Interferometric model for phase analysis in fiber couplers

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An interferometric model is proposed to estimate the phase differences in lossless, strongly coupled biconical fiber couplers. This approximate method is simpler than the traditional *s*-parameter network theory-based analysis technique and minimizes the number of unknowns. The phase difference between the transmitted and coupled light fields is directly related to the field interaction and can be estimated by employing the energy conservation and mode orthogonality principles. The maximum coupling coefficient and dependence of phase difference on coupling conditions can be analyzed for multiport single-mode fiber couplers. © 1996 Optical Society of America

1. Introduction

The phase differences between the transmitted and coupled light fields are an important parameter for the characterization of single-mode fiber couplers. Different types of fiber couplers have different phase characteristics, and controlling the phase difference is essential in various applications such as phase biasing, quadrature detection, and coherent communications.[1–3] Generally speaking, the phase difference in a fiber coupler is related to both the magnitude of the coupling coefficients and the insertion loss of the coupler. This parameter contains information related to the coupling characteristics of a fiber or optical waveguide device and may be used for device characterization and in device analysis. Network theory is the common way to determine phase shifts in a fiber coupler, provided the magnitudes of all the coupling coefficients are known.[4,5] For example, the 2×2 directional fiber coupler is the most widely used fiber coupler, with a phase difference of 90° between the transmitted and coupled fields. The scattering matrix method is a widely accepted technique for coupler phase analysis in which the coupler is treated as a black box. For this reason, the method is too general for analyzing multiport fiber couplers. An approximate but efficient way to

estimate the phase difference in a coupler is based on some assumptions related to the interaction of the fields in the coupler. Because tapered single-mode fiber couplers have very small core diameters within the tapered region, the fields of different fibers may be expected to extend from the core region into the entire cross section of the taper, a situation referred to as total field overlap in the following paragraphs. When this assumption is justified, a simple interferometric method can be applied to estimate the relative phases in such couplers.

For simplicity, only lossless fiber couplers are considered in this paper. From energy conservation, the light power in the coupling region is the same as the total output power from the coupler. As in the scattering matrix technique, a signal space can also be constructed by using the fields in different output fibers as space coordinates. The output states from each input fiber are vectors in the signal space, with elements representing the components of the field in the corresponding output fibers. The output vectors corresponding to different input fibers are orthogonal to each other for a lossless system, as can be proven by using network theory.[5] This is understandable because the mode fields must obey the general mode orthogonality principle of waveguides. In addition, symmetry arguments are often helpful for minimizing the number of unknowns. The properties of losslessness, orthogonality, and symmetry are all used to advantage in the interferometric model described here.

The 2×2 coupler, for which the phase shift between transmission and coupling coefficients is known to be $\pi/2$, is analyzed by using the interferometric tech-

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nique in Section 2. Section 3 contains general comments on multiport couplers, and the particular case of $1 \times N$ couplers is treated in Section 4. In Section 5, it is shown how the interferometric method, together with energy conservation and symmetry arguments, can be used to estimate bounds to the relative phase shift. A short summary is given in Section 6.

2. 2×2 Coupler

Equiphase planes of the fundamental mode in a single-mode fiber are perpendicular to the light propagation direction (fiber direction). It is assumed that the propagation modes of the two fibers are the fundamental modes, even in the taper region. The mode diameter of the fundamental mode will increase in the taper region because of the decreasing core diameter, but the extended mode field is still the eigenmode of the fiber. Because of the strong coupling in the taper region, part of the input light field will be transported into the other fiber to its eigenmode field, which is called the coupled-mode field. It is important to note that although there is strong coupling in the taper, the transmitted mode field is still considered as being bound to the input fiber whereas the coupled mode is considered as being bound to the other fiber. Total power transition is the common assumption in taper analysis, which means that the mode power can be totally transported from the taper to the corresponding fibers.

