Inertia-induced breakup of highly viscous drops subjected to simple shear

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Inertia-induced breakup of highly viscous drops subjected to simple shear[a)]

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We investigate the inertia-driven breakup of viscous drops suspended in a less viscous liquid under simple shear. For Stokes flow, it is known that there is a critical value of the viscosity ratio, beyond which breakup does not occur. We find that for viscosity ratios larger than this, inertia can be used as a mechanism of breakup. Inertia increases the angle of tilt of the drops and effectively leads to emulsification for a wider range of viscosity ratios than in Stokes flow. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1564825]

We address the effect of inertia on the deformation and breakup of a drop suspended in a matrix liquid. The drop-to-matrix viscosity ratio is denoted by $\lambda=\mu_d/\mu_m$. The case of equal viscosities is described in Refs. 1–3. At $\lambda=1$, inertia enhances the angle of tilt, so that the drop's tips experience higher velocity and elongation. In two-dimensions, the elongation of a circle (a circular cross section of a cylinder) increases with inertia,[4] and significant elongation is observed at higher viscosity ratios. A circle, however, does not break; it thins indefinitely. The aim of this Brief Communication is to show that in three-dimensions (3D), the addition of inertia increases the critical viscosity ratio λ_c (beyond which the drop does not break) from the Stokes flow value of 3.1. This, therefore, widens the range of viscosities at which shear-mixing takes place under simple shear.

In Stokes flow, the mechanism for achieving a stationary shape is that the drop rotates toward the direction of the flow, away from the axis of elongation which is at 45 degrees to the flow direction. Thus elongation is suppressed, and breakup does not occur.[5–9] Does the critical viscosity ratio increase when inertia is included? We denote the Reynolds number by $\mathrm{Re}=\rho\dot{\gamma}a^2/\mu_m$, and the capillary number by $\mathrm{Ca}=\mu_m\dot{\gamma}a/\sigma$. Here, a denotes the initial radius of the drop, ρ the density of the matrix liquid, $\dot{\gamma}$ the shear rate, and σ the surface tension coefficient. When the drop is subjected to simple shear, the capillary number represents the relative importance of viscous vs surface tension effects. When the capillary number is large enough, then capillary force becomes dominant only with the development of small length scales such as necks. However, until such length scales develop, viscous force dominates over capillary force. We can, therefore, examine the dependence of initial drop rotation on Re and obtain scalings for the dependence of λ_c on Re as follows. A drop under simple shear rotates clockwise with an angular velocity $\dot{\gamma}$. At the same time, the viscous force elongates it at a rate T/μ_d, where T denotes the magnitude of the stresses acting on a more viscous drop. The stress T is the product of viscosity and deformation rate. The drop cannot break when the rotation rate is larger than the deformation rate

$$\dot{\gamma}>\frac{T}{\mu_d}. \tag{1}$$

For example, in Stokes flow, T represents the viscous stress $T\sim\mu_m\dot{\gamma}$, so that Eq. (1) gives $\lambda_c\sim 1$.

Inertia in the matrix liquid changes the breakup scenario. For the case of large inertia, we can use equations governing an inviscid fluid to provide insight into these changes. In particular, Bernoulli's equation in the surrounding liquid yields a pressure gradient which sucks the drop to a higher tilt.[1,2] This Bernoulli pressure, or Reynolds stress, is $T\sim\rho U^2$, where U is the velocity scale ($U\sim\dot{\gamma}a$ in shear flow). Condition (1) then gives $\dot{\gamma}>\rho U^2/\mu_d$, or

$$\lambda_c\sim\mathrm{Re}. \tag{2}$$

These scalings and the numerical results presented below demonstrate that inertia can provide the force necessary to keep the drop aligned with the extensional axis. Thus, in the presence of inertia, very viscous drops can be broken.

