Analytical Method for Turbine Blade Temperature Mapping
to Estimate a Pyrometer Input Signal

by

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(ABSTRACT)

The purpose of this thesis is to develop a method to estimate local blade temperatures in a gas turbine for comparison with the output signal of an experimental pyrometer. The goal of the method is to provide a temperature measurement benchmark based on a knowledge of blade geometry and engine operating conditions. A survey of currently available methods is discussed including both experimental and analytical techniques.

An analytical approach is presented as an example, using the output from a cascade flow solver to estimate local blade temperatures from local flow conditions. With the local blade temperatures, a grid is constructed which maps the temperatures onto the blade. A predicted pyrometer trace path is then used to interpolate temperature values from the grid, predicting the temperature history a pyrometer would record as the blade rotates through the pyrometer line of sight. Plotting the temperature history models a pyrometer input signal.
Acknowledgements

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Thanks also to my teachers and committee members, Dr. Henry Wood and Dr. Hal Moses. Dr. Wood’s office is the best learning spot in Randolph, I hope I didn’t take up too much of his time there.

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I also owe thanks to my co-workers who helped directly with this work; thanks to , and and

, though both wacky and zany, were invaluable in criticizing my writing.

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Introduction

The benefits that radiation pyrometers can provide for gas turbine operation forecast their use as an integral part of many engines in the future. The primary benefit of a pyrometer for monitoring turbine blade temperatures is that it is a non-contacting, accurate feedback for fuel control systems, providing immediate performance improvements by allowing a given turbine to operate closer to its potential. Pyrometers can be used for condition monitoring as well. For example, the information gained can be used for better prediction of the need for hot-section overhauls. Pyrometers can also be used to detect abnormally hot blades, perhaps caused by blocked cooling passages, in time to prevent an actual failure. Measuring blade temperature directly is an additional advantage, especially in an era of cooled turbine blading.

There is a current need to adapt experimental pyrometers to production engines and to prove their performance and reliability. This thesis work is part of a pyrometer research program by the Center for Turbomachinery and Propulsion
Research at Virginia Tech, supported by the Rosemount Aerospace Division. Durability and lens optics cleansing are primary development areas being investigated. The key to the acceptance of the pyrometer, however, is in proving the accuracy of its temperature measurement. The focus of this paper is the temperature accuracy issue.

The current need is to provide a benchmark for local blade temperature. To prove the accuracy of the pyrometer it is essential to have an alternative temperature indicating technique for comparison with the pyrometer-measured temperature signal. Pyrometer input signals for an operating gas turbine must be determined to evaluate pyrometer performance such as accuracy, response time, and precision.

The ideal comparison model would be an accurate map of the turbine blade temperature at a given operating condition. The temperature at any given point on the blade, in coordinates of percent axial chord vs. percent blade height provides the basis for modeling an input signal. A geometric analysis, albeit a complex one, could determine the line (or swath) the pyrometer focus spot traces across the blade as the blade rotates through the line of sight of the detector. Transforming the trace path into coordinates of percent axial chord and percent blade height, and then plotting the corresponding blade temperatures would provide an excellent input signal model.

The extremely harsh environment of the turbine rotor, with its high temperatures, corrosive gases and severe centrifugal stresses, makes measurement of the local blade temperature difficult. There are several methods of temperature in-
ication available today, including intermediate turbine temperature (ITT) gages, thermal paints and thermocouples. These methods may help evaluate pyrometer performance, but each has its shortcomings.

An ITT gage allows engine operators to watch for hot starts and to judge optimum throttle position. A typical gage derives ITT by using an analog thermocouple circuit to add three times the temperature rise across the fan to the exhaust gas temperature (EGT), based on the assumption that the temperature drop through the two low pressure turbine stages is approximately three times the fan temperature rise (for a 3 to 1 bypass ratio engine) [1]. The ITT gage, though relatively inaccurate, is a good general indicator of the temperature conditions within the turbine and provides an ample safety margin in a relatively simple, low cost manner. However, this method estimates average gas temperature, rather than blade temperature, and thus is of limited use in estimating blade temperatures.

An engine may be satisfactorily operated with this type of gage, but for pyrometer development it is only a general indicator of turbine temperature. Unfortunately, it cannot be used to directly estimate the temperature distribution on a turbine blade.

Thermal paints indicate blade temperature directly, but are limited in application. The paints change either color or luster at a certain temperature. Available paints are limited to 14 to 28 degree (C) increments and can only be checked in post-run inspections. It is planned to use thermal paints as an additional check, but they are impractical for real time measurement.
Imbedding thermocouples into turbine blades is another alternative which others have tried for research and development purposes [2]. This approach is practical in wind tunnel test rigs, but poses several difficulties in an actual gas turbine installation. Instrumented blades are costly to produce and compromised in strength. Their questionable reliability is a problem due to the additional expense incurred when their replacement requires a hot section tear-down. Thermocouples are also sensitive to conduction problems as they are composed of materials different from those of the turbine blades [2]. Furthermore, the slip ring system for transmitting thermocouple voltage from the rotor to the controller poses severe problems owing to the high wheel velocities in many turbines. This latter problem may be eliminated with further development of the Fiber Optic Rotating Joint [3], however this may also prove costly and fragile.

Conventional measurement techniques are currently incapable of providing a reasonable comparison for a pyrometer in an actual engine. Thus an analytical approach seems to be a most promising avenue along which to pursue turbine blade temperature research.

