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motion, a sequence that mirrors directly the Rayleigh–Bénard problem for a high Prandtl number fluid.

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A simple model for the average local entrainment rate

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A new expression for the mean local entrainment rate in a turbulent intermittent flow is obtained. This expression is used to obtain a simple model for the entrainment rate assuming that an indicator function can be defined for the flow, that the interface defined by the indicator function is homogeneous in two directions, and that the turbulent Reynolds number is very large. A particularly simple form is obtained if the intermittency is in a scalar imbedded in a turbulence field and the correlation coefficient for the indicator function is self-similar. The final expression compares favorably with the limited existing data and with other model expressions. Recommendations for experimental verification are presented.

Most free turbulent shear flows exhibit an intermittent behavior, where for a certain realization of the experiment the fluid at location (x_i,t) is characterized by fluctuating vorticity, while for a different realization of the same experiment the flow is fluctuating but irrotational at the same space and time point. In order to describe such flows conditioned statistics are used. These are generated by introducing an indicator function to condition the fluid mechanical variables. The intermittency factor γ, which is the ensemble average of the indicator function, plays an important role in such flows and in the conditioned zone description, since it represents the probability of occurrence of turbulence at (x_i,t) and is an essential variable in the mathematical relationships between the unconditioned variables and the separate conditioned zone counterparts.

The majority of scientific papers concerning intermittency have appeared in the last ten to fifteen years and have been experimental studies, primarily. There have been fewer papers dealing with theoretical analysis of intermittent flows,[1–11] the most recent efforts of which are those of Kollman[10] and Pope.[11]

All of the papers that propose modeling methods for the prediction of the intermittency factor and conditioned zone statistics require modeling a number of unknown quantities in terms of other variables for which equations exist. One important term requiring modeling is the entrainment term, also called the production rate of intermittency. This term appears explicitly in the equation for the evolution of the intermittency factor,

$$\gamma_{,t} + (\gamma \langle U_i \rangle_T)_{,i} = \langle \dot{\omega} \rangle \,. \tag{1}$$

In this equation and throughout this Letter angle brackets indicate ensemble averages and Cartesian tensor notation is employed. Here $\langle \dot{\omega} \rangle$ represents the mean rate of production of γ and has dimensions of volume per unit volume per unit time. Equation (1) has been derived rigorously by Dopazo.[5]

The present Letter reports some recent work on the modeling of $\langle \dot{\omega} \rangle$, which is based upon Dopazo's rigorous form, retaining its convenient physical interpretation.

Dopazo has developed an expression for $\langle \dot{\omega} \rangle$ that is the starting point for the modeling; the equation requires no modeling assumptions (is not restricted to turbulent/nonturbulent intermittency) and can be considered to be without any prohibitive approximations:

$$\langle \dot{\omega} \rangle = \lim_{\Delta V \to 0} \left\langle \frac{1}{\Delta V} \int_s v_r \, dS \right\rangle . \tag{2}$$

This expression for $\langle \dot{\omega} \rangle$ is given a slightly more convenient notation and interpretation by employing the mean value theorem for integrals and taking the limit to obtain

$$\langle \dot{\omega} \rangle = \langle v_r A \rangle \,, \tag{3}$$

where v_r represents the speed of advance of an element of the surface at the point (x_i,t) relative to the fluid and A is interpreted as the interfacial area per unit volume at (x_i,t). A "spikey" function, A, gives $\dot{\omega}$ the pulse train nature discussed by Libby.[1] A more useful form for our purpose is obtained by reconsidering Eq. (2) and writing the right-hand side as follows:

$$\langle \dot{\omega} \rangle = \lim_{\Delta V \to 0} \left\langle v_r \frac{\Delta S}{\Delta V} \middle| S(x_i,t) \right\rangle P_s(x_i,t) \,, \tag{4}$$

where the notation $\langle v_r \, \Delta S / \Delta V | S \rangle$ is used to indicate a conditional expectation. In this case, the first factor represents the expected value of the quantity $v_r\ \Delta S/\Delta V$, given that an interfacial crossing of S at (x_i,t) has occurred. The second factor, P_s, represents the probability of such crossings at (x_i,t) in the volume ΔV.

