A Comparison of Optimum Grain Hedging Strategies Using Commodity Options and Futures Contracts: An Application of Portfolio Theory

by

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(ABSTRACT)

Commodity options add a new dimension to grain farmers' marketing alternatives. Producers of grain can now effectively ensure themselves a floor price without the risk of production shortfalls resulting in losses due to overhedged positions.

The purpose of this study was to determine optimum hedge levels using both commodity options and futures contracts and then compare the hedging tools given various location, crop mix, and levels of financial leverage. The study used portfolio theory where hedging strategies were simulated over time and minimum-variance hedge levels determined. Crop diversification and financial leverage were addressed using Quadratic Programming techniques. Selected strategies were tested over a new data set.

Commodity options are superior to futures contracts as a hedging tool for early season hedges. This was particularly true for crops with highly variable yields. The results also indicate that commodity options are a viable alternative for reducing long-run income variation and that crop diversification reduced income variation but did not reduce the overall need to hedge.

The study presented here is unique in a number of ways. Initially, very little if any published work is available on hedging grain with commodity options contracts. Secondly, the study addresses hedging strategies under the realm of production uncertainty. Finally, the study demonstrates there are E-V efficient alternatives to strict cash sales.

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Lastly, I acknowledge the pioneering work of previous researchers of this thesis topic. May we all work together to support and improve the livelihood of those we admire most, the men and women who feed and clothe us.

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Chapter 1. Introduction

Product marketing is now viewed by many farmers as their number one problem. This is evidenced by the considerable number of hedging studies whose purpose was to develop strategies to reduce price risk (Kenyon, 1984a). Marketing strategy research indicates that marketing is a complex discipline with no simple set of solutions. A farmers' approach to marketing depends on a multitude of factors which include crop mix, location, storage availability, cash flow needs, financial strength, risk preference and overall level of interest in marketing. Most producers, however, become frustrated with the complexity of marketing their product and subject themselves to volatile swings in market prices. This problem can only be solved through extensive market research and farmer education programs.

Grain producers have relied primarily on two alternative markets for pricing their products. They use the cash markets with its assortment of spot pricing, forward cash contracts, delayed pricing, price later contracts, basis contracts, and etc.; or, they hedge their products in the futures market. The cash markets have been the primary markets used by grain producers to price their products. A Commodity Futures Trading Commission survey by Helmuth indicated that only 11.9% of the grain producers surveyed had hedged their crop with futures during the period 1972 to 1976.

Futures markets have lacked support from farmers due to lack of understanding of the markets, lack of operating capital and/or adequate financial credit needed to maintain margin requirements (Harris and Baker), or lack of need due solely to the producer's price-yield relationship (Rolfo, McKinnon). Futures contracts, however, can provide producers with a flexible alternative to strict cash sales. Futures act as a forward pricing mechanism independent of local cash conditions (Carlton). Cash markets force producers to accept a spot price or a deferred price at a stated basis. A hedge allows the producer to set the general price level of the commodity without commitment to a basis level. In addition to possible basis appreciation, futures markets are liquid and allow producers to lift hedges in anticipation of a future price increase. The flexibility, of pricing in futures, however, is not gained without risk of losses. Producers stand to lose substantial amounts of equity while attempting to lift and replace hedges during volatile market conditions. Once hedges have been lifted, farmers subject themselves to price variability. There is evidence, however, which indicates that above normal profits can be obtained through selective hedging using technical analysis (Franzmann, Brown and Purcell; Shafer, Kenyon and Cooper; Irwin). A more detailed discussion of research contrary to the "Efficient Market Hypothesis" is presented later in the study.

The problems associated with lack of price flexibility in the cash market along with margin requirements and extreme volatility of the futures markets can be overcome with the use of commodity options (Kenyon, 1984b; Futures Magazine). Options offer a new addition to the producer's marketing alternatives. As described by Gardner, options can be viewed as insurance against a price decline. Producers can forward price with commodity options and effectively set a floor price for the commodity hedged.

The advantage of a commodity option as a potential hedging tool is the absence of margin calls, price flexibility and known price risks. The major concern with options is the relatively high cost of the option premium which must be paid to the option writer to assume price risks.

Commodity options can also be viewed as a source of additional income for those producers willing to write options on grain in storage or grain yet to be harvested. The very structure of the options market with its numerous strike prices, settlement dates, and corresponding put and call positions provide virtually an unlimited assortment of marketing strategies when combined with futures and cash market alternatives.

Neither of the three markets, cash, futures, or options can be adequately discussed unless the combination of price and production risk is incorporated into the analysis. Kenyon (1984b) and Gardner in separate studies both demonstrated income levels for different price-yield scenarios for hedges in these three markets. For instance, if a grain producer knew future production with certainty and expected a price increase, the cash market would lend the least expensive means of capturing the higher price. A certain price decline could best be avoided by an offsetting position in the futures markets. If, however, the producer feels the present price is sufficient and would like to insure against a price decline and not restrict upside price potential, then a put option should be purchased. Production uncertainty places the producer in the dilemma of an overhedged or an underhedged position. An overhedged futures position in the face of a declining futures' price brings great financial rewards to the producer; whereas, an overhedged futures position with increased futures prices can be financially devastating. The level of expected production to hedge and the hedging tool to use is a complex decision for a producer.

The problem faced by an individual farmer is that to achieve maximum income stability from hedging he/she needs to know the relationship between cash price received, production, futures price, and the resultant changes in income. For example, prices may rise in years when individual's yields are reduced; but, the decision on whether to hedge depends on whether total revenues are reduced or increased. Income stability would be greater with a strict cash position should revenues increase when quantity is reduced.

An approach to analyzing hedging strategies when price and output are stochastic is portfolio theory (Markowitz, 1952). Numerous studies have used the portfolio approach, (McKinnon; Peck; Johnson; Stein; Black; Berck; Ward and Fletcher; and Heifner). Portfolio theory has been used to analyze various areas of the futures markets from simultaneous pricing of futures, options, and forward contracts to individual producer hedging decisions. Studies related to optimum hedging strategies when prices and yields are uncertain are discussed later in detail.

Portfolio theory in its simplest form can perhaps explain the reluctance on the part of grain producers to hedge in futures markets (McKinnon, Rolfo). Assume a grain producer is located in a major agricultural producing area and experiences an abnormal weather condition which affects output. Most likely other producers of the product will be likewise affected. If price advances when production declines, or vice versa, income may be relatively stable. There would be no need to hedge. In such a situation, hedging to lock in a price when output is variable would increase the variability of income.

Portfolio theory can be used to explain the effects of crop diversification on overall farm risks and the effect of financial leverage. For example, a producer might possibly reduce risk exposure by producing two or more crops whose average returns are positive over time yet negatively correlated. Through crop diversification the producer may effectively eliminate the need to hedge. Likewise a producer holding half the farm assets in the form of cash or other secured assets may well favor holding a very "high-risk, high-return" cash position in the market.

A description of hedging is summed up very well by Fisher Black. "Commodity holdings are assets that form part of investors portfolios, either directly or indirectly. -- The notion that commodity holdings are priced like other assets means that investors who own commodities are able to diversify away risks. One way this can happen is through futures markets: ---".

Hedging with futures and options contracts is only one of many ways a producer can reduce risks. The benefit from hedging is that it can be a very inexpensive means of reducing risk which otherwise cannot be diversified away. Portfolio theory will be used here as the decision criterion to determine how much to hedge with each of these hedging tools.

1.1 Problem Statement

Hedging to the individual producer is a complex decision. The need to hedge and the optimum level to hedge depends on the producer's price-yield relationship, future's price, and the resultant changes in income from the cash and hedged positions. The decision on the correct hedging tool and use in appropriate combination with a cash position is a major problem for most producers.

1.2 Purpose

The purpose of this study was to compare optimum hedging strategies using commodity options and commodity futures contracts to determine which hedging tool is risk efficient for a given production, grain market situation, and producer risk preference. A second goal was to determine the level of expected production that should be hedged with each of these tools for a given set of price and yield expectations.

Final results of the study could be used to help producers, extension educators, and researchers determine the appropriate hedging tool and level of use for a given production and market situation. The study would also give policymakers additional information for the possible use of commodity options as an alternative price risk management tool to the current target price and loan programs.

1.3 Objectives

The major objective of the study was to compare commodity options and commodity futures contracts as hedging tools in the context of price and yield risk. Risk efficiency comparisons of futures and options were made under a mean-variance framework.

Minor objectives include:

1. Determination of optimal hedging strategies using commodity options and futures for single and multiple crop situations, various locations, varying levels of risk preference, and financial leverage.

2. Evaluation of the hedging tools within the optimal hedging framework to determine risk efficiency by simulating the decision making process over a new time series and comparing risk efficiency.

1.4 Review of Related Literature

Early theories of hedging viewed the activity as a transfer of risk from hedgers to speculators. J. M. Keynes in his Treaties on Money developed the theory of "normal backwardation" where he hypothesized that futures prices are discounted to present prices due to hedgers willingness to pay risk premiums for speculators to assume the price risk over a period of time. Holbrook Working viewed hedging as an act of arbitrage between the futures and the cash markets. Working looked at the role of hedging not as a means to avoid price risk but as a profit motive stemming from favorable price movements between spot and futures markets.

Modern theories of real world hedging activity include the fact that some handlers of commodities are completely hedged, whereas, others are only partially hedged. Handlers and merchants of commodities hedge at levels which reduce their risk exposure relative to how close the value of their inventory varies with an equivalent short futures position. The percentage of inventory they hedge will vary depending on relative price changes in the two markets and their price expectations.

An early proponent of this portfolio type of approach was Leland Johnson. He theorized that hedging was actually a portfolio decision where a handler attempts to reduce the variance of return relative to a specified level of return. He postulated the following expression: minimize

$$V(R) = X_i^2 \sigma_i^2 (1 - \rho_{ij}^2) \tag{1.1}$$

where,

$V(R)$ = the variance of returns,

$X_i^2\sigma_i^2$ = the variance of returns from holding X_i units,

ρ = the correlation coefficient between i and j, and

i,j = the cash and futures positions.

If the price movements are perfectly correlated, ρ is equal to 1 and the overall price risk is reduced to zero. Johnson continued the analysis comparing degrees of hedging and speculation using indifference curve techniques comparing expected returns for risk utility functions.

Ronald McKinnon was an early user of portfolio theory to describe hedging activities for primary producers. His article "Futures Markets, Buffer Stocks, and Income Stability for Primary

Producers" was an analysis to determine whether futures could replace buffer stocks as an income stabilizer. A major contribution, however, was his micro model of hedging which has been the base of several hedging studies including this one.

McKinnon viewed the farmer's hedging decision in a mean-variance framework. He attempted to minimize variance by determining the optimal percentage of the crop to hedge. The formulation is as follows: minimize

$$\sigma_y^2 = E(\tilde{Y}^2) - [E(\tilde{P}\tilde{X})]^2, \text{ w.r.t. } X_f \tag{1.2}$$

where

$$E(\tilde{Y}) = E(\tilde{P}\tilde{X}) + X_f E(\tilde{P}_f - P), \tag{1.3}$$

$\tilde{P}$ = distribution of cash price,

$\tilde{P}_f$ = distribution of futures price,

$\tilde{X}$ = distribution of output,

P = hedged price,

X_f = the amount to hedge,

E = the expectations variable, and

σ_y^2 = the variance of income.

The theoretical conclusion of this study proved that when both price and output are stochastic, the optimal hedge for a risk averse producer may be less than unity.

Additional early work which used portfolio theory as a basis for hedging activities included an article by Ward and Fletcher and a separate article by Anne Peck. Both articles assumed only price as the relevant stochastic variable. Ward and Fletcher developed a theoretical model of optimal decisions in cash and futures positions for farmers and marketing firms. The analysis was primarily a theoretical comparison of futures and forward contracting under a mean-variance criterion and their similarities and differences discussed.

Anne Peck used portfolio theory and examined the hedging of egg production. She hypothesized that price expectations and related forecast errors rather than historical prices and related variances more clearly explained hedging activities. The work was set in an optimal hedge setting and demonstrated that optimal hedge levels for egg producers ran consistently between 75% and 95% of production. The results indicated that optimal hedging strategies, derived from a portfolio approach, markedly reduced the producer's exposure to unpredictable price variation.

A later study to use portfolio theory and the mean-variance approach was performed by Jacques Rolfo for the case of cocoa producers. Rolfo basically used the McKinnon approach but maximized expected utility with respect to the amount of the crop hedged. His formulation is as follows: maximize

$$EU = E(\tilde{W}) - m[\mathrm{var}(\tilde{W})], \text{ w.r.t. to } n, \tag{1.4}$$

where,

$$\tilde{W} = \tilde{P}\tilde{Q} + n(f - \tilde{P}_f), \tag{1.5}$$

$\tilde{W}$ = distribution of income from the cash and hedged position,.

$\tilde{P}\tilde{Q}$ = distribution of gross returns from the distribution of cash price and quantity produced,

n = amount hedged,

f = forward price when the hedge is placed,

$\tilde{P}_f$ = distribution of futures prices when the hedge is lifted, and

m = farmers risk parameter.

The optimal hedge position is:

$$n^* = \mathrm{cov}(\tilde{P}\tilde{Q}, \tilde{P}_f)/\mathrm{var}(\tilde{P}_f) + [f - E(\tilde{P}_f)]/2m\,\mathrm{var}(\tilde{P}_f) \qquad (1.6)$$

In addition to the mean-variance framework, Rolfo used a logarithmic (Bernoulli) utility function which allowed for the more realistic assumption of decreasing absolute risk aversion and constant relative risk aversion.

A major contribution of Rolfo's study is the empirical application of portfolio theory. The optimum hedges for all four primary cocoa producing countries were less than one. The percent hedged was less for the African countries which were in the heart of the cocoa producing area relative to Brazil where production and price are less correlated. Rolfo concluded that producers who were not risk averse may not hedge at all or may even speculate. Producers would increase speculation should the forward price be higher than the expected price.

A recent study to determine optimum hedges given yield and price uncertainty was performed by Peter Berck. Berck used portfolio theory with quadratic programming to determine optimum hedges and crop mix for preplant decisions given a diversified portfolio of crop alternatives. The empirical results included a description of the mean-variance frontier, the choice of an efficient plan, and the benefits of diversification and the effects of diversification on hedging.

Berck makes several assumptions which have a major effect on the results of his study. He allows farmers to make simultaneous decisions on crops and futures, evaluates losses differently from gains, and used a credit constraint on futures holdings. He also allowed forecasting of gains on futures and speculation on these forecasts. The use of routine speculation under the set of assumptions had profound effects on the empirical results.

Berck's findings on the benefits of diversification and the effects of diversification on hedging are very useful. Crop diversification reduced the variance of returns almost a third. The use of futures hedging with crop diversification reduced the variance by two-thirds. These levels of reduced variances are possible even when only approximately 10% of the crop mix is routinely hedged.

Bercks' major conclusions were that the cost of hedging and the opportunity to diversify risk through other crops substantially change the optimal hedge. He suggests the study of this question should be placed in a dynamic setting where the optimal path of a hedge between planting and harvest is determined. Even though this study will not be set in an optimal control framework, it will address some of the issues of within-crop year hedging.