With cascade flow solvers becoming readily available (see Literature Review) [4,5], it is logical to take an aerothermodynamic approach to estimate local blade temperature from local flow conditions. The following section outlines the aerothermodynamic procedure, and subsequent sections describe an example procedure applied to a current research project.
Literature Review

The literature supporting this work comes from several branches of gas turbine research. Papers concerning radiation pyrometry, turbine blade heat transfer, and cascade flow solvers all helped focus and support this work.

The early pyrometer papers focused on the theory behind pyrometer applications, possible benefits, and early designs for gas turbine pyrometers. Barber’s paper of 1969 is one of the best of the early papers. Radiation theory, design requirements and early problems are all covered. Especially noteworthy is the fact that radiative energy emitted by a body ideally varies by the fourth power of temperature, hence the accuracy potential of the radiation pyrometer is high indeed. Initial pyrometers were strictly analog systems, fuel cooled, and designed to measure average blade temperature for the entire disk [6].

Advances in pyrometer systems in the seventies were dramatic. The frequency response of detector systems increased to the point where rough individual blade temperature profiles could be monitored. The introduction of dual
spectrum pyrometers allowed emissivity and reflection effects to be accounted for in signal processing circuits. Atkinson and Strange present the radiation theory behind dual spectrum pyrometry and confirm the theory with experimental data in their report [7]. With modern filtering systems, hot particle flashes could be rejected. In addition, fiber optics allowed the silicon transducer to be removed from the hot section area and eliminated the need for external cooling systems. Frequency response and necessary signal processing techniques for a detection system based on considerations of optical efficiency, spatial resolution, temporal resolution, and temperature range are detailed in the extensive work of Douglas [8]. Benyon’s paper neatly summarizes the 'ground rules' for a pyrometer system design and installation for use by new and prospective users of turbine pyrometry [9]. The radiation pyrometer has been proven under laboratory conditions, but the need for further development and testing of advanced systems for in-flight use in actual engines is apparent.

Most turbine blade heat transfer work is concentrated on cooled blades. Several heat transfer papers were useful in confirming the adiabatic wall assumption and for suggesting future improvement areas. These papers are referenced in the text where used.

A look at cascade flow solver literature is valuable for comparing the output of the available solver to results for similar geometries in published material. Holmes and Tong apply their solver to turbine blades and present results and comparisons to experimental data that are similar to the results obtained in the present work [5]. A paper presented by Denton and others also presents relevant
results (both experimental and computational) and discusses the nature of turbine passage shocks [10]. The results of these papers helped give confidence in the flow solver output of this project.

The reviewed material is just a small sample of the flow solvers available today. The abundance of computational flow solvers are an indication that the solvers necessary to carry out the following temperature indication scheme are readily available. Other computational fluids papers highlighted problems with the trailing edge flow and outlined solution techniques that were adopted for this work. These are also referenced as they are used in the text.
Aerothermodynamic Approach

The Literature Review contains references to several examples of the cascade solvers currently available. The codes available today are capable of handling transonic two-dimensional flows with a good degree of accuracy and relatively short computer run times. A typical cascade solver takes geometric boundary conditions and inlet aerothermodynamic data (inlet velocities, temperatures, and flow angles) and solves for the velocity distribution within the cascade. Some solvers also need exit conditions to obtain the interior solution. The velocity field can then be used to estimate wall temperature.

All solvers require blade geometry as input. By breaking the two-dimensional channel that models the rotor into a reasonable number of finite areas for numerical analysis, smoothness is sacrificed to keep run times reasonable. A turbine airfoil broken into fifty axial points per surface limits accurate modeling of leading and trailing edge details, but provides an overall velocity distribution with acceptable accuracy. Blade geometry must be given by the manufacturers,
measured from an example blade, or estimated in some manner. Of course, a knowledge of the blade shapes is also essential to calculate the pyrometer trace path.

Any geometry available must be rotated into a coordinate system compatible with the solver code used. The same must also be done with aerothermodynamic boundary conditions.

Matching cascade solver results to conditions within the actual gas turbine poses an additional problem. Possessing cycle data for the engine for a variety of operating points alleviates much of the problem. With this data it is only necessary to determine the operating point of the actual test engine, and to correct for current ambient conditions. Otherwise, several assumptions must be made. Given design conditions, off-design conditions must be either calculated from a cycle analysis or estimated based on information from conventional engine instrumentation and several assumptions. The assumptions of constant gas angles for the turbine inlet nozzles and constant relative exit angles for the rotor, allow the off-design stage temperature ratio to be calculated using an analytical method [11]. This method and experimental data are combined to carry out an iterative procedure that satisfies the velocity triangle constraints, the conditions of continuity, and the choked turbine non-dimensional flow rate. The details of this procedure are discussed in a following section presenting off-design work.

Accurate passage flow modeling must include the variations in flow conditions from blade root to tip. This information may also be provided, or it can...
be estimated using a free-vortex rotor design assumption to calculate the radial
distribution of aerodynamic boundary conditions from given mid-passage data.

Once the boundary conditions are decided and velocity solutions are com-
puted, a local wall temperature distribution can be derived from the velocity field.
The preferred method would be an interactive boundary layer solution that
produced wall temperature directly, based on an assumed wall heat flux bound-
ary condition. Most available codes are still inviscid solvers, thus a temperature
recovery scheme is a logical approach to the problem. The simplest method is the
adiabatic wall recovery relation with a constant recovery factor.

