Three Essays on Dynamic Games with Incomplete Information and Strategic Complementarities

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(ABSTRACT)

This dissertation consists of three essays that adopt both theoretical and empirical methods of analysis to study certain economies in which the incomplete information and the strategic complementarities between players are important. Chapter 1 explains the topics discussed in the subsequent chapters and gives a brief survey on the literature.

In Chapter 2, I revise a traditional global game model by dividing the continuum of players into a group of speculators and a group of stakeholders. It is found that the uniqueness property remains in the new game. Then I extend the static game to a two-stage game and investigate the efficacies of certain label changing mechanisms proposed by the authority to stabilize the regime in the dynamic context. It is shown that a label changing mechanism allowing for downward social mobility may not work, whereas a label changing mechanism allowing for upward social mobility generally makes the regime more stable.

In Chapter 3, I add a speculator and an authority to a bank-run model to investigate how the speculator endangers a business or an economy, and what the authority can do about it. In particular, I show that the speculator can increase the financial system's vulnerability by serving as a coordinating device for the investors and thus triggering the crisis. It is further shown that deterring the speculator may not undo the speculator's impact because of multiplicity problem; rewarding holding investors is useless; and eliminating the preemption motives among investors works given enough effort. A discussion of the 1997 Asian financial crisis and the IMF's role in it is also included.

Chapter 4 develops a repeated beauty-contest game to investigate the effect of previous winners' actions on the spread of subsequent players' actions. I first characterize the unique equilibrium of the game. Then I focus on the equilibrium dynamics of several variances depicting different forms of action variability. It is found that whether or not a specific variance diminishes over time depends on the relative precision of public and private signals. To illustrate the theoretical results, I conduct an empirical study on the Miss Korea contest. It is found that the contestants' faces have been converging to the "true beauty" overall, but diverging from each other over the last 20 years. Chapter 5 concludes.

To my parents, Xianliang Yi and Shubin Wang,

without whom I would never have the chance to pursue a doctoral degree.

To my wife, Lei Chen,

without whom this dissertation might have been completed earlier but my life would be

meaningless.

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Chapter 1

Introduction

One ordinary morning in 356 BC, standing aside a 7-meter-long slender wood at the south gate of the capital of the State of Qin, Shang Yang, the then chief advisor of the state, announced to a crowd of citizens that the government would give 3.8 kg brass as a reward to the first man who managed to move the wood from the current position to the north gate of the city. The task was of course not so easy that everybody could finish it without any difficulty, nor was it too difficult to achieve for any well-trained manual worker. The reward, on the other hand, was tremendous, considering that brass was the most precious hard currency at that time. However, no volunteers appeared: Although the reward seemed compelling, the crowd believed that the announcement was a prank played by some noble to make fun of them, and the incredibly huge reward could never be realized.

Widely regarded as the most successful reformer in Chinese history, Shang Yang did not play the game for fun. Prior to the scene above, with support from the Duke Xiao of Qin, Shang Yang proposed a reform plan, which consisted of a package of new laws that encouraged agricultural production, provided promotions and rewards based on contributions rather than kinship, reduced privilege items of the nobility, allowed for change of social class, opened the market for land transactions, etc. The main goal of the reform was to stimulate

ordinary citizens to contribute in both internal developments and external wars, through devolving power and vested interests from the nobility. This kind of reform usually would be opposed by most of the nobility, which explained why some much milder reforms carried out previously (for instance, Wu Qi's reform in the State of Chu) eventually failed.[1] Knowing this, Shang Yang realized that he needed support from the ordinary citizens to help his reform survive the attack by the nobility. To achieve this goal, the first thing Shang Yang had to do was to build the trust between the reformers and citizens. By rewarding any man who finished such a non-difficult task, his plan was to make the citizens believe that the government would always commit itself to its own promises. Observing that nobody would like to go for the reward, Shang Yang reannounced that the government was serious about this rewarding policy and the reward increased to 19 kg brass. Finally, a volunteer came out and successfully moved the wood to the north gate. Shang Yang rewarded him the 19 kg brass immediately. The package of reform plans was then enacted and announced to the public. Because of the previously established trust in the government, nobody doubted its resolve in implementing the new laws. Finally, although the reform was a very thorough and extreme one, it was successfully carried out and indeed built the foundation that enabled the State of Qin to eliminate all other states of China, and to unite the whole country for the first time as the Qin dynasty.

Although the story above is not perfectly depicted by any game-theoretical model included in this dissertation, it does illustrate the two themes of the present works: (1) In scenarios where a significant change happens if and only if a critical mass of individuals agree on it, there usually exist multiple subgroups whose interests are in conflict with each other. (2) In such environments, public signals matter. Chapter 2 is mainly devoted to the first theme. It compares the efficacies of different intervention policies aiming to stabilize a regime in which there are both stakeholder and speculator groups. Chapter 3 deals with both themes. We study a model with the coexistence of investors, a speculator, and an authority, with a focus

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[1]See Sima (2011) for the details of Wu Qi's reform as well as Shang Yang's.

on the signaling effects of the authority's and the speculator's actions. Chapter 4 investigates a game with features belonging to the second theme. It checks whether or not the publicly observed actions taken by the winners of previous stages push players' equilibrium actions to converge in a repeated game.

Although the specific economies investigated in the subsequent chapters are different from each other, they do have a remarkable feature in common: the strategic complementarities between individuals. In the regime-changing context, the larger the mass of attacking individuals, the safer it is for an individual to attack. In the context of financial crises, the more holding investors there are, the more attractive it is for any investor to hold; and given more others withdrawing, it is less attractive for an investor to hold. In the beauty-contest scenario, e.g., in a stock market, it is rational for an investor to take her action based on some noisy but publicly observed signals (act as a momentum investor) if others are mainly momentum investors, and to behave according to her understanding of the fundamental of the corresponding company (act as a fundamentalist) if others are mainly fundamentalists.

For games with strategic complementarity under complete information, Topkis (1979) and Vives (1990) show the existence of pure strategy equilibria and that the equilibrium set is indeed a lattice with a maximum element and a minimum element, which are often different. As a result, there usually are multiple equilibria, and the outcome of the game is crucially determined by which equilibrium is chosen by individuals to coordinate on. Diamond and Dybvig (1983) use such a model to describe the scenario that a bank run happens just because depositors coordinate on a socially inferior equilibrium, rather than because of a deterioration in the fundamental.

By absorbing the techniques and treatments used in the global games investigated by Carlsson and van Damme (1993), Morris and Shin (1998, 2000, 2002) introduce incomplete information to games with strategic complementarities, and construct their forms of global games suitable for studying scenarios such as regime changes, financial crises, price bubbles,

and so on. Instead of making the fundamental variable (e.g., the strength of the authority in the regime-changing model, the profitability of the investment portfolio in the bank-run model, the "true beauty" in the beauty-contest model) perfectly observable to all players, their models assume that all players receive noisy and private signals about the fundamental variable. They further argue that their settings have several merits. Firstly, it is more realistic to introduce the incomplete information to the model, since individuals often cannot observe perfectly the underlying fundamental, but instead have to deduce it based on their private information. Secondly, after introducing the incomplete information, under parsimonious conditions, there exists a unique equilibrium. What is more, the result of the game is determined by the underlying fundamental, e.g., a financial crisis happens if and only if the fundamental deteriorates to some threshold. This thus provides a more reasonable explanation for the occurrence of financial crises, and enables the researchers to investigate effects of other parameters and policies through comparative statics analysis. These models are further discussed and summarized in Frankel et al. (2003) and Morris and Shin (2003), and are widely adopted in the literature to explain regime changes, bank runs, financial crises, etc. The examples include Morris and Shin (2004); Rochet and Vives (2004); Goldstein and Pauzner (2005); Corsetti et al. (2006); Edmond (2013).

Global games have also been extended to contexts with dynamic settings. These works can mainly be split into two categories. In some of them, e.g., Allen et al. (2006) and Amador and Weill (2006), the authors want to depict the equilibrium dynamics of some variables. As a result, games admitting a unique equilibrium are used in these works. The work in Chapter 4 belongs to this category. In the other papers, the main objective of study is to investigate the informational roles of the endogenously formed public signals. Many of them regain the multiplicity in the equilibrium structure, and thus re-explain the occurrence of crises and bank runs as mis-coordinations between individuals. Examples in this strand include Atkeson (2000); Angeletos et al. (2006, 2007); Angeletos and Werning (2006); Costain (2007). The dynamic games investigated in Chapter 2 belong to the latter category. The work in Chapter

3 shares some common features with many papers in the second category. For instance, we focus on the speculator's and the authority's signaling effects on the investors' strategies, and obtain the multiplicity property for some cases and use it to explain the inefficacy of certain intervention policies designed to prevent a financial crisis from happening. However, it also differs from those works, in that we are not merely interested in whether or not introducing endogenous public signals helps regain the multiplicity. Actually, we refine the initial perfect Bayesian equilibrium set using some selection rules and focus on solutions that are the most likely to occur and interest us the most.

In Chapter 2, we investigate global games for regime changes. Observing that different social classes are often treated differently by the distribution rule embedded in a given regime, we divide the traditional continuum of players in the benchmark model into a stakeholder group and a speculator group. Compared to the speculators, stakeholders receive extra payoffs, the vested interests, if the regime survives the attack in the end. It is found that a unique equilibrium exists in the revised game. Comparative statics analyses are then adopted to investigate how changes in the fraction of stakeholders, variances of the noises, and the potential punishment for attacking stakeholders affect the vulnerability of the regime.

The static game is then extended to two-stage ones in which the authority has the chance to stabilize the regime through intervention policies that allow for changes of players' labels. Specifically, we consider two forms of label changing mechanisms: one allowing for downward social mobility and the other one that allows for upward social mobility. It is shown that although the former mechanism helps stabilize the regime in the first stage, it may increase the vulnerability of the regime in the second stage because of the multiplicity problem. In contrast, the latter mechanism generally decreases the vulnerability compared to the benchmark model.

Chapter 3 investigates global games in a bank-run context. The speculators have long been criticized for making the financial system more vulnerable, and many suggestions about what

should be done to deal with it have been put forward in both the literature and the mass media. However, little effort has been given to formalizing theses statements and concerns in a game-theoretical model. The work in this chapter tries to address this issue. We first add a speculator to a benchmark model. It is found that in the resulting two-stage game, the existence of the speculator increases the vulnerability of the financial system, especially when the attacking cost is relatively low.

To counteract this effect, we compare three different intervention policies imposed by the authority: deterring the speculator, rewarding holding investors, and eliminating the pre-emption motives among investors. We argue that the first method may not work because of the multiplicity problem; the second one is almost useless when a crisis is about to happen; the last tool works given enough effort. We then use a discussion of different intervention polices employed by governments during the 1997 Asian financial crisis to illustrate the theoretical results. Specifically, we explain why the capital regulation policies did not work, why interest rate defense policies could not stop capital flights, how Hong Kong's intervention worked, and how the IMF programs failed to rescue the participating countries by impeding them from eliminating the preemption motives.

In Chapter 4, we build a dynamic game consisting of a continuum of players to investigate the effect of previous winners' actions on the spread of subsequent players' actions. In each stage, besides the private signal, each player also observes actions taken by the winners of all previous stages as public signals. A unique equilibrium of the game is found and characterized. We then define variances of three forms of gap: variance of the gap between the average play and the underlying fundamental value, variance of the gap between a generic player's action and the average play, and the variance of the gap between a generic player's action and the winner's play. By checking their dynamics in the equilibrium, it is shown that the accumulation of private signals always reduces the first variance and the accumulation of the public signals always reduces the second variance. However, the accumulation of public signals reduces the first variance if and only if the public signals are sufficiently precise

compared to the private ones, and the accumulation of private signals reduces the second variance if and only if the private signals are sufficiently precise compared to the public ones. We also show that the third variance is a weighted sum of the other two, which turns out to be useful in applications where the fundamental is unobservable.

Based on the theoretical results, we conduct an empirical study on the last twenty editions of the Miss Korea contest. We find a descending trend in the variance of the gap between the average face and the underlying "true beauty" face over these years. Moreover, this process is accompanied by ascending trends in the other two variances, indicating that contestants' faces have been converging to the "true beauty" overall but diverging from each other over the two decades.

Chapter 2

Stabilizing a Regime: Kicking out Traitors or Absorbing Supporters?

2.1 Introduction

A regime in this chapter refers to the set of social rules and and norms that depict the form of a government.[1] A change of regime can happen for both internal and external reasons. For the former, we have seen political changes such as the collapse of the Soviet Union, the reform of China since 1978, the Arab spring revolutions, and the recently passed Obamacare, in which the pressures for regime changes came mainly from the internal participants, i.e., the citizens, of the regime. The latter cases refer to regime changes that happen mainly because of foreign interferences, such as the collapses of the Taliban regime in Afghanistan and the Saddam regime in Iraq. In this work, we ignore external aspects and focus on regime changes with internal pressures.

Once a regime has been chosen for a society, each individual in it gets her payoff according to her behavior as well as the distribution rules embedded in the regime. That explains why

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[1]Although the word "regime" has been widely used by mass media to refer to or imply an authoritarian government, especially a dictatorship, we take the more general definition of it in the current chapter.

there may exist internal pressures for a regime change: If an individual believes that she can get higher payoff by successfully attacking the current regime, she may have the incentive to do so. But even if the payoff after the regime change is higher, attacking the current one involves two kinds of costs. The first one is the attacking cost, which measures the opportunity cost of attacking. The second one is the cost of risk, which occurs because of the possibility of an unsuccessful attack. Usually a significantly large number of individuals is required to make a regime change happen, so a single individual's payoff on attacking depends also on others' actions and strategic supermodularities exist here: The larger the fraction of other attacking individuals, the smaller the cost of risk for an individual to attack. A rational individual must take into account the tradeoff between potential gains and underlying risks while making her decision on whether or not to attack the current regime.

In any given regime, it is usually the case that some individuals benefit more from the underlying distribution rules and others who benefit less or are even harmed, are taken advantage of to support the regime. That is to say, we have different social classes within the regime. Individuals in the upper classes, the stakeholders, can enjoy their vested interests as long as the regime remains. For this reason, stakeholders may be less willing to attack the current regime compared to others. What is more, individuals usually make their decisions repeatedly in reality and their labels could be changed because of their actions in previous stages. This could be because of the authority's effort to stabilize the current regime: It punishes stakeholders who attacked the regime by kicking them out of the interest group, or rewards previous supporters by absorbing them into the interest group so they can enjoy vested interests in the future. Although these two strategies have been widely observed and rigorously used by the authority whenever a regime was about to happen, e.g., right before the collapses of Chinese dynasties over the last 3000 years, differences in their functions haven't been checked formally yet. This chapter addresses this concern by using global game models that are suitable to the economic contexts of interest as depicted above.

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A global game is an incomplete information game in which the payoff structure is indexed by some unobservable fundamental random variables. The players can only observe private signals of the realization of the fundamental with noises. After observing their signals, players must take into account all possibilities of payoff structures based on their posterior distributions of the fundamental variable. This is exactly where the term "global" comes from. Carlsson and van Damme (1993) first define and investigate the general two-by-two global games. In their seminal paper, the authors' purpose of research is quite similar to that of Rubinstein's (1989) electronic mail game: For a game admitting multiple equilibria under complete information, how will the equilibrium structure change if we disturb the game a little such that the resulting incomplete information game is arbitrarily close to the complete information one? Their result is striking: It is found that a two-by-two global game essentially admits a unique equilibrium as the noises vanish, and the particular unique equilibrium is independent of the distributions of noises. Moreover, the uniquely selected equilibrium conforms to the risk dominance criterion of two-by-two game as in Harsanyi and Selten (1988). Frankel, Morris and Pauzner (2003) extend the initial two-by-two model to general global games with strategic complementarities. Morris and Shin (1998, 2000, 2003) initiate the application of global games to currency attack problems; a continuum of players and a binary action set are usually used throughout their works. With the existence of private signals, the uniqueness property is obtained and a regime change is due to a critical change in the fundamental.[2] It is thus beneficial for players to avoid the miscoordination problem, and for the government to set optimal policy through comparative static analysis. Morris and Shin's approach has become a benchmark model in the literature of global games as well as regime change models, for both theoretical and applied studies.

This work first investigates a static model by dividing the continuum of players into two groups: the traditional speculator group and the stakeholder group. The stakeholder group is defined as a group of players who can get extra payoffs if they refrain from attacking the

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[2]There are however, other approaches which take the regime change as the result of shifting between multiple equilibria, see, e.g., Angeletos, Hellwig and Pavan (2006, 2007) for detailed discussion.

status quo and the regime change does not happen in the end. This distinction between the two groups is based on the observation of the existence of stakeholders in reality as discussed above. It is found that the partition of the set of players does not change the classical uniqueness property of static global games, although speculators behave differently from the stakeholders in the unique equilibrium because of differences in their payoff settings.

The static game is then extended to two-stage games that allow for different relabeling schemes. Under dynamic settings, it is shown that we may gain multiplicity again with the existence of relabeling mechanisms. Moreover, absorbing speculators makes the current regime more stable in the whole game. However, although kicking out stakeholders makes the regime less vulnerable compared to the static case in the first stage, it may make it more vulnerable in the second stage compared to the benchmark case. To avoid this policy trap, it is crucial to remain all non-attacking stakeholders in the vested interest group in the second stage.

The rest of this chapter is organized as follows. Section 2.2 defines the static global game with speculators and stakeholders. Section 2.3 defines the solution concept and characterizes the unique equilibrium of the static game. Comparative static analyses based on the uniqueness are conducted in section 2.4. Section 2.5 extends the static game to a general two-stage game that allows for a variety of relabeling mechanisms. In section 2.6, we introduce several relabeling mechanisms aiming to stabilize the current regime and compare their effects on the equilibrium results. Section 2.7 concludes.

2.2   The Static Model

We first construct the basic static game in this section. The model is later extended to a two-stage game.

Players and payoffs. There are two groups of players, a continuum of speculators with

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measure $1 - \lambda$, and a continuum of stakeholders with measure λ. Throughout this work, we use i as a generic speculator, and j as a generic stakeholder. Each player has to decide whether or not to attack the status quo. Denote the mass of attacking agents by M, and the strength of the status quo by θ. The attack is successful (regime changes) if and only if $M \geq \theta$ holds. It is straightforward to see that the status quo is perfectly vulnerable for $\theta \leq 0$ because a regime changes happens for sure regardless of the mass of attacking players, and the status quo is perfectly invulnerable for $\theta > 1$ since $M < \theta$ always holds. Denote the result of the game by $A \in \{0, 1\}$, where $A = 0$ means that the status quo remains (the attack fails) and $A = 1$ means that the status quo is abandoned (successful attack).

Table 2.1: Payoff structures.

(a) Payoff for speculators		
	$A = 1$	$A = 0$
Attack	$1 - c$	$-c$
Do Not Attack	0	0

(b) Payoff for stakeholders		
	$A = 1$	$A = 0$
Attack	$1 - c$	$\omega - c$
Do Not Attack	0	γ

The stakeholders are different from the speculators in that they get extra payoffs if they choose to maintain the status quo and the total attack fails. In this sense the stakeholders could be viewed as having a vested interest in the status quo. As Table 2.1 shows, if the attack is successful, each agent gets payoff $1 - c$ for attacking and 0 for not attacking, where $c \in (0, 1)$ stands for the cost of attacking. But if the status quo remains, payoffs are different for the two groups: Each attacking speculator in this case receives payoff $-c$, meaning that the cost of attacking becomes a fine if the attack fails. For an attacking stakeholder, her payoff in this case is $\omega - c$ with $\omega \geq 0$. And for agents who choose not to attack, a speculator gets payoff 0 and a stakeholder gets $\gamma \geq 0$. Here both γ and ω refer to the vested interests for stakeholders under the current regime. The difference is that an attacking stakeholder can get vested interests ω, and γ is the vested interest for a non-attacking stakeholder. For this reason, we assume that the inequalities $\gamma \geq \omega \geq 0$ hold, meaning that a non-attacking stakeholder gets at least as much vested interests as an attacking stakeholder can have if the status quo remains. In other words, there might be a punishment for the action *attack*, and

the magnitude of the punishment is denoted by the term $\gamma - \omega \geq 0$.

Information. From the payoff structures depicted above, it is straightforward to check that under complete information, all agents will choose to attack, if $\theta \leq 0$ and not to attack if $\theta > 1$. But for $\theta \in (0, 1]$, the game admits multiple equilibria (attacking for all or not attacking for all) and all speculators and stakeholders thus suffer from mis-coordination risks.

Throughout this work, however, it is assumed that the exact value of status quo strength θ is unobservable to all agents. Instead, they observe θ with private noises. For speculator i, her private signal is $x_i = \theta + \sigma \epsilon_i$, where $\sigma > 0$ is a constant measuring the precision of signals and noise ϵ_i is a realization of a smooth and symmetric distribution characterized by density function $f(\cdot)$ and cumulative distribution function $F(\cdot)$, with zero mean. ϵ_i's are *i.i.d.* across all speculators. For stakeholder j, her private signal is $y_j = \theta + \delta \eta_j$, where $\delta > 0$ stands for precision of the noise, and the noise term η_j is a realization of a smooth and symmetric distribution characterized by density function $g(\cdot)$ and cumulative distribution function $G(\cdot)$, with zero mean. η_j's are i.i.d. across all stakeholders, η_j's are independent of ϵ_i's.

The prior for θ is simply set to be the "uniform distribution" over $\mathbb{R}$.[3] After receiving private signals, each agent can then form the posterior distribution for θ conditional on his private signal. Note that, the smaller σ (δ) is, the more precise speculators' (stakeholders') signals are. In the special case of $\sigma, \delta \to 0$, signals become arbitrarily precise and the posteriors for θ become Dirac delta functions around the observed signals. No matter how close to zero σ and δ are, as long as they are positive, the realization of θ is not common knowledge among the agents. This could be clearly seen especially in the case where both noises are with bounded supports. For example, let $F(\cdot)$ and $G(\cdot)$ both be the uniform distribution over

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[3]This is an improper and uninformative distribution which expresses the idea that "the fundamental variable could take any value". Although it is not well-defined as a distribution, it could be regarded as the limit of a sequence of well-defined uniform priors. More importantly, the limit of the resulting corresponding posteriors is also well defined with cumulative distribution function given by (2.22), which we shall use in the sequel. See Appendix 2.8.1 for details. We take this setting of the prior because we only care about the posteriors and this setting simplifies and eases our analysis considerably.

$[-\frac{1}{2}, \frac{1}{2}]$, and $\sigma = \delta = 1$. Then, for any agent i with private signal $x_i < -1$, she knows for sure that the realization of θ is lower than 0, but this statement is never a common knowledge: She may not know that every other agent knows the statement, even it is the case, she may not know that every other agent knows that every other agent knows the statement, and so on. And this general result holds for all positive σ and δ. For this reason, no matter how precise the signals are, the information structure of the incomplete information game will always be qualitatively different from that under complete information. And this important difference in information structure can lead to significant change in the equilibrium result, such as uniqueness of equilibrium, as the traditional literature of global games does, and could be seen immediately in the following section.

Each agent's strategy is a mapping from the signal space to the action set. Namely, strategies are defined as $s_i, s_j : \mathbb{R} \to \{attack,\ not\ attack\}$. The solution concept used is Bayes-Nash equilibrium, and because of the symmetry in the model setting, we consider only symmetric[4] Bayes-Nash equilibria.

2.3   Equilibrium

In this section, we characterize the equilibrium structure of the static game defined above. We first show that the game admits a trigger strategy equilibrium, then it is further proved that this trigger strategy equilibrium is also the unique equilibrium for the game. Based upon the uniqueness, we discuss the effects of existence of stakeholders and vested interests through comparative statics analysis in the subsequent section.

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[4]In the sense that strategies are identical for players in the same group.

2.3.1 Uniqueness

A trigger strategy is a cut-off rule, such that for any private signal no higher than a given threshold, the agent chooses to attack; otherwise the agent chooses not to attack. It is assumed here that all agents use trigger strategies and we investigate the resulting equilibria.

Definition 2.1. *A trigger strategy profile consists of two thresholds, x^* and y^*, such that each speculator i attacks the status quo if and only if $x_i \leq x^*$, each stakeholder j attacks the status quo if and only if $y_j \leq y^*$.*

To solve for an equilibrium profile (x^*, y^*), let us consider a speculator with signal $\hat{x}$, the conditional probability for regime change under strategies x^* and y^* is:

$$
\begin{aligned}
Prob(regime\ change|\hat{x}) &= Prob(M \geq \theta|\hat{x}) \\
&= Prob\{[(1-\lambda)Prob(x_i \leq x^*|\theta) + \lambda Prob(y_j \leq y^*|\theta)] \geq \theta|\hat{x}\} \\
&= Prob\left\{\left[(1-\lambda)F\left(\frac{x^*-\theta}{\sigma}\right) + \lambda G\left(\frac{y^*-\theta}{\delta}\right)\right] \geq \theta|\hat{x}\right\}.
\end{aligned}
$$

Now consider the inequality:

$$
(1-\lambda)F\left(\frac{x^*-\theta}{\sigma}\right) + \lambda G\left(\frac{y^*-\theta}{\delta}\right) \geq \theta. \tag{2.1}
$$

The left-hand-side of the inequality is decreasing in θ and the R.H.S. is strictly increasing in θ. For sufficiently small θ the L.H.S. is greater than the R.H.S., and for sufficiently large θ the reverse statement holds. So there exists a unique θ^* such that

$$
(1-\lambda)F\left(\frac{x^*-\theta^*}{\sigma}\right) + \lambda G\left(\frac{y^*-\theta^*}{\delta}\right) = \theta^*. \tag{2.2}
$$

Based on the definition above, we have inequality (2.1) hold for any $\theta \leq \theta^*$. We can thus

rewrite the conditional probability of a successful attack as

$$Prob(regime\ change|\hat{x}) = Prob\left\{\left[(1-\lambda)F\left(\frac{x^*-\theta}{\sigma}\right) + \lambda G\left(\frac{y^*-\theta}{\delta}\right)\right] \geq \theta|\hat{x}\right\}$$

$$= Prob(\theta \leq \theta^*|\hat{x})$$

$$= F\left(\frac{\theta^*-\hat{x}}{\sigma}\right),$$

where the last equality uses the fact that $F(\cdot)$ is symmetric with mean zero. And clearly the conditional probability of regime change is decreasing in $\hat{x}$. Note that x^* and y^* constitute an equilibrium, and according to the payoff structures in Table 2.1, a speculator attacks if and only if the conditional probability of a regime change is at least c. We thus have

$$F\left(\frac{\theta^*-x^*}{\sigma}\right) = c, \tag{2.3}$$

and analogously

$$G\left(\frac{\theta^*-y^*}{\delta}\right) = \frac{c+\gamma-\omega}{1+\gamma-\omega}. \tag{2.4}$$

Using the fact that both $F(\cdot)$ and $G(\cdot)$ are symmetric with zero mean again and substituting equations (2.3) and (2.4) into (2.2) yield the solution:

$$\theta^* = (1-c)\frac{1+\gamma-\omega-\lambda(\gamma-\omega)}{1+\gamma-\omega}, \tag{2.5}$$

$$x^* = \theta^* - \sigma F^{-1}(c), \tag{2.6}$$

$$y^* = \theta^* - \delta G^{-1}\left(\frac{c+\gamma-\omega}{1+\gamma-\omega}\right). \tag{2.7}$$

It is straightforward to see from definition that the solution is indeed an equilibrium: Given y^*, x^* is speculators' best response in trigger strategies, and vice versa. And since (2.2), (2.3) and (2.4) are necessary conditions for x^* and y^* to be an equilibrium, we have that the

solution above must be the unique trigger strategy equilibrium. What is more, the following proposition thus holds.

Theorem 2.1. *The unique trigger strategy equilibrium of the game characterized by x^* and y^* defined in (2.5), (2.6) and (2.7) is also the unique Bayesian equilibrium of the game, and it is dominance solvable.*

The proof of the theorem is provided in the appendix. We first construct sequences $\overline{x}^n$ and $\underline{x}^n$ ($\overline{y}^n$ and $\underline{y}^n$) for speculators (stakeholders), such that in elimination round n, it is strictly dominant for a speculator (stakeholder) to attack if $x_i < \underline{x}^n$ ($y_i < \underline{y}^n$), and not to attack if $x_i > \overline{x}^n$ ($y_i > \overline{y}^n$). Then we show sequences $\overline{x}^n$ and $\underline{x}^n$ ($\overline{y}^n$ and $\underline{y}^n$) converges to x^* (y^*) as defined above. the uniqueness of the equilibrium structure thus enables us to adopt comparative statics analysis in the next section.

2.3.2   An Example

To illustrate the processes of iterative elimination of dominated strategies, let us consider the following example: $c = 0.2$, $\gamma = 3$, $\omega = 1.5$, $\lambda = 0.1$, $\sigma = 0.2$, $\delta = 0.1$, and $F(\cdot) = G(\cdot) = \Phi(\cdot)$, where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Given this special case, we can calculate the threshold bundles $(\underline{x}^n, \overline{x}^n)$ and $(\underline{y}^n, \overline{y}^n)$ for $n = 0, 1, 2, 3, \ldots$.

Then each bundle is plotted in the same graph as shown in Figure 2.1. After $n + 1$ rounds of elimination, each speculator (stakeholder) chooses action *not attack* if her signal is higher than $\overline{x}^n$ ($\overline{y}^n$), and chooses to attack if her signal is less than or equal to $\underline{x}^n$ ($\underline{y}^n$). And for any signal between these two thresholds, the speculator's (stakeholder's) action is indeterminate. For this reason, we shade the area between $\overline{x}^n$ and $\underline{x}^n$ ($\overline{y}^n$ and $\underline{y}^n$) to highlight the regions of signals which lead to indeterminate actions after n rounds of eliminations.

As shown in Figure 2.1, as the process of iterative elimination of dominated strategies pro-
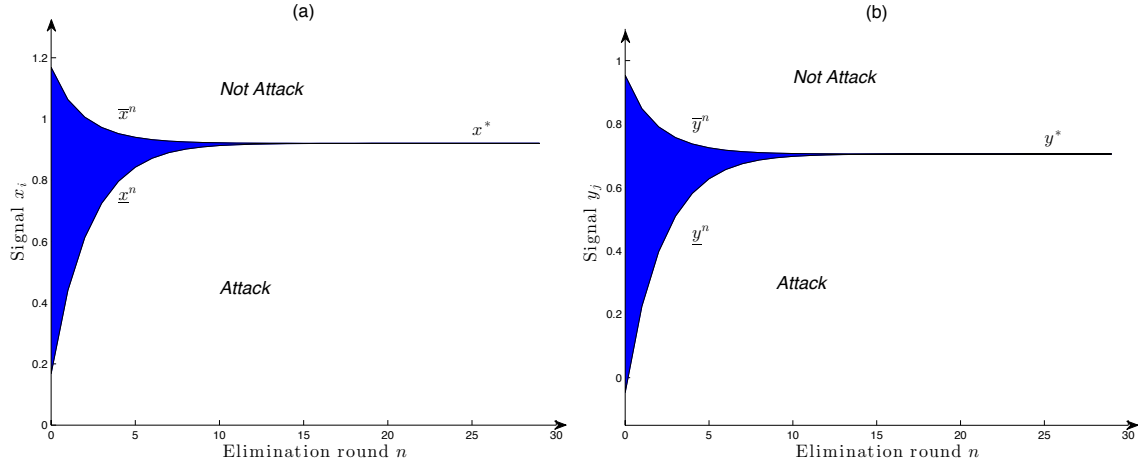
Figure 2.1: An example with $F(\cdot) = G(\cdot) = \Phi(\cdot), c = 0.2, \gamma = 3, \omega = 1.5, \lambda = 0.1, \sigma = 0.2, \delta = 0.1$. (a) Expansion of dominance regions for speculators. (b) Expansion of dominance regions for stakeholders.

ceeds, each shaded area becomes smaller and smaller. And the shaded area turns out to be a horizontal line after several rounds of elimination, meaning that the dominance regions expand to the whole signal space. The uniqueness property is thus obtained.

For this example, the unique equilibrium (x^*, y^*) is (almost) reached after 22 rounds of elimination, with $x^* = \overline{x}^* = \underline{x}^* \approx 0.92$ and $y^* = \overline{y}^* = \underline{y}^* \approx 0.71$.

2.4   Comparative Statics

The uniqueness property obtained in the preceding section enables us to investigate the effects of different variables and distributions on the equilibrium profile through comparative statics analysis. In this section, we conduct analyses with a focus on the effects of existence of the stakeholder group, heterogeneous distributions between two groups, as well as the asymptotic equilibrium when noises are vanishing.

Recall the unique equilibrium of the static game: $x^* = \theta^* - \sigma F^{-1}(c)$ and $y^* = \theta^* - \delta G^{-1}\left(\frac{c+\gamma-\omega}{1+\gamma-\omega}\right)$, where $\theta^* = (1-c)\frac{1+\gamma-\omega-\lambda(\gamma-\omega)}{1+\gamma-\omega}$, with $c \in (0, 1)$, $\lambda \in [0, 1]$, and $\gamma \geq \omega \geq 0$.

Proposition 2.1. *If $\lambda = 0$, the game becomes a classical one: All players are speculators.*

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The unique equilibrium is characterized by $x^* = \theta^* - \sigma F^{-1}(c)$ where $\theta^* = 1 - c$.

This is exactly the same as the result of the currency attack model first investigated in Morris and Shin (1998), summarized later in Morris and Shin (2003), and accepted as a benchmark model of global games for regime change thereafter. This result should not be surprising at all: With $\lambda = 0$, there is no need for the speculators to consider coordinating with stakeholders. The model thus degenerates to the classical one, in which all players are speculators observing *i.i.d.* private signals.

Now it is of interest to study effects of the existence of a stakeholder group.

Proposition 2.2. *With $\lambda \in (0,1)$, The existence of a punishment for attacking stakeholders makes speculators less aggressive.*

To see this, just check that $(1-c)^{\frac{1+\gamma-\omega-\lambda(\gamma-\omega)}{1+\gamma-\omega}} < (1-c)$ always holds as long as $\lambda \in (0,1)$ and $\gamma > \omega \geq 0$. In other words, the threshold for the equilibrium trigger strategy for speculators with existence of punishment is always lower than that without it. Speculators become less aggressive because they choose to attack if and only if they observe relatively lower signals. In other words, speculators become more cautious about choosing action *attack*. This result intuitively makes sense: Compared to speculators, the stakeholders are less willing to attack because of the punishment $\gamma - \omega$ under the status quo. The existence of the punishment thus reminds a speculator of the fact that she has to coordinate with not only all other speculators, but also a group of stakeholders which are less willing to attack. Then a speculator's risk of miscoordination with other players will increase if she still behaves as if there were no stakeholders. The fear of miscoordination of speculators in turn pushes them to become more cautious and thus less aggressive to attack the status quo.

Note, however, that the statement "a speculator will consider the stakeholders as less willing to attack" does not necessarily imply that in the equilibrium profile, stakeholders' threshold is lower then the speculators'. Actually, from the expression of the unique equilibrium, the

exact values of thresholds x^* and y^* depend crucially on the specific settings of variables and distributions, and it is possible that under appropriate setting we can have $y^* > x^*$. To make it clear, under the existence of the stakeholder group, it is the fear of miscoordination, rather than the condition that stakeholders have a lower threshold than the speculators, that makes the speculators less aggressive[5].

Another interesting result related to this proposition is that when there is no punishment for attacking stakeholders, namely in the case $\gamma = \omega \geq 0$, the equilibrium strategy threshold for speculators becomes $x^* = 1 - c - \sigma F^{-1}(c)$. The speculators behave as if there were no stakeholder group. This underlines again the proposition above: It is indeed the coexistence of punishment and stakeholder group, rather than merely the existence of stakeholder group, that makes the speculators more cautious on attacking.

Proposition 2.3. *For $\lambda \in (0,1)$, with the existence of a positive punishment, the larger the proportion of stakeholders, the less aggressive are all speculators and all stakeholders. The larger the punishment, the less aggressive are all speculators and all stakeholders.*

For the former part, just check that under $\lambda \in (0,1)$ and $\gamma - \omega > 0$, we have both $\frac{\partial x^*}{\partial \lambda} < 0$ and $\frac{\partial y^*}{\partial \lambda} < 0$. The reason for x^* being decreasing in λ is straightforward and related to the proposition above: the larger the value of λ, the larger the group of players less willing to attack for any speculator, so the higher the intensity of fear of miscoordination put on a speculator. This in turn makes a speculator behave more cautiously. The logic for y^* to decrease in λ is similar and symmetric. For stakeholders, the group of speculators are considered as players that are more willing to attack the status quo, the fear of miscoordination with speculators thus pushes stakeholders to become more willing to attack (y^* increases) if there are more speculators (λ decreases) and less willing to attack (y^* decreases) if there are fewer speculators (λ increases).

