

# MAGNETO-ELASTIC INTERACTIONS IN A CRACKED FERROMAGNETIC BODY

by

Satenik Harutyunyan

Thesis submitted to the faculty of the  
Virginia Polytechnic Institute and State University  
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE  
in  
MATERIALS SCIENCE AND ENGINEERING

Approved by:

Dr. William T. Reynolds, Chair  
Dr. Davresh Hasanyan  
Dr. Yu Wang

November 30, 2006  
Blacksburg, Virginia

**Keywords:** Magnetoelasticity, Ferromagnetic, Nonlinear Law of Magnetization, Crack

# **MAGNETO-ELASTIC INTERACTIONS IN A CRACKED FERROMAGNETIC BODY**

Satenik Harutyunyan

## **ABSTRACT**

The stress-strain state of ferromagnetic plane with a moving crack has been investigated in this study. The model considers a soft magnetic ferroelastic body and incorporates a realistic (nonlinear) susceptibility. A moving crack is present in the body and is propagating in a direction perpendicular to the magnetic field. Assuming that the processes in the moving coordinates are stationary, a Fourier transform method is used to reduce the mixed boundary value problem to the solutions of a pair of dual integral equations yielding to a closed form solution. As a result of this investigation, the magnetoelastic stress intensity factor is obtained and its dependency upon the crack velocity, material constants and nonlinear law of magnetization are highlighted. It has been shown that stress result around the crack essentially depend on external magnetic field, speed of the moving crack, nonlinear law of magnetization, and other physical parameters. The results presented in this work show that when cracked ferromagnetic structure is under the influence of magnetic field it is necessary to take into account the interaction effects between deformation of the body and magnetic field and that such interaction can bring to a new conditions for strengthening the materials. Closed form solutions for the stress-strain state are obtained, graphical representations are supplied and conclusions and prospects for further developments are outlined.

## **ACKNOWLEDGMENTS**

Thanks to the NSF GRFP for providing me with the Fellowship for 2004-2007 academic year to pursue my graduate study at Virginia Tech.

This work would not have been possible without the help of many people who it is my pleasure to thank here:

I would like to thank my advisor, Dr. William Reynolds, for allowing me to work with him, for his long hours of valuable discussions, constant support and guidance throughout my research.

I am grateful to my co-advisor, Dr. Davresh Hasanyan, for his constant support and above all for his confidence in me. Without his help I would not have finished this work.

I would also like to express my gratitude to my committee member, Dr. Yu Wang, for agreeing to be on my committee and also I am thankful for his helpful discussions.

Many thanks to Mariko Reynolds, for her emotional and enthusiastic support that made me feel at home in the United States.

Thanks to Narmin Aloyan for being always ready to share and reason my concerns.

I am grateful to all of my friends, for their invaluable friendship that went beyond boundaries of language and geography.

Many thanks to my sister, Ruzanna Kazharyan, for her continuous love and support throughout my life.

Foremost appreciation goes to my Mom, Hratsin Yeritsyan. This work is dedicated to her, for her unconditional love and patience.

# TABLE OF CONTENTS

<b>Abstract</b>	ii
<b>Acknowledgements</b>	iii
<b>Table of Contents</b>	iv
<b>List of Figures</b>	vi
<b>Magneto-Elastic Interactions in a Cracked Ferromagnetic Body</b>	1
<b>Objective</b>	1
<b>1. Introduction</b>	1
<b>2. Basic Equations and Boundary Conditions of Ferromagnetic Body</b>	4
<i>§2.1 Magnetostatics and Maxwell's Stress Tensor</i>	4
<i>§2.2 The General Equations in an Euler System of Coordinates</i>	9
<i>§2.3 The General Equations in Lagrange System of Coordinates</i>	11
<i>§2.4 The Linearized Equations and Boundary Conditions</i>	14
<i>§2.5 Different Magnetic Susceptibilities of a Soft Ferromagnetic Materials</i>	18
<b>3. Stress-Strain State of Ferromagnetic Plane with a Moving Crack</b>	21
<i>§3.1 Mathematical Modeling</i>	21
<i>§3.2 Governing Equations and Boundary Conditions</i>	22
<i>§3.3 Solution Methodology</i>	25
<i>§3.4 Magnetoelastic Stresses and Stress Intensity Factor</i>	26
<b>4. Numerical Computations and Discussions</b>	30
<i>§4.1 Stress Intensity Factor</i>	30
<i>§4.2 Magnetoelastic Stresses</i>	40
<b>Conclusions</b>	47

<b>Potential Work</b>	48
<b>References</b>	49
<b>Appendix A: <i>Nomenclature</i></b>	52
<b>Appendix B: <i>The Solution of Dual Integral Equations</i></b>	54
<b>Appendix C: <i>Magnetization Laws in Terms of Normalized Parameters</i></b>	56
<b>Vita</b>	58

## LIST OF FIGURES

<b>Figure 1:</b> <i>A volume element separated by a surface of discontinuity</i>	7
<b>Figure 2:</b> <i>Geometry of the deformed body in a magnetic field</i>	9
<b>Figure 3:</b> <i>Dependence of magnetic susceptibility versus magnetic field for pure iron</i>	19
<b>Figure 4:</b> <i>Ferromagnetic plane with a crack in a magnetic field</i>	22
<b>Figure 5:</b> <i>Dependence of normalized stress intensity factor on dimensionless magnetic field for various values of parameters with the magnetization expressed in a) Dreifous form</i>	32
<b>Figure 6:</b> <i>Dependence of normalized stress intensity factor on dimensionless magnetic field for various values of parameters with the magnetization expressed in b) Linear form</i>	34
<b>Figure 7:</b> <i>Dependence of normalized stress intensity factor on dimensionless magnetic field for various values of parameters with the magnetization expressed in c) Rayleigh form</i>	36
<b>Figure 8:</b> <i>Dependence of normalized stress intensity factor on dimensionless magnetic field for various values of parameters with the magnetization expressed in d) Magneto rigid form</i>	38
<b>Figure 9:</b> <i>Normalized magnetoelastic shear stress versus dimensionless magnetic field for various values of parameters</i>	41
<b>Figure 10:</b> <i>Normalized magnetoelastic tensile stress versus dimensionless magnetic field for various values of parameters</i>	44

# **MAGNETO-ELASTIC INTERACTIONS IN A CRACKED FERROMAGNETIC BODY**

## **OBJECTIVE**

The purpose of this research is to investigate the effect of a magnetic field on the stress-strain state around a crack in a ferromagnetic material.

Specifically, we wish to determine if the stress distribution at a crack tip can be altered by an applied magnetic field and to quantify the magnetic contribution to the brittle failure criteria.

## **1. INTRODUCTION**

Ferromagnetic materials are considered good candidates for the first wall in magnetic fusion energy (MFE) reactor. Being in presence of harsh loading conditions, including thermal and mechanical loads and of strong magnetic field the first wall of MFE reactor could be subjected to stress concentration around structural defects or cracks. It clearly appears that such a possibility could constitute an obstacle toward the development of this energy source.

The recent years have witnessed increased interest in investigations of the magnetoelasticity for ferromagnetic materials. The general theory of magnetoelasticity for ferromagnetic solids is developed by many authors [1-2, 4-5, 7, 12-13, 15, 17, 21]. The theory suggested by [1, 15] can be applied to the investigation of the phonon-magnon coupling (magnetoacoustic resonance). The coupling is pronounced when the wave frequency is near or above the magnetic resonance frequency, which is usually

higher than  $10^9 Hz$  (for example magnetoacoustic resonance in a material Yttrium-Iron-Garnett is observed when the frequency of spine waves  $\approx 10^{10} Hz$ ). In the papers [2, 3, 8, 11] a theory is presented for the magnetoelastic interactions in a soft ferromagnetic material of multi-domain structure when magnetic field intensity vector  $\vec{H}$  and magnetization vector  $\vec{M}$  are parallel in a rigid body state:  $\vec{M} = \chi(|\vec{H}|)\vec{H}$  ( $\chi$  is called the magnetic susceptibility). Notice that a soft magnetic material is characterized by small hysteresis losses (narrow hysteresis loop for  $H - M$  curves) and low remnant magnetization. Many Nickel-Iron alloys used widely as core materials for motors, generators, inductors and transformers are of this type.

The results related to the magnetoelastic stability and vibrations of thin-walled bodies, the stress deformation state (SDS) and also the wave propagation of ferromagnetic materials with linear law of magnetization are of special interest [10, 16, 19-20].

In a large number of works related to the theory of magnetoelasticity of ferromagnetic materials, it has been supposed that the magnetic field is linearly connected with the magnetization of the material, e.g. a soft ferromagnetic with a linear law of magnetization was considered [1,10,13-15, 19-23]. However, this assumption is valid for the ferromagnetic in the domain of very weak magnetic field or for most of nonferromagnetics in the strong magnetic field. With the application of ferromagnetic materials and structures in a strong magnetic field, i.e. higher than 1 Tesla (such as the structure of first wall in a fusion reactor), the magnetic field generated in the structures may be near to the region of saturation of the constitutive relation of magnetic field. In such cases to consider non-linear dependence between the magnetization and the magnetic field become imperative. That is why it becomes interesting to consider non-



linear dependence between the magnetization and the magnetic field as well as their influence on various physical processes (the buckling of thin ferromagnetic plate; the influence of nonlinearity of magnetization on stress -strain state; wave propagation and etc.).

In this work the problem of stress-strain state of ferromagnetic plane with a moving crack is discussed in detail.

## 2. BASIC EQUATIONS AND BOUNDARY CONDITIONS OF ELASTIC FERROMAGNETIC BODY

In this chapter we will present all necessary equations and boundary conditions for the elastic soft ferromagnetic body in an external magnetic field.

### §2.1 MAGNETOSTASICS AND MAXWELL'S STRESS TENSOR

We denote the magnetic field generated by a permanent magnetic or an induction coil in a vacuum by the magnetic induction vector  $\vec{B}^0$  with units Weber/meter<sup>2</sup> in MKSA system. When a magnetizable, elastic solid is placed in the field, magnetic moments are induced in the body. The magnetic moment per unit volume of a deformed body is called the magnetization and it is denoted by  $\vec{M}$  (Amper/meter). Inside the body, the magnetic induction is  $\vec{B}$  which is not necessarily to be equal to  $\vec{B}^0$ .

The induced magnetization is related to  $\vec{B}$  by  $\vec{B} = \mu_0(\vec{H} + \vec{M})$  where  $\vec{H}$  (Amper/meter) is called the magnetic intensity and  $\mu_0 = 4\pi \times 10^{-7}$  Newton/Amper<sup>2</sup> is a universal constant. The  $\vec{H}$  vector is introduced for convenience. In a vacuum,  $\vec{M} = 0$ , and  $\vec{B} = \mu_0\vec{H}$ ; in a body  $\vec{H}$  is related to  $\vec{M}$  by a constitutive law. The  $\vec{B}$  satisfies Gauss's law for magnetism and solenoid, and  $\vec{H}$  is governed by Ampere's law.

For a rigid solid, the constitutive law is written as  $\vec{H} = \vec{H}(\vec{M})$  which assumes a particular simple form, and  $\vec{H} = \chi^{-1}\vec{M}$  for a paramagnetic or diamagnetic isotropic material. The dimensionless constant  $\chi$  is called the magnetic susceptibility. For a deformable solid,  $\vec{H}$  depends on  $\vec{M}$  as well as on the displacement gradient of the body.

When a body is moving in a magnetic field or when the field is changing in time, an electric field  $\vec{E}$  and a current  $\vec{J}$  are generated. The  $\vec{E}$  is related to the  $\vec{B}$  through Faraday's law. For a rigid solid with electric conductivity  $\sigma$ ,  $\vec{J} = \sigma \vec{E}$ . Vectors  $\vec{E}$ ,  $\vec{B}$ ,  $\vec{J}$ , and  $\vec{H}$  are coupled and complying Maxwell's equations [1-5, 7-8, 10-15, 17, 19-23]:

$$\text{curl} \vec{E} = \frac{\partial \vec{B}}{\partial t} \Rightarrow \text{Faraday's Law}$$

$$\text{curl} \vec{H} = \vec{J} \Rightarrow \text{Ampere's Law}$$

$$\text{div} \vec{B} = 0 \Rightarrow \text{Gauss' Law}$$

$$\vec{J} = \sigma \left( \vec{E} + \frac{\partial \vec{v}}{\partial t} \times \vec{B} \right) \Rightarrow \text{Generalized Ohm's Law}$$

where

$$\vec{v} = \frac{\partial \vec{u}}{\partial t}; \vec{u} \text{ is a displacement vector of material particles.}$$

In this investigation we consider only a slowly moving body in a stationary magnetic field. Thus, the coupling of the induced current  $\vec{J}$  and the deformation will not be considered here. This is certainly valid for a magnetic insulator ( $\sigma = 0$ ). For paramagnetic conductors, this omission is not justified, as the effect of induced current is more pronounced than that of induced magnetization. However, for some of soft ferromagnetic materials, as a first approximation, we may omit the induced current effect. The effect of induced current in the form of a Lorentz force (body force) on an elastic waves has been investigated by [1,15]. Many papers on that subject are reviewed by [1,13,15].

Referring the volume  $V$  occupied by a body at any instant to a Cartesian coordinate system, we denote the position of a material particle at time  $t$  by  $\vec{R}(X_1, X_2, X_3)$ . In the presence of a magnetic induction  $\vec{B}^0 = \mu_0 \vec{H}^0$ , the total force  $\vec{F}$  and torque  $\vec{L}$  acting on a body are [1-3, 4, 7, 12, 13, 15, 17]:

$$\vec{F} = \mu_0 \int_V \vec{M} \cdot \nabla \vec{H} dV \quad (2.1)$$

$$\vec{L} = \mu_0 \int_V [\vec{R} \times (\vec{M} \cdot \nabla \vec{H}) + \vec{M} \times \vec{H}] dV \quad (2.2)$$

However, the above expressions do not imply that within a volume element  $dV$ , there is a body force per unit volume  $\mu_0 \vec{M} \cdot \nabla \vec{H}^0$  or a body couple per unit volume  $\mu_0 \vec{M} \times \vec{H}^0$  because of the additional interaction between the magnetization within the element  $dV$  and the surrounding magnetized material. Furthermore, by applying various vector formulas and magnetostatic equations, the above integrals can be transformed into surface integrals or volume integrals with entirely different integrals. Thus on an element with volume  $dV$  and surface  $dS$ , the net forces exerted by externally applied field and surrounding magnetized materials can not be uniquely specified.

The magnetic body force  $\vec{f}$  and the body couple  $\vec{c}$  acting on the mass within volume  $dV$  are:

$$\vec{f} = \mu_0 \vec{M} \cdot \nabla \vec{H}, \quad \vec{c} = \mu_0 \vec{M} \times \vec{H} \quad (2.3)$$

In the above equations the  $\vec{M}$  and  $\vec{H}$  inside the magnetized body are related to the external magnetic induction  $\vec{B}_0$  through a set of magnetoelastic field equations, constitutive equations and boundary conditions as given in the usual mechanical tractions to form a stress vector  $\vec{t}^{(n)}$ .

With these understandings, a magnetoelastic stress tensor  $t_{ij}$  may be defined as

$$t_{ij} n_i = t_j^{(n)} \quad (2.4)$$

where  $\vec{n}$  is a unit vector normal to the surface  $dS$  upon which  $\vec{t}^{(n)}$  is acting. Because of the assumed extrinsic body couple, the magnetoelastic stress tensor  $t_{ij}$  is not symmetric.

The balance equation for linear momentum then can be simply stated as

$$\int_S t_{ij} n_j dS + \mu_0 \int_V M_i H_{j,i} dV = \frac{d}{dt} \int_V \rho v_j dV, \quad \text{where } H_{j,i} = \frac{\partial H_j}{\partial X_i} \quad (2.5)$$

The local balance equation is given in the next paragraph. The usual mechanical body force is not considered here.

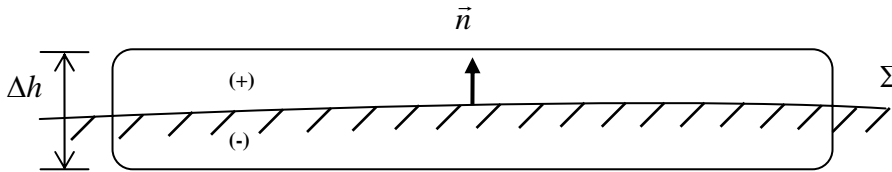
When the volume integral of the body force in (2.5) encloses a region which is separated by a surface of discontinuity  $\Sigma$  (Figure 1), the body force is discontinuous at this surface on account of the existence of a surface magnetization  $M^s$  which is defined as

$$\vec{n} \cdot (\vec{H}^+ - \vec{H}^-) = \vec{n} \cdot \vec{M}^s = M_n^s$$

where (+) and (-) indicate the two sides of  $\Sigma$ . Thus,

$$\begin{aligned} \mu_0 \int_V \vec{M} \cdot \nabla \vec{H} dV &= \mu_0 \int_{\Sigma} dS \int_{\Delta h} \vec{M} \cdot \vec{n} \frac{\partial \vec{H}}{\partial n} dn = \mu_0 \int_{\Sigma} dS \int_{\Delta h} M_n dH \\ &\xrightarrow{\Delta h \rightarrow 0} \mu_0 \int_{\Sigma} \frac{1}{2} (M_n^+ + M_n^-) (H^+ - H^-) dS \\ &= \mu_0 \int_{\Sigma} \frac{1}{2} (M_n^+ + M_n^-) M_n^s n dS \end{aligned} \quad (2.6)$$

where use has been made of the fact that the tangential components of  $\vec{H}$  are continuous across the surface  $\Sigma$ .