For simplicity assume the indicator function interface is homogeneous in the x_1, x_3 directions. In this case, $\langle \dot{\omega} \rangle$ is a function of x_2 and time only and Eq. (4) can be written as shown below under the assumption that $P_s(x_2,t)$ can be written as $B(x_2,t)\Delta x_2$, where B is the probability density function (pdf). The conditioned expectancy is also a function of x_2—the dependency on x_1 and x_3 having been removed by the homogeneity assumption:

$$\langle \dot{\omega} \rangle = \left\langle v_r \frac{\Delta S}{(\Delta S_p \cdot \Delta x_2)} \middle| S \right\rangle B\, \Delta x_2 \, . \tag{5}$$

Here ΔS_p represents the projected area of the interface per crossing [on the (x_1,x_3) plane]. Removing Δx_2 from the expectancy one obtains

$$\langle \dot{\omega} \rangle = \langle v_r \beta \,|S \rangle B \, , \tag{6}$$

where β is the ratio of the interfacial area per crossing to the projected area per crossing (on x_1,x_3). Applying a Reynolds decomposition to v_r and β where a prime denotes a fluctuation about the conditioned mean, Eq. (6) becomes

$$\langle \dot{\omega} \rangle = \{ \langle v_r |S \rangle \langle \beta \,|S \rangle + \langle v_r' \beta' |S \rangle \} B \, . \tag{7}$$

Using reasoning similar to that above one can show that the average interfacial area per unit volume is given by

$$\langle A \rangle = \langle \beta \,|S \rangle B \, , \tag{8}$$

so that the first term on the right-hand side of (7) represents the product of the average speed of the interface relative to the fluid at that point (when the surface is present, of course) and the average interfacial area per unit volume, which is a physically pleasing interpretation.

For Eqs. (6) or (7) to be useful a form for B is needed in terms of available variables. It is possible to obtain an expression for B in terms of γ and a two-point correlation function of the indicator function, I. The result is given by Paizis and Schwarz[12]:

$$B = \gamma_{,2} - 2 \langle I(x_2') I(x_2) \rangle_{,2'} \Big|_{x_2' \to x_2} \, , \tag{9}$$

where $x_2' = x_2 + \Delta x_2$ and $\langle I(x_2') I(x_2) \rangle$ is the correlation between the indicator function evaluated at two separated points in the inhomogeneous, x_2, direction.

Equation (9) applies to both single and multi-valued interfaces; the only restriction being that the interface only occur once in the interval Δx_2 as $\Delta x_2 \to 0$. The single-valued result is well known and can be obtained easily from (9). Unfortunately, the single-valuedness assumption for the surface location (and therefore B) has been shown to be incorrect in a number of important turbulent flows. Here we propose a model for B which does not necessarily result in the single-valued form.

Employing a Reynolds decomposition on the indicator function, and introducing the correlation coefficient

$$\rho(x_2,x_2') = \frac{\langle i(x_2) i(x_2') \rangle}{\langle i_2(x) i(x_2') \rangle^{1/2}} \, ,$$

one obtains an alternative expression after substitution in (9):

$$B = -2\gamma(1-\gamma)\rho(x_2,x_2')_{,2'} \Big|_{x_2' \to x_2} \, . \tag{10}$$

In this form the functional dependence of B and therefore $\langle A \rangle$ on γ is noteworthy, and one recognizes that the length scale characterizing $\langle A \rangle$ and $\langle \dot{\omega} \rangle$ is obtained from B through the derivative of the correlation coefficient.

Equation (10) involves essentially no approximations as long as the interface is homogeneous in x_1 and x_3. It is applicable to both single-valued and multi-valued interfaces.