Two recent applied studies using quadratic programming (QP) to analyze production and marketing decisions were on feeder calf backgrounding systems. Bobst, Greenewald, and Davis used portfolio theory and QP to evaluate 16 production-marketing alternatives which included both hedged and cash market positions. The problem was formulated to solve for the optimum hedge but selected the production-marketing alternatives which were risk efficient. Again, the optimum strategy was to hedge less than 100% of production. The optimum strategy suggested was to routinely hedge 76% of production.

Thomas Sporleder and Jim Winder recently published a technical article on the performance of live cattle options in a hedging portfolio. They used the alternatives of purchasing put options or writing call options to establish a short hedge position and determined optimum hedge levels using quadratic programming techniques. The results indicate that net returns increased while variability of returns declined when minimum variance hedge levels with options were used versus strict cash sales. Minimum variance hedges reduced the coefficient of variation of net returns by 38% with a 13% increase in net returns. Most of the returns and income stability came from writing call options rather than purchasing puts. Since writing calls is an initial long rather than a short position, questions would have to be raised as to whether these are true hedged positions. Writing options showed positive returns because the analysis was performed over the period 1980-1984 when cattle prices were relatively stable.

Simulation has been used in numerous hedging studies including this one. The process includes a broad spectrum of research techniques. Simulation in hedging studies has generally been used to simulate a production process through time comparing hedging strategies in a mean-variance framework using historical yields and prices. The means and variances of a number of strategies are compared to each other and to a cash position and superior strategies selected.

Hedging strategies have generally been classified into three basic groups; routine hedges, selective hedges, and multiple selective hedges. Routine hedges are usually placed prior to the production process and are intended to reduce price risk. Empirical results show that income loss associated with risk reduction is beyond what most producers see as acceptable (McCoy and Price, Kenyon and Cooper, Eddleman and Moya-Rodriguez). Selective and multiple selective hedging strategies are primarily concerned with timing of the hedge. Selective hedging usually involves placing the hedge once before or during the production process based on some decision rule such as profitability. Multiple selective hedging strategies involve placing and lifting hedges depending on price expectations. Most multiple hedging strategies used technical trading techniques. (Kenyon and Shapiro; Shafer, Griffin, and Johnston; Leuthold and Modler; Franzmann; Brown and Purcell).

Chapter 2. Theoretical Models of Producer Hedging

To choose among risky prospects, we assume one must have utility criteria which would allow the decision maker to choose between the perceived amount of uncertainty and the satisfaction derived from additional income. Criteria for choosing among risky prospects were developed by Von Neumann and Morgenstern in a set of axioms which identifies ordering, transitivity, substitution, and certainty equivalence among choices. The axioms provide a basis for choosing among risky prospects by establishing a weighting scheme identified as the "expected utility model". The amount of income and uncertainty associated with risky choices as valued by the independent decision maker is summarized by the individual's expected utility function for risk.

Two special cases of the expected utility function are the quadratic and the negative exponential function with the assumption of a normal distribution of expected returns. The assumptions are that the first two moments, mean and variance, are adequate in representing the decision makers attitudes toward risk. The negative exponential function is generally preferred to the quadratic due to its constant rather than increasing absolute risk aversion if normality is assumed.

Portfolio theory originated with the work of Harry Markowitz and his development of the "efficient frontier" (E-V analysis). Markowitz in 1959, proposed that decision makers establish decision rules where variance of profit is minimized for any given level of profit. His principles were expanded upon by Jean and Sharpe and are relevant to any enterprise, policy, or strategy mix where risk and return are taken into consideration.

The general use of the model is to determine a series of investments, strategy mix, or etc., which give the maximum return for various levels of risk (variance). These optimum choices can be plotted and are regarded as the efficient set or the "efficient frontier". Figure 1.

Any risky alternative which falls below AB would offer a lower expected income for a given level of risk and therefore is considered "risk inefficient". Given a set of the most efficient investments, strategies, etc., an individual can select the combination of return and variance which satisfies his/her own risk preference. This can be represented as alternative "C" which is the intersection of ones isoutility curve for risk and the "efficient frontier".

Hedging can be viewed from a portfolio framework where a producer attempts to combine the commodity (physical asset) with the futures (financial asset) so as to give maximum return for a level of risk. The producer attempts to find alternative "C" which may be a combination of cash and futures positions which satisfies the desired level of risk and return.

Portfolio theory provides the basis by which a decision maker can choose among choices; i.e., a decision rule. The decision rule consists of an orderly framework for evaluating courses of action. Most decision rules fall in the following categories: expected profit maximization, mean-variance analysis, mean-absolute deviation analysis, mean-semivariance analysis, and safety-first analysis.

The mean-variance approach assuming a negative exponential utility function was used in this study. Portfolio theory with a simple expected utility optimization procedure was used to evaluate single crop hedges while quadratic programming was used in the multiple crop case. The objective was to use the optimum hedge approach so that the producer maintains an efficient portfolio of cash, futures, or options throughout time.

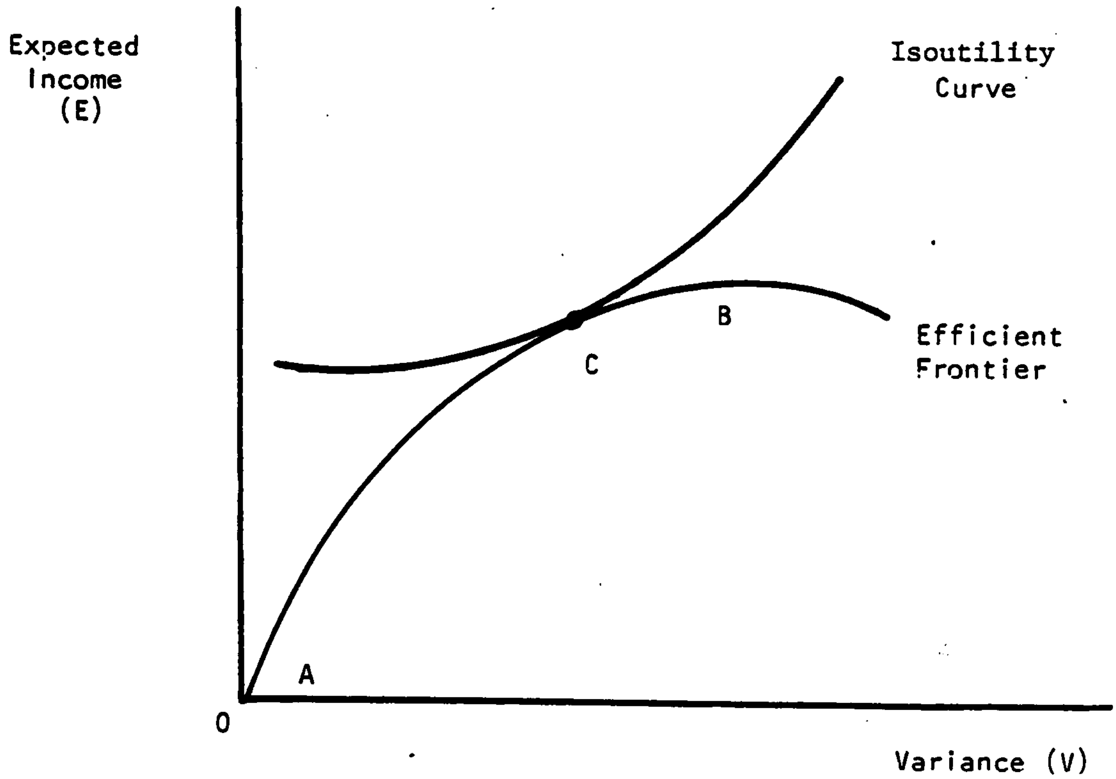

Figure 1. An Illustration of the "Efficient Frontier."

If we assume that profits for one or more crops are normally distributed by assumption, empirical verification, or application of the central limit theorem for the sum of several random variables (Goldberger) then we can assume a decision maker's risk utility function can be express by the exponential function

$$U = 1 - e^{a\pi} \tag{2.1}$$

where

U = utility of profits,

a = Arrow-Pratt risk aversion coefficient, and

π = profit.

Freud extended the exponential function to show that given a normal distribution of profits with mean μ and variance σ^2, the exponential utility function can be a mean-variance utility function.

$$E[U(\pi)] = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{+\infty}(1 - e^{-a\pi})e^{-(\pi-\mu)^2/2\sigma^2}d\pi \tag{2.2}$$

$$= 1 - \frac{1}{\sigma\sqrt{2}}\int_{-\infty}^{+\infty}e^{-[2\sigma^{2a\pi} + (\pi-\mu)^2]/2\sigma^2}d\pi \tag{2.3}$$

Now

$$2\sigma^2 a\pi + (\pi - \mu)^2 = (\pi - \mu + a\sigma^2)^2 + 2a\sigma^2(\mu - \frac{a}{2}\sigma^2); \tag{2.4}$$

Therefore (2.2) becomes

$$E[U(\pi)] = 1 - \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{\frac{-1}{2\sigma^2}(\pi-\mu+a\sigma^2)^2}e^{-a(u-\frac{a}{2}\sigma^2)}d\pi \tag{2.5}$$

$$= 1 - \frac{1}{\sigma\sqrt{2\pi}}e^{-a(u-\frac{a}{2}\sigma^2)}\int_{-\infty}^{+\infty}e^{1/2\sigma^2(\pi-\mu+a\sigma^2)^2}d\pi \tag{2.6}$$

Since, $\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{\frac{-1}{2}\sigma^2(\pi-\mu+a\sigma^2)^2}d\pi$ represents a normal density with mean $\mu - a\sigma^2$, the expression is equal to 1.

From (2.6) it is apparent that maximizing $E[U(\pi)]$ is equivalent to maximizing

$$\mu - \frac{a}{2}\sigma^2, \text{ so } E[U(\pi)] = \mu - \frac{a}{2}\sigma^2, \text{ and } E[U(\pi)] = E(\pi) - \frac{a}{2}(\mathrm{Var}\pi) \tag{2.7}$$

2.1 The Single Crop Case

Freud has shown that maximization of expected utility of profits can be expressed in a mean-variance framework. In that case, we can assume that given some profit function where profits are derived from a combination of cash and futures positions that expected utility is maximized at some optimum hedge level. The following sections demonstrate derivations of optimum hedge levels for futures, options, and multiple crop situations.

2.1.1 Optimum Hedge Levels With Commodity Futures

Rolfo developed a model where a producer maximizes expected utility in a mean-variance framework. We will assume a single crop case prior to harvest where futures price and production are unknown.

The derivation of the Rolfo model begins with:

the end-of-period revenue distribution, $\tilde{W}$, which is the cash distribution of revenue adjusted for the distribution of futures gains and losses.

$$\tilde{W}_f = \tilde{P}\tilde{Q} + n_f\overline{Q}(f - \tilde{P}_f) \tag{2.8}$$

The expected utility is equal to the expected end-of-period distribution of income adjusted for risk which is identified as a risk parameter times the variance of income. Expected utility is

$$EU = E(\tilde{W}_f) - m[\mathrm{var}(\tilde{W}_f)] \tag{2.9}$$

where,

$$\mathrm{var}(\tilde{W}_f) = E[\tilde{W}_f - E(\tilde{W}_f)]^2. \tag{2.10}$$

The variables are identified as follows:

$\tilde{Q}$ = distribution of output,

$\bar{Q}$ = predetermined level of output,

$\tilde{P}$ = distribution of cash prices,

$\tilde{P}_f$ = distribution of futures prices,

n_f = percentage of the predetermined level of output hedged,

$\tilde{P}\tilde{Q}$ = end of period distribution of cash income,

$\tilde{W}_f$ = end of period distribution of income adjusted for the futures hedge,

f = futures price accepted prior to harvest,

E = expectations operator,

EU = expected utility from the risky prospect, and

m = the risk parameter which in this case is $\alpha/2$.

The variance of the adjusted income distribution, $\mathrm{var}(\tilde{W})$is:

$$E[\tilde{P}\tilde{Q} + n_f\bar{Q}(f - \tilde{P}_f)] - \{E[\tilde{P}\tilde{Q} + n_f\bar{Q}(f - E(\tilde{P}_f))]\}^2 \tag{2.11}$$

$$= E[\tilde{P}\tilde{Q} + n_f\bar{Q}(f - \tilde{P}_f) - E(\tilde{P}\tilde{Q}) - n_f\bar{Q}(f - E(\tilde{P}_f))]^2 \tag{2.12}$$

$$= E[\tilde{P}\tilde{Q} + n_f\bar{Q}f - n_f\bar{Q}\tilde{P}_f - E[\tilde{P}\tilde{Q}] - n_f\bar{Q}f + n_f\bar{Q}E(\tilde{P}_f)]^2 \tag{2.13}$$

$$= E[\tilde{P}\tilde{Q} - E[\tilde{P}\tilde{Q}] - n_f\bar{Q}\tilde{P}_f + n_f\bar{Q}E(\tilde{P}_f)]^2 \tag{2.14}$$

$$= E[\tilde{P}\tilde{Q} - E(\tilde{P}\tilde{Q})]^2 + n_f^2\bar{Q}E[\tilde{P}_f - E\tilde{P}_f]^2 - 2n_f\bar{Q}E[\tilde{P}\tilde{Q} - E(\tilde{P}\tilde{Q})][\tilde{P}_f - E(\tilde{P}_f)] \tag{2.15}$$

$$= \mathrm{var}\tilde{P}\tilde{Q} + n_f^2\bar{Q}^2\mathrm{var}\tilde{P}_f - 2n_f\bar{Q}\mathrm{cov}(\tilde{P}\tilde{Q}, \tilde{P}_f) \tag{2.16}$$

By substituting (2.8) and (2.16) into (2.9), expected utility (EU) becomes

$$E[(\tilde{P}\tilde{Q} + n_f\bar{Q}(f - \tilde{P}_f)] - m[\mathrm{var}\tilde{P}\tilde{Q} + n_f^2\bar{Q}^2\mathrm{var}\tilde{P}_f - 2n_f\bar{Q}\mathrm{cov}(\tilde{P}\tilde{Q}, \tilde{P}_f)] \tag{2.17}$$

$$= E(\tilde{P}\tilde{Q}) + n_f\bar{Q}(f - E\tilde{P}_f) - m\mathrm{var}\tilde{P}\tilde{Q} - mn_f^2\bar{Q}^2\mathrm{var}\tilde{P}_f + 2mn_f\bar{Q}\mathrm{cov}(\tilde{P}\tilde{Q}, \tilde{P}_f) \tag{2.18}$$

To find the optimum percentage to hedge, maximize (2.18) w.r.t. n_f

$$\partial EU/\partial n_f = \bar{Q}(f - E\tilde{P}_f) - 2mn_f\bar{Q}^2\mathrm{var}\tilde{P}_f + 2m\bar{Q}\mathrm{cov}(\tilde{P}\tilde{Q}, \tilde{P}_f) = 0 \tag{2.19}$$

$$\partial^2 EU/\partial^2 n_f = -2m\bar{Q}^2\mathrm{var}\tilde{P}_f \tag{2.20}$$

If we assume the first derivative is positive and $2m\bar{Q}^2\mathrm{var}\tilde{P}_f$ is positive then the second derivative is negative and we have a maximum. Solving for n_f

$$2mn_f\bar{Q}^2\mathrm{var}\tilde{P}_f = \bar{Q}(f - E\tilde{P}_f) + 2m\bar{Q}\mathrm{cov}(\tilde{P}\tilde{Q}, \tilde{P}_f) \tag{2.21}$$

$$n_f = (f - E\tilde{P}_f)/2m\bar{Q}\mathrm{var}\tilde{P}_f + \mathrm{cov}(\tilde{P}\tilde{Q}, \tilde{P}_f)/\bar{Q}\mathrm{var}\tilde{P}_f \tag{2.22}$$

$$n_f = [(f - E(\tilde{P}_f))/2m\mathrm{var}\tilde{P}_f + \mathrm{cov}(\tilde{P}\tilde{Q}, \tilde{P}_f)/\mathrm{var}\tilde{P}_f][1/\bar{Q}] \tag{2.23}$$

The total amount to hedge can be determined by multiplying both sides of the equality by $\bar{Q}$.

$$n_f^* = (f - E(\tilde{P}_f))/2m\mathrm{var}\tilde{P}_f + \mathrm{cov}(\tilde{P}\tilde{Q}, \tilde{P}_f)/\mathrm{var}\tilde{P}_f \tag{2.24}$$

An inspection of equation (2.24) reveals that the first expression $(f - E(\tilde{P}_f))/2m\mathrm{var}\tilde{P}_f$ is proportional to the bias in the futures markets or the difference between the forward price and the expected futures price. The expected gain in the futures is inversely related to the level of risk aversion. Hedge levels would then increase with expected futures gains and decrease with increased levels of risk aversion. Should one accept the "efficient market hypothesis" which states that all relevant market information is in the current price, then the first expression drops out leaving the quantity to hedge dependent on the relationship of the covariance of cash income and futures price relative to the variance of futures price.

