Accurate Frequency Estimation with Phasor Angles

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(Abstract)

A power system should always operate in a balanced and stable condition at its designed frequency. Any significant upset of this balance will produce a change in the power system frequency. It is the responsibility of the monitoring and protective devices to detect and restore the system to the equilibrium operating condition at the nominal frequency as soon as it is practical to do so. An accurate measurement of both frequency deviation and rate of change of frequency will greatly facilitate the restoration process. In this thesis, a recursive algorithm for precise frequency and rate of change of frequency measurement is presented. The algorithm consists of three major steps. First, a rough frequency estimation for a data window is computed using a second order least error square approximation on the phasor angles of the input waveform. Then, a resampling based on the rough frequency estimation is carried out, followed by another second order least error square approximation to obtain the final results. The results of simulations using this approach are provided.
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Chapter One
Introduction

The theory of conservation of energy states that the total input energy to a system is equal to the total output energy plus energy consumed or lost within the system. In a power system, this implies that the total generated power at normal frequency being equal to the sum of all connected loads and all real power losses within the system. Any significant upset of this balance will be revealed by a change in the power system operating frequency.

When the power consumed by loads and losses is greater than the generated power, the system operating frequency will decrease, resulting in a situation classified as the underfrequency condition. This can happen when there is a loss of one or more generators, or when there is an increase in power demand during certain peak period of the day. When loads are suddenly lost in a system, input power will be greater than the consumed power. The extra power generated will be absorbed by the rest of the loads and the generator inertia, resulting in an increase in the operating frequency. This is classified as the overfrequency condition.

The above phenomenon can be verified by the simplified second order differential equation used to describe the motion of a generator rotor[WiAr]

\[
P_T = J\omega \frac{d^2 \delta}{dt^2} + D\omega \frac{d\delta}{dt} + \frac{E_s E_t}{X_s} \sin \delta
\]  

(1-1-1)

where \( P_T \) is the total input mechanical power into the system
\( J \) is the total moment of inertia of the rotor masses
\( D \) is the generator damping factor
\( E_s \) is the field generated voltage
\( E_t \) is the generator terminal voltage
\( X_s \) is the generator synchronous reactance and
δ is the phase angle by which \( E_g \) leads \( E_i \).

During steady state operation, the total input mechanical power, \( P_r \), is exactly equal to the electrical output power, \( \frac{E_g E_i}{X_g} \sin \delta \), hence δ is a constant. However, if the electrical output power is suddenly increased or decreased, δ will change accordingly. This means that the rotor rotational speed will decrease or increase respectively, resulting in a frequency change.

Efficient and safe operation will be jeopardized when the power system operates at off nominal frequencies significantly different from normal frequency. The performance of the power system will deteriorate and the whole system may eventually collapse if this is sustained. During underfrequency condition, the motor-driven auxiliaries will slow down, reducing generator output. Safety margins in generator cooling and bearing lubrication systems may become dangerously small. One of the most disastrous consequences of abnormal underfrequency operation is the accidental coincidence of the turbine blades' resonance frequency with the off-nominal frequency. This may damage the turbine blades, or result in their loss of life.

When the system is exposed to under-frequency condition, selective load shedding is employed to return the frequency back to its nominal value. During over-frequency condition, the output of one or more generators in the system must be reduced to restore the frequency to the nominal value, and then adjusted as necessary to maintain the nominal frequency.

As mentioned above, load unbalance is always faithfully portrayed by a change in the operating frequency. With an accurate system frequency measurement, one will know how much load or generated power needs to be shed during load unbalance in a power system. Moreover, if the rate of change of frequency is known, more accurate adjustment can be made to restore the operating frequency back to normal in the shortest period.

In the past thirty years, various techniques have been developed to measure power system frequency. With the introduction of digital computers, more sophisticated and
accurate algorithms are being developed to take advantage of the modern powerful high speed digital computers.

In this thesis, a technique of precise power system frequency and rate of change of frequency measurement will be presented. Chapter 2 takes a closer look at various frequency estimation algorithms developed in the past. Chapter 3 discusses in detail the new technique proposed here. The practical implementation of this algorithm will be reviewed in Chapter 4 and simulation results will be presented in Chapter 5. Chapter 6 includes a list of conclusions, and possible directions for further work in this field.
Chapter Two
Techniques of Frequency Estimation in Power Systems

2.1 Overview

A power system is a complex and interlocked structure. There are numerous factors that determine and dominate how it behaves. Moreover, the result of one factor may start a chain event which, if not carefully analyzed and corrected, will lead to catastrophic outcomes. In this section, the characteristics of the power system frequency deviation and major factors that affect its accurate estimation will be examined.

In order to maintain a stable, reliable and economical operating condition, various power systems are usually interconnected to form larger power networks. Each interconnected system is responsible for meeting its own obligations, with matters of mutual obligation or concern being governed by the operating guides provided by the North American Electric Reliability Council. The load on any interconnected power system is changing continuously and independently of other power systems in the network. The changing load is reflected by the fluctuating instantaneous frequency of the individual power system. Hence, the instantaneous frequencies at different points in the network at the same instant may be different. However, since all power systems are interconnected together, the frequency being measured here is actually the average system frequency. Thus, in practice, the frequencies measured at different points in the network will be essentially the same except during severe local transients.

From the above analysis, it should be clear that even though the system frequency may be low, it does not necessarily imply that the generation within the local system is insufficient to meet the area load. Conversely, there will be times when the system frequency is high but a local need for load reduction exists. Hence, after the frequency of the network has been estimated accurately, the Automatic Generation Control (AGC) is called upon to determine what actions should be taken.
Adams and McIntyre[AdMc] have given a detailed discussion of the characteristics of frequency deviation on the Eastern Interconnection. From their paper, it is seen that the power system frequency does not hover around 60 Hz, but crosses above and below this value so as to keep the running average close to nominal frequency. More importantly, they have shown that frequency deviation in power systems is a random parameter with a Gaussian like distribution. The Gaussian like probability density function strays to the right during low load periods and leans towards left under heavy system load.

Noise has been the ultimate challenge for precision measurement since it introduces errors in the measured quantities. Hence, the elimination or an attempt to reduce the influence of noise in a measurement technique has been a key objective.

A study published by the Electric Power Research Institute[EPRI] has classified power system noise as a combination of

- white noise
- harmonic noise
- random spike noise

White noise is defined as noise that has a flat power density spectrum containing all frequency components with equal power weighting. Since frequency measurements are restricted to finite bandwidths, a subset of white noise called band-limited white noise will be employed in this analysis.

Harmonic noise is due to the harmonic components of the system frequency.

Random spike noise is usually produced by external factors such as capacitor switching or random voltage noise induced by lightning.

Throughout the past thirty years, various techniques for measuring power system frequency have been developed. Computations are usually done on voltage waveforms because voltage waveforms, unlike current, are reasonably unchanging during normal
operating conditions. Generally speaking, the frequency measurement techniques can be divided into four basic categories:

1. Zero Crossing Techniques
2. Least Error Square Techniques
3. Frequency Domain Analysis
4. Kalman Filter Techniques

In this chapter, each technique will be examined in detail. Both the advantages and the disadvantages of each technique will also be discussed.

2.2 Zero Crossing Techniques

The most straightforward method for computing power system frequency is probably the zero crossing technique. Basically, the technique uses a counter to determine the number of counts between two consecutive positive going zero crossings of a voltage signal. The period of the voltage signal can be found by using the relationship

\[
\frac{1}{2} T = \frac{C_m}{C_{f_0}} \frac{1}{f_0}
\]

(2-2-1)

where \( T \) is the period of the voltage signal
\( C_m \) is the number of counts recorded for the voltage signal
\( C_{f_0} \) is the number of counts recorded for the signal operating at nominal frequency and
\( f_0 \) is the nominal frequency

Most of the existing frequency relays utilize the zero crossing technique. A rather simple circuit is needed for its implementation. This implies that the cost of the production is low. The technique also has the advantage of imposing minimum computation burden. However, it is rather vulnerable in the presence of noise. There is a
guard device present in relays using zero crossing technique. Whenever a zero crossing is detected at unreasonable premature instants (e.g., when noise spike crosses the zero crossing line), the whole cycle will be discarded. This means that the technique will function rather poorly in a noisy environment. Moreover, only two or three points in each voltage cycle are used to compute the frequency, i.e., very little signal information is being used. A new frequency can only be obtained at every cycle. The rate of change of frequency can be computed only through the gradient of the computed frequency.

Many variations of the zero crossing technique have been developed. Some variations take a counter window of more than two cycles to minimize the effect of noise. Some other variations transform the voltage signal to a rectangular wave several times the length of the period and determines the frequency from the rectangular waveform[GuPa].

In the next few paragraphs, a technique called level crossing technique[NgSr] will be examined in detail. Instead of measuring frequency at only the zero-crossings of a signal, the level crossing technique measures frequency at every incoming sample. The basic idea underlying this technique is using the slope of linearly interpolated samples to compute the frequency deviation.

Figure 2.2.1 shows a typical voltage waveform. It is seen that the amount of deviation from nominal frequency can be portrayed by a deviation in the period of the waveform. If the input signal is a pure sinusoid, the time deviation is really a ratio of the difference between voltage magnitude one nominal cycle apart and the difference of two consecutive voltage samples. This is represented by the following equations:

\[ n\tau - T = \frac{V(t) - V(t - n\tau)}{V(t) - V(t - \tau)} \tau \]  
\[ (2-2-2) \]

and

\[ n\tau - T = \frac{V(t) - V(t - n\tau)}{V(t - n\tau + \tau) - V(t - n\tau)} \tau \]  
\[ (2-2-3) \]

where \( T \) is the period of the waveform and
\( \tau \) is the sampling time interval.

\[
\begin{align*}
\text{Figure 2.2.1 Illustration of Level Crossing Technique}
\end{align*}
\]

Averaging Eq.(2-2-2) and (2-2-3) gives Eq.(2-2-4)

\[
\begin{align*}
n\tau - T &= \frac{[V(t) - V(t - n\tau)][V(t) - V(t - \tau) + V(t - n\tau + \tau) - V(t - n\tau)]}{2[V(t) - V(t - \tau)][V(t - n\tau + \tau) - V(t - n\tau)]} \tau \\
&= (2-2-4)
\end{align*}
\]

Eq.(2-2-4) is the underlying basic equation. Here, four samples are used to compute frequency at each incoming sample. The technique can be extended to more than four samples by using a weighted mean of several individual estimates. The weighted mean should have the maximum weight at samples where estimation will be more accurate.
(near zero crossing) and minimum weight at the least precise instants (near peaks and troughs). The authors suggested the following weight

\[ W(t) = [V(t) - V(t - \tau)][V(t - n\tau + \tau) - V(t - n\tau)] \]  (2-2-5)

In addition to meeting the above requirements, the weighting factor represented by Eq.(2-2-5) also has a constant sum over each complete half cycle in time.

Now, Eq.(2-2-4) can be extended to

\[ (m-1) \sum_{n}^{(m-1)r} (n\tau - T)W(t) = \frac{(m-1)r}{\sum_{t=0}^{(m-1)r} W(t)} \]  (2-2-6)

The level crossing technique is based on rather simple and straightforward mathematics. Hence, the computation is not really a burden. Also, the circuitry is relatively simple. Thus, the cost of manufacturing such a device will be relatively low. Each new frequency estimation can now be obtained in a fraction of a cycle instead of one frequency estimation per cycle as in the case of the zero crossing technique. Just like the zero crossing technique, this method has a good measurement dynamic, i.e., it can faithfully portray the frequency right after a major system disturbance since the initializing only takes one cycle.

However, since the Level Crossing Technique is doing a linear interpolation between every two samples, over and under-estimation will inadvertently be introduced. This is shown in Figure 2.2.2.
The Level Crossing Technique bases the whole computational procedure on the relative difference between amplitudes of difference cycles. This means that if the amplitude of the waveform is changing in a non-periodic way, such as when there is a decaying DC offset, or when the harmonic contents of the waveform are varying, or random noise is present in the system, the accuracy will be greatly affected. There is a weight factor used in this algorithm. However, the weight factor is not designed for lessening the above effects. It does, nevertheless, alleviate some effects caused by random noise and high frequency variations by taking an average of the computed frequencies. However, by introducing the weight factor, Nguyen and Srinivasan directly assume that the waveform is sinusoidal. In real power systems, it may not always be the case.

