ANTI-SWAY CONTROL OF A CONSTRUCTION CRANE MODELED AS A TWO-DIMENSIONAL PENDULUM

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(ABSTRACT)

Cranes are an indispensable aid to the construction industry, and much responsibility with regard to performance has been placed in the hands of the operator. The problem of controlling sway of the load due to crane motion, or wind effects must be solved dynamically by the operator to increase productivity and maintain safety. At the hands of inexperienced operators safety is sometimes sacrificed in order to expedite the required task. In an effort to minimize the loss of life and equipment, and to maximize productivity a system for actively damping the crane load has been developed.

This paper discusses an active damping system using state feedback control for a crane load modeled as a two-dimensional pendulum. Mathematical analysis indicates that the control theory used to damp the sway in the pendulum may be extended linearly into three dimensions. Thus, two control algorithms, operating independently, can be used to damp sway in two horizontal dimensions.

The designed system responds to sensed displacements of the load from equilibrium. It employs a control arm positioned a small distance below the boom tip that applies a force to the cable to damp the sway of the load. This system is intended to allow less experienced operators to work more efficiently and safely, decreasing training time and increasing overall productivity.
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DEFINITION OF SYMBOLS

L = length of cable from boom tip to payload center of mass

\( g \) = acceleration due to gravity, 386 in/sec

\( z_1 \) = length from boom tip to control arm

\( G = \frac{g}{L - z_1} \) = square of controlled pendulum's natural frequency

\( z \) = vertical displacement of load measured from equilibrium

\( y \) = horizontal displacement of load measured from equilibrium

\( y_1 \) = horizontal displacement of control actuator

\( \Phi \) = force on control actuator

\( x \) = state vector consisting of position and velocity in one dimension = \[
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
\]

\( A \) = continuous time, open loop state matrix

\( B \) = continuous time input matrix

\( C \) = output matrix

\( s \) = Laplace variable

\( \lambda \) = system poles in the \( s\)-plane

\( F \) = feedback gain matrix, \( 1 \times 2 = [f_1 \ f_2] \)

\( u \) = control input, displacement of control actuator

\( K \) = estimator gain matrix, \( 2 \times 1 = [k_1 \ k_2] \)

\( \hat{y} \) = estimated load position

\( \hat{y} \) = estimated load velocity

\( pe \) = magnitude of double estimator pole

\( T \) = discrete time sample interval

\( S \) = the matrix solution to the Riccati equation
CHAPTER 1:

INTRODUCTION

The problems encountered by operators of mobile cranes often result from uncontrolled motion of the load, which can be caused by crane motion, load shifting, or wind. Often, excessive motion of the load leads to overturning, resulting in injuries to operators and surrounding workers. In an attempt to limit the loss of life and equipment and increase productivity, the possibility of actively damping the crane load was investigated.

In industries such as construction, where cranes are used for a variety of tasks, the skill of the operator is relied upon for performance and safety, especially when excessive winds are present. The damping system presented here is designed to assist the operator by automatically damping the sway of the load resulting from crane motion, load shifts, wind, or other disturbances. The system does not attempt to replace the operator, but to make his task easier, allowing less experienced operators to perform more efficiently.

Much work has been done to control linear motion and overhead cranes in order to prevent oscillation of their loads. Most of these systems profile the speed trajectory of the crane based on the load characteristics to accomplish sway-free transport. For many production line applications this provides effective control, since the trajectory is known to the controller beforehand, the motion is linear, the payload is uniform for each move, and the environment has few disturbances. For a construction mobile crane, motion is in two or three dimensions, and the trajectory, length of cable, and payload may differ with
each move. For this reason trajectory control has not been investigated here, but another technique, discussed below, will be used to damp oscillations and limit load sway.

For the purposes of testing the control technique, the crane load was modeled as a pendulum hanging from a fixed point, and a physical model was built. The method of effecting control is to place a yoke around the hoist cable a small distance below the boom tip, which is attached to the boom as shown in Fig. 1. The yoke applies a force to the cable, inducing a slight bend. By defining the magnitude of the control displacement as a function of load position and velocity, effective damping can be accomplished.

Since this technique of effecting control has not previously been attempted on a crane, little was known about its feasibility. Therefore, it was decided to develop and implement a controller capable of damping a one dimensional system for test purposes.

The following steps outline the procedure used in the development of the damping system that was successfully tested.

1. A physical pendulum was built, and a manual control strategy was developed.
2. Data was collected from experiments using manual damping.
3. The pendulum system was analyzed mathematically, and a state space model was developed, which was used to develop a state feedback controller.
4. Constraints were applied which forced the controller to more accurately approximate the manual control and stay within necessary physical boundaries.
5. An automatic damping system was built and tested on the physical pendulum.

Many constraints of the full-scale physical system were included in the design process to ensure that a practical controller would be developed. Limited actuation distance and force, as well as sensor availability and system complexity were considered. The large inertia of the system limits the system response time, and the structural integrity of the boom limits the amount of actuation available.
The purpose of this paper is to prove the feasibility of the control technique presented. This was accomplished by designing and implementing a damping system in one dimension, and showing mathematically that the two horizontal dimensions are uncoupled. Many of the parameters of the controller would be specific to a particular implementation, but the theory behind the design would generalize to other applications.
Figure 1: Diagram of Pendulum Model
CHAPTER 2:

LITERATURE REVIEW

2.1 Introduction

Cranes are indispensable material handling machines used by many industries. They perform a variety of lifting and moving tasks which would be very difficult, or impossible for humans. In most crane applications, movement of the suspended load induces a pendulum motion. Waiting for the pendulation to die down before performing another move slows productivity considerably, since little natural damping is present. Excessive pendulation may damage surrounding equipment, or the crane itself, and injure surrounding workers.

Efforts to eliminate sway during and at the end of motion have led to several types of crane control systems, which include open loop trajectory control, linearizing feedback, intelligent operator interfaces, and robot-crane systems. This paper focuses on developing a system for the construction industry, which has a great need for anti-sway control due to its many crane uses.

2.2 Crane Safety

The construction industry is one of the most hazardous to its workers. A National Safety Council report indicates that 60 deaths and 57 disabling injuries occur per 1000 workers in the construction industry. For all other industries combined, only 14 deaths and 25 disabling injuries occur [Hester 1981]. The causes of accidents vary, but many could no doubt be avoided if a safer crane system were available, since construction cranes
provide many opportunities for injuries due to swinging loads and the chance of overturning.

A study conducted at Stanford University found that construction accidents may cost in excess of 3% of a project's total budget and up to 10% of total labor costs [Levitt 1981]. Adding automation to a site can reduce the insurance premiums that take up between 2% and 10% of the project's cost. These numbers suggest that any system which adds to the safety of the construction site would reduce the total project cost by a significant percentage.

2.3 Crane Productivity

When placing a load at a specific target, the crane operator is usually not aware of the exact location of the final position and is guided by a signalman and a tagman, who also help in stabilizing the load after the position is fixed. Repeated booming up and booming down needs to be done before the load is exactly positioned at its final destination. At the end of each try, the load is stabilized by the other crew members. The considerable time spent carefully lowering and stabilizing the load is wasted for all but the final move.

Wherever cranes are used, the skill of the operator is relied upon for safety and productivity. Studies show that accuracy and swiftness in the placing of objects will greatly improve the overall productivity on the construction site [Everett 1993]. A crane control which allows a novice operator to perform as well as a more experienced one will improve performance on a construction site. In the above example, an experienced operator would require less time to stabilize the load and fewer tries to position it accurately.
2.4 Stability of Mobile Cranes

A survey conducted by the Building Research Station in England found that overturning occurred in 23% of the mobile crane accidents which were studied, and accounted for large percentages of personal injuries and repair costs [Eden 1985]. A crane will overturn when a tipping moment greater than the restoring moment is present for a period of time. The dynamic effects of lifting or transporting a load, as well as other external factors, have a great effect on crane stability.

The "Dynamic behavior of a load lifted by a mobile construction-type crane" was investigated by Ito et al [Ito 1978]. The equations of motion which define the dynamic behavior of the load are derived and experimentally verified. This is the fifth report in a series that treats dynamic loading and forces encountered on a mobile crane. It is determined that the dynamic effects of crane loading can be quite large and can contribute to overturning.

The study of dynamic stability of mobile cranes was performed by Eden et al [Eden 1985]. They found that a number of static and dynamic factors, acting alone or in combination, contribute to the overturning of a mobile crane. These include swinging loads, centrifugal force, and wind, among others. This work suggests that one of the most prevalent causes of overturning is working out-of-level. By making operators aware of the effects of this and other dynamic factors, a safer working environment can be achieved.

2.5 Trajectory Control of Linear-Motion Cranes

The possibility of trajectory control for swing-free moves in linear motion robots was studied by Starr [1985]. His work produced a path-controlled robot manipulator capable of moving model artillery shells, suspended from a hook by a bolt, with limited sway during motion, and a swing-free stop. This system implements open loop control of
the robot's velocity, and requires a transport device capable of constant straight-line motion and rapid changes in speed.

Jones et al [Jones 1988] shows that there exists a family of trajectories that result in sway-damped motion and swing-free stops. Those discussed are variations of controlled acceleration of the transporting device. The system considered is open loop, and requires all information concerning the load to be known for effective control. Also, the acceleration time of the transport device is assumed to be much less than the period of the load.

A more general treatment of the topic of trajectory control is presented by Strip [1989]. He found it was possible to control swing with actuator accelerations of arbitrary length. By including a sensor to detect when the hook was below its support point, this controller is able to estimate the length of pendulum, and adjust the trajectory accordingly. This system is not a closed loop controller, but some data concerning the pendulum enters the control algorithm. The accelerations of the load are assumed to be small compared to gravity, the load mass is considered uniform, and it is assumed that the period of the pendulum can be approximated accurately.

Ridout [1989] presents an overview of existing trajectory control systems, and presents a new one, using linear state feedback control. The work is a treatment of trajectory control of linear cranes, and discusses the shortcomings of several open loop systems before presenting a closed loop approach. The closed loop system requires a sensor to measure the load angle. Several implementation issues are discussed, as are performance considerations.

While investigating the use of intelligent machines for the handling of nuclear waste, Noakes et al [Noakes 1990] used constant acceleration and deceleration to incorporate a swing-free trajectory in a U-shaped path. He demonstrates oscillation
damped algorithms on a full-size crane, using servo-motors, the accuracy of which is essential to the controller's operation. The system must be capable of adequate position sensing and resolution for this approach to be effective.

Moustafa and Ebeid [Moustafa 1992] added a degree of freedom to the problem of controlling overhead cranes. They develop a nonlinear model of overhead cranes moving in simultaneous travel and transverse, which induce sway in both horizontal dimensions. By using feedback of the hoist rope angles and their derivatives, a sway-free trajectory is obtained. The equations of motion are linearized to develop a controller which specifies input torque to the crane motors.

Open loop systems can be effective in assembly-line applications, where the length of the pendulum and desired trajectory are known, but would have difficulty with varied pendulum lengths and trajectories. Several aspects of closed loop systems seem to be applicable to the problem encountered by the construction industry, although linear motion cranes are seldom used.

2.6 Control of Rotary and Jib Cranes

Rotary cranes, with and without jibs attached, are commonly used by the construction industry. Most works concerning the control of rotary or jib cranes consider the complexities of modeling the highly nonlinear crane-load system. Several have implemented control algorithms to reduce sway and position errors in these types of cranes.

Kazakov [1985] investigates the effects that the arrangement of the hoisting ropes had on the load's dynamic behavior during swinging movements. Instead of assuming that a single rope supports the load, he investigated the effect that the geometry of the hoist ropes has on the load's dynamic behavior. Many crane hoists use a pulley system with two
or three hoist ropes, and he discovered that the geometry of the ropes can be used to limit the amount of load sway away from the plane of the boom.

A series of papers have been published concerning the control of rotary cranes by researchers at Osaka University in Japan. Sakawa and Nakazumi [Sakawa 1981] investigated the optimal control of a rotary crane. A control sequence was devised which allows the crane to hoist and rotate to pick and place a load with limited swing during, and at the end of, the transfer. It requires 70 to 150 seconds of CPU time to define the optimal trajectory, which would be too slow for a real time implementation.

This work was followed by several papers [Sakawa 1985, Sato 1988, Yoshimoto 1989] defining the use of an open loop control to move the load close to its destination with limited sway, and a closed loop controller to perform the final positioning, and eliminate any residual sway. The observer works for the entire trajectory, but the control loop is not closed until the load is close to its intended position. The crane boom was initially modeled as a rigid body, but the analysis was expanded in 1988 to include flexible booms and cables, which included the initial assumption as a special case.

Hara et al [Hara 1989] use the jib actuators to control load swing in the vertical plane including the crane boom. They assume that the displacement angle of the load is small, and that motion of the load is induced by extending or shrinking the boom. A semi-optimal control is developed which translates the system from its initial to its final state in minimum time while limiting sway. The system is open loop, and requires the initial and final position of the jib be known before the move is executed.

