Analysis of Phased Array Antenna Radiation Patterns Including

Mutual Coupling

by

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(ABSTRACT)

Methods of expressing the radiation patterns of phased arrays in closed form that include the effects of radiated mutual coupling are investigated. The two basic methods considered are the classical array analysis method and the active element pattern methods. The theoretical derivations of the active element pattern methods are presented and the various types of active element patterns are defined. Also, a new method based on active element patterns, the hybrid active element pattern method, is introduced which accurately predicts the patterns of small and moderately-sized arrays of equally-spaced elements. Arrays of center-fed dipoles are considered in this study since dipole arrays can be fully characterized, including mutual coupling, using modern numerical electromagnetic analysis codes, thus allowing verification of the array analysis methods presented here. The results are general, however, and may be applied to arrays of any type of element. The array patterns computed using the classical analysis method and the active element pattern methods are compared to those computed using ideal array analysis and the highly-accurate numerical codes.
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1. Introduction

Phased array antennas are becoming increasingly popular for use in applications that require control of radiation patterns, including radar and space communications. This popularity exists primarily because the radiation pattern of a phased array can be shaped and controlled by a proper choice of element feed voltages. The analysis and design of phased arrays is complicated, however, by the fact that the elements in the array do not behave independently of each other but rather interact electromagnetically. This electromagnetic interaction between elements is called \textit{mutual coupling}.

A great deal of effort has been spent developing analysis methods which account for the effects of mutual coupling in the array environment, and many of these methods have evolved into very accurate and reliable analysis tools, most notably numerical methods based on the method of moments. However, several fundamental questions remain regarding the closed-form mathematical expression of array radiation patterns including radiated mutual coupling effects \cite{1}. Array pattern expressions are useful analysis tools because they can often provide more insight into the behavior of the array pattern than other analysis methods.

The simplest method of expressing the radiation pattern of an array in closed form assumes that no mutual coupling exists in the array environment and that all of the element feed currents are proportional to the array excitation. This analysis method, referred to here as the \textit{ideal array analysis method}, is given by (see Section 2.2)
where $F_{un}(\theta, \phi) = g(\theta, \phi) \sum_{n=1}^{N} I_n e^{j\beta \hat{r} \cdot r_n}$

\[ (1-1) \]

is the unnormalized radiation pattern of the $N$-element array in the direction $(\theta, \phi)$, $g(\theta, \phi)$ is the ideal pattern of an isolated array element, the $\{I_n\}$ are the ideal feed currents, and the $e^{j\beta \hat{r} \cdot r_n}$ terms give phase information related to the locations of the individual elements with respect to one another. Practical arrays, however, are usually excited by a set of equivalent voltage sources with non-zero source impedances. Also, mutual coupling is present in a practical array and causes the element input impedances to vary from element to element (see Section 2.5). The resulting feed currents $\{I_n^*\}$ are therefore not proportional to the excitation voltages in a practical array. If the $\{I_n^*\}$ are substituted for the $\{I_n\}$ in the ideal analysis method (1-1), the method given by

\[ F_{un}(\theta, \phi) = g(\theta, \phi) \sum_{n=1}^{N} I_n^* e^{j\beta \hat{r} \cdot r_n} \]  

\[ (1-2) \]

results. This method has been widely applied for many years, and is therefore referred to here as the classical array analysis method. The $\{I_n^*\}$ can be computed using a number of different approaches, the earliest of which was described by Carter [2].

The classical array analysis method has long been widely applied to include mutual coupling in array pattern analysis, and countless examples of its use have appeared in the literature (for example, see Kraus [3]). A limitation of classical analysis, however, is that it assumes that all of the elements in the array are similar; that is, the elements must be of the same size, shape, orientation and relative polarization for classical array analysis to be used successfully. The method also requires that the element current distributions (or aperture field distributions, in the case of elements such as horns or reflectors) be identical within a complex multiplicative scaling factor. These limitations preclude the use of classical analysis to analyze many arrays, such as arrays of dissimilar elements or arrays whose element current distributions vary significantly from element to element [4].
More recently, closed-form array pattern expressions that employ individual active element patterns in their formulation have been suggested which successfully predict the effects of mutual coupling on the radiation patterns of almost any type of array [5-10]. An active element pattern is the pattern produced by an array when only one element is excited and the other elements are loaded with their respective generator impedances (see Section 4.3). Unfortunately, a clear theoretical derivation of these array pattern expressions based on active element patterns has not appeared in the general literature. Also, a clear distinction between the various types of active element patterns does not exist.

This study presents the derivation of several array pattern expressions based on active element patterns which include the effects of radiated mutual coupling. Feed network coupling is not considered. Beginning with the basic integral equation which governs the current distribution along wire antennas, the derivation builds a theoretical foundation for several different methods of expressing the radiation patterns of phased arrays. In the process of deriving these array pattern expressions, the various types of active element patterns are defined, and the differences between the element patterns are clarified. In addition, the radiation patterns of several test bed arrays computed using the active element pattern analysis methods are compared to patterns computed using the more widely applied classical analysis method and to highly-accurate patterns computed using numerical techniques. Arrays of wire antennas were selected for the test bed because they can be fully characterized (including mutual coupling effects) using modern numerical method codes. Though the derivation of the array pattern expressions is developed specifically for arrays of center-fed dipole elements, the array pattern expressions can be generalized to arrays comprised of elements of any type.

To provide a starting point for the discussion of array pattern expressions, the fundamentals of ideal array analysis, which does not account for the effects of mutual coupling, are reviewed in Chapter 2. Also in Chapter 2, some of the effects mutual coupling may have on array patterns and element impedances are discussed. Chapter 3 describes the Electromagnetic Surface Patch (ESP) code developed at the Ohio State University Electroscience Laboratory. ESP is a numerical
electromagnetics analysis program based on the method of moments and is used to generate the active element patterns employed in this study. ESP is also used to compute accurate patterns of the fully-excited testbed arrays for comparison against the patterns computed using the methods discussed in this report. Chapter 4 reviews the derivation of the classical array analysis method and presents the derivation of the methods based on active element patterns. In addition, Chapter 4 discusses the proper manner in which the active element patterns should be computed or measured. At the end of the chapter a new method is presented which allows the efficient and accurate analysis of moderately-sized, equally-spaced arrays. Chapter 5 presents the patterns of several testbed arrays computed using the classical array analysis method and the active element pattern methods. Chapter 5 also compares the patterns computed using these methods to the patterns computed using ESP for the same arrays. Conclusions are presented in Chapter 6. Two computer programs, PATMULT and ACTEL, were written to implement the ideal and classical analysis methods and the active element pattern methods, respectively. These programs are described in the two appendices.

As a final introductory note, this work will be placed into its historical perspective. The original problem was that of finding techniques to synthesize array radiation patterns in the presence of mutual coupling. This included synthesis of low sidelobe patterns as well as scanned beams. The research program began at the New Mexico State University Physical Science Laboratory while Dr. Warren Stutzman was on leave from Virginia Tech; Dr. Keith R. Carver and Russel P. Jedlicka of NMSU were co-researchers. This work evolved into a four-year effort funded by the Army Research Office. This project had two parts: a theoretical investigation into the synthesis problem and experimental confirmation. Two master's theses at Virginia Tech [1,11] laid out the methods of synthesis. Experimental confirmation did not provide convincing results, largely because of the choice of array realization. Microstrip elements were used because they could be made with close spacings leading to high mutual coupling to enhance the effects for verification purposes. The results showed, however, that microstrip arrays are difficult to characterize. Also, during the lengthy
study it was apparent that there are some deficiencies in classical treatments of array theory that attempt to include mutual coupling effects. These results prompted the study leading to this report.
2. Fundamentals of Array Antenna Analysis

The theory of array antennas was initially developed by assuming that array elements do not interact with one another. This ideal method of array analysis assumes that the current distributions on all of the elements are identical within a complex multiplicative constant, regardless of the geometry or feed configuration of the array. Mutual coupling is completely ignored in ideal array analysis. The purpose of this chapter is to review ideal array analysis, to present some examples of its use, and to point out some of its limitations with regard to predicting the effects of mutual coupling on the radiation patterns produced by arrays.

2.1 Radiation Patterns of Arrays

The electric field radiated by a distribution of current located arbitrarily in free space is given by [12]

\[
E = -j\omega \mu A + \frac{\nabla (\nabla \cdot A)}{j\omega t} \tag{2.1 - 1}
\]
where $\mu$ and $\varepsilon$ are the permeability and permittivity of free space, respectively, and

$$A(r) = \frac{1}{4\pi} \int \int \int_{\mathbb{R}^3} J(r') \frac{e^{-j\beta|r-r'|}}{|r-r'|} \, dv'$$  \hspace{1cm} (2.1 - 2)

The quantity $A(r)$ is called the magnetic vector potential. The quantity $\beta$ is the free space phase constant, equal to $\frac{2\pi}{\lambda}$. The vector $r$ represents the location of the observation point in space at which the radiated electric field is measured. Primed coordinates and vectors represent locations of source points on the antenna, and $J(r')$ is the current distribution on the antenna. The complete coordinate system used in this analysis is shown in Figure 2.1-1.

The quantity $|r-r'|$ is the distance between a given source point and the observation point. If the observation point is located sufficiently far away from the antenna, which is near the origin, such that $r' \ll r$, then $|r-r'|$ may be approximated with a simpler expression. Using the relation $r^2 = x^2 + y^2 + z^2$, the expression for $|r-r'|$ becomes

$$|r-r'| = \left[ r^2 + (x'^2 + y'^2 + z'^2) - 2(xx' + yy' + zz') \right]^{1/2} \approx \left[ r^2 - 2(xx' + yy' + zz') \right]^{1/2}$$  \hspace{1cm} (2.1 - 3)

where use has been made of the inequality

$$|x'^2 + y'^2 + z'^2| \ll |2(xx' + yy' + zz')|$$  \hspace{1cm} (2.1 - 4)

Using the binomial theorem,

$$(a + b)^n \approx a^n + na^{n-1}b$$  \hspace{1cm} (2.1 - 5)

with $a = r^2$, $b = -2(xx' - yy' - zz')$ and $n = 1/2$, the approximation for $|r-r'|$ becomes [11]

2. Fundamentals of Array Antenna Analysis
Figure 2.1-1. Coordinate system used in analysis.
\[ |r - r'| \approx r - \frac{xx' + yy' + zz'}{r} \]
\[ \approx r - \frac{r \cdot r'}{r} \]
\[ \approx r - \hat{r} \cdot r' \]  
(2.1 - 6)

where \( \hat{r} \) is the unit vector in the direction of \( r \). The use of this approximation for the term \(|r - r'|\) allows the expression for the magnetic vector potential in (2.1-2) to simplify to

\[ A(r) = \frac{e^{-jBr}}{4\pi r} \int_{\Omega} \int_{\Omega'} J(r') e^{jBr} \hat{r} \cdot r' \, dv' \]  
(2.1 - 7)

This expression for \( A(r) \) is valid in the far field where the observation point is far away from the antenna and the origin, that is, whenever the inequality in (2.1-4) holds.

If the current distribution consists of an array of \( N \) discrete, very thin, \( z \)-directed elements of length \( 2h \), the expression for the current distribution on the antenna can be written

\[ J(r') = \hat{z} \sum_{n=1}^{N} J_n(z'_n) \delta(x' - x_n) \delta(y' - y_n) \]  
(2.1 - 8)

where \((x_n, y_n, z_n)\) is the location of the center of the \( n \)th element, and a new shifted coordinate, \( z'_n = z' - z_n \), is introduced which is defined to be the coordinate along the \( n \)th element, referenced to the center of the element, such that \(-h \leq z'_n \leq h\) for each element. The quantity \( J_n(z'_n) \) is the current distribution along the \( n \)th element. The coordinate system used to describe the array geometry is shown in Figure 2.1-2. Note that the elements are assumed to be thin enough that the current distribution on each element can be approximated by an infinitely-thin filament of current.

After substituting (2.1-8) into (2.1-7), the expression for the magnetic vector potential becomes

2. Fundamentals of Array Antenna Analysis
Figure 2.1-2. Coordinate system used in array analysis.
\[ A(r) = \hat{\mathbf{\hat{r}}} \frac{e^{-j\beta r}}{4\pi r} \sum_{n=1}^{N} \left( \int_{-h}^{h} J_n(z_n') e^{j\beta \left[ x_n \sin \theta \cos \phi + y_n \sin \theta \sin \phi + (z_n + z_n') \cos \beta \right]} dz_n' \right) \] (2.1-9)

Equation (2.1-9) can be rewritten

\[ A(r) = \hat{\mathbf{\hat{r}}} \frac{e^{-j\beta r}}{4\pi r} \sum_{n=1}^{N} e^{j\beta \hat{r} \cdot r_n} \int_{-h}^{h} J_n(r) e^{j\beta r} \cos \theta \, dr \] (2.1-10)

where the less cumbersome dummy variable \( \ell \) has replaced \( z_n' \), and

\[ r_n = \hat{x} x_n + \hat{y} y_n + \hat{z} z_n \] (2.1-11)

For \( z \) – directed current distributions, the expression for the electric field shown in (2.1-1) simplifies in the far field to

\[ E = \hat{\mathbf{\hat{r}}} j\omega \mu \sin \theta A_z \\
= \hat{\mathbf{\hat{r}}} j\omega \mu \frac{e^{-j\beta r}}{4\pi r} \sum_{n=1}^{N} G_n(\theta, \phi) e^{j\beta \hat{r} \cdot r_n} \] (2.1-12)

where \( \hat{\mathbf{\hat{r}}} \) is the unit vector in the \( \phi \) – direction and

\[ G_n(\theta, \phi) = \sin \theta \int_{-h}^{h} J_n(r) e^{j\beta r \cos \theta} \, dr \] (2.1-13)

Note that the far electric field produced by an array of \( z \) – directed elements has only a \( \theta \) component; the electric field has no \( r \) or \( \phi \) components. The corresponding far magnetic field radiated by the array is given by
\[
H = \hat{\phi} \frac{E_\theta}{\eta}
\]  

(2.1 - 14)

where \( \hat{\phi} \) is the unit vector in the \( \phi \) direction and \( \eta \) is the intrinsic impedance of free space, 120\( \pi \) ohms.

The variation in strength of the electric field with direction from the array can be plotted using (2.1-12) with \( r \) held constant. Such a plot of radiated field strength versus direction from an antenna is called a radiation pattern. Usually a plot of the relative strength of the radiated field is required rather than a plot of the absolute strength; therefore, the factor \( j\omega \mu \frac{e^{-j\beta r}}{4\pi r} \) may be omitted from (2.1-12) when computing the radiation pattern since this factor remains constant for constant \( r \), regardless of the direction from the array. The unnormalized far-field radiation pattern of the array, \( F_{un}(\theta, \phi) \), can then be written as

\[
F_{un}(\theta, \phi) = \sum_{n=1}^{N} G_n(\theta, \phi) e^{j\beta \hat{r} \cdot r_n}
\]  

(2.1 - 15)

Note that the notation \( F_w(\theta, \phi) \) is often used in the literature to represent the magnitude pattern of an antenna. As used here, however, \( F_w(\theta, \phi) \) represents the complex-valued pattern of an antenna array, so that, in general, \( F_w(\theta, \phi) \) has a magnitude and phase associated with each direction \((\theta, \phi)\).
2.2 Ideal Array Analysis

The quantity $G_n(\theta, \phi)$, introduced in (2.1-13) as

$$G_n(\theta, \phi) = \sin \theta \int_{-h}^{h} J_n(\ell') e^{-j\beta \ell' \cos \theta} d\ell'$$  \hspace{1cm} (2.2 - 1)

is dependent upon the variation of the current along each wire. Conventionally, $G_n(\theta, \phi)$ is called the isolated element pattern for the $n$th element. If all of the elements in the array are similar, that is, if all of the elements are of the same length and the current distributions on the elements are all of the same shape but differ only by a scaling factor, then the current distribution on each similar element can be written as

$$J_n(\ell) = I_n \tilde{J}(\ell)$$  \hspace{1cm} (2.2 - 2)

where $\tilde{J}(\ell)$ is the normalized current distribution and is the same for each similar element. The quantity $I_n$ is a complex scaling factor and represents the element feed current, assuming each element has only one feedpoint. The feed current $I_n$ has amplitude $A_n$ and phase $\phi_n$, and can therefore be represented by

$$I_n = A_n e^{j\phi_n}$$  \hspace{1cm} (2.2 - 3)

After substituting (2.2-2) into (2.2-1), $G_n(\theta, \phi)$ for each element can be expressed as

$$G_n(\theta, \phi) = I_n g(\theta, \phi)$$  \hspace{1cm} (2.2 - 4)

where
\[ g(\theta, \phi) = \sin \theta \int_{-h}^{h} \tilde{J}(t) e^{j\beta t \cos \theta} \, dt \]  

(2.2 - 5)

Note that the term \(g(\theta, \phi)\) is the same for all elements in the array. The radiation pattern of the array of similar elements can now be written as

\[ F_{un}(\theta, \phi) = g(\theta, \phi) \sum_{n=1}^{N} I_n e^{j\beta \hat{r} \cdot r_n} \]  

(2.2 - 6)

Consider the special case of an array consisting of a single element (such as an isolated half-wave dipole). The element is located at the origin of the coordinate system and is fed with current \(I_n = 1.0 \angle 0^\circ\). From (2.2-6), the radiation pattern of such an antenna is

\[ F_{un}(\theta, \phi) = g(\theta, \phi) \]  

(2.2 - 7)

For the special case of a single element, the summation in (2.2-6) equals unity, and the radiation pattern of the isolated element is simply the factor \(g(\theta, \phi)\). Thus, \(g(\theta, \phi)\) represents the radiation pattern produced by a single element in the array and is therefore called the element pattern.

Equation (2.2-6) may be rewritten as

\[ F_{un}(\theta, \phi) = g(\theta, \phi) f(\theta, \phi) \]  

(2.2 - 8)

where

\[ f(\theta, \phi) = \sum_{n=1}^{N} I_n e^{j\beta \hat{r} \cdot r_n} \]  

(2.2 - 9)
The quantity \( f(\theta, \phi) \) is called the *array factor*, and represents the pattern that would be produced by an array of isotropic elements, for which \( g(\theta, \phi) = 1.0 \) in all directions \((\theta, \phi)\). The representation of the radiation pattern of the array as a product of an element pattern \( g(\theta, \phi) \) and an array factor \( f(\theta, \phi) \), as shown in (2.2-8), is called the *principle of pattern multiplication* [3].

Pattern multiplication provides the basis for the ideal analysis of arrays. In ideal array analysis, the radiation characteristics of the array are determined solely by the amplitudes and phases of the element feed currents and by the physical locations of the elements relative to each other. The element pattern is assumed to remain unchanged regardless of the number, locations and feed configuration of the array elements. Ideal array analysis does not account for many of the factors which can cause degradation of the performance of the array, most important of which is the effect of mutual coupling. Instead, the array is assumed to be made up of "ideal" similar elements with ideal element patterns which are unaffected by mutual coupling. In an actual array, mutual coupling effects cause element patterns to differ among elements in the array, even if the elements are of the same size and shape. Therefore, ideal array analysis provides an approximation to the actual radiation pattern of an array. In spite of its shortcomings, however, ideal array analysis is still useful as a tool for predicting the general shapes of radiation patterns and in some cases may produce very accurate results.

### 2.3 Ideal Analysis of Arrays of Center-Fed Dipoles

When arrays of center-fed dipole elements are analyzed using ideal array analysis, the current distributions on the elements are usually assumed to be sinusoidal. The current amplitude is zero at the ends of the element and varies sinusoidally along the element to the center feed point. The phase of the current is assumed to remain constant along the entire length of the element. Ideal sinusoidal
current distributions for 0.5-, 0.75- and 1.0-wavelength center-fed dipole elements are shown graphically in Figure 2.3-1.

The unnormalized ideal sinusoidal current distribution along a center-fed dipole element of length $2h$ can be expressed mathematically as [12]

$$\tilde{J}(\rho) = \sin \beta(h - |\rho|)$$  \hspace{1cm} (2.3 - 1)

where $\rho$ is the coordinate along the length of the element, such that $-h \leq \rho \leq h$. Actual dipole elements have non-sinusoidal current distributions, though for elements 0.5 wavelength long or shorter the sinusoidal distribution represented by (2.3-1) provides a very close approximation to the actual current distribution on the element [4]. For elements longer than 0.5 wavelength the sinusoidal distribution becomes a very poor approximation to the actual current distribution on the element. For the purpose of computing the radiation patterns of arrays of dipoles that are 0.5 wavelength long or less, the sinusoidal current approximation yields very good results, as will be shown in Chapter 5.

2.4 Examples of Ideal Array Analysis

Three examples of phased array antennas will now be considered to illustrate the application of ideal array analysis to typical arrays. These examples represent three different applications of phased arrays: scanning, sidelobe level control and null positioning. Each array analyzed here using ideal analysis techniques will be analyzed again in Chapter 5 using analysis techniques which include mutual coupling effects. Note that, though the array theory discussed in the previous sections assumed that the array elements were $z$-directed dipoles, the arrays considered in this section all consist of $x$-directed dipoles. This change was made because the computer program used to
Figure 2.3-1. Ideal sinusoidal current distributions for 0.5-wavelength, 0.75-wavelength and 1.0-wavelength center-fed dipoles.
generate the radiation pattern plots presented in this section requires that the array consist of \( x \)-
directed elements (see Appendix A).

### 2.4.1 Scanning Linear Array

Scanning arrays find use principally in radar applications. The geometry of the scanning array
considered here is shown in Figure 2.4-1. The array consists of five \( x \)-directed, uniformly-excited
half-wave dipoles with 0.5-wavelength spacing between elements. The elements are all centered on
the \( z \)-axis, with the central element located at the coordinate origin. The spatial phase term re-
quired for each element in the array pattern expression (2.2-6) can therefore be written

\[
e^{j\beta r \cdot r_n} = e^{j\beta s_n \cos \theta}, \quad n = 1, 2, 3, 4, 5 \tag{2.4 - 1}
\]

Since the element centers are located at \( z = -\lambda \) to \( z = \lambda \) in half-wavelength increments, the spatial phase term can be rewritten as

\[
e^{j\beta r \cdot r_n} = e^{j\pi(n - 3) \cos \theta}, \quad n = 1, 2, 3, 4, 5 \tag{2.4 - 2}
\]

If the feed current is expressed in its amplitude and phase components, then, using (2.2-6), the ra-
diation pattern produced by the array is

\[
F_{un}(\theta, \phi) = g(\theta, \phi) \sum_{n=1}^{5} A_n e^{j\pi(n - 3) \cos \theta} \tag{2.4 - 3}
\]

Since the array is uniformly excited, \( A_n = 1.0 \) for each element in the array.
Figure 2.4-1. Geometry of five-element scanning linear array of center-fed dipoles.
The element pattern for an ideal half-wave dipole element oriented in the \( x \) - direction is well-known and is given by [12]

\[
g(\theta, \phi) = \frac{\cos\left(\frac{\pi}{2} \sin \theta \cos \phi\right)}{\sqrt{1 - \sin^2 \theta \cos^2 \phi}} \tag{2.4 - 4}
\]

The \( E \)- and \( H \)-planes of the element pattern are shown in Figure 2.4-2. The \( E \)-plane (\( xy \) - plane or \( xz \) - plane) pattern has a figure-eight shape with a half-power beamwidth of 78 degrees. The \( H \)-plane (\( yz \) - plane) pattern is omnidirectional.