To see the latter part of the proposition, we have $\frac{\partial x^*}{\partial(\gamma-\omega)}$, $\frac{\partial y^*}{\partial(\gamma-\omega)} < 0$. The economic intuition

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[5]For detailed discussion on fear of miscoordination in global games, see Chassang (2010).

behind this result is as follows. Recall that $\gamma - \omega$ denotes the punishment for attacking stakeholders. An increase in the value of punishment makes the stakeholders less willing to attack. Knowing this, speculators also become less willing to attack since it is now more difficult for them to make a successful attack, this change again makes the stakeholders even less willing to attack, and so forth. The simplest and perhaps most intriguing scenario to illustrate the proposition, is to consider the limit result as the punishment becomes arbitrarily large: $(\gamma - \omega) \to \infty$. In this case, stakeholders choose action *not attack* for any signal. This is because as long as there is a positive probability for the status quo to remain, which is always the case when the realization of the fundamental θ is not common knowledge, the stakeholders' expected payoff for action *not attack* is always higher than that of action *attack*. Although the speculators are aware that all stakeholders refrain from attacking, they might still choose to attack if their private signals are sufficiently low, meaning that they might still cause a successful regime change *on their own*. Formally, under situation $(\gamma - \omega) \to \infty$, we have $y^* \to -\infty$ and $x^* \to (1-c)(1-\lambda) - \sigma F^{-1}(c)$.

Proposition 2.4. *As all players' signals become perfectly precise, that is, under $\sigma, \delta \to 0$, we have $x^*, y^* \to \theta^* = (1-c)\frac{1+\gamma-\omega-\lambda(\gamma-\omega)}{1+\gamma-\omega}$.*

This result accords with Rubinstein's (1989) emphasis on the significant difference between common knowledge and "almost common knowledge" and Carlsson and van Damme's (1993) argument of considering the asymptotic result of global games as a method for equilibrium selection: As $\sigma, \delta \to 0$, the fundamental variable θ becomes arbitrarily close to common knowledge to all players, but it never is – as long as $\sigma, \delta > 0$ holds, no matter how small they are. All players still need to take into account all kinds of game structures indexed by the value of θ. So they can never behave as if they were in a complete information game. This incomplete information structure ensures the uniqueness property.

Proposition 2.5. *The equilibrium threshold for each group does not depend on the distribution and precision of noises of the other group.*

The validity of the proposition above is quite obvious from the expression of the unique equilibrium: A speculator's (stakeholder's) equilibrium strategy depends exclusively on the cost of attacking c, the proportion of stakeholders λ, the punishment $\gamma - \omega$, the signal precision σ (δ), and the distribution $F(\cdot)$ ($G(\cdot)$). Any other information is irrelevant in determining the exact value for her threshold. This result arises directly from the assumption that both $F(\cdot)$ and $G(\cdot)$ are symmetric with zero mean. As seen in subsection 2.3.1, it is exactly the symmetry and zero mean properties of the two distributions that remove θ^*'s dependence on the two distributions in equation (2.2). Otherwise, each group's threshold could depend on the distribution of the other group's noises.

This proposition, however, inspires an important question. Since each group's equilibrium threshold does not depend on the noise structure of the other group, if each group of players do not know the distribution and precision of the noises for the other group, can we still obtain the same results? The answer is negative: Note that in the proof of uniqueness of equilibrium, the noise structures of both groups must be common knowledge. Otherwise the process of iterative elimination of dominated strategies could not proceed. As a result, although the noise structure of one group of players has nothing to do with the equilibrium threshold of the other group, the noise structures of both groups are required to be common knowledge for the previous results to hold.

2.5 Two-Stage Games

In the static model above, all speculators and stakeholders have only one chance to decide if they will attack in the one-shot game. What is more, once assigned to either the speculator group or the stakeholder group, they do not have the opportunity to change their labels. But in reality, players usually act multiple times (e.g., year after year), and there usually exist mechanisms and rules through which a player may have her label changed, probably based on her previous actions. Taking into account those considerations, we build in this section

a general two-stage game which incorporates both the dynamic aspect and a mechanism for the changing of players' labels.

There are two stages, $T \in \{0, 1\}$. In stage 0, stakeholders and speculators observe their initial labels and receive private and noisy signals about the fundamental θ. The information structure is the same as that in the static game. Then all players move simultaneously and get their instantaneous payoffs for stage 0 according to Table 2.1 with $\gamma > \omega > 0$. Denote by $A^0 \in \{0, 1\}$ the outcome of play in stage 0, and $M^0 \in [0, 1]$ the mass of attacking players in stage 0. The regime is abandoned in stage 0 ($A^0 = 1$) if and only if $M^0 < \theta$. The game ends if the regime is abandoned in stage 0, otherwise it moves on to stage 1.

We use i to refer to a generic player who is a speculator in stage 0, and use j to refer to a generic player whose stage-0 label is stakeholder. Unlike in stage 0 where it is common knowledge that the mass of stakeholders is λ and the mass of speculators is $1 - \lambda$, these proportions in stage 1 become endogenous variables and probably random variables to players. For each player, her label may be changed at the beginning of stage 1. To depict the changing mechanism of labels, we define here two **labeling functions** $L_x(\cdot)$ and $L_y(\cdot)$. Then for speculator i in stage 0 choosing action $a_i^0 \in \{Attack, \ Not \ Attack\}$, function $L_x(a_i^0)$ tells the probability of her being relabeled as a stakeholder in stage 1, and $1 - L_x(a_i^0)$ is the probability of her label remaining the same in stage 1. Similarly, $L_y(b_j^0)$ is the probability of a stakeholder j in stage 0 choosing action b_j being relabeled as a speculator in stage 1. Denote by 0 the action *Attack* and by 1 the action *Not Attack*. We require $L_x(0) \geq L_x(1)$ and $L_y(1) \geq L_y(0)$. That say, an attacking speculator in stage 0 is less likely to be relabeled as a stakeholder in stage 1, and an attacking stakeholder in stage 0 is more likely to be relabeled as a speculator in stage 1. To understand these properties, recall that based on the payoff settings, it is always better to be in the stakeholder group than in the speculator group. So a non-attacking speculator is more likely to be absorbed into the interest group for supporting the regime, and an attacking stakeholder is more likely to be kicked out for being a traitor of the interest group.

Depending on the specific forms of the labeling functions, a player's label in stage 1 may or may not change. We use set $\{\ominus, \oplus\}$ to depict players' statuses in stage 1, where element $\ominus$ means the label in stage 1 remains the same as that in stage 0 and element $\oplus$ means a label change. Further define a set $\Delta_x \subseteq \{\ominus, \oplus\}$ to denote all possible stage-1 statuses of speculators in stage 0, given function L_x. Formally speaking, status $\oplus$ belongs to Δ_x if and only if there exists an action $a \in \{0, 1\}$ such that $L_x(a) > 0$; status $\ominus$ belongs to Δ_x if and only if there exists an action a' such that $L_x(a') < 1$. Analogously we define Δ_y as the set of all possible status for stakeholders in stage 1 based on function L_y. It is straightforward to see from the definitions above that $\Delta_x \neq \emptyset$ and $\Delta_y \neq \emptyset$ always hold.

If the game continues after stage 0, at the beginning of stage 1, every player is relabeled, probably randomly, according to the labeling functions L_x and L_y. Each player i observes her stage-1 status $\ell_i \in \Delta_x$, each player j observes her stage-1 status $\ell_j \in \Delta_y$. Then All players choose their actions simultaneously again from set $\{Attack, \ Not \ Attack\}$. The regime is abandoned ($A^1 = 1$) in stage 1 if the attacking mass $M^1 \geq \theta$. Based on A^1 and their new labels, players receive instantaneous payoffs for stage 1 according to table 2.1. Finally, all players value stage-1 payoff by a discount parameter $\beta > 0$.

We take a short summary here to make it clear how much information each player possesses in the current context. Firstly, the prior distribution of the fundamental, the distributions of the noises, the payoff structures, the labeling function forms, and the order of play, are common knowledge to all players. If the game continues after stage 0, $A^0 = 0$ becomes common knowledge to all players in stage 1. Secondly, a player i (player j) in stage 0 observes x_i (y_j) and takes action a_i (b_j) in stage 0. Given the game continues, each player i (player j) remembers her private signal received and action taken in stage 0, and knows her new label for stage 1 represented by ℓ_i (ℓ_j).

The equilibrium concept used in the rest of the chapter is defined below.

Definition 2.2. *An equilibrium is a perfect Bayesian threshold equilibrium consisting of*

thresholds $x^{01*} = (x^{0*}, \{x^{\tau*}\}_{\tau \in \Delta_x})$, $y^{01*} = (y^{0*}, \{y^{\iota*}\}_{\iota \in \Delta_y})$, and beliefs characterized by cumulative distribution functions $\pi_i^0(\theta|x_i)$, $\pi_j^0(\theta|y_j)$, $\pi_i^1(\theta|x_i, A^0 = 0)$, and $\pi_j^1(\theta|y_j, A^0 = 0)$, such that player i attacks in stage 0 if and only if $x_i \leq x^{0*}$, attacks in stage 1 if and only if $x_i \leq x^{\tau*}$ for $\ell_i = \tau$, player j attacks in stage 0 if and only if $y_j \leq y^{0*}$, attacks in stage 1 if and only if $y_j \leq y^{\iota*}$ for $\ell_j = \iota$, and

$$x^{01*} \in \arg\max_{x \in \bar{\mathbb{R}}^{1+|\Delta_x|}} E_{\pi_i^0(\theta|x_i)} \left[\mu_i^0(x, x^{01*}, y^{01*}) + \mathbb{1}_{A^0=0} \cdot \beta\mu_i^1(x, x^{01*}, y^{01*}, L_x, L_y) \right], \forall x_i \in \mathbb{R}, \quad (2.8)$$

$$y^{01*} \in \arg\max_{y \in \bar{\mathbb{R}}^{1+|\Delta_y|}} E_{\pi_j^0(\theta|y_j)} \left[\mu_j^0(y, x^{01*}, y^{01*}) + \mathbb{1}_{A^0=0} \cdot \beta\mu_j^1(y, x^{01*}, y^{01*}, L_x, L_y) \right], \forall y_j \in \mathbb{R}, \quad (2.9)$$

$$x^{\tau*} \in \arg\max_{x' \in \bar{\mathbb{R}}} E_{\pi_i^1(\theta|x_i, A^0=0)} \mu_i^1(x', \ell_i = \tau, x^{01*}, y^{01*}, L_x, L_y), \ \forall x_i \in \mathbb{R}, \ \forall \tau \in \Delta_x, \quad (2.10)$$

$$y^{\iota*} \in \arg\max_{y' \in \bar{\mathbb{R}}} E_{\pi_j^1(\theta|y_j, A^0=0)} \mu_j^1(y', \ell_j = \iota, x^{01*}, y^{01*}, L_x, L_y), \ \forall y_j \in \mathbb{R}, \ \forall \iota \in \Delta_y, \quad (2.11)$$

π_i^0, π_j^0, π_i^1, and π_j^1 are updated according to Bayes' Rule whenever applicable, (2.12)

where $\bar{R}$ is the extended real line $[-\infty, +\infty]$; the cardinality function $|\cdot|$ counts the number of elements of a set; $\mathbb{1}_{(.)}$ is the characteristic function equaling 1 if the statement is correct and 0 otherwise; $\mu(\cdot)$'s are the instantaneous payoff functions, $\mu_i^0(x, x^{01*}, y^{01*})$ gives player i's stage-0 payoff if she is behaving according to the threshold strategy characterized by x and all other players are behaving according to threshold strategies characterized by x^{01*} and y^{01*}, respectively; other instantaneous payoff functions are analogously defined.

Conditions (2.8) and (2.9) require that it is optimal for player i and player j, respectively, to behave according to the the threshold strategies as defined, as long as everyone else is doing so. Equations (2.10) and (2.11) impose the sequential rationality requirements. The consistence requirement on the belief system is stated in condition (2.12).

Lemma 2.1. *For any two-stage game defined above, the following results hold:*

(i) Given an equilibrium, there exists a unique $\theta^{0*} \in [0, 1]$, such that the regime is abandoned in stage 0 if and only if $\theta \leq \theta^{0*}$.

(ii) Every game has an equilibrium in which we have $x^{\tau*} = y^{\iota*} = -\infty$ for all $\tau \in \Delta_x$ and all $\iota \in \Delta_y$. As a result, nobody attacks in stage 1, and the regime is abandoned in stage 1 if and only if $\theta \leq \theta^{1*} = 0$.

(iii) For any equilibrium with thresholds x^{01*} and y^{01*}, if there exists a θ^{1*} such that the regime is abandoned in stage 1 if and only if $\theta \leq \theta^{1*}$, we must have either $\theta^{1*} > \theta^{0*}$ or $\theta^{1*} = 0$.

Once the thresholds of an equilibrium is given, the uniqueness of θ^{0*} stems from the fact that equation (2.2) admits a unique solution. If the game continues in stage 1, information $\theta > \theta^{0*} \geq 0$ becomes public knowledge to all players. So given everybody else chooses not to attack in stage 1, it is optimal for a player to behave the same because she knows the regime will remain in the end for sure. There could not be an equilibrium in which we have $\theta^{1*} \in (0, \theta^{0*}]$. Because $\theta > \theta^{0*}$ is publicly known in stage 1, and if all other players are following the equilibrium strategies such that $\theta^{1*} \leq \theta^{0*}$, each player knows that the attacking could not succeed in stage 1, and thus chooses not to attack regardless of her private signal. As a result, in the equilibrium nobody attacks in stage 1, which contradicts $\theta^{1*} > 0$. So we must have either $\theta^{1*} > \theta^{0*}$ or $\theta^{1*} = 0$.

We further make the following assumption for the rest of the chapter.

Assumption 2.1. *Denote the densities of x_i and y_j conditional on θ by $p(x_i|\theta)$ and $q(y_j|\theta)$, respectively. It is assumed that $p(x|\theta)$ and $q(y|\theta)$ satisfy the Monotone Likelihood Ratio Property (MLRP henceforth) in x and y, respectively:*

$$\frac{p(\hat{x}|\hat{\theta})}{p(\hat{x}|\theta)} \geq \frac{p(x|\hat{\theta})}{p(x|\theta)}, \quad \frac{q(\hat{y}|\hat{\theta})}{q(\hat{y}|\theta)} \geq \frac{q(y|\hat{\theta})}{q(y|\theta)}, \quad \forall \hat{x} > x, \hat{y} > y, \hat{\theta} > \theta$$

Given a greater fundamental value, the ratio between the density of the private signal at a higher value and the density at a lower value is also greater. That is to say, the larger the observed value, the more likely it is generated based on a larger fundamental θ rather than

on a lower one.

Lemma 2.2. *In an equilibrium with thresholds x^{01*} and y^{01*}, given Assumption 2.1, functions $\pi_i^1(\theta|x, A^0 = 0)$, and $\pi_j^1(\theta|y, A^0 = 0)$ are decreasing in x and y, respectively.*

This property stems from the relationship between MLRP and the *Decreasing Hazard Rates* property.[6] It simply states that, given the fact that the regime survives stage-0 attacking, the higher the private signal, the lower the probability of θ being smaller than a given value. This monotonicity of stage-1 posterior distributions of θ in private signals makes our analysis in the next section more tractable.

2.6   Regime Stability in Dynamic Settings

Based on the general model built in the preceding section, we investigate the stability of a regime with both speculators and stakeholders in different dynamic settings. We are especially interested in comparing the efficacies of different label changing mechanisms aiming at stabilizing the regime. It is worth noting that the label changing mechanisms (L_x and L_y), once defined, are exogenous in our context. So it is different from works that treat their mechanisms as endogenous policy choices made by an authority, and focus on the signaling role of these mechanisms.[7]

We investigate first the benchmark case where no label changes could happen in stage 1, and then the two widely observed mechanisms: kicking out attacking stakeholders and absorbing non-attacking speculators.

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[6]See Barlow et al. (1963).
[7]See, for instance, Angeletos et al. (2006).

2.6.1 No Label Changes

In this case, a player is assigned to a group in stage 0 and stays in it if the game continues in stage 1, regardless of her and other players' actions taken in the previous stage. It is used to represent the scenario of a rigid social class system where the society consists of stratified classes with no social mobility. The labeling functions are defined as:

$$L_x(a_i^0) = 0, \ \forall a_i^0, \tag{2.13}$$

$$L_y(b_j^0) = 0, \ \forall b_j^0. \tag{2.14}$$

Based on these labeling functions, we have $\Delta_x = \{\ominus\}$ and $\Delta_y = \{\ominus\}$. It is commonly known in stage 0 that there will be no labels being changed in stage 1, and the only information added to all players in stage 1 is the common knowledge $A^0 = 0$. It will be interesting to compare the results here with that of the static game, recall that the unique equilibrium of the latter is characterized by θ^*, x^*, and y^* defined in (2.5), (2.6) and (2.7). We characterize the equilibrium of the dynamic game in the following proposition.

Proposition 2.6. *For the game with labeling functions defined in (2.13) and (2.14), there exists a unique equilibrium with thresholds $x^{0*} = x^*$, $y^{0*} = y^*$, $x^{\ominus*} = -\infty$ and $y^{\ominus*} = -\infty$. As a result, $\theta^{0*} = \theta^*$ and $\theta^{1*} = 0$.*

In the unique equilibrium, the regime could never be abandoned in stage 1 as long as it survives the stage-0 attack. Because it is common knowledge that nobody's label will be changed in stage 1 regardless of their actions in stage 0, they do not need to think about the effects of their actions on payoffs in the next stage. So their strategies in the earlier stage must constitute an equilibrium as in the static game. After observing $A^0 = 0$ in stage 0, players infer that $\theta > \theta^{0*}$ and thus coordinate on this information by not attacking regardless of their private signals. The regime is abandoned in the whole game if and only if $\theta \leq \theta^*$, the stability of the regime is thus the same as that in the static one.

Our model here admits a unique solution where the regime in stage 1 is less vulnerable than in stage 0. The results are different from those in Angeletos et al. (2007). Unlike in the present work where players receive private signals only once, in their work players receive new private signals in every stage. Their beliefs on θ become more and more precise. As a result, the informational role of the public signal that the regime survives in the previous stage becomes indeterminate and multiplicity occurs.

2.6.2 Kicking Out Stakeholders

Another label changing mechanism considered here is to relabel stakeholders in stage 0 to speculators. This happens when the interest group punishes members by keeping them out of the group. The labeling functions are defined as

$$L_x(a_i^0) = 0, \ \forall a_i^0, \tag{2.15}$$

$$L_y(b_j^0) = \begin{cases} k \in (0,1], \ if \ b_j^0 = 1, \\ k' \in [0,k), \ if \ b_j^0 = 0. \end{cases} \tag{2.16}$$

We have $\Delta_x = \{\ominus\}$ and $\Delta_y = \{\ominus, \oplus\}$ for this case. All speculators in stage 0 keep the same label in stage 1. Any attacking stakeholder in stage 0 will be kicked out of the interest group; and they turn into a speculator with probability k in stage 1, and this probability for a non-attacking stakeholder in stage 0 is $k' < k$. Since no speculators in stage 0 can be relabeled as stakeholders in stage 1, this scenario case best represents a society which has downward social mobility but does not allow for upward social mobility.

Proposition 2.7. *For the game with labeling functions defined in* (2.15) *and* (2.16)*, we have the following results:*

(i) In any equilibrium, $\theta^{0*} < \theta^*$.

(ii) Besides the equilibrium with $\theta^{1*} = 0$, there may exist an equilibrium in which $\theta^{1*} > \theta^{0*}$.

Moreover, with $k' = 0$, for any equilibrium we have $\theta^{1*} < \theta^*$.

(iii) For $k' > 0$, there may exist an equilibrium in which $\theta^{1*} > \theta^*$, and such equilibria exist at least for large enough k and λ, and small enough β and c.

The label changing mechanism could stabilize the regime in stage 0 by deterring some stakeholders from attacking in the earlier stage. Because it is always better off to be in the interest group, stakeholders in stage 0 behave less aggressively with the hope of getting the extra payoff γ or ω if the regime survives in the whole game.

But the effects of the mechanism on the vulnerability of the regime in the whole game are rather indeterminate. If all players are behaving according to the equilibrium with $\theta^{1*} = 0$, the mechanism stabilizes the regime, compared to the benchmark case. But as stated above, there may exist another equilibrium where it is not the case that all players coordinate on signal $A^0 = 0$ and do not attack anyway in stage 1. Instead, because of the disappearance of extra payoffs for being supportive in stage 1, stakeholders may behave more aggressively. If this destabilizing effect outweighs the stabilizing effect of the public signal $A^0 = 0$, there may exist another equilibrium such that $\theta^{1*} > \theta^{0*}$. When the attacking cost c is small enough such that the stakeholders behave rigorously aggressive after being kicked out of the group, and the discount parameter β is small enough such that the label changing mechanism's effects on deterring the stakeholders and on the signaling role of the signal $A^0 = 0$ are too small, with $k' > 0$ we may even have $\theta^{1*} > \theta^*$. If players behave according to this equilibrium, the regime is more vulnerable in this game than in the benchmark one.

Furthermore, the multiplicity feature causes mis-coordination problems and makes result of the game unpredictable. If lacks additional coordination devices, one cannot tell which equilibrium strategy a player is following. It thus becomes difficult to predict the result of the game for intermediate fundamental values between the two possible thresholds. If every player is behaving according to some equilibrium strategy, the mechanism always outperforms the benchmark case by making the regime less vulnerable only if it kicks out

just the stakeholders who attack in stage 0, i.e., $k' = 0$.

2.6.3 Absorbing Speculators

We now consider the scenario in which the interest group recruits some speculators in stage 0. This case represents a society allowing for upward social mobility. The labeling functions are defined as

$$L_y(b_j^0) = 0, \ \forall b_j^0, \tag{2.17}$$

$$L_x(a_i^0) = \begin{cases} h \in (0, 1], \ if \ a_i^0 = 0, \\ h' \in [0, h), \ if \ a_i^0 = 1, \end{cases} \tag{2.18}$$

All stakeholders in stage 0 remain their labels in stage 1. With probability h, a non-attacking speculator is relabeled as stakeholder in stage 1, and this probability for a speculator who attacks in stage 0 is $h' < h$. $\Delta_x = \{\ominus, \oplus\}$, $\Delta_y = \{\ominus\}$.

Proposition 2.8. *For the game with labeling functions defined in* (2.17) *and* (2.18)*, we have the following results:*

(i) In any equilibrium, $\theta^{0*} < \theta^*$.

(ii) Besides the equilibrium with $\theta^{1*} = 0$, there may exist an equilibrium in which $\theta^{1*} > \theta^{0*}$, and $\theta^{1*} < \theta^*$ always holds.

Compared to the benchmark model, speculators behave less aggressively because of the possible changes in their labels if they do not attack in the earlier stage. So the regime is less likely to be abandoned in stage 0. But in stage 1 this incentive disappears, so remaining speculators may behave more aggressively now. As a result there may exist an equilibrium such that $\theta^{1*} > \theta^{0*}$. But we must have $\theta^{1*} < \theta^*$ because every player behaves less aggressively in stage 1 than in the static game.

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Although the multiplicity and mis-coordination problem may still occur in this case, we have $\max\{\theta^{0*}, \theta^{1*}\} < \theta^*$ for every equilibrium. So this label changing mechanism stabilizes the regime compared to the benchmark game, as long as every player is behaving according to some equilibrium strategy.

2.7 Concluding Remarks

It is shown in the present work that, the uniqueness property of the benchmark model of global games remains when the set of players is partitioned into a speculator group and a stakeholder group. Comparative static analysis results suggest that both the existence of the added stakeholder group and the punishment for attacking stakeholders can (to some extent) deter the speculators from attacking the status quo by making them less aggressive. After extending the static game to a two-stage one, it is shown that the vulnerability remains the same in the benchmark case where there is no label changing mechanism. For the mechanism allowing to kick out stakeholders, we show that although it helps stabilize the regime in the first stage, it may make the regime more vulnerable in the whole game if some stakeholders who do not attack in stage 0 still get kicked out with positive probability. For the label changing mechanism which absorbs speculators into the stakeholder group, we show that although the multiplicity problem may still exist, it generally makes the regime less vulnerable compared to the benchmark case.

2.8 Appendix

2.8.1 Appendix A: The Improper Prior

We illustrate here that the improper prior used in the chapter can be treated as the limit of a sequence of proper ones, and the posterior distribution based on it is well defined as the limit of the resulting sequence of posteriors.

For each natural number $n = 1, 2, 3, \cdots$, denote by $U[-n, n]$ the uniform distribution on the interval $[-n, n]$. Obviously, we have density $\frac{1}{2n}$ for each point on this interval. Denote the cumulative distribution function and the probability density function of x_i conditional on θ by $\Psi(x_i|\theta)$ and $\psi(x_i|\theta)$, respectively. According to the setting of private signals, it is then straightforward to see that $\Psi(x_i|\theta) = F\left(\frac{x_i - \theta}{\sigma}\right)$ and $\psi(x_i|\theta) = \frac{1}{\sigma} f\left(\frac{x_i - \theta}{\sigma}\right)$. Further denote by $\varphi^n(\theta|x_i)$ the probability density function of the posterior distribution of θ given private signal x_i and prior $U[-n, n]$. Then applying Bayes' theorem here for any $\theta \in [-n, n]$, we have

$$
\begin{aligned}
\varphi^n(\theta|x_i) &= \frac{\frac{1}{2n}\psi(x_i|\theta)}{\int_{-n}^{n}\frac{1}{2n}\psi(x_i|\theta')\,\mathrm{d}\theta'} \\
&= \frac{f\left(\frac{\theta - x_i}{\sigma}\right)}{\sigma \int_{\frac{-n-x_i}{\sigma}}^{\frac{n-x_i}{\sigma}} f\left(\frac{\theta' - x_i}{\sigma}\right)\mathrm{d}\left(\frac{\theta' - x_i}{\sigma}\right)},
\end{aligned}
\tag{2.19}
$$

where the second equality uses the property that the distribution $F(\cdot)$ is symmetric with zero mean. Because $f(\cdot)$ is the density function of a smooth distribution on $\mathbb{R}$, we have

$$
\lim_{n \to \infty} \int_{\frac{-n-x_i}{\sigma}}^{\frac{n-x_i}{\sigma}} f\left(\frac{\theta' - x_i}{\sigma}\right)\mathrm{d}\left(\frac{\theta' - x_i}{\sigma}\right) = 1.
\tag{2.20}
$$

As a result, for any $\theta \in \mathbb{R}$, we have

$$\lim_{n \to \infty} \varphi^n(\theta|x_i) = \frac{1}{\sigma} f\left(\frac{\theta - x_i}{\sigma}\right), \tag{2.21}$$

$$\lim_{n \to \infty} \Phi^n(\theta|x_i) = F\left(\frac{\theta - x_i}{\sigma}\right), \tag{2.22}$$

where $\Phi^n(\theta|x_i)$ denotes the posterior cumulative distribution function of θ based on signal x_i and prior $U[-n, n]$. We thus have shown that, although an improper prior is used in the chapter, the resulting limit posterior distribution is well defined.

2.8.2 Appendix B: Proofs

Proof of Theorem 2.1.

We prove it by constructing dominance regions and then extending them to the whole signal space. Consider a speculator. If her signal x_i is quite low so that the conditional probability of event $\{\theta \leq 0|x_i\}$ is great enough, then she should always choose to attack regardless of other agents' choices. Formally, we can define a threshold $\underline{x}^0$ such that

$$Prob(\theta \leq 0|\underline{x}^0) = F\left(\frac{-\underline{x}^0}{\sigma}\right) = c. \tag{2.23}$$

Then for any speculator with signal below $\underline{x}^0$, it is always optimal for her to attack since the expected payoff for attacking is higher. Note that this statement holds true not matter what strategies other agents follow. For this reason, any strategy for speculators which assigns action *not attack* to some signal lower than $\underline{x}^0$ is strictly dominated. Similarly, we can define a threshold $\underline{y}^0$ by

$$Prob(\theta \leq 0|\underline{y}^0) = G\left(\frac{-\underline{y}^0}{\delta}\right) = \frac{c + \gamma - \omega}{1 + \gamma - \omega}. \tag{2.24}$$

Thresholds $\underline{x}^0$ and $\underline{y}^0$ together define the initial dominance regions: $(-\infty, \underline{x}^0]$ and $(-\infty, \underline{y}^0]$. Any strategy surviving the first round elimination of dominated strategies must assign action *attack* to signals in these regions.

Now as rational players, each speculator and stakeholder can calculate the values of $\underline{x}^0$ and $\underline{y}^0$. Namely, the initial dominance region thresholds are known to all agents. Then for a speculator with signal $\underline{x}^0$, her conditional probability for θ being non-positive is c as defined, but the conditional probability for regime change is larger than c: There is a positive probability that θ is larger than zero and the mass of attacking agents is greater than θ if the speculators take into account the two initial dominance regions. Then the thresholds for dominance regions could change and the second round elimination of dominated strategies happens.

To see this point clearly, define $\underline{\theta}^1$ solving:

$$(1-\lambda)F\left(\frac{\underline{x}^0 - \theta^1}{\sigma}\right) + \lambda G\left(\frac{\underline{y}^0 - \theta^1}{\delta}\right) = \underline{\theta}^1. \tag{2.25}$$

Note that the equation above admits a unique solution for $\underline{\theta}^1 \in (0,1)$, and the L.H.S. of (2.25) does not stand for the mass of attacking agents if the realization of the status quo strength is $\underline{\theta}^1$: The L.H.S. measures the mass of players whose signals fall into the dominance regions defined by $(-\infty, \underline{x}^0]$ and $(-\infty, \underline{y}^0]$, which means that the probability of regime change is *at least* as great as the value of L.H.S. And for any realization of θ smaller than $\underline{\theta}^1$, the dominance regions $(-\infty, \underline{x}^0]$ and $(-\infty, \underline{y}^0]$ make sure that the L.H.S. of (2.25) is greater than the R.H.S., regime change thus happens.

Then we can define $\underline{x}^1$ and $\underline{y}^1$:

$$Prob(\theta \leq \underline{\theta}^1 | \underline{x}^1) = F\left(\frac{\underline{\theta}^1 - \underline{x}^1}{\sigma}\right) = c, \tag{2.26}$$

$$Prob(\theta \leq \underline{\theta}^1 | \underline{y}^1) = G\left(\frac{\underline{\theta}^1 - \underline{y}^1}{\delta}\right) = \frac{c + \gamma - \omega}{1 + \gamma - \omega}. \tag{2.27}$$

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Because $\underline{\theta}^1 > 0$, we must have $\underline{x}^1 > \underline{x}^0$ and $\underline{y}^1 > \underline{y}^0$. And for any signal below $\underline{x}^1$, the conditional probability of regime change is larger than c. This implies that all strategies for speculators assigning action *not attack* to signals no greater than $\underline{x}^1$ are deleted in the second round of elimination of strictly dominated strategies. The analogous result also applies to the strategies for stakeholders. Now the dominance regions expand to $(-\infty, \underline{x}^1]$ and $(-\infty, \underline{y}^1]$.

Using the same logic as described above, we can define the general process of iterative elimination of strictly dominated strategies by solving dominance region thresholds $\underline{x}^n$ and $\underline{y}^n$ for $n = 0, 1, 2, \cdots$, where the thresholds are defined as:

$$
\begin{cases}
(1 - \lambda)F\left(\frac{\underline{x}^n - \underline{\theta}^{n+1}}{\sigma}\right) + \lambda G\left(\frac{\underline{y}^n - \underline{\theta}^{n+1}}{\delta}\right) = \underline{\theta}^{n+1} \\
F\left(\frac{\underline{\theta}^{n+1} - \underline{x}^{n+1}}{\sigma}\right) = c \\
G\left(\frac{\underline{\theta}^{n+1} - \underline{y}^{n+1}}{\delta}\right) = \frac{c + \gamma - \omega}{1 + \gamma - \omega}
\end{cases}
$$

Since we already have $\underline{x}^1 > \underline{x}^0$ and $\underline{y}^1 > \underline{y}^0$, and for each n, $\underline{\theta}^{n+1}$ is strictly increasing in $\underline{x}^n$ and $\underline{y}^n$, we thus have:

$$\underline{x}^0 < \underline{x}^1 < \underline{x}^2 < \underline{x}^3 < \cdots$$

$$\underline{y}^0 < \underline{y}^1 < \underline{y}^2 < \underline{y}^3 < \cdots$$

The thresholds then define the expansion of dominance regions of action *attack*: Any strategy assigning action *not attack* to some signals below $\underline{x}^n$ ($\underline{y}^n$) cannot survive the *(n+1)th* round of elimination of dominated strategies.

The case for the expansion of dominance regions of action *not attack* is similar: We first define $\bar{x}^0$ and $\bar{y}^0$ by the equations $F\left(\frac{\bar{x}^0 - 1}{\sigma}\right) = 1 - c$ and $G\left(\frac{\bar{y}^0 - 1}{\delta}\right) = \frac{1 - c}{1 + \gamma - \omega}$. $[\bar{x}^0, +\infty)$ and $[\bar{y}^0, +\infty)$ thus are the initial dominance regions of action *not attack*. To see this, it suffices to check that for any signal greater than $\bar{x}^0$ ($\bar{y}^0$), the posterior probability of the event $\theta > 1$ is so large that each speculator (stakeholder) chooses action *not attack* no matter what

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strategies the other players follow.

Then we can depict the process of iterative elimination of strictly dominated strategies through defining $\bar{x}^n$ and $\bar{y}^n$ for $n = 0, 1, 2, \cdots$ as follows:

$$
\begin{cases}
(1-\lambda)F\left(\frac{\bar{x}^n - \bar{\theta}^{n+1}}{\sigma}\right) + \lambda G\left(\frac{\bar{y}^n - \bar{\theta}^{n+1}}{\delta}\right) = \bar{\theta}^{n+1} \\
F\left(\frac{\bar{x}^{n+1} - \bar{\theta}^{n+1}}{\sigma}\right) = 1 - c \\
G\left(\frac{\bar{y}^{n+1} - \bar{\theta}^{n+1}}{\delta}\right) = \frac{1-c}{1+\gamma-\omega}
\end{cases}
$$

And here we have:

$$\bar{x}^0 > \bar{x}^1 > \bar{x}^2 > \bar{x}^3 > \cdots$$

$$\bar{y}^0 > \bar{y}^1 > \bar{y}^2 > \bar{y}^3 > \cdots$$

Any strategy assigns action *attack* to some signal above $\bar{y}^n$ ($\bar{x}^n$) cannot survive the *(n+1)th* round of deletion of strictly dominated strategies.

Obviously, according to the definitions of $\bar{x}^0$, $\bar{y}^0$, $\underline{x}^0$ and $\underline{y}^0$, we have $\bar{x}^0 > \underline{x}^0$ and $\bar{y}^0 > \underline{y}^0$. These results imply that each expansion process described above has a limit, otherwise there exists some signal which is in the dominance region of both actions. Denote the limit for the former process by $(\underline{x}^*, \underline{y}^*)$, and the limit for the latter process by $(\bar{x}^*, \bar{y}^*)$. Obviously we must have: $(\underline{x}^*, \underline{y}^*) \le (\bar{x}^*, \bar{y}^*)$.

Now since $(\underline{x}^*, \underline{y}^*)$ and $(\bar{x}^*, \bar{y}^*)$ are limits of the expansion processes of dominance regions for two actions, according to the definition of the two processes, the two limits should be fixed points. And because functions F and G are both symmetric with mean zero, it is

straightforward to see that $(\underline{x}^*, \underline{y}^*)$ and $(\bar{x}^*, \bar{y}^*)$ both solve the following equation system:

$$\begin{cases} (1-\lambda)F\left(\frac{x-\theta}{\sigma}\right) + \lambda G\left(\frac{y-\theta}{\delta}\right) = \theta \\ F\left(\frac{\theta-x}{\sigma}\right) = c \\ G\left(\frac{\theta-y}{\delta}\right) = \frac{c+\gamma-\omega}{1+\gamma-\omega} \end{cases}$$

It has already been shown that this system admits a unique solution θ^*, x^* and y^* described by (2.5), (2.6) and (2.7). So we have $\underline{x}^* = \bar{x}^* = x^*$ and $\underline{y}^* = \bar{y}^* = y^*$, which completes the proof that the game admits a unique equilibrium which is the trigger strategy equilibrium (x^*, y^*). $\qquad \square$

Proof of Lemma 2.1.

For statement (i), given the equilibrium thresholds x^{0*} and y^{0*} in stage 0, applying here the analogous logic of the static game we have the equation

$$(1-\lambda)F\left(\frac{x^{0*} - \theta^{0*}}{\sigma}\right) + \lambda G\left(\frac{y^{0*} - \theta^{0*}}{\delta}\right) = \theta^{0*} \qquad (2.28)$$

admits a unique solution θ^{0*}, and the game ends in stage 0 as long as $\theta \le \theta^{0*}$ because of opposite monotonicities of the two sides of the equation.