Direct numerical simulations are conducted with the volume-of-fluid method and a parabolic reconstruction of surface tension (VOF-PROST). Our algorithms and implementation are described in Refs. 3, 10–19. In particular,

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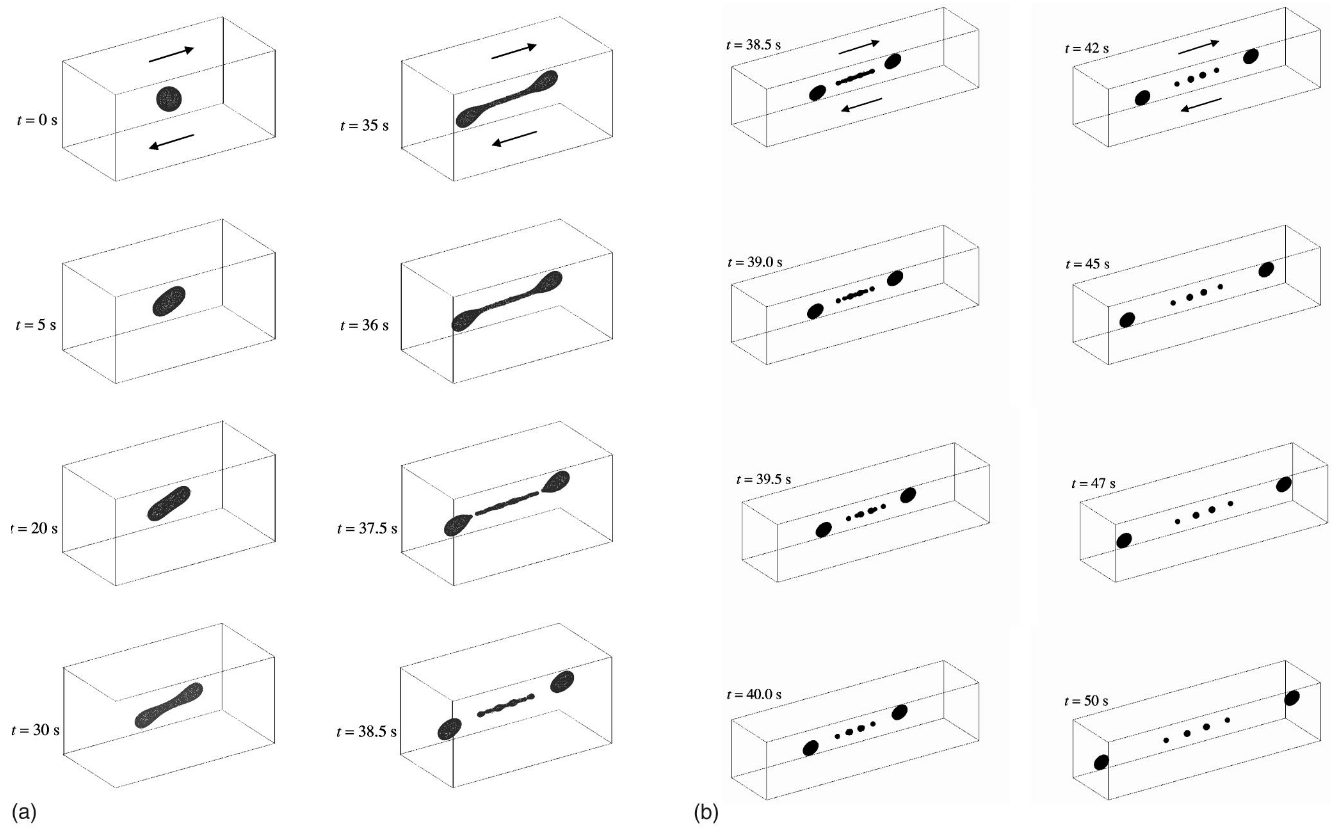

FIG. 1. Snapshots of drop shape for $\lambda=5$, $\mathrm{Re}=10$, $\mathrm{Ca}=0.17$. VOF-PROST algorithm: Computational domain $32a\times 8a\times 8a$, mesh $\Delta x=\Delta y=\Delta z=a/12$, $\Delta t=0.002\dot{\gamma}^{-1}$. The first daughter drops are formed by end-pinching. Subsequently, fragmentation of the filament yields smaller satellites.

