Control Power Requirements
for the
Velocity Vector Roll

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(ABSTRACT)

A method for determining the maximum control moments required for an aircraft to perform a velocity vector roll is investigated. The velocity vector roll is assumed to occur at constant angle of attack, constant velocity, and zero sideslip.

A simplified set of equations is developed for the non dimensional control moments about the three principal body axes. These equations take on a form well suited for numerical optimization methods. The Schittkowski SQP optimization code is used to provide fast, accurate solutions. The numerical method also shows the advantage of being adaptable to changing the airframe and flight performance parameters.

An exercise to find the global control moment maxima was performed for a an F-18 with constant aerodynamic derivatives and a load factor of one. The optimization was run for a range of discrete steady state roll rates, roll mode time constants and velocities. The results showed trends for the maxima to occur at the highest steady state roll rate parameter, smallest roll mode time constant and lowest velocity. Each control axis maximum is specific to a particular orientation and angle of attack. For the roll axis, the maximum occurs at nearly zero angle of attack and 270 of wind axis bank angle. The yaw axis maximum occurs at the largest angle of attack (70°) and 90 of wind axis bank angle. The pitch maximum occurs near 270 of wind axis bank and 55 angle of attack, but is highly sensitive to the selection of \( C_{ma} \). All control moment maxima occur at a flight path angle of 0°. The roll and yaw control moment maxima occur upon a maximum roll input starting from rest at the specified orientation and angle of attack. The pitch control maximum occurs at the steady state roll rate when the proper orientation and angle of attack is encountered.
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1.0 Introduction

A velocity vector roll can be defined as an aircraft rolling about its velocity vector at constant angle of attack and zero sideslip. After one revolution, an imaginary line from the plane's center of gravity to the nose will have traced out a cone with the velocity vector lying along the axis. This coning motion is accentuated as the angle of attack gets large. Historically, the majority of tactical maneuvering in the lateral-directional axes has been done at small angles of attack due mainly to concerns over maintaining a high specific energy and preventing departure. Advancements in technology of the planes and their weapons systems has prompted interest in studying aircraft agility at high angles of attack (\( \alpha \)). NASA is currently doing this type of flight research with the X-31 and F-18 HARV (High Alpha Research Vehicle, an F-18 modified with thrust vectoring).

Much of the research on high alpha maneuvering has come from studies using piloted simulations. Currently at NASA Langley there are two programs developing design guidelines for future attack/fighter aircraft. HANG (High Alpha Nose down Guidelines) is investigating the longitudinal axis requirements and HAIRRY (High Alpha Investigation of Requirements for Roll and Yaw) is studying the lateral-directional requirements [2]. Additionally, airframe contractors have been doing their own studies as part of the development process for their proposed next generation designs.

The simulator has several advantages that make it an ideal tool for this kind of research. Changes in the parameters affecting the performance of the aircraft can be implemented quickly, thus allowing the pilot to see a wide performance spectrum in a short period of time. A variety of targets and opponents is available to simulate virtually any tactical scenario. The output of data from the simulator is comprehensive and normally in a form suitable for quick reduction using one of many available plotting programs.
The current emphasis is on defining the agility metrics for the high alpha regime based on pilot evaluations. These metrics will be used to determine the new design requirements in the form of parameters like steady state roll rate and roll mode time constant. Another area of emphasis is the development of new technologies to provide advanced control capability in the high alpha regime. Thrust vectoring and nose strakes are two examples. A quandary arises in that the design requirements will specify performance criteria (roll rates and time constants), but the aircraft controls designer thinks in terms of generating moments. The performance guidelines will then have to be translated back into moment criteria to design the controls. The research presented here provides a tool to do this translation and determine the maximum control moment coefficient that could be encountered in velocity vector roll at the specified steady state roll rate, roll mode time constant and velocity. Since this tool is fast and easily adaptable to changes in the airframe or the performance parameters, it should bridge a part of the gap between the simulation research and the controls designer.
2.0  Nomenclature and Background

2.1  Nomenclature

Symbols

\( b \)  Span of airplane
\( \bar{c} \)  Length of mean aerodynamic chord
\( C_{rp} \)  Coefficient of rolling moment due to roll rate
\( C_{rp} \)  Coefficient of rolling moment due to yaw rate
\( C_{rcontrol} \)  Coefficient of rolling moment due to the roll controls
\( C_{mo} \)  Coefficient of pitching moment at reference condition
\( C_{m\alpha} \)  Coefficient of pitching moment due to pitch rate
\( C_{mcontrol} \)  Coefficient of pitching moment due to the pitch controls
\( C_{ra} \)  Coefficient of yawing moment due to angle of attack
\( C_{nr} \)  Coefficient of yawing moment due to roll rate
\( C_{nr} \)  Coefficient of yawing moment due to yaw rate
\( C_{rcontrol} \)  Coefficient of yawing moment due to the yaw controls
\( g \)  Acceleration due to gravity
\( I_{xp}, I_{yp}, I_{zp} \)  Moments of Inertia in principal body axes
\( L, M, N \)  Aerodynamic and Propulsive moment components in body axes
\( m \)  Mass of the airplane
\( n_z \)  Load factor, ratio of lift to weight
\( p, q, r \)  Components of the angular velocity about the x, y and z axes respectively. Subscript will determine coordinate system
\( \dot{p} \)  Roll rate acceleration. Subscript will determine coordinate system
\( \rho_{ss} \)  Steady state roll rate in wind axis coordinate system
\( q \)  Dynamic pressure
\( S \) Wing area
\( t^* \) Non-dimensional time parameter
\( V \) Magnitude of airplane’s velocity
\( X, Y, Z \) Aerodynamic and Propulsive force components in body axes
\( \alpha \) Angle of attack
\( \delta_R \) Generic roll controller, ±1
\( \gamma \) Flight path angle, wind axis
\( \mu \) Bank angle, wind axis
\( \rho \) Air density
\( \tau \) Roll mode time constant

**Subscripts**

b principal body axis coordinate system
w wind axis coordinate system

### 2.2 Background

Control power is the ability of an airplane’s controls to alter its trajectory by generating moments about the center of gravity. Normally an axis system fixed with respect to the aircraft is used to describe these moments. While an airplane may have many controls, with some capable of producing moments about more than one axis, we will for the sake of analysis assume that they can be reduced to one independent control for each of the roll, pitch and yaw axes.

This work is a continuation of the work done by Dr. W. Durham, Dr. F. Lutze and Dr. W. Mason at Virginia Polytechnic Institute and State University [1]. This paper has established analytical solutions for the maximum total body axis moments that could be encountered in a velocity vector roll. This section will list the equations from Reference 1 that will be used in this research. The general coordinate system rule will be that moments and moment of inertias will be represented in the principal body axis system while rates and accelerations will be given in wind axis coordinate system. A linear approximation to the roll response of an aircraft is given by:

\[
\dot{\phi}_w = -\frac{p_w(t)}{\tau} + \frac{p_{ss} \delta_R}{\tau}
\]  

(2.1)

where \( p_{ss} \) is the steady state roll rate in the wind axis coordinate system. Assume a unit step input, \( \delta_R(t) = 1, \ t \geq 0 \). Then:

\[
p_w(t) = p_w(0)e^{-t/\tau} + p_{ss}(1 - e^{-t/\tau})
\]  

(2.2)
By choosing the step input to be +1, we have arbitrarily chosen a roll to the right. Another necessary equation is a relation for the yaw rate:
\[ r_w(t) = (g / V) \cos \gamma(t) \sin \mu(t) \] (2.3)
and for the pitch rate
\[ \dot{\phi}_b = -\left( \frac{Z + mg \cos \mu \cos \gamma}{mV} \right) \] (2.4)

Under suitable assumptions, the equations for the total principal body axis moments for a velocity vector roll are given as:
\[ L_b \equiv I_{xp} \cos \alpha \dot{\phi}_w \]
\[ +(g / V)(l_{yp} - l_{xp}) \cos \mu \cos \gamma \sin \alpha \dot{\phi}_w(t) \] (2.5)
\[ - (g / V)(l_{yp} - l_{xp}) n_z \sin \alpha \dot{\phi}_w(t) \]
\[ M_b \equiv \frac{1}{2} (l_{xp} - l_{zp}) \sin 2 \alpha \dot{\phi}_w(t)^2 \]
\[ +(g / V)(l_{xp} - l_{zp}) \sin \mu \cos \gamma \cos 2 \alpha \dot{\phi}_w(t) \] (2.6)
\[ +(g / V)l_{yp} \sin \mu \cos \gamma \dot{\phi}_w(t) \]
\[ N_b \equiv I_{yp} \sin \alpha \dot{\phi}_w \]
\[ +(g / V)(l_{xp} - l_{zp}) \cos \mu \cos \gamma \cos \alpha \dot{\phi}_w(t) \] (2.7)
\[ -(g / V)(l_{xp} - l_{zp}) n_z \cos \alpha \dot{\phi}_w(t) \]

Several assumptions have been made in the derivation of these equations which must hold throughout subsequent derivations. The maneuver is restricted to a "perfect" velocity vector roll. This requires the speed and angle of attack to remain constant with zero sideslip throughout the maneuver. The constant speed and angle of attack assumptions cause the lift and drag to be constant, ignoring the higher order pitch rate effect. The component of thrust in the \( z \) wind axis direction will be neglected so that \( Z = -Lift = -n_z \cdot W = \text{const} \). If we assume the weight is constant over the short time period of the maneuver, then the load factor will be constant. Later we will choose a load factor of 1.0. It will be shown that this is tantamount to requiring the aircraft to have a thrust to weight ratio of at least 1.0. It is assumed that the controls have no capability of generating side force, i.e. \( Y = 0 \). Finally, to derive the simplified moment equations (2.5, 2.6 & 2.7), all terms involving \( \left( \frac{g}{V} \right)^2 \) were neglected.
3.0 Derivation of the Maximum Control Moment Equations

3.1 Derivation of the Roll Equations

The analysis starts from the total rolling moment equation (2.5). This equation relates the total rolling moment in the principal body axes to the plane's orientation, roll rate, roll acceleration, velocity, load factor and moment of inertia's. Since this equation is derived purely from the physics in the equations of motion, it can be labeled as the "trajectory" equation:

\[ L_{\text{trajectory}} = L_b \equiv L_{x_b} \cos \alpha \dot{\gamma}_w \]
\[ + \left(g / V\right)(l_{zp} - l_{x_b}) \cos \mu \cos \gamma \sin \alpha p_w(t) \]
\[ - \left(g / V\right)(l_{yp} - l_{x_b}) n_z \sin \alpha p_w(t) \]  \hspace{1cm} (3.1)

The total rolling moment can also be expressed in terms of aerodynamic derivative terms. In this approach a particular planform is tested in the wind tunnel (or by CFD code) to determine what forces and moments are generated in response to angular rates and attitudes imposed on the aircraft model. In this way the total moment can be "built up" from each contributing aerodynamic, thrust and control derivative component. This analysis will be general by lumping all the potential controls for each body axis into one generic control term that will produce the required moment. Assuming a negligible rolling moment contribution due to angle of attack and recalling that the sideslip angle is zero, the buildup equation for the rolling moment is:

\[ L_{\text{buildup}} = L_b = L_{x_b} \dot{p}_b + L_{r_b} + L_{\text{control}} = L_{\text{trajectory}} \]  \hspace{1cm} (3.2)

The aerodynamic derivatives are known in a wind tunnel coordinate system which we will assume is coincident with the principal body axis. We need to express the principal axis angular rates in terms of the known wind axis angular rates. This can be done by a transformation of \( \alpha \) about the y axis \([4]\).