If the system frequency has decreased by more than one sampling interval, the technique will be doing extrapolation instead of interpolation. This will further decrease the accuracy of the algorithm.

2.3 Least Error Square Approximation
Taylor Series Expansion technique has been widely used for solving various nonlinear equations. Hence, it may be intuitive to measure the sinusoidal waveform frequency using Taylor Series Expansion.

Sachdev and Giray[SaGi] give a rather detailed analysis of this technique. A single frequency sinusoidal wave can be represented as

\[ v(t) = A_v \sin(2\pi ft + \theta) = A_v \cos \theta \sin(2\pi ft) + A_v \sin \theta \cos(2\pi ft) \]  

(2-3-1)

Expanding \( \sin(2\pi ft) \) and \( \cos(2\pi ft) \) into Taylor series centered around the nominal frequency, \( f_0 \), and taking the first three terms, Eq.(2-3-2) is obtained.

\[ v(t) \approx a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 \]  

(2-3-2)

where \( x_1 = A_v \cos \theta \), \( x_2 = (f - f_0)A_v \cos \theta \), \( x_3 = A_v \sin \theta \), \( x_4 = (f - f_0)A_v \sin \theta \),

\[ x_5 = \left(-\frac{(2\pi)^2}{2}f^2 + (2\pi)^2f f_0 - \frac{(2\pi)^2}{2}f_0^2\right)A_v \cos \theta \]

\[ x_6 = \left(-\frac{(2\pi)^2}{2}f^2 + (2\pi)^2f f_0 - \frac{(2\pi)^2}{2}f_0^2\right)A_v \sin \theta \]

and \( a_{11} = \sin(2\pi f_0 t_1) \), \( a_{12} = 2\pi f_1 \cos(2\pi f_0 t_1) \), \( a_{13} = \cos(2\pi f_0 t_1) \), \( a_{14} = 2\pi f_1 \sin(2\pi f_0 t_1) \), \( a_{15} = t_1^2 \sin(2\pi f_0 t_1) \), \( a_{16} = t_1^2 \cos(2\pi f_0 t_1) \)

Similarly, \( v(t_2) \approx a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + a_{26}x_6 \)

\[ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \]

\[ v(t_m) = a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + a_{m4}x_4 + a_{m5}x_5 + a_{m6}x_6 \]

All values of \( a's \) can be computed from the nominal frequency, \( f_0 \), and sampling instant, \( t \). \( v(t_1) \ldots v(t_m) \) are measured inputs. The only unknowns are values of \( x's \).
To find the values of $x$s, Eq.(2-3-2) is first written in its matrix form

$$AX = V$$     \hspace{1cm} (2-3-3)

where $A$ is $m \times 6$ and $X$ is $6 \times 1$ and $V$ is $m \times 1$.

Since there are 6 unknowns, $m$ must be greater or equal to 6 in order to obtain a solution. To solve for $x$ when $m > 6$, the least error square estimation is used

$$X = \left[\left(A^TA\right)^{-1} A^T\right]V$$     \hspace{1cm} (2-3-4)

After the solution for the vector $X$ is found, the deviated frequency can be computed using Eq.(2-3-5) when $\theta$ is small and Eq.(2-3-6) when $\theta$ is large

$$\frac{x_2}{x_1} = \frac{(f - f_0)A_c \cos \theta}{A_c \cos \theta} = f - f_0$$     \hspace{1cm} (2-3-5)

$$\frac{x_4}{x_3} = \frac{(f - f_0)A_c \sin \theta}{A_c \sin \theta} = f - f_0$$     \hspace{1cm} (2-3-6)

Another alternative is to combine Eq.(2-3-5) and (2-3-6) to obtain

$$(f - f_0)^2 = \frac{x_2^2 + x_4^2}{x_1^2 + x_3^2}$$     \hspace{1cm} (2-3-7)

The matrix $\left[\left(A^TA\right)^{-1} A^T\right]$ plays a rather important role in the whole estimation scheme. It is the least error square approximator. The accuracy of the whole algorithm relies heavily on the performance of this matrix. A least error square approximator can be simply viewed as a filter and whether the filter will amplify or suppress noise depends on its coefficients. If the magnitude of the eigenvalues of $\left(A^TA\right)^{-1}$ of the filter is less than 1.0, noise in the input to the filter will be reduced in the output. However, if the sum is greater than 1.0, noise will be amplified.
There are a number of factors that affect the pseudo-inverse matrix. Since all elements of the matrix are functions of time \( t \), it is obvious that by changing the sampling rate, one can change the value of every element in the matrix. From experimental data shown in [SaGi], it is seen that increasing the sampling rate improves the noise immunity characteristic.

The data window size also plays a significant part in determining the pseudo-inverse matrix. One may see intuitively that when more data is used in a least error square approximation, the error of the estimation will be reduced. This is partially true. The longer the data window used, the smaller the effect of noise on the output. However, one has to remember that the accuracy of this technique does not depend solely on how much noise is smoothed. The foundation of this algorithm is not the least error square approximation, but the approximation of the input waveform with the Taylor Series Expansion. The least error square approximation only builds upon the results of the Taylor Series Expansion. Hence, error is being introduced from the start. When a larger window is used, more expansion terms will also need to be employed to give an accurate approximation of the longer waveform. Thus, increasing the data window size alone will not increase the accuracy. One needs to increase both the window size and the number of Taylor Series terms to get a better result. Also, when approximating the sinusoidal waveform with Taylor Series Expansion, an assumption is made that the frequency within the data window is constant. This may not be true. If the window size is increased when the waveform frequency is not constant, the frequency deviation within the data window will be even larger. This will introduce additional errors.

More Taylor Series Expansion terms also imply more unknowns and bigger pseudo-inverse matrix. These all add up to a heavier computation burden. When a relay system is being designed, one of the most important considerations is its speed. In the frequency relay case, this means that the algorithm must be fast enough to determine the correct frequency and make appropriate decisions accordingly. Thus, Sachdev and Giray propose a time reference solution where the property of the waveform symmetry is being used to reduce the number of computations.
Recognizing the necessity for more accurate algorithm, Giray and Sachdev published another paper [GiSa], advancing one step further, detailing a way to improve the accuracy of frequency estimation using the above algorithm.

Eq. (2-3-6) states that frequency deviation is the ratio of term $x_4$ and $x_3$. Hence, the accuracy of the frequency estimation depends upon the accuracy of terms $x_4$ and $x_3$. Eq. (2-3-4) shows that accuracy of $x$s really depends on the values of pseudo-inverse of matrix $A$. As mentioned above, the pseudo-inverse of $A$ can be viewed as a filter. Now, let $|H_1(f)|$ be the frequency response to $x_3$, $|H_2(f)|$ be the frequency response to $x_4$ and $\tilde{f}$ be the estimated frequency, the estimated frequency deviation can be written as

$$|\Delta \tilde{f}| = |\tilde{f} - f_0| = \frac{|H_2(f)|}{|H_1(f)|} \frac{|H_2(f)|}{\Delta f} \frac{1}{|H_1(f)|} \Delta f$$

(2-3-8)

From Eq. (2-3-8), it is clear that $|\Delta \tilde{f}| = \Delta f$ if and only if $\frac{|H_2(f)|}{\Delta f} \frac{1}{|H_1(f)|} = \pm 1$. With this in mind, both $\frac{|H_2(f)|}{\Delta f}$ and $|H_1(f)|$ are plotted against frequency and it is seen in [GiSa] that for larger frequency deviations, the error introduced will also be larger. Hence, Giray and Sachdev suggest using a lookup table to compensate this error. Since both $\frac{|H_2(f)|}{\Delta f}$ and $|H_1(f)|$ are not time-variant values (they are functions of frequency), they can be computed beforehand and stored in a table. Thus, whenever a frequency has been computed, its correction factor will be looked up in the table. Using the correction factor, it is shown that the estimation error, especially when the frequency deviation is greater than 4 Hz, will be reduced.

2.4 Frequency Domain Analysis

2.4.1 Technique Using Leakage Effect [GiHa]
From the definition of Fourier transform, one knows that for a periodic waveform, the Fourier transform of any one complete cycle will produce an identical result in the frequency domain as that produced by its Fourier series. If, however, only part of a cycle is being used to compute the Fourier transform, there will be a discrepancy between the result of this Fourier transform and the function's Fourier series. This discrepancy is the leakage effect. With the above concept in mind, the leakage coefficient is written

$$\eta = \frac{\sum_{k=0}^{N-1} |X(k)| - |X(1)|}{|X(1)|}$$

(2-4-1)

where $X(k)$ are the Discrete Fourier Transform terms, and $X(1)$ corresponds to the fundamental component.

It is found[GiHa] that for small frequency deviation ($\pm 5$Hz), the relationship between the leakage coefficient, $\eta$, and the frequency deviation, $\Delta f$, for a sinusoidal waveform, is linear. In fact, the equation governing the relationship is

$$|\Delta f| = \frac{\eta}{0.095584345}$$

(2-4-2)

One thing needs to be noticed here is that the above relationship only holds when fundamental frequency is the only frequency component present, i.e., the leakage coefficient defined here is the leakage coefficient of the fundamental frequency only. This point will be further illustrated later in this section.

For a cosinusoidal waveform, the above relationship will not hold. Hence, a zero-crossing detector must be employed to detect positive going zero-crossing to yield the sinusoidal waveform.

Close examination of Eq.(2-4-1) reveals that the leakage coefficient will always be positive regardless of whether the frequency has increased or decreased. Thus, a method
for distinguishing between these two conditions must be developed. In [GiHa], Girgis and Ham propose a check on the fundamental component, \( X(1) \). If \( \text{Re}(X(1)) \) is positive, then the fundamental component has increased. A negative \( \text{Re}(X(1)) \) indicates a decrease in fundamental component.

Since the deviated frequency can only be found using sinusoidal waveform in the above technique, the introduction of zero-crossing detector will most likely introduce errors. Since no zero-crossing-detector is perfect, a time error \( \Delta t \) will always be introduced. This implies that the sine wave being used will possess a phase delay. Hence, the Fourier transform will not be a Fourier transform of a perfect sine wave. As the whole algorithm is based on the assumption that a perfect sine wave is being analyzed, this will definitely introduce an error in the computed frequency deviation.

The leakage coefficient is defined to be contributed by the fundamental frequency only. However, a real power voltage waveform contains many frequency components other than the fundamental component. For example, when the waveform is experiencing amplitude swinging, (which is equivalent to saying that the waveform is undergoing amplitude modulation), low frequency components will be introduced. During or immediately after a transient, the Fourier transform of a signal will contain various frequency components. Also, a power system contains various forms of noise and they will all show up in the frequency domain in addition to the fundamental frequency component. The presence of non-fundamental frequency components will definitely impair the technique's performance.

2.4.2 Technique Using Phasor Angle[PTA]

One of the most useful tools introduced in the power system analysis is phasors. Phasors have been used in many power system applications, including state estimation and system stability studies.
In this section, frequency measurement technique proposed by Phadke[PTA] using the voltage phasor angles will be briefly described.