VOF-PROST implements a sharp-interface surface tension force algorithm described in Ref. 20. We refer the reader to these references for tests conducted on efficiency and accuracy. The computational domain is chosen sufficiently large so that the simulations concern a drop in an infinite bath. In Fig. 1, for example, tests determine that xyz dimensions of $32a\times 8a\times 8a$, where a is the initial radius, suffice. Figure 1 shows the case $\lambda=5$, $\mathrm{Re}=10$. After the first daughter drops pinch off, the rest of the drop lies in a weak velocity field and surface tension effects are important in the subsequent fragmentation. This viscosity ratio is less than critical for $\mathrm{Re}=10$: The extensional component of shear has enough time to elongate the drop and to break it. In Fig. 2, the angle of inclination Θ of stationary drops just below criticality is plotted. This shows that the orientation angle decreases with increasing viscosity ratio, at fixed Re. The angle increases as Re increases at fixed viscosity ratio.

Figure 3 provides a comparison of the critical curves for several Reynolds numbers. According to a phenomenological description of near-critical drop behavior in Stokes flow, there is a vertical asymptote[21]

$$\mathrm{Ca}_c \sim (\lambda_c-\lambda)^{-1/2}, \quad \lambda\to\lambda_c, \quad \lambda_c\approx 3.1. \tag{3}$$

Figure 3 shows that when Re is increased, inertia shifts the asymptote to higher values of λ. This is consistent with the singular behavior (3) with λ_c growing with the Reynolds number (2). Figure 4 for $\mathrm{Re}=1$ shows that the numerical results for critical capillary numbers are fitted well by (3) with $\lambda_c\approx 6.5$.

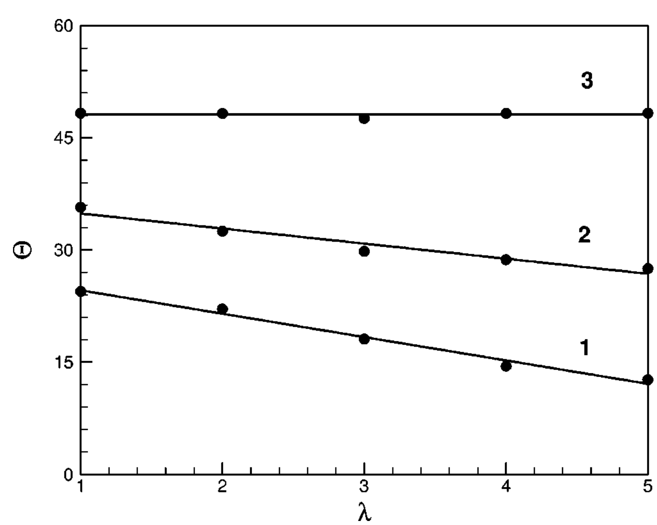

FIG. 2. Drop orientation as a function of the viscosity ratio λ for near-critical stationary states. Parameters: (1) $\mathrm{Re}=1$, $\mathrm{Ca}=0.46$; (2) $\mathrm{Re}=10$, $\mathrm{Ca}=0.139$; (3) $\mathrm{Re}=50$, $\mathrm{Ca}=0.05$.