\[
\begin{bmatrix}
\dot{p}_b \\
\dot{q}_b \\
\dot{r}_b
\end{bmatrix} =
\begin{bmatrix}
\cos \alpha & 0 & -\sin \alpha \\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
\dot{p}_w \\
\dot{q}_w \\
\dot{r}_w
\end{bmatrix}
\]  \hspace{1cm} (3.3)
Substituting the new form of the roll and yaw rates (3.3) into the appropriate spot in the buildup equation (3.2) gives:

\[
L_{\text{buildup}} = L_p \left( p_w \cos \alpha - r_w \sin \alpha \right)
+ L_r \left( p_w \sin \alpha + r_w \cos \alpha \right) + L_{\text{control}}
\]

(3.4)

Since the total rolling moment must be the same regardless of whether it's calculated from the trajectory equation (3.1) or the buildup equation (3.4), we can set these equations equal and solve for the control moment term.

\[
L_{\text{control}} = l_{\text{xp}} \cos \alpha \dot{p}_w
+ \left( \frac{g}{V} \left( l_{\text{xp}} - l_{z \text{p}} - l_{x \text{p}} \right) \cos \mu \cos \gamma \sin \alpha \right) p_w(t)
- \left( \frac{g}{V} \left( l_{\text{yp}} - l_{z \text{p}} \right) n_z \sin \alpha \right) p_w(t)
- L_p \left( p_w \cos \alpha - r_w \sin \alpha \right)
- L_r \left( p_w \sin \alpha + r_w \cos \alpha \right)
\]

(3.5)

Now that we have an expression for the roll control moment, we would like to obtain an analytic solution for the maximum roll control moment and orientation at which it will occur. The most desirable solution would be one which is not airframe dependent. If we take a given flight condition, then the roll rate and roll rate acceleration time histories, velocity, and gravity constant are all that are known. Clearly, the aircraft parameters, in particular the moments of inertia and aerodynamic derivatives are going to be the biggest drivers in where the maximum occurs, as we might expect. However, we will proceed in the most general form for as long as possible before becoming airframe specific.

To derive a form of the control moment equation that is suitable for finding the maximum, it's convenient to put the equation in the form

\[
L_{\text{control}} = C_1 \dot{p}_w + C_2 p_w + C_3
\]

(3.6)

where from (3.5) we see

\[
C_1 = l_{\text{xp}} \cos \alpha
\]

\[
C_2 = \left( \frac{g}{V} \left( l_{\text{xp}} - l_{z \text{p}} - l_{x \text{p}} \right) \cos \mu \cos \gamma \sin \alpha \right)
- \left( \frac{g}{V} \left( l_{\text{yp}} - l_{z \text{p}} \right) n_z \sin \alpha - L_p \cos \alpha - L_r \sin \alpha \right)
\]

(3.7)

\[
C_3 = L_p \dot{r}_w \sin \alpha - L_r r_w \cos \alpha
\]
But with the relation for the yaw rate (2.3), the coefficient $C_3$ becomes:

$$C_3 = L_p \left( \frac{g \cos \gamma \sin \mu \sin \alpha}{v} \right) - L_q \left( \frac{g \cos \gamma \sin \mu \cos \alpha}{v} \right)$$  \hfill (3.8)

Recalling the expressions for roll acceleration (2.1) and roll rate (2.2)

$$\dot{\rho}_w(t) = -\frac{P_w(t)}{\tau} + \frac{P_{ss}}{\tau}$$  \hfill (2.1)

$$\rho_w(t) = \rho_w(0) e^{-\tau} + P_{ss} \left( 1 - e^{-\tau} \right)$$  \hfill (2.2)

Or,

$$\dot{\rho}_w(t) = \left[ \frac{P_{ss} - P_w(0)}{\tau} \right] e^{-\tau}$$  \hfill (3.9)

Substituting these into the roll control moment equation (3.6):

$$L_{control} = C_1 \left[ \frac{P_{ss} - P_w(0)}{\tau} \right] e^{-\tau}$$

$$+ C_2 \left[ \rho_w(0) e^{-\tau} + P_{ss} \left( 1 - e^{-\tau} \right) \right] + C_3$$  \hfill (3.10)

Here again it's convenient to regroup, this time with respect to the exponential term as:

$$L_{control} = C_4 + C_5 e^{-\tau}$$  \hfill (3.11)

So the factors are as follows:

$$C_4 = C_2 P_{ss} + C_3$$

$$C_5 = C_1 \left[ \frac{P_{ss} - P_w(0)}{\tau} \right] + C_2 \left[ \rho_w(0) - P_{ss} \right]$$

$$= \left[ \frac{C_1}{\tau} - C_2 \right] \left[ P_{ss} - P_w(0) \right]$$  \hfill (3.12)

The exponential term takes a special form when the time is either zero or infinity (a fully developed roll). At time equal to zero, the exponential term equals one and the control moment is the sum of the two constants $C_4$ and $C_5$. At time equal to infinity, the exponential term is zero and the control moment is equal to the constant $C_4$. Before pursuing these specific cases further, it's worth investigating the possibility of the maximum occurring at a general time, i.e. in the interval between zero and infinity. For this to occur, the derivative with respect to time must be zero:

$$\frac{dL_{control}}{dt} = 0 = -\frac{C_5}{\tau} e^{-\tau}$$

$$\Rightarrow C_5 = 0$$  \hfill (3.13)
This implies:
\[ L_{\text{control}}(t) = C_4 \]  
(3.14)

But this is the condition for the control moment at time infinity. Since \( t = \infty \) is outside the interval being considered, the maximum cannot occur here. It must then occur at one of the endpoints, either \( t = 0 \) or \( t = \infty \). The corresponding equations are:
\[ L_{\text{control}}(t = 0) = C_4 + C_5 \]
\[ L_{\text{control}}(t = \infty) = C_4 \]  
(3.15)

While the time interval has decreased, the number of equations has increased from one to two. The sign of \( C_5 \) cannot be predetermined because it's a non-trivial function of the three unknown angles (\( \mu, \gamma \) & \( \alpha \)). Therefore we can't predict which endpoint will cause the maximum. The global maximum will be the maximum of the two endpoint conditions.
\[ \max|L_{\text{control}}(t)| = \max(L_{\text{control}}(0), L_{\text{control}}(\infty)) \]
\[ = \max(|C_4 + C_5|, |C_4|) \]  
(3.16)

For the case of \( t=0 \), substituting back in the expressions for the constants (3.7), (3.8) and (3.12) into (3.11) gives:
\[ L_{\text{control}}@t=0 = \dot{p}_w l_{xp} \cos \alpha - (g / V) \sin \mu \cos \gamma [-L_p \sin \alpha + L_r \cos \alpha] \]  
(3.17)

However, from the roll acceleration equation (2.1), for a roll starting from rest \( (p_W(0)=0) \), so (2.1) becomes
\[ \dot{p}_w = \frac{P_{ss}}{\tau} \]  
(3.18)

Substituting this new expression for roll acceleration at time zero (3.18) into the roll control moment at time zero equation (3.17) produces
\[ L_{\text{control}} @ t=0 = \frac{P_{ss}}{\tau} l_{xp} \cos \alpha - (g / V) \sin \mu \cos \gamma [-L_p \sin \alpha + L_r \cos \alpha] \]  
(3.19)

For the \( t = \infty \) case, substituting in the expressions for the constants and arranging:
\[ L_{\text{control}} @ t=\infty = p_w(t) \left\{ (g / V) \left[ (l_{yp} - l_{zp} - l_{xp}) \cos \mu \cos \gamma \sin \alpha 
- (g / V) (l_{yp} - l_{xp}) n_z \sin \alpha - L_p \cos \alpha - L_r \sin \alpha \right] 
- (g / V) \sin \mu \cos \gamma [-L_p \sin \alpha + L_r \cos \alpha] \right\} \]  
(3.20)
However, at \( t = \infty \) the roll rate has built up to its steady state value \( (p_{W(\infty)} = p_{ss}) \), so

\[
L_{control t=\infty} = p_{ss} \left\{ \left( \frac{g}{V} \right) \left( l_{yp} - l_{xp} - l_{xp} \right) \cos \mu \cos \gamma \sin \alpha \\
- \left( \frac{g}{V} \right) l_{xp} \sin \alpha - L_p \cos \alpha - L_r \sin \alpha \right\} - \left( \frac{g}{V} \right) \sin \mu \cos \gamma \left[ -L_p \sin \alpha + L_r \cos \alpha \right]
\]  
(3.21)

Finally, we will non-dimensionalize the roll control moment equations.

Since

\[
C_{l, control} = \frac{L_{control}}{qSb}
\]  
(3.22)

Each equation for the roll control moment can be divided by \( qSb \) and then regrouped. For the \( t=0 \) case:

\[
C_{l, control \ t=0} = \frac{L_{control \ t=0}}{qSb} = \frac{1}{qSb} \left\{ \frac{p_{ss}}{\tau} l_{xp} \cos \alpha \\
- \left( \frac{g}{V} \right) \sin \mu \cos \gamma \left[ -L_p \sin \alpha + L_r \cos \alpha \right] \right\}
\]  
(3.23)

It's desirable to have the aerodynamic derivative terms expressed in their non-dimensional form.

\[
C_{l, p} = \frac{L_p}{qSb} = \frac{L_p}{qSb \left( \frac{b}{2V} \right)}
\]  
(3.24)

\[
C_{l, r} = \frac{L_r}{qSb} = \frac{L_r}{qSb \left( \frac{b}{2V} \right)}
\]  
(3.25)

So, regrouping (3.23)

\[
C_{l, control \ t=0} = \frac{1}{qSb} \left\{ \frac{p_{ss}}{\tau} l_{xp} \cos \alpha \\
- \left( \frac{bg}{2V^2} \right) \sin \mu \cos \gamma \left[ -C_{l, p} \sin \alpha + C_{l, r} \cos \alpha \right] \right\}
\]  
(3.26)

One more regrouping brings out the velocity dependence.

\[
C_{l, control \ t=0} = \frac{1}{V^2} \left\{ \frac{2}{\rho Sb} \frac{p_{ss}}{\tau} l_{xp} \cos \alpha \\
- \left( \frac{bg}{2} \right) \sin \mu \cos \gamma \left[ -C_{l, p} \sin \alpha + C_{l, r} \cos \alpha \right] \right\}
\]  
(3.27)
Now the same process for the \( t = \infty \) equation

\[
C_{i\text{control } @ t=\infty} = \frac{P_{s s}}{q S b} \left\{ \left( \frac{g}{V} \right) (l_{y p} - l_{z p} - l_{x p}) \cos \mu \cos \gamma \sin \alpha \right. \\
\left. - \left( \frac{g}{V} \right) (l_{y p} - l_{z p}) n_z \sin \alpha - \frac{1}{q S b} \left( \frac{g}{V} \right) \sin \mu \cos \gamma \left[ -L_p \sin \alpha + L_r \cos \alpha \right] \right\}
\]

(3.28)

Regrouping to non-dimensionalize aerodynamic derivative terms

\[
C_{i\text{control } @ t=\infty} = \frac{P_{s s}}{q S b} \left\{ \left( \frac{g}{V} \right) (l_{y p} - l_{z p} - l_{x p}) \cos \mu \cos \gamma \sin \alpha \right. \\
\left. - \left( \frac{g}{V} \right) (l_{y p} - l_{z p}) n_z \sin \alpha \right\}
\]

