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Resonant manifestation of intrinsic nonlinearity within electroelastic micropower generators

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This letter investigates the nonlinear response of a bimorph energy harvester comprised of lead zirconate titanate (PZT-5A) laminates. For near resonant excitations, we demonstrate significant intrinsic nonlinear behavior despite geometrically linear motion. Fourth order elastic and electroelastic tensor values for PZT-5A are identified following methods recently published concerning a PZT-5H bimorph. A response trend indicative of a nonlinear dissipative mechanism is discussed as well as the inadequacy of linear modeling. The PZT-5A bimorph exhibits an increased softening frequency response in comparison to PZT-5H. The results contained herein are also applicable to electroelastic sensor and actuator technologies. © *2010 American Institute of Physics*. [doi:10.1063/1.3530449]

Hysteretic and constitutive nonlinearities comprise an important aspect of the dynamic interaction between the intensive (electric field, stress, and temperature) and extensive variables (polarization, strain, and entropy) characteristic of electroelastic materials. Curiously, nonlinear ferroelectric properties are almost uniformly presumed insignificant within a wide body of piezoelectric energy harvesting literature with the exception of a few recent studies.[1–4] To set the context for this letter, Fig. 1(a) illustrates the mechanism by which a piezoelectric cantilever bimorph generates energy in the first vibration mode. Stress along the length of the beam in response to ambient excitation yields a potential difference across the electroelastic laminates such that an electrical current may be conditioned to recharge a battery, power a sensor, or provide energy to a more complex power storage network. Electroelastic energy harvesters are typically designed to operate at resonance in order to take advantage of a significant rise in the mechanical response due to external harmonic excitation. However, the soft nature of many piezoelectric materials may invigorate nonlinear behavior despite small-amplitude motion as a consequence of the combined effects of increased stress and electric fields. Furthermore, a trend toward purposeful exploitation of nonlinear restoring forces for broadband performance employs dynamics on high-energy manifolds typified by large-amplitude motion.[5–14] Nonlinear material effects will become increasingly significant as these fields mature and have already been demonstrated within Refs. 3 and 4 to be worthy of future research and consideration. Accordingly, the contents of this letter are inspired by three major objectives: (1) to experimentally demonstrate the effect of material nonlinearity in another common lead zirconate titanate ceramic (PZT-5A), (2) employ a framework for nonlinear modeling, and (3) identify higher-order tensor values for a PZT-5A and compare to existing results for PZT-5H as studied in Ref. 3.

An electric enthalpy function due to the cubic theory of electroelasticity is utilized to model the effects of nonlinear ferroelectric behavior in a bimorph cantilever beam.[3,4,15–19] Higher-order stiffness and coupling effects are presumed to contribute to the nonlinear response while purely dielectric nonlinearity is neglected due to the weak electric fields. An illustration of this modeling presumption is shown in Fig. 1(b), where a Taylor series expansion of the linear constitutive relations for piezoelectric materials incorporates curva-

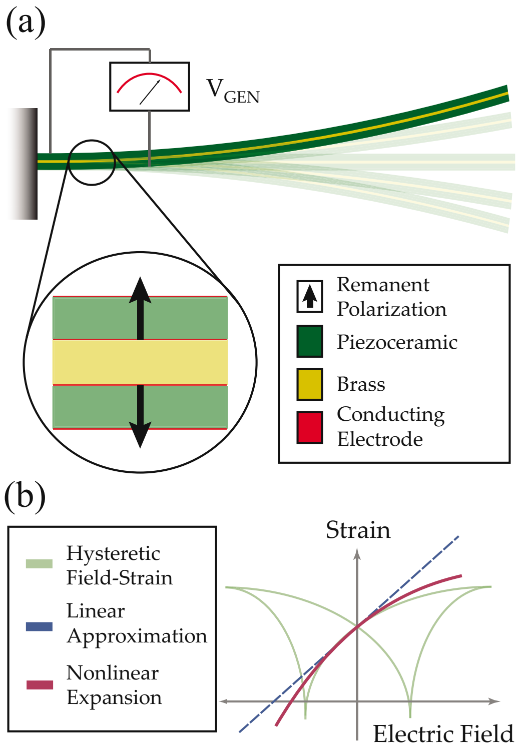

FIG. 1. (Color online) (a) Mechanism of piezoelectric power generation. A piezoelectric bimorph oscillates in response to a base excitation, which in turn yields a charge separation across the electrode layers. Due the series connection between laminates, the remanent polarization of each bimorph device is illustrated. (b) Illustration of the cubic theory of electroelasticity in relation to the linear approximation and the general butterfly hysteresis curve for ferroelectric materials.