We must invoke modeling assumptions in order to make (10) useful. Unfortunately there is extremely little experimental data on $\langle \dot{\omega} \rangle$, B, or $\rho(x_2,x_2')$. No information on the general behavior of ρ is provided in the literature. We will simplify (10) by imposing the obvious parametrization of the derivative of ρ by the inverse of a length scale. To make this more explicit we assume ρ can be written in a self-similar form. That is,

$$\rho(x_2,x_2') = \rho(\xi_2' - \xi_2) \, .$$

The following is one possibility:

$$\xi_2 = \int_0^{x_2} \frac{dx_2}{L(x_2)} \, ,$$

where $L(x_2)$ is the length scale that provides self-similar behavior. With these assumptions (10) can be written as

$$B = 2\gamma(1-\gamma)|R\,|/L(x_2) \, . \tag{11}$$

Here R is the slope of ρ at the origin $(x_2' = x_2)$; γ and L must be regarded as functions of x_2, and possibly time, for evolving flows.

Integrating B from zero to infinity in x_2 one obtains

$$m = \int_0^\infty B\, dx_2 = 2|R\,| \int_0^\infty \frac{\gamma(1-\gamma)}{L(x_2)} dx_2 \, , \tag{12}$$

where m is the expected-valuedness of the interface. Paizis and Schwarz obtain $m = 2.35$ for a plane wall jet at a particular x_1 and x_3. Since

$$\int_0^\infty \gamma(1-\gamma) dx_2$$

is on the order of the width of the intermittency layer, l, we expect $|R\,|/L$ to be $O(1/l)$ and therefore B scales with the inverse of the width of the layer. We have then

$$\langle A \rangle = \langle \beta /S \rangle 2\gamma(1-\gamma)/\hat{L} \, , \tag{13}$$

$$\langle \dot{\omega} \rangle = (\langle v_r |S \rangle \langle \beta \,|S \rangle + \langle v_r' \beta' |S \rangle) 2\gamma(1-\gamma)/\hat{L} \, , \tag{14}$$

where $\hat{L}$ is expected to be $O(l)$. If we assume $\langle \beta \,|S \rangle$ is $O(1)$ for a high Reynolds number, the average interfacial area per unit volume scales with the inverse of the width of the layer.

We consider now two simplifying assumptions: (a) the second term in the brackets on the right-hand side of Eq. (14) is assumed small in the case of scalar intermittency when the turbulent Reynolds number, R_l, is very large and the Schmidt/Prandtl number is $O(1)$ or greater and (b) under these same conditions, $\langle \beta \,|S \rangle$ is assumed to be only weakly dependent on x_2 and is taken as constant as an approximation. With these assumptions the final model is

$$\langle \dot{\omega} \rangle = 2C \langle v_r |S \rangle \gamma(1-\gamma)/\hat{L} \, , \tag{15}$$

where C is taken to be an empirically determined dimensionless constant and $\hat{L}$ is a length of $O(l)$. The term $\langle v_r | S \rangle$ is discussed shortly.

Unfortunately no measurements of $\langle \dot{\omega} \rangle$ have been made, so that direct comparison with Eq. (15) is not possible. However, measurements of $B(x_2)$ have been reported and since the behavior of B is vital to the model we can at least compare the model for B with experiment. Comparison of Eq. (15) for $\langle \dot{\omega} \rangle$ must await further experimentation.

Paizis and Schwarz have measured B in a two-dimensional wall jet and we can consider a comparison of the prediction based on (11) with their data. The wall jet is not homogeneous in (x_1, x_3), of course, but it is slowly developing in the x_1 direction so that the homogeneity assumption serves as an approximation in this case. Unfortunately no measurements of $\rho(x_2, x_2')$ are published and of course self-similarity and the corresponding distribution of the scaling length cannot be ascertained. This being the case, we will assume the simplest possible distribution of length scale—assume it to be a constant. If this assumption is made we are essentially comparing the model expression

$$B = c\gamma(1-\gamma) \tag{16}$$

to the experimental data, where c is adjusted to match the data and γ is given in the literature.

In Fig. 1 we have plotted Paizis and Schwarz's data (symbols) together with the results of (16) (solid line) and results for a single-valued interface (broken line). We adjusted c in order to match the maximum measured value of B. (There is no adjustable parameter in the single-valued case.) Neither model performs well in the tails, but the multi-valued model does surprisingly well, considering the great simplification imposed on $L(x_2)$.

