THREE-DIMENSIONAL VELOCITY EXTRACTION USING LASER DOPPLER VIBROMETRY

by

Jeffry J. Abel

This thesis is submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE in
MECHANICAL ENGINEERING

APPROVED:

L. D. Mitchell, Chairman

A. L. Wicks

R. L. West

February 1993

Blacksburg, Virginia
THREE-DIMENSIONAL VELOCITY EXTRATION USING LASER DOPPLER VIBROMETRY

by

Jeffry J. Abel

(ABSTRACT)

In the analysis of plates and beams, in-plane velocities have been assumed to be small and negligible. This was nearly an unavoidable assumption due to the fact that the in-plane velocity was near impossible to determine accurately with conventional techniques. This assumption needs to be checked experimentally. In addition, general engineering structures, such as machines, TV towers, buildings, etc., have major in-plane motions that are actually out-of-plane motions as viewed from another vantage point. These also need to be measured. Now with the use of a Laser Doppler Vibrometer the development of a method to measure three-dimensional velocities has provided the ability to measure in-plane velocities accurately. This thesis outlines the methods used for such three-dimensional extraction and gives an example of its use.

Not only is the final three-dimensional method described, but the whole process of developing the method is outlined. This will hopefully provide insight into the difficulties associated with this method as well as prevent other researchers from following similar fruitless approaches.
ACKNOWLEDGMENTS

I would like to acknowledge all the help and direction my committee members, Dr. A. L. Wicks and Dr. R. L. West, have given me throughout my work. Not only have they provided useful information with my research, they have given me a perspective beyond academics.

My deepest gratitude goes out to my advisor, Dr. L. D. Mitchell. His patience and direction was greatly appreciated to get through the many headaches and 'Mystery Problems' that occurred during my research. I would also like to recognize all the advice and guidance Dr. Mitchell has given me.

I would also like to acknowledge all the work David Montgomery has done with the Four Point Registration Method development. His work with the development of the Mathematica code in Appendix A was greatly appreciated.

I want to acknowledge the support and understanding of my parents, Jim and Dianne Abel. For they gave me the freedom and support to pursue my goals. Most of all, I want to acknowledge the support of my wife, MarySue. Her support was invaluable. She kept me going through it all.

Finally, I want to thank all my colleagues for all they have done.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>INTRODUCTION</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I.A.</td>
<td>Background</td>
<td>1</td>
</tr>
<tr>
<td>I.A.1.</td>
<td>The Problem</td>
<td>1</td>
</tr>
<tr>
<td>I.A.2.</td>
<td>Accelerometer Technology</td>
<td>2</td>
</tr>
<tr>
<td>I.A.3.</td>
<td>Laser Technology</td>
<td>4</td>
</tr>
<tr>
<td>I.B.</td>
<td>Literature Review</td>
<td>6</td>
</tr>
<tr>
<td>I.C.</td>
<td>Purpose of Research</td>
<td>10</td>
</tr>
<tr>
<td>Chapter II</td>
<td>ANALYSIS</td>
<td></td>
</tr>
<tr>
<td>II.A.</td>
<td>Introduction</td>
<td>13</td>
</tr>
<tr>
<td>II.B.</td>
<td>Laser Positioning</td>
<td>13</td>
</tr>
<tr>
<td>II.C.</td>
<td>Transformation</td>
<td>15</td>
</tr>
<tr>
<td>II.D.</td>
<td>Laser Position Velocity Error Indicator</td>
<td>18</td>
</tr>
<tr>
<td>Chapter III</td>
<td>TEST SET-UPS and EXPERIMENTAL PROCEDURE</td>
<td>22</td>
</tr>
<tr>
<td>III.A.</td>
<td>Introduction</td>
<td>22</td>
</tr>
<tr>
<td>III.B.</td>
<td>Test Set-ups</td>
<td>23</td>
</tr>
<tr>
<td>III.B.1</td>
<td>Two-Dimensional Set-ups</td>
<td>23</td>
</tr>
<tr>
<td>III.B.2</td>
<td>Final Two-Dimensional Set-up</td>
<td>35</td>
</tr>
<tr>
<td>III.B.3</td>
<td>Three-Dimensional Set-up Modifications</td>
<td>41</td>
</tr>
<tr>
<td>III.B.4</td>
<td>Final Three-Dimensional Test Set-up</td>
<td>47</td>
</tr>
<tr>
<td>Chapter IV</td>
<td>RESULTS</td>
<td></td>
</tr>
<tr>
<td>IV.A.</td>
<td>Two-Dimensional Test</td>
<td>51</td>
</tr>
<tr>
<td>IV.B.</td>
<td>Three-Dimensional Test</td>
<td>61</td>
</tr>
<tr>
<td>IV.C.</td>
<td>Three-Dimensional Confirmation</td>
<td>70</td>
</tr>
<tr>
<td>Chapter V</td>
<td>DISCUSSION of RESULTS</td>
<td>78</td>
</tr>
<tr>
<td>V.A.</td>
<td>Two-Dimensional Test</td>
<td>78</td>
</tr>
<tr>
<td>V.B.</td>
<td>Three-Dimensional Test</td>
<td>80</td>
</tr>
<tr>
<td>V.C.</td>
<td>Three-Dimensional Confirmation Test</td>
<td>81</td>
</tr>
<tr>
<td>Chapter VI</td>
<td>CONCLUSION and RECOMMENDATIONS</td>
<td>95</td>
</tr>
<tr>
<td>VI.A.</td>
<td>Conclusions</td>
<td>95</td>
</tr>
<tr>
<td>VI.B.</td>
<td>Recommendations</td>
<td>96</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
<td>98</td>
</tr>
<tr>
<td>APPENDIX A</td>
<td></td>
<td>99</td>
</tr>
<tr>
<td>APPENDIX B</td>
<td></td>
<td>102</td>
</tr>
<tr>
<td>VITA</td>
<td></td>
<td>104</td>
</tr>
</tbody>
</table>
## FIGURE LIST

<table>
<thead>
<tr>
<th>Figure number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Huffaker's Test Set-up</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>Donovan's Right Pyramid Configuration</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>Plate Dimension and Unit Vector Direction</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>Arbitrary Three-Dimensional Unit Vectors</td>
<td>17</td>
</tr>
<tr>
<td>5</td>
<td>Donovan's 'Likely Use' Configuration</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>Equipment Diagram</td>
<td>25</td>
</tr>
<tr>
<td>7</td>
<td>3-D View of Turntable Set-up</td>
<td>26</td>
</tr>
<tr>
<td>8</td>
<td>Top View of First Turntable Set-up</td>
<td>27</td>
</tr>
<tr>
<td>9</td>
<td>Turntable, Shaker Set-up with Rotated Coordinate System</td>
<td>30</td>
</tr>
<tr>
<td>10</td>
<td>Turntable, Shaker Set-up with Two Accelerometers</td>
<td>31</td>
</tr>
<tr>
<td>11</td>
<td>Turntable and Plate Set-up</td>
<td>34</td>
</tr>
<tr>
<td>12</td>
<td>Final 2-D Set-up</td>
<td>36</td>
</tr>
<tr>
<td>13</td>
<td>Two-Dimensional Laser Positions</td>
<td>37</td>
</tr>
<tr>
<td>14</td>
<td>Front View of Plate Used in 2-D Test</td>
<td>38</td>
</tr>
<tr>
<td>15</td>
<td>Alignment for X Axis</td>
<td>40</td>
</tr>
<tr>
<td>16</td>
<td>Three-Dimensional Set-up</td>
<td>42</td>
</tr>
<tr>
<td>17</td>
<td>Front View of Plate Used for 3-D Test</td>
<td>44</td>
</tr>
<tr>
<td>18</td>
<td>FRF's for Three-Dimensional Test</td>
<td>45</td>
</tr>
<tr>
<td>19</td>
<td>Final Three-Dimensional Set-up</td>
<td>49</td>
</tr>
<tr>
<td>20</td>
<td>X Magnitude Graph 2-D Test</td>
<td>52</td>
</tr>
<tr>
<td>21</td>
<td>Y Magnitude Graph 2-D Test</td>
<td>53</td>
</tr>
<tr>
<td>22</td>
<td>X Phase Graph 2-D Test</td>
<td>54</td>
</tr>
<tr>
<td>23</td>
<td>Y Phase Graph 2-D Test</td>
<td>55</td>
</tr>
<tr>
<td>24</td>
<td>X Magnitude Graph 2-D/3-D Test</td>
<td>57</td>
</tr>
<tr>
<td>25</td>
<td>Y Magnitude Graph 2-D/3-D Test</td>
<td>58</td>
</tr>
<tr>
<td>26</td>
<td>X Phase Graph 2-D/3-D Test</td>
<td>59</td>
</tr>
<tr>
<td>27</td>
<td>Y Phase Graph 2-D/3-D Test</td>
<td>60</td>
</tr>
<tr>
<td>28</td>
<td>Laser Locations in Space for 345Hz Test</td>
<td>62</td>
</tr>
<tr>
<td>29</td>
<td>X Magnitude Graph 3-D 345Hz Test</td>
<td>63</td>
</tr>
<tr>
<td>30</td>
<td>Y Magnitude Graph 3-D 345Hz Test</td>
<td>64</td>
</tr>
<tr>
<td>31</td>
<td>Z Magnitude Graph 3-D 345Hz Test</td>
<td>65</td>
</tr>
<tr>
<td>32</td>
<td>X Phase Graph 3-D 345Hz Test</td>
<td>66</td>
</tr>
<tr>
<td>33</td>
<td>Y Phase Graph 3-D 345Hz Test</td>
<td>67</td>
</tr>
<tr>
<td>34</td>
<td>Z Phase Graph 3-D 345Hz Test</td>
<td>68</td>
</tr>
<tr>
<td>35</td>
<td>Laser Locations in Space for 205Hz Test</td>
<td>71</td>
</tr>
<tr>
<td>36</td>
<td>X Magnitude Graph 3-D 205Hz Test</td>
<td>72</td>
</tr>
<tr>
<td>37</td>
<td>Y Magnitude Graph 3-D 205Hz Test</td>
<td>73</td>
</tr>
<tr>
<td>38</td>
<td>Z Magnitude Graph 3-D 205Hz Test</td>
<td>74</td>
</tr>
<tr>
<td>Figure number</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>39</td>
<td>X Phase Graph 3-D 205Hz Test</td>
<td>75</td>
</tr>
<tr>
<td>40</td>
<td>Y Phase Graph 3-D 205Hz Test</td>
<td>76</td>
</tr>
<tr>
<td>41</td>
<td>Z Phase Graph 3-D 205Hz Test</td>
<td>77</td>
</tr>
<tr>
<td>42</td>
<td>345Hz Temperature Plot</td>
<td>82</td>
</tr>
<tr>
<td>43</td>
<td>X Accelerometer Magnitude Graph 205Hz Test</td>
<td>84</td>
</tr>
<tr>
<td>44</td>
<td>Y Accelerometer Magnitude Graph 205Hz Test</td>
<td>85</td>
</tr>
<tr>
<td>45</td>
<td>Z Accelerometer Magnitude Graph 205Hz Test</td>
<td>86</td>
</tr>
<tr>
<td>46</td>
<td>X Accelerometer Magnitude Graph 345Hz Test</td>
<td>87</td>
</tr>
<tr>
<td>47</td>
<td>Y Accelerometer Magnitude Graph 345Hz Test</td>
<td>88</td>
</tr>
<tr>
<td>48</td>
<td>Z Accelerometer Magnitude Graph 345Hz Test</td>
<td>89</td>
</tr>
<tr>
<td>49</td>
<td>Accelerometer's Threshold Plots</td>
<td>90</td>
</tr>
<tr>
<td>50</td>
<td>205Hz Temperature Plot</td>
<td>94</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

I.A. Background

In this section the background of three areas will be discussed: the problem, accelerometer technology and laser technology. The main objective of this chapter is to describe the existing technology used to measure three-dimensional motion and show the problems associated with this technology. Also, a new technology will be presented and its possible application to replace current technology discussed.