After substituting (2.4-4) into (2.4-3), combining the exponential terms and recalling that the \( \{A_n\} \) are equal to unity, the radiation pattern becomes

\[
F_{un}(\theta, \phi) = \frac{\cos\left(\frac{\pi}{2} \sin \theta \cos \phi\right)}{\sqrt{1 - \sin^2 \theta \cos^2 \phi}} \sum_{n=1}^{5} e^{j(x_n + \pi(n - 3) \cos \theta)} \tag{2.4 - 5}
\]

The element pattern remains unchanged regardless of the locations and feed configuration of the elements; however, the array factor is affected by the phase of the element currents. Inspection of (2.4-5) shows that the array factor will have a maximum in the direction \( \theta = \theta_e \) if the current phases \( \{a_n\} \) are adjusted such that

\[
a_n + \pi(n - 3) \cos \theta_e = 0, \quad n = 1, 2, 3, 4, 5 \tag{2.4 - 6}
\]

In other words, the elements must be fed with currents of phase

\[
x_n = \pi(3 - n) \cos \theta_e , \quad n = 1, 2, 3, 4, 5 \tag{2.4 - 7}
\]

to produce a pattern maximum in the \( \theta = \theta_e \) direction. For example, to produce a pattern maximum in the \( \theta = 60^\circ \) direction, the phases of the element currents in radians must be
Figure 2.4-2. Patterns of ideal 0.5-wavelength dipole: (a) E-plane and (b) H-plane.
\[ \begin{align*}
\alpha_1 &= \pi \\
\alpha_2 &= \frac{\pi}{2} \\
\alpha_3 &= 0 \\
\alpha_4 &= -\frac{\pi}{2} \\
\alpha_5 &= -\pi 
\end{align*} \tag{2.4 - 8} \]

The absolute phase of the current is unimportant; the phase difference between elements determines the pattern produced by the array. Therefore, the current phase distribution

\[ \begin{align*}
\alpha_1 &= 0 \\
\alpha_2 &= -\frac{\pi}{2} \\
\alpha_3 &= -\pi \\
\alpha_4 &= -\frac{3\pi}{2} \\
\alpha_5 &= 0 
\end{align*} \tag{2.4 - 9} \]

would produce the same pattern as the phase distribution shown in (2.4-8).

The E-plane and H-plane patterns are shown in Figures 2.4-3 and 2.4-4 for arrays phased for scan angles (\(\theta_s\)) of 0 degrees and 90 degrees. The patterns produced by the arrays are simply the superposition of the array factor on the element pattern. The H-plane patterns are identical to those that would have been produced by an array of isotropic elements because the element pattern of a dipole element is omnidirectional in the H-plane. The effect of the element pattern on the total array pattern can be seen in the E-plane, where the total array pattern has nulls off the ends of the dipole elements (in the direction of the \(x\) - axis).
Figure 2.4-3. Patterns of five-element linear array of 0.5-wavelength center-fed dipoles scanned to $\theta = 0$ degrees: (a) E-plane and (b) H-plane. The element spacing is 0.5 wavelength.
Figure 2.4-4. Patterns of five-element linear array of 0.5-wavelength center-fed dipoles scanned to $\theta = 90$ degrees: (a) E-plane and (b) H-plane. The element spacing is 0.5 wavelength.
2.4.2 Dolph-Chebyshev Array

Dolph-Chebyshev arrays comprise a special class of arrays that allow control of sidelobe level through feed current amplitude control. The theory of Dolph-Chebyshev arrays is thoroughly documented in the literature and need not be discussed here, but these arrays exploit the well-known technique of reducing sidelobe levels in arrays by tapering the current amplitude distribution along the array [12]. The feed current amplitude is maximum for the central element or elements and tapers off for elements toward the ends of the array.

A five-element Dolph-Chebyshev array of half-wave dipoles with 0.5-wavelength spacing between elements will produce a radiation pattern with -20 dB sidelobes if fed with currents [12]

\[
I_1 = I_5 = 0.51801 \angle 0^\circ \\
I_2 = I_4 = 0.83272 \angle 0^\circ \\
I_3 = 1.0000 \angle 0^\circ 
\]  \hspace{1cm} (2.4 - 10)

As with the scanning array, the elements in the Dolph-Chebyshev array are all \(x\) - directed and lie along the \(z\) - axis; the geometry of the Dolph-Chebyshev array is identical to that of the scanning linear array studied earlier. The element pattern for the array is therefore given by (2.4-4). With the current phases \(\{\alpha_i\}\) all set to zero to produce a broadside pattern, the radiation pattern of the Dolph-Chebyshev array is given by

\[
F_{\text{un}}(\theta, \phi) = \frac{\cos\left(\frac{\pi}{2} \sin \theta \cos \phi\right)}{\sqrt{1 - \sin^2 \theta \cos^2 \phi}} \sum_{n=1}^{5} A_n e^{j\pi(n-3) \cos \theta} 
\]  \hspace{1cm} (2.4 - 11)

where the amplitudes \(\{A_n\}\) are given in (2.4-10). The H-plane pattern of the array is plotted in Figure 2.4-5. Note that all of the sidelobes in the pattern are down 20 dB from the main lobe. This
Figure 2.4-5. H-plane pattern of five-element Dolph-Chebyshev array of 0.5-wavelength center-fed dipoles designed for -20 dB sidelobe level. The element spacing is 0.5 wavelength.
illustrates the major advantage of Dolph-Chebyshev arrays; the sidelobes can all be set to the same level merely by adjusting the element feed current amplitude taper.

As in the case of the scanning array, the radiation pattern shown in Figure 2.4-5 is an ideal pattern and does not include effects caused by mutual coupling. As will be shown in Chapter 5, the most deleterious effect of mutual coupling on the performance of Dolph-Chebyshev arrays is the degradation of the sidelobe level. The beamwidth of the main lobe may also be significantly affected by mutual coupling.

2.4.3 Difference Pattern Array

Difference pattern arrays are designed to produce a radiation pattern null in a desired direction. One major application of difference pattern arrays is in radio direction finding; signal directions are much more easily found by using a steep null in a pattern rather than the broad peak of a main lobe. A difference pattern may be produced broadside to a linear array by feeding half of the elements in the array 180° out of phase with the other half of the elements in the array.

The geometry of a four-element difference pattern array of half-wavelength dipoles is shown in Figure 2.4-6. The elements are spaced 0.4 wavelength apart and are fed with currents

\[
I_1 = I_2 = 1.0 \angle 0^\circ \\
I_3 = I_4 = 1.0 \angle 180^\circ
\]  

(2.4 - 12)

The element pattern is given by (2.4-4) since the array consists of half-wave dipoles oriented in the x-direction. The radiation pattern of the array is therefore given by
Figure 2.4-6. Geometry of four-element difference pattern array of center-fed dipoles.
\[ F_{\text{un}}(\theta, \phi) = \frac{\cos\left(\frac{\pi}{2} \sin \theta \cos \phi\right)}{\sqrt{1 - \sin^2 \theta \cos^2 \phi}} \sum_{n=1}^{4} A_n e^{j\pi(0.8n - 2) \cos \theta} \] (2.4 - 13)

where

\[ A_n = \begin{cases} +1, & n = 1, 2 \\ -1, & n = 3, 4 \end{cases} \] (2.4 - 14)

The radiation pattern produced by the four-element difference array is plotted in Figure 2.4-7. Under ideal conditions the array radiates (and receives) no power in the broadside direction because of the pattern null. However, mutual coupling could affect the radiation pattern of the actual array by causing the null to "fill in," thus degrading the performance of the array. Null-filling is especially serious in adaptive arrays which employ null positioning to reduce interference caused by signals coming from undesired directions [13,14].

2.4.4 Summary

The preceding examples illustrate the use of ideal array analysis to predict the radiation patterns produced by arrays of dipole elements. The patterns computed using ideal array analysis are those that would be produced by arrays operating in ideal environments where mutual coupling is not present, a situation that does not occur in practice. Analysis methods exist, however, that account for mutual coupling. These methods will be examined in Chapter 4. The next section discusses some of the effects mutual coupling may have on the radiation patterns of phased arrays and on the input impedances of the array elements.
Figure 2.4-7. H-plane pattern of four-element difference pattern array of 0.5-wavelength center-fed dipoles designed for broadside null. The element spacing is 0.4 wavelength.
2.5 Effects of Mutual Coupling on Array Performance

The ideal method of array analysis discussed so far does not exactly predict the performance of actual arrays encountered in practice. This is because array elements operated in the array environment behave differently than elements operated alone in free space. In an array environment, the fields radiated by a given element produce changes in the current distributions of the other elements (or in the aperture field distributions in the case of aperture antenna elements such as horns). Likewise, the fields radiated and scattered by the other elements affect the current distribution (or aperture field) of the original element. Additionally, reflections may occur in the feed network of the array due to impedance mismatches between the array elements and their corresponding feedlines. This interaction between elements, either through radiation effects or through reflections in the feed network, is called mutual coupling and can have a profound effect on array performance since the pattern radiated by an array depends entirely upon the current distributions on the array elements. The two main mechanisms through which mutual coupling occurs in arrays, radiated coupling and feed network coupling, are illustrated in Figure 2.5-1.

The amount of radiated coupling between array elements depends primarily on the spatial orientation of the elements with respect to one another and the scan angle of the array main beam (or null in the case of difference arrays or null-positioning arrays). Elements that are located in the direction of each other's near field pattern maxima will couple much more strongly than elements located in the direction of near field minima. Thus, parallel elements will show much more coupling than collinear elements. Elements in a staggered (echelon) arrangement will experience an amount of coupling between that of parallel elements and collinear elements. Element spacing affects the strength of mutual coupling between elements. Elements that are widely spaced generally couple less strongly than elements that are closely spaced; however, the strength of coupling does not decrease monotonically with increased element spacing [4]. The direction of scan affects the amount of coupling experienced by array elements since elements that lie in the general direction of a main
Figure 2.5-1. The two mechanisms of mutual coupling in arrays: coupling caused by radiation effects and coupling caused by reflections in the feed network.
beam will be illuminated more strongly, and thus will experience greater coupling, than elements that lie away from the direction of the main beam. For example, the elements in a linear array that is phased for endfire scan will experience greater coupling than the elements in a similar array phased for broadside scan.

Mutual coupling is also greatly affected by the polarization of the array elements. Though most of the arrays considered in this study consist of parallel linearly-polarized elements, which are “co-polarized”, many array designs contain “cross-polarized” elements. For example, arrays can be designed with dipole elements that are arranged perpendicularly with respect to one another [15]. The relative polarization between linearly-polarized elements affects the degree to which the elements couple; co-polarized elements couple strongly whereas cross-polarized elements couple very little [16]. It is important to note that while relative polarization affects the mutual coupling between elements, the converse is also true; mutual coupling can cause the relative polarization of the array elements to change. Though changes in element polarization are usually very small in arrays of wire antennas in which all of the elements are oriented in the same direction, polarization can change significantly in arrays of aperture antennas where mutual coupling distorts the aperture fields of the elements [17,18].

Finally, mutual coupling can occur through the feed network of the array. Part of the coupled energy received by an array element can travel through the feed network and appear at the terminals of other elements, thus changing the excitation of the other elements. Though coupling through the feed network can have a significant effect on the performance of an array, only radiated field coupling is considered in this study. The treatment of feed network coupling can be found elsewhere in the literature [1,11,19].

It has already been pointed out that mutual coupling manifests itself in a phased array through altered distributions of currents (or aperture fields) on the array elements. Both the amplitude and phase distribution of the element current may be affected by mutual coupling. Experience has shown that, for arrays of parallel, center-fed, half-wavelength long dipoles, mutual coupling affects
the amplitude distribution of the current along the element very little but can produce a large shift in the current phase distribution compared to that obtained for an isolated dipole [12]. However, for arrays of collinear dipoles or dipoles arranged in echelon, mutual coupling can cause significant asymmetry in the amplitude and phase distributions of the current along the elements, as well as a large phase distribution shift, compared to the current distribution on an isolated element [4].

According to (2.1-12), which is an exact expression, the far-field pattern radiated by an array depends upon the current distribution along each element; therefore, changes to the element current distributions caused by mutual coupling can cause the resulting performance of the array to deviate significantly from the desired performance. For example, mutual coupling can cause degradation of the array pattern including increased sidelobe level, filling of pattern nulls, broadening of the main beam and a shift of the main beam pointing angle from the desired direction. Mutual coupling can also cause a reduction in the gain of the array.

The changes in the amplitude and phase distributions of the element currents caused by mutual coupling also cause the input impedances of the elements to change from that of an isolated element. Coupled elements are therefore very difficult to match to the feed network, meaning that energy will be reflected back from the element input terminals, thus compounding the problem of feed network coupling. The input impedance will change with scan angle, since the amount of coupling experienced by an element depends on the strength of the radiated field in which it is immersed, so that even if changes in input impedance are compensated for in one scan direction, the compensation may not work in another scan direction.

The dependence of the element input currents on mutual coupling can be expressed in the form of network equations. The input currents must satisfy the equation [2]

$$
\begin{bmatrix}
V_1 \\
\vdots \\
V_N
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & \cdots & Z_{1N} \\
\vdots & \ddots & \vdots \\
Z_{N1} & \cdots & Z_{NN}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
\vdots \\
I_N
\end{bmatrix}
$$

(2.5 - 1)
where $V_m$ is the voltage applied to the $m$th element and $Z_{mn}$, called the *mutual impedance* between the $m$th and $n$th elements, is defined by

$$Z_{mn} = \frac{V_m}{I_n} \quad (2.5 - 2)$$

where $I_n$ is measured at the terminals of the $n$th element when the $m$th element is excited and all other elements are open-circuited. If $m = n$, then $Z_{nn}$ is called the *self-impedance* of the $m$th element. The self-impedance of an array element differs slightly from the input impedance of an isolated element, since the self-impedance includes the slight mutual coupling effects of the other "split" (open-circuited) elements in the array. The mutual impedances defined by (2.5-2) contain all of the effects of radiated mutual coupling, and therefore depend upon the array geometry and scan angle.

For (2.5-1) to be useful, the mutual impedances must somehow be computed. One of the first methods devised to compute the $(Z_{mn})$, called the *induced-EMF method*, is described by Carter [2]. Other methods were later developed to compute the $(Z_{mn})$, including numerical methods based on the method of moments [20].

The input impedance of a given element in the fully-excited array, called the *active impedance* of the element, can be computed once the mutual impedances and input currents are known. The expression for the active impedance $Z_{am}$ of the $m$th element is found by dividing the appropriate equation in (2.5-1) by $I_m$ to give [21]

$$Z_{am} = \frac{V_m}{I_m} = Z_{mm} + \sum_{n=1}^{N} \frac{I_n}{I_m} Z_{mn} \quad (2.5 - 3)$$

Note that the $n = m$ term is excluded from the summation. This equation explicitly shows the dependence of the active impedance of a given array element on mutual coupling effects. Thus, the
active impedances depend on the scan angle and the relative orientation of the array elements with respect to one another.

Many methods of analysis have been developed in an effort to predict the effects of mutual coupling on array performance. One of the early efforts, the induced-EMF method, has already been mentioned. More recently, numerical techniques based on the method of moments have been developed to solve array analysis problems. One such tool, the Electromagnetic Surface Patch (ESP) code, will be described in Chapter 3. ESP was used to analyze all of the arrays discussed in this study.
3. Numerical Techniques for Accurate Analysis of Wire Antennas

The ideal method of array analysis, described in the previous chapter, does not adequately account for the mutual coupling effects in many practical arrays. Mutual coupling effects are usually important, as was discussed in Section 2.5. Other techniques must be employed to determine the effect mutual coupling has on element current distribution, radiation pattern and feed point impedance. One numerical analysis technique which accurately accounts for mutual coupling effects is the solution of an integral equation for the element currents by the method of moments. Many computer codes exist which employ moment methods to analyze wire antennas. The code used in this study is the Electromagnetic Surface Patch (ESP) code, which was written at the Ohio State University Electroscience Laboratory.
3.1 The ESP Moment Method Wire Antenna Code

The ESP code computes radiation patterns and scattering patterns for arbitrary combinations of wires and plates. The ESP code was used in this study because of its ability to compute the radiation patterns produced by wire antennas of arbitrary geometry, fed by any number of discrete voltage sources. ESP solves the reaction integral equation, developed by Rumsey [22] for electromagnetic scattering problems, for the current distribution along the antenna wires. The reaction integral equation, in the general form used by ESP, is given by [23]

\[- \int \int_{S} (J_{r} \cdot E_{r} - M_{r} \cdot H_{r}) \, ds = \int \int \int_{V} (J_{r} \cdot E_{r} - M_{r} \cdot H_{r}) \, dv\]  \hspace{1cm} (3.1 - 1)

where $J_{r}$ and $M_{r}$ are sources located in a volume $V$, outside a scatterer (the antenna). The sources $J_{r}$ and $M_{r}$ give rise to the incident field $(E_{r}, H_{r})$. The quantities $E_{r}$ and $H_{r}$ are fields due to the hypothetical test sources $J_{r}$ and $M_{r}$, which are located within the surface $S$ surrounding the scatterer. The quantities $J_{r}$ and $M_{r}$ are the unknown equivalent scatterer surface currents found using the field equivalence theory of Schelkunoff [24]. If the antenna is assumed to be a perfect electric conductor, then $M_{r} = 0$.

The ESP code uses the method of moments to solve (3.1-1) for the unknown current distribution $J_{r}(\mathbf{r}')$, where $\mathbf{r}'$ is the coordinate along the wire. The method of moments solves linear operator equations of the form

\[L(f) = g\]  \hspace{1cm} (3.1 - 2)

where $L$ is a known linear operator, $f$ is the unknown function to be solved for, and $g$ is a known excitation function [20]. In the application of the moment method to solve (3.1-1), the current distribution is approximated by a series of $N$ overlapping expansion modes [25] (also called expansion functions or basis functions) such that

3. Numerical Techniques for Accurate Analysis of Wire Antennas
\[ J_s(r') = \sum_{n=1}^{N} I_n f_n(r') \quad (3.1-3) \]

where the \( \{ I_n \} \) are complex coefficients for the expansion modes \( \{ f_n(r') \} \). (Note that the \( \{ I_n \} \) of (3.1.3) are different from the \( \{ I_n \} \) of Chapter 2, which represent feed currents to array elements.)

Each expansion mode is defined over two straight wire segments which are joined at the point \( r = r_n \) and whose outer ends are located at points \( r = r_{n-1} \) and \( r = r_{n+1} \), as shown in Figure 3.1.1.

Note that these three points need not be collinear. The \( n \)-th piecewise sinusoidal expansion mode, which exists over the two segments joined at the point \( r_n \), is defined by the equation

\[ f_n(r') = \frac{1}{2\pi a} \left[ \hat{r}_{n-1} \frac{\sin \beta |r' - r_{n-1}|}{\sin \beta |r_n - r_{n-1}|} + \hat{r}_n \frac{\sin \beta |r_{n+1} - r'|}{\sin \beta |r_n + 1 - r_n|} \right] \quad (3.1-4) \]

where \( a \) is the wire radius, \( \beta \) is the free space wavenumber, and

\[
\begin{align*}
P_{n-1} &= \begin{cases} 
1, & r_{n-1} < r' < r_n \\
0, & \text{elsewhere}
\end{cases} \\
P_n &= \begin{cases} 
1, & r_n < r' < r_{n+1} \\
0, & \text{elsewhere}
\end{cases}
\end{align*} \quad (3.1-5)\]

Unit vectors \( \hat{r}_{n-1} \) and \( \hat{r}_n \) point in the direction of each wire segment. A graphical representation of a typical expansion mode is shown in Figure 3.1.2. Note that the expansion mode may be asymmetrical; that is, \( |r_{n+1} - r_n| \) does not necessarily equal \( |r_n - r_{n-1}| \).

Using (3.1.3) and recalling that \( M_s = 0 \), (3.1.1) may be rewritten as

\[ - \sum_{n=1}^{N} I_n \int_{r_n}^{r_{n+1}} E_t \cdot f_n \, ds = \int_{V} \int_{V'} (J_t \cdot E_t - M_t \cdot H_t) \, dv \quad (3.1-6) \]

3. Numerical Techniques for Accurate Analysis of Wire Antennas
Figure 3.1-1. Wire segmentation used in ESP.
Figure 3.1-2. Typical sinusoidal expansion mode used by ESP [25].
where the \( n \) under the double integral sign indicates that the surface integral is evaluated over the \( n \)th expansion mode.

ESP uses \( N \) electric test sources \( \{J_m\} \) to form \( N \) different versions of (3.1-6). The \( \{J_m\} \) are of the same functional form as the expansion functions defined by (3.1-4), but are located a small distance \( \delta \) within the surface \( S \). The limiting form of (3.1-6) is taken as \( \delta \) approaches zero. No magnetic test sources are considered to exist within the surface \( S \); therefore, \( M_m = 0 \) for all \( m \). Because the testing functions \( \{J_n\} \) are identical to the expansion functions \( \{f_n\} \), the moment method formulation used in ESP is a Galerkin’s method.

The evaluation of (3.1-6) for each of the \( N \) testing functions forms the \( N \times N \) system of linear equations

\[
\sum_{n=1}^{N} I_n Z_{mn} = V_m \quad (3.1 - 7)
\]

where

\[
Z_{mn} = - \int \int \mathbf{E}_m \cdot \mathbf{f}_n \, ds \quad (3.1 - 8)
\]

\[
V_m = \int \int \int_{S_r} (J_i \cdot \mathbf{E}_m - M_i \cdot \mathbf{H}_m) \, dv \quad (3.1 - 9)
\]

The quantities \( \mathbf{E}_m \) and \( \mathbf{H}_m \) represent the fields due to the \( m \)th electric test source. Note that, though the same symbology is used, there is a profound difference between the \( \{Z_{mn}\} \) of (3.1-8) and the \( \{Z_{mn}\} \) of (2.5-1) in Section 2.5. The \( \{Z_{mn}\} \) of (3.1-8) represent mutual impedances between segments on either the same wire or on two different wires, whereas the \( \{Z_{mn}\} \) of (2.5-1) represent mutual impedances between two whole dipoles, referenced to their feed points. The system of equations in (3.1-7) may be represented in matrix form as

3. Numerical Techniques for Accurate Analysis of Wire Antennas
\[ [Z_{mn}] \{I_n\} = \{V_m\} \quad (3.1-10) \]

where brackets indicate a matrix quantity and braces indicate a vector. Because of the similarity in form between (3.1-10) and Kirchhoff’s network equations, the matrix \([Z_{mn}]\) and the vector \([V_m]\) are often called the generalized impedance matrix and generalized voltage vector, respectively. Note that these names are used because of the analogy to the network equations and that in general the entries of \([Z_{mn}]\) and \([V_m]\) do not represent actual impedances and voltages in the traditional circuit analysis sense.

