_{ij} = a_i E_{ij} \]  

(B1)

where \( a_i \) is the wire linearity constant. The cooling velocity is related to the wire orientation by the modified cosine law:

\[ Q_{ij} = Q_o \left( \cos^2 \phi_{ij} + k_i^2 \sin^2 \phi_{ij} \right)^{1/2} \]  

(B2)

where:

- \( Q_o \) = velocity magnitude (normalized by \( Q_{ref} \))
- \( \phi_{ij} \) = angle between the wire normal and the flow direction (wire yaw angle)
- \( k_i^2 \) = yaw constant for wire i.

For a given wire orientation, the wire yaw angle, \( \phi_{ij} \), can be expressed using the sine law for spherical trigonometry [18] as:

\[ \sin \phi_{ij} = \cos \alpha_{o_{ij}} \cos \alpha_r \cos (\alpha_p + \alpha_{w_{ij}}) + \sin \alpha_r \sin \alpha_{o_{ij}} \]  

(B3)

where the angles \( \alpha_o, \alpha_w, \alpha_r, \alpha_p \) are defined (for a vertical-stemmed probe) in Fig. B1. The wire angles, \( \alpha_{o_{ij}} \), and wire rotational positions, \( \alpha_{w_{ij}} \), are known quantities specifying the wire position relative to the fixed coordinate frame.
Fig. B1. Geometry for Mean Velocity Solution, Vertical-Stemmed Probe
The solution consists of iterating on the flow angles \( \alpha_p \) and \( \alpha_r \) in equations (B3), and solving for the value of \( Q_o \) predicted by each orientation:

\[
Q_{o_{ij}} = \frac{Q_{ij}}{(\cos^2 \phi_{ij} + k_i^2 \sin^2 \phi_{ij})^{1/2}}
\]  

(B4)

and evaluating the sum of the squares of the errors in \( Q_{o_{ij}} \):

\[
\Sigma_i \Sigma_j (Q_{\text{mean}} - Q_{o_{ij}})^2
\]

(B5)

where \( Q_{\text{mean}} \) is the average of all the \( Q_{o_{ij}} \)'s. The iteration proceeds until expression (B5) is minimized.

Once \( Q_o, \alpha_r, \) and \( \alpha_p \) are known, the mean velocity components are calculated as:

\[
U = Q_o Q_{\text{ref}} \cos \alpha_r \cos \alpha_p
\]

\[
V = -Q_o Q_{\text{ref}} \cos \alpha_r \sin \alpha_p
\]

\[
W = -Q_o Q_{\text{ref}} \sin \alpha_r
\]

(B6)
Appendix C. Solution for Reynolds Stresses

1) Development of Linearized Response Equations

Consider a hot-wire sensor with attached coordinate system \((\text{Ax}, \text{N}_1, \text{N}_2)\) as shown in Fig. C1. The instantaneous cooling velocity \((Q + q)\) can be resolved into its components in the wire coordinate directions by the modified cosine law as:

\[
(Q + q) = \left[ (Q + q)^2_{n1} + (Q + q)^2_{n2} + k^2 (Q + q)^2_{ax} \right]^{1/2}.
\] (C1)

These can be related to the instantaneous velocities in the fixed coordinate frame \((U + u, V + v, W + w)\) by:

\[
(Q + q)_{ax} = a_1 (U + u) + b_1 (V + v) + c_1 (W + w)
\]

\[
(Q + q)_{n1} = a_2 (U + u) + b_2 (V + v) + c_2 (W + w)
\]

\[
(Q + q)_{n2} = a_3 (U + u) + b_3 (V + v) + c_3 (W + w)
\] (C2)

where \(a_1, b_1, c_1, \ldots\), are the appropriate direction cosines (see below under ii to relate to known wire angles).

