

## Temperaturejump problem with arbitrary accommodation

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# Temperature-jump problem with arbitrary accommodation

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A concise and accurate result for the temperature-jump coefficient based on the linearized BGK model and arbitrary accommodation is reported. The jump coefficient is expressed as a power series in  $(1-\alpha)$ , and values of the expansion coefficients are given.

We wish to give a rearranged version of some previous results<sup>1</sup> for the temperature-jump coefficient. As discussed earlier<sup>1</sup> the temperature-jump coefficient can be expressed as

$$\epsilon' = \left( \frac{2-\alpha}{\alpha} \right) \frac{5\sqrt{\pi}}{8} \epsilon_0 + \Delta(\alpha), \quad (1)$$

where  $\alpha$  is the accommodation coefficient for the linearized BGK model,

$$\epsilon_0 = \frac{8}{5\sqrt{\pi}} \begin{vmatrix} 1 \\ 0 \end{vmatrix}^t \mathbf{H}_1^{-t} \mathbf{H}_2^t \begin{vmatrix} 1 \\ -\sqrt{\frac{2}{3}} \end{vmatrix}, \quad (2)$$

and

$$\Delta(\alpha) = \frac{(1-\alpha)}{\alpha\sqrt{\pi}} \begin{vmatrix} 1 \\ 0 \end{vmatrix}^t \mathbf{H}_1^{-t} \int_0^\infty \mathbf{H}^{-1}(\eta) \begin{vmatrix} 1 & 0 \\ 0 & \sqrt{\frac{2}{3}} \end{vmatrix} \times \mathbf{A}(\eta) \eta \exp(-\eta^2) d\eta. \quad (3)$$

Here,  $\mathbf{H}(\mu)$  is the  $2 \times 2$   $\mathbf{H}$  matrix introduced by Kriese *et al.*,<sup>2</sup>

$$\mathbf{H}_B^t = (\pi)^{-1/2} \int_0^\infty \mathbf{H}^t(\mu) \mathbf{Q}^t(\mu) \mathbf{Q}(\mu) \mu^B \exp(-\mu^2) d\mu, \quad (4)$$

$$\mathbf{Q}(\mu) = \begin{vmatrix} \sqrt{\frac{2}{3}}(\mu^2 - \frac{1}{2}) & 1 \\ \sqrt{\frac{2}{3}} & 0 \end{vmatrix}, \quad (5)$$

and the vector  $\mathbf{A}(\eta)$  is the solution of the Fredholm equation

$$\mathbf{A}(\eta) = (\alpha - 2) \mathbf{F}(\eta) + (1 - \alpha) (\mathbf{T}\mathbf{A})(\eta), \quad \eta > 0. \quad (6)$$

In addition, we use the superscript  $t$  to denote the transpose operation and the superscript  $-t$  to denote the transpose-inverse operation. In Eq. (6)

$$\mathbf{F}(\eta) = \eta \exp(-\eta^2) \mathbf{R}(\eta) \mathbf{H}^{-t}(\eta) \mathbf{H}_1^t \begin{vmatrix} \sqrt{\frac{3}{2}} \\ -1 \end{vmatrix}, \quad (7)$$

where

$$\mathbf{R}(\eta) = \frac{1}{N(\eta)} \begin{vmatrix} \sqrt{\frac{3}{2}} N_{22}(\eta) & -N_{12}(\eta) \\ -\sqrt{\frac{3}{2}} N_{12}(\eta) & N_{11}(\eta) \end{vmatrix}. \quad (8)$$

We note that Kriese *et al.*<sup>2</sup> have given  $N_{ij}(\eta)$  and  $N(\eta)$  in terms of Dawson's integral. The operator  $\mathbf{T}$  in Eq. (6) is such that

$$(\mathbf{T}\mathbf{X})(\eta) = \frac{\eta \exp(-\eta^2)}{\sqrt{\pi}} \mathbf{R}(\eta) \mathbf{H}^{-t}(\eta) \times \int_0^\infty \mathbf{H}^{-1}(\mu) \begin{vmatrix} \sqrt{\frac{3}{2}} & 0 \\ 0 & 1 \end{vmatrix} \mathbf{X}(\mu) \exp(-\mu^2) \mu \frac{d\mu}{\mu + \eta}. \quad (9)$$

We have computed the Hilbert-Schmidt<sup>3</sup> norm of the kernel  $\mathbf{K}$  of the operator  $\mathbf{T}$ , where

$$\mathbf{K}(\eta, \mu) = \frac{\eta \exp(-\eta^2)}{\sqrt{\pi}} \mathbf{R}(\eta) \mathbf{H}^{-t}(\eta) \times \mathbf{H}^{-1}(\mu) \begin{vmatrix} \sqrt{\frac{3}{2}} & 0 \\ 0 & 1 \end{vmatrix} \frac{\mu \exp(-\mu^2)}{\eta + \mu}. \quad (10)$$