For a unit amplitude input field injected into one of the input fibers, the coupled-mode field will be excited in the other fiber. The transmission coefficient, $\tilde{T}$, is defined as the phasor of the transmitted mode field, and the coupling coefficient, $\tilde{C}$, is defined as the phasor of the coupled-mode field. Because the input field intensity is assumed to be unity, the two fields defined above are consistent with the general definition of transmission and coupling coefficients, which means that the magnitude of the coefficients are the ratio of the magnitude of the output field to the input field. In the taper region, the two fields $\tilde{T}$ and $\tilde{C}$ overlap and interfere. Because their equiphase planes are parallel to each other and perpendicular to the light propagation direction, the phase difference is independent of the mode field distribution. The light transverse intensity distribution at any position x along the taper can be written as

$$I_{\text{taper}}(r, x) = |\tilde{T}(r, x) + \tilde{C}(r, x)|^2 = T^2(r, x) + C^2(r, x)$$
$$+ 2 \operatorname{Re}[\tilde{T}(r, x)\tilde{C}^*(r, x)], \quad (1)$$

where r is a vector in the transverse field plane, and dependence on r and x are separable. Note that because the mode fields have parallel equiphase planes, the third term can be written as $2T(r, x)C(r, x)\cos(\phi)$, where ϕ is the relative phase between the coupling and transmission coefficients ($\phi = \arg \tilde{T} - \arg \tilde{C}$) and is independent of the transverse field distribution. Here T and C are the magnitudes of phasors $\tilde{T}$ and $\tilde{C}$, respectively. Total field overlap mean that the transmitted field and the coupled field are coaxial, which is a valid assumption for tapered couplers be-

cause the core diameter is small and the distance between fiber cores is much smaller than the size of the field distribution in the coupling region. Under this assumption the transverse field distribution of all the modes can be considered as identical, and consequently the dependence on r of all the coefficients can be neglected. If the fields are not coaxial, only part of the fields will effectively overlap and interfere. In the following paragraphs, this situation is called partial field overlap. In a lossless linear system, the total power is constant in any cross section along x. Thus according to energy conservation, we have

$$P_{\text{output}} = P_{\text{fiber 1}} + P_{\text{fiber 2}}$$

$$= P_{\text{taper}} = \int T^2(r, x)\mathrm{d}s + \int C^2(r, x)\mathrm{d}s$$

$$+ \int 2T(r, x)C(r, x)\cos \phi \mathrm{d}s$$

$$= P_{\text{fiber 1}} + P_{\text{fiber 2}} + \int 2T(r, x)C(r, x)\cos \phi \mathrm{d}s, \quad (2)$$

where the integral is over any cross section. Mode orthogonality is required to satisfy Eq. (2). In other words, the two mode fields, $\tilde{C}$ and $\tilde{T}$, must be in phase quadrature. If we chose $0 \leq \phi \leq \pi$, then the phase difference will be

$$\cos \phi = 0, \quad \text{or } \phi = \pi/2. \quad (3)$$

Equation (2) indicates that the phase difference is always $\pi/2$ because the total overlap assumption was not employed in Eq. (2). For multiport fiber couplers, an explicit determination of the phase differences requires numerical calculations and integrations based on mode field distributions, which are in general difficult to obtain in the taper region. Instead, a uniform field distribution with radius r_0 [rectangular field distribution, $\Pi(r/r_0)$] is assumed here for estimating the phase differences in lossless multiport fiber couplers. It is a valid approximation because the equiphase planes of all the mode fields are parallel. For partially overlapping couplers, this uniform field model gives approximate trends and bounds for the phase changes. For example, if only a fraction α of the total fields overlap and interfere, then from the uniform field assumption, one has for the lossless 2×2 coupler

$$P_{\text{total}} = \int (T^2 + C^2)\mathrm{d}s$$

$$= P_{\text{taper}} = \int [|\sqrt{\alpha}\tilde{T} + \sqrt{\alpha}\tilde{C}|^2 + (1-\alpha)T^2$$

