APPENDIX
LDV SCANNER CALIBRATION: ADDITIONAL MATERIAL

This appendix discusses additional scanner calibration material. Specifically, it presents 1) the transformation equation that transformed the laser virtual origin from the LDV coordinate system to the digitizer coordinate system and 2) the raw calibration data from which the scan angle/DAC and scan angle/analog voltage relationships were obtained.

Nomenclature

$\boldsymbol{c}$	vector
$\boldsymbol{d}$	vector
$\boldsymbol{i}_D, \boldsymbol{j}_D, \boldsymbol{k}_D$	spatial intermediate coordinate system unit vectors
$\boldsymbol{i}_I, \boldsymbol{j}_I, \boldsymbol{k}_I$	spatial intermediate coordinate system unit vectors
$\boldsymbol{i}_L, \boldsymbol{j}_L, \boldsymbol{k}_L$	spatial LDV coordinate system unit vectors
l	direction cosine
P_1, P_2, P_3	pin labels
$\boldsymbol{r}_D$	vector to arbitrary point in digitizer coordinate system
$\boldsymbol{r}_I$	vector to arbitrary point in intermediate coordinate system
$\boldsymbol{r}_L$	vector to arbitrary point in LDV coordinate system
$[\boldsymbol{R}]_{I \to D}$	rotation matrix from intermediate coordinate system to digitizer coordinate system
$[\boldsymbol{R}]_{I \to L}$	rotation matrix from intermediate coordinate system to LDV coordinate system
$\boldsymbol{T}_{O_I \to \boldsymbol{o}_D}$	translation vector from intermediate coordinate system to digitizer coordinate system
$\boldsymbol{T}_{O_I \to \boldsymbol{o}_L}$	translation vector from intermediate coordinate system to LDV coordinate system
x_I, y_I, z_I	spatial intermediate Cartesian coordinate system coordinates, spatial intermediate Cartesian coordinate system components
x_L, y_L, z_L	spatial LDV Cartesian coordinate system coordinates; spatial LDV Cartesian coordinate system components
Δ	difference between coordinates; difference between vectors

Transformation Equation Development

An aluminum plate with three cylindrical steel pins was inserted between the LDV and tripod mounting surface. The plate dimensions and pin locations are illustrated by Fig. A1. The pins helped define a transformation equation which transformed the LDV virtual origin from LDV to digitizer coordinates.