To prove statement (ii), recall the informational role of public signal $A^0 = 0$. Observing this signal, every player knows $\theta > 0$. The regime remains if nobody attacks. So there is no incentive for a player to attack in stage 1 if everybody else withdraws from attacking then. The sequential rationality is satisfied.To show that there exists an equilibrium assuring no attacking happens in stage 1, notice that the labeling functions are common knowledge, so each player can calculate her expected payoff in stage 1 conditional on her action a_i^0 in stage 0 and the games continues in stage 1. Specifically, suppose there exists an equilibrium in which the regime is abandoned in stage 0 if and only if $\theta \le \theta^{0*}$. Then if nobody attacks in

stage 1, player i's stage-0 expected total payoff with action $a_i^0 = 1$ is

$$F\left(\frac{\theta^{0*} - x_i}{\sigma}\right)(1 - c) + \left[1 - F\left(\frac{\theta^{0*} - x_i}{\sigma}\right)\right](-c + \beta L_x(1) \cdot \gamma), \tag{2.29}$$

and her stage-0 expected total payoff with action $a_i^0 = 0$ is

$$\left[1 - F\left(\frac{\theta^{0*} - x_i}{\sigma}\right)\right]\beta L_x(0) \cdot \gamma. \tag{2.30}$$

Equalizing the two actions' payoffs we get the equation to solve for x^{0*}

$$F\left(\frac{\theta^{0*} - x^{0*}}{\sigma}\right) = \frac{c + \beta\gamma\left(L_x(0) - l_x(1)\right)}{1 + \beta\gamma\left(L_x(0) - l_x(1)\right)}. \tag{2.31}$$

Because the RHS of the equation is a constant and properly defined as a probability because $L_x(0) \geq L_x(1)$, the monotonicity of the LHS assures a unique solution x^{0*} given θ^{0*}. Analogously we have the equation to solve for y^{0*}

$$G\left(\frac{\theta^{0*} - y^{0*}}{\delta}\right) = \frac{c + \gamma - \omega + \beta\gamma\left(L_y(1) - l_y(0)\right)}{1 + \gamma - \omega + \beta\gamma\left(L_y(1) - l_y(0)\right)}. \tag{2.32}$$

Using the same logic in the static game, functions system (2.28), (2.31) and (2.32) admits a unique solution, which combined with $x^{\tau*} = y^{\iota*} = -\infty$ for all $\tau \in \Delta_x$ and $\iota \in \Delta_y$ constitute the threshold of an equilibrium of the whole game.

Statement (iii) says we cannot have an equilibrium in which $0 < \theta^{1*} \leq \theta^{0*}$. To see this, suppose there exists an equilibrium such that the inequalities hold. If all players behave according to the equilibrium strategies, the regime gets abandoned in stage 1 only if $\theta \leq \theta^{1*}$. But this event has probability 0 given the signal $A^0 = 0$ which conveys the public information $\theta > \theta^{0*}$, because $\theta^{1*} \leq \theta^{0*}$. But $\theta^{1*} > 0$ means that we could not have $x^{\tau*} = y^{\iota*} = -\infty$ for all $\tau \in \Delta_x$ and $\iota \in \Delta_y$. Without loss of generality, assume $x^{\tau*} > \infty$ for some τ. Then it is not rational for players in this subgroup to adopt such a strategy, because given that

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the conditional probability of a regime change in stage 1 is zero, they should not attack regardless of their private signals. This contradicts the assumption $0 < \theta^{1*} \leq \theta^{0*}$. □

Proof of Lemma 2.2.

We prove here the monotonicity of $\pi_i^1(\theta|x, A^0 = 0)$. The proof of the other is essentially the same. From the MLRP stated in Assumption 2.1, we have the inequality

$$p(\hat{x}|\hat{\theta})p(x|\theta) \geq p(x|\hat{\theta})p(\hat{x}|\theta), \quad \forall \hat{x} > x, \hat{\theta} > \theta. \tag{2.33}$$

Integrating the inequality above from x with respect to $\hat{x}$, we have

$$\int_x^\infty p(\hat{x}|\hat{\theta})p(x|\theta)\, d\hat{x} \geq \int_x^\infty p(x|\hat{\theta})p(\hat{x}|\theta)\, d\hat{x}, \tag{2.34}$$

which in turn gives us

$$p(x|\theta)(1 - P(x|\hat{\theta})) - p(x|\hat{\theta})(1 - P(x|\theta)) \geq 0, \forall x, \hat{\theta} > \theta, \tag{2.35}$$

where $P(\cdot|\cdot)$ is the conditional cumulative distribution function with $p(\cdot|\cdot)$ as the corresponding probability density function.

For any given equilibrium, we know from the statement (i) of lemma 2.1 that there exists a unique θ^{0*} and public information $A^0 = 0$ tells everybody $\theta > \theta^{0*}$ in stage 1. So for $\theta' \leq \theta^{0*}$ we must have $\pi_i^1(\theta'|x, A^0 = 0) = 0$. But for $\theta' > \theta^{0*}$ we have

$$\pi_i^1(\theta'|x, A^0 = 0) = \frac{F\left(\frac{\theta'-x}{\sigma}\right) - F\left(\frac{\theta^{0*}-x}{\sigma}\right)}{1 - F\left(\frac{\theta^{0*}-x}{\sigma}\right)}, \quad for\ \theta' > \theta^{0*}. \tag{2.36}$$

From the first order condition, we have that $\pi_i^1(\theta'|x, A^0 = 0)$ is decreasing in x for all $\theta' > \theta^{0*}$

if and only if

$$f\left(\frac{\theta^{0*}-x}{\sigma}\right)\left[1-F\left(\frac{\theta'-x}{\sigma}\right)\right]-f\left(\frac{\theta'-x}{\sigma}\right)\left[1-F\left(\frac{\theta^{0*}-x}{\sigma}\right)\right]\le 0. \qquad (2.37)$$

Now let $\theta=\theta^{0*}$ and $\hat\theta=\theta'$ in (2.35). because of the specific settings of the prior and noise distributions, we have

$$p(x|\theta^{0*})=-f\left(\frac{\theta^{0*}-x}{\sigma}\right)\ ,\qquad p(x|\theta')=-f\left(\frac{\theta'-x}{\sigma}\right), \qquad (2.38)$$

$$1-P(x|\theta')=F\left(\frac{\theta'-x}{\sigma}\right)\ ,\qquad 1-P(x|\theta^{0*})=F\left(\frac{\theta^{0*}-x}{\sigma}\right). \qquad (2.39)$$

Substituting the equations above into (2.35) gives us

$$f\left(\frac{\theta^{0*}-x}{\sigma}\right)F\left(\frac{\theta'-x}{\sigma}\right)-f\left(\frac{\theta'-x}{\sigma}\right)F\left(\frac{\theta^{0*}-x}{\sigma}\right)\le 0. \qquad (2.40)$$

Because $\theta' > \theta^{0*}$, we also have the following inequality

$$\frac{1-F\left(\frac{\theta'-x}{\sigma}\right)}{F\left(\frac{\theta'-x}{\sigma}\right)}\le \frac{1-F\left(\frac{\theta^{0*}-x}{\sigma}\right)}{F\left(\frac{\theta^{0*}-x}{\sigma}\right)}. \qquad (2.41)$$

Applying (2.41) into (2.40) gives us exactly (2.37). So $\pi_i^1(\theta|x, A^0=0)$ is decreasing in x. $\quad\square$

Proof of Proposition 2.6.

It is easy to check $y^{0*}=y^*$ and $y^{0*}=y^*$: As an infinitesimal agent, any player's instantaneous payoff in stage 1 has nothing to do with her action in stage 0. So players do not take into account the next stage when they make their stage-0 strategies, and the logic of the static game's solution applies here.

To prove the uniqueness, suppose there exists another equilibrium such that $\theta^{1*} > \theta^*$. Then for player i, her posterior probability of a regime change in stage 1 is $\pi_i^1(\theta^{1*}|x, A^0=0)$, which

is decreasing in x as stated in Lemma 2.2. The monotonicity thus gives us the equation to solve for x^{1*}:

$$\pi_i^1(\theta^{1*}|x^{1*}, A^0 = 0) = \frac{F\left(\frac{\theta^{1*}-x^{1*}}{\sigma}\right) - F\left(\frac{\theta^{0*}-x^{1*}}{\sigma}\right)}{1 - F\left(\frac{\theta^{0*}-x^{1*}}{\sigma}\right)} = c. \tag{2.42}$$

The equation above in turn gives us

$$F\left(\frac{\theta^{1*}-x^{1*}}{\sigma}\right) = c + (1-c)F\left(\frac{\theta^{0*}-x^{1*}}{\sigma}\right) \geq c. \tag{2.43}$$

Similarly we have

$$G\left(\frac{\theta^{1*}-y^{1*}}{\delta}\right) = \frac{c+\gamma-\omega}{1+\gamma-\omega} + \left(1 - \frac{c+\gamma-\omega}{1+\gamma-\omega}\right)G\left(\frac{\theta^{0*}-y^{1*}}{\delta}\right) \geq \frac{c+\gamma-\omega}{1+\gamma-\omega}. \tag{2.44}$$

Note that θ^{1*} is the unique solution to equation

$$(1-\lambda)F\left(\frac{x^{1*}-\theta^{1*}}{\sigma}\right) + \lambda G\left(\frac{y^{1*}-\theta^{1*}}{\delta}\right) = \theta^{1*} \tag{2.45}$$

Using the symmetry property of the noise distributions and combine the three equations above with equations (2.2), (2.3) and (2.4), we must have $\theta^{1*} \leq \theta^*$, contradiction. So the equilibrium depicted in the proposition is the unique one. $\qquad\square$

Proof of Proposition 2.7.

Given an equilibrium with thresholds $x^{01*} = (x^{0*}, x^{\ominus*})$ and $y^{01*} = (y^{0*}, y^{\ominus*}, y^{\oplus*})$, θ^{0*} is uniquely defined as the solution to equation

$$(1-\lambda)F\left(\frac{x^{0*}-\theta^{0*}}{\sigma}\right) + \lambda G\left(\frac{y^{0*}-\theta^{0*}}{\delta}\right) = \theta^{0*}. \tag{2.46}$$

We also have the following equations to solve for players thresholds:

$$F\left(\frac{\theta^{0*} - x^{0*}}{\sigma}\right) = c, \tag{2.47}$$

$$G\left(\frac{\theta^{0*} - y^{0*}}{\delta}\right) = \begin{cases} \frac{c+\gamma-\omega+\Omega}{1+\gamma-\Omega}, & if \ \theta^{1*} = 0, \\[2ex] \min\left\{1, \frac{c+\gamma-\omega+(1-\pi_j^0(\theta^{1*}|y^{0*}))\cdot\Omega}{1+\gamma-\omega}\right\}, & if \ \theta^{1*} > \theta^{0*}, \end{cases} \tag{2.48}$$

where $\Omega = \beta(k - k')\left(\mathbb{1}_{y^{0*}\leq y^{\ominus*}}\cdot\omega + \mathbb{1}_{y^{0*}>y^{\ominus*}}\cdot\gamma\right)$. The LHS of (2.48) is always greater than $\frac{c+\gamma-\omega}{1+\gamma-\Omega}$. Combining the three equations above and compare them with equations (2.2), (2.3) and (2.4) in the static game, we have $\theta^{0*} < \theta^*$.

For the second statement, to see that there may exist an equilibrium in which $\theta^{1*} > \theta^{0*}$, notice that given $\lambda \to \infty$ and a big enough β, we have $y^{0*} = -\infty$ and $\theta^{0*} = 0$, but $\theta^{1*} > 0$. So continuity of strategies in the parameters tells us that there may exist such equilibria depending on the parameter values. We then show that with $k' = 0$, as long as there exists such an equilibrium, we must have $\theta^{1*} < \theta^*$. For an equilibrium with $\theta^{1*} > \theta^{0*}$, we have

$$\theta^{1*} = \lambda \cdot Prob(y^{0*} < y \leq y^{\ominus*}|\theta^{1*})(1 - k') + \lambda \cdot Prob(y^{0*} < y \leq y^{\oplus*}|\theta^{1*})k'$$

$$+\lambda \cdot Prob(y \leq y^{\oplus*}, y \leq y^{0*}|\theta^{1*})k + \lambda \cdot Prob(y \leq y^{\ominus*}, y \leq y^{0*}|\theta^{1*})(1 - k)$$

$$+(1 - \lambda)F\left(\frac{x^{\ominus*} - \theta^{1*}}{\sigma}\right), \tag{2.49}$$

$$\pi_i^1(\theta^{1*}|x^{\ominus*}, A^0 = 0) = \frac{F\left(\frac{\theta^{1*}-x^{\ominus*}}{\sigma}\right) - F\left(\frac{\theta^{0*}-x^{\ominus*}}{\sigma}\right)}{1 - F\left(\frac{\theta^{0*}-x^{\ominus*}}{\sigma}\right)} = c, \tag{2.50}$$

$$\pi_j^1(\theta^{1*}|y^{\oplus*}, A^0 = 0) = \frac{G\left(\frac{\theta^{1*}-y^{\oplus*}}{\delta}\right) - G\left(\frac{\theta^{0*}-y^{\oplus*}}{\delta}\right)}{1 - G\left(\frac{\theta^{0*}-y^{\oplus*}}{\delta}\right)} = c, \tag{2.51}$$

$$\pi_j^1(\theta^{1*}|y^{\ominus*}, A^0 = 0) = \frac{G\left(\frac{\theta^{1*}-y^{\ominus*}}{\delta}\right) - G\left(\frac{\theta^{0*}-y^{\ominus*}}{\delta}\right)}{1 - G\left(\frac{\theta^{0*}-y^{\ominus*}}{\delta}\right)} = \frac{c + \gamma - \omega}{1 + \gamma - \omega}. \tag{2.52}$$

Lemma 2.2 assures all the posterior distributions in stage 1 decreases in the corresponding private signals. Applying the logic of the proof of that lemma here, the symmetric settings

of the noises ensures that the RHS of (2.49) is decreasing in θ^{1*}. So as long as we can solve equations (2.46)-(2.52) and get $\theta^{1*} > \theta^{0*}$, all the rationality requirements of an equilibrium are satisfied. And we know from the monotonicity of posteriors $y^{\oplus*} > y^{\ominus*}$. With $k' = 0$, we have

$$RHS \ of \ (2.49) \leq \lambda \cdot Prob(y^{0*} < y \leq y^{\ominus*}|\theta^{1*}) + \lambda \cdot Prob(y \leq y^{\oplus*}, y \leq y^{0*}|\theta^{1*})$$
$$+(1-\lambda)F\left(\frac{x^{\ominus*} - \theta^{1*}}{\sigma}\right). \quad (2.53)$$

If $y^{\ominus*} \geq y^{0*}$, we have

$$RHS \ of \ (2.49) \ \leq \ \lambda G\left(\frac{y^{\ominus*} - \theta^{1*}}{\delta}\right) + (1-\lambda)F\left(\frac{x^{\ominus*} - \theta^{1*}}{\sigma}\right)$$
$$< \ \lambda\left(1 - \frac{c+\gamma-\omega}{1+\gamma-\omega}\right) + (1-\lambda)(1-c) = \theta^*, \quad (2.54)$$

where the last inequality comes from (2.50) and (2.52). So the second statement is true for case $\theta^{1*} > \theta^{0*}$. Now if $\theta^{1*} \leq \theta^{0*}$, we have

$$RHS \ of \ (2.49) \ \leq \ \lambda G\left(\frac{\min\{y^{\oplus*}, y^{0*}\} - \theta^{1*}}{\delta}\right) + (1-\lambda)F\left(\frac{x^{\ominus*} - \theta^{1*}}{\sigma}\right)$$
$$< \ \lambda\left(1 - \frac{c+\gamma-\omega}{1+\gamma-\omega}\right) + (1-\lambda)(1-c) = \theta^*. \quad (2.55)$$

To get the last inequality, we apply (2.50), (2.52) and $y^{\oplus*} > y^{\ominus}$ to it if $y^{\oplus*} \leq y^{0*}$, and apply (2.48), (2.52) and $\theta^{1*} > \theta^{0*}$ to it if $y^{\oplus*} > y^{0*}$. Statement (ii) is proved.

To prove the third statement, note that with $k' > 0$, there will always be a mass of stakeholders being relabeled as speculators in stage 1 even if they do not attack in stage 0. With $c \to 0$, $k \to 1$ and $\beta \to 0$, we have $\theta^{0*} \to \theta^*$, and $x^{0*}, x^{\ominus*}, y^{\oplus*} \to \infty$. So given $\theta = \theta^*$, the total attacking mass in stage $M^0 \to \theta^*$. All speculators and stakeholders who attack in stage 0 will also attack in stage 1 because $c \to 0$ and $k \to 1$.and there will be a positive mass of stakeholders who do not attack in stage 0 but are relabeled as speculators in stage

1. As a result, the regime could be abandoned in stage 1 for some $\theta > \theta^*$ and we thus have $\theta^{1*} > \theta^*$. The continuity of the equation system (2.46)-(2.52) thus guarantees that there exists an equilibrium with $\theta^{1*} > \theta^*$ for small enough β and c, and large enough k. Statement (ii) is proved. $\qquad\square$

Proof of Proposition 2.8.

The proof of the first statement is essentially the same to the proof of the first statement in the previous proposition, we only need to consider the motive for speculators in stage 0 to behave less aggressively because of the potential extra payoff received in stage 1. And this gives us $\theta^{0*} < \theta^*$.

For the second statement, similar to the proof used in the previous proposition, we have here there may exist equilibria such that $\theta^{1*} > \theta^{0*}$ at least for cases with large enough β and small enough λ. Suppose there exists such an equilibrium, we have the following equations determining players' stage-1 strategies:

$$\theta^{1*} = (1-\lambda) \cdot Prob(x^{0*} < x \le x^{\oplus*}|\theta^{1*})h + 1 - \lambda) \cdot Prob(x^{0*} < x \le x^{\ominus*}|\theta^{1*})(1-h)$$

$$+(1-\lambda) \cdot Prob(x \le x^{0*}, x \le x^{\ominus*}|\theta^{1*})(1-h') + (1-\lambda) \cdot Prob(x \le x^{0*}, x \le x^{\oplus*}|\theta^{1*})h$$

$$+\lambda G\left(\frac{y^{\ominus*} - \theta^{1*}}{\delta}\right), \quad (2.56)$$

$$\pi_i^1(\theta^{1*}|x^{\ominus*}, A^0 = 0) = \frac{F\left(\frac{\theta^{1*}-x^{\ominus*}}{\sigma}\right) - F\left(\frac{\theta^{0*}-x^{\ominus*}}{\sigma}\right)}{1 - F\left(\frac{\theta^{0*}-x^{\ominus*}}{\sigma}\right)} = c, \quad (2.57)$$

$$\pi_i^1(\theta^{1*}|x^{\oplus*}, A^0 = 0) = \frac{F\left(\frac{\theta^{1*}-x^{\oplus*}}{\sigma}\right) - F\left(\frac{\theta^{0*}-x^{\oplus*}}{\sigma}\right)}{1 - F\left(\frac{\theta^{0*}-x^{\oplus*}}{\sigma}\right)} = \frac{c+\gamma-\omega}{1+\gamma-\omega}, \quad (2.58)$$

$$\pi_j^1(\theta^{1*}|y^{\ominus*}, A^0 = 0) = \frac{G\left(\frac{\theta^{1*}-y^{\ominus*}}{\delta}\right) - G\left(\frac{\theta^{0*}-y^{\ominus*}}{\delta}\right)}{1 - G\left(\frac{\theta^{0*}-y^{\ominus*}}{\delta}\right)} = \frac{c+\gamma-\omega}{1+\gamma-\omega}. \quad (2.59)$$

From the monotonic properties of the posteriors, we have $x^{\ominus *} > x^{\oplus *}$, and

$$
\begin{aligned}
RHS \ of \ (2.56) \ &\leq \ (1-\lambda)\cdot Prob(x^{0*} < x \leq x^{\ominus *}|\theta^{1*}) + (1-\lambda)\cdot Prob(x \leq x^{0*}, x \leq x^{\ominus *}|\theta^{1*}) \\
&\qquad + \lambda G\left(\frac{y^{\ominus *} - \theta^{1*}}{\delta}\right) \\
&= \ (1-\lambda)F\left(\frac{x^{\ominus *} - \theta^{1*}}{\sigma}\right) + \lambda G\left(\frac{y^{\ominus *} - \theta^{1*}}{\delta}\right) \\
&< \ \lambda(1-\lambda)(1-c) + \left(1 - \frac{c+\gamma-\omega}{1+\gamma-\omega}\right) = \theta^*.
\end{aligned}
\tag{2.60}
$$

The proposition is proved. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Chapter 3

Speculator-Triggered Crisis and Interventions

3.1 Introduction

The success of a business or an economy usually depends crucially on the stability and sustainability of the investments, in a variety of forms such as deposits, loans, debts, equities, financed by many investors. From time to time, based on the fundamentals of the business or economy, investors decide whether or not to hold their investments through keeping deposits in the bank, rolling over the debts, etc. A remarkable feature underlying these scenarios is the strategic complementarity between investors: The more holding investors there are, the more attractive it is for any investor to hold; and given more others withdrawing, it is less attractive for an investor to hold. Furthermore, there generally exist multiple equilibria under complete information,[1] and it requires coordination between investors to achieve the optimal result. Otherwise we may observe the occurrence of a crisis as the result of coordination failure, as in Diamond and Dybvig (1983).

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[1]See Topkis (1979) and Vives (1990): The equilibrium set for a supermodular game consists of a lattice with a maximum and a minimum elements, which are usually different.

Crises happen when a critical mass of investors decide to withdraw their investments and thus make remaining investments with the business or the economy immediately non-profitable. The course of a crisis usually starts with a turmoil of massive investment withdrawals which cause serious insolvency problems to the business or economy, and ends with disastrous results such as bankruptcy of a company or bank, collapse of a currency, and even the breakdown of a previously booming economy. The good news is that the crises, as extreme cases of coordination failures, seldom happen; nor do they occur in a random way. Based on a generalization of the global games in Carlsson and van Damme (1993), Morris and Shin (1998, 2000, 2003) argue that investors indeed receive heterogeneous and noisy private signals about the underlying fundamental, and this incomplete information feature resolves the multiplicity problem.[2] As a result, a crisis does not happen unless the fundamental deteriorates to reach some threshold. This explains why we do not observe crises quite often and makes comparative statics analysis for policies possible. Similar settings have been widely used in the literature afterwards.

The threshold of fundamental below which a crisis happens could depend on many factors. For example, the existence of a few speculators may make crises more likely to happen. Because of the widely existing short selling mechanisms through which a speculator can make a huge profit if the crisis happens, she is willing to trigger the crisis through attacking the status quo and letting investors coordinate on her action to withdraw their investments. By doing so, the speculator triggers a self-fulfilling prophecy and makes the crisis happen even for some fundamentals under which the business or economy could have been going well if there were no speculators.

In spite of a great number of studies and comments criticizing the speculators for making the financial system unstable by triggering crises,[3] little effort has been given to investigating the

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[2]The change to the equilibrium set structure brought about by incomplete information, no matter how infinitesimal the noises are, is a key characteristic in global games. This property can be tracked back to Rubinstein (1989); Harsanyi (1973).

[3]For example, Krugman (1999) considers Soros a speculator who "not only move money in anticipation of a currency crisis, but actually do their best to trigger that crisis for fun and profit".

speculator's effects in formal models; even less answered is what can be done to counteract these effects. This chapter is devoted to these concerns. We first add a speculator to a benchmark global game model and compare its results with that of the benchmark model. Then another player, the authority of a business or an economy, is added to the model. We investigate and compare the efficacies of three different interventions imposed by the authority on counteracting the speculator's effects. That is to say, we are not interested in whether or not an intervention can prevent a crisis from happening for all fundamentals, but rather would like to check if it works for some fundamentals such that the speculator cannot trigger crises any more: Given a fundamental value, a crisis happens here only if it also happens in the benchmark model.

When the attacking cost for the speculator is relatively small, she triggers a crisis for some mild or even fairly good fundamentals. What is more, the unique threshold equilibrium depicts a self-fulfilling prophecy of crisis in these cases. However, if the authority tries to deter the speculator by increasing the attacking cost, the intervention may or may not work because of multiple equilibria. To understand this, although the intervention successfully stops the speculator from attacking, as an endogenously determined action, it also sends investors a signal that the underlying fundamental is really weak; otherwise there is no need to impose such a policy. Then investors could coordinate either on the signal that the speculator decides not to attack, or on the signal that an intervention is imposed because of the weak fundamental. As a result, the intervention does not work if the latter case happens.

We also check the intervention policy that promises a reward to all holding investors if the crisis does not happen in the end. This also depicts scenarios in which the reward is promised unconditionally to all holding investors but they believe it is unrealizable unless the crisis does not happen. We show that this intervention never works no matter how great the reward is. It is because that, although the reward makes investors more inclined to hold under a sound fundamental, it cannot function in the same way when the fundamental is relatively weak and a crisis is about to happen. In this case, the speculator attacks first. After observing

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the speculator's move, investors deduce that the reward is unrealizable because a crisis will happen. So they ignore the reward while making decisions, and the intervention does not work at all.

The third form of intervention, eliminating the preemption motives, refers to scenarios where the authority covers a mass of withdrawals either by taking them over directly or by making it easier for them to be taken over by other market participants. We find this intervention works because it makes sure that an holding investor's payoff will no longer (or less) be harmed by other investors' withdrawals. As a result, the investors do not panic to withdraw their investments even after observing the speculator attack, and the crisis is thus prevented.

Our results suggest several policy implications. First, although the authority aims to prevent a crisis triggered by the speculator, to achieve this goal, she should focus on the investors, rather than on the speculator. If she resolves the preemption motives among investors, the speculator also retreats from attacking. If, however, she decides to impose policies preventing the speculator from attacking, it may not help avoid a crisis even if the speculator is deterred. Second, if the crisis is about to happen, it is futile for the authority to promise a reward that is conditional on whether or not the crisis is eventually prevented, since nobody takes it serious and realizable at the onset of a crisis.

This work has been inspired by some recent crises triggered by speculators, as well as the authorities' reactions in the course of the turmoils. We include in this chapter a discussion on the 1997 Asian financial crisis to illustrate our results and arguments in the model. Different from a vast literature that analyzes how the moral hazard problems, special cultural and political characteristics, and economic changes in other countries such as U.S. and China, added to the instability of the financial systems and economies of the countries and regions seriously harmed by the crisis, this work takes all these factors and the resulting bad fundamentals as given, and focuses instead on investigating if intervention policies successfully prevented a speculator-triggered crisis.

Malaysia's capital controls did not work because deterring the speculator from attacking cannot prevent a crisis, as argued in our model. Although interest rate defense policies proved effective when fundamentals were strong, they did not work in a crisis because a manual increase in the interest rate in this case was not because of an improvement in the future profitability and thus delivered no confidence to investors at all. As a result, investors figured out that the promised higher interest rate on loans and debts was unrealizable because of the expected insolvency of the companies and banks. The interest rate defense policies in this case can at best work by complementing other useful policies. Hong Kong's intervention worked because it successfully eliminated the preemption motives among investors. Although Thailand, Indonesia, South Korea, and the Philippines received a vast amount of rescue money through the IMF programs, the strict requirements of contractionary policies attached to the programs impeded these countries from pumping sufficient liquidity to companies and banks which were suffering from massive investment withdrawals. As a result, the authorities failed to eliminate the preemption motives among investors and capital flight continued. Our model (partly) explains the inefficacy of the IMF programs.

For other related studies, Corsetti et al. (2004) also argue that the existence of a large speculator, the "Soros", could make the status quo more vulnerable. But that work differs from our model, in that the large speculator in their paper functions more like a larger investor who can cause more withdrawals than others, whereas the speculator's interest is different from the investors' in our context. What is more, their work does not involve intervention policies imposed by authorities to counteract the speculator's effects, which is a main concern our work tries to address.

By formalizing the "twin crises" phenomenon described in Kaminsky and Reinhart (1999), Goldstein (2005) also depicts a vicious circle between the currency depreciation because of speculations and the massive withdrawals of investments, and moves on to suggest that it is effective to deal with each crisis to resolve both. However, our work argues that the vicious circle between the evolutions of the two crises does not imply parity between intervention

policies that handle different crises. Dealing with the withdrawal crisis proves effective to resolve the currency crisis, but policies designed in the opposite direction may not work. This difference results from the fact that the intervention imposed by the authority serves as an endogenous signal, but its informational function is not formally considered in that paper.

An important strand of the literature of global games, first suggested by Atkeson (2000), tries to regain multiple equilibria after introducing endogenous information, and explains crisis as coordination failures again.[4] Although we emphasize the importance of endogenous information as well, and Proposition 3.5 shares similarities with the results of Angeletos et al. (2006), the current work should not be categorized as one in that strand. Given the coordination feature of the game, it should not be surprising that multiplicity is regained after introducing endogenous information as public signals. However, since we are investigating effects of potential interventions under extremely volatile circumstances where the speculator is considerably aggressive and investors are quite sensitive to the speculator's actions, we use these properties to refine the initial perfect Bayesian equilibrium set and focus on solutions that are the most likely to occur and interest us the most. In other words, rather than being interested only in the equilibrium structure, we sacrifice some explainability to improve the solvability of our model.[5]

The rest of this chapter is organized as follows. We introduce the speculator into a benchmark global game model in the subsequent section. Section 3.3 gives the equilibria of the game and analyzes the effects of the existence of a speculator. Section 3.4 adds the authority as

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[4]See, for example, Angeletos et al. (2006, 2007), Angeletos and Werning (2006), Costain (2007), and Wang (2010).

[5]As stated in Haller (2000), "This paper points out tradeoffs between additional explanations and solvability of games ...literature on equilibrium selection and a host of refinements aims at narrowing the solution set (fewer phenomena explained, more accurate predictions) ...examples ...are payoff dominance, risk dominance, stability robustness, evolutionary stability, forward induction, backward induction, perfection, sequentiality, properness, diviness, intuitiveness, renegotiation-proofness, stationarity, symmetry, cheap talk, and many others. Another strand allows for broadening of the solution set (more phenomena explained, vaguer predictions). ...concepts allow for mixed strategies, correlated strategies, rationalizable sets, curb sets, craziness, cheap talk again, and ...non-additive beliefs."

a player to the model and investigates the efficacies of three general forms of intervention polices. In Section 3.5, we include a case study on the 1997 Asian financial crisis to illustrate our theoretical arguments. Section 3.6 concludes.

3.2 The Model

We take the bank run model in Morris and Shin (2000) as the benchmark.[6] While there are only investors who observe (exogenous) private signals in their setting, another player, the speculator, is added to our model. The speculator can attack the status quo of an authority, and her action is observable to all investors. In this way, a part of the investors' signals becomes *endogenous* and public. In what follows, we first introduce the benchmark model and then explain our additions and revisions to it.

3.2.1 Benchmark Model

Payoffs and timing. There is a continuum of investors with measure 1, indexed by $i \in [0, 1]$. There are three stages in the game, denoted by $T = \{0, 1, 2\}$. In stage 0, each investor has one unit of investment in an entity, e.g., a bank or a country. In stage 1, all investors choose their actions simultaneously from the set $\{W, \ H\}$, where action W stands for withdraw and H for hold. Let ℓ denote the mass of investors choosing to withdraw in stage 1. If an investor chooses W, i.e., to withdraw her money from the authority, she gets the one unit of investment back in stage 2 without any risk. If, instead, she chooses to wait, her realized monetary payoff depends on the authority's profitability in stage 2.

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[6]This stylized model is different from another stream of bank run models as in Diamond and Dybvig (1983) and Goldstein and Pauzner (2005), etc., in that these models assume local strategic complementarity because of the existence of the collapse of a bank, whereas our model admits global strategic complementarity. Although the collapse scenarios are not explicitly modeled in our setting, it depicts better the idea that the withdrawal of investors will reduce the remaining investors' payoffs, which is a reasonable assumption and widely observed in reality.

All investments in the entity are put into an asset portfolio which exhibits increasing returns to scale and does not mature until stage 2. In order to satisfy the withdrawal requests, if any, some immature investments have to be liquidated with costs. This harms the profitability of the portfolio. For this reason, the monetary payoff for an investor choosing to hold depends on both ℓ and the fundamental of the investment portfolio, denoted by θ. We simply let the monetary payoff for a holding investor be $e^{\theta-\ell}$. The utility function of investors is set to be $\ln(\cdot)$. So for a withdrawing investor, her payoff is $\ln(1) = 0$, and a holding investor gets payoff $\ln(e^{\theta-\ell}) = \theta - \ell$. Let $Z \in \{0, 1\}$ denote the outcome of the game, where $Z = 1$ means $\theta - \ell < 0$ and a crisis happens, $Z = 0$ means $\theta - \ell \geq 0$ and the crisis is prevented. We assume that an investor chooses to hold if the two actions give her the same payoff.

Timing and information. It is straightforward to see that under complete information, there are multiple equilibria for $\theta \in (0, 1)$: Either all depositors run on the bank, or they all hold their savings in the bank in stage 1. However, it is assumed throughout this work that the investors can only observe signals of θ with noise.

The fundamental θ is realized in stage 0 when the investment portfolio is chosen and remains fixed thereafter. We require that θ can only fall into a reasonable interval $(-M, M)$. But no investor can know the exact value of θ until her payoff is realized in stage 2. In stage 1, each investor i observes a private signal $x_i = \theta + \epsilon_i$, where ϵ_i's are i.i.d., independent of θ, and drawn from the uniform distribution on $(-\epsilon, \epsilon)$ with $0 < \epsilon < \frac{1}{2}$. The common prior for θ is a uniform distribution $U(-M, M)$ with $M \gg 1 + 2\epsilon$. This inequality makes the prior an uninformative one and simplifies our analysis. The noise structure, prior distribution, process of the game, and payoff functions are all common knowledge to all investors.

Note that given θ and the same strategy of all investors, strictly speaking, the mass of withdrawing investors ℓ is still a random variable. But under the continuum setting of investors, the "law of large numbers" on the empirical distribution of private signals makes sure that ℓ is equal to the probability of any investor choosing to withdraw based on the

strategy. For this reason, we take ℓ as a constant under this circumstance. This idea is formalized in the following assumption.

Assumption 3.1 (Judd (1985)). *Given a fundamental θ, the empirical distribution of all realized private signals always exists, and equals the uniform distribution on interval $(\theta - \epsilon, \theta + \epsilon)$.*

The assumption above is common knowledge to all players, and is made throughout the chapter. The benchmark model admits a unique equilibrium as stated below.

Proposition 3.1 (Morris-Shin). *There exists a unique Bayesian equilibrium in which each investor i chooses to withdraw if and only if $x_i < \frac{1}{2}$. In this equilibrium, the crisis happens $(Z = 1)$ if and only if $\theta < \frac{1}{2}$.*

The existence of noises eliminates the multiplicity problem, and whether or not the crisis happens completely depends on the value of fundamental θ.

3.2.2 Speculator and Our Model

Another player, the speculator, is now added to the game. The speculator can attack the status quo of the entity with the hope that her action could cause the entity to collapse and thus make her profits in the future. This kind of scenarios are widely observed in our economy. For instance, a speculator may attack a currency by large short positions of the currency through futures or forward contracts, and this behavior can trigger the investors in the country to withdraw their investments because of the fear of a sharp devaluation of the currency in the future. Finally, the speculator can benefit from the attack if enough investors choose to withdraw and the central bank has to devaluate its currency because it is out of foreign-exchange reserves. Similarly, a speculator can attack a bank by selling short the bank's shares in the futures market and underselling the shares in the stocks market at the same time. By doing so, the speculator can make great profits from the short selling if

the bank finally collapses because of a bank run. A speculator may also attack a company by releasing bad news towards its future earnings. This can lead a sufficient number of investors of the company to withdraw their investments, or reduce the company's ability to raise new debts in the market. As a result, the company may undergo a serious decrease in its share price because of a lack of capital or even a debt default, and the speculator can thus purchase the company's shares or even acquire it at a lower price in the future.

In stage 0, once the fundamental θ has been realized, the speculator observes it without any noise, and chooses her action from the set $\{A, N\}$. Action A stands for attacking the status quo and N means not to do so.[7] After the speculator chooses her action in period 0, investors observe it as public information and move simultaneously in stage 1. If the speculator refrains from attacking, her payoff is fixed as $\ln(1) = 0$. If action A is chosen, the speculator's payoff depends on both her own action and the profitability of the asset portfolio in stage 2. More specifically, the payoff of an attacking speculator is assumed to be the logarithmic form of the rate of return: $\ln \left(e^{\theta - \ell} e^c \right)^{-1} = \ell - \theta - c$, where θ and ℓ are the same as defined above in the benchmark model and $c \geq 0$ stands for the attacking cost.

Let us use the case of a currency attack to illustrate the speculator's payoff settings. Suppose in stage 0 the currency's exchange rate is P_0, and the speculator short sells the currency in stage 0 by contracts which take stage 2 as the strike date and P_0 as the strike price. In stage 2, the investments mature and the profitability of the authority is thus perfectly observable to everybody. We adopt the efficient-market hypothesis (EMH) and let $P_2 = P_0 e^{\theta - \ell}$ be the exchange rate in stage 2.[8] Then based on the futures contract, we have that the rate of return is $\frac{P_0}{P_2}$ if the speculator can buy the specific amount of the currency at P_2 in stage 2. However, there is a possibility that the central bank imposes capital control policies and make it more costly for the speculator to realize her profits from the contract. To measure

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[7] The speculator is usually an individual or a company which can access many information resources and sustain a good research facility, and is thus capable of knowing the exact fundamental value.

[8] This simple form has the merits of always generating a positive P_2 , as well as making the speculator's payoff easy to study as shown later.

this risk, we introduce attacking cost $c > 0$ into the model and make the rate of return as $\frac{P_0 e^{-c}}{P_2}$. The logarithmic form of it is thus $\ell - \theta - c$ as stipulated above. Attacking cost in other speculator-investors alike scenarios, such as the underselling losses in the bank run scenario, or the counterattack losses in the company collapse scenario, can also be well depicted by c.