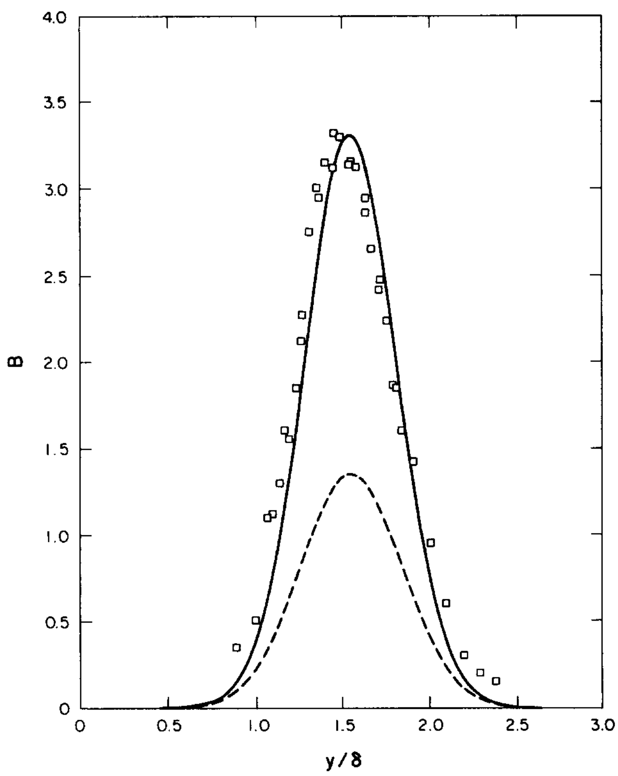

FIG. 1. Probability density of the interface position. Variables are those of Ref. 12. Data of Ref. 12; solid line, Eq. (16); broken line, single-valued approximation.

A new expression for the local mean entrainment rate has been derived [Eq. (4)] that leads with several simplifying assumptions to a simple model for the entrainment rate. The present development assumes homogeneity of the interface in two directions but does not impose any assumption of single-valuedness in the interface. Therefore the model is suitable as an approximation for slowly developing 2-D turbulent flows. If the correlation coefficient for the indicator function for the inhomogeneous direction is self-similar, one can argue that the length scale describing the average interfacial surface area per unit volume and the mean entrainment rate scales with the width of the intermittent layer. Furthermore, if the turbulent Reynolds number is very large and one restricts consideration to scalar intermittency with Prandtl/Schmidt number of $O(1)$ or greater the final simple model (Eq. 15) is obtained as a first approximation. Comparison of the model for the pdf of the interfacial position compares favorably with the limited data. No data is available on $\langle \dot{\omega} \rangle$ for comparison.

If a comparison of the models for $\langle \dot{\omega} \rangle$ which are given in several of the previously cited papers is made with Eq. (15) one observes a striking similarity in all essential features, the only significant difference being in the choice of the velocity scale in the model. (Several of the cited papers employ additional terms in their model to include other entrainment or destruction of entrainment effects which are not similar in form to the one given here.) In the present model the speed $\langle v_r / S \rangle$ represents the average speed of advancement of the interface relative to the surrounding fluid at the same point. It is reasonable to consider this speed to be $O(v_k)$, where v_k is the Kolmogorov velocity scale. In this case, Eq. (15) appears as the exact counterpart in nonreacting intermittent flow to the mean reaction rate model given by Libby[13] for the wrinkled laminar flame model of fast reactions.

There is a need for more experimental information relating to the behavior of the intermittency interface, the local mean entrainment rate and the behavior of the indicator function in turbulent flows. In order to better test the present model two different measurements (both of which appear possible with current technology) are suggested. Measurements of information that permits the determination of $\langle \dot{\omega} \rangle$ are possible in both shear flow turbulence and scalar intermittency situations. These measurements would provide the distribution of $\langle \dot{\omega} \rangle$ in the dominant inhomogeneous direction and in the slowly evolving downstream direction of thin layers. These measurements would require detailed measurements throughout a steady flow field of the intermittency and the conditioned velocities with sufficient accuracy and resolution to permit the evaluation of the derivatives in Eq. (1). Then $\langle \dot{\omega} \rangle$ is computed from the equation. Such measurements are possible and the author is pursuing this work. The second measurement proposal would permit a better assessment of the current model, but is also more difficult. In a thin layer turbulent flow, measurement of the correlation coefficient of the fluctuation of the indicator function as a function of spatial position is proposed. These measurements could be used to test the assumption of self-similarity and to obtain the distribution of the scaling length. This measurement is best performed using a passive tracer to produce the indicator function. Two fine resistance temperature probes in a thermally seeded flow could be used. As the model involves the derivative at the origin the required measurements are more difficult.