**Figure 1: A volume element separated by a surface of discontinuity**

Another way to evaluate the effect of surface magnetization is to introduce a tensor  $T_{ij}$  such that

$$\nabla T \equiv \bar{f}$$

where

$$T_{ij} = B_i H_j - \frac{1}{2} \mu_0 H_k H_k \delta_{ij} \quad (2.7)$$

We then can apply the divergence theorems to change the volume integral of  $f_i$  to a surface integral of  $n_i T_{ij}$  and evaluate the jump of the normal components of  $T_{ij}$  across the surface  $\Sigma$ , which is just the integrand in the final expression of (2.6). The quantity  $T_{ij}$  is known as a Maxwell's stress tensor.

With the magnetic force and moments introduced as in (2.3), there is obviously an energy supply to the medium, which equals to work done by these forces. This has been carefully evaluated in [12-13, 21]. The final result for the rate of change of energy per unit volume is

$$\varepsilon = \mu_0 M_i \frac{\partial H_j}{\partial x_i} v_j + \mu_0 \rho H_i \frac{d}{dt} \left( \frac{M_i}{\rho} \right) \quad (2.8)$$

where  $d/dt = (\partial/\partial t) + v_j(\partial/\partial X_j)$  and  $v_j = dX_j/dt$ . The first term is clearly the rate of work done by the body force  $f_j$ ; the second term is the rate of work done by the body couple combined with the rate of change of the magnetization with respect to axes that translate and rotate with the magnetic moment vector.

## §2.2 THE GENERAL EQUATIONS IN AN EULER SYSTEM OF COORDINATES

Let a particle of deformed solid originally at  $x_j$  ( $j = 1,2,3$ ) is moved, after deformation to  $X_J$  ( $J = 1,2,3$ ) at time  $t$  (Figure 2). The Cartesian System of coordinates  $X_J$  is called moving or Euler coordinates and  $x_j$  nonmoving or Lagrange system of coordinates.

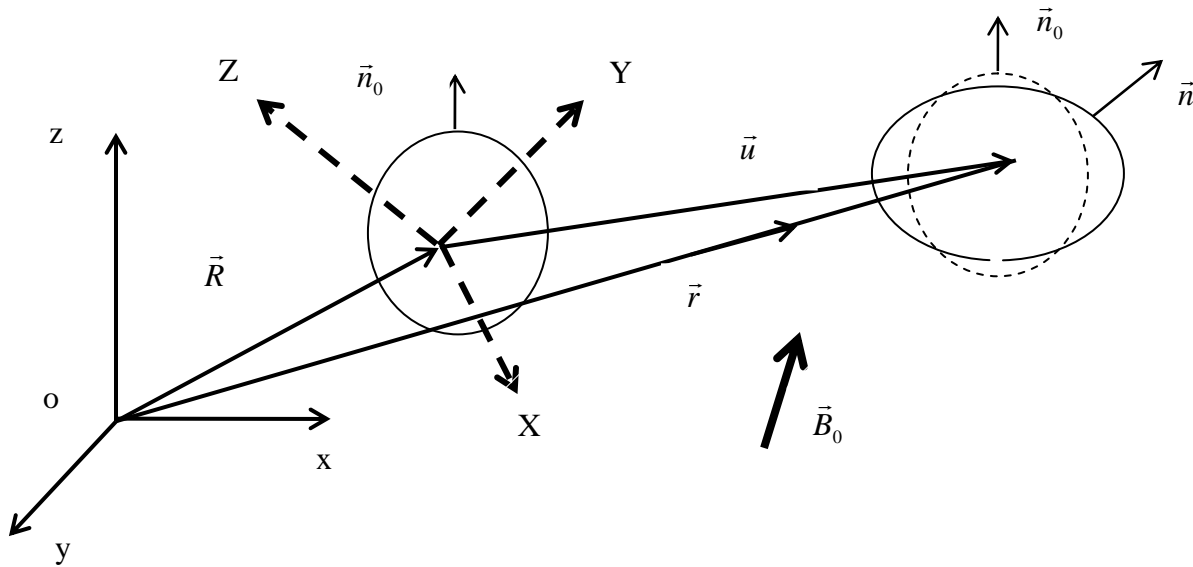


Figure 2: Geometry of the deformed body in a magnetic field

Then the functions

$$x_i = x_i(X_1, X_2, X_3, t) \quad (2.9)$$

or their inverses describe the deformation for the body as a whole and the partial derivatives  $\partial x_i / \partial X_j$  or  $\partial X_J / \partial x_j$  describe it locally. The deformation as described by the

transformations (2.9) is to be determined from a set of field equations, boundary conditions, and constitutive equations.

The general field equations of magnetoelasticity are derived substituting the body force  $f_i$ , body couple  $c_i$  as defined in (2.3) and the rate of energy supply  $\varepsilon$  in (2.8) into the usual equations of balance of linear momentum, angular momentum, and energy, respectively. The results, expressed in terms of the current position vector  $X_i$  are:

$$\textbf{Mass conservation: } \frac{d\rho}{dt} + \rho \frac{\partial v_i}{\partial X_i} = 0 \quad (2.10)$$

$$\textbf{Linear Momentum: } \frac{\partial t_{ij}}{\partial X_i} + \mu_0 M_i \frac{\partial H_j}{\partial X_i} = \rho \frac{dv_j}{dt} \quad (2.11)$$

$$\textbf{Angular Momentum: } t_{[jk]} + \mu_0 M_{[j} H_{k]} = 0 \quad (2.12)$$

$$\textbf{Energy change: } \rho \frac{dU}{dt} = t_{ij} \frac{\partial v_j}{\partial X_i} + \mu_0 \rho H_i \frac{d}{dt} \left( \frac{M_i}{\rho} \right) \quad (2.13)$$

In the above,  $\rho$  is the mass density,  $t_{ij}$  is the stress tensor in (2.4), and  $U$  is the internal energy per unit mass. Since the energy flow due to heat conduction is not considered here, the rate of change of  $U$  is the same as the rate of change of the free energy  $F$  as discussed in [7] and [17]. The indices with brackets represent the antisymmetric part, i.e.

$$t_{[ij]} = (t_{ij} - t_{ji})/2.$$

To the above equations, we add the magnetic field equations:

$$\textbf{Definition: } B_i = \mu_0 (H_i + M_i) \quad (2.14)$$

$$\textbf{Gauss' Law: } \partial B_i / \partial X_i = 0 \quad (2.15)$$

$$\textbf{Ampere's Law: } \partial H_{[j} / \partial X_{k]} = 0 \quad (2.16)$$

All magnetic quantities are to be evaluated within a body at the deformed configuration.



Across a surface of discontinuity of a deformed body, the boundary conditions for  $\vec{B}$  and  $\vec{H}$  are the same as the usual ones in magnetostatics. With a unit normal  $n_i$  drawn from the negative side (-) to the positive side (+) of the deformed surface of discontinuity,

***the boundary conditions are:***

$$n_i[B_i] = n_i(B_i^+ - B_i^-) = 0 \quad (2.17)$$

$$n_{[j}[H_{k}] = n_{[j}(H_{k]}^+ - H_{k]}^-) = 0 \quad (2.18)$$

***The boundary conditions for stresses are:***

$$n_i[t_{ij} + T_{ij}] = n_i[t_{ij} + B_i H_j - \frac{1}{2} \mu_0 H_k H_k \delta_{ij}] = g_j \quad (2.19)$$

where  $g_j$  is a components of surface mechanical force.

The theory is completed by adding to it constitutive equations for a soft ferromagnetic material.

## **§2.3 THE GENERAL EQUATIONS IN LAGRANGE SYSTEM OF COORDINATES**

Let an elastic dielectric medium with ordered magnetic structure be in an external stationary magnetic field with the magnetic intensity  $\vec{H}^0$  and the magnetic induction vectors  $\vec{B}^0 = \mu_0 \vec{H}^0$ . The medium surrounding the body is supposed to be a vacuum.

Under the influence of the magnetic field  $\vec{H}^0$  the total force (per unit volume)  $\vec{f}$  and body couple (per unit volume)  $\vec{c}$  acting on a body are:

$$\vec{f} = \mu_0 (\vec{M} \cdot \nabla) \vec{H}, \quad \vec{c} = \mu_0 \vec{M} \times \vec{H} \quad (2.20)$$

The motion of the deformable ferromagnetic body under the action of the given magnetic field will be described in a moving rectangular coordinate system  $X_1X_2X_3$ . Initial position of the body points in chosen coordinate system are defined by the Cartesian coordinates  $(x_1, x_2, x_3)$  which we later on used as the Lagrange (nonmoving) coordinates to the assess the moving coordinates of the medium points. Under the action of the volume forces and the volume moments (2.20) as well as surface forces of nonmagnetic origin, the medium is deformed and its motion in a system of coordinates  $(x_1, x_2, x_3)$  will be described by the following equations [1, 15, 21]

$$\frac{\partial}{\partial x_i} \left[ t_{im} \left( \delta_{mk} + \frac{\partial u_k}{\partial x_m} \right) \right] + f_k = \rho_0 \frac{\partial^2 u_k}{\partial t^2}, \quad (2.21)$$

$$e_{imk} t_{mk} + c_i = 0, \quad f_k = \frac{\partial T_{mk}}{\partial x_m}, \quad (i, m, k = 1, 2, 3) \quad (2.22)$$

and boundary conditions

$$\bar{n} \cdot [\bar{B} - \bar{B}^{(e)}] = 0 \quad (2.23)$$

$$\bar{n} \times [\bar{H} - \bar{H}^{(e)}] = 0 \quad (2.24)$$

$$\left[ t_{km} \left( \delta_{mi} + \frac{\partial u_i}{\partial x_m} \right) \right] n_k^0 = G_i + [T_{km}^{(e)} - T_{km}] \left( \delta_{mi} + \frac{\partial u_i}{\partial x_m} \right) n_k^0 \quad (2.25)$$

Detailed derivations of Eqs. (2.21) - (2.25) are given in [1, 15, 21].

In Eqs. (2.21) - (2.25)  $u_k$  are the displacement vector components,  $t_{im}$  is the tensor of the Lagrange's magnetoelastic stresses;  $e_{imk}$  is the permutation symbol with  $e_{ijk} = 1$  or  $-1$  according to whether the indices are in a cyclic or an anticyclic order, respectively, and  $e_{ijk} = 0$  otherwise;  $\delta_{mk}$  is the Kronecker symbol with  $\delta_{mk} = 1$  when  $m = k$  and  $\delta_{mk} = 0$

otherwise;  $\rho_0$  is the mass density of the medium before the deformation;  $n_k^0$  and  $n_k$  – are components of the external normal to the undeformed and deformed surfaces of the body respectively;  $T_{km}$  are the components of the Maxwell's stress tensor

$$T_{mk} = B_m H_k - \frac{1}{2} \mu_0 \delta_{mk} H^2 \quad (2.26)$$

The index "e" and "i" here and later on denotes accessory of the considered quantity to external and internal mediums respectively. Summation takes place by the recurring indices. It is obvious from the Eq. (2.23) that the tensor  $t_{im}$  in general cases are nonsymmetrical. It becomes symmetric only if the magnetic moments vanish ( $c_k = 0$ ). Substituting the values of  $c_k$  from the Eq. (2.20) into the Eq. (2.23) the last of mentioned equations yields

$$e_{imk} (t_{im} + \mu_0 M_i H_m) = 0 \quad (2.27)$$

whence the symmetry of the tensor  $t_{im} + \mu_0 M_i H_m$  follows. It is easy to see that the system of Eqs. (2.21) - (2.25) is not closed and it is necessary to add to them the constitutive equations of the magnetoelastic medium, which connect characteristics of the deformable state (elastic deformations, stresses, the magnetic field intensity, and the induction of the magnetic field). The constitutive equations have the form [1, 7, 15, 21]

$$t_{ij} = \rho \alpha_{ik} \alpha_{je} \frac{\partial U}{\partial \varepsilon_{ke}} + M_j \frac{\partial U}{\partial \mu_i}, \quad \alpha_{ij} = \delta_{ij} - u_{i,j} \quad (2.28)$$

$$\mu_0 H_i = \frac{\partial U}{\partial \mu_i}, \quad \mu_i = M_i / \rho \quad (2.29)$$

where  $U$  is the inner specific energy per unit mass;  $\varepsilon_{ke}$  are the components of the Green deformation tensor

$$\varepsilon_{ke} = \frac{1}{2} \left( \frac{\partial u_k}{\partial x_e} + \frac{\partial u_e}{\partial x_k} + \frac{\partial u_m}{\partial x_k} \cdot \frac{\partial u_m}{\partial x_e} \right) \quad (2.30)$$

$\rho$  is the mass density of the deformable medium. Using Eq. (2.29), the Eq. (2.28) can be written in a form

$$t_{ij} = \rho \alpha_{ik} \alpha_{je} \frac{\partial U}{\partial \varepsilon_{ke}} + \mu_0 M_j H_i \quad (2.31)$$

Taking into account the symmetry of the first term of the expression (2.31) one can see that the symmetry conditions (2.27) owing to Eq. (2.31) are identically satisfied. The expression for the specific intrinsic energy of the deformable elastic nonconductive ferromagnetic body, according to [1, 7, 15, 21], is chosen in the following form

$$U(\varepsilon_{ij}, M_i) = U^{el}(\varepsilon_{ij}) + U^m(M_i) \quad (2.32)$$

where  $U^{el}$  and  $U^m$  are an elastic and magnetic energy, respectively.

The simple variant of ferromagnetic body is the case, when the vectors  $\vec{H}$  and  $\vec{M}$  are parallel (i. e. magnetosoft ferromagnetic).

## §2.4 THE LINEARIZED EQUATIONS AND BOUNDARY CONDITIONS

Let us turn to the linearization of the main equations and boundary conditions describing the behavior of the deformable ferromagnetic body with nonlinear characteristic between magnetic field and magnetization which are given in the previous paragraph adopting the main assumptions of the small deformation theory. With that purpose, we represent the magnetic field characteristics in a following form [1, 7, 15, 21]

$$\vec{B} = \vec{B}_0 + \vec{b}, \quad \vec{H} = \vec{H}_0 + \vec{h}, \quad \vec{M} = \vec{M}_0 + \vec{m} \quad (2.33)$$

Here  $\vec{B}_0$ ,  $\vec{M}_0$  and  $\vec{H}_0$  are the magnetic induction vector, the magnetization and the magnetic field intensity in a rigid state, respectively;  $\vec{b}$ ,  $\vec{m}$  and  $\vec{h}$  are the additives (perturbations) to mentioned quantities caused by the deformation of the body. As defined above, the quantities  $\vec{B}_0$ ,  $\vec{M}_0$  and  $\vec{H}_0$  are determined from the solution of the following magnetostatic's problem:

**a) Equations in a domain occupied by the body (internal domain)**

$$\text{curl}\vec{H}_0 = 0 \quad \text{div}\vec{B}_0 = 0 \quad (2.34)$$

where  $\vec{B}_0 = \mu_0(\vec{H}_0 + \vec{M}_0) = \mu_0[1 + \chi(H_0)] \cdot \vec{H}_0$

**b) Equations in a domain outside of the body (external domain)**

$$\text{curl}\vec{H}_0^{(e)} = 0 \quad \text{div}\vec{B}_0^{(e)} = 0 \quad (2.35)$$

where

$$\vec{B}_0^{(e)} = \mu_0\vec{H}_0^{(e)} \quad \text{and} \quad \vec{M}_0^{(e)} = 0$$

**c) Conditions on the surface of the nondeformed body**

$$\vec{n}_0 \cdot [\vec{B}_0 - \vec{B}_0^{(e)}] = 0, \quad \vec{n}_0 \times [\vec{H}_0 - \vec{H}_0^{(e)}] = 0 \quad (2.36)$$

**d) Conditions at infinity**

$$\vec{B}_0^{(e)} \rightarrow \vec{B}^0, \quad \text{when } r = (x_1^2 + x_2^2 + x_3^2)^{1/2} \rightarrow \infty \quad (2.37)$$

The characteristics of the stress-strain state of the body (the displacement vector components  $u_k$  and stresses tensor components  $S_{im}$ ) and the quantities  $\vec{b}$ ,  $\vec{m}$  and  $\vec{h}$  are being determined from equations and boundary conditions (2.20) - (2.32) for the deformable ferromagnetic body. Assuming that the deformations and the module of

quantities  $\bar{b}$ ,  $\bar{m}$  and  $\bar{h}$  are small, one can linearized these equations and boundary conditions similarly to [1, 7, 15, 21].

