Development of a taper equation for *Pinus oocarpa* Schiede in natural stands of central Honduras

by

Darlin Noe Perez Regalado

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APPROVED:

Harold E. Burkhart, Chairman	Timothy G. Gregoire
John A. Scrivani	Charles T. Stiff

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(ABSTRACT)

Nine taper equations were tested to predict diameters inside bark along the stem for *Pinus oocarpa* Schiede trees growing in natural stands of central Honduras.

A five parameter submodel predicted as well as an eight parameter model proposed by Kozak, 1988. Taper variation was explored between two geographic regions from which trees with different taper were suspected. Results showed that different site classes, not fully accounted for in the model, might have an effect on the prediction of taper in each region. Also, the effect of crown class and live crown ratio on prediction was evaluated. The model selected exhibited different prediction patterns for dominant and suppressed trees. On the other hand, live crown ratio did not appear to affect prediction

A computer program was written to use the taper equation developed to compute total and merchantable volume to different top diameter limits.

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Chapter I

INTRODUCTION

Since the early nineteenth century, forest researchers have looked for methods to express tree form and taper in a meaningful and practical way. These extensive efforts are justified due to the fact that many other tree characteristics of interest to the forest manager are highly dependent on, and closely related to, the form of the main stem. Some of these include stem volume, bole surface area, height to a specified diameter, and diameter to a given commercial height.

As time elapsed, many empirical observations gave rise to the general assumption that different sections of the tree stem closely resemble the form of well known geometric solids (Husch et al. 1982). With this notion in mind, the development of techniques to examine tree form have steadily progressed from graphical analysis to numerical methods (Gray, 1956). During the last two decades, the latter approach has evolved from the use of simple equations to more complex segmented polynomial models, which have been intended to describe the taper of different tree sections separately.

The general trend has been the development of mathematical curves as the principal representation of stem form. To date, one of the aspects widely recognized by most researchers is the extreme variation in tree form which makes it difficult to formulate general rules readily applicable to a single species, or even to all the stems in a single stand (Larson, 1963).

The purpose of this study is the development of a simple taper equation based on the underlying theories proposed for describing the form of excurrent tree stems. In particular, this study explores in detail the development of a taper equation for trees of *Pinus oocarpa* Schiede, naturally growing in the central region of Honduras, Central America. Although Honduras has both subtropical and pine forests, most of the management activities are concentrated in the pine forest, and wood exports are almost entirely of softwood species. *Pinus oocarpa* Schiede is the most important pine species in the country, both in terms of its geographic distribution and economic contribution to the national economy.

To date, no research has been done on taper relationships for the species, and therefore the results of this study are expected to be directly useful in the current implementation of national or regional forest management activities.

Objectives:

The objectives of this research were to:

1. Develop a single unbiased taper equation for *Pinus oocarpa* Schiede in natural stands of central Honduras.
2. Evaluate the accuracy and precision of the fitted equation by comparison with other equations found in the literature.
3. Investigate taper variation with changes in geographic region and individual tree characteristics.

Chapter II

LITERATURE REVIEW

The increasing utilization of tree boles of different sizes as raw material for many types of wood products has stimulated the rapid development of techniques to express tree taper. Taper equations describe the average stem profile of a forest species in terms of easily measured tree variables, and in doing so supply users with the most complete information regarding tree form (Meyer, 1953; James and Kozak, 1984). The approach is attractive because it can provide reliable estimates of tree dimensions for a mix of products, without the need to collect additional data even in the event of changes in utilization standards (Amidon, 1984; Brink and Von Gadow, 1983).

In addition, taper equations are very useful to forest managers because most of them can be integrated to provide estimates of total or merchantable volume to any specified top diameter. The estimates of volume generated in this way might not be identical to those derived from volume equations, but they can still yield accurate results (Cao et al., 1980; Walters and Hann, 1986). Also, taper equations are very flexible allowing easy transformations to yield commercial heights and upper diameters based on commonly collected tree measurements (Byrne and Reed, 1986). As a result, the study of tree taper has received a great deal of attention through the years.

As cited by Larson (1963), the earliest studies on the distribution of growth on the tree stem and its resultant form are those by botanists: Heinrich Cotta, Theodore Hartig, and H.Nordlinger. These early studies and scientific observations were used as building blocks by other scientists like Pressler, Metzger, Schwartz, and Hohenadl to publish laws regarding the growth patterns along the stem which influence stem form. Since then many theories have been proposed to account for the natural variation of stem form.

One particular theory that has evoked much consideration is the mechanistic theory. As one of its early advocates, Metzger recognized that both the weight of the stem itself and the horizontal force imposed upon the stem by the wind were two important mechanical forces influencing the main stem. He demonstrated that the cubic paraboloid conforms to the assumption that the stem was a beam of uniform resistance to bending. Some years later, Gray (1956) refuted this idea and claimed that the quadratic paraboloid, with 20% less volume, would satisfy the mechanical requirements of the erect stem (Newnham, 1965; Wilson and Archer, 1979).

Simple taper equations

At the beginning of this century, many European researchers attempted to use the equation of solids of revolution to express the taper of trees. Unfortunately, deviations from these geometric forms take place very often in nature. Such form deviations have been attributed to factors like heredity (Squillace and Silen, 1962), age (Bickerstaff, 1946), silvicultural treatments (Myers, 1963; Newnham, 1965), and dominance (Horn, 1961). Despite these results, the expression of tree taper using a single equation has, for practical purposes, proven to be satisfactory.

According to Behre (1923), the Swedish forester A.G. Hojer was the first to use an analytical approach to develop a mathematical equation to express tree taper. Later, Behre (1927) proposed a hyperbolic equation that related tree diameter from tip to measuring point. Matte (1949) found that tree profile above breast height could be described by the equation:

$$y = x\sqrt{b_o + b_1 x + b_2 x^2}$$

where:

$$y = \frac{d}{Dbh_{ib}},$$

d = diameter inside bark along the stem,

Dbh_{ib} = diameter at breast height inside bark,

$$x = \frac{p}{(H - 4.5)},$$

p = distance from tip to measuring point,

H = total tree height.

Other equations were developed by Gray (1956) and Heger (1965). Kozak and Smith (1966) analyzed the use of multivariate techniques and concluded that the simple models worked best. Munro (1966) proposed a simple equation to predict diameter at any height along the stem:

$$d = Dbh \sqrt{b_o + b_1 \frac{h}{(H - 4.5)}}$$

where:

Dbh = diameter outside bark at breast height,

h = partial height above ground.

Later he found that upper stem diameters could be estimated with a lower standard error with the taper equation (Munro, 1968):

$$\frac{d^2}{Dbh^2} = b_o + b_1(\frac{h}{H}) + b_2(\frac{h}{H})^2$$

Kozak et al. (1969) conditioned this equation so that the predicted diameter would equal zero at the tree tip and obtained the following form:

$$\frac{d^2}{Dbh^2} = b_1(\frac{h}{H} - 1) + b_2(\frac{h^2}{H^2} - 1)$$

Ormerod (1971) proposed the nonlinear model:

$$d = D_i \left[\frac{H - h}{H - i} \right]^{b_1}$$

where:

D_i = diameter measured at height i

Bennett and Swindel (1972) used third-degree polynomials to estimate tree taper for planted slash pine (*Pinus elliottii* Engelm.). A two-variable model to predict underbark diameter for five conifer species in California was proposed by Amidon (1984). Reed and Byrne (1985) used the nonlinear model developed by D.W. Ormerod and modified it to account for taper variation between trees of different sizes.

The search for a more accurate single taper equation with desirable properties has continued today. More recently, Kozak (1988) introduced what he calls a "variable form" taper equation. The model is used to predict tree profile as a function of height, diameter at breast height, and relative height. The equation describes the shape of the stem as one continuous function, with a changing exponent from ground to top to compensate for the form of different tree sections.

The final equation has the following form:

$$d = b_o\, Dbh^{b_1}\, b_2^{Dbh}\; [\, X\,]^{\,C}$$

where:

d = diameter inside bark at height h from ground,

$$X = \left[\frac{1-\sqrt{z}}{1-\sqrt{I}} \right]$$

$$C = \left[b_3 z^2 + b_4 \ln(z + 0.001) + b_5\sqrt{z} + b_6 e^z + b_7\left(\frac{Dbh}{H}\right) \right]$$

$$I = \frac{HI}{H},$$

HI = height of the inflection point from ground,

$$z = \frac{h_i}{H}.$$

Although the equation looks very complex, it is basically a simple power function,

$$Y = mx^c$$

where m is an appropriate constant. Kozak's variable form equation uses as 'm' the diameter at the inflection point expressed as a function of Dbh. In his equation the inflection point, the point where the taper curve changes from neiloid to paraboloid form, is assumed to occur at 25% of total height. As stated by Grosenbaugh (1966), and by Furslund (1982) the power function can express taper by modifying the value of the exponent 'c' to account for the shapes of the lower, middle and upper sections of the stem.

Complex taper equations

In the search for taper models that would predict accurately and precisely the stem profile of commercial forest species, extensive research has also been conducted towards the development of more complex mathematical formulations. Bruce et al. (1968) derived an equation to estimate the ratio of squared upper diameter inside bark to squared Dbh as a function of Dbh, total height, and the 3/2, 3rd, 32nd, and 40th powers of relative height. He used an extension of the methods developed by Matte (1949) and Giurgiu (1963).

Other models have been derived for different tree species growing under diverse environmental conditions. Some instances are the models published by Ek and Kaltenberg

(1975) and Hilt (1980). Martin (1981) compared several previously derived models in an attempt to find the best taper prediction for selected commercial Appalachian hardwood species. Very often these equations have failed to adequately describe the butt and top sections of the tree. Demaerschalk and Kozak (1977) recognized that even the most complicated equation may fit one portion of the tree well but show considerable bias elsewhere. Grosenbaugh (1966) opined that taper functions in order to be accurate would have to allow for many inflection points along the stem.

Today many researchers contend that the cut portions of the stem resemble different forms depending on their original relative position in the tree. As a result, their work has been directed toward the consideration of submodels to describe the taper of tree segments. These submodels are then grafted at the join points by imposing restrictions on the overall model. Some examples in the literature of this approach are the models by Max and Burkhart (1976), Demaerschalk and Kozak (1977), Bennett et al. (1978), Cao et al. (1980) and Farrar (1987). These models tend to have good predictive ability, especially in the butt region of the stem (Byrne and Reed, 1986).

In another line of research, Fries (1965) and Fries and Mattern (1966) introduced the multivariate technique to derive taper curves for species growing in Sweden and British Columbia. One of the subfields of multivariate statistical analysis is called principal component analysis. The properties of this procedure and their applications to the definition of taper are summarized by Liu (1976), and Liu and Keister (1978).

Compatible taper and volume equations

A desirable attribute among equations that are mathematically related is analytic compatibility (Green and Reed, 1985). For instance, a taper equation is called analytically compatible when its definite integration derives a volume equation. Similarly, volume equations can be converted to compatible taper equations. Volume estimation systems derived from integration of taper equations are compatible when the coefficients of the derived volume equation can be written in terms of the taper equation coefficients (Byrne and Reed, 1986). The precision as well as accuracy of the volume estimates obtained this way, are dependent upon how well the particular taper equation fits the tree profile. The basic rationale behind the design of compatible systems of taper and volume equations, is that these two tree characteristics should not be considered independently, but on the contrary should be analyzed as tree attributes that are mathematically and biologically related (Munro and Demaerschalk, 1974).

The concept of compatibility was initially elaborated by J.P. Demaerschalk at the Faculty of Forestry, University of British Columbia. In his initial publications, Demaerschalk (1971, 1972, 1973) showed how the most commonly used total volume equations can be converted to compatible taper equations whenever taper data are available. He presented several taper equations derived from existing well-known total volume equations. The main advantage of compatibility is that discrepancies are avoided between volume estimates derived from taper equations and those obtained directly from existing volume equations.

Based on the concepts developed by Demaerschalk, many other scientists decided to make their own specific applications. For instance, Goulding and Murray (1976) developed a fifth-degree polynomial compatible taper equation for radiata pine *(Pinus radiata* D. Don) growing in New Zealand. Clutter (1980) further extended the concept by deriving a taper equation compatible with a variable-top merchantable volume equation.

