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# Simple model of convectiondiffusion coupling near flames

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$$\rho(x) = \int_0^\infty dx' \, K(x,x')\rho(x') + S(x), \qquad (5)$$

where

$$K(x,x') = a\pi \int_0^\infty dc \exp(-c^2)c^3\sigma^2(c)P(c,x,x')$$

and

$$S(x) = \int_0^\infty dc \exp(-c^2)c^4\left(c^2 - \frac{5}{2}\right)\left\{E_2\left(\frac{\sigma(c)}{c}x\right) - E_4\left(\frac{\sigma(c)}{c}x\right)\right\} .$$

note that

$$\lim_{x\to\infty} q(x) = A_T$$

and is known as the thermal creep slip.

We have solved Eq. (5) by techniques described in the earlier work. Results for $q(x)$ and $A_T$ for $\sigma(c)=1$ (BGK model) and a $\sigma(c)$ appropriate for rigid sphere molecules are reported in the Table I. Note that the present results for the BGK model are in excellent agreement with the "exact" results of Ref. 7. Also, note that for $A_T$, for diffuse reflection, the variational expression is given as[3,4]:

$$A_T = -\frac{1}{2}\left[\frac{\int_0^\infty dc \exp(-c^2)c^6\phi_p(c)\phi_t(c)}{\int_0^\infty dc \exp(-c^2)c^6\phi_p(c)} + \int_0^\infty dc \exp(-c^2)c^5\phi_t(c)\right] . \qquad (6)$$

Now, for the model equation (1) we have

$$\phi_t(c) = B - [1/\sigma(c)](c^2 - \tfrac{5}{2}), \quad \phi_p(c) = -1/\sigma(c). \qquad (7)$$

Thus, calculations show that the variational results for the BGK model $[B=0,\ \sigma(c)=1]$ and Eq. (1) with a collision frequency appropriate to rigid sphere molecules are 0.75 and 0.611, respectively. These are in agreement with the numerical results.

It should be noted, however, that if one used the correct $\phi_t(c)$ for rigid sphere molecules[3] expression (6) gives $A_T = 0.6584$, i.e., the results of Table I are not exactly representative of rigid spheres. Rather, they could be off by as much as 7% or 8% from the true results for rigid sphere molecules [Eq. (1) is only a modeled equation]. Nevertheless, the results of Table I indicate that the velocity profile for the rigid sphere molecules is flatter than that obtained by the use of the BGK model. This effect is quite similar to the effect observed in the Kramers' problem,[10] and it appears that velocity profile measurements for the thermal creep problem would be of some interest.


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# Simple model of convection-diffusion coupling near flames


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A Couette-flame sheet flow model is analyzed to study combined indraft convection-thermal diffusion effects on oxygen distribution near intense fires. These effects enhance the local oxygen concentration, thermal diffusion causing local overshoots relative to ambient conditions.


The region surrounding an intense fire is one not only of large radiant heating, but also of high conductive and diffusional transport combined with appreciable thermal buoyancy-induced convection. It has been speculated that the propagation of fires and the spontaneous ignition of combustible objects in their neighborhood is significantly influenced by the coupling of these processes. The present work theoretically investigates this interplay of indraft convection and diffusion by a simple analytical model that crudely simulates the physics of the environment near a fire. Specifically, a Couette-flame sheet flow model is studied wherein not only an indraft is simulated, but also the usually neglected transport mechanism of thermal diffusion. This model has the advantage of yielding closed form analytic solutions for the temperature and oxygen concentration profiles.

In a first approximation, the gas is treated as a binary mixture of oxygen and a heavier gas consisting of $N_2$ plus combustion products such as $CO_2$ and steam, so that only one specie conservation equation is required. Then, the diffusion flux vector $j_i$ in such a mixture including thermal diffusion can be written[1]

$$j_i = -\rho D_{ij}[\nabla C_i + (M_i M_j/\bar{M}^2)K_t \nabla \ln T] \qquad (1)$$

where $C_i$, $M_i$, and $D_{ij}$ are the concentration, the molecular weight, and the ordinary binary diffusion coefficients, respectively, $\bar{M}$ is the mean molecular weight, and $K_t$ is the dimensionless so-called thermal diffusion ratio (see Ref. 2 for a detailed discussion). Physically, thermal diffusion is related to the phenomenon of persistance of molecular velocity and is a complicated function of temperature, composition, species molecular weight ratio, and intermolecular force law. Since $K_t$ can be either positive or negative, the convention