The aforementioned systems were designed primarily to damp sway resulting from crane motion. While this is an important task, the construction industry requires a more generalized system. Namely, one capable of damping oscillations in all dimensions when the intended trajectory is not known to the controller.
2.7 Advanced Control Techniques

Since the dynamics of linear-motion cranes have been studied extensively, several modern control techniques have been applied. Among these are linearizing feedback, and on-line parameter estimation, which rely on some form of feedback to allow measurement of the load state to enter the control algorithm.

Ridout [1989] improved the time response of his linear feedback controller by including variable damping as a function of the system states, a technique usually used only on second-order systems. A contour map approach is used to shape the damping function, and a general tuning strategy is developed. Variable damping always improved the response time of the system, but at the cost of a more complex control algorithm.

Sagara and Ohkawa [Sagara 1990] implement a discrete time model reference adaptive control system (MRACS) on a linear motion crane both in the air and under the water. The dynamic characteristics are not constant, and may be unidentifiable under the water. Their experimental results show that the MRACS algorithms can control the crane effectively.

Virkkunen [1991] studied the anti-sway problem with a speed-controlled model gantry crane. The root locus method is used to tune a PID controller, which worked well for constant rope length. A control law implementing adaptive pole placement is successfully tested for varying rope lengths.

Fliess et al [Fliess 1991] develop a general state-space model defined from La Grange analysis to model a linear-motion crane. Linearizing feedback is applied, and computer simulations showed that a swing-free trajectory can be followed. To ensure proper controller operation, the reference trajectory of the controller must be fourth-order continuous.
Boustany and d'Andrea-Novel [Boustany 1992] propose an indirect adaptive scheme for mechanical systems such as overhead cranes where the number of actuators is less than the degrees of freedom. Dynamic state feedback can produce full linearization when all parameters are known. Adaptive estimation and control is used in the case of unknown parameters. Global stability is discussed, and simulation results are presented.

Several advanced control techniques can be used to improve the system performance, through adaptively estimating system parameters, or by adapting the control algorithm itself. In a perfect world, where high-speed computers are available on-board cranes, several of these techniques could be used to provide a control algorithm.

2.8 Operator Emulation

Almost every paper that deals with crane automation or control mentions the operator's natural ability to control the complex dynamics of the system without fully understanding them. Through experience, the operator learns the system's response to his inputs, and thereby develops a control law based on his perception of the system's state. By incorporating the operator's knowledge and experience into the control technique, new problems can be solved.

The most direct application of this phenomenon is offered by Javed and Sanders [Javed 1991], who propose using a neural net to control a gantry crane with human expertise. The system is trained with operator commands in order to learn the operational routines of the crane. It uses multi-layer perceptrons with an adaptive learning algorithm which allows the net to implement control based on its experience while continuing the learning process.

A model of the skilled operator in a crane system was developed and validated with simulations by Sutton et al [Sutton 1986]. The skilled operator was found to function as a "complex and versatile information processor" [Sutton 1986]. The crane is
modeled as a multi-loop man-machine system, and the skill development of an operator is studied. Sutton et al suggest that this operator model could be implemented into the crane control, thereby aiding the operator by alleviating the task of sway damping.

2.9 Intelligent Operator Systems

In the modern quest for automation, several researchers have investigated the possibilities of fully automated cranes, or robot-crane interfaces. In cases where the human operator is not considered expendable, the task is made less difficult with the aid of intelligent man-machine interfaces.

Work at the National Institute of Standards and Technology [Dagalakis 1992] has produced a robot crane called the SPIDER (Stewart Platform Instrumented Drive Environmental Robot). It can manipulate a tool in 6 degrees of freedom with great accuracy using a Stewart platform parallel link manipulator. An elaborate operator interface displays all important information, and a 3-D animation window aids in tracking the robot crane's movements. This may be the prototype for the robot crane of the future which will perform human-like tasks on a larger scale.

The benefits of an intelligent man-machine interface have expanded due to improved computing and data transmission technology. With the move to digital crane drives [Haas 1988] it is possible to incorporate the operator's skill into the control technique, which will allow operators to work more efficiently with less training and experience. Problems that are constantly present, such as sway damping, can be controlled by the computer, while those that vary with each move, such as positioning, can be controlled by the operator with assistance from the computer.

Pieska [1988] introduces an interactive control system for an iron scrap handling system. The control of the heavy-duty machinery puts constant mental and physical strain
on the operator. The designed controller uses laser positioning in three dimensions and contains a traditional operator panel with a graphics interface.

Harrigan [1990] discusses the use of model-based robot control of industrial robots. He investigates both automatic control to induce swing-free transport and computer assisted manual control. The computer assists the operator by modifying commands to achieve the intention of the operator. This can be used to avoid obstacles and increase overall safety in the robot's operational area.

2.10 Conclusion

Research to develop safer, more productive cranes has been extensive. However, little has been done to aid the construction industry with its various crane uses. Many controllers attempt to prevent sway, which would otherwise result from crane movement. In a noisy environment the focus must be on damping sway which will be present not only from crane movement, but from external disturbances as well. Instead of trying to remove the operator, one must seek to assist him, creating a more productive and safer crane system. The remainder of this paper discusses a new method which assists the operator by automatically damping load sway resulting from any disturbance.
3.1 Introduction

The first step discussed in the development of the control system concerned the building of a model for testing different control strategies. Experiments implementing manual damping with varied model parameters provided a large data set which was used to verify assumptions and to aid in the definition of necessary control attributes. By scaling the model, it was possible to verify the capability of the proposed system to damp the oscillations of a crane load with physically realizable forces and displacements.

3.2 Definition of Pendulum Model

It was necessary to build a physical model in order to take advantage of human intuitive control capability. The crane load was modeled as a pendulum hanging from a fixed point, as shown in Fig. 1 (p 4). This model provided a basis for defining system attributes as well as control techniques.

The model consisted of a 163 inch long steel cable suspended from a beam with a payload varying from 53 to 300 pounds. The control height (z1) was adjustable from 8 to 14.25 inches, measured from the top of the pendulum. Position and force measuring devices were mounted on the control arm to monitor the magnitude of the force applied and cable displacement at the point of actuation. An ultrasonic distance meter (UDM) was positioned part way down the cable to sense its displacement, and trigonometry was used to calculate the displacement of the payload from the two position measurements. The UDM was not placed at the load itself due to limitations on its effective range. The
load cell and potentiometric distance meter (PDM) were calibrated to increase accuracy and verify their linear behavior in the operating range.

3.3 Developing Manual Control Strategy

Prior research has shown that humans have an ability to control complex systems, even when they possess incomplete knowledge of the system dynamics. Through experience and trial and error, crane operators become capable of damping the oscillations of the load very effectively [Sutton 1986]. This manual control strategy could be translated into a control algorithm implemented with computer logic.

Before a manual control strategy was defined, several manual tests were performed on the physical system to determine its reaction to various control forces. The strategy that performed best was used consistently in the remainder of the tests. During initial testing, the operator found the control actuation counter-intuitive, but after a few tries, he was able to quickly damp the oscillations of the load. For purposes of consistency, one person (Michael Holk) performed all the manual damping experiments.

A verbal description of the strategy follows, and an illustration of this strategy can be found in Fig. 2. Starting with the pendulum at rest at one apex, and the control arm in the zero position, the operator would move the control arm toward the pendulum. When the pendulum was directly below the control arm, the force on the controller was at a maximum. Then the operator would gradually lessen the force on the control arm by allowing it to follow the pendulum to its other apex with a phase lag. When the pendulum reached this apex, the control arm would still be moving in the same direction. The control actuation had the same period as the pendulum, but its phase lag produced the damping effect. A plot showing the load and control displacement with manual damping is contained in Fig. 3.
Figure 2: Illustration of Manual Control Strategy
L = 163 in \hspace{1cm} z1 = 11 in \hspace{1cm} w = 240 lb

x(0) = [ -20.7 \quad 0 ]

Figure 3: Pendulum and Input Displacement with Manual Damping
3.4 Data Collection

The collection of data using manual damping and various model parameters was the next step in the control design. The manual damping strategy provided an example of a working control and a standard against which the performance of other systems could be measured.

A DAS-8 data acquisition card was used in conjunction with Labtech Notebook software to collect data. All three transducers were sampled every 0.2 seconds, producing approximately twenty samples per period of the pendulum. Ten samples per period is generally considered fast enough for accurate sensing, but it was believed that a faster rate would yield better results.

Those parameters varied in the tests were the height of the control arm, the weight of the payload, and the maximum value of the control actuation. The length of the pendulum and the initial conditions were held constant. For each set of parameters the pendulum was released at rest from a fixed, initial, horizontal displacement of twenty inches. Four trials were performed for each set of variables, and the resulting data was averaged to limit the effects of noise and measurement errors. An example of the output using manual damping is shown in Fig. 3.

3.5 Data Analysis

The extensive testing using manual damping performed on the system served several purposes. It provided an extensive database against which to test system approximations and the performance of the computer-based controller. It determined the type of control strategy that was implemented manually, and checked whether it was consistent with what was desired. This data also defined the relationships between model parameters and the maximum actuation force required, that were used to verify theoretical scale factors and to determine the feasibility of this overall control system on real cranes.
The transducer output was scaled to physical quantities and then the position and velocity (estimated) of the load were calculated. The control position was found to resemble saturated state feedback, using position and velocity of the load as states, which was expected, since the operator applied forces relative to his visual perception of the load's state. Saturation was induced in some tests by limiting the actuation distance mechanically, but it was present in all tests. The operator was not able to apply force beyond a certain limit, a characteristic that must be included in the real crane control algorithm due to limited actuation energy.

3.6 Model Scaling

The relationships between control and model parameters indicated how the model would scale, and helped determine its feasibility. Mathematical scale factors were calculated, and compared to those determined by the data. By mathematically examining the system, the theoretical force needed to displace the control arm was found. The estimated time to sufficiently control the load for longer cable lengths was also investigated.

3.6.1 Assumptions

To approximate the force at the control point, the following assumptions were made:

1. The angles in the cable are small.
2. Bend and stretch in the cable are negligible, as is its mass.
3. Tension in the cable is constant throughout.

3.6.2 Derivation of the Approximate Force Equation

The following free body diagrams were used to evaluate forces in the cable and on the control actuator.
Neglecting the effects due to angular velocity and acceleration (by assumption 1), one can define the tension in the cable as
\[
\tau = m \cdot g \cdot \cos \theta
\]  
(3.1)

Summing forces at the control point in the horizontal direction
\[
\Phi - \tau \cdot \sin \alpha + \tau \cdot \sin \theta = 0
\]  
(3.2)

leads to the following force equation
\[
\Phi = m \cdot g \cdot \cos \theta \cdot (\sin \alpha - \sin \theta)
\]  
(3.3)

Recalling Fig. 1 and including the small angle assumption leads to
\[
\sin \alpha = \alpha = \frac{y_1}{z_1}, \quad \sin \theta = \theta = \frac{y - y_1}{L - z_1}, \quad \text{and} \quad \cos \theta = 1
\]

which reduces Eq. (3.3) to
\[
\Phi = m \cdot g \cdot \left( \frac{y_1 - y}{z_1} \right)
\]  
(3.4)

Eq. (3.4) shows that the force should scale linearly as a function of the load weight multiplied by an angle measurement.
3.6.3 Verification of Force Equation

The measured control force was then defined as a function of the model parameters, using Eq. (3.5) as a guide. Performing a least-squares curve fit on the collected data showed that the actuation force was in fact linearly related to the weight of the load. Moreover, the force used was between 34 and 25 percent of the load weight, which was encouraging for implementation issues. The equations derived from the experimental data approximated the theory well enough to indicate that the control forces and displacements would scale linearly.

3.6.4 Settling Time

The settling time achieved with manual damping varied, depending on several model parameters. On the average, it was between three and four time periods of the pendulum. The time to damp the pendulation would not linearly extend to larger systems, but the number of periods should remain constant. Therefore, when longer cable lengths are used, the time to control the pendulum from the same initial displacement angle would increase.

3.7 Conclusion

Since this technique of effecting control had not previously been attempted on a crane, it was important to know the magnitude of the control necessary to achieve damping. The experiments with manual damping indicated that the required force and position of the controller may be realizable on a construction crane. Therefore, the data collected was very useful in defending the proposed system's feasibility. Much of the development of the control algorithm was based on the manual damping since it provided an example of a working control system.
CHAPTER 4:
STATE FEEDBACK CONTROLLER

4.1 Introduction
Since the data supported the assumption that this controller would be a feasible addition to construction cranes, further analysis of the system was performed to define the controller analytically. Several, necessary assumptions were made, then a detailed mathematical analysis was performed, and finally a feedback controller was designed using estimates of unmeasurable states.

4.2 Assumptions
Several aspects of the control problem were difficult to mathematically model, and some that were modeled physically were difficult to mathematically capture. Therefore, several assumptions concerning the system were made.