The same approach was used by Brister et al. (1982) to derive taper equations for plantations of slash pine *(Pinus elliottii* Engelm.) in Georgia and Florida. Other volume and taper compatible systems that appear in the literature are those by Van Deusen et al.(1982), Reed and Green (1984), Green and Reed (1985), McClure and Czaplewski (1986), and Lenhart et al. (1987).

When fitting a system of compatible tree volume and taper equations, a coefficient estimation procedure to ensure numerical consistency becomes necessary. Sullivan and Clutter (1972) suggested that one way to provide numerical consistency, is to fit one of the component equations and then solve algebraically for the coefficients of the other equation. However, Burkhart and Sprinz (1984) showed that the coefficient estimates resulting from such procedure are not efficient. Reed (1982) and Reed and Green (1984) employed an alternative method of simultaneously fitting all the equations in the system. They observed that the overall system estimation error was reduced substantially by using this procedure, as opposed to fitting the taper equation and then algebraically solving for the coefficients of the total and volume ratio equations.

Cao et al. (1980) made a very interesting comparison of several taper as well as taper and volume compatible models. They concluded that in general some precision in the estimation of diameters is apparently sacrificed to ensure the compatibility of a taper equation. The results demonstrated that, at least for the models compared, a single model did not consistently perform best for several related prediction objectives.

Chapter III

METHODS AND PROCEDURES

DATA

The data available for this study were collected as part of a cooperative research project between the College of Forestry, Wildlife and Range Sciences of the University of Idaho, and the Escuela Nacional de Ciencias Forestales (ESNACIFOR) based in Siguatepeque, Honduras. The research was aimed primarily at the collection and analysis of information on soils and over/understory vegetation for site quality classification of *Pinus oocarpa* Schiede in central Honduras. One hundred ninety five plots were located in natural homogeneous stands covering a wide range of sites, aspects, slopes and elevations (Table 1). All plots were located in undisturbed pure or mixed stands covering the provinces of Comayagua, La Paz, Francisco Morazan and Intibuca, in central Honduras (Fig. 1).

Table 1. Number of sampled plots by site index class for various levels of slope, aspect and elevation[1].

Levels of Slope, Aspect and Elevation	Site Index Class[2] (m)					
	5	10	15	20	25	Total
Slope (%)						
1-10	1	11	13	6	1	32
11-20	0	13	19	8	0	40
21-30	1	13	16	7	0	37
31-40	3	10	17	4	0	34
41-50	2	5	8	2	1	18
51-60	2	7	6	0	0	15
> 60	2	12	5	0	0	19
Aspect						
N	2	11	10	4	0	27
NE	4	9	15	3	0	31
E	3	15	8	2	0	28
SE	1	7	9	4	1	22
S	1	8	15	4	0	28
SW	0	8	11	3	1	23
W	0	5	8	2	0	15
NW	0	8	8	5	0	21
Elevation (m)						
651-850	1	5	3	0	0	9
851-1050	5	22	20	9	1	57
1051-1250	2	22	27	6	1	58
1251-1450	1	5	16	4	0	26
1451-1650	0	9	5	6	0	20
1651-1850	2	7	11	1	0	21
> 1851	0	1	2	1	0	4

[1] Taken from: Stiff, C.T., D.N. Perez, and F.D. Johnson, 1987.

[2] Index age: 15 years.

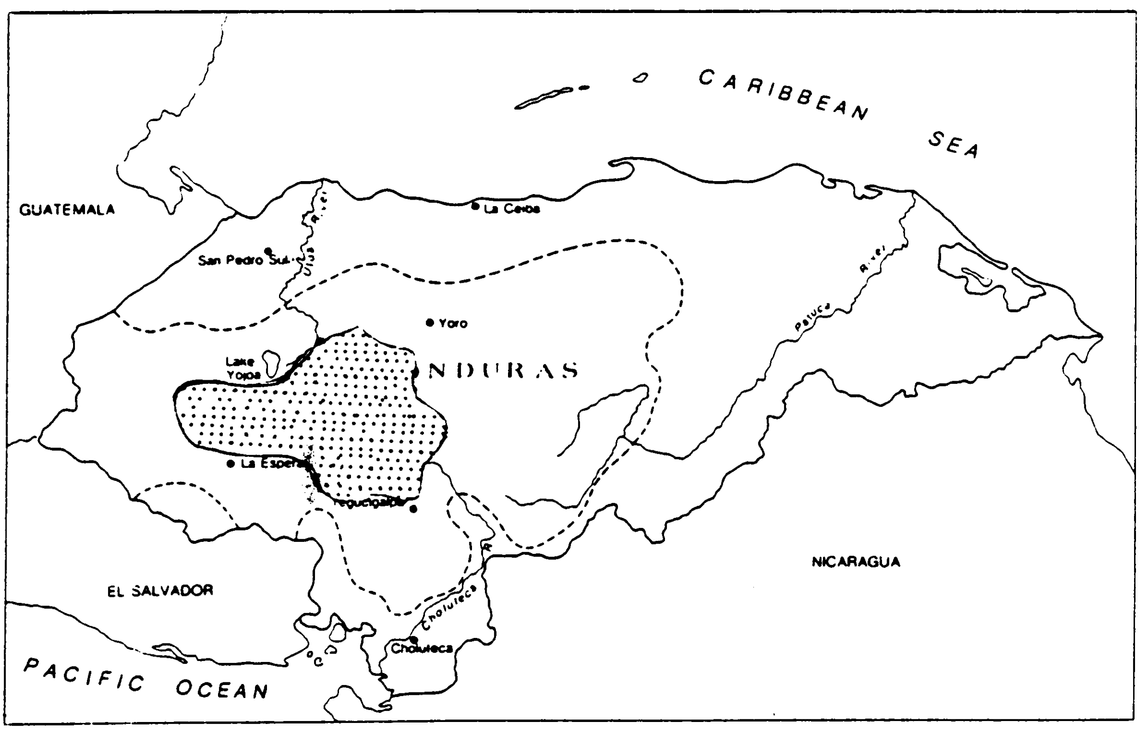

Figure 1. **Study area in Honduras (shaded), centering near Siguatepeque, headquarters for ESNACIFOR:** Dotted line denotes approximate limits of interior pine forests which are dominated by **Pinus oocarpa**, but include **Pinus tecunumanii** and **Pinus maximinoi** at higher elevations, and **Pinus caribaea** in interior dry valleys.

The plots were clusters of five points arranged in a circular or linear pattern, located by a combination of aerial photo interpretation and field observations. Each plot was placed in reasonably well-stocked stands with trees in fairly good condition. Two dominant or codominant trees and one intermediate or suppressed tree, all with Dbh $>$ 10 cm., were felled and measured for total height and crown length.

The sample trees were cut leaving a 0.3 meter stump and then sectioned at 0.8 and 1.3 meters. Above 1.3 meters, trees were sectioned into 10 equal-length segments. Two measures of diameter outside bark and two measures of diameter inside bark to the nearest centimeter, were taken using calipers or rulers at the bottom of the 12 resulting sections. The average diameter outside and inside bark of each of the sections was calculated as the geometric mean of two measurements taken at right angles across the geometric center. The diameter at breast height outside bark (Dbh) used in the analysis was that measured with a diameter tape to the nearest centimeter, before the sample tree was felled. The double bark thickness was calculated as the difference between the average diameter outside and inside bark as measured with calipers. This value was subtracted from Dbh to obtain Dbh_{ib} . A summary of stand and site variables across all plots is presented in Table 2.

ANALYSIS

The approach of this study was first to fit and assess the prediction potential of some simple models available in the literature, and later repeat the process with more complex

Table 2. **Summary of stand and site characteristics across all plots[1].**

Variable	Mean	Minimum	Maximum
Site index (m)	14.0	5.0	26.0
Plot age (yrs)[2]	36.0	12.0	120.0
Basal area (m^2/ha)	22.0	6.0	40.0
Number of trees/ha	536.0	71.0	1374.0
Slope (%)	31.0	0.0	90.0
Aspect (radians)	2.5	0.0	5.5
Elevation (m.a.s.l.)	1212.0	600.0	1900.0
Annual precipitation (cm)	136.0	90.0	200.0
Annual temperature (°C)	20.0	17.0	24.0
Soil depth (dm)	9.0	1.0	15.0

[1] Taken from: College of Forestry, Wildlife and Range Sciences, University of Idaho, and Escuela Nacional de Ciencias Forestales, 1985.

[2] Based on three destructively sampled trees per plot.

ones. Several transformations were introduced to some simple models to explore whether improvements in fit and prediction were possible.

A desired property of the equations selected was that of yielding a zero diameter at the top of the tree. The parameter estimates were obtained using ordinary least squares procedures. These procedures are based upon the following assumptions (Myers, 1986; Daniel and Wood, 1971):

1. The regressor variables are nonrandom and measured without error. Any error in the measurement is assumed to be small compared to the range.
2. The model errors, i.e., the departures of the responses from expectation, are random with mean zero and constant variance, σ_ϵ^2.
3. The errors are independent, normally distributed, uncorrelated from observation to observation and with the regressor variables.

The preceding assumptions are seldom fully satisfied in reality. In this study, a departure from assumption 3 is inevitable as several diameter observations were recorded from the same tree. This is generally the case when stem analysis data are used in many forest applications. Under these circumstances, the use of the least squares procedure is likely to have the following consequences (Neter and Wasserman, 1974):

1. Although the coefficient estimates are still unbiased, their variances tend to be inflated.
2. The usual estimate of the mean square error may seriously underestimate the variance of the error terms, σ_ϵ^2.

3. Confidence intervals and tests based on the **t** and **F** distributions are no longer strictly applicable, because actual probability levels, say x^{x} , will not be equal to the nominal levels, x.

A major cause of the correlated error term involves the omission of several key variables from the model. Thus, this study sought the best form of the taper equation in models developed based on past research and therefore expected to be closer to the true model. In addition, the primary objective of this study was the selection or development of a taper model with good predictive ability using validation data.

Notation

The following notation will be used:

b_i = estimated regression coefficients.

Dbh = dbh outside bark in centimeters.

Dbh_{ib} = dbh inside bark in centimeters.

H = total tree height in meters.

d = diameter inside bark at height h, in centimeters.

h = partial height above ground, in meters.

h_1 = lower limit of integration with respect to h.

h_2 = upper limit of integration with respect to h.

HI = height of the inflection point from ground.

DI = diameter inside bark at the inflection point.

$z = \frac{h}{H}$

$I = \frac{HI}{H}$

$K = \frac{\pi}{4} (100)^{-2} = 7.854 \times 10^{-5}$

V = total cubic volume inside bark (m^3) above stump (0.3 m).

V' = merchantable cubic volume above stump to some top diameter or height limit.

Description of the taper equations selected

All taper equations are expressions of d^2 or d in terms of h, H, and Dbh. Taper equations can be integrated to produce total or merchantable volume. As such, any taper equation has an intrinsically defined volume equation. Total volume (including stump) is found by evaluating the integral from ground level to H:

$$\int_0^H K\{ d(h) \}^2 \, dh$$

Similarly, an expression for merchantable volume can be found by changing the limits of the integral with the desired values for h_1 and h_2.

The following taper equations were selected from the literature and fitted to the sample data:

Model 1.

The quadratic polynomial model proposed by Kozak et al. (1969) was used,

$$\frac{d^2}{Dbh^2} = b_1(z-1) + b_2(z^2-1) \qquad [6.0]$$

Total volume can be found by evaluating the integral from point **a** to any point along the stem **b**, where **b** = $\frac{h_2}{H.}$

$$V = K\,Dbh^2 H \left[\frac{b_1 z^2}{2} + \frac{b_2 z^3}{3} - (b_1 + b_2)\,z \right]_a^b \qquad [1.1]$$

Merchantable volume to any desired partial height can be obtained by substituting **a** with $\frac{h_1}{H}$. To find **b** in terms of a diameter limit d, equation (1.0) can be solved as

$$b = \frac{h_2}{H} = \frac{1}{2b_2}\left[-b_1 \pm \sqrt{b_1^2 + 4b_2\,(b_1 + b_2 + \frac{d^2}{Dbh^2})} \right], \quad 0 \leq \frac{h_2}{H} \leq 1$$

Model 2.