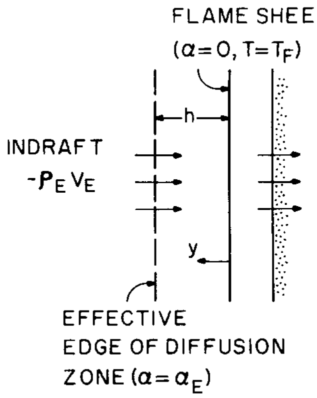


FIG. 1. Idealized convection-diffusion model.

used is that a positive value signifies that component 1 tends to move into the cooler region and component 2 toward the warmer region. For the lighter specie (here oxygen, mass fraction $\alpha$) at all temperatures of practical interest, $K_t$ is always negative and proportional to the product $\alpha(1-\alpha)$ so that it vanishes in either a completely oxygen-free or oxygen-pure environment

$$K_t \simeq -C_t(1-\alpha)\alpha \quad , \tag{2}$$

typical values of the constant of proportionality $C_t$ being on the order of 0.15 to 0.25 for the present binary mixture model $C_t$.[2] Since $K_t$ is always negative for all values of $0 \leq \alpha \leq 1.0$, it is clear that according to the sign convention the thermal diffusion effect causes oxygen to diffuse from cooler to hotter regions.

To simulate the environment near a fire, we consider the simple laminar diffusion convection flow model illustrated in Fig. 1. A normal cross current toward the flame is imposed to represent (roughly) the induced indraft. We then seek the temperature and oxygen concentration fields extending from the flame out to some distance $y=h$ representing the effective ambient environment $T=T_e$, $\alpha=\alpha_e$. In so doing, we include the thermal diffusion effect but neglect the very much smaller inverse back effect on the temperature profile (the so-called diffusion-thermo or Dufour effect[3]). Moreover, to bring out the essential physics without undue complication, the Prandtl, Schmidt, and Lewis numbers are each assumed constant as are the density–viscosity product $\rho\mu$ and the specific heats. The specific heat differences are also neglected. Finally, we imagine the flame to be a stoichiometric one [oxygen completely consumed at its face, i.e., $\alpha(y=0^+)=0$ at some average flame temperature $T_f \gg T_e$.]

Setting the lateral pressure gradient $\partial p/\partial y=0$, $u=\partial/\partial x=0$, and $\rho V=\text{const}=\rho_e v_e$ (the indraft mass flux), introducing the Howarth–Dorodnitzyn transformation $dY=\rho dy/\rho_e$, the Schmidt number $S_c=\mu/\rho D$, and the Prandtl number $\Pr \equiv \mu C_p/\lambda$ and setting $\rho\mu=\text{const}=\rho_e\mu_e$ together with the equation of state $\rho T=\text{const}=\rho_e T_e=\rho_f T_f$, the transport of heat and oxygen mass across the region above the flame sheet is governed by the equations

$$-\,\mathrm{Le}\,R_v\,(dT/d\eta)=d^2T/d\eta^2 \quad , \tag{3}$$

$$-R_v\frac{d\alpha}{d\eta}=\frac{d}{d\eta}\left[\frac{d\alpha}{d\eta}-C_t\,\alpha(1-\alpha)\frac{d\ln T}{d\eta}\right] \quad , \tag{4}$$

where $\eta \equiv Y/H$, Lewis number $\mathrm{Le}=\Pr/\mathrm{Sc}$, and $R_v \equiv \mathrm{Sc}\,\rho_e v_e H/\mu_e$ is the indraft Reynolds number based on $H$. The corresponding boundary conditions are that $T=T_f$, $\alpha=0$ at the flame $\eta=0$ and $T=T_e$, $\alpha=\alpha_e$ at $\eta=1$.