1. The motion of the pendulum was assumed to be in its linear range. The maximum displacement used in testing was slightly less than 7 degrees. In real cranes, displacements of this magnitude can be dangerous. The maximum allowable displacement angle has been specified as 3.5 degrees [Eden] for mobile cranes. In this operating range the angles of deflection and their derivatives will be small, and the vertical acceleration of the weight will be small compared to gravitational acceleration.

2. The mass of the cable was neglected, as was its stretch. The cable's mass is a negligible percentage of the payload, therefore, it does not enter any calculations. Documentation suggests that stretch in a cable becomes constant after a few loadings
Dynamic stretch in a 100 foot length of cable would be on the order of inches, not enough to influence calculations.

3. Except for the point of control actuation, bending of the cable was also neglected. The weight of the payload and stiffness of the cable will both limit bending, providing a near ideal cable.

4.3 Mathematical Analysis

Before a feedback control could be implemented, the system dynamics were modeled mathematically. This was accomplished by using Lagrange's equations with the assumptions stated previously. The resulting nonlinear equations of motion were then linearized about the pendulum's stable equilibrium by using Taylor's theorem. The resulting fourth-order system consisted of two uncoupled systems, each in a single dimension. A more detailed description of the analysis is provided in Appendix A.

4.3.1 State Space Model

The system in one dimension was represented by the following second-order state space model,

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-G & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
G
\end{bmatrix} u
\]

\(y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\) (4.1)

where

\[G = \frac{g}{L - z_1} = \omega_n^2\]

The state variables were position (\(x_1\)) and velocity (\(x_2\)) of the payload, and the input (\(u\)) was the position of the control arm. The mathematical model neglected natural damping, since it proved to have little effect on the pendulum's behavior, especially when larger payloads were used. The natural frequency of the model (\(\omega_n\)) was dependent on the length of the pendulum below the control arm. In essence, the application of a control
input changed the pendulum from one suspended from a fixed point to a somewhat shorter pendulum suspended from a moving point.

4.3.2 Verification

In order to prove the validity of the mathematical model, the position output of the model in simulation was compared to that of the pendulum. The math model was constructed in Simulink and provided with the same initial conditions as the pendulum. The sampled manual control position used on the pendulum was then input open loop to the math model in a real-time simulation. The results showed that the math model approximated the behavior of the pendulum well. The simulated output is compared with the measured output in Fig. 4.

4.4 Feedback Controller Design

Once the mathematical modeling was complete, the control design could be started. The inputs to the control algorithm were the positions of the load and controller, which were the system's measurable quantities. The results of the manual damping experiments showed that a form of state feedback could be used to damp the pendulum, so an analytical derivation of the required control was performed.

The poles of the model were on the imaginary axis (due to the negligible natural damping) and the controller must push these into the left half of the $s$-plane to induce stability. The controllability matrix of the system was of full rank; therefore, state feedback could be used to place the closed loop poles anywhere in the $s$-plane. The resulting closed loop system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -G - G \cdot f_1 & -G \cdot f_2 \end{bmatrix} \cdot x \quad (4.2)$$

was found by substituting

$$u = -F \cdot x$$

into Eq. (4.1).
L = 163 in  
z1 = 11 in  
w = 240 lb

\[ x(0) = \begin{bmatrix} -20.7 & 0 \end{bmatrix} \]

Figure 4: Pendulum and Model Displacement with Manual Damping
Instead of choosing controller poles and determining the gains to produce them, the feedback gains were chosen by several methods, and the resulting poles calculated only to ensure stability. The feedback gains placed the poles of the closed loop system according to the following equation,

$$\lambda_{cl} = \frac{-G \cdot f2 \pm \sqrt{(G \cdot f2)^2 - 4 \cdot (G + G \cdot f1)}}{2}$$ (4.3)

which was derived from Eq. (4.2). Stability placed the restriction $f2 > 0$ on the feedback matrix, since $G$ was a positive number. Increasing $f2$ pushed the poles to the left and away from the real axis. To bring the poles closer to the real axis, one must increase $f1$ negatively.

Feedback of the velocity of the load must be included in the control to induce stability, because the real parts of the system poles depend on $f2$. Since the velocity of the pendulum was not directly measurable, an estimator based on the mathematical model was designed.

### 4.5 Estimator Design

The load position measurement contained considerable noise due to the indirect way it was obtained. The system's observability matrix was of full rank, indicating the system was completely observable. A reduced-order estimator for velocity could have been built if the position measurement had contained less noise. Instead, a full-order estimator was designed because of its ability to filter the noise in the position measurement and prevent it from corrupting the velocity estimate. The full-order estimator gives the designer flexibility to choose the estimator poles which most effectively reduce the effects of system noise. Preliminary tests on the pendulum showed that the controller worked well when the estimator poles were placed at -2, but theoretical verification was necessary before the value was accepted as the best pole choice.
4.5.1 Estimator Derivation

The open loop observer system matrices were written as

\[
\hat{x} = \begin{bmatrix}
    0 & 1 \\
    -G & 0
\end{bmatrix} \cdot \dot{x} + \begin{bmatrix}
    0 \\
    G
\end{bmatrix} \cdot u
\]

(4.4)

\[
\hat{y} = [1 \ 0] \cdot \dot{x}
\]

Since the system output \( y \) is the only measurable quantity, define \( e = y - \hat{y} \) as the error signal. Including error correction in the estimator system matrices

\[
\hat{x} = A \cdot x + B \cdot u + K \cdot e
\]

(4.5)

\[
= A \cdot x + B \cdot u + K \cdot (y - \hat{y})
\]

and substituting

\[
\hat{y} = C \cdot \hat{x}
\]

yields the closed loop estimator system matrices.

\[
\hat{x} = A \cdot x + B \cdot u + K \cdot y - K \cdot C \cdot \hat{x}
\]

(4.6)

\[
= [A - K \cdot C] \cdot \dot{x} + B \cdot u + K \cdot y
\]

The 2x1 matrix \( K \) must be chosen such that the estimator state matrix \( [A - K \cdot C] \) had poles which were further left in the s-plane than those of the closed loop controller.

4.5.2 Pole Placement

The estimator's characteristic equation was used to place the estimator poles at the desired position in the s-plane.

\[
\Delta_{\text{cl}} = \lambda^2 + k_1 \cdot \lambda + G + k_2
\]

(4.7)

For simplicity, a double estimator pole was chosen at \(-pe\) (\(pe > 0\))

\[
\Delta_{\text{cl}} \text{des} = \lambda^2 + 2 \cdot \lambda \cdot pe + pe^2
\]

(4.8)

where \( \lambda \) denotes the closed loop estimator double pole.

The estimator gains required to place the poles correctly were found by equating the coefficients of the actual and desired characteristic equations. The following values

\[
k_1 = 2 \cdot pe \\
k_2 = pe^2 - G
\]

(4.9)
place both estimator poles at $\lambda = -pe$, a value that has yet to be determined.

### 4.5.3 Choice of Poles

The pendulum can only move at its natural frequency, regardless of the input applied. Therefore, all useful information concerning the position or velocity of the controlled pendulum will be at a specific frequency $\omega_n = \sqrt{G}$. For this application the estimator poles had to allow information at the frequency of the pendulum to pass. The ideal estimator would pass signals at this frequency and attenuate all others.

The estimator's transfer function from input to position estimate

$$\hat{y} = \frac{G \cdot u}{s^2 + 2 \cdot pe \cdot s + pe^2} + \frac{(2 \cdot pe \cdot s + pe^2 - G) \cdot y}{s^2 + 2 \cdot pe \cdot s + pe^2} \quad (4.10)$$

was found in order to place the estimator poles using frequency domain analysis. The control input $u$ was subjected to a second order lowpass filter (regardless of pole choice) to eliminate high-frequency noise. The position $y$ was subjected to a combination of lowpass and bandpass response, which attenuated both constant and high frequency signals.

Further analysis of the second part of Eq. (4.10) led to a method for optimal pole choice. Equating the numerator and denominator showed that the estimator would have unity gain and zero phase shift at the pendulum's natural frequency for any poles. By choosing the estimator pole at $pe = \sqrt{G}$, the lowpass characteristic was eliminated. Thus the noisy position measurement was subjected to a second order bandpass filter before it affected the position estimate and entered the controller.

Bode analysis of the system with the poles placed as above showed the frequency response of the estimator. The transfer function from input to velocity estimate was not used for pole placement, but Figs. 5 and 6 show that its frequency response is that of the derivative of the position estimate.
$G = 2.5395 \text{ rad/sec}^2$  \hspace{1cm} \text{pe} = \sqrt{G}$

Figure 5: Estimator Frequency Response to Control Input
\[ G = 2.5395 \text{ rad/sec}^2 \quad \text{pe} = \sqrt{G} \]

**Figure 6: Estimator Frequency Response to Load Displacement**
4.5.4 Discretization of Estimator

The estimator required discretization for implementation in a computer-based control algorithm. The zero-order-hold (ZOH) discretization algorithm
\[
Ad = e^{Ac \cdot T} \\
Bd = Ac^{-1} \cdot [Ad - I] \cdot Bc
\] (4.11)
defined by Bingulac and VanLandingham [Bingulac] was followed to perform this operation. The estimator state matrix from Eq. (4.6) was substituted for \( \text{Ac} \), and \( \text{Bc} \) was defined as \([B \ K]\), the estimator input matrix multiplying \( u \) and \( y \).

The matrix exponential was calculated using the Cayley Hamilton Theorem, and the discretized estimator system matrices in terms of the system pole (-pe) were found.
\[
Ad = \begin{bmatrix}
1 - pe \cdot T & T \\
- pe^2 \cdot T & 1 + pe \cdot T
\end{bmatrix} \cdot e^{-pe \cdot T}
\]

\[
Bd = \frac{e^{-pe \cdot T}}{pe^2} \cdot \begin{bmatrix}
G \cdot \left(-1 + pe \cdot T + e^{-pe \cdot T}\right) \\
G \cdot pe \cdot \left(-2 + pe \cdot T + 2 \cdot e^{-pe \cdot T}\right)
\end{bmatrix}
\]

\[
= \frac{\left(1 - pe \cdot T + e^{-pe \cdot T}\right)}{pe^2} \cdot \left(-3 + pe \cdot T + e^{-pe \cdot T}\right) + G \cdot \left(1 + pe - e^{-pe \cdot T}\right) \\
+ G \cdot pe \cdot \left(2 + pe \cdot T - 2 \cdot e^{-pe \cdot T}\right)
\] (4.12)

By finding the roots of its characteristic polynomial, one can find the poles of the discrete time estimator are at \( e^{-pe \cdot T} \).

The resulting discrete time estimator system with error correction can be written as
\[
\hat{x}(k+1) = Ad \cdot \hat{x}(k) + Bd \cdot \begin{bmatrix} u(k) \\ y(k) \end{bmatrix}
\]
\[
\begin{bmatrix}
\hat{y}(k+1) \\
\hat{y}(k+1)
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \hat{x}(k+1)
\] (4.13)

for implementation in a computer algorithm.
4.6 Control Law Using Estimated States

The separation principle states that the controller and estimator poles may be chosen independently, since their dynamics are uncoupled [Brogan]. Because the estimator dynamics will be added to the controller dynamics, the choice of $K$ such that the error converges faster than the closed loop system is critical. Analysis of the estimator showed that its poles should be placed at $-\sqrt{G}$, and from Eq. 4.3 the real parts of the controller poles are

$$\lambda_r = \frac{-G \cdot f_2}{2}$$

Through simulation and testing it was determined that placing the estimator poles at $-\sqrt{G}$ would not slow the estimator response to the point where the error would not converge for the range of $f_2$ studied.

4.7 Conclusion

The analytic derivation of the mathematical model and state feedback controller was important because it allows the user to vary model parameters and determine their effect on the system through testing in simulation mode. The symbolic derivation of the discrete time estimator and control law enabled coding of the control algorithm, permitting it to be used to damp the oscillations of the actual pendulum, as opposed to its linear model. It was expected that the choice of feedback gains and estimator poles would be critical for robustness and stability of the system.
CHAPTER 5

SELECTION OF FEEDBACK GAINS

5.1 Introduction

Once the decision to use state feedback was made, the next step was to select appropriate feedback gains. Stability enforces only one constraint, therefore, a wide range of gains could theoretically be used to control the pendulum. In practice, the poles cannot be placed arbitrarily, due to force constraints, inertia, and nonlinearities. The ideal gains would provide effective damping without exceeding the force constraints on the actuator.

Table 1 (p 39) contains a representative set of feedback gains for all the techniques of gain selection discussed in this chapter. All were computed with the control height set at eleven inches. The sets all have similar values for $f_2$, which implies that they should produce similar settling times when applied to the system.

5.2 Manual Control Gains

The gains used by the manual control were found by using a least-squares curve fit on the data collected previously. The control position was defined as a function of the load position and velocity (estimated). For each test a subset of the data was used, where the estimator had converged and the motion of the pendulum was appreciable. While the magnitude of the gains defined the magnitude of the control input, their ratio determined the phase lag between the load and controller.