$$+ (1-\alpha)C^2]\mathrm{d}s, \quad (4)$$

which, as Eq. (2), leads to the condition stated in Eq. (3) again. Equations (2) and (4) are expressed in terms of power; the integration symbol can be elimi-
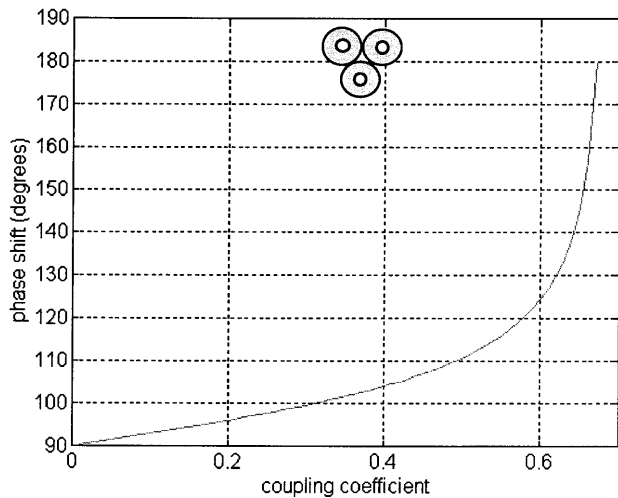
Fig. 1. Dependence of the phase difference between transmission and coupling coefficients on the magnitude of the coupling coefficient in a symmetric 3×3 fiber coupler.

nated for simplicity if we keep in mind that each term represents power in the corresponding mode.

The phase shift of $\pi/2$ can also be derived from output orthogonality. With the input in fiber 1, the output vector is $(\tilde{T}, \tilde{C})$, whereas with the input in fiber 2, it is $(\tilde{C}, \tilde{T})$. Orthogonality implies a phase difference of $\pi/2$ between $\tilde{C}$ and $\tilde{T}$. If the coupler is lossy, Eq. (2) will not hold and the phase difference is in general different from 90°.

3. Multiport Couplers

The interferometric model can also be applied to multiport fiber couplers. For example, a symmetric 3×3 coupler as shown in the insert of Fig. 1 consists of three fibers arranged in cross section in an equilateral triangle. The coupling coefficients between neighboring fibers are equal because of symmetry. Again calling the transmission and coupling coefficients $\tilde{T}$ and $\tilde{C}$, respectively, we see that energy conservation leads to

$$|\tilde{T} + 2\tilde{C}|^2 = T^2 + 2C^2 = 1. \qquad (5)$$

As justified at the end of the previous paragraph, the integration symbol is neglected in Eq. (5) and in the following equations for simplicity. It is valid because total mode field transition is assumed for the taper. To satisfy Eq. (5), the relative phase difference between the transmission and the coupling coefficients must satisfy

$$\cos \phi = -\frac{C}{2T}. \qquad (6)$$

If in particular the input to one fiber is uniformly distributed among the three outputs, then $C = T$ and $\phi = 2\pi/3$. This phase shift has been used in fiber gyroscopes for phase biasing.[6] In contrast with the 2 × 2 coupler, the phase difference between the transmitted and coupled fields in a 3×3 coupler depends on the magnitude of the coefficients. This dependence of phase difference ϕ on C is shown in Fig. 1. The phase shift varies from $\pi/2$ for weak coupling ($C \ll 1$) to π for strong coupling ($C = C_{\max} = 2T_{\min}$). Because the magnitude of $\cos \phi$ should be less than 1, and $T^2 + 2C^2 = 1$, the minimum transmission coefficient of a 3×3 coupler is $T_{\min} = 1/3$. This means that at least 11% of the input power will remain in the input fiber and total power transfer is not possible in a 3×3 coupler, a result also pointed out by Birks.[7]

There are usually different possible configurations for multiport fiber couplers. Another configuration for a 3×3 coupler is the linear array, often referred to as a 1×3 fiber coupler.[8] Here the input fiber is at the center of the array and the other two fibers are on both sides of the input fiber. The phase shift of the 1×3 linear array coupler depends on the interaction between the two nonadjacent fibers. If the coupling region is not tapered, the overlap between the two nonadjacent fiber fields can be neglected. Under such conditions the field of the input fiber interacts symmetrically with both neighboring fibers, and the coupler can be modeled as two simple 2×2 couplers. Hence the phase difference between the transmission and coupling coefficients is $\pi/2$, and total power transfer from one fiber to the others is possible. Assuming that the power of the input fiber interacts equally with the fields in each neighbor, we see that the interferometric model leads to

$$I_{\text{taper}} = 2|\tilde{T}/\sqrt{2} + \tilde{C}|^2 = T^2 + 2C^2 = 1, \qquad (7)$$

which again leads to condition $\phi = \pi/2$.