The nonlinear model proposed by Ormerod (1971,1973) was considered.

$$d = D_{1.3}\left[\frac{H-h}{H-1.3}\right]^{b_1}$$

where:

$D_{1.3}$ = Dbh, if d is diameter outside bark

= Dbh_{ib} , if d is diameter inside bark.

Model 2 was fitted using the observed Dbh_{ib} resulting

$$d = Dbh_{ib} \left[\frac{H-h}{H-1.3} \right]^{b_1} \qquad [2.0]$$

where:

Dbh_{ib} = observed diameter at breast height inside bark.

Volumes can be obtained by evaluating the integral using the equation

$$V = \frac{K\, Dbh_{ib}^2}{(2b_1+1)\,(H-1.3)^{2b_1}} \left[(H-h)^{2b_1+1} \right]_{h_1}^{h_2} \qquad [2.1]$$

When merchantable volume to a given top diameter is desired, then h_2 can be obtained from equation 2.0,

$$h_2 = (H-1.3)\left(\frac{d}{Dbh_{ib}} \right)^{\frac{1}{b_1}}, \quad 0 \le h_2 \le H.$$

Model 2a.

An equation to express Dbh_{ib} as a function of Dbh measured with diameter tape was developed. Equation [2.0] was then fitted using the estimated Dbh_{ib}.

$$d = \widehat{Dbh}_{ib} \left[\frac{H-h}{H-1.3} \right]^{b_1} \qquad [2.2]$$

where:

$\widehat{Dbh}_{ib}$ = estimated diameter at breast height inside bark.

Model 3.

The third model included in the analysis is the one developed by Amidon (1984), for mixed-conifer species in California. Amidon based his work on a four variable single equation used to predict taper above breast height proposed by Bennett and Swindel (1972). The original equation was modified to predict taper along the entire stem, and found to predict very well with only two parameters. The final equation is

$$d = b_1 \frac{Dbh\,(H-h)}{(H-1.3)} + b_2 \frac{(H^2-h^2)\,(h-1.3)}{H^2} \qquad [3.0]$$

Similarly, this taper equation is useful for estimating stem merchantable volume between any two points, h_1 and h_2, in the range of breast height to total height (Amidon, 1984). Integration of equation 3.0 gives

$$V = \frac{\pi}{100^2} \Bigg[\mathrm{T}_0^2\,(h_1-h_2) + \mathrm{T}_0\mathrm{T}_1\,(h_1^2-h_2^2) + \frac{(\mathrm{T}_1^2+2\mathrm{T}_0\mathrm{T}_2)\,(h_1^3-h_2^3)}{3} \qquad [3.1]$$

$$+ \frac{(\mathrm{T}_0\mathrm{T}_3+\mathrm{T}_1\mathrm{T}_2)\,(h_1^4-h_2^4)}{2} + \frac{(\mathrm{T}_2^2+2\mathrm{T}_1\mathrm{T}_3)\,(h_1^5-h_2^5)}{5}$$

$$+ \frac{(T_2T_3)(h_1^6 - h_2^6)}{3} + \frac{T_3^2(h_1^7 - h_2^7)}{7}]$$

where:

$$T_0 = \frac{b_1 Dbh(H)}{2(H-1.3)} - \frac{1.3b_2}{2} \qquad T_1 = \frac{b_2}{2} - \frac{b_1\,Dbh}{2(H-1.3)}$$

$$T_2 = \frac{1.3b_2}{2H^2} \qquad T_3 = -\frac{b_2}{2H^2}$$

Model 4.

One of the complex models also considered was the one developed by Max and Burkhart (1976). This model divides the stem in three sections , each of which is described with a different submodel. These submodels are then grafted together at two join points using the regression technique described by Fuller (1969) and Gallant and Fuller (1973).

$$\frac{d^2}{Dbh^2} = \left[b_1(z-1) + b_2\{z^2-1\} + b_3(a_1-z)^2Q_1 + b_4(a_2-z)^2Q_2\right] \qquad [4.0]$$

where:

$$Q_i = \begin{cases} 1 \;\; if\; z \le a_i \\ 0 \;\; if\; z > a_i; \;\; i = 1,2. \end{cases}$$

a_1 and a_2 in equation 4.0 are two join points to be estimated. The equation to estimate total volume as presented by Byrne and Reed (1986), is as follows:

$$V = \alpha Dbh^2 H \qquad [4.1]$$

where:

$$\alpha = K\left[(\frac{b_2}{3}) + (\frac{b_1}{2}) - (b_1 + b_2) + (\frac{b_3}{3})a_1^3 + (\frac{b_4}{3})a_2^3 \right]$$

As demonstrated by Cao (1978), the diameters at the join points, a_1 and a_2, can also be estimated from equation 4.0.

$$For\ a_1,\quad d_1 = Dbh\sqrt{b_1(a_1 - 1) + b_2(a_1^2 - 1)}$$

$$For\ a_2,\quad d_2 = Dbh\sqrt{b_1(a_2 - 1) + b_2(a_2^2 - 1) + b_3(a_1 - a_2)^2}$$

Merchantable volume can be obtained by evaluating the equation

$$V' = KDbh^2H\left[\frac{b_1}{2}z^2 + \frac{b_2}{3}z^3 - (b_1 + b_2)z - \frac{b_3}{3}(a_1 - z)^3Q_1 - \frac{b_4}{3}(a_2 - z)^3Q_2 \right]_{h_1}^{h_2} \qquad [4.2]$$

Model 5.

The equation developed by Kozak (1988) was also included in the analysis. It describes tree taper with a continuous function using a changing exponent to compensate for the form of different tree sections.

$$d = b_o\, Dbh^{b_1}\, b_2^{Dbh}\ X^{\ C} \qquad [5.0]$$

where:

$$X = \left[\frac{1-\sqrt{z}}{1-\sqrt{I}} \right]$$

$$C = \left[b_3 z^2 + b_4 \ln(z + 0.001) + b_5 \sqrt{z} + b_6 e^z + b_7 \left(\frac{Dbh}{H} \right) \right]$$

Equation 5.0 was linearized using logarithmic transformation.

$$\ln(d) = \left[\ln(b_o) + b_1 \ln(Dbh) + \ln(b_2) Dbh + b_3 \ln(X) z^2 + b_4 \ln(X) \ln(z + .001) \right. \qquad [5.1]$$

$$\left. + b_5 \ln(X) \sqrt{z} + b_6 \ln(X) e^z + b_7 \ln(X) \left(\frac{Dbh}{H} \right) \right]$$

This model is also conditioned so that diameter d = 0 at the top of the tree. In addition, d_i is equal to the estimated diameter at the inflection point, when $\frac{h_i}{H} = I$. Demaerschalk and Kozak (1977) found that for 32 commercial species in British Columbia, Canada, the inflection point (I) occurred at almost a constant relative height, regardless of size classes. Therefore, model 5 was fitted assuming that for *Pinus oocarpa* the inflection point occurs at a constant relative height of 0.25.

When compared to the other equations included in this study, model 5 has two weaknesses:

1. The taper equation cannot be integrated to calculate volume, or the resulting volume equation is too lengthy or complicated to be of practical use.
2. Computer iterations must be used to find merchantable height to a given top diameter.

Models 6 and 7.

Other models included in the analysis were those obtained by making transformations on the variables of simple models. For example, nonlinear least squares procedures were used to explore the possibility of improving fit and prediction for models 1 and 3. In both cases, this was done by estimating the exponent values with the equation:

$$d = b_1 (X_1)^{b_2} + b_3 (X_2)^{b_4}$$

Therefore, the exponent values for model 6 were found by fitting,

$$\frac{d^2}{Dbh^2} = b_1(z^{b_2} - 1) + b_3(z^{b_4} - 1) \qquad [6.0]$$

Similarly, for model 7

$$d = b_1 \left[\frac{Dbh\,(H-h)}{(H-1.3)} \right]^{b_2} + b_3 \frac{(H^{b_4} - h^{b_4})\,(h-1.3)}{H^{b_4}} \qquad [7.0]$$

Models 8.

The presence of seven independent variables in model 5, gave the indication that perhaps strong linear dependencies existed among them. When this condition is present, the coefficients using least squares procedures can be imprecisely estimated, thus producing areas in the regressor space where prediction could be poor. Several criteria like Variance Inflation Factors (VIF) and the eigenvalue spectrum, were used as diagnostic tools for multicollinearity. Variance inflation refers to the increase in the variance of the coefficient b_i, due to the linear dependency between the i^{th} regressor variable and the other ones present in the model. As a result, the higher the correlation or dependency, the lower the precision in the least squares estimate of coefficient b_i .

Another tool used to detect collinearity is the magnitude of the eigenvalues of the correlation matrix. The closer a particular eigenvalue is to zero, the greater the strength of the dependency. The condition number, that is the ratio between the largest and the smallest eigenvalue, is often used to measure collinearity. Large condition numbers are evidence that the coefficient estimates are unstable (Myers, 1986).

One means to combat collinearity is to eliminate variables to the point where the quality of fit is not severely compromised. With these considerations in mind, the possibility of obtaining a simpler model was fully explored.

Several criteria were used for the selection of a more parsimonious version of model 5 that would fit the data just as well. The following criteria were used in the selection of the "best" reduced model:

1. Mean Square Error (s^2):

$$s^2 = \frac{\sum_{l=1}^{n} (y_l - \hat{y}_l)^2}{n-p}$$

where:

i = 1,2,...,n,

n = number of diameter pairs compared for each model,

y_i = $\ln(d_i)$

$\hat{y}_i$ = estimated response,

p = number of parameters estimated in model.

2. Coefficient of Determination (R^2):

$$R^2 = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

where:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

3. Prediction Sum of Squares (PRESS):

Conceptually, the PRESS statistic is computed by fitting the candidate model while withholding one single observation each time. Then, the deleted response is estimated each time resulting in **n** prediction errors. The PRESS statistic is then defined as

$$PRESS = \sum_{i=1}^{n} (d_i - \hat{d}_{i,-i})^2$$

where:

$\hat{d}_{i,-i}$ = estimated response without the i^{th} observation.

4. Mallows' C_p Statistic:

The C_p statistic is a very useful criterion when one is discriminating between models. C_p for a regression model with p-parameters is expressed as

$$C_p = p + \left[\frac{(s^2 - \hat{\sigma}^2)(n-p)}{\hat{\sigma}^2} \right]$$

where:

$\hat{\sigma}^2$ = residual mean square for full model (Model 5).

The best model was selected from a group of promising candidates. The chosen ones had either the highest R^2 value, or the lowest value for one or more of the other criteria. Before selecting the "best" candidate model, each was fitted to the sample data and carefully scrutinized using tools like residual plots, partial plots, and collinearity diagnostics.

Prediction criteria

For analysis purposes, the data were randomly divided into fitting and validation subsets. Seventy percent of the sample, or 405 trees, were used for fitting the taper equations, and the remaining trees were considered as an independent data set representing the population and later used for testing.

In this study much emphasis was placed on the selection from a set of candidate models the one that will best predict inside bark diameters along the stem. Model validation is an important tool available to the researcher, because it places emphasis on prediction capability. The nine models selected were tested using the validation procedure to determine the "best" model for predicting taper. For this purpose, predicted diameters inside bark were compared to observed diameters. The comparison was based on the difference between these two diameters using the expression,

$$D_i = d_i - \hat{d}_i$$

where:

i = 1,2,...,n,

n = number of diameter pairs compared for each model,

d_i = observed diameter inside bark,

$\hat{d}_i$ = predicted diameter inside bark.

The following criteria were used to evaluate the models in terms of diameters.

1. Mean Bias ($\overline{B}$):

$$\overline{B} = \frac{1}{n} \sum_{i=1}^{n} D_i$$

2. Standard Deviation of the Differences (S_D):

$$S_D = \sqrt{\frac{1}{n-1} \left[\sum_{i=1}^{n} D_i^2 - \frac{(\sum_{i=1}^{n} D_i)^2}{n} \right]}$$

Model comparison involved the quality of prediction of the model for the population represented by the validation data set. The mean bias measures the accuracy of prediction, and the standard deviation of the differences measures the precision of prediction. To each model, ranks were assigned based on its performance in terms of the evaluation criteria. Rank number one corresponded to the "best" model in that criterion. Ranks given were then added to compute an overall performance value of a particular model when compared to the others.