As a result, according to the equations (2.33) - (2.37) we receive the following linear equations and boundary conditions for the magnetoelastic characteristics of the deformed body in the form:

$$t_{ji,j} + \mu_0(M_{0j}H_{0i,i} + M_{0j}h_{i,j} + m_jH_{0j,i}) - \mu_0M_{0j}H_{0i,k}u_{k,j} - \bar{t}_{ij,k}u_{k,j} = \rho_0 \frac{\partial^2 u_i}{\partial t^2} \quad (2.38)$$

$$e_{ijk}[h_{k,j} - H_{0j,m}u_{m,k}] = 0, \quad b_{i,i} - B_{0j,k}u_{k,j} = 0, \quad (i, j, k = 1, 2, 3) \quad (2.39)$$

where

$$t_{ij} = \sigma_{ij} + \mu_0 H_{0i} M_{0j} + \mu_0 (H_{0i} m_{0j} + H_{0j} m_{0i})$$

$$\sigma_{ij} = \lambda \delta_{ij} u_{k,k} + \mu (u_{i,j} + u_{j,i}), \quad \bar{t}_{ij} = \mu_0 H_{0i} M_{0j} \quad (2.40)$$

**The boundary conditions can be written in the form**

$$n_{0i}[t_{ij} + t_{ij}^M] = 0; \quad n_{0i}[b_i] - u_{m,i} n_{0m}[B_{0i}] = 0, \quad [\Phi] + \frac{\partial[\Phi^0]}{\partial X_i} \cdot u_i = 0 \quad (2.41)$$

where  $[A] = A^+ - A^-$  is a jump of the quantity  $A$  across a surface of discontinuity;

$\bar{h}^{(i,e)} = grad\Phi^{(i,e)}$ ;  $\bar{H}^{0(i,e)} = grad\Phi^{0(i,e)}$ ;  $\Phi^{(i,e)}$  and  $\Phi^{0(i,e)}$  are potential of perturbed and nonperturbet magnetic fields respectively;

$$t_{ij}^M = B_{0i}H_{0j} + B_{0i}h_j + B_{0j}h_i - 0.5\mu_0\delta_{ij}(H_{0k}^2 + 2H_{0k}h_k) \quad (2.42)$$

The dependence of  $\bar{b}$  and  $\bar{m}$  on  $\bar{h}$  can be obtained in the form [2]

$$\bar{m} = \hat{a} \cdot \bar{h} \quad \bar{b} = \mu_0(\hat{I} + \hat{a}) \cdot \bar{h} \quad (2.43)$$

where  $\hat{I}$  is the identity matrix and elements of the matrix  $\hat{a}$  are defined as

$$a_{ii} = \chi + \frac{H_{0i}^2}{H_0} \frac{d\chi}{dH_0} \quad a_{ij} = \frac{H_{0i}H_{0j}}{H_0} \frac{d\chi}{dH_0} \quad (i \neq j) \quad (2.44)$$

Then, for the magnetoelastic media with a nonlinear law of magnetization  $\vec{M} = \chi \langle \vec{H} \rangle \vec{H}$ , the motion equations (2.38) - (2.39) can be presented in the following form

$$\nabla^2 \vec{u} + \frac{1}{1-2\nu} \nabla(\nabla \cdot \vec{u}) + \frac{\mu_0}{\mu} \vec{f} = \frac{\rho_0}{\mu} \frac{\partial^2 \vec{u}}{\partial t^2} \quad (2.45)$$

$$\text{div} [(\hat{I} + \hat{a})\vec{h}] - B_{0j,k} u_{k,j} = 0 \quad (2.46)$$

$$e_{ijk} [h_{k,j} - H_{0j,m} u_{m,k}] = 0 \quad (2.47)$$

The components of the vector  $\frac{\mu_0}{\mu} \vec{f}$  are given:

$$\begin{aligned} \frac{\mu_0}{\mu} f_1 = & b_{11} h_{1,1} + b_{12} h_{2,2} + b_{13} h_{3,3} + b_{14} h_{1,2} + b_{15} h_{1,3} + b_{16} h_{2,3} + \\ & + \frac{\mu_0}{\mu} [2M_{0j} H_{01,j} + H_{0j} M_{0j,j}] - \frac{1}{\mu} \bar{t}_{1j,k} u_{k,j} \end{aligned} \quad (2.48)$$

where

$$b_{11} = \chi H_{01} + 2a_{11} H_{01}; \quad b_{13} = \chi H_{01} + a_{13} H_{03}; \quad b_{14} = \chi H_{02} + 3a_{12} H_{01} + a_{11} H_{02};$$

$$b_{15} = \chi H_{03} + 3a_{13} H_{01} + a_{11} H_{03}; \quad b_{16} = 2a_{23} H_{01} + a_{13} H_{02} + a_{12} H_{03}; \quad b_{12} = a_{22} H_{01} + a_{12} H_{02}$$

Expressions for  $\frac{\mu_0}{\mu} f_2$  and  $\frac{\mu_0}{\mu} f_3$  can be derived from  $\frac{\mu_0}{\mu} f_1$  by cyclic

permutation  $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$ . In the boundary conditions (2.41) the relation (2.42) will be taken into account also. From the motion equations (2.45) - (2.47) and the boundary conditions (2.41) for the magnetosoft ferromagnetic body with the linear law of magnetization can be obtained equations of motion and boundary conditions respectively.

Notice that, for the domain outside of the body (coinciding with the vacuum) the equations governing the magnetic field are:

$$\text{curl } \vec{h}^{(e)} = 0, \quad \text{div } \vec{h}^{(e)} = 0 \quad (2.49)$$

## §2.5 DIFFERENT MAGNETIC SUSCEPTIBILITIES OF A SOFT FERROMAGNETIC MATERIALS

Magnetic field  $\vec{H}_0$ , magnetization  $\vec{M}_0$  and magnetic induction  $\vec{B}_0$  characterizing undeformed state of a body are given as:

$$\vec{M}_0 = \chi(H_0) \vec{H}_0, \quad \vec{B}_0 = \mu_0 [1 + \chi(H_0)] \vec{H}_0 \quad (2.50)$$

The function  $\chi(H_0) = \mu_r(H_0) - 1$  is the magnetic susceptibility of a material. In general, for magnetosoft materials with nonlinear dependence (Figure 3) the susceptibility of a material  $\chi$  depends only upon the module of a magnetic field:  $H_0 = |\vec{H}_0|$ .

Experimental investigations [5, 11] show that the magnetic susceptibilities  $\chi(H_0)$  of different magneto soft ferromagnetic materials depending on the module of value of the magnetic field intensity  $H_0 = |\vec{H}_0|$  can be approximated using the main curve of magnetization with sufficiently high precision by the following formula:

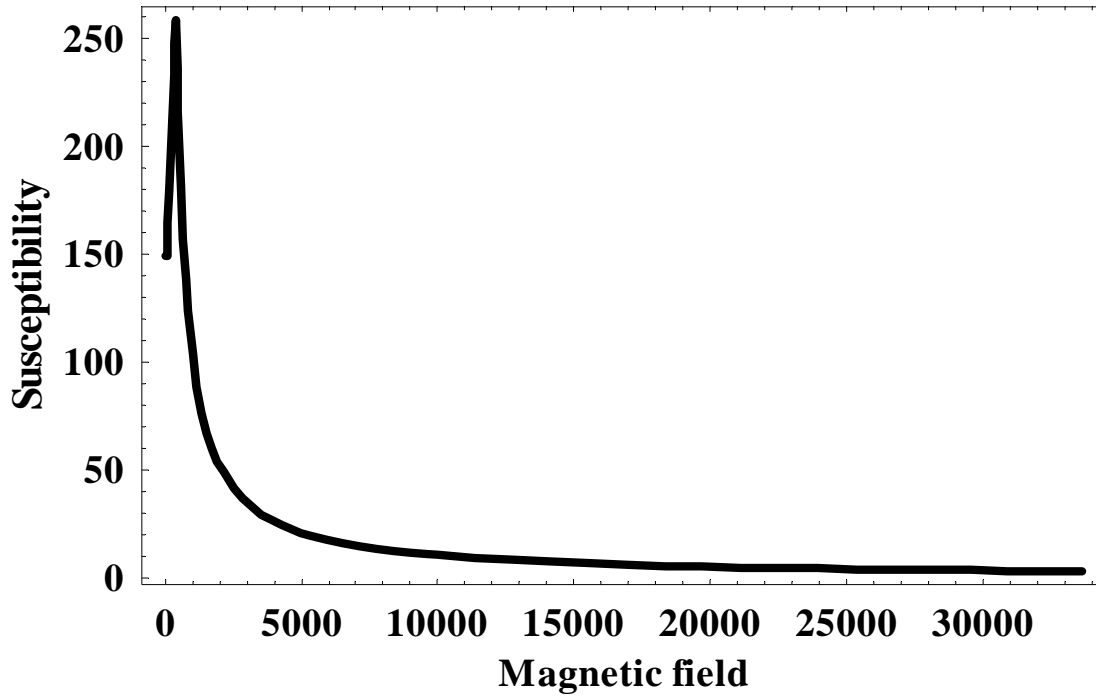
### 1) Dreifous form:

$$\chi(H_0) = (\beta / \mu_0 H_0) \text{arctg}(\alpha H_0) \quad (2.51)$$

where

$$\beta = 2B_s / \pi, \quad \alpha = (\mu_{ri} - 1)\mu_0 / \beta$$





**Figure 3: Dependence of magnetic susceptibility versus magnetic field for pure iron**

Here  $B_s$  denotes the induction saturation;  $\mu_{ri}$  is initial relative magnetic permeability of the material. The equation (2.51) are used by [2, 8] in the constitutive equations of magnetosoft materials with nonlinear law of magnetization.

**2) Rayleigh form:**

$$\chi(H) = \kappa_0 + b_r H \tag{2.52}$$

which is the linear approximation of the Dreifous formula and is applicable if  $H < H_c$ ,

where  $H_c$  is the coercitive force [2, 3].

**3) Linear dependence:**

If the coefficient of nonlinearly  $b_r = 0$ , then the dependence

$$\chi(H) = const = \mu_r - 1 \tag{2.53}$$

can be obtained from (2.52). It is often used for weak magnetic field in the constitutive equations of magnetosoft ferromagnetic materials with linear characteristic [1, 10,13-15,19-23].

**4) Magneto rigid form** (magnetically saturated materials):

$$\chi(H) = M_s / H \quad (2.54)$$

where  $M_s = B_s / \mu_0$  is the saturation magnetization.

The numerical values of coefficients  $\alpha$ ,  $\beta$ ,  $M_s$ ,  $\kappa_0$ ,  $b_r$ ,  $\mu_0$ ,  $\mu_r$  and  $\mu_{ri}$  for different ferromagnetic materials are given in [2]. The Eqs. (2.51) - (2.54) are consistent with the following Maxwell's equations and corresponding conditions:

**Equations in the internal domain:**

$$\text{curl } \vec{H} = 0, \quad \text{div } \vec{B}_0 = 0 \quad (2.55)$$

$$\vec{B}_0 = \mu_0(\vec{H}_0 + \vec{M}_0) = \mu_0[1 + \chi(H_0)]\vec{H}_0, \quad \vec{H}_0 = |\vec{H}_0| \quad (2.56)$$

**Equations in the external domain:**

$$\text{curl } \vec{H}_0^{(e)} = 0, \quad \text{div } \vec{B}_0^{(e)} = 0 \quad (2.57)$$

$$\vec{B}_0^{(e)} = \mu_0 \vec{H}_0^{(e)} \quad \vec{M}_0^{(e)} = 0 \quad (2.58)$$

**Conditions on the surface of the undeformed body:**

$$\vec{n}_0 \cdot [\vec{B}_0] = 0, \quad \vec{n}_0 \times [\vec{H}_0] = 0 \quad (2.59)$$

**Condition at infinity:**

$$\vec{B}_0^{(e)} \rightarrow \vec{B}_0, \quad \text{when } r = (x_1^2 + x_2^2 + x_3^2)^{1/2} \rightarrow \infty \quad (2.60)$$

### 3. STRESS-STRAIN STATE OF FERROMAGNETIC PLANE WITH A MOVING CRACK

Based on the equations and the boundary conditions derived in chapter 2 the problem of stress-strain state of ferromagnetic plane is discussed here. To such purpose, a soft magnetic ferroelastic body accounting a nonlinear law of magnetization and immersed in a magnetic field perpendicular to a crack line is considered. Assuming that the processes in a moving coordinates are stationary, Fourier transform method is used to reduce the mixed boundary value problem to the pairs of dual integral equations that are solved analytically. The magnetoelastic stress intensity factor is obtained and its dependency upon the crack velocity, material constants and nonlinear law of magnetization is presented.

#### § 3.1 MATHEMATICAL MODELING

A magnetoelastic plane with a finite crack of length  $2a$  is located in a magnetic field  $\vec{H}_0 = (0, H_0, 0)$  with  $H_0 = const$ . Crack is moving with the constant velocity  $V < c_R$  where  $c_R$  the speed of propagation of Raleigh waves, and is located in a plane  $(x_1, x_2)$  along a line  $x_2 = 0$ , and  $-a + Vt < x_1 < a + Vt$  (Figure 4).

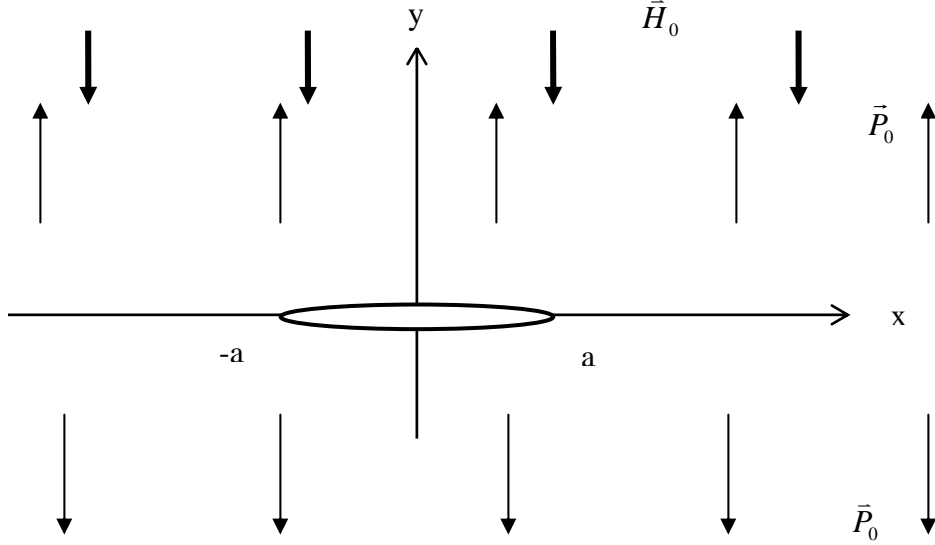
In-plane nontrivial displacements are represented as:

$$u_1 = u_1(x_1, x_2, t), u_2 = u_2(x_1, x_2, t), u_3 = 0 \quad (3.1)$$

where a set of moving coordinate system  $(x, y, z)$  attached at the center of the moving crack is chosen such that:

$$x = x_1 - Vt, y = x_2, z = x_3, t = t_1 \quad (3.2)$$

As a basic assumption, the crack propagation occurs during an interval of time when in the moving reference system of coordinates the magnetoelastic state is time-invariant.



**Figure 4: Ferromagnetic plane with a crack in a magnetic field**

### §3.2 GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

#### *Governing Equations*

The governing equations of magnetoelastic media with a nonlinear law of magnetization

$\vec{M} = \chi(|H_0|)\vec{H}$  can be expressed in the form (Chapter 2, Eqs. (2.45), (2.48)):

$$\Delta u_i + \frac{1}{1-2\nu} u_{j,ij} + \frac{\mu_0}{\mu} f_i = \frac{\rho}{\mu} \frac{\partial^2 u_i}{\partial t^2} \quad (3.3)$$

$$\text{div}[(\hat{I} + \hat{a})\vec{h}] - B_{0j,k} u_{k,j} = 0 \quad (3.4)$$

$$e_{ijk} [h_{k,j} - H_{0j,m} u_{m,k}] = 0 \quad (3.5)$$

Assuming that

$$u_1 = \varphi_{,x_1} + \psi_{,x_2}, \quad u_2 = \varphi_{,x_2} - \psi_{,x_1}, \quad \vec{h} = H_0 \text{grad } \Phi \quad (3.6)$$

from Eq. (3.3) we can get the following two decoupled equations in terms of  $\varphi$  and  $\psi$

$$c_1^2 \nabla^2 \varphi - \varphi_{,t_1 t_1} + \delta_1 \Phi_{,x_2} = 0 \quad c_2^2 \nabla^2 \psi - \psi_{,t_1 t_1} + \delta_2 \Phi_{,x_1} = 0 \quad (3.7ab)$$

Herein,  $\varphi$  and  $\psi$  are displacement potential functions.