We used the definition

$$\|\mathbf{T}\| = \left\{ \int_0^\infty d\eta \int_0^\infty d\mu \operatorname{tr}[\mathbf{K}^t(\mu, \eta) \mathbf{K}(\mu, \eta)] \right\}^{1/2}, \quad (11)$$

where

$$\operatorname{tr}[\mathbf{K}^t \mathbf{K}] = K_{11}^2 + K_{12}^2 + K_{21}^2 + K_{22}^2. \quad (12)$$

We find that  $\|\mathbf{T}\| \approx 0.04$ ; that is, for

$$|(1 - \alpha)| \lesssim (0.04)^{-1},$$

or  $\alpha \lesssim 25$ , it should be possible to obtain a convergent Neumann-Liouville iterative solution of Eq. (6). In fact for physically interesting cases  $\alpha \leq 1$  and in this range, one can expect a very rapidly convergent iterative solution.

Thus, we now express an iterative solution of Eq. (6) as

$$\mathbf{A}(\eta) = (\alpha - 2) \left[ \mathbf{F}(\eta) + \sum_{m=1}^{\infty} (1 - \alpha)^m (\mathbf{T}^m \mathbf{F})(\eta) \right]; \quad (13)$$

then, we can write the jump coefficient as

$$\epsilon' = \frac{5\sqrt{\pi}}{8} \left( \frac{2 - \alpha}{\alpha} \right) \left[ \epsilon_0 + \sum_{m=1}^{\infty} (1 - \alpha)^m \epsilon_m \right], \quad (14)$$

where

$$\epsilon_m = -\frac{1}{\sqrt{\pi}} \frac{8}{5\sqrt{\pi}} \begin{vmatrix} 1 \\ 0 \end{vmatrix}^t \mathbf{H}^{-t} \mathbf{W}_{m-1} \mathbf{H}_1^t \begin{vmatrix} \sqrt{\frac{3}{2}} \\ -1 \end{vmatrix}, \quad m > 0, \quad (15)$$

$$\mathbf{W}_m = \int_0^{\infty} \mathbf{H}^{-1}(\eta) \begin{vmatrix} 1 & 0 \\ 0 & \sqrt{\frac{2}{3}} \end{vmatrix} (\mathbf{T}^m \mathbf{B})(\eta) \eta \exp(-\eta^2) d\eta, \quad (16)$$

and

$$\mathbf{B}(\eta) = \eta \exp(-\eta^2) \mathbf{R}(\eta) \mathbf{H}^{-t}(\eta). \quad (17)$$

Equation (11) is a convenient result because the  $\epsilon_m$  do not depend on  $\alpha$ . We have evaluated Eqs. (2) and (15) numerically to find the results given in Table I. A Gaussian quadrature scheme was used to evaluate all integrals, and the number of quadrature points was increased until no change in Table I was observed. The final calculation utilized 80 Gauss-Legendre points<sup>4</sup> in the interval (0, 1), which was mapped onto (0,  $\infty$ ) according to the transformation  $\mu = \mu'/(1 - \mu')$ . The accuracy of the present results to the significant digits reported here is verified by the fact that Eqs. (15) give,

$$\sum_{i=0}^{10} \epsilon_i = 1.000000;$$

that is, in the limit  $\alpha \rightarrow 0$ , from Eq. (11) we get

$$\epsilon' = \frac{5\sqrt{\pi}}{8} \left( \frac{2 - \alpha}{\alpha} \right).$$

TABLE I. Numerical values for  $\epsilon_m$  as computed from Eq. (15).

$m$	$\epsilon_m$
0	1.17597
1	$-1.60683 \times 10^{-1}$
2	$-1.37349 \times 10^{-2}$
3	$-1.38665 \times 10^{-3}$
4	$-1.44586 \times 10^{-4}$
5	$-1.52085 \times 10^{-5}$
6	$-1.60481 \times 10^{-6}$
7	$-1.69582 \times 10^{-7}$
8	$-1.79329 \times 10^{-8}$
9	$-1.89710 \times 10^{-9}$
10	$-2.00735 \times 10^{-10}$

which is an exact result in this limit. Since in view of the expansion in  $(1 - \alpha)$  one would expect the maximum numerical error at  $\alpha \rightarrow 0$ , and since in this limit our result is exact to the number of significant digits quoted, it is clear that the present series expansion should provide exact values of  $\epsilon'$  to the number of significant digits quoted here.

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<sup>1</sup>J. R. Thomas, Jr., *Phys. Fluids* **16**, 1162 (1973).

<sup>2</sup>J. T. Kriese, T. S. Chang, and C. E. Siewert, *Int. J. Eng. Sci.* **12**, 441 (1974).

<sup>3</sup>F. Riesz and B. Sz-Nagy, *Functional Analysis* [(Translated from the 2nd French edition by L. Boron) Ungar, New York, 1955].

<sup>4</sup>A. H. Stroud and D. Secrest, *Gaussian Quadrature Formulas* (Prentice-Hall, Englewood Cliffs, N.J., 1966).