I.A.1. The Problem

A problem exists in accurately determining the three-dimensional dynamic motion of a point on a structure. Currently, the most widely used and accepted method for measuring three-dimensional motion is with the use of a triaxial accelerometer array. The use of accelerometers can, however, alter the dynamics of a structure, thus the measurements would not be representative of the actual motion of the structure. The problems associated with the use of accelerometers will be discussed in the Accelerometer Technology section.
With the advancements in single-dimensional vibrometers, a method has been proposed\textsuperscript{*} to measure three-dimensional motion with lasers. Although the method has been proposed, it has not been experimentally tested or perfected.

This thesis proposes a method that uses laser Doppler techniques to, in some cases, replace current usage of accelerometers. How the laser Doppler works will be briefly discussed in the Laser Technology section.

I.A.2. Accelerometer Technology

First, accelerometers can alter the dynamics of a structure. The dynamic changes are caused by mass loading, increased local stiffness and added interface damping. These problems cannot be avoided. No matter how small and compact accelerometers become, they still add mass to the structure at the measurement position. In structures that are large and/or very heavy, the mass loading effect may be negligible. However, this is not true for all structures. For structures that have less mass or a lower effective mass at the measurement point, the accelerometer may have a very significant effect on the dynamics. The mass loading can alter both the frequency and magnitude of the resonance response. Therefore, the measured response from the accelerometer is not the same as that of the actual structure.

The increase in local stiffness and addition of interface damping are dependent on the way the accelerometers are attached to the structure and are 'more variable' than mass loading. This means that these properties change depending on whether the accelerometers are

\textsuperscript{*} Superscript corresponds to the reference listed on page 98.
attached by wax, anaerobic cement, screws or a magnet. With each of these attachment methods, there are a number of factors influencing the stiffness and damping, but the mass loading is only affected by one factor, the amount of mass added to the structure. Thus, it is more difficult to determine how much effect the change in stiffness and damping will have on the structure.

Second, all accelerometers have some degree of cross-axis sensitivity. Cross-axis sensitivity is the percent of the transverse motion that will show up in the direct, on-axis response of the accelerometer. The cross-axis sensitivity for accelerometers is generally less than five percent. This is a source of error since the accelerometer is assumed to measure along its axis only. Therefore, the cross-axis sensitivity does not cause a problem in the measurement if the accelerometer has motion only along its axis. In any three-dimensional situation, where an accelerometer triad is being used, two or all three accelerometers are moving transverse or normal to their axes. In these situations there would be error on two or three of the accelerometer signals caused by the cross-axis sensitivity. One may ask if it is possible for the cross-axis sensitivity to be removed. The cross-axis sensitivity is inherent in the design of the accelerometer and cannot be removed completely. Some accelerometers, such as the Kistler PiezoBeam, have greatly reduced the cross-axis sensitivity, but it still exists.

The problem of the dynamic changes and cross-axis sensitivity exist when a single accelerometer is used, but when an accelerometer triad is used, these problems are amplified. The mass loading is worsened by the addition of two accelerometers and a mounting block. As described earlier, cross-axis sensitivity will effect at least two of the accelerometer measurements when the triad is used. These two problems are not the only
problems accelerometers may encounter but they are problems that, with the use of the laser system, will be avoided.

1.A.3. **Laser Technology**

Laser measurement technology is not new, but its application to structures is relatively new. Laser measurements were originally applied to measurement of velocities within a fluid flow. One of the most common laser measurement systems is the dual-beam system. The dual-beam system allows for the velocity of a fluid to be measured without disturbing or interrupting the flow. Not only is the dual-beam system capable of measuring the magnitude of the velocity, but with certain configurations it can measure the two- and three-dimensional velocities within a fluid. The capability to measure multi-dimensional velocities has expanded the usefulness of the dual-beam system. There is another laser method called Laser Doppler Vibrometry. This method has made it possible to easily obtain velocity measurements from solid structures. The Doppler phenomenon occurs, in this case, when the laser light (at a given frequency) is reflected from a moving target. The light that is reflected is shifted in frequency. This is known as the Doppler effect. The velocity of the target is then proportional to the change in frequency or otherwise known as the Doppler frequency.

The dual-beam system uses a single laser and splits the beam into two. The two laser beams are then directed into the fluid such that the beams cross within the fluid. At the intersection of the two beams an interference pattern is formed. The interference pattern is a series of light and dark bands of light. To measure the velocity of the fluid, the fluid is seeded with particles. When a particle passes through the interference pattern, it is
illuminated in a regular pattern as it passes through the bright areas. The illuminations of the particle are detected and the frequency of illumination is determined. With the spacing of the bright areas in the interference pattern known, the velocity is proportional to the observed frequency.

The Laser Doppler Vibrometer systems works in a different method. With this system, the laser beam is split into two beams. One is stored in the laser head (the reference beam) and the other is directed towards the target structure. When the laser beam comes in contact with a surface, it is reflected off of the surface at a shifted frequency. This signal is then recombined with the reference signal that was stored in the laser head. The resulting signal is a modulated signal. This signal is now proportional to the velocity of the structure.

The laser Doppler system used here was produced by Ometron, Inc. (London, England). This system is a reference-beam or laser Doppler Vibrometer system. This means that the laser and the laser beam detectors are in the same unit. This system measures velocities in its 'line-of-sight'. This means that the velocity measured is the velocity along the laser beam.

The Ometron laser system has a pair of mirrors within the laser head. The mirrors allow the laser beam to be directed to various points without having to physically move the laser. The mirrors are connected to and controlled by galvanometers, which are controlled electronically and are monitored by rotary angular transducers. The mirrors and the ability to position and monitor them accurately are an integral part of the laser system.
The laser eliminates the problems that the accelerometers encounter. The laser system eliminates mass loading by being a non-contacting measurement system. In other words, no part of the measurement system touches the structure during any part of the measurement process. This is significant because the original structure (system) will remain undisturbed while the measurements occur. Thus, the response that is measured is the actual response from the structure. There are no mass loading, local stiffening or local damping effects.

The laser system does not encounter cross-axis sensitivity because the laser does not physically touch or vibrate with the structure to measure its dynamics. The laser measures that component of the velocity that it can 'see'. The laser can only measure one-dimensional velocities. Therefore, the velocity component the laser can 'see' is the component of a general three-dimensional structural velocity that is in the direction of the laser beam. The measurement is not affected by the presence of motion normal to the laser beam. Therefore, the laser measurement has a zero cross axis sensitivity.

I.B. Literature Review

A literature search was performed to find information that would relate to the use of lasers for the measurement of three-dimensional velocities. There was not a lot of information on the use of a reference-beam system in measuring three-dimensional structural velocities. There was a great deal of information gathered on three-dimensional velocities in fluids. The methods varied greatly with the type of laser system and its experimental methods. Although, many of the methods and techniques worked in fluids, they were not readily applicable to the use with structures.
Although the majority of the three-dimensional velocity systems that measure fluids have claimed to be successful, some of the systems have difficulty producing good results. J.F. Meyers performed a comparison between a reference beam set-up and various fringe methods. The fringe methods required that the fluid be seeded with particles. These particles are what is actually being measured but because the particles are moving the same as the fluid, the velocity of the fluid was being measured. When the fringe method was considered for use on structures, it could be seen that there were many problems with trying to convert the method. The largest problem was that a particle had to move through the fringe area to be able to determine its velocity. This was done by having the particle becoming illuminated and darkened as it passed through the fringe. If this method would be used on a structure, a surface would be moving through the fringe. It would become increasingly difficult to measure the illuminations at the surface. One reason is because the light from the lasers could cause a blooming effect so that the fringe pattern could not be seen. Therefore, the fringe method would be difficult to use on structures.

Another three-dimensional method was proposed by J.H. Churnside and H.T. Yura. They used a single laser and multiple detectors to get the three-dimensional velocities. The target that was being measured was a moving belt. This method appeared to be able to work. Thus, this was considered to be a possible method. With further investigation it was determined that the method would be difficult to expand to include a scanning capability.

L.Z. Kennedy and J.W. Bilbro had proposed a totally different method. Their work centered on the two-dimensional measurement of wind velocity. A single-dimension Laser
Doppler Velocimeter was used in this method to get one velocity measurement, and a backscatter Doppler method was used to get the second velocity measurement. This worked by focusing the laser beam such that it was 1 cm in diameter in the target region. The seed particles could then be tracked as they traverse this region, and their velocity could be calculated. This method was not pursued due to the fact that a particle and not a surface could be measured in the transverse direction. Also, only two directions could be measured and a third could not be easily included.

One of the few papers found on measuring structures was by J.A. Caeo, J.R. Rieker, M.W. Trehewey and H.J. Sommer III. They were able to measure three degrees of freedom, DOF, but not the three translational DOF. They measured one translational and two rotational DOF. They also had a laser system that measured the displacement of the reflected laser beam instead of measuring the velocity at a point.

R. M. Huffacker had a paper that provided insight into a possible method. Huffacker used a laser Doppler system to measure the three-dimensional velocity of a gas. To measure the velocity he had one laser shooting into the gas and three laser detectors measuring the velocities. His set-up is shown in Fig. 1. The velocities were then transformed into an orthogonal coordinate system. The detectors were mounted in a very specific and precise manner. Each had an effective separation angle of half of the 8° 33' actual separation angle with respect to the laser beam, and were spaced at 120° increments around the laser beam. His method would not directly work on a structure. This was because his method depends upon the transparency of the gas. Most structures are opaque, thus blocking the laser light.
J.B. Donovan\textsuperscript{1} modified Huffaker's method for use on structures. Donovan used an Ometron laser system, very similar to the one described previously. His approach used one laser that took all three velocity measurements. The laser had to be physically moved to specifically defined locations separately. The locations were based on Huffaker's detector orientation to locate the laser positions in space, but the positions were not located around the laser beam as in Huffaker's test, but were located around the $Z$ axis of the reference coordinate system. See Fig. 2. In Donovan's work the $Z$ axis was the outward normal of the structure to be measured. This orientation is referred to as the right pyramid configuration. Donovan's work was completely analytical. He showed the transformation and sensitivity analysis, but did not show any experimental results.

It should be noted that the reference beam system Meyers\textsuperscript{4} used was much like Huffaker's\textsuperscript{8}. Meyers acknowledged the need for very precise positioning and that the accuracy of the positioning was difficult to achieve. Thus, the difficulty in accurately positioning the laser has been recognized.

I.C. Purpose of Research

J.B. Donovan\textsuperscript{8} has defined an analytical method for determining three-dimensional velocities with a series of single-dimensional laser system. As of yet, his method has not been tested in the lab. Therefore, the purpose of this research is to develop a practical and usable method to extract three-dimensional velocities from a structure using Donovan's basic concepts. The work reported here will take on many aspects. The first goal is to modify Donovan's\textsuperscript{1} method so that it can be easily applied to laboratory and industrial
measurement problems. The second goal is to test the procedure to determine the accuracy of the method. The third goal is to quantify the accuracy of the procedure.

This research will be primarily experimental. With the experimental work, it will be attempted to characterize the ability of the laser system to produce and reproduce accurate data.

