THE APERIODICALLY DAMPED LOUDSPEAKER SYSTEM

by

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(ABSTRACT)

The theory of the aperiodically damped loudspeaker system is derived and investigated in both the time and frequency domains. The advantages of the aperiodic configuration over sealed box loudspeaker systems are discussed.

The aperiodically damped loudspeaker system is described by the parameters used for sealed box loudspeaker systems, with some additions. A real system is constructed which takes full advantage of aperiodic configuration, and its performance is compared with a sealed box system of the same dimensions.

Complete derivations of the theory for both the sealed box and aperiodically damped systems are included, as well as methods for measuring all required parameters.

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CHAPTER 1: INTRODUCTION

The pursuit of high fidelity began shortly after the second world war, when relative economic prosperity in conjunction with offshoots of wartime technology made the home stereo an average consumer possibility. Radio broadcasting and disc recording spurred the growth of what was then an infant industry, and the market grew apace. Research uncovered quantifiable forms of distortion and methods of measuring these distortions at such a rate that by the late fifties the qualities of excellent electronics were well understood and incorporated into the better products of the day. By 1965, high fidelity was no longer a specialist persuit, and a quality music reproduction system was present in most American homes.

The area of music reproduction which has undergone the most scrutiny and conjecture is the loudspeaker system. Although there are other forms, the most popular type of loudspeaker is the moving cone. This system was well understood by the experts in the field as early as the 1950's, but it was not until the early 70's that acceptable theory on loudspeaker system design was fully developed. A method of predicting the performance of the sealed box loudspeaker from a few easily measured parameters was developed by Richard Small in 1971[1,2], and in 1973, working from the nearly complete (although largely unread) theory developed by Neville Thiele in 1961[3], Richard Small also brought vented box theory into usable form[4]. By 1980 the parameters required to predict system response (the Thiele/Small parameters) were published by most manufacturers for their low frequency drivers.

The sealed box loudspeaker system is commonplace, and offers satisfactory performance at a reasonable price. Also, it is not as difficult to design and implement as its vented counterpart, and with correct driver choice can provide excellent low frequency extension and well controlled transient response. However, variations within the same driver line, possibly due to poor quality control, almost always lead to systems which do not perform as the designer intended. Significant differences between the published and measured parameters of many drivers is not uncommon, but to keep cost down most systems are manufactured from nominal specifications. This leads to performance differences between production runs and, in some cases, even individual speakers of a stereo pair. In other words, it is not uncommon for the left speaker to sound quite different from the right. For proper musical reproduction, including timbre, correct localization of instruments, and image dimensions, the stereo pair must be extremely well matched[5].

It is also true that due to volume or cost constraints (as well as parameter variation) many drivers are not operated in their optimum environment. This leads to poor response in both the frequency and time domain, resulting in poor performance.

The question of desirable characteristics of a loudspeaker system is a topic of endless debate, but subjective tests[6] indicate it is frequency response descrepancies which makes systems sound most different, and correct transient response which makes them sound realistic. This explains why few vented systems are considered musically accurate, since in most cases their transient response is not nearly as good as a properly designed sealed system, and why accurate loudspeakers in pairs require higher quality control and, consequently, sell for a higher price.

The subject of this thesis is a method of loudspeaker system construction similar to the sealed box, but augmented by pressure relief. The modification takes the form of a large vent in the box which is densly filled with open-celled foam, dacron fiber, or dense fabric. The vent allows the system to breathe slightly, equalizing large pressure differences between the interior and exterior of the box. The damping due to the vent is largely frequency independent in the range of interest, hence the term "aperiodic."

The amount of pressure relief can be controlled by varying the amount and density of the vent filling, depending on system requirements. The flexibility of this parameter allows the principle to be successfully applied under a wide range of conditions, with predictable results.

Proper implementation of the aperiodic damping technique improves both frequency and transient response characteristics of many loudspeaker systems, offering the designer the ability to fine-tune the sealed box system to the desired specifications, or to design for and take advantage of this augmentation at the outset. The principles can also be applied to midrange and high frequency units, which would often benefit from its application.

The aperiodically damped system retains most of the advantages of the sealed box, and in many cases yields superior results.

CHAPTER 2: DEFINITIONS

The terms and symbols used will be the same as those of Small[7], with some additions.

DEFINITIONS OF TERMS

Baffle	a structure on which the driver is mounted, usually the front board of a speaker system.
Driver	a loudspeaker driver, which is the transducer element alone, including the magnetic system, frame, cone, voice coil, etc.
Frequency Response	this is more correctly termed power vs frequency response, and represents the acoustic power output due to constant voltage magnitude input, as a function of frequency
Low Frequencies	all frequencies for which $w < 2c/d$, where d is the driver diameter and c is the speed of sound in air at STP. This condition can also be stated as the wavelength being greater than the driver circumference.
Speaker	driver.
Speaker System	a completed system, including the driver, box, crossover network, damping material, etc.

Variovent a commercial product manufactured by Dynaudio GmbH in Denmark, consisting of a plastic basket 25mm deep and 125mm in diameter, in which a user specified amount of damping material may be inserted to control flow resistance.

DEFINITIONS OF SYMBOLS

B magnetic flux density of driver in air gap

c velocity of sound in air (=344 m/sec at STP)

C_{ab} acoustic compliance of air in enclosure

C_{as} acoustic compliance of driver suspension

C_{ms} mechanical compliance of driver suspension(=Cas/Sd)

C_{mes} electrical capacitance due to driver mass

$(=M_{as}S_d{}^2/B^2l^2)$

e_g open circuit output voltage of source

f natural frequency variable

f_s speaker resonance frequency

f_c system resonance frequency

$G(s)$ response function

l length of voice coil conductor in magnetic field

L_{ces} electrical inductance due to driver compliance

$(=C_{as}B^2l_2/S_d{}^2)$

M_{as} acoustic mass of driver diaphram assembly, including air load

M_{ms} mechanical mass of driver diaphram assembly, including air load (=$M_{as} S_d^2$)

P_a acoustic output power

P_e nominal electrical input power

Q Ratio of reactance to resistance (series circuit) or resistance to reactance (parallel circuit)

Q_{ec} Q of driver at f_c considering electrical resistance (R_e + R_g) only

Q_{es} Q of driver at f_s considering electrical resistance R_e only

Q_{mc} Q of driver at f_c considering system nonelectrical resistances only

Q_{ms} Q of driver at f_s considering driver nonelectrical resistances only

Q_{ts} total Q of driver at f_s including all system resistances

Q_{tc} total Q of driver at f_c including all system resistances

R_{ab} acoustic resistance of enclosure losses due to internal energy absorbtion

R_{al} acoustic resistance of enclosure losses due to leakage

R_{as} acoustic resistance of driver suspension losses

R_{at} acoustic resistance of total driver circuit losses

R_e DC resistance of driver voice coil

R_{es}	electrical resistance due to driver suspension losses ($=B^2 l^2 \backslash S_d{}^2 R_{as}$)
R_g	output resistance of source or amplifier
R_{ms}	mechanical resistance of driver suspension losses
R_{ar}	acoustic radiation resistance
s	complex frequency variable (=jw in this case)
S_d	effective projected surface area of driver diaphragm
T	time constant (=1/w =1/2pi f)
u	linear velocity
U	volume velocity
V_{as}	volume of air having same acoustic compliance as driver suspension ($=p_o c^2 C_{as}$)
V_{ab}	volume of air having same acoustic compliance as enclosure ($=p_o c^2 C_{ab}$)
$Z_{vc}(s)$	driver voice coil impedence function
n	efficiency
p_o	density of air ($=1.18kg/m^3$)
w	radian frequency variable
w_c	radian frequency of system resonance
w_s	radian frequency of speaker resonance
&	compliance ratio, C_{as}/C_{ab}
$@_{ls}$	loss factor at f_s
$@_{lc}$	loss factor at f_c

The author must apologize for the use of some non-standard symbols in this paper. The methods used in chapters 3, 4, and 5 are those of electrical engineering theory, where the symbol "**j**" is equivalent to the physics symbol "**i**", the square root of -1.

Another non-standard symbol is "**pi**", used throughout the text. "**pi**" = 3.14159.

CHAPTER 3: GENERAL ANALYSIS

The analysis of a driver-box combination is well covered elsewhere[8,9], and is only briefly discussed here.

THE ELECTRICAL SYSTEM

The loudspeaker electrical system consists of an amplifier, wiring, and a voice coil. All elements contribute electrical resistance to the circuit, while the voice coil also has an inductance. The amplifier and wire resistance are combined for convenience, and all elements are in series (fig. 1a). The following definitions are used:

e_g	source (amplifier) output voltage
R_g	sum of amplifier and wire resistance
R_e	driver voice coil resistance
L_{vc}	driver voice coil inductance.

Since we are concerned with frequencies near the driver resonance only, the reactance of L_{vc} is much smaller than the associated resistances, and is neglected.

The current through the voice coil is responsible for all acoustic output. It is often more convenient to consider the amplifier as its Thevenin equivalent current source (fig. 1b), with output $i_g = e_g/(R_g + R_e)$.

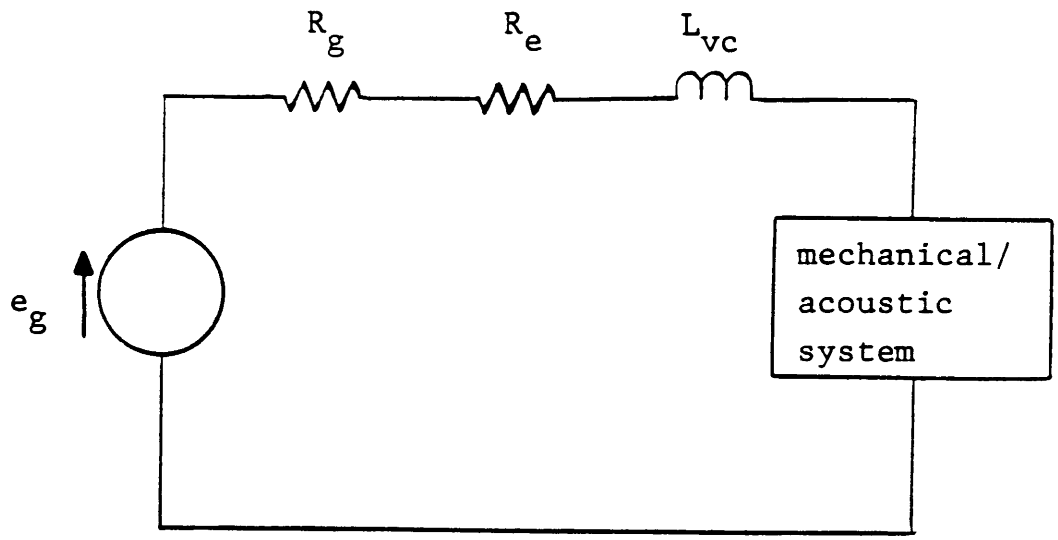


Fig. 1a   Driver electrical system

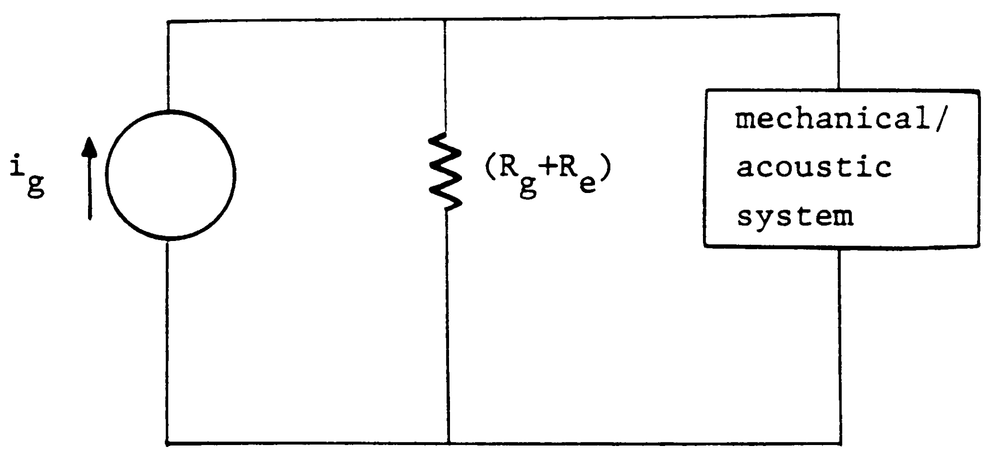


Fig. 1b Thevenin equivalent of driver electrical system

THE MECHANICAL AND ACOUSTICAL SYSTEMS

A driver mechanical system is essentially a motor consisting the voice coil wrapped around a cylindrical former situated in a permanent magnetic gap (fig. 2). Current in the voice coil is always perpendicular to the magnetic field, creating a force on the coil assembly along its axis. If th current is oscillatory, so is the resultant motion.

The cone is attached to the former at its apex, while the base is attached to an edge suspension which in turn is attached to the frame. A center suspension between the former and frame is also common. The cone, coil, former, and one edge of each suspension are considered to move rigidly as a whole, while the frame and attached box is assumed to be immobile. The suspensions contribute both compliance and resistive losses to the system, while all moving elements contribute moving mass.

The cone is coupled internally to the box volume, with the volume contributing a compliance, absorbtion losses, and leakage losses. Radiation resistance is also present on both sides of the cone. If the pressure in the box is p, the force induced on the moving system is

$$F = pS_d \qquad (1)$$

where S_d is the effective cone area. The cone then acts as a transformer from the mechanical to acoustic system, with a turns ratio of $1:S_d$.

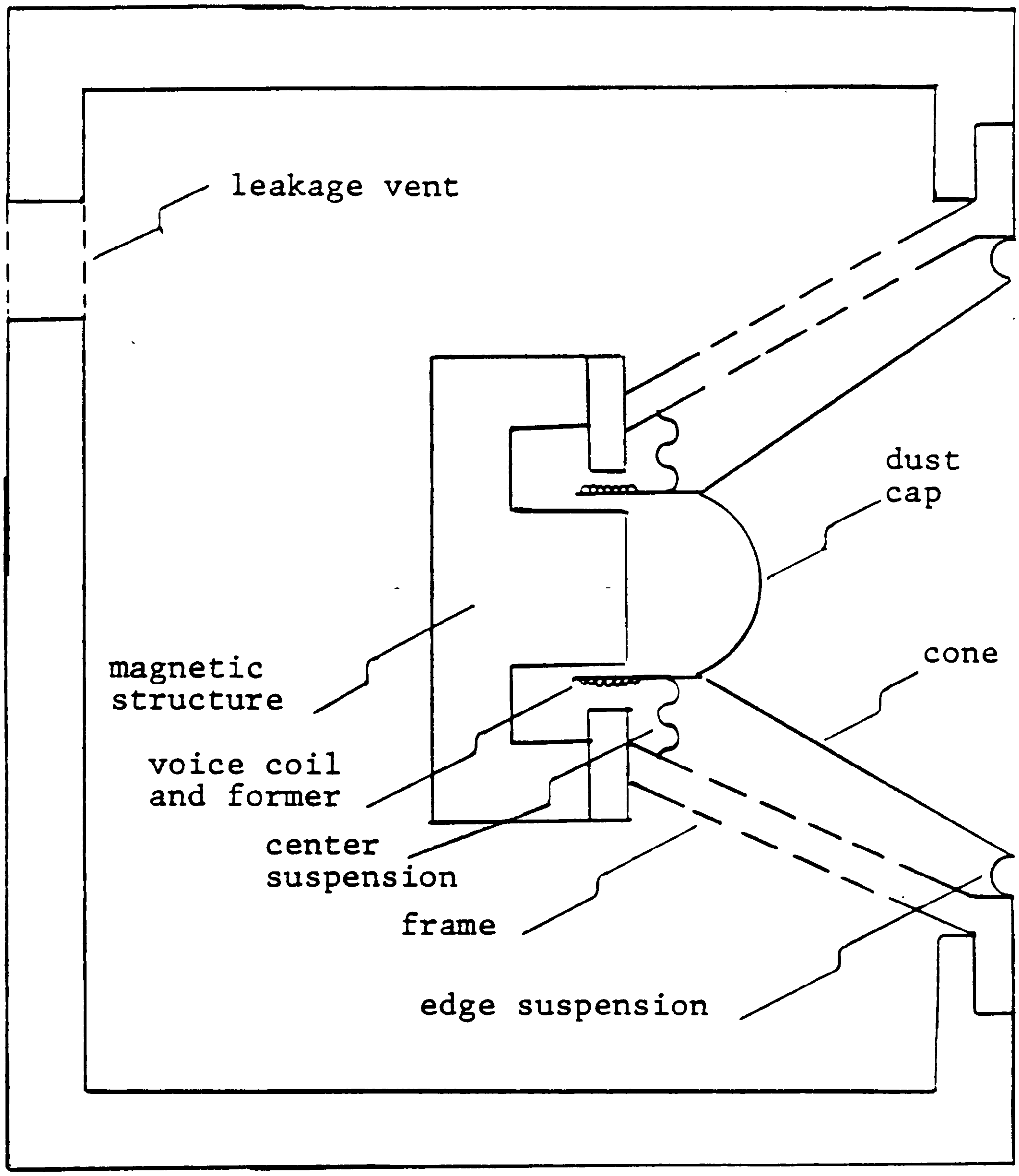


Fig. 2 Dynamic loudspeaker

The mechanical system can be drawn as in figure 3, with the associated elements defined below. The system is a variation of the forced harmonic oscillator. The (simplified) equation of motion is

$$F = M\, d^2x/dt^2 + R\, dx/dt + (1/C)x \qquad (2)$$

where

F is force

M is moving mass

R is motional resistance (damping)

C is compliance (C = 1/k, where k is the spring constant)

x is position

The equation of motion is identical to that of an electrical RLC circuit, with the following substitutions:

force	=	voltage
mass	=	inductance
damping	=	resistance
compliance	=	capacitance
position	=	charge

Complete analysis of the system is possible using electrical circuit theory. Elements under common velocity are series elements, while those under common force are parallel. For the acoustic system, the analogy is the same except that common pressure elements are in parallel while common volume velocity elements are in series (fig. 4).

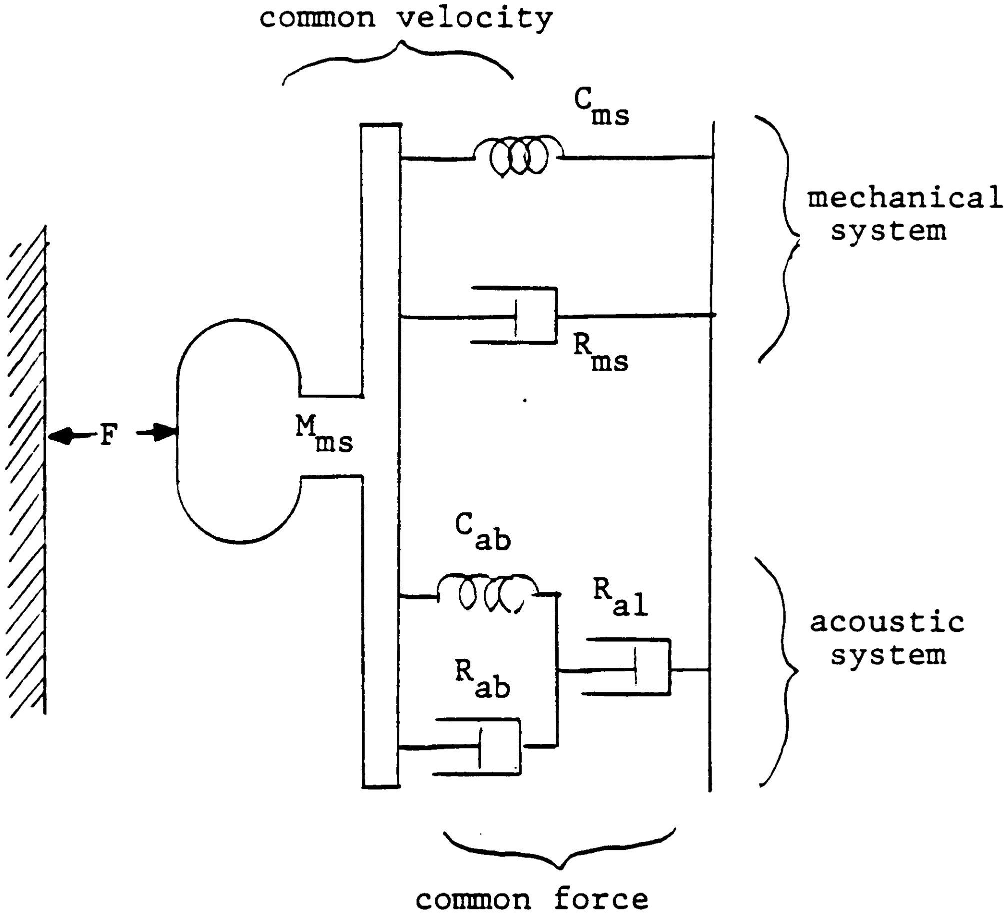


Fig. 3 Loudspeaker mechanical and acoustic systems

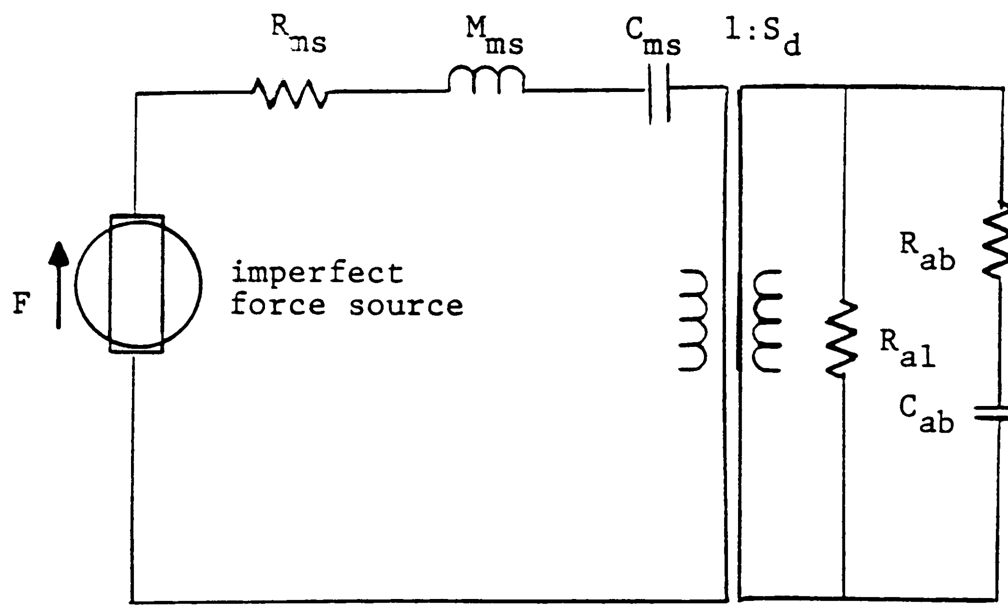


Fig. 4 Loudspeaker mechanical/acoustic circuit

With the cone acting as a transformer, all mechanical elements can be transformed into acoustic elements, and the associated force source into a pressure source (fig. 5). The transformation is accomplished through

$$C_{as} = C_{ms} S_d^2 \tag{3a}$$

$$M_{as} = M_{ms} / S_d^2 \tag{3b}$$

$$R_{as} = R_{ms} / S_d^2 \tag{3c}$$

$$p_g = F_g / S_d \,. \tag{3d}$$

The circuit is now easily analyzed in either its mechanical or acoustical form.

COUPLING OF THE ELECTRICAL AND MECHANICAL CIRCUITS

The electrical and mechanical (hence the acoustical) systems are coupled through the driver voice coil. To simplify the argument, the D.C. resistance of the voice coil is considered separate from the coil itself. The mechanical force on the cone, and the subsequent acoustical pressure, due to electrical current through the voice coil is

(mechanical) (acoustical)

$$F = Bli, \qquad p = Bli/S_d \tag{4}$$

where

F is mechanical force

B is the driver air gap magnetic flux density

l is the length of voice coil conductor in the magnetic field

i is the current through the voice coil

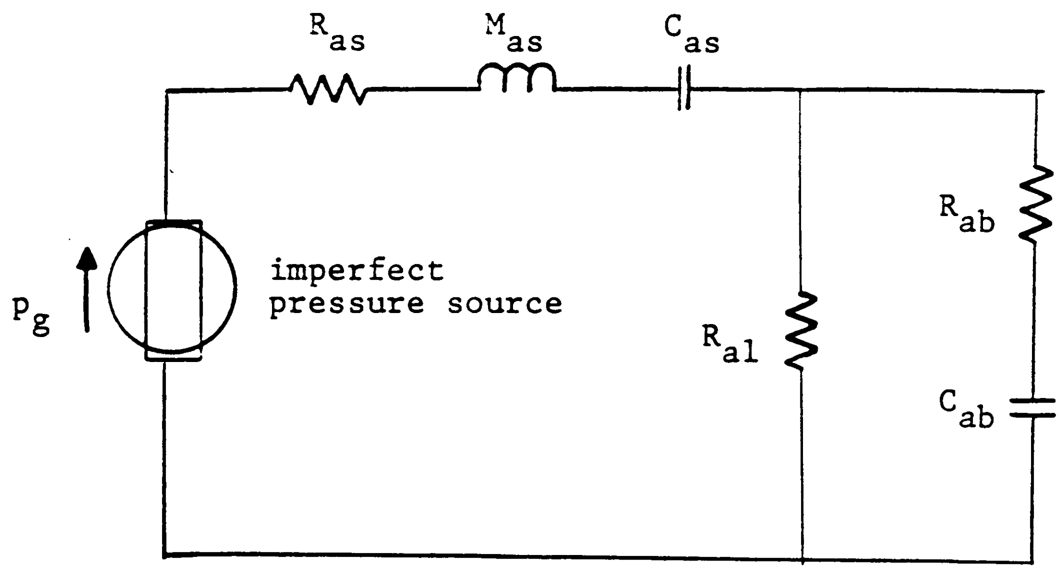


Fig. 5 Loudspeaker equivalent acoustic circuit

S_d is the effective cone area.

In addition, if the cone moves with linear velocity u or acoustic volume velocity U, the back E_{mf} generated in the voice coil is

$$E_{mf} = Blu \qquad\qquad E_{mf} = BlU/S_d. \tag{5}$$

With

$$Z_e = E_{mf}/i \quad \text{(electrical impedence)} \tag{6}$$

$$Z_m = F/u \quad \text{(mechanical impedence)} \tag{7}$$

$$Z_a = p/U \quad \text{(acoustical impedence)} \tag{8}$$

we have

$$Bl = E_{mf}/u = F/i \qquad\qquad Bl/S_d = E_{mf}/U = p/i \tag{9}$$

then

$$(Bl)^2 = FE_{mf}/ui \qquad\qquad (Bl/S_d)^2 = pE_{mf}/Ui. \tag{10}$$

Substitution of the equations 6, 7, and 8 yields

$$(Bl)^2 = Z_m Z_e \qquad\qquad (Bl/S_d)^2 = Z_a Z_e. \tag{11}$$

Therefore,

$$Z_e = (Bl)^2/Z_m \qquad\qquad Z_e = (Bl/S_d)^2/Z_m \tag{12}$$

or

$$Z_m = (Bl)^2/Z_e \qquad\qquad Z_a = (Bl/S_d)^2/Z_e. \tag{13}$$

Thus, impedance elements on one side of the circuit manifest on the other, but the relationship is of an inverted nature.

EQUIVALENT CIRCUIT ELEMENTS

A) RESISTANCE

The mechanical and acoustic equivalents of an electrical resistance R_e are

$$R_{me} = (Bl)^2/R_e \qquad R_{ae} = (Bl/S_d)^2/R_e. \qquad (14)$$

The electrical equivalents of a mechanical resistance R_m or acoustical resistance R_a are

$$R_{em} = (Bl)^2/R_m \qquad R_{ea} = (Bl/S_d)^2/R_a. \qquad (15)$$

B) MASS ELEMENTS

The electrical equivalents of mechanical and acoustic mass are

$$C_{em} = M_m/(Bl)^2 \qquad C_{ea} = M_a/(Bl/S_d)^2. \qquad (16)$$

Note that the mechanical and acoustical masses, which are analogous to inductances on their natural side, become capacitance on the electrical side.

C) COMPLIANCE ELEMENTS

The electrical equivalents of mechanical compliance C_m and acoustical compliance C_a are

$$L_{em} = (Bl)^2 C_m \qquad L_{ea} = (Bl/S_d)^2 C_a \qquad (17)$$

Note that mechanical and acoustical compliances, which are analogous to capacitances on their natural side, become inductances on the electrical side

D) ELECTRICAL CAPACITANCE ELEMENTS

The mechanical and acoustical equivilents of electrical capacitance C_e are

$$M_{me} = (Bl)^2 C_e \qquad M_{ae} = (Bl/S_d)^2 C_e . \quad (18)$$

The electrical capacitance becomes a mass element on the mechanical and acoustical sides.

E) ELECTRICAL INDUCTANCE ELEMENTS

The mechanical and acoustical equivalents of electrical inductance L_e are

$$C_{me} = L_e/(Bl)^2 \qquad C_{ae} = L_e/(Bl/S_d)^2 . \quad (19)$$

The electrical inductance becomes a compliance element on the mechanical and acoustical sides.

F) MECHANICAL SYSTEMS CONSTRAINED TO COMMON VELOCITY

Mechanical elements constrained to common velocity are analogous to common current elements. Therefore, their mechanical impedances add as series elements

With

$$Z_{mt} = Z_{m1} + Z_{m2} + Z_{m3} + \ldots .$$

then

$$Z_e = (Bl)^2/(Z_{m1} + Z_{m2} + Z_{m3} + \ldots)$$

implies

$$1/Z_e = (Z_{m1} + Z_{m2} + Z_{m3} + \ldots)/(Bl)^2$$

$$= Z_{m1}/(Bl)^2 + Z_{m2}/(Bl)^2 + Z_{m3}/(Bl)^2 + \ldots$$

or

$$1/Z_e = 1/Z_{em1} + 1/Z_{em2} + 1/Z_{em3} + \ldots \quad (20)$$

which is the form of electrical elements in parallel. Consequently, series elements in the mechanical circuit manifest as parallel elements in the electrical circuit. The same derivation holds for acoustical elements, with the substitution of Bl/S_d for Bl and volume velocity for linear velocity.