In Eqs. (3.7ab)

$$\nabla^2 = \partial^2 / \partial x_1^2 + \partial^2 / \partial x_2^2, \quad \delta_1 = \frac{\gamma^2 \lambda_2 - \lambda_1}{\gamma^2 - 1} c_1^2, \quad \delta_2 = \frac{\gamma^2 (\lambda_1 - \lambda_2)}{\gamma^2 - 1} c_2^2, \quad c_1^2 = \frac{2(1-\nu)}{1-2\nu} c_2^2,$$

$$c_2^2 = \mu / \rho,$$

$$\lambda_1 = 2\chi(H_0)[1 + \chi(H_0)]h_c^2,$$

$$\lambda_2 = \lambda_1 \{2[\chi(H_0) + H_0 \chi'(H_0)] / \chi(H_0) - H_0 \chi'(H_0) / [1 + \chi(H_0)]\} / 2,$$

$$h_c^2 = \mu_0 H_0^2 / \mu$$

Magnetic potential  $\Phi$  is determined from (3.4) - (3.5)

$$\gamma^2 \Phi_{,x_1 x_1} + \Phi_{,y_1 y_1} = 0 \quad (3.8)$$

where

$$\gamma^2 = (1 + \chi(H_0)) / (1 + \chi(H_0) + H_0 \chi'(H_0))$$

Using coordinate transformations (3.2), the Eqs. (3.7ab) and (3.8) become:

$$s_1^2 \varphi_{,xx} + \varphi_{,yy} + r_1 \Phi_{,y} = 0 \quad (3.9a)$$

$$s_2^2 \psi_{,xx} + \psi_{,yy} + r_2 \Phi_{,y} = 0 \quad (3.9b)$$

$$\gamma^2 \Phi_{,xx} + \Phi_{,yy} = 0 \quad (3.9c)$$

where

$$s_i^2 = 1 - M_i^2, M_i = V / c_i, r_i = \delta_i / c_i^2, (i = 1, 2)$$

The Mach numbers  $M_i < 1$ , since we assume that the crack is propagated at subsonic speed.

### **Boundary Conditions**

On the surface  $y = 0$  the following boundary conditions should satisfy

$$u_y(x, 0) = 0, \quad \text{for } |x| > a, \quad (3.10a)$$

$$\Phi(x, 0) = 0, \quad \text{for } |x| > a, \quad (3.10b)$$

$$\Phi_{,x}(x, 0) + d_1 u_{y,x}(x, 0) = 0, \quad \text{for } |x| < a, \quad (3.11a)$$

$$\frac{2\nu}{1-2\nu} u_{x,x}(x, 0) + \frac{2(1-\nu)}{1-2\nu} u_{y,y}(x, 0) - e_1 \Phi_{,y}(x, 0) = -P_{0mec} + P_{0mag} = -P_0, \quad \text{for } |x| < a \quad (3.11b)$$

$$u_{y,x}(x, 0) + u_{x,y}(x, 0) + L \cdot \Phi_{,x} = 0, \quad \text{for } |x| < \infty, \quad (3.12)$$

where

$P_{0mag} = h_c^2 (\chi(H_0))^2 / 2$ ;  $P_{0mec} / \mu$  is nondimensional mechanical force acting on the surface of the crack.

Notice that the boundary conditions (3.10a) and (3.10b) are consequence of symmetry of the displacements and the magnetic potential.

In Equations (3.8) - (3.12),

$$e_1 = h_c^2 [\chi(H_0)(1 + H_0 \chi'(H_0)) + (\chi(H_0) - 1)\chi(H_0)],$$

$$d_1 = -\chi(H_0), \quad L = h_c^2 (1 + \chi(H_0)).$$

### §3.3 SOLUTION METHODOLOGY

Applying a Fourier transform method to Equations (3.9) the potential functions are readily obtained

$$\varphi(x, y) = \frac{1}{\pi} \int_{-\infty}^{\infty} \left\{ A(\alpha) \exp[-\beta_1 y] + \frac{\delta_1}{c_1^2} \frac{|\alpha| \gamma}{\alpha^2 \gamma^2 - \beta_1^2} C(\alpha) \exp[-\gamma|\alpha|y] \right\} \exp[-i\alpha x] d\alpha \quad (3.13)$$

$$\psi(x, y) = \frac{1}{\pi} \int_{-\infty}^{\infty} \left\{ B(\alpha) \exp[-\beta_2 y] + \frac{\delta_2}{c_2^2} \frac{i|\alpha|}{\alpha^2 \gamma^2 - \beta_2^2} C(\alpha) \exp[-\gamma|\alpha|y] \right\} \exp[-i\alpha x] d\alpha \quad (3.14)$$

$$\Phi(x, y) = \frac{1}{\pi} \int_{-\infty}^{\infty} C(\alpha) \exp[-\gamma|\alpha|y] \exp[-i\alpha x] d\alpha \quad (3.15)$$

where

$$\beta_i^2 = \alpha^2 (1 - M_i^2)$$

In Eqs. (3.13) - (3.15)  $A, B$  and  $C$  are unknown functions to be determined from the boundary conditions (3.10) - (3.12). The unknowns  $A, B$  and  $C$  can be rewritten as a function of a new quantity  $D(\alpha)$ :

$$A(\alpha) = \frac{(1 + s_2^2)(1 + d_1 Q_2^0 + d_1(\gamma Q_1^0 - Q_2^0 - L))}{|\alpha| s_1 (1 - s_2^2)} D(\alpha) \quad (3.16)$$

$$B(\alpha) = \frac{[d_1(\gamma Q_1^0 + Q_2^0 - L) + 2]}{i\alpha(1 - s_2^2)} D(\alpha) \quad (3.17)$$

$$C(\alpha) = -d_1 D(\alpha) \quad (3.18)$$

In Eqs. (3.16) and (3.17)

$$Q_1^0 = \frac{\delta_1 \gamma}{c_1^2(\gamma^2 - s_1^2)} + \frac{\delta_2 \gamma}{c_2^2(\gamma^2 - s_2^2)}, \quad Q_2^0 = -\frac{\delta_1 \gamma^2}{c_1^2(\gamma^2 - s_1^2)} - \frac{\delta_2}{c_2^2(\gamma^2 - s_2^2)} \quad (3.19)$$

It is easy to show that based on the boundary conditions (3.10)-(3.12), the determination of unknown  $D(\alpha)$  yields to the following dual integral equations:

$$\frac{1}{\pi} \int_{-\infty}^{\infty} D(\alpha) \exp[-ix\alpha] d\alpha = 0, \quad |x| > a \quad (3.20)$$

$$\frac{1}{\pi} \int_{-\infty}^{\infty} |\alpha| D(\alpha) \exp[-ix\alpha] d\alpha = -\frac{P_0}{R_0}, \quad |x| < a \quad (3.21)$$

Herein,

$$R_0 = (1 + s_2^2)R_1 - 2s_2R_2 - d_1 \left( -\frac{2\nu}{1-2\nu} Q_1^0 - \frac{2(1-\nu)}{1-2\nu} \gamma Q_2^0 + e_1 \gamma \right) \quad (3.22a)$$

$$R_1 = \frac{(1 + s_2^2)(1 + d_1 Q_2^0) + d_1(\gamma Q_1^0 - Q_2^0 - L)}{s_1(1 - s_2^2)} \quad (3.22b)$$

$$R_2 = \frac{d_1 \gamma Q_1^0 + d_1 Q_2^0 - d_1 L + 2}{1 - s_2^2} \quad (3.22c)$$

### §3.4 MAGNETOELASTIC STRESSES AND STRESS INTENSITY FACTOR

Inserting Eqs. (3.6), (3.13)-(3.15) into Eq. (2.40) from Chapter 2, the magnetoelastic

$$\text{stresses } t_{yy}^T(x, y) / \mu = t_{yy}^E(x, y) / \mu + t_{yy}^M(x, y) / \mu$$

and

$$t_{xy}^T(x, y) / \mu = t_{xy}^E(x, y) / \mu + t_{xy}^M(x, y) / \mu$$

we can express through  $D(\alpha)$  as follows:



$$t_{yy}^T(x, y)/\mu = \frac{1}{\pi} \int_{-\infty}^{+\infty} \{(s_2^2 + 1)R_1 \exp[-|\alpha|s_1 y] - 2s_2 R_2 \exp[-|\alpha|s_2 y] + d_1 R_3 \exp[-\gamma|\alpha|y]\} |\alpha| D(\alpha) \exp[-i\alpha x] d\alpha \quad (3.23)$$

$$t_{xy}^T(x, y)/\mu = \frac{1}{\pi} \int_{-\infty}^{+\infty} \{2s_1 R_1 \exp[-|\alpha|s_1 y] - (s_2^2 + 1)R_2 \exp[-|\alpha|s_2 y] - d_1 R_4 \exp[-\gamma|\alpha|y]\} i\alpha D(\alpha) \exp[-i\alpha x] d\alpha \quad (3.24)$$

where

$$R_3 = \frac{2\nu}{1-2\nu} Q_1^0 + \gamma \frac{2(1-\nu)}{1-2\nu} Q_2^0 + \gamma e_0, \quad R_4 = Q_1^0 \gamma - Q_2^0 - L$$

$$e_0 = h_c^2 [1 + 2\chi(H_0) + H_0 \chi'(H_0)]$$

The solution of dual integral equations (3.20)-(3.21) may be represented in a form (Appendix B):

$$D(\alpha) = \frac{1}{\alpha} \int_{-a}^a E(s) \exp[-i\alpha s] ds \quad (3.25)$$

where

$$E(s) = \frac{P_0}{2i R_0} \frac{s}{\sqrt{a^2 - s^2}} \quad (3.26)$$

Substitution (3.25)-(3.26) into Eqs. (3.23)-(3.24) gives us a final representation of magnetoelastic stresses:

$$t_{yy}^T(x, y)/\mu = -\frac{P_0}{\pi R_0} \int_{-a}^a \frac{s}{\sqrt{a^2 - s^2}} \left\{ (1 + s_1^2) R_1 \frac{s-x}{s_1^2 y^2 + (s-x)^2} - \right. \\ \left. - 2s_2 R_2 \frac{s-x}{s_2^2 y^2 + (s-x)^2} + d_1 R_3 \frac{s-x}{\gamma^2 y^2 + (s-x)^2} \right\} ds \quad (3.27)$$

$$t_{xy}^T(x, y)/\mu = -\frac{P_0}{\pi R_0} \int_{-a}^a \frac{s}{\sqrt{a^2 - s^2}} \left[ 2s_1 R_1 \frac{s_1 y}{s_1^2 y^2 + (s-x)^2} - \right. \\ \left. - (s_2^2 + 1) R_2 \frac{s_2 y}{s_2^2 y^2 + (s-x)^2} - d_1 R_4 \frac{\gamma y}{\gamma^2 y^2 + (s-x)^2} \right] ds \quad (3.28)$$

For the special case when  $y = 0$  the magnetoelastic stress  $t_{yy}^T(x, y)/\mu$  have form

$$t_{yy}^T(x, 0)/\mu = -\frac{P_0}{\pi R_0} \left\{ (1 + s_1^2) R_1 - 2s_2 R_2 + d_1 R_3 \right\} \int_{-a}^a \frac{s}{\sqrt{a^2 - s^2}} \cdot \frac{1}{s-x} ds = \\ = -\frac{P_0}{\pi R_0} \left\{ (1 + s_1^2) R_1 - 2s_2 R_2 + d_1 R_3 \right\} \begin{cases} 1 - \frac{x}{\sqrt{x^2 - a^2}}, & x > a \\ 1 + \frac{x}{\sqrt{x^2 - a^2}}, & x < -a \\ 1, & |x| \leq a \end{cases} \quad (3.29)$$

The normalized stress intensity factor can be expressed as

$$K^I = \lim_{x \rightarrow a+0} \frac{\sqrt{x^2 - a^2}}{a} \frac{t_{yy}^c(x, 0)}{\mu P_0} = \frac{1}{\pi R_0} \left\{ (1 + s_1^2) R_1 - 2s_2 R_2 + d_1 R_3 \right\} \quad (3.30)$$

or

$$K^I = \frac{1}{\pi} \frac{F_1^*(M_1^2, M_2^2)}{F_2^*(M_1^2, M_2^2)} \quad (3.31)$$

where

$$F_1^*(M_1^2, M_2^2) = (2 - M_1^2) R_1 - 2\sqrt{1 - M_2^2} R_2 + d_1 R_3, \quad (3.32)$$

$$F_2^*(M_1^2, M_2^2) = (2 - M_2^2)R_1 - 2\sqrt{1 - M_2^2}R_2 + d_1\left(\frac{2\nu}{1 - 2\nu}Q_1^o + \frac{2(1 - \nu)}{1 - 2\nu}\gamma Q_2^o - e_1\gamma\right). \quad (3.33)$$

From Eq. (3.31), when  $V = 0$ , e.g. the crack is immovable, we can get

$$K^I \Big|_{V=0} = \frac{F_0^* + \gamma e_0}{F_0^* + \gamma e_1} \quad (3.34)$$

Herein,

$$F_o^* = 1 + \frac{c_2^2}{c_1^2}\tilde{Q}_2^0 + \gamma\tilde{Q}_1^0 + \frac{2\nu}{1 - 2\nu}\tilde{Q}_1^0 + \frac{2(1 - \nu)}{1 - 2\nu}\gamma\tilde{Q}_2^0 + \tilde{Q}_2^0 - L, \quad (3.35)$$

$$\tilde{Q}_1^0 = \frac{\delta_1\gamma}{c_1^2(\gamma^2 - 1)} + \frac{\delta_2\gamma}{c_2^2(\gamma^2 - 1)}, \quad \tilde{Q}_2^0 = -\frac{\delta_1\gamma^2}{c_1^2(\gamma^2 - 1)} - \frac{\delta_2}{c_2^2(\gamma^2 - 1)}.$$

In a case of magnetosoft material with a linear law of magnetization ( $\chi(H_0) = \chi = \text{const}$ )

from (3.34) directly follows that

$$K^I \Big| \approx [1 - \chi(1 - \nu)b_c^2]^{-1} \quad (3.36)$$

where

$$b_c^2 = (\chi + 1)^2 h_c^2$$

Result (3.36) coincides with the result derived by [10, 19-20]. Notice that, for

magnetosoft material with a linear law of magnetization, the intensity factor  $K^I \rightarrow \infty$

when  $b_c^2 \rightarrow 1/\chi(1 - \nu^2)$ .

## 4. NUMERICAL COMPUTATIONS AND DISCUSSIONS

This Chapter is devoted to the numerical discussions of stress intensity factor and magnetoelastic stresses of soft ferromagnetic plane with a moving crack in a transversal magnetic field. The computational results are based on Eqs. (3.31), (3.27) and (3.28). Notice that for numerical calculations the nonlinear laws of magnetization are taken in terms of nondimensional parameters (Appendix C).

### §4.1 STRESS INTENSITY FACTOR

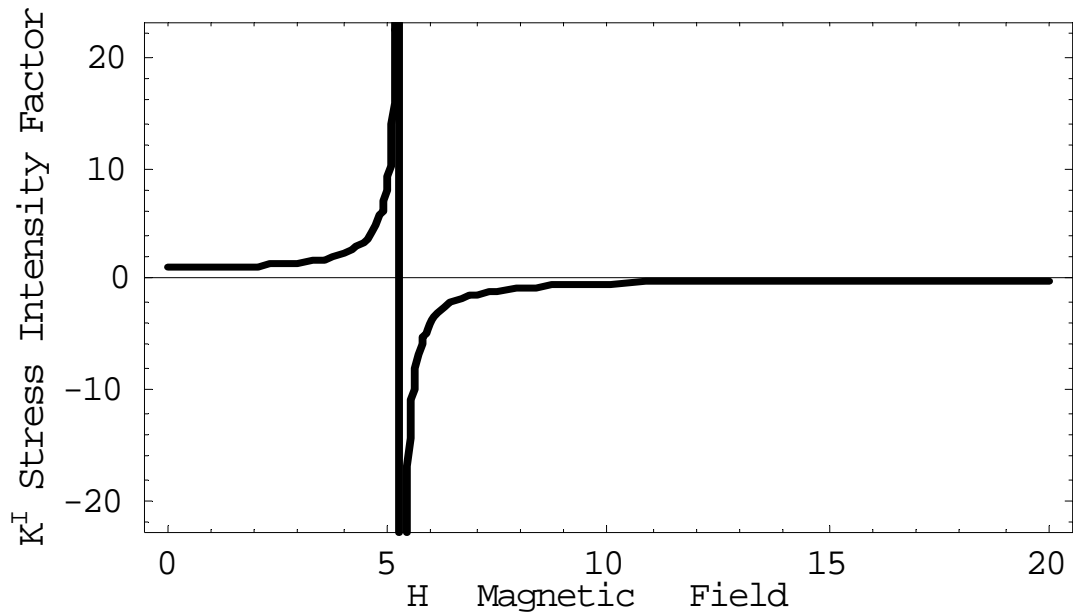
Numerical results have been calculated for the normalized stress intensity factor  $K^I$  and in particular the dependency of the stress intensity factor upon magnetic field  $h_c^2 = \mu_0 H_0^2 / \mu$  and normalized velocity of the crack  $\bar{V} = V^2 / c_1^2$  has been highlighted. From Eq. (3.31) it is possible to conclude that for  $h_c^2 \neq 0$  the intensity factor  $K^I \rightarrow \infty$  when  $V \rightarrow V_* < c_2$  (where,  $V_*$  is the maximum velocity valid within magneto-elastic assumption, leading the denominator of Eq. (3.31) to zero). These primary conclusions show that the intensity factor essentially depends on an external magnetic field, speed of moving crack and also upon physical parameters of a problem. Further numerical results are carried out for superpermalloy ( $\nu = 0.25$ ,  $\rho_0 = 8.77 \times 10^3 \text{ kg / m}^3$ ).

In Figures 5 - 8 the effects of normalized speed of moving crack  $\bar{V} = V^2 / c_1^2$  on the normalized intensity factor  $K^I$  for various values of normalized magnetic fields  $H = 10^9 h_c^2$  are shown.