The solution of Eq. (3) for the temperature profile is

$$\frac{T_f-T}{T_f-T_e}=\frac{\exp(-\,\mathrm{Le}\,R_v\eta)-1}{\exp(-\,\mathrm{Le}\,R_v)-1} \quad . \tag{5}$$

In the weak indraft limit of very small $R_v$, this gives a linear temperature profile whereas in the opposite extreme of large indraft Reynolds number (as perhaps would be the case for very intense fires), there is a "boundary layer" behavior with the temperature gradient confined to a thin region adjacent to the flame, as follows:

$$\begin{aligned}&\eta > 1/R_v : (T_f-T)\,(T_f-T_e) \approx 1-0[\exp(-R_v\,\mathrm{Le})]\\ &\eta \ll 1/R_v : (T_f-T)/(T_f-T_e) \approx \mathrm{Le}\,R_v\,\eta - 0[\exp(-R_v)] \;.\end{aligned} \tag{6}$$

Note that in the absence of thermal diffusion, the oxygen concentration profile for $\mathrm{Le}\approx 1$ also has the same form as the temperature

$$\frac{\alpha}{\alpha_e} \simeq \frac{1-\exp(-R_v\,\eta)}{1-\exp(-R_v)} \qquad (C_t=0). \tag{7}$$

This predicts, as expected, that a large indraft situation crowds all the diffusion into a thin sublayer region adjacent to the flame.

Now, consider the solution of Eq. (4) *including* thermal diffusion. It can be integrated once to obtain

$$\frac{d\alpha}{d\eta}=-\left[R_v-C_t(1-\alpha)\frac{d\ln T}{d\eta}\right]\alpha + C \quad , \tag{8}$$

where $C=(d\alpha/d\eta)_{\eta=0}$ is the oxygen diffusion flux into the flame. Note here that since $\partial T/\partial\eta<0$ in the region under consideration, the thermal diffusion term inside the bracket is positive. Further note that $\partial\ln T/\partial\eta$ in this term is a *known* coefficient function of $\eta$. Now strictly speaking, Eq. (8) is nonlinear in $\alpha$ and has the form of a Ricatti equation. However, if a reasonable approximation for the $(1-\alpha)\,d\ln T/d\eta$ term is supplied, it may be more expediently treated as a "quasi-linear" nonhomogeneous differential equation and solved by use of the standard integrating factor method. Thus, it can be integrated to yield

$$\frac{\alpha}{\alpha_e}=\frac{1-\{\exp[-R_v\eta-C_tI(\eta)]\}\{1+\int_0^\eta\exp(R_v\eta)\,d[\exp(C_tI)]\}}{1-\{\exp[-R_v-C_tI(1)]\}\{1+\int_0^1\exp(R_v\eta)\,d[\exp(C_tI)]\}}\,, \tag{9a}$$

where

$$I(\eta)\equiv-\int_0^\eta(1-\alpha)\frac{d\ln T}{\eta}\,d\eta=\ln\left(\frac{T_f}{T(\eta)}\right)+\int_0^\eta\frac{dT/d\eta}{T}\,d\eta \;. \tag{9b}$$

Making the substitution $d\eta=d\alpha/(d\alpha/d\eta)$ and using the zero thermal diffusion solution for $\alpha$ plus the $\mathrm{Le}\simeq 1$ temperature profile, it can be shown, after considerable analysis, that the integrals in Eqs. (9) can be approximated by the closed form expressions

$$I_t(\eta)\simeq\left(1-\frac{T(\eta)}{T_f}\right)\alpha^*+(1-\alpha^*)\ln\left(\frac{T_f}{T(\eta)}\right)\,, \tag{10a}$$

$$\int_0^\eta\exp(R_v\eta)\,d[\exp(C_tI)] \simeq\left(\frac{C_t\,\tilde{\alpha}_e}{\tilde{\alpha}_e-\alpha^*}\right)\exp(C_tI)\ln\frac{(1-\alpha/\alpha_e)^{1-\tilde{\alpha}_e}}{(1-\alpha/\alpha^*)^{1-\alpha^*}}\;, \tag{10b}$$

where $\alpha^*\equiv T_f\,\alpha_e/(T_f-T_e)$ and $\tilde{\alpha}_e\equiv\alpha_e/[1-\exp(-R_v)]$.