Next equations (C2) are substituted into (C1) to give:

\[
Q + q = [a_4(U^2 + u^2 + \alpha Ju) + b_4(V^2 + v^2 + 2v\overline{v})
\]

\[
+ c_4(W^2 + w^2 + 2W\overline{w}) + d_4(UV + uv + Uv + Vu)
\]
Fig. C1. Geometry for Evaluating Direction Cosines in Solution for Reynolds Stresses, Vertical-Stemmed Probe
where the constants are expressed in terms of previous constants as:

\[ a_4 = a_2^2 + a_3^2 + k^2 a_1^2 \]

\[ b_4 = b_2^2 + b_3^2 + k^2 b_1^2 \]

\[ c_4 = c_2^2 + c_3^2 + k^2 c_1^2 \]

\[ d_4 = 2(a_2 b_2 + a_3 b_3 + k^2 a_1 b_1) \] \hspace{1cm} (C4)

\[ e_4 = 2(a_2 c_2 + a_3 c_3 + k^2 a_1 c_1) \]

\[ f_4 = 2(b_2 c_2 + b_3 c_3 + k^2 b_1 c_1) . \]

Next, (C3) is linearized by factoring \( \sqrt{a_4} (U + u) \) from the right hand side, expanding by

\[ (1 + x)^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8} + \ldots , \]

and neglecting terms of third and higher order to get:
\[Q \cdot q = a_5 (U + u) + a_6 (V + v) + a_7 (W + w)\]
\[+ a_8 \left( \frac{v^2}{U} + \frac{w^2}{U} \right) + a_9 \left( \frac{w^2}{U} + \frac{v^2}{U} \right) + a_{10} \left( \frac{VW}{U} + \frac{Wv}{U} \right)\]
\[+ a_{11} \frac{Wv}{U} + a_{12} \frac{Ww}{U} + a_{13} \left( \frac{VW}{U} + \frac{Wv}{U} \right) + O(\varepsilon). \quad (C5)\]

The constants are the following combinations of previous constants:

\[a_5 = \sqrt{a_4}\]
\[a_6 = d_4/2 \sqrt{a_4}\]
\[a_7 = c_4/2 \sqrt{a_4}\]
\[a_8 = \sqrt{a_4} \left[ \frac{b_4}{2a_4} - \frac{1}{8} \left( \frac{d_4}{a_4} \right)^2 \right]\]
\[a_9 = \sqrt{a_4} \left[ \frac{c_4}{2a_4} - \frac{1}{8} \left( \frac{e_4}{a_4} \right)^2 \right]\]
\[a_{10} = \sqrt{a_4} \left[ \frac{f_4}{2a_4} - \frac{1}{4} \left( \frac{d_4}{a_4} \right) \left( \frac{e_4}{a_4} \right) \right] \quad (C6)\]
\[a_{11} = \sqrt{a_4} \left[ \frac{b_4}{a_4} - \frac{1}{4} \left( \frac{d_4}{a_4} \right)^2 \right]\]
\[a_{12} = \sqrt{a_4} \left[ \frac{c_4}{a_4} - \frac{1}{4} \left( \frac{e_4}{a_4} \right)^2 \right]\]
\[a_{13} = a_{10}. \]
Time-averaging (C5) results in:

\[ Q = a_5 U + a_6 V + a_7 W + a_8 \left( \frac{V^2}{U} + \frac{W^2}{U} \right) \]
\[ + a_9 \left( \frac{W^2}{U} + \frac{W^2}{U} \right) + a_{10} \left( \frac{VW}{U} + \frac{WV}{U} \right) . \]  

(C7)

An expression for the instantaneous fluctuation, \( q \), is obtained by subtracting (C7) from (C5):

\[ q = a_5 u + a_6 v + a_7 w + a_8 \left( \frac{v^2}{u} - \frac{v^2}{u} \right) \]
\[ + a_9 \left( \frac{w^2}{u} - \frac{w^2}{u} \right) + a_{10} \left( \frac{vw}{u} - \frac{vw}{u} \right) + a_{11} \frac{Vv}{U} \]
\[ + a_{12} \frac{Ww}{U} + a_{13} \left( \frac{Vw}{U} + \frac{Wv}{U} \right) . \] 