The excitation vector \([V_n]\) is constructed by ESP for antenna problems using the “delta gap” generator model. Each entry in the excitation vector corresponds to a wire segment. In the case of a transmitting array, the excitation vector represents the incidental electric field that results when the feed voltages are applied to the discrete feed points. The entries in the excitation vector that correspond to segments containing feed points are filled with the non-zero values of \(V_n\) evaluated over the extent of the infinitely thin delta gaps. For segments containing no feed points, the corresponding entries in the excitation vector are filled with zeroes, since no incident field is assumed to exist over these other segments, and consequently \(V_n = 0\).

Since the matrix \([Z_{mn}]\) and the vector \([V_m]\) are known, (3.1-10) can be solved for the expansion mode coefficients \([I_n]\). The approximation to the current distribution on the antenna is then obtained using (3.1-3). The current distribution computed by ESP includes the effects of mutual coupling due to radiation (but not feed network coupling) between antenna elements and between sections of individual elements; therefore, the radiation pattern computed by ESP from the current distribution also includes all mutual coupling effects due to radiation. Consequently, the radiation pattern computed by ESP for an array should predict very closely the pattern radiated by the actual operating array.
3.2 Verification of the Accuracy of the Moment Method

Solution

ESP is a fully-tested, standard code. The piecewise sinusoidal Galerkin's method employed in ESP has proven to be an accurate and numerically efficient means of analyzing wire antennas in studies performed by other investigators [26]. Therefore, it is assumed in this study that the ESP program produces reliable, accurate pattern data for the dipole arrays that are investigated. Two examples will be presented in this section to further demonstrate the ability of the ESP code to analyze wire antenna systems accurately. The first example tests the ability of ESP to compute accurate values for the input impedance and directivity of a half-wave dipole antenna. The second example tests the ability of ESP to compute an accurate solution for the current distribution on a full-wave dipole. The computed distribution is compared to the current distribution measured along an actual full-wave dipole.

3.2.1 Input Impedance Convergence Test

The computed expansion mode coefficients \( \{I_n\} \) are included in the output data generated by ESP. The input impedance of a dipole antenna is easily calculated by dividing the value of the feedpoint excitation voltage by the coefficient of the center expansion mode, since this number represents the element's input current. A 0.476-wavelength long center-fed dipole was analyzed by ESP with varying numbers of equal-length expansion modes used to approximate the current distribution. The length of 0.476 wavelength was chosen because the antenna was found to resonate at that length. The radius of the wire from which the dipole is made is 0.001 wavelength.
The accuracy of the solution computed by ESP depends primarily upon the number of expansion modes used to approximate the current distribution along the dipole. Theoretically, the larger the number of expansion modes used, the more accurately the computed current distribution represents the true distribution on the antenna. It will be shown, however, that the accuracy of the solution computed by ESP deteriorates if too many expansion modes are used to approximate the current distribution. The variation of the computed input impedance and directivity versus the number of expansion modes used is shown in Table 3.2-1.

The impedance and directivity data converge to the values of 72 ohms and 2.14 dBi, respectively, as the number of expansion modes used to approximate the current distribution reaches approximately 20. These values differ slightly from the theoretical values of 70 ohms and 2.15 dBi [12], because the actual current distribution on the antenna is only approximately sinusoidal, not exactly sinusoidal. The computed input impedance and directivity remain at their converged values as the number of expansion modes increases to approximately 40. When the number of modes exceeds 40, the impedance and directivity data begin to diverge again. The reason for this divergence is related to the thin-wire approximation [27]; when more than 40 expansion modes are used to approximate the current, the length of the wire segments will approach the order of the radius of the wire, thus violating the assumption that the segment length is much greater than the wire radius.

3.2.2 Current Distribution Convergence Test

The previous example tested the ability of ESP to compute accurate solutions for the input impedance and directivity of a half-wave dipole. In some respects this was an easy test, since the current distribution on a half-wave dipole is fairly well-behaved. This section presents ESP with the more difficult task of computing a solution for the current distribution on a full-wave dipole. The current distribution on a half-wave dipole is closely approximated by the ideal sinusoidal distrib-
Table 3.2-1. Input impedance and directivity of 0.476-wavelength center-fed dipole vs. number of expansion modes used to approximate the current distribution along the dipole. The wire radius is 0.001 wavelength.

<table>
<thead>
<tr>
<th>NUMBER OF SEGMENTS</th>
<th>NUMBER OF WIRE MODES</th>
<th>INPUT IMPEDANCE (OHMS)</th>
<th>GAIN (dBi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>63.4 - j3.19</td>
<td>2.11</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>69.2 - j3.63</td>
<td>2.12</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>70.3 - j2.45</td>
<td>2.13</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>70.8 - j2.47</td>
<td>2.13</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>71.1 - j2.48</td>
<td>2.13</td>
</tr>
<tr>
<td>20</td>
<td>19</td>
<td>71.8 + j0.00</td>
<td>2.14</td>
</tr>
<tr>
<td>30</td>
<td>29</td>
<td>72.3 + j0.00</td>
<td>2.12</td>
</tr>
<tr>
<td>40</td>
<td>39</td>
<td>72.2 + j1.26</td>
<td>2.14</td>
</tr>
<tr>
<td>50</td>
<td>49</td>
<td>73.1 + j0.00</td>
<td>2.09</td>
</tr>
<tr>
<td>60</td>
<td>59</td>
<td>73.7 + j1.29</td>
<td>2.06</td>
</tr>
</tbody>
</table>
ution of (2.3-1). However, as the length of the dipole increases beyond 0.5 wavelength, the actual current distribution deviates significantly from the ideal sinusoidal approximation. When the length of the dipole reaches a full wavelength, the current distribution on the antenna looks nothing like the ideal sinusoidal distribution shown in Figure 2.3-1.

A center-fed, 1.0-wavelength long dipole antenna was analyzed by ESP using several different numbers of expansion modes. The radius of the antenna wire is 0.007022 wavelength. The approximation to the current distribution on the antenna was constructed by substituting the values of the expansion mode coefficients ($L_j$), included in the output data produced by ESP, into (3.1-3). ESP converged quickly to a solution as the number of expansion modes was increased. The amplitude and phase distributions of the current computed by ESP using 19 expansion modes (20 segments) are shown in Figure 3.2-1a. For comparison, measurements made by Mack [4] of the current distribution on a full-wave dipole of the same wire radius are shown in Figure 3.2-1b. Inspection of Figure 3.2-1 shows that the approximation to the current distribution computed by ESP generally agrees with the distribution measured by Mack. There is a slight discrepancy between the two distributions in the height of the “spike” located at the center of the amplitude distribution. The presence of this spike, which is typical of dipoles longer than 0.5 wavelength, explains why it is very difficult to compute the input impedance of “long” dipoles accurately; the input impedance is very sensitive to the length of the antenna and the manner in which the current distribution is modeled. These results also show that the sinusoidal approximation shown in Figure 2.3-1 for a full-wave dipole gives a very poor approximation to the actual current distribution, since the sinusoidal approximation for a full-wave dipole has a current null located at the feed point and assumes that the phase distribution of the current along the dipole is constant. Based on the results of this section and those of Section 3.2.1, in addition to results obtained by others [28], 15 expansion modes per half wavelength were used to approximate the element currents of the arrays analyzed in this study.

The deviation of the actual current distribution from the ideal sinusoidal approximation causes the radiation pattern computed by ESP to differ from the radiation pattern computed using the
Figure 3.2-1. Current distributions for a 1.0-wavelength center-fed dipole (a) computed by ESP using 19 equal-length expansion modes, and (b) measured by Mack [4]. The wire radius is 0.007022 wavelength.
sinusoidal current approximation. The E-plane radiation patterns computed for the full-wave dipole using both ESP and the sinusoidal approximation are shown in Figure 3.2-2. The solid curve represents the pattern computed using the sinusoidal approximation of (2.3-1) and the dashed curve represents the pattern computed by ESP. The difference between the two patterns is small but noticeable, especially near the direction off the ends of the dipole, which is oriented along the $x$ – axis.

3.3 Examples of Radiation Patterns Computed by ESP

As mentioned in Section 3.1, radiation patterns computed by ESP include the effects of radiated mutual coupling and therefore represent very closely the patterns that would be produced by actual operating arrays. The patterns computed by ESP for phased arrays show the null-filling, changes in sidelobe level and other forms of pattern degradation caused by mutual coupling in the array environment. To demonstrate this pattern degradation, ESP was used to compute the radiation patterns of the three arrays analyzed in Sections 2.4.1 through 2.4.3.

The H-plane patterns computed by ESP for the five-element scanning array of 0.476-wavelength dipoles phased for 0-degree (endfire) and 90-degree (broadside) scan, which was discussed in Section 2.4.1, are shown in Figures 3.3-1a and b, respectively. The patterns computed using ideal array analysis, which were presented earlier in Figures 2.4-3b and 2.4-4b, are included in Figure 3.3-1 for comparison. The solid curves represent the ideal patterns and the dashed curves represent the patterns computed by ESP. The pattern degradation caused by mutual coupling is obvious but not severe. Interestingly, the plot shows that the level of the sidelobes in the direction of the $z$ – axis for the broadside array is actually less than the level predicted by ideal array analysis. Mutual coupling sometimes produces beneficial effects such as this, but not without some cost, as inspection
Figure 3.2-2. E-plane pattern of 1.0-wavelength center-fed dipole antenna computed using the sinusoidal current approximation of (2.3-1) (solid curve), and computed using ESP (dashed curve). The wire radius is 0.007022 wavelength.
Figure 3.3-1. H-plane patterns of (a) endfire array of Figure 2.4-3b and (b) broadside array of Figure 2.4-4b computed using ideal array analysis, which does not include mutual coupling effects (solid curves), and computed using ESP, which does include mutual coupling effects (dashed curves). The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.
of Figure 3.3-1b shows that the pattern suffers from significant null-filling and a slightly broadened main beam.

Figure 3.3-2 compares the H-plane pattern computed by ESP for the five-element Dolph-Chebyshev array with half-wavelength spacing described in Section 2.4.2 with the ideal pattern presented earlier in Figure 2.4-5 for the same array. Mutual coupling causes the level of the first sidelobe to increase slightly above the design value of -20 dB; however, the level of the second sidelobe decreases in the presence of mutual coupling. This effect, which was noted in the preceding discussion of the five-element scanning array, degrades the performance of the array by causing the main beam to broaden slightly. Mutual coupling can cause even more severe degradation in the performance of Dolph-Chebyshev arrays designed for sidelobe levels less than -20 dB.

The four-element difference pattern array analyzed in Section 2.4.3 using ideal array analysis was analyzed again using ESP. The H-plane pattern of the array computed using ideal analysis was shown earlier in Figure 2.4-7. The pattern is plotted again as a solid curve in Figure 3.3-3, and the pattern computed by ESP is plotted as a dashed curve in the same figure. Since the purpose of a difference pattern array is to produce a deep, sharp null in a desired direction, the most serious deleterious effect of mutual coupling on difference pattern arrays is null-filling, but the difference pattern array considered here is still able to retain a sharp null in the broadside direction in spite of mutual coupling effects due to the antisymmetry of the array excitation. However, if the pattern null is steered 30 degrees off broadside by feeding the array with the following voltages:

\[
I_1 = 1.0 \angle 0^0 \\
I_2 = 1.0 \angle -72^0 \\
I_3 = 1.0 \angle 36^0 \\
I_4 = 1.0 \angle -36^0
\]  

(3.3 - 1)

then mutual coupling will cause serious null-filling to occur. The radiation pattern produced by the array phased for a null 30 degrees off broadside is shown in Figure 3.3-4. The null in the pattern

3. Numerical Techniques for Accurate Analysis of Wire Antennas
Figure 3.3-2. H-plane pattern of five-element Dolph-Chebyshev array of 0.476-wavelength dipoles of Figure 2.4-5 computed using ideal array analysis (solid curve), and computed using ESP (dashed curve). The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.
Figure 3.3-3. H-plane pattern of four-element difference pattern array of 0.476-wavelength dipoles phased for broadside null computed using ideal array analysis, shown previously in Figure 2.4-7 (solid curve), and computed using ESP (dashed curve). The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.

<table>
<thead>
<tr>
<th>El.</th>
<th>Mag</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>180</td>
</tr>
</tbody>
</table>

\[ Zg = 50 \text{ ohms} \]
Figure 3.3-4. H-plane pattern of four-element difference pattern array of 0.476-wavelength dipoles with null steered to $\theta = 60$ degrees computed using ideal array analysis (solid curve), and computed using ESP (dashed curve). The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
computed by ESP is less than 20 dB deep, meaning that mutual coupling effects cause the performance of the array to degrade considerably.
4. Methods for Including Mutual Coupling in Phased Array Pattern Analysis

The simplest method for analyzing array radiation patterns is the ideal array analysis technique described in Chapter 2. In ideal array analysis, the radiation pattern produced by an array can be expressed using the principle of pattern multiplication, introduced in (2.2-8) as

\[ F_{\text{un}}(\theta, \phi) = g(\theta, \phi) f(\theta, \phi) \]  \hspace{1cm} (4 - 1)

where \( g(\theta, \phi) \) is the ideal element pattern, assumed to be identical for each element in the array, and \( f(\theta, \phi) \) is the array factor, introduced in (2.2-9) as

\[ f(\theta, \phi) = \sum_{n=1}^{N} A_n e^{j\alpha_n} e^{j\beta \hat{r} \cdot r_n} \]  \hspace{1cm} (4 - 2)

The array factor is determined completely by the ideal amplitudes \( \{A_n\} \) and phases \( \{\alpha_n\} \) of the feed excitations and the locations \( \{r_n\} \) of the element centers. Though it provides a fast and simple method for predicting the radiation patterns of arrays, ideal array analysis ignores the effects of mutual coupling on the element input impedances and the individual element patterns, and therefore offers no way to account for the degradation of the array pattern caused by mutual coupling.
The purpose of this chapter is to derive and differentiate between methods of array analysis which account for mutual coupling effects. The two basic types of array pattern analysis methods described here are the \textit{classical array analysis method} and the \textit{active element pattern methods}. The classical array analysis method assumes that the shapes of the current distributions on the array elements do not change in the presence of mutual coupling and that all radiated mutual coupling effects are contained within the set of element feed currents. Active element pattern methods assume that all radiated mutual coupling effects are contained within the set of individual active element patterns (to be defined later). In the active element pattern methods the true element current distributions are accounted for implicitly. Before discussing these array pattern analysis methods, however, some comments must be made regarding the feeding of array elements.

\section{4.1 Element Excitation Models}

An important consideration in the analysis of phased arrays is how to model the excitation of array elements. In practice, power is usually distributed to the elements in a phased array through a feed network. The feed network consists of one input port (for a transmitting array) to which the transmitter is connected and \( N \) output ports, one for each of the \( N \) elements in the array. The feed network performs the necessary power-splitting and phasing to properly excite each array element. Most array analysis techniques, however, cannot represent the actual network used to feed the elements in an array. Instead, the elements are assumed to be fed by a set of \( N \) independent generators, one generator for each element.

The two approaches that are generally used to model the excitation of the elements in an array are called the \textit{forced excitation model} and the \textit{free excitation model} \cite{5}. In the forced excitation approach, shown in Figure 4.1-1a, each element is fed directly by an ideal constant-voltage source.
Figure 4.1-1. Forced excitation array feed model (a) and its equivalent circuit (b) [5]. The voltage sources are ideal independent generators and supply constant voltage to their respective array elements.

\[ I = I_n \quad \text{if} \quad Z_{an} = Z_a \quad \text{(constant)} \]
\[ I = I'_n \quad \text{otherwise} \]
which maintains a specified voltage across the feed terminals of the element regardless of mutual coupling effects. The equivalent circuit representation of forced excitation is shown in Figure 4.1-1b. The generator voltage \( V_g \) is impressed directly across the element input impedance \( Z_{en} \); therefore, the forced excitation model is sometimes called the constant-voltage approach [5].

The forced excitation approach is unrealistic since it does not take into account the output impedances of the feed network branches which supply power to the array elements. A somewhat more realistic representation of the feed network is provided by the free excitation approach shown in Figure 4.1-2a. In free excitation each element is fed by a constant-voltage source through a generator impedance \( Z_g \). The value of \( Z_g \) is usually assumed to be the same for each generator and represents the impedance seen by the element looking back into its corresponding port in the feed network, with all of the other ports loaded with the active impedance of their respective elements. Typically, \( Z_g \) is assumed to have a value of 50 ohms. (Note that feed network coupling is not considered here.) In the case of an active array, where each element is fed by its own generator, \( Z_g \) represents the output impedance of each generator. The equivalent circuit representation of free excitation is shown in Figure 4.1-2b. The generator impedance \( Z_g \) and the element input impedance \( Z_{en} \) form a voltage divider, so that changes in \( Z_{en} \) caused by mutual coupling will cause the voltage \( V_{en} \) across the element terminals to change accordingly. Free excitation represents the case when constant incident power is supplied to the element and therefore is sometimes called the constant-incident power approach [5]. Note that the free excitation model reduces to the forced excitation model when \( Z_g = 0 \).

In both the forced excitation approach and the free excitation approach, each generator voltage is set to a value that is proportional to the input current desired for that element. For example, if a broadside pattern were desired from a uniformly-excited array, each generator voltage would be set to \( V_g = 1.0 \angle 0^\circ \) volt. If the array environment were free of mutual coupling effects and the elements had identical input impedances regardless of the array excitation, each element feed current would then be directly proportional to its respective excitation voltage. In other words, the element feed currents would all follow the relationship

4. Methods for Including Mutual Coupling in Phased Array Pattern Analysis
Figure 4.1-2. Free excitation array feed model (a) and its equivalent circuit (b) [5]. The voltages \( \{ V_{an} \} \) that appear across the element terminals depend on the generator impedance \( Z_g \) and the active element impedances \( \{ Z_{an} \} \).

\[
\begin{align*}
I &= I_n \text{ if } Z_{an} = Z_a \text{ (constant)} \\
I &= I_n' \text{ otherwise}
\end{align*}
\]
\[ I_n = \frac{V_n}{Z_a}, \quad \text{forced excitation} \]
\[ I_n = \frac{V_n}{Z_a + Z_g}, \quad \text{free excitation} \]  \hspace{1cm} (4.1 - 1)

where \( Z_e \) is the element input impedance, assumed in this case to be identical for every element in the array. In (4.1-1), the \( \{I_n\} \) are often called \textit{ideal feed currents} (since they are the feed currents that would result if no coupling were present) and are all proportional to the \( \{V_n\} \). However, because mutual coupling causes the input impedance of one element to change differently than the input impedance of another element, as explained in Section 2.5, the element feed currents will not each follow the same relationship to their respective excitation voltages. Instead, the feed currents will follow the relationships

\[ I_n' = \frac{V_n}{Z_{an}}, \quad \text{forced excitation} \]
\[ I_n' = \frac{V_n}{Z_{ae} + Z_g}, \quad \text{free excitation} \]  \hspace{1cm} (4.1 - 2)

where the \( \{Z_{ae}\} \) are the individual active element input impedances, which in general are different for each element in the array. The \( \{I_n'\} \) are the feed currents that result when mutual coupling is present in the array. In general, the \( \{I_n'\} \) are not proportional to the \( \{V_n\} \).

The preceding discussion illustrates how mutual coupling can affect the radiation pattern of a phased array by altering the element feed currents. Even though the generator voltages may be set to the proper values required to produce a desired pattern, mutual coupling will cause the amplitude and phase relationships of the element feed currents to deviate from the amplitude and phase relationships of the excitation voltages, thereby causing the array excitation to deviate from the desired excitation. As explained in Section 2.5, a change in the array excitation will result in degradation of the radiation pattern.
4.2 The Classical Array Analysis Method

One of the major effects of mutual coupling on arrays of center-fed dipoles is to alter the current distribution on each element from the distribution found on an isolated dipole. Both the amplitude distribution and the phase distribution of the current along each element can be altered due to mutual coupling. For an array of dipole elements 0.5 wavelength long or shorter, however, the current distribution along each element in the array differs little from the nearly sinusoidal distribution found on an isolated element. Also, as mentioned in Section 2.5, though the phase distribution may be shifted from that of an isolated element, the shape of the phase distribution on an electrically short dipole element changes very little. Therefore, the current distribution on each element in the array can be expressed as

\[ J_n(t) = I_n \cdot \tilde{J}(t) \]  

(4.2 - 1)

where \( \tilde{J}(t) \) is given by (2.3-1). The coefficients \( \{ I_n' \} \), which in general are complex-valued, represent the input current to each element in the fully-excited array with mutual coupling present. The \( \{ I_n' \} \) can be computed using either equation in (4.1-2) if the active element input impedances are known. Alternatively, the currents can be measured directly. Since the current distributions on the elements are nearly sinusoidal, each element in the array can be assumed to have the same ideal element pattern. The array pattern can therefore be approximated by

\[ F_{un}(\theta, \phi) \simeq g(\theta, \phi) \sum_{n=1}^{N} I_n' e^{j\beta \cdot \hat{r} \cdot r_n} \]  

(4.2 - 2)

where the element pattern \( g(\theta, \phi) \), which is given by (2.2-5), is the isolated element pattern. Note that (4.2-2) is analogous to the array pattern expression (2.2-6) used in ideal array analysis. The only difference between the two equations is that the \( \{ I_n \} \) used in (2.2-6) are proportional to the generator voltages applied to each element, whereas the \( \{ I_n' \} \) used in (4.2-2) are the actual element feed
currents, which contain the effects of mutual coupling and, in general, are not proportional to the generator voltages. The unnormalized isolated element pattern \( g(\theta, \phi) \) is found by substituting (2.3-1) into (2.2-5), yielding the expression

\[
g(\theta, \phi) = \frac{\cos(\beta h \sin \theta \cos \phi) - \cos \beta h}{\sqrt{1 - \sin^2 \theta \cos^2 \phi}}
\]

(4.2 - 3)

where \( h \) is the length of one half of a dipole element. The approximation in (4.2-2) has seen widespread use for many years and hence is referred to here as the classical array analysis method. This method has been used to include mutual coupling in array analysis by many investigators, including Kraus [3], Allen [6], Lee and Chu [19], Deshpande and Bailey [29], and Pozar [30].

Care must be taken to use the proper element excitation model when determining the feed currents \( \{I_n\} \). If the feed currents are measured, the measurements must be made at the terminals of each element with the complete feed network in place and the array fully excited. If the feed currents are computed using a numerical analysis code such as ESP, the free excitation model should be used with the generator impedance for each element set equal to the output impedance that element sees looking back into its corresponding port in the feed network. Note that, because of the presence of mutual coupling in the feed network, the output impedance of each port may change as the array is scanned.

The element pattern described by (4.2-3) is computed by assuming that an ideal sinusoidal current distribution exists on each element. As mentioned earlier in this section, this assumption is generally valid when the dipole elements are 0.5 wavelength long or shorter. As was shown in Section 3.2, however, the actual current distribution along an element longer than 0.5 wavelength may differ significantly from the ideal sinusoidal approximation. This suggests that a more accurate element pattern than that given by (4.2-3) may be needed to use the classical analysis method with arrays of long wire elements. On the other hand, Figure 3.2-2 shows that, for the case of a full-wave dipole, the pattern computed using (4.2-3) does not differ greatly from the actual pattern radiated by the

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dipole. Therefore, even though the sinusoidal current approximation is a grossly inaccurate method of predicting the current distribution along electrically long elements, (4.2-3) can still be used to compute a fairly accurate element pattern for long elements.