(3.29)

\[-p_{s s} \frac{b}{2V} \left[ C_{i p} \cos \alpha + C_{i r} \sin \alpha \right]
\]

\[-\frac{b g}{2V^2} \sin \mu \cos \gamma \left[ -C_{i p} \sin \alpha + C_{i r} \cos \alpha \right]
\]

Simplifying

\[
C_{i\text{control } @ t=\infty} = \frac{P_{s s}}{q S b} \frac{g}{V} \sin \alpha \left\{ (l_{y p} - l_{z p} - l_{x p}) \cos \mu \cos \gamma - (l_{y p} - l_{z p}) n_z \right\}
\]

\[+ \frac{b}{2V} \left\{ -p_{s s} C_{i r} \sin \alpha - p_{s s} C_{i p} \cos \alpha
\]

\[\left. - \left( \frac{g}{V} \right) \sin \mu \cos \gamma \left[ -C_{i p} \sin \alpha + C_{i r} \cos \alpha \right] \right\}
\]

(3.30)

3.2 Derivation of the Yaw Equations
The yaw equations have an almost identical form to the roll equations so the derivation is similar. The approach starts as before with the "trajectory" equation (2.7)

\[
N_{\text{trajectory}} = N_b \equiv l_{z p} \sin \alpha \dot{p}_w
\]

\[+ \left( \frac{g}{V} \right) (l_{y p} - l_{z p} + l_{x p}) \cos \mu \cos \gamma \cos \alpha \dot{p}_w(t) \]

(2.6)

\[- \left( \frac{g}{V} \right) (l_{x p} - l_{y p}) n_z \cos \alpha \dot{p}_w(t)
\]

Take the buildup equation to be:

\[
N_{\text{buildup}} = N_p P_b + N_r f_b + N_{\text{control}}
\]

(3.31)
Expressing the principal axis rates in terms of the known wind axis rates (3.3), then substituting these back into the buildup equation (3.31)

\[ N_{\text{buildup}} = N_p \left( p_w \cos \alpha - r_w \sin \alpha \right) + N_r \left( p_w \sin \alpha + r_w \cos \alpha \right) + N_{\text{control}} \]  

(3.32)

Setting the trajectory equation (2.7) and the buildup equation (3.32) equal and then solving for the control moment term

\[ N_{\text{control}} = I_{zp} \sin \alpha \dot{p}_w \]

\[ + \left( \frac{g}{V} \right) \left( l_{xp} - I_{yp} + I_{zp} \right) \cos \mu \cos \gamma \cos \alpha \rho_w(t) \]

\[ - \left( \frac{g}{V} \right) \left( l_{xp} - I_{yp} \right) n_z \cos \alpha \rho_w(t) \]

\[ - N_p \left( p_w \cos \alpha - r_w \sin \alpha \right) \]

\[ - N_r \left( r_w \cos \alpha + p_w \sin \alpha \right) \]

(3.33)

Reorganizing to the form

\[ N_{\text{control}} = C_1 \dot{p}_w + C_2 p_w + C_3 \]  

(3.34)

where from (3.33) we see

\[ C_1 = I_{zp} \sin \alpha \]

\[ C_2 = \left( \frac{g}{V} \right) \left( l_{xp} - I_{yp} + I_{zp} \right) \cos \mu \cos \gamma \cos \alpha \]

\[ - \left( \frac{g}{V} \right) \left( l_{xp} - I_{yp} \right) n_z \cos \alpha - N_p \cos \alpha - N_r \sin \alpha \]

\[ C_3 = N_p r_w \sin \alpha - N_r r_w \cos \alpha \]

(3.35)

Substituting the relation for the yaw rate (2.3), the \( C_3 \) factor becomes:

\[ C_3 = N_p \left( \frac{g}{V} \right) \cos \gamma \sin \mu \sin \alpha \]

\[ - N_r \left( \frac{g}{V} \right) \cos \gamma \sin \mu \cos \alpha \]  

(3.36)

Now substitute the relations for the roll acceleration (2.1) and roll rate (2.2) into the \( N_{\text{control}} \) equation (3.34)

\[ N_{\text{control}} = C_1 \left[ \frac{p_w(t) - p_w(0)}{\tau} \right] e^{-t/\tau} \]

\[ + C_2 \left[ p_w(0) e^{-t/\tau} + p_w(1 - e^{-t/\tau}) \right] + C_3 \]  

(3.37)

Regrouping this expression to the form

\[ N_{\text{control}} = C_4 + C_5 e^{-t/\tau} \]  

(3.38)

the new factors are

\[ C_4 = C_2 \rho_s + C_3 \]

\[ C_5 = \left[ \frac{C_4}{\tau} - C_2 \right] \left[ p_w(t) - p_w(0) \right] \]  

(3.39)
By an identical procedure to the roll formulation, it can be shown that the maximum yaw control moment can only occur on the endpoints of the time interval, i.e. at $t = 0$ or $t = \infty$ (the roll is fully developed). Subsequently, there is an equation for each condition. For the $t = 0$ case, we assume as always a roll starting from rest ($p_w(0) = 0$). The roll acceleration is then given by (3.18)

$$\dot{p}_w = \frac{P_{ss}}{\tau}$$

Substituting the constants (3.35), (3.36) and (3.39), and the roll acceleration at time zero (3.18) into the control moment equation (3.38)

$$N_{control} \Big|_{t=0} = \frac{P_{ss}}{\tau} l_{xp} \sin \alpha - (g / V) \sin \mu \cos \gamma \left[-N_p \sin \alpha + N, \cos \alpha \right]$$

(3.40)

For the $t = \infty$ case, the roll must be fully developed so that $p_w(\infty) = p_{ss}$ and there is no roll acceleration.

$$N_{control} \Big|_{t=\infty} = p_{ss} \left\{ (g / V) (l_{xp} - l_{yp} + l_{xo}) \cos \mu \cos \gamma \cos \alpha \right.$$

$$- (g / V) (l_{xp} - l_{yp}) n_z \cos \alpha - N_p \cos \alpha - N, \sin \alpha \}$$

$$- (g / V) \sin \mu \cos \gamma \left[-N_p \sin \alpha + N, \cos \alpha \right]$$

(3.41)

Non-dimensionalizing proceeds just as in the roll control moment derivation. For $t = 0$:

$$C_{n_{control}} \Big|_{t=0} = \frac{1}{V^2} \left\{ \frac{2}{\rho S b} \frac{P_{ss}}{\tau} l_{xp} \sin \alpha \right.$$

$$- \left( \frac{b g}{2} \right) \sin \mu \cos \gamma \left[-C_{n_p} \sin \alpha + C_{n_r} \cos \alpha \right] \}$$

(3.42)

and for the $t = \infty$ case:

$$C_{n_{control}} \Big|_{t=\infty} = \frac{P_{ss}}{q S b} \frac{g}{V} \cos \alpha \left\{ (l_{xp} - l_{yp} + l_{xo}) \cos \mu \cos \gamma - (l_{xp} - l_{yp}) n_z \right.$$ $$+ \left( \frac{b}{2V} \right) - p_{ss} C_{n_r} \sin \alpha - p_{ss} C_{n_p} \cos \alpha \frac{C_{n_r} \cos \alpha}{\frac{C_{n_r}}{\gamma}}$$

(3.43)
3.3 Derivation of the Pitch Equation

The formulation of the maximum pitch control moment differs from the roll and yaw formulation. The process still starts off with the trajectory equation (2.6)

\[ M_{\text{trajectory}} = M_b = \frac{1}{2} (l_{xp} - l_{zp}) \sin 2\alpha p_w^2(t) + (g/V) (l_{xp} - l_{zp}) \sin \mu \cos \gamma \cos 2\alpha p_w(t) + (g/V) l_{yp} \sin \mu \cos \gamma p_w(t) \]

The corresponding buildup equation is:

\[ M_{\text{buildup}} = M_0 + M_q q_b + M_\alpha \alpha + M_{\text{control}} \]  \hspace{1cm} (3.44)

Assume that the wind tunnel (zero lift) coordinate system is coincident with the principal axis system. The transformation matrix is simply a rotation of \( \alpha \) about the y axis. So since \( q_b = q_w \), the buildup equation is in the proper form. Now, setting the trajectory (2.6) and buildup equation (3.44) equal and solving for the control term:

\[ M_{\text{control}} = \frac{1}{2} (l_{xp} - l_{zp}) \sin 2\alpha p_w^2(t) + (g/V) (l_{xp} - l_{zp}) \sin \mu \cos \gamma \cos 2\alpha p_w(t) + (g/V) l_{yp} \sin \mu \cos \gamma p_w(t) - M_0 - M_q q_b - M_\alpha \alpha \]  \hspace{1cm} (3.45)

The relation for \( q_b \) is known from (2.4) as

\[ q_b = -\left( \frac{Z + mg \cos \mu \cos \gamma}{mV} \right) \]

This can be simplified somewhat by recalling the assumption we made for \( Z \)

\[ Z = -\text{Lift} = -n_z \cdot W = -n_z \cdot mg = \text{const} \]  \hspace{1cm} (3.46)

Substituting the simplified \( Z \) force equation (3.46) into the pitch rate equation (2.4)

\[ q_b = (g/V) [n_z - \cos \mu \cos \gamma] \]  \hspace{1cm} (3.47)

Substituting the new pitch rate expression (3.47) into the pitch control moment equation (3.45)

\[ M_{\text{control}} = \frac{1}{2} (l_{xp} - l_{zp}) \sin 2\alpha p_w^2(t) + (g/V) (l_{xp} - l_{zp}) \sin \mu \cos \gamma \cos 2\alpha p_w(t) + (g/V) l_{yp} \sin \mu \cos \gamma p_w(t) - M_q (g/V) [n_z - \cos \mu \cos \gamma] - M_0 - M_\alpha \alpha \]  \hspace{1cm} (3.48)

Chapter 3: Derivation of the Equations
The maximum must occur at a stationary point or on a boundary. For equation (3.48) to be stationary with respect to \( p_w(t) \), \( \frac{\partial M_{\text{control}}}{\partial p_w} = 0 \). This requires the roll acceleration \( \dot{p}_w(t) = 0 \). Since we start rolling from rest, the only possibility for \( p_w(t) \) is the steady state roll rate, i.e. \( p_w(t) = p_{ss} \).

\[
M_{\text{control}} = \frac{1}{2} \left( I_{xp} - I_{zp} \right) \sin 2 \alpha p_{ss}^2 \\
+ (g / V)(I_{xp} - I_{zp}) \sin \mu \cos \gamma \cos 2 \alpha p_{ss} \\
+ (g / V)I_{yp} \sin \mu \cos \gamma p_{ss} \\
-M_q (g / V) [n_z - \cos \mu \cos \gamma] \\
-M_0 - M_\alpha \alpha 
\]

(3.49)

Herein lies the difference with the roll and yaw equations. The stipulation that the solution must occur at the steady state roll rate reduces the time interval to the case of \( t = \infty \). Thus there is only one equation for the pitch control moment rather than two for the roll and yaw cases. Now it’s a matter of determining the orientation and angle of attack (\( \mu, \gamma \& \alpha \)) at this flight condition that produces the maximum pitch control moment. Before doing that it is beneficial to put the equation into a non-dimensional form.

\[
C_{m\text{control}} = \frac{M_{\text{control}}}{qSc} 
\]

(3.50)

\[
C_{m_0} = \frac{M_0}{qSc} 
\]

(3.51)

\[
C_{m_\alpha} = \frac{M_\alpha}{qSc} 
\]

(3.52)

\[
C_{m_q} = \frac{M_q}{qSc\tilde{t}} = \frac{M_q}{qSc \left( \frac{\bar{c}}{2V} \right)} 
\]