If one assumes the cash and futures prices tend to move together, then the sign of the second expression depends on the relative changes in the individual's production and the futures price. If a producer's output and price are inversely related, quantity hedged would most likely be reduced. Quantity hedged given an efficient market can be zero if P and Q are perfectly negatively correlated.

A thorough evaluation of the equation extends into individual supply elasticities as described by McKinnon. The covariance is between cash income and futures prices. If the individual producer's price-quantity relationship is negatively correlated, and if price changes are greater than changes in the individual producers output, the amount hedged would be negative. If the individual's output changes are greater than price changes (which might be expected) then the amount hedged would be positive. The optimal expression (2.9) can be viewed as a Lagrangian expression where m is the marginal utility of the variance of income which we assume to be negative.

A graphical analysis of the optimal market position for a short hedger using the portfolio approach was performed by Ward and Fletcher. They used an iso-variance curve which included all combinations of cash and futures positions yielding a constant risk value. The range varied from pure speculation to an all cash position. Iso-income lines were used to determine optimum hedges. Readers are referred to the Ward and Fletcher article for detail.

To represent the previous micro model in a portfolio setting, one needs to represent the variables in terms of income gains and losses rather than price-quantity relationships. Using the original equation (2.9), $\overline{Q}$ is multiplied through the futures transactions. The stochastic price-quantity relationship is then described as revenue or profit variables if cost-of-production is incorporated into the analysis. Then,

$$EU = E(\tilde{W}_f) - m[\mathrm{var}(\tilde{W}_f)] \tag{2.9}$$

where

$$(\tilde{W}_f) = \tilde{P}\tilde{Q} + n_f(f\overline{Q} - \tilde{P}_f\overline{Q}), \text{ or} \tag{2.25}$$

$$(\tilde{W}_f) = \tilde{\pi}_c + n_f(\tilde{\pi}_f) \tag{2.26}$$

$$\mathrm{var}(\tilde{W}_f) = E(\tilde{W}_f - E(\tilde{W}_f))^2 \tag{2.27}$$

$$= E[\tilde{\pi}_c + n_f(\tilde{\pi}_f) - E(\tilde{\pi}_c) - n_f E(\tilde{\pi}_f)]^2 \tag{2.28}$$

$$= E[\tilde{\pi}_c - E(\tilde{\pi}_c)]^2 + n_f^2 E[(\tilde{\pi}_f) - E(\tilde{\pi}_f)]^2 + 2n_f E[\tilde{\pi}_c - E(\tilde{\pi}_c)][(\tilde{\pi}_f) - E(\tilde{\pi}_f)] \tag{2.29}$$

$$\mathrm{var}(\tilde{\pi}_f) = \mathrm{var}\tilde{\pi}_c + n_f^2 \mathrm{var}\tilde{\pi}_f + 2n_f \mathrm{cov}(\tilde{\pi}_c, \tilde{\pi}_f). \tag{2.30}$$

Substituting equation (2.26) and (2.30) into (2.9), yields

$$EU = [E(\tilde{\pi}_c) + n_f(\tilde{\pi}_f) - m[\mathrm{var}\tilde{\pi}_c + n_f 2\mathrm{var}\tilde{\pi}_f + 2n_f \mathrm{cov}(\tilde{\pi}_c, \tilde{\pi}_f)]] \tag{2.31}$$

$$EU = [E(\tilde{\pi}_c) + n_f E(\tilde{\pi}_f) - m\mathrm{var}\tilde{\pi}_c - mn_f^2 \mathrm{var}\tilde{\pi}_f - 2n_f m\mathrm{cov}(\tilde{\pi}_c, \tilde{\pi}_f)], \tag{2.32}$$

Take the first derivative w.r.t. n_f, the optimal hedge becomes

$$\partial EU/\partial n_f = E(\tilde{\pi}_f) - 2mn_f \mathrm{var}\tilde{\pi}_f - 2m\mathrm{cov}(\tilde{\pi}_c, \tilde{\pi}_f) = 0 \tag{2.33}$$

The second derivative is

$$\partial^2 EU/\partial^2 n_f = -2m\mathrm{var}\tilde{\pi}_f \tag{2.34}$$

which indicates that EU is maximized. Since

$$2mn_f \operatorname{var}\tilde{\pi}_f = E(\tilde{\pi}_f) - 2m\operatorname{cov}(\tilde{\pi}_c, \tilde{\pi}_f), \text{ then} \tag{2.35}$$

$$n_f = E(\tilde{\pi}_f)/2m\operatorname{var}\tilde{\pi}_f - \operatorname{cov}(\tilde{\pi}_c, \tilde{\pi}_f)/\operatorname{var}\tilde{\pi}_f. \tag{2.36}$$

An analysis of the above formulation gives the same basic information as (2.24). The first expression to the right of the equality describes hedging assuming some bias in the futures markets. If no bias exists, the first expression is zero. The second expression is the ratio of covariance to variance. Should the covariance between the cash position and the futures position be negatively correlated, as one might expect, the percentage hedged will be positive.

The formulation just completed can be transformed into investment theory.

$$\text{if } n_f = E\tilde{\pi}_f/2m\operatorname{var}\tilde{\pi}_f - \operatorname{cov}(\tilde{\pi}_c, \tilde{\pi}_f)/\operatorname{var}\tilde{\pi}_f, \tag{2.37}$$

and for an efficient market

$$n_f = -\operatorname{cov}(\tilde{\pi}_c, \tilde{\pi}_f)/\operatorname{var}\tilde{\pi}_f, \tag{2.38}$$

then n_f is essentially the "Beta" coefficient in investment theory which is a measure of the individual security's return relative to the market returns. In this case the "Beta" would represent the relationship of the cash position to the futures position. The n_f is the same as the "Beta" coefficient in a regression analysis where cash returns are the dependent variable and hedge returns the independent variable. In this analysis, the "Beta" coefficient would give the ratio of hedge returns that would minimize the variance of hedge and cash returns. The hedge ratio n_f would be the minimum variance hedge ratio and can be tested statistically using standard t and F tests.

2.1.2 Optimum Hedge Levels With Commodity Options

Commodity options can be used as a hedging tool in much the same manner as a futures contract. A put option gives the purchaser the right but not the obligation to exchange the put option for a short futures position. A put option represents a short position the same as an initial sale in the futures markets. The value of the commodity option, the premium, reflects changes in the futures price, therefore, increases or decreases in futures prices result in increases or decreases in the option premium. Other variables also affect the value of the option such as length of time until day of expiration, volatility in futures prices, and the current rate of interest.

The difference between commodity futures and commodity options is in level of known risk. A producer can reap unlimited gains and essentially unlimited losses with futures. An option contract, however, gives the purchaser a level of known risk because the value of the option can only fall to zero. At that time the option becomes worthless and the purchaser simply lets the option expire.

One might justifiably question the normality assumption underlying the negative exponential utility function and mean-variance approach as it relates to hedging with commodity options. One would assume that returns on a put option would be skewed toward gains due to the price of the option never going below zero. Similarly, one would have to question whether commodity prices and yields also have lower bounds of zero, but most economist assume normality in studies evaluating prices and yields. Porter and Gaumnitz performed a study on normality assumptions where they compared mean-variance alternatives to those selected using stochastic dominance. and determined the results are not seriously affected. Paul Wilson in a similar study simulated normal and beta distributions for 1,181 swine operations and concluded that the differences between the simulations were not statistically different and stochastic dominance switching did not take place. In general, there is a close relationship between mean-variance and more general risk efficient sets.

A thorough examination of hedging returns involving commodity futures and options is presented in Chapter 3.

Using the same approach as with futures, the derivation of the single crop optimum hedge using commodity options is as follows:

$$EU = E(\tilde{W}_o) - m(var\tilde{W}_o) \tag{2.39}$$

where,

$$\tilde{W}_o = \tilde{P}\tilde{Q} + n_o(S - \tilde{P}_e - C_p) \tag{2.40}$$

then,

$$\tilde{W}_o = \tilde{\pi}_c + n_o(\tilde{\pi}_o) \tag{2.41}$$

where variables are defined as follows:

$\tilde{W}_o$ = end of period distribution of income adjusted for the options hedge,

S = the options strike price,

$\tilde{P}_e$ = end of period distribution of exercise prices,

C_p = cost or premium of the put option,

n_o = amount hedged using options,

$\tilde{\pi}_c$ = returns from the cash position,

$\tilde{\pi}_o$ = returns from the options position, and

all other variables are as previously defined.

The variance of W_o is:

$$\mathrm{var}(\tilde{W}_o) = E(\tilde{W}_o - E\tilde{W}_o)^2 \tag{2.42}$$

where

$$\tilde{W}_o = \tilde{\pi}_c - n_o(\tilde{\pi}_o) \tag{2.43}$$

$$\mathrm{var}(\tilde{W}_o) = E[\tilde{\pi}_c + n_o(\tilde{\pi}_o) - E\tilde{\pi}_c - n_oE(\tilde{\pi}_o)]^2 \tag{2.44}$$

$$= E[\tilde{\pi}_c - E\tilde{\pi}_c + n_o(\tilde{\pi}_o) - n_oE(\tilde{\pi}_o)]^2 \tag{2.45}$$

$$= E[\tilde{\pi}_c - E\tilde{\pi}_c]^2 + [n_o(\tilde{\pi}_o - E(\tilde{\pi}_o))]^2 \tag{2.46}$$

$$\mathrm{var}(\tilde{W}_o) = \mathrm{var}\tilde{\pi}_c + n_o^2\mathrm{var}\tilde{\pi}_o + 2n_o\mathrm{cov}(\tilde{\pi}_c, \tilde{\pi}_o). \tag{2.47}$$

Substituting (2.41) and (2.47) into (2.39) gives

$$EU = [E(\tilde{\pi}_c + n_o(\tilde{\pi}_o)) - m[\mathrm{var}\tilde{\pi}_c + n_o^2\mathrm{var}\tilde{\pi}_o + 2n_o\mathrm{cov}(\tilde{\pi}_c, \tilde{\pi}_o)]] \tag{2.48}$$

$$EU = E(\tilde{\pi}_c) + n_oE(\tilde{\pi}_o) - m\mathrm{var}\tilde{\pi}_c - n_o^2m\mathrm{var}\tilde{\pi}_o - 2n_om\mathrm{cov}(\tilde{\pi}_c, \tilde{\pi}_o) \tag{2.49}$$

Taking the first partial derivative of expected utility with respect to n_o gives

$$\partial EU/\partial n_o = E(\tilde{\pi}_o) - 2n_om\mathrm{var}\tilde{\pi}_o - 2m\mathrm{cov}(\tilde{\pi}_c, \tilde{\pi}_o) = 0 \tag{2.50}$$

The second partial derivative is

$$\partial^2EU/\partial^2n_o = -2m\mathrm{var}\tilde{\pi}_o \tag{2.51}$$

and since

$$2n_om\mathrm{var}\tilde{\pi}_o = E(\tilde{\pi}_o) - 2m\mathrm{cov}(\tilde{\pi}_c,\tilde{\pi}_o) \tag{2.52}$$

$$n_o = E(\tilde{\pi}_o)/2m\mathrm{var}\tilde{\pi}_o - \mathrm{cov}(\tilde{\pi}_c, \tilde{\pi}_o)/\mathrm{var}\tilde{\pi}_o \tag{2.53}$$

The proportion to hedge can be determined by dividing both sides by some predetermined expected output.

$$n_o{}^* = E(\tilde{\pi}_o)/2m\mathrm{var}\tilde{\pi}_o\overline{Q} - \mathrm{cov}(\tilde{\pi}_c, \tilde{\pi}_o)/\mathrm{var}(\tilde{\pi}_o)\overline{Q} \tag{2.54}$$

An interpretation of the options model is essentially the same as with futures. The first expression is related to the bias in the market or the producers expected price relative to the futures as reflected in options premiums and the second expression relates the profit changes between cash and options. It is not possible theoretically to determine whether options would be preferred to futures by hedgers. A put option purchased at-the-money[1] would have negative returns by the amount of the premium given an efficient market. This would cause (n_o) to be reduced. The covariance of the second expression would be strictly negative or zero with a possible smaller variance of $\tilde{\pi}_o$. The second expression would most likely cause (n_o) to be higher with options than futures.

2.2 Optimum Hedge Levels Given Crop Diversification

As described earlier, hedging is only one of several ways to reduce the producer's overall level of risk. A popular means is through crop diversification. Berck's research indicates that crop diversification substantially changes the optimum hedge.

A popular belief is that crop diversification reduces income risk as well as increase crop yields through crop rotation. Diversification will reduce income risk if the income from two or more crops are inversely related. Should two crops be affected by the same weather conditions, insect damage, or, etc., they may in fact be a riskier combination and would benefit through increased hedging. In either case multiple crops might have an impact on the optimum hedge level and will be taken into consideration in the following section.