If it is still deemed necessary to obtain a more accurate isolated element pattern for long elements, a better approximation to the actual current distribution on the elements than that provided by the sinusoidal distribution of (2.3-1) can be used. The current distribution for a typical element can be computed using a moment method formulation such as ESP, or by using an improved approximation to the current distribution on an isolated long element such as one of the approximations derived by King [4] or Shen [31]. The new element pattern is then found from the more realistic approximation to the current distribution along the long element. Note, however, that the accuracy gained by using this more complicated approach to compute the element pattern may be insignificant. Since mutual coupling effects can cause the shape of the current distributions on long dipole elements located in an array to differ significantly from element to element, the accuracy gained using an improved approximation to the current distribution could be offset by the error introduced in assuming that the current distributions on the elements in the fully-excited array are nearly identical. For the reasons outlined above, the form of the classical analysis method employed here uses (4.2-3) to compute the element patterns.

4.3 Active Element Pattern Methods

The classical array analysis method relies on the assumption that every element in an array of similar elements has an identical ideal element pattern \( g(\theta, \phi) \), regardless of the excitation of the array or the physical locations of the array elements. The mutual coupling effects are considered to be contained in the feed currents \( \{I_n\} \). Alternate methods, referred to here as active element pattern
methods, can be used to account for the effects of radiated mutual coupling on the array pattern. In the active element pattern methods, a unique element pattern is computed or measured for each element which contains all of the mutual coupling effects associated with feeding that element. As will be shown later, active element pattern methods do not consider the effect of mutual coupling on the element feed currents, but instead assume that the array excitation is proportional to the incident excitation voltages.

Active element patterns have long been employed to include the effects of mutual coupling in closed-form array pattern expressions; however, a theoretical derivation of the active element pattern array analysis methods has not appeared in the general literature. Also, the sometimes subtle differences between the active element pattern methods have not been explained. The purpose of this section is to provide a theoretical basis for the various active element pattern methods, to define the various types of active element patterns, and to point out the differences between the methods. The discussion of active element pattern methods that follows focuses mainly on arrays of wire elements located in free space; however, the results can be generalized to arrays of any type of element located in linear media.

4.3.1 The Complete Active Element Pattern Method

The derivation of the active element pattern methods for wire elements begins by considering the integral equation which defines the current distribution on the wire elements. The form that the integral equation takes depends on the geometry of the antenna and whether the problem to be solved is a radiation problem or a scattering problem. Regardless of the problem to be solved, however, the integral equation will always have the general form
where \( J(l') \) is the current distribution along the antenna wires, \( K(l, l') \) is a kernel determined by the geometry of the antenna, and \( E(l) \) is the incident excitation field. The coordinates \( l \) and \( l' \) refer to locations along the antenna wires, which in general may be oriented in any direction.

The excitation field \( E(l) \) in (4.3-1) represents the feed excitation in antenna problems and an incoming wave in scattering problems. For the case of a transmitting array of dipole elements, \( E(l) \) is the incident field produced on the wire surfaces when feed voltages are applied to the input terminals of the array elements. The incident field is cancelled by the radiated or scattered field so that the boundary condition of zero tangential electric field on the conducting surfaces is satisfied. Because the array is fed by a number of discrete sources, the excitation field may be expressed as a sum of individual components where each component corresponds to the feed voltage applied to a specific element. If \( E_n(l) \) is defined as the component of the excitation field due to the voltage applied to the terminals of the \( n \)th element, then by superposition the complete excitation field can be expressed as

\[
E(l) = \sum_{n=1}^{N} E_n(l)
\]  

(4.3 - 2)

where \( N \) is the number of elements in the array.

Substituting (4.3-2) into (4.3-1) yields

\[
\int_{array} J(l') \ K(l, l') \ dl' = \sum_{n=1}^{N} E_n(l)
\]  

(4.3 - 3)
which suggests that each field component \( E_n(l) \) contributes to part of the current distribution on each element in the array. The part of the current distribution due to the \( n \)th feed voltage, \( J_{e,n}(l) \), which is defined over all of the array elements, is given by

\[
\int_{\text{array}} J_{a,n}(l') K(l, l') \, dl' = E_n(l) \quad (4.3 - 4)
\]

A version of (4.3-4) can be written for each element in the array. The sum of the \( N \) versions of (4.3-4) yields

\[
\sum_{n=1}^{N} \int_{\text{array}} J_{a,n}(l') K(l, l') \, dl' = \sum_{n=1}^{N} E_n(l) \quad (4.3 - 5)
\]

which, using (4.3-2), can be rewritten

\[
\int_{\text{array}} \sum_{n=1}^{N} J_{a,n}(l') K(l, l') \, dl' = E(l) \quad (4.3 - 6)
\]

Comparison of (4.3-6) with (4.3-1) then gives the result

\[
J(l) = \sum_{n=1}^{N} J_{a,n}(l') \quad (4.3 - 7)
\]

Note the difference between the array current distributions \( \{J_{e,n}(l')\} \) described above and the individual element current distributions \( \{J_n(l')\} \) described in Section 4.2. As shown in Figure 4.3-1, the \( \{J_{e,n}(l')\} \) are components of the total current distribution on the array, and each component is de-
Figure 4.3-1. Comparison of (a) the array current distribution $J_{\alpha, n}(l')$ to (b) the element current distribution $J_{n}(l')$. 

4. Methods for Including Mutual Coupling in Phased Array Pattern Analysis
fined over all of the array elements. The \( \{J_n(r)\} \) of Section 4.2 represent current distributions along individual elements when the array is fully excited. In other words, \( J_{n0}(r) \) represents the current distribution on all elements when only the \( n \)th element is excited, whereas \( J_n(r) \) represents the current distribution on only the \( n \)th element when all of the array elements are excited.

The relationship shown in (4.3-7) is an important result, since it implies that the total current distribution on an array is the sum of the current components that result from the excitation of individual array elements. Recall that the far field pattern radiated by an array can be computed from the current distribution \( J(r') \) on the antenna using the expression for the magnetic vector potential, introduced in (2.1-7) as

\[
A(r) = \frac{e^{-j\beta r}}{4\pi r} \iint \int_j J(r') e^{jk \cdot r} \, dv' \tag{4.3-8}
\]

The expression for \( A(r) \) is then substituted into (2.1-1) to obtain the radiated electric field. If (4.3-7) is substituted into (4.3-8), the total pattern \( F_{\text{tot}}(\theta, \phi) \) radiated by the array can be expressed as

\[
F_{\text{tot}}(\theta, \phi) = \sum_{n=1}^{N} G_{an}(\theta, \phi) \tag{4.3-9}
\]

where \( G_{an}(\theta, \phi) \) is the pattern produced by the array current distribution component \( J_{n0}(r) \). The quantity \( G_{an}(\theta, \phi) \) is referred to here as the complete active element pattern and represents the pattern radiated by the array when the \( n \)th element is excited by its respective excitation voltage and the other elements are loaded with their respective generator impedances. Note that \( G_{an}(\theta, \phi) \) is a complex-valued function, and thus has an amplitude and a phase associated with each direction \((\theta, \phi)\). The \( \{G_{an}(\theta, \phi)\} \) contain all of the effects of radiated mutual coupling, and therefore (4.3-9) is an exact expression for the array pattern. An example of the application of (4.3-9) to analyze an array has not appeared in the general literature.
Note that the use of the array pattern expression of (4.3-9) is not limited to arrays of dipole elements; the complete active element pattern method can be applied to arrays comprised of any type of element, including aperture elements such as horns. In any type of array, there will be a radiated field component corresponding to the excitation applied to each element in the array. The total radiated field of the array is the superposition of these components; this is stated in (4.3-9). Also note that (4.3-9) applies not only to arrays located in homogeneous media, but also to arrays located in inhomogeneous environments as long as all regions consist of linear media.

There is an important difference between the element pattern \( g(\theta, \phi) \) used in the classical analysis method described in Section 4.2 and the complete active element patterns \( \{ G_{ae}(\theta, \phi) \} \) described in this section. In the classical analysis method, the element pattern \( g(\theta, \phi) \) represents the ideal pattern radiated by an isolated element. The effects of the other nearby array elements on the element pattern are neglected. An active element pattern, on the other hand, represents the pattern radiated by the entire array with only one element directly excited and the other elements parasitically excited by the active element. The active element pattern also includes all radiated mutual coupling effects associated with exciting the active element.

The complete active element patterns \( \{ G_{ae}(\theta, \phi) \} \) can be either measured directly or computed using an analysis code such as ESP. The excitation voltage applied to the active element should have the same relative magnitude and phase as it would in the fully-excited array; the feed voltage should not be normalized. The inactive elements should be loaded with their respective generator impedances. Note that the active element patterns must be recomputed every time the array geometry or excitation changes. For this reason the complete active element patterns \( \{ G_{ae}(\theta, \phi) \} \) do not provide a very useful tool for array analysis. Further refinements of the method described by (4.3-9) are necessary to produce a practical analysis method based on active element patterns. These refinements will be discussed in the remaining sections of this chapter.
4.3.2 The Unit-Input Active Element Pattern Method

The excitation field component $E_n(l)$ that is produced when only the $n$th element is excited and the other array elements are loaded with their generator impedances may be expressed as the product of a scaling factor and a "unit excitation" field, so that

$$E_n(l) = V_n \, e_n(l)$$

(4.3 - 10)

where $V_n$ is the generally complex voltage of the generator connected to the terminals of the $n$th element, and $e_n(l)$ is the unit excitation field, defined as the incident field that exists on the surface of the array elements when the feed voltage $V_n$ equals unity. As shown in Figure 4.3-2, the unit excitation field gives rise to a corresponding current distribution $\tilde{J}_{u,n}(\ell)$ defined over the entire array such that

$$e_n(l) = \int_{\text{array}} \tilde{J}_{u,n}(\ell') K(l, \ell') \, d\ell'$$

(4.3 - 11)

It is important to remember that the current distribution $\tilde{J}_{u,n}(\ell)$ is not the current distribution on only the $n$th element, but rather the current distribution defined over the entire array that results when the $n$th element is excited with unit voltage; the quantity $\tilde{J}_{u,n}(\ell)$ encompasses not only the current distribution along the excited element, but also the current distributions along the other parasitically-excited elements. It is apparent from (4.3-4), (4.3-10) and (4.3-11) that

$$J_{a,n}(\ell) = V_n \tilde{J}_{a,n}(\ell)$$

(4.3 - 12)
Figure 4.3-2. Array current distribution $\tilde{J}_{a,n}(l')$ due to unit feed voltage applied to the $n$th element.
which implies that the component of the array current distribution that results when excitation voltage \( V_n \) is applied to the \( n \)th element is simply the current component \( J_{n}(\theta) \) scaled by the factor \( V_n \).

From (4.3-12), the complete active element pattern for the \( n \)th element can be expressed as

\[
G_{an}(\theta, \phi) = V_n g_{an}(\theta, \phi)
\]

where \( g_{an}(\theta, \phi) \), referred to here as the unit-input active element pattern, is the active element pattern that corresponds to the unit-input array current distribution component \( J_{n}(\theta) \), and is the pattern radiated by the array when the \( n \)th element is fed with an incident excitation of \( V_n = 1.0 \angle 0^\circ \) volt. Using (4.3-13) in (4.3-9), the expression for the array pattern becomes

\[
F_{un}(\theta, \phi) = \sum_{n=1}^{N} V_n g_{an}(\theta, \phi)
\]

The unit-input active element pattern method in the form of (4.3-14) has not appeared in the general literature. Kahn [32] used an array pattern expression that has a form similar to that of (4.3-14) but fundamentally constitutes a different method. Kahn's array pattern expression is given by

\[
F_{un}(\theta, \phi) = \sum_{n=1}^{N} J_{n}(\theta) \bar{f}(\theta, \phi) I_n
\]

where the \( \{I_n\} \) as defined by Kahn are the element feed currents that include the effects of mutual coupling, and hence are the same as the \( \{I'_n\} \) described in Section 4.1, and \( f_{1}(\theta, \phi) \) is the open-circuit element field pattern, which is the array pattern that results when the \( n \)th element is fed with unit current and all of the other elements are open-circuited. Kahn does not provide a derivation of (4.3-15).

4. Methods for Including Mutual Coupling in Phased Array Pattern Analysis
The result shown in (4.3-14) is significant because it implies that the pattern radiated by the array can be computed for any set of feed voltages using only one set of unit-input active element patterns. In other words, the \( \{g_m(\theta, \phi)\} \) need only be computed once for each element, after which the array pattern can be computed for any excitation simply by using the appropriate set of feed voltages \( \{V_n\} \) in (4.3-14). As shown in Figure 4.1-2b, the feed voltages \( \{V_n\} \) are the Thevenin equivalent generator voltages used in the free excitation model, and in general are not related to the element feed currents \( \{I_n\} \) in a simple manner, since the feed currents depend upon the values of the active element input impedances \( \{Z_m\} \), as shown in (4.1-2). The \( \{g_m(\theta, \phi)\} \) remain unchanged as long as the array geometry does not change. Consequently, the array analysis method of (4.3-14) is far more useful than that of (4.3-9). If the array geometry changes, however, a new set of unit-input active element patterns must be calculated or measured. Note that the expression for the array pattern in (4.3-14) is still exact; no approximations have yet been made.

4.3.3 The Phase-Adjusted Active Element Pattern Method

The expressions for the array pattern presented so far in (4.3-9) and (4.3-14) do not explicitly show any dependence of the array pattern on the physical locations of the array elements; that is, in contrast to the ideal array pattern expression of (2.2-6), the spatial phase term \( e^{j\beta \hat{r} \cdot r_n} \) is not explicitly present in either (4.3-9) or (4.3-14). Instead, the spatial phase information attributable to the element locations is implicitly contained in the active element patterns. A different active element pattern, referred to here as the phase-adjusted active element pattern, can be defined which does not contain the spatial phase information. Using the phase-adjusted active element patterns \( \{g_m(\theta, \phi)\} \), the array pattern can be expressed as

\[
F_{un}(\theta, \phi) = \sum_{n=1}^{N} V_n g_{pn}(\theta, \phi) e^{j\beta \hat{r} \cdot r_n}
\]

(4.3 - 16)
The phase-adjusted active element pattern method in the form of (4.3-16) has appeared in work by Allen [6], Aumann, Fenn and Willwerth [7] and Parad [8], through Parad assumes the forced excitation feed model in his formulation, whereas the free excitation model is assumed here.

The physical interpretation of the phase-adjusted active element pattern is very similar to that of the unit-input element pattern discussed in Section 4.3.1 with one important difference. As with the unit-input element pattern, the phase-adjusted element pattern is the pattern that results when the active element is excited with unit voltage and the other elements are loaded with their respective generator impedances; however, the resulting pattern is that of the array shifted in space (translated but not rotated) so that the active element is located at the origin of the array coordinate system. This spatial translation is later accounted for when \( g_{an}(\theta, \phi) \) is multiplied by the spatial phase term \( e^{-j \hat{e} \cdot \hat{r}_n} \) in (4.3-16). The unit-input element pattern, on the other hand, is determined for each element with the array stationary. It is obvious from (4.3-14) and (4.3-16) that

\[
    g_{an}(\theta, \phi) = g_{pu}(\theta, \phi) e^{-j \hat{e} \cdot \hat{r}_n}
\]

Note that in a practical antenna test range the phase component of the phase-adjusted active element pattern can be measured using a probe antenna located near the array. The phase difference between the probe and the array element under test leads to the required phase information. This method of phase measurement could prove unsatisfactory, however, since the coupling between the array elements and the probe may corrupt the measured phase data. Of course, the most practical method of obtaining the phase-adjusted element pattern for an array element is to measure the unit-input element pattern and then factor out the spatial phase term using

\[
    g_{pu}(\theta, \phi) = g_{an}(\theta, \phi) e^{-j \hat{e} \cdot \hat{r}_n}
\]

which is simply a rearrangement of (4.3-17).

As with the other active element patterns presented so far in this chapter, the phase-adjusted active element pattern is a complex-valued function of \( \theta \) and \( \phi \) and contains both amplitude data and

4. Methods for Including Mutual Coupling in Phased Array Pattern Analysis
phase data. Since (4.3-16) is a weighted sum of these complex-valued functions, the phase data in the \( \{g_m(\theta, \phi)\} \) must be retained in order to compute the proper array pattern. Blank [9], however, has had some success using only the amplitude part of the phase-adjusted element patterns, \( |g_m(\theta, \phi)| \), and ignoring the phase information. Though Blank has obtained good results, in general the exclusion of the phase data from the element patterns may cause serious errors in predicted array patterns.

It is possible to infer from (4.3-16) that, because the spatial phase term has been extracted from the active element pattern, (4.3-16) can be used to compute array patterns that would result if elements were moved from their original positions without having to compute new element patterns. This, of course, cannot be done, since the phase-adjusted element pattern will change whenever the array geometry changes. Therefore, nothing seems to have been gained by spending the extra effort to derive the phase-adjusted element pattern method. It will be shown in the next section, however, that the concept of the phase-adjusted active element pattern provides a useful stepping-stone for the development of some approximate array analysis methods.

4.3.4 The Average Active Element Pattern Method

As the number of elements in an equally-spaced array of similar elements increases, the active element patterns associated with the interior elements (the elements located away from the edges of the array) become more and more alike; the interior elements all see essentially the same environment in a large equally-spaced array and thus behave in a similar manner. The active element patterns of the edge elements, however, will continue to differ significantly from those of the interior elements [17]. As the array becomes larger the ratio of the number of interior elements to edge elements will increase, and the interior elements will dominate the behavior of the array. For a large array the \( \{g_m(\theta, \phi)\} \) in (4.3-16) may be approximated by the active element pattern of a "typical"
interior element, allowing the active element pattern to be factored out of (4.3-16). The array pattern can then be expressed as

\[
F_{un}(\theta, \phi) \approx g_{av}(\theta, \phi) \sum_{n=1}^{N} V_n e^{jB_r \cdot r_n}
\]  
(4.3 - 19)

where \( g_{av}(\theta, \phi) \) is called the average active element pattern. The transition from (4.3-16) to (4.3-19) for large, equally-spaced arrays has been pointed out by many authors, including Allen [6], Aumann, Fenn and Willwerth [7] and Parad [8].

Array pattern expressions similar in form to (4.3-19) appear often in the literature, but usually with feed currents specified inside the summation rather than feed voltages as shown here. Such expressions constitute fundamentally different approaches to expressing array patterns than that of (4.3-19). For (4.3-19) to remain valid, the Thevenin equivalent generator voltages \( \{V_n\} \) (as defined in Figure 4.1-2) must be used inside the summation. Feed currents can be used in (4.3-19) provided they are proportional to the \( \{V_n\} \); these feed currents would then correspond to the ideal feed currents \( \{I_n\} \) described in Section 4.1. If the \( \{I_n\} \) are used, then (4.3-19) gives the array pattern scaled by \( Z_a \), the ideal active element input impedance, which is assumed to be the same for each element.

Theoretically, (4.3-19) exactly expresses the pattern radiated by an equally-spaced array of an infinite number of elements, since the phase-adjusted active element patterns associated with each of the elements in an infinite array are indeed identical. Array analysis methods which exploit this fact and model large arrays as infinite arrays have appeared often in the literature, including much work by Oliner and Malech [5] and Wasylkiwskyj and Kahn[10].

For the special case of circular arrays consisting of a finite number of equally-spaced elements, (4.3-19) is an exact result, except that \( g_{av}(\theta, \phi) \) must be "rotated" for each element to give the proper active element pattern [4]. An example of a circular array is illustrated in Figure 4.3-3. The array
Figure 4.3-3. (a) Geometry of a twelve-element, uniformly-spaced circular array. The dots indicate element positions. (b) Active element pattern of the first element. (c) Active element pattern of the third element. The element pattern of the third element is identical to the element pattern of the first element rotated 60 degrees.
consists of twelve parallel, equally-spaced dipole elements. The locations of the elements are indicated in Figure 4.3-3a. Assume that the active element pattern associated with element #1 is that shown in Figure 4.3-3b. Because of the radial symmetry of the array, the active element pattern for any other element in the array is identical to that of element #1, but rotated by an amount corresponding to the angular position of the element relative to element #1. For example, element #3, which is located 60 degrees clockwise with respect to element #1, has the active element pattern shown in Figure 4.3-3c. Thus, (4.3-19) gives the exact pattern radiated by the circular array, as long as \( g_m(\theta, \phi) \) is rotated appropriately for each element.

Some authors, including Lo and Vu [33], have suggested stabilizing the mutual coupling behavior of the elements in equally-spaced arrays by using match-loaded "guard" or "dummy" elements at the ends of the array. The purpose of the guard elements is to provide an environment for the edge elements that is similar to that seen by the interior elements. An example of the placement of two guard elements on each end of a five-element linear array is shown in Figure 4.3-4. All of the active elements, including the active edge elements, should have roughly the same element patterns with the guard elements in place. The disadvantage of modifying the array in this manner is that the match-loaded guard elements serve no purpose other than to stabilize the behavior of the active elements, and actually absorb power in their matched loads that otherwise could be radiated. This array is an example of a small finite array that could be analyzed using the average active element pattern method.

4.3.5 The Hybrid Active Element Pattern Method

The element-specific active element pattern methods (the complete, unit-input and phase-adjusted element pattern methods, each of which associates a unique element pattern with each element) described so far are only suitable in practice for the analysis of small arrays, since the active element
Figure 4.3-4. Example of the use of guard elements to stabilize the mutual coupling behavior of a five-element linear array of parallel dipoles.
pattern data needed to use these methods occupies a very large amount of computer memory. On the other hand, the average active element pattern method of (4.3-19) is only suitable for very large arrays. Arrays that contain a medium number of elements (tens to hundreds of elements) can present a special analysis problem; it is both too time-consuming and too memory-consuming to use an element-specific active element pattern method to analyze the array, but the average element pattern method cannot accurately predict radiation pattern details for medium-sized arrays. However, a hybrid active element pattern analysis method is introduced here that can be used to analyze medium-sized, equally-spaced arrays accurately and efficiently.

The derivation of the hybrid method begins by considering the phase-adjusted active element pattern method of (4.3-16). The array elements are divided into an interior element group and an edge element group. The edge elements are typically taken to be the first two or three elements on each end of a linear array, or the first two or three rows and columns of elements on the sides of a planar array. The number of interior elements is \( N_i \), and the number of edge elements is \( N_e \), so that the total number of array elements is \( N = N_i + N_e \). The array pattern computed using phase-adjusted element patterns can therefore be expressed as

\[
F_{un}(\theta, \phi) = \sum_{n=1}^{N_i} V_n \mathcal{E}_{pn}(\theta, \phi) e^{j\beta \mathbf{r} \cdot \mathbf{r}_n} + \sum_{m=1}^{N_e} V_m \mathcal{E}_{pm}(\theta, \phi) e^{j\beta \mathbf{r} \cdot \mathbf{r}_m} \tag{4.3-20}
\]

where the \( n \) subscripts refer to interior elements and the \( m \) subscripts refer to edge elements. For an equally-spaced array, the phase-adjusted element patterns of the interior elements should be similar enough so that (4.3-20) can be rewritten
\[ F_{un}(\theta, \phi) = g^i_{av}(\theta, \phi) \sum_{n=1}^{N_i} V_n e^{j\beta \hat{r} \cdot \hat{r}_n} + \sum_{m=1}^{N_r} V_m g_{pm}(\theta, \phi) e^{j\beta \hat{r} \cdot \hat{r}_m} \]  

(4.3 - 21)

where \( g^i_{av}(\theta, \phi) \) is the average active element pattern of the interior elements. If it is more convenient to do so, (4.3-17) can be used in (4.3-21) to obtain the alternate expression

\[ F_{un}(\theta, \phi) \approx g^i_{av}(\theta, \phi) \sum_{n=1}^{N_i} V_n e^{j\beta \hat{r} \cdot \hat{r}_n} + \sum_{m=1}^{N_r} V_m g_{am}(\theta, \phi) \]  

(4.3 - 22)

Either (4.3-21) or (4.3-22) should predict fairly accurately the pattern radiated by a medium-sized, equally-spaced array. The number of element patterns that must be stored in memory is dictated by the number of elements that are considered to be edge elements. For example, if the first two elements on each end of a linear array are considered to be edge elements, then only three element patterns need be stored in memory: the average pattern of the interior elements and the two edge element patterns at one end of the array. By symmetry, the element patterns for the two edge elements at the other end of the array are mirror images of the two edge element patterns stored in memory.