Another criterion used to compare and select models in terms of prediction, was the total squared error (TSE). TSE is defined as the sum of the squared mean bias and the variance of the differences (S_D^2). This final value was then compared assigning rank 1 to the model with the lowest overall value. In addition, residual plots were used to detect gross violation of assumptions or difficulties with the models.

Finally, the models were fitted to all the observations in order to obtain the overall coefficient estimates.

Evaluating taper variation

Due to the wide geographic distribution of the species, it has been well recognized that variation in growth patterns and specific adaptations to different environmental growing conditions are very common (Robbins and Hughes, 1983). Taper variation was explored between trees from two distinctive geographic regions from which trees with different taper characteristics were suspected. These regions are in the vicinity of La Esperanza, Intibuca and Siguatepeque, Comayagua. In addition, taper variation possibly due to different tree characteristics like the live crown ratio and the crown class was also evaluated.

Statistical test

In order to test for significant differences between taper of trees growing in the two geographic regions, an **F** test was conducted. This was done by fitting the model selected to the data from each region separately, and to the data combined. Both fitting and validation data sets were used. The test procedure is based on the assumption of equal variances in the two data groups.

H_o : regression estimates from both regions are the same.

H_1 : not H_o.

Test Statistic:

$$F_{obs} = \left[\frac{\dfrac{(SSE_f - SSE_r)}{(df_f - df_r)}}{MSE_f} \right]$$

where:

$SSE_f =$ sum of squared errors for full model (data combined),

$SSE_r = SSE_1 + SSE_2$,

$SSE_1 =$ sum of squared errors for region 1,

$SSE_2 =$ sum of squared errors for region 2,

$df_f =$ error degrees of freedom for full model,

$df_r = df_1 + df_2$,

$MSE_f =$ mean squared error for full model.

Reject H_o if F_{obs} exceeds the tabulated F value for $\alpha = .01$, (df_f - df_r), df_r.

Evaluation of practical differences

For evaluation of practical differences, the data sets from both regions were analyzed combined and separately using the selected taper model with the following criteria:

1. Mean Weighted Difference ($\overline{D}_w$):

$$\overline{D}_w = \frac{1}{n} \sum_{i=1}^{n} \frac{(d_i - \hat{d}_i)}{d_i}$$

2. Mean Absolute Weighted Difference ($\overline{|D_w|}$):

$$\overline{|D_w|} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{(d_i - \hat{d}_i)}{d_i} \right|$$

3. Residual plots.

Values for these criteria and residual plots were obtained and evaluated across the data set and for each of the 12 equally-spaced diameter observations along the stem. The procedure was repeated, using the entire validation data set, to evaluate possible taper variation due to live crown ratio and crown class.

Chapter IV

RESULTS AND DISCUSSION

The data set available for this study contained the observations of 600 destructively sampled trees, but only 578 trees were used in the analysis. The remaining trees were excluded because of incomplete or suspected erroneous measurements. In the trees included, total height ranged from 8.2 to 34.4 meters, diameter at breast height from 10.0 to 59.0 cm., and age from 11 to 174 years.

Linear and nonlinear regression techniques were used to fit all nine taper models selected. Table 3 shows the coefficient estimates and their standard errors for the models fitted using all the observations. Other fit statistics are listed in appendix B. Before proceeding any further, the selection procedure and the final form of models 6,7 and 8 will be discussed.

Table 3. Coefficient estimates and their standard errors for models to predict diameter inside bark (n = 6936).

MODEL	b_o[1]	b_1	b_2[1]	b_3	b_4	b_5 (a_1)	b_6 (a_2)	b_7
1		-0.995317 (0.008717)[2]	0.238889 (0.007273)					
2		0.662995 (0.001860)						
2a		0.703633 (0.001899)						
3		0.830181 (0.001140)	0.605086 (0.005242)					
4		-3.853070 (0.581173)	1.869216 (0.319268)	-2.106622 (0.308963)	21.077190 (1.662507)	0.796632 (0.020645)	0.105851 (0.004079)	
5	-0.684788 (0.043985)	1.138908 (0.019668)	-0.004298 (0.000759)	0.543205 (0.120686)	-0.036841 (0.027232)	-0.240226 (0.252752)	0.088589 (0.137796)	0.150991 (0.004706)
6		-0.855762 (0.004852)	0.111176 (0.003610)					
7		0.835239 (0.001086)	0.438276 (0.003643)					
8	-0.432876 (0.014480)	1.027609 (0.004523)	0.546840 (0.007340)	-0.048952 (0.003047)	0.141629 (0.004104)			

[1] b_o and b_2 for model 5 and b_o for model 8 are given in the natural log scale.

[2] The values in parenthesis are the standard errors of the coefficients.

Selection procedure for models 6 and 7.

Variable transformations were attempted in models 1 and 3, in order to obtain models 6 and 7 respectively. Nonlinear least squares procedures were used to estimate the exponent values for the two variables already present in the models mentioned. For both models, analysis showed that models 1 and 3 would fit the data better by changing the exponent value of the second variable from 2 to 4.

Therefore, model 6 was a result of this transformation on model 1. A slight reduction in the standard error of estimate from 1.8 cm to 1.7 cm was obtained. the final form of model 6 was:

$$\frac{d^2}{Dbh^2} = b_1(z-1) + b_2(z^4-1)$$

On the other hand, model 7 resulted from the transformation in model 3. Similarly, a modest reduction in the value of the standard error of estimate from 1.7 cm to 1.6 cm was obtained. Model 7 was fitted to the sample data as:

$$d = b_1 \frac{Dbh\,(H-h)}{(H-1.3)} + b_2 \frac{(H^4-h^4)\,(h-1.3)}{H^4} \qquad [3.0]$$

Selection procedure for model 8.

Model 8 is a version of model 5. The independent variables of model 5 will be denoted by:

$$X_1 = ln\,(Dbh)$$
$$X_2 = \text{Dbh}$$
$$X_3 = ln\,(X)z^2$$
$$X_4 = ln\,(X)ln\,(\,z + 0.001)$$
$$X_5 = ln\,(X)\sqrt{z}$$
$$X_6 = ln\,(X)e^z$$
$$X_7 = ln\,(X)(\frac{Dbh}{H})$$

where:

$$X = \left[\frac{1-\sqrt{z}}{1-\sqrt{I}}\right]$$

The first step in the analysis was to produce a record of all possible regressions for the seven variables listed above. Thus, there were 2^7 or 128 possible individual models that could be derived. However, as expected, only a small portion of them provided useful results. All candidate models were then evaluated and ranked using the diagnostic criteria. A subset of the best candidate models and their corresponding diagnostic criteria is listed in appendix C.

Next, candidate models that seemed promising in terms of mean squared error, PRESS, R^2, and C_p were further examined in detail. Multiple linear regression with a full set of collinearity diagnostics and partial plots as options was used.

High variance inflation factors and condition numbers showed that, in general, variables X_1 and X_2 as well as X_3, X_5, and X_6 were involved in heavy linear dependencies. Therefore, it appeared that basically these variables were offering redundant information.

Among a set of potential models with comparable fitting quality, selection preference was given to the one with the least number of parameters. Finally, the best candidate model, selected as model 8, had the form:

$$d = b_o\, Dbh^{b_1}\, X^{\,C}$$

where:

$$X = \left[\frac{1-\sqrt{z}}{1-\sqrt{I}} \right]$$

$$C = \left[b_2 z^2 + b_3 \ln(z + 0.001) + b_4 \left(\frac{Dbh}{H} \right) \right]$$

Model 8 was fitted using logarithmic transformation.

$$\ln(d) = \left[\ln(b_o) + b_1 \ln(Dbh) + b_2 \ln(X) z^2 + b_3 \ln(X) \ln(z + .001) + b_4 \ln(X) \left(\frac{Dbh}{H} \right) \right]$$

The regressor variables in model 8 did not show strong linear dependencies and, as a result, their coefficient estimates are expected to be more stable because of much less inflated variances. It seemed that the elimination of variables X_1,X_5, and X_6 had helped the fitting process and resulted in a simpler equation form.

Estimating Dbh inside bark from Dbh

Model 2a is based on the estimation of diameter inside bark at breast height from outside bark diameter. A simple linear regression was found to be satisfactory for the estimation.

$$Dbh_{ib} = b_o' + b_1' \, Dbh$$

The independent variable is the diameter outside bark as measured with the diameter tape. five hundred-seventy-eight independent observations of diameter at breast height were used to estimate the coefficients. These estimates were: b_o' = -1.088933 and b_1' = 0.894763, with standard errors of 0.172173 and 0.006003, respectively.

The standard error of estimate was 1.3 cm and the R^2 value was 0.975.

Quality of taper prediction

The mean bias and the standard deviation or variance of the differences between observed and predicted diameters, were the criteria used to judge the adequacy of prediction. Table 4 displays the values and ranks assigned to each model using the validation data set. Ranks for the mean bias were assigned according to the values in absolute terms. Rank total was obtained by summing the individual ranks for both criteria.

For convenience, the models evaluated are listed below:

Model	Source
1	Kozak et al. (1969)
2	Ormerod (1973)
2a	Ormerod (1973) Dbh_{ib} estimated.
3	Amidon (1984)
4	Max and Burkhart (1976)
5	Kozak (1988)
6	Kozak et al. (1969) estimated exponents
7	Amidon (1984) estimated exponents
8	Kozak (1988) reduced number of variables.

According to Table 4, models 5 and 8 have the best prediction ability. Both have the same rank value. The same table also shows a big gap between the total rank value for these models and model 4, which was ranked third. Model 1 was ranked as the worst predictor.

One of the disadvantages of assigning ranks to each individual criterion, is that rank values are insensitive to the relative difference between the criterion value for two given models. Table 5 displays almost the same information but the bias values are squared and added to the variance to give a total squared error. Ranks are then assigned based on the magnitude of this sole criterion. This procedure is simple and indicative of overall performance.

Table 5 shows that model 5 is the best predictor. Despite the multicollinearity present in the model, prediction was not severely affected. The diagnostic tools used to detect collinearity and its effect in the least squares procedure do not extend easily to show the impact on prediction. However, the presence of multicollinearity suggests areas in the regressor space where prediction is likely to be poor, especially when extrapolation outside the data range becomes necessary or another data set becomes available.

Model 8 was ranked second and compares very well with model 5, despite the fact that it is fitted with three fewer parameters. Both models, 5 and 8, were fitted assuming that the inflection point occurs at 25% of total height. The location of the inflection point was set at 15%, 20%, 30% and 35% of total height but these changes had little effect on the predictive properties of the resultant equations.

Model 2 appears to be an excellent taper equation, being ranked third. On the other hand, model 2a was ranked next to last, pointing to the loss in precision due to the estimation of Dbh_{ib} as a function of Dbh. Max and Burkhart's model ranked fourth consistently throughout the evaluation. It predicted diameter near the tree tip relatively poorly, which tended to obscure its good prediction ability for the butt and middle sections of the tree.

Table 4. Ranks of taper models based on each prediction criterion.

Model	Mean bias		Standard deviation of the differences		Total rank
	Value	Rank	Value	Rank	
1	-0.2646	8	1.6841	9	17
2	0.5450	9	1.3497	1	10
2a	-0.0790	3	1.6757	8	11
3	-0.1236	4	1.5782	6	10
4	-0.1441	5	1.4497	4	9
5	-0.0217	2	1.4075	2	4
6	-0.2189	7	1.6366	7	14
7	-0.1951	6	1.5361	5	11
8	-0.0094	1	1.4293	3	4

Table 5. Ranks of taper models based on a single criterion.

Model	Mean bias squared	Variance of the differences	Total squared error	Rank
1	0.0700	2.8361	2.9061	9
2	0.2970	1.8217	2.1187	3
2a	0.0062	2.8079	2.8141	8
3	0.0153	2.4906	2.5059	6
4	0.0208	2.1018	2.1226	4
5	0.0005	1.9815	1.9815	1
6	0.0479	2.6784	2.7263	7
7	0.0381	2.3596	2.3977	5
8	0.0001	2.0428	2.0429	2

In general, residual plots showed that most of the models do not perform well at the butt section of the large trees in the data set. As suggested by the final report of the site productivity study (College of Forestry, Wildlife and Range Sciences of the University of Idaho, and ESNACIFOR, 1985), it was difficult to include in the sample large trees from the best sites because these sites are usually the first to be harvested. As a consequence, many of the large trees selected were very old and of poor form which makes them rather unrepresentative of the target population the models are intended to describe.