5.3 Optimality

Since the actuation force and distance will be strictly limited on a crane, optimal control was investigated. The controller must be able to force the system to its
equilibrium in minimal time without exceeding the maximum allowable actuation force or displacement. Only constant gain feedback control was investigated because a simple control algorithm was important to the final design. The manual control results showed that state feedback could be used to control the sway of the pendulum. This reduced the problem of optimality to defining a cost function which included the desired system and control constraints.

5.4 Time Optimal Control

Time optimal control could be applied in simulation with limited actuation distance. This control defines a switching boundary, which would be impractical in real cranes due to the force required by this control and the considerable inertia of the crane system. This control technique was not studied further due to its impracticality.

5.5 Linear Quadratic Regulator (LQR) Control

The primary objective of the controller is to damp oscillations that result from external disturbances, which are assumed to be persistently exciting. Since the plant dynamics are assumed to be linear, a quadratic cost function was defined and the solution obtained from a linear regulator problem. The general LQR cost function can be defined as

$$J = \frac{1}{2} x^T(tf) H x(tf) + \frac{1}{2} \int_{0}^{Tf} \left( x(t)^T Q x(t) + u(t)^T R u(t) \right) dt \quad (5.1)$$

where $H$, $Q$, and $R$ are appropriate weight matrices [Kirk].

For this particular problem, the general function may be simplified. Forcing the system toward its equilibrium can be considered an infinite time process. The initial time was set to zero, and $x(\infty)$ was assumed to be zero due to the effects of natural damping. $Q$, and $R$ were chosen such that they were both positive definite and symmetric, ensuring that the cost function would have a minimum. The simplified cost function was defined as
\[ J = \frac{1}{2} \int_0^\infty \left( x^T \cdot Q \cdot x + u^T \cdot R \cdot u \right) dt \]  

(5.2)

For this infinite time process, the solution is given by the algebraic Riccati equation

\[ \dot{S} = 0 = A^T \cdot S + S \cdot A + Q - S \cdot B \cdot R^{-1} \cdot B^T \cdot S \]  

(5.3)

which was used to define the optimal regulator feedback gain matrix

\[ F = R^{-1} \cdot B^T \cdot S \]  

(5.4)

The feedback control law was then defined as

\[ u = -F \cdot x = -R^{-1} \cdot B^T \cdot S \cdot x \]  

(5.5)

[Kirk]. The LQR algorithm in Matlab was used to find the regulator gains for a number of different weight matrices.

The regulator's feedback gains differed quite a bit from those used manually, which indicated that another cost function may be used to define a different feedback gain matrix.

5.6 Force Minimization

The control force available on a crane will be strictly limited, due to limited actuation energy, and the structural integrity of the crane boom and yoke. Therefore, a cost function was defined which would minimize the force applied to control the load. It was realized that this would not produce a time optimal regulator, but it should provide control with considerably less force being applied to the actuator.

It was necessary to define the force on the control as a function of the positions of the control arm and the load. The dynamic effects of load motion were not included since the accelerations of the load have already been assumed small with respect to gravity. Eq. (3.1) (with \( u = y^1 \)) previously defined this relationship. Since both positive and negative forces had to be controlled, the force equation was squared to produce the following quadratic form,
\[ \Phi^2 = \left[ \frac{m \cdot g}{z_1 \cdot (1 - z_1)} \right]^2 \cdot \left( z_1^2 \cdot y^2 + 2 \cdot z_1 \cdot y \cdot u + u^2 \right) \]  \hspace{1cm} (5.6)

which has a single minimum.

Setting \( u = \frac{z_1}{l} \cdot y \) produced zero force for all \( y \), but implemented no control. It has been shown that in order to induce stability, the input must contain a factor of velocity, which can be added to Eq. (5.6). Incorporating the assumptions used for the LQR produced the following cost function

\[ J = \int_0^\infty \left( z_1^2 \cdot y^2 + 2 \cdot z_1 \cdot y \cdot u + u^2 + w \cdot \dot{y}^2 \right) dt \]  \hspace{1cm} (5.7)

where \( w \) was a positive weight. The constant terms have been eliminated since the actual "cost" of the control was not important, only the feedback gain matrix determining its minimum.

The force minimizing feedback matrix was found by solving the algebraic Riccati equation, which was adjusted due to the cross term between \( y \) and \( u \). The cost function was rewritten symbolically as

\[ J = \int_0^\infty \left( x^T \cdot Q \cdot x + u^T \cdot R \cdot u + 2 \cdot x^T \cdot N \cdot u \right) dt \]  \hspace{1cm} (5.8)

where

\[ x = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}, \quad Q = \begin{bmatrix} z_1^2 & 0 \\ 0 & w \end{bmatrix}, \quad R = I^2, \quad \text{and}, \quad N = \begin{bmatrix} -I \cdot z_1 \\ 0 \end{bmatrix} \]

The Riccati equation became

\[ \dot{S} = 0 = A^T \cdot S + S \cdot A + Q - (S \cdot B + N) \cdot R^{-1} \cdot (B^T \cdot S + N^T) \]  \hspace{1cm} (5.9)

and the feedback matrix for \( u = -F \cdot x \) was defined as

\[ F = R^{-1} \cdot \left( B^T \cdot S + N^T \right) \]  \hspace{1cm} (5.10)
Again, the LQR algorithm in Matlab was used to solve the Riccati equation for the constant feedback matrix minimizing the cost function in Eq. (5.8).

The analysis of the force minimizing feedback gains was interesting due to its simplicity. The gain multiplying the load position ($f1$) was $-\frac{z1}{l}$, which when fed back induced no force, but imparted no control. Stability analysis placed no restrictions on $f1$, so it could be chosen arbitrarily without affecting stability. For the system to be asymptotically stable, and an effective regulator, $f2$ must be greater than zero. Increasing the weight $w$ in the cost function produced a larger value for $f2$, which provided more control and a faster system response.

Substituting the feedback control

$$u = \frac{z1}{l} \cdot y - f2 \cdot \dot{y}$$

(5.11)

into Eq. (3.1) resulted in the following equation defining the control force.

$$\Phi = -\frac{m \cdot g \cdot l \cdot f2}{z1 \cdot (1 - z1)} \cdot \dot{y}$$

(5.12)

As shown, the control force increased linearly with $f2$, therefore, $f2$ must be chosen to provide effective control without requiring excessive force, or attempting to overpower the formidable inertia of the system.

### 5.7 Comparison of Feedback Gains

By examining Table 1, one can see that the manual controller used gains similar to those which would minimize the force needed to damp the oscillations of the pendulum. This comparison was skewed some when a hard saturation non-linearity limited the actuation available. In general, the manual gains seem to be closer to the force minimizing gains. It is most likely that the operator used gains as close as possible to the force minimizing control. The computer should be able to eliminate human errors and improve the efficiency of the force minimizing control.
Table 1

Representative Sets of Feedback Gains

<table>
<thead>
<tr>
<th>R</th>
<th>f1</th>
<th>f2</th>
<th>w</th>
<th>f1</th>
<th>f2</th>
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<tr>
<td>80</td>
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<td>0.1319</td>
<td>100</td>
<td>-0.0675</td>
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<td>-0.0675</td>
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<td>120</td>
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<td>-0.0675</td>
<td>0.1063</td>
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<td>180</td>
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<td>0.088</td>
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</table>

Manual Gains

<table>
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<th>weight</th>
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<th>f1</th>
<th>f2</th>
</tr>
</thead>
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<td>300</td>
<td>2.65</td>
<td>-0.1318</td>
<td>0.0455</td>
</tr>
</tbody>
</table>
5.8 Conclusion

The feedback gains affected both the magnitude and phase of the resulting control strategy. The phase seemed to be more sensitive, since a slight phase shift caused ineffective control actuation, or even unstable behavior. Robustness and the ability to induce damping with the estimated states was important to the final design. Therefore, it was decided that the behavior of the pendulum, subjected to a range of gains, would be used to decide which should be implemented. Testing in simulation was used to verify the effectiveness of gains and determine their scaling to longer pendulums.
CHAPTER 6

EFFECTS OF SATURATION

6.1 Introduction

As stated earlier, saturation of the control input was present in all tests using manual control. It was mechanically induced in some tests, but was present in all due to limits on the force the operator was willing to apply. Simulations showed that decreasing the feedback gains to the point where the maximum control input was comparable to that used by the manual control resulted in poor system settling time. By implementing a saturation function in the feedback loop and increasing the gains, the system settling time was improved without increasing the maximum control input.

6.2 Improvement in Response Time

When a saturation was implemented with optimal feedback gains, true optimality was lost, but system response was greatly improved for a fixed actuation limit. One can incorporate the saturation of the input into the development of the optimal control sequence by using Pontryagin's minimum principle. In this case, since the control input does not saturate frequently, or for extended periods of time, steady state feedback gains were used and the control input was subjected to a hard saturation.

If the feedback gains are increased enough, the control technique approaches the time optimal switching control. For this system the time optimal control would produce a single switching boundary in the phase plane. By implementing a saturation with feedback gains, this boundary was split in two, and a gradual transition of the control input between its extremes was enabled. The definition of the feedback gains determined the rate of the
transition, and the magnitude of the saturation determined the number of switchings required before the system reached its equilibrium. Due to system inertia and force constraints, time optimal control would not be feasible, but by incorporating an aspect of it the system settling time was improved for a fixed maximum actuation.

6.3 Stability Concerns

When the nonlinearity is added to the feedback loop, linear system theory can no longer be used to guarantee stability. Saturation tends to be an inherently stable function; therefore, its addition to the system should cause no stability problems. Since the nonlinear system was of second order, its stability was examined using phase plane analysis. It was possible to show that the system will be stable about its equilibrium point, provided the feedback gains are chosen such that the closed loop poles of the unsaturated system are in the left half of the s-plane.

6.3.1 Definition of Switching Boundaries

Consider the control input

\[
    u = \begin{cases} 
    +\text{sat} & -F \cdot x > \text{sat} \\
    -F \cdot x & |F \cdot x| < \text{sat} \\
    -\text{sat} & -F \cdot x < -\text{sat} 
    \end{cases} \tag{6.1}
\]

which is the result of the saturated state feedback control input. Saturating the input results in switching boundaries in the phase plane, which are defined by the lines where the system changes from linear feedback to constant (saturated) feedback. The switching boundaries in the phase plane are defined by

\[
    \dot{y} = \begin{cases} 
    \frac{\text{sat} - f_1 \cdot y}{f_2} & \text{sat} > f_1 \cdot y > -\text{sat} \\
    \frac{-\text{sat} - f_1 \cdot y}{f_2} & \text{sat} < f_1 \cdot y < -\text{sat} 
    \end{cases} \tag{6.2}
\]

The control input is constant outside the switching boundaries and defined by linear feedback between them. Therefore, the system matrices can be written as
region I
\[
\begin{bmatrix}
\dot{y} \\
\ddot{y}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-G & 0
\end{bmatrix}
\begin{bmatrix}
y \\
\dot{y}
\end{bmatrix} +
\begin{bmatrix}
0 \\
G \cdot \text{sat}
\end{bmatrix}
\]
outside the boundaries

region II
\[
\begin{bmatrix}
\dot{y} \\
\ddot{y}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-G - G \cdot f1 & -G \cdot f2
\end{bmatrix}
\begin{bmatrix}
y \\
\dot{y}
\end{bmatrix}
\]
between the boundaries

6.3.2 Solution of System Equations

The system can be solved outside the boundaries using Laplace techniques for solving linear second-order differential equations. For saturated input, the state space equations can be written in the following form.
\[\ddot{y} + G \cdot y = \pm G \cdot \text{sat}\]
For the following solution, let
\[
t = \text{time} \\
to = \text{time the present region was entered} \\
y(to) = y_0 \\
\dot{y}(to) = \dot{y}_0
\]
Summing the homogeneous solution
\[y = C1 \cdot \sin(\sqrt{G}(t - to)) + C2 \cdot \cos(\sqrt{G}(t - to))\]
and the particular solution
\[y = \pm \text{sat}\]
and incorporating the boundary conditions results in the final solution
\[y = \frac{\dot{y}_0}{\sqrt{G}} \cdot \sin(\sqrt{G}(t - to)) + y_0 \cdot \cos(\sqrt{G}(t - to)) \pm \text{sat} \quad (6.4)\]

One can see that the solution is purely oscillatory outside the switching boundaries, indicating a marginally stable system.