Investments chosen	Speculator observes θ	Investors observe x_i and a then simultaneously choose	Investments mature
θ drawn	and chooses action $a \in \{A, N\}$	actions from $\{W, H\}$	Players get payoffs

stage 0   stage 1   stage 2
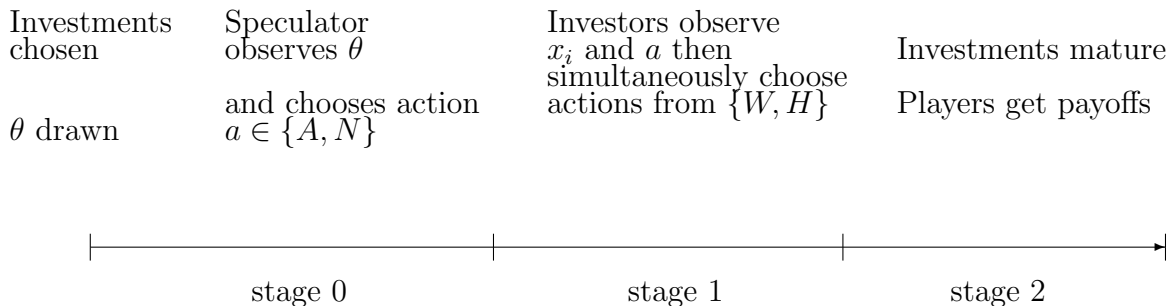
Figure 3.1: Order of events.

Before moving on to the next section, let us summarize the model. As shown in Figure 3.1, at the beginning of stage 0, the portfolio of investments is chosen and the profitability θ is thus realized and observable to the speculator. After observing θ , the speculator decides whether or not to attack in stage 0. In stage 1, all investors observe the speculator's action as public information and their private signals x_i's, and take their actions from $\{W, H\}$ simultaneously. Then all players' payoffs are realized in stage 2. For investors, action W always gives 0 payoff, and action H leads to payoff $\theta - \ell$. For the speculator, her payoff is 0 if she does not attack and $\ell - \theta - c$ if she attacks. ℓ and c represent the mass of withdrawing investors and attacking cost for the speculator, respectively.

3.3 Equilibrium

In this section we solve the game with the speculator and investigate her effects on the economy. The solution concept used here, *threshold equilibrium*, is defined as follows.

Definition 3.1. *A threshold equilibrium is a perfect Bayesian equilibrium consisting of a strategy* $s : (-M - \epsilon, M + \epsilon) \times \{A, N\} \longrightarrow \{W, H\}$ *for all investors, a strategy* $v :$

$(-M, M) \longrightarrow \{A, N\}$ *for the speculator, and beliefs depicted by probability density functions* $\pi_\theta(\cdot|x_i, a)$ *for all investors, where* $a \in \{A, N\}$, *such that* $v(\theta) = A$ *if and only if* $\theta < \hat{\theta}$, $s(x_i, A) = W$ *if and only if* $x_i < \hat{x}^A$, $s(x_i, N) = H$ *if and only if* $x_i > \hat{x}^N$, *and*

$$\hat{\theta} \in \arg\max_{\rho \in \mathbb{R}} \left(\ell(\theta, A|s) - \theta - c\right) \cdot \mathbb{1}_{\theta < \rho}, \quad \forall \theta \in (-M, M), \tag{3.1}$$

$$\hat{x}^a \in \arg\max_{x \in \mathbb{R}} E_{\pi_\theta(\cdot|x_i, a)} \left(\theta - \ell(\theta, a|s)\right) \cdot \mathbb{1}_{x_i > x}, \quad \forall x_i \in B_{M+\epsilon}(0), \ \forall a \in \{A, N\}, \tag{3.2}$$

$$\pi_\theta(\theta^*|x_i, a) = \begin{cases} \frac{1}{m(B_\epsilon(x_i) \cap v^{-1}(a))} & for \ \theta^* \in B_\epsilon(x_i) \cap v^{-1}(a) \\ 0 & otherwise \end{cases} \quad if \ B_\epsilon(x_i) \cap v^{-1}(a) \neq \emptyset, \tag{3.3}$$

$$\pi_\theta(\theta^*|x_i, a) = \begin{cases} \frac{1}{2\epsilon} & for \ \theta^* \in B_\epsilon(x_i) \\ 0 & otherwise \end{cases} \quad if \ B_\epsilon(x_i) \cap v^{-1}(a) = \emptyset, \tag{3.4}$$

where $B_\alpha(\beta)$ *represents the interval* $(\beta - \alpha, \beta + \alpha)$ *with* $\alpha > 0$, $m(\cdot)$ *is the Lebesgue measure on* $\mathbb{R}$, *characteristic function* $\mathbb{1}_\triangle$ *takes value* 1 *if statement* $\triangle$ *is true and* 0 *otherwise, and* $\ell(\theta, a|s)$ *is the mass of withdrawing investors given fundamental* θ, *the speculator's action* a, *and that all investors behave according to strategies* $s(\cdot)$.

Conditions (3.1) and (3.2) require that it is optimal for the speculator and investors to follow strategies characterized by thresholds $\hat{\theta}$, $\hat{x}^A$, and $\hat{x}^N$, given all other players doing so. Condition (3.3) makes sure that the beliefs for investors are updated by Bayes' rule whenever applicable. The belief system is thus *consistent* on the equilibrium path. We further put a requirement on the investors' beliefs, as stated in (3.4): If investor i's two signals conflict with each other and Bayes' rule is thus inapplicable, she always relies exclusively on her private signal x_i, and does not take into consideration the public signal $a \in \{A, N\}$. That is to say, when investor i observes that the speculator is deviating from the equilibrium strategy, she just treats the public signal as an uninformative one, and updates her posterior distribution of θ using only her private information. Also note that according to the definition, an investor

chooses to withdraw if and only if $x_i < \hat{x}^A$ given A observed, but chooses to withdraw if and only if $x_i \leq \hat{x}^N$ given N observed.

Lemma 3.1. *In any threshold equilibrium of the game, we must have $\hat{\theta} \leq 1 - c$, $\hat{x}^A \geq \frac{1}{2}$, and $\hat{x}^N \leq \frac{1}{2}$.*

Compared to the unique equilibrium in the benchmark model, now the investors are more willing to withdraw if the speculator chooses to attack and more willing to hold if N is chosen by the speculator, given their private signals fixed.

We are concerned about how the equilibria look like after the addition of the speculator. More importantly, we want to know under what conditions a crisis arises in the new context. The following results answer these questions.

Lemma 3.2. *A threshold equilibrium always exists, and all equilibria are characterized below:*

(i) For $0 \leq c \leq \frac{1}{2} + \epsilon$, there exists a unique threshold equilibrium, in which we have $\hat{\theta} = 1 - c$, $\hat{x}^A = 1 - c + \epsilon$, $\hat{x}^N = \frac{1}{2}$ if $0 \leq c \leq \frac{1}{2} - \epsilon$, and $\hat{x}^N = 1 - c - \epsilon$ if $c > \frac{1}{2} - \epsilon$. As a result, $Z = 1$ if and only if $\theta < 1 - c$.

(ii) For $\frac{1}{2} + \epsilon < c \leq 1$, there are multiple threshold equilibria. Specifically, we have the following results:

- *There exists a threshold equilibrium, in which $\hat{\theta} = 1 - c$, $\hat{x}^A = \hat{x}^N = \frac{1}{2}$.*

- *There exists a threshold equilibrium, in which $\hat{\theta} = 1 - c$, $\hat{x}^A = \frac{1}{2}$, and $\hat{x}^N = 1 - c - \epsilon$.*

- *There exists a threshold equilibrium, in which $\hat{\theta} = 1 - c$, $\hat{x}^A = \frac{1}{2}$, $\hat{x}^N \in (1 - c - \epsilon, 1 - c + \epsilon)$ and $\hat{x}^N$ solves equation $\frac{1 - c + \hat{x}^N + \epsilon}{4} = \frac{(\hat{x}^N - 1 + c + \epsilon)^2}{(\hat{x}^N - 1 + c + 3\epsilon)^2}$.*

(iii) For $1 < c < 1 + \epsilon$, there exists a threshold equilibrium with $\hat{\theta} = 1 - c$, and $\hat{x}^A = \hat{x}^N = \frac{1}{2}$. It is the unique one if we further have $c - 1 > \sup\{\frac{(n+1)\epsilon}{2} - \frac{2(n+1)^2}{(n+3)^2} : n \in (-1, 1)\}$. Otherwise there exists other threshold equilibria characterized by $\hat{\theta} = 1 - c$, $\hat{x}^A = \frac{1}{2}$, $\hat{x}^N \in (1 - c - \epsilon, 1 - c + \epsilon)$ and $\hat{x}^N$ solves equation $\frac{1 - c + \hat{x}^N + \epsilon}{4} = \frac{(\hat{x}^N - 1 + c + \epsilon)^2}{(\hat{x}^N - 1 + c + 3\epsilon)^2}$.

(iv) For $c \geq 1 + \epsilon$, there exists a unique threshold equilibrium, which is characterized by $\hat{\theta} = 1 - c$, $\hat{x}^A = \hat{x}^N = \frac{1}{2}$.

Recall that the speculator's payoff upon attacking is given by $\ell - \theta - c$ with $\ell \in [0, 1]$, so any strategy for the speculator that prescribes action A for $\theta \geq 1 - c$ is strictly dominated. And we can see from the results above that, in any equilibrium, the speculator attacks if and only if $\theta < \hat{\theta} = 1 - c$. This suggests that the speculator behaves as aggressively as possible: As long as the greatest possible payoff of attacking is higher than the payoff of action N, which is fixed at 0, she attacks. The reason for such aggressive behaviors is that in each equilibrium, the greatest possible payoff for action A is always achieved, as stated below.

Proposition 3.2. *For any $c \geq 0$, on each equilibrium path, all investors choose to withdraw ($\ell = 1$) after observing the speculator choosing action A, and the crisis thus happens. Furthermore, for $c \leq \frac{1}{2} + \epsilon$, the speculator can trigger a panic crisis: If the speculator chooses to attack, all investors withdraw ($\ell = 1$) and the crisis happens ($Z = 1$); if the speculator does not attack, all investors hold ($\ell = 0$) and the crisis is prevented ($Z = 0$).*

For relatively large c, only with small enough θ values would the speculator choose to attack. It is thus not striking to see that all investors choose to withdraw after observing A. Because their private signals will also be small enough such that they should have chosen to withdraw even without the existence of a speculator. To see this, just check that we must have $\theta < 1 - c$ for investors to observe A. Specifically, for $c > \frac{1}{2} + \epsilon$, inequality $\theta < \frac{1}{2} - \epsilon$ must hold for investors to observe public signal A. With such small fundamental values, inequality $\theta + \epsilon_i < \frac{1}{2}$ holds for any investor i. Recall that $\frac{1}{2}$ is the threshold in the benchmark model, combined with the fact, as stated in Lemma 3.1, that the speculator's action A makes investors more aggressive, we have that all investors withdraw after observing A on the equilibrium path.

However, the scenario for a relatively small attacking cost $c \leq \frac{1}{2} + \epsilon$ is rather different. The speculator's action, either A or N, not only serves as a public signal but also tells information

about the fundamental θ to all investors. As a result, investors coordinate on the speculator's action. Take action A for instance, after observing it, each investor can deduce $\theta < 1 - c$ as long as her information set is on the equilibrium path. What is more, she can further deduce that all other investors' information sets are also on the equilibrium path, and if they all play W, she should also join the group. As a result, the crisis happens. Similarly, public signal N tells investors $\theta \geq 1 - c$. On the equilibrium path, all investors coordinate on this signal to hold their investments. Eventually, the crisis is prevented. Note that on the equilibrium path, each investor's choice is independent of her own signal.

For $\theta < 1 - c$, the scenario above depicts a *panic* crisis.[9] Public signal A causes a fear of a subsequent crisis to all investors, this fear make them panic to withdraw all their investments. Although it is often argued that a panic is because of "irrational fears" in some contexts, it is clearly rational in the current work—it is actually prescribed by an equilibrium. Especially, based on argument (i) of Lemma 3.2, the panic crisis always happens for $\theta < 1 - c$ in the *unique* equilibrium. So with relatively small attacking cost c, the fundamental still exclusively determines whether or not a crisis happens: The essential reason a panic crisis happens is that the fundamental deteriorates. But this could not be true for all cases. With moderate values of the attacking cost, as shown in Lemma 3.2, the result of the game becomes ambiguous for a range of θ because of multiple equilibria. Specifically, we have the following proposition.

Proposition 3.3. *For $c \geq 0$, let θ_c denote the fundamental threshold such that a crisis happens ($Z = 1$) if and only if $\theta < \theta_c$. Then $\theta_c = 1 - c$ if $c \leq \frac{1}{2} + \epsilon$, θ_c is indeterminate for $c \in (\frac{1}{2} + \epsilon, 1 + \epsilon)$ because of the (potential) multiplicity, and $\theta_c = \frac{1}{2}$ for $c \geq 1 + \epsilon$.*

We are more interested in the case $c \leq \frac{1}{2} + \epsilon$, not only because it admits a unique solution

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[9]The terms crisis and panic are sometimes interchangeable, in the sense that a financial collapse often happen in the mood of a panic. However, they are treated differently in the current work. A crisis stands for a sufficiently low return of investments because of either a bad fundamental or the insolvency of the entity; a panic represents the scenario that all investors choose to withdraw whatever, because of the fear of a crisis. A crisis need not be in a panic, and a panic leads to a crisis for relatively low fundamentals.

and thus provides predictable results, but also for the reason that it better depicts reality, as we will argue later. To check the effects of the existence of a speculator on the game, we compare our results here with that in the benchmark model, in the following proposition.

Proposition 3.4. *Compared to the benchmark model, if $c < \frac{1}{2}$, the existence of a speculator increases the vulnerability by triggering a panic crisis for $\theta \in [\frac{1}{2}, 1 - c)$; if $c \in (\frac{1}{2}, \frac{1}{2} + \epsilon]$, the existence of a speculator reduces the vulnerability by stabilizing the status quo for $\theta \in [1-c, \frac{1}{2})$; if $c = \frac{1}{2}$, the existence of a speculator does not change the result (Z) for any θ.*

It should not be surprising to see that the speculator can both increase and reduce the vulnerability of the entity, depending on different fundamental values and attacking costs: Her action A makes investors more aggressive, and action N makes them less aggressive. Recall that, on the equilibrium path, all investors coordinate on the speculator's action if $c \leq \frac{1}{2} + \epsilon$. What is more, investors are more likely to observe and coordinate on A with smaller c, and more likely to observe and coordinate on N with larger c. So the speculator's effects on the economy vary for different cases.

With $c < \frac{1}{2}$ and $\theta \in [\frac{1}{2}, 1 - c)$, the existence of a speculator triggers a panic crisis which could have been prevented if she is eliminated and the model becomes just the same as the benchmark. We thus wonder if the authority can impose interventions to undo the speculator's impact in this case. That question is discussed in the subsequent section.

3.4   Interventions

The speculator's attacking cost, e.g., loss of interests, risk of capital regulation, spending on research and information, is usually quite small or even negligible compared to its size of business. The result that the speculator can trigger a panic crisis for relatively small c is widely observed in reality: a sharp depreciation of a currency or abandoning of a pegged exchange rate system after an attack in the foreign exchange market; a catastrophic drop

in the share price of a company's stocks after the release of a negative report on it; and a sudden bank run after a devaluation of the bank's assets by credit rating agencies.

In this section, we add the authority of the entity, e.g., a central bank, a bank, the board of a company, as a player in the model. Our main goal is to survey if the authority can *offset* the effects of the existence of a speculator using certain intervention policies. Namely, we are not trying to find the optimal intervention policy for all cases, but only interested in whether or not the authority's intervention can prevent a panic crisis which is about to happen, but would have been prevented if there were no speculator in the model.

Throughout this section, we take the attacking cost $c = 0$ for simplicity.[10] Without interventions, the existence of a speculator expands the region of θ for the happening of a crisis from $\theta < \frac{1}{2}$ in the benchmark model to $\theta < 1$. We thus focus on possible intervention policies for fundamental value $\theta \in [\frac{1}{2}, 1)$. Without loss of generality, let $\{0, 1\}$ denote the authority's action set, where action 0 represents no intervention imposed and 1 means the authority intervenes. For each $\gamma \in \{0, 1\}$, there is an intervention cost, $L(\gamma)$, for the authority. We assume $L(0) = 0$ and $0 < L(1) < 1$. Denote the authority's payoff function by

$$G(\theta, \ell, \gamma) = \begin{cases} 0 - L(\gamma) & for \ \ \theta < \frac{1}{2}, \\ 1 - \tilde{Z}(\theta, \ell, \gamma) - L(\gamma) & for \ \ \theta \in [\frac{1}{2}, 1), \\ 1 - L(\gamma) & for \ \ \theta \geq 1, \end{cases} \tag{3.5}$$

where the result $\tilde{Z}$ equals 1 (0) if the crisis happens (is prevented), note that $\tilde{Z}$ also depends on γ in an equilibrium because the authority's intervention has impacts on investors' actions. From the settings above, it is straightforward to see that it is dominant for the authority not to intervene ($\gamma = 0$) for $\theta \geq 1$ or $\theta < \frac{1}{2}$. She is merely interested in correcting deviations

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[10]In the case that the authority imposes a policy deterring the speculator, we also take into account positive attacking costs. But in these scenarios, the attacking cost could be regarded as the sum of $c = 0$ and Δc, where Δc is the magnitude of the intervention. In this sense, condition $c = 0$ actually means that there is no attacking cost before intervention for all cases.

resulted by the existence of a speculator. But for $\theta \geq 1$ ($\theta < \frac{1}{2}$), the model with existence of a speculator leads to the same result, $\tilde{Z} = 0$ ($\tilde{Z} = 1$), as in the benchmark model, so there is no need for the authority to intervene in these cases. We can interpret the argument in this way: It is so expensive to save the system given a sufficiently low fundamental that the authority will not do it; and it is unnecessary for the authority to intervene if the fundamental is strong enough and the status quo is invulnerable. However, for $\theta \in [\frac{1}{2}, 1)$, the scenario is different. As stated in Proposition 3.4, the speculator increases the vulnerability of the entity by triggering a panic crisis. So the authority may have an interest to undo this effect through interventions. Condition $L(1) < 1$ guarantees that it is always beneficial for the authority to impose a successful intervention if a panic crisis is about to happen.

Denote by $H(\theta, \ell, \gamma)$ and $I(\theta, \ell, \gamma)$ the payoff for an attacking speculator and the payoff for any holding investor, respectively. The payoffs for a non-attacking speculator and a withdrawing investor are still 0. For the information structure, we assume that the authority also observes the exact value of θ, and her action is a public signal to all other players. The speculator's action is still publicly observable to all investors. All other settings of the previous model are also inherited here.

Investments chosen	Authority observes θ	Speculator observes θ and γ	Investors observe x_i, a, and γ	Investments mature
θ drawn	chooses action $\gamma \in \{0, 1\}$	chooses action $a \in \{A, N\}$	simultaneously choose actions from $\{W, H\}$	Players get payoffs

| stage 0 | stage 1 | stage 2 | stage 3 |
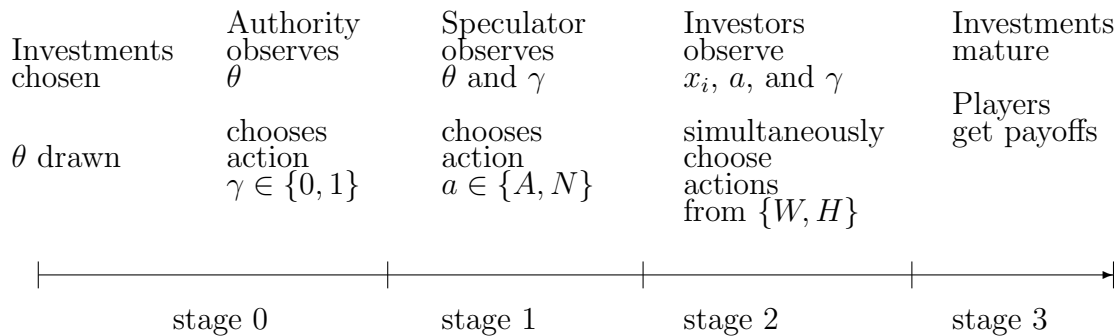
Figure 3.2: Order of events, with the authority added to the model.

The game proceeds as illustrated in Figure 3.2. In stage 0, fundamental θ is realized and observed by the authority, who then decides whether to impose the intervention or not. In stage 1, the speculator observes both θ and γ, and then decides whether or not to attack. In stage 2, each investor observes a private signal of θ with noise ϵ_i, and actions taken by the

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authority and the speculator as public signals. All investors move simultaneously. All players get payoffs in stage 3. Similar to the threshold equilibria used in the preceding section, we focus here on *intervention equilibria* defined as follows.

Definition 3.2. *An intervention equilibrium is a perfect Bayesian equilibrium consisting of a strategy $s : (-M - \epsilon, M + \epsilon) \times \{A, N\} \times \{0, 1\} \longrightarrow \{W, H\}$ for all investors, a strategy $v : (-M, M) \times \{0, 1\} \longrightarrow \{A, N\}$ for the speculator, a strategy $\mu : (-M, M) \longrightarrow \{0, 1\}$ for the authority, beliefs depicted by probability density functions $\pi_\theta(\cdot|x_i, a, \gamma)$ for all investors, and thresholds $\tilde{\vartheta}$, $\tilde{\theta}^\gamma$, $\tilde{x}^{a,\gamma}$, where $a \in \{A, N\}$ and $\gamma \in \{0, 1\}$, such that $\mu(\theta) = 1$ if and only if $\theta \in [\tilde{\vartheta}, 1)$, where $\tilde{\vartheta} \in [\frac{1}{2}, 1]$, $v(\theta, \gamma) = A$ if and only if $\theta < \tilde{\theta}^\gamma$, $s(x_i, a, \gamma) = W$ if and only if $x_i < \tilde{x}^{a,\gamma}$, and*

$$\mu(\theta) \in \arg \max_{\sigma \in \{0,1\}} G(\theta, \ell(\theta, \sigma|s, v), \sigma), \quad \forall \theta \in (-M, M), \tag{3.6}$$

$$\tilde{\theta}^\gamma \in \arg \max_{\rho \in \mathbb{R}} H(\theta, \ell(\theta, a, \gamma|s), \gamma) \cdot \mathbb{1}_{\theta < \rho}, \quad \forall \theta \in (-M, M), \tag{3.7}$$

$$\tilde{x}^{a,\gamma} \in \arg \max_{x \in \mathbb{R}} E_{\pi_\theta(\cdot|x_i, a, \gamma)} I(\theta, \ell(\theta, a, \gamma|s), \gamma) \cdot \mathbb{1}_{s(x_i, a, \gamma, \tilde{x}^{a,\gamma}) = H}, \quad \forall x_i \in B_{M+\epsilon}(0), \tag{3.8}$$

$$\pi_\theta(\cdot|x_i, a, \gamma) \text{ is obtained by Bayes' rule whenever applicable}, \tag{3.9}$$

$$\left.\begin{array}{c} if \ \mu^{-1}(1) = \emptyset, \ investors \ treat \ signal \ \gamma = 1 \ as \ message \ \theta \in [1/2, 1), \\ if \ (\gamma, a) \ conflict \ with \ x_i, \ investors \ update \ beliefs \ using \ x_i \ and \\ also \ take \ signal \ a \ into \ account \ whenever \ applicable, \end{array}\right\} \tag{3.10}$$

$$\tilde{\theta}^\gamma \text{ solves } H(\tilde{\theta}^\gamma, \ell = 1, \gamma) = 0, \tag{3.11}$$

where characteristic function $\mathbb{1}_\triangle$ takes value 1 if statement $\triangle$ is true and 0 otherwise, $B_\alpha(\beta)$ means neighborhood $(\beta - \alpha, \beta + \alpha)$ for $\alpha > 0$, and $\ell(\theta, \gamma|s, v)$ is the mass of withdrawing investors given fundamental θ, the authority's action γ, and that all investors and the speculator behave according to strategies $s(\cdot)$ and $v(\cdot)$, respectively.

Conditions (3.6), (3.7), (3.8), and (3.9) make sure that the strategies and beliefs depicted

above constitute a perfect Bayesian equilibrium. Conditions (3.10) and (3.11) serve as *selection rules* imposed on the equilibrium profile. Rather than considering in all perfect Bayesian equilibria, we are only interested in those that satisfy the two requirements. To understand their meanings, let us recall the case of $c = 0$ without the existence of an authority. As shown in the preceding section, in this case the speculator attacks as aggressively as possible ($\hat{\theta}$ solves $1 - \hat{\theta} = 0$) and all investors coordinate on the speculator's action. This in turn causes speculator-triggered crises as well as a call for authority interventions. The selection rules require that these sentiments do not change after the introduction of the authority as a new player: Rule (3.11) says that the speculator still behaves as aggressively as possible, and (3.10) requires that investors take into account the informational role of the speculator's action as much as possible. If the speculator behaves aggressively and investors trust the speculator in the about-to-happen trigger crisis, an addition of the authority's action as a public signal is unlikely to destroy either the speculator's pugnacity or investors' trust in the speculator. So equilibria that passes these selection rules are more likely to happen in reality. Based on the authority's payoff function, it is absolutely irrational for her to intervene for $\theta < \frac{1}{2}$ or $\theta \geq 1$, so if $\mu^{-1}(1) = \emptyset$ and $\gamma = 1$ happens, investors only take θ values in $[\frac{1}{2}, 1)$ seriously since rationality of the authority is implicitly assumed and common knowledge to all players. What is more, condition (3.10) is partly justified by (3.11). We usually have the speculator's and the authority's payoffs negatively dependent on or even opposite to each other. Signal $\gamma = 1$ and signal A thus may conflict with each other. But given requirement (3.11), investors know that the speculator will never deviate from the equilibrium strategy by attacking for some fundamental value that is high enough to make her attack nonprofitable for sure. For this reason, investors may infer that the conflict of information comes from the authority's deviation from the equilibrium path. As a result, they ignore the informational role of the authority's action, and take only signal A into account.

The strategies of all players in an intervention equilibrium can be characterized by thresholds $\tilde{\vartheta}$, $\tilde{\theta}^{\gamma}$, and $\tilde{x}^{a,\gamma}$. Recall that in our context the authority is only interested in preventing crises

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caused by the existence of the speculator, and according to (3.5) any strategy that prescribes action 1 for $\theta > 1$ or $\theta < \frac{1}{2}$ is strictly dominated. For this reason, we only need to investigate the authority's strategy for $\theta \in [\frac{1}{2}, 1)$, and again we focus on threshold strategies. Also note that with $\tilde{\vartheta} = 1$, we have $\mu(1) = \emptyset$ and the authority does not intervene for any θ. We further denote by $\tilde{\theta}_c$ the fundamental threshold such that the crisis happens ($\tilde{Z} = 1$) if and only if $\theta < \tilde{\theta}_c$. Now we move on to different interventions.

3.4.1 Deterring the Speculator

Because the authority wants to prevent a crisis triggered by the speculator, an option naturally making sense to her is to deter the speculator from attacking so investors do not observe signal A and the crisis thus does not happen. Specifically, she incurs an attacking cost, Δc, for the speculator. Now the speculator's payoff for attacking becomes $\ell - \theta - \Delta c$ if the authority intervenes and $\ell - \theta$ if the authority does not intervene. Without loss of generality, we consider $\Delta c \leq \frac{1}{2}$.[11] If the attacking cost is exogenously set as $c = \Delta c$, from Proposition 3.3 we have that the crisis does not happen for $\theta \geq 1 - \Delta c$. But in our context, the attacking cost is endogenously determined by the authority's action, and the efficacy of the intervention becomes ambiguous because of multiple equilibria.

Proposition 3.5. *In a game with settings $\tilde{Z}(\theta, \ell, \gamma) = \mathbb{1}_{\theta < \ell}$, $H(\theta, \ell, \gamma) = \ell - \theta - \Delta c \cdot \mathbb{1}_{\gamma=1}$, and $I(\theta, \ell, \gamma) = \theta - \ell$, where $\Delta c \in (0, \frac{1}{2}]$, there are multiple intervention equilibria.*

(i) *There is an intervention equilibrium in which $\tilde{\vartheta} = 1 - \Delta c$, $\tilde{\theta}^1 = 1 - \Delta c$, $\tilde{\theta}^0 = 1$, $\tilde{x}^{A,0} = 1 + \epsilon$, $\tilde{x}^{A,1} = 1 - \Delta c + \epsilon$, $\tilde{x}^{N,1} = \min(\frac{1}{2}, 1 - \Delta c - \epsilon)$, and $\tilde{x}^{N,0} = \frac{1}{2}$. As a result, $\tilde{\theta}_c = 1 - \Delta c$ and the speculator-triggered crisis is prevented for $\theta \in [1 - \Delta c, 1)$.*

(ii) *There is an intervention equilibrium in which $\tilde{\vartheta} = 1$, $\tilde{\theta}^1 = 1 - \Delta c$, $\tilde{\theta}^0 = 1$, $\tilde{x}^{A,0} = 1 + \epsilon$,*

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[11]Although our results still remain for all $\Delta c > \frac{1}{2}$, we adopt this inequality to avoid multiplicity possibilities that arise because of specific speculating cost values, as shown in Lemma 3.2. So the multiplicity in Proposition 3.5 has nothing to do with the specific value of the speculating cost.

$\tilde{x}^{A,1} = 1 - \Delta c + \epsilon$, $\tilde{x}^{N,1} = 1 + \epsilon$, and $\tilde{x}^{N,0} = \frac{1}{2}$. *As a result, the authority does not intervene at all and* $\tilde{\theta}_c = 1$.

The authority is capable of deterring the speculator: In both equilibria, the speculator does not attack for $\theta \geq 1 - \Delta c$ after the intervention. Recall that the speculator's action N tells investors that the fundamental is not weak enough for her to attack ($\theta > 1 - \Delta c$). However, the intervention tells investors that the fundamental value θ is not great enough ($\theta < 1$), otherwise it is unnecessary for the authority to intervene. After observing the two signals, if investors coordinate on the former one, we have the equilibrium in (i) and the intervention works; if investors coordinate on the latter signal, we have equilibrium (ii) in which the authority never intervenes, and even if she does, it cannot prevent any about-to-happen crisis triggered by the speculator. Because of the multiplicity problem, it is unclear whether or not an intervention that increases the speculator's attacking cost can help reduce the economy's vulnerability.

3.4.2 Rewarding Holding Investors

Another form of intervention considered here, is to promise the holding investors a reward if the status quo eventually remains. Denote by $\Delta w > 0$ the magnitude of the reward. If the authority intervenes and the crisis does happen in the end, a holding investor's payoff changes to $\theta - \ell + \Delta w$. Payoffs for all other players still remain the same as before. Because the crisis happens only when enough investors decide to withdraw, the authority may want to make action H more compelling by providing a reward to holding investors. Although this plan may sound effective at first sight, we argue in the proposition below that it does not work at all, no matter how large the reward is.

Proposition 3.6. *In a game with settings* $\tilde{Z}(\theta, \ell, \gamma) = \mathbb{1}_{\theta < \ell}$, $H(\theta, \ell, \gamma) = \ell - \theta$, *and* $I(\theta, \ell, \gamma) = \theta - \ell + \Delta w \cdot \mathbb{1}_{\gamma=1} \cdot (1 - \tilde{Z})$, *where* $\Delta w > 0$, *there is a unique intervention equilibrium in which* $\tilde{\vartheta} = 1$, $\tilde{\theta}^1 = \tilde{\theta}^0 = 1$, $\tilde{x}^{A,0} = \tilde{x}^{A,1} = 1 + \epsilon$, *and* $\tilde{x}^{N,1} = \tilde{x}^{N,0} = \frac{1}{2}$. *As a*

result, $\tilde{\theta}_c = 1$ and all investors and the speculator behave as if there were no authority in the model.

If a crisis is about to happen and the authority promises a reward to holding investors only if they help prevent the crisis, this intervention cannot be helpful. Because a single investor cannot change the result, and given that all others are going to withdraw, an investor figures out that the reward could never be realized, and still withdraws regardless of the magnitude of the reward. Although the reward is only realized if the crisis does not happen in our context, it can also depict the scenarios where the authority promises a reward to holding investors no matter what the result is, but investors believe that the authority is either unable or unwilling to do so if the crisis eventually happens, and thus consider the promise a conditional one as in our model.

Note that the uniqueness stems from requirements (3.10) and (3.11). Without these selection rules, there may still exist other equilibria in which the intervention can still work in certain cases. However, the two rules screen out these equilibria because they are believed to be less likely to happen: They either require a destruction of the trust between investors and the speculator, or demand a less aggressive speculator. But we are specifically interested in the question whether or not and how the authority can prevent a crisis, given an aggressive speculator trusted by all investors. This argument also applies to the results of the subsequent subsection.

3.4.3   Eliminating Preemption Motives

We now check the efficacy of another form of intervention which tries to prevent a crisis by eliminating preemption motives among the investors. Recall that although the reward policy above aims to makes it more attractive for all investors to hold in order to get the reward, the preemption motives among investors impede it from doing so: If an investor chooses to hold in stage 2, she must be afraid of the result that a great portion of other investors withdraw

and make the asset portfolio less profitable or even insolvent in stage 3. To avoid this risk, she withdraws her investment in stage 2 and gets 0 payoff for sure. The authority may want to stabilize the status quo by eliminating the preemption motives among investors. To do so, the authority can promise the investors that, even if some investors withdraw, the authority can buy out their shares so there is no need to liquidize any immature investments and the profitability in stage 3 is not affected.

We denote by $\Delta p \leq \frac{1}{2}$ the magnitude of the intervention policy.[12] If $\ell \leq \Delta p$, i.e., the whole withdrawing mass is covered by the policy, a holding investor gets payoff θ in stage 3. If $\ell > \Delta p$, i.e., the policy can cover only a part of the withdrawing mass, the authority has to liquidize some immature investments and the profitability is harmed. In this case a holding investor gets payoff $\theta - (\ell - \Delta p)$. With an intervention, the crisis happens if the mass of uncovered withdrawing investors exceeds the fundamental, i.e., $\tilde{Z} = 1$ if and only if $\ell - \Delta p > \theta$. The speculator's payoff for attacking, initially $\ell - \theta$, also changes accordingly after intervention, with ℓ replaced by the mass of uncovered withdrawing mass, $\ell - \Delta p$.

Proposition 3.7. *In a game with settings $\tilde{Z}(\theta, \ell, \gamma) = \mathbb{1}_{\theta < \ell} \cdot \mathbb{1}_{\gamma=0} + \mathbb{1}_{\theta < \ell - \Delta p} \cdot \mathbb{1}_{\gamma=1}$, $H(\theta, \ell, \gamma) = \ell \cdot \mathbb{1}_{\gamma=0} + (\ell - \Delta p) \cdot \mathbb{1}_{\gamma=1} \cdot \mathbb{1}_{\ell > \Delta p} - \theta$, and $I(\theta, \ell, \gamma) = \theta - \ell \cdot \mathbb{1}_{\gamma=0} - (\ell - \Delta p) \cdot \mathbb{1}_{\gamma=1} \cdot \mathbb{1}_{\ell > \Delta p}$, where $\Delta p \in (0, \frac{1}{2}]$, there exists a unique intervention equilibrium in which $\tilde{\vartheta} = 1 - \Delta p$, $\tilde{\theta}^1 = 1 - \Delta p$, $\tilde{\theta}^0 = 1$, $\tilde{x}^{A,0} = 1 + \epsilon$, $\tilde{x}^{A,1} = 1 - \Delta p + \epsilon$, $\tilde{x}^{N,1} = \frac{1}{2} - \Delta p$, and $\tilde{x}^{N,0} = \frac{1}{2}$. As a result, $\tilde{\theta}_c = 1 - \Delta p$ and the speculator-triggered crisis is prevented for $\theta \in [1 - \Delta p, 1)$.*

Because of the intervention, the investments of a subgroup of investors' with mass up to Δp will be covered if they withdraw. Knowing this, investors figure out that the portfolio's profitability in stage 3 will not be harmed at all as long as the withdrawing mass is not too much. For this reason, the preemption motives among investors are curtailed (if not eliminated), and it is less necessary for them to panic. As a result, for $\theta \in [\frac{1}{2}, 1)$, the crisis

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[12]Based on the setting of the authority's payoff, she is interested in preventing a crisis only if the fundamental value is good enough. So there is no need for the authority to impose an intervention with magnitude greater than $\frac{1}{2}$, and it suffices to check the efficacy of an intervention by focusing on $\Delta p \leq \frac{1}{2}$.

cannot happen as long as the magnitude of intervention is great enough ($\Delta p \geq 1 - \theta$).

3.5   1997 Asian Financial Crisis and the IMF

Although our model can apply to many crisis scenarios, the 1997 Asian financial crisis serves as an ideal example to illustrate the main arguments and results in the preceding section: It recorded different intervention policies taken by countries and regions involved, so we can compare their efficacies.