This thesis will not only include the successful approach, but will also contain many of the attempts made before developing the final approach. This will hopefully provide insight into the difficulties associated with this method as well as to prevent other researchers from following similar fruitless approaches.
CHAPTER II

ANALYSIS

II.A. Introduction

The information in the next three sections are primarily regarding the three-dimensional velocity extraction tests. Each section is written for the three-dimensional test, and the two-dimensional application is not discussed. It is assumed that the two-dimensional cases can be understood from the three-dimensional descriptions.

II.B. Laser Positioning

The positioning of the laser in space is integral in the method of being able to determine the three-dimensional velocity. The laser position must be determined as accurately as possible to reduce the possibility of position errors in the transformation algorithm. The main objective of determining the laser position is to identify the unit vector that points in the direction of the laser beam. The unit vector points from the structure to the laser head. See Fig. 3. The unit vector is expressed in the reference coordinate system.

To locate the laser in space, a computer algorithm called the Four Point Registration Method is used. The algorithm was written by David Montgomery in Mathematica and is listed in Appendix A. This method requires that the structure have four points marked with their exact coordinates known with respect to the structure's reference coordinate system. See Fig. 3. These four points are called registration points. At each of the three
Figure 3 – Plate Dimension and Unit Vector Direction
laser positions (A, B, and C) the laser beam needs to be moved to the first registration point. This is not done by moving the laser itself, but instead by electronically moving the two internal scanning mirrors that steer the laser beam. Once the laser beam is directly on the registration point, the two scanning mirror angles, $\Theta_X$ and $\Theta_Y$, are recorded. This process is repeated to get the scan angle pairs for the other three registration points. With the four pairs of scan angles, the laser's coordinate system can be located in space, but another angle pair is needed to get the unit vector. This is the scan angle pair that points to the target position where the data is to be taken. Therefore, at each new laser position, there are five scan angle pairs required. The scan angle pairs will be used to calculate a unit vector from the target position to the laser origin. In other words, the unit vector is in the same direction as the laser beam. The unit vector is the final result from the Four Point Registration Method that is listed in Appendix A.

To use the program in Appendix A, the following information is needed: the coordinates of the registration points, the scan angle pair that corresponds to each registration point and the scan angle pair for the target position. In the program, all of the previous data is stored in data files. Once all the data is inputted through the data files, the program will run and the unit vector, as defined in the structure's coordinate system, will be output. This unit vector can then be directly placed into the transformation equation eq.(1) which will be discussed in the following section.

II.C. Transformation

The velocity transformation converts the three laser measured velocities into three orthogonal velocities. The velocity transformation is derived and discussed in depth by
Donovan. In short, the transformation works by converting a triad of velocities as measured in a non-orthogonal coordinate system to a set of velocities in an orthogonal system. There are two coordinate systems used in this work: laser and structure. In this thesis, the reference coordinate system will imply the structure's reference coordinate system. The laser coordinate system will be referred to explicitly. The laser measures the velocity of the target position in a non-orthogonal coordinate system, which is defined by the arbitrary placement of the laser in the three positions. The arbitrary unit vectors that define the non-orthogonal system are shown in Fig. 4. For the transformation, the velocity and the unit vectors are needed at each laser position. With this information the transformation can be performed to calculate the components of the three-dimensional velocity in the reference coordinate system. The transformation is shown below.

\[
\begin{bmatrix}
V_x \\
V_y \\
V_z
\end{bmatrix}_{\text{Ref}} = \begin{bmatrix}
R_{Ax} & R_{Ay} & R_{Az} \\
R_{Bx} & R_{By} & R_{Bz} \\
R_{Cx} & R_{Cy} & R_{Cz}
\end{bmatrix}^{-1}
\begin{bmatrix}
V_A \\
V_B \\
V_C
\end{bmatrix}_{\text{Laser}}
\]  

(1)

where

\( V_x \) - X component of the transformed velocity
\( V_y \) - Y component of the transformed velocity
\( V_z \) - Z component of the transformed velocity

and

\( R_{Ax}, R_{Ay}, R_{Az} \) - Components of the unit vector for position A
\( R_{Bx}, R_{By}, R_{Bz} \) - Components of the unit vector for position B
\( R_{Cx}, R_{Cy}, R_{Cz} \) - Components of the unit vector for position C

and

\( V_A \) - Laser velocity as seen from position A
\( V_B \) - Laser velocity as seen from position B
\( V_C \) - Laser velocity as seen from position C
Figure 4 – Arbitrary Three-Dimensional Unit Vectors
It should be noted that all of the velocities \( (V_x, V_y, V_z, V_A, V_B, \text{ and } V_C) \) are complex numbers. This is to preserve the magnitude and phase information. The program used to transform all the data is listed in Appendix B. This program was also written in Mathematica.

II.D. **Laser Position Velocity Error Indicator**

One of the main modifications made to Dorovan's method is to allow the user to use any three non-coplanar, with respect to the target point, laser positions to take data. Although this allows the user greater flexibility and freedom in laser placement, it can cause problems in the transformed velocities. There are relative laser positions that can potentially amplify small errors in the measured velocities when taken together and run through the transformation. In general, widely separated laser positions A, B and C are best but this descriptor is too qualitative to be very useful. To be able to determine if the three laser positions used can potentially cause the large amplifications an indicator was developed. This indicator is called the laser position velocity error indicator, but will be considered as the potential amplification indicator or index for short. This index is a measure of the potential for measurement errors to be amplified through the transformation process. The potential amplification index is not an indicator of data quality, but is a measure of the potential for amplification of any measurement errors in \( V_A, V_B, \text{ and } V_C \) to even larger errors in the transformed velocities, \( V_x, V_y, \text{ and } V_z \). A potential amplification index would indicate a high potential for erroneous results. The word potential is used because if the velocities were measured perfectly by the laser in all positions, A, B, and C, then the transformation results will be perfect even in the presence of a high potential amplification index. The derivation is as follows.
The derivation of the potential amplification index starts with the transformation equations.

\[
\begin{bmatrix}
V_x \\
V_y \\
V_z
\end{bmatrix} =
\begin{bmatrix}
R_{by} R_{cz} - R_{bz} R_{cy} & R_{Az} R_{cz} - R_{Ay} R_{cy} & R_{Ay} R_{bz} - R_{Az} R_{by} \\
R_{Bz} R_{Cz} - R_{Bx} R_{Cy} & R_{Az} R_{Cz} - R_{Ay} R_{Cy} & R_{Ay} R_{Bz} - R_{Az} R_{Bx} \\
R_{Bz} R_{Cz} - R_{Bx} R_{Cy} & R_{Az} R_{Cz} - R_{Ay} R_{Cy} & R_{Ay} R_{Bz} - R_{Az} R_{Bx}
\end{bmatrix}
\begin{bmatrix}
V_A \\
V_B \\
V_C
\end{bmatrix}
\] (2)

The denominator in eq.(2) will be referred to as DEN. On each of the equations, we assume a potential error on each of the velocity measurements. For simplicity, only the $V_x$ will be expanded here.

\[
V_x \pm \Delta V_x = \frac{1}{\text{DEN}} \left[ \left( R_{by} R_{cz} - R_{bz} R_{cy} \right) (V_A \pm \Delta V_A) + \left( R_{Az} R_{cz} - R_{Ay} R_{cy} \right) (V_B \pm \Delta V_B) \right. \\
+ \left. \left( R_{Ay} R_{bz} - R_{Az} R_{by} \right) (V_C \pm \Delta V_C) \right]
\] (3)

Using a Taylor's Series Expansion for all three equations generates an error descriptor for each orthogonal velocity prediction\(^9\).

\[
V_x \pm \Delta V_x = V_x \pm \Delta V_A \left( \frac{R_{by} R_{cz} - R_{bz} R_{cy}}{\text{DEN}} \right) \pm \Delta V_B \left( \frac{R_{Az} R_{cz} - R_{Ay} R_{cy}}{\text{DEN}} \right) \\
+ \Delta V_C \left( \frac{R_{Ay} R_{bz} - R_{Az} R_{by}}{\text{DEN}} \right)
\] (4)
The absolute maximum error can now be expressed.

\[
|\Delta V_x| = |\Delta V_A\left(\frac{R_{By}R_{Cz} - R_{Bz}R_{Cy}}{DEN}\right) + |\Delta V_B\left(\frac{R_{Ay}R_{Cy} - R_{Ay}R_{Cz}}{DEN}\right) + |\Delta V_C\left(\frac{R_{Ay}R_{Bz} - R_{Az}R_{By}}{DEN}\right) | \]

(5)

To get a normalized potential amplification index, the potential errors are assumed as \(\Delta V_A = \Delta V_B = \Delta V_C = 1\). Now \(|\Delta V_x|\) becomes the potential amplification index under this normalization function.

\[
PAI_{Vx} = \sqrt{\left( \frac{R_{By}R_{Cz} - R_{Bz}R_{Cy}}{DEN} \right)^2 + \left( \frac{R_{Ay}R_{Cy} - R_{Ay}R_{Cz}}{DEN} \right)^2 + \left( \frac{R_{Ay}R_{Bz} - R_{Az}R_{By}}{DEN} \right)^2} 
\]

(6)

It should be noted that the terms in eq.(6) are the same as the corresponding row in the transformation matrix, eq.(2). This means that the potential amplification index is actually the RMS value of each row in the transformation matrix.

The potential amplification index can be thought of as a relative error amplification factor. This simply means that an error of 1 is assumed on each of the laser measurements and the potential amplification index shows how much that unit error, in the worst case, will be amplified into the resulting velocity components. The index cannot predict if there will be errors, but can give a relative measure of how much the measurement errors will be amplified. In some cases, the measurement error on each laser velocity measurement \((A, B, C)\) are not equal. In these cases it is helpful to look at the coefficient terms in eq.(5). These are the amplifications of each laser position velocity \((V_A, V_B \text{ and } V_C)\) as it affects the
particular reference velocity ($V_x$, $V_y$ or $V_z$). Through this observation we can see which laser measurement affects the particular transformed velocity in what way. These coefficient terms in eq. (5) can be thought of as individual potential amplification indices. They can be very helpful in determining if a certain laser position (A, B, or C) has a higher than normal measurement error in the velocity.

The potential amplification index is only activated when there is measurement error present in one or all of the $V_a$, $V_b$, and $V_c$ measurements. If there is no measurement error present, any three non-coplanar positions can be used. Practically, however, there will exist some measurement error. Sometimes it may be negligible, but other times it may be higher or become higher after transformation amplification. Therefore, the potential amplification index can be used to determine if the three laser positions have high or low potential for measurement error propagation. If the potential is high it may be desired to choose other positions that may lower the potential. In general, the potential will be lower the closer the three non-coplanar laser positions (A, B, C) are to being an orthogonal triad. This condition is seldom realized in practical applications. Thus, there is a strong need for the potential amplification index to be used.
CHAPTER III

TEST SET-UPS and EXPERIMENTAL PROCEDURE

III.A. Introduction

In this section two-dimensional and three-dimensional set-ups and experiments will be discussed. Two- and three-dimensional tests were planned so that difficulties could be progressively faced to eliminate problems in an effective manner. Thus, a two-dimensional test was designed and modified until a stable test structure was found. This gave a good indication as to where problems could arise and provided enough information to be able to design an effective three-dimensional test.

The objective of the various tests shown here is to be able to determine a component axis velocity within +/- 5%. The % error is defined as (transformed velocity - reference velocity)/reference velocity. This is calculated for each of the component axis velocities ($V_x$, $V_y$, $V_z$) and not for the overall vector quantity. This is because it is desired to determine the component velocities accurately and no matter how small they are with respect to the absolute velocity magnitude. The phase should be less than 2°.