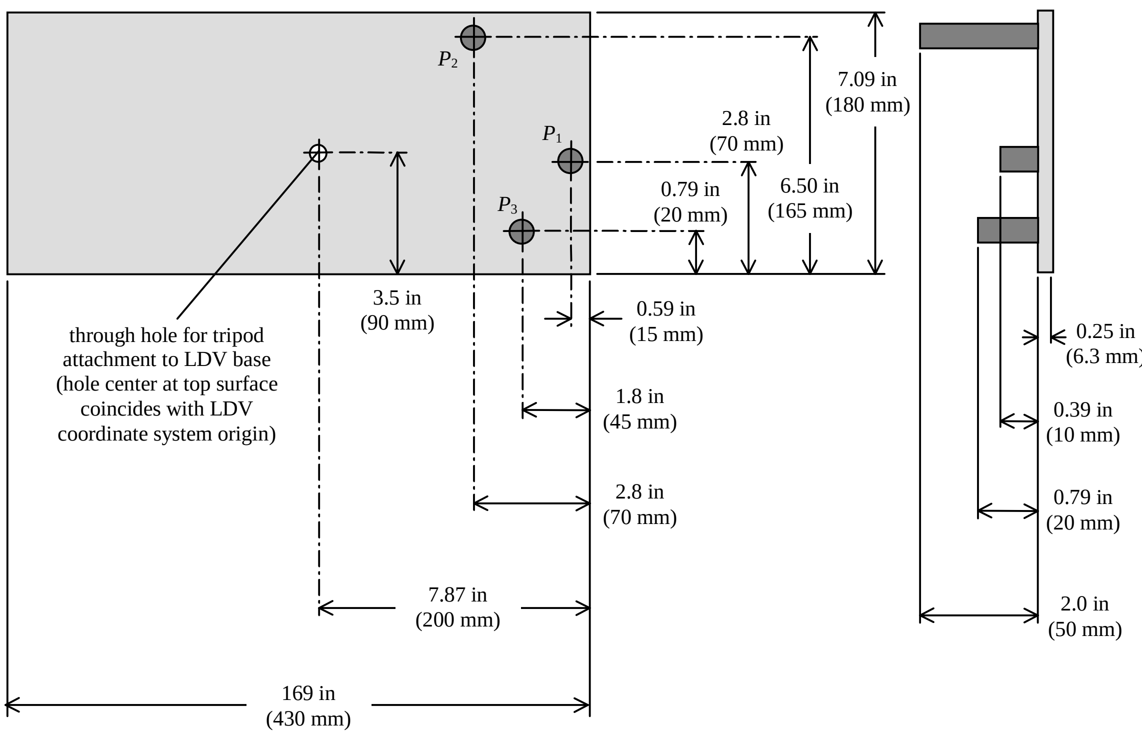

Figure A1. Plate dimensions and pin locations

First, an intermediate coordinate system was defined. This coordinate system was important since it provided a reference coordinate system for the LDV and digitizer coordinate systems. The centers of the pin tips were designated P_1, P_2 and P_3 as shown in Fig. A1. Point P_1 was selected as the origin and the vector from P_1 to P_2 was designated the x_I-axis. Thus, the unit vector that defined the x_I-axis was

$$\boldsymbol{i}_I = \frac{\boldsymbol{r}_{\boldsymbol{P}_2} - \boldsymbol{r}_{\boldsymbol{P}_1}}{\left|\boldsymbol{r}_{\boldsymbol{P}_2} - \boldsymbol{r}_{\boldsymbol{P}_1}\right|}, \tag{A1}$$

where $\boldsymbol{r}_{\boldsymbol{P}_1}$ and $\boldsymbol{r}_{\boldsymbol{P}_2}$ were specified using digitizer coordinates. The points P_1 and P_3 defined another vector:

$$\boldsymbol{c} = \boldsymbol{r}_{\boldsymbol{P}_3} - \boldsymbol{r}_{\boldsymbol{P}_1}, \tag{A2}$$

where $\boldsymbol{r}_{\boldsymbol{P}_1}$ and $\boldsymbol{r}_{\boldsymbol{P}_3}$ were also specified using digitizer coordinates. The vector cross-product between $\boldsymbol{i}_I$ and $\boldsymbol{c}$ yielded another vector perpendicular to the plane defined by points P_1, P_2 and P_3:

$$\boldsymbol{d} = \boldsymbol{i}_I \times \boldsymbol{c}. \tag{A3}$$

This vector was the z_I-axis; hence, the unit vector that defined the z_I-axis was

$$\boldsymbol{k}_I = \frac{\boldsymbol{d}}{|\boldsymbol{d}|}. \tag{A4}$$

The vector cross-product between $\boldsymbol{k}_I$ and $\boldsymbol{i}_I$ yielded another unit vector that defined the y_I-axis; it was

$$\boldsymbol{j}_I = \boldsymbol{k}_I \times \boldsymbol{i}_I. \tag{A5}$$

The unit vectors $\boldsymbol{i}_L$, $\boldsymbol{j}_L$ and $\boldsymbol{k}_L$ completely defined the intermediate coordinate system.

Second, the following transformation equation between the intermediate coordinate system and the LDV coordinate system was developed:

$$\boldsymbol{r}_L = [\boldsymbol{R}]_{I \to L}\boldsymbol{r}_I + \boldsymbol{T}_{O_I \to O_L}, \tag{A6}$$

where $\boldsymbol{r_L}$ was the location of an arbitrary point in the LDV coordinate system, $\boldsymbol{r_I}$ was the location of the same point in the intermediate coordinate system, $[\boldsymbol{R}]_{I \to L}$ was the rotation matrix from the intermediate coordinate system to the LDV coordinate system and $\boldsymbol{T}_{\boldsymbol{O_I} \to O_L}$ was the translation vector from the intermediate coordinate system origin, point P_1, to the LDV coordinate system origin. Considering Fig. A1, the translation vector is

$$\boldsymbol{T}_{\boldsymbol{O_I} \to O_L} = \begin{Bmatrix} P_{1_{x_L}} \\ P_{1_{y_L}} \\ P_{1_{z_L}} \end{Bmatrix}. \tag{A7}$$