Now the analysis will proceed to region II. By showing the system is stable between the switching boundaries, one can prove that the system will approach the origin, the stable equilibrium of the pendulum. Between the boundaries, the system equation becomes
\[ \ddot{y} + G \cdot f2 \cdot \dot{y} + (G + G \cdot f1) \cdot y = 0 \]

The poles of which are found to be
\[ \lambda = \frac{-G \cdot f2}{2} \pm \sqrt{d}, \quad \text{where} \quad d = \frac{(G \cdot f1)^2}{4} - G - G \cdot f1 \]

After incorporating the boundary conditions, the solution can be written as
\[ y = e^{-\frac{G \cdot f2 \cdot (t - t_0)}{2}} \cdot \left\{ \left( \frac{\dot{y}_0}{\sqrt{d}} + \frac{G \cdot f2 \cdot y_0}{\sqrt{d}} \right) \cdot \sin\left( \sqrt{d} \cdot (t - t_0) \right) + y_0 \cdot \cos\left( \sqrt{d} \cdot (t - t_0) \right) \right\} \]

(6.6)

6.3.3 Illustrated Example

Fig. 7 contains the system response for the following parameters, illustrating the stability of the system. The switching boundaries are drawn to more clearly indicate their presence.

\( y(0) = -20 \)
\( \dot{y}(0) = 10 \)
\( \text{sat} = 1.5 \)
\( F = [f1 \ f2] = [-0.0675 \ 0.1] \)
\( G = 2.5395 \)

6.3.4 Results

Provided that \( f2 > 0 \), the system will be asymptotically stable between the switching boundaries. Since it is marginally stable outside the boundaries, for any initial conditions the states will eventually converge to the point where the input no longer saturates. Together, these conditions imply global stability of the system. However, the mathematical model is only valid when the pendulum is within its linear range.
\[ L = 163 \text{ in} \quad \text{z1} = 11 \text{ in} \]
\[ F = \begin{bmatrix} -0.0675 & 0.1 \end{bmatrix} \]
\[ \text{sat} = 1.5 \text{ in} \]
\[ x(0) = \begin{bmatrix} -20 & 10 \end{bmatrix} \]

Figure 7: Phase Plane of Math Model
6.4 Conclusion

A real crane control system would have an absolute limit on the available actuation displacement, due to yoke size or force constraints. Modeling this as a saturation allowed the user to improve the system response time for a fixed actuation limit. In short, the feedback control was enhanced by subjecting it to a saturation function. Stability was maintained, and the system time response was improved.
CHAPTER 7:

DEFINITION OF TEST MODELS

7.1 Introduction

The feedback control system was tested both in simulation and on a physical pendulum to determine its parameters and capabilities. Simulations allowed observation of ideal time responses and alerted the designer to instabilities, while testing on a pendulum highlighted the effects of nonlinearities, modeling errors, and sensor noise. Both methods of testing provided valuable knowledge concerning the controller's operation. This chapter discusses the systems used for testing both in simulation and on the pendulum defined earlier.

7.2 Discrete Time System Identification Algorithm

The large amount of data taken from manual testing was used in conjunction with an algorithm capable of defining discrete time system matrices from input-output samples [Bingulac]. A Matlab m-file (Appendix B) was written to define the linear discrete time approximation and simulate it with the input sequence from the manual test. The simulated output was compared to the pendulum output and the least-squares error was calculated for a number of different pseudo-observability indices. The discrete time ZOH approximations of the pendulum were used in later simulations.

The pendulum was defined as a single-input single-output (SISO) system, and its control input and measured load displacement were used for the identification. Systems of order two through ten were defined by the identification algorithm, and in almost every one of sixty tests, the third-order system best approximated the pendulum's behavior.
Furthermore, the second order system was usually the next best approximation, based on the norm of the error between the measured and simulated system output.

All the identified systems contained a set of dominant eigenvalues near the unit circle and in some cases outside it. The third order approximation was always stable, but those of higher order often had one pair of eigenvalues outside the unit circle. Providing the algorithm with too many system modes caused it to produce an unstable approximation. In all cases the dominant eigenvalues were very near those defined by the second order approximation, which showed that the dominant system poles were those contained in the mathematical model.

Further analysis of the systems defined by the identification algorithm was not performed, since they were used only to help verify the performance of the controller. A more detailed analysis may have produced the reason that the third order system was the best approximation.

7.3 Testing in Simulation Mode

Preliminary simulations were used to check the operation of the controller after each step of the design process. They determined the mathematical model's response and were very useful in comparing time responses when various feedback gains were used. Testing of the final control system was also performed in simulation. The controller was applied to both the math model and the discrete time approximation to evaluate its effectiveness. The system contained in Figs. 8 to 10 was used to perform the simulations using Simulink in Matlab. For all simulations the estimator was built in discrete time to imitate its implementation in computer code.
Figure 8: System for Testing in Simulation Mode with Simulink
Figure 9: Estimator Subsystem
Figure 10: Pendulum Subsystem
7.3.1 Math Model

For simulations the math model was built as a continuous time system to represent the analog nature of the pendulum. Adjusting $G$ in the pendulum block provided the ability to test with various control heights. Noise present in the actual position signal was found to be Gaussian, by analyzing the transducer outputs in the absence pendulum motion. Its standard deviation was determined and a Gaussian noise signal with the same standard deviation was added to the position signal in simulation. This was used to show the effect the noise in the pendulum measure would have on the estimator, and to determine whether the poles chosen would be effective.

7.3.2 Discrete Time Approximation

A single change in the simulation included the system's discrete time approximation. The math model in the pendulum block was replaced by the discrete time system matrices. The initial conditions of the matrices were set to correspond to those defined by the identification algorithm. For most simulations, the third-order approximation was used, but some larger order systems were included to further test the controller's ability to perform in the presence of modeling errors.

7.4 Testing with the Pendulum Model

The controller's performance on the identified system increased confidence in its robustness and overall damping capabilities, but before the system could be implemented, testing had to be performed on the pendulum model defined earlier. A power actuator and computer interface were designed, and the control algorithm was coded for use with a computer.
7.5 Actuator Design

The actuator used for the control tests would not scale to the size needed for applications in real cranes, but it provided a way to test the control theory on a physical model and study its performance.

The actuator was designed to impart a purely horizontal force, as was the original control arm. A ball screw which operated at an efficiency above 90% [Ball Screws and Actuators Co.] and a DC motor, mounted on a wooden frame were chosen to replace the manually operated control arm on the original pendulum. The sizing of both components was based on speed and force requirements derived from the manual control tests. A diagram of this actuator design is shown in Appendix C.

Attaching the screw to the cable prevented its rotation, so its position could be changed by rotating the shaft. A 90 volt, .5 horsepower, 1725 rpm DC motor was chosen to provide the necessary rotation speed and torque. The force was applied to the cable in the direction of the control actuation only, so that no transverse motion of the load would be induced.

The maximum force applied to the cable was found from the motor torque by the following equation, taken from the ball screw manufacturer's specifications [Ball Screws and Actuators Co.].

\[
\text{Force(lbs)} = \frac{\text{Torque(in-lbs.)}}{.177 \cdot \text{lead(in)}} \quad (7.1)
\]

The rated motor torque was 18.26 in lbs and the lead of the ball screw was .2 in; therefore, when the motor was operated at full speed, the ball nut was moved with a force of 515.8 lbs. The force capacities of both the ball screw and motor were oversized to allow testing with large loads.
7.6 Power Circuit

The controller required motor operation in both forward and reverse; therefore, two 90 volt power supplies were connected such that ±90 volts was attainable from a neutral line. Two relays were used to regulate the polarity and duration of the voltage applied to the motor. These were necessary to isolate the electronics from the power circuitry. A power shutoff switch was added to prevent unwanted motion of the motor during testing of the control software.

For several reasons, on-off control of the motor was used. It was the simplest to build, and most closely approximated the ZOH discrete time model which was used for the discretization of the estimator. Power resistors were added to the circuit for resistive braking, and allotments were made in the control code to limit overshoot. A diagram of the power circuit is contained in Appendix C.

7.7 Computer Interface

As before, three transducers determined the system state, but a DAS-16 data acquisition card was used due to its output capability. Two channels of output were required, one for each relay. The output power capacity of the card was increased using op-amps to boost voltage, and transistors to boost current in order to effectively operate the relays.

Special wiring configurations and additional circuit elements were necessary to limit noise and protect the more delicate components of the system. The diagram of the computer interface in Appendix C shows all the hardware connections as built.

7.8 Hardware Modeling

As expected, the hardware was non-ideal, so it was modeled and incorporated into the overall system design. The behavior of the hardware was studied by examining the difference between the desired and measured control input positions in preliminary tests of
the controller. The hardware was found to act as a 0.4 second time delay, with some low-pass characteristics.

Several ways of modeling the hardware were attempted. A first-order low-pass model could not compensate for the considerable delay present at the pendulum's frequency. A continuous time model of a pure delay also fell short of rectifying the problem.

The final decision was to model the hardware dynamics as a second order discrete time delay, since the delay time was twice the sample frequency, and design a separate feedback loop to correct the problem. The state variable representation of this system was

\[
\begin{bmatrix}
    u_a(k+1) \\
    u_a(k+2)
\end{bmatrix} = \begin{bmatrix}
    0 & 1 \\
    0 & 0
\end{bmatrix} \begin{bmatrix}
    u_a(k) \\
    u_a(k+1)
\end{bmatrix} + \begin{bmatrix}
    0 \\
    1
\end{bmatrix} u_d(k)
\]

(7.2)

where \( u_a \) denotes the actual control input and \( u_d \) the desired control input. Feedback was used to implement a tracking system since both the desired and actual input were measurable. Feeding back the measured input, \( u_a \), through a gain \( f_3 \) placed both hardware poles at \( \sqrt{f_3} \), but caused a non-unity gain in the overall transfer function. Therefore, a gain was added in the input path to achieve accurate tracking, as can be seen in Fig. 11. The final closed loop state space representation of the hardware correction system was

\[
\begin{bmatrix}
    u_a(k+1) \\
    u_a(k+2)
\end{bmatrix} = \begin{bmatrix}
    0 & 1 \\
    \frac{-f_3}{1+f_3} & 0
\end{bmatrix} \begin{bmatrix}
    u_a(k) \\
    u_a(k+1)
\end{bmatrix} + \begin{bmatrix}
    0 \\
    \frac{1}{1+f_3}
\end{bmatrix} u_d(k)
\]

(7.3)

which used feedback of the actual control position to achieve accurate tracking of the desired input command.
Figure 11: Block Diagram of Input Correction System
7.9 Coding Of Control Algorithm

The control algorithm worked well in simulations, but adjustments were necessary before it could be coded and implemented. The choice of hardware defined the necessary control signals, but other decisions had to be made concerning timing and corrections for non-ideal responses and delays. As stated earlier, any phase shift, possibly caused by a delay, would greatly degrade the controller's effectiveness. Also, since various feedback gains were to be tested, it was necessary that the actuator be positioned accurately.

The main part of the control algorithm that required coding was the discrete time estimator. The inputs to the estimator were the position of the load and actuator, and its outputs were the estimated position and velocity of the load at the next sample interval.

The estimated states were used to define the desired position of the actuator, which was subtracted from its measured position to find a net displacement. The hardware correction loop was then applied to the signal to compensate for actuator delays. The net position command was then conditioned to define signals to control the relays which supplied power to the motor.

The following control algorithm was implemented with the BASIC code contained in Appendix D.

1. Get present input and load position
2. Estimate load position and velocity at next sample interval
3. Compute distance to move control arm
4. Apply tracking algorithm
5. Define signals to control relays
6. At correct time, move actuator
7. At end of sample interval, go to 1.
7.10 Conclusion

By comparing the test results from the controller's operation in simulation and on the pendulum, observations were made concerning the accuracy of the math models for both position and force. Although the controller performed well in simulation, testing with the pendulum model was required to define the system parameters and verify the system's ability to perform in the presence of nonlinearities and modeling errors. The simulations supplied a range of gains which provided feasible control, but many of the real world effects could not be included in the simulation. For this reason testing performed with the pendulum was used to determine the system parameters which produced the most effective controller.
CHAPTER 8:

RESULTS OF TESTING

8.1 Introduction

The most important result of simulations and testing with the pendulum was the verification of modeling techniques. The mathematical models of both the pendulum system and actuator force required examination in the presence of the working controller. The control actuator and hardware design would not scale to real cranes, but the theory presented could be used to design crane damping systems provided the mathematical modeling was accurate.

The control parameters specific to this application were also defined during testing with the pendulum, but these may change depending on the specific requirements of an implementation, such as actuator speed, stroke length, and sensor noise. The parameters used in the final testing were the result of preliminary experimentation which determined the best controller behavior, given restrictions on stroke length and actuation force.

8.2 Estimator Poles

The estimator was an integral part of the control algorithm in simulation and in practice. The effectiveness of any controller would be greatly reduced, if not negated, by poor or noisy measurement of the system state. Theoretical analysis showed that the estimator poles should be placed at $-\sqrt{G}$ rad for good estimator performance.

Final testing on the pendulum was performed before the frequency response of the estimator was fully understood. The frequency range of the test pendulum was 1.578 to 1.611 rad, and little noise entered the estimates when the poles were set at $-2$ rad. In
applications where the exact pendulum frequency would be unknown, this type of robustness is a welcome quality.