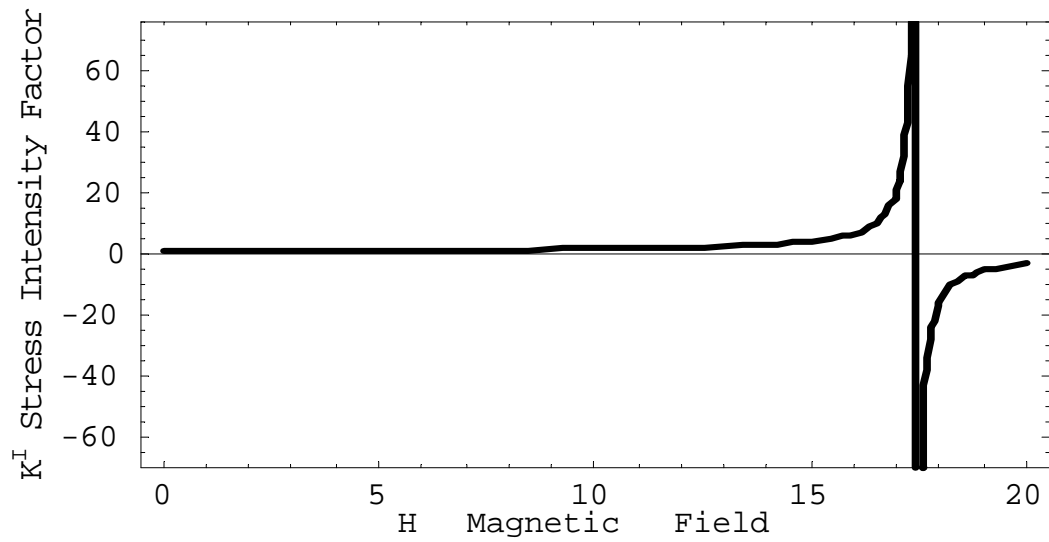
The analog calculation is carried out using various laws of magnetization in Figures 5-8. For example, in Figure 5, it is visible that the external magnetic field essentially change the value  $K^I$  compare with a purely elastic case (magnetic field  $h_c^2 = 0$ ).

In all cases the normalized intensity factor  $K^I$  is essentially depends upon the enclosed magnetic field and the speed of the crack. The numerical calculations show also: a) for  $V^2/c_1^2 = \bar{V} < 0.28$ , the intensity factor  $K^I$  first increases with increasing magnetic field then sharply decreases and passes through zero, becomes negative and finally increases sharply, b) when  $V^2/c_1^2 = \bar{V} \rightarrow 0.3$  (i. e. the speed of the crack is close to the propagation speed of Rayleigh waves), the intensity factor  $K^I \rightarrow \infty$  with the increase of magnetic field. Note, that the linear law expressed in Eq. (2.53) takes place at rather weak magnetic fields. The nonlinear laws (2.51), (2.52) and (2.54) can be used for strong magnetic fields. Synergistic implications of the interaction of magneto-elastic fields on the intensity factor  $K^I$  for different law of magnetizations are displayed in Figures 5-8.

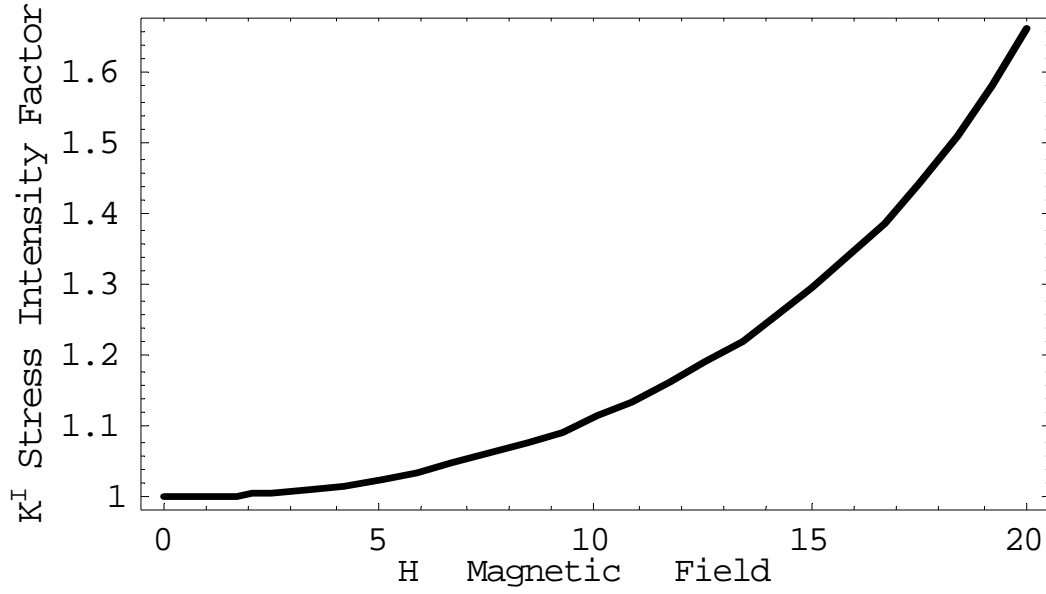
**DREIFOUS FORM:**  $(\chi(H) = (b/H)\text{arctg}(\kappa H/b))$



**Figure 5a:** Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $b = 10^6$ ,  $\kappa = 10^5$  and  $\bar{V} = 0.2$

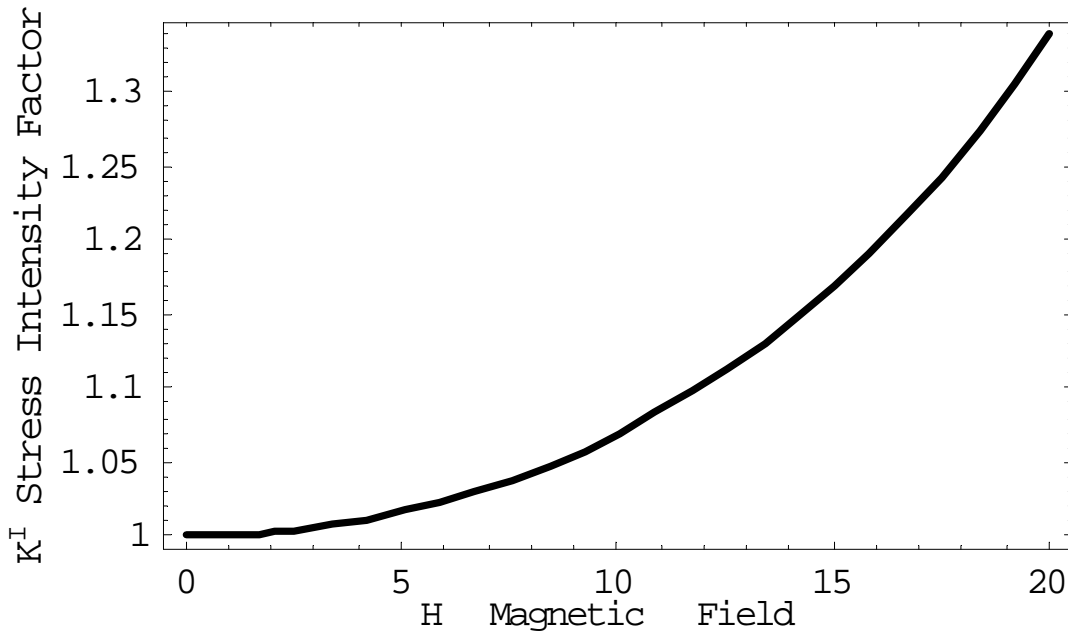


**Figure 5b:** Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $b = 10^6$ ,  $\kappa = 10^5$  and  $\bar{V} = 0.18$



**Figure 5c: Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters**

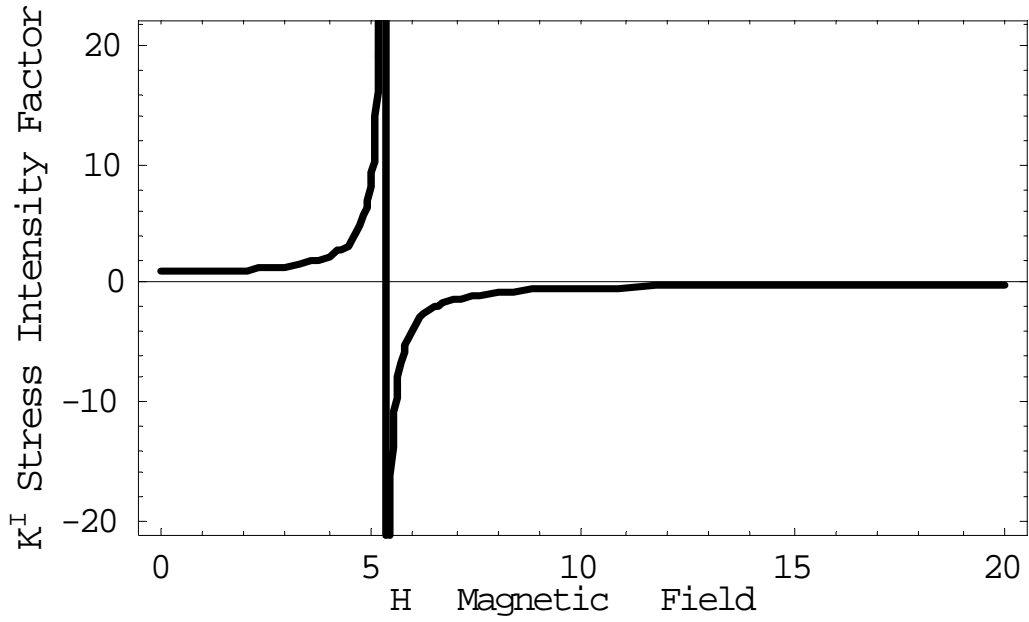
$b = 10^6$ ,  $\kappa = 10^5$  and  $\bar{V} = 0.1$



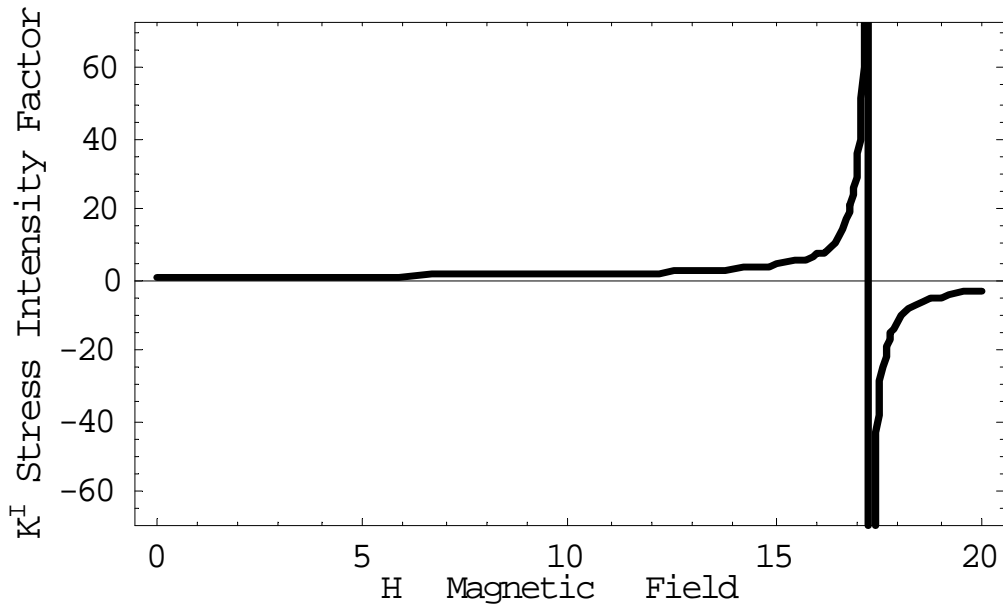
**Figure 5d: Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters**