If the linear array coupler is tapered, the field overlap between the two nonadjacent fibers increases, and the phase shift depends on the coupling coefficient in a manner similar to the plots shown in Fig. 1. The phase shift of an arbitrary linear array 3×3 fiber coupler should therefore be within the region bounded by the line $\phi = \pi/2$ and the curve in Fig. 1, depending on the field overlap. By assuming total field overlap, we can determine an upper bound of the phase difference for a number of coupler configurations. This upper bound is a good approximation to the phase difference for tapered fiber couplers in which the fields nearly totally overlap. Some examples are given in Section 4.

4. Symmetric 1×N Couplers

1×N fiber couplers with uniform power distribution are of interest in communication systems. Two typical configurations are shown in Fig. 2, where fiber 1 is the input fiber and the coupling to the other fibers is equal. The symmetry for 1×4 fiber couplers can be guaranteed by inserting three dummy fibers between the three coupling fibers, 2, 3, and 4, shown in Fig. 2.[9] For tapered couplers, the field overlap between all the fibers is almost complete and the phase shift can be determined by using the interferometric model together with energy conservation. For a lossless 1×N fiber coupler with total overlap, we have

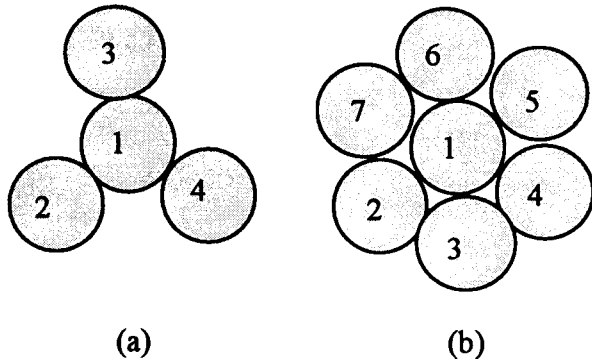$$|\tilde{T} + (N-1)\tilde{C}|^2 = T^2 + (N-1)C^2 = 1, \qquad (8)$$

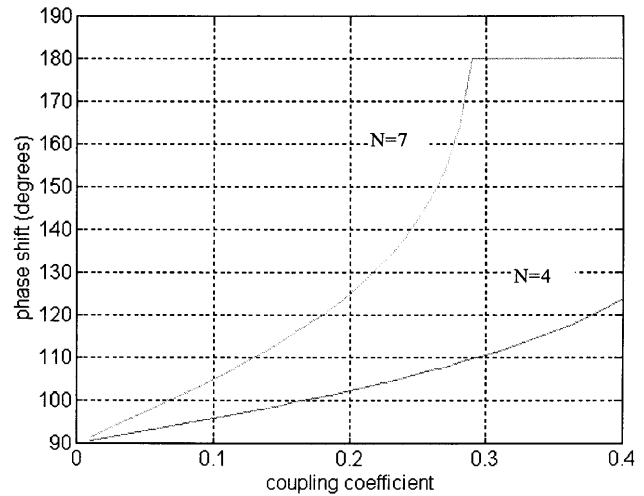Fig. 2. Fiber configurations of the symmetric 1×4 and 1×7 fiber couplers.

Fig. 3. Dependence of the phase difference between transmission and coupling coefficients on the magnitude of the coupling coefficient in the symmetric 1×4 and 1×7 fiber couplers.

where again $\tilde{T}$ and $\tilde{C}$ are the transmission and coupling coefficients, with magnitudes T and C, respectively. Relative phase ϕ of the coefficients obtained from Eq. (8) is given by

$$\cos \phi = -(N-2) \frac{C}{2T} \quad (N > 2), \qquad (9)$$

where $T = [1 - (N-1)C^2]^{1/2}$.