(C8)

Squaring (C8) and taking the time-average results in the linear equation for \( \overline{q^2} \) in terms of the Reynolds stresses:

\[ \overline{q^2} = a_5^2 \overline{u^2} + [a_6^2 + a_{11}^2 \left( \frac{V^2}{U} \right)^2 + a_{13}^2 \left( \frac{W^2}{U} \right)^2 + 2a_6a_{11} \left( \frac{V}{U} \right) \]
\[ + 2a_6a_{13} \left( \frac{W}{U} \right) + 2a_{11}a_{13} \left( \frac{VW}{U^2} \right) ] \overline{v^2} + [a_7^2 + a_{12}^2 \left( \frac{W}{U} \right)^2 \]
\[ + a_{13}^2 \left( \frac{V}{U} \right)^2 + 2a_7a_{12} \left( \frac{W}{U} \right) + 2a_7a_{13} \left( \frac{V}{U} \right) + 2a_{12}a_{13} \left( \frac{VW}{U^2} \right) \] \overline{w^2} + \]
If (C8) for one wire (A) is multiplied by the corresponding equation for a second wire (B) and time-averaged, the following equation for the mean product of the two wire signals results:

\[ q_A q_B = a_5 b_5 \bar{u}^2 + [a_5 b_6 + a_{11} b_{11} (V/U)^2 + a_{13} b_{13} (W/U)^2] \bar{v} \]

\[ + (a_6 b_{11} + a_{11} b_6) \frac{V}{U} + (a_6 b_{13} + a_{13} b_6) \frac{W}{U} + (a_{11} b_{13} + a_{13} b_{11}) \frac{VW}{U^2} \bar{v}^2 \]

\[ + [a_7 b_7 + a_{12} b_{12} (W/U)^2 + a_{13} b_{13} (V/U)^2 + (a_7 b_{12} + a_{12} b_7) \frac{W}{U}] \bar{v} \]

\[ + (a_7 b_{13} + a_{13} b_7) \frac{V}{U} + (a_{12} b_{13} + a_{13} b_{12}) \frac{VW}{U^2} \bar{w} \bar{v} + [a_5 b_6 + a_6 b_5] \]

\[ + (a_5 b_{11} + a_{11} b_5) \frac{V}{U} + (a_5 b_{13} + a_{13} b_5) \frac{W}{U} \bar{v} \bar{w} + [a_5 b_7 + a_7 b_5] \]

\[ + (a_5 b_{12} + a_{12} b_5) \frac{W}{U} + (a_5 b_{13} + a_{13} b_5) \frac{V}{U} \bar{w} \bar{v} + [a_6 b_7 + a_7 b_6] \]

\[ + (a_6 b_{12} + a_{12} b_6 + a_7 b_{13} + a_{13} b_7) \frac{W}{U} + (a_6 b_{13} + a_{13} b_6 + a_7 b_{11} + a_{11} b_7) \frac{V}{U} + \]
where $b_5, \ldots, b_{13}$ are the coefficients for wire B corresponding to $a_5, \ldots, a_{13}$ in (C6). A third equation for $q_B^2$ is obtained as was (C9). The mean velocities ($U, V, W$) are first solved for by the method of Appendix B.

**ii) Relating Wire and Fixed Coordinates**

Here the direction cosines in (C2) are expressed in terms of known angles, $\alpha_o$ and $\alpha_w$ (see Section IX-D-1), for a vertical-stemmed probe.