Calculating the local blade temperatures based on the local flow variables for
each of the cross sections leads to a set of surface blade temperature profiles lay-
ing on the blade axially at various heights corresponding to the cross sections fed
into the flow solver. Transforming the cross section blade heights into non-
dimensional coordinates of percent blade height, and also, the axial temperature
points into percent axial chord, creates a temperature grid that maps onto the
blade surface.

A geometric analysis on a computer-aided design (CAD) system can deter-
mine the coordinates of the pyrometer trace path in the same system of percent
blade height and percent axial chord. The calculated intersection coordinates can
then be used as the input to interpolate for the temperatures the pyrometer
would see during a blade passage.
The pyrometer line-of-sight coordinates in terms of time can call blade temper- 
atures from the grid map to indicate pyrometer temperature input vs. time for 
a given operating condition.

The following chapters briefly describe the pyrometer project and outline the 
aerothermodynamic procedure used for the project. Some of the problems en- 
countered may be unique to this project, but the application of the procedure 
demonstrates the feasibility, merit, and limitations of the method, while at the 
same time showing the theory and modeling necessary to carry out the entire 
procedure.
Current Research Program

Pyrometer development for gas turbine applications has progressed to the point where testing in actual engines is now essential (see Literature Review). The Center for Turbomachinery and Propulsion Research at Virginia Tech is currently involved in a pyrometer installation research project. The project entails installing a pyrometer in a gas turbine and extensively testing and developing the pyrometer system for future use as a control feedback element.

The Pratt & Whitney JT15D-1A turbofan currently occupying the Virginia Tech Airport test cell serves as an excellent pyrometer test bed, in part because its uncooled first stage turbine lends is well-suited to analytical temperature estimates. If the pyrometer proves accurate on the uncooled blades, it can then be confidently applied to cooled blades.

The JT15D is a small 2000 lb. thrust class, 3.3 bypass ratio, twinspool engine commonly installed in tandem on the Cessna Citation or Aerospatiale Corvette business jets [1]. The first stage turbine of this engine has seventy-one blades and
a design speed of over 30,000 rpm [12]. An engine cross section schematic is shown in Figure 1, and the pyrometer penetration in Figure 2. Unfortunately, the folded burner adds complexity to the installation, and limits direct viewing to only the suction side of passing blades. The line of sight intersects the blade primarily in the mid-section to tip region and is blind to the leading edge. Parallel CAD work is being performed using wireframe surface modeling to determine the exact path the pyrometer sight beam inscribes.

In the adjacent control room, an IBM PC-based data acquisition system supports a conventional aircraft cockpit display of engine operating information. Information logged, such as inlet total temperature and pressure, high and low spool percent speed, compressor discharge pressure, ITT, and fuel flow rate will serve as inputs to locate the engine operating point. The operating point will in turn determine the input to the cascade solver, ultimately leading to a model pyrometer signal. Operating in parallel is an IBM PC AT linked to a LeCroy high-speed analog-to-digital converter dedicated to handling the pyrometer output.

The blade temperature calculation method presented supports the project and is related to data gathered at the facility. The method is unique to the JT15D project in several ways, but illustrates an example of the aerothermodynamic approach outlined previously and provides example results.
Figure 1. JT15D-1 Cross section schematic
Example Procedure - Initial work

To derive blade surface temperature from local flow conditions it is necessary to solve the flow field as accurately as possible. This section details the preliminary work done to take the available information and prepare it for use in the cascade flow solver.

The approach for calculating the turbine blade temperatures is dictated to some degree by the limited resources available. Relative leading and trailing edge aerothermodynamic data for five sections corresponding to 0%, 25%, 50%, 75%, and 100% of the blade height at design conditions are given. Geometric data is available for cross sections at 6% (section A-A), 28% (B-B), 49% (C-C) and 92% (D-D) blade height [13]. Unfortunately, the geometric data is not given in great detail--the pressure and suction surfaces were defined by only seventeen points each.

The two dimensional cascade solver used was written by G. Micklow [14]. The code is a non-orthogonal finite area potential flow solver that is based on the
integral continuity equation for compressible, isentropic flow. The inviscid code is a portion of in his work that will later mesh with a boundary layer code and an unsteady flow code. Though originally conceived to handle slightly transonic compressor cascades, the code was modified to include the mildly supersonic turbine case. Micklow, by referring to the research done for this thesis, was able to stretch and confirm the range of applicability of his solver. Micklow’s code has been confirmed on several geometries given in a paper by Caspar [15]. Micklow also certifies, based on a review of the results, that the solutions are accurate, subject to the inviscid isentropic assumptions and the limited geometric data available.

The next step in the procedure was to model a turbine blade using the thirty-four points given. The original attempt involved the use of a simple cubic spline. However, this method distorted the trailing edges of the blades abnormally. Instead, a parametric spline combined with a judiciously placed circle for the trailing edge radius is used to generate additional points. The low number of points given effectively limits the justifiable number of additional points that may be accurately added by interpolation. Using a finer mesh may lead to a perceived increase in accuracy of the solution; however, the results would be based on geometric boundary conditions that are not necessarily more accurate. This fact, coupled with the desire to keep the computer run times reasonable, leads to a practical ceiling of fifty axial points per surface.