In summary, the evaluation points very favorably to the selection of either model 5 or 8. This study recommends model 8 because it is simpler and predicts just as well as model 5. Figure 2, 3 and 4 show the way model 8 approximates the taper of a small, medium and large size trees selected at random from the validation data set. Figure 5 displays how models 2 and 4, which were ranked third and fourth respectively, predict the taper for the medium-sized tree.

Figure 6 shows the mean bias per section along the stem for three of the models with the best prediction. Sections 1,2 and 3 refer to diameter measurements at 0.3,0.8 and 1.3 meters above ground. Sections 4 through 12 refer to nine equally-spaced observations, each at $\frac{(H-1.3)}{10}$ successive points up the stem. The mean values were computed using 173 observations, or the number of trees present in the validation data set. It is clear that both model 5 and 8 have basically the same prediction ability along the stem. Model 8 shows less bias in the first three sections but then tends to overpredict diameters from breast height up to section 7. On the other hand, Model 4 tends to overpredict diameters above section 9.

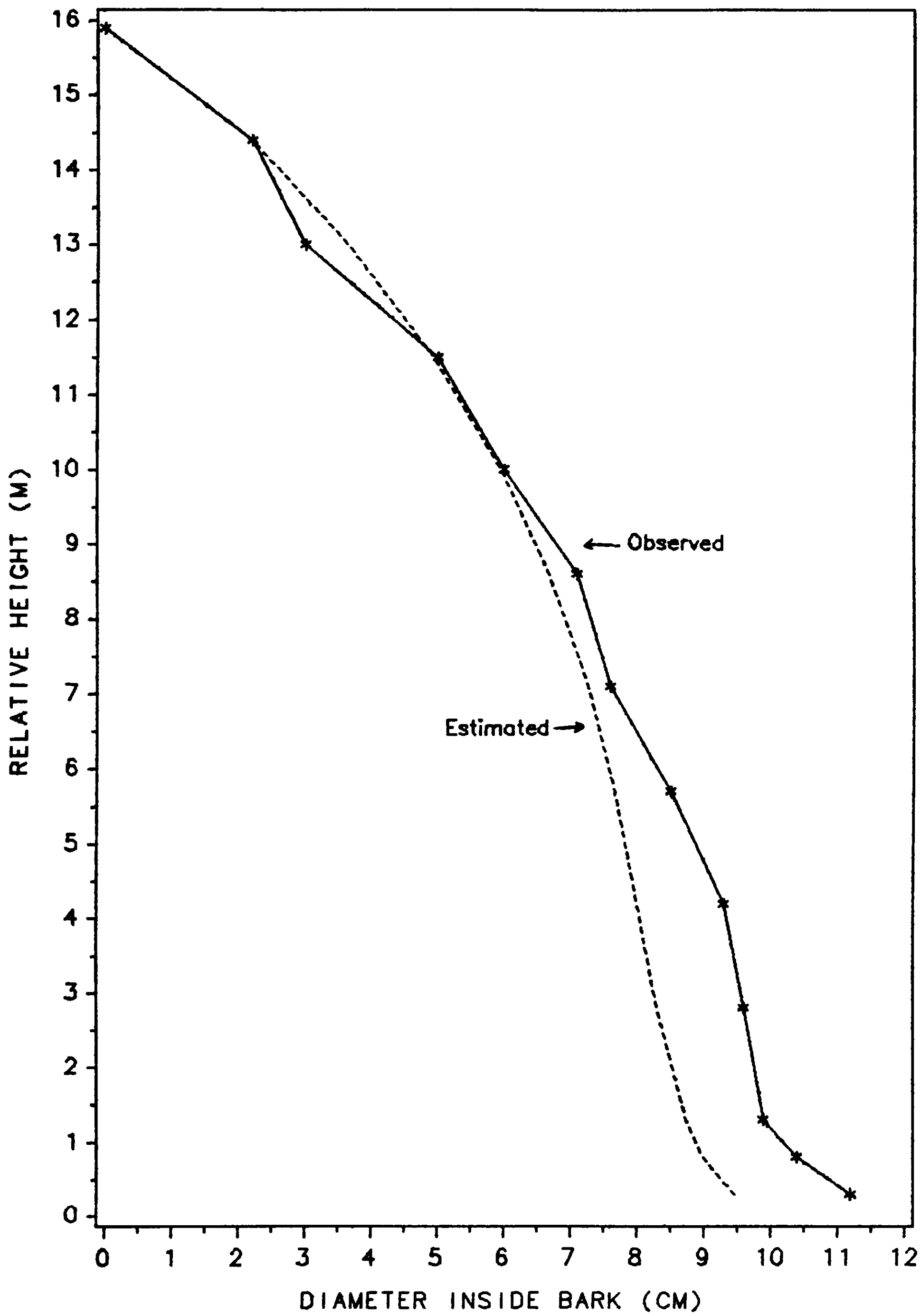

Figure 2. **Observed and estimated tree profile using model 8 for a small size tree:** (Dbh = 11.6 cm.; H = 15.9 m.; Age = 12 years).

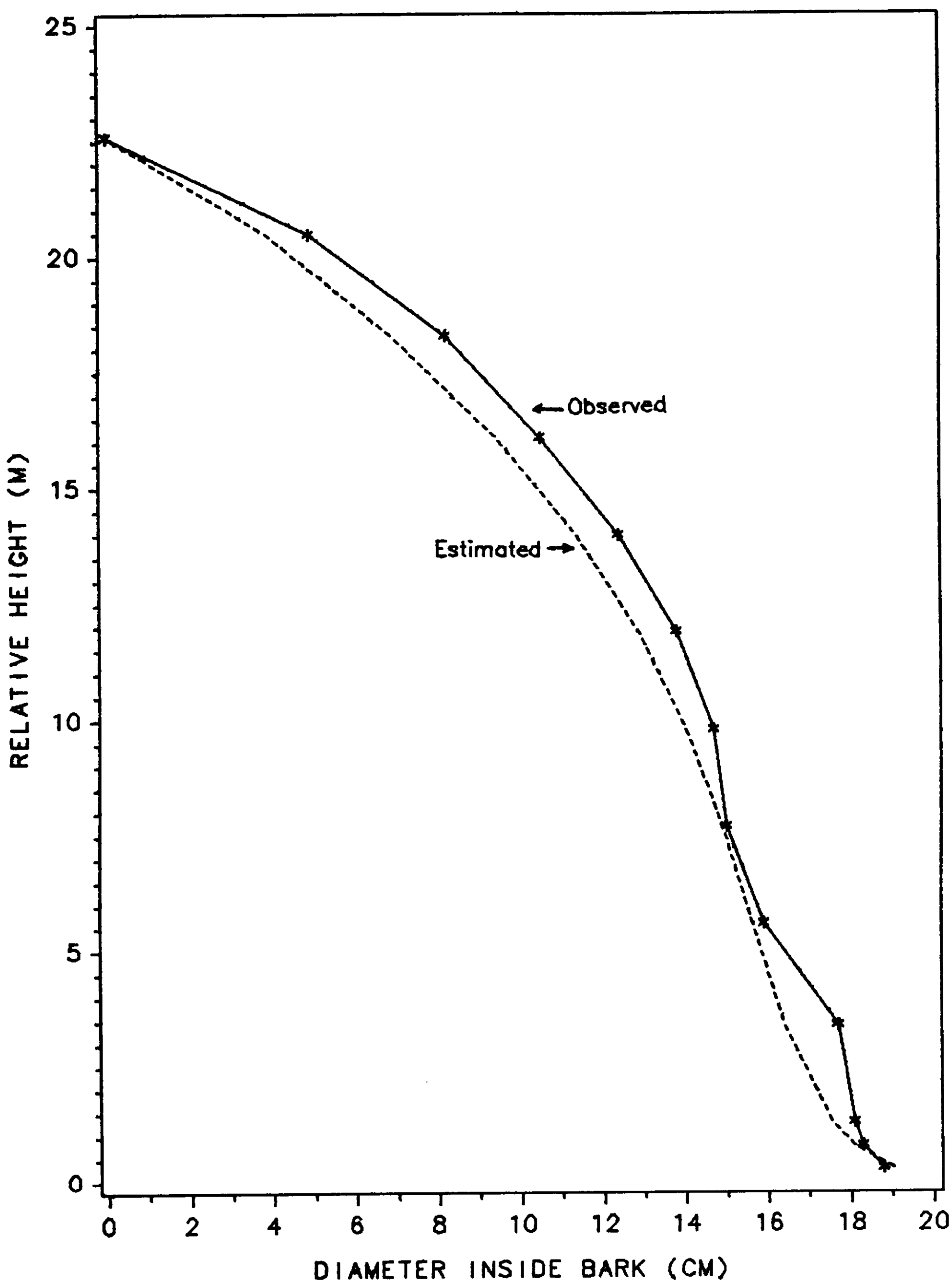

Figure 3. **Observed and estimated tree profile using model 8 for a medium size tree:** (Dbh = 22.1 cm.; H = 22.6 m.; Age = 16 years).

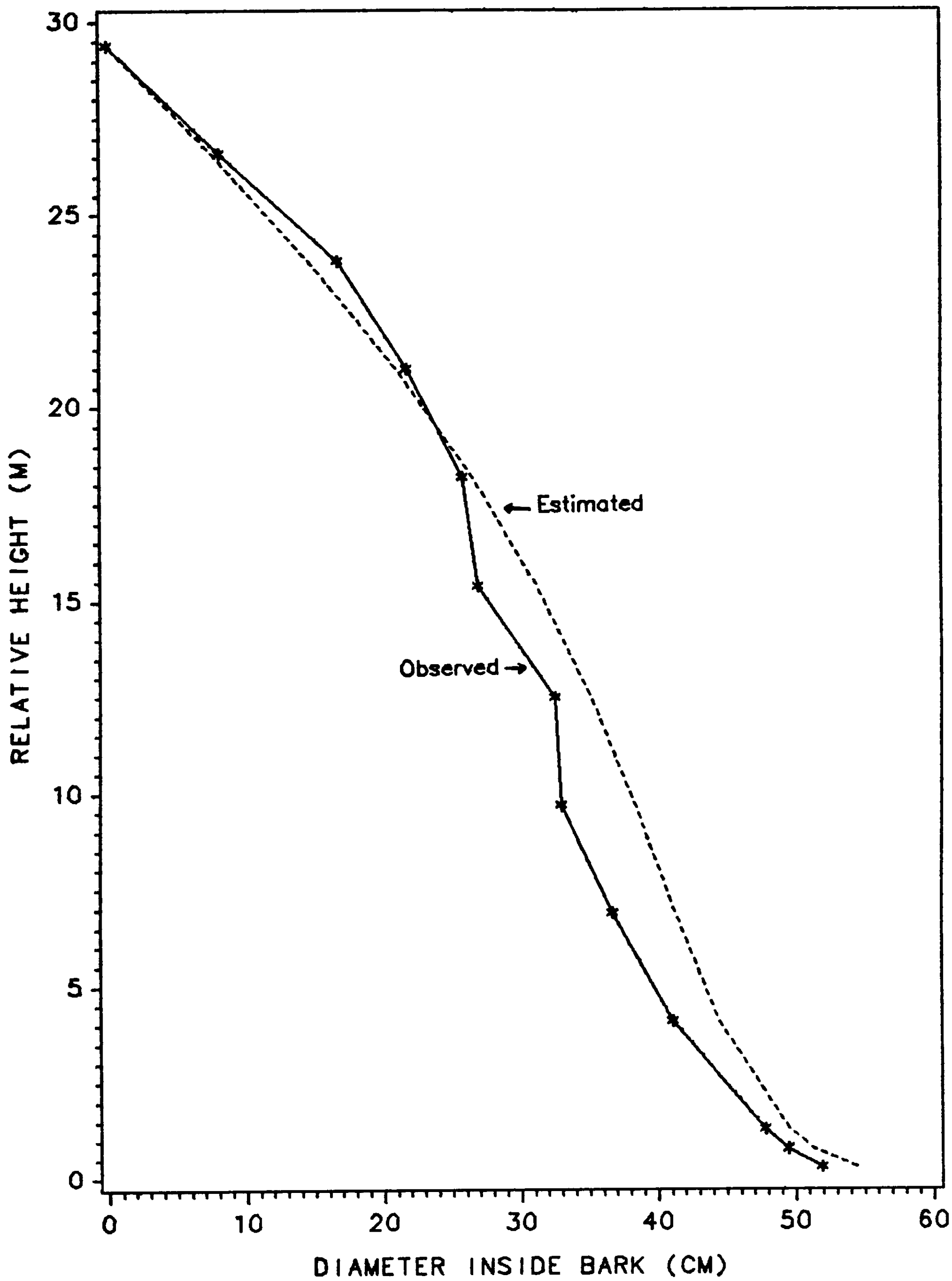

Figure 4. **Observed and estimated tree profile using model 8 for a large size tree:** (Dbh = 56.3 cm.; H = 29.4 m.; Age = 93 years).