Recently, Kokotoff, Auckland and Bornholdt [34] have proposed an array analysis method that is similar to the hybrid active element pattern method presented here. Their method, called the sub-array dilation method, allows the efficient analysis of very large uniformly-spaced arrays (arrays with elements numbering into the thousands), especially planar arrays, which are too large to analyze in practice. In the subarray dilation method, active element impedances and patterns are computed for elements located in a "subarray" that is similar in design to the original large array, but contains fewer elements; the smaller subarray consists of only tens or hundreds of elements but still has the
same element spacing and element orientation as the original array. Impedance and element pattern data for single elements in the subarray are then "mapped" to corresponding groups of elements located in similar regions in the large original array. For example, all of the elements near the center of the large array are assumed to have the same active impedance and active element pattern as those of the center element of the subarray. Figure 4.3-5 shows a simple example of how the individual elements of a $3 \times 3$ element subarray would be mapped to corresponding elements located in a $6 \times 8$ element large array. The subarray dilation method and the hybrid active element pattern method presented here differ in that the hybrid element pattern method uses patterns computed for elements located in the original full-sized array, whereas the subarray dilation method uses patterns computed for elements located in a subarray. The hybrid active element pattern method thus works well for moderately-sized arrays, and the subarray dilation method works well for very large arrays, where full-sized array computations would be intractable.

4.4 Summary

Several closed-form array pattern expressions have been derived in this chapter that account for the effects of mutual coupling. Table 4.4-1 lists each array analysis method discussed in this chapter and summarizes the inherent advantages and disadvantages of each method. Table 4.4-1 also indicates the level of approximation implicit in each method. These array analysis methods will be used in Chapter 5 to analyze several testbed arrays.
Figure 4.3-5. Illustration of a $3 \times 3$ subarray dilation into a $6 \times 8$ array [34].
Table 4.4-1. Summary of array analysis methods which include the effects of mutual coupling.

<table>
<thead>
<tr>
<th>METHOD, DEFINING EQUATION AND LEVEL OF APPROX.</th>
<th>ADVANTAGES</th>
<th>DISADVANTAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carter's Method (4.2-2) (Approximate)</td>
<td>- requires very little computer memory</td>
<td>- not accurate for arrays of large elements</td>
</tr>
<tr>
<td></td>
<td>- uses same algorithm as classical anal.</td>
<td></td>
</tr>
<tr>
<td>Complete AEP (4.3-9) (Exact)</td>
<td>- yields exact pattern expression for any configuration of any type of element</td>
<td>- requires large amount of memory</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- must recompute AEP's when scan dir. changes</td>
</tr>
<tr>
<td>Unit-Input AEP (4.3-14) (Exact)</td>
<td>- same as complete AEP but same set of AEP's can be used for any scan dir.</td>
<td>- requires large amount of memory</td>
</tr>
<tr>
<td>Phase-Adjusted AEP (4.3-16) (Exact)</td>
<td>- same as unit-input AEP method</td>
<td>- requires large amount of memory</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- AEP's difficult to compute or measure</td>
</tr>
<tr>
<td>Average AEP (4.3-19) (Approximate)</td>
<td>- requires very little memory</td>
<td>- can only use with large, equally-spaced arrays</td>
</tr>
<tr>
<td></td>
<td>- exact method for circular arrays</td>
<td>- cannot predict null-filling</td>
</tr>
<tr>
<td>Hybrid AEP (4.3-21) and (4.3-22) (Approximate)</td>
<td>- requires little memory</td>
<td>- can only use with equally-spaced arrays</td>
</tr>
<tr>
<td></td>
<td>- accounts for effects of edge elements efficiently</td>
<td>- complex algorithmic bookkeeping</td>
</tr>
</tbody>
</table>

4. Methods for Including Mutual Coupling in Phased Array Pattern Analysis
5. Investigations of Testbed Arrays

In the previous chapter, the derivations of several array analysis methods that account for mutual coupling effects were presented. In this chapter, the results of computer experiments that verify the various analysis methods of Chapter 4 are presented. Two computer programs were written for this purpose. The program PATMULT implements either the ideal array analysis method described in Chapter 2, which does not include mutual coupling effects, or the classical array analysis method described in Section 4.2, which does include mutual coupling effects. PATMULT is very similar to the ARRPAT program written by Stutzman [12]. The program ACTEL implements the active element pattern methods described in Section 4.3. The patterns computed using PATMULT and ACTEL are compared to those computed using the ESP code. Arrays of wire elements were chosen for the testbed arrays because of the high degree of accuracy that can be obtained from moment method solutions, which include radiated mutual coupling effects. The array designs considered in this chapter were chosen to show how well each analysis method predicts pattern degradation effects caused by mutual coupling, including null-filling, change in sidelobe level and beam broadening. Some of the arrays investigated are practical designs and are widely applied; others, however, have little practical value but were included to demonstrate specific capabilities of the analysis methods to account for mutual coupling effects.
5.1 Application of the Classical Array Analysis Method

The program PATMULT, which is described in detail in Appendix A, computes radiation patterns using either the ideal array analysis method, introduced in (2.2-6) as

\[ F_{\text{un}}(\theta, \phi) = g(\theta, \phi) \sum_{n=1}^{N} I_n e^{j\beta \hat{r} \cdot r_n} \quad (5.1-1) \]

or the classical array analysis method, introduced in (4.2-2) as

\[ F_{\text{un}}(\theta, \phi) = g(\theta, \phi) \sum_{n=1}^{N} I'_n e^{j\beta \hat{r} \cdot r_n} \quad (5.1-2) \]

The isolated element pattern \( g(\theta, \phi) \), which appears in both (5.1-1) and (5.1-2), is given by

\[ g(\theta, \phi) = \frac{\cos(\beta h \sin \theta \cos \phi) - \cos \beta h}{\sqrt{1 - \sin^2 \theta \cos^2 \phi}} \quad (5.1-3) \]

for center-fed dipole elements, where \( h \) is the length of one half of a dipole element. The element pattern given by (5.1-3) was introduced previously in (4.2-3).

Inspection of (5.1-1) and (5.1-2) indicates that the same computer algorithm can be used to implement either pattern analysis method. Thus, PATMULT can compute patterns using either the ideal analysis method or the classical analysis method. If ideal feed currents \( \{I_n\} \) are supplied to PATMULT, then PATMULT produces results based on (5.1-1). If feed currents \( \{I'_n\} \), which in-
clude the effects of mutual coupling, are supplied, then PATMULT in effect implements (5.1-2) to compute the array pattern. The \( \{I'_r\} \) can be measured directly or can be computed from active element impedances calculated using either a numerical code such as ESP or another method such as the induced-EMF method described by Carter [2]. If the ESP code is used, the values of the \( \{I'_r\} \) can be obtained directly from the set of expansion mode coefficients, which are included in the output produced by ESP; the feed current for a given element is simply the coefficient associated with the expansion mode located at the feedpoint of the element. The ideal analysis method and the classical analysis method were used to analyze all of the testbed arrays considered in this study.

5.1.1 Arrays of Half-Wave Dipole Elements

As explained in Section 4.2, the classical array analysis method relies on the assumption that the current distribution along each element in a dipole array does not change in shape in a mutual coupling environment, but instead is only scaled in amplitude and shifted in phase. All mutual coupling effects are assumed to be contained in the values of the element feed currents, so that the classical analysis method can be used to accurately predict array patterns. The examples presented in this section will show that these assumptions are valid when the dipole elements are approximately 0.5 wavelength long or shorter. The next section, however, will show that the mutual coupling information contained in the element feed currents alone cannot be used to accurately predict array patterns for arrays of "long" dipoles (dipoles longer than 0.5 wavelength).

The first array analyzed using the classical analysis method was a seven-element scanning linear array composed of elements 0.476 wavelength long and spaced 0.4 wavelength apart. The geometry of the array is shown in Figure 5.1-1. The H-plane patterns radiated by the array when scanned to 0, 30, 60 and 90 degrees were computed using the classical analysis method and the ideal analysis method. For each scanning direction, ESP was used to compute the input currents \( \{I'_r\} \), which
Figure 5.1-1. Geometry of seven-element scanning linear array of 0.476-wavelength center-fed dipoles. The wire radius is 0.001 wavelength.
include the effects of mutual coupling, using the free excitation model with each generator impedance set to 50 ohms. The computed patterns are shown as solid curves in Figures 5.1-2 through 5.1-5. The "a" part of the figures shows patterns computed using ideal analysis and the "b" part of the figures shows patterns computed using classical analysis. The patterns computed by ESP for the array scanned to each direction, which are assumed to be the actual patterns radiated by the array, are shown as dashed curves in the same figures. The feed currents \( \{I_a\} \) and \( \{I'_a\} \), which were supplied to PATMULT to calculate the patterns shown, are also included in the figures. Figures 5.1-2 through 5.1-5 show that the classical analysis method predicts the actual patterns radiated by the seven-element array much better than the ideal analysis method. Although the patterns computed using the ideal analysis method agree fairly well with the actual patterns radiated by the array, they do not show the severe null-filling that occurs due to mutual coupling effects. The classical analysis method, on the other hand, predicts this null-filling very accurately.

The five-element Dolph-Chebyshev array of half-wave dipoles, analyzed using the ideal analysis method in Section 2.4.2, was also analyzed using classical analysis. The geometry of the array was shown in Figure 2.4-1. The H-plane pattern computed using ideal analysis was shown in Figure 2.4-5, and is repeated for convenience in Figure 5.1-6a. The pattern computed using the classical analysis method (with the feed currents \( \{I'_a\} \) again calculated using ESP) is plotted as a solid curve in Figure 5.1-6b. The pattern computed using ESP is plotted as a dashed curve in both figures. The agreement between the pattern computed using the classical analysis method and the pattern computed using ESP is again much better than the agreement between the pattern computed using ideal analysis and the pattern computed using ESP.

The four-element difference pattern array of 0.476-wavelength dipoles with 0.4-wavelength spacing of Figure 2.4-6, which was discussed in Sections 2.4.3 and 3.3, was analyzed using the classical analysis method to demonstrate the effectiveness of the method in predicting null depth in null-steering arrays. The patterns computed using the ideal analysis method and the classical analysis method are shown as solid curves in Figures 5.1-7a and b, respectively. The pattern computed using ESP is shown as a dashed curve in both figures for comparison. As with the scanning array and the

5. Investigations of Testbed Arrays
Figure 5.1-2. Patterns computed for seven-element array of 0.476-wavelength dipoles scanned to $\theta = 0$ degrees (endfire) using (a) the ideal analysis method of (5.1-1) and (b) the classical analysis method of (5.1-2) (solid curves). The dashed curves represent the pattern computed using ESP. The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
Figure 5.1-3. Patterns computed for seven-element array of 0.476-wavelength dipoles scanned to \( \theta = 30 \) degrees using (a) the ideal analysis method of (5.1-1) and (b) the classical analysis method of (5.1-2) (solid curves). The dashed curves represent the pattern computed using ESP. The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
Figure 5.1-4. Patterns computed for seven-element array of 0.476-wavelength dipoles scanned to $\theta = 60$ degrees using (a) the ideal analysis method of (5.1-1) and (b) the classical analysis method of (5.1-2) (solid curves). The dashed curves represent the pattern computed using ESP. The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
Figure 5.1-5. Patterns computed for seven-element array of 0.476-wavelength dipoles scanned to $\theta = 90$ degrees (broadside) using (a) the ideal analysis method of (5.1-1) and (b) the classical analysis method of (5.1-2) (solid curves). The dashed curves represent the pattern computed using ESP. The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
Figure 5.1-6. Patterns computed for five-element Dolph-Chebyshev array of 0.476-wavelength dipoles using (a) the ideal analysis method of (5.1-1) and (b) the classical analysis method of (5.1-2) (solid curves). The dashed curves represent the pattern computed using ESP. The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.
Figure 5.1-7. Patterns computed for four-element difference pattern array of 0.476-wavelength dipoles with null steered to $\theta = 60$ degrees using (a) the ideal analysis method of (5.1-1) and (b) the classical analysis method of (5.1-2) (solid curves). The dashed curves represent the pattern computed using ESP. The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
Dolph-Chebyshev array discussed earlier, classical analysis closely predicts the actual pattern radiated by the array. The most important feature to note in Figure 5.1-7b is how well classical analysis predicts the depth of the desired null located at \( \theta = 60 \) degrees. The depth of the null located at \( \theta = 140 \) degrees is accurately predicted by classical analysis as well.

### 5.1.2 Arrays of Long Dipole Elements

The patterns obtained thus far using the classical analysis method to analyze arrays of half-wave dipole elements agree well with the patterns computed using ESP. This good agreement results primarily because the current distributions along half-wave dipole elements in an array remain very well-behaved despite mutual coupling effects. Even the phase distribution remains nearly constant along the elements, though it may be shifted by a significant amount. Because the current distribution near the feed point is so well-behaved, it is relatively easy to compute the feed currents for a half-wave element accurately. As was shown in Section 3.2.2, however, the current distribution near the center of an element longer than 0.5 wavelength varies rapidly with position along the element, making the computation of the feed current very difficult using available analysis methods. Since the feed currents for an array of long dipoles cannot be calculated accurately, the pattern computed for such an array using classical analysis will deviate considerably from the actual pattern radiated by the array.

To illustrate this problem a five-element scanning linear array of 1.0-wavelength center-fed dipoles was analyzed using both the ideal analysis method and the classical analysis method. The geometry of the array is shown in Figure 5.1-8. The H-plane patterns computed using both methods for the array scanned to 0, 30, 60 and 90 degrees are shown as solid curves in Figures 5.1-9 through 5.1-12, respectively. The "a" part of the figures shows the patterns computed using ideal analysis and the "b" part of the figures shows the patterns computed using classical analysis. The patterns computed
Figure 5.1-8. Geometry of five-element scanning linear array of 1.0-wavelength center-fed dipoles. The wire radius is 0.001 wavelength.
Figure 5.1-9. Patterns computed for five-element array of 1.0-wavelength dipoles scanned to \( \theta = 0 \) degrees (endfire) using (a) the ideal analysis method of (5.1-1) and (b) the classical analysis method of (5.1-2) (solid curves). The dashed curves represent the pattern computed using ESP. The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.
Figure 5.1-10. Patterns computed for five-element array of 1.0-wavelength dipoles scanned to $\theta = 30$ degrees using (a) the ideal analysis method of (5.1-1) and (b) the classical analysis method of (5.1-2) (solid curves). The dashed curves represent the pattern computed using ESP. The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.
Figure 5.1-11. Patterns computed for five-element array of 1.0-wavelength dipoles scanned to \( \theta = 60 \) degrees using (a) the ideal analysis method of (5.1-1) and (b) the classical analysis method of (5.1-2) (solid curves). The dashed curves represent the pattern computed using ESP. The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.
Figure 5.1-12. Patterns computed for five-element array of 1.0-wavelength dipoles scanned to $\theta = 90$ degrees (broadside) using (a) the ideal analysis method of (5.1-1) and (b) the classical analysis method of (5.1-2) (solid curves). The dashed curves represent the pattern computed using ESP. The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.
by ESP are shown as dashed lines in the same figures. The element feed currents \( I_j' \) were computed by ESP using the free excitation model with each generator impedance set to 50 ohms.

Comparison of the "a" and "b" parts of Figures 5.1-9 through 5.1-12 shows that the ideal analysis method (which excludes mutual coupling effects) yields a better approximation to the actual array patterns than the classical analysis method (which includes mutual coupling effects). To explain the poor results obtained using classical analysis, the amplitudes and phases of the expansion mode coefficients computed by ESP for each element of the array scanned to 30 degrees are shown in Table 5.1-1. These coefficients give the value of the computed current at the center of each expansion mode. The coefficients for only one side of each element are shown, since the current distributions are symmetrical about the feed points. Mode location 1 corresponds to the end of a given element and mode location 15 corresponds to the center of a given element.

The largest coefficients (those coefficients with relative amplitudes of 0.5 or greater) generally lie in the region bounded by and including mode locations 3 and 11; the coefficients for mode 15, which correspond to the element feed currents, are also large. The regions that contribute most to the radiation from an array of full-wave dipole elements are therefore the regions located away from the element ends and the element feed points. In these regions of high current, the relative amplitudes of the coefficients for each element at a given mode location are all within ten percent of one another. Also, the phase progression from one element to an adjacent element at a given mode location within these high current regions ranges from -150 to -155 degrees. In contrast, the amplitudes of the computed feed currents differ by as much as 29%, and the phase progression of the computed feed currents ranges from -161 to -165 degrees from element to element.

Since it was desired to steer the main beam of the array pattern to \( \theta = 30 \) degrees, the excitation voltages applied to the array elements all have unity amplitude and an element-to-element phase progression of -156 degrees, which is found from [12]

\[
\Delta \alpha = - \beta d \cos \theta_o
\]

\[
(5.1 - 4)
\]
Table 5.1-1. Expansion mode coefficients for five-element array of 1.0-wavelength center-fed dipoles scanned to $\theta = 30$ degrees. Mode 1 corresponds to the end of a dipole and mode 15 corresponds to the feedpoint. The current distribution is symmetrical about the feedpoint.

<table>
<thead>
<tr>
<th>MODE NO.</th>
<th>REL AMP</th>
<th>REL PHASE (DEG)</th>
<th>REL AMP</th>
<th>REL PHASE (DEG)</th>
<th>REL AMP</th>
<th>REL PHASE (DEG)</th>
<th>REL AMP</th>
<th>REL PHASE (DEG)</th>
<th>REL AMP</th>
<th>REL PHASE (DEG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.284</td>
<td>-91</td>
<td>0.276</td>
<td>113</td>
<td>0.270</td>
<td>-43</td>
<td>0.264</td>
<td>162</td>
<td>0.263</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>0.497</td>
<td>-90</td>
<td>0.482</td>
<td>114</td>
<td>0.470</td>
<td>-42</td>
<td>0.462</td>
<td>163</td>
<td>0.461</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>0.679</td>
<td>-90</td>
<td>0.655</td>
<td>114</td>
<td>0.639</td>
<td>-41</td>
<td>0.628</td>
<td>164</td>
<td>0.632</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>0.824</td>
<td>-90</td>
<td>0.791</td>
<td>115</td>
<td>0.770</td>
<td>-40</td>
<td>0.760</td>
<td>165</td>
<td>0.770</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>0.927</td>
<td>-89</td>
<td>0.886</td>
<td>116</td>
<td>0.861</td>
<td>-39</td>
<td>0.852</td>
<td>167</td>
<td>0.870</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>0.987</td>
<td>-89</td>
<td>0.937</td>
<td>117</td>
<td>0.909</td>
<td>-38</td>
<td>0.902</td>
<td>169</td>
<td>0.931</td>
<td>16</td>
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<tr>
<td>7</td>
<td>1.000</td>
<td>-88</td>
<td>0.941</td>
<td>118</td>
<td>0.912</td>
<td>-36</td>
<td>0.909</td>
<td>171</td>
<td>0.949</td>
<td>18</td>
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<tr>
<td>8</td>
<td>0.966</td>
<td>-83</td>
<td>0.900</td>
<td>119</td>
<td>0.871</td>
<td>-33</td>
<td>0.873</td>
<td>174</td>
<td>0.927</td>
<td>21</td>
</tr>
<tr>
<td>9</td>
<td>0.886</td>
<td>-86</td>
<td>0.814</td>
<td>122</td>
<td>0.787</td>
<td>-30</td>
<td>0.797</td>
<td>178</td>
<td>0.866</td>
<td>26</td>
</tr>
<tr>
<td>10</td>
<td>0.763</td>
<td>-85</td>
<td>0.688</td>
<td>125</td>
<td>0.666</td>
<td>-25</td>
<td>0.687</td>
<td>-176</td>
<td>0.771</td>
<td>32</td>
</tr>
<tr>
<td>11</td>
<td>0.601</td>
<td>-82</td>
<td>0.528</td>
<td>131</td>
<td>0.516</td>
<td>-17</td>
<td>0.554</td>
<td>-166</td>
<td>0.654</td>
<td>41</td>
</tr>
<tr>
<td>12</td>
<td>0.407</td>
<td>-76</td>
<td>0.346</td>
<td>144</td>
<td>0.361</td>
<td>2</td>
<td>0.423</td>
<td>-147</td>
<td>0.534</td>
<td>56</td>
</tr>
<tr>
<td>13</td>
<td>0.197</td>
<td>-55</td>
<td>0.206</td>
<td>-175</td>
<td>0.275</td>
<td>42</td>
<td>0.359</td>
<td>-113</td>
<td>0.455</td>
<td>81</td>
</tr>
<tr>
<td>14</td>
<td>0.158</td>
<td>38</td>
<td>0.288</td>
<td>-113</td>
<td>0.375</td>
<td>86</td>
<td>0.440</td>
<td>-77</td>
<td>0.483</td>
<td>113</td>
</tr>
<tr>
<td>15</td>
<td>0.521</td>
<td>74</td>
<td>0.640</td>
<td>-87</td>
<td>0.706</td>
<td>112</td>
<td>0.738</td>
<td>-51</td>
<td>0.716</td>
<td>144</td>
</tr>
</tbody>
</table>
where \( d \) is the element spacing and \( \theta_o \) is the desired scan direction. Therefore, the feed currents \( \{I_n\} \) supplied to PATMULT to implement the ideal analysis method were

\[
\begin{align*}
I_1 &= 1.0 \angle 0^\circ \\
I_2 &= 1.0 \angle -156^\circ \\
I_3 &= 1.0 \angle -312^\circ \\
I_4 &= 1.0 \angle -108^\circ \\
I_5 &= 1.0 \angle -264^\circ
\end{align*}
\]

\( (5.1 - 5) \)

The feed currents \( \{I'_n\} \) supplied to PATMULT to implement the classical analysis method on the other hand were (from ESP)

\[
\begin{align*}
I_1' &= 0.521 \angle 74^\circ \\
I_2' &= 0.640 \angle -87^\circ \\
I_3' &= 0.706 \angle 112^\circ \\
I_4' &= 0.738 \angle -51^\circ \\
I_5' &= 0.716 \angle 144^\circ
\end{align*}
\]

\( (5.1 - 6) \)

Note that these feed currents are the expansion mode coefficients for the modes located at the element feed points. Careful comparison of the feed currents shown in (5.1-5) and (5.1-6) with the expansion mode coefficients located in the high current regions of the actual operating array (see Table 5.1-1) shows that the relationships between the currents \( \{I_n\} \) used in the ideal method match the relationships between the high current regions on each element much more closely than the relationships between the currents \( \{I'_n\} \) used in the classical analysis method. In other words, the amplitudes and phases of the \( \{I_n\} \) are related to one another in a manner very similar to the way the amplitudes and phases of the expansion mode coefficients located in the high current regions are related; both sets of complex values are close to one another in amplitude and have phase progressions from element to element ranging from -150 to -156 degrees. The \( \{I'_n\} \), however, have amplitudes that vary considerably from element to element and phase progressions that range from -161 to -165 degrees. Consequently, the ideal analysis method gives a better approximation to the actual pattern radiated by this array than the classical analysis method, even though the ideal anal-

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ysis method does not include mutual coupling effects. These results are somewhat fortuitous and do not prove that ideal analysis always yields more accurate pattern predictions for arrays of long dipoles than classical analysis, but rather show that the classical analysis method cannot be expected to give any better results than ideal analysis when applied to arrays of electrically large elements.

