Erratum: ''Derivation of amplitude equations and analysis of sideband instabilities in twolayer flows'' [Phys. Fluids A 5, 2738 (1993)]

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Corrigendum for "Derivation of amplitude equations and analysis of sideband instabilities in two-layer flows" [Phys. Fluids A 5, 2738 (1993)]

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In the amplitude equations, one further term should have been taken into account. This term is formally of higher order when the original scaling of variables is used. However, the analysis of sideband instabilities involves various rescalings, and for one of the cases the term becomes of the same order as others. It therefore affects the criteria for sideband instability.

The following changes should be made. The second equation of (1) should have an additional term

$$\epsilon^2[\delta^*(A\bar{A}_\eta)_\eta+\overline{\delta^*}(\bar{A}A_\eta)_\eta]. \tag{C1}$$

In order to derive this term, the following changes should be made in Sec. III. On the right-hand side of (57), add the terms

$$\begin{aligned}&\epsilon A\bar{A}_\eta\{\pi_0 Q_2[D_\zeta;\mathbf{a}_1\exp(i\zeta),\overline{\mathbf{a}_1}\exp(-i\zeta)]\\&\quad+2\pi_0 L_3(R_c,0,0)\phi_4\\&\quad+2\pi_0 N_2[R_c,k_0D_\zeta,0;\mathbf{a}_1\exp(i\zeta),\overline{\psi_2}]\}+\text{c.c.}\\&\quad=:\epsilon A\bar{A}_\eta F+\text{c.c.}\end{aligned} \tag{C2}$$

Here c.c. stands for the complex conjugate of the terms indicated. To (58), add the definition

$$\phi_5=-L(R_c,0,0)^{-1}F. \tag{C3}$$

To the right-hand side of the last equation in (59), add the term $\epsilon A\bar{A}_\eta\phi_5+\text{c.c.}$ To the right-hand side of (60), add the terms

$$\begin{aligned}&\epsilon^2\{[\mathbf{r}_2^*(R_c),\phi_5](A\bar{A}_\eta)_\eta\\&\quad+A\bar{A}_{\eta\eta}(\mathbf{b}_0,Q_2[D_\zeta;\mathbf{a}_1\exp(i\zeta),\overline{\psi_2}\exp(-i\zeta)])\\&\quad+A_\eta\bar{A}_\eta(\mathbf{b}_0,Q_2[D_\zeta;\psi_2\exp(i\zeta),\overline{\mathbf{a}_1}\exp(-i\zeta)])+\text{c.c.}\}.\end{aligned} \tag{C4}$$

Similarly, in (65), we get the additional term

$$\begin{aligned}&\epsilon^2\{[\mathbf{w}_2^*(R_c),\phi_5](A\bar{A}_\eta)_\eta\\&\quad+A\bar{A}_{\eta\eta}(\tilde{\mathbf{b}}_0,Q_2[D_\zeta;\mathbf{a}_1\exp(i\zeta),\overline{\psi_2}\exp(-i\zeta)])\\&\quad+A_\eta\bar{A}_\eta(\tilde{\mathbf{b}}_0,Q_2[D_\zeta;\psi_2\exp(i\zeta),\overline{\mathbf{a}_1}\exp(-i\zeta)])+\text{c.c.}\}.\end{aligned} \tag{C5}$$

As a result of carrying these additional terms, we get the additional term given by (C1) in Eq. (1). The analysis of sideband instabilities is affected as follows. In the last row of (93), the first two entries become

$$\tilde{\delta}i\hat{\nu}A_0-\overline{\delta^*}\theta^2A_0 \tag{C6}$$

and

$$\tilde{\delta}i\hat{\nu}A_0-\delta^*\theta^2A_0. \tag{C7}$$

Instead of (95), we find the condition that the eigenvalues of the matrix

$$\begin{pmatrix}-\kappa\theta^2+\beta A_0^2 & \beta A_0^2 & \delta A_0\\ \bar{\beta}A_0^2 & -\bar{\kappa}\theta^2+\bar{\beta}A_0^2 & \bar{\delta}A_0\\ -\overline{\delta^*}A_0\theta^2 & -\delta^*A_0\theta^2 & -\tilde{\kappa}\theta^2\end{pmatrix} \tag{C8}$$

must have negative real parts for all real values of θ. If κ_r and $\tilde{\kappa}$ are positive, then this is the case for large θ, and hence we only need to check that there are no purely imaginary eigenvalues (other than the zero eigenvalue for $\theta=0$). The characteristic equation for (C8) is of the form

$$-\lambda^3+p_1(\theta^2)\lambda^2+p_2(\theta^2)\lambda+\theta^2\tilde{p}_2(\theta^2)=0, \tag{C9}$$

where the coefficients are polynomials in θ^2 of the degree indicated by the index. Imaginary eigenvalues are ruled out if $\tilde{p}_2$ has no positive real roots and, in addition, either $\theta^2\tilde{p}_2+p_1p_2$ has no positive real roots or p_2 is positive where such a root occurs.

The rest of the analysis can be completed as before, except that some additional terms appear in (96) and (97).

Another correction arises in the final section. In this section, it was argued on the basis of generic transversality conditions that the heteroclinic solutions found at $\epsilon=0$ should persist for small ϵ. It turns out that for the solution (126) these generic transversality conditions do not hold; the reason for this is not fully understood. The failure of transversality follows from the fact that the solution (126) is a member of a one-parameter family of heteroclinic solutions which was found by Bekki and Nozaki.[1] Hence, we cannot, in general, expect heteroclinic solutions close to (126) for small nonzero ϵ. In the special case $\tilde{\gamma}=0$, the full system (115) retains the spatial reflection symmetry of the reduced system for $\epsilon=0$ [Eq. (119)], and in this case, we can expect heteroclinic solutions close to (126) which have the same spatial reflection symmetry. Hence, heteroclinic solutions close to (126) are expected to exist if both ϵ and $\tilde{\gamma}$ are small. We are grateful to Arjen Doelman for bringing this point to our attention.

[1] N. Bekki and K. Nozaki, "Formations of spatial patterns and holes in the generalized Ginzburg–Landau equation," Phys. Lett. A 110, 133 (1985).

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