In the search for a suitable test set-up, it was found that Huffaker's' set-up (Fig. 1) would be possible. Another possible set-up is one that was proposed by J.B. Donovan which stemmed from Huffaker's work. Donovan actually proposed two different set-ups, but they both work the same way with different variables used to locate the laser beams. The first set-up is shown in Fig. 2. This is considered the Right-Pyramid configuration. The
other is shown in Fig. 5. He refers to this as a 'Likely Use' configuration. Both Huffaker's and Donovan's methods allow for a single point on a structure to be measured. Huffaker's and Donovan's Right-Pyramid configuration use a separation angle from a central axis to locate the laser, this will be modified to be used in the two-dimensional tests.

III.B. Test Set-ups

For all of the experimental set-ups, the basic test analysis equipment remains the same. The equipment is shown in Fig. 6. A Zonic 6080 FFT and a Zonic 6081Z are used to collect and manipulate all the data. All the accelerometers and the force transducer run to PCB ICP (Integrated Circuit Power) amplifiers and then to the Zonic FFT system. The excitation signal is generated by a Hewlett-Packard 3324A Synthesized Signal/Sweep Generator. The signal is amplified by a Harman/Kardon hk770 power amplifier and is then connected to a Ling Shaker. The Hewlett Packard 340 workstation is connected to the laser controller to control the laser mirrors. The laser velocity and the laser Doppler signal are monitored on a Tektronix 2214 oscilloscope. The velocity signal is then run to the Zonic FFT system. These components are used throughout all of the test set-ups. All the components are not necessarily used simultaneously.

III.B.1 Two-Dimensional Test Set-ups

The first two-dimensional test set-up used a shaker, a machinist's turntable, one accelerometer and a fixed laser. The set-up is shown in Figs. 7 and 8. The accelerometer was mounted directly to a shaker and oriented along its axis. The shaker was then mounted to the turntable such that the very top-center of the accelerometer was aligned
Figure 6 – Equipment Diagram
Figure 7 – 3-D View of Turntable Set-up
Figure 8 – Top View of First Turntable Set-up
with the axis of rotation for the turntable. Also, the shaker's primary axis is mounted parallel with the turntable top. The point to be measured, the target position, is the very center of the top of the accelerometer. The laser was then mounted and aimed at the target point with the axis of the shaker and the laser beam being coincident.

With the use of the turntable it is very easy to get separation angles accurate to within 0.25°. To get the desired separation angle the turntable is rotated to the desired position and data can be taken. This procedure proved to be very quick and easy.

To run the test, the turntable was placed in the 0° position and then the laser was placed such that the laser beam was pointed directly down the axis of the accelerometer. Once positioned, the laser was not moved again for the remainder of the test. Next, the shaker was driven at a single frequency for the duration of the test. During the test, the turntable was rotated to 10 different angular positions (-45°, -38°, -30°, -20°, -10°, 10°, 20°, 30°, 38° and 45°) where the velocity was measured. During the test the reference velocities were measured at 0° and +/-90°.

To transform the velocities, only five combinations are used. The combinations are +/-45°, +/-38°, +/-30°, +/-20° and +/-10°. The results for the Y velocities were very good. The Y component direction magnitude error was well under 1% and the phase error was well below 1°. The X velocities did not turn out so well. The first problem was getting a reference velocity. To get the Y reference velocity, the turntable was rotated to 0° and the velocity recorded. To get the X reference velocity, the turntable was rotated +90° and the velocity recorded. This is where a problem existed because when the turntable was rotated -90° the velocity was 20% to 40% different from the +90° velocity. The phase
difference of $180^\circ$ was expected, but the magnitude was changing. Even if only the transformed velocities are considered, without comparing them to the reference velocities, the transformed velocities in the X direction varied 10% to 20% with respect to the maximum X velocity and had a $2.5^\circ$ phase difference. This would seem to show that there was a problem. It should be noted that the magnitude of the X velocity was approximately 1% of the Y velocity.

It is believed that the difference in magnitude between the X and Y velocities could be causing the variation in the transformed velocity data. To check this, the reference coordinate system was rotated $45^\circ$ as shown in Fig. 9. The rotated coordinate system makes both the reference velocities roughly equal in magnitude. The test was then run in the same manner as the previous test. This test showed remarkable improvement in the results. The magnitude of the transformed reference velocities had errors less than 1% and phase differences less than $1^\circ$. Even though the test had made an improvement in the results, the problem of the variation in the transformed and reference velocities from the last test had not been solved. With the reference velocities being much closer in magnitude, the changing transverse motion of the accelerometer is very small in comparison. Therefore, the error caused by the non-stationary behavior of the test set-up is negligible when measured in this configuration. Although the percent error has been reduced by rotating the coordinate system, the source of the error has not been corrected but the error is now a much smaller percent of the reference velocity.

To find the source of the variation, a second accelerometer was attached to the shaker as shown in Fig. 10. The new accelerometer was a Kistler PiezoBeam. Its cross-axis sensitivity was claimed to be <1% and would allow the very small velocities in the X
Figure 9 – Turntable, Shaker Set-up with Rotated Coordinate System
Laser Area of Movement

Y

X

Accelrometer

Mounting Block

Shaker

Accelrometer (PiezoBeam)

Turntable

Figure 10 – Turntable, Shaker Set-up with Two Accelerometers
direction to be measured in the presence of a large Y direction velocity. The measurements from the accelerometers are for relative comparison as a function of time only. They are not used as the reference velocities. With the new set-up, the turntable was rotated at given increments through the 180° and accelerometer readings are taken at each position. The accelerometer readings are integrated to get velocity. The results are shown in Table 1. It was found that the magnitude of the velocities, from the Kistler accelerometer, were fluctuating as the turntable was rotated. When the same test was repeated, the Kistler repeated the same trend. This means that at a given angular position on the turntable the readings are always high or low. It was shown that when the test was run again that the trend was repeatable. The magnitude was not repeatable, but did follow the trend. This would seem to suggest that the shaker was influenced as it rotated.

Table 1 - Kistler Accelerometer Variations for Rotation of the Shaker on a Rotary Turntable

<table>
<thead>
<tr>
<th>Table Angle</th>
<th>Velocity in/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>-90°</td>
<td>1.65x10⁻³</td>
</tr>
<tr>
<td>-80°</td>
<td>1.66x10⁻³</td>
</tr>
<tr>
<td>-70°</td>
<td>1.62x10⁻³</td>
</tr>
<tr>
<td>-60°</td>
<td>1.77x10⁻³</td>
</tr>
<tr>
<td>-50°</td>
<td>2.04x10⁻³</td>
</tr>
<tr>
<td>-40°</td>
<td>2.60x10⁻³</td>
</tr>
<tr>
<td>-30°</td>
<td>2.05x10⁻³</td>
</tr>
<tr>
<td>-20°</td>
<td>2.20x10⁻³</td>
</tr>
<tr>
<td>-10°</td>
<td>2.37x10⁻³</td>
</tr>
<tr>
<td>0°</td>
<td>2.52x10⁻³</td>
</tr>
<tr>
<td>+10°</td>
<td>2.37x10⁻³</td>
</tr>
<tr>
<td>+20°</td>
<td>2.40x10⁻³</td>
</tr>
<tr>
<td>+30°</td>
<td>2.13x10⁻³</td>
</tr>
<tr>
<td>+40°</td>
<td>1.82x10⁻³</td>
</tr>
<tr>
<td>+50°</td>
<td>1.81x10⁻³</td>
</tr>
<tr>
<td>+60°</td>
<td>1.85x10⁻³</td>
</tr>
<tr>
<td>+70°</td>
<td>1.69x10⁻³</td>
</tr>
<tr>
<td>+80°</td>
<td>1.61x10⁻³</td>
</tr>
<tr>
<td>+90°</td>
<td>1.51x10⁻³</td>
</tr>
</tbody>
</table>
Because the shaker seemed to change its transverse dynamics as it was rotated on the turntable, a different test structure was sought. The new test structure will also use the turntable to get the accurate separation angles. The new structure is an 1/8th inch thick steel plate mounted vertically as shown in Fig. 11. The plate is mounted on the turntable with the accelerometers being attached at the center by the use of wax. The accelerometers were placed at the center initially for convenience, but it was desired that the point would produce approximately the same velocity for each accelerometer. The plate will be excited by a suspended shaker attached in the upper corner. The shaker was suspended so that it can be moved as the plate was rotated such that there was no moment entered into the plate. Also it allowed for the force to remain constant as the plate rotated. No force transducer was used.

Initially, with this new set-up, the turntable would be rotated at 10° increments and the accelerometers would be monitored to check the stability for the structure as it was rotated. The accelerometer's signal would be integrated to get velocity. The turntable top will be clamped to the base to remove any play from the rotation gearing. The results were similar to those found with the shaker mounted directly on the turntable. As the turntable was rotated the X accelerometer's velocity varied by 10% in magnitude. The Y accelerometer's velocity varied by 1%. The X accelerometer velocity was approximately 10% of the Y accelerometer velocity. Thus, each varied about the same in absolute velocity terms.

Through further investigation it was determined that there are two definite positions that produce low X velocity readings and two positions that produce high X velocity readings. Various clamping schemes were used but the variations could not be eliminated. Without
Figure 11 - Turntable and Plate Set-up
being able to eliminate the variation the test would and could not be successful. To modify the set-up again the whole turntable was removed. This lead to the final two-dimensional set-up which will be described in detail in the following section.

III.B.2 Final Two-Dimensional Set-up

The two-dimensional test uses the 1/8th inch thick steel plate mounted vertically to a test bed. See Fig. 12. The accelerometers are mounted in the center of the plate. The plate has four registration points marked on it in known locations and is excited by a suspended shaker attached to the upper left hand corner of the plate. To determine the excitation frequency, the FRF's from both of the accelerometers are analyzed. A frequency that has approximately the same magnitude on both FRF's would be a suitable frequency. Unfortunately, the FRF's were unable to be printed. For this test 45Hz was chosen. The target point was the point to be measured and, in this case, it was the center of the Y accelerometer top.

The test was started by getting the X and Y reference velocities. The laser will be moved to four different coplanar angular positions where data will be taken, two positions on both sides of the Y axis. The final locations of the laser positions are shown in Fig. 13. The reference velocities that will be used for comparison with the transformed velocities will be taken with the laser. To get the reference velocities, the laser must be aligned with each axis. The X and Y are both done in different manners. To get alignment with the Y axis, the laser is put in its home position and pointed at the target position. The laser beam is then scanned up and down then left and right to the corresponding registration points. Each registration point is at the same distance from the target point. See Fig. 14.
Figure 12 – Final 2-D Set-up
Figure 13 – Two-Dimensional Laser Positions
Figure 14 - Front View of Plate Used in 2-D Test
If the scan angle is the same up as it is down, then the laser is aligned vertically with the target point. The laser is aligned horizontally when the left and right scan angles are equal to the horizontal registration points. With the alignment for the Y axis complete, the velocity measurement was taken from the laser as the Y reference velocity.