To further demonstrate the problems that can be encountered when applying the classical analysis method to arrays of long dipoles, the method was used to analyze a five-element Dolph-Chebyshev array of 1.0-wavelength dipoles. This array has the same geometry as the five-element scanning linear array just discussed. The patterns computed using the ideal analysis method and the classical analysis method are shown as solid curves in Figures 5.1-13a and b, respectively. The dashed curves in both figures represent the array pattern computed by ESP. As with the scanning array just discussed, the pattern computed using ideal analysis is closer to the actual array pattern than the pattern computed using classical analysis.

The results presented in this section show that classical analysis often fails to accurately predict the patterns radiated by arrays of long dipoles because it is usually not possible to characterize the relationships between the element current distributions simply by examining the feed currents alone. Also, it is difficult to compute the feed currents of long elements accurately. Similar problems may arise if classical analysis is used to analyze arrays of other types of elements that have electrically large dimensions, such as yagis or horns. The current (aperture) distributions along such elements can change significantly in an array environment because of mutual coupling effects.
Figure 5.1-13. Patterns computed for the five-element Dolph-Chebyshev array of 1.0-wavelength dipoles using (a) the ideal analysis method of (5.1-1) and (b) the classical analysis method of (5.1-2) (solid curves). The dashed curves represent the pattern computed using ESP. The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.
5.2 Application of Active Element Pattern Methods

The active element pattern array analysis methods described in Chapter 4 are implemented by the program ACTEL, which is described in detail in Appendix B. By supplying an appropriate parameter to ACTEL through its input file, ACTEL can be made to implement either the complete active element pattern method of (4.3-9),

\[ F_{un}(\theta, \phi) = \sum_{n=1}^{N} G_{an}(\theta, \phi) \]  

(5.2 - 1)

the unit-input active element pattern method of (4.3-14),

\[ F_{un}(\theta, \phi) = \sum_{n=1}^{N} V_{n} g_{an}(\theta, \phi) \]  

(5.2 - 2)

the phase-adjusted active element pattern method of (4.3-16),

\[ F_{un}(\theta, \phi) = \sum_{n=1}^{N} V_{n} g_{pn}(\theta, \phi) e^{j\beta \hat{t} \cdot r_{n}} \]  

(5.2 - 3)

the average active element pattern method of (4.3-19),

\[ F_{un}(\theta, \phi) \approx g_{av}(\theta, \phi) \sum_{n=1}^{N} V_{n} e^{j\beta \hat{t} \cdot r_{n}} \]  

(5.2 - 4)
or the hybrid active element pattern method of (4.3-21),

\[ F_{un}(\theta, \phi) = \delta_{av}(\theta, \phi) \sum_{n=1}^{N_f} V_n e^{j\beta \hat{r} \cdot \hat{r}_n} + \sum_{m=1}^{N_x} V_m \delta_{pm}(\theta, \phi) e^{j\beta \hat{r} \cdot \hat{r}_m} \quad (5.2 - 5) \]

The data required to implement these methods is supplied to ACTEL in two input files. The first file contains program control parameters, the magnitudes and phases of the element excitation voltages and the \(x-, y-\) and \(z-\) coordinates of the element locations. The second file contains the appropriate active element patterns, which are computed using ESP. ACTEL also reads a third input file containing the pattern of the fully-excited array, which is also computed using ESP. For comparison, the pattern of the fully-excited array is plotted on the same graph as the pattern computed using the selected active element pattern method.

### 5.2.1 Element-Specific Active Element Pattern Methods

The element-specific methods are the complete active element pattern method, the unit-input active element pattern method and the phase-adjusted active element pattern method. According to the theory presented in Section 4.3, an array pattern computed using any one of these three methods exactly equals the actual array pattern. Therefore, there should be no discernable difference between the pattern computed using one of these methods and the pattern computed using ESP. For example, the unit-input active element pattern method of (5.2-2) was used to analyze the seven-element scanning linear array of 0.476-wavelength dipoles of Figure 5.1-1 scanned to \(\theta = 60\) degrees.

The H-plane pattern computed using the unit-input active element pattern method is shown as a solid curve in Figure 5.2-1, and the pattern computed using ESP is shown as a dashed curve in the
### Figure 5.2-1.

Pattern of seven-element array of 0.476-wavelength dipoles scanned to $\theta = 60$ degrees computed using the unit-input active element pattern method (solid curve) and computed using ESP (dashed curve). Note that the two curves coincide with one another. The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.

<table>
<thead>
<tr>
<th>El.</th>
<th>Mag</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>-72</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>-144</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>-216</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>-288</td>
</tr>
<tr>
<td>6</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1.0</td>
<td>-72</td>
</tr>
</tbody>
</table>

$Zg = 50$ ohms
same figure. Only one pattern appears to be plotted in Figure 5.2-1 because both patterns are exactly the same. Comparison of Figure 5.2-1 to Figure 5.1-4 shows that the pattern computed using the unit-input element pattern method is indeed the same as the pattern computed using ESP.

The two curves plotted in Figure 5.2-1 are represented in tabular form in Table 5.2-1. The two sets of pattern data are almost identical, confirming that the unit-input active element pattern analysis method yields an exact expression for the actual array pattern. The small discrepancies between the data most likely result from numerical errors. The complete active element pattern method of (5.2-1) and the phase-adjusted active element pattern method of (5.2-3) were also used to analyze the array; the patterns computed by both methods exactly match the patterns represented by Figure 5.2-1 and Table 5.2-1.

The unit-input active element pattern method is the most practical of the three element-specific active element pattern methods. As explained in Section 4.3, the complete active element pattern method is unattractive for practical use because a new set of complete active element patterns must be computed every time the scan angle changes. With either the unit-input element pattern method or the phase-adjusted element pattern method, however, the same set of active element patterns that are used to compute the pattern of an array scanned to one direction can be used to compute the pattern of the array scanned to any other direction. Therefore, only the input file supplied to ACTEL that contains the excitation voltages and element locations would need to be edited to compute the array pattern for a new scan angle using either of these two methods; the input file containing the individual active element patterns would remain unchanged. The unit-input element pattern method is more practical than the phase-adjusted element pattern method, though, because the spatial phase terms \( e^{i\theta \cdot r_n} \) need not be considered in the unit-input method.
Table 5.2-1. Pattern of seven-element array scanned to $\theta = 60$ degrees in tabular form.

<table>
<thead>
<tr>
<th>THETA (DEGREES)</th>
<th>ESP PATTERN (dBi)</th>
<th>UNIT-INPUT AEP METHOD (dBi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-12.8907</td>
<td>-12.8906</td>
</tr>
<tr>
<td>10</td>
<td>-13.0018</td>
<td>-13.0017</td>
</tr>
<tr>
<td>20</td>
<td>-14.4032</td>
<td>-14.4030</td>
</tr>
<tr>
<td>30</td>
<td>-22.4546</td>
<td>-22.4522</td>
</tr>
<tr>
<td>40</td>
<td>-10.5225</td>
<td>-10.5224</td>
</tr>
<tr>
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<td>-2.5749</td>
</tr>
<tr>
<td>60</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>70</td>
<td>-2.7093</td>
<td>-2.7093</td>
</tr>
<tr>
<td>80</td>
<td>-18.1601</td>
<td>-18.1614</td>
</tr>
<tr>
<td>90</td>
<td>-12.2596</td>
<td>-12.2594</td>
</tr>
<tr>
<td>100</td>
<td>-19.8979</td>
<td>-19.8974</td>
</tr>
<tr>
<td>110</td>
<td>-16.0133</td>
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</tr>
<tr>
<td>120</td>
<td>-18.3563</td>
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<td>130</td>
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<td>-19.9511</td>
</tr>
<tr>
<td>170</td>
<td>-17.8941</td>
<td>-17.8945</td>
</tr>
<tr>
<td>180</td>
<td>-17.1071</td>
<td>-17.1075</td>
</tr>
</tbody>
</table>
5.2.2 Average Active Element Pattern Method

The first array analyzed using the average active element pattern method of (5.2-4) was the seven-element scanning linear array of 0.476-wavelength dipoles spaced 0.4 wavelength apart, which was analyzed using the classical array analysis method in Section 5.1.1. The patterns computed using the average active element pattern method for the array scanned to 0, 30, 60 and 90 degrees are shown as solid curves in Figures 5.2-2 through 5.2-5, respectively. The center element of the array was chosen to supply the "average" active element pattern. The patterns computed using ESP for the same array are shown as dashed curves in the same figures. Also, the excitation voltages applied to the array elements for each scan direction are listed in each figure. Comparison of Figures 5.2-2 through 5.2-5 with the "a" part of Figures 5.1-2 through 5.1-5 shows that the average active element pattern method does not seem to predict the patterns radiated by the array any better than the ideal analysis method. Most notably, the method fails to predict the null-filling that is present in the actual array pattern. The failure to predict the pattern nulls occurs because the average element pattern method is essentially identical to the ideal analysis method. The only difference between the two methods is that the ideal element pattern \( g(\theta, \phi) \) used in the ideal method is replaced by \( g_{ae}(\theta, \phi) \) in the average element pattern method. The computed pattern is therefore merely the ideal array factor superimposed on the average element pattern, which means that the average element pattern method can never predict any null-filling effects caused by mutual coupling.

The array pattern computed using the average element pattern method for the five-element Dolph-Chebyshev array of 0.476-wavelength dipoles discussed in Section 5.1-1 is shown in Figure 5.2-6. As with the scanning linear array just discussed, comparison of Figure 5.2-6 with Figure 5.1-6a shows that the average active element pattern method predicts the actual array pattern no better than the ideal analysis method.

With the hope that the average element pattern method might be able to accurately predict array patterns produced by arrays of long dipole elements, the method was used to compute the pattern...
Figure 5.2-2. Pattern of seven-element array of 0.476-wavelength center-fed dipoles scanned to $\theta = 0$ degrees (endfire) computed using the average active element pattern method of (5.2-4) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.

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Figure 5.2-3. Pattern of seven-element array of 0.476-wavelength center-fed dipoles scanned to $\theta = 30$ degrees computed using the average active element pattern method of (5.2-4) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
### Figure 5.2-4.

Pattern of seven-element array of 0.476-wavelength center-fed dipoles scanned to $\theta = 60$ degrees computed using the average active element pattern method of (5.2-4) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.

<table>
<thead>
<tr>
<th>El.</th>
<th>Mag</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
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<td>2</td>
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<td>-72</td>
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<tr>
<td>3</td>
<td>1.0</td>
<td>-144</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>-216</td>
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<tr>
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<td>1.0</td>
<td>-288</td>
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<tr>
<td>6</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1.0</td>
<td>-72</td>
</tr>
</tbody>
</table>

$Z_g = 50$ ohms
Figure 5.2-5. Pattern of seven-element array of 0.476-wavelength center-fed dipoles scanned to $\theta = 90$ degrees (broadside) computed using the average active element pattern method of (5.2-4) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
Figure 5.2-6. Pattern of five-element Dolph-Chebyshev array of 0.476-wavelength dipoles computed using the average active element pattern method of (5.2-4) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.8 wavelength and the wire radius is 0.001 wavelength.
radiated by the five-element scanning linear array of 1.0-wavelength dipoles spaced 0.5 wavelength apart, which was discussed in Section 5.1.2. The patterns computed using the average active element pattern method for the array scanned to 0, 30, 60 and 90 degrees are shown in Figures 5.2-7 through 5.2-10, respectively. Though the average element pattern method produces more satisfactory results for arrays of long dipoles than the classical analysis method, comparison of Figures 5.2-7 through 5.2-10 with Figures 5.1-9 through 5.1-12 shows that the patterns computed using the average element pattern method are still no more accurate than those computed using the ideal analysis method.

The examples presented in this section show that the average element pattern method is not worth applying to small arrays of either short or long dipoles since ideal analysis yields comparable results with less computational or measurement effort. Other work that has appeared in the literature, however, shows that the method does work well when applied to larger arrays [5-8,10].

5.2.3 Hybrid Active Element Pattern Method

The hybrid active element pattern method introduced in Section 4.3.4 was applied to the seven-element scanning linear array discussed in the previous section. The patterns computed using the method are shown in Figures 5.2-11 through 5.2-14 for scan angles of 0, 30, 60 and 90 degrees, respectively. The center three elements were considered to be "interior" elements and the two elements at each end of the array were considered to be "edge" elements in the implementation of the method for this example. The transition point between the interior elements and the edge elements was determined by trial and error; the use of two edge elements permitted the most accurate array pattern prediction while still minimizing the number of element patterns stored in computer memory. (Note that, for a seven-element array, if the three elements on each side of the array are defined to be edge elements, then the center element would be the only interior element, and the hybrid ele-
Figure 5.2-7. Pattern of five-element scanning linear array of 1.0-wavelength dipoles scanned to $\theta = 0$ degrees (endfire) computed using the average active element pattern method of (5.2-4) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.

Generator Voltages

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$Z_g = 50$ ohms
Figure 5.2-8. Pattern of five-element scanning linear array of 1.0-wavelength dipoles scanned to $\theta = 30$ degrees computed using the average active element pattern method of (5.2-4) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.

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$Z_g = 50 \text{ ohms}$
Generator Voltages

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</thead>
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</table>

Zg = 50 ohms

Figure 5.2-9. Pattern of five-element scanning linear array of 1.0-wavelength dipoles scanned to $\theta = 60$ degrees computed using the average active element pattern method of (5.2-4) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.
Table 5.2-10. Generator Voltages

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</thead>
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<td>1.0</td>
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<tr>
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<tr>
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<td>0</td>
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</tbody>
</table>

\[ Z_g = 50 \text{ ohms} \]

Figure 5.2-10. Pattern of five-element scanning linear array of 1.0-wavelength dipoles scanned to \( \theta = 90 \) degrees (broadside) computed using the average active element pattern method of (5.2-4) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.5 wavelength and the wire radius is 0.001 wavelength.

5. Investigations of Testbed Arrays
### Generator Voltages

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<th>Phase</th>
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<td>1.0</td>
<td>-144</td>
</tr>
</tbody>
</table>

*edge elements

$Z_g = 50 \text{ ohms}$

---

**Figure 5.2-11.** Pattern of seven-element scanning linear array of 0.476-wavelength dipoles scanned to $\theta = 0$ degrees (endfire) computed using the hybrid active element pattern method of (5.2-5) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
Figure 5.2-12. Pattern of seven-element scanning linear array of 0.476-wavelength dipoles scanned to $\theta = 30$ degrees computed using the hybrid active element pattern method of (5.2-5) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
Figure 5.2-13. Pattern of seven-element scanning linear array of 0.476-wavelength dipoles scanned to $\theta = 60$ degrees computed using the hybrid active element pattern method of (5.2-5) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
Figure 5.2-14. Pattern of seven-element scanning linear array of 0.476-wavelength dipoles scanned to $\theta = 90$ degrees (broadside) computed using the hybrid active element pattern method of (5.2-5) (solid curve) and computed using ESP (dashed curve). The element spacing is 0.4 wavelength and the wire radius is 0.001 wavelength.
ment pattern method would effectively become the phase-adjusted element pattern method.) The
determination of the optimum transition point between the interior elements and the edge elements
that allows accurate patterns to be calculated while holding down the number of element patterns
stored in computer memory could be a topic of further research (see Section 6.2).

The agreement between the patterns computed using the hybrid element pattern method and the
patterns computed using ESP is very good. Unlike the average element pattern method, the hybrid
method closely predicts the null depths and sidelobe levels for each scan angle, as well as the shapes
of the main beams. Note that the patterns computed for this array using the classical analysis
method, shown in Figures 5.1-2 through 5.1-5, are slightly more accurate than the patterns com-
puted using the hybrid method. For arrays of electrically large elements, however, the hybrid
method would likely yield results far more accurate than those obtained using classical analysis,
since the hybrid method accounts for changes caused by mutual coupling in the current distribution
(or aperture field distribution) of large elements.
6. Conclusions

This study has focused on the problem of including mutual coupling effects in the formulation of closed-form expressions for the radiation patterns of phased arrays. Several methods of expressing array patterns have been presented along with examples of the application of each method. This chapter summarizes the results presented in this study and gives recommendations for future work.

6.1 Summary of Results

The major results of this study are presented in this section. Though this work has considered arrays of center-fed dipoles exclusively, the results obtained are general and can be applied to arrays of any type of element.
6.1.1 Theoretical Derivation of Array Pattern Expressions (Section 4.3)

The theoretical derivation of several array analysis methods that account for the effects of mutual coupling has been presented. This derivation begins by considering the basic integral equation which governs the current distribution along wire antennas and then proceeds with the formulation of exact array pattern expressions based on element-specific active element patterns (the complete, unit-input and phase-adjusted active element pattern methods, described in Sections 4.3.1 through 4.3.3, respectively). The derivation of an approximate analysis method based on average active element patterns, which can be applied to large equally-spaced arrays, has also been presented (Section 4.3.4). The use of these array analysis methods is not limited to free-standing arrays located in homogeneous media; the analysis methods can be applied to arrays located in inhomogeneous environments as well as long as all regions consist of linear media. The effects of feed network coupling are not included in the array analysis methods derived here. A treatment of feed network coupling can be found elsewhere [1].

6.1.2 Definitions of Active Element Patterns (Section 4.3)

As part of the process of deriving the analysis methods based on active element patterns, the various types of element patterns have been defined and the differences between the element patterns have been explained. Also, the manner in which the various active element patterns must be measured or computed has been explained.
6.1.3 Hybrid Active Element Pattern Method (Section 4.3.5)

A new method for approximating the patterns of moderately-sized, equally-spaced arrays has been introduced. This array analysis method, called the hybrid active element pattern method, uses a combination of average element patterns and element-specific element patterns to approximate the array pattern expression. The method accurately predicts array patterns while using much less computer memory than the analysis methods based on element-specific active element patterns alone.

6.1.4 Comparison of Array Analysis Methods (Chapter 5)

Several examples were presented which demonstrate the application of the classical array analysis method and the active element pattern methods to analyze array radiation patterns. The examples also demonstrate the degree of accuracy with which each method can predict array patterns. These examples show that the classical analysis method of (5.1-2) yields accurate results only for arrays of similar elements with very well-behaved current distributions, such as half-wave center-fed dipole elements. The classical analysis method is inconvenient since the element feed currents \( I_n \) must be recalculated every time the feed excitation changes. The complete active element pattern of (5.2-1) predicts array patterns exactly regardless of element type, but requires that the element patterns be recalculated whenever the array excitation changes. The unit-input element pattern method of (5.2-2) and the phase-adjusted element pattern method of (5.2-3), on the other hand, predict array patterns exactly but do not require extensive recalculation whenever the excitation changes. One significant advantage of the classical analysis method over the active element pattern methods, however, is that the classical analysis method uses very little computer memory, whereas the active element pattern methods can use enormous amounts of computer memory to store element patterns. The average active element pattern method of (5.2-4) was found to be an ineffective technique.
for expressing the patterns of small, equally-spaced arrays; however, the hybrid active element pattern method of (5.2-5) provided very accurate pattern expressions for these same arrays. Many of the inherent advantages and disadvantages of the various array analysis methods considered in this study are summarized in Table 4.4-1 at the end of Chapter 4.

6.2 Recommendations for Future Work

Several recommendations are presented in this section for possible future work which would expand on the ideas presented in this study.

6.2.1 Hybrid Active Element Pattern Method

The hybrid active element pattern analysis method introduced in this study should be examined more thoroughly to investigate its potential to accurately predict array patterns of moderately-sized, equally-spaced arrays. Specifically, the selection of an optimum transition point from "interior" element to "edge" element can be determined that yields sufficient accuracy while holding down the number of element patterns needed to implement the method. In addition, the method could be applied to arrays of electrically large elements to determine whether the computed array patterns are more accurate than those obtained using the ideal analysis method or the classical analysis method. Finally, the hybrid method could be applied to the analysis of planar arrays.
6.2.2 Near Field Active Element Pattern Methods

The work presented in this study deals exclusively with predicting far field array patterns; however, active element pattern methods could also be derived for near field pattern analysis. The near field derivation would closely follow the derivation presented in Section 4.3 for far field analysis.

6.2.3 Pattern Synthesis Methods Including Mutual Coupling

Pattern synthesis methods based on active element patterns could be devised. These methods would implicitly account for radiated mutual coupling effects through the element patterns, thus yielding the excitation voltages necessary to produce a desired pattern in a mutual coupling environment. Specifically, the work on pattern synthesis presented by Takamizawa [1] could be modified to make use of the active element pattern methods.

6.2.4 Scan Blindness Predictions

In many practical arrays mutual coupling can cause the active input impedances of the array elements to attain values at certain scan angles such that the net power supplied to the array, and hence the net power radiated, equals zero. This phenomenon is called scan blindness [35]. The active element pattern methods described in this report could be investigated to determine whether they can predict the onset of scan blindness in arrays.
References


Appendix A. Computer Program PATMULT

The defining equation for the ideal array analysis method, (5.1-1), and the defining equation for the classical array analysis method, (5.1-2), have the same basic form; therefore, the same algorithm can be used to compute array radiation patterns using either method. The program PATMULT was specifically written to compute the radiation patterns of arrays of parallel $x$-directed dipoles of arbitrary length and location using these two array analysis methods, and is very similar to the ARRPAT program written by Stutzman [12]. PATMULT implements the ideal analysis method if the ideal feed currents ($I_n$) are supplied to the program and implements the classical analysis method if the true feed currents ($I'_n$), which contain the effects of mutual coupling, are supplied to the program. The program also compares the array pattern computed using ideal or classical analysis to the pattern computed using ESP by plotting the two patterns on the same graph and tabulating the patterns in an output file. PATMULT is written in the FORTRAN computer language.
A.1 Input Files

PATMULT reads two input files and writes one output file when it is executed. The version of PATMULT used in this study also includes a subroutine which produces radiation pattern plots using the plotting facilities available at Virginia Tech (Versatec plotter). The first input file supplied to PATMULT, ARRAY INPUT, contains program control parameters and the locations and excitations of the array elements. The second input file supplied to the program, FULL INPUT, is generated by ESP and contains the pattern computed by ESP for the fully-excited array. This pattern is plotted along with the pattern computed by PATMULT for comparison purposes. PATMULT is set up to read FULL INPUT, which is created by ESP, without any assistance from the user. However, the user must create ARRAY INPUT.