The crisis began in Thailand, with a speculation on the Thai baht which exhausted the foreign currency reserve held by the Thai government and finally led to a collapse of the Baht in July 1997. It then kept spreading over most Asian countries through financial contagions, evolving from an initial financial crisis to a disastrous economic crisis in real sectors. The crisis destroyed both the currencies and the economies of countries such as Thailand, Indonesia, Philippines, Malaysia, and South Korea. From June 1997 to July 1998, the Thai Baht devalued (pegged to U.S. dollar) by 40.2%, the devaluation for the Indonesian Rupiah is 83.2%, the Philippine Peso devalued by 37.4%, the Malaysian Ringgit devalued by 39%, and the South Korean Won devalued by 34.1%. The GNP during the same time period dropped 40% for Thailand, 83.4% for Indonesia, 37.3% for the Philippines, 38.9% for Malaysia, and 34.2% for South Korea. Eventually, the nominal GDP of the whole ASEAN (Association of Southeast Asian Nations) fell by 31.7% in 1998. Other countries and regions, such as Hong Kong, Laos, Taiwan, Singapore, and China, were also affected by the crisis, although with smaller damages.[13]

Although there are a variety of explanations of the crisis which emphasize different attributes

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[13]For detailed depictions of the crisis, see Radelet and Sachs (2000), Cheetham (1998), and Goldstein (1998).

of countries in the region,[14] it is widely accepted that all these countries had something—highly leveraged debts—in common, and this made their economies extremely vulnerable to financial shocks. The economic take-off of these countries relied heavily on investments, which were intensively financed by inflow of foreign capitals in the form of debts and equities.[15] To facilitate foreign borrowings, these authorities pegged their currencies to U.S. dollar (or a basket of currencies that mainly consists of U.S. dollar) at fixed exchange rates, in order to reduce foreign creditors' risks from fluctuations in the exchange market. Given their extraordinary economic growth in the 1980s, foreign creditors formed solid confidence in these countries. Combined with the relatively high rate of returns in these regions, east Asia became the most attractive place for investments. As of 1997, almost half of the total capital inflow into developing countries went to Asia every year.

But problems came thereafter. Because it was always easy to borrow money from overseas, the companies and banks lost their caution of potential risks. They became more inclined to invest in high-profit but high-risk projects and loans, and once the investments failed, they could cover the loss by issuing new debts and investing even more adventurously. This scheme made the size of foreign debts held by them considerably large over years, and it worked because there were overheating sectors, such as the real estate and the stock markets, which could support the high rate of returns while issuing new debts. But the accumulation of debts made the companies and banks very vulnerable to devaluations of the local currencies. Because they borrowed and had to pay back the debts in U.S. dollars, if there was a sharp devaluation of the local currency, they would become insolvent and bankrupt immediately, no matter how sound their investments and loans were. The possibility of a devaluation emerged: There had been deficits in the current accounts of the ASEAN countries since

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[14]Hughes (1999) points out the important role the crony capitalism played in Malaysia. Some economists, such as Moreno (1998) and Palma (2002), believe that the devaluation of Chinese Renminbi and the Japanese Yen, together with rising interest rates in U.S., deteriorated the current accounts of the ASEAN countries and caused outflow of capitals from them. Feenstra et al. (2002) attribute the crisis in South korea to the chaebol politics.

[15]The case is different for China. Almost all of China's foreign investments at that time were in the form of factories on ground, which were rather different to withdraw in a short time, this partly explains why the crisis could hardly spread to China.

mid-1990s. The foreign exchange reserves held by them, although still keeping increasing, were insufficient to protect the local currencies from devaluation pressures.

Now we have scenarios here similar to that depicted in our models above. The authority here is the government whose goal is to maintain the profitability of the companies and banks in the country or region, investors here are the equity holders (both foreign and domestic) of the investments. If an investor withdraws, she can sell her shares and change the local currencies to U.S. dollars at the current exchange rate. Furthermore, the withdrawing investors harm holding investors' payoffs in two ways: The withdrawals cause both a pressure on liquidating immature investments which decreases the authority's profitability, and a devaluation pressure on the local currencies which reduces the remaining investors' revenue in real terms.

Although a crisis could still happen without the existence of a speculator if enough investors realized the risk in the exchange rate and withdrew their investments, it is usually hard for them to coordinate to make a significant withdrawing mass. The benchmark model tells us that the status quo could still hold as long as the fundamental was not too bad, and this preserved the possibility and gave opportunity for the authority to make structural changes in the economy to achieve a "soft landing". But as shown above, the existence of a speculator makes the economy more vulnerable, and the possibility of a "soft landing" is perished by the speculator-triggered crisis even for some "fairly good" fundamentals.[16] George Soros, with his Soros Fund Management, is widely believed as the speculator who triggered the Asian financial crisis.[17]

The authorities thus had to intervene in the market in order to counteract the specula-

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[16]Strictly speaking, the scenario here is slightly different from our theoretical model, in the sense that the speculator here can also affect the profitability of the economy in a *direct* way by imposing a devaluation pressure on the local currency. But the size of the short selling of the local currency or dumping of stocks is relatively small or even negligible compared to the authority's foreign exchange reserve or the economy size. As a result, the speculator makes profits only if it manages to trigger the crisis by causing enough investors to withdraw. For this reason, we argue that our model still fits this case study.

[17]The then prime minister of Malaysia Mahathir bin Mohamad accused Soros of making the crisis, so does Krugman (1999).

tor's impacts. Although these countries and areas had the same foreign debts problem and shared similar fundamentals, different interventions taken by them led to different results, as explained below.

3.5.1 Capital Regulation

Unlike other countries, such as South Korea, Indonesia, and Thailand, which sought help from the IMF, Malaysia reacted to the crisis in a different way—imposing strict capital controls—because of the belief that the crisis was made by Soros.[18] The intervention reintroduced a fixed peg against U.S. dollar at 3.80, put stricter conditions on capital outflows, and abandoned oversea trades in the Malaysian ringgit and other ringgit assets. These rules made it extremely risky and costly for the speculators to keep attacking the local currency. Although the authority successfully deterred the speculators, the intervention did not make the situation any better. After the implementation of the intervention policies in January 1998, Malaysia's economy declined into a serious recession sustaining for many years. Construction, manufacturing, and agricultural sectors all contracted significantly, the GDP plunged 6.2% in the same year. More importantly, even the initial peg turned out to be untenable and devaluations of the local currency continued because of tremendous capital outflows.

Despite many existing works on how and why the capital controls did not work for Malaysia,[19] we present here a novel and intuitive explanation based on our theoretical results above: The authority managed to deter the speculators from attacking, speculators thus could no longer trigger the crisis by informing the investors that the fundamental was really bad. However,

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[18]Soros (1998) defended himself by arguing that his fund was actually a buyer rather than a seller of the ringgit when the crisis started, because of a fear of potential capital regulation policies. However, he also admitted that the short selling was made several months prior. This still makes him a speculator properly defined as in our specific context. Because in the model the speculator only needs to make investors impatient and thus triggers the crisis, it is unnecessary for him to behave as a larger investor to push the status quo down.

[19]See Kaplan and Rodrik (2002), Johnson and Mitton (2003).

the intervention policy did so: Investors treated the authority's intervention as a signal that the fundamental was so bad that the authority could not bear attacking on the ringgit any more. Losing confidence in the profitability of their investments, investors withdrew as soon as possible, otherwise their payoffs became lower because other withdrawing investors would add to the pressure on the current peg and further harmed the profitability. Massive withdrawals thus happened because of the preemption motives among investors. Finally, the intervention did not work because investors were coordinating according to the second equilibrium of Proposition 3.5, but the authority falsely believed that the investors would respond to the intervention as in the first equilibrium of Proposition 3.5.

3.5.2 Interest Rate Defense

Most countries and regions involved in the crisis raised their interest rates as a response. This is especially the case for the Philippines and Indonesia. The Philippine central bank raised interest rates by 1.75% in May 1997 and by 2% again in June 1997. Its overnight rate even reached 32% (from 15%) in mid July 1997 after the crisis originated in Thailand. Indonesia also increased the overnight rate to as high as 65% in 1998.

The logic of raising interest rates stems from a traditional instrument, the interest rate defense. Authorities using this instrument usually want to attract more capital inflow and enhance its economic fundamental by maintaining relatively high interest rates. A higher interest rate makes investing in the country more attractive as well as conveys a confidence in the economy. This explains why high interest rates were suggested when the countries went to the IMF for help.[20] However, the capital flight kept happening, and the interest rate defense policies did not work.[21]

Different from works that criticize interest rate defense policies by emphasizing the trade-

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[20]See Fischer (2002) for instance.

[21]Baig and Goldfajn (2002) conclude that there is no empirical correlation between the high interest rates and changes in exchange rates, for five countries during the crisis.

off between strengthening the fundamental and contracting the economy, and thus question whether it is worth imposing the policies,[22] we argue here that, in these specific situations, interest defense policies have *no merit* at all: An interest rate defense policy essentially promises holding investors a reward which could be realized only if the local currency did not collapse eventually. But as shown in Proposition 3.6, given the strategies in the unique intervention equilibrium, the speculator still attacks as if there were no intervention, and investors infer that the reward is unrealizable and thus coordinate on the speculator's action by withdrawing their investments.

The conventional two merits of interest rate defense policies—attracting capitals and conveying confidence in the economy—both disappear when a crisis is about to happen. For the former one, how could an investor put investments into a country whose future is worried about by the whole world? For the latter, the intervention could not convey any confidence at all under this extreme circumstance. What it does convey is the message that the fundamental is too bad that the authority has to intervene. Those who believe a higher interest rate in this case conveys confidence to the public may have misunderstood the causality between a higher interest rate and a higher rate of return: It is the higher rate of return that usually leads to a higher interest rate, not the inverse. So if the rate of return itself is high, a higher interest rate will reinforce this belief of investors. However, if all investors have seen high risks in the financial system and their expected profitability for investment is really low, how could a sudden increase in interest rates regain any confidence?[23]

Our argument of the inefficacy of an interest rate defense policy also applies to other urgent scenarios where a crisis is about to happen, such as the *1992 Black Wednesday*.[24]

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[22]See Bensaid and Jeanne (1997), Lahiri and Végh (2003), andFlood and Jeanne (2005) for example.

[23]Kindleberger and Aliber (2011) and Radelet and Sachs (2000) also make similar statements arguing that raising interest rates will not work at all at the onset of a turmoil.

[24]The British government announced a rise in the base interest rate, from 10% to 12%, on 16 September. After several hours,realizing that the previous adjustment did not work, the government raised the interest rate again, to 15%. But the even higher interest rate still could not work. In the same evening, the then Chancellor Norman Lamont announced failure of the status quo, Britain left the ERM (European Exchange Rate Mechanism) and the interest rate went back to 10% on the next day.

3.5.3 Intervention in Hong Kong

Among all countries and regions intensively involved in the crisis, Hong Kong is widely regarded as the only region that successfully kept its currency peg and saved its economy from a disastrous collapse, although it had to pay the price of a recession lasting for several years.[25]

The case of Hong Kong was different from other Southeast Asian countries in that it did not had imbalance problems in its current account. However, the overheating property and equity markets still made the economy vulnerable to speculative attacks and the following massive withdrawals of investments. The attacks mainly happened in August 1998, when international speculators dumped massive stocks in the SEHK (Hong Kong Stock Exchange), hoping to break down both the current currency peg and the stock market by triggering massive withdrawals so they could make tremendous profits through their previously arranged short positions in the futures market. The authority of Hong Kong intervened promptly and resolutely. Starting from 14 August 1998, with the cooperation of several main financial institutes of mainland China, the Hong Kong Monetary Authority (HKMA) entered the stock market and bought all the shares, mainly blue chip ones, dumped by the speculators and panic investors. Meanwhile, HKMA put the Hong Kong dollars it held because of capital outflows back into the financial system, such that the increased liquidity went to companies that were suffering from withdrawal of equities other than stocks. Eventually, by 28 August, the Hang Seng Index even increased by 17.55%, 4% market value of the whole SEHK was held by the authority, and the speculators quitted with terrible losses caused by their short positions in the futures market.

The Hong Kong authority's intervention functioned exactly as a policy eliminating the pre-emption motive among investors as in Proposition 3.7. Its operations in the stock market and the financial system assured any investor that her investment's payoff would not be

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[25] Jao (2001) provides thorough discussions on how the authority intervened and what Hong Kong had suffered during the succeeding recession.

harmed by withdrawals of other investors. So the preemption motives were eliminated and investors did not have to panic. Instead, they regained confidence in the economy and held their investments in it. Although this intervention was then criticized for going against the basic principles of a laissez-faire market, it was later widely accepted that the authority had to impose the extreme policy under the extreme circumstance and it proved effective. The speculators also admitted that they made a mistake at that time.[26]

3.5.4 The IMF's Role

Hong Kong was able to impose such an aggressive intervention mainly because of its sufficient foreign exchange reserves and the support from mainland China. But for other countries and regions that had already been suffering from imbalance of payment problems, they had to seek for help abroad. The major *lender of last resort* in this case, is the International Monetary Fund (IMF).

Four of the five countries most afflicted by the crisis obtained help through signing the IMF programs.[27] The Philippines received a package more than $1 billion on July 18 1997, Thailand got a $17.2 billion standby arrangement on August 20, Indonesia got a $40 billion package on October 31, and South Korea got a $57 billion standby on December 4. None of these programs worked functionally to achieve the previously set objectives. Even after the new letters of intent were signed with Thailand, Korea, and Indonesia in December 1997 and January 1998, the situation continued deteriorating and local currencies had to keep devaluating.

The reason for the inefficacy of the IMF programs lies in the programs themselves. The rescue money did not come for free. As a prerequisite to have the programs approved, these countries had to promise to conform to a list of requirements, which included keeping a

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[26]See, for example, Lanman (19 May 2009) and Nip (05 July 2007).

[27]Malaysia refused to participate in the program because of the reform requirements attached to those programs.

fiscal surplus, allowing bank closures, raising interest rates, tightening credit, etc. There was no doubt that the fundamental reasons for the crisis were over-expansionary policies and irresponsible regulations. But was it appropriate to correct all these mistakes abruptly, in the midst of the crisis? We argue that the contractionary policy requirements embedded in the programs impeded them from functioning.

Note that although the crisis could be triggered by the speculator, once she has released her action as a public signal, the focus of intervention should be given to the investors, because it is actually the investors who cause massive withdrawals and make the crisis happen.[28] As stated in Proposition 3.7, the key is to eliminate the preemption motives among investors. However, the intervention is only half-finished after receiving sufficient rescue money from the IMF. To further eliminate the preemption motives using the rescue money, the authority still needed to assure investors that she would take over the withdrawals. But the policies in the second step could not be carried out under contractionary policy requirements. Sudden bankruptcies of so many banks pushed the investors and depositors of the remaining ones to believe that their banks would be the next one and thus caused the traditional bank runs; tightened credit and higher interest rates made the liquidity even less possible to be pumped to the companies that needed to prove profitability to its remaining investors, and this in turn added to new withdrawals. Eventually, the authorities could not make full use of their rescue money from the IMF, and it is meaningless to borrow enough money just to satisfy the outflow requests in the exchange market while all investor lost their confidence and decided to leave anyway.

The crisis is of course not ignited by the IMF, and those countries could still have failed to eliminate the preemption motives because of their inherent social or political problems even without the contractionary policy requirements implemented in the rescue programs. It, however, still makes much sense to call the Asian financial crisis an "IMF crisis", in that

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[28]As Donald Tsang Yam-Kuen, the then Hong Kong Financial Secretary, said, "We knew also that if we were not able to remove the incentives in the securities market, it would be a futile exercise to fight the speculators off just in the currency market.

the IMF brought sufficient rescue money to those countries but prevented the rescue money from functioning properly to resolve the crisis.

3.6 Concluding Remarks

The existence of a speculator can make a financial system more vulnerable, because of the short selling mechanism. The speculator's action causes a panic among investors who then withdraw their investments; as a result, the crisis happens for some mild or even good fundamentals. We examine different intervention policies which aim to counteract the speculator's effects, so the authority can prevent the crisis from happening for fundamentals that are not so bad, and plans a "soft landing" through enhancing the fundamental in the future. It is found that deterring the speculator may or may not work because of the multiple equilibria, depending on how investors coordinate their plays. Promising a reward to holding investors if the crisis does not happen proves useless, because they can figure out that the reward is unrealizable and disregard it. Eliminating the preemption motives among investors, by covering a proportion of withdrawals, works because the policy makes it no long necessary for investors to worry about other investors' withdrawals. As a result, they do not panic and the status quo remains.

We use our results to explain why different interventions led to different results in 1997 Asian financial crisis. Specifically, we argue that Malaysia's capital controls did not work because investors were in a panic and took the intervention as a signal of weak fundamentals. The interest rate defense policies did not work at all because when a crisis was about to happen, no investor took the reward caused by higher interest rates as realizable. Interest rate defense policies did not convey confidence to investors at all under this extreme circumstance. We use Hong Kong's intervention to illustrate how eliminating the preemption motives worked. It is also pointed out that although the IMF programs provided sufficient rescue money to Thailand, Indonesia, South Korea, and the Philippines, the contractionary policy require-

ments implemented in the programs prevented the authorities from imposing active policies which could eliminate the preemption motives among investors. As a result, those programs did not achieve their scheduled objectives, and the interventions failed.

Although the application of our theoretical results in this work is based on the 1997 Asian financial crisis, our model could fit and explain a much wider range of similar scenarios, such as the 1992 Black Wednesday, the 2008 Troubled Asset Relief Program (TARP), the crashes of Chinese companies triggered by the Muddy Waters Research, etc.

3.7 Appendix: Proofs

Proof of Proposition 3.1.

For an investor i with signal x_i, her posterior distribution of θ becomes a uniform distribution on interval $(x_i - \epsilon, x_i + \epsilon) \cap (-M, M)$. And recall $M \gg 1 + 2\epsilon$, so for $x_i \in [1, 1+\epsilon]$, we have the expected value of θ as $E(\theta|x_i) = x_i \geq 1$. And for $x_i > 1 + \epsilon$, $E(\theta|x_i) \geq \frac{x_i - \epsilon + M}{2} > 1 + \epsilon > 1$. Because the expected payoff for an investor is $E(\theta - \ell)$, $\ell \leq 1$ always holds, and an investor holds her money if two actions give the same payoff, we have that any investor must hold if her signal is greater than or equal to 1, regardless of other investors' actions. Analogously, $E(\theta|x_i) < 0 \leq \ell$ for $x_i < 0$. So any investor should withdraw if her signal is lower than 0, regardless of other investors' actions. Define $\bar{x}(1) = 1$ and $\underline{x}(1) = 0$, we have our first dominance regions for the two actions: Any strategy s_1 that prescribes action W for $x_i \geq \bar{x}(1)$ is strictly dominated. To see this, just consider another strategy s_2 which prescribes action H for $x_i \geq 1$ and prescribes the same actions for $x_i < 1$ as in s_1. It is then clear that s_2 strictly dominates s_1. Analogously, all strategies which prescribe action H for $x_i < \underline{x}(1)$ are also dominated. We have illustrated so far the first round of elimination of dominated strategies.

Because the noises and payoffs are common knowledge, every investor can exclude the strategies which are dominated in the first round into consideration, and this also becomes common knowledge. We can then move on to subsequent rounds of elimination of dominated strategies. Recall that for any $x_i \in (0, 1)$, the posterior distribution for θ is the uniform distribution $U(x_i - \epsilon, x_i + \epsilon)$. Base on her private signal, investor i forms distribution of x_j, the private signal observed by other investors, as a triangular distribution:

$$\begin{cases} f_{x_j}(x|x_i) = \frac{x-x_i+2\epsilon}{4\epsilon^2}, \ F_{x_j}(x|x_i) = \frac{(x-x_i+2\epsilon)^2}{8\epsilon^2} & for \ x \in (x_i - 2\epsilon, x_i], \\[2mm] f_{x_j}(x|x_i) = \frac{x_i+2\epsilon-x}{4\epsilon^2}, \ F_{x_j}(x|x_i) = 1 - \frac{(x-x_i-2\epsilon)^2}{8\epsilon^2} & for \ x \in (x_i, x_i + 2\epsilon), \\[2mm] f_{x_j}(x|x_i) = 0, F_{x_j}(x|x_i) = 0 & for \ x \leq x_i - 2\epsilon, \\[2mm] f_{x_j}(x|x_i) = 0, F_{x_j}(x|x_i) = 1 & for \ x \geq x_i + 2\epsilon, \end{cases} \tag{3.12}$$

where $f_{x_j}(\cdot|x_i)$ and $F_{x_j}(\cdot|x_i)$ are the probability density function (PDF) and the cumulative distribution function (CDF) of x_j, conditional on x_i, respectively. For $n > 1$, denote $\bar{x}(n)$ as the unique solution to equation $F_{x_j}(\bar{x}(n-1)|\bar{x}(n)) = \bar{x}(n)$, we have that inequalities $\frac{1}{2} \leq \bar{x}(n) \leq \bar{x}(n-1)$ always hold, and strictly hold if $\bar{x}(n-1) \neq \frac{1}{2}$. Knowing that all other investors will surely choose to hold if their signals are above $\bar{x}(n-1)$, an investor with signal x_i can induce that the mass of withdrawing investors is at most $F_{x_j}(\bar{x}(n-1)|x_i)$. And for $x_i > \bar{x}(n)$, we have $E(\theta - \ell|x_i) \geq x_i - F_{x_j}(\bar{x}(n-1)|x_i) > \bar{x}(n) - F_{x_j}(\bar{x}(n-1)|\bar{x}(n)) = 0$. So all strategies which prescribe action W to signals greater than or equal to $\bar{x}(n)$ is dominated and thus eliminated in nth round. Similarly, for $n > 1$, we define $\underline{x}(n)$ as the unique solution to equation $F_{x_j}(\underline{x}(n-1)|\underline{x}(n)) = \underline{x}(n)$. We also have $\underline{x}(n-1) \leq \underline{x}(n) \leq \frac{1}{2}$, and the equalities are strict for $\underline{x}(n-1) \neq \frac{1}{2}$. Again, for an investor with signal x_i, the mass of withdrawing investors is at least $F_{x_j}(\underline{x}(n-1)|x_i)$. Then for any $x_i < \underline{x}(n)$, we have $E(\theta - \ell|x_i) \leq x_i - F_{x_j}(\underline{x}(n-1)|x_i) < \underline{x}(n) - F_{x_j}(\underline{x}(n-1)|\underline{x}(n)) = 0$. Any strategy that prescribes action H to signals lower than $\underline{x}(n)$ is strictly dominated and thus eliminated in the nth round of elimination of dominated strategies.

The sequence $\{\bar{x}(1), \bar{x}(2), \bar{x}(3), \cdots\}$ is decreasing and sequence $\{\underline{x}(1), \underline{x}(2), \underline{x}(3), \cdots\}$ is increasing, and they both converge to $\frac{1}{2}$. Finally, all strategies prescribing action H to any $x_i < \frac{1}{2}$ or prescribing action W to any $x_i \geq \frac{1}{2}$ are eliminated as dominated strategies. So the only strategy surviving iterative elimination of dominated strategies is to withdraw for $x_i < \frac{1}{2}$ and to hold for $x_i \geq \frac{1}{2}$ foe each investor. To see this strategy indeed forms an equilibrium profile, it suffices to check the expected payoff of action H, given signal x_i and the

fact that all other investors are using the strategy above, $E(\theta - \ell | x_i) = x_i - F_{x_j}\left(\frac{1}{2} | x_i\right)$. It is straightforward to see that the expected payoff of H is below 0 if and only if $x_i < \frac{1}{2}$. So there exists a unique equilibrium, in which all investors adopt the strategy surviving iterative elimination of dominated strategies. Given this equilibrium, it is then straightforward to see that the crisis happens $(\theta - \ell < 0)$ if and only if $\theta < \frac{1}{2}$. $\qquad\square$

Proof of Lemma 3.1.

To check the first argument, recall that the speculator knows the exact value of θ. Her payoff is given by $\ell - \theta - c$, and $\ell \in [0, 1]$ always holds. Then for any $\theta > 1 - c$, it is dominant for the speculator to choose action N since it gives payoff 0 whereas action A leads to negative payoff. If we have $\hat{\theta} > 1 - c$, then according to the definition of a threshold equilibrium, the speculator should choose to attack as long as she observes $\theta \in (1 - c, \hat{\theta})$. But we have already shown that A is strictly dominated by N, contradiction. We thus have $\hat{\theta} \leq 1 - c$.

Then assume $\hat{x}^A < \frac{1}{2}$ in an equilibrium. Consider private signal $x_i = \hat{x}^A$, if $B_\epsilon(\hat{x}^A) \cap v^{-1}(A) = \emptyset$, we have that the probability density $\pi_\theta(\gamma | \hat{x}^A, A)$ equals $\frac{1}{2\epsilon}$ if $\gamma \in B_\epsilon(\hat{x}^A)$ and equals 0 otherwise. So the expected value for θ is $E(\theta | \hat{x}^A, A) = \hat{x}^A < \frac{1}{2}$, but from the proof above we have that the distribution of other investors' signals, x_j, is a triangular distribution, and $E(\ell(\theta, A) | \hat{x}^A, A) = E_{\pi_\theta(\cdot | \hat{x}^A)} \ell(\theta, A) = F_{x_j}(\hat{x}^A | \hat{x}^A, A) = \frac{1}{2}$. Then $E(\theta - \ell(\theta, A) | \hat{x}^A, A) < 0$ suggests that an investor with private signal $x_i = \hat{x}^A$ and public signal A should choose action W, which contradicts the definition of a threshold equilibrium with threshold $\hat{x}^A < \frac{1}{2}$. Now consider $B_\epsilon(\hat{x}^A) \cap v^{-1}(A) \neq \emptyset$. Recall that in a threshold equilibrium, $v^{-1}(A) = (-M, \hat{\theta})$, this gives us $B_\epsilon(\hat{x}^A) \cap v^{-1}(A) = (\hat{x}^A - \epsilon, \min(\hat{x}^A + \epsilon, \hat{\theta}))$, $E(\theta | \hat{x}^A, A) = \frac{\hat{x}^A - \epsilon + \min(\hat{x}^A + \epsilon, \hat{\theta})}{2} < \hat{x}^A < \frac{1}{2}$, and $E(\ell(\theta, A) | \hat{x}^A, A) > \frac{1}{2}$. Again, we have that in this case, an investor with signals $x_i = \hat{x}^A$ and A should choose to withdraw, rather than to hold as prescribed by the equilibrium with threshold $\hat{x}^A$, contradiction. We have finished the proof of $\hat{x}^A \geq \frac{1}{2}$.

Analogously, suppose there exists an equilibrium in which $\hat{x}^N > \frac{1}{2}$. Recall that according

to the definition of a threshold equilibrium, we have $v^{-1}(N) = [\hat{\theta}, M)$. Then we have $E(\theta - \ell(\theta, N)|\hat{x}^N, N) > 0$ always holds, and this inequality suggests an investor with $x_i = \hat{x}^N$ should hold. But according to the definition of a threshold equilibrium, an investor with signals $x_i = \hat{x}^N$ and N should choose to withdraw, contradiction. So we must have $\hat{x}^N \le \frac{1}{2}$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ □

Proof of Lemma 3.2.

We prove (i) first. Since action N always gives 0 payoff to the speculator, no matter how the investors behave, we have that $\hat{\theta}$ only depends on $\hat{x}^A$, and first consider the investors' equilibrium strategies after observing A. With $c \le \frac{1}{2} + \epsilon$, we have $1 - c + \epsilon \ge \frac{1}{2}$. And we must have $\hat{x}^A \le 1 - c + \epsilon$ in any threshold equilibrium. To see this, just assume we have in some equilibrium $\hat{x}^A > 1 - c + \epsilon \ge \frac{1}{2}$. As stated in Lemma 3.1, we always have $\hat{\theta} \le 1 - c$, recall that $v^{-1}(A) = (-M, \hat{\theta})$ as defined in the threshold equilibrium, so with we must have $B_\epsilon(\hat{x}^A) \cap v^{-1}(A) = \emptyset$, and the posterior distribution of θ, based on $x_i = \hat{x}^A$, is defined by $\pi_\theta(\gamma|\hat{x}^A, A) = \frac{1}{2\epsilon}$ for $\gamma \in B_\epsilon(\hat{x}^A)$. We thus have $E(\theta|\hat{x}^A, A) = \hat{x}^A \ge \frac{1}{2}$, $E(\ell(\theta, A)|\hat{x}^A, A) = \frac{1}{2}$, and $E(\theta - \ell(\theta, A)|\hat{x}^A, A) > 0$. Note that $B_\epsilon(x) \cap v^{-1}(A) = \emptyset$ holds true for any $x \in (1 - c + \epsilon, \hat{x}^A]$, and $E(\theta - \ell(\theta, A)|x, A)$ is continuous in x on this region, we must have some $x < \hat{x}^A$ such that $E(\theta - \ell(\theta, A)|x, A) > 0$ holds. Which means that given other players are following the strategies prescribed by the equilibrium, it is optimal for an investor with private signal x so hold, but according to the definition of a threshold equilibrium any investor with private signal lower than $\hat{x}^A$ should withdraw, contradiction. So we must have $\hat{x}^A \le 1 - c + \epsilon$. Combining with Lemma 3.1, we have that $\hat{x}^A \in [\frac{1}{2}, 1 - c + \epsilon]$ holds in any equilibrium.

Then for any $x \in [\frac{1}{2}, 1 - c + \epsilon]$, denote by $\theta^*(x)$ the threshold of the best response of the speculator. That is to say, given that all investors are following the threshold strategy $\hat{x}^A = x$, it is optimal for the speculator to attack if and only if $\theta < \theta^*(x)$. Since the speculator knows

the exact value of θ, given the investors' strategy, she can calculate the mass of withdrawing investors for any given θ value. And $\theta^*(x)$ must solve equation $\frac{x-\theta^*(x)+\epsilon}{2\epsilon} - \theta^*(x) - c = 0$, which gives us $\theta^*(x) = \frac{x-2\epsilon c+\epsilon}{1+2\epsilon}$. Note that for $x \in [\frac{1}{2}, 1-c+\epsilon]$, we always have $\theta^*(x) < x$ and $\theta^*(x) \leq 1 - c$.

For $c = \frac{1}{2} + \epsilon$, we have $1 - c + \epsilon = \frac{1}{2}$, $\hat{x}^A = \frac{1}{2}$, and $\theta^*(\hat{x}^A) = \frac{1}{2} - \epsilon = 1 - c$.

For $c < \frac{1}{2} + \epsilon$, $\hat{x}^A = 1 - c + \epsilon$ still holds. To see this, assume $\hat{x}^A \in [\frac{1}{2}, 1-c+\epsilon)$. We thus have $v^{-1}(A) = (-M, \theta^*(\hat{x}^A))$, where $\theta^*(\hat{x}^A) < \hat{x}^A$ and $|\theta^*(\hat{x}^A) - \hat{x}^A| = \frac{2\epsilon(c+\hat{x}^A)-\epsilon}{1+2\epsilon} < \epsilon$ for $\hat{x}^A < 1 - c + \epsilon$. Then there must exist some $\Delta > 0$, such that for all $x \in B_\Delta(\hat{x}^A)$, $B_\epsilon(x) \cap v^{-1}(A) \neq \emptyset$ holds. $\pi_\theta(\cdot|x, A)$ is thus defined as $\frac{1}{m(B_\epsilon(x)\cap v^{-1}(A))}$ on $B_\epsilon(x) \cap v^{-1}(A)$ and 0 otherwise, and $B_\epsilon(x) \cap v^{-1}(A) = (x - \epsilon, \theta^*(\hat{x}^A))$ in this case. Then we can calculator the posterior distribution of θ and thus the distribution of other investors signals x_j for an investor with private signal $x \in B_\Delta(\hat{x}^A)$ and get $E(\theta|x, A) = \frac{x-\epsilon+\theta^*(\hat{x}^A)}{2}$ and $E(\ell(\theta, A)|x, A) = F_{x_j}(\hat{x}^A|x, A) = Tri(\hat{x}^A|x - 2\epsilon, \theta^*(\hat{x}^A) + \epsilon, E(\theta|x, A)) = 1 - \frac{(\theta^*(\hat{x}^A)+\epsilon-\hat{x}^A)^2}{(\theta^*(\hat{x}^A)+\epsilon-x+2\epsilon)(\theta^*(\hat{x}^A)+\epsilon-E(\theta|x,A))}$, where $Tri(\cdot|a, b, c)$ denotes the CDF of a triangular distribution with parameters a, b, and c. Note that $E(\theta|x, a)$ is strictly increasing in x and $E(\ell(\theta, A)|x, a)$ is strictly decreasing in x for $x \in [\frac{1}{2}, 1-c+\epsilon)$. What's more, they are both continuous in x on this set. For this reason, if there exists an equilibrium with $\hat{x}^A \in [\frac{1}{2}, 1-c+\epsilon)$, we must have $E(\theta - \ell(\theta, A)|\hat{x}^A, A) = 0$, otherwise there exists some $x < \hat{x}^A$ such that $E(\theta - \ell(\theta, A)|x, A) > 0$, but according to the definition of an equilibrium, an investor with a private signal lower that $\hat{x}^A$ must choose to withdraw. So $\hat{x}^A$ must be the solution to equation

$$1 - \frac{2(1 + \epsilon - x - c)^2}{(2 + 3\epsilon - x - c)^2} = \frac{(1 + \epsilon)x - \epsilon c - \epsilon^2}{1 + 2\epsilon}. \tag{3.13}$$

Now define $k = 1 - c + \epsilon - x$, for $x \in [\frac{1}{2}, 1-c+\epsilon)$ and $c \in [0, \frac{1}{2}+\epsilon)$, we have $k \in (0, \frac{1}{2}+\epsilon-c)$. Then the equation is transformed to

$$\frac{2k^2}{(k + 1 + 2\epsilon)^2} = \frac{(1 + \epsilon)k + c(1 + 2\epsilon)}{1 + 2\epsilon}, \tag{3.14}$$

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which is then equivalent to

$$(1 + \epsilon)k^3 + (1 + 2\epsilon)(c + 2\epsilon)k^2 + (1 + 2\epsilon)^2(2c + 1 + \epsilon)k + (1 + 2\epsilon)^3 c = 0. \qquad (3.15)$$

It is then obvious to see that (3.15) admits no solutions for $k > 0$, contradicting the assumption that there exists some threshold equilibrium with $\hat{x}^A < 1 - c + \epsilon$.

We have shown that for $c \leq \frac{1}{2} + \epsilon$, in any threshold equilibrium, we must have $\hat{x}^A = 1 - c + \epsilon$. As a result, $\hat{\theta} = \theta^*(\hat{x}^A) = 1 - c$. To see they together actually could be part of an equilibrium, it suffices to check that $E(\theta - \ell(\theta, A)|x, A) = x - \frac{1}{2} \geq 0$ holds if $x \geq \hat{x}^A$, and we have $E(\theta - \ell(\theta, A)|x, A) < 1 - c - 1 \leq 0$ for $x < \hat{x}^A$. given $\hat{x}^A = 1 - c + \epsilon$ and $\hat{\theta} = 1 - c$. Also, it is straightforward to see that, given the investors' strategy threshold $\hat{x}^A = 1 - c + \epsilon$, it is optimal for the speculator to attack if and only if $\theta < 1 - c$.

Now we move on to $\hat{x}^N$. We have already proved $\hat{\theta} = 1 - c$, which gives us $v^{-1}(N) = [\hat{\theta}, M) = [1 - c, M)$.

If $0 \leq c \leq \frac{1}{2} - \epsilon$, we have $1 - c \geq \frac{1}{2} + \epsilon$. Recall that $\hat{x}^N \leq \frac{1}{2}$ must hold as stated in Lemma 3.1. And for any $x \leq \frac{1}{2}$, $B_\epsilon(x) \cap v^{-1}(N) = \emptyset$ holds. Suppose $\hat{x}^N < \frac{1}{2}$, then because of the continuity of an investor's payoff function, there exists some $x > \hat{x}^N$ such that $E(\theta - \ell(\theta, N)|x, N) < 0$, contradiction. So the only case left is $\hat{x}^N = \frac{1}{2}$. To see that this threshold could indeed be part of an equilibrium, just recall that $E(\theta - \ell|x, N)$ is strictly increasing in x and equals 0 for $x = \frac{1}{2}$, given $\hat{x}^N = \frac{1}{2}$.