A pure sinusoidal waveform can be expressed as

\[ x(t) = \sqrt{2} X \sin(\omega t + \phi) \] (2-4-3)

After sampling Eq.(2-4-3) with a sampling frequency N times per cycle, Eq.(2-4-3) can be rewritten into

\[ x_k = \sqrt{2} X \sin\left(\frac{2\pi}{N} k + \phi\right) \] (2-4-4)

The Discrete Fourier Transform of Eq.(2-4-4) is given by

\[ \bar{X} = \frac{1}{\sqrt{2}} \left( X_c + jX_s \right) \] (2-4-5)

where

\[ X_c = \frac{2}{N} \sum_{k=0}^{N-1} X_k \cos\left(\frac{2\pi}{N} k\right) \] and

\[ X_s = -\frac{2}{N} \sum_{k=0}^{N-1} X_k \sin\left(\frac{2\pi}{N} k\right) \]

Next, the recursive phasor computation technique is used to obtain the rth phasor, which is given below

\[ \bar{X}^{(r)} = \bar{X}^{(r-1)} + j \frac{1}{\sqrt{2}} \frac{2}{N} (x_{N+r} - x_r) \exp\left(-j \frac{2\pi}{N} (r-1)\right) \] (2-4-6)

From Eq.(2-4-6), the phasor of a sinusoidal wave at a frequency \( f = f_0 + \Delta f \) can be found to be:

\[ \bar{X}^{(r)}_{f_0+\Delta f} = \bar{X}^{(0)}_{f_0} \frac{\sin\left(\frac{N\Delta\omega\Delta t}{2}\right)}{N \sin\left(\frac{\Delta\omega\Delta t}{2}\right)} \exp\left(jr\Delta\omega\Delta t\right) \exp\left(j(N-1)\frac{\Delta\omega\Delta t}{2}\right) \]
\[ + \bar{X}^{(0)}_{f_0} \frac{\sin \left( \frac{N(\omega + \omega_0)\Delta t}{2} \right)}{N \sin \left( \frac{(\omega + \omega_0)\Delta t}{2} \right)} \exp \left( -j \frac{(N-1)(\omega + \omega_0)\Delta t}{2} \right) \exp(-jr(\omega + \omega_0)\Delta t) \]

(2-4-7)

where \( \bar{X}^{(0)}_{f_0} \) is the initial computation of the phasor at \( f_0 \) and 

\( \Delta t \) is time interval between sampled data

As seen in Eq.(2-4-7), if \( \Delta f \) is relatively small compared to \( f_0 \), the second term of Eq.(2-4-7) will be close to zero and we can simplify the above equation to

\[ \bar{X}^{(r)}_{f_0 + \Delta f} = \bar{X}^{(0)}_{f_0} \frac{\sin \left( \frac{N\Delta \omega \Delta t}{2} \right)}{N \sin \left( \frac{\Delta \omega \Delta t}{2} \right)} \exp( jr \Delta \omega \Delta t ) \exp \left( (N-1) \frac{\Delta \omega \Delta t}{2} \right) \]

(2-4-8)

In order to examine the relationship between the phasor and the waveform frequency, the computed phasor is broken down into its magnitude and angle parts. The magnitude of the new phasor is the product of the magnitude of the original phasor and a magnitude factor,

\[ |\bar{X}^{(r)}_{f_0 + \Delta f}| = \left| \bar{X}^{(0)}_{f_0} \right| \frac{\sin \left( \frac{N\Delta \omega \Delta t}{2} \right)}{N \sin \left( \frac{\Delta \omega \Delta t}{2} \right)} \]

(2-4-9)

The existence of this magnitude factor provides the basis for measuring frequency using frequency leakage technique described earlier.

By denoting the angle of the new phasor as \( \psi_r \) and the angle of the previous phasor as \( \psi_{r-1} \), one can approximate the new phasor angle as

\[ \psi_r = \frac{\Delta f}{f_0} \frac{2\pi}{N} r + \psi_{r-1} \]

(2-4-10)
Now, using the definition of derivatives,

\[ \frac{d\psi}{dt} = \lim_{t \to t_0} \frac{\psi_t - \psi_{t-1}}{t} \approx \frac{\psi_t - \psi_{t-1}}{1/Nf_0} = 2\pi \Delta f \]  

(2-4-11)

The system frequency can now be computed with

\[ f = f_0 + \Delta f = f_0 + \frac{1}{2\pi} \frac{d\psi}{dt} \]  

(2-4-12)

and the rate of change of frequency is

\[ \frac{df}{dt} = \frac{1}{2\pi} \frac{d^2\psi}{dt^2} \]  

(2-4-13)

By computing the phasor, the harmonic noise can be partially eliminated. With the help of a low pass filter, the spike voltage noise will not interfere with the frequency computation. With the help of least error square computation on redundant number of phasor angles, the accuracy of frequency computation and the rate of change of frequency computation will be greatly improved. Since the technique is based on sampled phasor data and most microcomputer-based power system devices installed already have the ability to compute phasors, the method can be added for low incremental cost to microcomputer-based devices that are performing other functions, without additional hardware.

The whole estimation, however, solely depends on the accuracy of the phasor angle measurement. It is seen that the second term of Eq.(2-4-7) is neglected to obtain the relationship between phasor angle and deviated frequency. This approximation will inadvertently introduce some error in the estimation. One way to alleviate the above
approximation error is to use the positive sequence voltage waveform as the input instead of the phase voltage waveform since computing the positive sequence quantities will introduce an automatic filtering effect. This filtering effect makes the second term of Eq.(2-4-7) identically zero.

Phasor measurement has been one of the major interests throughout the power industry. A great amount of researches have been dedicated to precision phasor estimation in the power industry. Since the accuracy of this technique depends highly on the accuracy of the waveform's phasor measurement, as the phasor measurement becomes more and more accurate, the frequency measurement utilizing phasor angles will also become more and more precise.

2.5 Kalman Filter Technique

2.5.1 Review of Kalman Filtering[GiBr]

Kalman filter technique has been widely used for conditioning data and estimating variables from noisy measurements. The technique is also suitable for estimating several variables from sampled data. The technique can be designed to filter random as well as harmonic noise.

The basic equations for Kalman filter are state equations

\[ x_{k+1} = \phi_k x_k + w_k \]  \hspace{1cm} (2-5-1)

where \( x_k \) is the state vector at step \( k \)
\( \phi_k \) is the state transition matrix
\( w_k \) is the process noise vector with a covariance matrix
\[ E[w_k w_j^T] = Q_k \] where \( Q_k = 0 \) if \( k \neq j \)
and measurement equation

$$z_k = H_k x_k + v_k \quad (2-5-2)$$

where \( z_k \) is the measurement vector at step \( k \)
\( H_k \) is the measurement matrix
\( v_k \) is the random measurement error vector with covariance
\[ E[v_k v_j^T] = R_k \] where \( R_k = 0 \) if \( k \neq j \)

Having the a priori estimate \( \hat{x}_{k/k-1} \) and \( P_{k/k-1} \) and assuming that \( E[w_k v_j^T] = 0 \) for all values of \( k \) and \( j \), the Kalman recursive equations can be set up.

The Kalman gain matrix is defined as

$$K_k = P_{k/k-1} H_k^T (H_k P_{k/k-1} H_k^T + R_k)^{-1} \quad (2-5-3)$$

The error covariance matrix can be updated with

$$P_k = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] = (I - K_k H_k) P_{k/k-1} \quad (2-5-4)$$

The measurement residual can be used to update the estimate by

$$\hat{x}_k = \hat{x}_{k/k-1} + K_k (z_k - H_k \hat{x}_{k/k-1}) \quad (2-5-5)$$

To project ahead, the following two equations are used

$$\hat{x}_{k+1/k} = \phi_k \hat{x}_k \quad (2-5-6)$$
$$P_{k+1/k} = \phi_k P_k \phi_k^T + Q_k \quad (2-5-7)$$
2.5.2 Two-State Kalman Filter [GiHw]

The two-state Kalman filter technique is actually a technique to compute the voltage phasor, not frequency. The two states, \( x_1 \) and \( x_2 \), of the Kalman filter are real and imaginary parts of a phasor respectively. Once phasor values are known, the relationship between the phasor angle and deviated frequency explained in [PTA] is used to compute the waveform frequency as described earlier in the Frequency Domain Analysis Technique.

The equation of a waveform can be written as

\[
v(t) = A_v \exp(j\omega t + \phi) = x_1 \cos \omega_0 t - x_2 \sin \omega_0 t
\]  

(2-5-8)

With the assumption that the amplitude of the signal has a Rayleigh distribution while the phase angle of the waveform has a uniform distribution, i.e., the real and imaginary parts of the phasor are both constants with a Gaussian noise, the state transition matrix can be defined as

\[
\Phi_k = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}
\]  

(2-5-9)

In this case,

\[
x_k = \begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix}
\]  

(2-5-10)

The measurement equation assumes the form

\[
z_k = \begin{bmatrix} \cos(\omega_0 k \Delta t) & -\sin(\omega_0 k \Delta t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + v_k
\]  

(2-5-11)

where \( v_k \) is the measurement error vector.
After knowing the initial estimation error covariance $P_o$ and the initial estimate $\hat{x}_o$, Eq.(2-5-1) through (2-5-7) are used to solve for real time phasors recursively.

Once the phasor values are known, the phasor angles can be calculated and the frequency deviation is computed using the equation \[
\frac{d\phi}{dt} \approx 2\pi\Delta f
\] where $\phi$ is the phasor angle in radians, just as described in the previous section.

It can be seen that the two-state Kalman filter technique utilizes the same relationship as the technique proposed in [PTA]. The only difference is that the phasors are computed using Kalman filter, which performs better only in the presence of noise with variance which changes with time. However, since the principles of the two techniques are basically the same, they both suffer from the same problem: a small error in phasor angle computation will produce a large error in frequency estimation.

2.5.3 Three-State Kalman Filter[GiHw]

The use of a three-state extended Kalman filter model to compute power system frequency is a rather interesting proposal. In this technique, the frequency deviation is represented by a third state variable, $x_3$. In this form, the frequency can be computed directly using Kalman Filter. The detailed mathematical derivation is given below.

Eq.(2-5-8) can be rewritten as

\[
\nu(t) = A_x e^{i[(\omega+\Delta\omega)t+\theta]}
\]
\[
= x_1 \cos(\omega_o t + 2\pi x_3 t) - x_2 \sin(\omega_o t + 2\pi x_3 t)
\]

(2-5-12)

where $x_1$ and $x_2$ still represent the real and imaginary components of the phasor respectively while $x_3$ represents the frequency deviation.

From Eq.(2-5-12), the Kalman measurement equation can be written as
\[ z_k = h_k(x) + v_k \]  \hspace{1cm} (2-5-13) \]

where

\[ h_k(x) = x_i \cos(\omega_0 k \Delta t + 2 \pi x_{3_k} \Delta t) - x_{2_k} \sin(\omega_0 k \Delta t + 2 \pi x_{3_k} \Delta t) \]  \hspace{1cm} (2-5-14) \]

The state transition matrix here will be a 3 x 3 Identity Matrix.

The recursive Kalman filter gain is

\[ K_k = P_{k-1} H_k^T \left( H_k (\hat{x}_{k-1}) P_{k-1} H_k^T (\hat{x}_{k-1}) + R_k \right)^{-1} \]  \hspace{1cm} (2-5-15) \]

where \( H_k (\hat{x}_{k/k-1}) = \frac{\partial h_k(x_k)}{\partial x_k} \bigg|_{x_k = \hat{x}_{k/k-1}} \) and from Eq (2-5-14), one obtains

\[ H_{11k} = \cos(\omega_0 k \Delta t + 2 \pi x_{3_k} - k \Delta t) \]
\[ H_{12k} = \sin(\omega_0 k \Delta t + 2 \pi x_{3_k} - k \Delta t) \]
\[ H_{13k} = 2\pi \left( H_{12k} \hat{x}_{1k-1} - H_{11k} \hat{x}_{2k-1} \right) \]

The state estimate update is

\[ \hat{x}_k = \hat{x}_{k/k-1} + K_k [z_k - h_k(\hat{x}_{k/k-1})] \]  \hspace{1cm} (2-5-16) \]

and the error covariance update is

\[ P_k = [I - K_k H_k (\hat{x}_{k/k-1})] P_{k/k-1} \]  \hspace{1cm} (2-5-17) \]

The results obtained from Kalman filter technique and the Frequency Analysis technique have been studied and compared in great detail by Girgis[GrHw]. The conclusion he obtained is that the Kalman filter technique is a very accurate technique, especially in the presence of noise.
The most significant advantage of the Kalman filter technique is its ability to filter out variable covariance noise. In fact, the Kalman filter technique produces results that are least affected by noise than those of other algorithms described in this thesis. Another advantage of the extended Kalman filter technique is that it allows different model representations, such as random constant, random walk or a combination of both, for the frequency deviation.

The Kalman filter technique has one major drawback: the whole technique is based on the assumption that the probability density functions of the state vectors are known. However, this is not always true. For example, during or right after a transient has occurred, the state vectors may not have a pure Gaussian probability density function. This will definitely affect the accuracy of the estimation. Moreover, since the purpose of the relay is to detect abnormality, the moment right after transient is usually the most crucial moment. Also, in order to minimize the noise effect on the measurement, high values for the variance of the measurement noise need to be used. At the same time, high variance will introduce unnecessary delay in obtaining the estimates, i.e., the response time will be even longer.