G) MECHANICAL ELEMENTS UNDER COMMON FORCE

Elements under common force in the mechanical circuit are parallel elements, and their net mechanical impedance is

$$1/Z_{mt} = 1/Z_{m1} + 1/Z_{m2} + 1/Z_{m3} + \ldots$$

then

$$Z_e = (Bl)^2(1/Z_{m1} + 1/Z_{m2} + 1/Z_{m3} + \ldots$$
$$= (Bl)^2/Z_{m1} + (Bl)^2/Z_{m2} + (Bl)^2/Z_{m3} + \ldots$$

or

$$Z_e = Z_{em1} + Z_{em2} + Z_{em3} + \ldots \qquad (21)$$

Therefore, elements under common force in the mechanical circuit manifest as series elements in the electrical circuit. The same holds for acoustical elements, with the appropriate substitutions for Bl, force, and velocity.

H) SERIES AND PARALLEL ELECTRICAL ELEMENTS

The discussion above for series and parallel mechanical elements holds equally well with electrical elements, with the substitution of electrical for mechanical and mechanical for electrical subscripts. The result is that parallel electrical elements manifest as series mechanical elements, while series electrical elements appear as parallel mechanical elements.

I) MECHANICAL EQUIVALENT OF ELECTRICAL VOLTAGE AND CURRENT

With

$$E_{mf} = Blu \qquad\qquad E_{mf} = BlU/S_d$$

we find

$$u = E_{mf}/Bl \qquad\qquad U = E_{mf}S_d/Bl. \qquad (22)$$

Thus, electrical voltage becomes mechanical or acoustical velocity, and a voltage source becomes a velocity or volume velocity source.

With

$$F = Bli \qquad\qquad p = Bli/S_d \qquad (23)$$

electrical current becomes mechanical force or acoustical pressure, and a current source becomes a force or pressure source.

The inverse relationship also holds in the converse case, such that a force becomes current, velocity becomes voltage, etc. With these relationships, it is possible to predict the velocity of or force on a mechanical element by measuring the voltage across or current through its electrical equivilent.

THE COMPLETE CIRCUITS

With the relationships above, the complete equivalent acoustic circuit can be drawn as in figure 6, with the following definitions:

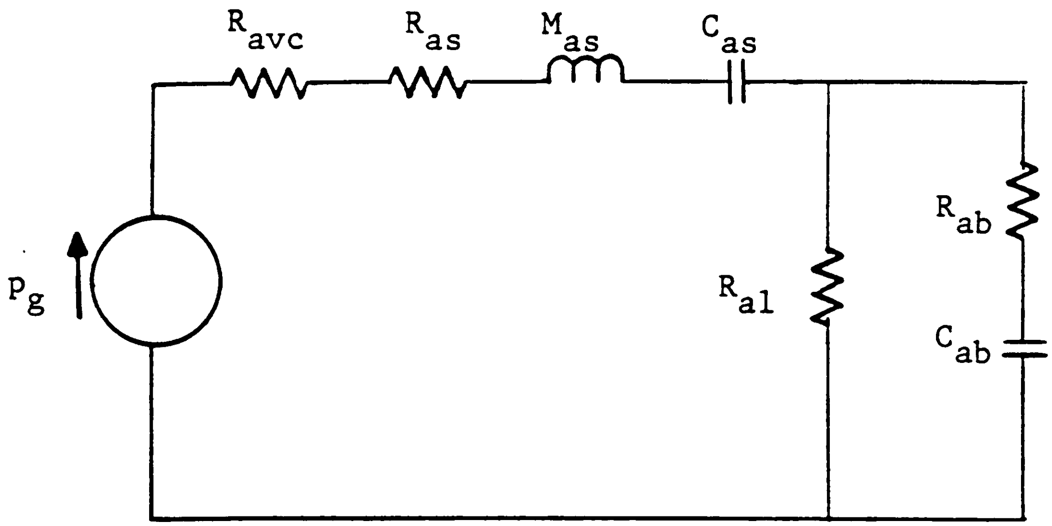


Fig. 6 Complete loudspeaker acoustic circuit

P_g is the pressure source due to the amplifier Thevenin equivalent current source.

R_{avc} is the acoustic equivalent of the amplifier and driver voice coil resistances.

R_{as} is the driver suspension losses.

C_{as} is the driver suspension compliance.

M_{as} is the driver moving mass.

C_{ab} is the box compliance.

R_{ab} is the box losses.

R_{al} is the leakage losses.

U_s is volume velocity in the driver branch.

U_b is volume velocity in the box branch.

U_l is volume velocity in the leakage branch.

The radiation resistance R_{ar}, which would manifest in each radiating branch, has been ignored because it is usually less than 1% of the system losses[10], even though it is responsible for all of the acoustic output.

The complete equivalent electrical circuit can be drawn as in figure 7, where:

e_g is the amplifier output voltage.

R_g is the amplifier output resistance.

R_e is the driver voice coil resistance.

C_{mas} is the equivalent of driver moving mass.

L_{cas} is the equivalent of driver suspension compliance.

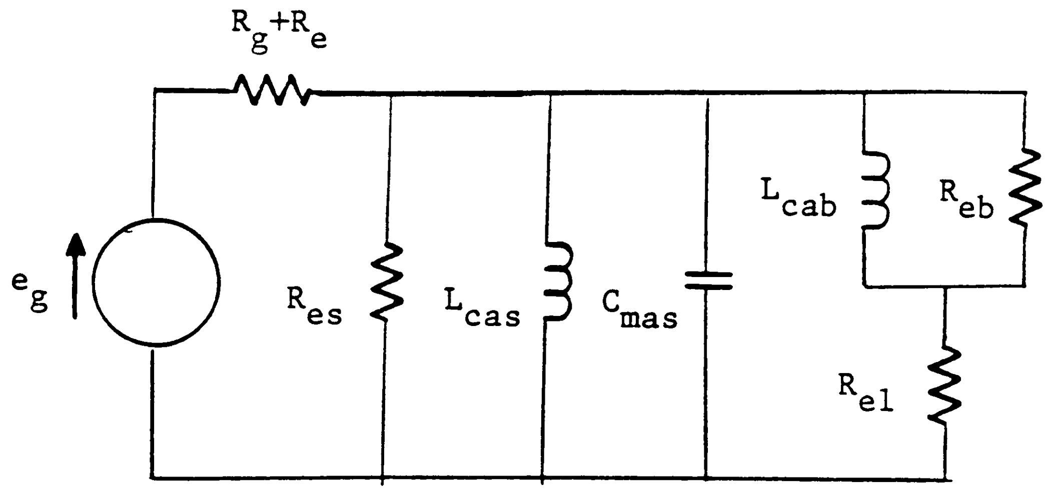


Fig. 7 Complete loudspeaker electrical circuit

R_{es} is the equivalent of driver suspension losses.

L_{cab} is the equivalent of box compliance.

R_{eb} is the equivalent of box losses.

R_{el} is the equivalent of leakage losses.

THE GENERAL TRANSFER FUNCTION

The acoustic circuit of figure 6 can be drawn as the general circuit of figure 8, where:

Z_{as} is the net impedance of the driver branch

Z_{ab} is the net impedance of the box branch

Z_{al} is the net impedance of the leakage branch

U_d is the driver branch volume velocity

U_b is the box branch volume velocity

U_l is the leakage branch volume velocity

The effeciency relationship of the system is[11]

$$n = P_a/P_e \qquad (24)$$

where P_a is the radiated output power, and P_e is the nominal electrical losses. The (nominal) electrical power lost to the driver is

$$P_e = i^2 R_e = e_g{}^2 R_e/(R_g + R_e)^2 \qquad (25)$$

while the acoustic output power is

$$P_a = |-U_d + U_l|^2 R_{ar}. \qquad (26)$$

$$U_d = U_b + U_l. \qquad (27)$$

Therefore,

$$U_d - U_l = U_b,$$

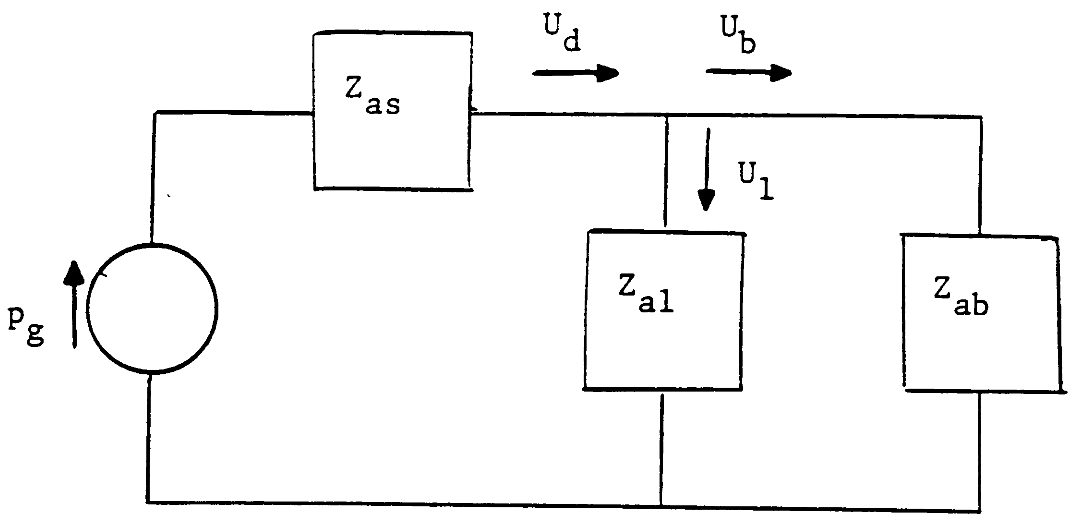


Fig. 8 General loudspeaker acoustic circuit

and consequently

$$|-U_d + U_l|^2 = |-U_b|^2 = |U_b|^2. \qquad (28)$$

The effeciency function can then be expressed as

$$n = R_{ar}|U_b|^2(R_g + R_e)^2/e_g{}^2 R_e. \qquad (29)$$

Further analysis of figure 8 reveals

$$U_d = p_g/(Z_{as} + Z_{ab} + Z_{as}Z_{ab}/Z_{al}). \qquad (30)$$

At low frequencies, the radiation resistance is independent of aperture size and is given by

$$R_{ar} = p_o w^2/2\text{pi}\,c. \qquad (31)$$

Combining equations 29, 30, and 31, we find

$$n = (Bl/S_d)^2(p_o/2\text{pi}\,cR_eM_{as}{}^2) *$$

$$(|sM_{as}/(Z_{as} + Z_{ab} + Z_{as}Z_{ab}/Z_{al}|)^2 \qquad (32)$$

$$= (Bl/S_d)^2(p_o/2\text{pi}\,cR_eM_{as}{}^2) * |G(s)|^2 \qquad (33)$$

where G(s) is the loudspeaker transfer function. Note that all frequency dependent terms are included in G(s), while the terms to the left amount to a constant.

MEANING OF THE TRANSFER FUNCTION

The generalized transfer function G(s) holds for all direct radiator loudspeakers. It is an expression of the acoustic output given input of constant voltage magnitude, at low frequencies. To within a constant, the transfer function G(s) is exactly the frequency response of the loudspeaker system.

CHAPTER 4: THE SEALED BOX LOUDSPEAKER SYSTEM

The sealed box loudspeaker system is easily analyzed through electrical circuit analogies of the acoustic system.

ANALYSIS OF THE SEALED BOX ACOUSTIC CIRCUIT

In practice, the sealed box suffers very little from leakage losses, and Ral is ignored. Box losses, which are caused by structural bracing and filling the enclosure interior with sound absorbent material (usually dacron or long fiber wool) are quite small when compared to suspension losses at low frequencies, and are also ignored[12]. From figure 6, with R_{al} opened and R_{ab} shorted, the sealed box acoustic circuit can be drawn as in figure 9, where C_{as} and C_{ab} have been combined into C_{at}.

The circuit is simple RLC, with the reactive elements in series. As with electrical circuits, the resonance frequency and quality factors are defined as:

$$T_c^2 = 1/w_c^2 = C_{at}M_{as} \quad \text{(system resonance)} \qquad (34)$$

$$Q_{mc} = 1/w_c C_{at} R_{as} \quad \text{(mechanical system Q)} \qquad (35)$$

$$Q_{ec} = 1/w_c C_{at} R_{avc} \quad \text{(electrical system Q)} \qquad (36)$$

$$Q_{tc} = 1/w_c C_{at}(R_{as} + R_{avc}) = Q_{ec}Q_{mc}/(Q_{ec} + Q_{mc}) \qquad (37)$$

(total system Q)

Notice that if the box was not present, the only element which would change is C_{at}, ie. $C_{at} = C_{as}$. The driver free air parameters are then defined as:

$$T_s^2 = 1/w_s^2 = C_{as}M_{as} \quad \text{(free air resonance)} \qquad (38)$$

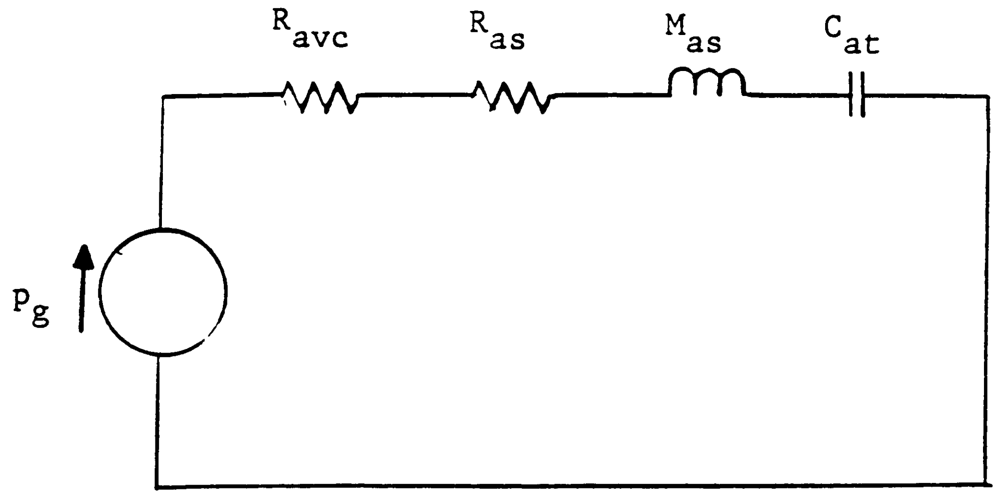


Fig. 9 Simplified sealed box loudspeaker system acoustic circuit

$$Q_{ms} = 1/w_s C_{as} R_{as} \quad \text{(mechanical free air Q)} \tag{39}$$

$$Q_{es} = 1/w_s C_{as} R_{avc} \quad \text{(electrical free air Q)} \tag{40}$$

$$Q_{ts} = 1/w_c C_{as}(R_{as} + R_{avc}) = Q_{ms}Q_{es}/(Q_{ms} + Q_{es}) \quad \text{(total system free air Q)} \tag{41}$$

The driver free air parameters Q_{ts}, w_s, and C_{as} are of fundamental importance because they control all of the sealed box alignments possible with a given driver.

MEANING OF THE QUALITY FACTOR

The parameter Q is the "quality factor" or "selectivity" of a resonance. It is calculated from the impedance (mechanical, electrical, or acoustical) of the system (see Appendix A), and describes the damping. As will be shown later, Q also controls the transfer function amplitude near resonance.

If the damped harmonic oscillator equation is written as[13]

$$d^2x/dt^2 + 2D\, dx/dt + w_o^2 x = 0, \tag{42}$$

the damping coefficient D can be expressed as

$$D = w_o/2Q. \tag{43}$$

The solution can then be expressed as

$$x(t) \simeq C \exp(-w_o t/2Q)\cos(w_d t + \Theta) \tag{44}$$

where

$$w_d = w_o(1 - 1/4Q^2)^{1/2}. \tag{45}$$

Then for large Q the system is poorly damped, and transient ringing over many cycles will result.

THE SEALED BOX TRANSFER FUNCTION

The transfer function in all cases is given by eq. 32:

$$G(s) = sM_{as}/(Z_{as} + Z_{ab} + Z_{as}Z_{ab}/Z_{al})$$

For the sealed box, Z_{al} = infinity and the transfer function reduces to

$$G(s) = sM_{as}/(Z_{as} + Z_{ab}). \qquad (46)$$

With fig. 9 further simplified to fig. 10 by

$$R_{atc} = R_{as} + R_{avc} \qquad (47)$$

$$C_{at} = C_{as}C_{ab}/(C_{as} + C_{ab}) \qquad (48)$$

then

$$Z_{as} = R_{atc} + sM_{as} + 1/sC_{as} \qquad (49)$$

$$Z_{ab} = 1/sC_{ab} \qquad (50)$$

Then

$$\begin{aligned} G(s) &= sM_{as}/\{R_{atc} + sM_{as} + 1/sC_{as} + 1/sC_{ab}\} \\ &= sM_{as}/\{R_{atc} + sM_{as} + [C_{ab} + C_{as}]/sC_{ab}C_{as}\} \\ &= sM_{as}/\{R_{atc} + sM_{as} + 1/sC_{at}\} \\ &= s^2C_{at}M_{as}/\{sC_{at}R_{atc} + s^2C_{at}M_{as} + 1\} \qquad (51) \end{aligned}$$

with

$$T_c^2 = 1/w_c^2 = C_{at}M_{as} \qquad (52)$$

$$Q_{tc} = 1/w_cC_{at}R_{atc} \qquad (53)$$

then

$$G(s) = s^2T_c^2/\{s^2T_c^2 + sT_c/Q_{tc} + 1\}. \qquad (54)$$

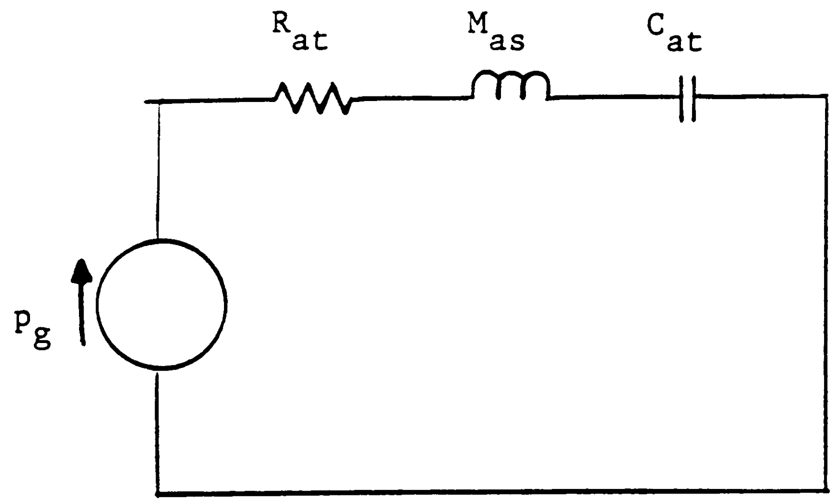


Fig. 10 Simplified sealed box loudspeaker system acoustic circuit, in minimum form

Normalized to $s_n = sT_c$, we have

$$G(s_n) = s_n^2/\{s_n^2 + s_n/Q_{tc} + 1\} \tag{55}$$

which is the transfer function of a normalized second order high pass filter of standard form, with shape constant $1/Q_{tc}$.

Graphs of $|G(s_n)|$ are shown in figures 11a,b for several values of Q_{tc}, plotted in dB vs. octaves above system resonance. Figures 12a,b are the pulse response for the same Q_{tc}'s as figure 11, in m^3/sec vs system period. Notice that as Q_{tc} increases the output peak near resonance increases in magnitude, while the pulse response displays delayed ringing.

DESIGN OF A SEALED BOX LOUDSPEAKER SYSTEM

From figure 10, with equation 47,

$$R_{atc} = R_{as} + R_{avc}$$

and defining

$$\& = C_{as}/C_{ab} \tag{56}$$

then

$$C_{at} = C_{as}C_{ab}/(C_{as} + C_{ab})$$

$$= C_{as}/(1 + \&). \tag{57}$$

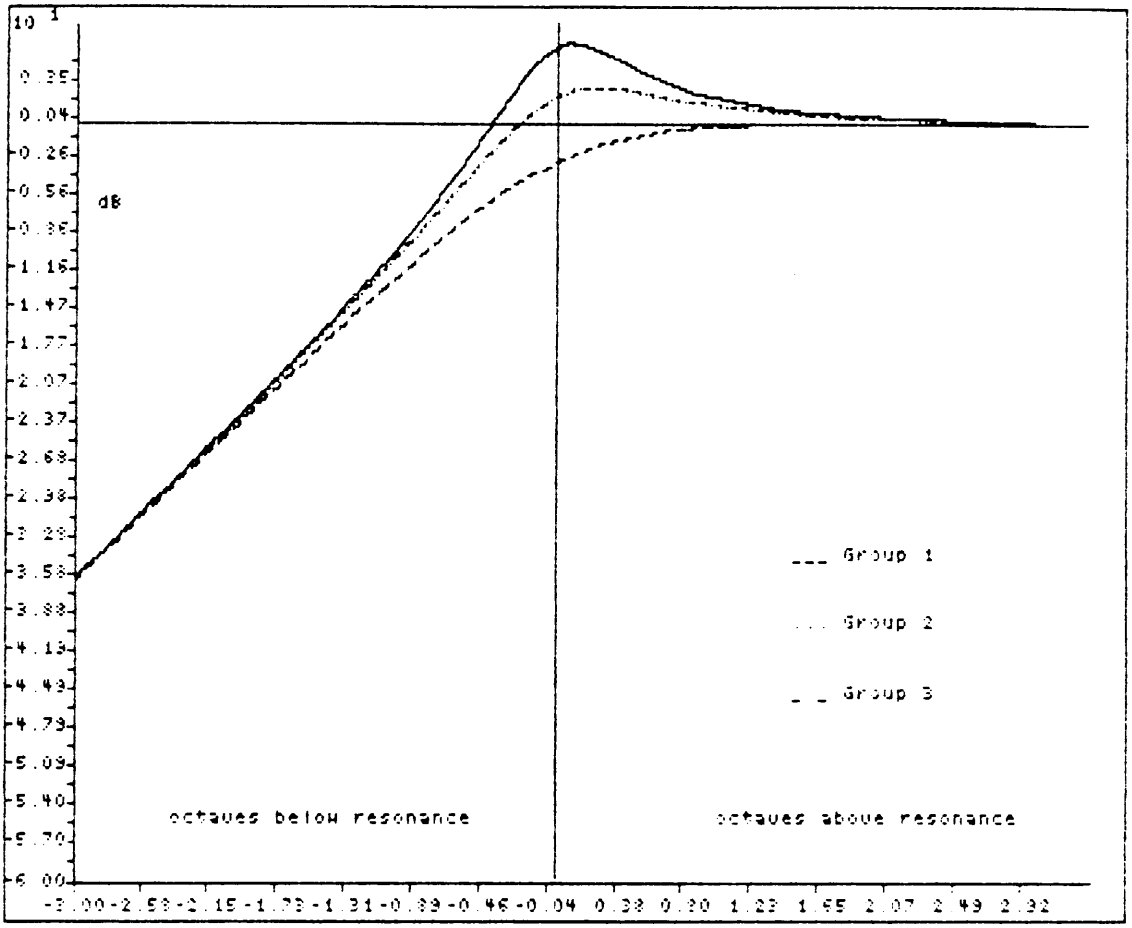
With the system Q defined as

$$Q_{tc} = 1/(w_c R_{atc} C_{at})$$

$$= (M_{as}/C_{at})^{1/2}\ 1/R_{atc} \tag{58}$$

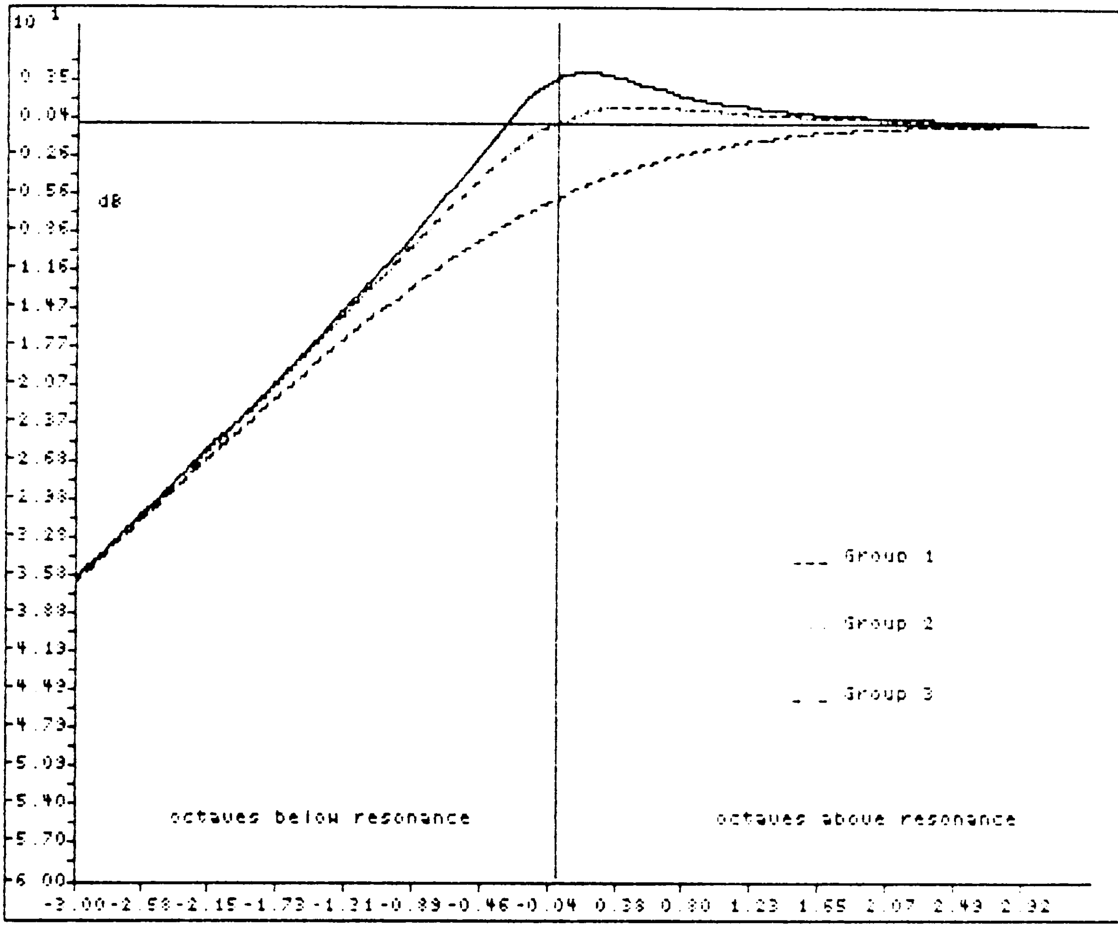
implies

$$1/C_{at} = Q_{tc}^2 R_{atc}^2/M_{as} = (1 + \&)/C_{as} \tag{59}$$



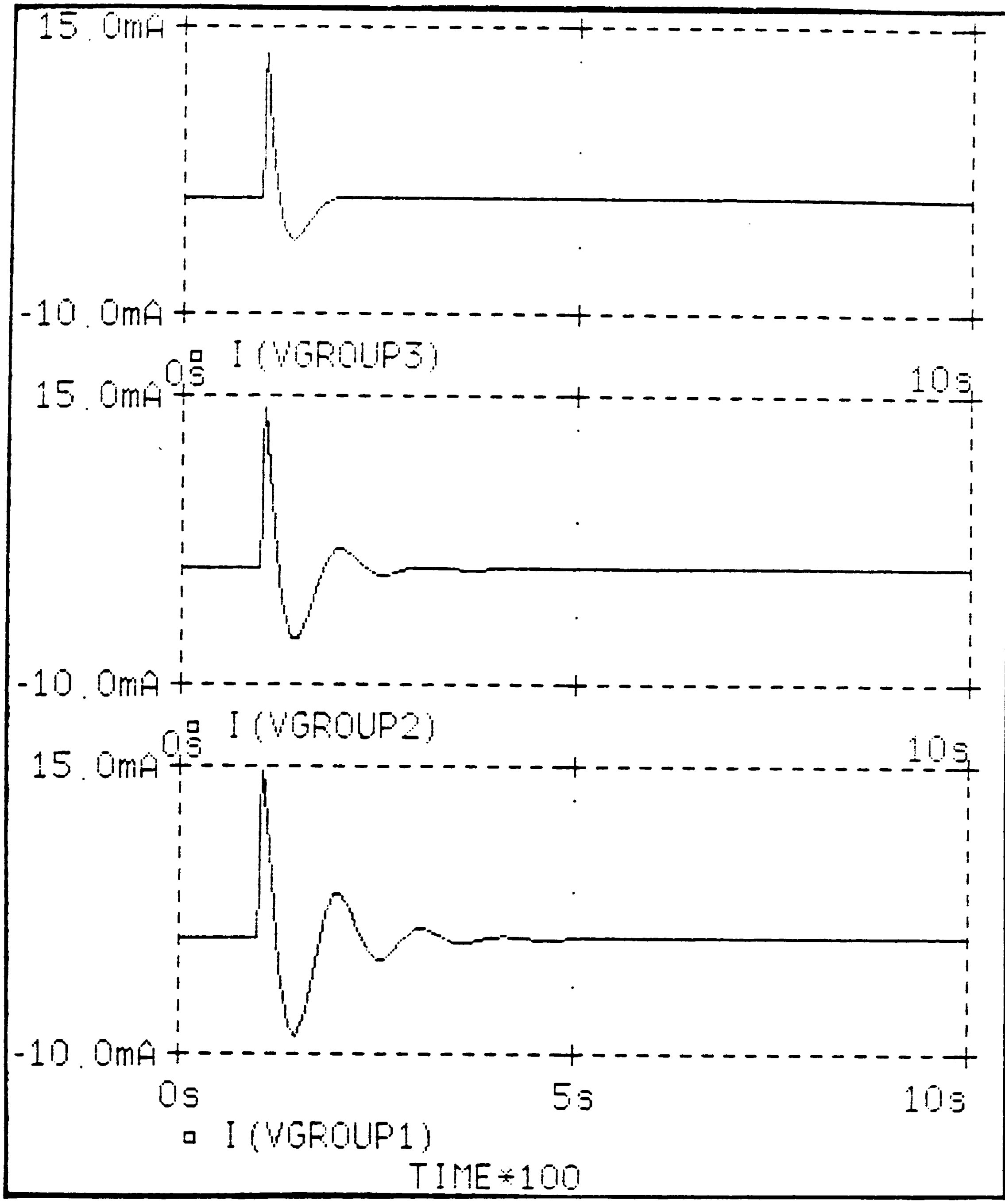
	Group 1	Group 2	Group 3
Qtc =	2.00	1.26	0.70