If $\frac{1}{2} - \epsilon < c \leq \frac{1}{2} + \epsilon$, we have $1 - c - \epsilon < \frac{1}{2}$. $\hat{x}^N < 1 - c - \epsilon$ cannot happen, otherwise $E(\theta - \ell(\theta, N)|\hat{x}^N, N) = \hat{x}^A - \frac{1}{2} < 0$ and because of continuity there exists some $x > \hat{x}^N$ such that $E(\theta - \ell(\theta, N)|x, N) < 0$, contradiction. Combining with Lemma 3.1, we get $\hat{x}^N \in [1 - c - \epsilon, \frac{1}{2}]$. Note that if $\hat{x}^N = \frac{1}{2}$, then $B_\epsilon(\frac{1}{2}) \cap v^{-1}(N) = (1 - c, \frac{1}{2} + \epsilon)$. So $E(\theta|\frac{1}{2}, N) = \frac{1 - c + \frac{1}{2} + \epsilon}{2} > \frac{1}{2}$, and $E(\ell(\theta, N)|\frac{1}{2}, N) = F_{x_j}(\frac{1}{2}|\frac{1}{2}, N) < \frac{1}{2}$. Because of continuity and monotonicity of the payoff function, there must exist some $x > \hat{x}^N = \frac{1}{2}$, such that $E(\theta - \ell(\theta, N)|x, N) < 0$,

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contradiction. So $\hat{x}^N$ cannot be $\frac{1}{2}$ in this case. Now suppose $\hat{x}^N \in (1-c-\epsilon, \frac{1}{2})$. For all $x \in (1-c-\epsilon, \frac{1}{2})$, we have $E(\theta - \ell(\theta, N)|x, N) = \frac{1-c+x+\epsilon}{2} - Tri(\hat{x}^N|1-c-\epsilon, x+2\epsilon, \frac{1-c+x+\epsilon}{2})$. Again, we have $E(\theta - \ell(\theta, N)|x, N)$ is continuous and strictly increasing in $x \in (1-c-\epsilon, \frac{1}{2})$, then we must $E(\theta - \ell(\theta, N)|\hat{x}^N, N) = 0$, which means that $\hat{x}^N$ solves equation

$$\frac{2(x-1+c+\epsilon)^2}{(x-1+c+3\epsilon)^2} = \frac{1-c+x+\epsilon}{2}. \tag{3.16}$$

But the equation above admits no solution for $x \in (1-c-\epsilon, \frac{1}{2})$. To see this, define $R(x) = \frac{2(x-1+c+\epsilon)^2}{(x-1+c+3\epsilon)^2} - \frac{1-c+x+\epsilon}{2}$. It is then easy to see that $R(x)|_{x \to (1-c-\epsilon)} < 0$ and $R(x)|_{x \to \frac{1}{2}} < 0$. Furthermore, we define the critical points x^* as solutions to $R'(x) = 0$, which gives us $\frac{8\epsilon(x^*-1+c+\epsilon)}{(x^*-1+c+3\epsilon)^3} = \frac{1}{2}$, so $R(x^*) = \frac{(x^*-1+c+\epsilon)(x^*-1+c-\epsilon)+8\epsilon(c-1)}{8\epsilon}$. Recall that we have $\epsilon < \frac{1}{2}$ as an assumption throughout the chapter, so $c - 1 \le \frac{1}{2} + \epsilon - 1 < 0$, $x^* - 1 + c - \epsilon < 0$ and $x^* - 1 + c + \epsilon > 0$ both hold if $x^* \in (1-c-\epsilon, \frac{1}{2})$. This tells us that either there is no critical point in this region or there exists some critical point x^* within it but $R(x^*) < 0$. We thus know that $R(x) < 0$ holds for all $x \in (1-c-\epsilon, \frac{1}{2})$ and equality 3.16 is never true solution within this region. Then we must have $\hat{x}^N = 1-c-\epsilon$ in this case. To see that this threshold could indeed constitute an equilibrium, just check that for all $x \le \hat{x}^N = 1-c-\epsilon$ we have $E(\theta - \ell(\theta, N)|x, N) \le x - \frac{1}{2} < 0$, and for all $x > \hat{x}^N$, $E(\theta - \ell(\theta, N)|x, N) > 1 - c - 0 > 0$. Proof of (i) is finished.

Now to prove (ii), (iii), and (iv). Given $c > \frac{1}{2} + \epsilon$, we have $1 - c + \epsilon < \frac{1}{2}$. And recall that $\hat{\theta} \le 1 - c$ and $\hat{x}^A \ge \frac{1}{2}$ must hold in any threshold equilibrium. Then $B_\epsilon(\hat{x}^A) \cap v^{-1}(A) = \emptyset$ is always true. For this reason, it is impossible to have $\hat{x}^A < \frac{1}{2}$, otherwise $E(\theta - \ell(\theta, A)|\hat{x}^A, A) = \hat{x}^A - \frac{1}{2} > 0$, but in the equilibrium an investor with signals $x_i = \hat{x}^A$ and A is suggested to withdraw, contradiction. So we must have $\hat{x}^A = \frac{1}{2}$, based on which $E(\theta - \ell(\theta, A)|x, A)$ equals 0 if $x = \frac{1}{2}$, is negative for $x < \frac{1}{2}$ and positive for $x > \frac{1}{2}$. Given $\hat{x}^A = \frac{1}{2}$ and $1 - c < \frac{1}{2} - \epsilon$, it is optimal for the speculator to attack if and only if $\theta < 1 - c$. To see this, recall that $\hat{\theta} \le 1 - c$ always holds as stated in Lemma 3.1, and given that based on public signal A,

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al investors withdraw if and only if $x_i < \frac{1}{2}$, we have that the payoff of attacking for the speculator, $\ell - \theta - c$, is 0 if $\theta = 1 - c$ and positive if $\theta < 1 - c$. This suggests $\hat{\theta} = 1 - c$.

Then consider $\hat{x}^N$. Recall $v^{-1}(N) = [1 - c, M)$ and $1 - c < \frac{1}{2} - \epsilon$,so $\hat{x}^N = \frac{1}{2}$ can be part of an equilibrium. To see this, just check $B_\epsilon(\frac{1}{2}) \cap v^{-1}(N) = B_\epsilon(\frac{1}{2})$ and $E(\theta - \ell(\theta, N)|x, N)$ is 0 for $x = \hat{x}^N = \frac{1}{2}$, is negative for $x < \frac{1}{2}$ and positive for $x > \frac{1}{2}$.

From Lemma 3.1, $\hat{x}^N$ must hold, we further state $\hat{x}^N \in [1 - c - \epsilon, 1 - c + \epsilon) \cup \{\frac{1}{2}\}$. To see this, assume first $\hat{x}^N \in [1 - c + \epsilon, \frac{1}{2})$. Because $B_\epsilon(x) \cap v^{-1}(N) = B_\epsilon(x)$ for all $x \in [1 - c + \epsilon, \frac{1}{2})$, we have $E(\theta - \ell(\theta, N)|\hat{x}^N, N) = \hat{x}^N - \frac{1}{2} < 0$, then continuity suggests that there exists $x \in [1 - c + \epsilon, \frac{1}{2})$ such that $x > \hat{x}^N$ and $E(\theta - \ell(\theta, N)|x, N) < 0$, contradiction. Analogously, $B_\epsilon(x) \cap v^{-1}(N) = \emptyset$ for all $x < 1 - c - \epsilon$. If we have $\hat{x}^N < 1 - c - \epsilon$, continuity again suggests that there exists $x > \hat{x}^N$ such that $E(\theta - \ell(\theta, N)|x, N) < 0$, contradiction. So we $\hat{x}^N$ either equals $\frac{1}{2}$ or belongs to set $[1 - c - \epsilon, 1 - c + \epsilon)$.

Now consider case $c \geq 1 + \epsilon$, and assume $\hat{x}^N \in [1 - c - \epsilon, 1 - c + \epsilon)$. Then for all $x \in (\hat{x}^N, 1 - c + \epsilon)$, we have $E(\theta|x, N) = \frac{1 - c + x + \epsilon}{2} \leq 0$ and $E(\ell(\theta, N)|x, N) > 0$. This thus suggests an investor with signals $x > \hat{x}^N$ and N to withdraw, contradiction. So with $c \geq 1 + \epsilon$, the unique value of $\hat{x}^N$ is $\frac{1}{2}$. (iv) is proved.

Then for $1 < c < 1 + \epsilon$, assume $\hat{x}^N \in [1 - c - \epsilon, 1 - c + \epsilon)$. $\hat{x}^N$ cannot equal $1 - c - \epsilon$. To see this, just check $E(\theta - \ell(\theta, N)|x, N)|_{x \to (1 - c - \epsilon)^+} = 1 - c - 0 < 0$. Then consider $\hat{x}^N \in (1 - c - \epsilon, 1 - c + \epsilon)$. Again, given $\hat{x}^N$, continuity and strict monotonicity of the payoff function in private signals suggests $E(\theta, \ell(\theta, N)|\hat{x}^N, N) = 0$. Which means that $\hat{x}^N \in (1 - c - \epsilon, 1 - c + \epsilon)$ could be part of an equilibrium if and only if it solves equation

$$R(x) = \frac{1 - c + x + \epsilon}{2} - \frac{2(x - 1 + c + \epsilon)^2}{(x - 1 + c + 3\epsilon)^2} = 0 \tag{3.17}$$

Note that $R(x)|_{x \to (1 - c - \epsilon)^+} < 0$ and $R(x)|_{x \to (1 - c + \epsilon)^-} < 0$. Now for $n \in (-1, 1)$, we define $x = 1 - c + n\epsilon$ for all $x \in (1 - c - \epsilon, 1 - c + \epsilon)$, and get $R(x) = R(1 - c + n\epsilon) = 1 - c + \frac{(n+1)\epsilon}{2} - \frac{2(n+1)^2}{(n+3)^2}$.

So if $c - 1 \le \sup\{\frac{(n+1)\epsilon}{2} - \frac{2(n+1)^2}{(n+3)^2} : n \in (-1, 1)\}$, we have $\sup\{R(x) : x \in (1 - c - \epsilon, 1 - c + \epsilon)\} \ge 0$ and continuity thus suggests that $R(x) = 0$ admits at least a solution for $x \in (1 - c - \epsilon, 1 - c + \epsilon)$, otherwise $R(x) = 0$ has no solution for this region. (iii) is proved.

We finally consider $\hat{x}^N$ for case $\frac{1}{2} + \epsilon < c \le 1$. Recall that $\hat{x}^N \in [1 - c - \epsilon, 1 - c + \epsilon) \cup \{\frac{1}{2}\}$ must hold for all $c > \frac{1}{2} + \epsilon$. $\hat{x}^N = 1 - c - \epsilon$ could be part of an equilibrium in this case. To see this, given this threshold, for all $x \le \hat{x}^N = 1 - c - \epsilon$, we have $E(\theta - \ell(\theta, N)|x, N) \le x - \frac{1}{2} \le 1 - c - \epsilon - \frac{1}{2} < 0$. And $E(\theta - \ell(\theta, N)|x, N) > 1 - c - 0 \ge 0$ holds for all $x > 1 - c - \epsilon$. These two inequalities thus tell us that, given N, it is optimal for an investor to withdraw if $x_i \le 1 - c - \epsilon$ provided all other investors are doing so. Then for $\frac{1}{2} + \epsilon < c \le 1$, $\hat{x}^N$ could hold in an equilibrium. Now suppose $\hat{x}^N \in (1 - c - \epsilon, 1 - c + \epsilon)$, from the logic shown above, we know that this could happen in an equilibrium if and only if $R(x) = 0$ admits solutions for $x \in (1 - c - \epsilon, 1 - c + \epsilon)$. With $\frac{1}{2} + \epsilon < c \le 1$, we have here $R(x)|_{x \to (1-c-\epsilon)^+} > 0$ and $R(x)|_{x \to (1-c+\epsilon)^-} < 0$. Then continuity of function $R(x)$ thus suggests that there exists some $x^* \in (1 - c - \epsilon, 1 - c + \epsilon)$ and x^* solves $R(x) = 0$. Continuity and strict monotonicity of the payoff function for action H makes sure that $\hat{x}^N = x^8$ could be a the strategy threshold in an equilibrium. We have finished the proof of (ii) and thus proved the whole lemma. $\square$

Proof of Proposition 3.2.

It it straightforward to check that, for all $c \ge 0$ and in all equilibria characterized in Lemma 3.2, $\hat{\theta} = 1 - c$ always holds. And given $\theta < \hat{\theta}$, we have $x_i < \hat{x}^A$ for all investors. Then the mass of withdrawing investors must be 1 ($\ell = 1$). This in turn gives us $\theta - \ell < 1 - c - 1 \le 0$, the crisis thus happens. For $c \le \frac{1}{2} + \epsilon$, inequality $\min(\frac{1}{2}, 1 - c - \epsilon) \le \theta - \epsilon$ holds for every $\theta \ge 1 - c$. So $x_i > \hat{x}^N$ holds for each investor i if $\theta > 1 - c$, $\ell = 0$. For the case $\theta = 1 - c$, we still have that $x_i > \hat{x}^N$ is true almost everywhere in the continuum set of investors, $\ell = 0$ still holds. Finally, as long as the speculator does not attack, we must have $\theta \ge 1 - c$, and $\theta - \ell \ge 1 - c - 0 > 0$, so $Z = 0$. $\square$

Proof of Proposition 3.3.

Consider case $c \leq \frac{1}{2} + \epsilon$. Given $\theta < 1 - c$, we have $x_i < \hat{x}^A = 1 - c + \epsilon$ holds for each investor. So $\theta - \ell < 1 - c - 1 \leq 0$ must hold, and the crisis thus happens. Now given $\theta \geq 1 - c$, we have $x_i = \theta + \epsilon_i > \min(\frac{1}{2}, 1 - c - \epsilon)$ holds, so $\theta - \ell \geq 1 - c - 0 > 0$ and $Z = 0$. So $\theta_c = 1 - c$ in this case. For $c \geq 1 + \epsilon$, in the unique equilibrium, investors disregard the speculator's actions and behave the same as in the benchmark model, so $\theta_c = \frac{1}{2}$ holds. For $c \in (\frac{1}{2} + \epsilon, 1 + \epsilon)$, depending on different equilibria the players are behaving according to, the fundamental threshold could also be different. Specifically, θ_c could be either $\frac{1}{2}$, or $1 - c$, or greater than $1 - c$. $\qquad\square$

Proof of Proposition 3.4.

Recall that in the benchmark model, a crisis happens if and only if $\theta < \frac{1}{2}$. But with the existence of a speculator, if $c < \frac{1}{2}$, we have $\theta_c = 1 - c > \frac{1}{2}$. So for $\theta \in [\frac{1}{2}, 1 - c)$, inequality $\theta < \theta_c$ holds and the panic crisis happens. But the crisis is prevented for the same range of θ in the benchmark model. The status quo is more vulnerable after the addition of a speculator in the context.

Similarly, for $c \in (\frac{1}{2}, \frac{1}{2} + \epsilon]$, we have $\theta_c = 1 - c < \frac{1}{2}$. So with $\theta \in [1 - c, \frac{1}{2})$, $\theta \geq \theta_c$ holds and the status quo remains. But the crisis happens for the same range of θ in the benchmark model. The speculator stabilizes the system in this case. For case $c = \frac{1}{2}$, $\theta_c = \frac{1}{2}$, it is equal to the fundamental threshold in the benchmark model, so adding a speculator into the model does not change the result of the game. $\qquad\square$

Proof of Proposition 3.5.

For the first statement. Given $H(\theta, \ell, \gamma) = \ell - \theta - \Delta c \cdot \mathbb{1}_{\gamma=1}$, from condition (3.11) we

have $\tilde{\theta}^1 = 1 - \Delta c$ and $\tilde{\theta}^0 = 1$. To check other requirements, given others' strategies, the authority finds it irrational to intervene for $\theta < 1 - \Delta c$ since the crisis happens anyway for this region; she also finds it profitable to intervene for $\theta \in [1 - \Delta c, 1)$, because otherwise the crisis happens and by intervening she can prevent it. So $\tilde{\vartheta} = 1 - \Delta c$ satisfies condition (3.6). Now consider the speculator. Given $\gamma = 0$, for any $\theta < 1$ the speculator should attack because by doing so she gets positive payoff; given $\gamma = 1$, she does not attack for $\theta \geq 1 - \Delta c$ since this always leads to non-positive payoff, but for $\theta < 1 - \Delta c$ action A leads to positive payoff. So given $\tilde{\theta}^1 = 1 - \Delta c$ and $\tilde{\theta}^0) = 1$, condition (3.7) is satisfied. Now check conditions (3.8), (3.9), and (3.10) together. Signals $(A, 0)$ suggest $\theta < 1 - \Delta c$. Given all other investors following threshold $\tilde{x}^{A,0}$, for $x_i < 1 - \Delta c + \epsilon$, Bayes' rule is applicable, $E_{x_i}(\ell) = 1$, and $E_{x_i}(\theta) < 1 - \Delta c$; for $x_i \in [1 - \Delta c + \epsilon, 1)$, investor i updates her belief either on her private signal only or on both x_i and A, in both cases $E_{x_i}(\ell) = 1$ and $E_{x_i}(\theta) < 1$ hold. And for $x_i \geq 1$, investor i has to rely on her own private signal only, $E_{x_i}(\ell) = 0$, and $E_{x_i}(\theta) > 1$. So given the authority's and the speculator's strategies, if all other investors follow threshold $\tilde{x}^{A,0}$, it is also for any investor to do the same. The three requirements are thus satisfied for $\tilde{x}^{A,0}$. To justify $\tilde{x}^{A,1} = 1 - \Delta c + \epsilon$, note that signals $(A, 1)$ conflict with each other, and the investors rely on their own private signals as well as signal A whenever applicable. So for investor i with $x_i < 1 - \Delta c + \epsilon$, we have $E_{x_i}(\theta) < 1 - \Delta c < 1$, $E_{x_i}(\ell) = 1$, and she withdraws; if $x_i \geq 1 - \Delta + \epsilon$, she takes into account only x_i, $E_{x_i}(\theta) = X_i > \frac{1}{2}$, $E_{x_i}(\ell) \leq \frac{1}{2}$, so she holds. The three conditions are satisfied for $\tilde{x}^{A,1}$. To check $\tilde{x}^{N,1} = \min(\frac{1}{2}, 1 - \Delta c - \epsilon)$, note that signals $(N, 1)$ suggest $\theta \in [1 - \Delta c, 1)$. Given $\tilde{x}^{N,1}$, we have $E_{x_i}(\theta - \ell) \leq x_i - \frac{1}{2} < 0$ for $x_i < \tilde{x}^{N,1}$, $E_{x_i}(\theta - \ell) \geq \frac{1}{2} - \frac{1}{2} = 0$ for $x_i \geq \tilde{x}^{N,1}$ if $\Delta c + \epsilon \leq \frac{1}{2}$, and $E_{x_i}(\theta - \ell) \geq 1 - \Delta c - 0 > 0$ for $x_i \geq \tilde{x}^{N,1}$ if $\Delta c + \epsilon > \frac{1}{2}$. So $\tilde{x}^{N,1}$ passes the three requirements. We then justify $\tilde{x}^{N,0} = \frac{1}{2}$ and finish proof of the first statement. Signals $(N, 0)$ suggest $\theta \geq 1$. If $x_i > 1 - \epsilon$, Bayes' rule is applicable, investor i thus believes $\theta \geq 1$ and chooses to withdraw; if $x_i \leq 1 - \epsilon$, investor i updates her belief using only her private signal. It is straightforward to see that an investor with signals $(N, 0)$ withdraws if and only if $x_i < \frac{1}{2}$, provided all other investors are doing so.

We have shown that the thresholds as in the statement indeed pass all requirements in the definition of an intervention equilibrium. Based on these thresholds, it is obvious to see the crisis happens if and only if $\theta < 1 - \Delta c$. the first statement if proved.

For the second statement, condition (3.11) obviously is satisfied. Given other players' strategies, the crisis always happens for $\theta < 1$, no matter the authority intervenes or not. So $\mu^{-1}(1) = \emptyset$ and $\tilde{\vartheta} = 1$, condition (3.6) is thus satisfied. Given investors' strategies, the speculator's best response is to attack as long as $\theta < 1$ after observing signal $\gamma = 0$ and to attack as long as $\theta < 1 - \Delta c$ after observing $\gamma = 0$, by doing so she maximizes payoff for each θ. $\tilde{\theta}^1$ and $\tilde{\theta}^0$ meet requirement (3.7). Then to check requirements (3.8), (3.9), and (3.10) together for investors. $(A, 0)$ suggest $\theta < 1$, by the same logic in above paragraph, the three conditions get satisfied for $\tilde{x}^{A,0} = 1 + \epsilon$. Now check $\tilde{x}^{A,1} = 1 - \Delta c + \epsilon$, recall that signal $\gamma = 1$ delivers message $\theta \in [\frac{1}{2}, 1)$ in this case. For $x_i \in (\frac{1}{2} - \epsilon, 1 - \Delta c + \epsilon)$, Bayes' rule is applicable and $E_{x_i}(\theta - \ell) < 1 - \Delta c - 1 < 0$, for private signals which make Bayes' rule fails we have $E_{x_i}(\theta - \ell) \leq \frac{1}{2} - \epsilon - 1 < 0$ for $x_i \leq \frac{1}{2} - \epsilon$, and $E_{x_i}(\theta - \ell) \geq 1 - \Delta c + \epsilon - \frac{1}{2} > 0$ for $x_i \geq 1 - \Delta c + \epsilon$. So $\tilde{x}^{A,1}$ passes the three requirements. Now consider $\tilde{x}^{N,1} = 1 + \epsilon$, signals $(N, 1)$ suggest $\theta \in [1 - \Delta c, 1)$. We have $E_{x_i}(\theta - \ell) \geq 1 + \epsilon - \frac{1}{2} > 0$ for $x_i \geq 1 + \epsilon$, $E_{x_i}(\theta - \ell) < 1 - 1 = 0$ for $x_i \in (1 - \Delta c - \epsilon, 1 + \epsilon)$, and $E_{x_i}(\theta - \ell) \leq 1 - \Delta c - \epsilon - 1 < 0$ for $x_i \leq 1 - \Delta c - \epsilon$. $\tilde{x}^{N,1} = 1 + \epsilon$ is thus justified. Finally, we check $\tilde{x}^{N,0} = \frac{1}{2}$. Signals $(N, 0)$ suggest $\theta \geq 1$, and investor i will hold if $x_i > 1 - \epsilon$. For $x_i \leq 1 - \epsilon$, investor i updates her belief using x_i only. Given $\tilde{x}^{N,0} = \frac{1}{2}$, her payoff for holding strictly decreases in x_i and equals 0 at $x_i = \frac{1}{2}$. We have justified $\tilde{x}^{N,0}$ and proved that strategies characterized by the thresholds defined in the statement indeed could constitute an intervention equilibrium. It is then straightforward to see that in this equilibrium the authority never intervenes and the crisis happens if and only if $\theta < 1$. $\qquad\square$

Proof of Proposition 3.6.

To prove it is an intervention equilibrium. Given other players' strategies, a crisis happens anyway as long as $\theta < 1$, it is then optimal for the authority never to intervene. Given investors' strategy, the best response for the speculator is to attack if and only if $\theta < 1$, regardless of the authority's strategy. Requirements (3.6), (3.7), and (3.11) are satisfied. Now for (3.8), (3.9), and (3.10). Signals $(A, 0)$ suggest $\theta < 1$, by the same logic in above cases $\tilde{x}^{A,0}$ passes the three requirements. Signals $(N, 0)$ suggest $\theta \geq 1$, given other investors follow $\tilde{x}^{N,0} = \frac{1}{2}$, investor i's expected payoff for holding decreases in x_i and reaches 0 at $x_i = \frac{1}{2}$, $\tilde{x}^{N,0}$ is thus justified. $(A, 1)$ suggest $\theta < 1$. With $x_i < 1 + \epsilon$, investor i updates her belief such that $\theta < 1$ always holds Then $E_{x_i}(\ell) = 1 > \theta$ always happens and she knows the reward cannot be realized. So investor i withdraws for $x_i < 1 + \epsilon$ in this case. With $x_i \geq 1 + \epsilon$, investor i updates her belief using only her private signal, so $E_{x_i}(\ell) = 0 < \theta$, and $E_{x_i}(\theta - \ell + \Delta w) > 0$. $\tilde{x}^{A,1} = 1 + \epsilon$ is thus justified. Then for $\tilde{x}^{N,1} = \frac{1}{2}$, signals $(N, 1)$ suggests $\theta \geq 1$. If $x_i > 1 - \epsilon$, Bayes's rule is applicable and investor i infers all other investors will hold, so she should also hold to get positive investment return plus the reward. If $x_i \leq 1 - \epsilon$, she updates her belief on x_i, and the expected payoff for holding becomes negative if and only if $x_i < \frac{1}{2}$, $\tilde{x}^{N,1} = \frac{1}{2}$ is justified. So these thresholds indeed constitute an intervention equilibrium.

To show the uniqueness, note that to get requirement (3.11) satisfied, we must have $\tilde{\theta}^1 = \tilde{\theta}^0 = 1$. To have these two thresholds as part of an equilibrium, we must have $\tilde{x}^{A,1} \geq 1 + \epsilon$ and $\tilde{x}^{A,0} \geq 1 + \epsilon$, otherwise the speculator gets negative payoffs for some $\theta < 1$ and it is thus not optimal for her to adopt those two thresholds. Also note that for any investor with $x_i > 1 + \epsilon$, she knows $\theta > 1$ for sure and chooses to hold whatever. So we must have $\tilde{x}^{A,1} = \tilde{x}^{A,0} = 1 + \epsilon$. Given these thresholds, a crisis happens as long as $\theta < 1$, so the authority should never intervene, $\tilde{\vartheta} = 1$. Since $(N, 1)$ and $(N, 0)$ both suggest $\theta \geq 1$, and we only consider threshold strategies for investors, by logic in the proof of Lemma 3.2, it is easy to see that we must have $\tilde{x}^{N,1} = \tilde{x}^{N,0} = \frac{1}{2}$. Uniqueness is shown and the proof of the whole proposition is finished. $\qquad\square$

Proof of Proposition 3.7.

We first show that the thresholds defined in the proposition really constitute an intervention equilibrium. Given $H(\theta, \ell, \gamma)$ as defined in the proposition, condition (3.11) is obviously satisfied. Given other players' strategies, for $\theta \in [1 - \Delta p, 1)$, the crisis does not happen if the authority intervenes, otherwise it happens. For $\theta < 1 - \Delta p$, the speculator attacks anyway, and given $\tilde{x}^{A,0}$ and $\tilde{x}^{A,1}$, $\ell = 1$ and a crisis always happens, so the authority does not intervene in this case. $\tilde{\vartheta} = 1 - \Delta p$ meets requirement (3.6). Now for requirement (3.7), given investors' strategy, if $\gamma = 0$, for $\theta < 1$, the speculator finds it profitable to attack because by doing so she can make positive payoff; for $\theta \geq 1$, attacking always leads to negative payoff. If $\gamma = 1$, the payoff for attacking becomes $(\ell - \Delta p) \cdot \mathbb{1}_{\gamma=1} \cdot \mathbb{1}_{\ell > \Delta p} - \theta$. For $\theta < 1 - \Delta p$, if the speculator attacks, $\tilde{x}^{A,1}$ leads to $\ell = 1$ and $H(\theta, \ell, \gamma) = 1 - \Delta p - \theta > 0$. For $\theta \geq 1 - \Delta p$, if the speculator attacks, $\tilde{x}^{A,1}$ gives $H(\theta, \ell, \gamma) < 1 - \Delta p - \theta \leq 0$. So the best response for the speculator is to follow $\tilde{\theta}^1 = 1 - \Delta p$ and $\tilde{\theta}^0 = 1$, requirement (3.7) gets satisfied. Now to check (3.8), (3.9), and (3.10). For $\tilde{x}^{A,0} = 1 + \epsilon$, signals $(A, 0)$ suggest $\theta < 1 - \Delta p$. Given other investors following threshold $\tilde{x}^{A,0}$, investor i gets $E_{x_i}(I) = E_{x_i}(\theta - \ell) < 1 - 1 = 0$ for $x_i < 1 + \epsilon$, and $E_{x_i}(I) \geq 1 - \frac{1}{2} > 0$ for $x_i \geq 1 + \epsilon$. $\tilde{x}^{A,0}$ thus meet the three requirements. To check $\tilde{x}^{A,1} = 1 - \Delta p + \epsilon$. Signals $(A, 1)$ conflict with each other and the investors only take into account the speculator's action whenever applicable. For investor i, we have $E_{x_i}(I) = E_{x_i}(\theta) - (1 - \Delta p) < 0$ for $x_i < 1 - \Delta p + \epsilon$, and $E_{x_i}(I) > E_{x_i}(\theta) - (\frac{1}{2} - \Delta p) > 0$ for $x_i \geq 1 - \Delta p + \epsilon$. $\tilde{x}^{A,1}$ is thus justified. Now for $\tilde{x}^{N,1}$, recall that $\frac{1}{2} - \Delta p < 1 - \Delta p - \epsilon$ always holds and signals $(N, 1)$ suggest $\theta \in [1 - \Delta p, 1)$. Given other investors following threshold $\tilde{x}^{N,1} = \frac{1}{2} - \Delta p$, investor i gets $E_{x_i}(I) = E_{x_i}(\theta) > 0$ for $x_i > 1 - \Delta p - \epsilon$, $E_{x_i}(I) \geq E_{x_i}(\theta) - (\frac{1}{2} - \Delta p) \geq 0$ for $x_i \in [\frac{1}{2} - \Delta p, 1 - \Delta p + \epsilon)$ and $E_{x_i}(I) < E_{x_i}(\theta) - (\frac{1}{2} - \Delta p) < 0$ for $x_i < \frac{1}{2} - \Delta p$. So $\tilde{x}^{N,1}$ is justified. To justify $\tilde{x}^{N,0} = \frac{1}{2}$, signals $(N, 0)$ suggest $\theta > 1$, it is straightforward to see that an investor should follow threshold $\tilde{x}^{N,0} = \frac{1}{2}$ given others are doing so. Now we have shown conditions (3.8), (3.9), and (3.10) get satisfied. Based on these thresholds, it is

straightforward to see $\tilde{\theta}_c = 1 - \Delta p$.

Now check the uniqueness. Condition (3.11) requires $\tilde{\theta}^1 = 1 - \Delta p$ and $\tilde{\theta}^0 = 1$. To make the speculators' two thresholds part of an equilibrium, we must have $\tilde{x}^{A,1} \geq 1 - \Delta p + \epsilon$ and $\tilde{x}^{A,0} \geq 1 + \epsilon$. But it is strictly dominated for an investor to withdraw for some $x_i > 1 - \Delta p + \epsilon$ given $\gamma = 1$ and for some $x_i > 1 + \epsilon$ given $\gamma = 0$. $\tilde{x}^{A,1} = 1 - \Delta p + \epsilon$ and $\tilde{x}^{A,0} = 1 + \epsilon$ must hold. Without intervention, a crisis happens as long as $\theta < 1$. But given Δp, the uncovered withdrawing mass is at most $1 - \Delta p$, the authority knows for sure that the crisis cannot happen if she intervenes for $\theta > 1 - \Delta p$. This means $\tilde{\vartheta} \leq 1 - \Delta p$. The case $\tilde{\vartheta} < 1 - \Delta p$ cannot happen, otherwise for $\theta \in (\tilde{\vartheta}, \tilde{\theta}^1)$, we must have both the authority prevents a crisis and the speculator gets positive payoff through attacking, contradiction. So $\tilde{\vartheta} = 1 - \Delta p$ must hold. Given the intervention, an investor's payoff for holding is always positive for $\theta > 1 - \Delta p$, so $\tilde{x}^{N,1} \leq 1 - \Delta p - \epsilon$. And for $x_i < 1 - \Delta p - \epsilon$, the investor relies on her private signal only, and only threshold $\tilde{x}^{N,0}$ can survive the intervention equilibrium requirements. For $\tilde{x}^{N,0}$, same logic in previous proofs suggest only $\frac{1}{2}$ survives all requirements. Uniqueness is proved. $\square$

Chapter 4

Dynamic Beauty Contests: Learning from the Winners to Win?

4.1 Introduction

The metaphor "beauty contest", coined by Keynes (1936), refers to circumstances under which higher order beliefs play an important role for individuals. If a group of readers are asked to pick out the most beautiful face from several faces listed in a newspaper, and are told that only individuals who vote for the winning face get rewarded. Then a rational individual should not only care about which face she thinks is the most beautiful, but also take into account the tastes of the other readers, how individuals think of others' tastes, and so on. Keynes argued that the beauty contest resembled the financial market, and used this concept to explain price fluctuations in equity markets.

All kinds of beauty-contest games have been widely used and rigorously investigated.[1] Morris and Shin (2002) formalize the basic ideas in a static game where a continuum of players

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[1] The most well-known form among them, is perhaps the p-beauty contest game, which is mainly used in behavioral and experimental economics, as a tool to identify the depth of individuals' real cognitive power and to question classical assumptions on rationality. The current work does not belong to that line of research.

receive private signals about an underlying fundamental, and for each player there may be a trade-off between being accordant with the fundamental and being accordant with others' actions. They use the model to explain why public information may impede the price competently revealing the fundamental through gathering individuals' actions. That is because the importance of higher order beliefs and the cluelessness of other players' private information together coerce each player to place her action heavily based on the public signal. Eventually, the public signal drives the price away from the fundamental, e.g., a biased public news causes significant price changes (such as bubbles and crashes) which lack supports by the fundamental.[2]

Given that the price could not well reveal the underlying fundamental because of the existence of the public signal in the static model, it is of interest to investigate whether or not the average action of players reveals the fundamental better and better in a dynamic setting. This concern stems from the observation that beauty contest games are usually played repeatedly in reality. What is more, besides the private signals, i.e., private understandings on the underlying fundamental, players also observe previous winners' actions (with noises) as public signals: Investors learn about actions taken by "Buffett" and "Soros" either from mass media or through materials such as biographies and other "investment bible" publications; the previous winners' pictures are also well known to all contestants of a beauty pageant in the current year. Combining both the dynamic and winner-effect concerns depicted above leads to the initial motivation of this work.

Although the winner's action in each stage has to deviate more or less from the fundamental because a player outperforms all others only if she happens to do a better job in taking into account the biases embedded in previous public signals, the more signals of these actions

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[2]As the intuition stated in part II, chapter 12 of Keynes (1936), "It would be foolish ...to attach great weight to matters which are very uncertain. It is reasonable, therefore, to be guided to a considerable degree by the facts about which we feel somewhat confident, even though they may be less decisively relevant to the issue than other facts about which our knowledge is vague and scanty." Applying this argument to the model, with private signals as the uncertain matters and the public signal as the certain matter, explains the result.

observed by players, the better job the public information as a whole does in revealing the fundamental. One may thus presume that players' actions should converge to the fundamental in the dynamic context, because both the private and public information become more and more precise from stage to stage. But as shown in our work, that is not necessarily the case. For specific contexts in which the public signals are relatively precise compared to the private ones, e.g., in beauty pageants, the average actions converge to the fundamental because players are mainly learning the fundamental from the winners' actions and the inclusion of new public signals is beneficial in this process. However, the average actions may diverge from the fundamental in environments where the public signals are relatively noisy compared to the private ones (this is the case in stock markets for instance). The reason is that players are mainly learning about the fundamental from their private signals. Given the importance of higher order beliefs, the inclusion of new but noisy public signals forces players to give more weight to the worse signals while making decisions. The average actions thus diverge from the fundamental as a result. Besides the gap between the average action and the fundamental, we also investigate the gap between a generic player's action and the average action as well as the gap between a generic player's action and the winner's action. None of the latter two gaps are converging for sure in the equilibrium: Once again, the relative precisions of the public and private signals matter.

To illustrate and apply our theoretical results in reality, we construct an empirical study on the Miss Korea pageant between 1984-2013. It is chosen for its suitability to our model as well as data availability.[3] Although there has long been concerns and critiques as to the popularity of plastic surgeries that made all contestants of Miss Korea look alike, this issue first attracted the spotlight of the public in 2013 because of the explosively contagious circulations of photos of some contestants on the internet.[4] Our empirical study attempts to answer two questions

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[3]For instance, although we can have all daily prices for the 4-month lifetime of all NYMEX crude oil futures since 1985, it is impossible to have also the public signals that revealing the winners' actions.

[4]See, http://gawker.com/plastic-surgery-blamed-for-making-all-miss-korea-contes-480907455, for an instance of websites discussing about this issue. There are also prank "Link Up" games on it. See, for example, https://play.google.com/store/apps/details?id=com.xcc.MissKorealink.

raised about these articles and discussions online. The first is that it seemed self-evident to those authors that an option of plastic surgeries would undoubtedly make all contestants' faces look alike. But to our knowledge, such an option only gives contestants the ability to choose perhaps any type of face they want, and thus makes the contestants strategic players. As we discussed above, their equilibrium actions are not assured to converge in any of the three definitions of action gaps. Second, most articles did not notice that those pictures were actually taken for contestants of Miss Daegu, which is just a local level preliminary contest for the final Miss Korea contest.[5] Even if their statements are true locally, it is not necessarily true for the whole country and thus for the final contest. Our empirical study leads to findings somewhat contrary to those viewpoints. Although contestants' faces tended to converge to the underlying "true beauty", they have been getting far away from the average face over years. During the last 20 years, the faces of the Miss Korea contestants have actually become more and more diverse.

Allen et al. (2006) and Amador and Weill (2006) also investigate beauty-contest games in dynamic settings. Our model differs from those in several aspects. In the former work, the authors focus on how higher order beliefs work in an overlapping population setting and how the initial bias is gradually eliminated in the finite-time price calibration process. In contrast, the present work does not take the overlapping population and finite time assumptions that give birth to the bias elimination process, and focuses on relationship between the patterns of dynamics of different biases and the signals' relative precisions. The latter paper deals with some random neighbor-effect and focuses on the converging speed of the equilibrium beliefs. The present work instead investigates the specific winner-effect in the game.