The X axis alignment needs a completely different approach since the laser is looking at the edge of the plate. See Fig. 15. To get the alignment, the Y accelerometer had a mark placed at a distance of 1-5/8 inch from the surface of the plate and in the center of its side. The 1-5/8 inch is the distance from the surface of the plate to just short of the top of the Y accelerometer. Then a blue, transparent film was used to make an alignment shield. The shield was then mounted to the edge of the plate and a vertical cut was made approximately 1.5 inches long at a distance of 1-5/8 inch from the surface of the plate. A horizontal mark was then made on the shield that is in the same horizontal plane as the mark on the accelerometer. The laser could then be shot through the blue shield and onto the accelerometer. The marks could then be lined up and the alignment was complete. Data can be taken and the X reference velocity recorded. A small spacer had to be placed in the vertical cut in the shield to be able to take data. This allowed the laser beam to pass through the shield without being disturbed. If the spacer was not placed in the shield, noise would appear on the velocity signal due to the interference the shield causes.

During the two-dimensional test, at each of the angular positions, the laser was aligned so as to remain in a horizontal plane with the target position. This is accomplished by pointing the laser at the target position and scanning up and down. When the scan angles are equal then the laser is in the plane of the target. Also, the five scan angle pairs, four to the registration points and one to the target position, are recorded to locate the laser in
Figure 15 – Alignment for X Axis
space and to get the unit vector. The velocity data is then taken and recorded. The laser
is then moved to the next position and the process repeated.

With the scan angle pairs, velocity data and the coordinates of the registration points, the
velocities could be transformed to the orthogonal velocity pairs. First, the unit vector for
each position is calculated. Second, the velocities are run through the transformation.
The results are listed in the results section.

III.B.3. Three-Dimensional Set-up Modifications

The three-dimensional set-up is a direct extension of the successful two-dimensional set-
up. The set-up consists of three accelerometers mounted on a triaxial mounting block.
The block is mounted on the 1/8th inch thick steel plate. The plate is mounted vertically
to a test bed by way of clamps and is excited by a shock cord suspended shaker that is
attached to the upper left hand corner of the test plate. See Fig. 16 for set-up. The plate
also has four points marked on it at known locations. These are called the registration
points.

It was quickly realized that trying to use Huffaker's or Donovan's methods of velocity
extraction would be very difficult. First, the tolerances on the separation angles must be
tight. With their methods, it would be difficult to precisely locate the laser at the needed
locations. This was because the whole laser unit must be moved since the structure was
mounted to a fixed steel and concrete test bed. Second, even if it was possible to position
the laser accurately, it would be difficult, if not impossible, to maintain the right pyramid
configuration while trying to measure a fixed structure. Because of these limitations, it
Figure 16 – Three-Dimensional Set-up
was necessary to modify their methods or find a different method all together. The method was modified. In the modified method, the user is not restricted by specific laser configurations. The user can position the laser at any three positions in space. But, for the method to work, the laser must be located in space with respect to the reference coordinate system. To do this, the Four Point Registration Method is used. This method is described in depth in Section II.B. To reiterate, this method determines the unit vector for the particular laser position that will be used in the velocity transformation.

For this test it was desired to get the three reference velocities to be close in magnitude. The factors involved in this problem are the position of the accelerometers on the plate, the excitation frequency and excitation location. To narrow the problem down, the excitation location would remain in the upper corner. Next, the accelerometers were fixed in a position on the plate. See Fig. 17 for location. Finally, the appropriate shaking frequency was to be determined. To determine the frequency, an impact test was done on the plate to get the Frequency Response Function's, or FRF, of all three accelerometers in the reference directions. The FRF's are shown in Fig. 18. The top FRF is the X, the middle is the Y and the bottom is the Z. Acceptable frequencies are those frequencies that have response magnitudes that are about the same for all three directions. The FRF's in Fig. 18 are mobility functions since the response of the accelerometers have been integrated once to yield velocity. The velocity is ratioed to the excitation force to yield the mobility.

From the FRF, a frequency of 290Hz was chosen for the first test. During the test it was observed that the Y accelerometer velocity was varying >>10% from the starting magnitude. For the test to have any chance of success, the structure must produce a
Figure 17 – Front View of Plate Used for 3-D Test
Figure 18 – FRF’s for Three-Dimensional Test
stationary velocity signal. The cause of the variation was unknown, but it was suspected that small changes in temperature and humidity cause the dynamic characteristics of the structure to vary. Low damping structures are particularly susceptible to large changes in response near the resonance peaks. Notice that the X and Z accelerometers were near a saddle in their FRF's as shown in Fig. 18. Changes in the dynamic characteristics will have little effect on the response in this region. However, the Y direction accelerometer was operating in a high slope region between an anti-resonance and a resonance. Small changes in the dynamic resonant frequencies could have large effects on the response.

With the 290Hz excitation frequency not producing a stationary velocity on the structure, it was necessary to study the FRF again. The next frequency chosen was 205Hz. This frequency appeared to have all three magnitudes about the same but the frequency was at high slope area on the X accelerometer FRF. During this test the temperature in the plate was attempted to be controlled by turning the air conditioning on and off. This was done to eliminate the temperature variable in trying to determine the cause of the structural variation. This method helped control the temperature, but the temperature still varied more than 2°. This test had marginal stability, but was run completely. The results were very bad and the test could be considered a failure. It was suspected the instability was due to slight changes in resonance characteristics causing large changes in response since all three transducers were near a resonance condition. It was also found that the impedance matcher that was used with the signal generator had a loose connection and could cause a change in the signal's amplitude. The impedance matcher was removed and a different frequency was tested.
One last frequency was selected from the FRF, 345Hz. This frequency showed that the Y mobility would be between 4.5 and 7 times the magnitude of the X and Z mobilities, but with the frequency being in a saddle point. The responses should be minimally affected by structural dynamic property changes with time. A new method of controlling the temperature was developed. The new method allowed the temperature in the room to stabilize itself. The windows were closed and the air conditioning was not used. The idea was to allow the temperature to become steady state. This is the final test and will be described in more detail in the next section.

Along with the final three-dimensional test, a confirmation test was run. The test was another three-dimensional test run at 205Hz. The purpose of this test was to determine if the method would work at a 'non-ideal' frequency. This means, a frequency that may be more susceptible to room and environment changes. This was run under the same conditions as the 345Hz test. This test varies from the first 205Hz test by having a force transducer installed to monitor the force, a new temperature control method (same as the 345Hz test) and the impedance matcher removed.

III.B.4. **Final Three-Dimensional Test Set-up**

The three-dimensional set-up has been described in the previous section, but will be briefly described again. The 1/8th inch thick plate is mounted vertically and clamped to a fixed test bed. The plate is excited by a suspended shaker that is attached to the upper left hand corner of the plate by means of a force transducer. The force transducer was used to monitor the stability of the signal exciting the plate. A thermocouple monitors the temperature of the plate in the lower corner. This is because it is suspected that the
problem with the structure stability could be related to the fluctuating room temperature. Although the temperature was suspected, it was not proven. The accelerometer triad is mounted off center on the plate. Fig. 19 shows the final set-up. The plate will be excited at 345Hz.

Once the experiment was set-up and all the equipment was running with the structure being excited, the system was allowed to warm up for approximately one hour. This allows for the thermal stabilization of the system. Over the course of the experiment, the temperature would be monitored to try and reduce the temperature fluctuation. The air conditioning was turned off and the temperature stabilized itself. During the test the temperature was approximately 82°F (29°C). With the system stable, the three reference velocities, \( V_x, V_y \), and \( V_z \), are measured with the laser. The two in-plane velocities are measured on the edge of the accelerometer top. The \( X \) and \( Z \) are both measured the same way as the \( X \) was in the two-dimensional test. For the \( Z \), an additional shield was added to the top of the plate and the laser was mounted on a tripod such that the laser was able to hang over the plate to shoot down through the shield and onto the accelerometer. The out-of-plane, \( Y \), velocity was measured at the target position in the same manner as in the two-dimensional test (see Fig. 15). With the reference velocities measured, the various angular laser shots, \( V_A, V_B \), and \( V_C \), are taken. This was done by physically moving the laser to different positions in space. In this experiment a total of seven angular positions are taken. This would provide a good variation in position combinations available for analysis. At each of the seven locations the laser must be registered in space. This requires that the scan angle pairs to each of the four registration points be recorded, and also the scan angle pair to the target position. Now the accelerometers, force and velocity from the laser could be measured and recorded. The magnitude was measured from the
Figure 19 – Final Three-Dimensional Set-up
spectrum of the signal. The phase was determined by generating the cross spectrum between the signal and the signal from the Hewlett Packard Synthesized Signal/Sweep Generator. This allowed for the preservation of phase information. Thus all the velocities are complex numbers. The accelerometer's signal was integrated once to get the data in velocity units. This procedure was to be done at each of the seven different laser positions. With all the angular shots completed, the three reference velocities are again measured just to determine if the reference velocities have varied over the course of the test.
CHAPTER IV

RESULTS

IV.A. Two-Dimensional Test

The result of the two-dimensional test are shown in Figs. 20 - 23. The magnitude error results are shown in Figs. 20 and 21. These graphs are in percent error format. At each position, data was taken twice. Once with one average and another with ten averages. This is done to monitor the stability of the structure. The two-dimensional test took data at four laser positions. These four positions are taken in pairs of two. Thus, all possible combinations of data points are transformed. This will show the relative accuracy of various position combinations. The reference velocities, with units of in/s, used to calculate the percent error and its equation are shown below:

\[
\begin{align*}
X: & \quad 1.165 \times 10^{-2} \angle 91.98^\circ \\
Y: & \quad 3.268 \times 10^{-1} \angle -90.92^\circ
\end{align*}
\]

\[
\% \text{ Error } = \frac{\text{Transformed Velocity} - \text{Reference Velocity}}{\text{Reference Velocity}} \tag{7}
\]

The phase results are in Figs. 22 and 23. The phase graphs are reported in absolute difference. The equation is shown below in degrees.

\[
\text{Phase Difference} = \text{Transformed Phase} - \text{Reference Phase} \tag{8}
\]

The potential amplification indices for the two dimensional case are shown below.
Figure 20 - X Magnitude Graph 2-D Test
Figure 21 – Y Magnitude Graph 2-D Test
Figure 23 - Y Phase Graph 2-D Test
Table 2 - Potential amplification Indices 2-D test

<table>
<thead>
<tr>
<th>Potential amplification Index</th>
<th>2-D Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>Pos. 1,2</td>
<td>2.9040</td>
</tr>
<tr>
<td>Pos. 1,3</td>
<td>1.3924</td>
</tr>
<tr>
<td>Pos. 1,4</td>
<td>1.1156</td>
</tr>
<tr>
<td>Pos. 2,3</td>
<td>1.0781</td>
</tr>
<tr>
<td>Pos. 2,4</td>
<td>0.9326</td>
</tr>
<tr>
<td>Pos. 3,4</td>
<td>3.8100</td>
</tr>
</tbody>
</table>

With the data from the two-dimensional test it is possible to transform the data in a three-dimensional manner. The position combinations will be taken in groups of three, instead of two, and a three-dimensional transformation performed. The Z direction will not be used because of the amplification errors caused by the laser positions being almost coplanar. The potential amplification index will show that the Z direction will be unusable, but the X and Y directions should be usable. In this way the error from not remaining exactly in the horizontal plane would be accounted for in the three-dimensional transformation. The potential amplification indices for the two-dimensional case with the three-dimensional transformation is listed below:

Table 3 - Potential amplification Indices 2-D/3-D Case

<table>
<thead>
<tr>
<th>Potential amplification Index</th>
<th>2-D/3-D case</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>Pos. 1,2,3</td>
<td>1.1689</td>
</tr>
<tr>
<td>Pos. 1,2,4</td>
<td>1.0057</td>
</tr>
<tr>
<td>Pos. 1,3,4</td>
<td>2.1940</td>
</tr>
<tr>
<td>Pos. 2,3,4</td>
<td>4.9016</td>
</tr>
</tbody>
</table>

The graphs for the 2-D/3-D case are shown in Figs. 24 - 27. The magnitude plots, Figs. 24 and 25, are in the percent error format. The phase plots, Figs. 26 and 27, are in the absolute difference format.
Figure 24 - X Magnitude Graph 2-D/3-D Test
Figure 26 – X Phase Graph 2-D/3-D Test
IV.B. Three Dimensional Test

The unit vectors for the seven various angular positions for the 345Hz test is shown in Fig. 28. This will show the spatial relation of the different laser positions. The magnitude results are given in a percent error format in Figs. 29 - 31. The percent error is calculated using eq. (7). This will show how close the transformed velocities are to the reference. The reference velocities in the before and after cases were within 1% of each other so the before velocities were chosen as the reference velocities. The reference velocities are listed here in in/s:

\[ \begin{align*}
X & : 2.741 \times 10^{-2} \pm 126.3^\circ \\
Y & : 7.294 \times 10^{-2} \pm 123.2^\circ \\
Z & : 4.046 \times 10^{-2} \pm 123.3^\circ
\end{align*} \]

On each graph there are four lines plotted. This is because at each laser position, data was taken four times, and in this order: 1 ave, 10 ave, 1 ave and 10 ave. This gave a crude indication of how stable the structure was at each laser location. For most of the points all four data readings were very close. The phase results are shown in Figs. 32 - 34. The phase results are recorded in an absolute difference format. Eq.(8) shows how absolute difference is defined. Along the abscissa are the position combinations. These are the locations used to get the transformed velocities for that point.