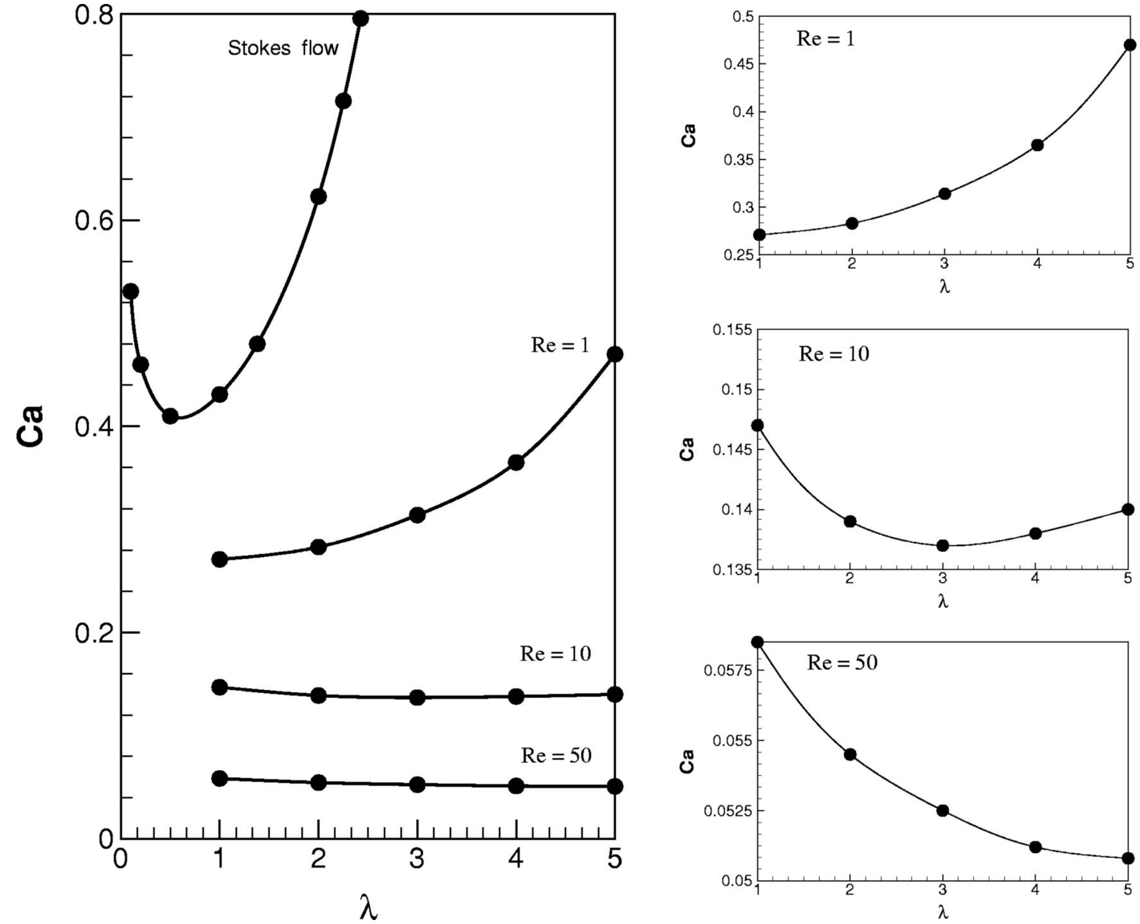

FIG. 3. Critical capillary number (left) as a function of the viscosity ratio λ for Re=0, 1, 10, 50, and magnifications (right). VOF-CSF algorithm, $\Delta x=\Delta y=\Delta z=a/8$, $\Delta t=0.001\dot{\gamma}^{-1}$, computational domain $16a\times 4a\times 8a$.

In Stokes flow, the critical capillary number hits the minimum value at around $\lambda=0.6$.[9,10] The magnifications in Fig. 3 show that inertia shifts the minima toward higher λ. For instance, the minimum at Re=10, is close to $\lambda=3$, while at Re=50, the minimum occurs for λ larger than 5. Correspondingly, λ_c increases with increasing Reynolds number.

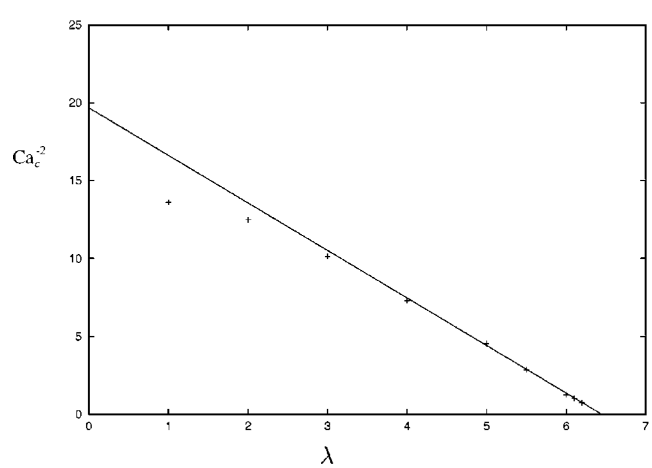

FIG. 4. Critical curve from simulations for Re=1 (symbols) compared with scaling $Ca_c=k/\sqrt{\lambda_c-\lambda}$, with $k=0.573$ and $\lambda_c=6.45$ (solid line).