(3.53)
So, substituting the pitch control moment (3.49) into the definition of the pitch control moment coefficient (3.50)

\[ C_{m_{\text{control}}} = \frac{M_{\text{control}}}{qSc} \]

\[ = \frac{1}{qSc} \left\{ \frac{1}{2} \left( I_{xp} - I_{zp} \right) \sin 2 \alpha p_{ss}^2 \right. \]
\[ + \left( g / V \right) \left( I_{xp} - I_{zp} \right) \sin \mu \cos \gamma \cos 2 \alpha p_{ss} \]
\[ + \left( g / V \right) I_{yp} \sin \mu \cos \gamma p_{ss} \]
\[ - M_s (g / V) [n_z - \cos \mu \cos \gamma] \]
\[ - M_0 - M_{a} \alpha \} \]

Regrouping to form the non-dimensional aerodynamic derivative coefficients (3.51), (3.52) and (3.53), the pitch control moment equation (3.54) becomes

\[ C_{m_{\text{control}}} = \frac{1}{qSc} \left\{ \frac{1}{2} \left( I_{xp} - I_{zp} \right) \sin 2 \alpha p_{ss}^2 \right. \]
\[ + \left( g / V \right) \left( I_{xp} - I_{zp} \right) \sin \mu \cos \gamma \cos 2 \alpha p_{ss} \]
\[ + \left( g / V \right) I_{yp} \sin \mu \cos \gamma p_{ss} \}
\[ - C_{mq} \left( \frac{\bar{C}}{2V} \right) \]
\[ - C_{mo} - C_{m_{a}} \alpha \}

3.4 Summary of the Control Moment Equations

Roll:

\[ C_{l_{\text{control}}} @ t = 0 = \frac{1}{V^2} \left\{ \frac{2}{\rho Sb} \frac{p_{ss}}{\tau} I_{xp} \cos \alpha \right. \]
\[ - \left( \frac{b g}{2} \right) \sin \mu \cos \gamma \left[ - C_{l_{p}} \sin \alpha + C_{l_{r}} \cos \alpha \right] \}

(3.56)

\[ C_{l_{\text{control}}} @ t = \infty = \frac{P_{ss}}{qSb V} \sin \alpha \left\{ (I_{yp} - I_{zp} - I_{xp}) \cos \mu \cos \gamma \right. \]
\[ - (I_{yp} - I_{zp}) n_z \}
\[ + \frac{b}{2V} \left\{ - p_{ss} C_{l_{p}} \sin \alpha - p_{ss} C_{l_{r}} \cos \alpha \right. \]
\[ - (g / V) \sin \mu \cos \gamma \left[ - C_{l_{p}} \sin \alpha + C_{l_{r}} \cos \alpha \right] \}

(3.57)
Pitch:
\[
C_{m_{\text{control}}} = \frac{1}{qSc} \left\{ \frac{1}{2} (I_{xp} - I_{z}) \sin 2\alpha \rho_{ss}^{2} \\
+ (g / V) (I_{xp} - I_{z}) \sin \mu \cos \gamma \cos 2\alpha \rho_{ss} \\
+ (g / V) I_{zp} \sin \mu \cos \gamma \rho_{ss} \right\} \\
-C_{m_{\alpha}} \left( \frac{b}{2V} \right) (g / V) [n_{z} - \cos \mu \cos \gamma] \\
-C_{m_{\alpha}} - C_{m_{\alpha}} \alpha
\] (3.58)

Yaw:
\[
C_{n_{\text{control}}} = \frac{1}{V^{2}} \left\{ 2 \frac{p_{ss}}{\rho Sb} I_{zp} \sin \alpha \\
- \left( \frac{bg}{2} \right) \sin \mu \cos \gamma \left[ -C_{n_{p}} \sin \alpha + C_{n_{r}} \cos \alpha \right] \right\} \\
C_{n_{\text{control}}} = \frac{p_{ss}}{qSb V} \cos \alpha \left\{ (I_{xp} - I_{yp} + I_{z}) \cos \mu \cos \gamma - (I_{xp} - I_{yp}) n_{z} \right\} \\
+ \frac{b}{2V} \left\{ -p_{ss} C_{n_{r}} \sin \alpha - p_{ss} C_{n_{p}} \cos \alpha \\
- (g / V) \sin \mu \cos \gamma \left[ -C_{n_{p}} \sin \alpha + C_{n_{r}} \cos \alpha \right] \right\} \\
\] (3.60)
4.0 Establishing the Parameters

The control moment equations at this point are functions of only one known parameter, the gravity constant. Now, we seek solutions to these equations for the maximum control moments. To do so, we must specify a sufficient number of the parameters by choosing an airframe and flight condition. For the airframe, the McDonnell Douglas F/A-18 will be used. The airframe choice specifies parameters such as the moments of inertia and the aerodynamic derivatives. The flight condition parameters are the steady state roll rate, roll mode time constant and velocity. With the airframe and flight condition parameters known, the equations can be solved for the maximum control moments, the orientation and the angle of attack (μ, γ & α) at which they occur. Before doing that, it's beneficial to make a few assumptions regarding the aircraft parameters.

4.1 The airframe parameters

When we choose an airframe we know the aerodynamic derivatives from wind tunnel tests, computational methods or actual flight data. Even though this analysis will be done based on the F-18, we still would like to keep it as general as possible to facilitate the analysis and comparison with other airframes. The moments of inertia are important more for their relative ratio of $I_{xp}:I_{yp}:I_{zp}$ than to the individual magnitudes. The F-18 has a ratio of 1:5.3:6.2. Factors such as the forebody vortex can cause distinct differences in the shapes of the plots for, say, $C_{i,p}$ vs $\alpha$ for two planforms while the median values of $C_{i,p}$ may be very close.

In this analysis, we will restrict the aerodynamic derivatives to constant values to generalize the results towards any fighter/attack aircraft and to simplify the optimization process somewhat. For the F-18, the non-dimensional
aerodynamic derivatives will be:

\[
C_{l_o} = -0.40
\]

\[
C_{l_i} = 0.20
\]

\[
C_{n_p} = 0.02
\]

\[
C_{n_t} = -0.35
\]

\[
C_{n_{10}} = -0.085
\]

\[
C_{m_{q}} = -5.02
\]

\[
C_{m_{a}} = -0.397
\]

Each constant was obtained by computing a median value over the range of angle of attack in the F-18 aerodynamic database. Now, all the airframe parameters are constant. The important physical F-18 parameters are:

\[
l_{xp} = 23,168 \text{ slug} \cdot \text{ft}^2
\]

\[
l_{yp} = 123,936 \text{ slug} \cdot \text{ft}^2
\]

\[
l_{zp} = 143,239 \text{ slug} \cdot \text{ft}^2
\]

\[
S = 400 \text{ ft}^2
\]

\[
\bar{c} = 11.52 \text{ ft}
\]

\[
b = 34.72 \text{ ft}
\]

4.2 The flight performance parameters

The flight performance parameters are not as predetermined as the aircraft parameters. Any airframe will have an infinite number of flight performance conditions to choose from and a different set of control moment maxima associated with each one. If the work was being driven by simulation results, then the flight performance parameters (steady state roll rate, roll mode time constant and velocity) would be known. Global control maxima are sought here, so a range of values for each flight performance parameter is chosen. This creates a "grid" of steady state roll rates, roll mode time constants and velocities. Preliminary results from the HAIRRY study suggest possible ranges for these parameters. For the steady state roll rate a range from 40°/sec to 160°/sec will be used. The HAIRRY study suggests roll rates based on angle of attack. However, here the angle of attack is one of the variables being solved for. It's likely that some of the maximum control moments will appear at maximum angle of attack. Roll rates of 80°/sec and above at the high angle of attacks have not
proven to be tactically desirable to the pilot due to the disorientation of the fast coning motion. The analysis will eliminate points from the grid for this reason. Time constants will range between .5 and 1.5 seconds. Finally, the velocity should range from its slowest feasible value (100 ft/sec) to slightly above the corner speed (400 ft/sec). The altitude will be taken as 10,000 ft, with a corresponding density of .001755 slug/ft$^3$.

The load factor will be assumed constant at 1.0. That is, we expect the plane to be maneuvering around a neutral flight condition, neither climbing or descending very rapidly. Earlier, in the Z-force equation assumptions, it was stated that the component of thrust in the z wind direction would be neglected. Figure 4.1 shows a free body diagram of the plane. The forces acting in the z wind direction are the lift, weight (level flight reference condition) and a component of the thrust. Requiring the load factor to be 1.0 would require the lift to equal the weight. At the slowest of the speeds to be considered here, we would not be able to produce enough lift to equal the weight. We would however have the component of the thrust we have otherwise neglected. The assumption of a constant load factor of 1.0 is equivalent to stating that the pilot will have enough throttle to make up for any deficiency in lift. Since we expect to have conditions of low lift at the slowest speeds and post stall alphas, we are in effect assuming the aircraft has a thrust to weight ratio close to one, which modern fighter/attack aircraft do.
Figure 4.1 Free Body Diagram of forces in the x-z plane
5.0 Numerical Optimization of the Control Moment Equations

The form of the problem that we'll be solving with the numerical routine is

\[
\max f(x) \quad \begin{cases} 
\mu \\ 
\gamma \\ 
\alpha 
\end{cases}
\]

(5.1)

where \( f(x) \) is the expression for the control moment coefficient as given by (3.56) thru (3.60) subject to the following constraints

\[
0^\circ \leq \mu \leq 360^\circ \\
-90^\circ \leq \gamma \leq 90^\circ \\
0^\circ \leq \alpha \leq 70^\circ
\]

(5.2)

With the specification of the airframe and flight performance parameters, a solution for the maximum control moments can be obtained. The answers are valid only for that airframe at that flight condition. As postulated earlier, it's desirable to be able to generate solutions for a variety of airframes at a variety of flight performance conditions, so the method of solution must be adaptable if it will be a useful tool. Optimization using a numerical routine has this advantage. Changing airframes and flight performance parameters is just a change to the data statements, the equations stay the same. Additionally, our equations are functions of sines, cosines and constants so the local maximums should be well defined. There are a number of routines currently available to do this kind of optimization, but the one used here is Schittkowski's SQP (Sequential Quadratic Programming) code [5].

A couple issues need to be considered when implementing the code. The optimization code is designed to search for minimums. To search for a maximum, one simply multiplies the equation by minus one and proceeds, i.e. \( \max f = \min(-f) \). But in the control case, we're really looking for the maximum absolute value since the sign is denoting the direction of the moment with respect
to the principal body axes. So to do this, we have to find the minimum and
maximum and then take the maximum of the absolute values of these.

Another factor we must take into account is that the optimization routine
works "locally". That is it assumes that the function has one minimum and it's
well defined. The routine starts at an initial condition and proceeds to find a
minimum. However, our functions will have more than one local minimum due to
the presence of sine and cosine terms. We then have to start the optimization at
different initial conditions to find all the local minima. Comparing the local minima
will determine the global minimum. A "grid" of initial conditions is used to search
for all the local minima. This grid consists of nu's from 0° to 360° by 10°,
gamma's from 0 to 90° by 10°, and alpha's of 0°, 35° and 70°. The grid has
come from experience that the initial conditions are most sensitive to μ and least
sensitive to α.