Also, in the extended Kalman filter technique, the Kalman gain matrix can no longer be computed off-line. This implies that a fast microprocessor or Digital Signal Processor is needed for its implementation. This problem can be partially solved by using a smaller sampling rate. It is seen in [GiHw] that lowering the sampling rate will increase the settling time of the estimated frequency deviation. However, the estimated magnitude and phase will be essentially unaffected.
Chapter Three
Accurate Frequency Estimation with Voltage Phasor Angles

3.1 Theoretical Concept

The technique proposed in this thesis is based on the fundamental principle described in section 2.4.2. It has been shown [PTA] that the time derivative relationship between phasor angle and waveform frequency becomes more accurate as the system frequency approaches the frequency assumed for establishing the sampling rate. Hence, after the first frequency approximation, a resampling scheme is proposed and new phasor angles of the resampled waveform are computed. From these angles, a final corrected frequency can be obtained.

In the following section, mathematical derivations for the relationship between phasor angles and system frequency will be illustrated in detail.

A real valued sinusoidal signal can be written as

\[
x = \text{Re}(\overline{X} \exp(j2\pi ft)) = \frac{\overline{X} \exp(j2\pi ft) + \overline{X}^* \exp(-j2\pi ft)}{2}
\]  

(3-1-1)

where \( \overline{X} \) is the phasor value of \( x \)
\( f \) is the signal frequency

The Discrete Fourier Transform of the rth sample of \( x(k\Delta T) \) can be represented as

\[
\overline{X}^{(r)} = \frac{2}{N} \sum_{k=r}^{r+N-1} x(k\Delta T) \exp(-j2\pi f_o k\Delta T)
\]

(3-1-2)

where \( \Delta T \) is the time between samples.
Substituting Eq. (3-1-1) into Eq. (3-1-2):

\[ \overline{X}^{(r)} = \frac{1}{N} \sum_{k=r-N}^{r-1} \left( \overline{X} \exp(j 2 \pi f_k \Delta T) + \overline{X}^* \exp(-j 2 \pi f_k \Delta T) \right) \exp(-j 2 \pi f_0 \Delta T) \]  

(3-1-3)

After changing the limits of summation

\[ \overline{X}^{(r)} = \frac{1}{N} \exp(j 2 \pi (f - f_0) r \Delta T) \sum_{k=0}^{N-1} \overline{X} \exp(j 2 \pi (f - f_0) k \Delta T) \]

\[ + \frac{1}{N} \exp(-j 2 \pi (f + f_0) r \Delta T) \sum_{k=0}^{N-1} \overline{X}^* \exp(-j 2 \pi (f + f_0) k \Delta T) \]  

(3-1-4)

Using the identity \( \sum_{j=0}^{N-1} \left( \exp(j \pi \zeta \Delta T) \right)^r = \frac{\sin \left( \frac{N \pi \zeta \Delta T}{2} \right)}{\sin \left( \frac{\pi \zeta \Delta T}{2} \right)} \), Eq. (3-1-4) can be rewritten as

\[ \overline{X}^{(r)} = \frac{\overline{X}}{N} \exp(j 2 \pi (f - f_0) r \Delta T) \frac{\sin \left( N \pi (f - f_0) \Delta T \right)}{\sin \left( \pi (f - f_0) \Delta T \right)} \exp(j (N - 1) \pi (f - f_0) \Delta T) \]

\[ + \frac{\overline{X}^*}{N} \exp(-j 2 \pi (f + f_0) r \Delta T) \frac{\sin \left( N \pi (f + f_0) \Delta T \right)}{\sin \left( \pi (f + f_0) \Delta T \right)} \exp(-j (N - 1) \pi (f + f_0) \Delta T) \]  

(3-1-5)

Now, let \( \overline{A} = \frac{\overline{X}}{N} \frac{\sin \left( N \pi (f - f_0) \Delta T \right)}{\sin \left( \pi (f - f_0) \Delta T \right)} \exp(j (N - 1) \pi (f - f_0) \Delta T) \) and

\( m = \frac{\sin \left( N \pi (f + f_0) \Delta T \right) \sin \left( \pi (f - f_0) \Delta T \right)}{\sin \left( \pi (f + f_0) \Delta T \right) \sin \left( N \pi (f - f_0) \Delta T \right)} \), Eq. (3-1-5) becomes

\[ \overline{X}^{(r)} = \overline{A} \exp(j 2 \pi (f - f_0) r \Delta T) \]

\[ + m \overline{A} \exp(-j 2 \pi (f + f_0) r \Delta T) \exp(-j 2 (N - 1) \pi f_0 \Delta T) \]  

(3-1-6)
Eq. (3-1-6) describes a planetary type of motion where the vectors $\vec{A}$ and $m\vec{A}^*$ rotate in opposite directions in the complex plane with $m\vec{A}^*$ rotates $f - f_0$ times faster than $\vec{A}$. Taking the time derivative of the phasor angle of Eq. (3-1-6):

$$
\frac{d\phi}{dt} = 2\pi \left( f \frac{1 - m^2}{1 + m^2 + 2m \cos(4\pi ft - 2\pi f_0 \Delta T + 2\pi Nf_0 \Delta T + 2\theta)} - f_0 \right) 
$$

(3-1-7)

where $\theta$ is the phasor angle of $\vec{A}$.

Here, it is seen that

$$
\frac{d\phi}{dt} = 2\pi \left( f \frac{1 - m^2}{(1 - m)^2} - f_0 \right) \quad \text{when} \cos(4\pi ft - 2\pi f_0 \Delta T + 2\pi Nf_0 \Delta T + 2\theta) = -1
$$

$$
\frac{d\phi}{dt} = 2\pi \left( f \frac{1 - m^2}{1 + m^2} - f_0 \right) \quad \text{when} \cos(4\pi ft - 2\pi f_0 \Delta T + 2\pi Nf_0 \Delta T + 2\theta) = 0
$$

$$
\frac{d\phi}{dt} = 2\pi \left( f \frac{1 - m^2}{(1 + m)^2} - f_0 \right) \quad \text{when} \cos(4\pi ft - 2\pi f_0 \Delta T + 2\pi Nf_0 \Delta T + 2\theta) = 1
$$

To understand the behavior of the relationship between the time derivative of the phasor angles and the waveform frequency, the characteristics of the factor $m$ is plotted against the waveform frequency in Figure 3.1.1.
From Figure 3.1.1, it is seen that when \( |f - f_0| \) is small, the factor \( m \) will also be relatively small (except when \( |f - f_0| = 0 \) where \( m \) becomes undefined. However, one can reach the exact relationship approximated by Eq.(3-1-8) by substituting \( f = f_0 \) in Eq.(3-1-6)) and we can approximate Eq.(3-1-7) as

\[
\frac{d\phi}{dt} \approx 2\pi(\Delta f) \tag{3-1-8}
\]

It is obvious from Figure 3.1.1 and Eq.(3-1-7) that the smaller the value of \( \Delta f \), the more accurate the estimation will be. Hence, the idea of resampling is introduced.

### 3.2 Proposed Technique Details[Phad]

The first step in this proposed technique is to compute the phasor. Let the sampling rate be \( N \) samples per cycle, the first phasor is computed using
\[ \bar{X} = \frac{1}{\sqrt{2}} \left( \frac{2}{N} \sum_{k=1}^{N} x_k \cos \left( \frac{2\pi k}{N} \right) - j \frac{2}{N} \sum_{k=1}^{N} x_k \sin \left( \frac{2\pi k}{N} \right) \right) \] (3-2-1)

Denoting \( \frac{2}{N} \sum_{k=1}^{N} x_k \cos \left( \frac{2\pi k}{N} \right) \) by \( X_c \) and \( -\frac{2}{N} \sum_{k=1}^{N} x_k \sin \left( \frac{2\pi k}{N} \right) \) by \( X_s \) and using the recursive formula derived in [Phad], each successive phasor can be computed with

\[
\begin{align*}
X_c^{(k+1)} &= X_c^{(k)} + \frac{2}{N} (x_{k+1} - x_{k+1-N}) \cos \left( \frac{2\pi}{N} k \right) \\
X_s^{(k+1)} &= X_s^{(k)} - \frac{2}{N} (x_{k+1} - x_{k+1-N}) \sin \left( \frac{2\pi}{N} k \right)
\end{align*}
\] (3-2-2)

Hence, a new phasor is available for every incoming sample after initialization (In actual simulation, as will be seen later, phasor computations are not carried out on every incoming sample). The angle of the \( k \)th phasor is given by

\[
\phi(k) = \tan^{-1} \frac{X_s^{(k)}}{X_c^{(k)}}
\] (3-2-3)

Assuming that voltage phasor angles vary as a quadratic function with respect to the sample number,

\[
\phi(k) = a_0 + a_1 k + a_2 k^2
\] (3-2-4)

Let a computation window of \( M \) phasor angles be used to estimate the value of coefficients \( a_0, a_1 \), and \( a_2 \),

\[
\begin{align*}
\phi(1) &= a_0 + a_11 + a_21^2 \\
\phi(2) &= a_0 + a_12 + a_22^2 \\
\vdots & \quad \vdots & \quad \vdots \\
\phi(M) &= a_0 + a_1M + a_2M^2
\end{align*}
\] (3-2-5)

Putting Eq. (3-2-5) in the form of matrices,
\[
\begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_M
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
1 & 2 & 2^2 \\
\vdots & \vdots & \vdots \\
1 & M & M^2
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
a_2
\end{bmatrix}
\text{ or }
\phi = Xa
\] (3-2-6)

The unknown matrix \( a \) can be solved using the least error square solution

\[
a = \left[ X^T X \right]^{-1} X^T \phi
\] (3-2-7)

Here, the pseudo-inverse matrix \( \left[ X^T X \right]^{-1} X^T \) is known as the gain matrix and can be computed off-line. One thing has to be remembered is that since the least error square approximation is actually doing a curve fit, all the phasor angles, \( \phi_s \), must be made monotonic, i.e., there should not be any wrap-around of angles. This can be easily done by appropriately adding \( \pm 2\pi \). Also, to keep the curve fitting uniform in all intervals and the arithmetic errors at a minimum, it is best to make \( \phi_1 \) to be zero. Hence, \( \phi_1 \) is subtracted from all subsequent phasor angles in the computation window.

Once the values of \( a_1 \) and \( a_2 \) are known, one can proceed to compute the frequency and the rate of change of frequency.

Taking the derivative of Eq.(3-2-4) with respect to \( k \)

\[
\frac{d\phi}{dk} = a_1 + 2a_2 k
\] (3-2-8)

The relationship between sample number and time can be expressed as

\[
k = Nf_0 t \quad \text{and} \quad \frac{dk}{dt} = Nf_0
\] (3-2-9)

where \( N \) is the number of phasor angles used in the computation per cycle
\[ f_0 \] is the nominal frequency.

Using Chain Rule,

\[
\frac{d\phi}{dt} = \frac{dt}{dk} \frac{d\phi}{dk} = Nf_0 (a_1 + 2a_z Nf_0 t)
\]

(3-2-10)

In section 3.1, it is shown that \( \frac{d\phi}{dt} \approx \Delta\omega \). Hence

\[
\Delta\omega \approx Nf_0 (a_1 + 2a_z Nf_0 t) \Rightarrow
\]

\[
\Delta f \approx \frac{1}{2\pi} Nf_0 (a_1 + 2a_z Nf_0 t)
\]

(3-2-11)

where \( t \) determines which instant inside the computation window the computed frequency corresponds to.