As a result of the limited number of axial points, modeling rounded trailing edges is impossible, and cusps must be used to cap slightly lengthened blades.
Each cusp is located such that the average of the upper and lower slopes nearly matches the trailing edge velocity angle. This method was successfully used in the work of Essers and Kafyeke [16]. Another major criterion for generating smooth blades is to keep slopes monotonically increasing up to the blade surface peak and monotonically decreasing from the peak to the trailing edge point. This condition helps eliminate a wavy surface boundary line that often leads to instability.

Since two of the typically five or seven streamlines between a blade row are the pressure and suction surfaces of adjacent blades, any abrupt changes in the geometry of these surfaces will have important effects. Sharp changes in surface slopes can lead to numerical instability, because of the induced rapid changes in the local flow variables. As a result, acceptable local convergence can be impossible and global convergence both difficult and costly. Future programs, not ready in time for this work, will include the boundary layers developed as streamlines, making the inviscid code less sensitive to blade geometry.

Geometric difficulties center at the leading and trailing edge regions where blade slopes vary rapidly. These changes in slopes cause the grid generator to form overlapping polygons which lead to negative areas and adversely affect convergence. Leading edge effects need be minimized only to help global convergence; the leading edge cannot be seen by the pyrometer. Simply increasing the number of axial points to the practical maximum of fifty best achieves this result. For the trailing edge, convergence is assured by making painstakingly certain that the slopes in the region are strictly monotonic and that the changes
in slope are averaged over several points. This procedure is roughly equivalent to making the second derivative of the surface line smooth.

Once the blade smoothness is satisfactory, it is necessary to match the given geometric cross sections to the available aerodynamic data. Because the blades were designed using a free vortex condition, with a nearly constant relative trailing edge angle, interpolation of proper aerothermodynamic data for the geometric sections introduces little, if any, error. To mesh with the grid generator of the cascade solver it is necessary to rotate all the aerodynamic data and geometry to the proper stagger angle for each geometric cross section.

The aerodynamic data given by the engine manufacturer was computed assuming constant relative total temperature along streamlines through the rotor, which does not conflict with the assumptions of Micklow's code. However, a problem that has to be addressed is the conflict between using an inviscid isentropic code and being given cycle-generated aerothermodynamic data that empirically includes total pressure losses and some underturning.

The solution is to use the known upstream conditions and generate isentropic exit conditions. The energy equation, the continuity equation, and the ideal gas equation of state can be solved for the three unknown exit conditions, given the upstream Mach number, temperature, and velocity angle. These equations do not yield a closed form solution, but rather are used to generate a table for exit velocity angle and temperature based on an assumed exit Mach number.

Starting with the continuity equation:

\[ \rho_1 V_{ax1} A_1 = \rho_2 V_{ax2} A_2 \] (3.1)
The annular entrance area $A_1$ equals the annular exit area $A_2$ for the rotor, and velocity angles can be introduced to obtain:

$$\rho_1 V_1 \cos \alpha_1 = \rho_2 V_2 \cos \alpha_2$$  (3.2)

Introducing the ideal gas equation of state to solve for $\rho$ and the definition of Mach number leads to:

$$\frac{P_1}{RT_1} M_1 \sqrt{\gamma RT_1} \cos \alpha_1 = \frac{P_2}{RT_2} M_2 \sqrt{\gamma RT_2} \cos \alpha_2$$  (3.3)

cancelling and grouping,

$$\cos \alpha_2 = \frac{P_1}{P_2} \frac{\sqrt{T_2}}{\sqrt{T_1}} \frac{M_1}{M_2} \cos \alpha_1$$  (3.4)

then using the isentropic relation $\frac{P_2}{P_1} = \left[ \frac{T_2}{T_1} \right]^{\frac{\gamma}{\gamma-1}}$ yields:

$$\cos \alpha_2 = \frac{M_1}{M_2} \left[ \frac{T_1}{T_2} \right]^{\frac{\gamma}{\gamma-1} - 1/2} \cos \alpha_1$$  (3.5)

Assuming $M_2$, the energy equation ($T_{total} = \text{constant}$) can be solved for $T_1$ and $T_2$ using the relation:

$$T = \frac{T_{total}}{1 + \frac{\gamma - 1}{2} M^2}$$  (3.6)
Substituting Eq. (3.6) into Eq. (3.5) allows a table based on assumed $M_2$ to be created. The table consists of a list of $M_2$ values and the corresponding isentropic exit velocity angles and exit local gas temperatures. The most logical choice is the solution with the exit Mach number that matches the given design exit Mach number. As would be expected, the turning angle is a degree or so greater for the isentropic case, but the cascade solver has better convergence using these values. In initial work, the flow modeled was trying to turn in the wake to match downstream conditions.

At this point all preliminary work to run the cascade solver is complete. The blade smoothing procedure and isentropic exit condition calculations are performed for each of the four geometric cross sections. The next section describes the use of the output of the cascade solver and the initial processing of the output data.
The cascade solver produces aerodynamic data corresponding to a grid which divides the cascade passage into an array (typically seven axial lines by thirty-five tangential lines). Figures 3 through 6 show the grid for each of the four cross sections available. Each point corresponds to the center of a polygon in the computational fluid dynamics code. For each point the associated axial velocity, tangential velocity, and relative Mach number are fed into arrays for further manipulation.