The rest of the chapter is organized as follows. Section 4.2 defines the dynamic beauty contest game as well as the solution concept. Section 4.3 characterizes the unique equilibrium of the game. Section 4.4 defines three gaps depicting players' equilibrium actions and checks their

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[5]Actually, this was pointed out in the initial post on this issue. See http://blog.livedoor.jp/kaikaihanno/archives/27092143.html. However, almost all following posts just ignored this and argued that those were the Miss Korea contestants' pictures.

dynamics. Section 4.5 includes the analysis of the Miss Korea pageant as an application to our theoretical results. Section 4.6 concludes.

4.2   The Model

The economy is populated by a continuum $[0, 1]$ of players. Time is discrete and there are infinitely many stages, $t \in \{1, 2, 3, \cdots\}$. Before the game begins, nature chooses a fundamental value $\theta \in \mathbb{R}$. Players cannot observe the true value of the fundamental directly. However, there is a public signal $y_1 \in \mathbb{R}$ released before the game starts, and all players form a common prior distribution of the fundamental as $\theta \sim \mathcal{N}(y_1, \frac{1}{\beta_1})$, i.e., it is commonly believed in stage 1 that the fundamental is normally distributed with mean y_1 and precision β_1. In each stage t, each player $i \in [0, 1]$ also observes a private signal $x_{it} = \theta + \varepsilon_{it}$, where ε_{it}'s are independently distributed from the identical normal distribution $\mathcal{N}(0, \frac{1}{\alpha_t})$. Furthermore, the noises are idiosyncratic in the following sense: For any players i and j, and any stages t and τ, ε_{it} and $\varepsilon_{j\tau}$ are independent as long as $(i, t) \neq (j, \tau)$.

Players discount future payoffs using the constant discount factor $b > 0$. In each stage, players take actions simultaneously. With action $a_{it} \in \mathbb{R}$, player i's instantaneous payoff for stage t is given as

$$U_{it} = -(1 - \gamma)(a_{it} - \theta)^2 - \gamma L_{it}, \tag{4.1}$$

where $\gamma \in (0, 1)$ and

$$L_{it} = \int_0^1 (a_{it} - a_{jt})^2 \, \mathrm{d}j. \tag{4.2}$$

A greater (smaller) γ means players are worried more about staying close to the average of the group (fundamental value).[6]

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[6]The instantaneous utility function defined in (4.1) is slightly different from that used by Morris and Shin (2002), who take $U_{it} = -(1 - \gamma)(a_{it} - \theta)^2 - \gamma(L_{it} - \bar{L}_t)$, where $\bar{L}_t = \int_0^1 L_{it} \, \mathrm{d}i$. Term $\bar{L}_t$ makes sure the sum of the second parts of the utility functions always equals zero. As a result, the social welfare depends

Besides the private signals, starting from stage 2, all players observe another public signal

$$y_t = a^*_{t-1} + \xi_t, \qquad\qquad (4.3)$$

where ξ_t's are distributed from a normal distribution with mean 0 and variance $\frac{1}{\beta_t}$ and independent across time and from all noises in the privates signals. a^*_{t-1} is the action taken by the (essentially unique) player who makes the highest instantaneous payoff in the previous stage: $a^*_{t-1} \in \{a_{i,t-1} | U_{i,t-1} \geq U_{j,t-1}, \forall j \neq i\}$. In other words, in each stage $t \geq 2$, all players observe, with some noise, the action taken by the previous stage's most successful player, as a public signal. For simplicity of expressions, we further assume that nobody knows whether or not she has been picked out as the most successful player in the previous stage.[7] The payoff structures, all noise structures, and order of play, are common knowledge between all players.

So far we have defined the complete game. For player i in stage t, her history is denoted by $h_{it} = (x_{i\tau}, y_\tau)_{\tau \leq t}$ if $t = 1$ and $h_{it} = (x_{i\tau}, y_\tau, a_{i,\tau-1})_{\tau \leq t}$ if $t > 1$. We now define the equilibrium concept for the game.

Definition 4.1. *An equilibrium is a symmetric Perfect Bayesian Equilibrium, consisting of a strategy $s_t(\cdot)$ and a belief $\pi_t(\theta|\cdot)$ for each stage t, such that $a_{it}(h_{it}) \equiv s_t(h_{it})$ and $\pi_{it}(\theta) \equiv \pi_t(\theta|h_{it})$ for all i and t, and*

$$(a_{i\tau})_{\tau \geq t} \in \arg \max_{(f_\tau(h_{i\tau}))_{\tau \geq t}} E_{it} \left(\sum_{\tau=t}^{\infty} b^{\tau-t} U_{i\tau}(f_\tau) \,\middle|\, \pi_{i\tau}(\theta), s_\tau(\cdot) \right), \quad \forall t = 1, 2, 3, \cdots, \quad (4.4)$$

every belief $\pi_t(\theta|h_{it})$ is updated according to Bayes' Rule. $\qquad (4.5)$

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exclusively on how close the actions are to the fundamental value in average. However, since we do not care about social welfares in the current work, term $\bar{L}_t$ becomes irrelevant here and is eliminated from the utility function.

[7]The main results of the work still hold without this assumption because of the continuum setting. We adopt the assumption only to simplify the notations in the rest of the chapter. If a player knows she has been picked out as the player and remembers her action in the previous stage, she can induce the true fundamental value. We then have to distinguish between these fully-informed players and other players when it comes to the strategies and beliefs, which makes the definitions and notations unnecessarily tedious.

Condition (4.4) above imposes the requirement of sequential optimality on strategies. (4.5) requires the use of Bayes' Rule. Because of the specific settings of the continuous random noises, there will be no information nodes that are off the equilibrium path and (4.5) is thus always applicable.

4.3 Equilibrium

We investigate the equilibrium structure of the game in this section. It is shown that there exists a unique equilibrium in which all players adopt a strategy linear in all public and private signals, and each player maximizes her expected instantaneous payoff in all stages. These results provide us foundations for further investigation of strategy gap dynamics in the subsequent section.

Recall that the instantaneous payoff function is defined in (4.1) and (4.2). Given the quadratic settings of (4.1), it is straightforward to have the following first-order condition to maximize the expected stage payoff:

$$a_{it} = (1-\gamma)E_{it}(\theta) + \gamma \, E_{it}\left(\int_0^1 a_{jt} \, \mathrm{d}j\right),\qquad(4.6)$$

where $E_{it}(\cdot)$ is short for $E_{it}(\cdot|h_{it})$.

Lemma 4.1. *In any equilibrium of the full game, for each stage t, strategies a_{it}'s are also prescribed by the equilibrium of the corresponding stage game, i.e., $a_{it} = (1-\gamma)E_{it}(\theta) + \gamma \, E_{it}\left(\int_0^1 a_{jt} \, \mathrm{d}j\right).$*

In any equilibrium, given other players' strategies, it is optimal for one to just maximize the expected instantaneous payoff in each stage. It is straightforward to check the validity of this lemma. Although the action of the current stage's most successful player will be part of the following stage's public signal, the event of being that one has zero measure for all

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players. As a result, nobody cares about the potential signaling effect of her current action in the future. So given other players' strategies, one must choose the action to maximize her expected instantaneous payoff for the current stage.

Assumption 4.1. *Suppose each player $i \in [0,1]$ receives the realization χ_{it} of a statistic X_{it} in stage t, and all X_{it}'s are identically and independently distributed from a normal distribution characterized by Cumulative Distribution Function P_t. Denote a generic path of all realizations by $D_t : [0,1] \to \mathbb{R}$ such that $D_t(i) = \chi_{it}$. Then for each $z \in \mathbb{R}$, the Lebesgue measure of set $\{i \in [0,1] \,|\, \chi_{it} \leq z\}$ exists and equals $P_t^{-1}(z)$. What is more, each D_t is a* **surjective** *function.*

The assumption above makes sure that the empirical distribution of signals are well depicted by the population distribution function and every real number must be observed by some players as a realization. To see that the assumption could be valid at least in some probability spaces, see Judd (1985) for the former statement and Haller and Yi (2013) for the latter. Assumption 4.1 makes the following analysis technically convenient and is thus made through the rest of the chapter.

Lemma 4.2. *In any equilibrium, we must have $a_{i1} = \frac{\beta_1 y_1 + (1-\gamma)\alpha_1 x_{i1}}{\beta_1 + (1-\gamma)\alpha_1}$ for each player i in stage 1. As a result, the most successful player in stage 1 receives the private signal $x_1^* = \theta + \frac{\beta_1}{\alpha_1}(\theta - y_1)$.*

Because all players need to maximize the current stage payoff as stated in lemma 4.1, in stage 1, they behave just as if they were in the static game of Morris and Shin (2002). Given the strategies adopted by all players, we can calculate the exact value of the private signal observed by the most successful player in stage 1, and Assumption 4.1 ensures the existence of such a player. Term $\frac{\beta_1}{\alpha_1}(\theta - y_1)$ is actually the noise realization in her private signal. If y_1 happens to equal θ, the player who happens to observe exactly the true fundamental makes the highest stage-1 payoff. Otherwise the biased prior comes into effect and the player who receives a private signal with best amount of deviation from the fundamental outperforms

others in the stage. Given a fixed deviation of y_1 from θ, the larger the relative precision of the prior β_1/α_1, the more is the most successful player's private signal deviating from the fundamental.

Now we move on to characterize the unique equilibrium for the game.

Proposition 4.1. *The game admits a unique equilibrium in which* $a_{it} = \frac{\tilde{\beta}_t \tilde{y}_t + (1-\gamma)\tilde{\alpha}_t \tilde{x}_{it}}{\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t}$, *where $\tilde{y}_t$ and $\tilde{x}_{it}$ are sufficient statistics summarizing information conveyed in the public and private signals, respectively. $\tilde{\beta}_t$ and $\tilde{\alpha}_t$ are precisions of the statistics $\tilde{y}_t$ and $\tilde{x}_{it}$, respectively. Furthermore, they are recursively defined as follows:*

(1) $\tilde{\alpha}_1 = \alpha_1$, $\tilde{x}_{i1} = x_{i1}$, $\tilde{\beta}_1 = \beta_1$, $\tilde{y}_1 = y_1$.

(2) For $t > 1$, $\tilde{\alpha}_t = \tilde{\alpha}_{t-1} + \alpha_t$, $\tilde{x}_{it} = (\tilde{\alpha}_{t-1}\tilde{x}_{i,t-1} + \alpha_t x_{it})/\tilde{\alpha}_t$.

(3) For $t > 1$, $\tilde{\beta}_t = \tilde{\beta}_{t-1} + \Omega_t \beta_t$, $\tilde{y}_t = (\tilde{\beta}_{t-1}\tilde{y}_{t-1} + \Omega_t \beta_t \Upsilon_t)/\tilde{\beta}_t$, where $\Omega_t = \left[\frac{(1-\gamma)(\tilde{\alpha}_{t-1} + \tilde{\beta}_{t-1})}{\tilde{\beta}_{t-1} + (1-\gamma)\tilde{\alpha}_{t-1}}\right]^2$ and $\Upsilon_t = \frac{[\tilde{\beta}_{t-1} + (1-\gamma)\tilde{\alpha}_{t-1}]y_t - \gamma\tilde{\beta}_{t-1}\tilde{y}_{t-1}}{(1-\gamma)(\tilde{\alpha}_{t-1} + \tilde{\beta}_{t-1})}$.

Corollary 4.1. *In the equilibrium, the most successful player of stage t observes $\tilde{x}_t^* = \theta + \frac{\tilde{\beta}_t}{\tilde{\alpha}_t}(\theta - \tilde{y}_t)$. For each i and t, $(a_{it} - \theta)$ is a normally distributed random variable with mean 0 and variance $\frac{\tilde{\beta}_t + (1-\gamma)^2 \tilde{\alpha}_t}{\left(\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t\right)^2}$.*

The uniqueness thus enables us to study the dynamics of statistics of interest in the equilibrium, as shown in the subsequent section.

4.4 Equilibrium Dynamics

This section investigates how the deviations of actions from some targets evolve by time in the equilibrium. We first check the deviation of the average action from the true fundamental θ to answer the question whether or not the players' actions converge on average to the fundamental. The deviation of a single player's action from the mean action is introduced

afterward, and a relationship between the dynamics of these two is given. Finally, we intro-duce the deviation of a single player's action from the action taken by the most successful player. A relationship between these three deviations is given, and thus serves as a useful tool for empirical studies in the subsequent section.

Denote the players' average action in stage t by $\bar{a}_t = \int_0^1 a_{it} \, \mathrm{d}i$. Given a player i, we further define

$$d_{1t} = \bar{a}_t - \theta, \tag{4.7}$$

$$d_{2t} = a_{it} - \bar{a}_t, \tag{4.8}$$

$$d_{3t} = a_{it} - a_t^*. \tag{4.9}$$

Because of the symmetries in both the information structure and equilibrium strategies of all players, the second and third deviations above are well and equivalently defined across all players. Nevertheless, equilibrium strategies are dependent on private and public signals, in which stochastic noises are enclosed. As a result, all three terms defined above should be random variables, and their distributions in the equilibrium are depicted in propositions below.

Proposition 4.2. *In the equilibrium, $d_{1t} \sim \mathcal{N}(0, V_{1t})$, where $V_{1t} = \frac{\tilde{\beta}_t}{\left(\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t\right)^2}$. As a result, $\partial V_{1t}/\partial \tilde{\alpha}_t < 0$ always holds and $\partial V_{1t}/\partial \tilde{\beta}_t < 0$ holds if and only if $\frac{\tilde{\beta}_t}{\tilde{\alpha}_t} > 1 - \gamma$.*

Recall that d_{1t} is the gap between the average play and the underlying fundamental in stage t. Given its mean is 0, V_{1t} thus measures how likely the average play in stage t deviates far away from the fundamental value θ. According to their definitions, $\tilde{\alpha}_t$ and $\tilde{\beta}_t$ both increase in t, i.e., $\tilde{\alpha}_{t+1} > \tilde{\alpha}_t$ and $\tilde{\beta}_{t+1} > \tilde{\beta}_t$ hold for all t. From stage t to stage $(t+1)$, the increase in the precision of summed private signals has an unambiguous effect on V_1: It makes $\bar{a}_{t+1}$ less likely (compared to $\bar{a}_t$) to deviate far away from θ. However, the effect of the improved precision of summed public signals is rather indeterminate: It decreases the variability of

$d_{1,t+1}$ only if the summed public signals ($\tilde{y}_t$) is precise enough compared to the summed private signals ($\tilde{x}_t$). For this reason, the comparison between V_{1t} and $V_{1,t+1}$ is indeterminate.

Because of the continuum setting of players and the symmetry in their private signals, taking average eliminates noises embedded in private signals. So the remaining variability of d_{1t} is solely caused by noises embedded in public signals. A more precise summed public signal leads to two consequences: making players assign more weights to public signals while making decisions, and reducing the variability of noises in the public signals. Although the latter effect tends to reduce V_{1t}, the former tends to increase V_{1t}. Trade-offs between them thus make changes in V_{1t} over time indeterminate, and it is decreasing in $\tilde{\beta}_t$ only if the summed public signals is relatively precise enough.

Proposition 4.3. *In the equilibrium, $d_{2t} \sim \mathcal{N}(0, V_{2t})$, where $V_{2t} = \frac{(1-\gamma)^2 \tilde{\alpha}_t}{\left(\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t\right)^2}$. As a result, $\partial V_{2t}/\partial \tilde{\beta}_t < 0$ always holds and $\partial V_{2t}/\partial \tilde{\alpha}_t < 0$ holds if and only if $\frac{\tilde{\beta}_t}{\tilde{\alpha}_t} < 1 - \gamma$.*

V_{2t} measures how likely a generic player's action deviates far away from the average play in stage t. From stage t to stage $(t+1)$, the increase in the precision of summed public signals has an unambiguous effect on V_2: It makes $d_{2,t+1}$ less likely (compared to d_{2t}) to deviate far away from 0. However, the effect of the improved precision of summed private signals is rather indeterminate: It decreases the variability of $d_{1,t+1}$ only if the summed private signals ($\tilde{x}_t$) is precise enough compared to the summed public signals ($\tilde{y}_t$). The comparison between V_{2t} and $V_{2,t+1}$ is thus indeterminate.

Recall that all players observe the same public signals, taking average does not eliminate noises embedded in these signals. So the variability of d_{2t} stems from noises embedded in the private signals in this case. Then an improvement in the precision of the summed private signals both makes players count more on the private signals and reduces the variability of the private noises. Again, the former tends to increase V_{2t} and the latter tends to decrease V_{2t}. The trade-off relationship between them thus leads to the result that the latter effect outperforms the former only if the relative precision of $\tilde{x}_t$ is great enough.

Proposition 4.4. *In the equilibrium, for* $t = 1, 2, 3, \cdots$, *given* $V_{1t} < V_{1,t+1}$, *we must have* $V_{2t} > V_{2,t+1}$; *and given* $V_{2t} < V_{2,t+1}$, *we must have* $V_{1t} > V_{1,t+1}$.

If the variability of d_1 increases from t to $t + 1$, i.e., the average play deviates more from the fundamental, it must be companied with the change that a generic player's action becomes closer to the average play, from t to $t + 1$.

Proposition 4.5. *In the equilibrium,* $d_{3t} \sim \mathcal{N}(0, V_{3t})$, *where* $V_{3t} = V_{2t} + (1 - \gamma)^2 V_{1t}$.

The proposition above is useful in empirical studies where we have only observations of all players' and the winners' actions, but have no idea what the fundamental value is. In this case, V_{1t} could be inferred using V_{2t} and V_{3t}.

4.5 An Empirical Study: Miss Korea Pageant

In this section, an empirical study is conducted to serve as an application to our theoretical results above. We collected 1125 pictures of the contestants of the Miss Korea beauty contest—the national beauty pageant selecting South Korea's representative to the Miss Universe pageant—from year 1994 to 2013. The reason we choose this specific country's beauty pageant within the given time period is that the popularity of plastic surgeries among the contestants of Miss Korea pageant since 1990s has been well known.[8] It is thus reasonable to consider the contestants as strategic players and treat their faces as actions.

Strictly speaking, the empirical case studied here is different from the theoretical model above in the sense that players play repeatedly in the model but contestants seldom attend the pageant for multiple times. However, if we just denote by $\tilde{x}_{it}$ the single private signal received by the i-th player in stage t with precision $\tilde{\alpha}_t = \tilde{\alpha}_{t-1} + \alpha_t$, we do not need to have parallel meanings of the decomposition treatment of $\tilde{x}_t$ as in the theoretical part, and α_t

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[8]See, for example, http://fashionista.com/2013/08/inside-miss-korea-scandal-glory-lee/.

here represents the improvement in the precision of private signals from year $t - 1$ to t. All contestants could also observe all winners of previous years, so $\tilde{y}_t$ and $\tilde{\beta}_t$ have equivalent settings and meanings as in the model. It is thus straightforward to see that the theoretical model above could fit the Miss Korea scenario well. We then move on to describe the data and methods used in the study.

4.5.1   Data

There are 47–64 contestants in each year, for each of them we choose a picture from the official website.[9] These pictures are basically taken with different background environments from year to year, and usually include the whole bodies or the upper bodies of the contestants. To eliminate the variations caused by background environments, clothes and body poses, we focus only on their faces. So each picture is cut into a smaller one (face for short henceforth) that contains only the face area of the contestant. What is more, we rescale them such that all faces have same dimensions. We also convert all the initially colored pictures to back-and-white ones, so each pixel of a face is represented by only one value between 0–255, where value 0 represents pure "black" and 255 for pure "white". Finally, each face is depicted by a 125×200 matrix, where 125 is the number of pixels representing the width of a face image, and 200 is the number of pixels for the height.

The first place award, also called "Gin", is given to only one contestant per year; the second place award, the "Line", is given to 2 contestants in each year; and the third place award, the "beauty", admits 3–6 receivers every time. In this study, we take the "Gin" winner as the most successful player of the corresponding year.

There are 20 years considered, denote year 1994 by $t = 1$ and year 2013 by $t = 20$. For each

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[9]See http://misskorea.hankooki.com/profile_search.php?mode=search. Almost all contestants have at least one picture available online, exceptions are 1 contestant's picture is missing for 1996, 2 missing for 2004, 1 missing for 2012, and 1 missing for 2013. For these contestants, we just exclude them from considerations. For contestants with multiple pictures available, we pick out the one that best depicts the contestant's facial characteristics, as well as has similar photo backgrounds with other contestants' pictures.

year t, denote by N_t the number of contestants. For $i \in \{1, 2, \cdots, N_t\}$, denote by A_{it}^0 the matrix depicting the face of the i-th contestant of year t. Now define operator $\Delta(\cdot)$ such that for any matrix $M = (M_1, M_2, \cdots M_n)$, we have $\Delta(M) = (M_1^T, M_2^T, \cdots, M_n^T)^T$. Then for each t and each i in year t, we define

$$A_{it} = \Delta(A_{it}^0). \tag{4.10}$$

Recall that each A_{it}^0 is a 125×200 matrix, so each A_{it} is a 25000×1 vector. And each face is fully depicted also by A_{it}: $\Delta^{-1}(A_{it})$ gives us the value of each pixel in the initial face image, i.e., matrix A_{it}^0.

All data processing and analysis in this section are realized in Matlab codes. Appendix 4.7.2 gives a link to the whole dataset and all codes used in this empirical study.

4.5.2 Methods

Those faces are then projected onto an *Eigenface* space constructed by eigenvectors of the empirical covariance matrix calculated based on the pixel values of all images, as introduced in Turk and Pentland (1991). Each face is thus represented as a vector of coordinates in the *Eigenface* space, and distances between faces are calculated by imposing the Euclidean metric between different vectors of coordinates. This process is depicted in details as follows.

The Eigenface method is essentially an alias for the Principal Component Analysis (PCA) in Image Recognition. The main idea is to first treat a given face as the sum of the average face and the deviation from the average, and then find vectors that best explain the variations in these deviations. We include below the treatments and properties without proofs used to impose the PCA method in our study. The details and proofs can be found in Chapters 1, 2, 3, and 6 of Jolliffe (2002).

We first define the matrix containing all faces. Define

$$A_t = (A_{1t}, A_{2t}, \cdots, A_{N_t,t}), \tag{4.11}$$

$$A = (A_1, A_2, \cdots, A_{20}). \tag{4.12}$$

So matrix A_t contains information of the faces of all contestant in year t, and matrix A depicts all 1125 faces. Further define the average faces

$$\bar{A}_t = \frac{1}{N_t} \sum_{m=1}^{N_t} A_{mt}, \tag{4.13}$$

$$\bar{A} = \frac{1}{1125} \sum_{t=1}^{20} N_t \bar{A}_t. \tag{4.14}$$

$\bar{A}_t$ is the average face of contestants in year t, and $\bar{A}$ is the average face of all contestants in recent 20 years. Further define the matrix of face gaps $B = A - \bar{A}$ where notation $(A - \bar{A})$ is short for $A - (\bar{A}, \bar{A}, \cdots, \bar{A})$, and the empirical covariance matrix

$$\Sigma^A = \frac{1}{25000 - 1} (A - \bar{A})^T (A - \bar{A}) = \frac{1}{25000 - 1} B^T B. \tag{4.15}$$

Now we would like to find a normalized eigenface, depicted by a 25000×1 vector ν, to maximize the variability of the projections of all face gaps on it, which is defined as

$$Var(B^T \nu) = \frac{1}{1125 - 1} \|B^T \nu - mean(B^T \nu)\|^2 = \frac{1}{1125 - 1} \nu^T B B^T \nu. \tag{4.16}$$

So we would like to solve the problem

$$\max_{\nu} \ \nu^T B B^T \nu, \quad s.t. \ \nu^T \nu = 1. \tag{4.17}$$

Imposing the Lagrangian method on the problem gives us the necessary condition

$$BB^T \nu = \lambda \nu, \qquad (4.18)$$

which is exactly the eigenvalue problem of matrix BB^T. Because of the definition of B, we have $rank(BB^T) = 1125 - 1 = 1124$. So there are essentially 1124 nontrivial solutions to problem (4.18). Rank the eigenvalues of problem (4.18) in descending order, i.e., $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{1124}$, and denote their associated eigenvectors by $\nu_1, \nu_2, \cdots, \nu_{1124}$. All eigenvalues are positive, since they are also eigenvalues of the covariance matrix Σ^A.[10] These eigenvectors are exactly the eigenfaces we use in this study. They also serve as an orthogonal basis for the eigenface space because BB^T is symmetric. Obviously, ν_1 solves problem (4.17). What is more, for any $n \geq 2$, vector ν_n solves problem

$$\max_{\nu} \ Var \left[(B^T - \sum_{k=1}^{n-1} B^T \nu_k \nu_k^T) \nu \right], \quad s.t. \ \nu^T \nu = 1. \qquad (4.19)$$

That is to say, after subtracting the first $(n-1)$ eigenfaces from the face gaps, compared to other remaining eigenfaces, ν_n still explains the most variability remaining in the new matrix of face gaps. Although there are 1124 eigenfaces in total, we may not want to use them all. Actually, one of the main goal of PCA is to reduce the number of dimensions needed to conduct the investigation. To illustrate this, for any $n < 1124$, define

$$S_n = (\nu_1, \nu_2, \cdots, \nu_n) \qquad (4.20)$$

as the basis of the space spanned by the first n eigenfaces. Each face A_{it} can thus be represented by a_{it}^n, the coordinates of its projection on the eigenface space, defined as

$$a_{it}^n = S_n^T A_{it}. \qquad (4.21)$$

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[10]Actually, because BB^T is a 25000×25000 matrix and the dimension of Σ^A is only 1125×1125, in practice we first calculate eigenvectors ω_n's for Σ^A and then calculate ν_n's by $\nu_n = B\omega_n / \|B\omega_n\|$.

Given a_{it}^n of a face, its projected image is given by $\Delta^{-1}(S_n S_n^T A_{it})$. We would like to find a proper n for this study. To do so, we pick up different alternative values for n and compare the different projected faces based on different S_n. As shown in Figure 4.1, from the second to the last column, the projected faces become closer and closer to the initial face images shown in the first column in each row. What is more, the projected faces in the last column are almost the same as shown in the first column, suggesting that $n = 700$ could be a good choice.

Figure 4.1: Projected faces using different numbers of eigenfaces. The first column shows the initial faces. From the second to the last column, the first 50, 100, 150, 200, 400 and 700 eigenfaces are used, respectively, to serve as the basis of space. Each row represents a given face.

To justify the choice $n = 700$, we define $\Gamma_n = \frac{\sum_{\ell=1}^n \lambda_\ell}{\sum_{k=1}^{1124} \lambda_k}$ for all $n < 1124$. It is known in PCA that Γ_n tells us the proportion of variability embedded in the face gap matrix that has been accounted for by the first n eigenfaces. The plot for function Γ_n is given in figure 4.2. The first 700 eigenfaces explain 99.6% of the variability, so there could be little gains from adding more eigenfaces into the basis.

Nevertheless, we want to exclude some eigenfaces associated with the greatest eigenvalues from the space basis. To understand this, recall that when talking about the differences in contestants' faces, we actually refer to differences in facial details, such as the sizes and positions of eyes, nose, mouth and forehead, the sharpness of jaw, and so on. But in reality, because the distributions of facial organs are somewhat similar across human faces, especially
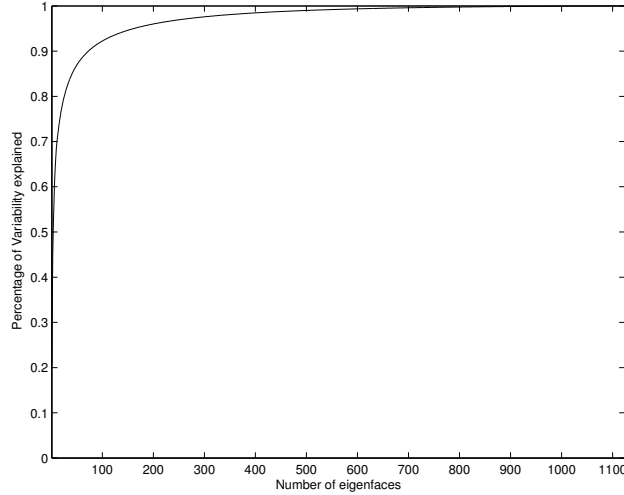
Figure 4.2: Plot of Γ_n: the proportion of variability embedded in the face gap matrix that has been accounted for by the first n eigenfaces.

across the faces of the contestants of beauty pageants, the first eigenfaces turn out to account for the variability caused by features other than facial characteristics, which include face angles, hair styles, earrings, etc. This is illustrated in Figure 4.3, where we include the associated images of eigenfaces ν_{10}, ν_{50}, ν_{100}, ν_{150}, ν_{200} and ν_{250}. The first three images are mostly depicting the non-facial features as discussed above. And images after them seem to be dealing with facial characteristics. So we exclude the first 149 eigenfaces from the basis.

Figure 4.3: Images of different eigenfaces. From left to right, images represented by the 10th, 50th, 100th, 150th, 200th and 250th eigenface, respectively.

Combining the two truncation rules explained above, we have the basis for the eigenface space used in this study as

$$S = (\nu_{150}, \nu_{151}, \cdots, \nu_{700}), \tag{4.22}$$

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and action of the i-th player in year t, i.e., the projection of face A_{it} on ν, is thus defined as

$$\mathring{a}_{it} = S^T A_{it}. \tag{4.23}$$

Now we have one final step left in data processing: brightness aligning. Although the environments in which pictures were taken are similar for contestants in the same year, they are rather different across years. For instance, in some years contestants were organized to take pictures in a studio with supplementary lighting facilities; whereas in other years pictures were mainly taken outdoors on a either sunny or cloudy day. This makes the initial face images brighter in some years and darker in others. Given a face A_{it}, we multiply all its elements by $\tau > 0$, then the image's brightness increases if $\tau > 1$, and decreases if $\tau < 1$.

Figure 4.4: Two faces in different brightness. The first column shows the initial images, $\tau = 0.8$ for the second column, $\tau = 1.2$ for the third column. Each row represents a given face.

As shown in Figure 4.4, the two contestants are from different years, and the first column clearly shows that the upper face is brighter than the lower one. Although a change in brightness does not necessarily reduce the recognizability of a face—the 3 faces in each

row of Figure 4.4 depict the same facial characteristics of the contestant—it does affect the validity of our analysis if we treat difference between the coordinate vectors of two faces on basis ν as the face gap. To see this, just note that for the i-th and j-th contestants in year t, we have

$$\|S^T \tau A_{it} - S^T \tau A_{jt}\| = \tau \|\mathring{a}_{it} - \mathring{a}_{jt}\|. \tag{4.24}$$

So if we treat $\mathring{a}_{it}$'s as contestants' actions, we are likely to overestimate the face gaps in the years when face images are brighter, and to underestimate the gaps in the years when face images are darker. To overcome this shortcoming, we normalize each $\mathring{a}_{it}$ and define the actions as

$$\hat{a}_{it} = \frac{\mathring{a}_{it}}{\|\mathring{a}_{it}\|} = \frac{S^T A_{it}}{\|S^T A_{it}\|}. \tag{4.25}$$

The treatment above gives us a_{it}'s that are insensitive to brightness changes and thus makes the statistics of face gaps in different years comparable. Based on the meanings of their counterparts in the theoretical part, we define here

$$\hat{V}_{2t} = \frac{\sum_{i=1}^{N_t} \|\hat{a}_{it} - \bar{\hat{a}}_t\|^2}{N_t - 1}, \tag{4.26}$$

$$\hat{V}_{3t} = \frac{\sum_{i=1}^{N_t} \|\hat{a}_{it} - \hat{a}_t^*\|^2}{N_t - 2}, \tag{4.27}$$

where the average action in year t is defined as $\bar{\hat{a}}_t = \sum_{i=1}^{N_t} \hat{a}_{it}/N_t$ and $\hat{a}_t^*$ is the action played by the winner of the "Gin" title in year t.

Recall that in the theoretical counterpart, the stage-t action a_{it} is simply a real number, but here we have $\hat{a}_{it}$, which is a multidimensional vector, to represent the actual action taken by the i-the contestant of year t. To make definitions in (4.26) and (4.27) meaningful and useful in our study, we make the following assumption.

Assumption 4.2. *Given year t, denote the bundle of all contestants' actions in the underlying game depicted in the theoretical model above by $a_t = (a_{it})_{i \in \{1,2,\cdots,N_t\}}$, denote by $\hat{a}_t = (\hat{a}_{it})_{i \in \{1,2,\cdots,N_t\}}$ the bundle of actions discovered in faces. For any two linear functions*

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$f(\cdot)$ and $g(\cdot)$, we have $|f(a_t) - g(a_t)| = \rho\|f(\hat{a}_t) - g(\hat{a}_t)\|$, where $\rho > 0$ is a constant.

That is to say, the contestants are actually playing according to an underlying game depicted by the theoretical model, but their actions detected in reality are expressed in vectors, rather than numbers as in the model. This assumption thus enables us to estimate the distance between the underlying actions by using euclidean distance between vectors defined in (4.25). ρ could be any positive scale, and is irrelevant to analyses below since we are mainly focusing on the time trends in the estimated variances. For this reason, we simply take $\rho = 1$ to simplify notations.

It is then straightforward to see that $\hat{V}_{2t}$ and $\hat{V}_{3t}$ are finite-version estimates of variances V_{2t} and V_{3t} defined in Propositions 4.3 and 4.5, respectively. What is more, based on Proposition 4.5, V_{1t} could be estimated by $\hat{V}_{1t}$ which is defined by

$$(1 - \gamma)^2 \hat{V}_{1t} = \hat{V}_{3t} - \hat{V}_{2t}. \tag{4.28}$$

4.5.3 Results

In the specific context of this study, the public signals refer to faces of the winners of previous years observed by all contestants in later years. These signals are rather precise because of the popularity of mass media as well as the improvements in photography over the time period of study, which together guaranteed the delivery of winners' faces with little distortions to the public, especially to contestants in the future. However, on the other hand, the private signals should be less precise compared to the public ones in the study. To understand this, note that private signals here refer to contestants' understandings of the underlying fundamental value, i.e., the "true beauty". These understandings consist of all kinds of related information the contestants have gathered in daily lives, such as discussions with friends, observations of faces that are thought of as beautiful by others, suggestions by experts, and so on. As a result, the generally existing heterogeneity in daily

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lives of contestants leads to remarkable differences between their understandings of the "true beauty". In other words, if we treat these understandings in the same year as realizations of independently and identically distributed random variables as introduced in the theoretical part, we must have that the private signals are relatively less precise compared to the public ones.
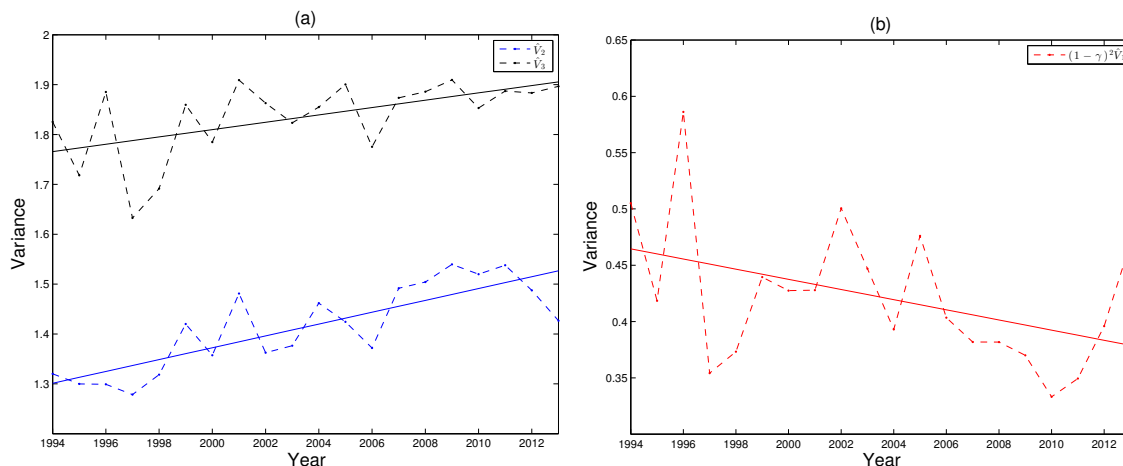

Figure 4.5: Plots of sample V_1, V_2 and V_3 of Miss Korea contests between 1994–2013.

The discussion above suggests a rather great ratio $\tilde{\beta}_t/\tilde{\alpha}_t$, which, combined with Proposition 4.2, further suggests a descending V_{1t} in the equilibrium of the model. This is confirmed by investigating $\hat{V}_{1t}$ defined in (4.28). Because the sample size for each year could not be considered as sufficiently large (only about 55 contestants in each year), we can foresee non-negligible gaps between underlying V_{1t} and $\hat{V}_{1t}$. For this reason, we do not expect a pure monotonicity of $\hat{V}_{1t}$ in t, even if that is the case for the underlying V_{1t}, but rather focus on whether or not we find an overall time-trend in $\hat{V}_{1t}$. This is done by running a simple regression of $\hat{V}_{1t}$ on t and then checking the sign and significance of the estimate of the slope parameter. As shown in (b) of Figure 4.5, where the dashed red curve represents the plot of $\hat{V}_{1t}$ and the solid red line is the fitted regression line, we do observe a descending trend in $\hat{V}_{1t}$ because the slope of the fitted regression is negative, with a p-value as low as 0.06, as shown in Table 4.1.