This vector was known; hence, the only unknown term in Eq. (A6) was the rotation matrix, $[\boldsymbol{R}]_{I \to L}$. Rearranging Eq. (A6) yielded

$$\boldsymbol{r}_L - \boldsymbol{T}_{O_I \to O_L} = [\boldsymbol{R}]_{\boldsymbol{I} \to \boldsymbol{L}} \boldsymbol{r}_I \,. \tag{A8}$$

The points P_1, P_2 and P_3 were all known in both the intermediate and LDV coordinate systems; therefore, at point P_2 with the LDV coordinate system origin translated to point P_1:

$$\left(P_{2_{x_L}} - P_{1_{x_L}}\right) \boldsymbol{i}_{\mathrm{L}} + \left(P_{2_{y_L}} - P_{1_{y_L}}\right) \boldsymbol{j}_{\mathrm{L}} + \left(P_{2_{z_L}} - P_{1_{z_L}}\right) \boldsymbol{k}_{\mathrm{L}} = P_{2_{x_I}} \boldsymbol{i}_{\mathrm{I}} + P_{2_{y_I}} \boldsymbol{j}_{\mathrm{I}} + P_{2_{z_I}} \boldsymbol{k}_{\mathrm{I}} \tag{A9}$$

or

$$\Delta P_{2 \to 1_{x_L}} \boldsymbol{i}_{\mathrm{L}} + \Delta P_{2 \to 1_{y_L}} \boldsymbol{j}_{\mathrm{L}} + \Delta P_{2 \to 1_{z_L}} \boldsymbol{k}_{\mathrm{L}} = P_{2_{x_I}} \boldsymbol{i}_{\mathrm{I}} + P_{2_{y_I}} \boldsymbol{j}_{\mathrm{I}} + P_{2_{z_I}} \boldsymbol{k}_{\mathrm{I}} \,, \tag{A10}$$

where

$$\begin{aligned} \Delta P_{2 \to 1_{x_L}} &= P_{2_{x_L}} - P_{1_{x_L}} \\ \Delta P_{2 \to 1_{y_L}} &= P_{2_{y_L}} - P_{1_{y_L}} \;. \\ \Delta P_{2 \to 1_{z_L}} &= P_{2_{z_L}} - P_{1_{z_L}} \end{aligned} \tag{A11}$$

Evaluating successive vector dot-products between Eq. (A10) and the unit vectors $\boldsymbol{i}_L$, $\boldsymbol{j}_L$ and $\boldsymbol{k}_L$ eventually yielded

$$\begin{aligned}\Delta P_{2\to 1 x_L} &= P_{2x_I}\boldsymbol{i}_{\mathrm{I}}\cdot\boldsymbol{i}_{\mathrm{L}} + P_{2y_I}\boldsymbol{j}_{\mathrm{I}}\cdot\boldsymbol{i}_{\mathrm{L}} + P_{2z_I}\boldsymbol{k}_{\mathrm{I}}\cdot\boldsymbol{i}_{\mathrm{L}}\\ \Delta P_{2\to 1 y_L} &= P_{2x_I}\boldsymbol{i}_{\mathrm{I}}\cdot\boldsymbol{j}_{\mathrm{L}} + P_{2y_I}\boldsymbol{j}_{\mathrm{I}}\cdot\boldsymbol{j}_{\mathrm{L}} + P_{2z_I}\boldsymbol{k}_{\mathrm{I}}\cdot\boldsymbol{j}_{\mathrm{L}}\ .\\ \Delta P_{2\to 1 z_L} &= P_{2x_I}\boldsymbol{i}_{\mathrm{I}}\cdot\boldsymbol{k}_{\mathrm{L}} + P_{2y_I}\boldsymbol{j}_{\mathrm{I}}\cdot\boldsymbol{k}_{\mathrm{L}} + P_{2z_I}\boldsymbol{k}_{\mathrm{I}}\cdot\boldsymbol{k}_{\mathrm{L}}\end{aligned} \qquad \text{(A12)}$$