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The ponderomotive force in a magnetized plasma: The effect of radio-frequency-induced magnetization

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The ponderomotive force acting on a magnetized plasma resulting from a radio-frequency (rf) electric field is studied both theoretically and experimentally. The fundamental difference between the predictions of the single-particle and fluid approaches is emphasized and its origin elucidated. An experiment is described that illustrates the collective interaction of a fluid with an rf field. This collective behavior manifests itself in the perpendicular ponderomotive force by the interaction of the rf-induced magnetization current with the magnetic field.

In the investigation of the nonlinear interaction of intense rf fields with a magnetized plasma, the time-averaged (ponderomotive) force has attracted considerable attention.[1] Among the various proposed applications, the ponderomotive force has recently been invoked to interpret the observed stabilization of low-frequency modes in mirror devices.[2–5] Several theoretical approaches, including single-particle, fluid, stress tensor, and kinetic, have been used to calculate the ponderomotive force. Each of these has found applications in its appropriate domain of validity. However, it has been noticed by numerous authors[1] that there are basic differences between the predictions derived from the various approaches. In particular, whereas in the single-particle approach the ponderomotive force can be expressed as the gradient of the ponderomotive potential,[6] in the fluid approach this is, in general, not possible.[7–9] In fact, in the fluid description of the ponderomotive force perpendicular to the magnetostatic field, an extra term arises as a result of the interaction of the rf-induced magnetization current with the magnetostatic field.

The purpose of this Letter is to elucidate the origin of the fundamental difference between the predictions of the single-particle and fluid approaches. An experiment is described that demonstrates the influence of the perpendicular ponderomotive force on the motion of an ion beam. It will be shown that the interpretation of the experimental results must take into account the collective interaction of the fluid with the rf field.

We consider a fluid consisting of a single species with charge q and mass m (either ions or electrons) immersed in a uniform external magnetic field $\mathbf{B}_0$. The fluid is subjected to a stationary electromagnetic field having an electric field component $\mathbf{E}(\mathbf{r},t) = \mathrm{Re}[\mathbf{E}(\mathbf{r})e^{-i\omega t}]$. The total time-averaged force acting on a fluid element can be written in the form[7–9]

$$\mathbf{F} = -\nabla p + (q/c)n\mathbf{v}\times\mathbf{B}_0 - n\nabla\Phi + \mathbf{B}_0\times(\nabla\times\mathbf{M}), \qquad (1)$$

where the ponderomotive potential Φ and induced magnetization $\mathbf{M}$ are given by

$$\Phi = (1/16\pi n)(\delta_{\alpha\beta} - \epsilon_{\alpha\beta})E^*_\alpha E_\beta, \qquad (2)$$

$$\mathbf{M} = \frac{1}{16\pi}\frac{\partial\epsilon_{\alpha\beta}}{\partial\mathbf{B}_0}E^*_\alpha E_\beta. \qquad (3)$$

Here $\boldsymbol{\epsilon}$ is the dielectric tensor for a cold, magnetized fluid. In writing Eq. (1) we have assumed that the time-averaged electric field is zero. It should be noted that the expression (2) yields formally the same ponderomotive potential as is derived from a single-particle approach.[6]

In the following, we adopt a cylindrical coordinate system r, θ, z with $\mathbf{B}_0 = B_0\hat{e}_z$ and assume that the amplitude of the oscillating field is a function of r only. For simplicity, we confine the present study to the case for which the fluid density and temperature are uniform. In a steady state, Eq. (1) then shows that the ponderomotive effects give rise to an azimuthal component of the momentum density of a fluid element:

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