The noise level used in simulation was calculated from measurements of the noise present in the load position measurement. The noise was all but eliminated by the estimator with its poles set at \(-\sqrt{G}\), and it barely affected the system response. Therefore, it was decided that the optimal choice of estimator poles is at \(-\sqrt{G}\).

8.3 Required Force

Analyzing tests performed with the pendulum was the only way to verify the validity of the force equation (Eq. 3.4), which was necessary to determine how the system would scale to the dimensions of construction cranes. The equation should hold as long as the pendulum stays within its linear range, regardless of the feedback gains used or the accuracy of the actuator position. The position of the control actuator was used to control the pendulum, and the force on the actuator was monitored with a load cell to measure the amount of force required to induce damping.

All tests with the pendulum could be used to verify the accuracy of the mathematical force equation, since the force on the actuator was measured by a strain gage. The actual control force used was very close to the theoretical force defined by Eq. (3.4), substituting \(\hat{Y}\) for the load position. Typical results are contained in Fig. 12, which shows the actual and calculated force for two different sets of feedback gains. The measured force was slightly less than the calculated value, most likely due to estimator convergence or small modeling errors.
Figure 12: Measured and Calculated Control Forces
In simulation, force was calculated by using Eq. (3.4). The calculated force was in fact minimized by the feedback gains developed earlier. Fig. 13 shows that the difference in the calculated input forces for the following sets of feedback gains

\[ F_{\text{reg}} = [0.005 \ 0.1180] \quad F_{\text{fmin}} = [-0.0675 \ 0.1180] \]

was not very substantial. As can be seen, these simulations used the same value for \( f_2 \), but \( f_1 \) was chosen according to Table 1. The dominant parameter in determining the amount of input force was \( f_2 \), which also determined the amount of damping applied to the pendulum. Since the input force was similar for the different gain matrices, other criteria must be used to determine the gains providing the best overall performance.

8.4 Saturation

As was stated earlier, saturation was used to increase the feedback gains without increasing the maximum control displacement. Due to problems with the actuator, a valid assessment of the role of saturation in the pendulum testing could not be found. However, its effect in simulation was studied. Fig. 14 plots the input and output used for saturation levels at 2 in and 1 in. The system response is almost identical, but comparison of the input force (Fig. 15) reveals that less maximum force is needed. Further decreasing the saturation hindered the system response. By limiting the maximum actuation distance, the maximum force requirement was decreased while sacrificing very little performance.
L = 163 in  \quad z1 = 11 \text{ in}  \quad w = 240 \text{ lb}

F_{\text{reg}} = [0.005 \quad 0.1180]

F_{\text{fmin}} = [-0.0675 \quad 0.1180]

Figure 13: Calculated Control Force for Different Feedback Gains
Figure 14: Model Input and Output for Different Saturation Levels
$L = 163 \text{ in}$ \quad $z_1 = 11 \text{ in}$ \quad $w = 240 \text{ lb}$

$sat1 = 1 \text{ in}$

$sat2 = 2 \text{ in}$

$F = \begin{bmatrix} -0.0675 & 0.1180 \end{bmatrix}$

Figure 15: Control Force for Different Saturation Levels
8.5 Hardware Correction

Simulations did not provide good modeling of the necessary hardware correction; therefore trial and error with the actuator was required to define the correct feedback gain. \( f_3 = .8 \) provided fast tracking convergence with limited overshoot for the tests performed. Although some overshoot was present, it was found that fast convergence was more important because the testing was designed to evaluate the performance of a particular feedback gain matrix. Any lag between the desired and actual input greatly affected the gains which were administered to the pendulum. It is assumed that more effective actuator design and control would be implemented in full-scale experimentation.

8.6 Control Gains

The control gains for a particular application will be limited by actuator speed, as well as force restrictions. Forces applied to the cable will be translated to the crane boom; therefore, the structural integrity of the boom will influence the amount of control force available. The maximum speed of the actuator also presents a factor limiting the magnitude of the applicable control gains.

8.6.1 Testing with the Pendulum

The actuator was designed to provide more speed than would be needed for reasonable feedback gains, but unforeseen delays caused by slow code execution enforced a speed limit which had to be addressed. Various control gains were tested on the pendulum model, but the actuator's performance (even with feedback correction) limited the accuracy of the actuator's position. To find the actual feedback gains applied to the pendulum, a least-squares curve fit was performed, using the estimated state variables and the measured control input. The curve was compared to the measured input position, and those tests which provided a good fit were used in evaluating the performance of various feedback gains.
The tests used to show the accuracy of the force equation will again be used to evaluate the performance of the minimum force gains. The curve fit revealed that the actual gains applied to the pendulum were

\[
F_{tr1} = \begin{bmatrix} -0.0512 & 0.1058 \end{bmatrix}
\]
\[
F_{tr2} = \begin{bmatrix} -0.0439 & 0.1641 \end{bmatrix}
\]

The following graphs show the effect that increasing \( f2 \) had on the input displacement, force, and the overall system response.

Fig. 16 shows the pendulum time response for both sets of gains with similar initial conditions. The added force and input displacement in \( tr2 \) leads to better damping. Fig. 17 contains the input displacement and force (measured) for the two tests. For a particular implementation, the available force would most likely dictate which feedback gains are used.

The response of the pendulum to the regulator gains was compared to that of the minimum force gains. The pendulum output with the following feedback matrix (from a curve fit)

\[
F_{tr3} = \begin{bmatrix} -0.0092 & 0.1637 \end{bmatrix}
\]

was compared to the output from \( tr2 \), which used a similar value for \( f2 \). Fig. 18 shows that the system response is almost identical. This figure also serves as proof that the mass of the pendulum does not affect its time response. The load used for \( tr2 \) was 240 lb and 53 lb for \( tr3 \). Once scaled appropriately, the forces were compared and were found to be almost equal in magnitude.

These tests confirmed previous results that only \( f2 \) affected the amount of damping and force applied to the pendulum.
\begin{align*}
L &= 163 \text{ in} \quad z1 = 11 \text{ in} \quad w = 240 \text{ lb} \\
F_{tr1} &= \begin{bmatrix} -0.0512 & 0.1058 \end{bmatrix} \\
F_{tr2} &= \begin{bmatrix} -0.0439 & 0.1641 \end{bmatrix}
\end{align*}

![Graph showing pendulum displacement for different feedback gains](image)

**Figure 16:** Pendulum Displacement for Different Feedback Gains
\[ L = 163 \text{ in} \quad z_1 = 11 \text{ in} \quad w = 240 \text{ lb} \]

\[ F_{tr1} = \begin{bmatrix} -0.0512 & 0.1058 \end{bmatrix} \]

\[ F_{tr2} = \begin{bmatrix} -0.0439 & 0.1641 \end{bmatrix} \]

**Figure 17:** Control Displacement and Force for Different Feedback Gains
\( L = 163 \text{ in} \quad z_1 = 11 \text{ in} \quad w = 240 \text{ lb} \)

\[
F_{tr2} = \begin{bmatrix} -0.0439 & 0.1641 \end{bmatrix}
\]

\[
F_{tr3} = \begin{bmatrix} -0.0092 & 0.1637 \end{bmatrix}
\]

**Figure 18**: Pendulum Displacement for Different Feedback Gains
8.6.2 Testing in Simulation Mode

The system's response to any set of feedback gains could be determined in simulation, and theoretical forces calculated. A wide range of feedback gains was implemented, and the system response studied. The effects that saturation had on both force and system response was also investigated further.

It was observed in simulation, that the optimal regulator gains induced almost no phase shift in the pendulum, but the minimum force gains did cause a phase shift. Fig. 19, shows the uncontrolled pendulum position output, as well as that of the pendulum controlled with regulator and minimum force gains. The phase shift is evident, but it did not affect the induced damping, as the figure illustrates.

The system performed equally well for any reasonable value of $f_1$. Increasing $f_2$ reduced settling time, but increased the input force. The simulated results verified earlier findings from testing with the pendulum. However, the question of robustness was studied more deeply. In the presence of large noise levels, the regulator gains provided slightly better performance.

The final test of robustness was devised by simulating the controller with the third order discrete time approximation of a test using manual damping with the control height at 10 in. The estimator parameters and feedback gains were determined with the control at 11 inches, about a 10% difference. Here, the regulator gains showed a marked advantage over the force minimizing gains. Fig. 20 compares the system output for the following gain matrices.

$$F_{reg} = \begin{bmatrix} 0.005 & 0.1180 \end{bmatrix} \quad F_{f_{min}} = \begin{bmatrix} -0.0675 & 0.1180 \end{bmatrix}$$

Fig. 21 shows the input displacement and force for a load of 240 lb. used to produce the result in Fig. 20.
Figure 19: Displacement of Controlled and Uncontrolled Model
to Illustrate Phase Shift
L = 163 in  
$z_1 = 11 \text{ in}$  
$w = 240 \text{ lb}$

$F_{\text{reg}} = [0.005 \ 0.1180]$  
$F_{\text{fmin}} = [-0.0675 \ 0.1180]$  

Figure 20: Simulation Output with Induced Modeling Errors
\[ L = 163 \text{ in} \quad z1 = 11 \text{ in} \quad w = 240 \text{ lb} \]

\[ F_{\text{reg}} = \begin{bmatrix} 0.005 & 0.1180 \end{bmatrix} \]

\[ F_{\text{fmin}} = \begin{bmatrix} -0.0675 & 0.1180 \end{bmatrix} \]

**Input Displacement**

![Graph of Input Displacement](image)

**Control Force**

![Graph of Control Force](image)

- - - Input for \( F_{\text{reg}} \)
- - - Input for \( F_{\text{fmin}} \)

**Figure 21:** Input Displacement and Force with Induced Modeling Errors
8.7 Scaling

When longer cables are used, the frequency of the pendulum is reduced. To achieve the same amount of damping, the value of \( f_2 \) must be increased because the real part of the controller poles

\[
\lambda_r = \frac{-G \cdot f_2}{2}
\]

depends upon the square of the pendulum frequency. To achieve the same damping for all pendulum frequencies, let

\[
f_2 = \frac{\text{Const}}{G} \quad (8.1)
\]

where the constant depends upon the amount of damping desired.

For a sinusoidal function, the maximum magnitude of the velocity equals the frequency multiplied by the maximum magnitude of the position. Therefore, the velocity is decreased as a percentage of the displacement for long pendulum lengths. The damping is dependent upon the velocity; therefore, a greater \( f_2 \) is required to achieve effective damping at lower frequencies.

A pendulum with the same scale as the test model was simulated with its dimensions in feet. The system output and input force for the following parameters:

- \( L = 163 \text{ ft} \)
- \( z_1 = 11 \text{ ft} \)
- \( F = \begin{bmatrix} 0.1180 & 1.1685 \end{bmatrix} \)
- \( \text{sat} = 1 \text{ ft} \)
- \( \text{weight} = 240 \text{ lb} \)

are shown in Fig. 22. Damping was achieved within two cycles of the pendulum, and the force was less than 10% of the load weight. Another test was done with the saturation at .5 ft. The decrease in force (4 lb) was not substantial, but the effective damping was noticeably worse.
L = 163 ft  
z_1 = 11 ft  
w = 240 lb  
sat = 1 ft  

$F = \begin{bmatrix} 0.1180 & 1.1685 \end{bmatrix}$

**Figure 22: Model Displacement and Control Force**
The ratio of the regulator gains for the longer pendulum also changes, as shown above. In order to provide a constant damping effect over a range of pendulum lengths, a design should choose the constant in Eq. (8.1), and calculate the regulator gains which define the correct value of f2. The controller would implement gains depending upon the measured length of the cable.

8.8 Conclusion

Testing of the control system shows that for an actuator with limited performance characteristics, if the dimensions of the pendulum are known to the controller, effective damping can be achieved. Simulations showed that the feedback matrix which minimizes the linear regulator cost function defined by Eq. (5.2) should be used by the controller. It provided damping with less actuation distance and was more robust than other gain matrices studied. The magnitude of the estimator poles for any application should be the natural frequency of the pendulum. As the pendulum length changes throughout a cycle, the feedback gains and estimator equations would require periodic updating.

For another implementation, the feedback gains may have to be adjusted depending on the amount of force, and actuator speed available. It is important to note that system response and force are inversely related. Therefore, force limitations will slow system response, so if force is extremely limited, then this controller may not be a feasible solution to the problem.
CHAPTER 9:

IMPLEMENTATION ON CONSTRUCTION CRANES

9.1 Introduction

The control theory and mathematical modeling techniques have been derived and can be used for implementation issues. For construction cranes the control theory and position measurements must be extended into two dimensions. Since a crane load is not a simple pendulum, compensations for additional degrees of freedom must also be included in the design. Due to the various uses of cranes, the system may require fine-tuning to best suit the particular needs of each industry.