$b = 10^6$ ,  $\kappa = 10^5$  and  $\bar{V} = 0.0001$

**Linear Law of Magnetization:** ( $\chi(H) = \kappa = \text{const}$ )

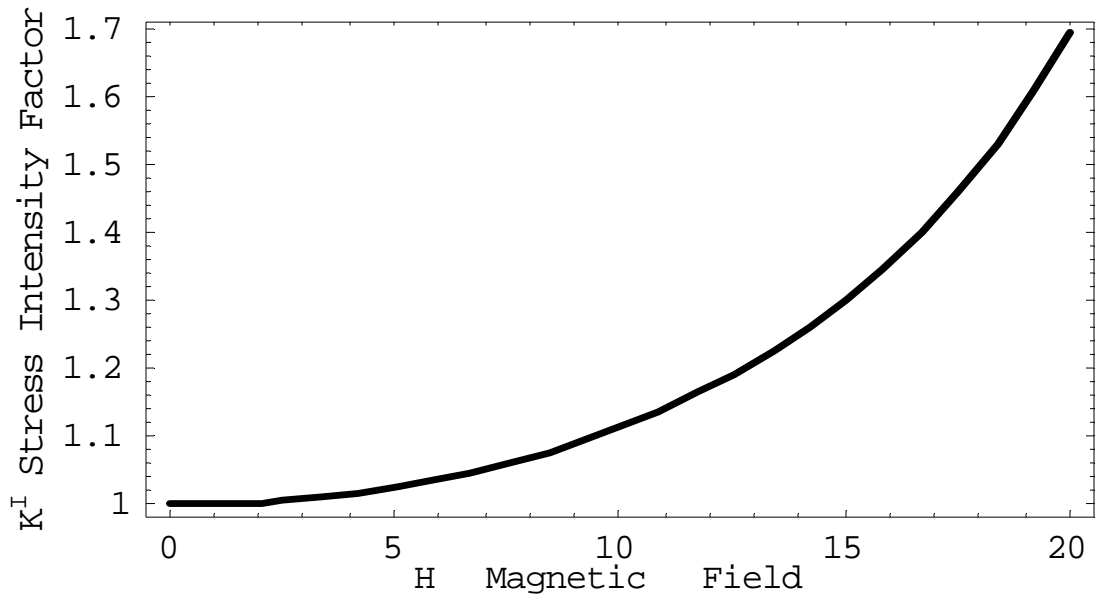


**Figure 6a:** Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $\kappa = 10^5$  and  $\bar{V} = 0.2$

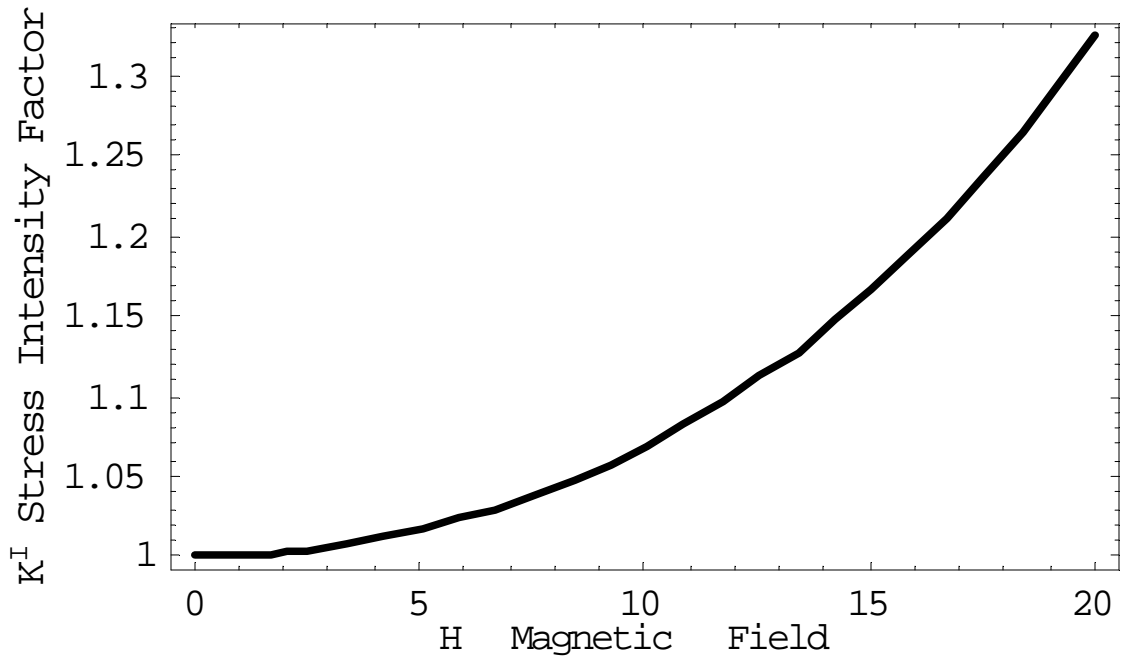


**Figure 6b:** Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $\kappa = 10^5$  and  $\bar{V} = 0.18$



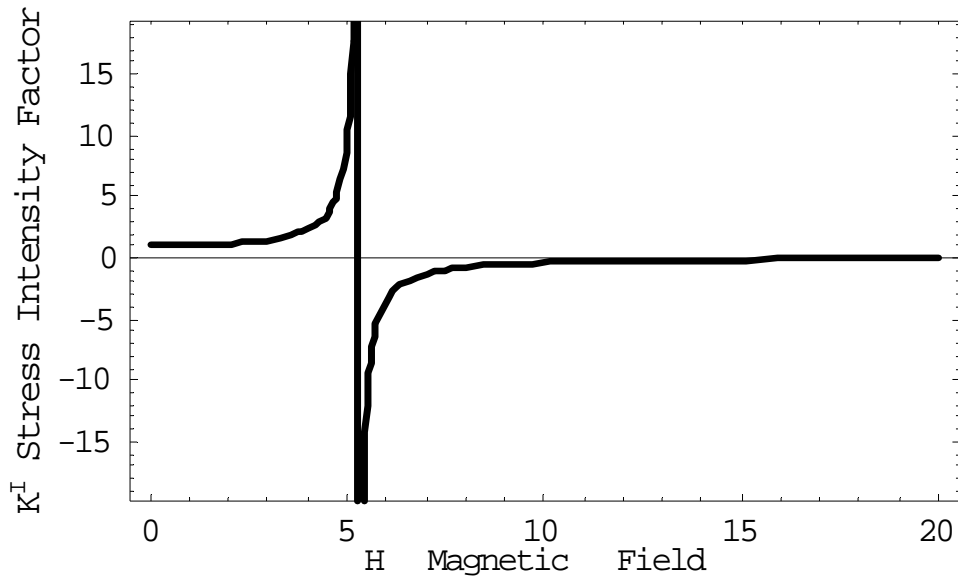


**Figure 6c: Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $\kappa = 10^5$  and  $\bar{V} = 0.1$**

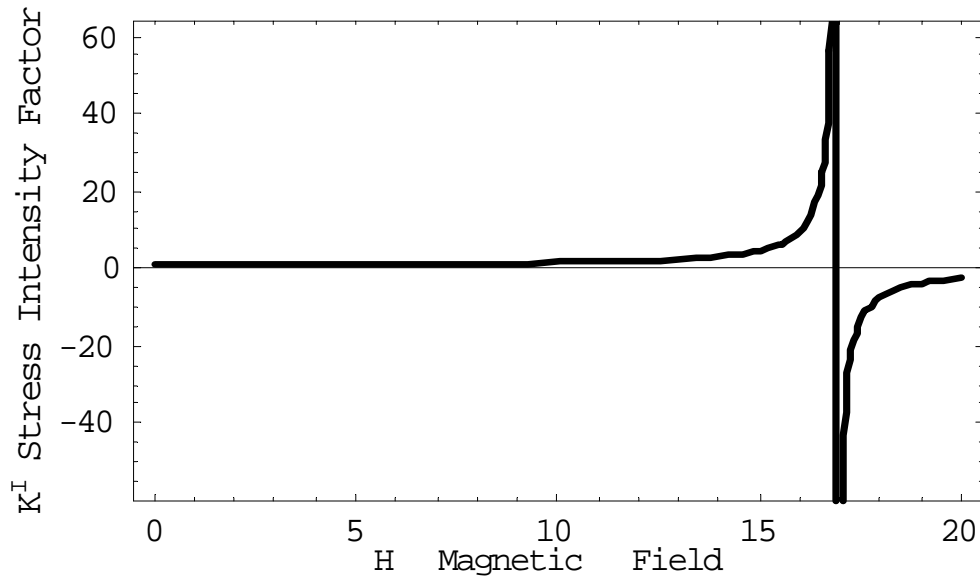


**Figure 6d: Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $\kappa = 10^5$  and  $\bar{V} = 0.0001$**

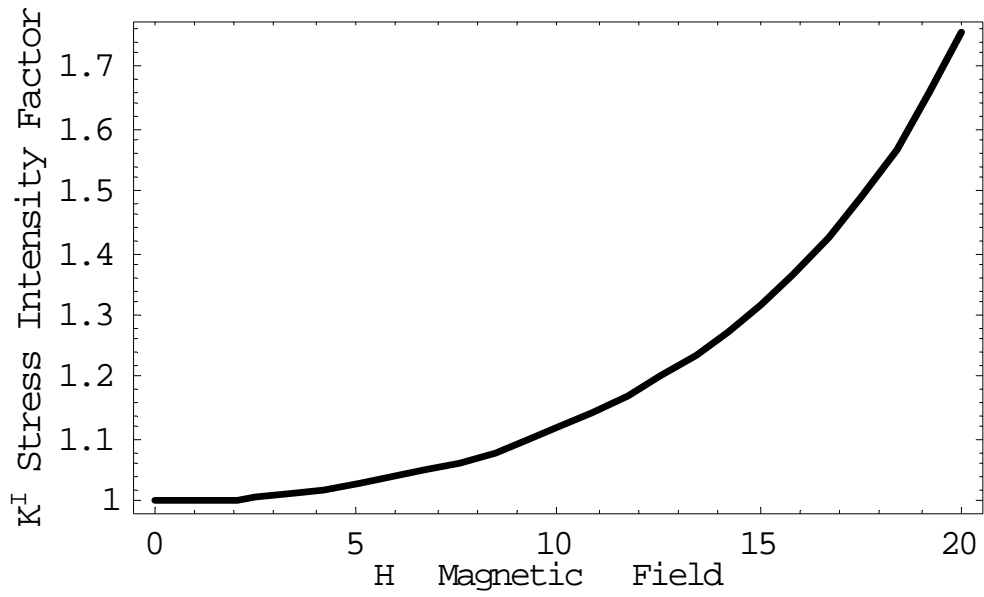
**Rayleigh form:** ( $\chi(H) = \kappa + bH$ )



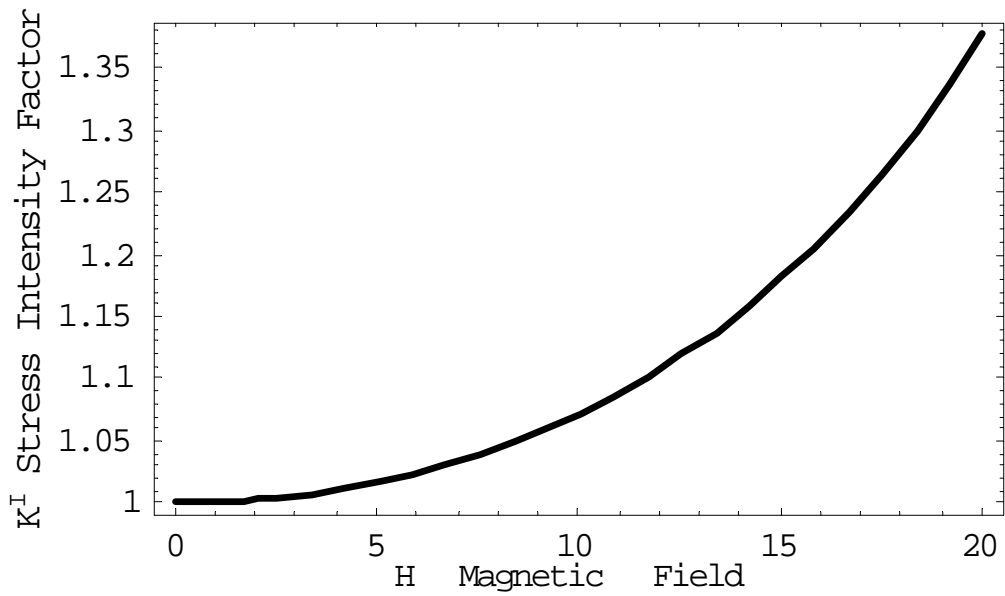
**Figure 7a:** Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $b_r = 10^3$ ,  $\kappa = 10^5$  and  $\bar{V} = 0.2$



**Figure 7b:** Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $b_r = 10^3$ ,  $\kappa = 10^5$  and  $\bar{V} = 0.18$

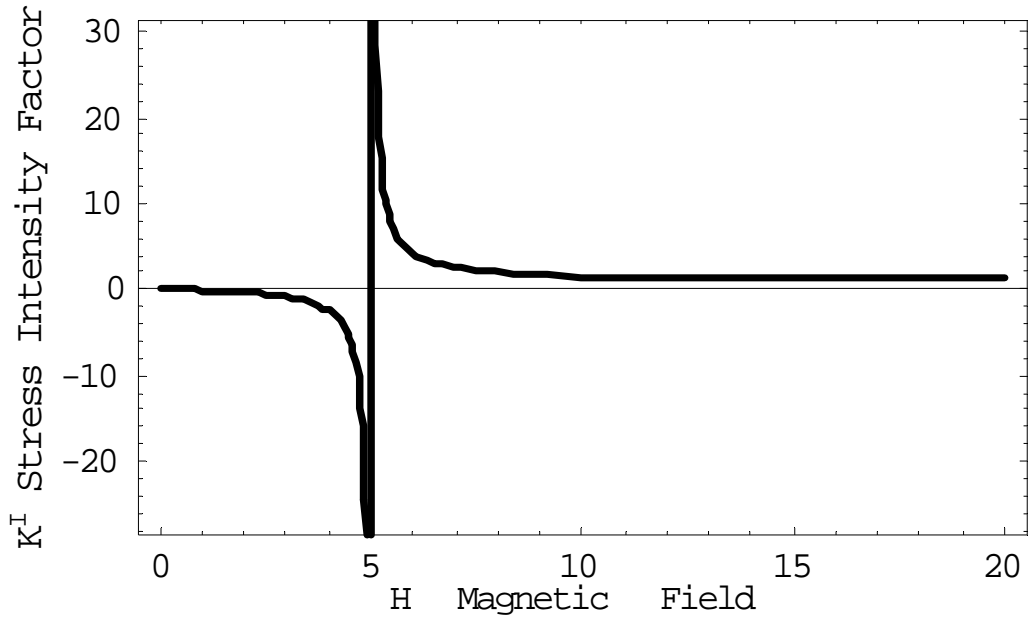


**Figure 7c: Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $b_r = 10^3$ ,  $\kappa = 10^5$  and  $\bar{V} = 0.1$**

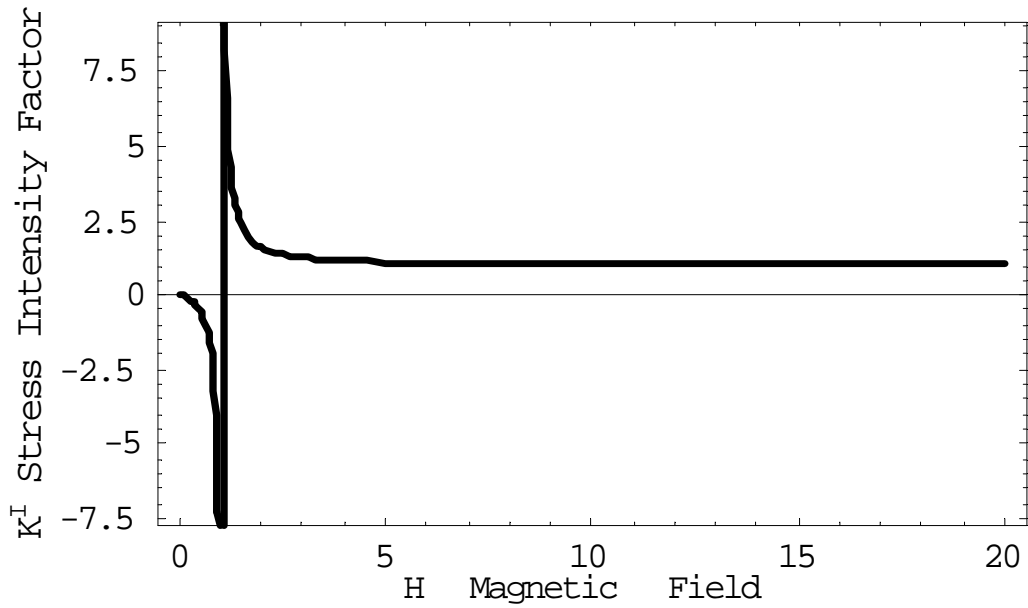


**Figure 7d: Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $b_r = 10^3$ ,  $\kappa = 10^5$  and  $\bar{V} = 0.0001$**

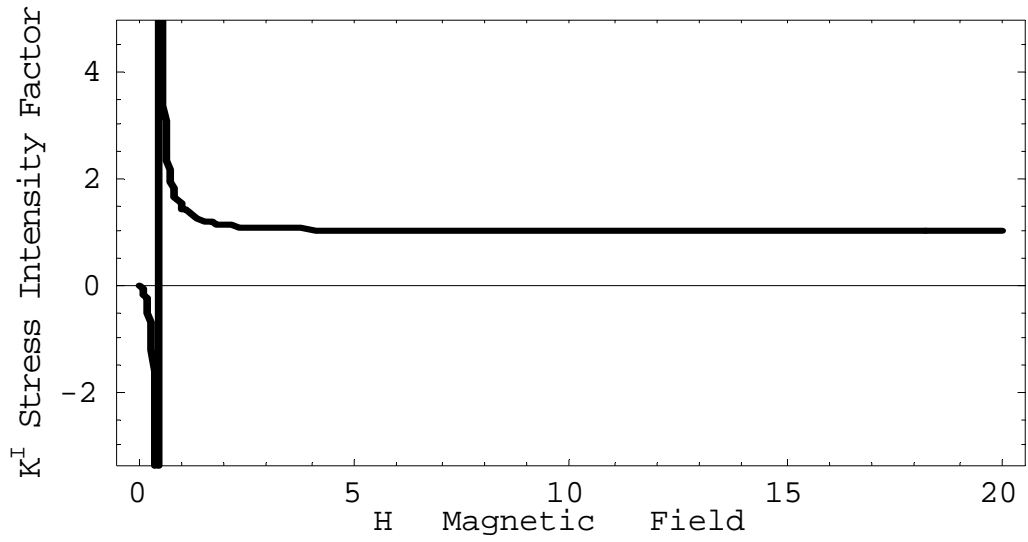
**Magneto rigid dependence:** ( $\chi(H) = M_{s1} / H$ )



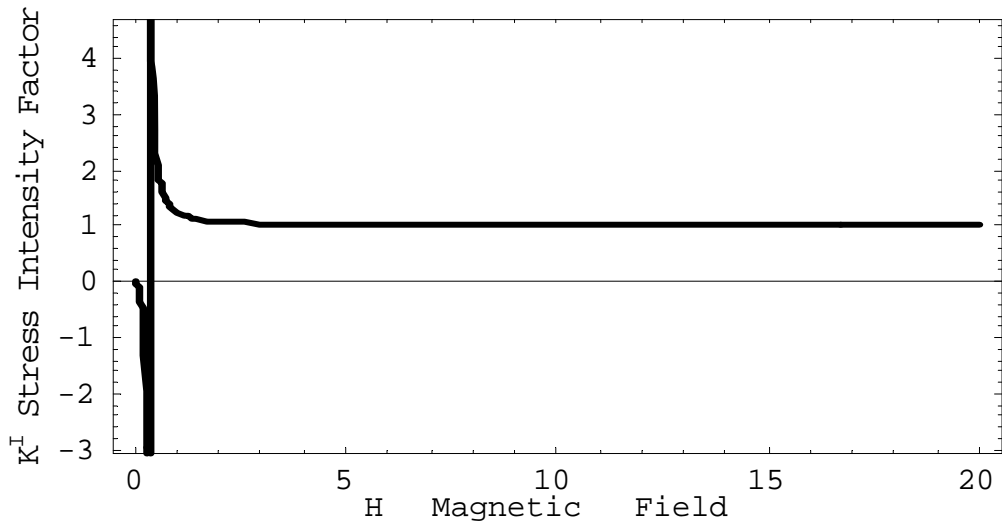
**Figure 8a:** Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $M_{s1} = 10^5$  and  $\bar{V} = 0.2$



**Figure 8b:** Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $M_{s1} = 10^5$  and  $\bar{V} = 0.18$



**Figure 8c: Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $M_{s1} = 10^5$  and  $\bar{V} = 0.1$**



**Figure 8d: Dependence of  $K^I$  on dimensionless magnetic field  $H$  for parameters  $M_{s1} = 10^5$  and  $\bar{V} = 0.0001$**

## §4.2 MAGNETOELASTIC STRESSES

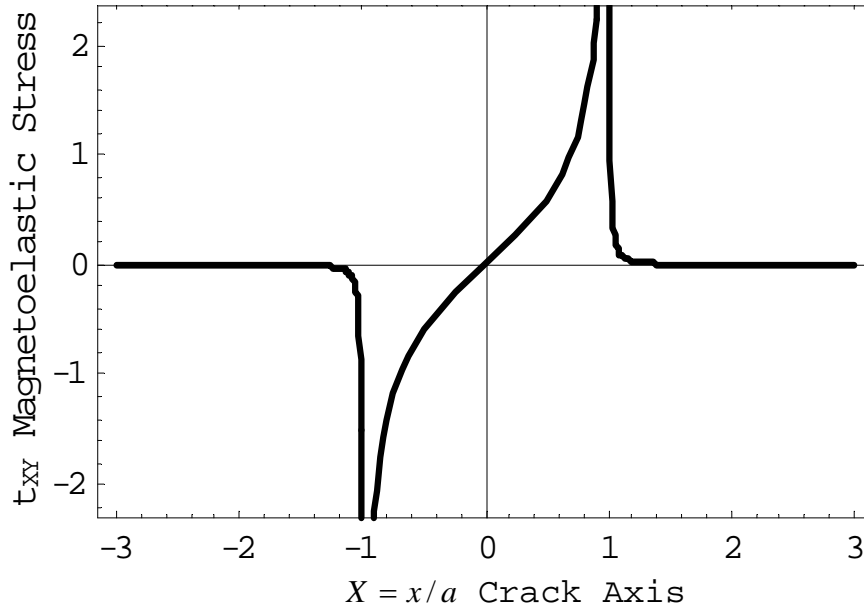
Magneto-elastic stresses have been calculated by using Eqs. (3.27)-(3.28). Dimensionless stresses  $t_{XY}(x, y) = t_{xy}^T(x, y)/(P_0\mu)$  and  $t_{YY}(x, y) = t_{yy}^T(x, y)/(P_0\mu)$  are shown in Figures 9-10 respectively. The stress distributions were essentially independent of the form assumed for the magnetization law, so only these calculated for the Dreifous form will be presented for illustration purposes.