Fig. 11a Sealed box loudspeaker system transfer functions for Qtc = 2.00, 1.26, 0.70



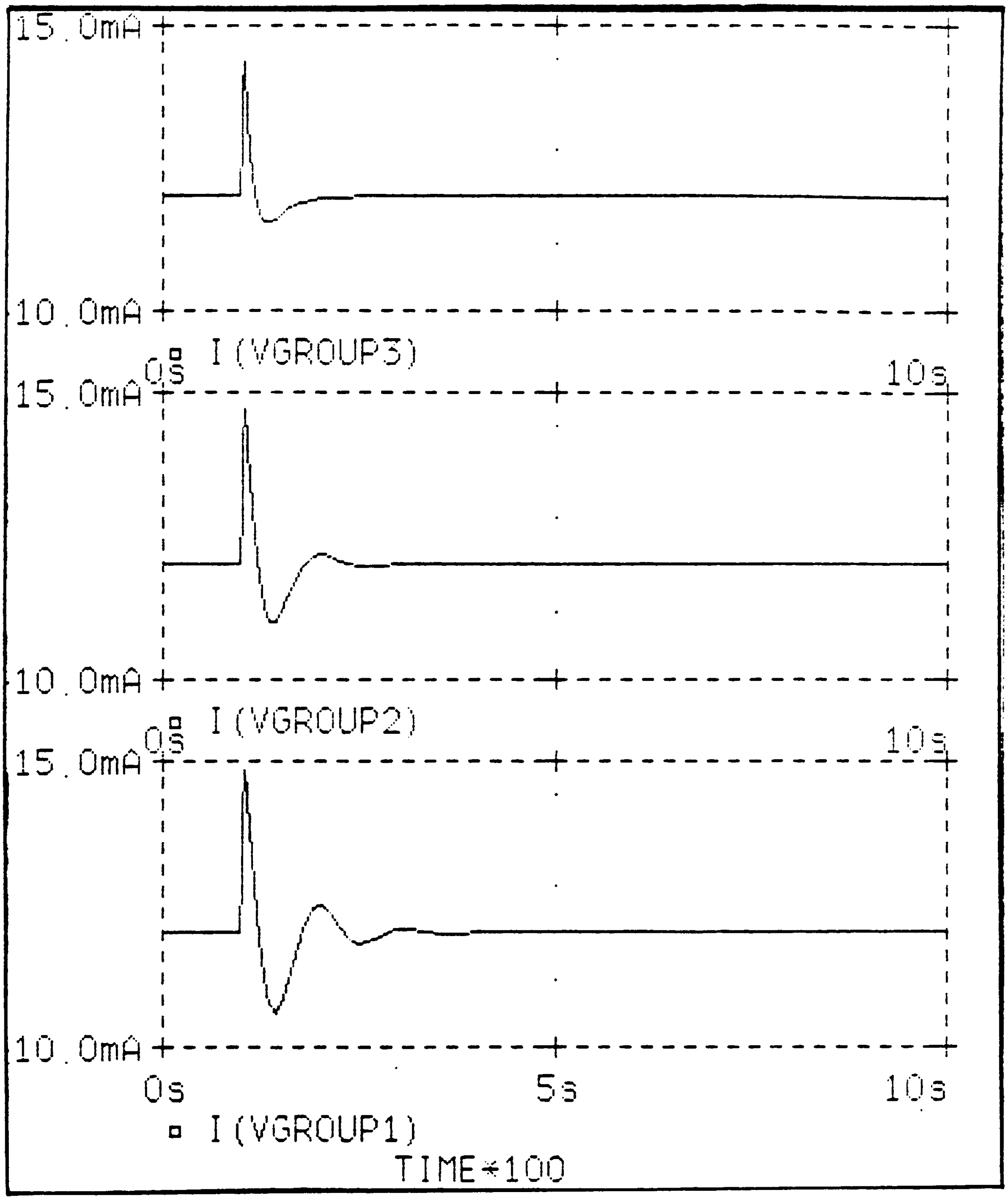
	Group 1	Group 2	Group 3
Qtc =	1.50	1.00	0.50

Fig. 11b Sealed Box loudspeaker system transfer functions for Qtc = 1.50, 1.00, 0.50



top : Q_{tc} = 0.70
middle: Q_{tc} = 1.26
bottom: Q_{tc} = 2.00

Fig. 12a Pulse response of sealed box loudspeaker systems



top : $Q_{tc} = 0.50$
middle: $Q_{tc} = 1.00$
bottom: $Q_{tc} = 1.50$

Fig. 12b Pusle response of sealed box loudspeaker systems implies

$$1 + \& = Q_{tc}^2 R_{atc}^2 C_{as}/M_{as}$$

$$= Q_{tc}^2 (R_{atc}^2 C_{as}^2 / M_{as} C_{as})$$

$$= Q_{tc}^2 (w_s^2 R_{atc}^2 C_{as}^2)$$

$$= Q_{tc}^2 / Q_{ts}^2 . \qquad (60)$$

The parameters V_{as} and V_{ab} are now introduced, where

$$V_x = p_o c^2 C_x .$$

V_x is the volume of air required to create an acoustic compliance C_x. Therefore,

$V_{as} = p_o c^2 C_{as}$ is the equivalent volume of the speaker suspension compliance (61)

$V_{ab} = p_o c^2 C_{ab}$ is the volume of the box having compliance C_{ab}. (62)

Then

$$\& = C_{as}/C_{ab} = V_{as}/V_{ab} . \qquad (63)$$

Substitution into 60 yields

$$V_{ab} = V_{as}/(Q_{tc}^2/Q_{ts}^2 - 1)$$

$$= p_o c^2 C_{as}/(Q_{tc} 2/Q_{ts}^2 - 1) . \qquad (64)$$

Further analysis of figure 10 reveals the system resonance relation

$$w_c^2 = 1/C_{at} M_{as} \qquad (65)$$

$$= (1 + \&)/C_{as} M_{as} \qquad (66)$$

$$= (Q_{tc}^2/Q_{ts}^2) w_s^2 . \qquad (67)$$

Therefore

$$w_c = (Q_{tc}/Q_{ts}) w_s . \qquad (68)$$

The required box volume V_{ab} for a target alignment Q_{tc} is given by eq. 64, and the resultant system resonance frequency w_c by eq. 68. Notice that a target $Q_{tc} < Q_{ts}$ results in a negative box volume, and $Q_{tc} = Q_{ts}$ implies an infinite box volume, which is the free air response of the driver.

DISCUSSION OF THE SEALED BOX LOUDSPEAKER SYSTEM

The sealed box loudspeaker system performance is easily predicted through its transfer function. The amplitude peak near resonance is controlled by Q_{tc}, and the output rolloff below about one half of the resonance frequency is 12 dB/octave. Inspeaction of figure 11 indicates for systems with $Q_{tc} > 1.0$, the rolloff immediately below resonance is slightly greater than 12 dB/octave, and for large Q_{tc} aproaches 18 db/octave, while systems with Q_{tc} less than 0.8 roll off at about 6 dB/octave in this region.

The transient response of systems with Q_{tc} shows considerable ringing (fig. 12), and for Q_{tc} greater than 1.0 this ringing is substantial over several cycles. The audible effect of transient ringing is a "punchy" bass response, and in conjunction with the frequency response peak of high Q_{tc} systems yields more apparent deep bass. This type of sound is preferred for most modern electronic music, the "natural" sound of which has never been heard. However, classical music, which is performed without electronic augmentation, sounds very unnatural on systems with high Q_{tc}. Since it can be compared directly to the original, one must assume that realistic reproduction of classical music is a measure of correct system performance.

Inspection of equation 64 indicates that for low Q_{tc}, either a driver with low Q_{ts} or a large box must be used, and in most cases, both. The first constraint, low Q_{ts}, requires a significant electromagnetic damping, implying a large magnetic system, increasing cost. The second constraint, large box volume, necessatates using designs with high panel surface area, any vibration of which adds considerable coloration to the system output. Adequate damping of panel resonances requires an additional layer of material to be added to the box walls, incurring additional expense. Displacement of panel resonances requires internal bracing, and, again, additional expense. The least expensive way to minimize panel output is by making a smaller cabinet, resulting in a higher Q_{tc}.

Perhaps the most attractive characteristic of the sealed box loudspeaker system is its ease of manufacture. The derivation of system design clearly shows a driver of any given Q_{ts} will work in a wide range of alignments, the only constraint being box volume. Since the manufacture of sealed boxes of the same volume is a simple matter, consistent results should be easily achieved. This is true only for drivers which have the same parameters, however.

Variation of Q_{ts} and C_{as} by as much as 75% can drastically change the system performance from speaker to speaker, resulting in poor pair mismatch. The only way to correct for this is to change the box volume, which is laborous and time consuming.

As will be shown below, pressure relief, i.e. aperiodic damping, can alleviate these problems to some extent. Systems with a driver of high Q_{ts}, therefore high Q_{tc}, can be made to perform like a system of much lower Q_{tc}, with no sacrifice to low bass extension. Systems in which a driver is required to perform in a small box volume, therefore high Q_{tc}, can be augmented to respond like a system with much lower Q_{tc}, but low bass loss due to the small box is not corrected. Finally, response matching of the system near resonance is easily accomplished by changing the amount of damping material in the leakage vent, which requires little time and labour.

CHAPTER 5: THE APERIODICALLY DAMPED LOUDSPEAKER SYSTEM

The aperiodically damped loudspeaker system is similar to the sealed box loudspeaker system, but is augmented through pressure relief to increase system damping. The basic design parameters of the sealed box loudspeaker system are also used to describe the aperiodically damped system, the only addition being a parameter defining leakage loss.

THE PRESSURE RELIEF ACOUSTIC CIRCUIT

The pressure relief acoustic circuit is shown in figure 13, where again R_{ab} is neglected. All parameters are fixed from the driver constants and box volume except R_{al}. The system is considered aperiodically damped because R_{al} is a frequency independent parameter, and the leakage time constant $R_{al}C_{ab}$ is not a function of driver or system resonance frequency.

THE APERIODICALLY DAMPED SYSTEM TRANSFER FUNCTION

From equation 32, the aperiodically damped system transfer function is given by

$$G(s) = sM_{as}/(Z_{as} + Z_{ab} + Z_{as}Z_{ab}/Z_{al})$$

where

$$Z_{as} = R_{atc} + sM_{as} + 1/sC_{as}$$

$$Z_{ab} = 1/sC_{ab}$$

$$Z_{al} = R_{al}. \qquad (69)$$

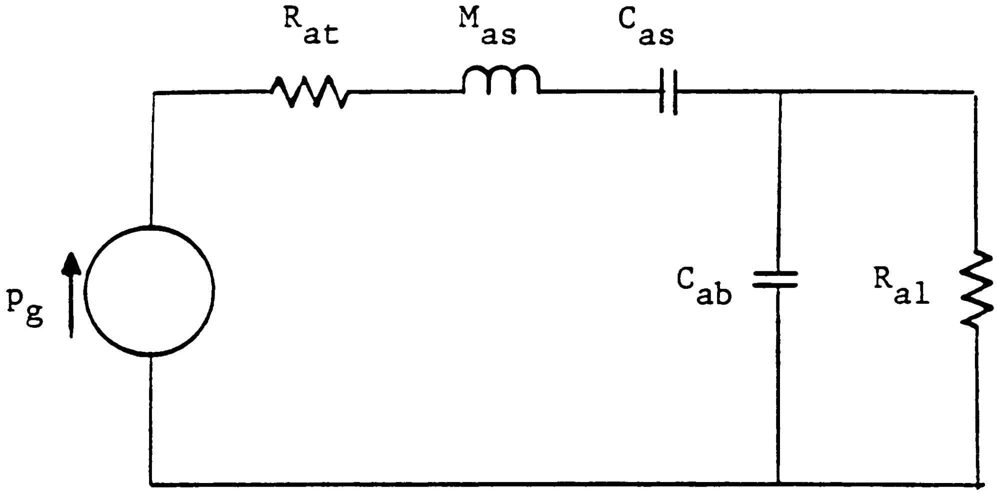


Fig. 13 Aperiodically damped loudspeaker system acoustic circuit

Then

$$G(s) = sM_{as}/\{\ R_{atc} + sM_{as} + 1/sC_{as} + 1/sC_{ab}$$
$$+ (R_{atc} + sM_{as} + 1/sC_{as})/sC_{ab}R_{al}\ \}$$
$$= s^2 C_{as}M_{as}/\{\ sC_{as}R_{atc} + s^2 C_{as}M_{as} + 1 + C_{as}/C_{ab}$$
$$+ (sC_{as}R_{atc} + s^2 C_{as}M_{as} + 1)/sC_{ab}R_{al}\ \}. \qquad (70)$$

With

$$T_s{}^2 = 1/w_s{}^2 = C_{as}M_{as}$$
$$Q_{ts} = 1/w_s C_{as} M_{as}$$
$$\& = C_{as}/C_{ab}$$
$$1 + \& = Q_{tc}{}^2/Q_{ts}{}^2$$

and defining

$$@l_s = w_s C_{ab} R_{al} \qquad (71)$$

we have

$$G(s) = s^3 T_s{}^3 @l_s/\{\ s^3 T_s{}^3 @l_s + s^2 T_s{}^2 (@l_s/Q_{ts} + 1)$$
$$+ sT_s(@l_s[Q_{tc}{}^2/Q_{ts}{}^2] + 1/Q_{ts}) + 1\ \} \qquad (72)$$

which is of third order and of non-standard form.

The transfer function as a function of system resonance frequency is achieved through

$$T_s = T_c Q_{tc}/Q_{ts} \qquad (73)$$
$$@l_s = (w_s/w_c)w_c C_{ab} R_{ab} = @l_c Q_{ts}/Q_{tc} \qquad (74)$$

then

$$G(s) = s^3 T_c{}^3 @l_c (Q_{tc}/Q_{ts})^2/\{\ s^3 T_c{}^3 @l_c (Q_{tc}/Q_{ts})^2$$
$$+ s^2 T_c{}^2 (Q_{tc}/Q_{ts})^2 (@l_c/Q_{tc} + 1)$$
$$+ sT_c(Q_{tc}/Q_{ts})(@l_c Q_{tc}/Q_{ts} + 1/Q_{ts}) + 1\ \}$$

or

$$G(s) = s^3 T_c^3 @_{1c} / \{ s^3 T_c^3 @_{1c} + s^2 T_c^2 (@_{1c}/Q_{tc} + 1)$$

$$+ sT_c(@_{1c} + 1/Q_{tc}) + Q_{ts}^2/Q_{tc}^2 \}. \quad (75)$$

In terms of the dimensionless variable

$$s_c = sT_c, \quad (76)$$

the transfer function can be expressed as

$$G(s_c) = s_c^3 @_{1c} / \{ s_c^3 @_{1c} + s_c^2 (@_{1c}/Q_{tc} + 1)$$

$$+ s_c(@_{1c} + 1/Q_{tc}) + Q_{ts}^2/Q_{tc}^2 \}. \quad (77)$$

Notice that for $@_{1c} >> 1$ (therefore $R_{a1} >> 1$, keeping C_{ab} constant), eq. 77 reduces to eq. 55, the sealed box transfer function.

ANALYSIS OF THE TRANSFER FUNCTION

The standard form for a third order high pass filter transfer functon is

$$G(s_n[3]) = s_n^3 @ / \{ s_n^3 @ + s_n^2 (@/Q + 1)$$

$$+ s_n(@ + 1/Q) + 1 \}. \quad (78)$$

The transfer function for the aperiodically damped loudspeaker system (eq. 77) is

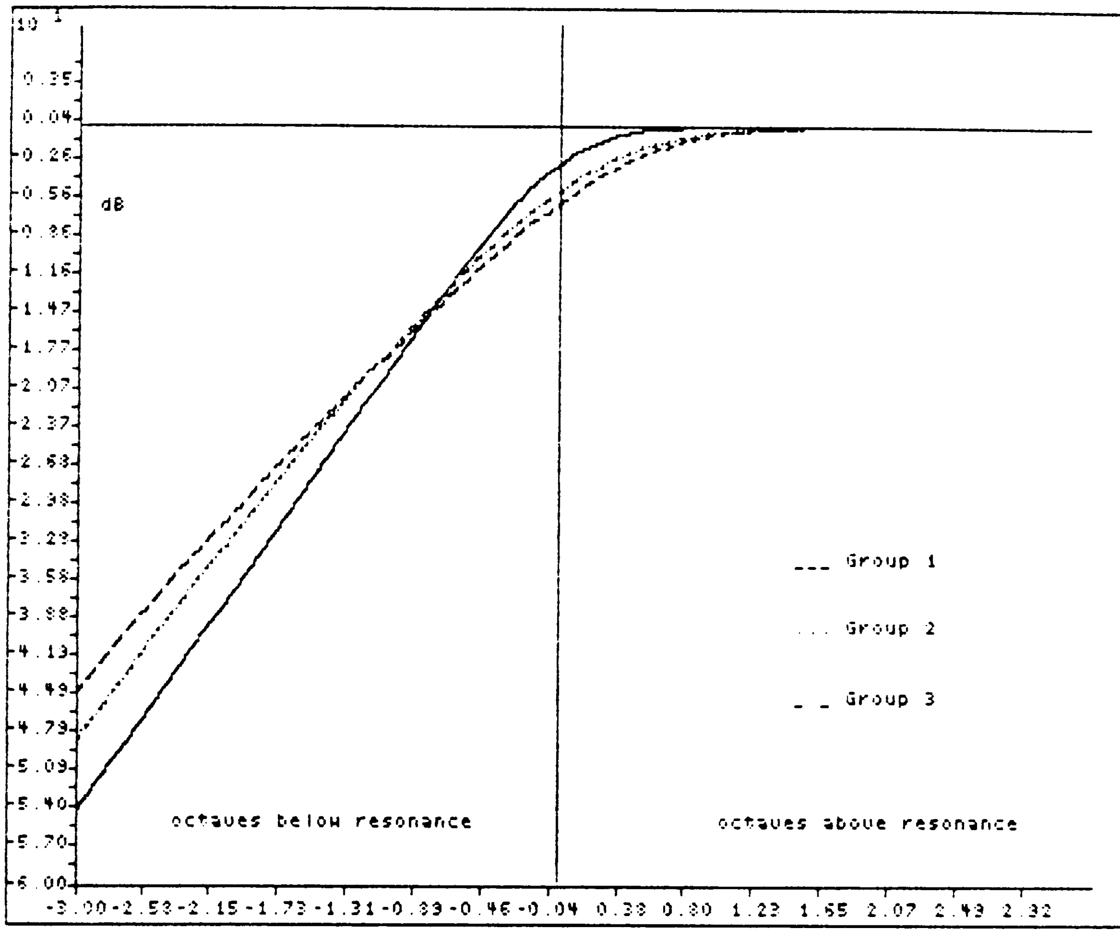
$$G(s_c) = s_c^3 @_{1c} / \{ s_c^3 @_{1c} + s_c^2 (@_{1c}/Q_{tc} + 1)$$

$$+ s_c(@_{1c} + 1/Q_{tc}) + Q_{ts}^2/Q_{tc}^2 \}.$$

The only term in (77) which does not have an exact match in (78) is the last, Q_{ts}^2/Q_{tc}^2. From the analysis of the sealed box, it was determined that $Q_{ts} < Q_{tc}$ in all cases, so $(Q_{ts}^2/Q_{tc}^2) < 1$ in all cases.

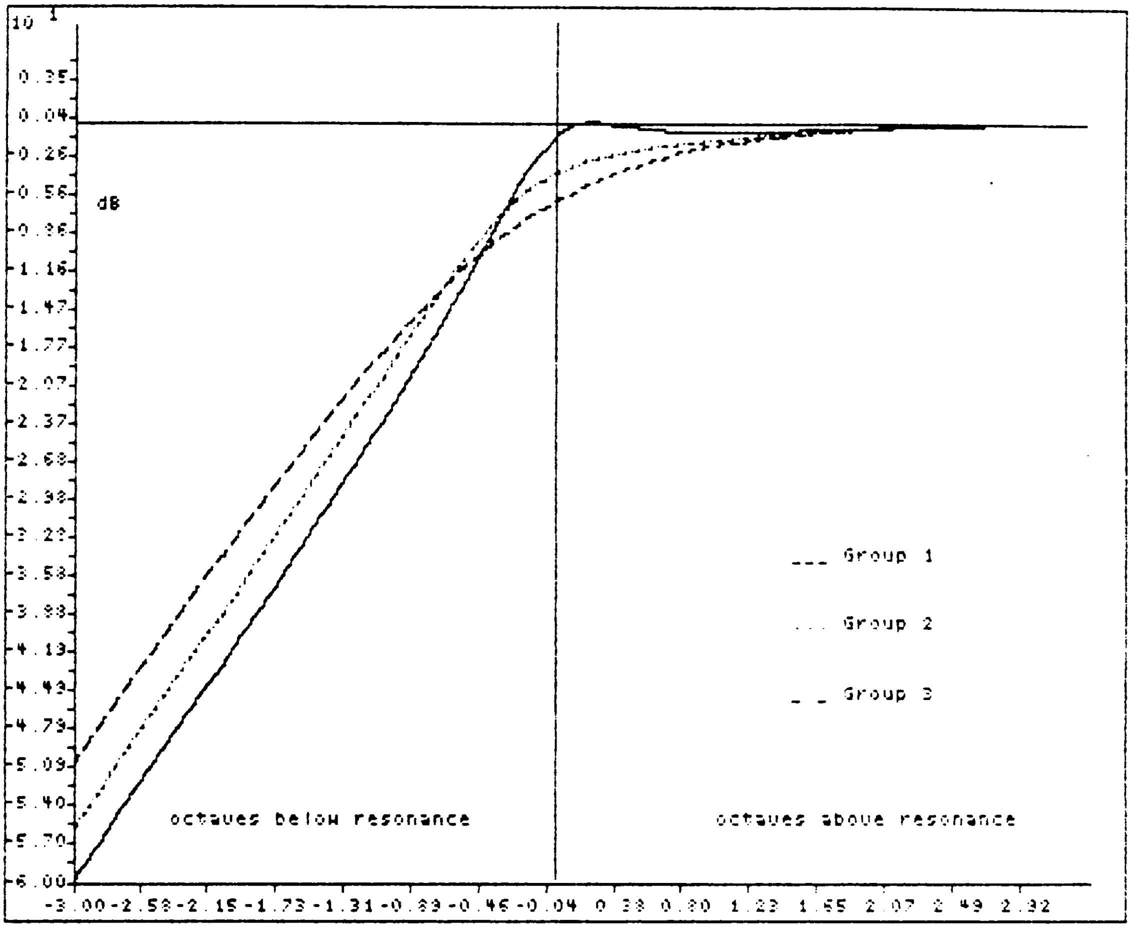
The transfer function $G(s_c)$ is plotted for several values of $@_{1c}$, Q_{tc}, and Q_{ts} in figures 14a-d. In each case the standard Chebyshev third order transfer function (i.e. $Q_{tc} = Q_{ts} = 1/@_{1c}$) is also included as a reference. Note that in all cases the standard transfer function exhibits a flat frequency response much closer to system resonance than the modified forms, but also in all cases the output rolloff is significantly steeper for the standard form than the modified form.

With the sealed box, high Q_{tc} lead to increased system effeciency near resonance, a sharp rolloff immediately below resonance, and poor transient response. The standard third order transfer functon exhibits similar characteristics in the frequency domain. Real exploration in the time domain is not possible because $Q_{tc} = Q_{ts}$ is not physically realizable. The system can be simulated, however, by creating the complete acoustic circuit analog on computer and plotting the output volume velocity for pulse response input. This was done using the PSpice circuit simulation program. The results for the $Q_{tc} = 1/@_{1c} = 1.26$ case is plotted in figure 15. Note that the transient ringing is of reduced magnitude and delay for $Q_{ts} < Q_{tc}$. Comparison of figures 15 with figures 12a,b and Appendix C (the sealed box pulse response) indicate the aperiodically damped box has far superior pulse response than the sealed box of the same Q_{tc}.



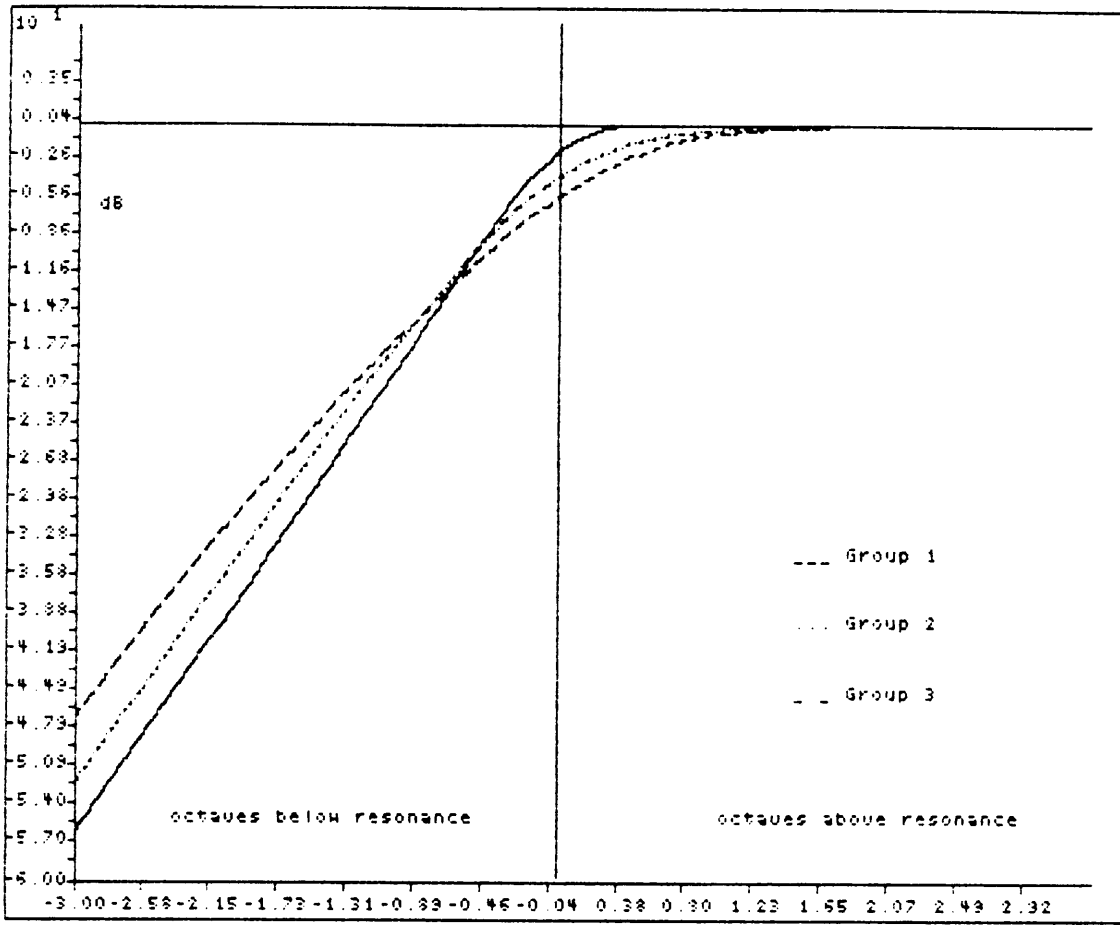
	Group 1	Group 2	Group 3
@lc =	1.00	1.00	1.00
Qtc =	1.00	1.00	1.00
Qts =	1.00	0.70	0.50

Fig. 14a **Standard and modified Chebyshev third order transfer functions, Qts = 1.00, 0.70, 0.50 Qtc = 1/@lc = 1.00**