Now,

\[
\frac{d\Delta\omega}{dt} = 2\pi \frac{d\Delta f}{dt} \approx 2\pi \frac{df}{dt} \approx \frac{d^2\phi}{dt^2} = \frac{d}{dt} \left( Nf_0 a_1 + 2(Nf_0)^2 a_z t \right) = 2(Nf_0)^2 a_z \Rightarrow
\]

\[
\frac{df}{dt} \approx \frac{1}{2\pi} 2(Nf_0)^2 a_z
\]

(3-2-12)

Eq.(3-2-11) and (3-2-12) are the main formulas used for calculating the frequency and the rate of change of frequency. It is seen from Eq.(3-2-12) that the rate of change of frequency is assumed to be constant within the computation window. It has to be remembered that the frequency estimation obtained at the end of the computation window is actually the frequency at the half cycle point before the end of the window. This is because the phasor computed at the end of one cycle actually represents the phasor at the center. The frequency and the rate of change of frequency estimations obtained above are already rather accurate. However, since more precise frequency estimation is needed, one more step is required.
It has been shown earlier that the frequency estimation will be more accurate when the actual frequency approaches the frequency established for the sampling rate. Hence, resampling the waveform with the estimated frequency obtained and using the new phasor to perform a final correction estimation will be a rather attractive solution to achieve precision measurement.

To explain the idea of resampling in details, assume the nominal frequency is 60 Hz and the sampling rate is 1440 samples per second, which corresponds to \( \frac{1440}{60} = 24 \) samples per cycle. When the frequency has changed to 55 Hz, each cycle will now have \( \frac{1440}{55} = 26.18 \) samples. By re-normalizing through resampling, one tries to do a mathematical interpolation so that there will always be 24 samples per cycle regardless of the waveform frequency.

![Diagram](image)

*Figure 3.2.1 Illustration of Resampling Technique*
Refer to Figure 3.2.1 for the resampling algorithm, the values of $z_1$ and $z_2$ can be represented by the following two equations

$$z_1 = Z_m \sin(\varphi)$$  \hspace{1cm} (3-2-13)

$$z_2 = Z_m \sin(\varphi + \alpha) = Z_m \sin \varphi \cos \alpha + Z_m \cos \varphi \sin \alpha$$  \hspace{1cm} (3-2-14)

where $Z_m$ is the amplitude of the waveform

$\varphi$ is a sample instant and is an arbitrary known value

$\alpha$ is the interval between two samples at the new frequency and is

$$2 \pi f_{new} / (Nf_0)$$

Combining Eq.(3-2-13) and Eq.(3-2-14) results in

$$Z_m \cos \varphi = \frac{(z_2 - z_1 \cos \alpha)}{\sin \alpha}$$  \hspace{1cm} (3-2-15)

Let $x$ be the fractional distance between $z_1$ and $z_2$, the resampled point $z'$ is then given by

$$z' = Z_m \sin (\varphi + x \alpha)$$
$$= Z_m \sin \varphi \cos x \alpha + Z_m \cos \varphi \sin x \alpha$$  \hspace{1cm} (3-2-16)
$$= z_1 \cos x \alpha + (z_2 - z_1 \cos \alpha) \frac{\sin x \alpha}{\sin \alpha}$$

After resampling points have been found, the phasor angles of the new sampled data are computed and another estimation using Eq.(3-2-7), Eq.(3-2-11) and Eq.(3-2-12) is made to obtain the correction frequency, $\Delta f'$, and the final rate of change of frequency measurements.
The rate of change of frequency computed here is the final estimation of the rate. The final frequency estimation is computed using

\[ f_{\text{final}} = f_0 + \Delta f + \Delta f' \]  \hspace{1cm} (3-2-17)

Two major assumptions have been made on the input waveform when performing resampling. They are

- the input waveform can be accurate resampled with a pure sinusoidal waveform (i.e., the input waveform is free from any major distortions)
- the input waveform can be accurately resampled with a single frequency waveform (i.e., frequency deviation within the resampling window is small)

As will be shown in the Chapter 5, theses two assumptions may not be always true and hence, a residual error will remain. However, the residual error will be shown to be negligible for frequency relaying applications.

### 3.3 Algorithm

The algorithm basically consists of three major steps:

1) computes rough frequency estimation using raw voltage phasors from sampled data

2) resamples the waveform with the resampling frequency (The choice of resampling frequency will be explained later)

3) computes the correction frequency using the new voltage phasors of the resampled waveform and obtains the final results.
Since a relay system is being studied here, computation time is a rather crucial criterion. If all sampled data is used, there will be a huge amount of information present. This will take a rather long time to evaluate. Hence, the actual implementation of the algorithm only utilizes certain selected phasors. This will speed up the computation time considerably.

The number of cycles used to compute a frequency estimate is assumed to be 6 nominal cycles. Number of samples per cycle used to compute a phasor is 24. Hence, after initialization, each new phasor will be obtained in every \( \frac{1}{24 \times 60} = 0.6944 \) millisecond. However, only 4 phasor angles per cycle (i.e., \( 4 \times 6 = 24 \) phasor angles for the computation window) are used to compute the coefficients \( a_0, a_1 \) and \( a_2 \) in Eq.(3-2-7). The whole estimation proceeds recursively. The sliding window is 8 cycles long with 6 cycles for estimating frequency. The choice for 6 cycle over 4 or 8 cycles will be discussed further in section 5.2. The reason to use 8 cycle sliding window is:

1) It takes one cycle of sampled data to compute the first phasor value.

2) When frequency is less than 60 Hz, we need more data to fulfill the 6 cycle criterion for resampling. For example, let us say that \( f = 55 \) Hz. Number of samples needed to compute the frequency will now be \( 24 \times 6 \times \frac{60}{55} \approx 158 \) samples. Hence, number of 60 Hz periods needed will be \( \frac{158}{24} \approx 7 \) cycles.

Implementing resampling can be tricky. Since the resampling and subsequent frequency deviation computation all build on the first approximate frequency estimation, it is necessary to locate the exact instant in the waveform where resampling frequency is used. This is crucial when waveform frequency is not a constant (e.g., when the waveform is undergoing frequency swings).

It has been stated in Section 3.1 that the measured frequency using the proposed technique will exhibit an oscillatory behavior due to the planetary behavior of its phasor
angles. In an effort to minimize this error, the average of the estimated frequency for the computation window is taken as the resampling frequency, i.e.,

\[
\frac{\sum_{k=1}^{24} f_k}{24} = f_r
\] (3-3-1)

where \( f_k \) is the first frequency estimate for \( k = 1 \ldots 24 \) as seen in Figure 3.3.1

\( f_r \) is the resampling frequency

It is assumed that the resampling frequency, \( f_r \), corresponds to the frequency at the end of two and a half cycle in the first computation window (This is not true when frequency is changing within the computation window. However, the power system frequency deviation within the window is always too small to contribute a significant frequency estimation error). Hence, the second estimation frequency has to be computed at the same instance, i.e.,

\[
\Delta f' \approx \frac{1}{2\pi} Nf_r \left( a_1 + 2a_2 C \right)
\] (3-3-2)

where \( C \) corresponds to the sample instant of the estimation and is equal to

\[
C = \left( N\left( \frac{6}{2} - \frac{1}{2} \right) - \left( 6N - 6N \frac{f_0}{f_r} \right) \left( \frac{f_r}{f_r} + \frac{N}{2} \right) \right)
\]

and the final estimated frequency is simply

\[
f_{\text{final}} = f_r + \Delta f'
\] (3-3-3)
Figure 3.3.1 Illustration of Frequency Estimation

The fact that the estimated frequency is at 3.5 nominal cycles before the current time implies that the measurement has a delay of 3.5 nominal cycle or $\frac{1}{60} \times 3.5 = 0.0583$ second. This delay can be altered easily by changing the frequency estimation instant, the value $\ell$ in Eq. (3-2-11)

The complete flowcharts of the algorithm used for simulation are shown in Figure 3.3.2a and Figure 3.3.2b.
Figure 3.3.2a  Flow Chart of the Algorithm (Main Body)
SUBROUTINE
Initialization

Read in Gain Matrix

Read in first cycle of data

Compute first phasor angle

Read in more data

Compute phasor using iterated procedure

Compute the phasor angle

end of 8th cycle

Yes

EXIT

No

Figure 3.3.2b  Flow Chart of the Algorithm (Initialization)
Chapter Four
Practical Implementation

4.1 Overview of A Practical System

A prototype frequency relay using the above described technique is currently being developed. The frequency range of interest is between 56 Hz and 64 Hz with the nominal frequency located at 60 Hz. The input to the relay is a single phase voltage waveform since voltage is approximately constant at power system buses. The initialization takes 8 nominal cycles and all frequency computations have a time delay of 0.0583 second or three and a half nominal cycles. Figure 4.1.1 shows the block diagram of the entire system.

It has been shown in section 2.4.2 that the oscillatory nature of the phasor angle can be partially eliminated when positive sequence phasor is used in the simulation. Hence, one may wonder why the technique proposed here and the prototype being produced only uses the single phase voltage quantity. The answer is cost. When positive sequence phasor is needed, all three phase voltages have to be known. This means that additional equipment, such as more voltage transformers and analog input circuitry, will be needed. All these will add up to cost. A practical relay is one that achieves all the requirements and at the same time, requires to the least cost to build. As a result, various software controls (e.g., taking average of rough estimated frequencies as the resampling frequency) are employed in an effort to eliminate the errors introduced by single phase measurements without imposing additional cost.

In the following section, the function of each block will be briefly discussed. The memory and speed requirements for the relay will also be looked into.
Figure 4.1.1 Overall Block Diagram of the Frequency Relay
4.2 Peripheral Devices

4.2.1 Surge Filter

There will be two surge filters. One for input signals and the other for power supply unit. The surge filters should effectively filter out all high voltage transients from external environment and thus prevent/minimize their impact on the circuit. The filters are usually built to meet the industry standard ANSI/IEEE C37.90.[SWC]

4.2.2 Isolation Interface for Signals

Typical voltage that microelectronics equipment accepts is around 10V and typical input voltage signal to relay is at least a few times higher. Applying too much voltage or current to the electronic equipment will of course damage it. Hence, an isolation interface will be needed to:

1) step down the transmission line voltage to one (such as 12V) that is suitable for electronic equipment.

2) create an isolation medium between relay and its environment so that no outside induced voltages will directly enter into the relay.

The signal after the isolation circuit must duplicate the signal on the input line. The isolation interface should introduce as little phase and signal distortion as possible in the frequency range of 56 to 64 Hz. Either transformers or isolation amplifiers can be used to accomplish this task. The most significant advantage of using transformers is that it is more economical compared to using isolation amplifiers since transformers cost a lot less than isolation amplifiers. But, a transformer is more likely to introduce distortions in
the waveform. If there exists a high DC offset, it is more likely to drive the transformer into saturation. Same holds when the input signal magnitude is too high.

4.2.3 AC/DC Power and Its Isolation and Regulating Circuit

This is the power that supplies the whole circuit. It has to go through some isolation interfaces and a regulator before it enters the relay. As mentioned in the previous section, the purpose of the isolation interface is to make sure that no outside induced voltages go directly into the circuit and damage the electronic components.

4.2.4 Signal Conditioning

A well designed relay has to be secure, i.e., it should not trip when it is not supposed to. One factor that prevents frequency relay from making accurate frequency measurement is high distortions of input signals. Hence, signal conditioning block is introduced. This is where the undervoltage detector for both the signal voltage and the power supply voltage will be located. When the input signal is below a preset limit (this can happen in the case of loss of the signal input, loss of power supply or internal power supply failure), the frequency relay will be inhibited since a condition exists when the measurement made by the frequency relay will no longer be accurate enough to make a secure decision.

Another circuit may be included here to communicate with other relays which may be used in a supervisory capacity.

4.2.5 Low Pass Filter
A Low Pass Filter is needed to eliminate high frequency components and act as an anti-aliasing filter. At this time, a Sigma Delta A/D Converter is being considered for the frequency relay. It usually comes with an anti-aliasing filter. This alleviates the stringent requirements for a very sharp rolloff frequency of the low pass filter.

4.2.6 Programmable Gain Amplifier (PGA)

This is only an option since it will increase the level of cost and complexity. However, if the accuracy of the estimated frequency does not reach one's expectation and a major error due to quantization is introduced, the inclusion of a PGA can be considered. Also, it seems that by using a PGA, a lower resolution A/D Converter can be employed to achieve the same accuracy.