The optimization program will have to be run several thousand times for
the global control maximum set due to the number of equations we're optimizing,
the grid of flight performance parameters (steady state roll rate, roll mode time
constant and velocity), and the grid of initial starting conditions for μ, γ and α.
Since the optimization code converges quickly in nearly every case, the total
computing time is reasonable (approximately 7 minutes for each control moment
equation on a Sun workstation). The results are the non-dimensional control
moment and the orientation of the plane in terms of μ, γ and α.
6.0 Results and Analysis

6.1 Analysis of the Global Maximum Control Moments

Running the Schittkowski SQP optimization program with the initial
guesses detailed in Chapter 5 for the grid of flight performance parameters from
Chapter 4 for the F-18 produced the global maximum control moment
coefficients, orientation angles $\mu$ and $\gamma$, angle of attack $\alpha$, the time the
maxima occurred and flight performance parameters listed in table 6.1. Trends
appear regarding the type of flight performance condition that produced the
maximum control moment. The maximum moments all occurred at the highest
steady state roll rate parameter and lowest speed. They would all occur at the
smallest time constant, except that the pitch control maximum is not a function of
the roll acceleration and hence the roll mode time constant.

The trends in the maximum control moment coefficients with respect to
the flight performance parameters are what we might have expected. The trend
toward the highest steady state roll rate parameter at the maximum control
coefficient is intuitive. We would expect the controls to have to produce a larger
moment to generate a higher roll rate. The trend toward the smallest time
constant is apparent when we consider the roll acceleration. Recalling
equation (3.18) for the roll acceleration starting from rest

$$\dot{p}_w = \frac{p_{ss}}{\tau}$$

we see that the greatest roll acceleration will occur for the greatest steady state
roll rate and smallest roll mode time constant. We might have expected that the
controls would have to produce larger moments to attain the higher roll rate
accelerations thus the trend for small roll mode time constant at the maxima.
The trend for smallest velocity at the maximum control moment coefficient is
apparent from the definitions of the control moment coefficients (3.22) and
(3.50).

$$C_{\text{control}} = \frac{L_{\text{control}}}{qSb}$$


\[ C_{m,control} = \frac{L_{control}}{\dot{q}S\bar{c}} \]

The non-dimensionalizing factor
\[ \left( \frac{1}{\dot{q}Sb} \right) \text{ or } \left( \frac{1}{\dot{q}Sc} \right) \]

is strongly influenced by the velocity due to the dynamic pressure. At constant altitude (density), the dynamic pressure is a function only of the square of the velocity. Since the velocity only appears in the control moment equations (3.19), (3.21), (3.40), (3.41) and (3.49) in the \((g/V)\) factors, we might expect that the maximum would occur when these factors are largest which would be at smallest velocity. In both the control moment equations (3.19), (3.21), (3.40), (3.41) and (3.49) and the non-dimensionalizing factor (6.1), the trend is for smaller velocities to increase the moment coefficient. But since the dynamic pressure introduced in the non-dimensionalization factor is a second degree function of the velocity, it will be more influential in the velocity trend than the control moment’s velocity dependency. High dynamic pressures will make the moment coefficient smaller since the non-dimensionalizing factor (6.1) in (3.22) and (3.50) is smaller. At the slowest velocities the non-dimensionalizing factor (6.1) is larger so the control moment coefficient is larger. Intuitively we know that the aerodynamic controls are more effective at higher dynamic pressures, it's the very slow speeds where we might have expected the most trouble generating each control moment and that is reflected in the control moment coefficients.

The trends in the maximum control moment coefficients with the flight performance parameters can be seen mathematically as well. Recalling the roll control moment coefficient equation at \(t = 0\) (3.56):

\[
C_{t,control}@t=0 = \frac{1}{V^2} \left\{ \frac{2}{\rho Sb} \left( \frac{p_{es}}{\tau} \right)_{xp} \cos \alpha 
- \left( \frac{bg}{2} \right) \sin \mu \cos \gamma \left[ -C_{t,p} \sin \alpha + C_{t,r} \cos \alpha \right] \right\}
\]

The velocity dependence is isolated in the outermost factor. Since the velocity appears in the denominator of this factor, the smallest velocity produced the maximum control moment coefficient.
Substituting the generalized F-18 parameters into the roll control moment equation (3.56):

$$C_{l_{\text{control}}} = 0.190 \frac{P_{ss}}{\tau} \cos \alpha - 0.0559 \sin \mu \cos \gamma \left[ 4 \sin \alpha + 2 \cos \alpha \right]$$

So, the first term of the equation will be dominant due to the magnitude of the inertia ($I_x = 23,168 \text{ slug} \cdot \text{ft}^2$). Maximizing the first term will cause the trends towards the highest steady state roll rate parameter and smallest roll mode time constant as well as influence the angle of attack. The ratio $\frac{P_{ss}}{\tau}$ is largest when the steady state roll rate parameter is at its maximum value (160°/sec = 2.79 rad/sec) while the time constant is at its minimum value (.5 sec), hence the trend for each. The $\cos \alpha$ term is largest when alpha is small.

The second term of equation (3.56) determines the angles $\mu$ and $\gamma$.

Since the first term will be a positive number, we want to make the second term the largest positive number it can be. Since alpha is small from maximizing the first term, the $\sin \alpha$ term is negligible leaving the second term as approximately

$$-\frac{1}{V^2} \left( \frac{bg}{2} \right) \sin \mu \cos \gamma C_l, \cos \alpha$$

(6.2)

For our positive value of $C_l$, the largest positive number will occur when:

$$\sin \mu \cos \gamma = -1$$

With the restriction on gamma

$$-90^\circ \leq \gamma \leq 90^\circ$$

This condition will only be met when $\gamma = 0^\circ$ and $\mu = 270^\circ$. So $C_l$, or more specifically the sign of $C_l$, is the major influence on the solution for $\mu$ and $\gamma$.

An important trend occurs in the roll control moment solutions over the range of roll mode time constants. As mentioned earlier, the solutions for the smallest time constant (.5 sec) come from the time zero equation (3.56). However, when the time constant is 1 or 1.5 second, then the maximum is generally a result of the time equal to infinity (steady state) equation. Recalling
the time infinity equation for the roll control moment coefficient (3.57)

\[ C_{r\text{control}}^{\text{at } t=\infty} = \frac{p_{ss} g}{q S b V} \sin \alpha \left\{ \left( l_{xp} - l_{zp} - l_{xp} \right) \cos \mu \cos \gamma - \left( l_{yp} - l_{zp} \right) n_z \right\} \]

\[ + \frac{b}{2V} \left\{ -p_{ss} C_{l_{r}}, \sin \alpha - p_{ss} C_{l_{p}} \cos \alpha \right. \]

\[ - \left( \frac{g}{V} \right) \sin \mu \cos \gamma \left[ -C_{l_{p}} \sin \alpha + C_{l_{r}} \cos \alpha \right] \]

The time infinity equation has two additional terms involving the aerodynamic derivatives and the angle of attack that the time zero equation did not have. While the moment of inertia terms are still largest, they are not as dominant as they were for the time zero case. When the time constant is small, the moment of inertia term in the time zero equation is large enough to outweigh the conglomeration of terms in the time infinity equation. Otherwise, as the time constant gets larger, the time zero inertia term decreases to the point where it is not large enough to outweigh the time infinity solution. The important thing to remember is that regardless of which equation produces the maximum, the other equation may produce a comparable moment at a different orientation and angle of attack.

A similar type of mathematical explanation of the trends can be done for the pitch axis results. Recalling the pitching moment coefficient equation (3.58)

\[ C_{m\text{control}} = \frac{1}{q S c} \left\{ \frac{1}{2} \left( l_{xp} - l_{zp} \right) \sin 2 \alpha p_{ss} \right. \]

\[ + \left( \frac{g}{V} \right) \left( l_{yp} - l_{zp} \right) \sin \mu \cos \gamma \cos 2 \alpha p_{ss} \]

\[ + \left( \frac{g}{V} \right) l_{yp} \sin \mu \cos \gamma p_{ss} \}

\[ - C_{m_{\ell}} \left( \frac{\bar{c}}{2V} \right) \left( \frac{g}{V} \right) \left[ n_z - \cos \mu \cos \gamma \right] \]

\[ - C_{m_{L}} - C_{m_{T}} \alpha \]

The pitch equation has its maximum at the steady state condition (time = \( \infty \)), which means that there are no roll acceleration terms. Since the time constant only appears as a result of the expression for the roll acceleration, the pitch control moment is not a function of the time constant. The velocity appears only in the denominator of terms again. Most commonly it appears in the \( (g/V) \) terms, but it is also in the non-dimensional time term \( \left( \frac{b}{2V} \right) \), and hidden in the dynamic
pressure. Each one of these terms is largest when \( V \) is smallest, hence the trend of maximum coefficient at smallest velocity.

The pitch equation has quite a few more terms than the roll equation did. It's not readily apparent which terms will be dominant, or even what sign some terms will have due to the greater dependency on the unknown angles \( \mu \) and \( \gamma \). The terms involving the moments of inertia have the potential to be dominant, but a balancing act involving the sines and cosines of the angles has to be performed to keep these terms from canceling each other out. This balance is done primarily through the selection of alpha, hence alpha came out being a non-intuitive number (51.01°). In the case of the roll, alpha was near its smallest value to make the motion nearly all body axis roll. For the yaw, alpha was driven to its largest value to make the motion as much body axis yaw as possible (90° would be all body axis yaw but we're limited to 70°). But the velocity vector roll is a lateral-directional maneuver and does not resolve into a pure longitudinal axis motion. Subsequently, the angle of attack is not at either of its extremes, but between them. Mathematically this point will be determined through the magnitudes of the inertia's and the pitch aerodynamic derivatives.

The yaw equation optimization is very similar to the case of the roll. The yaw control coefficient equation had it's maximum at time zero (3.59).

Recalling,

\[
C_{n_{\text{control}}} = \frac{1}{V^2} \left\{ \frac{2 p_{ss} l_{zp} \sin \alpha}{\rho S b} \right\} \left\{ -\left( \frac{bg}{2} \right) \sin \mu \cos \gamma \right\}
\]

Once again the velocity appears only in the denominator of the first factor so it should have its minimum value (100 ft/sec). Substituting in the generalized F-18 parameters and the velocity into equation (3.59)

\[
C_{n_{\text{control}}} = 1.175 \frac{p_{ss}}{\tau} \sin \alpha - 0.0559 \sin \mu \cos \gamma \left[ -0.2 \sin \alpha - 0.35 \cos \alpha \right]
\]

Again, the first term dominates this equation due to the magnitude of the moment of inertia. This case even more so since the inertia \( l_{zp} \) (143,239 slug ft²) is roughly six times greater than \( l_{xp} \) was. We want to maximize each factor in the first term. The steady state roll rate divided by the time constant is the expression for the roll acceleration for our roll starting from rest. The greatest
acceleration comes when the steady state roll rate parameter is its maximum value (160°/sec) and the time constant is its smallest value (.5 sec). Finally, \( \sin \alpha \) is largest when alpha is at its upper limit of 70°.

Some insight on the angles \( \mu \) and \( \gamma \) can be gotten by examining the second term of the equation. With the first term dominant and positive, the second term should be the largest positive number possible. The angle of attack is at its maximum (70°) from maximizing the first term, but the \( C_{n_r} \cos \alpha \) term will be larger than the \( C_{n_p} \sin \alpha \) term since \( |C_{n_r}| \equiv 17.5 |C_{n_p}| \). So the second term can be approximated by:

\[
\frac{1}{V^2} \left\{ -\frac{b g}{2} \sin \mu \cos \gamma C_{n_r} \cos \alpha \right\} \tag{6.3}
\]

and will be positive if \( C_{n_r} \) is negative and \( \sin \mu \cos \gamma = 1 \)

Since gamma is limited to \(-90° \leq \gamma \leq 90°\)

The optimal choice is \( \gamma = 0° \) and \( \mu = 90° \). So \( \mu \) and \( \gamma \) are determined primarily by the sign of \( C_{n_r} \).