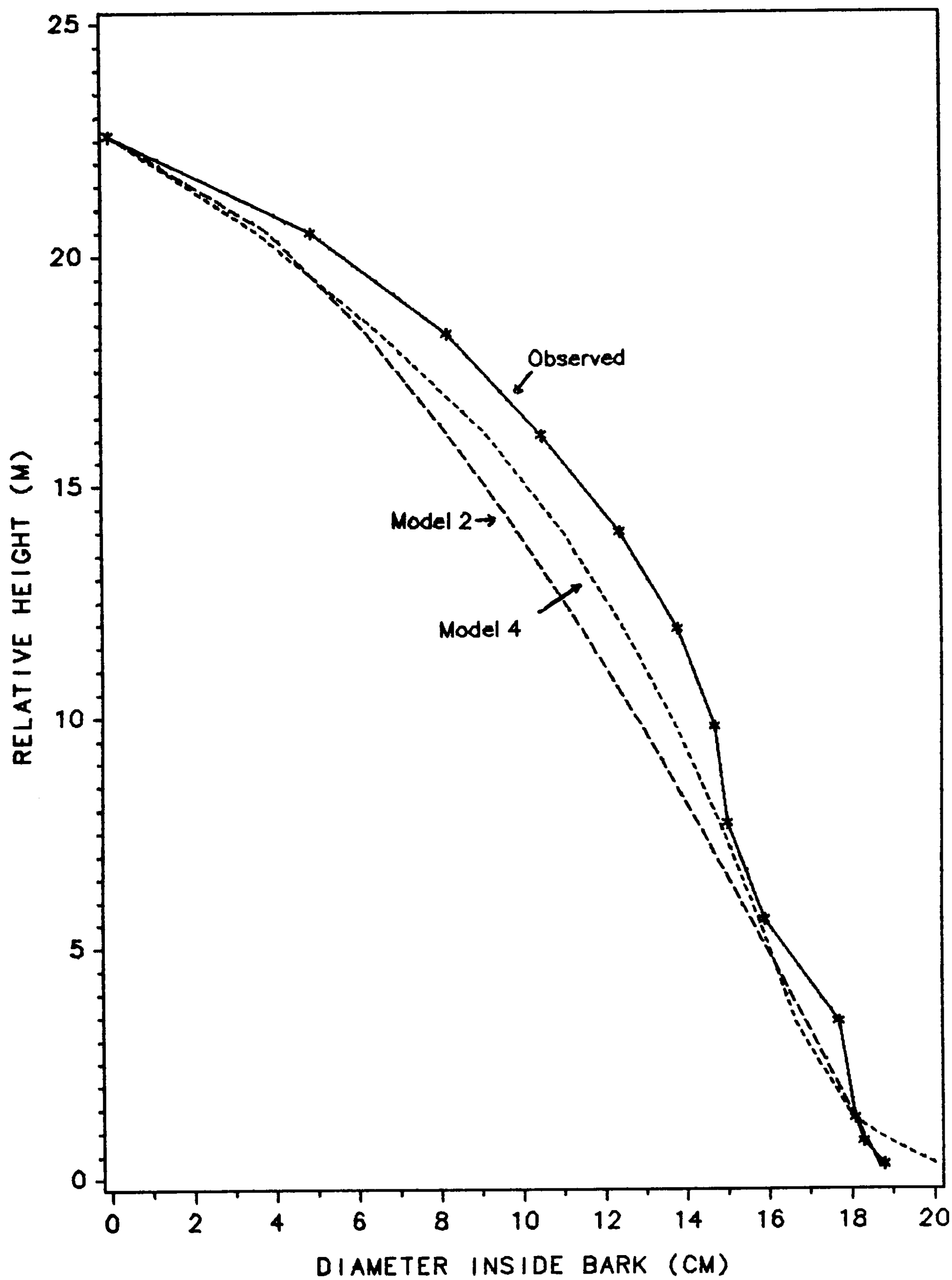

Figure 5. Observed and estimated tree profile using models 2 and 4 for a medium size tree: (Dbh = 22.1 cm.; H = 22.6 m.; Age = 16 years).

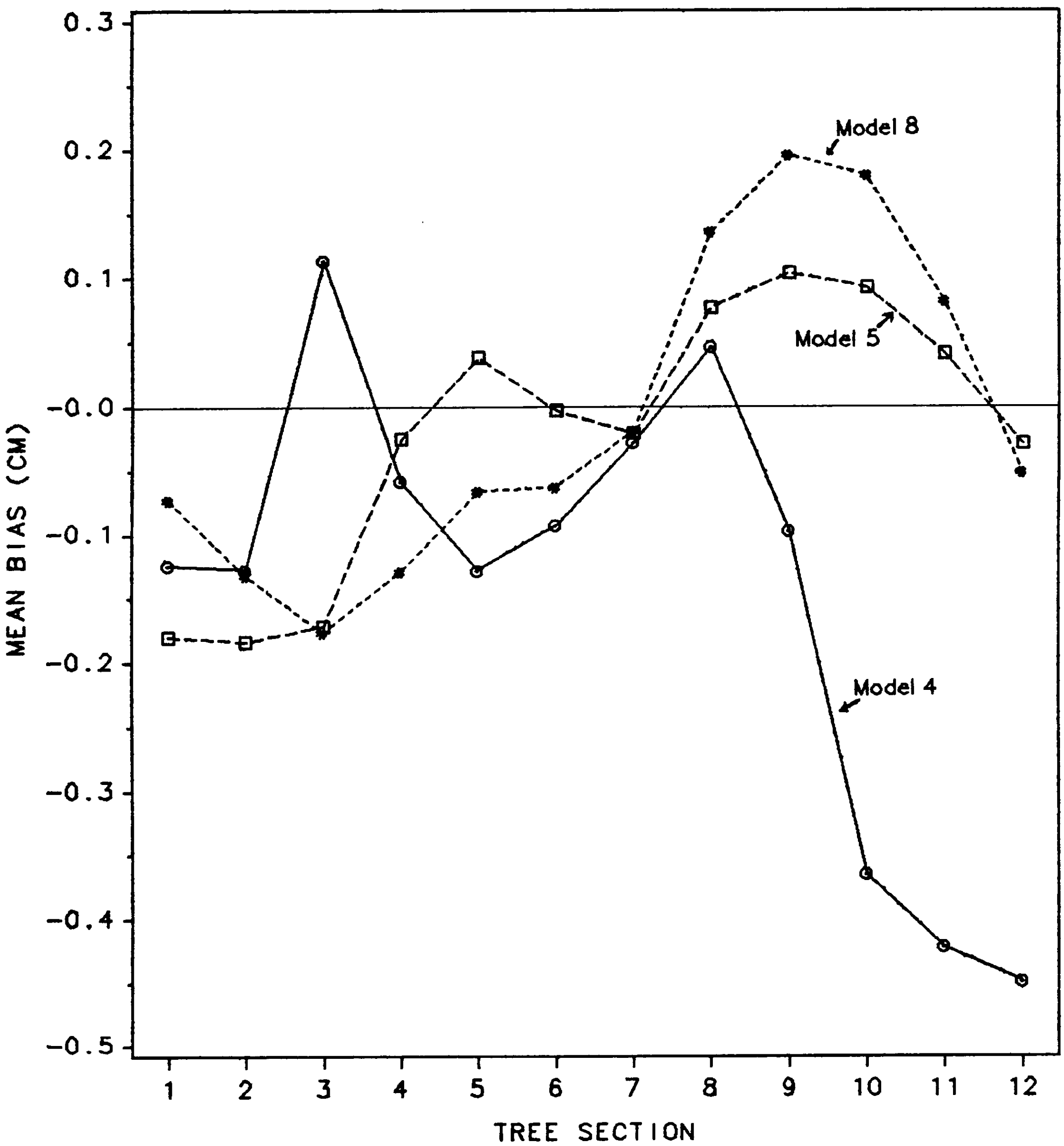

Figure 6. Mean bias along the stem for three of the models with best prediction in the study.

Taper variation

Variation by geographic region

Taper variation between trees from two important forest regions of Honduras was evaluated. These regions are only about 80 Km. apart, but there is on the average about 600 meters difference in elevation between them. This feature alone could have a distinctive effect on the environmental conditions affecting tree growth and development. A total of 108 trees were included from the forested region surrounding Siguatepeque, Comayagua and 97 trees were selected from the region of La Esperanza, Intibuca. Model 8 was used to predict tree taper in both regions. Linear regression techniques were used to fit the model to each data set separately and to both data sets combined.

An F test was conducted to test for significant differences in parameter estimates between the two geographic regions. The F_{obs} value was 8.12, therefore H_o is rejected. That is, there is sufficient evidence to state that the parameter estimates from both regions are significantly different at an approximate .01 probability level. The test is very likely to reject H_o at even low probability levels because of the high number of degrees of freedom for the error term, and the tendency of the mean square error to underestimate the actual variance, σ_t^2.

In order to explore in detail the possible taper differences between the two regions, the parameter estimates from both regions were carefully examined. Table 6 shows the parameters estimated by fitting model 8 to the observations from both regions sepa-

Table 6. Parameter estimates for two selected geographic regions.

Parameter estimate[1]	Geographic region	
	Siguatepeque	La Esperanza
b_o[2]	-0.463871	-0.505528
b_1	1.028970	1.048436
b_2	0.553862	0.600924
b_3	-0.067647	-0.067572
b_4	0.126413	0.097377

[1] Model: $\ln(d) = \left[\ln(b_o) + b_1 \ln(Dbh) + b_2 \ln(X) z^2 + b_3 \ln(X) \ln(z + .001) + b_4 \ln(X) (\frac{Dbh}{H}) \right]$

[2] values for b_o are given in the natural log scale.

rately. By simple observation, it appears that the parameter estimates b_2 and b_4 are, among the rest, slightly different in value for each region.

Partial regression leverage plots are used in multiple regression to reveal the true role and relative strength of individual regressors in the model. The partial plots for the variables of model 8 showed that the variables associated with the parameter estimates b_1, b_2, and b_4 had good explanatory power with points following an approximate straight line. In particular, the difference in value in the parameter estimate of b_4 for each region, might indicate that different Dbh and total height ratios hold for trees from those two regions, reflecting perhaps differences in growth patterns for tree characteristics closely related to taper.

The average site index based on an index age of 15 years for those plots established around La Esperanza was 12.26 meters with range between 6.64 and 18.38, while for those around Siguatepeque was 14.33 meters with range between 8.54 and 19.38 meters.

Test of practical differences

Although the test showed statistical difference between the parameter estimates of both regions, it is important to evaluate whether practical differences exist. The parameter estimates for model 8 obtained by using the fitting data set were employed. Trees in the validation data set were sorted by region, yielding a total of 27 trees from La Esperanza, and 39 trees from Siguatepeque to evaluate differences.

Table 7 shows the results for all observations. From the table, it is clear that overall model 8 tends to overpredict diameters in both regions but especially in the region of Siguatepeque. The same information is displayed in Figure 7a and 7b, but this time a distribution pattern per tree section can be readily observed. The pattern varies by geographic region, especially in the mid sections.

In summary, the user can expect results similar to those shown in Figure 7 when applying the parameter estimates of model 8 that appear in Table 3, to estimate diameters in each region. That is, greater mean bias due to overprediction can be expected for trees growing in good sites with lower Dbh to height ratios. This information can aid forest managers in making more sensible taper applications when dealing with data from either of the two regions.