The arrays are used to compute both the axial and tangential components of the Mach number. By combining these components with the blade geometry, scaled Mach number vector field plots are created for the entire grid. The Mach number field plots for the four blade sections are shown in Figures 7, 8, 9, and 10. The length of each arrow corresponds to the magnitude of the local Mach number and the arrow orientation indicates the local flow direction. These plots include a cross mark for any vectors corresponding to a Mach number greater
Figure 3. Grid for section A-A (6% blade height)
Figure 4. Grid for section B-B (28% blade height)
Figure 5. Grid for section C-C (49% blade height)
Figure 6. Grid for section D-D (92% blade height)
Figure 7. Mach number vector field plot for section A-A (6% height)
Figure 8. Mach number vector field plot for section B-B (28% height)
Figure 9. Mach number vector field plot for section C-C (49% height)
Figure 10. Mach number vector field plot for section D-D (92% height)
than one. The corresponding printed data from the solver shows the supersonic Mach numbers are higher as the blade tip is approached.

The Mach number vector plots are convenient for visualizing the output of the code. Whether the flow is qualitatively correct can be readily determined by looking at the plots. The plots also help confirm the validity of using isentropic exit conditions and sharp trailing edges. Though in reality separation occurs on blunt trailing edges, the inviscid code adheres to the contour. Therefore, as stated above, the cusped model of the trailing edges produces both better Mach number results and better convergence for an inviscid cascade solver. The plots also show the relative acceleration through the rotor and give a qualitative indication of shock regions.

A second set of plots produced for code development and future use are graphs of Mach number vs. gridline. The plots of Mach numbers along the streamlines through the channel are helpful in locating shock regions and are later used in the blade temperature programs. Mach number plots for the twenty to fifty potential lines crossing the streamlines both verify the uniform inlet and exit conditions and offer further insight into shock formation. Collectively, these plots also help make clear which effects result from geometry problems and which result from the aerodynamic boundary conditions.

Graphs of Mach number vs. percent axial chord along a streamline reveal some interesting points that become useful in the final temperature estimating scheme. The streamline plots for section D-D (92% height) are shown, as representative examples, in Figures 11 and 12. The numbers in the upper right hand
Figure 11. Mach number vs. percent axial chord for section D-D streamlines
Figure 12. Mach number vs. percent axial chord for section D-D streamlines
corner of each graph count the streamlines across the cascade from the suction surface to the pressure surface. The suction surface streamline corresponds to 1, the midstream line to 4, and the pressure surface to 7. (see Figure 10)

The streamline along the suction surface, the one of primary interest, is the least stable streamline. The imperfect blade smoothness is evident. A shock is predicted at 42% axial chord. The shock is localized to the region near the surface and may be emphasized because of geometric effects. It certainly weakens as mid-passage is approached and may be strictly local; in any case it is not very strong. At these low Mach numbers an isentropic shock model is valid. However, shocks of this nature are present in every cross section studied. The shock is weaker and occurs slightly further downstream as the hub is approached, indicating a part-passage three dimensional shock. Shocks are smeared and weakened as the number of axial divisions used in the cascade solver is decreased.

The sharp drop in Mach number at the trailing edge of the suction surface streamline is caused by the inviscid code following the trailing edge cusp. The pressure surface streamline plot (number 7) of Figure 12 shows a sharp drop in Mach number just before the trailing edge. The Mach number vector plot for this cross section, Figure 10, provides the explanation with reference to the vector near the trailing edge of the pressure surface. The inviscid flow follows the trailing edge contour, producing an unrealistically low velocity data point. The trailing edge cusp for section D-D is not as sharp as for the other sections, so this section represents a worst case. This problem clearly illustrates how pressure side
effects are numerically communicated to the suction side in computational modeling.

When defining the free stream boundary conditions for the temperature recovery routine, some of the mid-passage streamlines will be used to smooth the surface line values. The smoothing helps to eliminate unwanted effects of the inviscid assumption and lessens the effects of the rough blade surface contours. This technique is outlined in the following section.
Estimating Blade Temperature

The heart of the blade temperature estimation in this work is the temperature recovery relation for an adiabatic wall. In terms of the adiabatic wall temperature, the stagnation temperature, and the static temperature, the recovery factor is expressed as [17]:

\[
r = \frac{T_{aw} - T_{\infty}}{T_0 - T_{\infty}} \quad (5.1)
\]

The cascade solver assumes \( T_0 \) is constant, so this value is known for each cross section. Assuming an ideal gas, and given the local Mach number produced by the cascade solver, the local static temperature can be easily calculated using the relation:

\[
T_{\infty} = \frac{T_0}{1 + \frac{(\gamma - 1)}{2}M^2} \quad (5.2)
\]
It is assumed that $\gamma = 1.33$ for the temperature range encountered in the turbine. This assumption is confirmed by a look into the air tables, as $\gamma$ varies from only 1.326 to 1.336 in this application [18]. A variable ratio of specific heats can be added to the code if greater temperature ranges are met.