Figures 9-10 are plots of magnetic-induced stresses  $t_{XY}(x, y)$  and  $t_{YY}(x, y)$  with increasing magnetic field. By comparing Fig. 9b with Fig. 9c, it can be seen, that  $t_{XY}(x, y)$  abruptly changes sign at a dimensionless field between  $H = 0.09$  and  $H = 0.1$ .

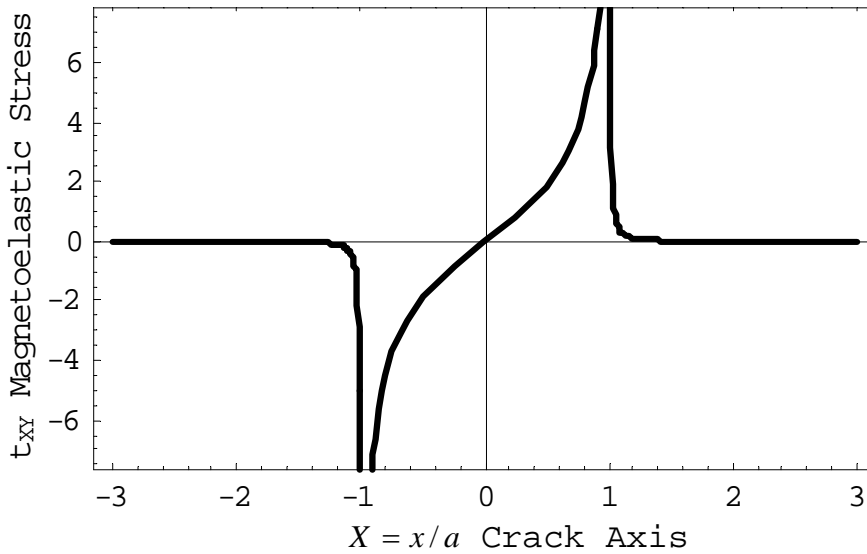
By comparing Figs. 10a, 10b and 10c,  $t_{YY}(x, y)$  goes from purely negative ahead of the crack tip to positive at  $H = 0.09$  to negative again of  $H = 0.1$ . That is, for a field of about  $H = 0.09$  stress around of the crack tip switches from tensile to compressive and then back to tensile again. This field strength is well beyond the saturation field for the material parameters assumed in the calculations.

For calculations we assumed also: normalized velocity of a crack:  $V^2/c_1^2 = \bar{V} = 0.1$ ; normalized distance:  $y/a = 0.01$  and the normalized parameter:  $X = x/a$  are changes in interval $[-3, 3]$ .

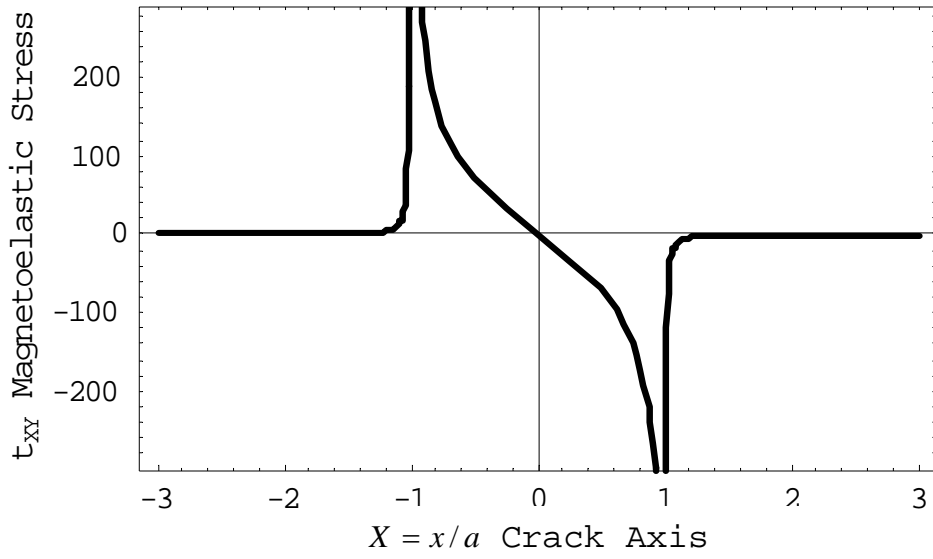
*Dimensionless magnetoelastic stresses  $t_{xy}$  and  $t_{yy}$  on the  $y/a = 0.01$  surface; nonlinear law of magnetization is in a Dreifous form  $\chi(H) = (b/H)\text{arctg}(\kappa H/b)$  for parameters  $b = 10^6, \kappa = 10^5$ ; normalized crack speed is  $V^2/c_1^2 = \bar{V} = 0.1$*



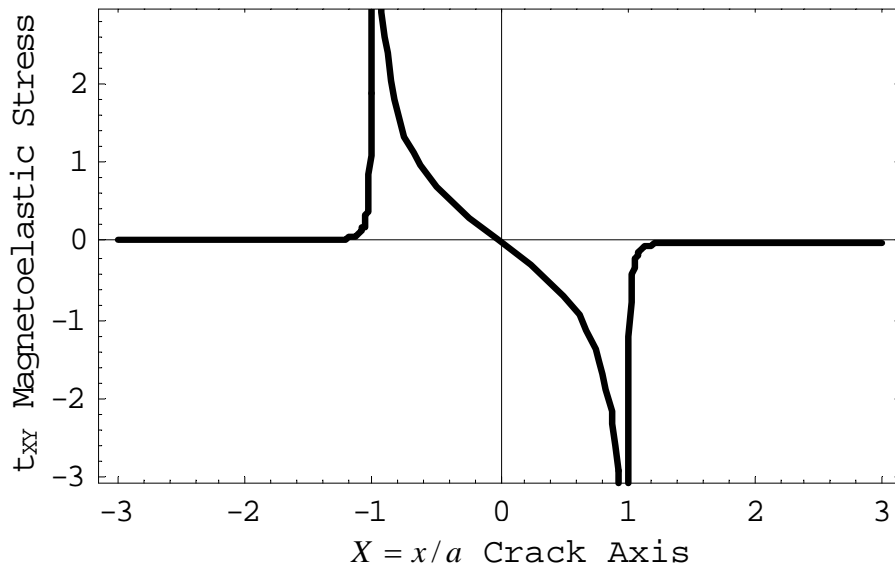
**Figure 9a: Dimensionless stress  $t_{xy}$  when  $H=0.001$**



**Figure 9b: Dimensionless stress  $t_{xy}$  when  $H=0.05$**

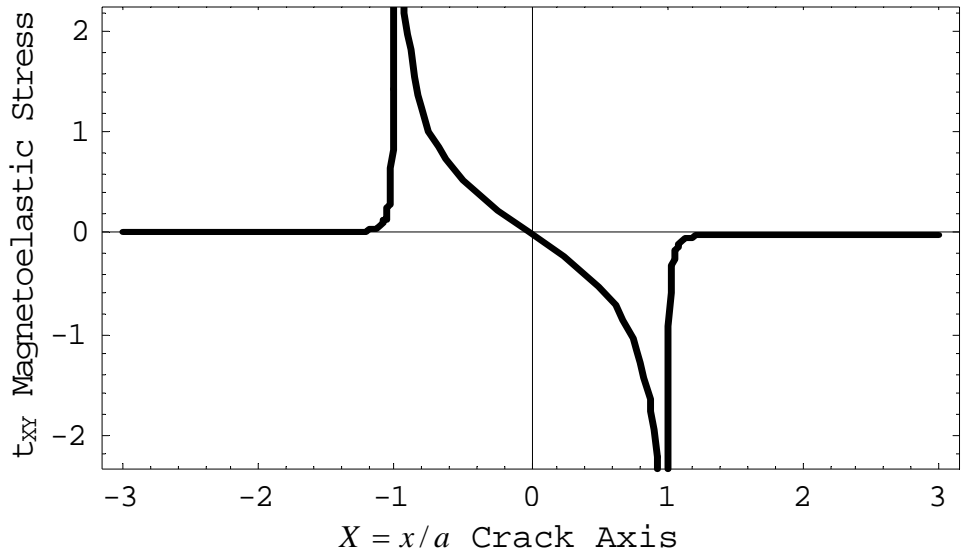


**Figure 9c: Dimensionless stress  $t_{xy}$  when  $H=0.1$**

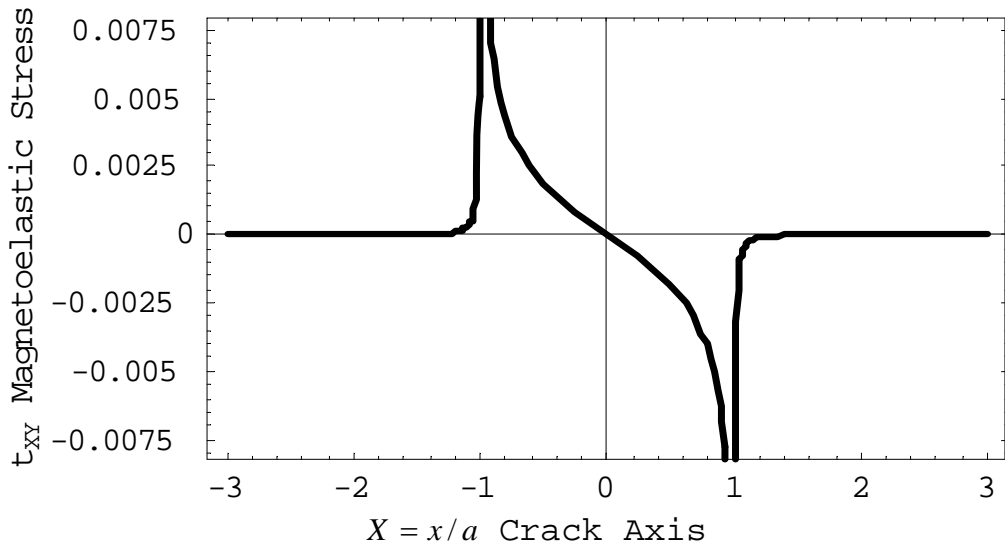


**Figure 9d: Dimensionless stress  $t_{xy}$  when  $H=2$**

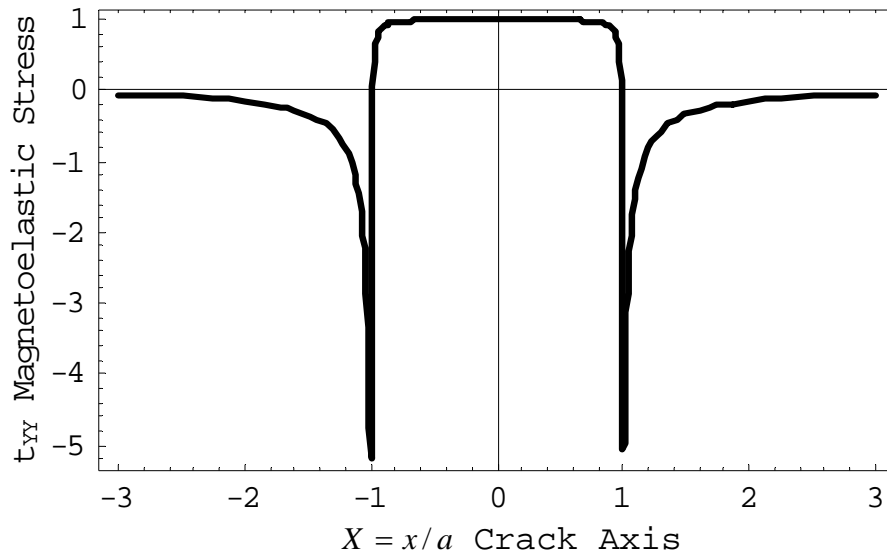




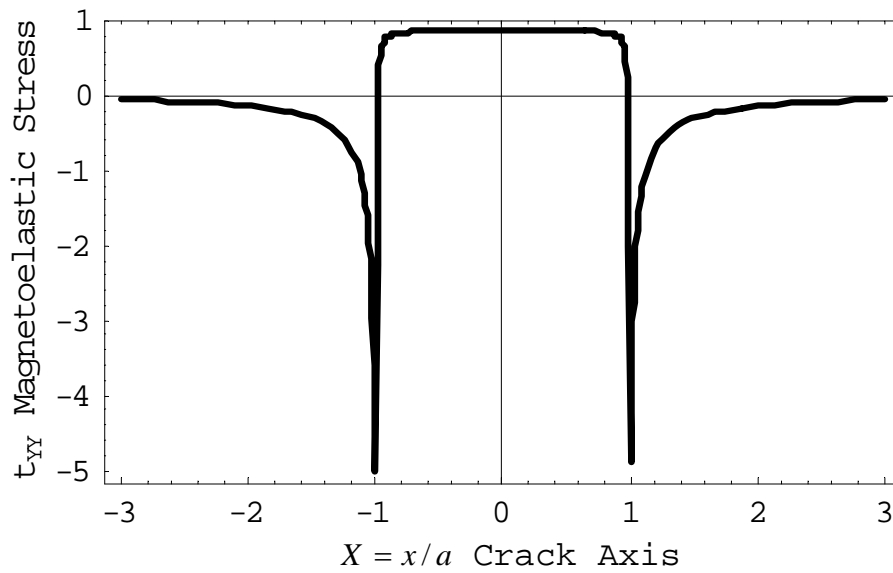
**Figure 9e: Dimensionless stress  $t_{xy}$  when  $H=10$**



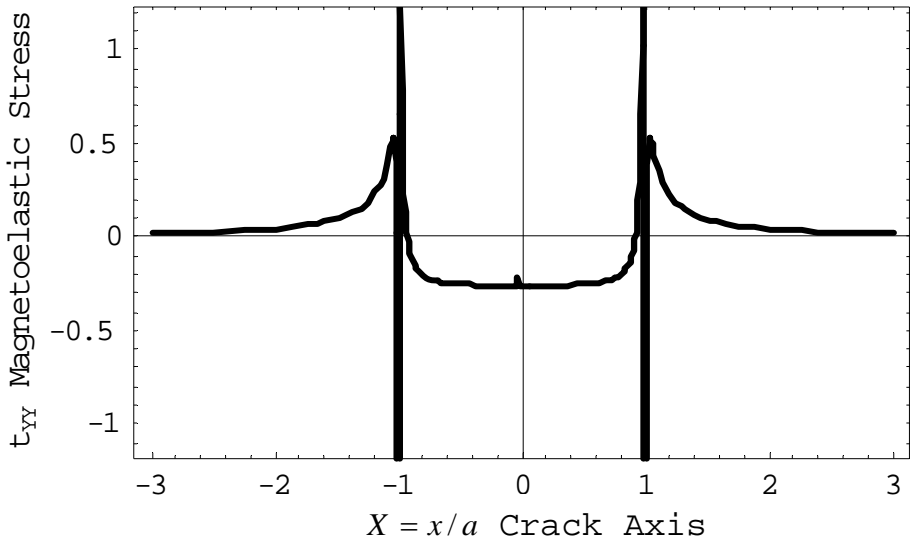
**Figure 9f: Dimensionless stress  $t_{xy}$  when  $H=900$**



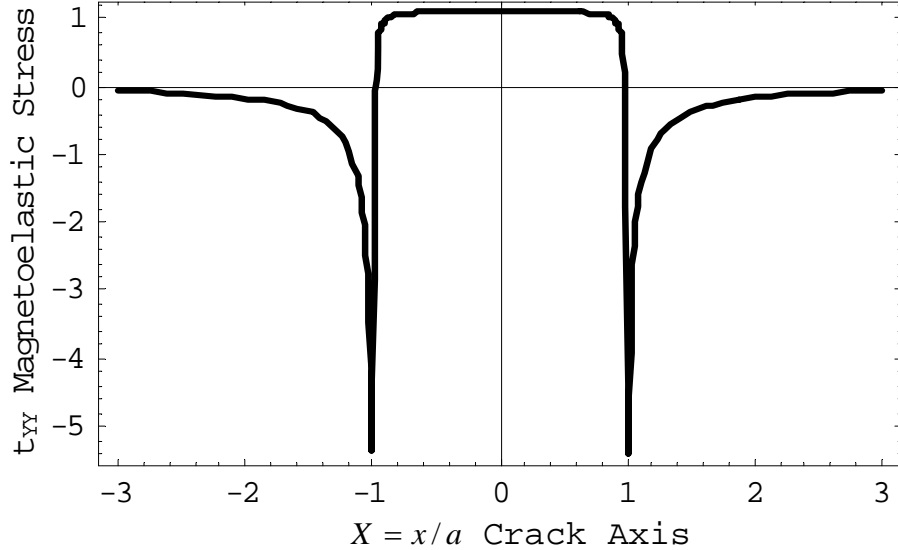
**Figure 10a: Dimensionless stress  $t_{yy}$  when  $H=0.001$**