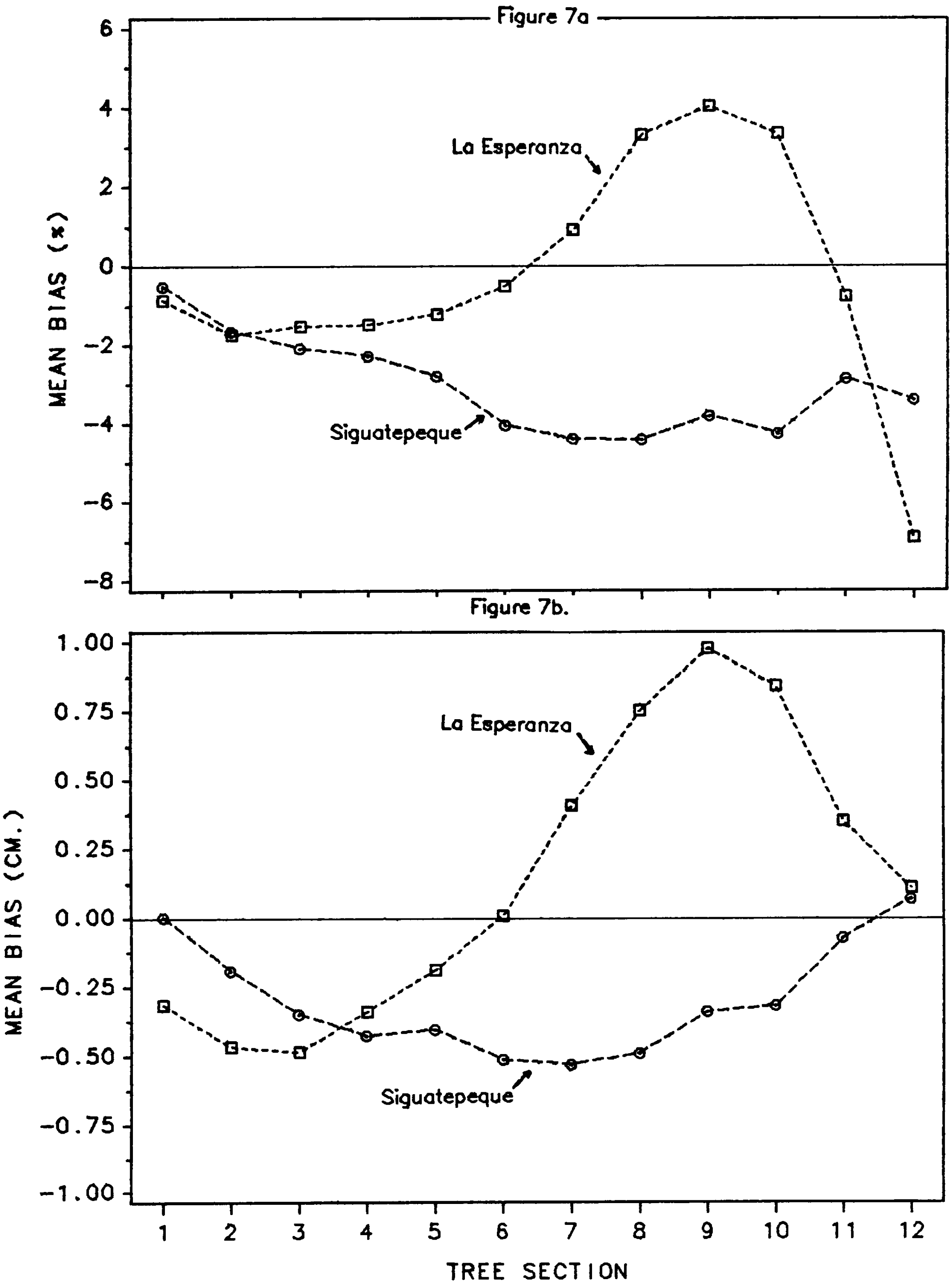

Figure 7. **Mean bias along the stem for trees from two different geographic regions:** 7a). As a percentage of observed diameters, and 7b). as a difference in cm.

Table 7. Mean bias expressed as a percentage of observed diameters and in centimeters for two selected geographic regions.

Geographic region	Criteria[1]	
	Mean bias	Mean absolute bias
Siguatepeque	-3.04	8.35
	(-0.30)	(1.11)
La Esperanza	-0.30	9.46
	(0.14)	(1.27)

[1] criteria values are given in percentage and (centimeters).

Variation by crown characteristics

The influences of crown class and live crown ratio on taper prediction was explored. In order to examine taper prediction as affected by crown class, trees in the validation data set were divided in two groups. One group contained the observations on 129 dominant and codominant trees, while the other group contained those on 44 trees in the intermediate and suppressed crown classes. In Table 8, they are denoted as class 1 and class 2 respectively. Table 8 shows overall values for both crown classes.

The examination of mean bias per section along the stem in Figure 8, shows very different patterns for both classes. It appears that for dominant and codominant trees, model 8 tends to overestimate diameters in the lower as well as the top sections, while for intermediate and suppressed trees the opposite is true. This result probably has to do with the effect that competition had on tree form. That is, dominant trees have the tendency to develop more conical rapidly tapering stem with pronounced butt swell. On

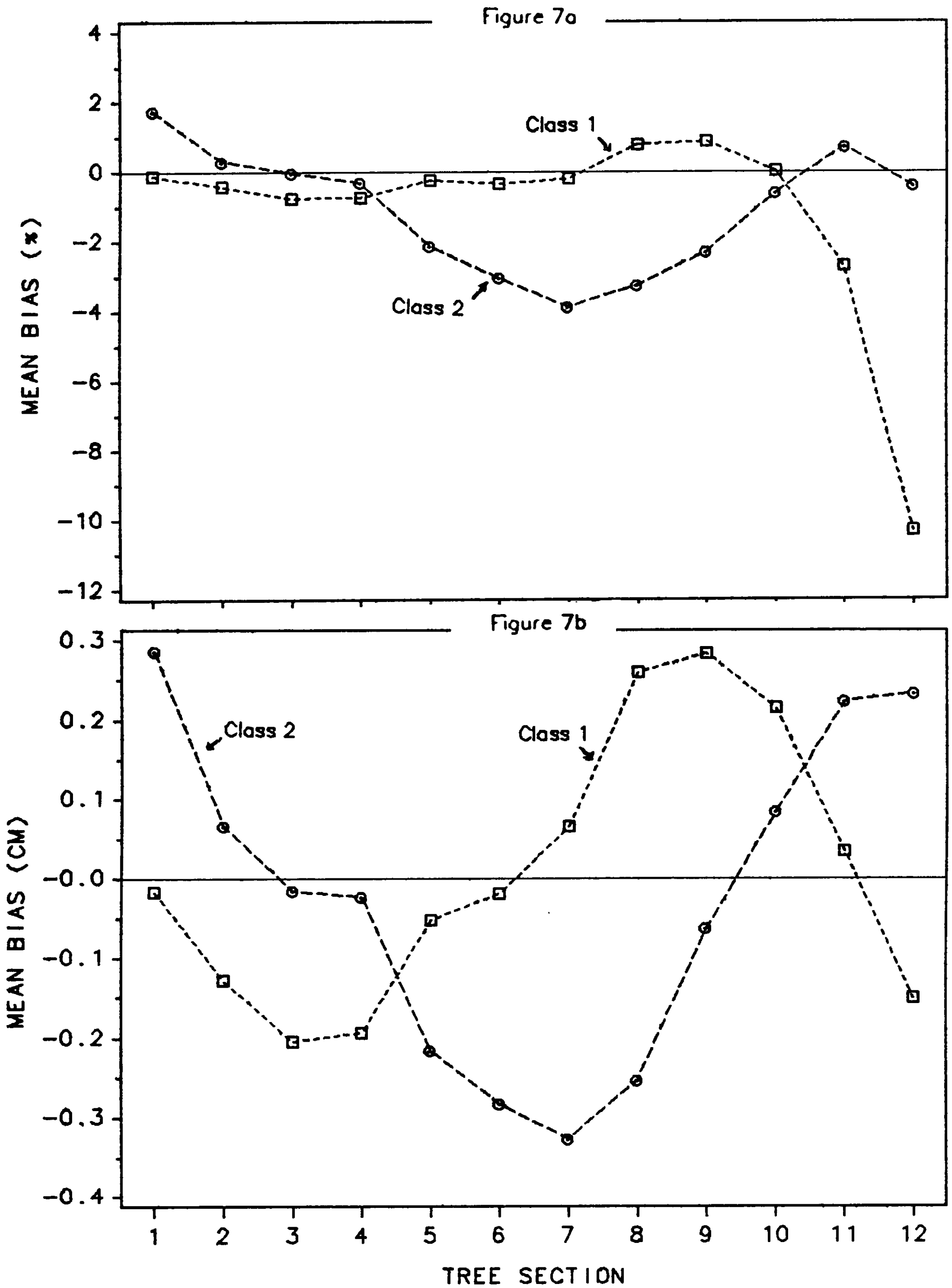

Figure 8. Mean bias along the stem for trees of different crown classes: 7a). As a percentage of observed diameters, and 7b). as a difference in cm.

Table 8. Mean bias expressed as a percentage of observed diameters and in centimeters for two crown classes.

Crown class	Criteria[1]	
	Mean bias	Mean absolute bias
Class 1	-1.18	8.21
	(0.01)	(1.17)
Class 2	-1.10	9.18
	(-0.02)	(0.77)

[1] criteria values are given in percentage and (centimeters).

Table 9. Mean bias expressed as a percentage of observed diameters and in centimeters for trees with different crown ratios.

Crown ratio	Criteria[1]	
	Mean bias	Mean absolute bias
LCR < =45	-2.25	8.39
	(-0.01)	(1.07)
45< LCR < =60	-2.12	8.13
	(-0.09)	(1.09)
LCR >60	-0.23	8.97
	(0.14)	(1.04)

[1] criteria values are given in percentage and (centimeters).

the contrary, suppressed trees tend to develop more cylindrical form, due primarily to an even distribution of diameter increment along the stem.

The effect of live crown ratio on taper variation was also explored. The validation data set was divided in three groups of trees with different live crown ratios (LCR) expressed in percent. Thus, 30 trees were placed in the class with LCR less or equal to 45%; 85 trees in the class with LCR greater than 45% but less or equal to 60%, and 58 trees with LCR greater than 60%. The cutoff points for the groups were chosen arbitrarily in order to give an even distribution and as such are not based on any biological consideration. It was found that only 8 trees in the validation set had LCR less than 36%, because most trees in the sample were dominant and codominant stems. Model 8 was again used in the prediction of diameters. Table 9 shows the overall results. The values obtained for mean bias per individual section did not show any noticeable trend or marked differences among the groups.

In summary, it appears that model 8 predicts differently for dominant and suppressed trees. On the other hand, live crown ratios did not appear to affect prediction substantially.

Chapter V

Conclusions and Recommendations

Although model 8 is the recommended model, the others can also be applied. As discussed in the preceding section, Ormerod's and Max and Burkhart's models also showed good predictive ability, especially the latter. In addition, these two models have the added advantage of providing estimates of volume directly by evaluating the corresponding integral.

The use of model 8 in the study of taper variation by geographic region showed that the inclusion of a variable to express site index would probably improve taper prediction. It should also be noticed that model 8 tends to overpredict diameters for trees growing in better sites.

Model 8 can be applied to any of the geographic regions discussed by using the parameter estimates for the region, or it can be applied by using the estimates obtained using the whole data set. If continued forest operations are planned for any of the regions, then perhaps the model with the parameter estimates for that specific region should be used.

Finally, the influence of crown class and live crown ratio on taper prediction was examined. Results showed that model 8 tends to predict differently for dominant and suppressed trees. Live crown ratio does not appear to have a substantial effect on taper prediction.

Estimation of Volumes

One of the most useful applications of taper equations is that they can be integrated to calculate volume. Because the model recommended does not have a closed form integral, a computer program written in BASIC was developed for volume computations. The program is interactive and allows the user to obtain total volume as well as merchantable volumes inside bark to any desired top diameter in centimeters. A copy of the program is listed in appendix A.