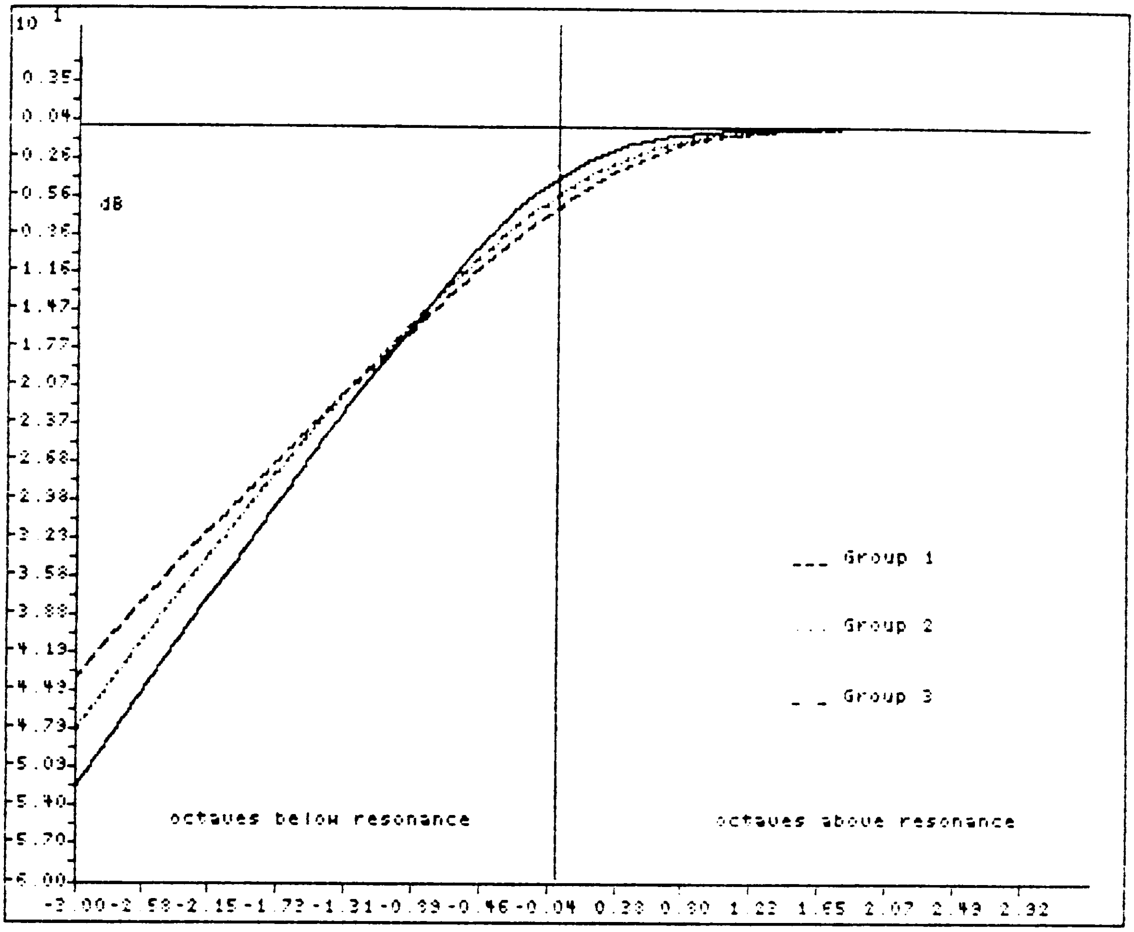
	Group 1	Group 2	Group 3
@lc =	0.53	0.53	0.53
Qtc =	1.90	1.90	1.90
Qts =	1.90	1.50	1.10

Fig. 14b Standard and modified Chebyshev third order transfer functions, Qts = 1.90, 1.50, 1.10
Qtc = 1/@lc => Qtc = 1.90, @lc = 0.53



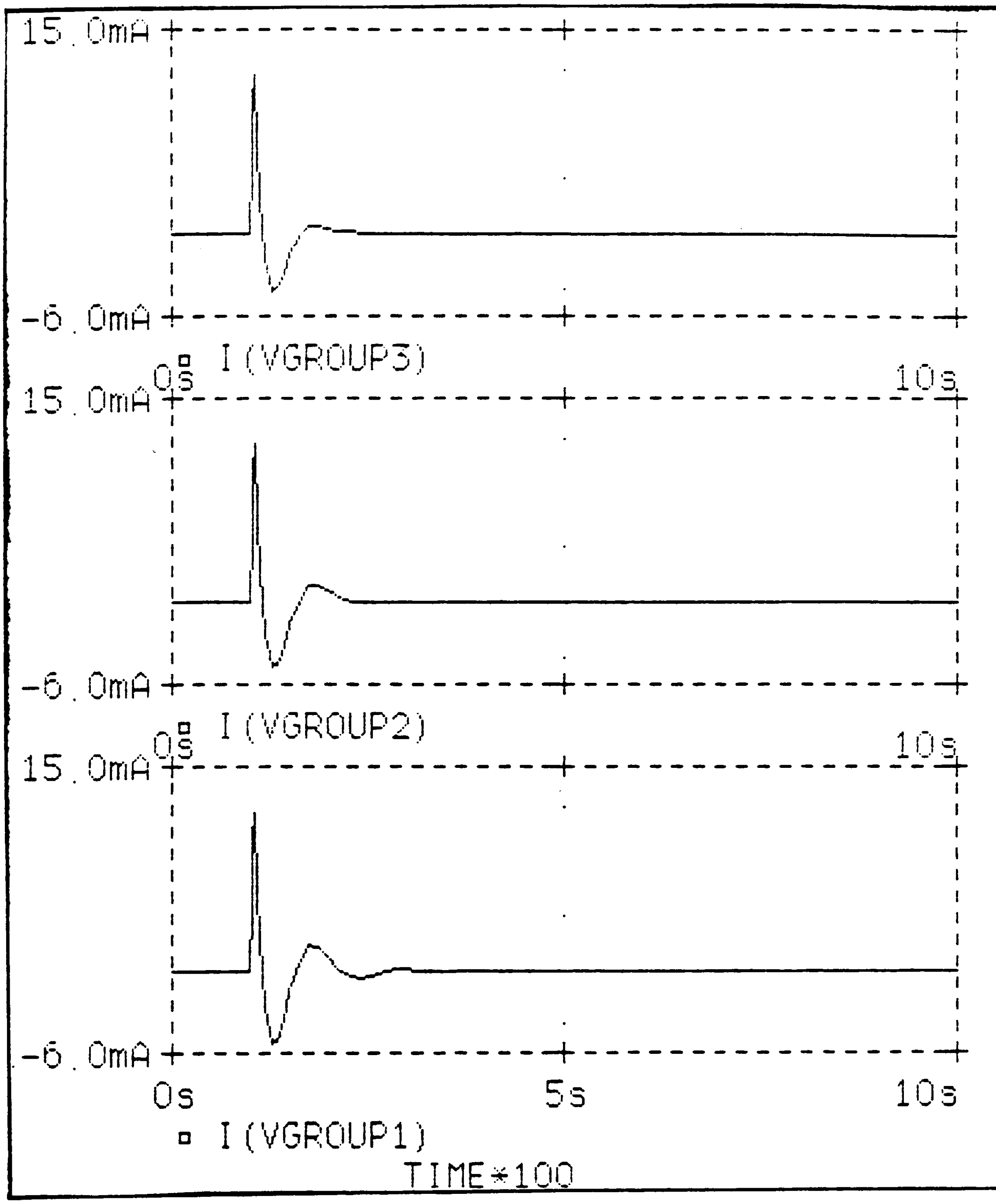
	Group 1	Group 2	Group 3
@lc =	0.80	0.80	0.80
Qtc =	1.26	1.26	1.26
Qts =	1.26	1.00	0.70

Fig. 14c Standard and modified Chebyshev third order transfer functions, Qts = 1.26, 1.00, 0.70
Qtc = 1/@lc => Qtc = 1.26, @lc = 0.80



	Group 1	Group 2	Group 3
@lc =	1.26	1.26	1.26
Qtc =	0.80	0.80	0.80
Qts =	0.80	0.60	0.40

Fig. 14d Standard and modified Chebyshev third order transfer functions, Qts = 0.80, 0.60, 0.40
Qtc = 1/@lc => Qtc = 0.80, @lc = 1.26



top : Q_{tc} = 1.26, Q_{ts} = 0.70, $@_{1c}$ = 0.80
middle: Q_{tc} = 1.26, Q_{ts} = 1.00, $@_{1c}$ = 0.80
bottom: Q_{tc} = 1.26, Q_{ts} = 1.26, $@_{1c}$ = 0.80

Fig. 15 Pulse response of standard and modified Chebyshev third order filter functions

For the standard and modified standard form of the aperiodically damped transfer function, the transient response is improved for $Q_{ts} < Q_{tc}$ and the output rolloff below system resonance is less steep, at the cost of flat frequency response.

DESIRABLE CHARACTERISTICS

Up to this point, the topic of desirable characteristics has not been considered. One would think that flat frequency response over the widest bandwidth would lead to the most accurate reproduction, but subjectively this is not true. Although smooth frequency response is required for accuracy, well controlled transient response is much more significant in creating a realistic sonic impression. This is the case because the ear is more sensitive to information in the time domain than the frequency domain[14]. For the sealed box, systems with Q_{tc} = 0.5 to 0.8 are considered much more musically accurate than those of higher Q_{tc}.

EXPLORATION OF THE APERIODICALLY DAMPED SYSTEM

The transfer function and transient response for the aperiodically damped system has been explored for standard and modified Chebyshev third order high pass filter functions. Transient response was improved and the stop band rolloff rate was decreased in all cases for the modified form, and frequency response was deminished near the system resonance. Although in the frequency domain this was a disadvantage for systems with flat response, sealed box alignments with Q_{tc} greater than 1.0 <u>always</u> have an output peak near resonance, and <u>always</u> exhibit poor transient behaviour. The addition of pressure relief to such systems can substantially improve both frequency and transient response.

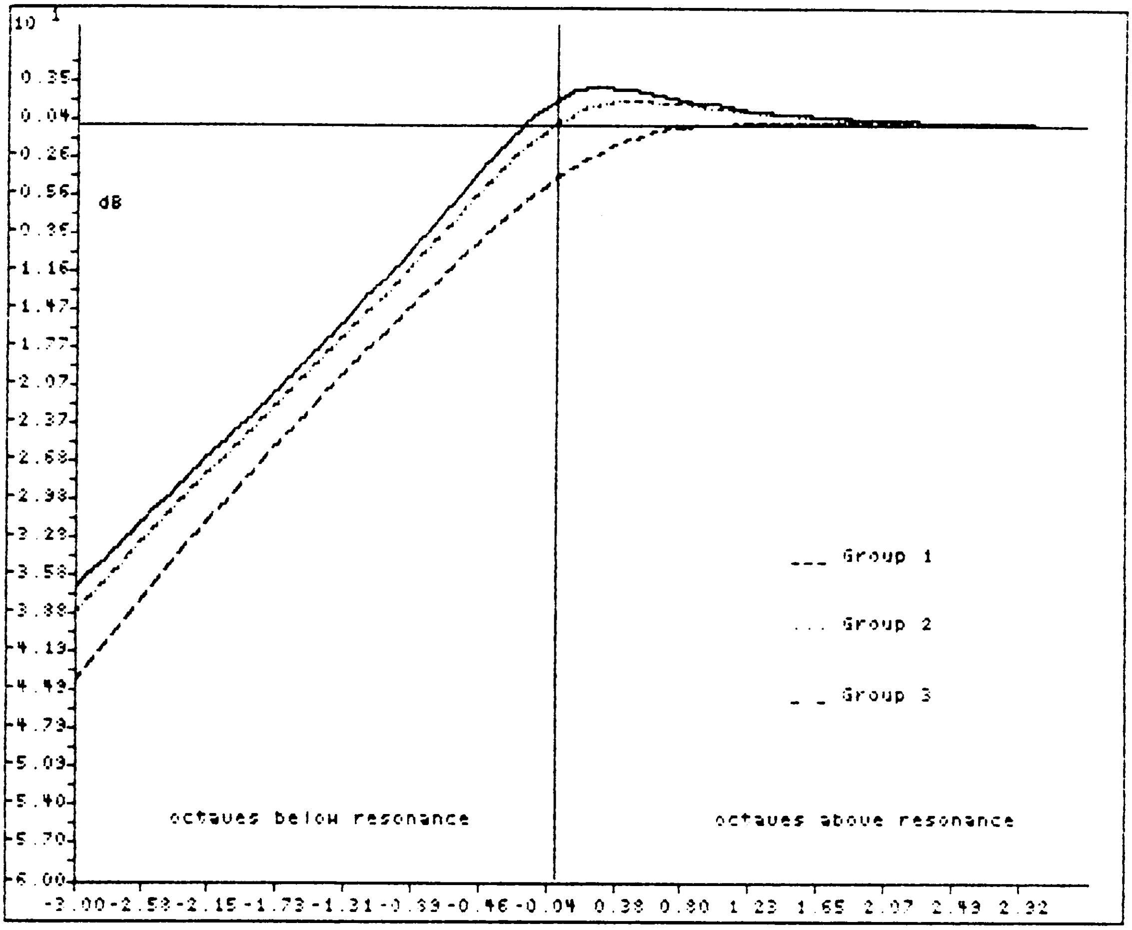
The transfer function for the pressure relief system has three degrees of freedom, Q_{ts}, Q_{tc}, and Q_{lc}, corresponding to driver free air Q, box volume, and leakage losses. A complete analysis of the system is possible through the transfer function and circuit simulation techniques. Since in practice Q_{lc} is limited to a minimum of about 1.0, and systems with a Q_{tc} less than 1.0 which require additional damping can generally be improved by tightly packing the enclosure with fiber, only situations which could benefit from the pressure relief technique will be considered here. Some general examples similar to common situations are explored below.

1) THE INFINITE BAFFLE/ACOUSTIC SUSPENSION TRANSITION

This situation occurs when $Q_{tc} = 2Q_{ts}$, and is considered the transition point because & $= C_{as}/C_{ab} = V_{as}/V_{ab} = 3.0$. This can be interpreted as the box volume contributing three times the restoring force than the driver has alone, or a compliance decrease by a factor of $1/4$. In general, systems with $Q_{tc} > 2Q_{ts}$ would benefit more from a larger box first and additional damping second, but this is not always possible.

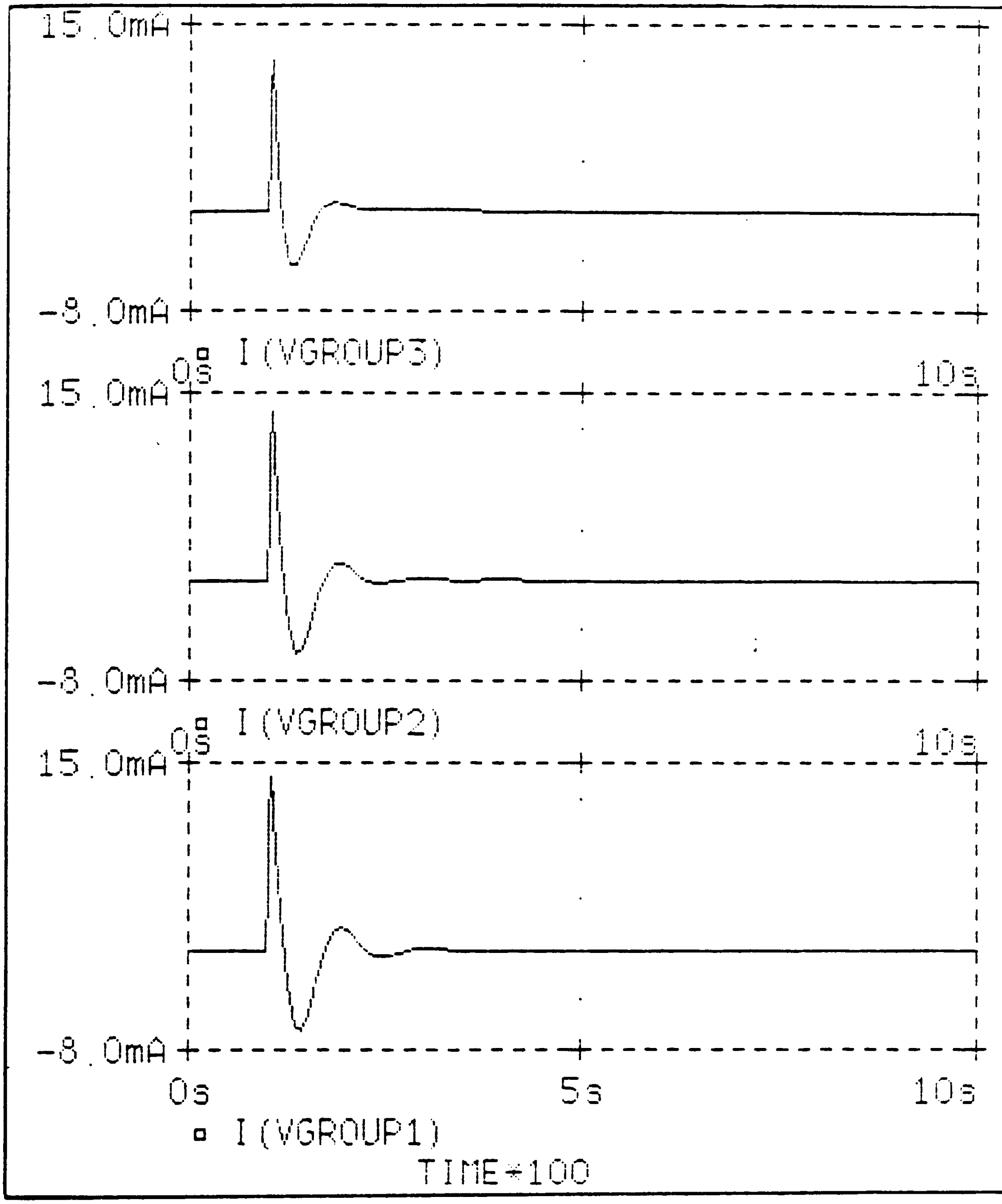
Figure 16a shows the frequency response for $Q_{tc} = 2Q_{ts} = 1.40$ and $Q_{lc} = 9.0$, 3.0, and 1.0, and the pulse response is displayed in 16b. The difference between infinite Q_{lc} and $Q_{lc} = 9$ is insignificant in terms of both frequency and transient response, and most real systems, no matter how well constructed, exhibit some minor leakage losses. $Q_{lc} = 9.0$ is a fair representation of this loss.

The most important feature the frequency response graph (fig. 16a) is that the severe peak near resonance is reduced for $Q_{lc} = 3.0$, and nearly eliminated for $Q_{lc} = 1.0$, while the rolloff rate below resonance is not greatly altered, remaining between 12 and 15 dB/octave. A small hump in the frequency response (about 0.5 dB) is evident at twice the system resonance for the $Q_{lc} = 1.0$ case, while the -3dB point is just above the resonance frequency. Comparison of the $Q_{lc} = 1.0$ curve with those of figure 11 reveals the system to respond approximately as a sealed box of $Q_{tc} = 0.7$. Analysis of the transient response reveals the system to be slightly more underdamped than an effective $Q_{tc} = 0.7$, closer to that of 0.8 (see Appendix C). The result is similar for the $Q_{lc} = 3$ case, where the frequency response is about that of an effective $Q_{tc} = 1.0$, while the transient response is closer to $Q_{tc} = 1.1$. In both cases, the



	Group 1	Group 2	Group 3
@lc =	9.00	3.00	1.00
Qtc =	1.40	1.40	1.40
Qts =	0.70	0.70	0.70

Fig. 16a Transfer functions for aperiodically damped loudspeaker system, Qtc = 1.40, Qts = 0.70 @lc = 9.00, 3.00, 1.00

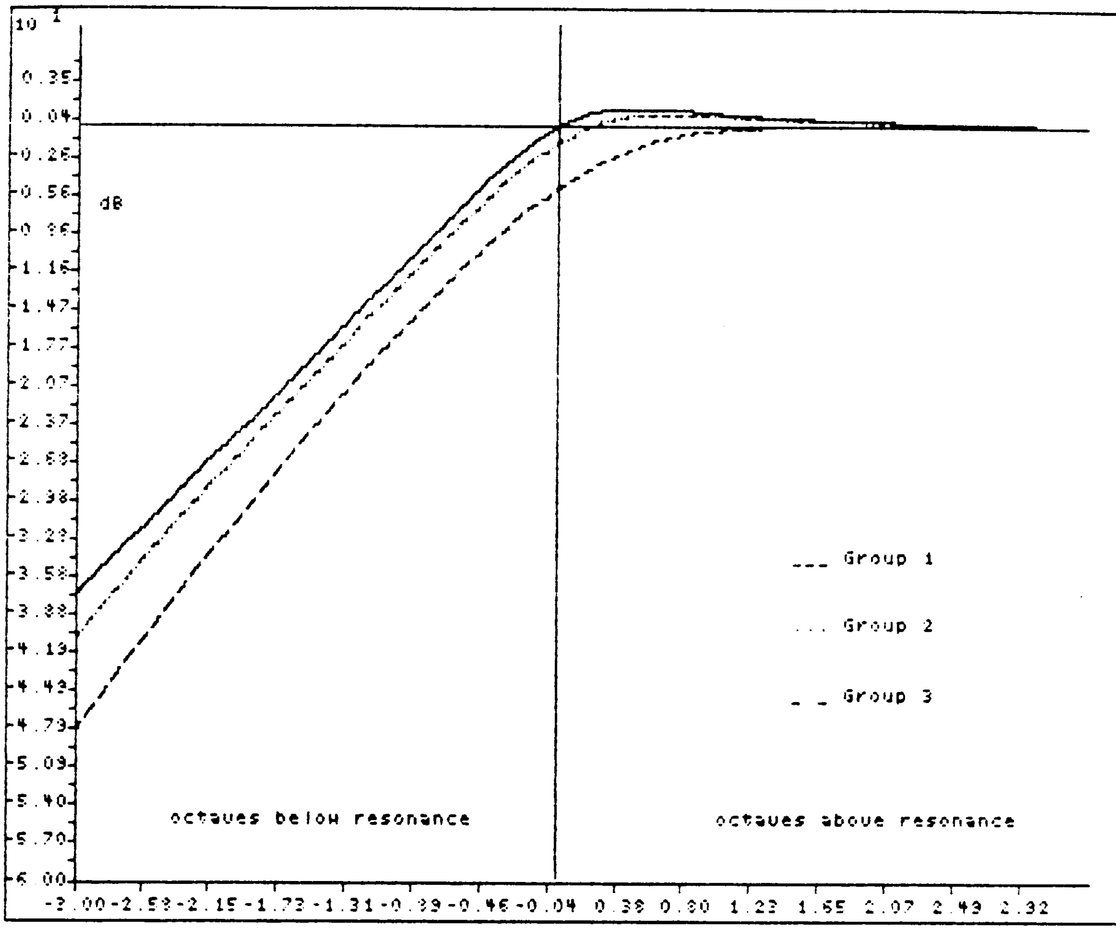


top : Q_{tc} = 1.40, Q_{ts} = 0.70, $@_{1c}$ = 1.00
middle: Q_{tc} = 1.40, Q_{ts} = 0.70, $@_{1c}$ = 3.00
bottom: Q_{tc} = 1.40, Q_{ts} = 0.70, $@_{1c}$ = 9.00

Fig. 16b Pulse response for loudspeaker system with $Q_{tc} = 2Q_{ts} = 1.40$, aperiodically damped frequency and transient response is significantly improved over the sealed box alone.

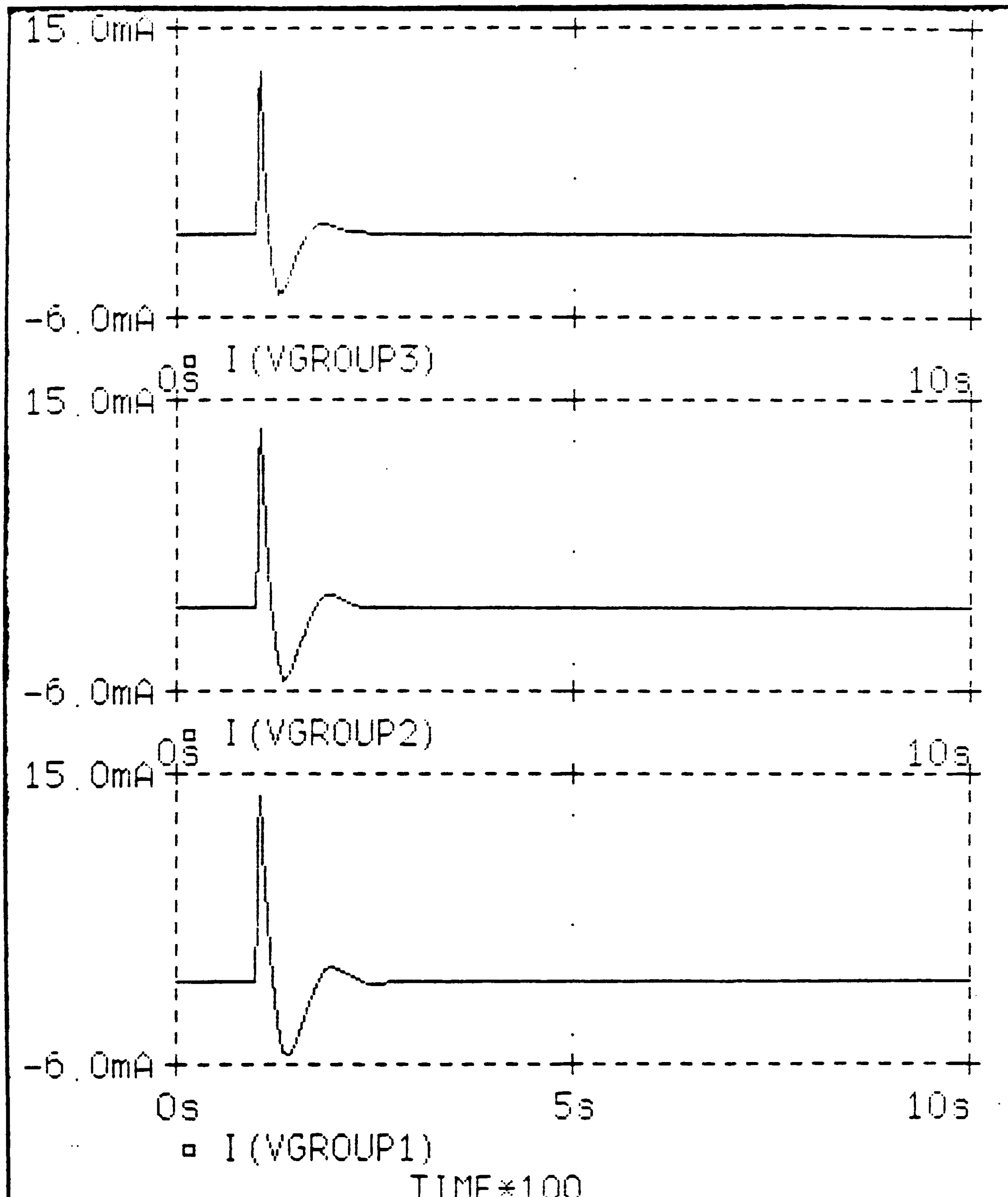
2) SYSTEMS WITH Q_{tc} = 1.5 Q_{ts}

This situation is more common than the above, and is fairly representative of most inexpensive sealed box loudspeaker systems, as well as many cone midrange units which are required to operate near their system resonance when housed in a subenclosure. Figures 17a,b are the results for a system of Q_{ts} = 0.7, Q_{tc} = 1.05, and Q_{lc} = 9.0, 3.0, and 1.0. The system transient response is improved in both cases, but in the case of Q_{lc} = 1.0, the frequency response is about 6dB down at resonance. The system can be fine tuned, however, by adjusting R_{al}. Figure 18a is the results for Q_{lc} = 2.5, 2.0, and 1.5. This is easily accomplished by changing the resistive padding in the leakage vent, and although the results are not dramatically different from those above they do allow the designer to optimize the system. For example, the system with Q_{lc} = 2.0 achieves a frequency response about the same as that of a sealed box with Qtc = 0.7, and a transient response (fig. 18b) of about Qtc = 0.8. This is a considerable improvement over the original sealed box system (Qtc = 1.05), and the system now has response characteristics similar to the driver alone, although with a higher resonance frequency.