The dependence of phase ϕ on coupling coefficient C is shown in Fig. 3 for $N = 4$ and $N = 7$. This phase shift is an upper bound that gives a good approximation to the actual phase shift for the case of a symmetric tapered fiber coupler.

Because $\cos \phi$ must be smaller than unity, the 1×N coupler with total overlap has a maximum coupling coefficient $C_{max} = 2/N$ or, equivalently, a minimum transmission coefficient $T_{min} = (N-2)/N$. For many applications, however, equal power distribution among all the outputs is desirable. This requires $C = T = 1/\sqrt{N}$. The two conditions are mutually compatible only for $N = 3$ (with $\phi = 2\pi/3$ for $C = T = 1/\sqrt{3}$) and $N = 4$ (with $\phi = \pi$ for $C = T = 1/2$). For the 1×7 coupler, $C_{max} = 2/7$, which means that $(5/7)^2 \approx 52\%$ of the injected power remains inside the input fiber. This drawback is the main reason why weak field interactions with long coupling lengths are found to be preferable for increasing the value of the maximum coupling coefficient in such couplers. By reducing the field overlap between nonadjacent fibers, the phase shifts become smaller than the upper bound shown in Fig. 3, and equal power distribution becomes possible. This is an example of how the phase analysis technique can be applied to coupler design and manufacturing.

Figure 3 shows upper bounds of the phase difference for the 1×4 and 1×7 couplers. The actual phase shift depends on the amount of field overlap, and the minimum value is the one that is determined by assuming no field overlap between nonadjacent fibers. In this low field interaction limit, the fields of different fibers are partially confined inside the individual fibers and only a fraction of the fields will effectively interfere with each other in the center input fiber. Assuming that a fraction α of the field intensity overlaps and interferes within the center

fiber, we see that energy conservation and the interferometric model gives for the 1×N coupler

$$|\tilde{T} + (N-1) \sqrt{\alpha}\tilde{C}|^2 + (N-1)(1-\alpha)C^2$$
$$= T^2 + (N-1)C^2 = 1, \quad (10)$$

where the definitions of the coefficients are the same as in Eq. (8). The phase difference between transmission and coupling coefficients can be derived from Eq. 10 and is given by

$$\cos \phi = -(N-2) \frac{\sqrt{\alpha}C}{2T}. \qquad (11)$$

Equations (10) and (11) reduce to Eqs. (8) and (9) under the total field overlap condition ($\alpha = 1$). When the field overlap decreases, the phase shift will also decrease according to Eq. (11). The coupling coefficient can still be made large by increasing the length of the coupling region, even though the field overlap is small. The smaller the field overlap, the longer the coupling region has to be. The minimum phase is $\pi/2$ for any 1×N couplers when the field interaction between fibers is very weak. The condition for equal power distribution in a 1×N coupler is $C = T = \sqrt{N}$. Equation (11) thus leads to a maximum field overlap $\alpha_{max} = (2/N - 2)^2$. For a 1×7 coupler, $C = T = 0.378$ and $\alpha_{max} = 0.16$. It may be difficult to realize such a coupler because the upper limit of α will require a long interaction region, which may lead to oscillations in the transfer function.

5. Asymmetric Couplers

Because there are at least two independent coupling coefficients in asymmetric couplers, mode field orthogonality is generally needed, in addition to energy conservation and symmetry arguments, to determine the phase shift. Figure 4 shows two different 1×4 couplers. The coupler of Fig. 4(a) is simple to analyze by using the mode orthogonality principle. As-
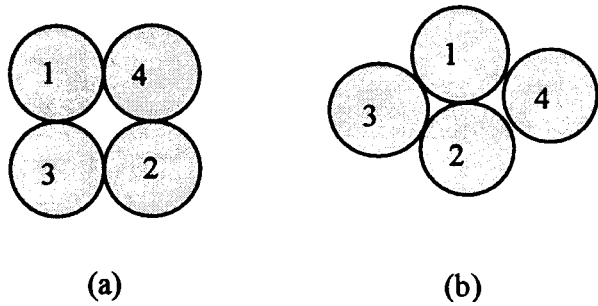
Fig. 4. Two other possible fiber configurations of a 1×4 fiber coupler.