The required structure of ARRAY INPUT is shown in Figure A-1a. The definitions of the variables on the first line of the file are:

N    Number of elements in array (must be less than or equal to 10).
IPLANE    Set to 1 for xz-plane pattern, 2 for yz-plane pattern or 3 for xy-plane pattern.
SPACE    Element spacing in wavelengths (not used with nonuniformly-spaced arrays; in such cases, a dummy value is supplied).
ELENGTH    Length of each array element in wavelengths.
IAEP    Not used by PATMULT; a dummy value is supplied.

The first line of ARRAY INPUT is followed by N more lines of input which supply the following information about the element locations and excitations:

X, Y, Z    x, y and z components of the location of the center of the element, in wavelengths.
A    Relative amplitude of the excitation coefficient for the element.
ALPHA    Relative phase of the excitation coefficient for the element, in degrees.

An example version of ARRAY INPUT constructed for the seven-element scanning linear array of 0.476-wavelength dipoles, whose geometry is shown in Figure 5.1-1, scanned to $\theta = 60$ degrees,
N IPLANE SPACE ELENGTH IAEP
X(1) Y(1) Z(1) A(1) ALPH(1)
X(2) Y(2) Z(2) A(2) ALPH(2)
X(3) Y(3) Z(3) A(3) ALPH(3)
...
X(N) Y(N) Z(N) A(N) ALPH(N)

(a)

7 2 .40 0.476 2
0.00 0. -1.20 1.000 0.00
0.00 0. -0.60 1.000 -72.00
0.00 0. -0.40 1.000 -144.00
0.00 0. 0.00 1.000 -216.00
0.00 0. 0.40 1.000 -288.00
0.00 0. 0.80 1.000 0.00
0.00 0. 1.20 1.000 -72.00

(b)

Figure A-1. (a) Structure of first input file supplied to PATMUL. (b) Example input file.
is shown in Figure A-1b. The input file is set up to implement the ideal analysis method. Note that the elements are spaced 0.4 wavelength apart and are all fed with unit excitation. The element-to-element phase progression is -72 degrees.

A.2 Program Output

The output file created by PATMULT echoes the input data supplied by the user for purposes of verification. Also, the pattern computed using ideal or classical analysis and the pattern computed using ESP are written in tabular form to the output file. Part of an example output file, which corresponds to the example input file shown in Figure A-1b, is shown in Figure A-2. The version of PATMULT used here contains a subroutine that plots the radiation pattern computed using the selected analysis method as a solid curve and the pattern computed using ESP as a dashed curve on the same graph using a Versatec plotter. The pattern plots for the example considered here are shown in Figure 5.1-4a. The program can easily be modified for use with other plotting facilities.

A.3 Source Code Listings

PATMULT shares several subroutines with the ACTEL program described in Appendix B. The source code for PATMULT, which includes the subroutines unique to the program but does not include the subroutines shared with ACTEL, is listed in Figure A-3. The source code for the subroutines that are shared between PATMULT and ACTEL is listed in Figure A-4.
### PATMULT OUTPUT

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**ELEMENT SPACING = 0.4000**
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**Figure A-2.** Example output file produced by PATMULT. This output file corresponds to the input file shown in Figure A-1b.

Appendix A. Computer Program PATMULT
C*******************************************************************************
C PATMULT
C*******************************************************************************
C THIS PROGRAM GENERATES A RADIATION PATTERN PLOT FOR AN ARRAY OF
C VARIABLE-LENGTH DIPOLES USING EITHER THE CLASSICAL ANALYSIS METHOD
C OR THE FEED-REFERENCED METHOD. THE PROGRAM ALSO READS IN PATTERN
C DATA GENERATED BY THE PROGRAM ESP, WHICH INCLUDES THE EFFECTS OF
C MUTUAL COUPLING, FOR THE SAME ARRAY. THE TWO RADIATION PATTERNS ARE
C PLOTTED ON ONE GRAPH AND TABULATED TOGETHER FOR EASE OF COMPARISON.
C
C THE PROGRAM REQUIRES THAT I/O UNIT NO. 5 BE SET ASIDE FOR PROGRAM
C READS AND I/O UNIT NO. 6 BE SET ASIDE FOR PROGRAM WRITES.
C
C TWO INPUT FILES ARE USED BY THE PROGRAM:
C
C ARRAY INPUT - CONTAINS INFORMATION ABOUT THE TYPES, EXCITATIONS AND
C POSITIONS OF THE ELEMENTS IN THE ARRAY.
C
C FULL INPUT - FILE GENERATED BY ESP WHICH CONTAINS THE PATTERN OF
C THE ARRAY INCLUDING MUTUAL COUPLING EFFECTS. THE
C PATTERN DATA SHOULD BE IN "LINEAR" FORMAT INSTEAD OF
C DECIBEL FORMAT.
C
C ONE OUTPUT FILE IS PRODUCED BY THE PROGRAM:
C
C PATMULT OUTPUT - CONTAINS ECHO OF INPUT DATA SUPPLIED IN FILE ARRAY
C DATA AND THE PATMULT PATTERN AND ESP PATTERN IN
C TABULAR FORM.
C
C THE PROGRAM ALSO PLOTS THE TWO PATTERNS USING THE VERSATEC PLOTTER.
C
C LATEST REVISION: FEBRUARY 4, 1990
C
C*******************************************************************************

REAL IDEAL(360),MC(360)
REAL X(10),Y(10),Z(10),A(10),ALPHA(10)
COMMON/BLOC/, X,Y,Z,A,ALPHA
COMMON/CDNS/, PI,DTR
COMMON/IDEAL/, IDEAL
COMMON/ESP/, MC
PI=3.141592654
DTR=PI/180.

C
C READ INPUT FILE 'ARRAY INPUT':
C
C N = NO. OF ELEMENTS IN ARRAY.
C IPLANEn = 1, FOR XZ-PLANE PATTERN.
C 2, FOR YZ-PLANE PATTERN.
C 3, FOR XY-PLANE PATTERN.
C SPACEn = SPACING BETWEEN ELEMENTS IN WAVELENGTHS (RELEVANT ONLY
C IF THE ARRAY IS UNIFORMLY-SPACED).
C LENEn = LENGTH OF ARRAY ELEMENTS IN WAVELENGTHS.
C IAEp = (NOT USED IN THIS PROGRAM.)
C X,Y,Zn = X,Y,Z COORDINATES OF THE CENTER OF THE ITH ELEMENT.
C A = AMPLITUDE OF THE EXCITATION FOR THE ITH ELEMENT.

Figure A-3. Source code listing for program PATMULT, not including subroutines shared with program ACTEL.
ALPHA = PHASE OF THE EXCITATION FOR THE I^TH ELEMENT.

CALL READFILE(N,IPLANE,SPACE,ELENGTH,IAEP)

GENERATE THE PATTERN RADIATED BY THE ARRAY USING THE PRINCIPLE OF
PATTERN MULTIPLICATION. THE PATTERN DATA IS STORED IN A DATA ARRAY
CALLED IDEAL.

CALL IDEALPAT(N,IPLANE,ELength)

NORMALIZE IDEAL PATTERN AND CONVERT TO DB.

CALL NORMAL(Ideal)

READ IN THE PATTERN DATA GENERATED BY THE PROGRAM ESP. THE PATTERN
DATA MUST BE IN 'LINEAR' FORM; THAT IS, NOT IN DB FORM.

CALL READESP

NORMALIZE PATTERN GENERATED BY ESP AND CONVERT TO DB.

CALL NORMAL(MC)

PLOT PATTERNS.

IPL = 1, FOR FULL CIRCULAR PLOT.

2, FOR SEMICIRCULAR PLOT WITH DB LEVELS LABELED.

IPL = 2
CALL DBLPLOT(IPL,IDEAL,MC,IPLANE,N,SPACE,ELength)
CALL PLOT(12.,0.,999)

WRITE ARRAY PATTERNS COMPUTED USING PATTERN MULTIPLICATION AND
COMPUTED USING ESP IN TABULAR FORM TO I/O UNIT 6.

CALL OUTDAT(N,ELength,SPACE,IPLANE)

STOP
END

*************************************************************************
C THIS SUBROUTINE CALCULATES THE PATTERN OF THE ARRAY USING THE
C PRINCIPLE OF PATTERN MULTIPLICATION.
*************************************************************************

SUBROUTINE IDEALPAT(N,IPLANE,ELength)
COMPLEX ELPat,CABS,TEMP,ARFACT
REAL IDEAL(360)
COMMON/IDEAL/ IDEAL
COMMON/CONS/ P1,DTR

IF(IPLANE.EQ.1)PHI=0.
IF(IPLANE.EQ.2)PHI=90.*DTR
IF(IPLANE.EQ.3)THETA=90.*DTR
TEMP=(0.,0.)
DO 10 IDEG=1,360
    IF(IPLANE.LE.2)THETA=IDEG*DTR
    IF(IPLANE.EQ.3)PHI=IDEG*DTR
    TEMP=ELPat(THETA,PHI,ELength)*ARFACT(N,THETA,PHI)
10 CONTINUE

Figure A-3. (continued)
```

10 CONTINUE
RETURN

C*****************************************************************************
C THIS SUBROUTINE READS THE PATTERN OF THE FULLY-EXCITED ARRAY GENERATED C
C BY THE PROGRAM ESP. THE PATTERN DATA IS STORED IN THE DATA ARRAY MCC.
C*****************************************************************************

SUBROUTINE READESP
REAL MC(360)
COMPLEX COMPMC, CABS, CMPLX
COMMON/ESP/ MC
COMMON/CONS/ PI, DTR
C NPATS = NO. OF PATTERN CUTS IN FILE (SHOULD EQUAL 1)
C NTYPE = 1, FOR RADIATION PATTERN PLOT (NTYPE SHOULD EQUAL 1)
C             2, FOR SCATTERING PATTERN PLOT
C FREQ = FREQUENCY IN MHZ
C IIA = 1, FOR AN ELEVATION PATTERN (XZ- OR YZ-PLANE)
C             2, FOR AN AZIMUTH PATTERN (XY-PLANE)
C NPTS = NO. OF PATTERN POINTS TO BE PLOTTED (SHOULD BE 361)
C ANGLE = FIXED ANGLE FOR PATTERN PLOT (E.G., ANGLE = PHI = 0 FOR AN
C             XZ-PLANE PATTERN PLOT)
C
READ(5,*) NPATS, NTYPE, FREQ
IF(NPATS.GT.1) STOP
READ(5,*) IIA, NPTS, ANGLE
IF(NPTS.NE.361) STOP
C READ IN THE PATTERN MAGNITUDES AND PHASES FOR THE TWO POLARIZATIONS.
C THE PATTERN DATA IS IN 'LINEAR' FORM INSTEAD OF DB FORM.
DO 40 IDEG=1, 361
    READ(5,*) AMAG1, ANG1, AMAG2, ANG2
    AREAL=AMAG1*COS(ANG1*DTR)+AMAG2*COS(ANG2*DTR)
    AIMAG=AMAG1*SIN(ANG1*DTR)+AMAG2*SIN(ANG2*DTR)
    COMPMC=CMPLX(AREAL, AIMAG)
    IF(IDEG.NE.1) MC(IDEG-1)=CABS(COMPMC)
40 CONTINUE
RETURN

C*****************************************************************************
C THIS SUBPROGRAM RETURNS THE ARRAY FACTOR VALUE OF A GENERAL ARRAY OF
C ISOTROPIC ELEMENTS AT THE ANGLE (THETA,PHI). THE ARRAY FACTOR IS
C UNNORMALIZED.
C*****************************************************************************

COMPLEX FUNCTION ARFACT(N, THETA, PHI)
COMPLEX TEMP, CEXP, IMAG, CABS, EXCIT, 9RDTR
REAL X(10), Y(10), Z(10), A(10), ALPHA(10)
COMMON/BLOC1/ X, Y, Z, A, ALPHA
COMMON/CONS/ PI, DTR
IMAG=(0., 1.)
TEMP=(0., 0.)
ST=SIN(THETA)
CT=COS(THETA)
SP=SIN(PHI)
CP=COS(PHI)
DO 10 I=1, N
    EXCIT=A(I)*CEXP(IMAG*ALPHA(I))
10 CONTINUE
RETURN

Figure A-3. (continued)

Appendix A. Computer Program PATMULT 145
```
BRDTR=CEXP(IMAG2.*PI*X(I)*XSTCP+1Y(I)*XSTSP+Z(I)*XCT))
TEMP=TEMP+EXCIT*BRDTR
10 CONTINUE
ADRFACT=TEMP
IF(CABS(ADRFACT).LT.1.E-06)ADRFACT=(0.,0.)
RETURN
END

C THIS SUBPROGRAM RETURNS THE ELEMENT PATTERN OF A DIPOLE ELEMENT IN
C THE DIRECTION (THETA, PHI). THE DIPOLE IS ASSUMED TO BE X-DIRECTED.

C******************************************************************************
FUNCTION ELPAI(THETA,PHI,ELENGT)
COMPLEX CMPLX,ARG,CABS,CEXP,SSUM
COMMON/CONS/ PI,DTR
SINCOS=SIN(THETA)*COS(PHI)
DENOM=SQRT(1.-SINCOS**2)
H=ELENGT/2.
COSBH=COS(2.*PI*H)
IF(DENOM.EQ.0.)THEN
ELPAI=(0.,0.,0.)
ELSE
ARGI=(COS(2.*PI*H*SINCOS)-COSBH)/(/DENOM*(1.-COSBH))
END IF
END

C******************************************************************************
C THIS SUBROUTINE WRITES THE PATTERN COMPUTED USING ESP AND THE PATTERN
C COMPUTED USING THIS PROGRAM TO A DISK FILE. INFORMATION ABOUT THE
C GEOMETRY AND EXCITATION OF THE ARRAY ARE ALSO WRITTEN TO THE FILE.

C******************************************************************************
SUBROUTINE OUTDAT(N,ELENGT,SPACE,IPLANE)
REAL IDEAL(360),MC(360),X(10),Y(10),Z(10),A(10),ALPHA(10)
REAL MCMAX,IMAX
COMMON/CONS/ PI,DTR
COMMON/BLOC/ X,Y,Z,A,ALPHA
COMMON/IDEAL/ IDEAL
COMMON/ESP/ MC

C WRITE OUT GEOMETRY AND EXCITATION DATA.
WRITE(6,20)N,ELENGT,SPACE,IPLANE
20 FORMAT(' PATMUST OUTPUT',/,' NO. OF ELEMENTS = ',I3,/, ' ELEMENT LENGTH = ',F8.4,/, ' ELEMENT SPACING = ',F8.4,/, ' IPLANE = ',I2,/) WRITE(6,21)
21 FORMAT(' EL NO.  X    Y    Z    AMP',/,' PHS',/)
DO 30 I=1,N
ALDEG=ALPHA(I)/DTR
WRITE(6,22)I,X(I),Y(I),Z(I),A(I),ALDEG
22 FORMAT(I6,8X,1G10.3,4X,1G10.3,3X,1G10.3,3X,1G10.3,3X,1G10.3,3X,1G10.3)
30 CONTINUE

C WRITE OUT DATA FOR BOTH PATTERNS.

Figure A-3. (continued)
C
WRITE(6,23)
23 FORMAT(//',' THETA IDEAL PAT. MC PAT.'//,//)
DO 50 I=1,360
  WRITE(6,24)I,IDEAL(I),MC(I)
24 FORMAT(1X,4,6X,F8.4,8X,F8.4)
50 CONTINUE
RETURN
END

Figure A-3.  (continued)
C THIS SUBROUTINE READS THE FILE 'ARRAY INPUT', WHICH CONTAINS PROGRAM
C CONTROL PARAMETERS AND THE LOCATIONS AND EXCITATIONS OF THE ARRAY
C ELEMENTS.
C
SUBROUTINE READFILE(N,IPLANE,SPACE,ELength,IAEP)
REAL X(10),Y(10),Z(10),A(10),ALPHA(10)
COMMON/BLOC1/X,Y,Z,A,ALPHA
COMMON/CONS/PI,DTR
READ(5,*),N,IPLANE,SPACE,ELength,IAEP
IF(N.GT.10)STOP
DO 10 I=1,N
   READ(5,*),X(I),Y(I),Z(I),A(I),ALPHA(I)
   ALPHA(I)=ALPHA(I)*DTR
10 CONTINUE
IF(IPLANE.LT.1.OR.IPLANE.GT.3)STOP
C WRITE INPUT DATA TO OUTPUT FILE FOR VERIFICATION.
WRITE(6,100)
100 FORMAT(1X,//13X,'ELEMENT LOCATIONS AND EXCITATIONS',//5X,
      *'I',5X,'X(I)',6X,'Y(I)',6X,'Z(I)',6X,'A(I)',6X,'ALPHA(I)'/)
DO 20 I=1,N
   ALPDEG=ALPHA(I)*DTR
   WRITE(6,110)I,X(I),Y(I),Z(I),A(I),ALPDEG
20 FORMAT(1X,I5,5F10.4)
RETURN
END

C THIS SUBROUTINE CONVERTS PATTERN DATA TO DB AND NORMALIZES THE DATA
C SO THAT IT LIES WITHIN THE RANGE -40 DB TO 0 DB.
C
SUBROUTINE NORMAL(DATA)
REAL DATA(360)
EMAX=0.
DO 1 I=1,360
   ABSDAT=ABS(DATA(I))
   IF(ABSDAT.GT.EMAX)EMAX=ABSDAT
1 CONTINUE
IF(EMAX.NE.0.)THEN
   DO 2 I=1,360
      TEMP=ABS(DATA(I))/EMAX
      IF(TEMP.NE.0.)DATA(I)=20.*ALOG10(TEMP)
      IF(TEMP.EQ.0.)DATA(I)=-40.
      IF(DATA(I).LT.-40.)DATA(I)=-40.
2 CONTINUE
ELSE
   DO 3 I=1,360
      DATA(I)=-40.
3 CONTINUE
END IF
RETURN
END

Figure A-4. Source code listing for subroutines shared between PATMULT and ACTEL.

Appendix A. Computer Program PATMULT
C******************************************
C THIS SUBROUTINE PLOTS TWO RADIATION PATTERNS ON THE SAME GRAPH, ONE
C PATTERN WITH A SOLID LINE AND THE OTHER WITH A DOTTED LINE. THE
C PATTERN DATA MUST BE IN DECIBEL UNITS, AND MUST BE NORMALIZED
C AND LIE WITHIN THE RANGE -40 DB TO 0 DB.
C
C IPILOT  = 1, FOR FULL CIRCULAR PLOT
C          = 2, FOR SEMICIRCULAR PLOT
C DATA1   = ARRAY CONTAINING THE RADIATION PATTERN TO BE PLOTTED WITH A
C          = ARRAY CONTAINING THE RADIATION PATTERN TO BE PLOTTED WITH A
C          = DOTTED LINE
C SPAC   = THE NUMBER OF ELEMENTS IN THE ARRAY
C SPACE   = SPACING BETWEEN ELEMENTS IN WAVELENGTHS
C LENGTH  = LENGTH OF EACH ELEMENT IN WAVELENGTHS
C********************************************************
SUBROUTINE DBLPLT(IPILOT,DATA1,DATA2,IPANE,N,SPACE,LENGTH)
    REAL DATA1(360),DATA2(360)
    DTR=3.14159265/180.
    CALL PLOTS(0,0,0)
    CALL NEWPEN(2)
    CALL PLOT(5.5,5.5,-3)
    R=2.0
    IF(IPILOT.EQ.1) THEN
        CALL PLOT(-R,0.,3)
        CALL PLOT(R,0.,2)
        CALL PLOT(0.,-R,3)
        CALL PLOT(0.,R,2)
    ELSE
        CALL PLOT(-R,0.,3)
        CALL PLOT(R,0.,2)
        CALL PLOT(0.,0.,3)
        CALL PLOT(0.,R,2)
    END IF
    CALL SEMI-CIRCLES
    DO 2 J=1,4
        RAD=R*(5.-J)/4.
        CALL PLOT(RAD,0.,3)
        DO 1 I=1,180
            X=RAD*COS(I*DTR)
            Y=RAD*SIN(I*DTR)
        1 CONTINUE
    2 CONTINUE

Figure A-4. (continued)
CALL PLOT(X,Y,2)
1 CONTINUE
2 CONTINUE
C LABEL DB POWER LEVELS
CALL SYMBOL(-1875,-25,.125, 1'-40 -30 -20 -10 0 DB',0,.21)
END IF
C LABEL THE AXES
IF(IPLAN.EQ.1)THEN
    CALL SYMBOL(-R,R+1.25,.25,'XZ-PLANE',0,.8)
    CALL SYMBOL(0.,R+25.,125, 'X',0.,1)
    CALL SYMBOL(R+25,0.,125, 'Z',0.,1)
END IF
IF(IPLAN.EQ.2)THEN
    CALL SYMBOL(-R,R+1.25,.25,'YZ-PLANE',0,.8)
    CALL SYMBOL(0.,R+25.,125, 'Y',0.,1)
    CALL SYMBOL(R+25,0.,125, 'Z',0.,1)
END IF
IF(IPLAN.EQ.3)THEN
    CALL SYMBOL(-R,R+1.25,.25,'XY-PLANE',0,.8)
    CALL SYMBOL(0.,R+25.,125, 'Y',0.,1)
    CALL SYMBOL(R+25,0.,125, 'X',0.,1)
END IF
C PRINT NUMBER OF ELEMENTS
REALN=FLOAT(N)
CALL SYMBOL(-R,-R-1.,125, 'NO. ELEMENTS = ',0.,14)
CALL NUMBER(-R-2,-R-1.,125,REALN,0.,-1)
C PRINT ELEMENT SPACING
CALL SYMBOL(-R,-R-1.25,.125, 'EL. SPACING (WVLENGTHS) = ',0.,26)
CALL NUMBER(-R-3.375,-R-1.25,.125,SPACE,0.,3)
C PRINT ELEMENT LENGTH
CALL SYMBOL(-R,-R-1.5,.125, 'EL. LENGTH (WVLENGTHS) = ',0.,25)
CALL NUMBER(-R-3.25,-R-1.5,.125,LENGTH,0.,3)
C CHECK FOR PROPER NORMALIZATION OF PATTERN DATA
DO 10 I=1,360
   IF(DATA1(I).GT.0. .OR. DATA1(I).LT.-40.)THEN
      WRITE(6,12)
      12 FORMAT(/IX,'SOLID LINE DATA NOT PROPERLY NORMALIZED',/)
      STOP
   END IF
   IF(DATA2(I).GT.0. .OR. DATA2(I).LT.-40.)THEN
      WRITE(6,13)
      13 FORMAT(/IX,'DASHED LINE DATA NOT PROPERLY NORMALIZED',/)
      STOP
   END IF
10 CONTINUE
C MODIFY DATA TO ENABLE PLOTTING
DO 20 I=1,360
   DATA1(I)=(DATA1(I)+40.)/40.
   DATA2(I)=(DATA2(I)+40.)/40.
20 CONTINUE
C IF(IPLAN.EQ.1)THEN
C PLOT PATTERNS ON FULLY-CIRCULAR GRAPH
IPEN=3
DO 30 I=1,360
   X=DATA1(I)*R*COS(I*DR)
30 CONTINUE
Y = DATA1(I) * R * SIN(I * DTR)
CALL PLOT(X, Y, IPEN)
IPEN = 2
30 CONTINUE
X = DATA1(I) * R * COS(DTR)
Y = DATA1(I) * R * SIN(DTR)
CALL PLOT(X, Y, IPEN)
IPEN = 3
IDOT = 0
DO 60 I = 1, 360
X = DATA2(I) * R * COS(I * DTR)
Y = DATA2(I) * R * SIN(I * DTR)
CALL PLOT(X, Y, IPEN)
IPEN = 2
IDOT = IDOT + 1
IF(IDOT .EQ. 3) IPEN = 3
IF(IDOT .EQ. 3) IDOT = 0
40 CONTINUE
IPEN = 2
X = DATA2(I) * R * COS(DTR)
Y = DATA2(I) * R * SIN(DTR)
CALL PLOT(X, Y, IPEN)
ELSE
C PLOT PATTERNS ON SEMI-CIRCULAR GRAPH
X = DATA1(360) * R * COS(360 * DTR)
Y = DATA1(360) * R * SIN(360 * DTR)
CALL PLOT(X, Y, 3)
DO 50 I = 1, 180
X = DATA1(I) * R * COS(I * DTR)
Y = DATA1(I) * R * SIN(I * DTR)
CALL PLOT(X, Y, 2)
50 CONTINUE
X = DATA2(360) * R * COS(360 * DTR)
Y = DATA2(360) * R * SIN(360 * DTR)
CALL PLOT(X, Y, 3)
IDOT = 0
IPEN = 2
DO 60 I = 1, 180
X = DATA2(I) * R * COS(I * DTR)
Y = DATA2(I) * R * SIN(I * DTR)
CALL PLOT(X, Y, IPEN)
IPEN = 2
IDOT = IDOT + 1
IF(IDOT .EQ. 3) IPEN = 3
IF(IDOT .EQ. 3) IDOT = 0
60 CONTINUE
END IF
C RETURN PATTERN DATA TO ORIGINAL RANGE (-40 DB TO 0 DB).
DO 70 I = 1, 360
DATA1(I) = 40. * DATA1(I) - 40.
DATA2(I) = 40. * DATA2(I) - 40.
70 CONTINUE
C STOP PLOTTING ROUTINE, THEN RETURN
CALL PLOT(12., 0., -999)
RETURN
END

Figure A-4. (continued)
Appendix B. Computer Program ACTEL

The program ACTEL was written to compute the radiation patterns of arrays of parallel $x$-directed dipoles of arbitrary length and location using the active element pattern array analysis methods described in Section 4.3. Specifically, these analysis methods include the complete active element pattern method of (4.3-9), the unit-input active element pattern method of (4.3-14), the phase-adjusted active element pattern method of (4.3-16), the average active element pattern method of (4.3-19) and the hybrid active element pattern method of (4.3-21). The program also compares the array pattern computed using the selected active element pattern method to the pattern computed using ESP by plotting the two patterns on the same graph and tabulating the patterns in an output file. ACTEL is written in the FORTRAN computer language.