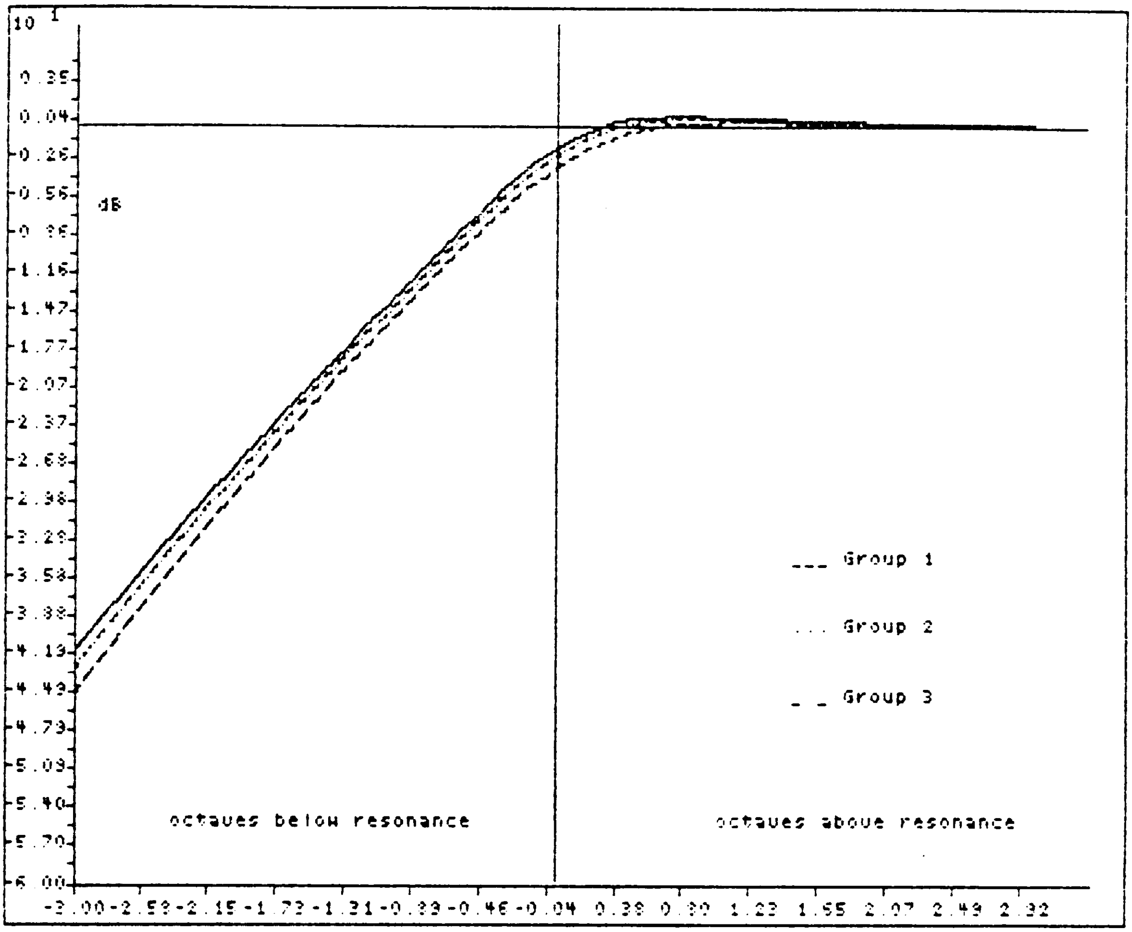
	Group 1	Group 2	Group 3
@lc =	9.00	3.00	1.00
Qtc =	1.05	1.05	1.05
Qts =	0.70	0.70	0.70

Fig. 17a Transfer functions for aperiodically damped loudspeaker system, Qtc = 1.05, Qts = 0.70 @lc = 9.00, 3.00, 1.00



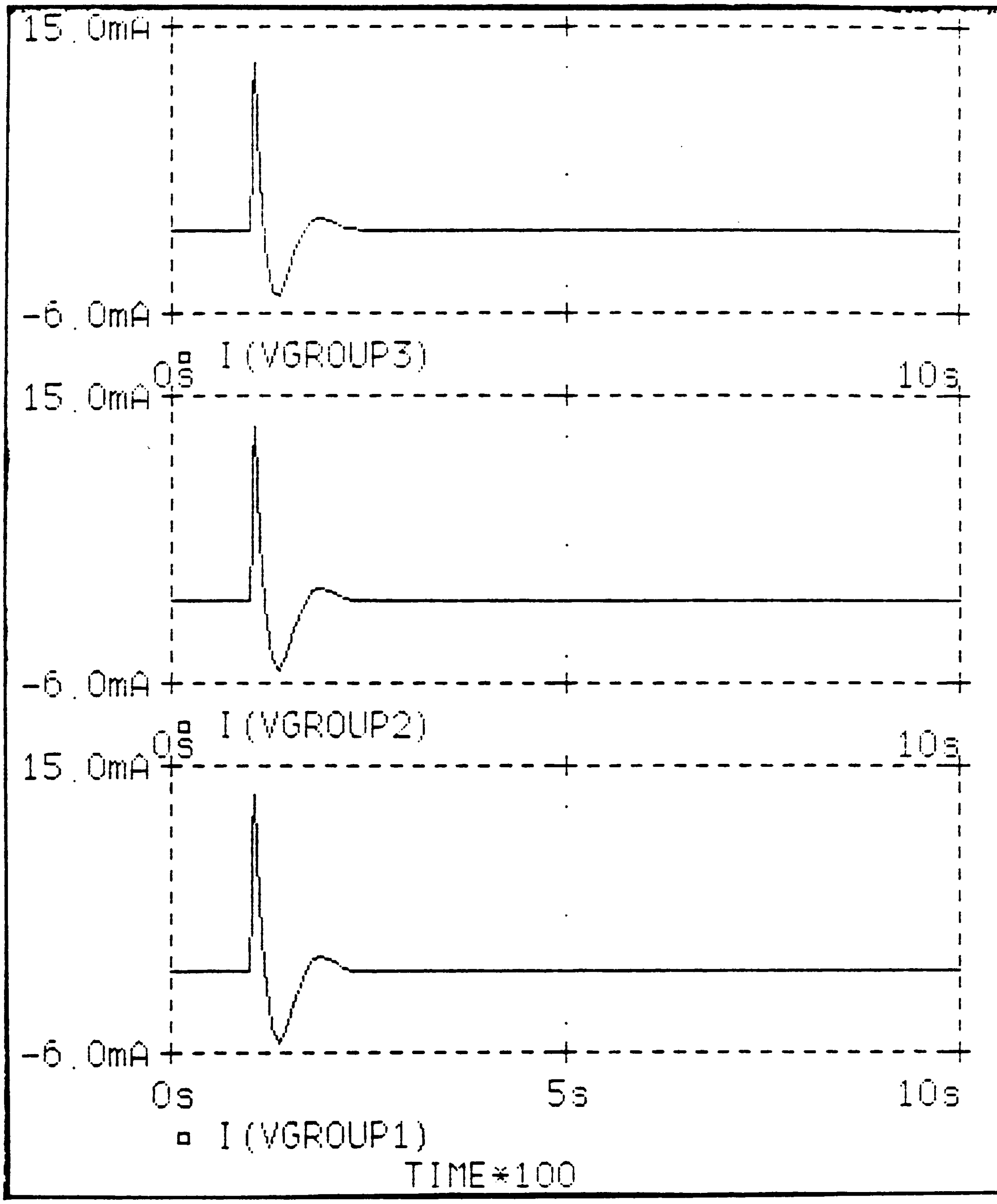
top   : Q_{tc} = 1.05, Q_{ts} = 0.70, $@_{1c}$ = 1.00
middle: Q_{tc} = 1.05, Q_{ts} = 0.70, $@_{1c}$ = 3.00
bottom: Q_{tc} = 1.05, Q_{ts} = 0.70, $@_{1c}$ = 9.00

Fig. 17b Pulse response for loudspeaker system with $Q_{tc} = 1.5Q_{ts} = 1.05$, aperiodically damped



	Group 1	Group 2	Group 3
@lc =	2.50	2.00	1.50
Qtc =	1.05	1.05	1.05
Qts =	0.70	0.70	0.70

Fig. 18a Transfer functions for aperiodically damped loudspeaker system, Qtc = 1.05, Qts = 0.70 @lc = 2.50, 2.00, 1.50



top : Q_{tc} = 1.05, Q_{ts} = 0.70, $@_{lc}$ = 1.50
middle: Q_{tc} = 1.05, Q_{ts} = 0.70, $@_{lc}$ = 2.00
bottom: Q_{tc} = 1.05, Q_{ts} = 0.70, $@_{lc}$ = 2.50

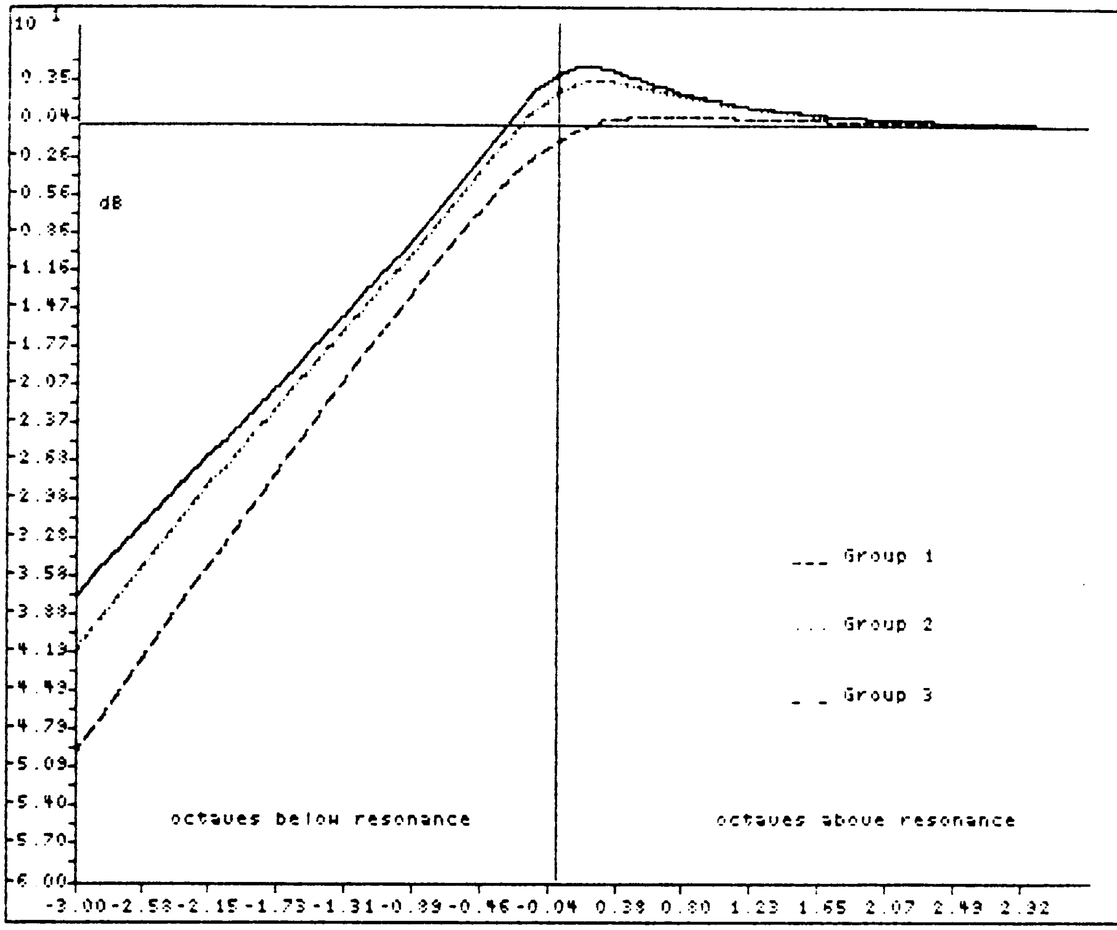
Fig. 18b Pulse response for loudspeaker system with $Q_{tc} = 1.5Q_{ts} = 1.05$, aperiodically damped

SYSTEMS WITH A DRIVER OF HIGH Q_{ts}

Although modern low frequency drivers are rarely designed with Q_{ts} larger than 1.0, this is not the case for many small cone midranges and midrange domes. The analysis above holds for wavelengths smaller than the driver circumference reguardless of what that circumference is, so the pressure relief technique can be applied. The case considered here is that of a driver with Q_{ts} = 1.3 in a system of Q_{tc} = 1.7. This could be interpreted as a midrange (or tweeter) dome in a small subenclosure. The frequency and transient characteristics are plotted in figures 19a,b. In this case, system response is dramatically improved for low Q_{lc}, making the driver far more capable of high quality performance. The transient response is still far from ideal, but in the case of Q_{lc} = 1.0 the frequency response is nearly flat. Furthermore, if this driver were used in the midrange the flat frequency response near resonance will yield far superior performance than the sealed enclosure case, as the midrange is usually crossed over to the low frequency driver in this region. Any peaks in midrange frequency response in this region will result in system response abberations.

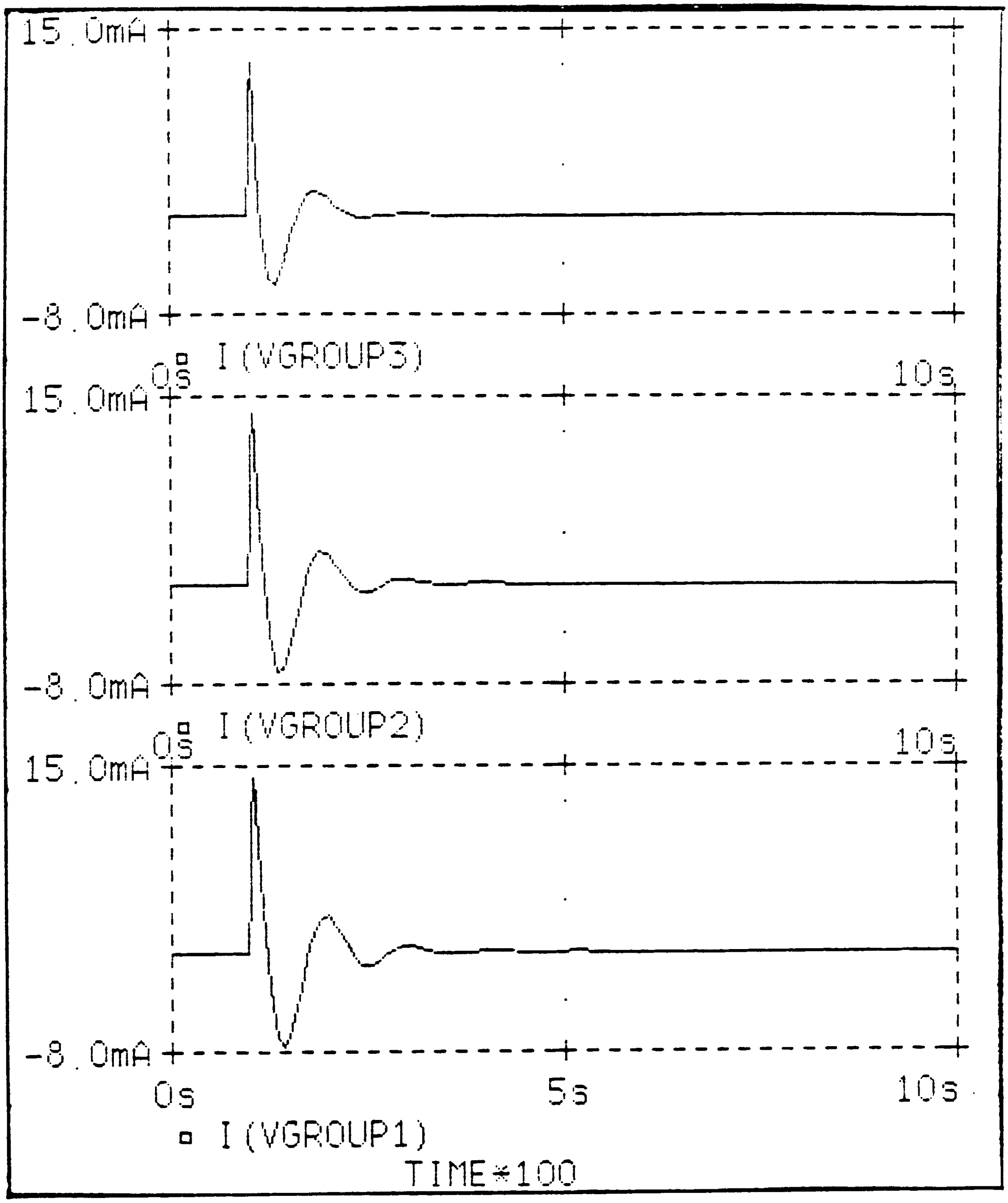
OUTPUT ROLLOFF BELOW RESONANCE

Careful analysis of the frequency response of a low Q_{lc} pressure relief system reveals the output to roll off at a



	Group 1	Group 2	Group 3
@lc =	9.00	3.00	1.00
Qtc =	1.70	1.70	1.70
Qts =	1.30	1.30	1.30

Fig. 19a Transfer functions for aperiodically damped loudspeaker system, Qtc = 1.70, Qts = 1.30 @lc = 9.00, 3.00, 1.00



top   : Q_{tc} = 1.70, Q_{ts} = 1.30, $@_{1c}$ = 1.00
middle: Q_{tc} = 1.70, Q_{ts} = 1.30, $@_{1c}$ = 3.00
bottom: Q_{tc} = 1.70, Q_{ts} = 1.30, $@_{1c}$ = 9.00

Fig. 19b Pulse response for loudspeaker system with high Q_{tc} and Q_{ts}, aperiodically damped slope greater than 12 dB/octave, and as Q_{ts} approaches Q_{tc} the slope reaches 18 dB/octave. Although this appears to be a disadvantage, careful inspection of the sealed box frequency response graphs (fig.11a,b, and Appendix C) indicate that for high Q_{tc} systems the rolloff for the octave immediately below resonance is also nearly 18 dB/octave. Since it is precisely these systems which benefit most from the pressure relief configuration, the trade-off is of importance only several octaves below system resonance, and in this region the system output is low enough compared to the passband (at least 12 dB down) that it is of little importance.

SYSTEM FINE TUNING

Small changes in Q_{lc} can result in altering the system frequency response by one or two decibels without greatly altering the system transient response characteristics (see figs. 17b and 18b). The easiest method of changing Q_{lc} is by adding layers of fabric to the leakage vent, the results easily measured through a frequency response measurement or the electrical impedance curve. Fine tuning of a sealed box system requires the system be opened and blocks of wood added to adjust the box volume. With aperiodically damped systems, the adjustments can be accomplished without removing the drivers or cabinet panels. This fine tuning is necessary for precise pair matching.

USING APERIODIC DAMPING IN A REAL SYSTEM

This work was originally undertaken to improve the response of an actual loudspeaker system already constructed, and this thesis is a product of attempts to explain the results. The system consists of a 30cm subwoofer in a 60 liter sealed box with a Q_{tc} of 0.75, a 17cm midrange/woofer in a 12 liter box, and a 2.5cm dome tweeter. The midrange/woofer is required to cover the bandwidth from 100 to 3200 Hz, the crossovers being located at 200 and 1600 Hz at 6 dB/octave. A maximum response deviation of ± 3 dB is accepted.

The free air and sealed box parameters for the 17 cm driver were measured as outlined in Appendix A. The results are:

<u>Free Air</u>

f_s = 89.0 Hz

Q_{ms} = 5.073

Q_{es} = 1.999

Q_{ts} = 1.434.

In the sealed box of volume 12 liters, the values are:

<u>Sealed Box</u>

f_c = 103.5 Hz

Q_{mc} = 6.393

Q_{ec} = 2.517

Q_{tc} = 1.806.

The other parameters are

w_s = 559.2/sec

w_c = 650.3/sec

M_{as} = 103 kg/m^4

R_{at} = 40185 Nsec/m^5

C_{ab} = $8.17 \cdot 10^{-8}$ m^5/N

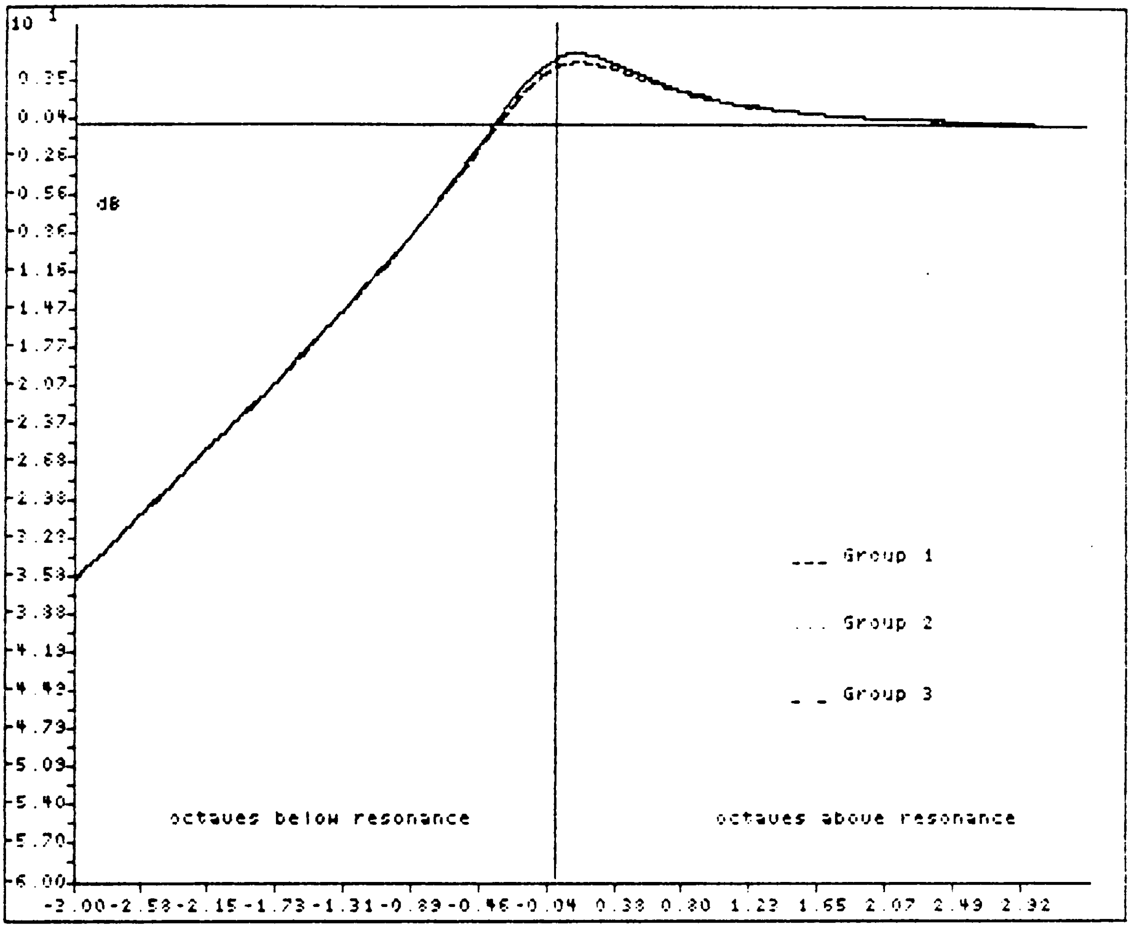
C_{as} = $3.10 \cdot 10^{-8}$ m^5/N

Bl = 4.3 Tm (manufacturer's specification)

S_d = .011 m^2 (manufacturer's specification).

The predicted response peak is 5.5 dB at 127 Hz (fig. 20). Actual measurement of the system revealed the peak to be +6 dB $\pm$1 dB somewhere between 116 and 180 Hz (fig. 21). Even when attenuated by the crossover, this output is at the level of the subwoofer, creating response abberations.

Working with the aperiodically damped system transfer function (eq. 77) indicated $@_{1c}$ between 1.0 and 1.5 would yield satisfactory results (fig. 22). A hole was cut in the back of the cabinet and the Dynaudio Variovent "soft" inserted. The resistive damping was varied by adding layers of felt and porous foam to a bonded dacron pad, all the damping material being tightly squeezed into the variovent and kept as uniform as possible. An @lc of 1.15 was obtained with three layers of felt in the variovent, while four layers resulted in $@_{1c}$ = 1.68. Three layers were used in the completed system.



	Group 1	Group 2	Group 3
Qtc =	1.81	1.66	1.66

Fig. 20 Transfer functions of 17cm drivers in 12 l sealed box, Qtc = 1.806, 1.663

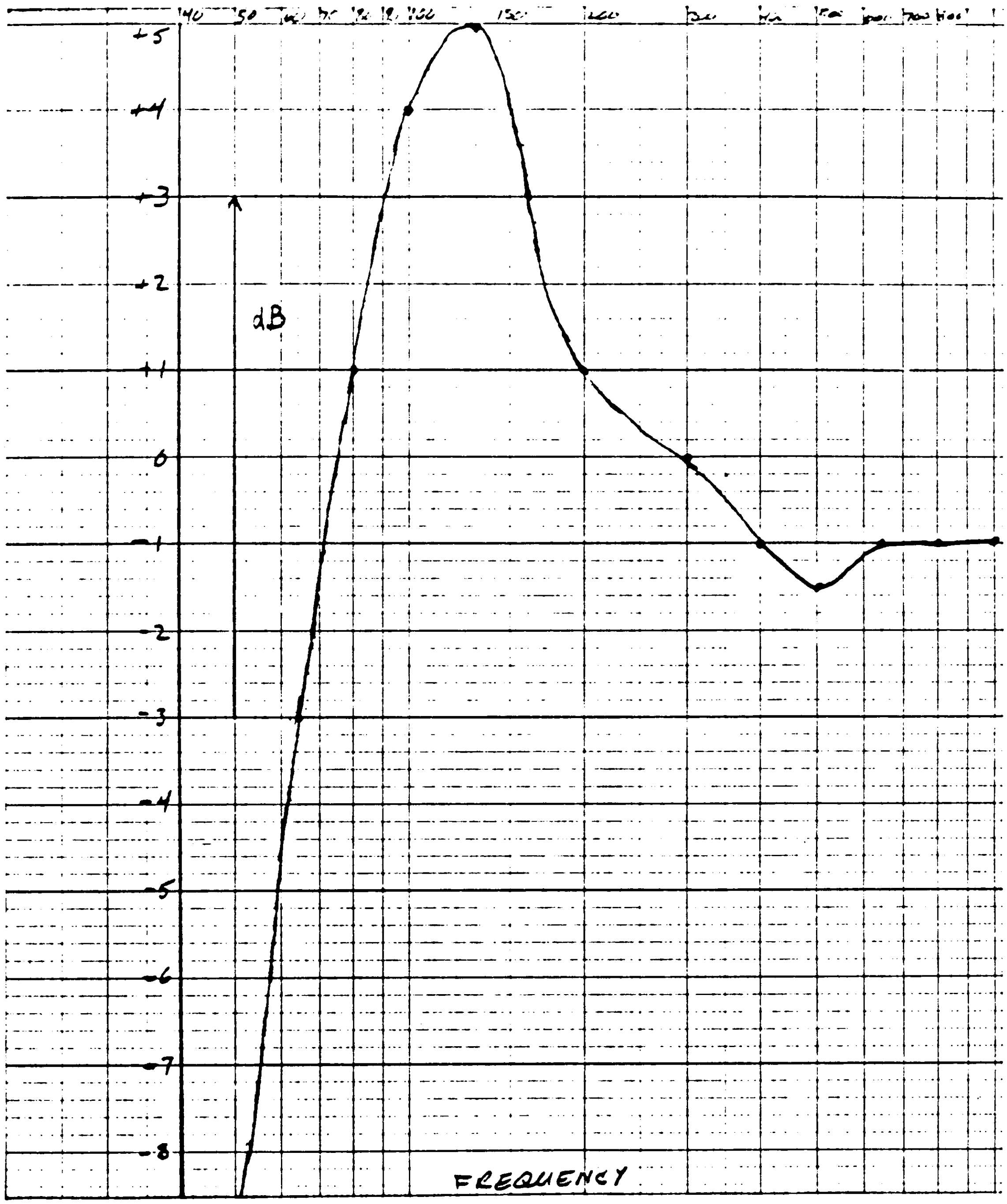
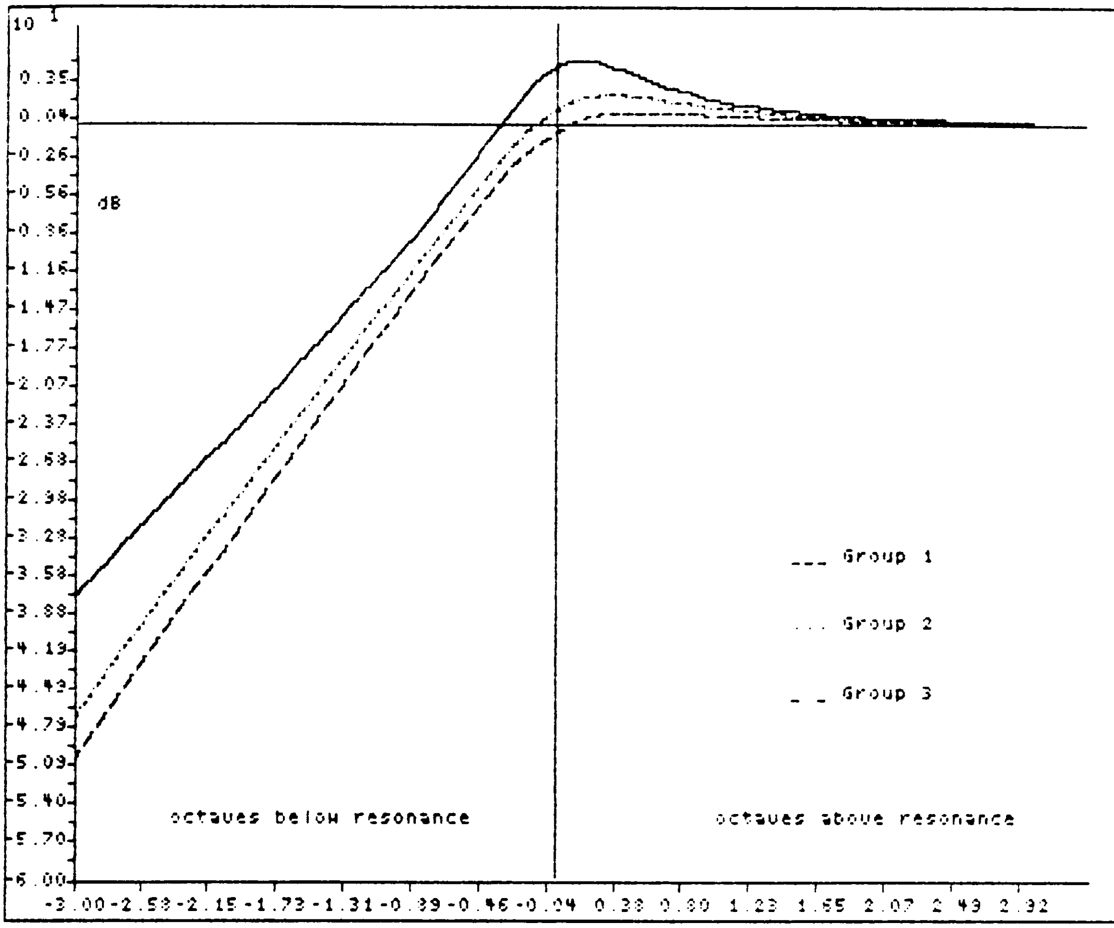


Fig. 21 Sealed box response for 17cm driver



	Group 1	Group 2	Group 3
@lc =	9.00	1.50	1.00
Qtc =	1.81	1.81	1.81
Qts =	1.43	1.43	1.43

Fig. 22 Transfer functions for 17cm driver in 12 l box, with and without pressure relief, @lc = 9.00, 1.50, 1.00

The resulting measured frequency response is given in figure 23. The frequency response peak is +1.2 dB ±1 dB at about 140 Hz. Attempts to make α_{lc} less than 1.15 lead to (measured) phase abberations and a very asymmetric impedence curve, indicating the presence of a box volume/leakage mass interaction, a situation to be avoided (see Appendix B).