Further simplification yielded

$$\begin{aligned}\Delta P_{2\to 1 x_L} &= P_{2x_I} l_{11_L} + P_{2y_I} l_{12_L} + P_{2z_I} l_{13_L}\\ \Delta P_{2\to 1 y_L} &= P_{2x_I} l_{21_L} + P_{2y_I} l_{22_L} + P_{2z_I} l_{23_L}\\ \Delta P_{2\to 1 z_L} &= P_{2x_I} l_{31_L} + P_{2y_I} l_{32_L} + P_{2z_I} l_{33_L}\end{aligned} \qquad \text{(A13)}$$

or in matrix form

$$\begin{Bmatrix}\Delta P_{2\to 1 x_L}\\ \Delta P_{2\to 1 y_L}\\ \Delta P_{2\to 1 z_L}\end{Bmatrix} = \begin{bmatrix} l_{11_L} & l_{12_L} & l_{31_L}\\ l_{21_L} & l_{22_L} & l_{23_L}\\ l_{31_L} & l_{32_L} & l_{33_L}\end{bmatrix}\begin{Bmatrix}P_{2x_I}\\ P_{2y_I}\\ P_{2z_I}\end{Bmatrix} \qquad \text{(A14)}$$

or

$$\begin{Bmatrix}\Delta P_{2\to 1 x_L}\\ \Delta P_{2\to 1 y_L}\\ \Delta P_{2\to 1 z_L}\end{Bmatrix} = [\boldsymbol{R}]_{I\to L}\begin{Bmatrix}P_{2x_I}\\ P_{2y_I}\\ P_{2z_I}\end{Bmatrix}, \qquad \text{(A15)}$$

where

$$[\boldsymbol{R}]_{\boldsymbol{I}\to\boldsymbol{L}} = \begin{bmatrix}\boldsymbol{l}_{11_L} & \boldsymbol{l}_{12_L} & \boldsymbol{l}_{31_L}\\ \boldsymbol{l}_{21_L} & \boldsymbol{l}_{22_L} & \boldsymbol{l}_{23_L}\\ \boldsymbol{l}_{31_L} & \boldsymbol{l}_{32_L} & \boldsymbol{l}_{33_L}\end{bmatrix} = \begin{bmatrix}\boldsymbol{i}_I\cdot\boldsymbol{i}_L & \boldsymbol{j}_I\cdot\boldsymbol{i}_L & \boldsymbol{k}_I\cdot\boldsymbol{i}_L\\ \boldsymbol{i}_I\cdot\boldsymbol{j}_L & \boldsymbol{j}_I\cdot\boldsymbol{j}_L & \boldsymbol{k}_I\cdot\boldsymbol{j}_L\\ \boldsymbol{i}_I\cdot\boldsymbol{k}_L & \boldsymbol{j}_I\cdot\boldsymbol{k}_L & \boldsymbol{k}_I\cdot\boldsymbol{k}_L\end{bmatrix}. \qquad \text{(A16)}$$

The rotation matrix $[\boldsymbol{R}]_{I\to L}$ elements were simply direction cosines among the unit vectors $\boldsymbol{i}_L$, $\boldsymbol{j}_L$ and $\boldsymbol{k}_L$. The rotation matrix was easily calculated since the unit vectors $\boldsymbol{i}_L$, $\boldsymbol{j}_L$ and $\boldsymbol{k}_L$ were all known. The translation vector and rotation matrix, $\boldsymbol{T}_{O_I\to O_L}$ and $[\boldsymbol{R}]_{I\to L}$

respectively, completely defined the transformation equation from the intermediate coordinate system to the LDV coordinate system.