9.2 Extension into Two Horizontal Dimensions

Recalling the linearized mathematical model developed earlier, and the analysis in Appendix A, it has been shown that the two horizontal dimensions are uncoupled in the linear range of the pendulum. Therefore, the load can be damped in two dimensions by identical controllers acting independently, provided the measurement of the system state can be broken down into components in each dimension. The one dimensional control arm was a special case of the original design that was capable of actuation in two dimensions.

9.2.1 Actuation in Two Dimensions

The original control arm designed by Dr. Yvan Beliveau was a yoke surrounding the cable, capable of actuation in two dimensions. By examining Fig. 23, and
Symbols:

\( x_0, y_0 \) = equilibrium position

\( x, y \) = displacement from equilibrium

\( P_1, P_2 \) = length of actuator position to command

Figure 23: Diagram Of Two Dimensional Yoke
using the Pythagorean Theorem, the following equations for actuator positions are derived.

\[
\begin{align*}
P1 &= \sqrt{x^2 + (y_0 + y)^2} \\
P2 &= \sqrt{y^2 + (x_0 + x)^2}
\end{align*}
\] (9.1)

The control signals \( x \) and \( y \) would be defined by two controllers acting independently. Actuation distance commands are found by trigonometry and would be specific to the geometry of the particular actuator used.

9.2.2 Measure of Load Position

The most obvious method of sensing the load position requires the use of accelerometers placed at the load. Solid-state sensor technology would enable this measurement scheme, but would require two integrators and be susceptible to drift. The control displacement could most likely be determined by a measure internal to the actuator position controller. However, the sensors would have to operate in harsh conditions which may damage the sensor itself or its connections to the control center, inhibiting the controller.

Another method to measure the load position is to measure the force on the controller. Assuming the actuator in positioned accurately, the measurement can be compared to the expected force, and an error signal defined. Hydraulic actuators presently used on some cranes have pressure sensors inside the cylinder which are used to determine the load weight. Provided that hydraulic actuation will be used for the controller, the same method may be used to determine the force on each piston in the yoke. The position of the load may be determined by the following equation, derived from Eq. (3.4), which has been shown to provide an accurate approximation of the control force.
\[ y = y_1 - \left( \frac{\Phi}{m \cdot g} \cdot \frac{y_1}{z_1} \right) \cdot (1 - z_1) \]  \hspace{1cm} (9.2)

By comparing the actual force in each piston to the measured force, it would be possible to determine the actual load position and input it to the estimator. This assumes that the quantities in Eq. (10.2) are reasonably well known, and the trigonometry involved in the measurement can be determined. This method does not require sensors to be exposed to the elements, since the pressure sensors would be located within the hydraulic cylinders.

### 9.3 Additional Degrees of Freedom

Since the controller is designed to respond only to sensed perturbations of the load, any unwanted movement of the controller would have a destabilizing effect. Flexing of the boom due to wind and prescribed movements of the load would change the position of the controller relative to the load, just as a defined control actuation would. Therefore, it is imperative that this movement be sensed and included in the control algorithm. Placing an accelerometer on the yoke bypasses modeling the complex dynamics of the crane boom. It allows the damper to monitor only their effect on the yoke, and make the necessary corrections to the position of the damper, thereby maintaining its position relative to the load as defined by the control algorithm. The control algorithm can be adjusted to account for yoke movement by adding an input to the open loop system as shown

\[ \dot{x} = \begin{bmatrix} 0 & 1 \\ -G & 0 \end{bmatrix} \cdot x + \begin{bmatrix} 0 & 0 \\ G & G \end{bmatrix} \cdot u \]  \hspace{1cm} (9.3)

where \( u_b \) represents the displacement of the yoke (double-integrated accelerometer output). This feature requires an external sensor, but if the movements of the boom and yoke are appreciable, damping may be impossible without knowledge of them.
9.4 Conclusion

The theory presented to this point defined the general development of the control algorithms necessary to design a damping system for a construction crane. Although several constraints expected to be found on cranes have been incorporated into the general design, it is inevitable that other adjustments must be made before a working damper is in production. Simulation results show that limits of actuation distance and/or force, which would be present on a real crane, do not appreciably hinder the damper's effectiveness. The control law parameters should be chosen as defined earlier, but may need to be adjusted, due to force limitations or other constraints. It has been shown that the technique of damping a crane load by way of a control arm is a viable method of increasing crane productivity and safety.
CHAPTER 10

CONCLUSIONS AND FUTURE WORK

10.1 Summary

The strategy of controlling the crane load with a yoke surrounding the cable has been shown as an effective way to damp oscillations. The accuracy of the mathematical models of the pendulum and the input force have been shown, and a working controller has been demonstrated in one dimension.

As built and in simulation, the controller is capable of damping the load's oscillations from an initial displacement of 3.5 degrees in 2 to 4 time periods of the pendulum, using a maximum of 10% of the load weight. Modeling and experimentation have shown that these quantities will scale linearly to the dimensions of construction cranes. Thus, the feasibility of the control strategy as defined has been demonstrated.

One possible setback is that the controller may use too much force to control the load. If a gain matrix could damp the sway in a similar time, while requiring about 5% of the load weight, the controller would be greatly improved. Testing on a full-scale crane, with control in both horizontal dimensions would be necessary before a marketable controller is designed.

10.2 Variable Damping

Variable damping as a function of the system state was studied by Ridout [1989]. It is possible that this technique was used unknowingly by the manual control operator. Using little damping when the load is out of control, and gradually increasing the damping when the load was brought under control could improve the system response. Instead of
saturating a state feedback control, the operator may have adjusted the feedback matrix depending on the position of the pendulum. The variable damping that results must be relatively simple to implement, due to limited computational power on a crane.

10.3 Three Dimensional Models

The project must also continue toward the development of a three dimensional controller. By building a physical model to control the two horizontal dimensions, or incorporating the coupling of the nonlinear equations of motion, the true three dimensional system may be studied. Due to the proprietary nature of crane booms, it is necessary to work closely with industry to build a full-scale model. Crane manufacturers could use their present specifications to guide designers as to the maximum control force available.

10.4 Conclusions

The system developed using linear feedback may be of use to other industries, but its current force requirement would probably disallow implementation in construction cranes due to present design criteria. The estimator design, or control law using saturation can be implemented in linear-motion cranes, or possibly some other applications where damping is necessary and more force is available.

When improving the design, it is important to consider that limited computational power is available on a crane, and any sensors necessary will have to perform in very diverse and harsh conditions. This may prove to eliminate the possibility of damping systems from cranes for now, but as more intricate controls are introduced, the damping mechanism discussed here may in time find its way onto construction cranes.
REFERENCES


Ball Screws and Actuators, Co. (1990) Catalog No. 91-2. San Jose, California.


Appendix A

DERIVATION OF MATHEMATICAL MODEL

A.1 Introduction

A mathematical model of the controlled pendulum was necessary for the derivation of the control and estimator algorithms. This appendix contains the derivation of the math model for the pendulum shown below. The notation used in this appendix may differ from that used in the rest of the paper due to the three dimensional model presented here.

Diagram of the Three Dimensional Pendulum
A.2 Symbols

\( x^1, y^1, z^1 \) = position of control arm

\( x, y, z \) = position of payload measured from origin

\( T \) = potential energy of the system

\( V \) = kinetic energy of the system

\( q_i \) = displacement variable in the \( i \) direction

A.3 Assumptions

1. The cable length, \( L \), is constant.

2. Bend and stretch in the cable are negligible.

A.4 Derivation

The following Lagrangian function, \( \mathcal{L} \), was used to define the equations of motion.

\[
\mathcal{L} = T - V \tag{A.1}
\]

where

\[
V = m \cdot g \cdot z
\]

and

\[
T = \frac{1}{2} m \left( \dot{x}^2 + \dot{y}^2 + \dot{z}^2 \right) \tag{A.2}
\]

represented the system's potential and kinetic energy, respectively.

Since \( V \) is not dependent upon velocity, the following form of Lagrange's equation was used.

\[
\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0 \tag{A.3}
\]

Substituting Eq. (A.2) into Eq. (A.3) results in the following equations of motion for the pendulum load.

\[
m \cdot \ddot{x} + m \cdot \ddot{z} \cdot \left( \frac{\partial z}{\partial x} \right) + m \cdot \ddot{z} \cdot \frac{d}{dt} \left( \frac{\partial z}{\partial x} \right) - m \cdot \dot{z} \cdot \frac{\partial^2 z}{\partial x^2} + m \cdot g \cdot \frac{\partial z}{\partial x} = 0
\]

\[
m \cdot \ddot{y} + m \cdot \ddot{z} \cdot \left( \frac{\partial z}{\partial y} \right) + m \cdot \ddot{z} \cdot \frac{d}{dt} \left( \frac{\partial z}{\partial y} \right) - m \cdot \dot{z} \cdot \frac{\partial^2 z}{\partial y^2} + m \cdot g \cdot \frac{\partial z}{\partial y} = 0 \tag{A.4}
\]

By the Pythagorean Theorem, \( z \) can be written as a quadratic function of \( x \) and \( y \).
\[ 0 = z^2 + 2 \cdot z \cdot (z_1 - L) - 2 \cdot L \cdot z_1 + x^2 - 2 \cdot x \cdot x_1 + y^2 - 2 \cdot y \cdot y_1 + 2 \cdot L \cdot \sqrt{x_1^2 + y_1^2 + z_1^2} \tag{A.5} \]

Applying the quadratic formula, and choosing the stable equilibrium of the pendulum (z = 0) as the root of interest, the following equation for z is developed

\[ z = L - z_1 - \sqrt{z_1^2 + L^2 - x^2 + 2 \cdot x \cdot x_1 - y^2 + 2 \cdot y \cdot y_1 - 2 \cdot L \cdot \sqrt{x_1^2 + y_1^2 + z_1^2}} = L - z_1 - \sqrt{C} \tag{A.6} \]

Taking time derivatives of \( z \) and substituting into Eq. (A.4) results in the following equations for the horizontal acceleration of the load

\[ \ddot{x} = C^{-2} \cdot \left( x_1 - x \right) \cdot \left( \frac{1}{4} \cdot \dot{C}^2 - \frac{1}{2} \cdot C \cdot \ddot{C} \right) + \frac{g \cdot (x_1 - x)}{\sqrt{C}} \]
\[ \ddot{y} = C^{-2} \cdot \left( y_1 - y \right) \cdot \left( \frac{1}{4} \cdot \dot{C}^2 - \frac{1}{2} \cdot C \cdot \ddot{C} \right) + \frac{g \cdot (y_1 - y)}{\sqrt{C}} \tag{A.7} \]

To linearize the system, take its partial derivatives with respect to \( x, \dot{x}, x_1, \dot{x}_1, y, \dot{y}, y_1, \dot{y}_1 \)

and evaluate at the pendulum's stable equilibrium point, where all these quantities and \( z \) equal zero.

The resulting linearized system in two dimensions becomes

\[
\begin{bmatrix}
\ddot{x} \\
\ddot{x} \\
\dot{y} \\
\ddot{y}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
-G & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -G & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x} \\
y \\
\dot{y}
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
G \\
0
\end{bmatrix} \begin{bmatrix}
x_1 \\
y_1
\end{bmatrix} \tag{B.8}
\]

where

\[ G = \frac{g}{L - z_1}. \]

As can be seen, the two dimensions are uncoupled, enabling the system to be split into two systems, each in a single dimension.
Appendix B

M-FILE TO IMPLEMENT IDENTIFICATION ALGORITHM

function [a,b,c,d,xo]=ident(data,no,m,p)
%
% Matrix data holds the input-output data in columns
% m = number of inputs
% p = number of outputs
% no = set of available pseudo-observability indices

%initialize output matrix and set ermin to arbitrarily high value
ermm = zeros(length(no),1);
ermin = 1000;

% find the size of the data matrix
[M,N]=size(data);

% create matrices that will enable construction of the Z matrix
Udata=data(:,1:m);
Ydata=data(:,m+1:m+p);

% Start loop over all available pseudoobservability indices
[len,nul] = size(no);
for l=1:len

% Set of pseudoobservability indices to use
pseu=no(l,:);

% Find rank n
n=0;
for j=1:p
n = n + pseu(j);
end;

% Define i for later use
i=norm(pseu,inf) + 1;

% Find the selection vectors Vi,VA from poi
vi=zeros(n,1);
va=ones(n,1);
counter=1;
flag=zeros(p,1);
for j=2:(i-1);
    for k=1:p
        if j <= pseu(k)
            vi(counter,1)=1;
            va(counter,1)=0;
        end;
        if flag(k,1)==1 counter=counter-1; end;
        if vi(counter,1)==0 flag(k,1)=1; end;
        counter=counter+1;
    end;
end;
vi(n,1)=0;
va(n,1)=1;

% Find the selection vectors Vli,Vld
vli=zeros(i*p,1);
vld=ones(i*p,1);

counter=1;
for k=1:i
    for j=1:p
        if pseu(j)>=k
            vli(counter,1)=1;
            vld(counter,1)=0;
        else
            vli(counter,1)=0;
            vld(counter,1)=1;
        end;
        if k>pseu(j)+1
            vld(counter,1)=0;
        end;
        counter=counter+1;
    end;
end;