For the roll case, we saw that the time constant determined whether the maximum solutions occurred at the time zero or time infinity condition. Since the roll and yaw equations have a similar form, it may be expected that the time constant will have a similar effect for the yaw results. It turns out that the magnitude of \( I_{zp} \) in the time zero equation is so large that the affects of the time constant changes are obscured until you begin considering time constants much larger than the largest one here. Therefore all of the yaw maximum solutions occur at time zero, and the corresponding time infinity moments are significantly smaller.

6.2 Maximums with Roll Rate Restricted by Angle of Attack

Table 6.1 presents the results for the optimization over the entire grid of flight performance parameters. Currently, steady state wind axis roll rates of 160°/sec are not obtainable nor tactically desirable at angles of attack above 40°. Therefore, the results presented for the pitch and the yaw are only of theoretical interest. However, we can selectively remove points from the flight performance grid that produce impractical solutions. Limiting the steady state roll rate to 80°/sec at angles of attack above 40° will be the criterion for
removing points from the flight performance grid. These revised global maxima will be more applicable to current generation fighter/attack aircraft. These results are presented in Table 6.2.

Restricting the steady state roll rate to 80°/sec or less at angles of attack above 40°, did not affect the trends. The global maxima still occurred at the lowest velocity, highest allowable steady state roll rate parameter, and smallest roll mode time constant (except for the pitch which is not a function of the time constant). The only differences are that the pitch and yaw maximum control moments are scaled down from the decrease in the steady state roll rate parameter. The roll maximum occurred at a small angle of attack so it is not affected. The pitch and yaw solutions are at virtually the same orientations as before.

Figures 6.1 and 6.3 show the non-dimensional moment coefficients for the roll and yaw respectively at the time of the control moment coefficient maximum condition and continuing for ten seconds after the control moment coefficient maximum. Figure 6.2 shows the pitching moment coefficients from ten seconds prior to the control moment coefficient maximum up to the control moment coefficient maximum. The data for each figure was generated by a program that numerically integrates the applicable equations for a velocity vector roll [3]. For the cases of the roll and yaw, the control moment maximum occurred at the instant of the maximum roll input and then decreased rapidly. The control moment runs nearly parallel to the total moment in these cases. The difference between the control moment and the total moment is a result of the aerodynamic derivative moments (C_{lp}^*p and C_{lr}^*\dot{r}). We require the initial roll rate to be zero but there are no restrictions on the initial yaw rate, hence the control moment and total moment are not identical at the maximum. This difference is small near time zero but increases as the roll rate builds up to its steady state value.

For the pitch (figure 6.2), the control moment maximum is cyclical, but builds more steadily to its maximum value than the roll and yaw cases. The control moment runs nearly parallel to the total moment. Again, the difference between the two is the result of the aerodynamic derivative moments. In this case one of the aerodynamic derivative moments is a constant (C_{ma} \cdot \alpha) and the other (C_{mq} \cdot q) is nearly constant, so the difference between the control moment
and the total moment does not vary as much with time. Although the $C_{ma} \cdot \alpha$ moment coefficient is a constant term, it is an important term because of its ability to directly shift the control moment up or down on the graph in figure 6.2. Since $C_{ma} = C_{La} (h - h_a)$ the $C_{ma} \cdot \alpha$ term can vary widely depending on the static margin $(h_{\text{ng}} - h)$ chosen. In this sense, the location of the center of gravity as it affects the $C_{ma}$ derivative becomes an important factor in determining the pitch control maximum. For a relaxed static stability aircraft, moving the center of gravity towards its aft limit makes the static margin smaller. This in turn reduces $C_{ma}$, which will shift the control moment curve upwards in figure 6.2, resulting in a lower control moment maximum. Moving the center of gravity forwards will cause the static margin to be larger, hence $C_{ma}$ will increase causing the maximum control moment to increase with it. In any case, the center of gravity location's affect on $C_{ma}$ causes the pitch moment coefficient solution to be unique to that particular geometry.

6.3 Starting Conditions for the Global Maximums

The roll and yaw control moment maxima both occurred at the time $= 0$ condition. Additionally, we assumed that the roll starts from rest (i.e. no initial roll rate). So the starting condition for these cases is to be stabilized (no initial roll rate) at the angles $\mu$, $\gamma$ and $\alpha$ as given in the control moment maximum solution (tables 6.1 and 6.2) and then apply a maximum roll acceleration $\left( \frac{p_{\text{as}}}{\tau} \right)$ input. The fact that the maximum roll and yaw control moments occur at the start of the roll is important from a practical standpoint. Since air combat maneuvering is a highly dynamic situation, it can be argued that the majority of velocity vector rolls initiated are small to moderate in amplitude change. Depending on the roll mode time constant, the steady state roll rate is reached for only a short period of time if at all. But even the shortest velocity vector roll has a starting point and for this reason the chance of encountering the roll and yaw control moment maximums becomes a very real possibility.

The pitch control moment maximum occurred at the steady state condition. Subsequently, the roll rate is at the steady state value and the roll acceleration is zero. When these conditions are met, the pitch control maximum is encountered every time the orientation (in terms of $\mu$, $\gamma$ and $\alpha$) coincides with the values in tables 6.1 or 6.2 for the global maximum solution. Since the
maximum occurred at the steady state condition, there are an infinite number of starting points leading up to the steady state. Figure 6.4 shows possible starting points (in terms of $\mu$ and $\gamma$) from ten seconds before the roll reached the condition for the maximum. This plot is generated by integrating backwards in time from the global control moment condition [3]. Since the roll rate has to build up to its steady state value, the practical starting points lie approximately three time constants or more prior to the maximum. In figure 6.4, the spike in the $\mu$ curve is a result of $\mu$ becoming undefined as $\gamma$ approaches 90°. While any time point on figure 6.4 represents a possible starting point for the global maximum, we expect the conditions where gamma is smallest to be most practical. For example, at a $\mu$ near 0°, a flight path angle ($\gamma$) of 75° combined with our 55° angle of attack puts the nose of the plane 40° past the vertical.

### 6.4 Comparison to the Total moment solutions

It's worthwhile to compare the results of Reference 1 to the results generated here. It isn't a direct comparison since we're not comparing exactly the same moment. Reference 1 finds the maximum total moment while the maximum control moment is found here. Still, we would like to see what the relationship is between the two. To do this the optimization program is run with the following changes to the parameters. The results of Reference 1 were for a steady state roll rate of 1.0 radian/sec (57.3°/sec), a roll mode time constant of 1.0 sec and a velocity of 100 ft/sec. Additionally, the aerodynamic derivative constants are slightly different. These new derivatives are:

\[
\begin{align*}
C_{lp} &= -.25 \\
C_{lr} &= .15 \\
C_{np} &= .02 \\
C_{nr} &= -.35 \\
C_{ma} &= 0.0 \\
C_{mq} &= -4.0 \\
C_{m\alpha} &= -.4
\end{align*}
\]  
(6.4)
The results for the control moment optimization as well as the results from Reference 1 are presented in table 6.3. Figures 6.5, 6.6, and 6.7 show plots of the moment coefficients versus time for each axis at the condition for the control moment coefficient maximum. As in the global maximum cases, the control moment differs from the total moment by the amount of the aerodynamic derivative moments. The lower steady state roll rate parameter and slower time constants has reduced the magnitudes of the control moment coefficients from the global maximum values. A word should be said about figure 6.5, the plot for the roll control moment coefficient maximum. From the discussion in section 6.1, we would expect the roll control moment maximum to occur at the time infinity (steady state) condition for a time constant of 1. It turns out that for the time constant of 1 the time zero and time infinity solutions are generally very close. When you change to the new derivatives $C_{IP}$ and $C_{IR}$ at this steady state roll rate parameter, the time zero solution is slightly larger than the time infinity solution.

The moment values from table 6.3 are close to each other in magnitude but differ somewhat in orientation. The roll control moment is 94% of the total moment but occurs at an additional 150° of $\mu$. The pitch control moment is 86% of the total pitch moment but occurred at nearly the same $\mu$. Remember that this is strongly dependent on $C_{ma}$. A better estimation of the maximum pitch control moment as a function of the maximum total pitching moment would take into account $C_{ma}$. In fact, by looking at figures 6.2 and 6.6, $C_{ma} \cdot \alpha$ makes up the majority of the difference between the total moment and the control moment. So a better approximation to the maximum pitch control moment would be the maximum total moment minus $C_{ma} \cdot \alpha$. Using this approximation, the estimated maximum pitch control moment turns out to be 90,327 slug ft² which is 98.7% of the calculated maximum control moment. The yaw control moment values are very close to each other. The estimated total moment and estimated control moments are within a half of a percent of the same magnitude. The estimated control moment is 92% of the actual total moment. A bigger difference is in the angle $\mu$ between these moments. The total moment maximum occurs at 150° of $\mu$ past the control moment maximum.
6.5 Pitch Moment Comparisons to the Pushover Maneuver

The results shown in tables 6.1, 6.2, and 6.3 show that a substantial pitching control moment coefficient can be required to perform the velocity vector roll for the global maximum condition. The pushover is a basic longitudinal maneuver used in the HANG study in which the pilot pushes full forward stick from a reference steady flight condition and evaluates the response, generally in terms of pitch rate and pitch acceleration. Knowing the pitch acceleration, the non-dimensional total pitching moment can be calculated as

\[ C_m = \frac{\dot{\theta}_{yp}}{q \bar{S} \bar{c}} \quad (6.5) \]

Choosing a value of 35.1 for the dynamic pressure will correspond to a velocity of 200 ft/sec at an altitude of 10,000 ft. Assuming a pitch rate acceleration of \(-.21 \text{ rad/sec}^2\) \((-12 \text{ deg/sec}^2\), the total pitching moment coefficient can be calculated from (6.5) as

\[ C_m = \frac{\dot{\theta}_{yp}}{q \bar{S} \bar{c}} = \frac{(-.21 \text{ rad/s}^2)(123,936 \text{ slug} \cdot \text{ft}^2)}{(35.1 \text{ slug ft/s}^2)(400 \text{ ft}^2)(11.52 \text{ ft})} = -.161 \]

The optimization routine was run to generate the maximum control moment coefficient for the velocity vector roll at the same dynamic pressure and with the steady state roll rate limited to 80°/sec at angle of attack above 40°. The resulting solution was \( C_{m control} = -.629 \) at \( pss = 40\% \text{s}, \mu = 98.82^\circ, \gamma = 0^\circ \) and \( \alpha = 70^\circ \). The moment coefficient required for the velocity vector roll is nearly four times that for the pushover. As the dynamic pressure increases, the difference between the pushover moment coefficients and the velocity vector roll coefficients increases. At a dynamic pressure of 79.0 \( \frac{\text{slug}}{\text{ft} \cdot \text{s}^2} \) (which corresponds to a velocity of 300 ft/sec at 10,000 ft), the pushover coefficient is \( C_m = -.072 \) and for the velocity vector roll it is \( C_{m control} = -.574 \) at \( pss = 40\% \text{s}, \mu = 103.1^\circ, \gamma = 0^\circ \) and \( \alpha = 70^\circ \). Now the velocity vector roll control moment is nearly eight times the pushover moment. The important thing to note is that the
pitching moment due mainly to roll and yaw coupling in a lateral-directional maneuver can outweigh the pitching moment needed to perform a strictly longitudinal maneuver.
Table 6.1 Solutions for the Global Maximum Control Moment Coefficient