4.2.7 Analog to Digital Converter

16 bit Delta Sigma A/D Converter (Crystal CDB5326) is being considered for this application. From the specifications, it seems to be adequate for the purpose. It also has an internal anti-aliasing filter. This lessens the burden on the analog low pass filter. A good A/D Converter is crucial to accurate frequency measurement. Since the whole technique is based on sampled data and estimated phasor angles, the stability of the oscillator will affect the estimation accuracy to a considerable extent.

4.2.8 Mass Memory
This is where data are stored permanently from the RAM. A hard drive should be able to provide the function.

4.2.9 Communication

A serial port may be implemented to facilitate the transfer of results to a remote site where they can be analyzed.

4.2.10 Relay Contact

After frequency and rate of change of frequency have been estimated, the results will be passed through a decision circuit where a decision will be made as to whether to trip the relay, or to provide an alarm based up on user settings. If it is decided to trip the relay, an indicator light will turn on and the trip information will be stored until the reset button has been pressed or some other pre-defined actions have been taken.

4.3 Main Processor

4.3.1 Digital Signal Processor

The main processor being tested with this relay system is the Texas Instrument's Digital Signal Processing Chip TMS320C26. A more detailed discussion of the DSP chip will be outlined in a later section.
4.3.2 Random Access Memory

This is where the program will be executed. A detailed discussion of relay RAM size will be included in later sections.

4.3.3 Read Only Memory

The program will be stored here. The table of gain matrix data and the table of trigonometric data can also be stored here.

4.3.4 Erasable Programmable ROM

User defined settings may be stored here. These data can be changed by user from time to time.

4.4 Memory Consideration

Throughout the rest of this chapter, all discussions are based on the 4 angles per cycle and 6 computation cycle scheme. The choice of using this scheme for the frequency relay will be explained in chapter 5.
The TMS320C26 has built in ROM and RAM[TMS]. The Read Only Memory is 256 words. The Random Access Memory is broken into four blocks, B0 to B3, with B2 acting as Data Memory. B0, B1 and B3 can be programmed to be either Data or Program Memory. The sizes of B0 to B3 are 512, 512, 32 and 512 words respectively.

An approximation of the amount of data memory needed is summarized in Table 4.4.1. The Program Memory basically stores the whole program, which is about 60000 bytes. Hence, the total RAM needed will be around 64000 bytes.

<table>
<thead>
<tr>
<th>Data Name</th>
<th>Amount of Data</th>
<th>Bytes/Data</th>
<th>Total Data in Bytes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain Matrix Coefficient</td>
<td>$3 \times 4 \times 6$</td>
<td>4</td>
<td>288</td>
</tr>
<tr>
<td>Sampled Data</td>
<td>$24 \times 8$</td>
<td>4</td>
<td>768</td>
</tr>
<tr>
<td>Phasor Values</td>
<td>$2 \times 7 \times 4$</td>
<td>4</td>
<td>224</td>
</tr>
<tr>
<td>Phasor Angles</td>
<td>$2 \times 6 \times 4$</td>
<td>4</td>
<td>192</td>
</tr>
<tr>
<td>Resampled Data</td>
<td>$24 \times 7$</td>
<td>4</td>
<td>672</td>
</tr>
<tr>
<td>Resampled Phasor Values</td>
<td>$2 \times 6 \times 4$</td>
<td>4</td>
<td>192</td>
</tr>
<tr>
<td>Resampled Phasor Angles</td>
<td>$2 \times 6 \times 4$</td>
<td>4</td>
<td>192</td>
</tr>
<tr>
<td>Temporary Storage for</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>results</td>
<td>$2 \times 4 \times 6$</td>
<td>4</td>
<td>192</td>
</tr>
<tr>
<td>Other data</td>
<td>100</td>
<td>4</td>
<td>400</td>
</tr>
<tr>
<td>Total Data Memory</td>
<td></td>
<td></td>
<td>3120</td>
</tr>
</tbody>
</table>

It has been mentioned earlier that the whole algorithm relies heavily on the accuracy of input waveform phasor angles. Hence, a high precision trigonometric table is

---

1Other Data includes control flags, user set limits and etc.
needed for sine, cosine and arc tangent functions. The precision has to be around 0.00001 radian. To create a table storing all these values will require a large amount of ROM. The alternative is to approximate the trigonometric function using polynomials.

4.5 Speed Consideration

Since a relay system is being studied here, everything happens in real time. Thus, every frequency estimation has to be computed in a rather stringent time period. When 4 phasor angles per cycle scheme is used, each new frequency has to be computed every \( \frac{1}{4 \times 60} \) second. This means that the first phasor angle computation, the first Least Error Square fit, the resampling, the second phasor angle computation and the second Least Error Square fit all have to be done within \( \frac{1}{4 \times 60} \) second. The simulation program written in C takes around four seconds to run a one second simulation on a 486 33 MHz machine. If the code is rewritten in assembly language and DSP chip is used, the computation time will be much shorter.

An approximation of how fast the algorithm executes can be done by counting the number of operations the program needs to execute. One can ignore the initialization part for the moment since this only has to be done once. A rough estimation of number of floating point operations used in the algorithm has been computed to be around 15000 operations per frequency estimation. This turns out to be \( 15000 \times 4 \times 60 \approx 3.6 \text{MFLOPS} \). The 486 machine that the simulation runs on has an operation speed of 1 MFLOPS (this basically explains why it took about 4 seconds to run a 1 second simulation).

Most operations in the frequency estimation algorithm are iterative summations. Hence, the speed of operation can be tremendously improved by performing the whole operation on a DSP chip. The Texas Instrument TMS320C30 claims an operation speed of 33 MFLOPS. The graph shown in the Texas Instrument TMS320C26 menu depicts
that TMS320C26's Fixed-Point Generation Speed is around 16 MFLOPS. This should be enough to solve the real time speed problem.

4.6 Other Hardware Considerations

A digital filter (such as a double differentiator and smoother) may be installed to eliminate DC and low frequency components in the waveform. This can be either a circuit design, or may be some data manipulation controlled by the software.

The whole technique depends heavily on the accuracy of the sampling instants to obtain the right phasor angles. Hence, stability of the A/D clock is crucial to the accurate frequency measurement.
Chapter Five
Simulation Results

5.1 Overview

In the first part of this chapter, simulations are performed on various window sizes using the proposed algorithm to determine the optimal number of phasor angles needed to obtain an accurate measurement. In the second part, the validity of the proposed technique in the presence of distortions and discontinuities is verified. The sampling rate used here is 24 samples per cycle. The required frequency estimation accuracy is ±0.005 Hz.

5.2 Determination of Computation Window Size

As mentioned before, a real-time relay system is being designed. Hence, the response time of the relay is rather crucial. This means that accurate frequency measurements should be obtained in the least amount of time. The speed of the estimation is directly affected by the amount of data used for analysis. Assuming a computation window size of 6 cycles, if all 24 phasor angles per cycle are used in the frequency computation, the total amount of data needs to be processed will be \(24 \times 6 = 144\) phasor angles. If, instead, one selectively uses only 4 phasor angles per cycle to compute the frequency, the amount of data needed to be processed will be reduced to \(4 \times 6 = 24\) phasor angles. This will speed up the estimation process tremendously.

Clearly, the smaller the amount of data processed, the faster the response time will be. In order to determine how much that one really needs to achieve the desired accuracy, a few simulations using various number of phasor angles and computation windows are
performed. At the same time, the mathematical background underlying the whole technique will be reviewed graphically to provide a clearer understanding.

Simulations have been carried out on a one second long pure sinusoidal waveform using 4, 6 and 8 computation cycle window with number of phasor angles per cycle being 1, 2, 4, 8, 12 and 24. The range of simulation is from 56 Hz to 64 Hz. The maximum frequency estimation error and the maximum time duration for each scheme are obtained from simulations and tabulated in Table 5.2.1. The maximum time duration is obtained from executing the program on an Intel 80486 33 MHz computer.

Table 5.2.1 Maximum Estimation Error and Computation Time (Simulation Performed on a 486 66 MHz PC) Using Different Schemes

<table>
<thead>
<tr>
<th></th>
<th>4 Cycles</th>
<th>6 Cycles</th>
<th>8 Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Error(Hz)</td>
<td>Time(sec)</td>
<td>Error(Hz)</td>
</tr>
<tr>
<td>1 Angle</td>
<td>0.001040</td>
<td>1.3</td>
<td>0.000581</td>
</tr>
<tr>
<td>2 Angles</td>
<td>0.000802</td>
<td>2.1</td>
<td>0.000230</td>
</tr>
<tr>
<td>4 Angles</td>
<td>0.000744</td>
<td>3.2</td>
<td>0.000010</td>
</tr>
<tr>
<td>8 Angles</td>
<td>0.00034</td>
<td>6.6</td>
<td>0.000010</td>
</tr>
<tr>
<td>12 Angles</td>
<td>0.000029</td>
<td>10.4</td>
<td>0.000009</td>
</tr>
<tr>
<td>24 Angles</td>
<td>0.000026</td>
<td>22.3</td>
<td>0.000009</td>
</tr>
</tbody>
</table>

The accuracy that needs to be achieved is within 0.005 Hz over the range of 56 to 64 Hz. Looking at the Table, it seems that all schemes are acceptable. However, one thing needs to be remembered here is that pure sinusoidal waveform without any distortions is being used as the input waveform. When dealing with real power system voltage waveforms, there will be various noises and distortions that will influence the accuracy. Hence, in choosing the scheme to be used in the frequency relay from this Table, one has to allow a considerable amount of margin for error. With this in mind, it is
clear that the 4 angles per cycle scheme exhibits a huge improvement over the other schemes. When using more than 4 angles per cycle, the time needed for frequency estimation increases significantly while the accuracy remains approximately the same. Since a scheme with the least computation window to achieve a certain accuracy is desired, the 4 angle 6 cycle scheme seems most appropriate.

It is intuitive that the longer the computation window, the better the results. However, it might be puzzling that estimation error of 1 angle per cycle scheme is much higher than 4 angles per cycle scheme while the accuracy difference between 4 angles per cycle and 24 angles per cycle scheme is negligible. To understand this, the nature of phasor angles needs to be examined.

Figure 5.2.1 through 5.2.3 show plots of phasor angles versus sample number when frequency is at 58 Hz. As explained before, the slope of the Least Error Square of these angles will be the estimated frequency deviation. Comparing these three graphs, one can clearly see that the actual angles of the phasor exhibit "declining sinusoidal" phenomenon, (which is a direct reflection of the cyclic nature of off-nominal frequency depicted in Eq.(3-1-7)). This is accurately portrayed by Figure 5.2.1 when all phasor angles are plotted. In Figure 5.2.2, the trends can still be seen. However, in Figure 5.2.3, the "declining sinusoidal" phenomenon is lost because too few phasor angles are used in constructing the plot.
Figure 5.2.1 Phasor Angles for 24 Angles per Cycle Scheme

Figure 5.2.2 Phasor Angles for 4 Angles per Cycle Scheme
Figure 5.2.3 Phasor Angles for 1 Angle per Cycle Scheme

Figure 5.2.4 shows a close up view of the "declining sinusoidal" behavior. Now, if 4 points shown in the Figure are chosen from this "declining sinusoid", the slope of the LES of these 4 points may be different from the slope of the LES for the total phasor angles. Hence, an error arises in frequency estimation. This whole phenomenon may be related to the Nyquist Sampling Theorem: one has to have more than two samples per cycle in order to reconstruct one cycle of a periodic waveform. The 2 angles per cycle scheme does not work because when frequency falls below 60 Hz, there will be no longer 2 points per cycle after resampling.
5.3 Tests with Off-Nominal Constant Frequency Waveform

It has been shown in the last section that the six cycle computation window and four phasor angles per cycle scheme should be satisfactory in both accuracy and speed. In the following section, real power system conditions will be introduced into incoming waveform with non-varying frequency to test the validity of the scheme.

5.3.1 Pure Sinusoidal With Constant Frequency Input

The maximum estimation error for the 4 phasor angles per cycle and 6 cycle computation window scheme has been shown in Table 5.2.1. In this section, the detailed plots of the estimation results when input are pure sinusoidal waveforms at constant
frequency are displayed in Figure 5.3.1a through 5.3.1e. With these figures, comparisons can be made later to show the effect of distortions on the scheme.