Next, the potential amplification indices are computed for all the possible combinations of viewpoints taken three at a time in space. These are presented in Table 4.
Figure 28 - Laser Locations in Space for 345Hz Test
Figure 30 - Y Magnitude Graph 3-D 345Hz Test
Figure 31 - Z Magnitude Graph 3-D 345Hz Test
Figure 32 – X Phase Graph 3-D 345Hz Test
Figure 34 - Z Phase Graph 3-D 345Hz Test
Table 4 - Potential amplification Indices - 3-D Case 345Hz

<table>
<thead>
<tr>
<th>Potential amplification Index 345Hz</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pos 1,2,3</td>
<td>1.3229</td>
<td>4.3685</td>
<td>4.6463</td>
</tr>
<tr>
<td>Pos 1,2,4</td>
<td>1.4448</td>
<td>3.1268</td>
<td>3.5172</td>
</tr>
<tr>
<td>Pos 1,2,5</td>
<td>1.3255</td>
<td>1.1092</td>
<td>1.4229</td>
</tr>
<tr>
<td>Pos 1,2,6</td>
<td>1.1920</td>
<td>1.4306</td>
<td>1.5289</td>
</tr>
<tr>
<td>Pos 1,2,7</td>
<td>1.1264</td>
<td>3.0092</td>
<td>2.6937</td>
</tr>
<tr>
<td>Pos 1,3,4</td>
<td>2.2149</td>
<td>3.0184</td>
<td>4.0898</td>
</tr>
<tr>
<td>Pos 1,3,5</td>
<td>2.2033</td>
<td>1.0865</td>
<td>1.4091</td>
</tr>
<tr>
<td>Pos 1,3,6</td>
<td>3.5195</td>
<td>2.8998</td>
<td>4.0795</td>
</tr>
<tr>
<td>Pos 1,3,7</td>
<td>13.492</td>
<td>27.550</td>
<td>34.034</td>
</tr>
<tr>
<td>Pos 1,4,5</td>
<td>11.168</td>
<td>4.1791</td>
<td>2.6645</td>
</tr>
<tr>
<td>Pos 1,4,6</td>
<td>4.4141</td>
<td>2.6090</td>
<td>6.1889</td>
</tr>
<tr>
<td>Pos 1,4,7</td>
<td>1.9411</td>
<td>2.8750</td>
<td>4.8568</td>
</tr>
<tr>
<td>Pos 1,5,6</td>
<td>1.3937</td>
<td>0.7547</td>
<td>1.7352</td>
</tr>
<tr>
<td>Pos 1,5,7</td>
<td>1.0438</td>
<td>0.8972</td>
<td>1.6752</td>
</tr>
<tr>
<td>Pos 1,6,7</td>
<td>1.9663</td>
<td>2.0848</td>
<td>3.4832</td>
</tr>
<tr>
<td>Pos 2,3,4</td>
<td>19.551</td>
<td>34.918</td>
<td>49.892</td>
</tr>
<tr>
<td>Pos 2,3,5</td>
<td>3.2604</td>
<td>1.2470</td>
<td>3.5031</td>
</tr>
<tr>
<td>Pos 2,3,6</td>
<td>1.8626</td>
<td>1.6892</td>
<td>1.5599</td>
</tr>
<tr>
<td>Pos 2,3,7</td>
<td>1.2701</td>
<td>2.7198</td>
<td>2.5567</td>
</tr>
<tr>
<td>Pos 2,4,5</td>
<td>1.8239</td>
<td>1.1351</td>
<td>2.2168</td>
</tr>
<tr>
<td>Pos 2,4,6</td>
<td>1.2544</td>
<td>1.1277</td>
<td>1.6845</td>
</tr>
<tr>
<td>Pos 2,4,7</td>
<td>0.8752</td>
<td>1.7414</td>
<td>2.2680</td>
</tr>
<tr>
<td>Pos 2,5,6</td>
<td>1.3940</td>
<td>0.8775</td>
<td>2.0906</td>
</tr>
<tr>
<td>Pos 2,5,7</td>
<td>0.9465</td>
<td>0.9000</td>
<td>1.8186</td>
</tr>
<tr>
<td>Pos 2,6,7</td>
<td>1.6283</td>
<td>2.3130</td>
<td>1.6978</td>
</tr>
<tr>
<td>Pos 3,4,5</td>
<td>3.2825</td>
<td>1.4095</td>
<td>2.4042</td>
</tr>
<tr>
<td>Pos 3,4,6</td>
<td>2.0886</td>
<td>1.6124</td>
<td>2.8274</td>
</tr>
<tr>
<td>Pos 3,4,7</td>
<td>1.5263</td>
<td>2.8149</td>
<td>4.5171</td>
</tr>
<tr>
<td>Pos 3,5,6</td>
<td>1.3938</td>
<td>0.7491</td>
<td>1.6651</td>
</tr>
<tr>
<td>Pos 3,5,7</td>
<td>0.9940</td>
<td>0.8520</td>
<td>1.7206</td>
</tr>
<tr>
<td>Pos 3,6,7</td>
<td>1.8633</td>
<td>2.1397</td>
<td>2.8166</td>
</tr>
<tr>
<td>Pos 4,5,6</td>
<td>1.3936</td>
<td>0.6999</td>
<td>2.3018</td>
</tr>
<tr>
<td>Pos 4,5,7</td>
<td>1.0485</td>
<td>0.8173</td>
<td>2.2920</td>
</tr>
<tr>
<td>Pos 4,6,7</td>
<td>2.1470</td>
<td>1.9427</td>
<td>5.4202</td>
</tr>
<tr>
<td>Pos 5,6,7</td>
<td>1.3908</td>
<td>4.0860</td>
<td>23.6948</td>
</tr>
</tbody>
</table>

69
IV.C. Three-Dimensional Confirmation Results

The unit vectors for the five various angular positions for the 205Hz test is shown in Fig. 35. All the results are presented in the same manner as the previous three-dimensional test. The magnitude error graphs for the 205Hz test are shown in Figs. 36 - 38. The phase results for the 205Hz test are shown in Figs. 39 - 41. The reference velocities, with units of in/s, are shown below.

\[
\begin{align*}
X: & \quad 3.356 \times 10^2 \angle -106.0^\circ \\
Y: & \quad 2.544 \times 10^2 \angle -99.76^\circ \\
Z: & \quad 3.046 \times 10^2 \angle -114.60^\circ
\end{align*}
\]

The potential amplification indices are shown in Table 5.

<table>
<thead>
<tr>
<th>Potential amplification Index</th>
<th>205Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Pos 1,2,3</td>
<td>1.8045</td>
</tr>
<tr>
<td>Pos 1,2,4</td>
<td>1.5725</td>
</tr>
<tr>
<td>Pos 1,2,5</td>
<td>1.1678</td>
</tr>
<tr>
<td>Pos 1,3,4</td>
<td>3.1826</td>
</tr>
<tr>
<td>Pos 1,3,5</td>
<td>8.8242</td>
</tr>
<tr>
<td>Pos 1,4,5</td>
<td>1.4559</td>
</tr>
<tr>
<td>Pos 2,3,4</td>
<td>3.2969</td>
</tr>
<tr>
<td>Pos 2,3,5</td>
<td>1.4690</td>
</tr>
<tr>
<td>Pos 2,4,5</td>
<td>1.3339</td>
</tr>
<tr>
<td>Pos 3,4,5</td>
<td>1.4128</td>
</tr>
</tbody>
</table>
Figure 35 - Laser Locations in Space for 205Hz Test
Figure 36 – X Magnitude Graph 3-D 205Hz Test
Figure 37 - Y Magnitude Graph 3-D 205Hz Test
X Phase Absolute Difference
3-D 205Hz Test

Figure 39 – X Phase Graph 3-D 205Hz Test
Figure 40 – Y Phase Graph 3-D 205Hz Test
Figure 41 – Z Phase Graph 3-D 205Hz Test
CHAPTER V
DISCUSSION of RESULTS

V.A. Two-Dimensional Test

The final locations of the laser positions are shown in Fig. 13. This figure shows the relative positions of each position and the angles to the Y axis. This will give an indication of the effective separation angle. The separation angle is a measure that Donovan used to indicate the possible quality of the data. The potential amplification index is a more general way of determining the possibility of quality data. It can be readily noticed in Figs. 20 and 21 that the Y velocity data is much better than the X velocity. This was expected because of the success of having measured the out-of-plane velocities previously.

On the whole, the results were good. Position combinations 1,3, 1,4 and 3,4 produced the best results. For each of these position combinations, the separation angle could be considered to be approximately 30°, 39° and 9° respectively. It can be noticed that a 9° separation angle is small and could lead to possible high potential for error amplification, but does not cause any problem in this case. This is highlighted in Donovan10. The potential amplification indices in Table 2 can be used to explain some of the good results. The indices for the 1,3 and 1,4 combinations are small. This indicates a low potential for error amplification. Thus, these points would have been expected to produce good results. But, the 3,4 combination had a higher potential amplification index. This would indicate that this combination would be much more sensitive to any measurement error. Since the results were good, it would be assumed that there is not much measurement
error on the 3 or 4 position measured velocity. From these results, the test was considered a success and the three-dimensional test could be attempted.

The 1,2, 2,3 and 2,4 position combinations in the two-dimensional test did not produce data that was as good as expected. One possible explanation is that the second laser position's measured velocity had a high measurement error. This would tend to propagate through the transformation and become evident in the transformed velocities. All the position combinations that have produced bad data are associated with position two, it would seem to be that this position is a bad data point. We can use the potential amplification index, Table 2, to help come to this conclusion. Position combinations 2,3 and 2,4 have as good or better potential amplification indices than combinations 1,3 and 1,4, yet they produce worse results. The potential amplification index is a measure of how much a unit error on the laser velocity measurements will be amplified into the transformed velocity. Therefore, to get a higher error, the measurement error must be higher on one or both measurements. In this case it was determined that positions 3 and 4 had low measurement errors. Thus, position 2 must have a high measurement error. This is a very good example to show that the potential amplification index is not a measure of data quality, but is the measure of potential error amplification.