% Create selector matrices
In=eye(n);
Si=[];Sa=[];
for j=1:n
    if vi(j,1)==1 Si=[Si,In(:,j)]; end;
end;
if va(j,1)==1  Sa=[Sa,In(:,j)] ; end;
end;

In=eye(i*p);
Sli=[];Sld=[];
for j=1:i*p
  if vli(j,1)==1  Sli=[Sli,In(:,j)] ; end;
  if vld(j,1)==1  Sld=[Sld,In(:,j)] ; end;
end;

% Initialize data for parameter determination
% use only the first 4/5 of the data for identification
q=round(4*M/5);
U=[];
Y=[];

% Create Zk
for j=1:i
  U=[U;Udata(:,j:q+j-1)];
  Y=[Y;Ydata(:,j:q+j-1)];
end;
Z=[U;Y];
Zk=[U;Sli*Y];
Y2k=Sld*Y;

% Find Br and Ar
V=Y2k*Zk*(inv(Zk'*Zk));
Br=V(:,1:(m*i));
Ar=V(:,(m*i+1):(m*i+n));

% Parameter identification

% First step is to identify Co
In=eye(n);
Co=In(1:p,:);
A2=In(p+1:n,:);

% 2nd step find Ao
Ao=Si*A2+Sa*Ar

% Steps 3 and 4 create Bc and Aro, Ar1
Bc=[];
for j=1:i
\[ Bc = [Bc; Br(:,(j-1)*m+1:j*m)]; \]
end;
Aro=Ar(:,1:p);
Ar1=Ar(:,p+1:n);

% Step 5 create Qc
Qc=[];
for j=1:i
    Qc=[Qc,(Ao)^(j-1)*Sa];
end;

% Step 6 Find Bo
Bo=Qc*Bc;

% Step 7 Find Do
Do = Co*inv(Ao)*Bo - inv(Aro)*Br(:,1:m);

% Step 8 Find X(0) initial conditions
% Find H, use selector to find H1
r=i-1;
H=[];
z=zeros(p,m);
for k=1:i
    Hc=[];
    for j=1:i
        if j==k
            Hc=[Hc;Do];
        elseif j<k
            Hc=[Hc;z];
        else
            dum=Co*Ao^(j-k)*Bo;
            Hc=[Hc;dum];
        end;
    end;
    H=[H Hc];
end;
H1=Sl*H;

% Find Zo and use eqn xo=yo-H1*uo
zo = Z(:,1);
uo = zo(1:m*i);
yo = zo(m*i+1:m*i+n);
Xo = yo - H1*uo;
% Simulate Discrete-time system
% First define input u
u = data(:,1:m);
ym = dtsim(Ao,Bo,Co,Do,Xo,u);

% Find error norm for all data
% Define actual output y
y = data(:,m+1:m+p);
err = y - ym;
ernm(l) = norm(err, 'fro')
if ernm(l)<ermin
  a=Ao; b=Bo; c=Co; d=Do; xo=Xo;
  ermin = ernm(l);
end;
end;
Appendix C

DIAGRAMS OF HARDWARE CONNECTIONS FOR AUTOMATIC CONTROLLER

Actuator Design
Power Circuit

Notation

Pgnd = power circuit ground
gnd = common ground
no = normally open relay contact
nc = normally closed relay contact
con = relay control
Rp = power resistor, for resistive braking, 25.7 ohms
R1 = gain resistor = 1 kohm
R2 = gain resistor = 3 kohm
D0, D1 = diodes to protect DAS-16
Appendix D

BASIC CODE TO IMPLEMENT CONTROL ALGORITHM

REM  Tom Ruddy
REM  1-dimensional control program in basic

OPEN "out.mat" FOR OUTPUT AS #1

REM  define constants
CONST G = 386  'gravitational acceleration (in/sec^2)'
CONST l = 163  'length of cable (inches)'
CONST x2 = 71  'distance to bucket (inches)'
CONST rps = 28.75  'motor 1725 rpm (rev/sec)'
CONST lead = .2  'lead of ball screw (in/rev)'

REM  dimension arrays
DIM dat%(5)  'data to and from DAS16'
DIM y%(161, 3)  'input from transducers'
DIM yyl(161)  'load position'
DIM udl(161)  'desired input position'
DIM ucl(161)  'corrected input position command'
DIM yhatl(161)  'estimated load position'
DIM ydoyhatl(161)  'estimated load velocity'

REM  main program starts here

REM  initialize DAS16
flag% = 0
mode% = 0  'mode 0, initialize'
dat%(0) = &H300  'base address 300 hex'
dat%(1) = 2  'interrupt level, not used'
dat%(2) = 1  'dma level, not used'
CALL das16(mode%, dat%(0), flag%)
IF flag% > 0 THEN
  cnum = 1
  GOTO 500
END IF

mode% = 1  'mode 1, setup'
dat%(0) = 0 'low mux limit'
dat%(1) = 2 'high mux limit'

CALL das16(mode%, dat%(0), flag%)
   IF flag% > 0 THEN
      cnum = 2
      GOTO 500
   END IF

mode% = 15 'mode 15 analog output (D/A)'
dat%(0) = 0 'set channel zero
dat%(1) = 0 'to zero'

CALL das16(mode%, dat%(0), flag%)
   IF flag% > 0 THEN
      cnum = 3
      GOTO 500
   END IF

mode% = 15 'zero channel 1'
dat%(0) = 1 'set channel 1'
dat%(1) = 0 'to zero'

CALL das16(mode%, dat%(0), flag%)
   IF flag% > 0 THEN
      cnum = 4
      GOTO 500
   END IF

REM zero the transducers
PRINT "Place the system in its equilibrium position"
PRINT "and press a key"
DO WHILE INKEY$ = "" LOOP

mode% = 3 3, data acquisition'
FOR j% = 1 TO 100
   FOR i% = 1 TO 3 'read 3 channels'
      CALL das16(mode%, dat%(0), flag%)
      IF flag% > 0 THEN
         cnum = 5
         GOTO 500
      END IF
      yl(i%, dat%(1)) = dat%(0) 'put data into correct var'
   NEXT i%
   FOR dtime% = 1 TO 2500 'wait loop'

100
NEXT dtime%= y1zero! = y1zero! + y1(j%, 0) y2zero! = y2zero! + y1(j%, 1) fzero! = fzero! + y1(j%, 2) NEXT j%

y1zero! = y1zero / 100 y2zero! = y2zero / 100 fzero! = fzero / 100 PRINT y1zero, y2zero, fzero

REM collect necessary input PRINT INPUT ; "Enter x1, length to control arm (inches). ", x1! PRINT INPUT ; "Enter T, sample interval (seconds). ", T!
PRINT INPUT ; "Enter sat, max stroke length in one direction (inches). ", sat!
PRINT INPUT ; "Enter F(1), feedback gain multiplying yhat. ", F1!
PRINT INPUT ; "Enter F(2), feedback gain multiplying ydothat. ", F2!
PRINT INPUT ; "Enter F(3), pole for hardware comp ( <1 ). ", F3!
F3! = F3 * F3 'simplifies later computations'
PRINT INPUT ; "Enter pole, estimator double pole ( <0 ). ", pole!
PRINT

REM define matrices for estimator Gr! = G / (1 - x1!) 'natural frequency of pendulum'
p! = -pole 'for gain calculation'
k1! = 2 * p 'define estimator gains'
k2! = p * p - Gr 'to place poles'
tmp1! = EXP(pole * T)

REM estimator A matrix in discrete time ad11! = (1 - pole * T - k1! * T) * tmp1 ad12! = T * tmp1 ad21! = (-Gr * T - k2! * T) * tmp1 ad22! = (1 - pole * T) * tmp1

m1! = 0 'm = inv(Ac) scaled'
m12! = -1
m21! = Gr + k2
m22! = -k1
n11! = ad11 - 1
n12! = ad12
n21! = ad21
n22! = ad22 - 1
b11! = 0
b12! = k1
b21! = Gr
b22! = k2
tmp2! = 1 / (Gr + k2)

REM estimator B matrix in discrete time
bd11! = tmp2! * (m11! * (n11! * b11! + n12! * b21!) + m12! * (n21! * b11! + n22! * b21!))
bd12! = tmp2! * (m11! * (n11! * b12! + n12! * b22!) + m12! * (n21! * b12! + n22! * b22!))
bd21! = tmp2! * (m21! * (n11! * b11! + n12! * b21!) + m22! * (n21! * b11! + n22! * b21!))
bd22! = tmp2! * (m21! * (n11! * b12! + n12! * b22!) + m22! * (n21! * b12! + n22! * b22!))
PRINT ; bd11; bd12; bd21; bd22

REM initialize states, u, and counter
yhat!(1) = 0
ydothat!(1) = 0
ud!(1) = 0
j% = 0

PRINT "Turn motor switch on and press a key to begin."
DO WHILE INKEY$ = ""
LOOP

REM start loop for control
100 start! = TIMER
      j% = j% + 1
      IF j% > 160 THEN GOTO 400
      mode% = 3
      FOR i% = 1 TO 3
          CALL das16(mode%, dat%(0), flag%)
          IF flag% > 0 THEN
              cnum = 6
              GOTO 500
          END IF
      NEXT i%
      y!(j%, dat%(1)) = dat%(0)
      NEXT j%
y(1, 0) = (y(1, 0) - ylzerol) * .002442 / .1301  'convert volts'
y(1, 1) = (y(1, 1) - y2zerol) * .002442 / .15456  'to inches'
yy(1) = y(1, 0) + (y(1, 1) - y(1, 0)) * (l - x1) / (x2 - x1)  'compute position y'

yhat(1 + 1) = ad11 * yhat(1) + ad12 * ydothat(1) + bd11 * y(1, 0)
              + bd12 * yy(1)                      

ydothat(1 + 1) = ad21 * yhat(1) + ad22 * ydothat(1) + bd21 * y(1, 0)
               + bd22 * yy(1)                      

udi(1) = -F1 * yhat(1 + 1) - F2 * ydothat(1 + 1)  'find control input'
       IF udi(1) > sat! THEN udi(1) = sat!     'implement saturation'
       IF udi(1) < -sat! THEN udi(1) = -sat!    

uc(1) = (1 + F3) * udi(1) - F3 * y(1, 0)  'tracking to limit'
          'hardware delay'
          'time to run motor'

utime! = (uc(1) - y(1, 0)) / rps / lead  
autime! = ABS(utime!)
IF utime! > 0 THEN dat!(1) = 4095 ELSE dat!(1) = 0  
mode% = 15  'forward, or reverse'
dat%(0) = 1  'mode 15, D/A output'
CALL das16(mode%, dat%(0), flag%)  
          'channel 1'
          'set forward or reverse'
          'add delay to fix'
          'hardware difficulties'

150

delay1! = TIMER
delay2! = TIMER
IF delay2! - delay1! < .01 THEN GOTO 150

dat%(0) = 0  
IF autime! <= .07 THEN GOTO 300

200

now! = TIMER
IF (now! - start!) >= (T - autime!) THEN  
   CALL das16(mode%, dat%(0), flag%)   
ELSE GOTO 200
END IF

300

dat%(1) = 0
now! = TIMER
IF (now! - start!) >= (T - .07) THEN  
   CALL das16(mode%, dat%(0), flag%)   
ELSE GOTO 300
END IF
350  now! = TIMER
    IF (now! - start!) >= T THEN
        GOTO 100
    END IF
       'loop at end of
       'sample interval'

400  dat%(0) = 1
    dat%(1) = 0
       'return relay 1 to'
       'norm closed position'
    CALL das16(mode%, dat%(0), flag%)  
    y%(j%, 0) = 0
    PRINT #1, y%(1, 0); 0; yy%(1); yhat%(1); ydothat%(1); 0
    FOR j% = 2 TO 160
        y%(j%, 2) = (y%(j%, 2) - fzero!) / 4.088 'force from volts'
        PRINT #1, y%(j%, 0); ud%(j% - 1); y%(j%); y%(j%, 2)
    NEXT j%
       'output y1 ud y what ydo that'
    CLOSE #1
    STOP

500  PRINT "An error has occurred at call ", cnum
    PRINT "flag = ", flag%
    mode% = 15
    dat%(1) = 0
       'return both relays to'
    dat%(0) = 0
       'norm closed positions'
    CALL das16(mode%, dat%(0), flag%)  
       'channel 0'
    dat%(0) = 1
    CALL das16(mode%, dat%(0), flag%)  
       'channel 1'
    STOP
VITA

The author was born on December 23, 1970 in Scranton, PA. He lived from childhood in Dunmore, PA, and in June of 1988 he was graduated from Dunmore High School. He began his undergraduate education at Lafayette College in the fall of that year and received his Bachelor of Science Degree in Electrical Engineering in May of 1992. The author enrolled at Virginia Polytechnic and State University in pursuit of a Master of Science Degree in the fall of 1992, specializing in control systems.

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