Third, a similar procedure established the following transformation equation between the intermediate coordinate system and the LDV coordinate system:

$$\boldsymbol{r}_D = [\boldsymbol{R}]_{\boldsymbol{I\rightarrow D}}\,\boldsymbol{r}_I + \boldsymbol{T}_{\boldsymbol{O_I\rightarrow O_D}}\,, \tag{A17}$$

where $\boldsymbol{r_D}$ was the location of an arbitrary point in the digitizer coordinate system, $\boldsymbol{r}_I$ was the location of the same point in the intermediate coordinate system, $[\boldsymbol{R}]_{I\rightarrow D}$ was the rotation matrix from the intermediate coordinate system to the digitizer coordinate system and $\boldsymbol{T}_{\boldsymbol{O_I}\rightarrow O_D}$ was the translation vector from the intermediate coordinate system origin, point P_1, to the digitizer coordinate system origin. The translation vector is known since the coordinates of point P_1, the origin of the intermediate coordinate system, were known in the digitizer coordinate system; symbolically the translation vector is

$$\boldsymbol{T}_{\boldsymbol{O_I}\rightarrow O_D} = \begin{Bmatrix} P_{1x_D} \\ P_{1y_D} \\ P_{1z_D} \end{Bmatrix}. \tag{A18}$$

Hence, the only unknown term in Eq. (A17) was the rotation matrix, $[\boldsymbol{R}]_{I\rightarrow D}$. Rearranging Eq. (17) yielded

$$\boldsymbol{r}_D - \boldsymbol{T}_{\boldsymbol{O_I\rightarrow O_D}} = [\boldsymbol{R}]_{\boldsymbol{I\rightarrow D}}\,\boldsymbol{r}_I\,. \tag{A19}$$

The points P_1, P_2 and P_3 were all known in both the intermediate and digitizer coordinate systems; therefore, at point P_2 with the digitizer coordinate system origin translated to point P_1:

$$\left(P_{2x_D} - P_{1x_D}\right)\boldsymbol{i}_D + \left(P_{2y_D} - P_{1y_D}\right)\boldsymbol{j}_\mathrm{D} + \left(P_{2z_D} - P_{1z_D}\right)\boldsymbol{k}_\mathrm{D} = P_{2x_I}\boldsymbol{i}_\mathrm{I} + P_{2y_I}\boldsymbol{j}_\mathrm{I} + P_{2z_I}\boldsymbol{k}_\mathrm{I} \tag{A20}$$

or

$$\Delta P_{2\to 1_{x_D}} \boldsymbol{i}_{\mathrm{D}} + \Delta P_{2\to 1_{y_D}} \boldsymbol{j}_{\mathrm{D}} + \Delta P_{2\to 1_{z_D}} \boldsymbol{k}_{\mathrm{D}} = P_{2_{x_I}} \boldsymbol{i}_{\mathrm{I}} + P_{2_{y_I}} \boldsymbol{j}_{\mathrm{I}} + P_{2_{z_I}} \boldsymbol{k}_{\mathrm{I}}, \tag{A21}$$

where

$$\begin{aligned} \Delta P_{2\to 1_{x_D}} &= P_{2_{x_D}} - P_{1_{x_D}} \\ \Delta P_{2\to 1_{y_D}} &= P_{2_{y_D}} - P_{1_{y_D}} \; . \\ \Delta P_{2\to 1_{z_D}} &= P_{2_{z_D}} - P_{1_{z_D}} \end{aligned} \tag{A22}$$

Evaluating successive vector dot-products between Eq. (A21) and the unit vectors $\boldsymbol{i}_D$, $\boldsymbol{j}_D$ and $\boldsymbol{k}_D$ eventually yielded