Consider Fig. C1, with wire coordinate directions ($\text{AX}, \text{NI}, \text{N2}$) and fixed coordinate directions ($i, j, k$). The two sets of unit vectors are related by:

\[
\text{AX} = \cos \alpha_o \cos \alpha_w i + \cos \alpha_o \sin \alpha_w j - \sin \alpha_o k
\]

\[
\text{NI} = -\sin \alpha_w i + \cos \alpha_w j + 0 k
\]

\[
\text{N2} = \sin \alpha_o \cos \alpha_w i + \sin \alpha_o \sin \alpha_w j + \cos \alpha_o k
\]

(C11)
This then defines the direction cosines of \((C2)\) as:

\[
\begin{align*}
    a_1 &= \cos \alpha \cos \omega \\
    b_1 &= \cos \alpha \sin \omega \\
    c_1 &= -\sin \alpha \\
    a_2 &= -\sin \omega \\
    b_2 &= \cos \omega \\
    c_2 &= 0 \\
    a_3 &= \sin \alpha \cos \omega \\
    b_3 &= \sin \alpha \sin \omega \\
    c_3 &= \cos \alpha .
\end{align*}
\]

iii) Relating Voltages to Velocities

The hot-wire output obtained from the linearizer was found to not follow a linear response curve, but actually a power law:

\[
E^a = Q
\]

where \(E\) is the wire voltage, \(Q\) is the fluid velocity normalized by \(Q_{\text{ref}}\), and \(a\) is the wire linearity constant (see Appendix A).

For an instantaneous signal:

\[
Q + q = (E + e)^a = E^a(1 + \frac{e}{E})^a .
\]

This is linearized by the expansion:

\[
(1 + x)^a = 1 + ax + \frac{a(a - 1)}{2} x^2 + ... .
\]

Letting \(\frac{e}{E} = x\) and neglecting higher-order terms, equation (1) becomes:
\[ Q + q = E^a \left[ 1 + a\left( \frac{e}{E} \right) \right] . \] (C15)

Squaring this equation gives:

\[ Q^2 + 2Qq + q^2 = E^{2a} \left[ 1 + 2a\left( \frac{e}{E} \right) + a^2\left( \frac{e}{E} \right)^2 \right] \] (C16)

which is time averaged to:

\[ Q^2 + \overline{q^2} = E^{2a} \left[ 1 + \frac{a^2}{E^2} \overline{e^2} \right] . \] (C17)

Subtracting (C17) from (C16) and neglecting \( q^2 - \overline{q^2} \) and \( e^2 - \overline{e^2} \) results in:

\[ 2Qq = E^{2a} \left[ 2a\left( \frac{e}{E} \right) \right] \] (C18)

which is rearranged using (C13) to give the following equation for instantaneous velocity fluctuation \( q \):

\[ q = aE^{a-1} e = L e . \] (C19)

Considering two wires subscripted A and B, the following response equations are written:

\[ \overline{q^2}_A = L^2_A \overline{e^2}_A \]
\[ \overline{q^2}_B = L^2_B \overline{e^2}_B \] (C20)
\[ q_{AqB} = L_A L_B e_A e_B. \]

The quantities \( L_A \) and \( L_B \) are known from the linearity constants \( a_A \) and \( a_B \) and the mean wire voltages, \( E_A \) and \( E_B \). The squares of the wire rms signals give \( e_A^2 \) and \( e_B^2 \). The mean product, \( e_A e_B \), is obtained by using the sum and difference circuits on the experimental correlator and is evaluated from:

\[ e_A e_B = \frac{1}{4} \left[ (e_A + e_B)^2 - (e_A - e_B)^2 \right]. \] (C21)

Equations (C20) are then used as the inputs to the linearized set of Reynolds stress equations to be solved, as developed in part 1.
Appendix D. Method of Least-Squares Fitting The Reynolds-Stress Equations

The system of equations obtained from the reduced fluctuating voltage measurements can be expressed in summation form as:

\[
\sum_{j=1}^{6} A_{ij} X_j = B_i \quad i = 1, n \quad (D1)
\]

where \(X_j\) are the six Reynolds stresses, \(A_{ij}\) are the coefficients in the linear equations, and \(B_i\) are the appropriate \(q_A^2, q_B^2, q_A^2 q_B\) quantities. For \(N\) probe angular settings \(n = 3 \times N\) equations are obtained.