The second driver of the pair had the following parameters:

Free Air	Sealed Box
f_s = 85.3 Hz	f_c = 102.0 Hz
Q_{ms} = 4.738	Q_{mc} = 5.918
Q_{es} = 1.845	Q_{ec} = 2.313
Q_{ts} = 1.328	Q_{tc} = 1.663

w_s = 536.0/sec

w_c = 640.9/sec

M_{as} = 99.36 kg/m^4

R_{at} = 40101 $Nsec/m^5$

C_{as} = $3.50 \cdot 10^{-8}$ m^5/N

C_{ab} = $8.17 \cdot 10^{-8}$ m^5/N

Simulations indicated α_{lc} = 1.40 would result in the frequency response of the second system being within 0.5 dB of the first. Working with the variovent, α_{lc} = 1.37 was obtained. The predicted frequency response of both systems is displayed in figure 24, and the measured responses in

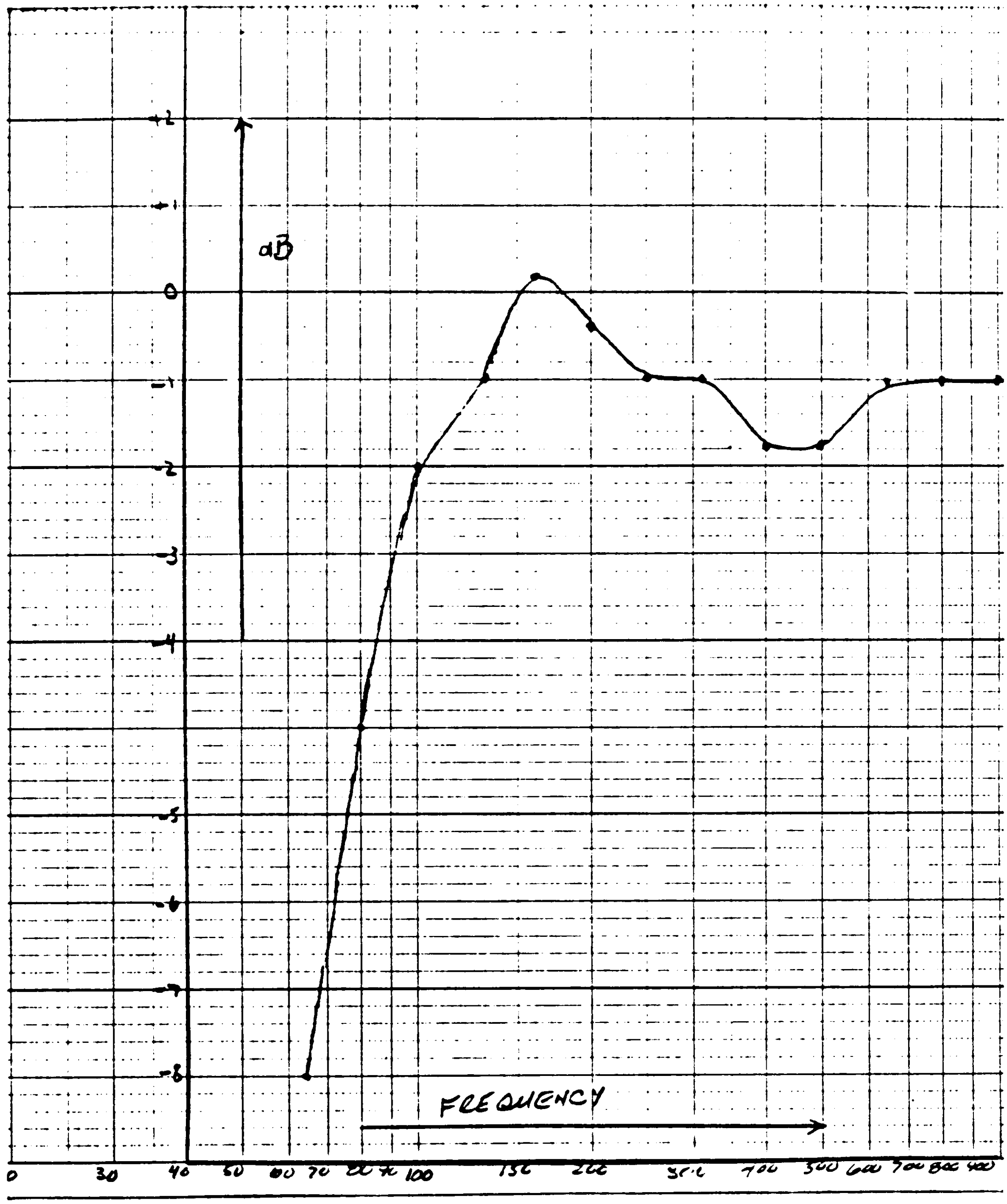
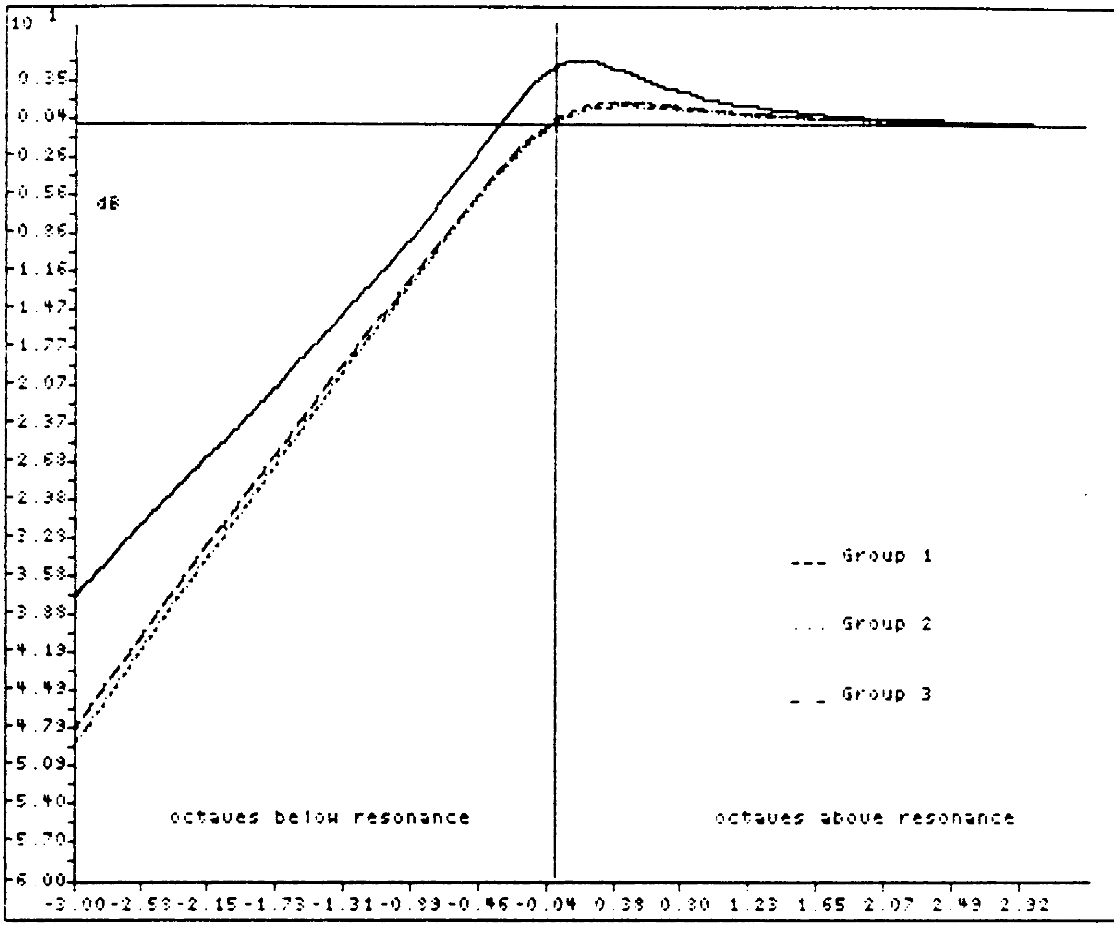


Fig. 23 Frequency response of 17cm driver in pressure relief box



	Group 1	Group 2	Group 3
@lc =	9.00	1.15	1.37
Qtc =	1.81	1.81	1.66
Qts =	1.43	1.43	1.33

Fig. 24 Transfer functions of 17cm matched drivers in aperiodically damped system, compared to their sealed box response

figure 25. The deviation between the two is within 1 dB, the measurement error of the instruments.

The predicted pulse response is given in figure 26 for the sealed and aperiodically damped systems. The predictions are far from ideal, but the system output is being attenuated below 200 Hz, so much of this response is of reduced magnitude. Although this is not verified experimentally, the aperiodically damped system is subjectively far superior in transient and frequency performance to the sealed box.

The completed system takes full advantage of the pressure relief configuration. The 17cm drivers, which were nearly unusable in the sealed box, displayed excellent response characteristics when the sysem was aperiodically damped. The frequency response of the pair was matched to within ± 0.5 dB, resulting in consistent image width and depth.

SUMMARY

Characteristics of the sealed box loudspeaker system are compared to those of the aperiodically damped loudspeaker system in table 1. The systems considered are based on a $Q_{tc} > 1.0$, the alignments which benefit from aperiodic damping. The excellent results which can be obtained using the pressure relief technique are evident in every category.

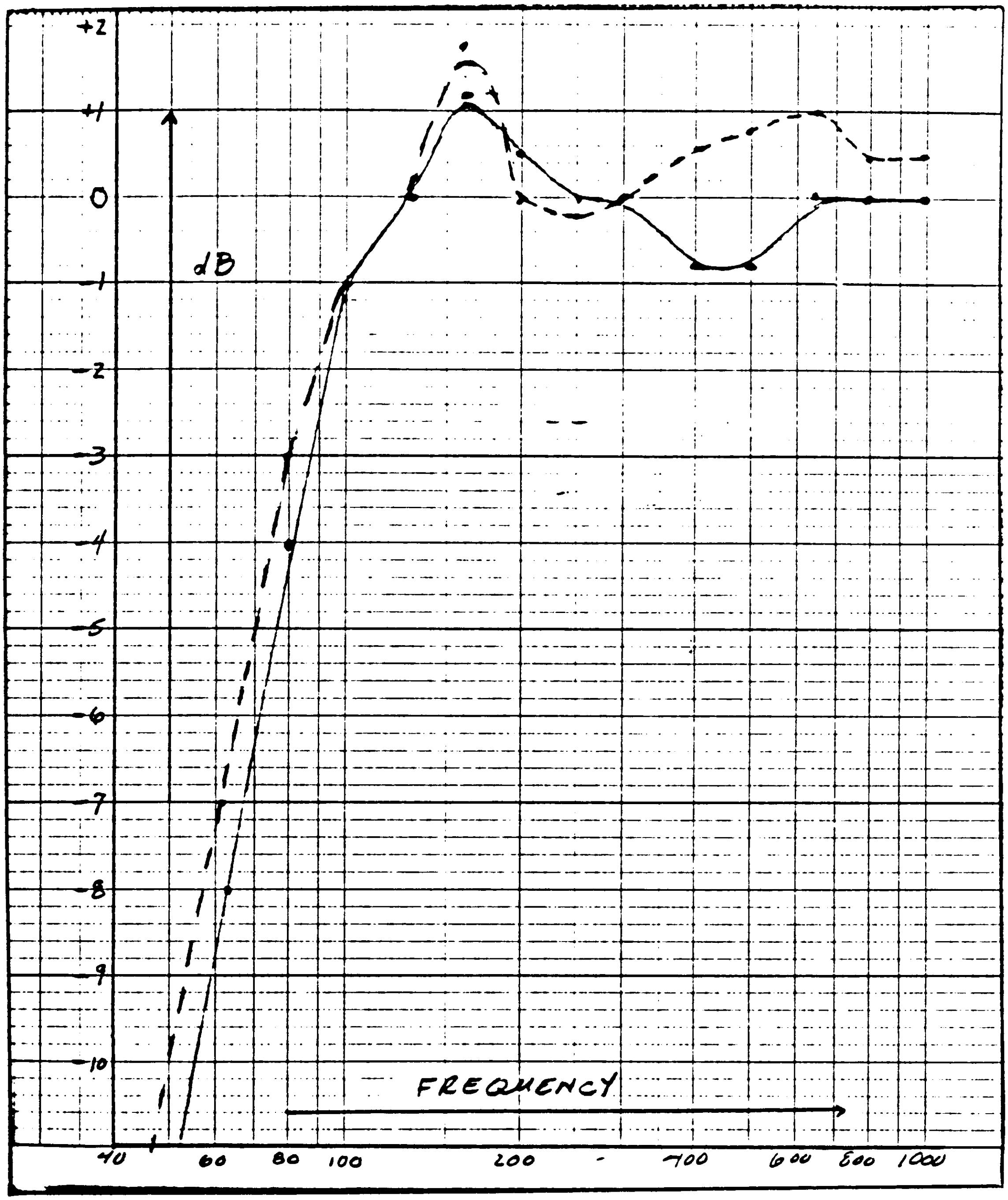
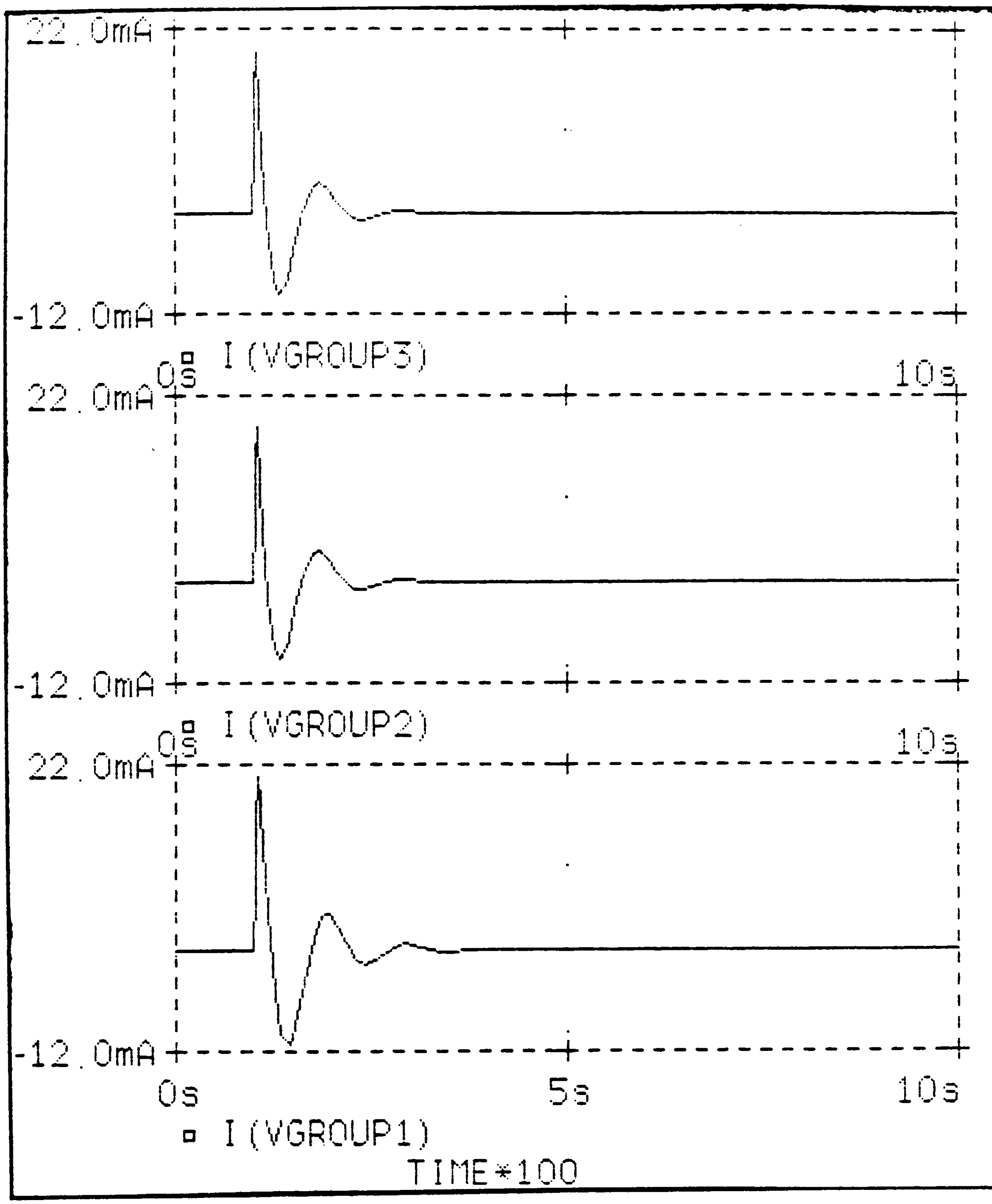


Fig. 25 Frequency response of matched 17cm systems

_ _ Q_{tc} = 1.81 ... Q_{tc} = 1.66



top : Q_{tc} = 1.81, Q_{ts} = 1.43, $@_{lc}$ = 1.15
middle: Q_{tc} = 1.66, Q_{ts} = 1.33, $@_{lc}$ = 1.37
bottom: Q_{tc} = 1.81, Q_{ts} = 1.43, $@_{lc}$ = 9.00

Fig. 26 Pulse response for 17cm loudspeaker systems, sealed box and aperiodically damped

Table 1 Comparison of the sealed box and aperiodically loudspeaker systems for Q_{tc} > 1.00

Quality	Sealed Box	Aperiodically Damped Box
Frequency Response	Peak due to underdamped resonance	Controlled through additional damping
Transient Response	Poor due to underdamped resonance	Controlled through additional damping
Stopband Rolloff	>12 dB/octave immediately below resonance, 12 dB/octave well below resonance	>12 db/octave
Pair Matching	difficult due to box volume constraints	easily done without opening the box

CHAPTER 6: CONCLUSION

The advantages of the aperiodically damped system over the sealed box loudspeaker system have been investigated, and a method for predicting system response has been derived. Several situations in which the pressure relief principle can be applied to improve system performance have also been explored.

Although the aperiodically damped system is more complicated than the sealed box, it is an easily implemented modification on most existing sealed box systems which require additional damping and/or better controlled frequency response. Furthermore, the value of the loss factor $@_{lc}$ is not that critical for correcting transient response, in that a 25% variation does not significantly change both transient delay and ring amplitude. The frequency response behaviour was strongly influenced near system resonance by $@_{lc}$, but the negative aspects are self cancelling. For example, systems with high Q_{tc} require low values of $@_{lc}$ to correct their transient response, and the response peak near resonance due to the high Q_{tc} is attenuated by the low $@_{lc}$. Systems with a lower Q_{tc} (Q_{tc} near 1.0) do not require much additional damping, so $@_{lc}$ need not be too low. Therefore, the attenuation from flat frequency response is not too severe.

Although no concrete methods for design of an aperiodically damped loudspeaker system have been presented, the theory derived here indicates the improvements which can be obtained using trial and error methods, and a real system has benefitted from the application.

The aperiodically damped loudspeaker system offers several advantages over many sealed box systems, and can be applied to all sealed box systems. Improved frequency response and well controlled transient response allow the designer to construct physically smaller systems while retaining the advantages of a larger box, to fine-tune existing systems, and to enhance the performance of low, mid, and possibly high frequency drive units. This represents a significant addition to existing loudspeaker system design theory.

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APPENDIX A : MEASUREMENTS

The principles discussed in the first part of this writing can be applied only if the parameters needed can be measured. This is easily accomplished through the electrical impedance curve since the mechanical and acoustic elements are coupled through the back Emf of the voice coil. The methods outlined here are those used for the analysis of this paper. Other methods can be found in reference 7.

The complete electrical circuit is given in figure 27.

BREAK IN

Prior to all measurements, the driver should be "broken in" by operating it several hours or overnight at a high enough power level and low enough frequency to cause considerable cone excursion. The method used for this project was to run the driver with a one third octave warble tone centered at 20 Hz at a power level of 15 watts for ten two hour periods, with three hour rests in between each session.

MAKING THE IMPEDANCE CURVE

The impedance curve can be taken under either constant current or constant voltage conditions. The constant current method was used throughout this work because it is easier to implement and requires less equipment than the constant voltage method.

Figure 28 shows the circuit necessary to make a constant current impedance curve. The value of the series

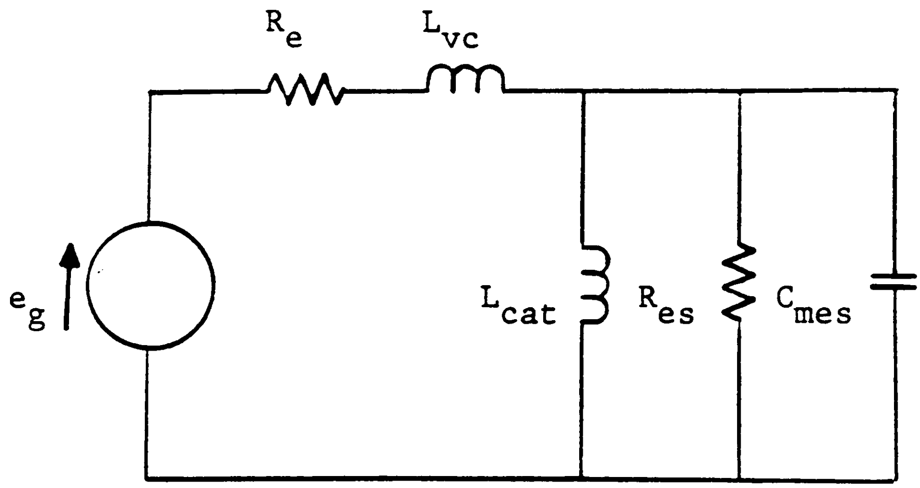


Fig. 27 Complete sealed box loudspeaker system electrical circuit

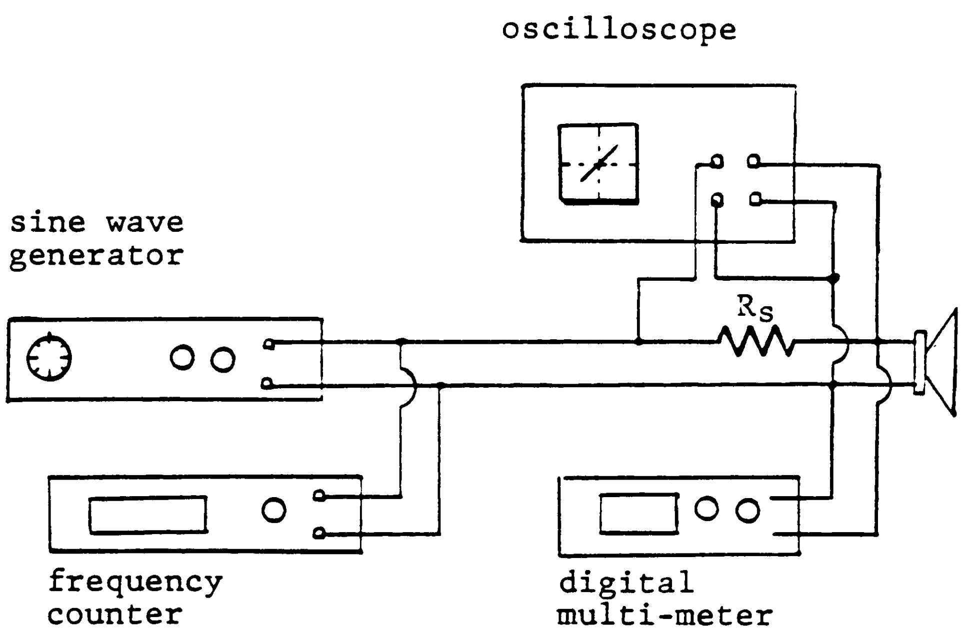


Fig. 28 Test set-up for constant current impedance measurement

resistor R_s should be at least 2000 ohms to keep the measurements within 2%. If higher accuracy is required, a larger resistor may be used. The frequency counter should be able to measure divisions of 0.5 Hz. The sine wave source must be capable of constant voltage magnitude output from 20 to 20,000 Hz into R_s. This is easily checked by moving the leads of the voltmeter to the generator a running a sweep test. The 10 ohm resistor R_t must be calibrated to within 1%, and the voltmeter accurate well into the millivolt range. A sample curve is given as figure 29.

Then, connect the circuit a shown and let everything warm up for at least an hour. Adjust the current through R_t until the voltage drop across it is exactly 10 millivolts. Replace R_t with wires which lead to the driver voice coil terminals. Run a sweep over the desired bandwidth, plotting the voltage measurement * 1000 as a function of frequency. Since i = 1 milliamp, the voltage across the driver terminals * 1000 is the impedance magnitude of the driver at that frequency, in ohms.

The phase measurement can be made on the oscilloscope by inspecting the Lissajous figure. However, although phase is of great importance when considering time delay or crossover networks, the phase measurement is not required for this analysis, as all calculations are based on impedance magnitude alone.

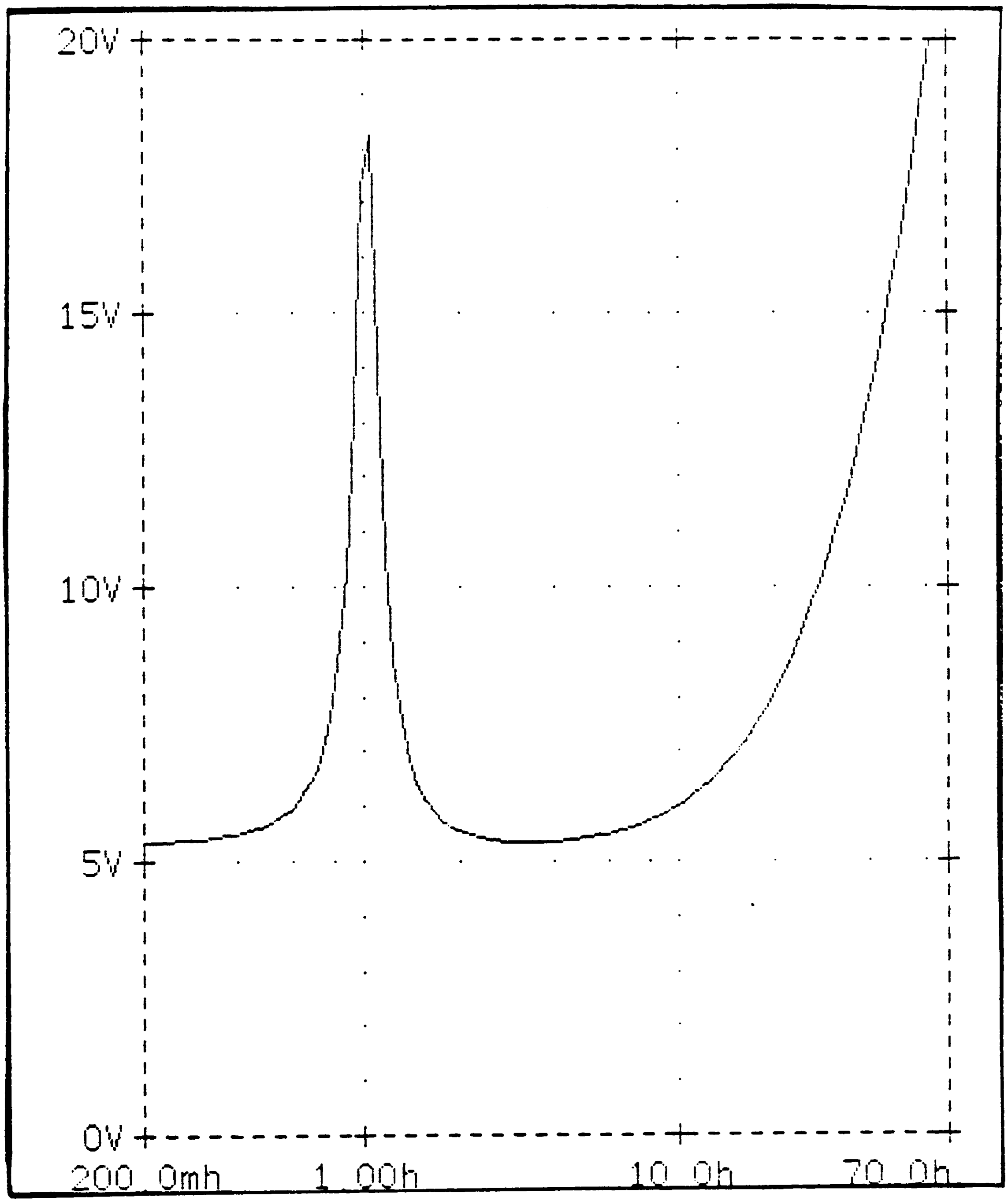


Fig. 29 Sample voice coil impedance curve

RESONANCE FREQUENCIES, f_s AND f_c

The point of highest impedance magnitude in the "humped" region is at the resonance frequency. This should be measured to within 0.5 Hz to minimize calculation errors. If the circuit was simple LRC, the Lissajous figure would also become a 45° line at this frequency. The presence of Lvc in the circuit (see figure 27) causes the zero phase condition to be noncoincident with the driver resonance, so the zero phase frequency should not be used as the resonance frequency.

DRIVER MASS AND COMPLIANCE

The mechanical moving mass M_{ms} and mechanical compliance C_{ms} are measured by changing the mass of the cone and finding the new resonance frequency. The cone mass is changed by adding clay or rope calk symmetrically around the dust cap. The new resonance should be at least 20% lower than the driver resonance without the clay. Then

$$M_{ms} = M_n/\{(f_s/f_n)^2 - 1\} \quad \text{(A1)}$$

and

$$C_{ms} = 1/\{4\pi^2 f_s^2 M_{ms}\} \quad \text{(A2)}$$

where

M_{ms} is the mechanical mass of the cone (kg)

M_n is the mass added to the cone (kg)

f_s is the original resonance of the driver

f_n is the resonance of the driver with added mass

C_{ms} is the mechanical compliance of the driver (in meters/Newton).

The acoustic parameters are then

$$C_{as} = C_{ms} S_d^2 \tag{A3}$$

$$M_{as} = M_{ms} / S_d^2 \tag{A4}$$

where

S_d is effective driver surface area (m^2)

C_{as} is the driver acoustic compliance (m^5/N)

Mas is the driver acoustic mass (kg/m^4)

A derivation of these relationships is in "Measurement Derivations 1" which follows this section.

MEASUREMENT OF Q's

The driver and system Q's are the most important parameters of the loudspeaker. Small[16] was the first to derive a simple method of measurement from the impedance curve. The method holds for both the free air and sealed box configurations. The free air configuration is used here as an example, with the sealed box configuration found by substitution of the sealed box for free air subscripts.