$$\begin{aligned} \Delta P_{2\to 1_{x_D}} &= P_{2_{x_I}} \boldsymbol{i}_{\mathrm{I}} \cdot \boldsymbol{i}_{\mathrm{D}} + P_{2_{y_I}} \boldsymbol{j}_{\mathrm{I}} \cdot \boldsymbol{i}_{\mathrm{D}} + P_{2_{z_I}} \boldsymbol{k}_{\mathrm{I}} \cdot \boldsymbol{i}_{\mathrm{D}} \\ \Delta P_{2\to 1_{y_D}} &= P_{2_{x_I}} \boldsymbol{i}_{\mathrm{I}} \cdot \boldsymbol{j}_{\mathrm{D}} + P_{2_{y_I}} \boldsymbol{j}_{\mathrm{I}} \cdot \boldsymbol{j}_{\mathrm{D}} + P_{2_{z_I}} \boldsymbol{k}_{\mathrm{I}} \cdot \boldsymbol{j}_{\mathrm{D}} \; . \\ \Delta P_{2\to 1_{z_D}} &= P_{2_{x_I}} \boldsymbol{i}_{\mathrm{I}} \cdot \boldsymbol{k}_{\mathrm{D}} + P_{2_{y_I}} \boldsymbol{j}_{\mathrm{I}} \cdot \boldsymbol{k}_{\mathrm{D}} + P_{2_{z_I}} \boldsymbol{k}_{\mathrm{I}} \cdot \boldsymbol{k}_{\mathrm{D}} \end{aligned} \tag{A23}$$

Further simplification yielded

$$\begin{aligned} \Delta P_{2\to 1_{x_D}} &= P_{2_{x_I}} l_{11_D} + P_{2_{y_I}} l_{12_D} + P_{2_{z_I}} l_{13_D} \\ \Delta P_{2\to 1_{y_D}} &= P_{2_{x_I}} l_{21_D} + P_{2_{y_I}} l_{22_D} + P_{2_{z_I}} l_{23_D} \\ \Delta P_{2\to 1_{z_D}} &= P_{2_{x_I}} l_{31_D} + P_{2_{y_I}} l_{32_D} + P_{2_{z_I}} l_{33_D} \end{aligned} \tag{A24}$$

or in matrix form

$$\begin{Bmatrix} \Delta P_{2\to 1_{x_D}} \\ \Delta P_{2\to 1_{y_D}} \\ \Delta P_{2\to 1_{z_D}} \end{Bmatrix} = \begin{bmatrix} l_{11_D} & l_{12_D} & l_{31_D} \\ l_{21_D} & l_{22_D} & l_{23_D} \\ l_{31_D} & l_{32_D} & l_{33_D} \end{bmatrix} \begin{Bmatrix} P_{2_{x_I}} \\ P_{2_{y_I}} \\ P_{2_{z_I}} \end{Bmatrix} \tag{A25}$$

or

$$\begin{Bmatrix} \Delta P_{2\to 1_{x_D}} \\ \Delta P_{2\to 1_{y_D}} \\ \Delta P_{2\to 1_{z_D}} \end{Bmatrix} = [\boldsymbol{R}]_{I\to D} \begin{Bmatrix} P_{2_{x_I}} \\ P_{2_{y_I}} \\ P_{2_{z_I}} \end{Bmatrix}, \tag{A26}$$

where

$$[\boldsymbol{R}]_{\boldsymbol{I}\rightarrow\boldsymbol{D}} = \begin{bmatrix} \boldsymbol{l}_{11_{\boldsymbol{D}}} & \boldsymbol{l}_{12_{\boldsymbol{D}}} & \boldsymbol{l}_{31_{\boldsymbol{D}}} \\ \boldsymbol{l}_{21_{\boldsymbol{D}}} & \boldsymbol{l}_{22_{\boldsymbol{D}}} & \boldsymbol{l}_{23_{\boldsymbol{D}}} \\ \boldsymbol{l}_{31_{\boldsymbol{D}}} & \boldsymbol{l}_{32_{\boldsymbol{D}}} & \boldsymbol{l}_{33_{\boldsymbol{D}}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{i}_I \cdot \boldsymbol{i}_D & \boldsymbol{j}_I \cdot \boldsymbol{i}_D & \boldsymbol{k}_I \cdot \boldsymbol{i}_D \\ \boldsymbol{i}_I \cdot \boldsymbol{j}_D & \boldsymbol{j}_I \cdot \boldsymbol{j}_D & \boldsymbol{k}_I \cdot \boldsymbol{j}_D \\ \boldsymbol{i}_I \cdot \boldsymbol{k}_D & \boldsymbol{j}_I \cdot \boldsymbol{k}_D & \boldsymbol{k}_I \cdot \boldsymbol{k}_D \end{bmatrix}. \qquad \text{(A27)}$$