<table>
<thead>
<tr>
<th></th>
<th>Non-dimensional control moment</th>
<th>Moment (slug \cdot ft^2)</th>
<th>μ (deg)</th>
<th>γ (deg)</th>
<th>α (deg)</th>
<th>Time (sec)</th>
<th>( p_{ss} ) (°/sec)</th>
<th>( \tau ) (sec)</th>
<th>V (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
<td>1.07</td>
<td>130,780</td>
<td>270.0</td>
<td>0.0</td>
<td>1.2</td>
<td>0</td>
<td>160.0</td>
<td>0.5</td>
<td>100.0</td>
</tr>
<tr>
<td>Pitch</td>
<td>-14.10</td>
<td>-570,270</td>
<td>271.61</td>
<td>0.0</td>
<td>51.01</td>
<td>( \infty )</td>
<td>160.0</td>
<td>---</td>
<td>100.0</td>
</tr>
<tr>
<td>Yaw</td>
<td>6.18</td>
<td>752,694</td>
<td>90.0</td>
<td>0.0</td>
<td>70.0</td>
<td>0</td>
<td>160.0</td>
<td>0.5</td>
<td>100.0</td>
</tr>
</tbody>
</table>
Table 6.2 Global Maximum Control Moment Coefficients
with steady state roll rate restricted to 80°/sec or less
for angles of attack above 40°

<table>
<thead>
<tr>
<th></th>
<th>Non-dim control moment</th>
<th>Moment (slug ft²)</th>
<th>μ (deg)</th>
<th>γ (deg)</th>
<th>α (deg)</th>
<th>Time (sec)</th>
<th>ρss (°/sec)</th>
<th>τ (sec)</th>
<th>V (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
<td>1.07</td>
<td>130,780</td>
<td>270.0</td>
<td>0.0</td>
<td>1.2</td>
<td>0</td>
<td>160.0</td>
<td>.5</td>
<td>100.0</td>
</tr>
<tr>
<td>Pitch</td>
<td>-4.00</td>
<td>-161,610</td>
<td>272.87</td>
<td>0.0</td>
<td>55.57</td>
<td>∞</td>
<td>80.0</td>
<td>—</td>
<td>100.0</td>
</tr>
<tr>
<td>Yaw</td>
<td>3.09</td>
<td>376,819</td>
<td>90.0</td>
<td>0.0</td>
<td>70.0</td>
<td>0</td>
<td>80.0</td>
<td>.5</td>
<td>100.0</td>
</tr>
</tbody>
</table>
Table 6.3 Comparison of Maximum Total Moment to Maximum Control Moment for
\( p_{ss} = 1 \) radian/sec, \( \tau = 1 \) sec and \( V = 100 \) ft/sec

<table>
<thead>
<tr>
<th></th>
<th>Moment (slug ft²)</th>
<th>( \mu ) (deg)</th>
<th>( \gamma ) (deg)</th>
<th>( \alpha ) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Roll</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total, estimated</td>
<td>23,200</td>
<td>–</td>
<td>–</td>
<td>0</td>
</tr>
<tr>
<td>Total, actual</td>
<td>25,800</td>
<td>120</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Control, estimated</td>
<td>24,249</td>
<td>270</td>
<td>0</td>
<td>4.0</td>
</tr>
<tr>
<td><strong>Pitch</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total, estimated</td>
<td>-111,300</td>
<td>270</td>
<td>0</td>
<td>61</td>
</tr>
<tr>
<td>Total, actual</td>
<td>-106,700</td>
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<td>-15</td>
<td>62</td>
</tr>
<tr>
<td>Control, estimated</td>
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<td>273</td>
<td>0</td>
<td>58</td>
</tr>
<tr>
<td><strong>Yaw</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total, estimated</td>
<td>134,600</td>
<td>–</td>
<td>–</td>
<td>70</td>
</tr>
<tr>
<td>Total, actual</td>
<td>147,900</td>
<td>242</td>
<td>-14</td>
<td>70</td>
</tr>
<tr>
<td>Control, estimated</td>
<td>135,543</td>
<td>90</td>
<td>0</td>
<td>70</td>
</tr>
</tbody>
</table>
Figure 6.1 Rolling Moment Coefficients following Input for Global Roll Control Maximum
Figure 6.2 Pitching Moment Coefficients leading to Pitch Control Moment Coefficient Moment Maximum for Roll Rate Restricted to 80°/sec
Figure 6.3 Yaw Moment Coefficients following Input for Global Yaw Control Moment Maximum with Roll Rate Restricted to 80°/sec
Figure 6.4 Potential Starting Conditions leading to Maximum Pitch Control Moment Coefficient with Roll Rate Restricted to $80^\circ$/sec
Figure 6.5 Roll Moment Coefficients following Roll Control Moment Coefficient Maximum
p_{ss}=1 \text{ rad/sec}, \tau=1 \text{ sec}, V=100 \text{ ft/sec}
Figure 6.6 Pitching Moment Coefficients leading to Maximum Pitch Control Moment Coefficient
\( \rho_s = 1 \text{ rad/sec, } \tau = 1 \text{ sec, } V = 100 \text{ ft/sec} \)
Figure 6.7 Yaw Moment Coefficients from Maximum Yaw Coefficient Moment Condition
$p_{ss}=1$ rad/sec, $\tau=1$ sec, $V=100$ ft/sec
7.0 Conclusions and Recommendations

7.1 Conclusions

The problem of determining the maximum control moment coefficients and the orientation at which they occur during a velocity vector roll was investigated. The results were a set of simplified equations suitable for optimization and applicable for any airframe at any flight condition. A numerical optimization scheme was chosen and applied in an exercise to determine the global control moment coefficient maxima for a generalized F-18.

The numerical optimization approach has a couple of important qualities. The numerical method is easily adapted to the problem. The equations presented in Chapter 3 for the control moment coefficients are not yet airframe specific. Changing the airframe and flight performance parameters (steady state roll rate, time constant and velocity) is simply a matter of changing input data to the program. Additionally, the numerical method is relatively fast in obtaining the solution. The adaptability and speed of the numerical method makes this an effective tool.

The results of the search for the global control moment coefficient maxima for the generalized F-18 show important trends. As expected, the maxima for roll, pitch and yaw all occurred at the highest allowed steady state roll rate, smallest roll mode time constant (except for the pitch which was not a function of time constant) and slowest velocity. The roll and yaw solutions occur at a maximum roll input starting from rest (no initial roll rate). The pitch solution came at a steady state rolling condition (no roll acceleration). All solutions occur at their own unique orientation in terms of mu, gamma and alpha. Mu and gamma are determined almost exclusively by the aerodynamic derivative terms. Gamma turns out to be zero in each case, but mu toggles between $90^\circ$ and $270^\circ$ depending on the sign of the aerodynamic derivative term. Alpha is mainly influenced by the moment of inertia terms. Generally, alpha is a result of the
velocity vector roll motion developing towards a pure body axis motion about the axis being considered. Subsequently, the roll control maximums occur when alpha is near zero, the yaw control maximums occur when alpha is maximum (70°) and the pitch control maximums occur at a point in between.

The maximum control moment and its orientation can be estimated for the generalized F-18 from the maximum total moment. These "back of the envelope" calculations are useful in engineering scenarios where time is limited, such as during a piloted simulation. The rules for taking the maximum total moment and estimating the maximum control moment and its orientation are as follows:

1. Gamma is always zero.
2. \( \alpha_{\text{control}} = \alpha_{\text{total}} \).
3. For Roll: \( C_{L_{\text{control}}} = 0.94 C_{L_{\text{total}}} \) and \( \mu_{\text{control}} = \mu_{\text{total}} + 150° \).
4. For Pitch: \( C_{M_{\text{control}}} = 0.86 C_{M_{\text{total}}} \) and \( \mu_{\text{control}} = \mu_{\text{total}} \) for \( C_{M_{\alpha}} = -0.4 \) or \( C_{M_{\text{control}}} = C_{M_{\text{total}}} - C_{M_{\alpha}} \cdot \alpha \) and \( \mu_{\text{control}} = \mu_{\text{total}} \) for any \( C_{M_{\alpha}} \).
5. For Yaw: \( C_{N_{\text{control}}} = 0.916 C_{N_{\text{total}}} \) and \( \mu_{\text{control}} = \mu_{\text{total}} - 150° \).

While it appears that these estimates would hold for other aircraft with similar ratio of inertia's \( I_{x_{p}} : I_{y_{p}} : I_{z_{p}} \) (1:5.3:6.2), it’s not been proven here for anything other than the generalized F-18.

### 7.2 Suggestions for Future Research

The most obvious area for continuation of this work would be in the relaxation of a few of the assumptions. The constant load factor and constant aerodynamic derivatives are likely first candidates. The data for these could be incorporated into subroutines in the form of either curve fitted equations or "look up" data tables. Relaxing the load factor assumption will require that the thrust component in the Z-wind direction should be accounted for.

The programming aspect of the work could become more sophisticated to increase the "user friendliness". Currently, the computer code is run for each control moment coefficient equation independently. That is, the program is run to get the roll control solution at time zero, then again for time infinity, then again for the yaw at time zero, and so on. Consolidating the process to solve for the entire maximum control moment coefficient set with one program run would make this an easier tool to use.
References


Appendix A: Sample Computer Code

Enclosed in Appendix A is an example of the code written and modified for the solution of the Control Power Requirements for the Velocity Vector Roll research. The code written for the roll control moment solution at time = infinity is shown here, but the code for the other control moment cases is nearly identical. The code consists of a brief main program and subroutines that interface with the Schittkowski SQP routine. The initial version of these programs was provided by Dr. E.M. Cliff of Virginia Polytechnic Institute and State University. Subsequent modifications were made to adapt them to the control moment problem. The main program contains the parameter settings, flight performance parameter grid, initial guess grid and output file control. The subroutines contain the control moment coefficient equation and the analytical expressions for the gradient which is used by the SQP routine.
Main program for use of Schittkowski's SQP code
Written by Dr. E.M. Cliff of Virginia Polytechnic Institute and State Univ. to interface with the Schittkowski SQP code. Modified Sept, 1993 by Patrick Ashley of Virginia Tech to solve the control moment equations for the velocity vector roll.