The 2-D/3-D transformation case is inserted primarily for comparison. First, Table 3 shows what happens to the potential amplification index if the three laser positions are close to being coplanar with the target position. The Z potential amplification is very high. This is because the laser measurements were taken in the X-Y plane and very little Z velocity data could be acquired. Second, the result graphs show more consistency in the
data. The data does not have any major spikes or variance. Finally, this transformation produced very good results overall. Now the three-dimensional test could be performed.

V.B. Three-Dimensional Test

From the error magnitude results, one of the most obvious items is the high errors in the 1,3,7 and 2,3,4 position combinations in the 345Hz case. See Figs. 29 - 31. Also, the 5,6,7 position combination has a large error in the Z magnitude. These errors can be partially explained by looking at the potential amplification index for each of these points in Table 4.

As seen in Table 4, at 1,3,7 and 2,3,4 and the Z of 5,6,7 the potential amplification index is very high. This would show that if there was any error in any of the laser measurements, such error would be greatly amplified and would present itself in the $V_x$, $V_y$ and $V_z$ predictions. Since every measurement contains a degree of error, the errors are high because of the amplification factor. This amplification is caused by the orientation of the laser positions with respect to each other. In the cases where the indices are high, it can be shown that the three positions are very close to being coplanar and have a low separation angle.

It can also be noted that there are other small peaks that approach 5% or greater in the error magnitude results. At many of these peaks, the corresponding position combination contains position 7. This appeared to be reason to suspect a possible problem with the measured velocity from position 7. When the magnitude results are studied, not all the combinations that contain position 7 show high error. So what is causing the errors? This
problem was investigated and it was determined that position 7 must have a higher than normal measurement error. To explain this conclusion, eq.(5) was investigated for all the position combinations that contained position 7. The result was that the individual potential amplification index that corresponded to position 7 varied from near zero to around 15. For the combinations that produced good or low percent errors all had individual potential amplification indices approaching zero for position 7. This would explain that if there was an error on position 7 and the individual potential amplification index is very near zero that the error would not be propagated but reduced or eliminated. The reverse is true for the positions that had the small error peaks. At these positions it was observed that the individual potential amplification index for position 7 was higher, about one or greater. Thus, a measurement error on position 7 would be passed along or magnified when transformed. By looking at the individual potential amplification indices it was possible to determine that position 7 has a higher than normal measurement error.

Another interesting point is that the phase error is very low. Even with the large errors in the magnitude, the phase errors remained relatively small. The largest error was less than 2° with the majority of the errors less than 0.5 degrees. This test was 'almost Ideal'. The structure remained very stable and stationary. The temperature variation was about 1°F (0.55°C). The temperature plot is shown in Fig. 42. The stability will be discussed more in the following section.

V.C. **Three-Dimensional Confirmation Test**

This test is very interesting for the fact that it is not an 'Ideal' test. The frequency of 205Hz can be seen of Fig. 18. Note the difference in the FRF's at each position, 205Hz
for this test and 345Hz for the previous test. The 345Hz is at a saddle point on all three FRF's. This would indicate that if a temperature variation or some other outside influence that may alter the resonance peaks were to occur, the response would have a minimal effect on the structure's measured velocity. On the other hand, the 205Hz frequency has very steep slopes on the X and Z response and is in a saddle for the Y. This would indicate that if an external influence were to shift the structure frequencies slightly, the X and Z would be effected much more than the Y for the 205Hz case. Meaning that the velocity of the structure would have a greater variation than the 345Hz test with the same outside influences. The difference in variation between the two tests can be seen in the graphs of the accelerometer magnitude readings over the duration of each test. The integrated X, Y and Z accelerometer reading for the 205Hz test are in Figs. 43 - 45. The graphs for the 345Hz test are shown in Figs. 46 - 48.

The uncertainty in the accelerometer data was calculated to determine if the variations seen in the accelerometer data could be caused by noise on the accelerometer signal. The noise is a concern because of the small velocities being measured by the accelerometers. Thus, the variations could be a signal-to-noise ratio problem. To get the noise floor for the accelerometers, a threshold was determined. "For voltage output transducers, threshold denotes the equivalent noise level of its built-in charge to voltage converter." To determine the threshold, the test procedure was obtained from a test engineer at Kistler. The procedure is to suspend the accelerometers from the edge of a bench. The accelerometers should remain motionless. The amplifiers are to be connected and set to the same gain as when the data was taken. In the tests the X gain was 10x, the Y gain was 100x and the Z gain was 10x. The signal was then analyzed to determine the magnitude of the signal at the certain frequencies (in this case 205Hz and 345Hz). The results are
Figure 43 – X Accelerometer Magnitude Graph 205Hz Test
Figure 44 – Y Accelerometer Magnitude Graph 205Hz Test
Figure 45 - Z Accelerometer Magnitude Graph 205Hz Test
Figure 46 - X Accelerometer Magnitude Graph 345Hz Test
Figure 47 – Y Accelerometer Magnitude Graph 345Hz Test
Figure 48 – Z Accelerometer Magnitude Graph 345Hz Test
Figure 49 - Accelerometer’s Threshold Plots
shown in Fig. 49. The top plot is the spectrum of the X accelerometer signal, the middle the Y accelerometer and the bottom is the Z accelerometer. The spectrum has been integrated to get the units in in/s. The uncertainty in the accelerometer signal at a given frequency is approximated by +/-2 x (the magnitude at the frequency). In Tables 6 and 7 the uncertainty for both the 345Hz test and the 205Hz test are listed.

Table 6 - Uncertainty for Accelerometers at 345Hz

<table>
<thead>
<tr>
<th>Accelerometer</th>
<th>Threshold (in/s)</th>
<th>Uncertainty (in/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>3.135x10^{-7}</td>
<td>+/-6.270x10^{-7}</td>
</tr>
<tr>
<td>Y</td>
<td>5.079x10^{-6}</td>
<td>+/-1.016x10^{-5}</td>
</tr>
<tr>
<td>Z</td>
<td>2.691x10^{-7}</td>
<td>+/-5.382x10^{-7}</td>
</tr>
</tbody>
</table>

Table 7 - Uncertainty for Accelerometers at 205Hz

<table>
<thead>
<tr>
<th>Accelerometer</th>
<th>Threshold (in/s)</th>
<th>Uncertainty (in/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>6.0x10^{-7}</td>
<td>+/-1.2x10^{-6}</td>
</tr>
<tr>
<td>Y</td>
<td>1.1x10^{-5}</td>
<td>+/-2.2x10^{-5}</td>
</tr>
<tr>
<td>Z</td>
<td>6.0x10^{-7}</td>
<td>+/-1.2x10^{-6}</td>
</tr>
</tbody>
</table>

It can be seen that from the uncertainty in the accelerometer measurements that the trends in the accelerometer magnitudes are real. In addition to the uncertainty in the accelerometers, the quantitization error has been calculated at each frequency (205Hz and 345Hz). The quantitization errors are shown in Table 8. It can be seen that the quantitization errors are larger than the uncertainty in the measurements. Therefore, the error in the accelerometers are dominated by the quantitization error. Even with the
quantization error being larger than the uncertainty, the trends in the accelerometers are real and must be caused by some outside influence.

Table 8 - Quantization Errors for 3-D Tests

<table>
<thead>
<tr>
<th></th>
<th>Quantization Errors (in/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>205Hz</td>
<td>+/-6.6060x10^{-6}</td>
</tr>
<tr>
<td>345Hz</td>
<td>+/-3.9254x10^{-6}</td>
</tr>
</tbody>
</table>

From the graphs, it can be seen that the Y and Z accelerometer, while having a trend, did not vary more than 2%. The X accelerometer did vary much more, about 8%. When the FRF's in Fig. 18 are inspected at 205Hz, it is noticed that the slope of the X is the greatest of the three. Therefore, if there were any influences on the system's dynamic characteristics, the X would be effected more than the Y or Z. This is consistent with what occurred.

From the accelerometer plots for the 345Hz test, it is noticed that all three accelerometers remained well within 1% variation. Thus, the structure would be believed to be very stationary. This could be because of the 345Hz test falling on the three saddle points on the FRF's. These saddles are relatively insensitive to small dynamic characteristics changes in the structure.

A possible contribution to the stability problem is the variance in the temperature. The temperature varied 2°F over the course of the test. The temperature plot is shown in Fig. 50. It is suspected that temperature has played a part in the instability of the structure, but
the exact cause of the instability has not been determined. The temperature data is shown for comparison with the temperature data in Fig. 42 and with the accelerometer magnitude plots, Figs. 43 - 48. Since the temperature has not been verified as causing the problem this data is shown for comparison only. Humidity may also have an influence on damping levels in the structure. Humidity was not monitored in any of the tests.
Temperature Plot
205Hz Test

Figure 50 - 205Hz Temperature Plot
CHAPTER VI

CONCLUSION and RECOMMENDATIONS

VI.A. Conclusions

The three shot method is good enough to give results that are as good or better than accelerometers, without the drawbacks that accelerometers encounter. For the method to be reliable, however, the velocity of the structure must be stationary with respect to time. Although the method can be used with any three arbitrary laser positions, it has been shown that those positions that produce smaller potential amplification indices have a better chance of giving accurate results. The potential amplification index is not an absolute measure of the validity of the results. It measures the potential of velocity error amplification in the prediction of the orthogonal velocity set. The most important factor in getting good data is the minimization of the measurement error. This may be difficult or impossible but it is a big factor in getting good data. If more than three laser data positions are used, it may be possible to determine which position has the larger measurement error. This can be done by using the individual potential amplification indices. This is done by searching the potential amplification indices for their minimum with respect to the various position combinations.

The method, however, is very sensitive to changing structural dynamic characteristics. As it can be seen from the confirmation test, a structure that changes over the course of taking data will produce bad data.
VI.B. Recommendations

Additional work should be done on trying to solve the problem involving high sensitivity to small dynamic changes in the structure. This may be difficult since the method depends upon the structure being exactly the same at three instances of viewpoint data.

Also, a full sensitivity analysis should be performed. The analysis should include information dealing with the sensitivity caused by position error. Along with the sensitivity analysis, an indicator could be developed that would indicate that the data at a given position is going to be bad instead of having an indicator that can be used to determine why the data was bad.

A study of what can cause structural changes and how to control them would be very helpful. Since the method is very dependent on the structure remaining stationary in time, it would be helpful to know how to maintain a structure in a stationary state.

Work should be done on trying to implement this method into a three-dimensional scanning mode. Thus making the method much more usable and lending itself to many more applications.

The threshold for the accelerometers should be calculated in a more actual situation. The accelerometers should be attached to the test structure and no excitation used. Then the threshold could be measured. This would give a more realistic measure of the noise floor since ambient floor vibration can excite the test structure if it is in a clamped state, such as
in the three-dimensional plate test. Such excitation could cause structural response that could raise the measurement threshold.