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Appendix A

Computer Program to Estimate Volumes

```
'              TOTAL AND MERCHANTABLE VOLUMES
'                   Species : Pinus oocarpa Schiede
'                  Program written by: Noe Perez R.
'                          (June 1988)
'
'

'  This program calculates total and merchantable cubic volume
' per tree for Pinus oocarpa Schiede in units of the metric system
' of measurements. It uses a taper equation (Model 8), from the
' M.S. Thesis by Perez (1988), with the parameter estimates
' calculated using the entire data set.
'  The data are read from a specified file created by the user which
' may contain up to 100 records. The file must not contain string
```

```
' characters. Only numbers are expected to be saved under the
' following format:
'
'                    "dbh   ht"
'
'  where:
'
'     dbh = diameter at breast height with bark in centimeters,
'       ht = total tree height in meters.
'
'
'     If merchantable volumes are desired, the user must specify the
' top diameter limit in centimeters while executing the program.
' Volumes are computed by summing the volumes of small tree
' segments whose lengths are taken to be the one hundredth
' fraction of total tree height. The Smalian formula is used to
' compute the volume of each segment.
'
'
' MAIN PROGRAM
'
DIM DBH#(100),HT#(100),D#(100),VOL#(100),VOLSEC#(100)
DIM VOLCO#(100)
option base 1
cls
```

```
call fileopen(infile$,top%,top1#) ' opens the file named by the user

Call readthedata(dbh#(),ht#(),nu%)  ' reads the data into arrays

Call Compute (dbh#(),ht#(),d#(),vol#(),volsec#(),volco#(),(nu%)_
        (top%),sec#,z#,x#,c#,length#,(top1#),ped#)
'    compute volumes

Call Display (dbh#(),ht#(),vol#(),volco#(),(nu%),(top%),(top1#),_
        (length#),(infile$))
'    display results

close #1
end

'******************Subroutines*******************
sub fileopen(infile$,top%,top1#)
cls
locate 12,10
input "TECLEE EL NOMBRE DEL ARCHIVO DE DATOS";infile$
open infile$ for input as #1
cls
locate 6,22
print "VOLUMENES COMERCIALES Y TOTALES"
locate 7,23
print "Especie: Pinus oocarpa Schiede"
```

```
locate 8,16
print "*******************************************"
locate 10,20
input "DESEA VOLUMENES COMERCIALES (S/N)";ANS$
ans$=ucase$(ans$)
print

'  loop to input top diameter limit
'  if commercial volumes are desired

while flag = 0
if ans$="S" then
 locate 12,16
 input "TECLEE EL DIAMETRO LIMITE SUPERIOR EN CM";top%
 locate 14,18
 print "EL DIAMETRO SUPERIOR TECLEADO ES";top%
 locate 16,20
 input "...ES EL VALOR CORRECTO (S/N)";answer$
 answer$=ucase$(answer$)
 top1# = top%/100
 locate 18,16
 print
 if answer$="S" then
   flag=1
 else
   cls
```

```
 end if
elseif ans$= "N" then
locate 14,17
print "SOLO VOLUMENES TOTALES SERAN CALCULADOS"
top% = 0
flag = 1
end if
wend

'   loop ends

call pause
end sub
'**************************************************
sub Readthedata(dbh#(1),ht#(1),nu%)
cls
nu% = 1

'   data input loop

while not eof(1)
input#1,dbh#(nu%),ht#(nu%)  'reads the records
nu% = nu% + 1
wend

'   loop ends
```

```
nu% = nu% - 1
end sub
'**************************************************
sub Compute (dbh#(1),ht#(1),d#(1),vol#(1),volsec#(1),volco#(1),nu%,_
        top%,sec#,z#,x#,c#,length#,top1#.ped#)_
pi#=3.14159/4
locate 12,28
print "Espere,...trabajando!!!"

'    outer loop reads individual trees
for m% = 1 to nu%
 diam% = top%
 sec# = 0
 z# = 0
 ped# = (ht#(m%) - 0.3)/100
 length#= (ht#(m%) - 0.3)/100
 x# = (1 - sqr(z#))/(1 - 0.5)
 ra#= dbh#(m%)/ht#(m%)
 c#=0.54684*z#**2-0.048952*log(z# + 0.001)+0.141629*ra#
 d#(1) = (0.648641 * dbh#(m%)**1.027609 * x#**c#)/100
 '    diameter at tree base

 '    volume of the stump
 sec# = sec# + 0.3
```

```
z# = sec#/ht#(m%)
x# = (1 - sqr(z#))/(1 - 0.5)
ra#= dbh#(m%)/ht#(m%)
c#=0.54684*z#**2-0.048952*log(z# + 0.001)+0.141629*ra#
d#(2) = (0.648641 * dbh#(m%)**1.027609 * x#**c#)/100
'  diameter at stump level
stump = pi# * ((d#(1)**2 + d#(2)**2)/2) * 0.3
vol#(m%) = stump

'  inner loop calculates and sums volume for each tree

for s% = 3 to 100
 sec# = sec# + length#
 z# = sec#/ht#(m%)
 x# = (1 - sqr(z#))/(1 - 0.5)
 c#=0.54684*z#**2-0.048952*log(z# + 0.001)+0.141629*ra#
 d#(s%) = (0.648641 * dbh#(m%)**1.027609 * (x#**c#))/100
 volsec#(s%) = pi# * ((d#(s% - 1)**2 + d#(s%)**2)/2)*ped#
 vol#(m%) = vol#(m%) + volsec#(s%)

if diam% <> 0 then

'   conditional loop for calculating commercial volumes

 if d#(s%) > top1# then
   volco#(m%) = volco#(m%) + volsec#(s%)
```

```
  else
    diffd# = d#(s% - 1) - d#(s%)
    diffd1# = d#(s% - 1) - top1#
    ito# = (ped# * diffd1#)/ diffd#
    volped# = pi# * ((top1#**2 + d#(s% - 1)**2)/2)*ito#
    volco#(m%) = volco#(m%) + volped#
    diam% = 0
  end if

'   conditional loop ends
 end if
next

'   inner loop ends

 volcono# = ((pi# * d#(s%)**2)/3) * ped#
 vol#(m%) = vol#(m%) + volcono#
'    sums the volume of segment at tree top
next
'   outer loop ends

end sub
'**************************************************
sub Display(dbh#(1),ht#(1),vol#(1),volco#(1),nu%,top%,_
        top1#,length#,infile$)
```

```
' on input :  dbh# = array containing the diameters at b.h.
'               ht# = array containing the total heights
'              vol# = array containing total volume per tree
'            volco# = array containing commercial volume per tree
'               nu% = total number of trees in datafile
'           infile$ = datafile name
'              top% = top diameter limit in centimeters
'              top1 = top diameter limit in meters
'           length# = length of segment

cls
locate 8,22
print "VOLUMENES TOTALES Y COMERCIALES"
print
print
print tab(19) "NOMBRE DE ARCHIVO DE DATOS : ";infile$
print
print tab(19) using "NUMERO DE ARBOLES EN ARCHIVO = ###";nu%
print
print tab(19) using "DIAMETRO LIMITE SUPERIOR = ##";top%
call pause
cls

'     displays total volume
```

```
locate 1,32
print "VOLUMEN TOTAL"
print
print
print spc(10) "ARBOL" spc(12) "DAP" spc(9) "ALTURA TOT.";_
    spc(8) "VOL. TOT."
print spc(11) "NO." spc(12) "(cm.)" spc(12) "(m.)"
    spc(14) "(m3)"
print tab(6) string$(65,240)
for c% = 1 to nu%
 contar% = 1
 print tab(11) using "###          ###.#          ##.#_
          #.####";c%,dbh#(c%),ht#(c%),vol#(c%)

 if c% = 20*contar% then
  while not instat
   locate 22,19
   print "por favor, presione una tecla para continuar"
  wend
  incr contar%
  s$ = inkey$
 end if

next
call pause
```

```
'     displays commercial volumes

if top% = 0 then
  cls
  locate 12,16
  print "...VOLUMENES COMERCIALES NO FUERON CALCULADOS!!"
  call pause
  cls
  locate 2,25
  print "...Presione <esc> para continuar"
  exit sub
else
  cls
  locate 1,29
  print "VOLUMEN COMERCIAL"
  locate 2,26
  print "Diametro Limite:";top%;" cm."
  print
  print
  print spc(10) "ARBOL" spc(12)"DAP" spc(9)"ALTURA TOT.";_
        spc(8) "VOL. COM."
  print spc(11) "NO." spc(12) "(cm.)" spc(12) "(m.)"_
        spc(14)"(m3)"
  print tab(6) string$(65,240)
 for c% = 1 to nu%
  contar% = 1
```

```
  print tab(11) using "###          ###.#          ##.#_
          #.####";c%,dbh#(c%),ht#(c%),volco#(c%)
 next

if c% = 20*contar% then
 while not instat
  locate 22,20
  print "por favor, presione una tecla para continuar"
 wend
 incr contar%
 s$ = inkey$
 end if
end if

'    sets screen environment

call pause
cls
locate 2,25
print "...Presione <esc> para continuar"
end sub
'**************************************************
' subroutine for screen control

sub pause
while not instat
```

```
 locate 22,17
 print "por favor, presione una tecla para continuar"
wend
a$ = inkey$
end sub

'*******END OF FILE******
```

Appendix B

Fit Statistics for Taper Models

Model	Source	Dependent variable[1]	R^2	MSE
1	Kozak et al. (1969)	$\frac{d^2}{Dbh^2}$	0.9741	0.005878
2	Ormerod (1973)	d	0.9475	0.022498
2a	Ormerod (1973) Dbh_{ib} estimated	d	0.9512	0.023432
3	Amidon (1984)	d	0.9913	2.984043
4	Max and Burkhart (1976)	$\frac{d^2}{Dbh^2}$	0.9783	0.004936
5	Kozak (1988)	ln (d)	0.9590	0.016843
6	Kozak et al. (1969) estimated exp.	$\frac{d^2}{Dbh^2}$	0.9737	0.005979
7	Amidon (1984) estimated exp.	d	0.9918	2.815085
8	Kozak (1988) reduced.	ln (d)	0.9587	0.016977

[1] All fit statistics are given in the units of the dependent variable and were computed using the fitting data set.

Appendix C

Diagnostic Criteria for Candidate Models

The independent variables of the candidate models are defined as:

$$X_1 = ln\,(Dbh)$$

$$X_2 = \text{Dbh}$$

$$X_3 = ln\,(X)z^2$$

$$X_4 = ln\,(X)ln\,(\,z + 0.001)$$

$$X_5 = ln\,(X)\sqrt{z}$$

$$X_6 = ln\,(X)e^z$$

$$X_7 = ln\,(X)(\,\frac{Dbh}{H}\,)$$

N^o	Candidate Model	PRESS	MSE	C_p	R^2
1	$X_1 X_2 X_3 X_4 X_5 X_7$	82.13	0.016841	6.48	0.959026
2	$X_1 X_2 X_3 X_4 X_6 X_7$	82.13	0.016842	6.78	0.959024
3	$X_1 X_2 X_3 X_5 X_6 X_7$	82.14	0.016843	7.08	0.959021
4[1]	$X_1 X_2 X_3 X_4 X_5 X_6 X_7$	82.15	0.016842	8.00	0.959030
5	$X_1 X_2 X_3 X_4 X_7$	82.20	0.016865	12.39	0.958960
6	$X_1 X_2 X_4 X_5 X_6 X_7$	82.33	0.016881	18.13	0.958928
7	$X_1 X_3 X_4 X_5 X_7$	82.64	0.016952	37.40	0.958748
8	$X_1 X_3 X_4 X_6 X_7$	82.64	0.016953	37.74	0.958746
9	$X_1 X_3 X_5 X_6 X_7$	82.65	0.016953	37.75	0.958746
10	$X_1 X_3 X_4 X_5 X_6 X_7$	82.66	0.016953	38.82	0.958753
11[2]	$X_1 X_3 X_4 X_7$	82.72	0.016977	43.85	0.958677
12	$X_1 X_4 X_5 X_6 X_7$	82.84	0.016990	48.51	0.958655
13	$X_1 X_2 X_3 X_6 X_7$	84.64	0.017358	154.62	0.957759
14	$X_1 X_3 X_6 X_7$	85.14	0.017466	184.84	0.957487
15	$X_1 X_2 X_3 X_5 X_7$	85.67	0.017573	216.53	0.957236

[1] Model 5 (full model)

[2] Model selected as Model 8.

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