We will begin by assuming a two crop case. The hedging tool could be futures contracts, options contracts, or a combination of the two. The profit functions for each crop are as follows:

$\tilde{W}_1 = \tilde{P}_1\tilde{Q}_1 + n_{1f}(f_1 - \tilde{P}_{f1})$, or in profit variables (2.55)

[1] Commodity options are traded over numerous exercise prices. The value of the option reflects differences between the exercise price and the current futures prices. Options at-the-money only reflect the time value until expiration.

$$\tilde{W}_1 = \tilde{\pi}_1 + n_{f1}(\tilde{\pi}_{f1}) \tag{2.56}$$

$$\tilde{W}_2 = \tilde{P}_2\tilde{Q}_2 + n_{f2}(f_2 - \tilde{P}_{f2}), \text{ or} \tag{2.57}$$

$$\tilde{W}_2 = \tilde{\pi}_2 + n_{f2}(\tilde{\pi}_{f2}) \tag{2.58}$$

The combined profit function is:

$$\tilde{W}_{12} = \alpha\tilde{P}_1\tilde{Q}_1 + n_{f1}(f_1 - \tilde{P}_{f1}) + (1 - \alpha)\tilde{P}_2\tilde{Q}_2 + n_{f2}(f_2 - \tilde{P}_{f2}) \tag{2.59}$$

or,

$$\tilde{W}_{12} = \alpha\tilde{\pi}_1 + n_{f1}(\tilde{\pi}_{f1}) + (1 - \alpha)\tilde{\pi}_2 + n_{f2}(\tilde{\pi}_{f2}). \tag{2.60}$$

The producer wishes to maximize utility in a mean-variance framework,

$$EU_{12} = E\tilde{W}_{12} - m[\mathrm{var}(\tilde{W}_{12})]. \tag{2.61}$$

To keep the derivations as simple as possible the functions are defined in profit variables.

$$\mathrm{var}(\tilde{W}_{12}) = E[\tilde{W}_{12} - E(\tilde{W}_{12})]^2, \tag{2.62}$$

$$\mathrm{var}(\tilde{W}_{12}) = \alpha^2\mathrm{var}(\tilde{\pi}_1) + n_{f1}^2\mathrm{var}(\tilde{\pi}_{f1}) + (1 - \alpha)^2\mathrm{var}(\tilde{\pi}_2)$$

$$+ n_{f2}^2\mathrm{var}(\tilde{\pi}_2) + 2\alpha(1 - \alpha)\mathrm{cov}(\tilde{\pi}_1, \tilde{\pi}_2) + 2\alpha n_{f1}\mathrm{cov}(\tilde{\pi}_1, \tilde{\pi}_{f1})$$

$$+ 2\alpha n_{f2}\mathrm{cov}(\tilde{\pi}_1, \tilde{\pi}_{f2}) + 2(1 - \alpha)n_{f1}\mathrm{cov}(\tilde{\pi}_2, \tilde{\pi}_{f1})$$

$$+ 2(1 - \alpha)n_{f2}\mathrm{cov}(\tilde{\pi}_2, \tilde{\pi}_{f2}) + 2n_{f1}n_{f2}\mathrm{cov}(\tilde{\pi}_{f1}, \tilde{\pi}_{f2}) + 2n_{f1}n_{f2}\mathrm{cov}(\tilde{\pi}_{f1}, \tilde{\pi}_{f2}). \tag{2.63}$$

Substituting (2.60) and (2.63) into (2.61)

$$EU(W_{12}) = \alpha\bar{\pi}_1 + n_{f1}(\bar{\pi}_{f1}) + (1-\alpha)\bar{\pi}_2 - n_{f2}(\bar{\pi}_{f2}) - m[\alpha^2 var(\bar{\pi}_1)$$

$$+ n_{f1}^2 var(\bar{\pi}_{f1}) + (1-\alpha)^2 var(\bar{\pi}_2) + 2\alpha(1-\alpha)cov(\bar{\pi}_1, \bar{\pi}_2)$$

$$+ 2\alpha n_{f1} cov(\bar{\pi}_1, \bar{\pi}_{f1}) + 2\alpha n_{f2} cov(\bar{\pi}_1, \bar{\pi}_{f2})$$

$$+ 2(1-\alpha)n_{f1} cov(\bar{\pi}_2, \bar{\pi}_{f1}) + 2(1-\alpha)n_{f2} cov(\bar{\pi}_2, \bar{\pi}_{f2})$$

$$+ 2n_{f1}n_{f2} cov(\bar{\pi}_{f1}, \bar{\pi}_{f2})]. \qquad (2.64)$$

The complexity of the problem grows enormously in a multiple crop situation. The expected utility of a producer not only depends on the expected gains from his/her cash and futures position but includes a complex area of risk which is composed of the variances and covariances of all relevant variables. This, in a way, relates the complexity of marketing to an individual producer.

The access to hedging could have an effect on a farmer's planting decision. A producer may increase or decrease acres planted to a crop depending on whether he or she is able to lock in what he/she believes to be a favorable price. The acreage allotted to two particular crops and the amount hedged can be determined as follows:

maximize EU w.r.t. $n_{f1}, n_{f2},$ and α

$$\partial EU/\partial n_{f1} = \bar{\pi}_{f1} - 2mn_{f1} var(\bar{\pi}_{f1}) - 2m\alpha cov(\bar{\pi}_1, \bar{\pi}_{f1}) - 2m cov(\bar{\pi}_2, \bar{\pi}_{f1})$$

$$+ 2\alpha m cov(\bar{\pi}_2, \bar{\pi}_{f1}) - 2mn_{f2} cov(\bar{\pi}_{f1}, \bar{\pi}_{f2}) = 0. \qquad (2.65)$$

$$\partial EU/\partial n_{f2} = \bar{\pi}_{f2} - 2mn_{f2} var(\bar{\pi}_2) - 2m\alpha cov(\bar{\pi}_{f1}, \bar{\pi}_{f2}) - 2m cov(\bar{\pi}_2, \bar{\pi}_{f2})$$

$$+ 2m\alpha cov(\bar{\pi}_2, \bar{\pi}_{f2}) - 2mn_{f1} cov(\bar{\pi}_{f1}, \bar{\pi}_{f2}) = 0. \qquad (2.66)$$

$$\partial EU/\partial \alpha = \bar{\pi}_1 - \bar{\pi}_2 - 2m\alpha var(\bar{\pi}_1) + 2m\alpha var(\bar{\pi}_2) + 2m(1-\alpha) var(\bar{\pi}_2)$$

$$- 2m\,cov(\tilde{\pi}_1, \tilde{\pi}_2) + 4m\alpha\,cov(\tilde{\pi}_1, \tilde{\pi}_2) - 2mn_{f1}cov(\tilde{\pi}_1, \tilde{\pi}_{f1})$$

$$- 2mn_{f2}cov(\tilde{\pi}_1, \tilde{\pi}_{f2}) + 2mn_{f1}cov(\tilde{\pi}_2, \tilde{\pi}_{f1}) + 2mn_{f2}cov$$

$$(\tilde{\pi}_2, \tilde{\pi}_{f2}) = 0. \tag{2.67}$$

The system now has three equations and three unknowns. The solution can be found using matrix algebra where determinants are found using cofactor expansion and Cramer's rule. The matrix design for the system can be seen in Table 1.

Table 1. Matrix Design for Multiple Crop Optimum Hedge Levels.

$$\begin{bmatrix} \text{(a)} & \text{(b)} & \text{(c)} \\ -2m \operatorname{var}(\pi_1) + 2m \operatorname{var}(\pi_2) & 2m \operatorname{cov}(\pi_2, \pi_{f1}) & 2m \operatorname{cov}(\pi_2, \pi_{f2}) \\ + 4m \operatorname{cov}(\pi_1, \pi_2) - 2m \operatorname{var}(\pi_2) & - 2m \operatorname{cov}(\pi_1, \pi_{f1}) & - 2m \operatorname{cov}(\pi_1, \pi_{f2}) \\ \text{(d)} & \text{(e)} & \text{(f)} \\ - 2m \operatorname{cov}(\pi_1, \pi_{f1}) + 2m \operatorname{cov}(\pi_2, \pi_{f1}) & - 2m \operatorname{var}(\pi_{f1}) & - 2m \operatorname{cov}(\pi_{f1}, \pi_{f2}) \\ \text{(g)} & \text{(h)} & \text{(i)} \\ - 2m \operatorname{cov}(\pi_1, \pi_{f2}) + 2m \operatorname{cov}(\pi_2, \pi_{f2}) & - 2m \operatorname{cov}(\pi_{f1}, \pi_{f2}) & - 2m \operatorname{var}(\pi_2) \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ \\ n_{f1} \\ \\ n_{f2} \end{bmatrix} = \begin{bmatrix} X \\ 2m \operatorname{cov}(\pi_1, \pi_2) - \pi_1 \\ + \pi_2 - 2m \operatorname{var}(\pi_2) \\ Y \\ - \pi_{f1} + 2m \operatorname{cov}(\pi_2, \pi_{f1}) \\ Z \\ - \pi_{f2} + 2m \operatorname{cov}(\pi_2, \pi_{f2}) \end{bmatrix}$$

To simplify the mechanics of the F.O.C.'s, the expressions are first designated by the letters a through i and X, Y, and Z. Later expressions are designated by the letters J through S.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} \alpha \\ n_{f1} \\ n_{f2} \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ n_{f1} \\ n_{f2} \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ n_{f1} \\ n_{f2} \end{bmatrix} = \begin{bmatrix} \frac{-JR}{L} + m & \frac{-JS}{L} + N & \frac{-J}{L} \\ -\frac{KR}{L} + P & \frac{-KS}{L} + Q & \frac{-K}{L} \\ \frac{R}{L} & \frac{S}{L} & \frac{1}{L} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

then,

$$\alpha = \left(\frac{-JRX}{L} + mx\right) + \left(\frac{-JSY}{L} + NY\right) + \left(\frac{-JY}{L}\right) \tag{2.68}$$

$$n_{f1} = \left(\frac{-KRX}{L} + PX\right) + \left(\frac{-KSY}{L} + QY\right) + \left(\frac{-KY}{L}\right) \tag{2.69}$$

$$n_{f2} = \left(\frac{RX}{L}\right) + \left(\frac{SY}{L}\right) + \left(\frac{Z}{L}\right) \tag{2.70}$$

where

$$J = \frac{-b}{a}\left(\frac{-dc + fa}{-db + ea}\right) + \frac{c}{a} \tag{2.71}$$

$$K = \frac{-dc + fa}{-db + ea} \tag{2.72}$$

$$L = -\left(\frac{-gb + ah}{a}\right)\left(\frac{-dc + fa}{-db + ea}\right) + \frac{-gc + ia}{a} \tag{2.73}$$

$$M = \frac{db}{a(-db + ea)} + \frac{1}{a} \tag{2.74}$$

$$N = \frac{-b}{-bd + ea} \tag{2.75}$$

$$P = \frac{-d}{-db + ea} \tag{2.76}$$

$$Q = \frac{a}{-db + ea} \tag{2.77}$$

$$R = \left(\frac{-gb + ah}{a}\right)\left(\frac{d}{-db + ea}\right) - \frac{g}{a} \tag{2.78}$$

$$S = \frac{gb - ah}{bd - ea} \tag{2.79}$$

The second order conditions are:

$$\partial^2 EU/\partial n_{f1}^2 = -2m\mathrm{var}(\pi f_1) \tag{2.80}$$

$$\partial^2 EU/\partial n_{f1}\partial\alpha = -2m\mathrm{cov}(\pi_1, \pi_{f1}) + 2m\mathrm{cov}(\pi_2, \pi_{f1}) \tag{2.81}$$

$$\partial^2 EU/\partial n_{f1}\alpha n_{f2} = 2m\mathrm{cov}(\pi_{f1}, \pi_{f2}) \tag{2.82}$$

$$\partial^2 EU/\partial n_{f2}^2 = -2m\mathrm{var}(\pi^2) \tag{2.83}$$

$$\partial^2 EU/\partial n_{f2}\partial\alpha = -2m\mathrm{cov}(\pi_1, \pi_{f2}) + 2m\mathrm{cov}(\pi_2, \pi_{f2}) \tag{2.84}$$

$$\partial^2 EU/\partial n_{f2}\partial n_{f1} = -2m\mathrm{cov}(\pi_{f1}, \pi_{f2}) \tag{2.85}$$

$$\partial^2 EU/\partial\alpha^2 = -2m\mathrm{var}(\pi_1) - 2m\mathrm{var}(\pi_2) + 4m\mathrm{cov}(\pi_1, \pi_2) \tag{2.86}$$

$$\partial^2 EU/\partial\alpha\partial n_{f1} = -2m\mathrm{cov}(\pi_1, \pi_{f1}) + 2m\mathrm{cov}(\pi_2, \pi_{f1}) \tag{2.87}$$

$$\partial^2 EU/\partial\alpha\partial n_{f2} = -2m\mathrm{cov}(\pi_1, \pi_{f2}) + \mathrm{cov}(\pi_2, \pi_{f2}) \tag{2.88}$$

The Hessian Matrix of second derivatives is then:

$$
\begin{bmatrix}
-2m \text{ var } (\pi_1) - 2m \text{ var } (\pi_2) + 4m \text{ cov } (\pi_1, \pi_2) & -2m \text{ cov } (\pi_1, \pi_{f1}) + 2m \text{ cov } (\pi_2, \pi_{f1}) & -2m \text{ cov } (\pi_1, \pi_{f2}) + \text{cov } (\pi_2, \pi_{f2}) \\
-2m \text{ cov } (\pi_1, \pi_{f1}) + 2m \text{ cov } (\pi_2, \pi_{f2}) & -2m \text{ var } (\pi_{f1}) & 2m \text{ cov } (\pi_{f1}, \pi_{f2}) \\
-2m \text{ cov } (\pi_1, \pi_{f2}) + 2m \text{ cov } (\pi_2, \pi_{f2}) & -2m \text{ cov } (\pi_{f1}, \pi_{f2}) & -2m \text{ var } (\pi_2)
\end{bmatrix}
$$

If the covariance terms are negative as we would expect, the Hessian matrix is negative semidefinite and expected utility would be maximized.

An examination of the multiple crop hedging model indicates that expected utility is maximized at some optimum allocation of acreage among crops and optimum levels of hedges for each of the crops. It is also apparent that the allocation of acreage among crops affects the level of hedging of crops and the pôssiblity of hedging can also affect planting decisions. An approximation of the optimal allocation of acreage and crops can be determined using quadratic programming and is discussed in the following section.

2.3 Portfolio Theory and Quadratic Programming

Quadratic programming is only one of a number of nonlinear programming formulations. It is perhaps the most common nonlinear technique and has been used extensively in portfolio analysis. Markowitz used quadratic programming (QP) to determine the efficient frontier of investment opportunities given limited funds. Berck used QP to determine optimum hedged positions using futures for a typical California cotton producer.

QP has primarily been used as the maximization of a concave or minimization of a convex objective function subject ot linear constraints. (Hartley) The general formulation for the maximization problem is as follows:

$$\text{maximize } \Sigma p_j x_j + XDX \quad (2.89)$$

$$\text{subject to } \Sigma a_{ij} x_j \leq b_i \quad i = 1, 2, \ldots m. \quad (2.90)$$

$$x_j \geq 0 \quad j = 1, 2, \ldots n. \quad (2.91)$$

where D is the variance - covariance matrix.