suming a coupling coefficient $\tilde{C}_1$ between any pair of adjacent fibers and $\tilde{C}_2$ between nonadjacent fibers, we see that the scattering matrix is

$$S = \begin{bmatrix} \tilde{T} & \tilde{C}_2 & \tilde{C}_1 & \tilde{C}_1 \\ \tilde{C}_2 & \tilde{T} & \tilde{C}_1 & \tilde{C}_1 \\ \tilde{C}_1 & \tilde{C}_1 & \tilde{T} & \tilde{C}_2 \\ \tilde{C}_1 & \tilde{C}_1 & \tilde{C}_2 & \tilde{T} \end{bmatrix}. \tag{12}$$

Orthogonality of the row vectors leads to the two relationships,

$$TC_2 \cos \phi_2 + C_1{}^2 = 0, \tag{13}$$

$$T \cos \phi_1 + C_2 \cos(\phi_1 - \phi_2) = 0, \tag{14}$$

where ϕ_1 and ϕ_2 are the relative phases between $\tilde{C}_1$ and $\tilde{T}$ and $\tilde{C}_2$ and $\tilde{T}$, respectively. Note that in this case, the interferometric model with the assumption of total field overlap leads to

$$|\tilde{T} + 2\tilde{C}_1 + \tilde{C}_2|^2 = T^2 + 2C_1{}^2 + C_2{}^2 = 1,$$

or

$$C_1{}^2 + TC_2 \cos \phi_2 + 2TC_1 \cos \phi_1 + 2C_1C_2$$
$$\times \cos(\phi_1 - \phi_2) = 0, \tag{15}$$

which is exact according to the orthogonality relationships of Eqs. (13) and (14).

The coupler of Fig. 4(b) is more complex because it involves a large number of independent coefficients of the scattering matrix. Useful approximate relationships can be derived from the interferometric model, however. First, the orthogonality of the two output fields obtained with input from fiber 1 and fiber 2 leads to Eq. (13), from which phase shift ϕ_2 can be determined if the magnitudes of coupling coefficients C_1 and C_2 are known. Equation (14) is not accurate for this coupler, but under the assumption of total field overlap Eq. (15) is a reasonable approximation coming from the interferometer model. From Eqs. (13) and (15), we can conclude that Eq. (14) is a reasonable approximation from which phase shift ϕ_1 can be estimated. Corrections to the phase shifts obtained in this limit can be made according to the amount of field overlap in a manner similar to that used to derive Eqs. (10) and (11).
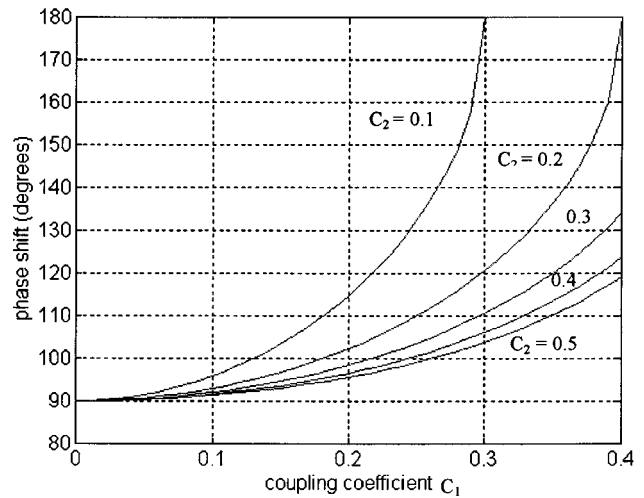

Fig. 5. Dependence of phase ϕ_2 on coupling coefficient C_1 for different values of C_2 (see text for definitions).