Finally, study the limits of accuracy expected between the largest and the smallest velocity component. Effectively determining what kind of accuracy, on all the velocity components, can be expected based on the maximum velocity component.
REFERENCES


APPENDIX A

Listing of Mathematica code for laser positioning:

```mathematica
Needs["Calculus\n\nVectorAnalysis"];
theta = \{\{ax Degree, ay Degree\}, \{bx Degree, by Degree\},
\{cx Degree, cy Degree\}, \{dx Degree, dy Degree\},
\{ex Degree, ey Degree\}\};
\r = \{ra, rb, rc, rd, re\};
structcoordfile = InputString["\n\nEnter filename of points A, B, C, D in structure coordinates: "];
pts = ReadList[structcoordfile, Table[Number, \{3\}]];
Print["\n\nGenerating equations...\n"];
p1p2 = pts[i, 1] - pts[i, 1];
p1p3 = pts[i, 2] - pts[i, 1];
p1p4 = pts[i, 3] - pts[i, 1];
p2p3 = pts[i, 3] - pts[i, 2];
p2p4 = pts[i, 4] - pts[i, 1];
p3p4 = pts[i, 4] - pts[i, 3];
lenp1p2 = Sqrt[Sum[p1p2[i, 1]^2, \{i, 1, 3\}]];
lenp1p3 = Sqrt[Sum[p1p3[i, 1]^2, \{i, 1, 3\}]];
lenp1p4 = Sqrt[Sum[p1p4[i, 1]^2, \{i, 1, 3\}]];
lenp2p3 = Sqrt[Sum[p2p3[i, 1]^2, \{i, 1, 3\}]];
lenp2p4 = Sqrt[Sum[p2p4[i, 1]^2, \{i, 1, 3\}]];
lenp3p4 = Sqrt[Sum[p3p4[i, 1]^2, \{i, 1, 3\}]];
\rz = p1p2 / lenp1p2;
\rxt = CrossProduct[p1p3, p1p2];
lenrxt = Sqrt[Sum[rxt[i, 2][1, 3] \{i, 1, 3\}]];
\rxt = \rz / lenrxt;
\ry = CrossProduct[\rz, \rxt];
\rot = \{\rz, \rxt, \ry\};
op[n_] := \{-5.315, 6.024 + 1.811 Sin[theta[n, 1]],
2.638 - 1.811 Cos[theta[n, 1]]\};
\oeln[n_] := \{\r[n] * Tan[theta[n, 2]]
/ Sqrt[1 + Tan[theta[n, 1]]^2 + Tan[theta[n, 2]]^2],
\r[n] * Tan[theta[n, 1]]
/ Sqrt[1 + Tan[theta[n, 1]]^2 + Tan[theta[n, 2]]^2],
\r[n] \}/
/ Sqrt[1 + Tan[theta[n, 1]]^2 + Tan[theta[n, 2]]^2]};
\pt[n_] := op[n] + oeln[n];
P1P2 = \pt[2] - \pt[1];
P1P3 = \pt[3] - \pt[1];
P1P4 = \pt[4] - \pt[1];
P2P3 = \pt[3] - \pt[2];
P2P4 = \pt[4] - \pt[2];
P3P4 = \pt[4] - \pt[3];
lenP1P2 = Sqrt[Sum[P1P2[i, 1]^2, \{i, 1, 3\}]];
```

99
lenP1P3 = Sqrt[Sum[P1P3[i][i]^2,{i,1,3}]];  
lenP1P4 = Sqrt[Sum[P1P4[i][i]^2,{i,1,3}]];  
lenP2P3 = Sqrt[Sum[P2P3[i][i]^2,{i,1,3}]];  
lenP2P4 = Sqrt[Sum[P2P4[i][i]^2,{i,1,3}]];  
lenP3P4 = Sqrt[Sum[P3P4[i][i]^2,{i,1,3}]];  
Rz = P1P2 / lenP1P2;  
Rx = CrossProduct[P1P3,P1P2];  
Ry = CrossProduct[Rz,Rx];  
rot2 = {Rz,Ry,Rz};  
error = (lenP1P2-lenpP1p2)^2 + (lenP1P3-lenpP1p3)^2 + (lenP1P4-lenpP1p4)^2 +  
       (lenP2P3-lenp2p3)^2 + (lenP2P4-lenp2p4)^2 + (lenP3P4-lenp3p4)^2;  

noofpts=Input["Enter number of positions to determine: "];  
ptno=0,  
"These are the normalized unit vectors">>u_vectors.result;  

While [ptno!=noofpts,  

anglepairsfile = InputString["Enter filename of points A, B, C, D (and E) in laser angles: "];  
If [anglepairsfile=="", Break[] ];  
angles = ReadList[anglepairsfile, Table[Number, {2}]];  
ax = angles[[1,1]];  
ay = angles[[1,2]];  
bx = angles[[2,1]];  
by = angles[[2,2]];  
cx = angles[[3,1]];  
cy = angles[[3,2]];  
dx = angles[[4,1]];  
dy = angles[[4,2]];  
ex = angles[[5,1]];  
ey = angles[[5,2]];  

minrange = Input["Enter minimum value for the range: "];  
guess1 = Input["Enter initial guess for range to a: "];  
guess2 = Input["Enter initial guess for range to b: "];  
guess3 = Input["Enter initial guess for range to c: "];  
guess4 = Input["Enter initial guess for range to d: "];  
maxrange = Input["Enter minimum value for the range: "];  

Print["Finalizing equations... "];  
objfunc = Simplify[N[(Sum[eqn[i][i]^2,{i,1,3}]+error).30]];  

Print["Solving equations via Minimization... "];  
errorflag = {1,1};  
errorcheck = Check[UseResult=FindMinimum[N[objfunc,30],  
{ra,guess1,minrange,maxrange},{rb,guess2,minrange,maxrange},  
{rc,guess3,minrange,maxrange},{rd,guess4,minrange,maxrange}],  
errorflag];
tmpranges = testresult[[2]]; 
Print["\nMinimization converged to the following error: "];
Print[testresult[[1]]];
If [testresult[[1]] > 1.0, 
Print["\nWARNING: Convergence value > 1"];
]

If errorcheck==errorflag,


g1 = ra /. tmpranges[[1]]; 
g2 = rb /. tmpranges[[2]]; 
g3 = rc /. tmpranges[[3]]; 
g4 = rd /. tmpranges[[4]]; 
Print["ra = ", g1, " rb = ", g2, " rc = ", g3, " rd = ", g4]; 
Print["\nNow solving equations via Newton's Method..."]; 
list = Simplify[N[Join[eqn, {error}], 30]]; 
ranges = FindRoot[N[list, 30], {ra, g1}, {rb, g2}, {rc, g3}, {rd, g4}, 
MaxIterations -> 30],

Print["\nOnly Minimization solver used"];
ranges = tmpranges
];

Print[ranges];

Print["\nNow solving equations for unit vector... "]; 
rotStoT = N[rot1, 30]; 
rotLtoT = N[rot2 /. ranges, 30]; 
rotLtoS = N[Inverse[rotStoT] . rotLtoT, 30]; 
re = 1; 
vvector = N[rotLtoS . (-1 oe[5]), 30]; 
Print["\nUnit vector of laser beam in structure coordinates: "]; 
Print[vvector];

uveclen=Sqrt[vvector[[1]]^2+vvector[[2]]^2+vvector[[3]]^2];
uvecor=N[uvector/uveclen,30];
Print[uveclen];
Print[uvecor];
tosavevec=InputString["Do you want to save this unit vector? (y/n) : "];

If[tosavevec=="Y" || tosavevec=="y", 
If[ptno==0, 

nuvecor->uvectors.result, 
nuvecor->uvoctors.result], 
ptno=ptno-1];

ptno=ptno+1;
];
Listing of Mathematica code for velocity transformation

```
uvecsfile=InputString["Enter filename of unit vectors (uvecors.result) : "];
uvecs=ReadList[uvecsfile,Expression];

lv1file=InputString["Enter filename of 1 ave laser velocity data: "];lv1=ReadList[lv1file,String];lv1=ToExpression[lv1];

lv10file=InputString["Enter filename of 10 ave laser velocity data: "];lv10=ReadList[lv10file,String];lv10=ToExpression[lv10];

resultfile=InputString["Enter the output filename (no extension): "];filetitle=InputString["Enter title for data: "];errorfile=StringJoin[{resultfile,".sen_error"}];Put[filetitle,resultfile];Put[filetitle,errorfile];PutAppend["This contains the inverse matrices and error sensitivities",errorfile];

tlv1={};tlv10={};

mtlv1={};mtlv10={};
noptx=Length[uvecs];

For[a=1,a<=noffx-2,++a,
   For[b=a+1,b<=noffx-1,++b,
      For[c=b+1,c<=noffx,++c,
         PutAppend["",resultfile];PutAppend["",resultfile];PutAppend["",resultfile];PutAppend["","Position Combination ",a,b,c,resultfile];

         PutAppend["",errorfile];PutAppend["",errorfile];PutAppend["",errorfile];PutAppend["","Position Combination ",a,b,c,errorfile];

         rau = {uvecs[[a,1]],uvecs[[a,2]],uvecs[[a,3]]};
         rbu = {uvecs[[b,1]],uvecs[[b,2]],uvecs[[b,3]]};
         rcu = {uvecs[[c,1]],uvecs[[c,2]],uvecs[[c,3]]};
         T = {rau,rbu,rcu};
```
Ts = Inverse[T];

PutAppend["",errfile];
PutAppend["",errfile];
PutAppend["The Inverse Matrix",errfile];
PutAppend[N[Transpose[MatrixForm,30],errfile];

PutAppend["",errfile];
PutAppend["",errfile];
PutAppend["Rel. Mag. Error X direction ",
Sqrt[Transpose[MatrixForm,30][1,1][1,2]+Transpose[MatrixForm,30][1,3][1,2]],errfile];

PutAppend["Rel. Mag. Error Y direction ",
Sqrt[Transpose[MatrixForm,30][2,1][2,2]+Transpose[MatrixForm,30][2,3][2,2]],errfile];

PutAppend["Rel. Mag. Error Z direction ",
Sqrt[Transpose[MatrixForm,30][3,1][3,2]+Transpose[MatrixForm,30][3,3][3,2]],errfile];

V1 = {lv1[[a]],lv1[[b]],lv1[[c]]};
V10 = {lv10[[a]],lv10[[b]],lv10[[c]]};
Vsl = Ts . V1;
Vsl0 = Ts . V10;

PutAppend["",resultfile];
PutAppend["",resultfile];
PutAppend["1 ave transformed velocity: Real + Imag and Mag +
Phase",resultfile];

PutAppend[Transpose[MatrixForm,30][1,1],resultfile];
PutAppend[Transpose[MatrixForm,30][1,3],resultfile];
PutAppend[Transpose[MatrixForm,30][2,1],resultfile];
PutAppend[Transpose[MatrixForm,30][2,3],resultfile];
PutAppend[Transpose[MatrixForm,30][3,1],resultfile];
PutAppend[Transpose[MatrixForm,30][3,3],resultfile];

PutAppend["",resultfile];
PutAppend["",resultfile];
PutAppend["10 ave transformed velocity: Real + Imag and Mag +
Phase",resultfile];

PutAppend[Transpose[MatrixForm,30][1,1],resultfile];
PutAppend[Transpose[MatrixForm,30][1,3],resultfile];
PutAppend[Transpose[MatrixForm,30][2,1],resultfile];
PutAppend[Transpose[MatrixForm,30][2,3],resultfile];
PutAppend[Transpose[MatrixForm,30][3,1],resultfile];
PutAppend[Transpose[MatrixForm,30][3,3],resultfile];

};
Vita

Jeffry J. Abel was born on July 17, 1969 in Warren, Michigan, and was raised in Beverly Hills, Michigan. He attended Seaholm High School and graduated cum laude in 1987. He then attended Michigan Technological University and graduated with his Bachelor of Science in Mechanical Engineering in 1991. During college he spent three summers interning with General Motors Chevrolet-Pontiac-Canada group. After graduating from Michigan Tech he proceeded to graduate school at Virginia Polytechnic Institute and State University in Mechanical Engineering.