If the constraints from the problem form a convex set, and if the matrix of the quadratic form is negative-definite, then the Kuhn-Tucker conditions are both necessary and sufficient for a global optimum.

The Kuhn-Tucker conditions are a means of determining optimality of a constrained maximization or minimization by examining the movement from an optimal point to see if it fails to increase the objective function or in fact is not feasible. If we define λ_i as a Lagrange multiplier for the ith constraint, these multipliers turn out to be the marginal values of small changes in the constraining variables. A binding constraint will be binding or exhausted at the optimal solution. The economic value of λ_i, is $\lambda_i \geq 0$ and the constraint g_i (x) is equal to zero.

If we define:

$\lambda = [\lambda_1, \lambda_2 \ldots \lambda_m]$ as the Lagrange multipliers for the constraints in (2.90); and:

$u = [u_1, u_2 \ldots u_m]$ as the Lagrange multipliers for th constraints in (2.91); then,

$$\partial f/\partial X_j = p_j + 2\Sigma\, d_{j,k} X_k, \text{ and} \quad (2.92)$$

$$\Sigma\, \lambda_i\, a_{ij} = u_j \quad j = 1, 2, \ldots n, \text{ combined give} \quad (2.93)$$

$$p_j + 2\Sigma d_{j,k} x_k - (\Sigma \lambda_i a_{ij} - u_j) = 0 \quad (2.94)$$

$j = 1,2 \ldots n.$

The economic interpretation states that the changes in the objective function relative to the constraint is equal to the summation of the marginal values of that constraint.

The Kuhn-Tucker conditions for a Quadratic Programming problem are as follows:

1. "D" must be negative-definite
2. $- 2\Sigma d_{jk}X_k + \Sigma a_{ij}\lambda_i - u_j = p_j \quad j = 1,2, \ldots m$
3. $\Sigma a_{ij}X_j + \gamma_i = b_i \quad i = 1,2, \ldots m$
4. $\lambda_i(\gamma_i) = 0 \quad i = 1,2, \ldots m$
5. $u_j(X_j) = 0 \quad j = 1,2, \ldots n$
6. $X_j, \lambda_i, u_j \geq 0$ for all i and j

where γ_i is defined as the slack, where

$- \gamma_i = \Sigma a_{ij}X_j - b_j$

and all other variables are as previously defined.

The general case for the portfolio problem seeking optimum hedges is as follows:

maximize:

$$\Sigma \pi_j x_j + \Sigma \pi_i x_i - m\Sigma\Sigma\sigma_{ij}x_j x_k \qquad (2.95)$$

subject to:

$$\Sigma x_j = 1 \qquad (2.96)$$

$$\Sigma x_j - \Sigma x_i \leq 0 \qquad (2.97)$$

$$x_j \geq 0 \qquad (2.98)$$

$$x_i \geq 0 \qquad (2.99)$$

The efficient portfolio would be determined giving the optimum allocation of land between crops and the optimum level to hedge of each of these crops. In this particular case, the hedge eliminates speculation by not allowing quantity hedged to exceed expected production on land allocated to each crop (see Berck, Musser for a discussion on speculation in a QP framework).

The portfolio selection for the two crop case is as follows:

$$\max: \pi_1 X_1 + \pi_2 X_2 + \pi_{f1} n_{f1} + \pi_{f2} n_{f2} - m[XDX] \tag{2.100}$$

$$\text{st. } X_1 + X_2 = 1 \tag{2.101}$$

$$n_{f1} \leq X_1 \tag{2.102}$$

$$n_{f2} \leq X_2 \tag{2.103}$$

The Q.P. matrix:

maximize:

$$\pi_1 X_1 + \pi_2 X_2 + \pi_{f1} n_{f1} + \pi_{f2} n_{f2} - m[XDX] \tag{2.104}$$

subject to constraints:

$$X_1 + 2 \quad = 1 \tag{2.105}$$

-1 1 < 0

-1 1 < 0

The Q.P. matrix labeled in detail is listed in Table 2.

2.4 Optimal Hedge Portfolios and Financial Leverage

The inclusion of a risk free asset or debt financing into portfolio analysis significantly changes the results. As stated in the Separation Theorem, leverage effectively eliminates the "efficient frontier" by replacing portfolio risk with financial risk. A graphical analysis is presented in Figure 2.

Leverage changes the menu of risk return tradeoffs into a linear combination when an efficient portfolio is included with a risk-free asset or financed through debt. Since the variance of a risk-free asset is zero, the efficient frontier becomes a linear combination of the risk-free asset and other portfolios. All points between R_f and C involve combinations of portfolio C and a risk-free asset for an investor with isoutility curves U_1andU_2. All points between C and G include combinations of portfolio C financed by various levels of debt, for an investor with isoutility curves U_3andU_4.

Both investors can attain higher levels of utility by moving from the traditional E-V frontier to the linear combinations.

Through the use of a risk-free asset, a producer could select portfolio C and some level of a risk-free asset to give portfolio A which is risk preferred to B. Likewise, through borrowings a

Table 2. Detailed Design of the Two Crop Q.P. Matrix.

Max

$$EU = (\pi_2)X_1 + (\pi_2)X_2 + (\pi_f)\ nf_1 + (\pi_2 f)\ nf_2 - r\begin{bmatrix} X_1 & X_2 & n_1 f & n_2 f \end{bmatrix}$$

$$\begin{bmatrix} var(\pi_1) & cov(\pi_2,\pi_1) & cov(\pi_1,n_1 f) & cov(\pi_1,n_2 f) \\ cov(\pi_2,\pi_1) & var(\pi_2) & cov(\pi_2,n_1 f) & cov(\pi_2,n_2 f) \\ cov(n_1 f,\pi_1) & cov(n_1 f,\pi_2) & var(n_1 f) & cov(n_1 f,n_2 f) \\ cov(n_2 f,\pi_1) & cov(n_2 f,\pi_2) & cov(n_{f1},n_{f1}) & var(n_2 f) \end{bmatrix} \quad \begin{bmatrix} X_a \\ X_b \\ n_{fa} \\ n_{fb} \end{bmatrix}$$

subject to:

$X_1 + X_2$				$= 1$
-1		1		≤ 0
	-1		1	≤ 0

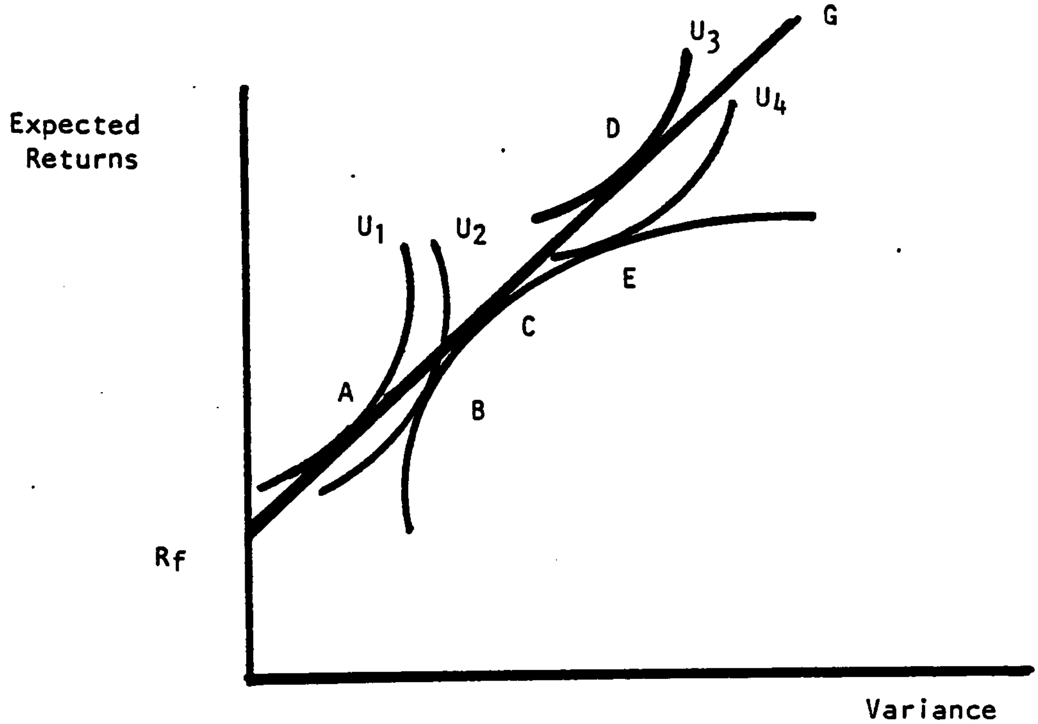

Figure 2. Producer Equilibrium Combining a Risk-Free Asset or Debt Financing With the Efficient Frontier.

producer could move from portfolio E to D which would give a higher return for the same level of risk. Theoretically, there is only one preferred portfolio on the efficient frontier and the producer would attain his/her risk preference by borrowing or holding risk-free assets. In essence, it becomes more risk efficient to substitute leverage for operating risks than to select other risky alternatives on the efficient frontier. Two farmers with identical resource restrictions and price-yield relationships would have identical crop plans even though their risk preferences are different. The size of their operations and degree of financial leverage, however, would be different.

2.5 Conceptualization of the Simulation Process

Simulation is an analytical technique which brings reality to a system. Its uses are varied and many with no set format or algorithm. A simulation model may be deterministic or stochastic, single or multiperiod, maximize or minimize a linear or non-linear objective function, search for an optimal solution, or be nonoptimizing, represent a part of a complex process, or be behavioral or mathematical (Walker and Helmers).

The steps in constructing simulation models are well defined. The steps include (1) model formulation, (2) synthesis, (3) verification/validation, and (4) experimentation. Formulating the model involves determination of the market structure which includes information flows, decision rules, feed back loops, and input-output requirements. The model should be designed to have key criteria testable by statistical analysis.

A flow chart of a typical decision making process for a summer crop can be seen in Figure 3. Preharvest hedging is actually a continuous process but for illustrative purposes the season is divided into six time periods. The periods can be described as the new crop marketing year, preplanting, post planting, midseason, late season, and harvest. During the new crop marketing season a producer's hedging depends on the average and variability of past yields, past prices,

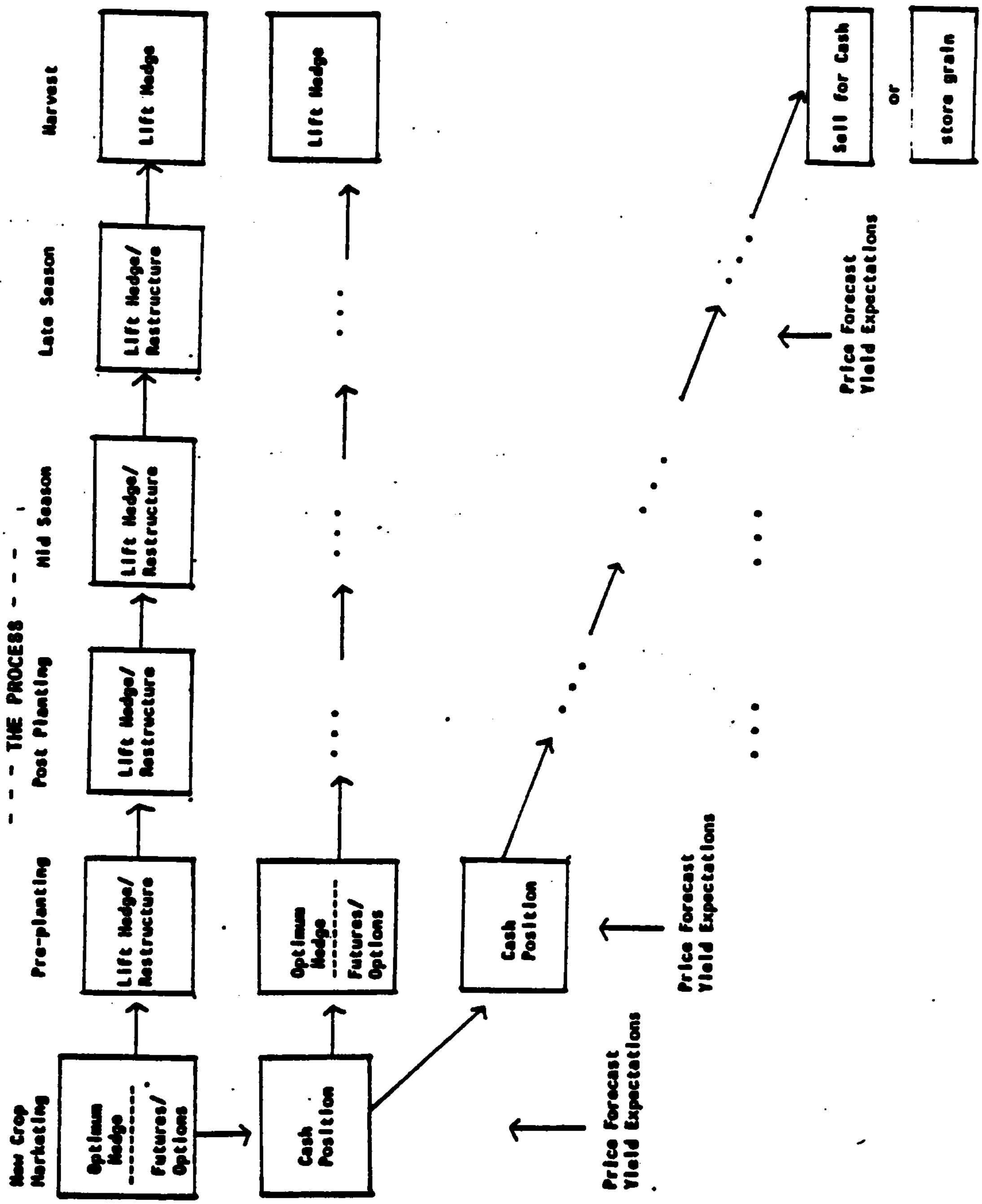

Figure 3. Flow Chart of Preharvest Hedging.

futures prices, expected yields, acreage and changes in government programs. Preplant hedging depends on the previous items listed and any adjustments to acreage planted and government programs. The USDA prospective planting reports are available in the pre-plant period and may also have a bearing on his/her decision. The post-planting time period verifies information on individual crop acreage and gives an initial indication of possible yields based on crop condition in the critical early growth stage. The producer should also receive the USDA planted acreage report. Midseason hedging decisions are based on futures prices and individual crop conditions. By late season the producer should have a good estimate of crop yield and be primarily hedging from new price information. The harvest period is when the product is sold or stored. During harvest the marketing decision would depend on the futures price, cash price, basis, harvest progress, storage capabilities, yield confirmation, and tax situation.