Figure 5 shows phase shift ϕ_2 as a function of C_1, for various values of C_2. For $C_1 = C_2$, one finds the previous result of the symmetric 1×4 coupler, and for $C_2 \sim 0$, the result of the symmetric 1×3 coupler. It is interesting to note that phase ϕ_1 is relatively insensitive to changes in coupling coefficient C_1 and increases almost linearly with C_2, as shown in Fig. 6. It can be verified that the relationship between ϕ_1 and coupling coefficient C_1 obtained from Eqs. (13) and (14) is very close to the curve shown in Fig. 3 for $N = 4$, which means that the phase difference is almost the same for any kind of fiber configuration if total field overlap is assumed.

6. Summary

We have shown that the phase shift in single-mode fiber couplers can be estimated by using an interferometric model. The model's validity relies on the total field overlap condition, which is a valid assumption for strongly coupled tapered fiber couplers. The phase
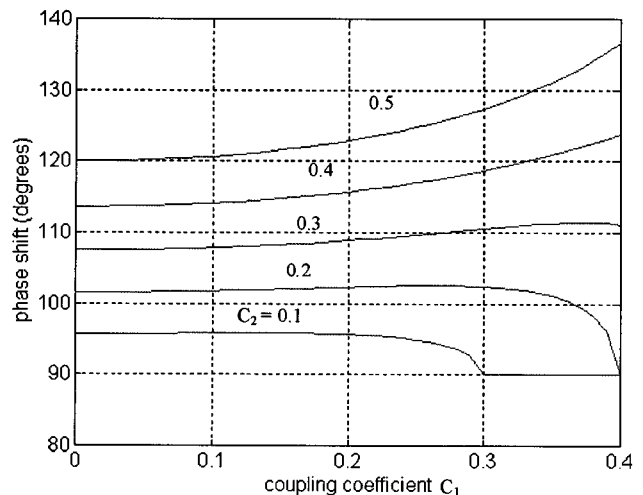

Fig. 6. Dependence of phase ϕ_1 on coupling coefficient C_1 for different values of C_2 (see text for definitions).

difference in weakly coupled couplers can also be estimated by the interferometric model by assuming partial field overlap and energy conservation. Estimating the phase differences between the transmitted and the coupled-mode fields is straightforward compared with the scattering matrix technique. The model also provides a clear physical meaning for the phase difference occurring in fiber couplers. Different single-mode fiber couplers have been analyzed as examples. The results are consistent with those determined by using network analysis and experiment.

References

1. S. K. Sheem, "Fiber-optic gyroscope with 3×3 directional coupler," Appl. Phys. Lett. **37,** 869–871 (1980).
2. K. P. Koo, A. B. Tveten, and A. Dandridge, "Passive stabilization scheme for fiber interferometers using 3×3 fiber directional couplers," Appl. Phys. Lett. **41,** 616–618 (1982).
3. R. Hereth and G. Schiffner, "Broad-band optical directional couplers and polarization splitters," J. Lightwave Technol. **7,** 925–930 (1989).
4. G. L. Abbas, V. W. S. Chan, and T. K. Yee, "A dual-detector optical hyterodyne receiver for local oscillator noise suppression," J. Lightwave Technol. **3,** 1110–1122 (1985).
5. J. Pietzsch, "Scattering matrix analysis of 3×3 fiber couplers," J. Lightwave Technol. **7,** 3030–3037 (1989).
6. H. Poisel, G. F. Trommer, W. Buhler, E. Hartl, and R. Muller, "Low-cost fiber-optic gyroscope," Electron. Lett. **26,** 69–70 (1990).
7. T. A. Birks, "Effect of twist in 3×3 fused tapered couplers," Appl. Opt. **31,** 3004–3014 (1992).
8. A. Niu, C. M. Fitzgerald, T. A. Birks, and C. D. Hussey, "1×3 linear array singlemode fiber couplers," Electron. Lett. **28,** 2330–2332 (1992).
9. D. B. Mortimore, J. W. Arkwright, and R. M. Adnams, "Monolithic wavelength-flattened 1×4 singlemode fused fiber coupler," Electron. Lett. **27,** 2252–2253 (1991).