The major problem with the adiabatic wall relation is knowing the proper value for $r$. Values range from 0.83 to 0.91 for flat plates, with the higher values applying to turbulent flow [17]. The recovery factor work of Kopelov and Gurov includes a study of experimental variation of $r$ with Reynolds number on turbine blades. They found that the recovery factor went up with Reynolds number and leveled off at a Reynolds number of $2 \times 10^5$ at values bounded by 0.78 and 0.95, with a mean of 0.87 [19]. The nominal chord-based Reynolds number for the blade is $5 \times 10^5$. Thus turbulent flow is assumed, especially since the leading edge is not a problem here. Assuming a constant $r = 0.89$ is reasonable, recalling the favorable pressure gradient in the stage. Consigney and Richards also used $r = 0.89$ in their turbine rotor heat transfer work with good results [20]. Fortunately, the equation is forgiving with respect to the $r$ value, as a 10% error in $r$ leads to about a 1% error in the value of $T_{aw}$ in this range, as:

$$T_{aw} = r(T_0 - T_\infty) + T_\infty$$  \hspace{1cm} (5.3)

Another judgement comes into play concerning the choice of local Mach number to use in calculating local temperature, $T_\infty$. The trouble stems from both the absence of boundary layer effects and from the local problems that arise due to digitized geometry. The most critical adjustment is to correct for the less
sharply cusped trailing edge of section D-D. Figure 13 shows how the slopes of the streamlines near the pressure surface change abruptly as they leave the cascade. Note the two streamlines closest to the pressure side trailing edge break upward substantially in a single axial step, following the trailing edge cusp. The solver turns the flow to follow the geometry faithfully, causing the unrealistic flow pattern mentioned previously. The grid outside the cascade has no geometric boundary conditions to induce turning, so no error is induced in this region. The imposed condition of periodicity that is necessary in an infinite cascade solver causes the suction side trailing edge to feel the effects of the pressure side trailing edge. In this region, geometric problems are compounded. The suction side trailing edge streamline has a distinct Mach number decrease as shown in plot 1 of Figure 11, even though the suction side surface is relatively smooth. The error introduced by using mid-passage streamlines as part of the free stream boundary in the trailing edge area is less than the geometrically-induced inviscid error inherent in the surface streamlines.

The method chosen to include the mid-passage streamlines uses the first four streamlines in percentages that vary from suction to mid-passage and according to percent axial chord ($X/C$). Smooth polynomials are used that include gradually more mid-stream information as the trailing edge is approached. This smoothing is done in a FORTRAN program loop of the temperature estimating program. The corrected Mach number plot for section D-D, for example, was calculated using information from the first four streamlines in fractions that varied as follows:
Figure 13. Trailing edge grid lines for section D-D
\[ \text{PART4} = 0.3 \times (X/C)^5 \]
\[ \text{PART3} = 0.25 \times (X/C)^4 \]
\[ \text{PART2} = 0.25 \times (X/C)^4 \]
\[ \text{PART1} = 1.0 - (\text{PART4} + \text{PART3} + \text{PART2}) \]

\( \text{PART1}, 2, 3, 4 \) correspond to the fraction of the respective streamlines used to define the free-stream Mach number for the corrected Mach number profile. The relevant Mach number streamlines range from 1 for the suction surface to 4 for the mid-passage streamline. This method corrects primarily the trailing edge data and allows the suction surface streamline to more nearly match the conditions which a viscous solution would provide. The simple fourth and fifth power polynomials cause negligible correction to be made before 50% axial chord. At this point over 95% of the surface streamline information is retained \( (\text{PART1} > 0.95) \). At 75% \( X/C \), about 7% each of the second to fourth streamlines are used, still retaining greater than 75% of the surface streamline information. This drops quickly to about 20% surface information at the trailing edge. The corrected Mach number profiles for all four cross sections are shown in Figures 14 and 15. Especially note the corrected Mach number plot for section D-D (section 4), as compared to the four uncorrected streamline plots for section D-D of Figure 11; the Mach number drop at the trailing edge is restricted while retaining the nature of the original curve. As a guideline in choosing the constant factors for the streamline fractions, an attempt was made to match the trailing edge Mach number to the given surface line value. Sections A-A, B-B, and C-C were smoothed in the same manner, however the polynomial constants varied to
include less midstream data as section D-D was the worst case. Actually, the smoothing method employed for developing the corrected Mach number curves of Figures 14 and 15 has little effect on the original curves, except near the trailing edge of the blades, where the unrealistic acceleration is reduced.

Once a satisfactorily smooth $M_\infty$ curve is created, a $T_\infty$ curve is computed for each of the four cross sections using the adiabatic wall recovery factor equation (5.3). As each cross section may have a different number of axial points, any of the axially less-populated curves are expanded to fifty axial points to create four fifty-point temperature lines. The right hand plots of Figures 14 and 15 consist of the temperature lines for each of the cross sections. The four lines form a four by fifty temperature grid on the blade surface. This array is used as a data base from which the trace path calls values. Each point in the array corresponds to a unique value of percent axial chord and one of the four possible percent blade heights in non-dimensional coordinates.

Ideally, the exact path the pyrometer reception 'spot' sweeps out along a blade in non-dimensional coordinates as the blade passes would be known in the non-dimensional coordinates. This is a rather complicated geometric problem being solved in a parallel effort using the Virginia Tech Mechanical Engineering Computer Aided Design Facility. Preliminary results from the CAD work of Williams [21] shows the path can be modeled by the curve $Y = .25 \times \left( \frac{X}{C} \right) + .611$ (with $Y$ being the percent blade height) shown, along with the positions of the four cross sections of available temperature information, in Figure 16. To generate the model signal, fifty points from the model
Figure 14. Corrected Mach number and Temperature profiles for A-A and B-B

Estimating Blade Temperature
Figure 15. Corrected Mach number and Temperature profiles for C-C and D-D
pyrometer intersection curve are used to linearly interpolate temperature values from the two nearest cross sections using axially corresponding points of the array. A model pyrometer input signal for design conditions (95% speed) is represented in Figure 17. This result is a culmination of all the previous curves and should prove to be a valuable tool in evaluating pyrometer performance.