The simulation used in this study was a simplified version of the previously described decision making process. The decision rule on the amount to hedge was determined by the minimum-variance or optimum hedge ratio developed in section 2.1 for single crop hedging and by quadratic programming for the allocation of hedges across the season and for the effects of diversification. For the single crop case, the optimum hedge ratios were determined and then simulated over the data set 1966-1980 to obtain average returns and variances at the optimum hedge levels. The results are, in fact, an E-V frontier to compare a single hedge decision across hedge periods. The assumptions are that all economic information affecting price is accurately reflected in the futures price and that yield predictions are reasonably estimated by the yield prediction equations.

The next procedure addressed the question of whether diversification of hedges, i.e., an allocation of hedges across the season, was superior to a single hedge decision. The result then was an optimal portfolio of hedges that were analyzed simultaneously and not sequentially across the season. To address the previously described hedging process in a sequential manner one would have to model price expectations into an optimal control type framework. In this analysis, individual period hedging was simulated over the period 1966-1980 and the average returns and the variance-covariance matrix was used in the Q.P. framework. The results are to be interpreted as a pre-plant decision where the plan would be that appropriate levels of the predicted yield would be hedged during the season as new yield predictions become available.

The results on the effects of crop and hedge diversification were obtained in like manner. Again, the optimal portfolio was determined simultaneously given the previously described assumptions. All optimum hedged ratios or portfolios were simulated over a new data set to determine E-V efficiency.

Chapter 3 Data Description and Simulation Process

The comparison of optimum hedging strategies was performed at three locations. Two locations were in Virginia and the third in the Midwest. Test data from Warsaw Virginia Agricultural Experiment Station was used as the primary test site with Orange Virginia Experiment Station and Allerton Trust Farms in Champaign, Illinois as secondary test sites. The Warsaw location was considered primary only in the sense that additional comparisons of options and futures were performed at that location.

The Warsaw Virginia Agricultural Experiment station is located in the Coastal Plain region known as the Tidewater area of Virginia. The Soil is of a deep sandy loam and considered fertile when adequate moisture is available. Due to the low moisture holding capacity of the soil, the area experiences periodic drought. Yields tend to be above average when compared to other areas of the state, but are highly variable.

The Orange Virginia Agricultural Experiment Station is in the Piedmont section of Virginia. The Experiment Station is located in an area where soil is classified as Davison soil which once was a prehistoric lakebed. The heavy clay soil is relatively fertile and produces above average and consistent yields compared to other areas of the Piedmont.

The Allerton Trust Farms are located near Champaign, Illinois and are comprised of seven farms totaling approximately 3000 acres. The glacial soils are fertile and the area generally receives adequate and consistent rainfall. Each farm is operated by a different farmer and the University records producer production practices, rainfall, and inputs used.

The simulation process outlined in this chapter was performed on all three locations and simulates cash, futures, and options returns over the period 1966-1980. The average returns, variances, and covariances of returns generated in the simulation process were used to derive optimum hedge levels which are presented in Chapter 4.

3.1 Prices, Yields, and Relevant Data

Average yields for Warsaw and Orange locations were the average yields of the various maturity groups as recorded in the variety tests for corn, soybeans, double-crop soybeans, and wheat. (Variety Performance Test, Agronomy Dept., VPI & SU.) The data was fairly consistent and available from year to year except for second crop soybean yields which were not available prior to 1972. Where missing data was a problem, the yields were extrapolated from county averages. Daily rainfall was recorded at each of the stations and reported in the U. S. Climatological Surveys.

The variety tests average yields were high compared to county averages and yields produced by average farmers. The test are of the plot design and do not represent commercial farming practices. Individual farmer yields would have been preferred for this study but none were available. County averages do not adequately reflect true variation in yields due to aggregation. Therefore, variety test data was selected due to the realistic variation in yields and a consistent set of rainfall data.

Data for the Allerton Trust Farms were provided courtesy of the University of Illinois at Champaign. Yield data used in the simulation process was average yields for all farms. Only corn and soybeans yields were used due to their predominance in the area. Rainfall was recorded on a daily basis during the Spring and Summer months. Yield data for all three locations are presented in Appendix Table 1.

Cash prices for commodities at the two Virginia locations were weekly cash prices for the Northern Neck and Fredricksburg areas as reported by the Virginia Department of Agriculture's Virginia Agricultural Commodity Newsletter. The cash price used was the closest price reported to the 15th of the month for the respective harvest months. Harvest months for corn, soybeans, and wheat, were September, October, and June, respectively. Midwestern cash prices were monthly average daily track country station bids reported by Stother Grain Co. and I. H. French & Co. both of Champaign, Illinois. Midwest harvest month for both corn and soybeans was October. Cash price data for all three locations are presented in Appendix Table 2.

Futures prices were collected for every hedging month and harvest month. Futures prices were daily closings as quoted in the Chicago Board of Trade's Statistical Annual for the 15th of each hedging month or the closest daily close after the 15th day. Futures prices used in the harvest month were closing prices the same day as the cash quote for the Virginia locations. Futures closings for the harvest month for the Midwest location were the 15th day or closest day to the 15th of each harvest month. See Appendix Table 3 for a listing of futures prices.

Rainfall data for the Virginia locations came from the National Weather Service's Climatilogical Surveys. Mid-month totals of rainfall for relevant months were collected. Since hedging decisions were made in the middle of each month, monthly total rainfall represented rainfall from the 15th of one month to the 14th of the following month. Rainfall for the Midwest location was rainfall collected on the farm and reported in an annual summary. Again, daily rainfall was accumulated for midmonth totals. See Appendix Table 4 for rainfall data.

Cost of production for each individual crop was obtained from U.S.D.A.'s "Economic Indicators of the Farm Sector-Cost of Production". Regional total cost of production for 1980 were deflated by the Producer Price Index for each year over the series 1966-1980 to provide a proxy for actual annual cost of production for each crop. See Appendix Table 5 for a listing of production costs for each crop at each location.

3.2 Determination of Commodity Option Premiums

Premiums for commodity options were determined by the Block commodity option pricing model. The model as described by Avner Wolf is as follows:

$$C = e^{-r\tau}[f_{tT}, N(d_1) - EN(d_2)],.$$

where

$$d_1 = [\ell n(f_t, \tau/E) + (S_f^2/2)\tau]/S_f\sqrt{\tau}\text{, and}$$

$$d_2 = d_1 - S_f\sqrt{\tau}$$

The value of the put option is

$$P = c - e^{-r\tau}(f_t, \tau - E)$$

where

c = call premium.

t = time when hedge is placed,

f_{tT}, = futures price on date t of maturing contract T,

S_f^2 = variance of the rate of change of the futures price,

S_f = standard deviation of the rate of change of the futures price,

E = strike price,

r = effective interest rate,

τ = duration of the option as a fraction of a year; $\tau = (T - t)/365$, and

P = put premium.

Commodity option premiums in this study were derived using the Block model as developed and published by the editors of Futures Magazine. The program was adapted to meet special needs of this and other studies within the Department of Agricultural Economics at Virginia Tech. Option premiums were determined for each hedging period where the strike price was set equal to the futures price. Option premiums were determined "at-the-money". The value of the premium at harvest reflected a strike price which was equal to the hedging month's futures price. Interest rates were monthly average rates for 91 day Treasury bills. (Conference Board Monthly Reports) Days to maturity were based on 365 trading days a year. Volatility was based on the previous month's daily futures closings. Volatility was determined by the formula:

$$V = \left[\left(\frac{252}{n-2}\right) \left(\sum_{t=1}^{T} \ln R_t - m \right)^2 \right]^{1/2} \text{ x100}$$

where

$$R_t = \frac{F_t}{F_{t-1}}$$

F_t = closing futures prices.

$$m = \frac{1}{T-1} \sum_{t=1}^{T} \ln R_t$$

Option premiums used in the study are presented in Appendix Table 6.

3.3 The Simulation Model

The simulation model was a series of accounting statements which combined cash returns and hedge returns to derive total net returns to hedging over the period 1966-1980. Returns were deflated so that statistical analysis could be performed on random variability of yields, prices, and income. The simulation process produced a listing of hedge returns for each individual hedge, descriptive statistics such as means, variances, standard deviations, ranges, etc. and covariances and corresponding correlation coefficients. The variance-covariance matrix derived in the simulation process was used in the quadratic programming analysis discussed in Chapter 4.

The general form of the simulation model for a futures hedge was as follows:

$$\pi_{N,i,n} = \{[\pi_{c,i,n} + \pi_{fi,t,n}]/PPI\}x100$$

$$\pi_{c,i,n} = (Y_{i,n})(P_{c,i,n}) - (C_{i,n})$$

$$\pi_{f,i,n} = (P_{t,i,n})(\hat{Y}_{t,i,n}) - (P_{H,i,n})(\hat{Y}_{t,i,n})$$

where

i = each individual crop,

n = year,

t = hedging period,

π_N = deflated net profit,

π_c = cash returns,

π_f = hedge returns,

PPI = Producer Price Index,

Y_i = actual yield,

P_c = cash price at harvest,

C_i = total cost of production in year n,

P_H = futures price at harvest,

$\hat{Y}$ = predicted yield, and

P_t = futures price when hedge is placed.

Profits from an option hedge is slightly different than a futures hedge. With options, hedge gain and losses are the difference between the original price paid for the put option and the selling price of the put option at harvest. It was assumed that the put premium at harvest reflected the ability to exercise that option. Therefore π_o would substitute for π_f, where π_o is:

$$\pi_{o,i,n} = (O_{H,i,n})(\hat{Y}_{t,i,n}) - (O_{t,i,n})(\hat{Y}_{t,i,n})$$

where

π_o = returns from the option hedge,

O_H = option premium at harvest,

O_t = option premium when hedge is placed, and all other variables are as previously described.

The simulation was performed using the software package Statistical Analysis System (SAS). Cash and futures prices, option premiums, cost-of-production, and acutal and predicted yields for all crops at all locations along with the producer price index were included in the data set. A series of transformation statements produced cash and hedge returns for each hedging period at 100% of predicted yield from 1966-1980. Returns were both actual and deflated.

The single crop optimum hedge ratios were determined by simple regression where deflated cash return was the dependent variable and hedge period return the independent variable. The optimum hedge or minimum-variance ratios were then entered into the transformation statements to adjust the predicted yield hedged for each period. The resulting simulation produced the returns and distribution of returns at the optimum hedge levels over the period 1966-1980.

The same set of transformation statements produced the variance-covariance matrix and mean returns needed to analyze the optimum allocation of hedges and the effects of diversification with the Q.P. framework.

Both the single crop optimum hedge ratios and the optimal organizations of crops and crop hedges were simulated over a new data set. The simulation procedure previously described was reproduced with only the data changed. Acreage allocated to each crop was incorporated into the simulation on new data.

3.4 Yield Prediction

Yield prediction equations were used to simulate a farmer's estimate of yield potential throughout the growing season. Improved yield estimates lessen the risks associated with overhedging or underhedging a crop. Lack of good yield estimates early in the growing season in particular could be a major reason why grain farmers do very little preharvest hedging with futures contracts.

Yield prediction equations used in this study use only rainfall as the relevant explanatory variable for yields. Other variables such as disease, weed competition, wind, and temperature affect crop yields but were not added due to the limited number of observations and the difficulty of measuring most of these variables quantitatively.

Estimation was performed using monthly rainfall in a polynomial specification where rainfall in critical growing months was squared to reflect diminishing yields associated with additional rainfall. Rainfall for each hedging month was included in the model due to the need for a new yield prediction for each hedging month. The need for monthly prediction created degrees of freedom problems due to only 15 yield observations over the data set 1966-1980. Data prior to 1966 were either missing or inconsistent.

Rainfall explained a high percentage of total variation in crop yields. The data, however, were plagued with collinearity problems so many individual parameter estimates were insignificant and often of the wrong sign. Ridge regression was performed on selected yield estimation equations for various K levels. New predicted values were determined and the sum of squared residuals (SSR) compared against OLS. In all cases OLS gave lower SSR's. Since the purpose was forecasting yields and not necessarily explaining each individual variable, the OLS estimates were selected. A thorough evaluation of individual yield estimates follows.

3.4.1 Yield Estimation

A linear specification, polynomial specification, and a ridge regression on the polynomial specification were performed at all locations. All three estimations are presented here for the Warsaw location. Similar results were obtained for Orange and the Midwest data. Due to the need for conciseness, only the final yield estimations are presented for Orange and Midwest.

Summary statistics for the linear specification at Warsaw are presented in Table 3. The corn model had a relatively good fit explaining 62% of total variation in corn yields. The signs of the coefficients appear correct with rainfall in the Spring and Fall months reducing yields and rainfall in the critical growing months of July and August are positive and significant. Collinearity diagnostics indicate that collinearity does not appear to be a problem. This is evidenced by no variance inflation factors greater than 10, no condition indices greater than 30, and only two variance proportions at the smallest eigenvalue greater than .50 which were April and May rainfall and May was only .56. The F value appears adequate considering a model of this type and there seems to be no serial correlation.

The soybean models do not perform quite as well as the corn model when comparing goodness of fit and F tests. Signs on the parameter estimates for the full season soybeans appear correct. Spring months coefficients are positive which probably indicates some soil moisture criteria. Critical months rainfall is positive but not significant. The harvest month variable is negative. The second crop soybean model has appropriate signs for rainfall with August being positive and significant. July rainfall is negative which probably indicates the effects of rainfall at planting. The models generally explain about 60% of the variation with relatively good predictions. There appears to be no serial correlation and collinearity does not seem to be a problem.

Table 3. Summary of Yield Estimation, Linear Specification Model, Warsaw Virginia Agricultural Experiment Station, 1966-1980.