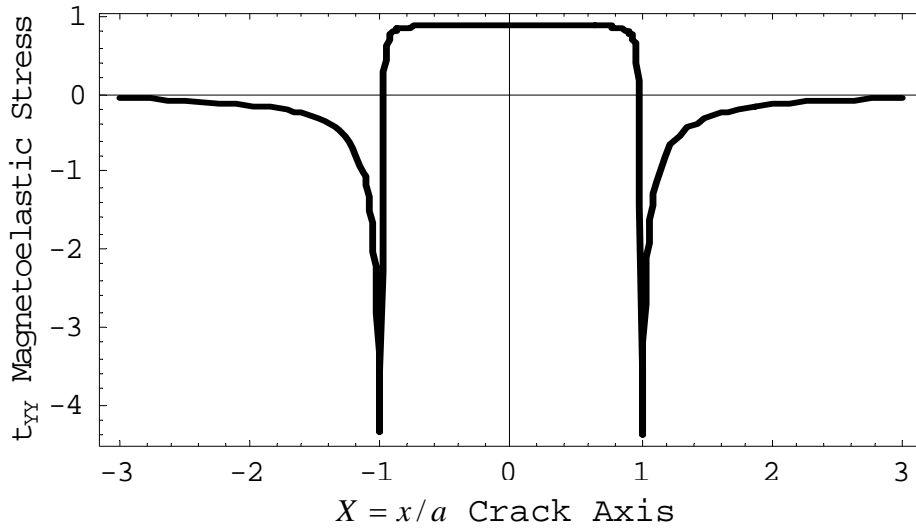
**Figure 10b: Dimensionless stress  $t_{yy}$  when  $H=0.05$**



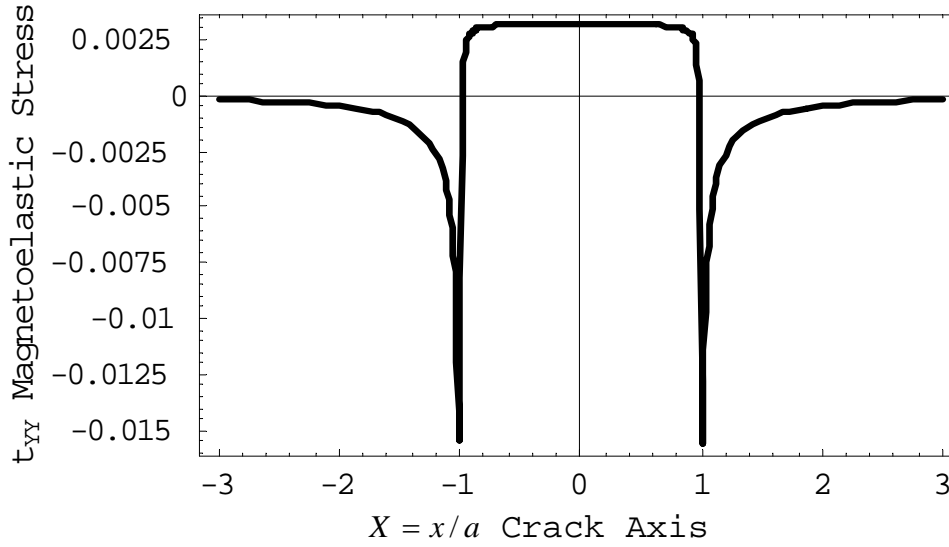
**Figure 10c: Dimensionless stress  $t_{yy}$  when  $H=0.09$**



**Figure 10d: Dimensionless stress  $t_{yy}$  when  $H=2$**



**Figure 10e: Dimensionless stress  $t_{yy}$  when H=10**



**Figure 10f: Dimensionless stress  $t_{yy}$  when H=900**

## CONCLUSIONS

In this work the equations of magnetoelasticity for magnetosoft materials with a nonlinear law of magnetization are presented. Based on the present model the stress-strain state of ferromagnetic plane with a moving crack in a transverse magnetic field has been analyzed. It has been shown that the nonlinear law of magnetization has qualitative and quantitative influence on the magnetoelastic quantities. Introducing a magnetic field into the elastic crack problem makes stress intensity velocity dependent and produces a singularity at velocities close to the Rayleigh velocity. Magnetic fields introduce anomalies in the stress distribution around a crack. The shear stress component changes sign at a critical field and the normal stress ahead of the crack tip switches from tensile to compressive and back to tensile again with increase field strength. The present model can be used in magnetoelasticity studies where soft ferromagnetic medium are considered and are under the presence of large magnetic fields. It can be useful for machining (raising intensity factor) or increasing toughness (making intensity factor negative).

## POTENTIAL WORK

- Confirm the analytical model with experiment. To do this, fracture toughness tests can be performed on an alloy that has both high susceptibility (or permeability) and low ductility. Candidate materials include High Purity Iron and Hyperco (Fe-49Co-2V), a relatively brittle ordered alloy with moderately high susceptibility.
- The model we have developed assumes a homogeneous magnetic field within the material. However, demagnetized ferromagnetic materials have a random distribution of magnetization direction arranged in a domain structure. An important modification of the current model would be to account for a distribution of magnetization directions (that is, a domain structure).

## REFERENCES

1. G.Y. Bagdasaryan, Vibrations and Stability of Magnetoelastic Systems (in Russian), Editorial Board of Yerevan State University, Yerevan (1999).
2. G.Y. Bagdasaryan, and D.J. Hasanyan, On a Magnetoelastic Equations of Ferromagnetic Body With a Nonlinear Law of Magnetization (in Russian), Proc. NAS Armenia, Physica, 30, 6, 253-260 (1995).
3. M.V. Belubekyan, and Yu.M. Khachatryan, On a Problem of Magnetoelastic Stability of Ferromagnetic Plate (in Russian), Proceedings of Academy of Science of BSSR, ser. Phys.-Math. Sci., 110, 110-116 (1984).
4. W.F. Brown Jr., Magnetoelastic Interactions, Springer-Verlag, New York (1966).
5. R.M. Bozorth, Ferromagnetism. Toronto-New-York-London (1951).
6. Yu. A. Brychkov, A.P. Prudnikov, Integral Transforms of Generalized Functions, Gordon & Breach Sci. Publ., New York, 1989.
7. A.C. Eringen, and G.A. Maugin, Electrodynamics of Continua Media, 1, 2, New York; Springer Verlag (1990).
8. A.R. Gachkevich, and M.J. Solodiak, Thermoelasticity Electroconducting Ferromagnetic solids, 3-All-Union Symp, "Theoretical Problems of Magnetoelasticity" (in Russian), Yerevan, 56-59 (1984).
9. F.D. Gakhov, Boundary Value Problems (I.N. Sneddon, Trans.), Addison-Wesley, Reading, MA, 1966, pp. 426-428.
10. D.J. Hasanyan, and G.T. Philiposyan, Modeling and Stability of Magnetosoft Ferromagnetic Plates in a Magnetic Field. Proceedings of the Royal Society London A, UK, 457, 2063-2077 (2001).

11. P.M. Kolesnikov, Introduction to Nonlinear Electrodynamics (in Russian), Minsk (1975).
12. L.D. Landau, and E.M. Lifshitz, Electrodynamics of Continuous Media. Oxford [England]: Butterworth-Heinemann (1995).
13. F.C. Moon, Magnetosolid Mechanics, Wiley, New York (1984).
14. F.C. Moon, and Y.-H. Pao, Vibration and Dynamic Instability of a Beam-Plate in a Transverse Magnetic Field, Trans. ASME , Journal of Applied Mechanics, 36, 92-100 (1969).
15. G.A. Maugin, Continuum Mechanics of Electromagnetic Solids, North-Holland (1988).
16. Y. Nishida, Y. Shindo, and A. Atsumi, Diffraction of Horizontal Shear Waves by a Moving Interface Crack, Acta Mechanica, 54, 23-34 (1984).
17. Y.-H. Pao, and C.S. Yeh, A Linear Theory for Soft Ferromagnetic Elastic Solids, J. Eng. Sci., 11, 415-435 (1973).
18. A.D. Polyanin, A.V. Manzhirov, Handbook of Integral Equations, CRC Press, 2000.
19. Y. Shindo, The Linear Magnetoelastic Problem for a Soft Ferromagnetic Elastic Solid with a Finite Crack, Trans. ASME, Journal of Applied Mechanics, 44, 47-51 (1977).
20. Y. Shindo, A. Ogihara, and K. Horiguchi, Dynamic Bending of a Magnetically Saturated Plate with a Thorough Crack in a Uniform Magnetic Field, Theoretical and Applied Mechanics, 49, 69-81 (2000).



21. H.F. Tiersten, Coupled Magnetomechanical Equations for Magnetically Saturated Insulators, *J. Math. Phys.*, 5, 1298-1318 (1964).
22. A.A.F. van de Ven, Magnetoelastic Buckling of Magnetically Saturated Bodies, *Acta Mechanica*, 47, 229-246 (1983).
23. X.-J. Zheng, and X. Wang, Analysis of Magnetoelastic Interaction of Rectangular Ferromagnetic Plates with Nonlinear Magnetization, *Int. J. of Solids and Structures*, 38, 8641-8652 (2001).

## **APPENDIX A: NOMENCLATURE**

$\vec{H}_0$  - Magnetic field characterizing undeformed state of a ferromagnetic body

$\vec{M}_0$  - Magnetization characterizing undeformed state of a ferromagnetic body

$\vec{B}_0$  - Magnetic induction characterizing undeformed state of a ferromagnetic body

$\vec{H}^0$  - Initial applied magnetic field

$\chi(H_0)$  - Magnetic susceptibility of the material

$2a$  - Length of a finite crack

$\mu_0$  - Magnetic permeability of the vacuum

$\vec{n}_0$  - Normal of a crack surface

$\rho$  - Mass density

$u_i$  - Displacement components

$f_i$  - Magnetic volume forces

$\mu$  - Lamé coefficient

$\mu_r$  - Magnetic permeability

$c_1, c_2$  - Longitudinal and transversal elastic waves speeds

$V$  - Velocity of a crack

$c_R$  - Speed of a propagation of the Raleigh waves

$M_i$  - Mach numbers

$\vec{h}(h_1, h_2)$  - Induced magnetic field

$\varphi, \psi$  - Potential functions characterizing displacements in a fixed rectangular coordinates

$\Phi$  - Magnetic potential

$\nabla^2 = \Delta$  - Laplace's operator

$t_{yy}^T(x, y), t_{xy}^T(x, y)$  - Magnetoelastic stresses

$t_{yy}^M(x, y), t_{xy}^M(x, y)$  - Maxwell's stress components

$t_{yy}^E(x, y), t_{xy}^E(x, y)$  - Pure elastic stresses

$\nu$  - Poisson's ratio

$P_{0mec}$  - Mechanical force acting on the surface of a crack

$P_{0mag}$  - Magnetic force

$P_0$  - Sum of the magnetic and mechanical forces

$K^I$  - Magnetoelastic stress intensity factor

## APPENDIX B: THE SOLUTION OF DUAL INTEGRAL EQUATIONS

The pair of dual integral equations of our consideration is:

$$\frac{1}{\pi} \int_{-\infty}^{\infty} D(\alpha) \exp[-ix\alpha] d\alpha = u_y(x,0) = 0, \quad |x| > a \quad (1)$$

$$\frac{1}{\pi} \int_{-\infty}^{\infty} |\alpha| D(\alpha) \exp[-ix\alpha] d\alpha = f(x), \quad |x| < a \quad (2)$$

where  $f(x)$  is known, and  $D(\alpha)$  is to be determined. To obtain the solution of the pair of equations (1) - (2) we define the function  $E(x)$  as follows:

$$\int_{-\infty}^{\infty} \alpha D(\alpha) \exp[-i\alpha x] d\alpha = E(x) \equiv \begin{cases} 0, & |x| > a \\ E(x), & |x| < a \end{cases} \quad (3)$$

Using the Fourier transformation theorem, from (3) we can get:

$$\alpha D(\alpha) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(x) \exp[-i\alpha x] dx \quad (4)$$

and using (1) the expression (4) can be rewritten in the following form:

$$\alpha D(\alpha) = \frac{1}{2\pi} \int_{-a}^a E(x) \exp[-i\alpha x] dx \quad (5)$$

Notice that the integral representation (5) is identically satisfying to Eq.(1).

Substituting (5) into Eq. (2) and using the result [9, 18]:

$$\int_{-\infty}^{\infty} \frac{|\alpha|}{\alpha} \exp[i\alpha t] dt = \frac{2}{t}, \quad (6)$$

we can get

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{|\alpha|}{\alpha} \exp[i\alpha x] \int_{-a}^a E(s) \exp[-i\alpha s] ds d\alpha =$$

$$\frac{1}{\pi} \int_{-a}^a E(s) \int_{-\infty}^{\infty} \frac{|\alpha|}{\alpha} \exp[i\alpha(x-s)] d\alpha d\alpha = \frac{1}{\pi} \int_{-a}^a E(s) \frac{2}{s-x} ds = \frac{2}{\pi} \int_{-a}^a \frac{E(s)}{s-x} ds = f(x) \quad (7)$$

Thus, we will solve the following singular integral equation:

$$\frac{2}{\pi} \int_{-a}^a \frac{E(s)}{s-x} ds = f(x) \quad \text{when} \quad |x| < a \quad (8)$$

In classes of unbounded functions the solution of (6) has a following form [9, 18]:

$$E(s) = \frac{1}{2\pi\sqrt{a^2-s^2}} \left[ c_0 - \int_{-a}^a \frac{f(\tau)\sqrt{a^2-\tau^2}}{\tau-s} d\tau \right] \quad (9)$$

where  $c_0 = const$  and is equal to zero because of condition

$$\int_{-a}^a E(s) ds = u_y(a,0) - u_y(-a,0) = 0.$$

In our case  $f(x) = -P_0 / R_0 = const$ , thus from (8) we can get:

$$E(s) = \frac{P_0}{2i R_0} \frac{s}{\sqrt{a^2-s^2}}, \quad |s| < a \quad (9)$$

**APPENDIX C: MAGNETIZATION LAWS IN TERMS OF NONDIMENSIONAL  
PARAMETERS**

**Dreifous form**

$$\chi(H_0) = \frac{\beta}{\mu_0 H_0} \operatorname{arctg} \left[ \frac{(\mu_{ri} - 1) \mu_0 H_0}{\beta} \right] \text{ in terms of dimensionless parameters are}$$

expressed as:

$$\chi(H_0) = \frac{\beta}{\mu_0 h_c \sqrt{\mu/\mu_0}} \operatorname{arctg} \left[ \frac{\kappa \mu_0 \mu_0 h_c \sqrt{\mu/\mu_0}}{\beta} \right],$$

where  $\kappa = \mu_{ri} - 1$  is an initial susceptibility of the material;  $h_c = H_0 \sqrt{\mu_0/\mu}$  is a

dimensionless magnetic field. For most ferromagnetic materials elastic modulus

$\mu \sim 10^{12} \text{ N/m}^2$ . From here follows that  $h_c \sim \sqrt{\frac{10^{-6}}{10^{12}}} H_0 \left[ \frac{\text{m}}{\text{A}} \right] \sim 10^9 H$ , where

$H = H_0 \times \left[ \frac{\text{m}}{\text{A}} \right]$  is a dimensionless magnetic field.

After all, the Dreifous form becomes:

$$\chi(H_0) = \frac{\beta}{10^{-9} H \sqrt{\mu_0 \mu}} \operatorname{arctg} \left[ \frac{\kappa 10^{-9} H \sqrt{\mu_0 \mu}}{\beta} \right] = \frac{b}{H} \operatorname{arctg} \left[ \frac{\kappa H}{b} \right],$$

where

$$b = 10^9 \beta \sqrt{\mu \mu_0} = 10^9 \frac{2B_s}{\pi} \sqrt{\mu \mu_0} \sim \frac{2B_s}{\pi} 10^6$$

Parameter  $b$  depends on induction saturation  $B_s$  and for most ferromagnetic materials  $b \sim 10^6$ . For superpermalloy the following parameters are :  $b \sim 10^6$  and  $\kappa = \mu_{ri} - 1 \sim 10^5$ .

**1) Rayleigh form**

$$\chi(H_0) = \kappa + b_r H_0 = \kappa + b_{r1} H,$$

where  $\kappa = \mu_{ri} - 1$  is an initial susceptibility of the material;  $b_r$  and  $b_{r1}$  are nonlinear coefficient and dimensionless nonlinear coefficient respectively expressed in Rayleigh form. For superpermalloy the following dimensionless parameters are:

$$b_{r1} \sim 10^3 \text{ and } \kappa = \mu_{ri} - 1 \sim 10^5.$$

**2) Magneto rigid form**

$$\chi(H_0) = M_s / H_0 = M_{s1} / H,$$

where  $M_s$  and  $M_{s1}$  are saturation magnetization and dimensionless saturation magnetization respectively.

## Vita

Satenik Harutyunyan was born and grown up in Armenia, one of the small republics of former Soviet Union. She earned her M.S. Degree in Mechanical Engineering and Applied Mathematics from the Yerevan State University of Armenia in 1995. Satenik worked as a Research Assistant at the Institute of Mechanics, where she has been conducting research on Plasticity and strengthening problems of Cylindrical and Conical tubes. Currently she is pursuing her Master's/Pd.D. Degree in Materials Science and Engineering Department at Virginia Tech and is advised by Dr. W. Reynolds. Her thesis focuses on Magneto-Elastic Interactions in a Ferromagnetic Materials.