A sample impedance curve with the appropriate labels is given in figure 30. Then

$$Q_{ts} = \{(f_1 f_2)^{1/2}/(f_2 - f_1)\}\{R_e/(R_e + R_{es})\}^{1/2} \tag{A5}$$

$$Q_{ms} = \{(f_1 f_2)^{1/2}/(f_2 - f_1)\}\{(R_e + R_{es})/R_e\}^{1/2} \tag{A6}$$

$$Q_{es} = Q_{ts} Q_{ms}/(Q_{ms} - Q_{ts}) \tag{A7}$$

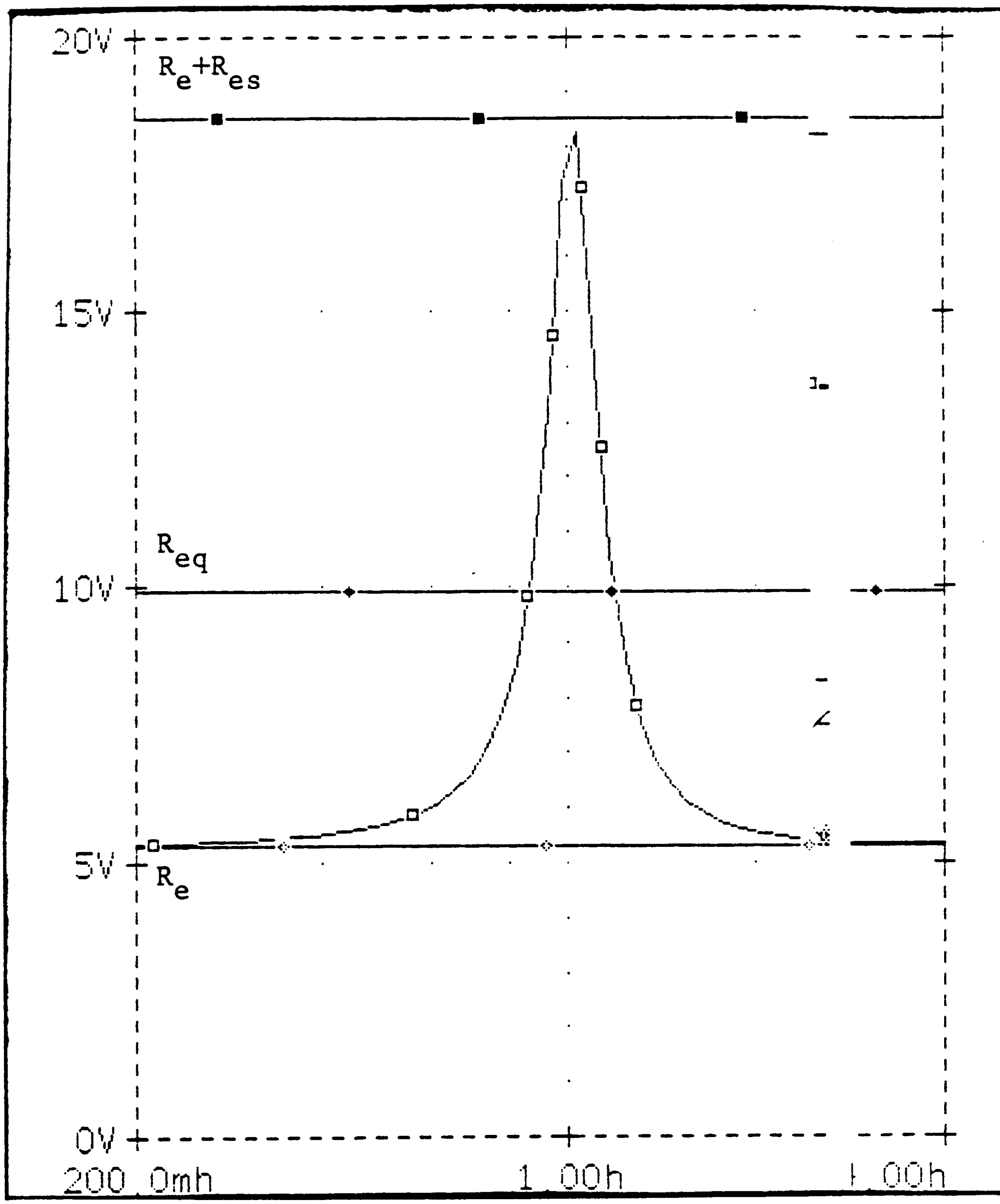


Fig. 30 Labelled voice coil impedance curve

where

$$R_{eq} = \{(R_e + R_{es})R_e\}^{1/2} \quad (A8)$$

$Z_{vc}(f)$ is the driver impedance as a function of frequency

Q_{ts} is the driver total Q at f_s

Q_{ms} is the driver mechanical Q at f_s

Q_{es} is the driver electrical Q at f_s

R_e is the driver free air resistance

R_{es} is the electrical equivalent of the driver mechanical losses R_{ms} or acoustical losses R_{as}

f_1 is the lower frequency for which $|Z_{vc}(f)| = R_{eq}$

f_2 is the higher frequency for which $|Z_{vc}(f)| = R_{eq}$

The derivation of these methods is in "Measurement Derivations 2,3,4,5" following this section.

OTHER PARAMETERS

The driver loss R_{as} is given by

$$R_{as} = (Bl/S_d)^2/\{(R_e + R_{es})-R_e\} \quad (A9)$$

where $(R_e + R_{es})$ the impedance magnitude at f_c

The box compliance C_{ab} is given by

$$C_{ab} = V_{as}/p_o c^2 \quad (A10)$$

MEASUREMENT OF @lc

The loss parameter @lc has the effect of reducing the impedance magnitude over the system resonance range, but reducing the resonance frequency only about 1%. To calculate $@_{lc}$, the sealed box measurements Q_{tc}, Q_{mc}, and Q_{ec} must first be made, and the leakage introduced after this has been done. Then a new impedance curve must be taken. Then

$$@_{lc} = xy/z - \{x^2y^2 - z(x^2(r_{lc}^2-1) + z)\}^{1/2}/z \qquad \text{(A11)}$$

and

$$R_{al} = @_{lc}/W_c C_{ab} \qquad \text{(A12)}$$

where

r_{lc} is $|Z_{vc}(f_c)|/R_e$ of the new impedance curve

$$x = (Q_{ts}/Q_{tc})^2 - 1 \qquad \text{(A13)}$$

$$y = r_{lc}^2/Q_{mc} - 1/Q_{tc} \qquad \text{(A14)}$$

$$z = (r_{lc}/Q_{mc})^2 - 1/Q_{tc}^2 \qquad \text{(A15)}$$

The derivation for this expression can be found in the "Measurement Derivations 6,7" following this section.

DERIVATIONS OF THE MEASUREMENT METHODS

1) Measurement of M_{ms} and C_{ms}

The driver resonance frequency is given by

$$1/w_s^2 = C_{ms} M_{ms} . \qquad \text{(A16)}$$

With an additional mass M_n added to the cone, the new resonance frequency is given by

$$1/w_n^2 = C_{ms} (M_{ms} + M_n). \qquad \text{(A17)}$$

Then

$$1/C_{ms} = w_s^2 M_{ms} = w_n^2 (M_{ms} + M_n) \qquad \text{(A18)}$$

and

$$(w_s/w_n)^2 = (f_s/f_n)^2 = 1 + M_n/M_{ms} \qquad \text{(A19)}$$

therefore

$$M_{ms} = M_n/\{(f_s/f_n)^2 - 1\}. \qquad \text{(A20)}$$

The driver mechanical compliance is given by

$$C_{ms} = 1/(w_s^2 M_{ms}) = 1/\{4pi^2 f_s^2 M_{ms} . \qquad \text{(A21)}$$

Two alternative methods for calculating C_{ms} are given in reference 7, pg. 73.

2) INVARIANCE OF Qe, Qm, and Qt

The complete acoustic circuit for the sealed box loudspeaker system is given in figure 31, and the electrical circuit in figure 32, where L_{vc} has been ignored. The values of the equivalent elements are:

$$R_{avc} = (Bl/S_d)^2/R_e \qquad \text{(A22)}$$

and

$$R_{es} = (Bl/S_d)^2/R_{as} \qquad \text{(A23)}$$

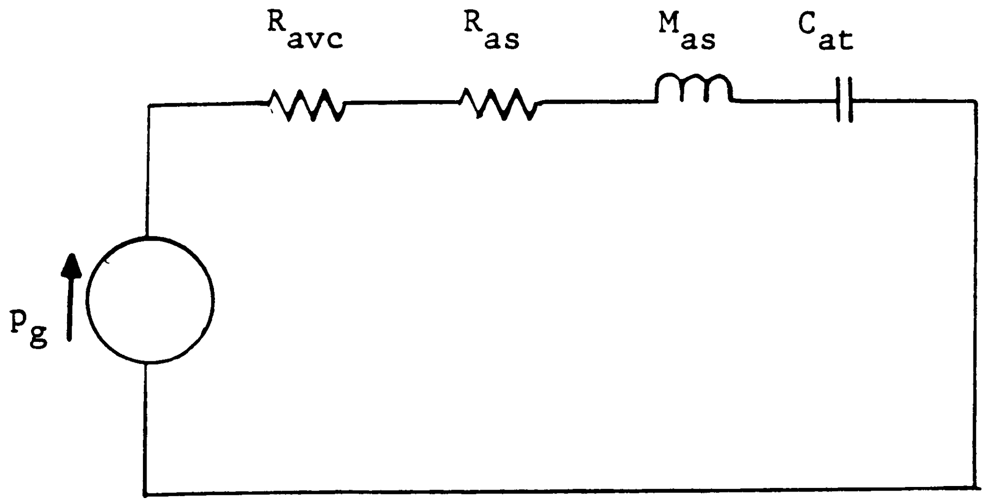


Fig. 31 Complete sealed box loudspeaker system acoustic circuit

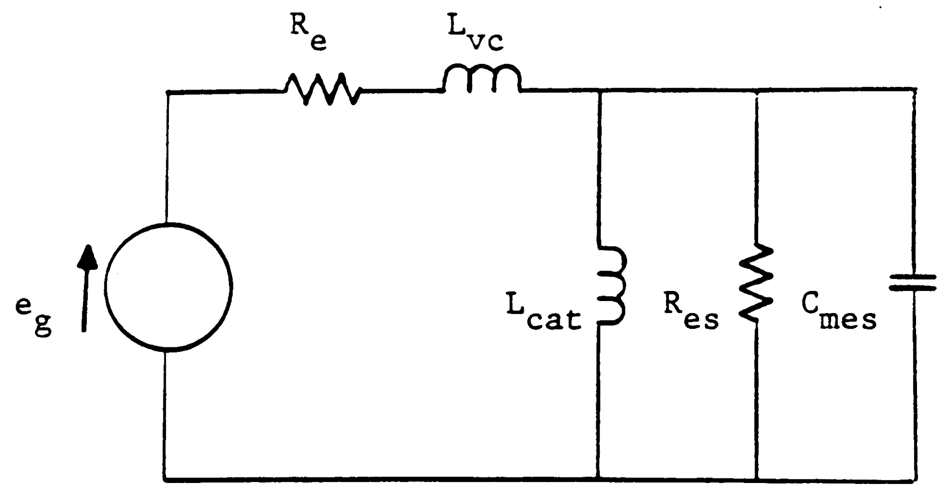


Fig. 32 Complete sealed box loudspeaker system electrical circuit

$$C_{mes} = M_{as}(S_d/Bl)^2 \tag{A24}$$

$$L_{cet} = C_{at}(Bl/S_d)^2 . \tag{A25}$$

$$w_c^2 = 1/C_{at}M_{as} = 1/C_{mes}L_{cet} \tag{A26}$$

For the acoustic circuit, the quality factors were defined as:

$$Q_{mc} = 1/w_c C_{at} R_{as} \tag{A27}$$

$$Q_{ec} = 1/w_c C_{at} R_{avc} \tag{A28}$$

$$Q_{tc} = 1/w_c C_{at}(R_{avc} + R_{as}) \tag{A29}$$

or

$$1/Q_{tc} = 1/Q_{ec} + 1/Q_{mc} . \tag{A30}$$

For the electrical circuit,

$$Q_{mc} = w_c C_{mes} R_{es} \tag{A31}$$

$$Q_{ec} = w_c C_{mes} R_e \tag{A32}$$

and

$$1/Q_{tc} = 1/Q_{mc} + 1/Q_{ec} . \tag{A33}$$

Then, using the definitions for the acoustic circuit,

$$\begin{aligned} Q_{mc} &= 1/w_c C_{at} R_{as} \\ &= 1/w_c L_{cet}(S_d/Bl)^2(Bl/S_d)^2(1/R_{es}) \\ &= (R_{es}/w_c L_{cet})[C_{mes}/C_{mes}] \\ &= w_c C_{mes} R_{es} \end{aligned}$$

which agrees with eq. A31,

$$\begin{aligned} Q_{ec} &= 1/w_c C_{at} R_{avc} \\ &= 1/w_c L_{cet}(S_d/Bl)^2(Bl/S_d)^2(1/R_e) \\ &= R_e/w_c L_{cet}[C_{mes}/C_{mes}] \\ &= w_c C_{mes} R_e \end{aligned}$$

which agrees with equation A32,

$$1/Q_{tc} = 1/Q_{ec} + 1/Q_{mc}$$

$$= (1/\omega_c C_{mes})(1/R_e + 1/R_{es})$$

$$= (1/\omega_c M_{as})(Bl/S_d)^2(S_d/Bl)^2(R_{avc} + R_{as})$$

$$= (R_{avc} + R_{as})/\omega_c M_{as}[C_{at}/C_{at}]$$

$$= \omega_c C_{at}(R_{avc} + R_{as})$$

which agrees with equation A29.

Therefore, Q_{ec}, Q_{mc}, and Q_{tc} are the same for both the complete electrical and complete acoustic equivalent circuits.

3) THE SEALED BOX SYSTEM VOICE COIL IMPEDANCE FUNCTION

From figure 32, the impedance at the driver terminals (ignoring L_{vc}) is:

$$Z_{vc}(s) = R_e + Z_p(s) \tag{A34}$$

where

$$1/Z_p(s) = sC_{mes} + 1/sL_{cet} + 1/R_{es} \tag{A35}$$

$$= (s^2 C_{mes} L_{cet} R_{es} + R_{es} + sL_{cet})/sL_{cet} R_{es}$$

$$= (s^2 T_c^2 + sL_{cet}/R_{es} + 1)/sL_{cet}.$$

Therefore,

$$Z_p(s) = sL_{cet}/\{s^2 T_c^2 + sL_{cet}/R_{es} + 1\}$$

$$= R_{es}\{(sL_{cet}/R_{es})/[s^2 T_c^2 + sL_{cet}/R_{es} + 1]\}$$

With

$$Q_{mc} = \omega_c C_{mes} R_{es} = R_{es}/\omega_c L_{cet}$$

then

$$Z_p(s) = R_{es}\{(sT_c/Q_{mc})/[s^2 T_c^2 + sT_c/Q_{mc} + 1]\}. \tag{A36}$$

Therefore,

$$Z_{vc}(s) = R_e + R_{es}\{(sT_c/Q_{mc})/[s^2T_c^2 + sT_c/Q_{mc} + 1]\} \quad (A37)$$

4) MEASUREMENT OF Q_e, Q_m, and Q_t from $Z_{vc}(s)$ (DERIVATION)

With the voice coil impedance function (eq. A37) and figure 32, and

$$Q_{mc} = w_c C_{mes} R_{es}$$

$$Q_{ec} = w_c C_{ces} R_e$$

then

$$Q_{mc}/Q_{ec} = R_{es}/R_e \quad (A38)$$

and with

$$r_c = (R_{es} + R_e)/R_e = 1 + Q_{mc}/Q_{ec} \quad (A39)$$

then

$$R_{es} = R_e(r_c - 1) \quad (A40)$$

$$Q_{ts} = Q_{es}Q_{ms}/(Q_{es} + Q_{ms}) = Q_{ms}/r_c. \quad (A41)$$

With

$$\begin{aligned} Z_{vc}(s) &= R_e + R_{es}\{(sT_c/Q_{mc})/[S^2T_c^2 + sT_c/Q_{mc} + 1]\} \\ &= R_e[1+(r_c-1)\{(sT_c/Q_{mc})/[s^2T_c^2 + sT_c/Q_{mc} + 1]\}] \\ &= R_e[1+\{(r_c sT_c/Q_{mc} - sT_c/Q_{mc})/[s^2T_c^2 + sT_c/Q_{mc} + 1]\}] \\ &= R_e[(s^2T_c^2 + r_c sT_c/Q_{mc} + 1)/(s^2T_c^2 + sT_c/Q_{mc} + 1)] \\ &= R_e[\{r_c + Q_{mc}(sT_c + 1/sT_c)\}/\{1+Q_{mc}(sT_c+1/sT_c)\}]. \end{aligned} \quad (A42)$$

With $s \to j\omega$,

$$Z_{vc}(j\omega) = R_e[\{r_c + jQ_{mc}(\omega/\omega_c - \omega_c/\omega)\}/$$
$$\{1 + jQ_{mc}(\omega/\omega_c - \omega_c/\omega)\}] \qquad (A43)$$

and

$$|Z_{vc}(j\omega)|^2 = R_e^2[\{r_c^2 + Q_{mc}^2(\omega/\omega_c - \omega_c/\omega)^2\}/$$
$$\{1 + Q_{mc}^2(\omega/\omega_c - \omega_c/\omega)^2\}]. \qquad (A44)$$

and

$$|Z_{vc}(j\omega)|^2/R_e^2 = \{r_c^2 + Q_{mc}^2[(\omega - \omega_c^2/\omega)/\omega_c]^2\}/$$
$$\{1 + Q_{mc}^2[\omega - \omega_c^2/\omega)/\omega_c]^2\} \qquad (A45)$$

For any two frequencies ω_1, ω_2, such that $\omega_1\omega_2 = \omega_c^2$, the magnitude of the voice coil impedance function is equal. The proof of this is in the following section, "Derivation of the Frequency Choice".

Then

$$|Z_{vc}(j\omega_1)|^2/R_e^2 = |Z_{vc}(j\omega_2)|^2/R_e^2. \qquad (A46)$$

Let $\omega = \omega_2$, then $\omega_c^2/\omega = \omega_1$, and

$$|Z_{vc}(j\omega_2)|^2/R_e^2 = \{r_c^2 + Q_{mc}^2[(\omega_2 - \omega_1)/\omega_c]^2/$$
$$\{1 + Q_{mc}^2[(\omega_2 - \omega_1)/\omega_c]^2. \qquad (A47)$$

Now, let $|Z_{vc}(j\omega_{1,2})| = r_1 R_e$. Then

$$r_1^2 + r_1^2 Q_{mc}^2[(\omega_2 - \omega_1)/\omega_c]^2$$
$$= r_c^2 + Q_{mc}^2[(\omega_2 - \omega_1)/\omega_c]^2$$

implies

$$r_c^2 - r_1^2 = Q_{mc}^2[(\omega_2 - \omega_1)/\omega_c]^2(r_1^2 - 1)$$

implies

$$Q_{mc}^2 = (\omega_c/[\omega_2 - \omega_1])^2\{[r_c^2 - r_1^2]/[r_1^2 - 1]\}$$

and therefore

$$Q_{mc} = (w_c/|w_2 - w_1|)\{[r_c^2 - r_1^2]/[r_1^2-1]\}^{1/2}. \quad \text{(A48)}$$

Now, if we define

$$r_1 = (r_c)^{1/2} \quad \{\to r_1 R_e = [(R_e + R_{es})R_e]^{1/2}\} \quad \text{(A49)}$$

then

$$Q_{mc} = (w_c/|w_2 - w_1|)\{[r_c^2 - r_c]/[r_c - 1]\}^{1/2}$$

$$= (w_c/|w_2 - w_1|)r_c^{1/2}. \quad \text{(A50)}$$

Since $r_c = (R_e + R_{es})/R_e$, and with $w_1 = 2\mathbf{pi}\, f_1$, $w_2 = 2\mathbf{pi}\, f_2$, then

$$Q_{mc} = (f_c/|f_2 - f_1|)[(R_e + R_{es})/R_e]^{1/2} \quad \text{(A51)}$$

and

$$Q_{tc} = Q_{mc}/r_c =$$

$$= (f_c/|f_2 - f_1|)[R_e/(R_e + R_{es})]^{1/2} \quad \text{(A52)}$$

Note that in the "Measurement Methods" section f_c was replaced by $(f_1 f_2)^{1/2}$. The theory derived here indicates they are the same, but in practice f_c is about 1% higher than the product. It is not clear which one is most correct, but f_c is located at a turning point on the impedance curve and is difficult to measure exactly, while both f_1 and f_2 are in regions of high slope, their measurement being much more accurate. With this in mind, it is suggested that the product be used.

5) DERIVATION OF THE FREQUENCY CHOICE

From eq. A44, the voice coil impedance function is

$$|Zvc(jw)|^2 = Re^2[\{rc^2 + Qmc^2(w/wc - wc/w)^2\}/$$
$$\{1 + Qmc^2(w/wc - wc/w)^2\}].$$

With $w1 < w2$, we need $|Zvc(jw1)| = |Zvc(jw2)|$.

Therefore (eliminating Re^2)

$$\{rc^2 + Qmc^2(w1/wc - wc/w1)^2\}/$$
$$\{1 + Qmc^2(w1/wc - wc/w1)^2$$
$$= \{rc^2 + Qmc^2(w2/wc - wc/w2)^2\}/$$
$$\{1 + Qmc^2(w2/wc - wc/w2)^2\} \qquad (A53)$$

implies

$$rc^2 + Qmc^2(w2/wc - wc/w2)^2$$
$$+ rc^2Qmc^2(w1/wc - wc/w1)^2$$
$$+ Qmc^4(w2/wc - wc/w2)^2(w1/wc - wc/w1)^2$$
$$= rc^2 + Qmc^2(w1/wc - wc/w1)^2$$
$$+ rc^2Qmc^2(w2/wc - wc/w2)^2$$
$$+ Qmc^4(w2/wc - wc/w2)^2(w1/wc - wc/w1)^2. \qquad (A54)$$

Eliminating common terms and dividing by Qmc^2,

$$(w2/wc - wc/w2)^2 + rc^2(w1/wc - wc/w1)^2$$
$$= (w1/wc - wc/w1)^2 + rc^2(w2/wc - wc/w2)^2 \qquad (A55)$$

implies

$$rc^2\{(w1/wc - wc/w1)^2 - (w2/wc - wc/w2)^2\}$$
$$= (w1/wc - wc/w1)^2 - (w2/wc - wc/w2)^2. \qquad (A56)$$

The two sides differ only by the multiplicative factor r_c. Since r_c has no specific value, the only choice is that both sides must equal zero. Thus

$$(w_1/w_c - w_c/w_1)^2 = (w_2/w_c - w_c/w_2)^2 \tag{A57}$$

implies

$$w_1^2/w_c^2 + w_c^2/w_1^2 = w_2^2/w_c^2 + w_c^2/w_2^2$$

and

$$w_c^4(1/w_2^2 - 1/w_1^2) = w_1^2 - w_2^2$$

then

$$w_c^4 = (w_1^2 - w_2^2)/\{1/w_2^2 - 1/w_1^2)$$

$$= w_1^2 w_2^2. \tag{A58}$$

Therefore,

$$w_c^2 = w_1 w_2. \tag{A59}$$

6) VOICE COIL IMPEDANCE FUNCTION OF THE PRESSURE RELIEF SYSTEM

The complete electrical circuit for the general aperiodically damped system is given in figure 33, where

$$R_{es} = (Bl/S_d)^2/R_{as} \tag{A60}$$

$$C_{mes} = (S_d/Bl)^2 M_{as} \tag{A61}$$

$$L_{ces} = (Bl/S_d)^2 C_{as} \tag{A62}$$

$$L_{ceb} = (Bl/S_d)^2 C_{ab} \tag{A63}$$

$$R_l = (Bl/S_d)^2/R_{al}. \tag{A64}$$

Then

$$Z_{vc}(s) = R_e + Z_p(s)$$

where

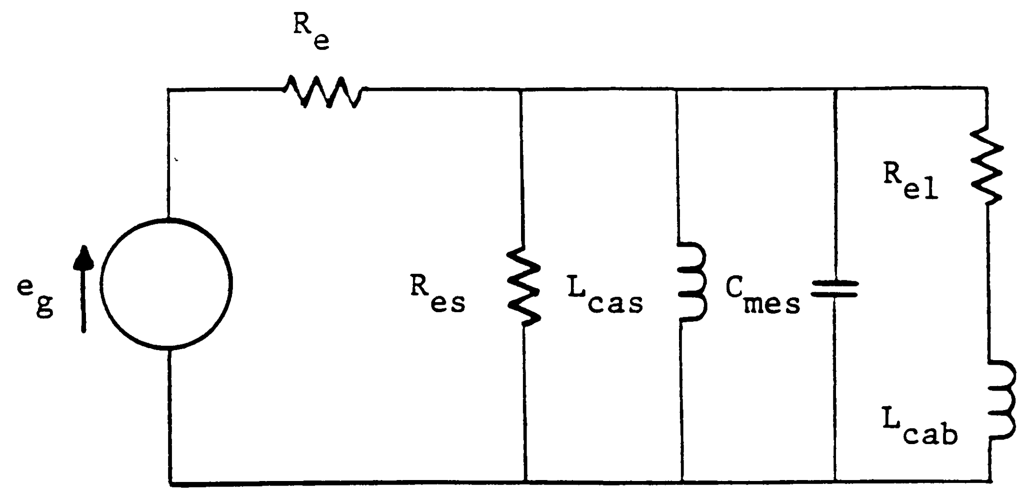


Fig. 33 Complete aperiodically damped loudspeaker electrical circuit

$$1/Z_p(s) = 1/R_{es} + sC_{mes} + 1/(sL_{ces}) + 1/[R_l(1 + sL_{ceb}/R_l)]$$

$$= [\{sL_{ces}/R_{es} + s^2 C_{mes} L_{ces} + 1\}/(sL_{ces})] + 1/[R_l(1 + sL_{ceb}/R_l)]. \quad (A65)$$

With

$$@_{ls} = w_s C_{ab} R_{al} = w_s L_{ceb}/R_l$$

$$Q_{ms} = R_{es}/(w_s L_{ces})$$

$$T_s{}^2 = C_{mes} L_{ces}$$

$$1/Z_p(s) = [\{s^2 T_s{}^2 + sT_s/Q_{ms} + 1\}/(sL_{ces})] + 1/[R_l(1 + sT_s @_{ls})]$$

$$= \{[s^2 T_s{}^2 + sT_s/Q_{ms} + 1][1 + sT_s @_{ls}] + sL_{ces}/R_l\}/ [sL_{ces}(1 + sT_s @_{ls})].$$

With

$$\& = C_{as}/C_{ab} = L_{ces}/L_{ceb}$$

$$1/Z_p(s) = \{s^3 T_s{}^3 @_{ls} + s^2 T_s{}^2 (1 + @_{ls}/Q_{ms}) + sT_s[1/Q_{ms} + (1 + \&)@_{ls}] + 1\}/ [sL_{ces}(1 + sT_s @_{ls})]$$

$$= \{s^3 T_s{}^3 @_{ls} + s^2 T_s{}^2 (1 + @_{ls}/Q_{ms}) + sT_s[1/Q_{ms} + (Q_{tc}/Q_{ts})^2 @_{ls}] + 1\}/ [sL_{ces}(1 + sT_s @_{ls})]. \quad (A66)$$

With

$$Q_{es} = w_s C_{mes} R_e = R_e/(w_s L_{ces}) \quad (A67)$$

$$Z_{vc}(s)/R_e = 1 + (sT_s/Q_{es})(1 + sT_s @_{ls})/D(s) \quad (A68)$$

where D(s) is the numerator of 1/Zp(s) in eq. A66.

Therefore,

$$Zvc(s)/Re = \{D(s) + (sTs/Qes)(1 + sTs/@ls)/D(s).$$

or

$$Zvc(s)D(s)/Re$$

$$= s^3 Ts^3 @ls + s^2 Ts^2 [1 + @ls(1/Qms + 1/Qes)]$$

$$+ sTs[1/Qms + 1/Qes + (Qtc/Qts)2@ls] + 1$$

$$= s^3 Ts^3 @ls + s^2 Ts^2 (1 + @ls/Qts)$$

$$+ sTs[1/Qts + (Qtc/Qts)^2 @ls] + 1.$$

(A69)

Now, the expression for the numerator of $Zvc(s)/Re$ has the exact same form as $D(s)$, except Qms replaced with Qts.

Let the numerator of $Zvc(s)/Re = N(s)$. Then

$$Zvc(s)/Re = N(s)/D(s). \qquad (A70)$$

To get both $N(s)$ and $D(s)$ in terms of the sealed box resonance frequency wc, we use

$$Ts = (Qtc/Qts)Tc$$

$$Qms = (ws/wc)wc\,Cmes\,Res = (Qts/Qtc)Qmc$$

$$@ls = (ws/wc)wc\,Lceb/Rl = (Qts/Qtc)Qlc \qquad (A71)$$

then

$$N(s) = [Qtc/Qts]^2\{s^3 Tc^3 @lc + s^2 Tc^2(1 + @lc/Qtc)$$

$$+ sTc(@lc + 1/Qtc) + Qts^2/Qtc^2\}$$

(A72)

and

$$D(s) = [Qtc/Qts]^2\{s^3 Tc^3 @lc + s^2 Tc^2(1 + @lc/Qmc)$$

$$sTc(@lc + 1/Qmc) + Qts^2/Qtc^2\}.$$

(A73)

Then

$$Z_{vc}(s) = R_e[N(s)/D(s)] \tag{A74}$$

7) MEASUREMENT OF $@_{1c}$

From equation A74, the normalized magnitude of the voice coil impedance function at w_c can be expressed as

$$\begin{aligned} &|Z_{vc}(jw_c)|/R_e \\ &= |\{[(Q_{ts}/Q_{tc})^2 - 1 - @_{1c}/Q_{tc}] + j[1/Q_{tc}]\} / \\ &\quad \{[(Q_{tc}/Q_{tc})^2 - 1 - @_{1c}/Q_{mc}] + j[1/Q_{mc}]\}|. \end{aligned} \tag{A75}$$

With

$$x = (Q_{ts}/Q_{tc})^2 - 1 \tag{A76}$$

then

$$\begin{aligned} &|Z_{vc}(jw_c)|/R_e \\ &= |\{[x - @_{1c}/Q_{tc}] + j[1/Q_{tc}]\}/ \\ &\quad \{[x - @_{1c}/Q_{mc}] + j[1/Q_{mc}]\}| \end{aligned} \tag{A77}$$

and

$$\begin{aligned} &|Z_{vc}(jw_c)|/R_e|^2 \\ &= \{[x - @_{1c}/Q_{tc}]^2 + 1/Q_{tc}^2\}/ \\ &\quad \{[x - @_{1c}/Q_{mc}]^2 + 1/Q_{mc}^2\} \\ &= r_{1c}^2 \end{aligned} \tag{A78}$$

where r_{1c} is the normalized voice coil impedance magnitude at w_c.

