A technique is proposed whereby a 2-D distribution is transmitted via a set of 1-D projections. The color-coded projections are produced by illuminating the object with the dispersed spectrum of a white light source. They are transmitted serially and reconstructed by backprojection.

I. Introduction

The transfer of images through perturbing media has been the object of much research. Of particular interest are the problems where it is not possible to define the propagation from object to image space as a one-to-one spatial mapping. Examples of such problems include imaging through fog or other hostile environments and image transfer through a single fiber or other light pipe.

Wavelength multiplexing techniques, in which the spatial coordinates of an image are encoded into wavelengths, are particularly appealing in the solution of such problems. This is because the spectral intensity of the light remains practically unaffected by perturbations which completely scramble the spatial distribution of intensity. Wavelength (or color) can be used to encode 1-D distributions without much difficulty. Techniques were demonstrated whereby the transmittance distribution to be encoded is illuminated by white light and imaged onto a slit through a dispersive element (prism or grating), or where a wedge interference filter was used to encode the object. The reconstructions of such color-coded 1-D signals are obtained with another dispersive element or wedge filter which remaps wavelength into a spatial distribution.

Extension of these techniques to include the transmission of 2-D images have also been proposed. The most immediate solution to this problem is to transmit one line at a time. This, however, requires precise synchronization of the encoder and decoder scans. Transmission of 2-D images through fibers can also be accomplished in parallel with a sheet of fibers, each carrying one color-coded line of the object, with a single fiber by encoding each line with a different part of the spectrum or via a spatial pulse modulation technique. Parallel transmission with a single channel is obviously limited to a number of image elements (pixels) not exceeding the total number of resolvable spectral elements of the dispersive instrument. The serial technique, although inherently slower, can use this same number of information cells for each line.

This paper explores the possibility of transmitting serially a set of projections of the object. These projections can be obtained by illuminating the object with the spectrum of a source of uniform spectral intensity. The orientation of that spectrum defines the orientation of the projection. Synchronization requires only that the angular speed of the projected spectrum be known. (If the angular range of the scan exceeds π, this speed, if constant, can also be found from the periodicity of the signal.) The reconstruction can be obtained by simply integrating the backprojections on film or by using any one of the many algorithms which have been developed for the purpose of reconstructing 2-D images from their 1-D projections in transaxial tomography.

It should be mentioned that, for equal image quality, all these techniques require the transmission of the same number of bits of data. The transmission of projections instead of lines, however, has the advantage common to most integral transform representation (e.g., holography) of partial redundancy. Unlike a missing line, which results in missing pixels, a missing projection produces only an extended low-intensity artifact.

II. Theory

Let the spectrum of a white light source illuminate a 2-D object with transmittance or reflectance distribution \( f' (x,y) = f(r, \theta) \). If the spectrum is displayed along an axis \( u \) (Fig. 1) tilted by an angle \( \phi \) with respect to the \( x \) axis, each line \( u = r \cos(\theta - \phi) \) is encoded into a specific wavelength \( \lambda \). The transmitted or reflected light has a spectral intensity:

\[
L'(\lambda) = L(\lambda) \delta(u - a\lambda) \int f(r, \theta) ds, \tag{1}
\]
polychromatic input

\[ f(r, \phi) \sim \frac{1}{r} \]  

where \( L(\lambda) \) is the source spectral intensity, \( a \) is a constant of proportionality, and the integral is a line integral along the path \( u = r \cos(\theta - \phi) \):

\[
\int_{-\infty}^{\infty} f(r, \phi) ds = \int_0^\pi \int_0^{2\pi} f(r, \phi) \delta(u - r \cos(\theta - \phi)) r dr d\theta = p(u; \phi).
\]

(2)

This is called a linear projection of the distribution \( f(r, \phi) \). Such projections, encoded in wavelength, can be transmitted by channels which do not conserve spatial information.