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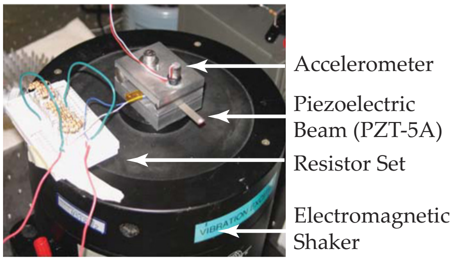

FIG. 2. (Color online) Experimental setup for testing the bimorph harvesters. The PZT-5A cantilever has a series capacitance of 3.83 nF and electromechanical coupling coefficient $\bar{e}_{31} = -10.4\ \mathrm{C/m^2}$ while the PZT-5H capacitance and coupling coefficient values are 7.41 nF and $-16.6\ \mathrm{C/m^2}$, respectively. Both piezoceramics have an elastic compliance of about $s_{11}^E = 16.5\ \mu\mathrm{m^2/N}$ and the mechanical damping ratio for the PZT-5A cantilever is 0.45%.

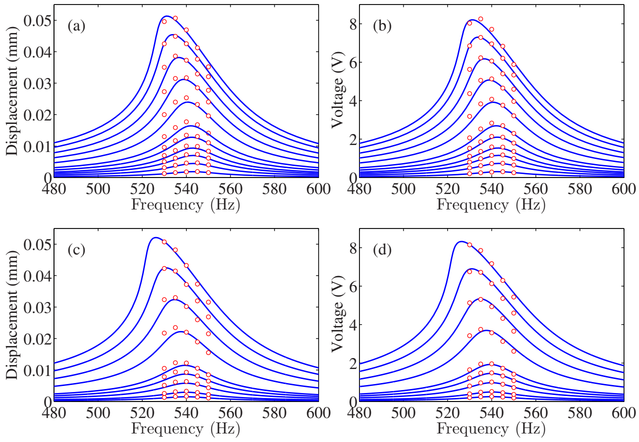

FIG. 3. (Color online) Experimental steady state oscillation amplitudes (○) with numerically identified frequency response curves (−) for the displacement at the location of the laser vibrometer reading (first column) as well as the voltage drop across the resistive load (second column). Graphs (a) and (b) are for PZT-5H while graphs (c) and (d) are for PZT-5A.

ture effects present in the more general hysteretic strain-field curve. Similarly to the linear relations, the nonlinear expansion presumes changes in strain, and polarization remains approximately reversible. This theory has been previously demonstrated to accurately model the nonlinear response of a piezoelectric bimorph and is applied in this letter to analyze a bimorph of comparable dimensions but with PZT-5A laminates instead. To enable parameter identification, an experiment was designed and is pictured in Fig. 2. A brass cantilever (length $L = 23.83$ mm) with symmetric PZT-5A laminates of respective thickness $h_p = 0.264$ mm was connected in series to a 100 kΩ resistive load and secured to an electromagnetic shaker. The overhang volume ($101.9\ \mathrm{mm^3}$) and mass per unit length (0.0334 kg/m) are similar to the PZT-5H cantilever in Ref. 3, which had an overhang volume of $103.2\ \mathrm{mm^3}$ and mass per unit length of 0.0335 kg/m. Also, both devices share an electromechanical resonance of about 540 Hz. Transient-free oscillation amplitudes in both the mechanical domain (as measured by a laser vibrometer) and electrical domain (voltage drop across a resistor) were recorded at discrete base excitation values of $0.05g$, $0.1g$, $0.2g$, $0.3g$, $0.4g$, $0.8g$, $1.2g$, $1.6g$, and $2g$, where $g = 9.8\ \mathrm{m/s^2}$. Near resonant excitation of a uniform electroelastic cantilever can be modeled by a system of nonlinear ordinary differential equations according to Ref. 3 (to which the reader is also referred for a full derivation based on variational methods) as

$$\ddot{x} + 2\mu\omega\dot{x} + \omega^2 x - \theta v = \gamma Z_e \cos \Omega t - \mu_a \dot{x}|\dot{x}| - \alpha x^3 + \varphi x^2 v, \qquad (1a)$$