Then

$$\begin{aligned} &r_{1c}^2\{[x - @_{1c}/Q_{mc}]^2 + 1/Q_{mc}^2\} \\ &= [x - @_{1c}/Q_{tc}]^2 + 1/Q_{tc}^2. \end{aligned} \tag{A79}$$

Manipulation of this expression yields

$$@_{lc}^{2}z - @_{lc}(2xy) + [x^{2}(r_{lc}^{2} - 1) + z] = 0 \qquad (A80)$$

where

$$y = (r_{lc}^{2}/Q_{mc}) - 1/Q_{tc} \qquad (A81)$$

$$z = (r_{lc}/Q_{mc})^{2} - 1/Q_{tc}^{2}. \qquad (A82)$$

Then

$$@_{lc} = xy/z - \{x^{2}y^{2} - z[x^{2}(r_{lc}^{2} - 1) + z\}^{1/2}/z \qquad (A83)$$

where the negative root is always taken. The positive root often yields a negative $@_{lc}$.

Then

$$R_{al} = @_{lc}/(W_{c}C_{ab}) \qquad (A84)$$

APPENDIX B: BOX VOLUME \ VENT MASS INTERACTION

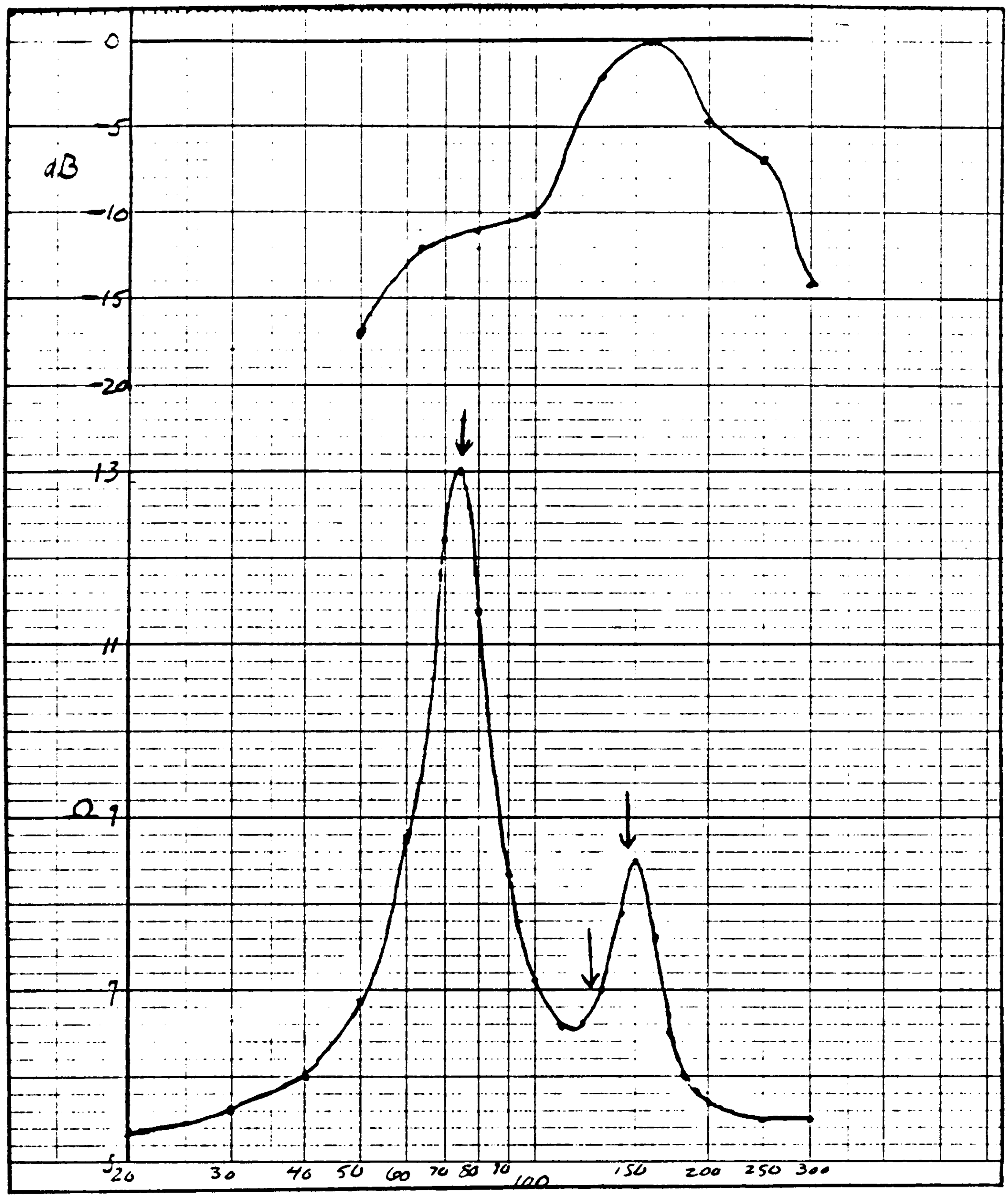
If the vent has acoustic mass M_{av}, a second system resonance will occur at

$$w_b^2 = 1/(C_{ab} M_{av}). \quad \text{(B1)}$$

It is this property which the bass reflex designs take advantage of, but for the aperiodically damped system it is a liability. The pressure relief vent is filled with dacron, open-celled foam, and/or fabric, yielding high internal damping. Air flow through the vent is not laminar, as the air must move through "tunnels" within the damping material. This greatly increases the inertia of the air trapped in the leakage vent, resulting in high acoustic mass. The result is a very low box/vent resonance which is extremely well damped.

In all the systems measured for this paper, this second resonance manifested only for leakage vents which were nearly undamped, a second hump being generated on the voice coil impedance curve. Since the measurements taken covered a full two octaves below the sealed box system resonance, and in well damped vents the second hump never occured, the evidence indicates the leakage vent has nearly infinite mass. Then for well damped vents the leakage is purely resistive, and the system aperiodically damped.

If the damping in the vent is insufficient, the system impedance curve will resemble figure 34, bottom trace, where a second hump is clearly visible. This situation occured with the 17cm system when the variovent was loaded with bonded dacron only. The frequency response is the top trace of figure 34. The scale used is one fifth the scale of figure 25. The output peak in the one third octave band centered at 130 Hz is nearly 10 dB above the midrange, and the rolloff below the upper resonance nowhere near smooth. Subjectively, the low frequecny output consisted of essentially one note with extremely poor transient response. This situation should be avoided.

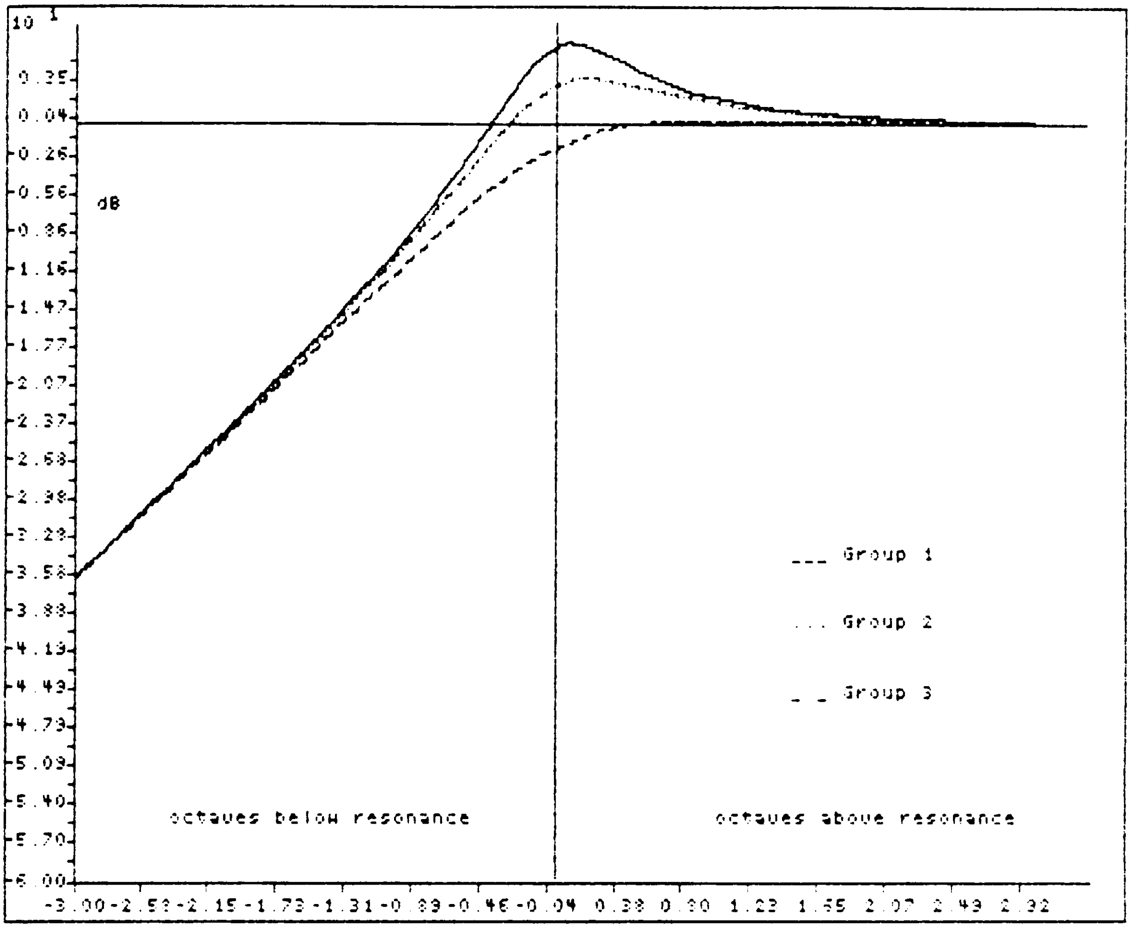


top : output(dB)
bottom: impedance

Fig. 34 Measurement of box/vent interaction due to inadequate damping in vent

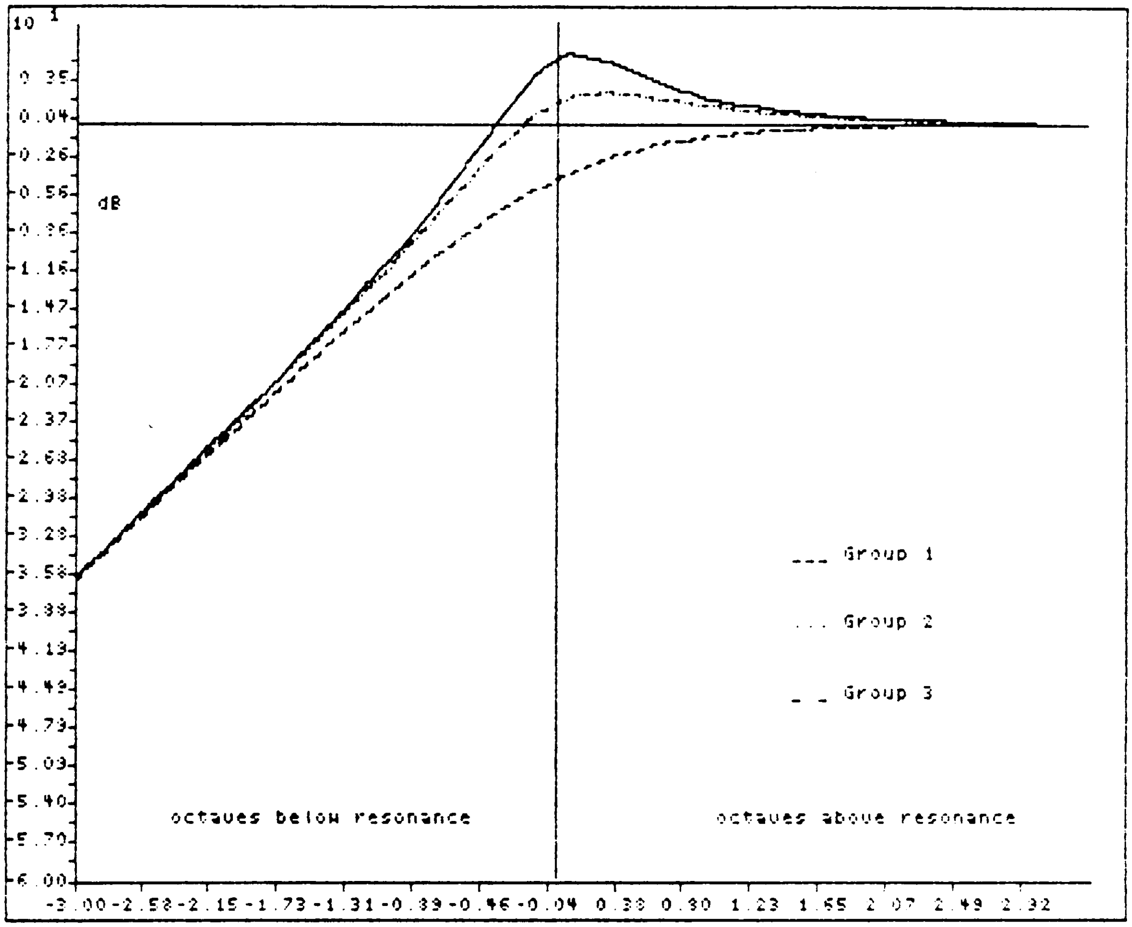
APPENDIX C: SEALED BOX SYSTEM RESPONSE

The following graphs represent the sealed box loudspeaker system frequency and transient response for the range Q_{tc} = 0.4 to 2.0.



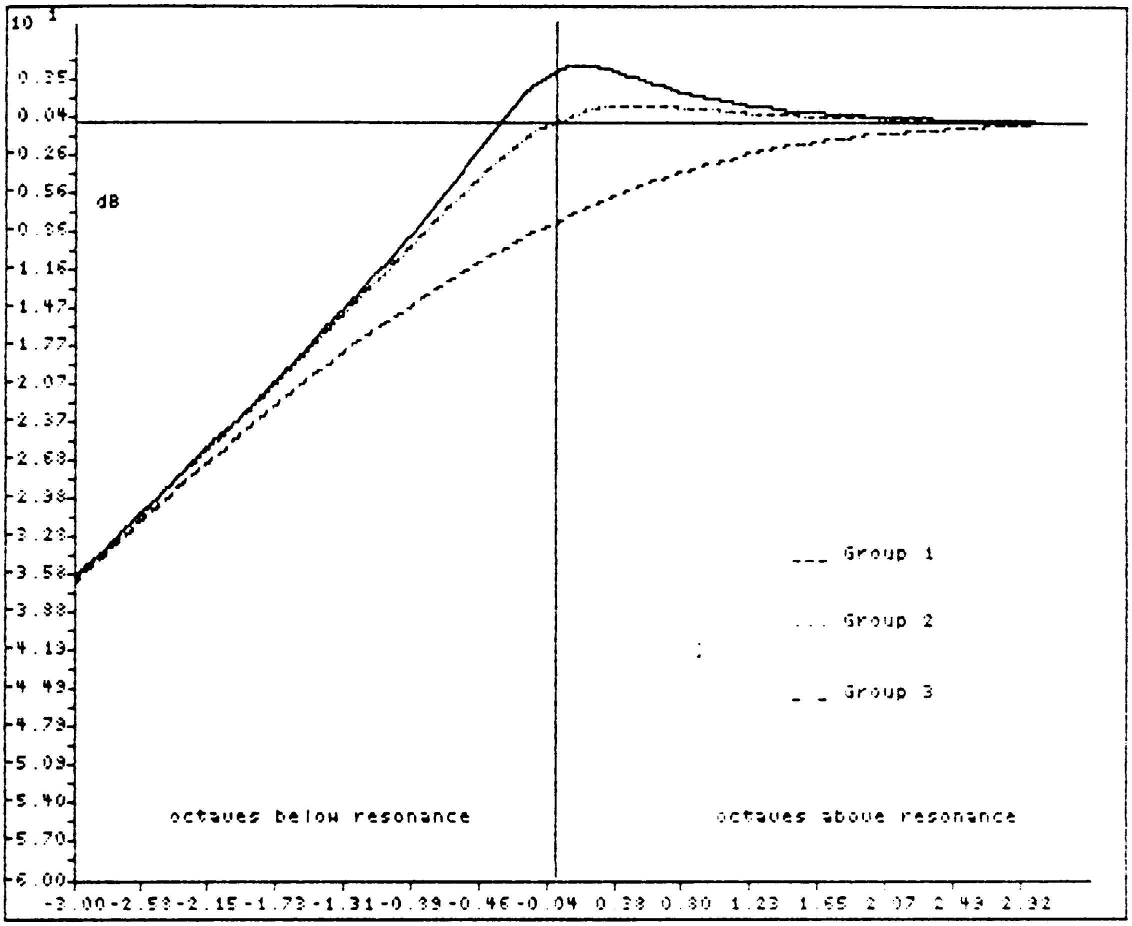
	Group 1	Group 2	Group 3
Qtc =	2.00	1.40	0.80

Fig. 35a Sealed box loudspeaker system transfer functions for Qtc = 2.00, 1.40, 0.80



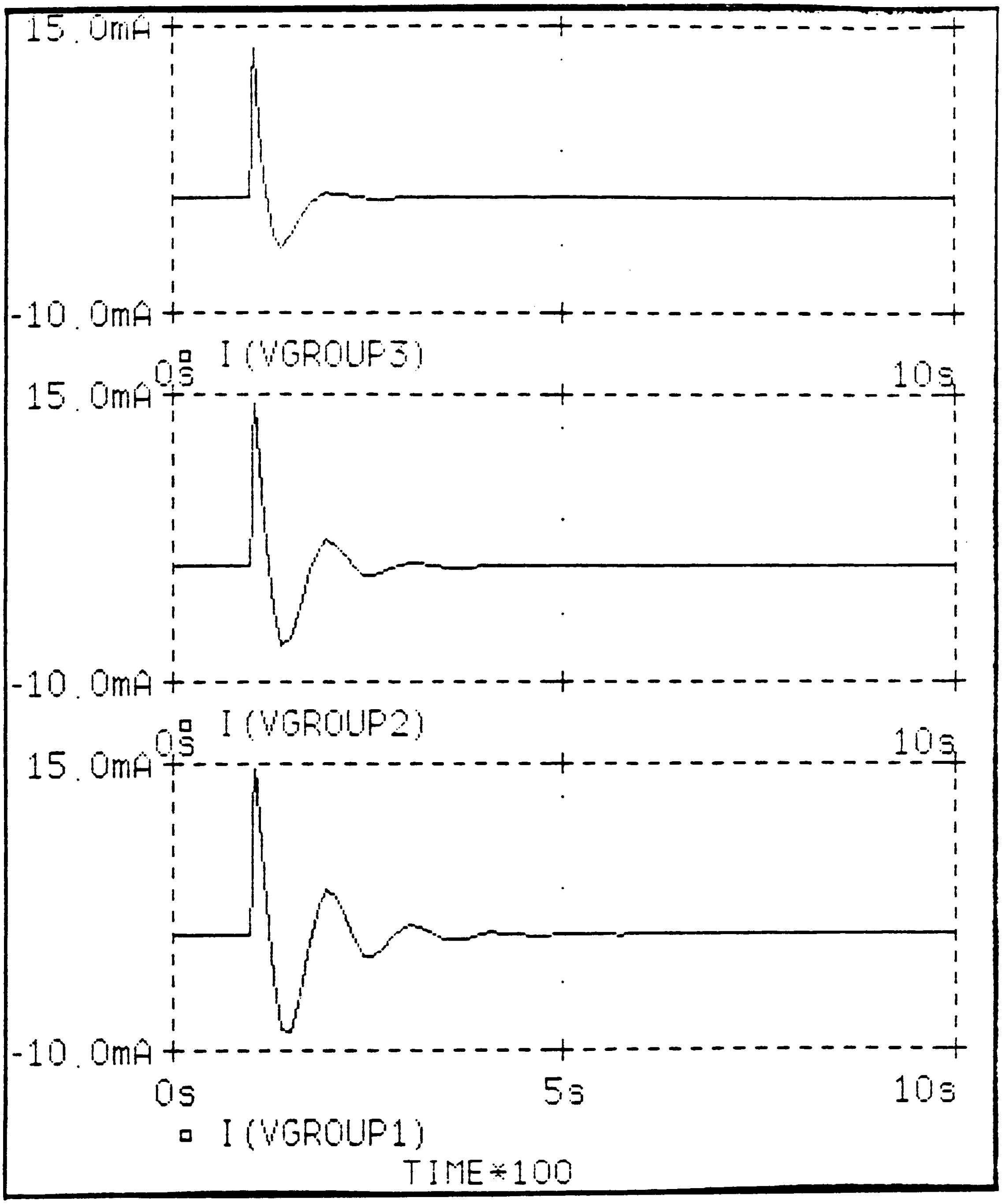
	Group 1	Group 2	Group 3
Qtc =	1.80	1.20	0.60

Fig. 35b Sealed box loudspeaker system transfer functions for Qtc = 1.80, 1.20, 0.60



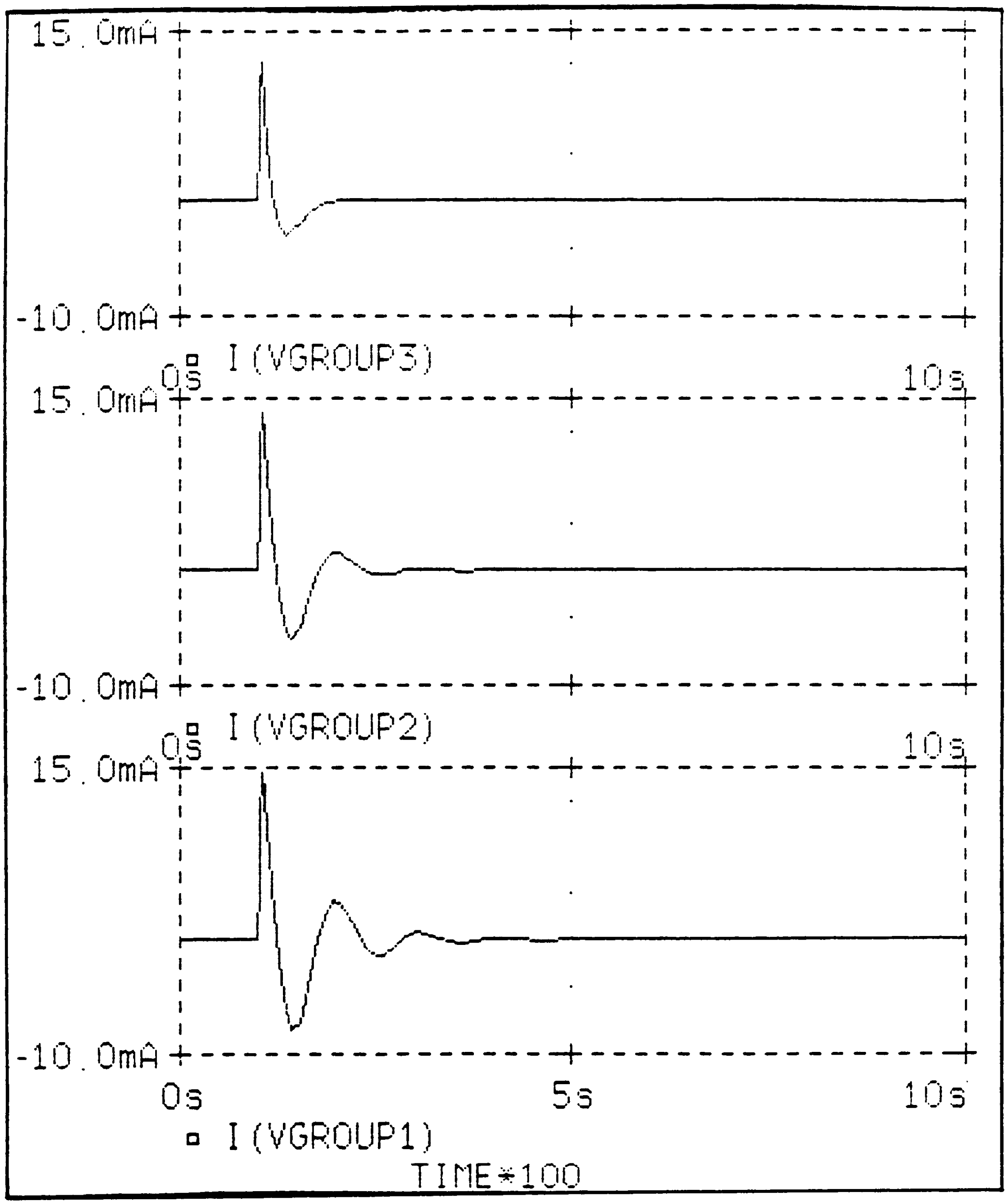
	Group 1	Group 2	Group 3
Qtc =	1.60	1.00	0.40

Fig. 35c Sealed box loudspeaker system transfer functions for Qtc = 1.60, 1.00, 0.40



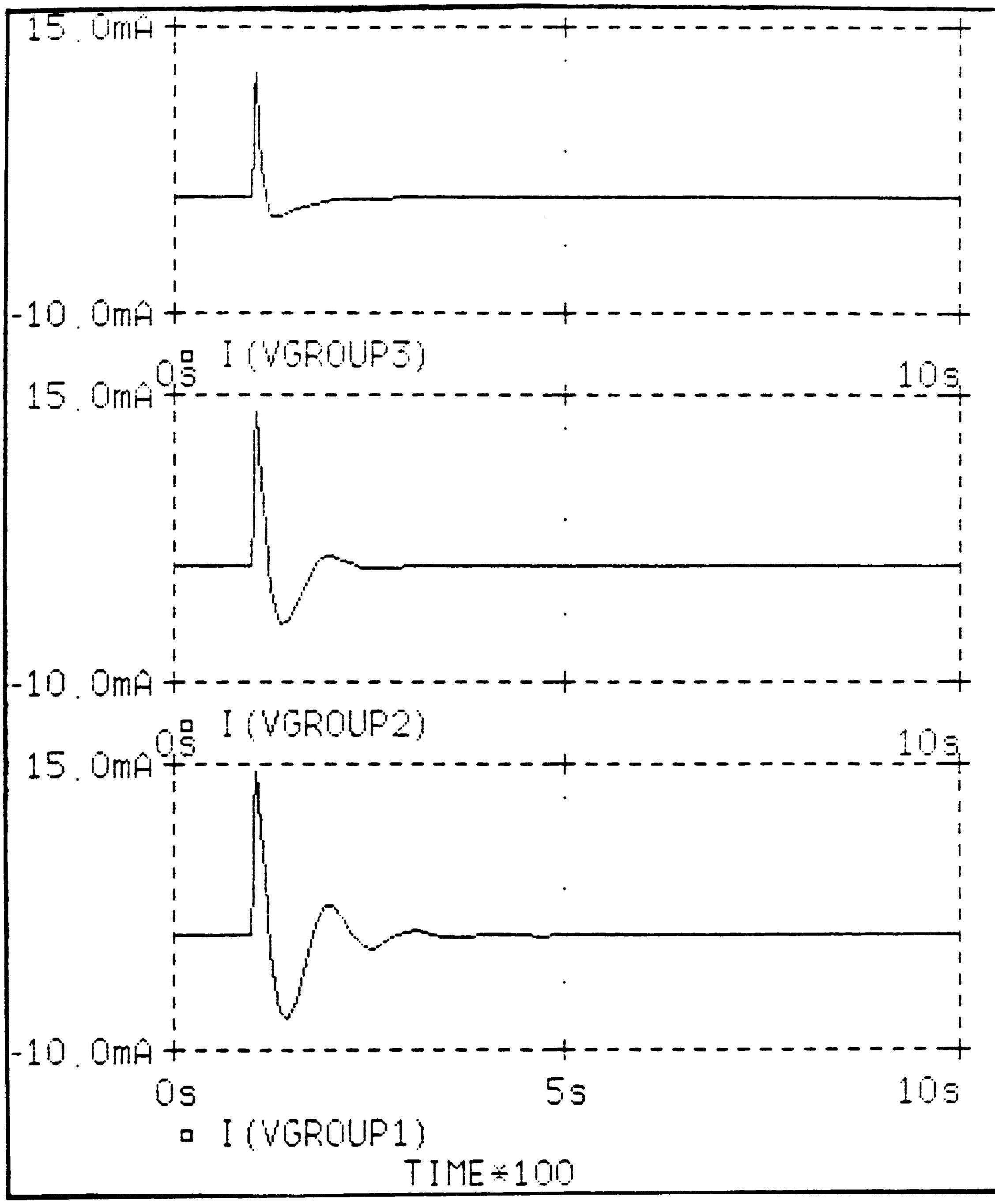
top : Q_{tc} = 0.80
middle: Q_{tc} = 1.40
bottom: Q_{tc} = 2.00

Fig. 36a Pulse response of sealed box loudspeakers



top : $Q_{tc} = 0.60$
middle: $Q_{tc} = 1.20$
bottom: $Q_{tc} = 1.80$

Fig. 36b Pulse response of sealed box loudspeakers



top : Q_{tc} = 0.40
middle: Q_{tc} = 1.00
bottom: Q_{tc} = 1.60

Fig. 36c Pulse response of sealed box loudspeakers

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