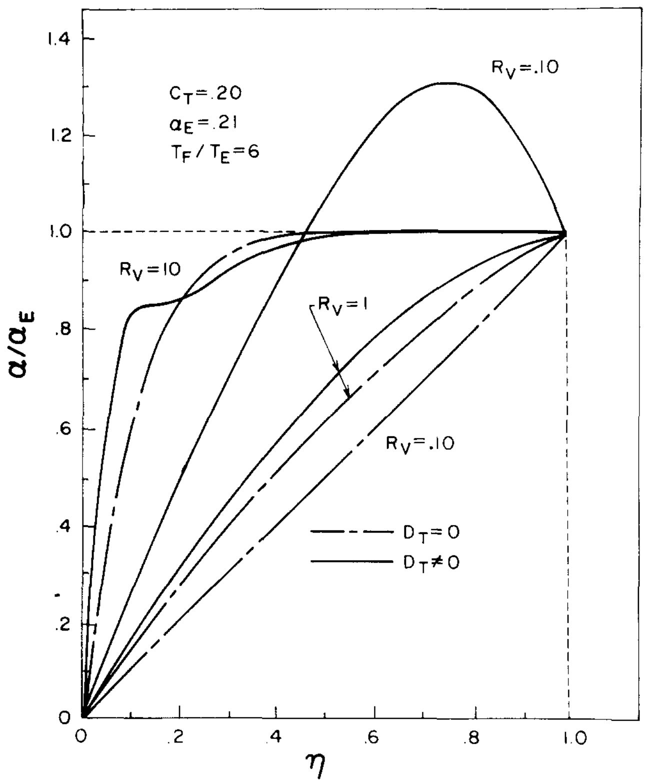


FIG. 2. Typical oxygen profile solutions.

Some typical oxygen diffusion profiles according to the foregoing solutions are presented in Fig. 2 as a function of the indraft Reynolds number $R_v$. The value of $T_f/T_e=6$ used here is typical of the intense fire situations encountered in practice. In the weak indraft limit $R_v \ll 1$ where the oxygen concentration would drop linearly toward the flame assuming $D_t=0$, it is seen that the large temperature gradient acting through the mechanism of thermal diffusion drives a strong oxygen flux toward the hot flame region and thereby significantly increases the local oxygen concentration in the outer half of the region above the flame sheet. Indeed, this effect is so strong in this case that a 10 to 20% overshoot in $\alpha_{02}$ occurs. For intermediate values of $R_v \simeq 0(1)$, this thermal diffusion effect diminishes; however, the local oxygen concentration is still increased by the larger indraft convection effect, which tends to sweep the ambient concentration closer to the flame before it is diminished by diffusion. In the large indraft limit $R_v \gg 1$, the indraft effect completely dominates ($\alpha \simeq \alpha_e$) except in a thin transport sublayer region adjacent to the flame when $T$ rises rapidly toward $T_f$. It is interesting to note that although $\alpha$ tends to also drop rapidly in this layer, the large thermal-diffusion current of oxygen that develops due to the large temperature gradients therein again causes a pronounced local overshoot in the concentration.

The present results suggest that the combined effects of large indrafts and thermal diffusion cause substantial local enhancement of the oxygen concentration in regions adjacent (but not necessarily close) to an intense flame. In particular, it appears that the latter diffusion mechanism may play a more important role in analyzing the gas dynamics of fires than heretofore expected. The occurrence of regions of unusually high oxygen concentration has serious implications concerning the propagation of industrial fires and of forest fire storms.

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# Unstable normal modes induced by magnetic shear

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A simple mathematical model is presented to describe instabilities resulting from the transferral of energy in different regions of an inhomogeneous plasma. The normal mode solution and the stability criterion are computed.

Microinstabilities, such as the universal and loss-cone instabilities, are believed to contribute to the enhanced plasma losses from confinement devices. The theoretical analyses of such instabilites and the means to suppress them by proper design (magnetic shear or finite length, for example) have led to a serious consideration of plasma instabilities of an inhomogeneous medium.[1-9] Subsequently, concepts such as "local dispersion relation" for localized normal modes,[1] convective wavepackets,[2,3] and nonlocal normal modes[4-9] have been developed. In an attempt to provide an understanding of the interrelation among these concepts for plasma instabilities in a nonuniform medium, Lau and Briggs[9] considered an initial value problem, employing a mathematical model for which an exact solution in closed form is obtained. Even though the model can be justified only under certain conditions, the solution which they obtained appears to be able to describe the evolution of disturbances in systems with and without localized normal modes. The solution also reduces to the standard form in the limit of an infinitely uniform medium.

From the case studies with respect to the particular model developed by Lau and Briggs,[9] it would appear that a necessary condition for the existence of an unstable normal mode (localized or nonlocalized) is the existence of some region which is locally unstable.

  