We then check whether or not there exist significant time trends in $\hat{V}_{2t}$ and $\hat{V}_{3t}$. Recall

Table 4.1: Parameter estimates of simple linear time-trend regressions.

Model	Intercept (p-value)	Slope (p-value)
$(1-\gamma)^2 \hat{V}_{1t} = c_1 + d_1 t + \eta_{1t}$	$0.47\,(10^{-12})$	$-0.0045\,(0.06)$
$\hat{V}_{2t} = c_2 + d_2 t + \eta_{2t}$	$1.28\,(2 \times 10^{-21})$	$0.012\,(10^{-5})$
$\hat{V}_{3t} = c_3 + d_3 t + \eta_{3t}$	$1.75\,(7.7 \times 10^{-22})$	$0.0074\,(0.01)$

that Proposition 4.3 suggests indeterminate changes in V_{2t} with a sufficient large $\tilde{\beta}_t/\tilde{\alpha}_t$, because in this case an improvement in the summed public signal decreases the variance but an improvement in the private signal increases the variance. Proposition 4.5 also does not predict clear trends in V_{3t}. However, as shown in (a) of Figure 4.5 and the last two lines of Table 4.1, we observe significant ascending trends in $\hat{V}_{2t}$ and $\hat{V}_{3t}$, with p-values of the positive slopes as low as 10^{-5} and 10^{-2}, respectively.

Our findings thus suggest that, from year 1994 to 2013, there has been a descending trend in the variability of the gap between the average face and the underlying fundamental, meaning that the contestants were in overall getting better and better understandings of the underlying "true beauty". However, we also find ascending trends in the variabilities of both the gap between a representative contestant's face and the average face, and the gap between a representative contestant's face and the winner's face. This thus indicates increasing diversities in faces of the contestants of Miss Korea over recent 20 years.

4.6   Conclusions

In the dynamic beauty contest game, it is obvious that players get more and more accurate understandings on the fundamental value, given the accumulation of both private and public signals. However, it is rather groundless to assert that players' actions in the equilibrium will also show a pattern of convergence over stages. As shown in the theoretical part of the chapter, dynamics of the three deviations are all indeterminate, and whether or not a deviation statistically decreases between two consecutive stages depends crucially on the

relative precisions of the public and private signals.

For the empirical study on the Miss Korea contests, we find a converging trend in the overall contestants' actions to the underlying "true beauty", which is consistent to our presumed hypothesis based on the theoretical results in the first half of this chapter. Strikingly, diverging trends in both the deviation of a generic contestant's face from the average face and the deviation of a generic face from the winner's face are found. This thus suggests increasing diversity in contestants' faces over years, and disagrees with the "same-looking-contestant" viewpoint widespread on internet and mass media, which should be faulted for using inappropriate data.

4.7 Appendix

4.7.1 Appendix A: Proofs

Proof of Lemma 4.1.

To prove it, let us assume there exists an equilibrium such that for some i and t we have $a_{it} \neq (1-\gamma)E_{it}(\theta) + \gamma E_{it}\left(\int_0^1 a_{jt}\,\mathrm{d}j\right)$. Let a'_{it} be defined as in equation (4.6), and substitute a_{it} with a'_{it} while keeping player i's other stage strategies as well as all other players' strategies the same as in the initial equilibrium. Because equation (4.6) is the first order condition for maximization of the expected stage payoff, player i's stage-t payoff increases after the substitution. This substitution has effects on player i's expected payoffs for future stages if and only if she turns out to be the most successful player of stage t, which is a zero-probability event according to the continuum setting of the player set. As a result, a_{it} cannot be part of an equilibrium, contradiction. So in any equilibrium, the equality must be satisfied for each player in each stage. $\qquad\square$

Proof of Lemma 4.2.

To see that it is optimal for player i to adopt $a_{i1} = \frac{\beta_1 y_1 + (1-\gamma)\alpha_1 x_{i1}}{\beta_1 + (1-\gamma)\alpha_1}$ given others are doing so, notice that the conditional distribution of θ on h_{i1} is

$$\theta|h_{i1} \sim \mathcal{N}\left(\frac{\beta_1 y_1 + \alpha_1 x_{i1}}{\beta_1 + \alpha_1}, \frac{1}{\beta_1 + \alpha_1}\right), \tag{4.29}$$

given all players are taking this stage strategy, we have

$$
\begin{aligned}
E_{i1}\left(\int_0^1 a_{j1}\,\mathrm{d}j\right) &= E_{i1}\left(\int_0^1 \frac{\beta_1 y_1 + (1-\gamma)\alpha_1 x_{j1}}{\beta_1 + (1-\gamma)\alpha_1}\,\mathrm{d}j\right) \\
&= E_{i1}\left(\frac{\beta_1 y_1 + (1-\gamma)\alpha_1 \theta}{\beta_1 + (1-\gamma)\alpha_1} + \frac{(1-\gamma)\alpha_1}{\beta_1 + (1-\gamma)\alpha_1}\int_0^1 \varepsilon_{j1}\,\mathrm{d}j\right) \\
&= \frac{\beta_1 y_1 + (1-\gamma)\alpha_1 E_{i1}(\theta)}{\beta_1 + (1-\gamma)\alpha_1}. \tag{4.30}
\end{aligned}
$$

So the RHS of equation (4.6) could be expressed as

$$(1-\gamma)E_{it}(\theta) + \gamma\,E_{it}\left(\int_0^1 a_{jt}\,\mathrm{d}j\right) = \frac{(1-\gamma)\left[\beta_1 + (1-\gamma)\alpha_1 + \gamma\alpha_1\right]E_{i1}(\theta) + \gamma\beta_1 y_1}{\beta_1 + (1-\gamma)\alpha_1}$$

$$= \frac{\beta_1 y_1 + (1-\gamma)\alpha_1 x_{i1}}{\beta_1 + (1-\gamma)\alpha_1}, \tag{4.31}$$

which is exactly the LHS of the equation under the stage strategy. So (4.6) is fulfilled.

We then adopt the method used in Morris and Shin (2002) to show that there are no other choices for a_{i1}. Define operator $\bar{E}_1(\theta) = \int_0^1 E_{j1}(\theta)\,\mathrm{d}j$ as well as

$$\bar{E}_1^n(\theta) = \underbrace{\bar{E}_1(\bar{E}_1(\cdots(\bar{E}_1(\theta)\cdots)))}_{n}. \tag{4.32}$$

The initial first order condition (4.6) can thus be rewritten as

$$a_{i1} = (1-\gamma)E_{i1}(\theta) + \gamma\,E_{i1}\left(\int_0^1 a_{j1}\,\mathrm{d}j\right)$$

$$= (1-\gamma)\sum_{n=0}^{\infty}\gamma^n E_{i1}\left(\bar{E}_1^n(\theta)\right). \tag{4.33}$$

Define $\mu_1 = \alpha_1/(\beta_1 + \alpha_1)$, based on (4.29) and (4.32), it is straightforward to have $\bar{E}_1^n(\theta) = (1-\mu_1^n)y_1 + \mu_1^n\theta$ and $E_{i1}(\bar{E}_1^n(\theta)) = (1-\mu_1^{n+1})y_1 + \mu_1^{n+1}x_{i1}$. Also notice that $\mu_1 < 1$ always holds, so the RHS of (4.33) is bounded and can thus be rewritten as

$$a_{i1} = (1-\gamma)\sum_{n=0}^{\infty}\gamma^n E_{i1}\left(\bar{E}_1^n(\theta)\right)$$

$$= (1-\gamma)\sum_{n=0}^{\infty}\left[(\gamma^n - \gamma^n\mu_1^{n+1})y_1 + \gamma^n\mu_1^{n+1}x_{i1}\right]$$

$$= (1-\gamma)\left[\left(\frac{1}{1-\gamma} - \frac{\mu_1}{1-\gamma\mu_1}\right)y_1 + \frac{\mu_1}{1-\gamma\mu_1}x_{i1}\right]$$

$$= \frac{\beta_1 y_1 + (1-\gamma)\alpha_1 x_{i1}}{\beta_1 + (1-\gamma)\alpha_1}. \tag{4.34}$$

This is the unique solution that satisfies condition (4.6), and it also makes sure every player

is maximizing her stage-1 payoff given others' strategies, so the first half of the statement is now proved.

For the second half, notice that for individuals $i \neq j$ we have $x_{i1} - x_{j1} = \theta + \varepsilon_{i1} - \theta - \varepsilon_{j1} = \varepsilon_{i1} - \varepsilon_{j1}$, so in the equilibrium play

$$a_{i1} - \theta = \frac{\beta_1(y_1 - \theta) + (1 - \gamma)\alpha_1\varepsilon_{i1}}{\beta_1 + (1 - \gamma)\alpha_1} \tag{4.35}$$

$$a_{i1} - a_{j1} = \frac{(1 - \gamma)\alpha_1(\varepsilon_{i1} - \varepsilon_{j1})}{\beta_1 + (1 - \gamma)\alpha_1}. \tag{4.36}$$

In the equilibrium, player i's stage-1 payoff thus becomes

$$
\begin{aligned}
U_{i1} &= -(1 - r)(a_{i1} - \theta)^2 - rL_{i1} \\
&= -\frac{(1 - \gamma)\left[\beta_1(y_1 - \theta) + (1 - \gamma)\alpha_1\varepsilon_{i1}\right]^2 + \gamma\left[(1 - \gamma)\alpha_1\right]^2 \int_0^1 (\varepsilon_{i1} - \varepsilon_{j1})^2 \, \mathrm{d}j}{(\beta_1 + (1 - \gamma)\alpha_1)^2}.
\end{aligned} \tag{4.37}
$$

Now suppose player i turns out to be the most successful player of stage 1, Assumption 4.1 makes sure that ε_{i1} is a maximizer of the RHS of (4.37) in $\mathbb{R}$. Further denote by ε_1^* the value of private noise of the first stage's most successful player, it thus solves the first order condition

$$\beta_1(y_1 - \theta) + (1 - \gamma)\alpha_1\varepsilon_1^* + \gamma\alpha_1 \int_0^1 (\varepsilon_1^* - \varepsilon_{j1}) \, \mathrm{d}j = 0, \tag{4.38}$$

which in turn gives us

$$\varepsilon_1^* = \frac{\beta_1}{\alpha_1}(\theta - y_1). \tag{4.39}$$

So the most successful player in stage 1 receives private signal $x_1^* = \theta + \varepsilon_1^* = \theta + \frac{\beta_1}{\alpha_1}(\theta - y_1)$. The lemma is proved. $\qquad\square$

Proof of Proposition 4.1.

We prove the proposition by induction. Lemma 4.2 has already assured the validity of the statement for the first stage. Now for $t \geq 2$, suppose the statement is true for stage $t - 1$,

it suffices to show the statement is also true for stage t. Furthermore, assume that in stage $t-1$ we have

$$\theta | h_{i,t-1} \sim \mathcal{N} \left(\frac{\tilde{\beta}_{t-1} \tilde{y}_{t-1} + \tilde{\alpha}_{t-1} \tilde{x}_{i,t-1}}{\tilde{\beta}_{t-1} + \tilde{\alpha}_{t-1}}, \frac{1}{\tilde{\beta}_{t-1} + \tilde{\alpha}_{t-1}} \right). \tag{4.40}$$

Then in stage t, there are two signals added into player i's history:

$$x_{it} = \theta + \varepsilon_{it}, \tag{4.41}$$

$$y_t = a^*_{t-1} + \xi_t, \tag{4.42}$$

where a^*_{t-1} is the action taken by the most successful player in stage $t-1$. Based on the validity of the statement in stage $t-1$, we can adopt the same logic of the proof for $x_1^* = \theta + \frac{\beta_1}{\alpha_1}(\theta - y_1)$ in Lemma 4.2, and get

$$\tilde{x}^*_{t-1} = \theta + \frac{\tilde{\beta}_{t-1}}{\tilde{\alpha}_{t-1}}(\theta - \tilde{y}_{t-1}). \tag{4.43}$$

Inserting the stage strategy $a_{i,t-1}$ and (4.43) into (4.42) yields

$$
\begin{aligned}
y_t &= \frac{\tilde{\beta}_{t-1} \tilde{y}_{t-1} + (1-\gamma)\tilde{\alpha}_{t-1} \tilde{x}^*_{t-1}}{\tilde{\beta}_{t-1} + (1-\gamma)\tilde{\alpha}_{t-1}} + \xi_t \\
&= \frac{\tilde{\beta}_{t-1} \tilde{y}_{t-1} + (1-\gamma)\tilde{\alpha}_{t-1}\theta + (1-\gamma)\tilde{\beta}_{t-1}(\theta - \tilde{y}_{t-1})}{\tilde{\beta}_{t-1} + (1-\gamma)\tilde{\alpha}_{t-1}} + \xi_t.
\end{aligned} \tag{4.44}
$$

Rearranging (4.44) gives us

$$\frac{\left(\tilde{\beta}_{t-1} + (1-\gamma)\tilde{\alpha}_{t-1} \right) y_t - \gamma\tilde{\beta}_{t-1}\tilde{y}_{t-1}}{(1-\gamma)(\tilde{\alpha}_{t-1} + \tilde{\beta}_{t-1})} = \theta + \frac{\tilde{\beta}_{t-1} + (1-\gamma)\tilde{\alpha}_{t-1}}{(1-\gamma)(\tilde{\alpha}_{t-1} + \tilde{\beta}_{t-1})}\xi_t. \tag{4.45}$$

Denote the LHS of (4.45) by Υ_t and clearly we have

$$\frac{\tilde{\beta}_{t-1} + (1-\gamma)\tilde{\alpha}_{t-1}}{(1-\gamma)(\tilde{\alpha}_{t-1} + \tilde{\beta}_{t-1})}\xi_t \sim \mathcal{N}\left(0, \frac{1}{\Omega_t \beta_t} \right), \tag{4.46}$$

where $\Omega_t = \left[\frac{(1-\gamma)(\tilde{\alpha}_{t-1}+\tilde{\beta}_{t-1})}{\tilde{\beta}_{t-1}+(1-\gamma)\tilde{\alpha}_{t-1}}\right]^2$. Player i can thus update her belief on θ using (4.41) and (4.45). The two noise terms in these are independent and both normally distributed, the conjugacy property of normal prior and posterior distributions gives us

$$\theta|h_{it} \sim \mathcal{N}\left(\frac{\tilde{\beta}_{t-1}\tilde{y}_{t-1} + \tilde{\alpha}_{t-1}\tilde{x}_{i,t-1} + \alpha_t x_{it} + \Omega_t\beta_t\Upsilon_t}{\tilde{\beta}_{t-1} + \tilde{\alpha}_{t-1} + \alpha_t + \Omega_t\beta_t}, \frac{1}{\tilde{\beta}_{t-1} + \tilde{\alpha}_{t-1} + \alpha_t + \Omega_t\beta_t}\right). \quad (4.47)$$

The proposition also defines $\tilde{\alpha}_t = \tilde{\alpha}_{t-1} + \alpha_t$, $\tilde{x}_{it} = (\tilde{\alpha}_{t-1}\tilde{x}_{i,t-1} + \alpha_t x_{it})/\tilde{\alpha}_t$, $\tilde{\beta}_t = \tilde{\beta}_{t-1} + \Omega_t\beta_t$, and $\tilde{y}_t = (\tilde{\beta}_{t-1}\tilde{y}_{t-1} + \Omega_t\beta_t\Upsilon_t)/\tilde{\beta}_t$. We thus rewrite (4.47) as

$$\theta|h_{it} \sim \mathcal{N}\left(\frac{\tilde{\beta}_t\tilde{y}_t + \tilde{\alpha}_t\tilde{x}_{it}}{\tilde{\beta}_t + \tilde{\alpha}_t}, \frac{1}{\tilde{\beta}_t + \tilde{\alpha}_t}\right). \quad (4.48)$$

We then impose here again the logic in the proof of Lemma 4.2 by analogously defining $\mu_t = \tilde{\alpha}_t/(\tilde{\beta}_t+\tilde{\alpha}_t)$ as well as getting $\bar{E}_t^n(\theta) = (1-\mu_t^n)\tilde{y}_t+\mu_t^n\theta$ and $E_{it}(\bar{E}_t^n(\theta)) = (1-\mu_t^{n+1})\tilde{y}_t+\mu_t^{n+1}\tilde{x}_{it}$. As a result, we must have in an equilibrium

$$a_{it} = \frac{\tilde{\beta}_t\tilde{y}_t + (1-\gamma)\tilde{\alpha}_t\tilde{x}_{it}}{\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t}. \quad (4.49)$$

So in the equilibrium, given the statement of the proposition as well as property (4.48) are both true for stage $t-1$, it must also be the case for stage t. And the validity of both the statement and the property for stage 1 are shown in Lemma 4.2, uniqueness is depicted by (4.49) and the proof is thus completed by induction. $\qquad\square$

Proof of Corollary 4.1.

For the first half of the statement, based on the inductive logic used in the proof above, the way we showed (4.43) also proves $\tilde{x}_t^* = \theta + \frac{\tilde{\beta}_t}{\tilde{\alpha}_t}(\theta - \tilde{y}_t)$. For the second half, notice that

$$a_{it} - \theta = \frac{\tilde{\beta}_t(\tilde{y}_t - \theta) + (1-\gamma)\tilde{\alpha}_t(\tilde{x}_{it} - \theta)}{\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t}. \quad (4.50)$$

We rewrite the two noise terms in the equation above as

$$
\begin{aligned}
\tilde{\alpha}_t(\tilde{x}_{it} - \theta) &= \tilde{\alpha}_{t-1}\tilde{x}_{i,t-1} + \alpha_t x_{it} - (\tilde{\alpha}_{t-1} + \alpha_t)\theta \\
&= \sum_{n=1}^{t} \alpha_n(x_{in} - \theta) \\
&= \sum_{n=1}^{t} \alpha_n \varepsilon_{in}, \qquad\qquad\qquad (4.51) \\
\tilde{\beta}_t(\tilde{y}_t - \theta) &= \tilde{\beta}_t \tilde{y}_{t-1} + \Omega_t \beta_t \Upsilon_t - (\tilde{\beta}_t + \Omega_t \beta_t)\theta \\
&= \beta_1(y_1 - \theta) + \sum_{n=2}^{t} \Omega_n \beta_n(\Upsilon_n - \theta) \\
&= \beta_1(y_1 - \theta) + \sum_{n=2}^{t} \sqrt{\Omega_n}\beta_n \xi_n. \qquad (4.52)
\end{aligned}
$$

The last equality of (4.52) results from (4.45). And we already know $\varepsilon_{in} \sim \mathcal{N}(0, 1/\alpha_n)$, $\sqrt{\Omega_n}\xi_n \sim \mathcal{N}(0, \Omega_n/\beta_n)$ for $n \geq 2$, and $(y_1 - \theta) \sim \mathcal{N}(0, 1/\beta_1)$. Because of the independence between noises, we thus have

$$
\begin{aligned}
Var[\tilde{\alpha}_t(\tilde{x}_{it} - \theta)] &= \sum_{n=1}^{t} \alpha_n^2 \cdot \frac{1}{\alpha_n} \\
&= \tilde{\alpha}_t, \qquad\qquad\qquad (4.53) \\
Var[\tilde{\beta}_t(\tilde{y}_t - \theta)] &= \beta_1 + \sum_{n=2}^{t} \beta_n^2 \cdot \frac{\Omega_n}{\beta_n} \\
&= \tilde{\beta}_t. \qquad\qquad\qquad (4.54)
\end{aligned}
$$

All the properties above together suggest $(a_{it} - \theta)$ is normally distributed with mean 0 and variance $\frac{\tilde{\beta}_t + (1-\gamma)^2 \tilde{\alpha}_t}{\left(\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t\right)^2}$. The whole statement is proved. $\qquad\square$

Proof of Proposition 4.2.

According to the definition of d_{1t}, we have

$$
\begin{aligned}
d_{1t} &= \frac{\tilde{\beta}_t(\tilde{y}_t - \theta) + (1 - \gamma)\int_0^1 \tilde{\alpha}_t(\tilde{x}_{it} - \theta)\,\mathrm{d}i}{\tilde{\beta}_t + (1 - \gamma)\tilde{\alpha}_t} \\
&= \frac{\tilde{\beta}_t(\tilde{y}_t - \theta) + (1 - \gamma)\int_0^1 \sum_{n=1}^t \alpha_n\varepsilon_{in}\,\mathrm{d}i}{\tilde{\beta}_t + (1 - \gamma)\tilde{\alpha}_t} \\
&= \frac{\tilde{\beta}_t(\tilde{y}_t - \theta) + (1 - \gamma)\sum_{n=1}^t \left[\int_0^1 \alpha_n\varepsilon_{in}\,\mathrm{d}i\right]}{\tilde{\beta}_t + (1 - \gamma)\tilde{\alpha}_t} \\
&= \frac{\tilde{\beta}_t(\tilde{y}_t - \theta)}{\tilde{\beta}_t + (1 - \gamma)\tilde{\alpha}_t}.
\end{aligned}
\tag{4.55}
$$

The second equality above results from (4.51) and the last equality is because $\int_0^1 \alpha_n\varepsilon_{in}\,\mathrm{d}i = 0$ for each n. Property (4.54) thus leads to

$$
d_{1t} \sim \mathcal{N}\left(0, \frac{\tilde{\beta}_t}{\left(\tilde{\beta}_t + (1 - \gamma)\tilde{\alpha}_t\right)^2}\right),
\tag{4.56}
$$

the first part is proved. It is then straightforward to see $\partial V_{1t}/\partial\tilde{\alpha}_t < 0$. To check the last property, we have

$$
\begin{aligned}
\frac{\partial V_{1t}}{\partial\tilde{\beta}_t} &= \frac{\left(\tilde{\beta}_t + (1 - \gamma)\tilde{\alpha}_t\right)^2 - 2\tilde{\beta}_t(\tilde{\beta}_t + (1 - \gamma)\tilde{\alpha}_t)}{\left(\tilde{\beta}_t + (1 - \gamma)\tilde{\alpha}_t\right)^4} \\
&= \frac{(1 - \gamma)^2\tilde{\alpha}_t^2 - \tilde{\beta}_t^2}{\left(\tilde{\beta}_t + (1 - \gamma)\tilde{\alpha}_t\right)^4},
\end{aligned}
\tag{4.57}
$$

so $\partial V_{1t}/\partial\tilde{\beta}_t < 0$ holds if and only if $\frac{\tilde{\beta}_t}{\tilde{\alpha}_t} > 1 - \gamma$. $\qquad\square$

Proof of Proposition 4.3.

The definition of d_{2t} gives us

$$
\begin{aligned}
d_{2t} &= \frac{\tilde{\beta}_t \tilde{y}_t + (1-\gamma)\tilde{\alpha}_t \tilde{x}_{it}}{\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t} - \frac{\tilde{\beta}_t \tilde{y}_t + (1-\gamma)\int_0^1 \tilde{\alpha}_t \tilde{x}_{jt}\,\mathrm{d}j}{\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t} \\
&= \frac{(1-\gamma)\tilde{\alpha}_t(\tilde{x}_{it} - \theta)}{\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t},
\end{aligned}
\tag{4.58}
$$

combining with (4.53) we have

$$
d_{2t} \sim \mathcal{N}\left(0\,,\ \frac{(1-\gamma)^2\tilde{\alpha}_t}{\left(\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t\right)^2}\right).
\tag{4.59}
$$

The first half is proved. For the second part of the statement, it is straightforward to see it is true by simply checking the signs of $\partial V_{2t}/\partial\tilde{\beta}_t$ and $\partial V_{2t}/\partial\tilde{\alpha}_t$. $\qquad\square$

Proof of Proposition 4.4.

Based on the previous two propositions we have

$$
V_{1t} + \frac{V_{2t}}{1-\gamma} = \frac{1}{\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t}.
\tag{4.60}
$$

Obviously, the RHS of the equation above is always decreasing in from t to $t' = t+1$ for any t. So as long as $V_{1t} < V_{1t'}$, we must have $V_{2t} > V_{2t'}$; and given $V_{2t} < V_{2t'}$, we must have $V_{1t} > V_{1t'}$. $\qquad\square$

Proof of Proposition 4.5.

In the equilibrium we have

$$
\begin{aligned}
d_{3t} &= \frac{(1-\gamma)\tilde{\alpha}_t(x_{it} - x_t^*)}{\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t} \\
&= \frac{(1-\gamma)\left[\tilde{\alpha}_t(x_{it} - \theta) + \tilde{\beta}_t(\tilde{y}_t - \theta)\right]}{\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t}.
\end{aligned}
\tag{4.61}
$$

d_{3t} is thus normally distributed with mean zero and variance

$$
V_{3t} = \frac{(1-\gamma)^2(\tilde{\alpha}_t + \tilde{\beta}_t)}{\left(\tilde{\beta}_t + (1-\gamma)\tilde{\alpha}_t\right)^2} = V_{2t} + (1-\gamma)^2 V_{1t}.
\tag{4.62}
$$

This completes the proof. $\qquad\square$

4.7.2   Appendix B: Data and Codes

All datasets and codes used in the empirical study can be downloaded from the following link:

https://www.dropbox.com/sh/atmw3p8ofl7i0ob/16FKekFBi3

To run the programs, you need to download the whole folder, have Matlab program with version later than R2012a installed on the computer, and follow the instructions in the "Read_Me" file.

Chapter 5

Conclusions

I include in this dissertation three works that mainly investigate the effects of public signals on the results of the games, while respecting the heterogeneity in players' roles in the economy. The games are all in dynamic contexts and exhibit strategic complementarities among players. As a result, abrupt and significant changes, such as the sudden change of a regime or the collapse of a financial system, may occur in the equilibrium. Although there are also other methods used in the literature to deal with such thorough changes,[1] we adopt the global games initiated by Morris and Shin (1998, 2000, 2002), for their suitability in embedding our specific questions and concerns in the model.

On the theoretical side, this dissertation develops dynamic games that formalize and add different aspects and features to the standard setting of global games. More specifically, Chapter 2 proposes a general framework to investigate the efficacy of an intervention attempting to stabilize a regime by carrying out a mechanism that changes individuals' social

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[1]For instance, another strand dealing with this kind of scenarios is the long-standing catastrophe theory. See Thom (1975) for initial contributions in this strand, Zeeman (1974) and Varian (1979) for applications in Economics, and Rosser Jr (2007) for a comprehensive literature review. The catastrophe theory and other dynamic systems methods are not suitable for our studies here because although they provide clear descriptions of the catastrophic scenarios at the macro-level, they do not take into account the micro-foundations of the economy; as a result, they are inappropriate in analyzing the interactions between different players and how these micro-level considerations contribute to the catastrophic changes.

class labels based on their previous actions; Chapter 3 provides a general model that is suitable for embedding a variety of intervention policies by the authority, and is thus capable of comparing their efficacies in a common economic environment; noticing that players often observe the previous winners' actions as public signals, Chapter 4 formalizes this phenomenon in a dynamic beauty-contest game.

On the empirical side, several questions of interest are answered in this dissertation. Regarding the problem of choosing between a mechanism kicking out traitors from the stakeholder group and a mechanism absorbing supporters from the speculator group, Chapter 2 suggests that the latter is generally a better choice. Although there have been criticisms of the speculators for triggering a financial crisis and discussions on different intervention policies leading to different results, Chapter 3 formalizes all these ideas in a game-theoretical model, and uses its results to explain the roles of the speculators and the authorities in the 1997 Asian financial crisis. By conducting an empirical study based on the results obtained in the theoretical part, Chapter 4 finds that the diversity in faces of the contestants of the Miss Korea pageant has actually been increasing over the last 20 years. As a result, the "same-looking-contestant" viewpoint widespread on internet and mass media should be faulted for using inappropriate data.

Although global games are mainly used as a theoretical tool for qualitative analysis in almost all other papers in the literature, I would like to continue my research in this strand in the future, by focusing more on applications and further exploring the suitability for empirical studies. The empirical work in Chapter 4 is my first step in this direction.

Bibliography

[1] Allen, F., Morris, S. and Shin, H. S. (2006). Beauty contests and iterated expectations in asset markets, *The Review of Financial Studies* **19**(3): 719–752.

[2] Amador, M. and Weill, P.-O. (2006). Learning by matching, *2006 Meeting Papers*, number 373, Society for Economic Dynamics.

[3] Angeletos, G.-M., Hellwig, C. and Pavan, A. (2006). Signaling in a global game: Coordination and policy traps, *Journal of Political Economy* **114**(3): 452–484.

[4] Angeletos, G.-M., Hellwig, C. and Pavan, A. (2007). Dynamic global games of regime change: Learning, multiplicity, and the timing of attacks, *Econometrica* **75**(3): 711–756.

[5] Angeletos, G.-M. and Werning, I. (2006). Crises and prices: Information aggregation, multiplicity, and volatility, *The American Economic Review* **96**(5): 1720–1736.

[6] Atkeson, A. (2000). [Rethinking multiple equilibria in Macroeconomic modeling]: Comment, *NBER Macroeconomics Annual* **15**: 162–171.

[7] Baig, T. and Goldfajn, I. (2002). Monetary policy in the aftermath of currency crises: The case of Asia, *Review of International Economics* **10**(1): 92–112.

[8] Barlow, R. E., Marshall, A. W. and Proschan, F. (1963). Properties of probability distributions with monotone hazard rate, *The Annals of Mathematical Statistics* **34**(2): 375–389.

[9] Bensaid, B. and Jeanne, O. (1997). The instability of fixed exchange rate systems when raising the nominal interest rate is costly, *European Economic Review* **41**(8): 1461–1478.

[10] Carlsson, H. and van Damme, E. (1993). Global games and equilibrium selection, *Econometrica* **61**(5): 989–1018.

[11] Chassang, S. (2010). Fear of miscoordination and the robustness of cooperation in dynamic global games with exit, *Econometrica* **78**(3): 973–1006.

[12] Cheetham, R. (1998). Asia crisis, *conference, US-ASEAN-Japan Policy Dialogue. School of Advanced International Studies of Johns Hopkins University, June*, pp. 7–9.

[13] Corsetti, G., Dasgupta, A., Morris, S. and Song Shin, H. (2004). Does one soros make a difference? a theory of currency crises with large and small traders, *The Review of Economic Studies* **71**(1): 87–113.

[14] Corsetti, G., Guimaraes, B. and Roubini, N. (2006). International lending of last resort and moral hazard: a model of IMF's catalytic finance, *Journal of Monetary Economics* **53**(3): 441–471.

[15] Costain, J. (2007). A herding perspective on global games and multiplicity, *The BE Journal of Theoretical Economics* **7**(1): 22.

[16] Diamond, D. and Dybvig, P. (1983). Bank runs, deposit insurance, and liquidity, *Journal of Political Economy* pp. 401–419.

[17] Edmond, C. (2013). Information manipulation, coordination, and regime change, *The Review of Economic Studies* **80**(4): 1422–1458.

[18] Feenstra, R., Hamilton, G. and Lim, E. (2002). Chaebol and catastrophe: A new view of the Korean business groups and their role in the financial crisis, *Asian Economic Papers* **1**(2): 1–45.

[19] Fischer, S. (2002). The Asian crisis: A view from the IMF, *Journal of International Financial Management & Accounting* **9**(2): 167–176.

[20] Flood, R. and Jeanne, O. (2005). An interest rate defense of a fixed exchange rate?, *Journal of International Economics* **66**(2): 471–484.

[21] Frankel, D., Morris, S. and Pauzner, A. (2003). Equilibrium selection in global games with strategic complementarities, *Journal of Economic Theory* **108**(1): 1–44.

[22] Goldstein, I. (2005). Strategic complementarities and the twin crises, *The Economic Journal* **115**(503): 368–390.

[23] Goldstein, I. and Pauzner, A. (2005). Demand–deposit contracts and the probability of bank runs, *The Journal of Finance* **60**(3): 1293–1327.

[24] Goldstein, M. (1998). The Asian financial crisis: Causes, cures, and systemic implications, *Policy Analyses in International Economics*, Vol. 55, Institute for International Economics.

[25] Haller, H. (2000). Non-additive beliefs in solvable games, *Theory and Decision* **49**(4): 313–338.

[26] Haller, H. and Yi, M. (2013). Paths of a continuum of independent random variables, *Working Papers, Virginia Polytechnic Institute and State University, Department of Economics* .

[27] Harsanyi, J. (1973). Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points, *International Journal of Game Theory* **2**(1): 1–23.

[28] Harsanyi, J. C. and Selten, R. (1988). *A General Theory of Equilibrium Selection in Games*, MIT Press Books, Cambridge: MIT Press.

[29] Hughes, H. (1999). Crony capitalism and the east Asian currency and financial 'crises', *Policy* **15**: 3–9.

[30] Jao, Y. (2001). *The Asian Financial Crisis and The Ordeal of Hong Kong*, Greenwood Publishing Group.

[31] Johnson, S. and Mitton, T. (2003). Cronyism and capital controls: Evidence from Malaysia, *Journal of Financial Economics* **67**(2): 351–382.

[32] Jolliffe, I. (2002). *Principal Component Analysis*, New York: Springer.

[33] Judd, K. L. (1985). The law of large numbers with a continuum of iid random variables, *Journal of Economic Theory* **35**(1): 19–25.

[34] Kaminsky, G. and Reinhart, C. (1999). The twin crises: the causes of banking and balance-of-payments problems, *The American Economic Review* pp. 473–500.

[35] Kaplan, E. and Rodrik, D. (2002). Did the Malaysian capital controls work?, *Preventing Currency Crises in Emerging Markets*, University of Chicago Press, pp. 393–440.

[36] Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*, London: Macmillan.

[37] Kindleberger, C. and Aliber, R. (2011). *Manias, Panics and Crashes: A History of Financial Crises*, Palgrave Macmillan.

[38] Krugman, P. (1999). *The Accidental Theorist: and Other Dispatches from the Dismal Science*, WW Norton.

[39] Lahiri, A. and Végh, C. (2003). Delaying the inevitable: Interest rate defense and balance of payments crises, *Journal of Political Economy* **111**(2): 404–424.

[40] Lanman, S. (19 May 2009). Greenspan says Hong Kong's Yam was right to buy stocks in 1998, *Bloomberg* .

[41] Moreno, R. (1998). What caused east Asia's financial crisis?, *FRBSF Economic Letter* **98**(24): 1–3.

[42] Morris, S. and Shin, H. (1998). Unique equilibrium in a model of self-fulfilling currency attacks, *The American Economic Review* **88**(3): 587–597.

[43] Morris, S. and Shin, H. (2000). Rethinking multiple equilibria in Macroeconomic modeling, *NBER Macroeconomics Annual* **15**: 139–161.

[44] Morris, S. and Shin, H. (2003). Global games: Theory and applications, *Advances in Economics and Econometrics: Theory and Applications, Eighth world Congress*, Vol. 1, Cambridge: Cambridge University Press, pp. 56–114.

[45] Morris, S. and Shin, H. S. (2002). Social value of public information, *The American Economic Review* **92**(5): 1521–1534.

[46] Morris, S. and Shin, H. S. (2004). Coordination risk and the price of debt, *European Economic Review* **48**(1): 133–153.

[47] Nip, A. (05 July 2007). Soros manager regrets attack on HK dollar, *The Standard* .

[48] Palma, G. (2002). The three routes to financial crises, *International Capital Markets: Systems in Transition* p. 297.

[49] Radelet, S. and Sachs, J. (2000). The onset of the east Asian financial crisis, *Currency crises*, University of Chicago Press, pp. 105–162.

[50] Rochet, J.-C. and Vives, X. (2004). Coordination failures and the lender of last resort: was Bagehot right after all?, *Journal of the European Economic Association* **2**(6): 1116–1147.

[51] Rosser Jr, J. B. (2007). The rise and fall of catastrophe theory applications in economics: Was the baby thrown out with the bathwater?, *Journal of Economic Dynamics and Control* **31**(10): 3255–3280.

[52] Rubinstein, A. (1989). The electronic mail game: Strategic behavior under "almost common knowledge", *The American Economic Review* **79**(3): 385–391.

[53] Sima, G. (2011). *Comprehensive Mirror to Aid in Government (in Chinese)*, Vol. 1, Beijing: Zhonghua Book Company.

[54] Soros, G. (1998). *The Crisis of Global Capitalism: Open Society Endangered*, New York: PublicAffairs.

[55] Thom, R. (1975). *Structural stability and morphogenesis: an outline of a general theory of models*, Advanced book program, W. A. Benjamin.

[56] Topkis, D. (1979). Equilibrium points in nonzero-sum n-person submodular games, *SIAM Journal on Control and Optimization* **17**(6): 773–787.

[57] Turk, M. and Pentland, A. (1991). Eigenfaces for recognition, *Journal of Cognitive Neuroscience* **3**(1): 71–86.

[58] Varian, H. R. (1979). Catastrophe theory and the business cycle, *Economic Inquiry* **17**(1): 14–28.

[59] Vives, X. (1990). Nash equilibrium with strategic complementarities, *Journal of Mathematical Economics* **19**(3): 305–321.

[60] Wang, C. (2010). Bailouts and bank runs: Theory and evidence from TARP, *Working Papers, HSBC Business School, Peking University* .

[61] Zeeman, E. C. (1974). On the unstable behaviour of stock exchanges, *Journal of Mathematical Economics* **1**(1): 39–49.