A typical pre-mixed emulsion contains drops of various sizes. After shearing, a number of those drops would be fragmented. We examine the analogous problem of numerically simulating the breakup of mother drops of varying volumes while keeping flow conditions the same. This requires that $Re=K Ca^2$ where K is a constant. For the case $\lambda=1$,[1–3,9] it is known that the daughter drop volume $V_D=(4/3)\pi D^3$ saturates to a fraction of the critical volume V_c as the mother drop size increases. In particular, at $\lambda=1$, inertia decreases the saturated V_D/V_c. For the case of λ larger than 1 (e.g., $\lambda=4$, and $K=7.5$, 525, and 18 852, corresponding to criticalities at Re=1, 10, 50, respectively), we again find that an increase in Reynolds number results in a decrease of this fraction. Figure 5 shows the effect of viscosity ratio for fixed Re on the saturated daughter volume. Here, the critical Reynolds numbers are 1 for (a) and 50 for (b), and λ is varied from 1 to 4. We see that V_D/V_c saturates at a slightly smaller value for larger λ. The effect of viscosity ratio for our data is, however, weak as observed for Stokes drops.[9]

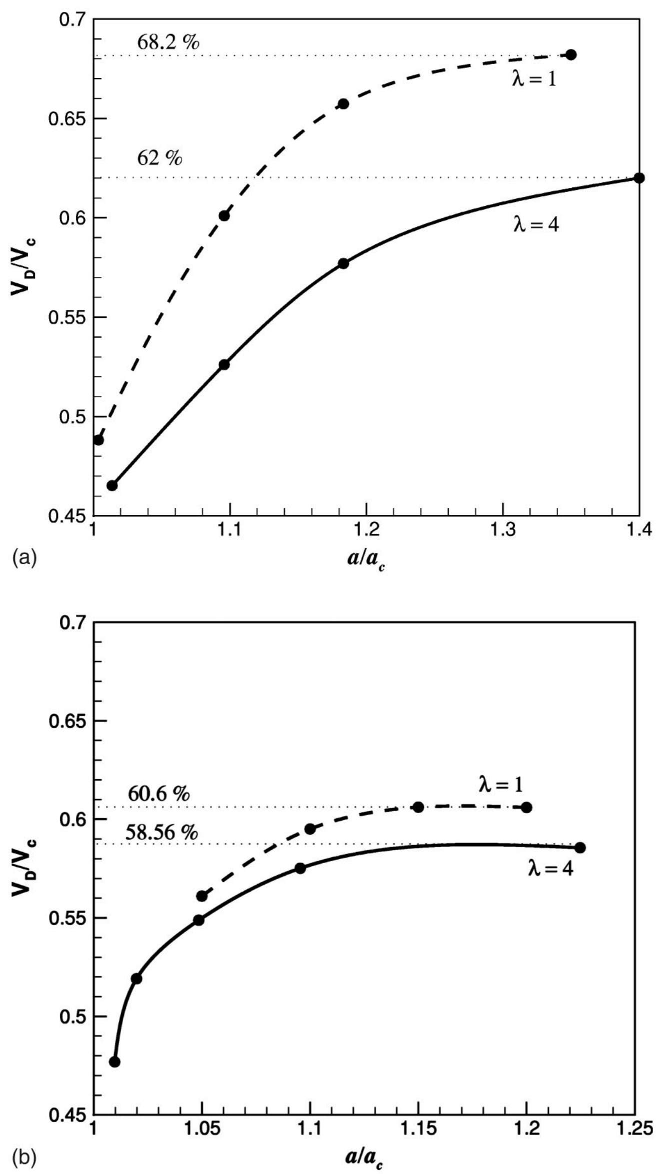

FIG. 5. Ratio of daughter volume to critical volume V_D/V_c vs ratio of mother drop radius to critical radius a/a_c. (a) $\mathrm{Re}_c=1$, $\lambda=1,4$, and (b) $\mathrm{Re}_c=50$, $\lambda=1,4$. At $\lambda=4$, $K=7.5$ (a), 18 852 (b) with $\mathrm{Re}=K\mathrm{Ca}^2$ along the curves.

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