To reconstruct the object, the light collected by a receptor is formed into a point source and sent into a dispersive system which displays the spectral intensity \( L'(\lambda) \) along a line \( u' \) making an angle \( \phi' \) with the \( x' \) axis and smeared uniformly in the direction perpendicular to \( u' \). The next projection, corresponding to angle \( \phi' + \Delta \phi \), is backprojected in a similar manner. It is well-known that the image obtained by simply summing together all the backprojected data is a blurred reconstruction of the original which can be written as a convolution:

\[
g(r, \phi) = f(r, \phi) * h(r, \phi),
\]

(3)

with a point spread function (PSF)

\[
h(r, \phi) \propto \frac{1}{r}.
\]

(4)

The long wings of this PSF are responsible for the blurred appearance of the reconstructed image. This problem has been extensively studied in the context of transaxial tomography for which many reconstruction algorithms using either digital or analog techniques exist. Several analog optical schemes could be used here. For example, the projection data displayed by the dispersive element of the reconstruction setup can be processed in a 1-D incoherent optical system. Two channels would be necessary since the required deblurring PSF is bipolar. It is also possible to record the blurred summation image and subsequently process it in a 2-D optical system, either coherent or incoherent. This additional step may not be welcome, however, and truncation of the backprojection before filtering may be a source of error because of the long-range effect of the \( 1/r \) PSF. A third possibility is to record the projections line by line in the form of a sinogram and to process it optically.

In addition to sharpening the reconstruction, the postprocessing step can also compensate for the non-uniform source spectral intensity and the nonuniform spectral absorption in the transmission channel.

### III. Experiment

For the purpose of demonstrating the technique, the system of Fig. 2 was used. A prism projects the spectrum of a xenon lamp on the input transparency. Light is then gathered after having gone through a spatially distorting medium and is transmitted by a small fiber bundle. The fibers end is imaged into a point source, and the spectrum of that source is spatially displayed by a second prism. The cylindrical optics produce a blurred appearance of the reconstructed image. This problem has been extensively studied in the context of transaxial tomography for which many reconstruction algorithms using either digital or analog techniques exist. Several analog optical schemes could be used here. For example, the projection data displayed by the dispersive element of the reconstruction setup can be processed in a 1-D incoherent optical system. Two channels would be necessary since the required deblurring PSF is bipolar. It is also possible to record the blurred summation image and subsequently process it in a 2-D optical system, either coherent or incoherent. This additional step may not be welcome, however, and truncation of the backprojection before filtering may be a source of error because of the long-range effect of the \( 1/r \) PSF. A third possibility is to record the projections line by line in the form of a sinogram and to process it optically.

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smearing of that spectrum in a direction perpendicular to its display. A rotation of the prism and cylindrical lens allows one to address and reconstruct the different projections sequentially. In the actual experiment the input and the film recording the backprojections were rotated instead of the optical elements.

Figure 3 shows some results; the transmission of a letter C using sixteen color-coded projections recorded with angular increments \( \pi/16 \). No attempt was made to filter or enhance the backprojected image. The blurring effect of the 1/r PSF is, therefore, strongly visible. Other possible sources of error in that experiment are the nonuniform spectral intensity of the source, the spectral response of the film, and the imperfect alignment of the rotating elements, all of which could be corrected.

### IV. Conclusions

A technique to transmit an image via its projections has been described. The 1-D projections are coded in wavelength and serially transmitted through an arbitrary distorting medium or a single transmission channel which scrambles spatial information. A blurred backprojected image can be obtained directly, or the data can be filtered for a more accurate reconstruction.

The integral form in which the data are transmitted offers, through partial redundancy, a better immunity to noise than, for example, the serial transmission of pixels or of color-coded lines. It is conceivable that, in some particular hostile environment, illumination with a rotated spectrum would be easier to achieve than a precise linear scan of the image. The technique may also find uses when, as in the recording of 2-D slices through a 3-D body, the raw data are already in the form of a projection.

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### References