![Frequency Estimation Graph](image1)

**Figure 5.3.1a** Measurement on Constant Frequency Waveform with No Distortions (f = 56 Hz)

![Rate of Change of Frequency Graph](image2)

![Frequency Estimation Graph](image3)

**Figure 5.3.1b** Measurement on Constant Frequency Waveform with No Distortions (f = 58 Hz)

57
Figure 5.3.1c  Measurement on Constant Frequency Waveform with No Distortions \((f = 60 \text{ Hz})\)

Figure 5.3.1d  Measurement on Constant Frequency Waveform with No Distortions \((f = 62 \text{ Hz})\)
Examining Figure 5.3.1a through 5.3.5e, it can be seen that the estimated frequency error and the rate of change error are almost the same whether the frequency is at 56 Hz, 58 Hz, 60 Hz, 62 Hz or 64 Hz. This is credited to the resampling technique used. As mentioned before, the frequency measurement using phasor angles technique becomes most accurate when the frequency approaches the frequency established for sampling. The resampling technique does just that. Hence, the result is very accurate throughout the frequency range between 56 Hz and 64 Hz.

Notice that the estimated frequency range plotted for each figure is $f_a \pm 0.0005$ Hz where $f_a$ is the actual frequency of the waveform while the rate of change of frequency range is $[df/dt]_a \pm 0.01$ Hz/sec where $[df/dt]_a$ is the actual rate of change of waveform frequency.

5.3.2 Effect of Harmonic Components

As outlined in [EPRI], power system noise can be viewed as composed of white noise, spike noise and harmonic noise. Both spike noise and white noise can be essentially
filtered by analog filters. Harmonic noise, however, has been a major challenge to most power system measurements. Low order harmonics will not be filtered by analog filters and they are usually present in the waveform entering the measuring device. In this section, simulations are performed on waveforms experiencing harmonic distortions while maintaining a constant operating frequency.

To generate a waveform with harmonic components present, Eq. (5-3-1) is used

\[ \nu(t) = \sin(2\pi ft) + \sum_{k=1}^{N} p_{2k+1} \sin(2\pi ((2k+1)f)t) \]  \hspace{1cm} (5-3-1)

where \( f \) is the fundamental frequency

\( p_{i} \) is the \( i \)th harmonic percentage

Higher harmonics become negligible after the waveform is passed through various filtering processes while the third harmonic, being the largest and most troublesome harmonic in a power system, may still be present. Since the third harmonic is the predominant harmonic that will influence the measurement, the input waveform is generated with a simplified equation

\[ \nu(t) = \sin(2\pi ft) + p_{3} \sin(2\pi (3f)t) \]  \hspace{1cm} (5-3-2)

\( p_{3} \) is set to 10\% since this is usually the highest harmonic percentage present in a power system during quasi steady state operation. Two cycles of the generated input waveform are shown in Figure 5.3.2.
Figure 5.3.2 Input Waveform with 3rd Harmonic Distortion

Figure 5.3.3a through 5.3.3e show the results of simulations.

Figure 5.3.3a Measurement on Constant Frequency Waveform with 10% 3rd Harmonic Distortions Present (f = 56 Hz)
Figure 5.3.3b Measurement on Constant Frequency Waveform with 10% 3rd Harmonic Distortions Present (f = 58 Hz)

Figure 5.3.3c Measurement on Constant Frequency Waveform with 10% 3rd Harmonic Distortions Present (f = 60 Hz)
Figure 5.3.3d Measurement on Constant Frequency Waveform with 10% 3rd Harmonic Distortions Present (f = 62 Hz)

Figure 5.3.3e Measurement on Constant Frequency Waveform with 10% 3rd Harmonic Distortions Present (f = 64 Hz)

It is obvious that the presence of harmonic distortions increases the estimation error at off-nominal frequency (Notice that the frequency range being plotted here is $f_a \pm 0.005$Hz and the rate of change range is $\pm 0.1$ Hz/Sec). The reason is that the amplitude of the waveform is changing rather rapidly and the computed phasor will no
longer be accurate. However, when the operating frequency is at 60 Hz, there is not any significant increase in the estimation error (Figure 5.3.3c). This is because 24 samples are used to compute the phasor for each cycle. When the operating frequency is at 60 Hz, there are exactly 24 samples per cycle. Hence, the filtering effect of the phasor computation effectively filters out all the third harmonic component. However, when the operating frequency is off-nominal, there will not be 24 samples per cycle. The presence of harmonic renders the phasor computation inaccurate. The larger the frequency deviation from the nominal frequency, the higher the effect will be felt by the presence of the harmonic components. Moreover, when performing resampling, the voltage waveform will no longer be able to be resampled accurately with a pure sinusoidal function at a fixed frequency. Hence, a significant error is being introduced. Figure 5.3.4 depicts this resampling error when frequency is at 56 Hz.

Figure 5.3.4 Resampling Error When Frequency is 56 Hz
In order to understand why the magnitude of oscillation error for 56 Hz is larger than that for 64 Hz, the resampled phasor angles are examined.

![Figure 5.3.5 Phasor Angle After Resample: (a) frequency = 56 Hz; (b) frequency = 64 Hz](image)

It is seen from Figure 5.3.5 that the phasor angles after resampling when frequency is at 56 Hz has less than two "oscillations" within its computation window while the phasor angles have more than two "oscillations" when frequency is at 64 Hz. Hence, the gradient of the LES angle fit is less susceptible to variation at 64 Hz than at 56 Hz. This translates to smaller oscillation errors at 64 Hz than 56 Hz as seen in Figure 5.3.3e and 5.3.3a. The phasor angle oscillations seen here is more likely to be attributed to the inaccuracy of phasor computation and resampling due to the presence of third harmonic component than the nature of phasor angle oscillations explained in Chapter 3.

5.3.3 Effect of Amplitude Swing
During a transient condition, the power system voltage waveform may experience certain kinds of amplitude swings. To see how this can affect the frequency measurement, input voltage waveform is generated with the following equation:

\[ v(t) = A(1 + m \sin(2\pi f_m t)) \sin(2\pi f_c t) \]  \hspace{1cm} (5-3-3)

where \( A \) is the amplitude of the waveform
\( m \) is the modulation index
\( f_m \) is the modulating frequency and
\( f_c \) is the power system frequency

For a typical power system, the modulation index will be less than 10% while the modulating frequency will be around 1 Hz. Hence, these criteria are used for the input waveform generation. The input waveform for one second duration is plotted in Figure 5.3.6 and the results of measuring \( f_c \) which range from 56 Hz to 64 Hz are shown in Figure 5.3.7a through 5.3.7e.
Figure 5.3.7a Measurement on Constant Frequency Waveform Experiencing 10% Amplitude Swing

(f = 56 Hz)

Figure 5.3.7b Measurement on Constant Frequency Waveform Experiencing 10% Amplitude Swing

(f = 58 Hz)
Figure 5.3.7c  Measurement on Constant Frequency Waveform Experiencing 10% Amplitude Swing
(f = 60 Hz)

Figure 5.3.7d  Measurement on Constant Frequency Waveform Experiencing 10% Amplitude Swing
(f = 62 Hz)
Figure 5.3.7e  Measurement on Constant Frequency Waveform Experiencing 10% Amplitude Swing

($f = 64$ Hz)

Figure 5.3.7a through 5.3.7e all exhibit about the same magnitude of error. Hence, the error introduced by the amplitude swing is not really affected by power system operating frequency, $f_c$. Rather, it is a function of modulation index and modulation frequency. The estimation error here is much less than the estimation error when harmonic distortions are present. This is due to the fact that only gradual change is present in the waveform here while in harmonic distortions case, distinct waveform amplitude change occurred at every single cycle. The gradual change in the one second period here enables the phasor computation to be more accurate than the case of harmonic distortion. The place where waveform amplitude changes the most is when time = 0, 0.5 and 1 second (refer to figure 5.3.6) and the place where waveform amplitude changes the least is around 0.25 and 0.75 second. Thus, when actual frequency is at 60 Hz, the maximum error occurs at 0, 0.5 and 1 second and the minimum error occurs at 0.25 and 0.75 second, as depicted in Figure 5.3.7c. However, when actual frequency is off nominal, it will be hard to predict exactly where the maximum and minimum points are since cancellations prevail as the phasor angles will assume more or less a ramp function. However, the maximum and minimum error should still be around the same places as depicted in Figure 5.3.7a, b, d and e.
5.4 Tests with Variable Frequency Waveform

It is not unusual for a power system to change its frequency gradually during quasi steady state operation. However, the frequency variation is generally small. The proposed technique will inadvertently introduce some error when doing measurements. This is due to:

- The frequency within the computation window will no longer be constant. Thus, a resampling error is expected since the assumption made on performing resampling is that the waveform within the computation window can be represented with a single frequency sinusoid. However, since the change in frequency is rather small within the computation window, this will not introduce a significant error.

- The resampling frequency is the average of the first approximations of frequencies from the beginning of half cycle before the computation window to the end of five and a half cycle of the computation window (see Figure 3.3.1). It is assumed that this frequency corresponds to the frequency at the end of two and a half cycle instant of the computation window. This is no longer true since frequency is changing within the window. Hence, when second frequency approximation is performed, there will exist a time difference which transforms into an error in the final estimated frequency.

- Since frequency is changing, there may be instants such that the waveform phasor angle within the computation window is not able to be faithfully represented by the second degree least error square fit. This, it is believed, accounts for major errors.

5.4.1 Effect of Constant Rate of Change of Frequency
In this section, the effect of waveform experiencing a constant rate of change of frequency is studied. Errors arising from the two points mentioned above will be examined in detail here.

The resampling error for one computation window is plotted in Figure 5.4.1. It is seen that the smallest error occurs at around the center of the window, where resampling frequency is closest to the actual frequency of the waveform, while maximum errors occur at 2 ends of the window, where the actual frequency of the waveform is greater or smaller than the resampling frequency. Since the rate of change of frequency is relatively small (4 Hz/Sec), it is seen that the resampling error is negligible (Max. Error < 6×10⁻⁵). Hence, resampling error will not contribute much to the frequency estimation error.

![Resampling Error](image)

**Figure 5.4.1 Resampling Error When Rate = -4 Hz/Sec**

The resampling frequency error for rate of change of frequency with a true value of -4, -2, 2 and 4 Hz/Sec is plotted in Figure 5.4.2. It is seen that there exists an offset in
all cases. The error offsets for -4 Hz/sec and 4 Hz/sec cases are symmetric about the x-axis and so are -2 Hz/sec and 2 Hz/sec cases. It is believed that this contributes to the final offset error shown in Figure 5.4.3 (note that final frequency estimation error for ±4 Hz, ±2 Hz have the same amount of offset if oscillations are ignored. This corresponds to the symmetry of the resampling frequency error). Also, since the resampling frequency when the rate is increasing is larger than actual frequency (as can be seen from the resampling frequency error graphs Figure 5.4.2a and b), the number of computation cycles used for final estimation will be less than 6 cycles. Hence, the gradient of LES fit will vary more easily, resulting in a larger fluctuation error than the cases when frequencies are decreasing. All these can be seen in the simulation results shown in Figure 5.4.3a through 5.4.3d. Another point worth mentioning is that as the magnitude of rate of change of frequency becomes larger, so is the offset error. This is simply due to the fact that larger rate of change of frequency implies a greater deviation of frequency within the computation window. Hence a larger resampling frequency error offset. The fact that as frequency deviates further from the nominal frequency, the oscillation error increases is a direct reflection of the increasing resampling frequency oscillation error shown in Figure 5.4.2.

![Resampling Frequency Error](image)

**Figure 5.4.2 Resampling Error:** (a) Rate = -4 Hz/Sec; (b) Rate = -2 Hz/Sec
Figure 5.4.2 Resampling Error: (c) Rate = 2 Hz/Sec; (d) Rate = 4 Hz/Sec

Figure 5.4.3a Measurement on Waveform Experiencing Constant Rate of Change of Frequency (Rate = -4 Hz/Sec)
Figure 5.4.3b Measurement on Waveform Experiencing Constant Rate of Change of Frequency
(Rate = -2 Hz/Sec)

Figure 5.4.3c Measurement on Waveform Experiencing Constant Rate of Change of Frequency
(Rate = 2 Hz/Sec)
5.4.2 Effect of Frequency Swing

Other than experiencing a constant rate of change of frequency, a power system also occasionally experiences frequency swing phenomena. During quasi steady state operation, the swing is gradual and mild.