As before, the rotation matrix $[\boldsymbol{R}]_{I\rightarrow D}$ elements were simply direction cosines among the unit vectors $\boldsymbol{i}_D$, $\boldsymbol{j}_D$ and $\boldsymbol{k}_D$. Again, the rotation matrix was easily calculated since the unit vectors $\boldsymbol{i}_D$, $\boldsymbol{j}_D$ and $\boldsymbol{k}_D$ were all known. The translation vector and rotation matrix, $\boldsymbol{T}_{O_I\rightarrow O_D}$ and $[\boldsymbol{R}]_{I\rightarrow D}$, respectively, completely defined the transformation equation from the intermediate coordinate system to the digitizer coordinate system.

Last, the two transformation equations, Eqs. (A6) and (A17), established the overall transformation equation from LDV to digitizer coordinates. The transformation equation from the intermediate coordinate system to the LDV coordinate system was Eq. (A6); therefore, the transformation equation from the LDV coordinate system to the intermediate coordinate system was

$$\boldsymbol{r}_I = [\boldsymbol{R}]^{-1}_{\boldsymbol{I}\rightarrow\boldsymbol{L}}\left(\boldsymbol{r}_{\boldsymbol{L}} - \boldsymbol{T}_{O_I\rightarrow O_L}\right). \qquad \text{(A28)}$$

Inserting Eq. (A29) into (A17) yielded the desired transformation equation from the LDV coordinate system to the digitizer coordinate system:

$$\boldsymbol{r}_{\boldsymbol{D}} = [\boldsymbol{R}]_{\boldsymbol{I}\rightarrow\boldsymbol{D}}\left([\boldsymbol{R}]^{-1}_{\boldsymbol{I}\rightarrow\boldsymbol{L}}\left(\boldsymbol{r}_{\boldsymbol{L}} - \boldsymbol{T}_{O_I\rightarrow O_L}\right)\right) + \boldsymbol{T}_{O_I\rightarrow O_D}. \qquad \text{(A29)}$$

This equation transformed the laser virtual origin from the LDV coordinate system to the digitizer coordinate system.

Calibration Data

The scan angle/DAC step and horizontal scan angle/analog voltage relationships specified by Eqs. (95), (96), (99) and (100) and Eqs. (102), (103), (106) and (107) were obtained by regressing the data listed in Table A1.

Table A1. Scan angle/DAC step and scan angle/analog voltage data

DAC Steps	Recorded Analog Voltage		Calculated Scan Angle	
	Horizontal Scanner (V)	Vertical Scanner (V)	Horizontal Scan Angle (mrad)	Vertical Scan Angle (mrad)
-3500	-4.38	-4.38	188	-193
-3150	-3.94	-3.94	168	-174
-2800	-3.50	-3.50	150	-154
-2450	-3.06	-3.06	131	-135
-2100	-2.63	-2.63	112	-116
-1750	-2.19	-2.19	93.7	-96.6
-1400	-1.75	-1.75	75.0	-77.1
-1050	-1.31	-1.31	56.4	-57.7
-700	-0.875	-0.875	37.4	-38.4
-350	-0.437	-0.437	18.8	-19.2
0	$\approx 8.0 \times 10^{-5}$	$\approx 4.5 \times 10^{-5}$	0	0
350	0.437	0.437	-18.9	19.2
700	0.875	0.875	-37.8	38.5
1050	1.31	1.31	-56.6	57.8
1400	1.75	1.75	-75.5	77.2
1750	2.19	2.19	-93.8	96.2
2100	2.63	2.63	-113	115
2450	3.06	3.06	-131	134
2800	3.50	3.50	-150	154
3150	3.94	3.94	-169	173
3500	4.38	4.38	-188	192