This program will control the assignment of the parameters, including the grid of flight performance parameters (steady state roll rate, time constant and velocity). It will control the initial guess grid of mu, gamma and alpha for calling the optimization routine. It will also control the output data files.

implicit none
implicit logical (a-z)
integer nmax
parameter (nmax = 5)

integer iprint, ibtype, maxsch, maxitn, m, me, n, h,i,j,k
integer ii, jj, kk
real*8 x, xguess, xl, xu, xscale, fval
  real*8 pss, tau, gs, V, lxp, lyp, lzp, rho, S, b, nz, Clp, Clr, rtd
  real*8 maxf, maxmu, maxgam, maxalp

ps = steady state roll rate
tau = roll mode time constant
gs = gravity constant
V = velocity
\[\text{lxp, lyp, lzp = principal moments of inertia}\]
\[\text{rho = density}\]
S = wing area
b = span
nz = load factor
C_Lp, C_Lr = aerodynamic derivatives with respect to roll moment
rtd = conversion factor for radians to degrees
maxf = maximum value of the function
maxmu, maxgam, maxalp = current global maximum values for mu, gamma and alpha

dimension x(nmax), xguess(nmax), xl(nmax), xu(nmax), xscale(nmax)
  character*1 done

x = array of variable being solved for (mu, gamma and alpha)
xguess = array of initial guesses for mu, gamma and alpha
xl = array of lower limit constraints
xu = array of upper limit constraints
xscale = array of scaling factors
done=character to control end of routine in interactive mode
common /f18/ pss,tau,gs,V,lxp,lyp,lzp,rho,S,b,Cip,C1r

i=print controls what information sqp code prints out
data i=print / 0 /
data ibtype / 0 /
maxsch controls the max number of searches the opt routine performs
data maxsch / 25 /
maxitn controls the maximum number of iterations the opt routine performs
data maxitn / 200 /

n=number of variables
m=number of constraint equations
me=number of equality constraint equations
data n, m, me / 3, 0, 0 /
data xl / 0.0d0, -1.5707963d0, 3*0.0d0 /
data xu / 6.2832d0, 1.5707963d0, 1.2217305d0, 2*1.0d0 /
data xscale / 5*1.0d0 /

rt=180.0/3.14159265

data pss,tau,gs,V,lxp,lyp,lzp,rho,S,b,Cip,C1r/
& 1.00d0, 1.0d0, 32.17d0, 100.0d0, 23168.0d0,
& 123936.0d0, 143239.0d0, 101755d0, 400.0d0, 34.72d0,
use this for the median values of Cip & C1r
& -.400d0,.200d0 /
use this Cip & C1r for comparisons with "Kinematics and Aerodynamics
& -.25d0,.15d0 /

data maxf,maxmu,maxgam,maxalp / 4*0.0d0 /

Open output files
this one contains the output messages for the sqp routine
open(unit=17, file='short.out')
this one contains a table of every initial guesses and the
  corresponding solution
open(unit=18, file='guesssoln.out')
this one contains a listing of the parameter data and
  corresponding solution for each max solution
open(unit=19, file='datsoin.out')
this is the most useful data file. It contains the max solutions
  in a table suitable for Kaleidagraph
open(unit=20, file='graph.out')
write header on check out file
write(18,1008)'imu','igam','ialp','mu','gamma','alpha','CLcntrl'

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write header on graph file
write(20,1025)'mu','gam','alp','pss','tau','V','CLcnnrl','Lcntrl'

On Screen Mode: Uncomment this to run the program in an interactive mode.

Entering initial guess in degrees

write (6,1001) n
read (5,1002) (xguess(i), i=1,n)

************** Nested Loops to run through the grid of V,pss & tau's

Velocity Loop
   do 45 kk=1,4
      V=100.0*real(kk)
   end

pss Loop
   do 45 jj=1,4
      pss=real(jj)*40.0/rtd
   end

tau Loop
   do 45 ii=1,3
      tau=real(ii)*.5
      Initialize global max to zero
      maxf=0.0
   end

************** Initial guess grid (mu, gamma and alpha)

Alpha loop
   do 30 h=0,70,35

Gamma loop
   do 31 j=0,9

Mu loop
   do 35 k=0,36
      xguess(1)=real(k)*10.00
      xguess(2)=real(j)*10.0
      xguess(3)=real(h)
   end

converting degrees back to radians for computation
   do 36 i=1,n
      xguess(i)=xguess(i)/rtd
      continue
   end

This is the main call to the opt routine
   call sqp(m,me,n,xguess,ibtype,xl,xu,
+ xscale,iprint,maxtn,maxsch,x,fval)

Looking for the global max, compare current value with max value
   if (abs(fval) .gt. abs(maxf)) then
maxf=fval
maxmu=x(1)
maxgam=x(2)
maxalp=x(3)

eendif

Write to short output file
write(17,*) 'Initial guess
Solution'
write(17,1006) 'mu =', xguess(1)*rtd,'deg','x(1)*rtd,' deg'
write(17,1007) 'gam =', xguess(2)*rtd,'deg','x(2)*rtd,' deg'
&
'Lntr=','fval'
write(17,1006) 'alp =', xguess(3)*rtd,'deg','x(3)*rtd,' deg'
write(17,*) '

Write to guesssoln file (initial guesses and Lvalue)
write(18,1009)xguess(1)*rtd,xguess(2)*rtd,xguess(3)*rtd,
&
x(1)*rtd,x(2)*rtd,x(3)*rtd,fval

Write to screen
write(6,*)
write(6,1004) 'mu =', x(1)*rtd, 'deg'
write(6,1004) 'gamma =', x(2)*rtd, 'deg'
write(6,1004) 'alpha =', x(3)*rtd, 'deg'
&
write('*,*)
write(6,1005) 'Lntr=','fval
write('*,*)

On screen mode: Repeat process for another initial guess?
write(6,*) 'Are you finished? (Type "y")'
read(5,1003) done
if (done.ne. 'y') goto 21

Automatic mode:
continue
continue
continue

c

c
write max and angles to datso1n file
write(19,*) ' t = infinity'
write(19,1020) ' p = ', pss, ' rad/s'
write(19,1020) ' tau = ', tau, ' sec^-1'
write(19,1021) ' V = ', V, ' ft/s'
write(19,1023) ' rho = ', rho, ' slug/ft^3'
write(19,1021) ' S = ', S, ' ft^2'
write(19,1021) ' b = ', b, ' ft'
write(19,1021) ' Clp = ', Clp
write(19,1021) ' Clr = ', Clr

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write(19,\*)
write(19,1024)' CLcntrmax = .maxf
write(19,1021)' Lcntrmax = .5*maxf*rho*(V**2)*S*b.
& ' slug*ft^2'
write(19,1022)' mu = .maxmu*rtd,'deg'
write(19,1022)' gam = .maxgam*rtd,'deg'
write(19,1022)' alp = .maxalp*rtd,'deg'
write(19,\*)
write(19,\*)

& Write to main graph file
write(20,1026)maxmu*rtd,maxgam*rtd,maxalp*rtd,pss,tau,
& V,maxf,.5*maxf*rho*(V**2)*S*b

45 continue

**************End of Nested Loop for flight parameter grid

Close files:
close(unit=17)
close(unit=18)
close(unit=19)
close(unit=20)

c stop

c Format statements
1001 format(5x,'Input initial guesses for the ','i2,',' parameters.'/
+ 5x,'One at a time (F10), in degrees (not radians)'//)
1002 format(f10.0)
1003 format(a1)
1004 format(a,f12.7,a)
1005 format(40x,a,f12.4)
1006 format(2x,a,f12.7,a,5x,f12.7,a)
1007 format(2x,a,f12.7,a,5x,f12.7,a,3x,a,f12.4)
1008 format(a5,1x,2(a4,1x),a10,1x,2(a9,1x),a11)
1009 format(f5.1,1x,2(f4.1,1x),f10.5,1x,2(f9.5,1x),f11.7)
1020 format(a,f4.1,a)
1021 format(a,f12.4,a)
1022 format(a,f10.5,a)
1023 format(a,f8.6,a)
1024 format(a,f11.7,a)
1025 format(a7,1x,2(a6,1x),a4,1x,a3,1x,a5,1x,a8,1x,a10)
1026 format(f7.2,1x,2(f6.2,1x),f4.1,1x,f3.1,1x,f5.1,1x,f8.4,1x,f10.2)

end
subroutine nlfunc(m,me,mmax,n,f,g,x,active)

The following subroutine is a part of the file sqp.f in the Va Tech
Schittkowski Sqp optimization routine. This subroutine will contain
the control moment coefficient equation to be maximized.

implicit none
implicit logical (a-z)

integer m, me, mmax, n
logical active
real*8 f, g, x
dimension g(mmax), x(n), active(mmax)

common /f18/ pss,tau,gs,V,lxp,lzp,lyp,lzo,rho,S,b,Clp,Clr

real*8 pss,tau,gs,V,lxp,lzp,lyp,lzo,nz,Clp,Clr
real*8 rho,S,b,qbar,tstar,rtd
real*8 c1,c2,c3,s1,s2,s3

Initialize

  c1=dcos(x(1))
  c2=dcos(x(2))
  c3=dcos(x(3))
  s1=dsin(x(1))
  s2=dsin(x(2))
  s3=dsin(x(3))

rtd=180.0/3.14159265
nz=1.0
dynamic pressure
qbar=.5*rho*(V**2)
non dimensional lat-directional time
tstar=b/(2.0*V)

Evaluate function to be minimized (max f = min -f)

f=((pss*gs/(qbar*S*b*V))*(lyp-lzp-lxp)*c1*c2*s3
 & -(pss*gs/(qbar*S*b*V))*(lyp-lzp)*nz*s3
 & -tstar*pss*Clp*c3
 & +tstar*pss*Clr*s3
 & -tstar*(gs/V)*s1*c2*(Clp*s3 + Clr*c3))

g is ignored since there aren't any constraints in this
problem
return
end
subroutine nlgrad(m,me,mmax,n,f,g,df,dg,x,active,wa)
   The following subroutine is a part of the file sqp.f in the Va Tech
   Schittkowski Sqp optimization routine. It will compute the gradients
   of the control coefficient equation as needed by the main sqp routine.

   implicit none
   implicit logical (a-z)

   Routine to compute gradients of \( F \)

   real*8 fp1
   parameter (fp1 = 1.0d0)

   integer m, me, mmax, n
   logical active
   real*8 f, g, x, df, dg, wa
   dimension g(mmax), x(n), active(mmax), df(n), dg(mmax,n), wa(mmax)
   integer i, j
   real*8 xs, delt, fp, fm, eps, dmax1, dsqrt, dabs
   intrinsic dmax1, dsqrt, dabs
   common /cmache/ eps
                     common /f18/ pss,tau,gs,V,lxp,lyp,lzp,rho,S,b,Clp,Clr

   real*8 pss,tau,gs,V,lxp,lyp,lzp,lterm,nz,Clp,Clr
   real*8 rho,S,b,qbar,tstar,rtd
   real*8 c1,c2,c3,s1,s2,s3

   Initialize
   c1=dcos(x(1))
   c2=dcos(x(2))
   c3=dcos(x(3))
   s1=dsin(x(1))
   s2=dsin(x(2))
   s3=dsin(x(3))

   lterm=lyp-lzp-lxp

   dynamic pressure
   qbar=.5*rho*(V**2)
   nondimensional lat-directional time
   tstar=b/(2.0*V)
   rtd=180.0/3.14159265
   nz=1.0
Analytic form of the gradient di

df/dmu

\[ df(1) = -(pss*gs/(qbar*S*b*V))*lterm*s1*c2*s3 \]
&
\[ -tstar*(gs/V)*c1*c2*(Clp*s3 + Clr*c3) \]

df/dgamma

\[ df(2) = -(pss*gs/(qbar*S*b*V))*lterm*c1*s2*s3 \]
&
\[ +tstar*(gs/V)*s1*s2*(Clp*s3 + Clr*c3) \]

df/dalpha

\[ df(3) = -(pss*gs/(qbar*S*b*V))*lterm*c1*c2*c3 \]
&
\[ -(pss*gs/(qbar*S*b*V))*(lzp)*nz*c3 \]
&
\[ + tstar*pss*Clp*s3 + tstar*pss*Clr*c3 \]
&
\[ - tstar*(gs/V)*s1*c2*(Clp*c3 - Clr*s3) \]

return
end
Vita

The author was born in Pontiac, Michigan on November 29, 1963 but grew up in Boulder, Colorado. He graduated in May of 1986 from the University of Colorado with a bachelor of science degree in Mechanical Engineering. He then worked for three and a half years as a designer in the Space Shuttle Orbiter Mechanical Ground Support Equipment Group with Rockwell's Space Systems Division in Downey, California. After that he began his graduate work at Virginia Polytechnic Institute and State University. During this time he interned for one year in NASA Langley's Flight Dynamics Branch. This thesis represents the final requirements towards the completion of the Master of Science degree in Aerospace and Ocean Engineering.