In this section, the algorithm is tested against a waveform experiencing frequency swing. To generate the input waveform with frequency swing, the following equation is used

\[ v(t) = A \sin \left( 2\pi \left( f_1 t - \frac{f_2 \cos(2\pi f_3 t)}{2\pi f_3} \right) \right) \]  

where \( A \) is the amplitude of the waveform
\( f_1 \) is operating frequency on which the swinging occurs
\( f_2 \) is the maximum frequency swing
\( f_3 \) is the number of complete swings per second
The frequency deviation is a function of time and is given by

$$\Delta f(t) = f(t) - f_0 = f_1 - f_0 + f_2 \sin(2\pi f_3 t)$$  \hspace{1cm} (5.3-3)

In the simulations that follow, $A$ is set to 1, $f_2$ is set to 0.1 Hz and $f_3$ is set to 1. The graph of actual frequency and the rate of change of frequency being measured is shown in Figure 5.4.4

![Graph of actual frequency and rate of change of frequency](image)

**Figure 5.4.4** Actual Input waveform Frequency and Rate of Change of Frequency
Figure 5.4.5a Measurement on Waveform Experiencing Frequency Swing

\( f_1 = 56 \text{ Hz} \)

Figure 5.4.5b Measurement on Waveform Experiencing Frequency Swing

\( f_1 = 58 \text{ Hz} \)
Figure 5.4.5c Measurement on Waveform Experiencing Frequency Swing

\( f_1 = 60 \, \text{Hz} \)

Figure 5.4.5d Measurement on Waveform Experiencing Frequency Swing

\( f_1 = 62 \, \text{Hz} \)
Figure 5.4.5c Measurement on Waveform Experiencing Frequency Swing 
\((f_e = 64 \text{ Hz})\)

Figure 5.4.5a through 5.4.5e depict the results of measurement. It is seen that the largest error occurs when frequency deviation is highest. To get an understanding of the frequency estimation error behavior here, the phasor angles of a waveform swinging at a center frequency of 60 Hz will be examined. Since the frequency is varying in a sinusoidal fashion, the waveform phasor angle will be varying in a cosinusoidal fashion.

Figure 5.4.6 and 5.4.7 display the phasor angles when time is around 0.25 second (when the frequency has swing to its highest point) and when time is around 0.50 sec (when frequency is at its center frequency) respectively.

When frequency deviates most from its center frequency, the LES fit for phasor angle before resampling gives a rather precise representation of the actual phasor angle behavior. However, since the phasor angle is changing in a cosinusoidal fashion, the computation window near the zero crossing point is not exactly a straight line. It looks more like a 3rd order polynomial function with the point of inflection near the center of the computation window. This is seen after resampling has been carried out (Figure 5.4.6b). The second order least error square fit is not able to faithfully portray this behavior. Hence, an error exists.
When frequency is crossing its center frequency, the LES fit for phasor angle before resampling has a rather poor performance. However, after resampling, the LES gives an exact representation of the actual phasor angle behavior (Figure 5.4.7b). Since the second approximation is used as a correction for the first approximation, if the LES is able to represent the phasor angles after resampling exactly, one is able to eliminate all the error obtained during the first approximation and hence, an exact measurement can be obtained.

![Graph 1](image1.png)

**Figure 5.4.6 Phasor Angle At 0.25 Second**

![Graph 2](image2.png)

![Graph 3](image3.png)

**Figure 5.4.7 Phasor Angle At 0.50 Second**
5.5 Effect of Discontinuity

One of the most fundamental requirements for a relay is that it has to be secure, i.e., it should false trip. The technique proposed here relies heavily on the smooth curve of phasor angles within the computation window while the smoothness of the phasor angles can be affected by various conditions: a sudden jump in voltage waveform as in the case when load changes, a sudden change in the phase of voltage waveform as in the case when a switching operation is performed and so on. In fact, any discontinuities in the input waveform will affect the phasor angle computation. To counteract these situations, a guard flag is introduced to inhibit the relay from tripping when certain conditions are met.

The principle of guard flag operation is illustrated in Figure 5.5.1. Referring to the Figure, the phasor angle gradient at earlier instants is used to project with a tolerance the range of values that the phase angle at the next instant would be if the waveform is continuous. If the next point is within the projection, as in the case of point 2 as predicted from point 1, the waveform is continuous. However, if the next point is outside the projection, as in the case of point 3 as projected from point 2, there must exist a discontinuity between the two points and the guard flag is turned on, inhibiting the relay from tripping.
In this section, the guard flag operation is tested. Simulations are carried out on waveforms when a fault has occurred. To generate the input voltage waveform, the Electromagnetic Transient Program (EMTP) is used to simulate a simple system shown in Figure 5.5.2. The parameters of the system are obtained from [Kong]. A three-phase fault is staged since three-phase fault produces the largest disturbance in a power system and the purpose of this simulation is to examine how the proposed frequency algorithm works during such a disturbance. The fault starts at 0.5 second and has a duration of 0.05 second. The breakers reclose at 1.05 second. The generator damping factors are left blank so that a prolonged post fault power swing can be observed. The monitoring device is placed on Bus 2. The voltage waveform is first stepped down and passed through a low pass filter having a cut off frequency at 360 Hz before feeding it to the frequency measurement algorithm. The low pass filter is a two-stage RC filter shown in Figure 5.5.3
Figure 5.5.2 One Line Diagram of the Simulation Network

Figure 5.5.3 Two Stage RC filter used in EMTP simulation
The rotor angle difference between machine 1 and 2 during the whole simulation period is shown in Figure 5.5.4. It is seen that once the fault has been cleared, the rotor angle difference starts to converge. Hence, this is a stable system. The rotor angle difference is also directly related to the system frequency.

![Rotor Angle Between Two Machines](image)

**Figure 5.5.4 Rotor Angle Difference**

During the period of waveform discontinuity due to fault and breaker operations, the frequency estimation is no longer valid and the guard flag is turned on. In order to approximate the frequency within this period, a simple quadratic fit for frequency is employed. It is known that the frequency has to be continuous on a power system waveform. The rate of change of frequency is assumed to be continuous at the instant when guard flag resets. Hence, the initial conditions are

\[ f(0^+) = f(0^-) \]  \hspace{1cm} (5-5-1)
\[ f(t_{\text{last}}) = f(t_{\text{last}}^+) \]  \hspace{1cm} (5-5-2)

and

\[ \left. \frac{df}{dt} \right|_{t = t_{\text{last}}} = \left. \frac{df}{dt} \right|_{t = t_{\text{last}}^+} \]  \hspace{1cm} (5-5-3)

where \( f(t) \) is the frequency as a function of time,

\( t = 0 \) is the time that guard flag is turned on and

\( t = t_{\text{last}} \) is the time that guard flag is turned off

With the above initial conditions, the coefficient of the quadratic fit \( f(t) = at^2 + bt + c \) can be computed as

\[ a = \frac{t_{\text{last}} \left. \frac{df}{dt} \right|_{t = t_{\text{last}}} - f(t_{\text{last}}) + f(0)}{t_{\text{last}}^2} \]  \hspace{1cm} (5-5-4)

\[ b = \frac{f(t_{\text{last}}^-) - \frac{1}{2} t_{\text{last}} \left. \frac{df}{dt} \right|_{t = t_{\text{last}}^-} - f(0)}{\frac{1}{2} t_{\text{last}}^2} \]  \hspace{1cm} (5-5-6)

\[ c = f(0) \]  \hspace{1cm} (5-5-7)

The result of a ten second simulation is plotted in Figure 5.5.5a with Figure 5.5.5b zoomed in on the time period when the guard flag is activated. Figure 5.5.6a and 5.5.6b depict the rate of change of frequency for the whole simulation and the time period when the guard flag is activated respectively.
As expected for a stable system, the frequency after the fault has been cleared is swinging back towards the nominal frequency, 60 Hz. The decaying frequency swing agrees with the rotor angle difference seen in Figure 5.5.4. Also, the guard flag operates
normally, inhibiting the relay when the presence of fault and breaker operations creates a discontinuity in the phasor angle.

To understand why a quadratic frequency fit is used for frequency estimation within the blocking window, the actual measurements obtained using the proposed technique during the discontinuities are plotted in Figure 5.5.7.

![Figure 5.5.7 Raw Measurement With Input Waveform From EMTP Simulation](image)

It is seen that during the period of discontinuity, the raw measurement results display large irregular frequency jumps. This is not the true frequency of the waveform. By introducing the quadratic frequency fit within the blocking period, a rather smooth frequency transition period is obtained.
Chapter Six
Conclusions

The six cycle computation window and four phasor angles per cycle scheme is very accurate for frequency and rate of change of frequency measurement. In fact, it is seen that for a pure sinusoidal waveform operating at constant frequency within the frequency range of 56 Hz to 64 Hz, the measurement error is only about 0.00001 Hz (section 5.1). Even when distortions are introduced, the estimation error is still much smaller than 0.005 Hz (section 5.3). The proposed algorithm also performs with great precision when measuring frequency variations (section 5.4).

It is believed that the estimation errors seen are due to the following two reasons in the algorithm:

- the inability to compute an accurate phasor of a distorted waveform and failure of resampling the input waveform with a single frequency sinusoid (in the case of harmonic distortions).

- the inability to faithfully portray the phasor angle behavior with second order polynomial least error square (in the case of frequency swing).

Nevertheless, all simulation errors are smaller than 0.005 Hz. Hence, the frequency measurement using phasor angles and resampling technique is very well suited for high precision measurement. However, if more accurate results are desired, one can attempt to eliminate the sources of error by

- introducing noise components while doing resampling to eliminate the resampling error.

- using a higher order polynomial to portray the frequency behavior within the computation window.
Even though the precision measurement here is done with four phasor angles per cycle scheme, it can be deduced from the explanations given in section 5.2 that three phasor angles per cycle scheme should work as well.

The proposed technique is rather robust: one is able to obtain the frequency at any instant within the computation window by changing the value of time $t$ in Eq. (3-2-10). Throughout the simulations done in Chapter 5, the algorithm takes 8 nominal cycles for initialization. After which, frequency at the beginning of the 3.5 nominal cycle before the present instant is being computed. This nominal delay of 3.5 cycles or $\frac{1}{60} \times 3.5 = 0.0583$ second can be altered easily by changing the frequency estimation instant $t$. Theoretically speaking, since the frequency computed at the end of a cycle is actually the frequency at the center of the cycle, the minimum delay that can be achieved with the proposed technique is 0.5 nominal cycle or $\frac{1}{60} \times 0.5 = 0.0083$ second. However, if one really needs to know the waveform frequency at the end of the computation window, a simple extrapolation scheme can be designed.

The proposed technique is able to perform accurately in both over and underfrequency conditions. Moreover, as shown in Section 4.5, the proposed technique is fast enough to be implemented on a practical relay. Hence, this technique can be used in both generation control and load shedding schemes.

In addition to high precision measurement and robustness, the technique imposes a low incremental cost to microcomputer based devices. This may be the biggest advantage in practical implementation. Moreover, as the processing speed of electronic equipment becomes faster, the frequency relay in the future may become both a relay and a monitoring device, i.e., its primary purpose is still system protection while at the same time, it will be given a time history ability to capture data during transient period and transmit them to the remote station for detailed analysis. The invention of the Phasor Measurement Unit (PMU) will greatly facilitate this feature.
During the past few years, most frequency measurement techniques proposed use the relationship between phasor angles and deviated frequency. Almost all of them attempt to achieve accuracy by eliminating error caused by noise (whereas the technique proposed here attempts to achieve a precision measurement by eliminating the fundamental error introduced from the relationship between phasor angles and system frequency through resampling). Those noise elimination techniques range from using a simple FIR filters to the employment of more sophisticated algorithms, such as Neural network[ChSl] and MUSIC[Ein]. However, since sophisticated techniques usually impose heavy computation burdens, it is not clear at the present moment whether all those techniques can be implemented to produce a practical relay that operates in real time. However, it is certain that as technology becomes more advanced, all these techniques can be added on to the technique proposed here to obtain the ultimate precision measurement.
References


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Vita

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