B.1 Input Files

ACTEL reads three input files and writes one output file when it is executed. The version of ACTEL used in this study also includes a subroutine which produces radiation pattern plots using
the plotting facilities available at Virginia Tech. The first input file supplied to ACTEL, ARRAY INPUT, is identical to the first input file supplied to PATMULT and contains program control parameters and the locations and excitations of the array elements; for instructions on how to construct ARRAY INPUT, see Section A.1. The input parameter IAEP, which appears on the first line of ARRAY INPUT and is not used by PATMULT, should be supplied to ACTEL. Parameter IAEP should be set to 1 if the complete active element pattern method is to be used, set to 2 if the unit-input active element pattern method is to be used, and set to 3 if either the phase-adjusted, average or hybrid active element pattern method is to be used. The second input file supplied to the program, AEP INPUT, contains the active element pattern data for each element in the array. The construction of this file is explained later. The third input file supplied to ACTEL, FULL INPUT, is generated by ESP and contains the pattern computed by ESP for the fully-excited array. This input file is identical to the second input file supplied to PATMULT. The pattern computed by ESP is plotted along with the pattern computed by ACTEL for comparison purposes.

The first and third input files used by ACTEL are identical to the two input files used by PATMULT, and are described in Appendix A. The second input file used by ACTEL, however, is unique to ACTEL and contains the active element patterns of the array elements. This file, AEP INPUT, must be constructed by the user. To construct AEP INPUT, an active element pattern for each element is created by using ESP to compute the radiation pattern of the array with the active element excited and all other elements loaded with their respective generator impedances. This results in \( N \) separate data files generated by ESP, where \( N \) is the number of array elements. The \( N \) data files each contain an active element pattern corresponding to an array element. The \( N \) data files are appended together without modification to form AEP INPUT. The order in which the active element patterns appear in AEP INPUT should match the order in which the element location and excitation data appear in ARRAY INPUT. Thus, for the example array corresponding to the input file shown in Figure A-1b, the first active element pattern to appear in AEP INPUT should be that of the element located at \( z = -1.2\lambda \), followed by the active element pattern of the element located at \( z = -0.8\lambda \), and so on.
ACTEL requires active element pattern data for every element in the array, even if the average active element pattern method or the hybrid active element pattern method is used. For the average active element pattern method, ESP is used to compute the element pattern of a “typical” element; this pattern then becomes the average element pattern. The file containing the average element pattern data is copied \( N - 1 \) times, and the resulting \( N \) copies of the pattern are appended together to form AEP INPUT. For the hybrid active element pattern method, active element patterns are computed for each “edge” element and for a typical “interior” element. \( N_i \) copies of the interior average element pattern are then created, where \( N_i \) is the number of interior elements. The files containing the edge element patterns and the \( N_i \) identical interior element patterns are appended together to form AEP INPUT, taking care that the order in which the active element patterns appear in AEP INPUT matches the order in which the element locations and excitation data appear in ARRAY INPUT. This repetition of average element pattern data is obviously inefficient but was tolerated for the simple examples presented in this study. For practical use, ACTEL should be modified to eliminate the need for multiple copies of the average element pattern data.

### B.2 Program Output

As with the PATMULT program, the output file created by ACTEL echoes the input data supplied by the user for purposes of verification. Also, the pattern computed using the selected active element pattern method and the pattern computed using ESP are written in tabular form to the output file. Part of an example output file, which corresponds to the example input file shown in Figure A-1b, is shown in Figure B-1. Note that IAEP is set to 2 in ARRAY INPUT; thus, ACTEL uses the unit-input active element pattern method to compute the array pattern. The version of ACTEL used here employs the plotting subroutine listed in Figure A-4 to plot the radiation pattern computed using the active element pattern method as a solid curve and the pattern computed using
ACTEL OUTPUT

NO. OF ELEMENTS = 7
ELEMENT LENGTH = 0.4760
ELEMENT SPACING = 0.4000
IPLANE = 2
IAEP = 2

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<thead>
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<th>EL NO.</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>AMP</th>
<th>PHS</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>0.000</td>
<td>0.000</td>
<td>-1.200</td>
<td>1.00000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.800</td>
<td>1.00000</td>
<td>-72.000</td>
</tr>
<tr>
<td>3</td>
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<td>0.000</td>
<td>-0.400</td>
<td>1.00000</td>
<td>-144.000</td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.00000</td>
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<tr>
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<td>0.000</td>
<td>0.400</td>
<td>1.00000</td>
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<td>0.000</td>
<td>0.800</td>
<td>1.00000</td>
<td>0.000</td>
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<tr>
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<td>0.000</td>
<td>0.000</td>
<td>1.200</td>
<td>1.00000</td>
<td>-72.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>THETA</th>
<th>FULL EXC.</th>
<th>AEP METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-12.8911</td>
<td>-12.8909</td>
</tr>
<tr>
<td>2</td>
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<td>-12.8920</td>
</tr>
<tr>
<td>3</td>
<td>-12.8942</td>
<td>-12.8941</td>
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<td>-12.9432</td>
</tr>
<tr>
<td>9</td>
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<td>-12.9682</td>
</tr>
<tr>
<td>10</td>
<td>-13.0018</td>
<td>-13.0017</td>
</tr>
<tr>
<td>11</td>
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<td>-13.0457</td>
</tr>
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Figure B-1. Example output file produced by ACTEL. This output file corresponds to the input file shown in Figure A-1b.
ESP as a dashed curve on the same graph using a Versatec plotter. The pattern plots for the example considered here are shown in Figure 5.2-1. The program can easily be modified for use with other plotting facilities.

**B.3 Source Code Listing**

The source code for ACTEL, which includes the subroutines unique to the program but does not include the subroutines shared with PATMULT, is listed in Figure B-2. The source code for the subroutines that are shared between PATMULT and ACTEL is listed in Figure A-4.
C******************************************************************************
C ACTEL
C
C THIS PROGRAM GENERATES A RADIATION PATTERN PLOT FROM ACTIVE ELEMENT
C PATTERN DATA COMPUTED BY THE PROGRAM ESP. THE RADIATION PATTERN
C FORMED FROM THE ACTIVE ELEMENT PATTERNS IS COMPARED TO THE PATTERN
C OF THE FULLY-EXCITED ARRAY, WHICH IS ALSO COMPUTED BY ESP.
C
C THE PROGRAM CAN BE CONFIGURED TO CALCULATE ARRAY PATTERNS FROM THE
C FOLLOWING THREE KINDS OF ACTIVE ELEMENT PATTERNS (AEP):
C
C 1. COMPLETE AEP - THE AEP COMPUTED BY FEEDING THE ACTIVE ELEMENT
C   WITH THE SAME EXCITATION THAT IS FED TO THE ELEMENT IN THE
C   FULLY-EXCITED ARRAY.
C 2. UNIT-INPUT AEP - THE AEP COMPUTED BY FEEDING THE ACTIVE ELEMENT
C   AN EXCITATION OF 1 VOLT AT 0 DEGREES PHASE.
C 3. PHASE-ADJUSTED AEP - THE AEP COMPUTED BY FEEDING THE ACTIVE
C   ELEMENT WITH 1 VOLT AT 0 DEGREES PHASE AND LOCATING THE ACTIVE
C   ELEMENT AT THE ORIGIN.
C
C I/O UNIT NO. 5 IS RESERVED FOR PROGRAM READS AND I/O UNIT NO. 6 IS
C RESERVED FOR PROGRAM WRITES.
C
C INPUT FILES (MUST BE SUPPLIED IN THIS ORDER):
C
C ARRAY INPUT - CONTAINS INFORMATION ABOUT THE GEOMETRY AND EXCITA-
C TION OF THE ARRAY.
C AEP INPUT - CONTAINS THE ACTIVE ELEMENT PATTERNS OF EACH ELEMENT
C IN THE ARRAY. THE ACTIVE ELEMENT PATTERNS ARE GENER-
C ATED BY ESP.
C FULL INPUT - CONTAINS THE PATTERN OF THE FULLY-EXCITED ARRAY,
C WHICH IS COMPUTED USING ESP.
C
C OUTPUT FILE:
C
C ACTEL OUTPUT - FILE TO WHICH ERROR MESSAGES, ECHO OF INPUT DATA AND
C TABULATED PATTERNS ARE WRITTEN.
C
C THE PROGRAM PLOTS THE PATTERN COMPUTED USING THE AEP METHOD AND THE
C PATTERN COMPUTED USING ESP ON THE SAME GRAPH USING THE VERSATEC
C PLOTTER.
C
C LATEST REVISION: FEBRUARY 4, 1990
C
C******************************************************************************

REAL APAT(360),FULL(360)
COMPLEX AEP(10,360)
REAL X(10),Y(10),Z(10),A(10),ALPHA(10)
COMMON/BLOC1/ X,Y,Z,A,ALPHA
COMMON/CONS/ PI,DTR
COMMON/FULL/ FULL
COMMON/AEPATS/ AEP
COMMON/APAT/ APAT
PI=3.141592654

Figure B-2. Source code listing for program ACTEL, not including subroutines shared with program PATMULT.
DTR=\pi/180.

CALL READFILE(N,IPLANE,SPACE,ELENGTH,IAEP)

CALL READAEP(N)

CALL AEPPAT(N,IPLANE,IAEP)

CALL NORMALIZE(IAEP)

CALL READFULL

CALL NORMALIZE(FULL)

CALL PLOT PATTERNS.

CALL DBLPLT(IPLOT,APAT,FULL,IPLANE,N,SPACE,ELENGTH)

CALL PLOT(12,0,.999)

CALL OUTDAT(N,ELength,SPACE,IPLANE,IAEP)

Figure B-2. (continued)
STOP
END

C******************************************************************************
C THIS SUBROUTINE READS IN THE ACTIVE ELEMENT PATTERNS GENERATED BY ESP
C FROM THE FILE 'AEP INPUT,' CONVERTS THE DATA FROM POLAR FORM TO
C RECTANGULAR FORM, AND LOADS THE DATA FOR THE PROPER POLARIZATION INTO
C THE PROGRAM ARRAY AEP(I,IDEG), WHERE I IS THE ELEMENT NUMBER AND IDEG
C IS THE DIRECTION IN DEGREES.
C******************************************************************************
SUBROUTINE READAEP(N)
    COMPLEX AEP(10,360),CMPLX
    COMMON/AEPATS/ AEP
    COMMON/CONS/ PI,DTR
    DO 1 I=1,N
        READ(5,*)NPATS, NTYPE, FREQ
        IF(NPATS.NE.1)STOP
        READ(5,*)IEA, NPTS, ANGLE
        IF(NPTS.NE.361)STOP
    C READ IN THE MAGNITUDES AND PHASES FOR THE TWO POLARIZATIONS OF THE
    C ACTIVE ELEMENT PATTERNS. THE PATTERN DATA IS IN 'LINEAR' FORM INSTEAD
    C OF DECIBEL FORM.
    DO 10 IDEG=1,361
        READ(5,*)AMAG1,ANG1,AMAG2,ANG2
        AREAL=AMAG1*COS(ANG1*DTR)+AMAG2*COS(ANG2*DTR)
        AIMAG=AMAG1*SIN(ANG1*DTR)+AMAG2*SIN(ANG2*DTR)
        IF(IDEG.NE.1)AEP(I,IDEG-1)=CMPLX(AREAL,AIMAG)
    10 CONTINUE
    1 CONTINUE
    RETURN
END

C******************************************************************************
C THIS SUBROUTINE CALCULATES THE PATTERN RADIATED BY THE ARRAY USING
C THE CHOSEN ACTIVE ELEMENT PATTERN METHOD.
C******************************************************************************
SUBROUTINE AEPPAT(N,IPLANE,IAEP)
    REAL APAT(360)
    COMPLEX AEP(10,360), SUM, CABS, EXCIT, PHASTERM
    COMMON/AEPATS/ AEP
    COMMON/APAT/ APAT
    COMMON/CONS/ PI, DTR

C THE NEXT 3 LINES ARE NEEDED ONLY BY THE PHASE-ADJUSTED AEP METHOD.
    IF(IPLANE.EQ.1)PHI=0.
    IF(IPLANE.EQ.2)PHI=90.*DTR
    IF(IPLANE.EQ.3)THETA=90.*DTR

    DO 10 IDEG=1,360
        SUM=(0.,0.)
        IF(IAPAT.EQ.1)SUM(1)=SUM+APAT(I)
        IF(IPLANE.EQ.2)THETA=IDEG*DTR
        IF(IPLANE.EQ.3)PHI=IDEG*DTR
    10 ENDIF
    DO 20 I=1,N
        IF(IAPAT.EQ.2)SUM=SUM+EAEP(I,IDEG)
        IF(IAPAT.EQ.3)SUM=SUM+EAEP(I,IDEG)*EXCIT(I)
        20 CONTINUE
    RETURN
END

Figure B-2.  (continued)
C PHASE-ADJUSTED, AVERAGE OR HYBRID AEP METHOD:
    IF(TAEP.EQ.3)THEN
        SUM=SUM+AEPI(I,IDEG)*EXCIT(I)*PHASTERM(I,THETA PHI)
    ENDIF
20 CONTINUE
    APAT(IDE)=CABS(SUM)
10 CONTINUE
RETURN
END

*******************************************************************************
C THIS SUBROUTINE READS THE FILE 'FULL INPUT,' WHICH CONTAINS THE
C PATTERN OF THE FULLY-EXCITED ARRAY, CONVERTS THE DATA TO RECTANGULAR
C FORM, AND LOADS THE DATA FOR THE PROPER POLARIZATION INTO THE PROGRAM
C ARRAY FULL(IDEG), WHERE IDEG CORRESPONDS TO THE DIRECTION IN DEGREES.
C 'FULL INPUT' IS GENERATED BY ESP, AND THE FOLLOWING DATA IS CONTAINED
C IN THE FILE, IN ADDITION TO THE PATTERN DATA:
C
C NPATS = NO. OF PATTERN CUTS IN FILE (SHOULD EQUAL 1)
C NTYPE = 1, FOR RADIATION PATTERN PLOT (NTYPE SHOULD EQUAL 1)
C 2, FOR SCATTERING PATTERN PLOT
C FREQ = FREQUENCY IN MHZ
C IEA = 1, FOR AN ELEVATION PATTERN (XZ- OR YZ-PLANE)
C 2, FOR AN AZIMUTH PATTERN (XY-PLANE)
C NPTS = NO. OF PATTERN POINTS TO BE PLOTTED (SHOULD BE 361)
C ANGLE = FIXED ANGLE FOR PATTERN PLOT (E.G., ANGLE = PHI = 0 FOR AN
C XZ-PLANE PATTERN PLOT)
*******************************************************************************
SUBROUTINE READFULL
    COMPLEX CMPLX,CABS,FULLCOMP
    REAL FULL(360)
    COMMON/FULL/ FULL
    COMMON/CONS/ PI,DTR
    READ(5,*)NPATS,NTYPE,FREQ
    IF(NPATS.GT.1)STOP
    READ(5,*)IEA,NPTS,ANGLE
    IF(NPTS.NE.361)STOP
C THIS SECTION PERFORMS THE SAME FUNCTION AS THE CORRESPONDING SECTION
C IN THE SUBROUTINE 'READAEP.'
DO 40 IDEG=1,361
    READ(5,*)AMAG1,ANG1,AMAG2,ANG2
    AREAL=AMAG1*COS(ANG1*DTR)+AMAG2*COS(ANG2*DTR)
    AIMAG=AMAG1*SIN(ANG1*DTR)+AMAG2*SIN(ANG2*DTR)
    FULLCOMP=CMPLX(AREAL,AIMAG)
    IF(IDEG.NE.1)FULL(IDEG-1)=CABS(FULLCOMP)
40 CONTINUE
RETURN
END

*******************************************************************************
C THIS FUNCTION CONVERTS THE COMPLEX VALUE OF THE EXCITATION COEF. (THE
C FEED CURRENT) FOR THE ITH ELEMENT FROM POLAR FORM TO RECTANGULAR FORM.
C THIS SUBROUTINE IS USED BY THE NORMALIZED AEP METHOD AND THE PHASE-
C ADJUSTED AEP METHOD.
*******************************************************************************
COMPLEX FUNCTION EXCIT(I)
    COMPLEX CMPLX

Figure B.2. (continued)
REAL X(10), Y(10), Z(10), A(10), ALPHA(10)
COMMON/BLOC1/ X,Y,Z,A,ALPHA
AREAL=A(I)*COS(ALPHA(I))
AIMAG=A(I)*SIN(ALPHA(I))
EXCT=CMPLX(AREAL,AIMAG)
RETURN

C***************************************************************************
C THIS FUNCTION RETURNS THE VALUE OF THE SPATIAL PHASE TERM FOR THE ITH
C ELEMENT IN THE DIRECTION (THETA,PHI). THIS SUBROUTINE IS USED ONLY
C WITH THE PHASE-ADJUSTED AEP METHOD.
C***************************************************************************

COMPLEX FUNCTION PHASTERM(I,THETA,PHI)
COMPLEX CEXP
REAL X(10),Y(10),Z(10),A(10),ALPHA(10)
COMMON/BLOC1/ X,Y,Z,A,ALPHA
COMMON/CONS/ PI,DTR
ST=SIN(THETA)
TEMP=X(I)*ST*COS(PHI)+Y(I)*ST*SIN(PHI)+Z(I)*COS(THETA)
PHASTERM=CEXP((0,1.)*2.*DPI*TEMP)
RETURN
END

C***************************************************************************
C THIS SUBROUTINE WRITES THE PATTERN COMPUTED USING ESP AND THE PATTERN
C COMPUTED USING THIS PROGRAM TO A DISK FILE. INFORMATION ABOUT THE
C GEOMETRY AND EXCITATION OF THE ARRAY ARE ALSO WRITTEN TO THE FILE.
C***************************************************************************

SUBROUTINE OUTDAT(N,ELNGTH,SPACE,IPLEANE,IAEP)
REAL FULL(360),APAT(360),X(10),Y(10),Z(10),A(10),ALPHA(10)
COMMON/CONS/ PI,DTR
COMMON/BLOC1/ X,Y,Z,A,ALPHA
COMMON/FULL/ FULL
COMMON/APAT/ APAT

C WRITE OUT GEOMETRY AND EXCITATION DATA.

WRITE(6,20)N,ELNGTH,SPACE,IPLEANE,IAEP
20 FORMAT(' ACTEL OUTPUT',//,' NO. OF ELEMENTS = ',I3,/,'
$ ELEMENT LENGTH = ',F6.4,/,'
$ ELEMENT SPACING = ',F6.4,/,'
$ IPANE = ',I2,/,'
$ IAEP = ',I2,/,)

WRITE(6,21)
21 FORMAT(' EL NO. X Y Z AMP',
$ PHS',/)
DO 30 I=1,N
ALDEG=ALPHA(I)/DTR
WRITE(6,22)I,X(I),Y(I),Z(I),A(I),ALDEG
30 CONTINUE

C WRITE OUT BOTH PATTERNS IN TABULAR FORM.

WRITE(6,23)
23 FORMAT(' THETA FULL EXC. AEP METHOD',/)
DO 50 I=1,360

Figure B-2. (continued)
WRITE(6,24)I,FULL(I),APAT(I)
24   FORMAT(1X,I4,6X,F8.4,8X,F8.4)
50 CONTINUE
   RETURN
END

Figure B-2. (continued)

Appendix B. Computer Program ACTEL
Vita

David F. Kelley was born to H. Frederick and Martha H. Kelley on November 15, 1964, in St. Petersburg, Florida. He graduated from Woodbridge Senior High School, Woodbridge, Virginia, in 1982 and attended Virginia Polytechnic Institute and State University, where he received the Bachelor of Science degree in electrical engineering in 1986 and the Master of Science degree in electrical engineering in 1990. While at Virginia Tech, Mr. Kelley worked as a graduate research assistant and as a graduate teaching assistant. He also worked part time at HY-Tech Research Corporation in Radford, Virginia. He now works in the Advanced Antenna Development Group at Atlantic Aerospace Electronics Corporation in Greenbelt, Maryland. Mr. Kelley is a member of the IEEE.

David F. Kelley