The pyrometer input signal model of Figure 17 represents a solution for design point conditions on a test stand. Off-design solutions could be easily produced if the aerodynamic data for the off-design speeds were available. With the lower Mach numbers of the off-speed case, the flow field is simpler for the cascade solver. Convergence time is reduced and shock effects lessen or disappear.

With no off-design conditions given for this stage, they are estimated incorporating several assumptions. To accurately model off-design conditions a complex cycle analysis would have to be done which would require performance maps for the various engine components. Unfortunately this information is probably more difficult to obtain from the manufacturer than off-design stage aerothermodynamic boundary conditions. A less complex method for estimating off-design conditions based on data obtained from engine tests is developed below. The off-design case presented is for an 85% speed case, as compared to the 95% speed (design) case presented previously, that used data given by the engine manufacturer. For free-vortex blading the temperature change across a turbine stage can be expressed by [11]:

---

Estimating Blade Temperature
Figure 16. Model pyrometer trace path curve and available cross sections
Figure 17. Model pyrometer input signal at design (95% speed)
\[
\frac{\Delta T_0}{T_{01}} = \frac{U^2}{c_p T_{01}} \left[ \frac{V_{ax}}{U} (\tan \alpha_1 + \tan \beta_2) - 1 \right]
\] (5.4)

This equation is derived from the velocity triangles and the energy equation for a turbine with constant gas angles for the trailing edge of the inlet nozzle vanes and rotor blades. By canceling, grouping constants, and then simplifying by assuming the domination of the squared term (which is later confirmed) yields:

\[ T_{01} - T_{02} \approx k U^2 \] (5.5)

The value of \( k \) may be determined for each cross section from the given design data. Using measured ITT as the stage exit total temperature \( (T_{02}) \) and the calculated \( k \), yields an off-design upstream total temperature \( (T_{01}) \). The validity of the assumption of using engine measured ITT as \( T_{02} \) was confirmed by extrapolating off-design engine measured data to the design point. For similar ambient conditions, the extrapolated ITT was within 20 degrees (R) of the given design point data value for \( T_{02} \).

From \( T_{01} \), the measured inlet total pressure (compressor discharge pressure), and inlet velocity triangle, an iterative procedure for the inlet aerothermodynamic conditions begins based on an assumed static temperature. The upstream conditions must match the choked turbine non-dimensional flow rate for the machine (Eq. 5.6).

\[ \frac{\dot{m} \sqrt{T_0}}{P_0} = c_1 \] (5.6)
Expanding the mass flow rate term and incorporating area into the constant,

\[
\frac{\rho V_{ax} \sqrt{T_0}}{P_0} = c_2
\]  

(5.7)

Calculating density, and grouping the gas constant with the flow constant puts the equation into the desired form.

\[
\frac{PV_{ax} \sqrt{T_0}}{TP_0} = c
\]  

(5.8)

The iteration begins by assuming an inlet static temperature and calculating the resulting upstream properties as follows. The upstream Mach number is found using Eq. 5.9.

\[
M = \frac{2}{\sqrt{\gamma - 1}} \left( \frac{T_0}{T} - 1 \right)
\]  

(5.9)

The Mach number can be converted to velocity and the axial velocity is found from the inlet air angle. The static pressure can be found using the isentropic relation between the total and static temperatures. A solution is reached when the non-dimensional flow rate for a choked turbine matches the design value.

From the data of Table 1, it can be seen that the mean-line \( \frac{V_{ax}}{U} \) varies only 6% for the speed change assumed, while \( U^2 \) varies by 25%. Thus, it can be seen that the principal variable influencing \( \Delta T_0 \) is the variation of \( U^2 \), justifying the original simplification of Eq. 5.4 to Eq. 5.5.
Table 1. Parameters for Off-Design Temperature Calculation

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<tr>
<th>Section</th>
<th>N</th>
<th>( T_1 )</th>
<th>( V_{rel} )</th>
<th>( U )</th>
<th>( M_1 )</th>
<th>( \beta_1 )</th>
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<td></td>
<td></td>
<td>( ^\circ R )</td>
<td>ft/s</td>
<td>ft/s</td>
<td>( ^\circ psia )</td>
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<td>A-A</td>
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<td>1981</td>
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<td>1260</td>
<td>.4766</td>
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<tr>
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<td>607</td>
<td>1341</td>
<td>.4003</td>
<td>42.39</td>
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<td>1423</td>
<td>.3415</td>
<td>32.04</td>
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<td>1416</td>
<td>.2823</td>
<td>9.54</td>
<td>53.7</td>
</tr>
</tbody>
</table>

Estimating Blade Temperature
Exit conditions are found from the inlet conditions through a similar iterative procedure that includes the assumptions of constant relative total temperature and pressure through the rotor. With the relative exit gas angles assumed constant, a solution is found for the guessed exit static temperature when the conditions for conservation of mass are met. The upstream and downstream conditions then serve as inputs to the flow solver as before.