$$C\dot{v} + R^{-1}v + \theta\dot{x} = -\varphi x^2 \dot{x}, \qquad (1b)$$

where x represents the first modal coordinate, v is the voltage, μ is the proportional damping ratio, ω is the short-circuit natural frequency of the cantilever, θ is the modal linear coupling, γ is a modal excitation term, Z_e is the base acceleration with angular frequency Ω, and C and R represent the PZT capacitance (series connection) and load impedance, respectively. In the mechanical domain, the nonlinear forces are comprised of nonlinear dissipative, stiffness, and voltage coupling terms proportional to the coefficients μ_a, α, and φ. The linear electrical network dynamics are augmented by a similar third-order modal coupling. Due to the small-amplitude motion, Eqs. (1a) and (1b) reflect a simplifying assumption of weak strain proportional to the second spatial derivative of the beam displacement in compliance with linear Euler–Bernoulli beam theory. Moderately large-amplitude motion considerations would yield nonlinear inertial contributions and hardening type spring forces (in the first vibration mode).[4,20,21] Fourier analysis of the experimental data reveals response signal energy primarily concentrated within the first mode and hence a single term harmonic balance analysis provides a sufficiently accurate basis for generating an analytical solution for the system. As such, a tenth order algebraic equation in the modal response was derived, which could be used in a nonlinear least-squares optimization algorithm to identify the nonlinear coefficients.[3]

The cantilever displacement and voltage response results are shown in Fig. 3. The numerically identified analytical solution yields good correlation with the experimental data in both the mechanical and electrical domains. Interestingly, despite the similarity in the beam dimensions, the PZT-5A bimorph presents a slightly increased softening response than the PZT-5H beam. What is more important for energy harvesting analysis, however, is that nonlinear responses in both beams manifest at base acceleration levels as low as $0.5g$—a common value utilized to validate numerical studies of harvester designs. Table I summarizes our numerical identification results for the higher-order elastic and electroelastic tensor components for both piezoceramics.

Considering only linear proportional damping is also a common modeling practice. Our analysis and experiment, however, indicate that as base acceleration levels increase, the frequency response is suppressed in both the response amplitude and frequency range. The effect of this simplification is shown in Fig. 4, where the linear damping model overshoots the experimental data whereas a nonlinear dissipation term is capable of modeling this result. For the

TABLE I. Identified higher-order elastic and electroelastic tensor values for PZT-5A and PZT-5H.

Parameter	PZT-5A	PZT-5H[a]
c_{1111}^p	$-3.9844 \times 10^{17}\ \mathrm{N/m^2}$	$-3.6673 \times 10^{17}\ \mathrm{N/m^2}$
e_{3111}	$1.7849 \times 10^{8}\ \mathrm{m/V}$	$1.7212 \times 10^{8}\ \mathrm{m/V}$

[a] Reference 3.

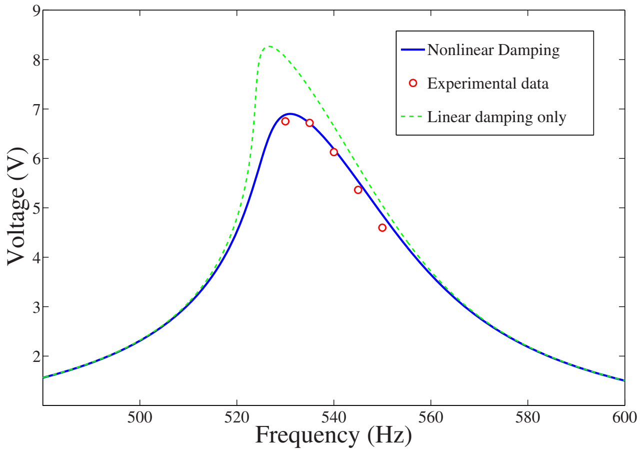

FIG. 4. (Color online) Illustration of a theoretical overshoot of experimental measurements when nonlinear dissipation is neglected. Data are for PZT-5A at a base acceleration of 1.6g.

PZT-5A bimorph, we identified $\mu_a \approx 1.5 \times 10^4$ (PZT-5H: $\mu_a \approx 1.4 \times 10^4$). The exact physical mechanism for this phenomenon may primarily be a consequence of air drag but is a subject of future research and may also reflect structural effects, dielectric phenomena, viscoelectroelasticity,[22] or even residual stress.[23] In particular, how this term scales with cantilever dimensions and proof masses is of interest for more general and accurate models.

The accuracy of a linear modeling framework at electromechanical resonance for both bimorphs is illustrated in Fig. 5. Linear piezoelectricity appears to be a valid presumption for base excitations up to about 0.5g, beyond which the qualitative and quantitative trends in the actual device response can only be accurately described by considering nonlinear effects. At our maximum test acceleration level of 2g, the linear model overpredicts the voltage response with an unacceptable error of nearly 25%–30%. It is worth emphasizing that our particular device has not been tuned to resonate at a lower frequency regime as is typically done. Hence, our devices are extremely stiff yet *still* generate nonlinear responses to base acceleration values studied in devices where linear piezoelectricity is presumed. More flexible devices will begin to demonstrate not only increased material softening effects, but begin to engage geometric nonlinearities as well. Our observations and modeling call into question the accuracy of purely numerical analyses presuming linear piezoelectric response with no experimental validation. This letter provides further experimental validation of the importance of considering nonlinear piezoelectricity in studying electroelastic power generation. To enable more accurate future analyses, fourth order elastic and electroelastic coefficients have been suggested for two common engineering piezoceramics. It is important to note that the mathematical model is for bimorph generators and that an asymmetric configuration or imperfections will yield quadratic nonlinearities in addition to cubic effects. As energy harvesting materials improve, increased elasticity and higher coupling coefficients will render these effects even more significant and future studies should adequately justify exclusion of nonlinear terms.