We wish to minimize the sum of the squares of the 'errors' predicted by each equation:

\[
\sum_{i=1}^{n} \left( \sum_{j=1}^{6} A_{ij} X_j - B_i \right)^2 \quad (D2)
\]

with respect to the unknown quantities \(X_k\), \(k = 1, 6\). This is done by setting the derivative with respect to each \(X_k\) equal to zero:

\[
\frac{\partial}{\partial X_k} \sum_{i=1}^{n} \left( \sum_{j=1}^{6} A_{ij} X_j - B_i \right)^2 = 0 \quad (D3)
\]

\[
\sum_{i=1}^{n} \left( \sum_{j=1}^{6} A_{ij} X_j - B_i \right) A_{ik} = 0 \quad k = 1, 6
\]
Reversing the order of summation, this becomes:

\[
\sum_{j=1}^{6} X_j \left( \sum_{i=1}^{n} A_{ik} A_{ij} \right) = \sum_{i=1}^{n} A_{ik} B_i \quad k = 1,6 \quad (D4)
\]

which is alternatively written as:

\[
\sum_{j=1}^{6} A'_{kj} X_j = B'_{k} \quad k = 1,6
\]

where

\[
A'_{kj} = \sum_{i=1}^{n} A_{ik} A_{ij} \quad (D6)
\]

\[
B'_{k} = \sum_{i=1}^{n} A_{ik} B_i \quad k = 1,6 .
\]

Hence, a system of six equations in six unknowns has been generated from the system of n equations and six unknowns.
Appendix E. Pipe Flow Relations

i) Calculation of Wall Shear Stress from Pressure Drop

For fully-developed incompressible pipe flow, there is no radial pressure gradient, and the forces on a control volume are as shown in Fig. E1. The axial pressure gradient and the wall shear stress are often assumed constant, resulting in:

\[ P(a^2) - (P + \Delta P)(a^2) + \tau_o(2\pi aL) = 0 \]  \hspace{1cm} (E1)

where the pipe radius is \( a \) and \( \Delta P \) is the static pressure drop (taken as positive) along length \( L \). The wall shear stress is then solved for as:

\[ \tau_o = \frac{a\Delta P}{2L} \]  \hspace{1cm} (E2)

resulting in the shear velocity:

\[ U_\tau = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{a\Delta P}{2\rho L}} \]  \hspace{1cm} (E3)

The density, \( \rho \), and pressure drop, \( \Delta P \), are measured parameters.

ii) Shear Stress Distribution Across the Pipe

In fully-developed incompressible pipe flow, a force balance on a differential annulus of flow results in
Fig. 8.1. Forces on Control Volume in Fully-Developed Pipe Flow
\[
\frac{\partial}{\partial r}(r \tau_{rz}) = \frac{\partial p}{\partial z} . \tag{E4}
\]

The axial pressure gradient is assumed constant, such that

\[
\frac{\partial p}{\partial z} = -\frac{\Delta p}{L} . \tag{E5}
\]

Integrating (E4) from 0 to \( r \) and using (E5) gives:

\[
\tau_{rz}(r) = -r \frac{\Delta p}{2 L} \tag{E6}
\]

which can be divided by the result at \( r = a \) (\( \tau_{rz}(a) = -\tau_o \)) to give

\[
\frac{\tau_{rz}(r)}{\tau_o} = -\frac{r}{a} . \tag{E7}
\]

The shear stress, \( \tau_{rz} \), is expanded into laminar and turbulent shear stresses as:

\[
\tau_{rz} = \mu \frac{\partial u_z}{\partial r} - \rho \frac{u_r u_z}{r} \approx -\rho \frac{u_r u_z}{r} \tag{E8}
\]

where the laminar shear stress is small compared to the turbulent shear stress. Substituting (E8) into (E7) and using \( u_r^2 = \tau_o / \rho \) results in

\[
\frac{u_r u_z}{u_r^2} = \frac{r}{a} \tag{E9}
\]
which shows a linear variation of \( \frac{u_r u_z}{U^2} \) across the pipe (from \( r = a \) to \( r = -a \)).
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