Corn

Variable [1]	Estimate	SE [2]	T [3]	Prob > |T|	VIF [4]	VP! [5]	CI [6]
Intercept	68.0275	40.85	1.66	.1398	0.00	.54	1.00
Time	3.7684	2.56	1.47	.1843	2.14	.37	4.21
Apr [7]	-9.8001	10.76	-.91	.3926	2.40	.81	4.99
May	-5.3671	6.03	-.89	.4033	1.71	.56	5.75
Ju	-3.7651	4.22	-.89	.4022	1.42	.02	6.15
Jl	10.1529	3.86	2.63	.0339	1.66	.15	8.10
Aug	12.1719	5.12	2.38	.0490	2.04	.21	13.45
Sep	-.9678	3.63	-.27	.7972	2.08	.28	18.26

R^2 = .6252 AR^2 = .2504 F = 1.668 Prob > F = .2579

DW = 2.336 SSR = 5990.11

Soybeans

Variable	Estimate	SE	T	Prob > |T|	VIF	VP	CI
Intercept	13.6554	10.86	1.26	.2555	0.00	.75	1.00
Time	.0200	.63	.03	.9755	2.44	.38	4.08
Apr	.9857	2.65	.37	.7224	2.77	.76	4.40
May	2.2582	1.45	1.56	.1706	1.88	.56	5.26
Ju	.6310	1.05	.60	.5703	1.68	.14	6.02
Jl	.5118	.98	.52	.6203	2.03	.24	7.16
Aug	.6585	1.20	.55	.6042	2.15	.02	10.76
Sep	.7290	.90	.81	.4501	2.46	.01	14.63
Oct	-.3904	.78	-.50	.6356	1.97	.30	21.98

R^2 = .5986 AR^2 = .0634 F = 1.119 Prob > F = .4586

DW = 1.923 SSR = 269.73

Table 3. (continued)

Second Crop Soybeans

Intercept	-5.7202	16.20	-.35	.7361	0.00	.75	1.00
Time	-0.0303	.93	-.03	.9752	2.45	.38	4.08
Apr	1.4560	3.95	.37	.7250	2.77	.76	4.40
May	3.2961	2.16	1.52	.1785	1.88	.56	5.26
Ju	.9420	1.57	.60	.5699	1.68	.14	6.02
Jl	-.6606	1.46	-0.45	.6673	2.03	.24	7.16
Aug	3.6249	1.80	2.12	.0901	2.15	.02	10.76
Sep	-.7326	1.35	-.54	.6060	2.46	.01	14.63
Oct	.9062	1.17	.78	.4670	1.97	.30	21.98

R^2 = .6226 AR^2 = .1195 F = 1.237 Prob > F = .4089

DW = 2.393 SSR = 599.93

Wheat

Intercept	71.6787	16.65	4.30	.0035	0.00	.95	1.00
Time	-.8381	.76	-1.11	.3054	1.44	.12	5.09
Jan	-.2568	2.16	-.12	.9089	1.88	.02	5.30
Feb	.3564	2.26	.16	.8793	1.17	.12	6.23
Mar	-1.1079	3.14	-.35	.7343	2.05	.16	7.29
Apr	-2.4442	3.17	-.77	.4662	1.60	.74	10.12
May	-2.4437	2.42	-1.01	.3464	2.10	.37	11.26
Ju	2.1910	1.76	1.24	.2531	1.89	.00	19.52

R^2 = .4854 AR^2 = −.0293 F = .943 Prob > F = .5298

DW = 1.902 SSR = 783.75

[1]The variables January through October are mid-month total rainfalls.
[2]Standard error.
[3]T-statistic.
[4]Variance inflation factors.
[5]Variance proportions.
[6]Condition indices.
[7]The abbreviated variables names identify mid-month total rainfall for the months January through October.

The wheat model does not perform very well with low R^2s and F values. Signs, however, appear correct showing negative relationships between yields and rainfall during the disease building early Spring months and positive with rainfall during the dough and fill stage from mid May to mid June. No parameter estimates, however, are significant. Again, collinearity and serial correlation do not appear to be a problem.

The overall problem with the linear specification is inability to predict extreme variation in yields. In particular, these models failed to predict yields during drought conditions thereby consistently over-predicting low yields. Due to the relative importance of low yields on hedging results, additional model specifications were investigated.

The polynomial specification, in general, improved the goodness of fit but introduced collinearity into the data (See Table 4.) Many variables are insignificant as evidenced by low t-statistics. Collinearity diagnostics demonstrate that squaring of terms introduced collinearity as evidenced by high VIF's, condition indices, and high variance proportions for the smallest eigenvalue. The polynomial specification did manage to identify the importance of rainfall in the critical growing months. As a result, the model more accurately predicted low yields in drought years as evidenced by lower sum of squares of residuals (SSR).

The corn model was improved as evidenced by higher goodness of fit criteria. Signs of parameter estimates appear correct with July and July squared variables being highly significant. The polynomial specification predicted yields better that the linear specification as evidenced by the SSR falling by one half. Problems with the corn model include a less significant F statistic and a negative intercept term which initially places the estimation at a low level.

Collinearity is present in the data used to estimate the polynomial specified corn model. Variance inflation factors for the critical growing months range from 32.95 to 79.13. Four condition indices are above 10. Variance proportions indicate that collinearity exists between the intercept term and June, July, June squared, and July squared. The question is whether collinearity is a problem. A small eigenvalue is associated with the existance of a non-orthogonal matrix. One small eigenvalue in relation to the others may indicate collinearity which is degrading. In the model four eigenvalues approach zero. It is obvious that collinearity is present and it is degrading; but, the degree of damage is not known.

It is not clear whether the linear or polynomial specification best represents the full-crop soybean model. The goodness of fit increased and the F-statistic declined. Negative coefficients for June and August rainfall are questionable. The July variables appear to be of the correct signs and magnitudes. Again, no variables were highly significant when comparing t-statistics. The sum of squared residuals, however, were reduced indicating a better prediction.

The double-crop soybean model improved greatly with the polynomial specification. The model clearly shows the effects of May, July and August rainfall on double-crop soybean yields. The R^2 increased from 62% to 92% and the F-statistic increased from 1.237 to 2.979. The SSR decreased from 599.93 to 133.33.

The wheat model is improved with the polynomial specification. The damage of excess rainfall becomes apparent, particularly, the rainfall from mid April to mid May. The R^2's increased from 48% to 78% and F-statistic increased from .943 to 1.439. The SSR was reduced from 783.75 to 331.25.

An attempt was made to reduce multicollinearity by using ridge regression. Ridge regression is a biased estimation technique where small values are added to the X'X matrix in an attempt to lessen the nonorthoganality problem. The goal is to reduce the mean square errors below that of the ordinary least squares estimates. Ridge regression estimates for various levels of K are presented in Table 5. In many cases the sign of the coefficients, general magnitude, and standard errors improved. Multicollinearity was also reduced as evidenced by lower VIF's. Forecast errors between the ridge estimates and the polynomial specification OLS models were compared. In all cases the SSR for the OLS models were less than for the ridge estimates. Ridge estimates and an analysis of forecast errors in similar processes were performed at all locations.

Table 4. Summary of Yield Estimation, Polynomial Specification Model, Warsaw Virginia Agricultural Experiment Station, 1966-1980.

Corn

Variable [1]	Estimate	SE [2]	T [3]	Prob > |T|	VIF [4]	VP [5]	CI [6]
Intercept	-120.644	129.45	-.93	.4041	0.00	.87	1.00
Time	5.2171	2.83	1.84	.1390	3.07	.14	2.88
Apr [7]	-3.0817	14.37	-.21	.8407	5.03	.00	4.13
May	1.5177	7.10	.21	.8413	2.78	.04	4.56
Ju	.0036	25.76	.00	.9999	62.01	.58	6.41
Jl	57.2040	24.63	2.32	.0809	79.13	.61	8.14
Aug	17.9814	19.01	.95	.3978	32.95	.15	9.55
JuSQ [8]	-.2929	2.63	-.11	.9168	64.33	.58	19.60
JlSQ	-3.8209	1.94	-1.97	.1202	78.71	.63	52.19
AugSQ	-.5057	2.02	-.25	.8145	43.16	.01	55.33
Sep	-.9282	4.18	-.22	.8351	3.25	.00	88.52

R^2 = .8173 AR^2 = .3605 F = 1.789 Prob > F = .3022

DW = 2.097 SSR = 2919.85

Soybeans

Intercept	9.8498	39.63	.25	.8198	0.00	.87	1.00
Time	.1571	.87	.18	.8706	3.23	.13	2.94
Apr	1.2218	4.40	.28	.7993	5.05	.00	4.20
May	2.9891	2.18	1.37	.2646	2.81	.04	4.35
Ju	-2.6757	7.97	-.33	.7593	63.78	.58	4.87
Jl	5.1702	7.55	.68	.5426	79.65	.61	7.30
Aug	-1.5538	5.91	-.26	.8095	34.06	.14	9.60
JuSQ	.3740	.81	.46	.6750	64.84	.59	11.64
JlSQ	-.3653	.59	-.62	.5815	78.81	.63	22.34
AugSQ	.2169	.64	.34	.7565	46.29	.06	53.97
Sep	.4150	1.43	.29	.7913	4.10	.00	58.51
Oct	.3938	1.00	-.39	.7213	2.14	.00	91.59

R^2 = .6958 AR^2 = −.4198 F = .624 Prob > F = .7553

DW = 1.959 SSR = 204.44

Table 4. (continued)

Second Crop Soybeans

Intercept	-73.8686	32.00	-2.31	.1042	0.00	.87	1.00
Time	.5228	.71	.73	.5181	3.23	.13	2.94
Apr	4.2205	3.55	1.19	.3205	5.05	.00	4.20
May	6.0036	1.76	3.40	.0423	2.81	.04	4.35
Ju	.5107	6.44	.08	.9418	63.78	.58	4.87
Jl	17.2924	6.10	2.84	.0658	79.65	.61	7.30
Aug	5.8942	4.77	1.24	.3045	34.06	.14	9.60
JuSQ	.0662	.65	.10	.9255	64.84	.59	11.64
JlSQ	-1.4534	.48	-3.03	.0561	78.80	.63	22.34
AugSQ	-.2285	.52	-.44	.6877	46.29	.06	53.97
Sep	-.5536	1.16	-.48	.6655	4.10	.00	58.51
Oct	.6487	.81	.80	.4825	2.14	.00	91.59

R^2 = .9161 AR^2 = .6086 F = 2.979 Prob > F = .2001

DW = 1.845 SSR = 133.33

Wheat

Intercept	129.8396	37.75	3.44	.0263	0.00	.60	1.00
Time	-1.1953	.86	-1.38	.2393	2.53	.27	3.16
Jan	-2.0042	2.04	-.98	.3815	2.26	.01	3.44
Feb	-.8244	2.04	-.40	.7074	1.29	.07	6.22
Mar	-3.3440	10.14	-.33	.7582	28.99	.19	7.59
Apr	-7.5547	20.43	-.37	.7303	89.62	.98	9.35
May	-20.0091	8.14	-2.45	.0699	32.16	.11	10.39
MarSQ	.0667	1.55	.04	.9677	35.05	.26	12.65
AprSQ	.6720	2.70	.25	.8157	83.37	.97	40.33
MaySQ	1.7914	.78	2.29	.0837	33.69	.00	52.48
Ju	2.8176	1.94	1.45	.2212	3.12	.08	106.50

R^2 = .7825 AR^2 = .2387 F = 1.439 Prob > F = .3876

DW = 2.100 SSR = 331.25

[1]The variables January through October are mid-month total rainfalls.
[2]Standard error.
[3]T-statistic.
[4]Variance inflation factors.
[5]Variance proportions.
[6]Condition indices.
[7]The abbreviated variable names identify midmonth rainfall totals from January through October.
[8]Variable names with "SQ" are that month's total rainfall squared.

Table 5. Ridge Regression Estimates for Yield Estimation, Warsaw Agricultural Experiment Station, 1966-1980.

K = .005	AVCL [1] =	-24.00 + 4.18337 Time - 4.4121 Apr [2] -.9879 May -6.1215 Ju + 34.3448 Jl + 14.5579 Aug + .3116 JuSQ [3] -2.0085 JlSQ - .2870 AugSQ -.77470 Sep
K = .05	AVCH =	66.0249 + 2.82011 Time -5.5616 Apr -3.6019 May -4.4543 Ju + 10.6525 Jl + 8.72502 Aug +.1596 JuSQ -.1691 JlSQ +.1417 AugSQ -.37151 Sep
K = .005	AVSL =	13.6346 + .09042 Time + 1.10664 Apr + 2.69517 May -1.9004 Ju + 3.2354 Jl - 1.0479 Aug + .2880 JuSQ -.2144 JlSQ + .1669 AugSQ .47019 Sep -.36809 Oct
K = .07	AVSH =	17.3641 + .06297 Time + .67697 Apr + 2.03748 May -.02887 Ju + .71702 Jl + .03251 Aug +.08669 JuSQ -.01669 JlSQ + .07225 AugSQ + .47382 Sep -.32081 Oct
K = .005	AVS2L =	-40.991 + .21228 Time + 3.46377 Apr + 5.03168 May -1.1190 Ju + 9.5701 Jl + 4.5238 Aug + .2310 JuSQ -.8374 JlSQ - .1169 AugSQ -.61879 Sep + .74351 Oct
K = .07	AVS2H =	-4.2037 + .04445 Time + 1.36749 Apr + 3.23849 May -.1038 Ju + 1.1745 Jl + 2.35129 Aug + .1155 JuSQ -.1475 JlSQ + .0873 AugSQ -.65852 Sep + .69069 Oct
K = .07	AVWH =	76.3857 - .91809 Time -1.4694 Apr -4.2338 May + 1.81234 Ju - .73105 Jan + .09683 Feb -1.3395 Mar + .10591 MarSQ - .08354 AprSQ +.26353 MaySQ
K = .005	AVWL =	112.937 - 1.1737 Time - 4.3553 Apr - 15.212 May + 2.48370 Ju - 1.6602 Jan - .51357 Feb -3.5701 Mar + .1985 MarSQ + .2625 AprSQ + 1.3416 MaySQ

[1]Variable names ending in L and H indicate low and high K values for estimating average yields of corn (AVC), soybeans (AVS), second crop soybeans (AVS2), and wheat (W).
[2]The abbreviated variable names identify midmonth rainfall totals from January through October.
[3]The term "SQ" represents midmonth total rainfall squared.

3.4.2 Predicted Yields

Yields were predicted using the polynomial specification estimated by OLS. These models were primarily selected due to lower SSR's. Since the models are used for prediction, forecasting ability rather than structural soundness was the overriding criteria for model selection. The polynomial specification was used for all crops for consistency in the simulation process and to avoid the injection of specification bias. In most all cases the polynomial specification also gave better parameter estimates. A summary of the prediction for all crops at all locations is presented in Table 6.

Predicted yields for each hedging period were determined using each month's current rainfall up to that hedging period and assuming average rainfall for all future periods. Appendix Table 7 presents hedging period predicted yields along with actual yields for each crop and location for the period 1966-80.

Table 6. Yield Prediction Equations Used in the Simulation Process, All Crops at All Locations, 1966 - 1980.

WARSAW

CORN =
(129.45) [1] (2.83) (14.37) (7.10)
-120.644 + 5.2171 Time - 3.0817 Apr [2] + 1.5177 May
(25.76) (24.63) (19.01) (2.63)
+ .0036 Ju + 57.2040 Jl + 17.9814 Aug - .2929 JuSQ [3]
(1.94) (2.02) (4.18)
- 3.8209 JlSQ - .5057 AugSQ .9282 Sep

R^2 = .8173 F = 1.789⅛ ⅛Prob > F = .3022 DW = 2.097

SOYBEANS =
(39.63) (.87) (4.40) (2.18)
9.8498 + .1571 Time + 1.2218 Apr +2.9891 May
(7.97) (7.55) (5.91) (.81)
- 2.6757 Ju + 5.1702 Jl - 1.5538 Aug + .3740 JuSQ
(.59) (.64) (1.43) (1.00)
- .3653 JlSQ + .2169 AugSQ +.4149 Sep - .3938 Oct

R^2 = .6958 F = .624 Prob > F = .7553 DW = 1.959

SECOND CROP SOYBEANS =
(32.00) (.71) (3.55)
- 73.8686 +5227 Time + 4.2205 Apr
(1.76) (6.44) (6.10) (4.77)
+ 6.0036 May + .5107 Ju + 17.2924 Jl + 5.8942 Aug
(.65) (.48) (.52) (1.16)
+ .0662 JuSQ - 1.4534 JlSQ - .2285 AugSQ - .5536 Sep
(.81)
+ .6487 Oct

R^2 = .9161 F = 2.979 Prob > F = .2001 DW = 1.845

WHEAT =
(37.75) (.86) (2.04) (2.04)
129.839 - 1.1953 Time - 2.0042 Jan - .8244 Feb
(10.14) (20.43) (8.14) (1.55)
- 3.3440 Mar - 7.5547 Appr - 20.0091 May + .0667 MarSQ
(2.70) (.78) (1.94)
+ .6720 AprSQ + 1.7914 MaySQ +2.8176 Ju

R^2 = .7825 F = 1.439 Prob > F = .3876 DW = 2.100

[1]Standard error of the parameter estimate.
[2]The abbreviated variables identify midmonth total rainfall for January through October.