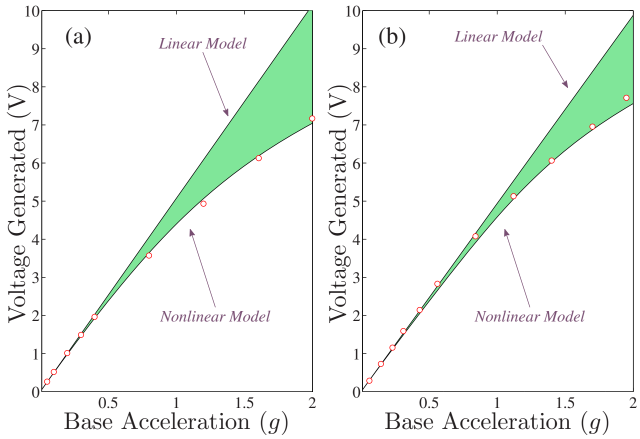

FIG. 5. (Color online) Deviation from true voltage output at electromechanical resonance (540 Hz) for varying base excitation as a consequence of neglecting material and dissipative nonlinearities for (a) PZT-5A and (b) PZT-5H with experimental data for each case (o).

[1] Y. Hu, H. Xue, and J. Yang, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **53**, 1387 (2006).
[2] A. Triplett and D. Quinn, J. Intell. Mater. Syst. Struct. **20**, 1959 (2009).
[3] S. Stanton, A. Erturk, B. Mann, and D. Inman, J. Appl. Phys. **108**, 074903 (2010).
[4] S. Stanton and B. Mann, "Nonlinear electromechanical dynamics of piezoelectric inertial generators: Modeling, experiment, and analysis," Nonlinear Dyn. (to be published).
[5] A. Arrieta, P. Hagedorn, A. Erturk, and D. Inman, Appl. Phys. Lett. **97**, 104102 (2010).
[6] A. Erturk, J. Hoffmann, and D. Inman, Appl. Phys. Lett. **94**, 254102 (2009).
[7] S. Stanton, C. McGehee, and B. Mann, Appl. Phys. Lett. **95**, 174103 (2009).
[8] S. Stanton, C. McGehee, and B. Mann, Physica D **239**, 640 (2010).
[9] B. Mann and N. Sims, J. Sound Vib. **319**, 515 (2009).
[10] S. Shahruz, J. Comput. Nonlinear Dyn. **3**, 041001 (2008).
[11] L. Gammaitoni, I. Neri, and H. Vocca, Appl. Phys. Lett. **94**, 164102 (2009).
[12] F. Cottone, H. Vocca, and L. Gammaitoni, Phys. Rev. Lett. **102**, 080601 (2009).
[13] H. Xue and H. Hu, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **55**, 1632 (2008).
[14] D. Barton, S. Burrow, and L. Clare, J. Vibr. Acoust. **132**, 021009 (2010).
[15] G. Maugin, *Nonlinear Electromechanical Effects and Their Application* (World Scientific, Singapore, 1985).
[16] J. Yang, *Analysis of Piezoelectric Devices* (Springer, New York, NY, 2005).
[17] H. Tiersten, J. Appl. Phys. **74**, 3389 (1993).
[18] U. von Wagner and P. Hagedorn, J. Sound Vib. **256**, 861 (2002).
[19] U. von Wagner and P. Hagedorn, Nonlinear Dyn. **31**, 133 (2003).
[20] M. Daqaq, C. Stabler, Y. Qaroush, and T. Seuaciuc-Osorio, J. Intell. Mater. Syst. Struct. **20**, 545 (2008).
[21] S. Mahmoodi, N. Jalili, and M. Daqaq, IEEE/ASME Trans. Mechatron. **13**, 58 (2008).
[22] S. Parashar and U. von Wagner, Nonlinear Dyn. **37**, 51 (2004).
[23] Y. Kim, L. Cai, T. Usher, and Q. Jiang, Smart Mater. Struct. **18**, 095033 (2009).