The off-design Mach number vector plots are shown for the four cross sections in Figures 18 through 21. The inlet angles of attack are higher, and the supersonic regions have become smaller or disappeared, as in the root section case (Figure 18). The streamline Mach number traces for section D-D are shown for the 85% speed case in Figures 22 and 23. As compared to the design point traces of Figures 11 and 12, the off-design flow has greater near-trailing-edge recompression. Trailing edge smoothing is still necessary, especially for section D-D.

The corrected surface Mach number graphs and corresponding temperature profiles are shown in Figures 24 and 25. Again, the off-design traces show more trailing edge recompression and lower Mach numbers. The predicted temperature profiles are similar in nature but are nearly 200 degrees (R) lower. The interpolated model pyrometer signal shown in Figure 26 reflects the recompression region in a flatter temperature trace. The temperature change shown is dramatic for a 10% wheel speed change. When the trace paths for the design and off-design case are plotted together to show their relative emissive power, Figure 27 results. This is a non-dimensional plot of ratio of the emissive power of the
two cases compared to a blade at a uniform temperature of 1800 degrees (R).
This Figure shows the sensitivity potential of a pyrometer.

The off-design results will be useful in pyrometer development, and are easily produced once the aerothermodynamic boundary conditions are known. In the following section possible improvement areas are targeted.
Figure 18. Off-design Mach number vector plot for section A-A (85% speed)
Figure 19. Off-design Mach number vector plot for section B-B (85\% speed)
Figure 20. Off-design Mach number vector plot for section C-C (85% speed)
Figure 21. Off-design Mach number vector plot for section D-D (85% speed)
Figure 22. Off-design Mach number vs. percent axial chord for D-D (85% speed)
Figure 23. Off-design Mach number vs. percent axial chord for D-D (85% speed)
Figure 24. Off-design corrected Mach number and Temperature profiles for A-A and B-B
Figure 25. Off-design corrected Mach number and Temperature profiles for C-C and D-D
Figure 26. Model pyrometer input signal for off-design (85% speed)
Figure 27. Relative emissive power comparison between 85% and 95% speed
Conclusions

The results of this thesis give qualitative and quantitative views of a pyrometer signal not readily available previously. The goal of producing a model signal was successfully met. At this point the details of the model signal will be impossible to capture with the pyrometer system. For this application, with a high blade passing frequency and moderate temperatures, the hope for distinctly separated blade signatures is weak. Even a pyrometer system with a 1MHz sampling frequency capability could only capture about five target spot points per blade. The path the pyrometer sight beam traces (see Figure 16) starts at the leading edge at mid-span and sweeps toward the tip region at the trailing edge. The trend for blade temperatures is to increase as blade height increases and to decrease as percent axial chord increases. These trends seem to cancel out to some degree for this trace path so the temperature input signal will tend to be rather flat. The shock structures may also be difficult to capture by the
pyrometer because of the unsteadiness of shocks and the temperature smearing caused by conduction.

While there is room for improvement in the aerothermodynamic approach, it already provides a model signal with greater detail than current pyrometers can capture. With better boundary conditions, both aerodynamic and geometric, this procedure should yield a model signal with sufficient accuracy to evaluate the performance of experimental pyrometer systems.
The work in this paper represents a preliminary method for estimating turbine blade temperatures from the flow conditions. There are several areas where improvements can be made in continuation of the project. This work used a two-dimensional code, but three-dimensional solvers are becoming prevalent and offer additional accuracy [4,5]. Much of the set-up work will be simplified when codes with interactive boundary layer methods are available. Solutions will be more accurate and geometric effects will be minimized. Advanced codes may soon be able to handle separation bubbles and the wake flow [22].

Including heat transfer effects is another area targeted for study. Certainly for any cooled blade stages, the code must take the cooling flows into account. Even the uncooled blades of this study have a certain degree of conduction to the turbine disk and from the suction to pressure side, that if included in the analysis, would improve the results further. Axial conduction and the unsteadiness of shock attachment may also smooth the temperature profiles to some degree. Se-
veral conduction models are presented by Maccallum which model the thermal response of blading to engine acceleration if modeling transients is desired [23]. The work of Brown and Martin suggests that the effects of secondary flows may increase heat-transfer coefficients, and should be included in rigorous analytical projects [24].

A detailed cycle analysis would improve the off-design aerothermodynamic boundary conditions. However, additional data from the manufacturer may be easier to obtain. The effects of ambient conditions on the turbine temperature may also be handled in a more sophisticated manner than a non-dimensional flow parameter. Variable specific heats may be added to the program to further improve accuracy, or to handle several different stages. In a case where substantial laminar regions exist, a relationship between the recovery factor and Reynolds number should improve the accuracy of the results, although at low Mach number recovery factor errors are inherently minimized as the difference in total temperature and adiabatic wall temperature is small.

An area that needs future attention is improvement in matching the actual engine operating point to the aerothermodynamic procedure to compare the experimental to the analytical data. High pressure turbine wheel speed, ambient conditions, ITT, and compressor discharge pressure are available and should be adequate for initial comparisons. Including fuel flow rate may be a useful addition to the input as an indicator for total temperatures.
The number of the suggested improvements to the procedure that are incorporated into a program will depend on the accuracy desired and availability of inputs and cascade solvers.
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