ORANGE

CORN =
(78.19) (1.79) (6.20) (6.30)
-20.4087 - .2710 Time + 3.0049 Apr + .6974 May
(19.59) (9.12) (19.56) (1.61)
+ 10.3488 Ju + 14.0238 Jl + 2.7135 Aug - .5945 JuSQ
(.59) (2.13) (3.95)
- .6327 JlSQ + .9130 AugSQ +3.7217 Sep

R^2 = .8089 F = 1.693 Prob > F = .3229 DW = 2.41

SOYBEANS =
(22.75) (.53) (1.78) (1.87)
28.9070 - .6246 Time - .5303 Apr - 1.3499 May
(6.13) (2.95) (6.35) (.48)
- 14.2101 Ju + 8.6749 Jl + 10.1400 Aug + .9951 JuSQ
(.19) (.70) (1.16) (.69)
- .5404 JlSQ - 1.266 AugSQ +2.4187 Sep + 1.8010 Oct

R^2 = .8421 F = 1.455 Prob > F = .4217 DW = 1.729

SECOND CROP SOYBEANS =
(9.27) (.22) (.73)
28.5208 + .2732 Time - 2.3729 Apr
(.76) (2.50) (1.20) (2.59)
- .0452 May - 10.7470 Ju + 7.5611 Jl + 5.094 Aug
(.20) (.08) (.28) (.47)
+ .7118 JuSQ - .4888 JlSQ - .7713 AugSQ +2.1115 Sep
(.28)
+ .7948 Oct

R^2 = .9727 F = 9.701 Prob > F = .0435 DW = 2.453

WHEAT =
(57.00) (1.83) (3.32) (5.24) (22.23)
86.9845 - .0608 Time - 3.246 Jan - 4.3614 Feb - 3.2555 Mar
(24.35) (27.90) (3.13) (2.90)
- 3.1303 Apr - 20.1189 May + .4840 MarSQ +1.4991 AprSQ
(4.00) (2.09)
+ 3.3817 MaySQ +1.3695 Ju

R^2 = .4753 F = .362 Prob > F = .9121 DW = 2.594

[3]Variable names with "SQ" are that month's total rainfall squared.

MIDWEST

CORN =
(53.90) (2.22) (5.42) (26.91)
78.4060 - .9089 Time + 2.2795 May + 30.8392 Ju
(14.23) (12.97) (12.65) (16.46)
+ 6.4041 Jl + 13.4388 Jl2 - 1.4634 Augl + 11.5887 Aug
(2.89) (1.20) (1.78) (11.29)
- 3.6683 JuSQ - 1.3129 JlSQ - .6975 AugSQ - 12.2185 Sep

R^2 = .8296 F = 1.328 Prob > F = .4569 DW = 2.336

SOYBEANS =
(15.82) (.65) (1.59) (7.90)
27.3003 + .8131 Time + 1.7437 May + 11.0491 Ju
(4.17) (3.80) (3.71) (4.83)
- 2.8896 Jl 6.2991 Jl2 - 1.4962 Augl - 4.3868 Aug
(.85) (.35) (.52)
- 1.1657 JuSQ - .0818 JlSQ + .4039 AugSQ
(3.31)
- 4.7734 Sep

R^2 = .8200 F = 1.242 Prob > F = .4830 DW = 2.182

3.5 Data Analysis

Minimum variance hedge levels depend upon the relationships between prices, yields, and resultant income changes associated with changes in futures prices. To address these relationships, various hedging strategies were simulated over the period 1966-1980. The minimum-variance hedge ratios were determined using simple regression with cash return the dependent variable and hedge return the independent variable. Hedging strategies using the minimum-variance ratios were simulated over the period and returns and variance of returns determined. The series of accounting statements provided a detailed summary of period returns.

The simulation process was used to determine returns to hedging for each hedging period and provide information needed to address multiple crop hedging in the Q.P. framework. A detailed summary of annual return to hedging and cash positions along with means, variances, ranges, covariances, correlation coefficients and other descriptive statistics were produced. In addition to descriptive statistics, variance-covariance matrices were developed and later used in the Q.P. analysis. Skewness and a test of normality for hedging period returns were also performed and used to provide information as to whether the returns are, in fact, non-normal. Should the distributions of returns be non-normal, then questions would have to be raised regarding whether the optimum hedge selections were misrepresented. A summary of cash prices, yields, and deflated cash returns for all crops at all three locations in presented in Appendix Figure 1.

Corn yields at Warsaw were more variable than at the other locations. Corn yields in the midwest showed the least amount of variation. The coefficients of variation for corn at Warsaw, Orange, and the Midwest were 27.38, 23.78, and 17.18, respectively. The correlation coefficients between price and yield were .38, .23, and -.27 for Warsaw, Orange, and the Midwest. The combined effects of yield variability and the relationship between prices received and yields result in income being much more variable at the Virginia locations than in the Midwest.

Soybean yields appeared to be less variable than corn yields at all three locations. Coefficients of variation for soybeans at Warsaw, Orange and the Midwest were 19.79, 18.49, and 15.37, respectively. Correlation coefficients between soybean price and yield were .09, -.06, and -.35 for Warsaw, Orange, and the Midwest location. Again, the negative relationship between soybean price and yield results in more stable returns in the Midwest.

Second crop soybean yields were variable ranging between 10 and 50 bushels per acre at the Virginia locations. Coefficients of variation were 37.88, and 22.80 for Warsaw and Orange. Wheat yields had coefficients of variation of 18.60 and 20.11 for Warsaw and Orange, respectively. Correlation coefficients between price and yield were .29 and .42 for second crop soybean and -.53 and -.02 for soft red winter wheat for Warsaw and Orange. Returns to second crop soybean were variable while wheat returns were relatively stable.

Negative relationships between cash and hedging returns indicate the need to hedge. Correlation coefficients for corn, first crop soybeans, and double crop soybeans generally indicate the need to hedge while wheat does not. The Virginia locations indicate a greater need to hedge than the midwest as indicated by the generally larger negative correlation coefficients between cash and hedging returns. (See Table 7 and 8).

The simulation process produced information describing the distribution of returns, prices, and yields. Normality is assumed under an E-V approach and is a basic assumption in portfolio theory. Many studies, some of which have already been discussed, have been performed evaluating normality and the effects on decision making. Most agree that lack of normality does not seriously affect the results. When multiple alternatives are analyzed as in the Q.P. analysis, aggregation of several distributions usually produce a normal distribution as described by the central limit theorem. The W statistic (Shapiro-Wilk) is used here to test for normality.

Skewness criteria and normality test for prices, yields, and returns for Warsaw, Ornage, and the Midwest are presented in Appendix Table 8. Crop yields tend to be skewed to the left and prices skewed to the right. Cash returns are slightly skewed to the right, futures returns to the left, and options returns to the right. In many cases the W statistics rejected normality especially for first and second soybean option hedges.[2] The W statistic is a distance measurement which falls within the range of 0 and 1 with small values leading to rejection of normality. A common practice is to reject normality for levels of significance of .01 or less. For 15 observations a W statistic less than .835 would result in rejecting normality at the .01 level. W statistics falling between .83 and .984 would represent a "Grey" area. All values of W above .984 would indicate normality could not be rejected at the .01 level. A summary of the distributions indicate that 25% of the price distributions were significantly non-normal and 75% in the "gray" area. No price distributions were statistically normal. Yields showed that 10%, 70%, and 20% were non-normal, "gray", or normal. Commodity futures fell 8% non-normal; 90% "gray", and 2% statistical normal. Distributions on commodity options had 44% non-normal, 54% "gray", and 2% statistically normal.

Crop diversification is viewed as an alternative way to reduce income variation. Diversification may also reduce the overall level of hedging (Berck). To address these issues, an indepth analysis of multiple crop alternatives were analyzed in the Q.P. framework and is presented in Chapter 4. A general evaluation of diversification among crops can be seen here with their correlation coefficients presented in Table 9. Corn and soybean returns were positively correlated with each other. Soybeans were negatively correlated with wheat.

The test of normality indicates that non-normal distributions exists especially when hedging soybeans with options. Non-normality may have an effect on the minimum-variance hedge ratios for first and second crop soybeans. Additional testing would have to be made to determine the effects on the magnitudes of the ratios. The multiple crop and crop hedge organizations would succumb to the central limit theorem and would most likely be normally distributed. It appears that the non-normality problems associated with any of the variables may well fall back to the non-normality of price.

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[2] Hedging returns for options are not skewed as far to the right as one might suspect. Years when prices advanced, prices were also volatile. The increased volatility caused put premiums to be relatively high when the hedge was placed. This tended to reduce returns to the option hedges.

Table 7. Correlation Coefficients Between Cash and Futures Hedging Returns at All Three Locations, 1966 - 1980.

Warsaw

Hedging [1] Period	Corn	Soybeans	Second Soybeans	Wheat
1	-.68	-.62	-.66	.55
2	-.75	-.66	-.74	.45
3	-.79	-.68	-.79	.35
4	-.71	-.50	-.67	.28
5	-.52	-.23	-.42	.35
6	.38	.06	-.19	.42
7		-.53	-.63	.29
8				.40

Orange

Hedging Period	Corn	Soybeans	Second Soybeans	Wheat
1	-.43	-.73	-.70	.31
2	-.48	-.69	-.71	.46
3	-.52	-.66	-.70	.43
4	-.47	-.38	-.70	.44
5	-.52	-.03	-.71	.33
6	.54	.38	-.70	.32
7		-.38	-.57	.30
8				.11

Midwest

Hedging Period	Corn	Soybeans
1	-.62	-.37
2	-.59	-.23
3	-.61	-.17
4	-.50	.23
5	-.40	.41
6	.75	.56

[1]Hedging periods 1 through 8 represent hedging periods March through August for corn, March through September for soybeans, and January through May for wheat.

Table 8. Correlation Coefficients Between Cash and Options Hedging Returns at All Three Locations, 1966 -1980.

Warsaw

Hedging [1] Period	Corn	Soybeans	Second Soybeans	Wheat
1	-.80	-.48	-.62	.42
2	-.78	-.39	-.59	.21
3	-.76	-.37	-.57	.27
4	-.67	-.36	-.49	.15
5	-.67	-.36	-.44	.11
6	.11	.07	-.15	.21
7		-.53	-.57	.01
8				.38

Orange

Hedging Period	Corn	Soybeans	Second Soybeans	Wheat
1	-.51	-.61	-.53	.26
2	-.52	-.47	-.41	.68
3	-.50	-.48	-.42	.57
4	-.43	-.49	-.41	.67
5	-.49	-.15	-.40	.36
6	.36	.42	.04	.37
7		-.53	-.67	.39
8				-.02

Midwest

Hedging Period	Corn	Soybeans
1	-.64	-.33
2	-.65	-.23
3	-.66	-.31
4	-.59	-.44
5	-.52	-.24
6	.65	.54

[1]Hedging periods 1 through 8 represent hedging periods March through August for corn, March through September for Soybeans, and January through May for wheat.

Table 9. Correlation Coefficients Among Crop Return at All Three Locations, 1966 -1980.

Warsaw

	Corn	Soybeans	Second Soybeans	Wheat
Corn	1			
Soybeans	.63	1		
Second Soybeans	.66	.91	1	
Wheat	-.01	-.54	-.44	1

Orange

	Corn	Soybeans	Second Soybeans	Wheat
Corn	1			
Soybeans	.52	1		
Second Soybeans	.35	.90	1	
Wheat	.27	-.11	-.11	1

Midwest

Corn-Soybeans	.55

Chapter 4. Comparison of Optimum Hedge Levels

Chapter 4 is the description of the analysis and a presentation of the results. The chapter is presented in three major sections. First, single-crop minimum-variance hedge ratios are discussed. The derivation of the minimum variance ratios was developed in Chapter 2 and extended in Chapter 3. Optimum hedge levels with diversification of crops and crop hedges are presented next using the Q.P. framework. The last section addresses the effects of financial leverage on optimum hedge levels and hedge selection.

4.1 Single-Crop Minimum-Variance Hedge Comparisons

The minimum-variance hedge ratio is synonymous with the parameter estimate in a regression where cash returns are the dependent variable and hedge returns the independent variable. The hedge ratio or the "Beta coefficient" gives the weight which minimizes the variance between cash and hedge returns. In traditional econometric terms, the parameter estimate gives the rate which best explains total variation in cash returns. The number of possible ratios or hedge levels which could be compared under a traditional simulation process is essentially infinite. The minimum variance approach reduces the set by identifying strategies which give minimum variance. Since no other ratio will give a lower level of variance, comparison of hedging tools can be made by comparing the levels of variance and the corresponding return. Analyzing the data from a minimum variance approach makes E-V comparison straightforward. The parameter estimates also give statistical information such as R^2 and F test statistics which provide information on how significant hedges were over time at offsetting variation in cash returns.

Minimum variance hedge ratios for hedges using futures and options contracts for all three locations are presented in Tables 10 and 11. Hedge and cash returns, standard deviations, ranges, and a goodness of fit for each hedge are provided. Minimum variation in income was obtained with hedging for all crops except wheat. Hedging wheat increased income variability which means cash positions give a more stable income over time. Hedge levels for corn and soybeans were higher for the Virginia locations than for the Midwest. This result is consistent with the previous theory that income is less variable for producers located in major production areas. Income losses associated with production shortfalls are offset by price increases. The results are consistent with most hedging studies using future contracts where lower levels of income variation can only be achieved with reduced levels of average returns. Hedging with options gave minimum-variance ratios which were E-V efficient to futures and cash for corn at all three locations. Results were mixed for first and second crop soybeans. In most cases option hedges resulted in higher average returns.

Minimum-variance hedge ratios for Warsaw corn ranged from 104.83 to 180.06 percent of predicted yield for March and July, respectively, when hedging with futures contracts. Average futures losses ranged from $4.92 to $19.41 and average returns ranged from $56.58 to $71.07 as compared to an average cash return of $76.00. Hedging reduced variation of returns from 15 to 39 percent. The periods 1 through 6 represent hedging decisions made in March through August and can be viewed as an E-V frontier where each is a separate strategy. The optimum allocation of hedges in presented in the following section. Appendix Table 9 presents the range of futures and options returns and average return for each hedging strategy.

Hedging soybeans with futures contracts produced negative futures returns for early hedges and positive returns for late season hedges. Hedging losses were reduced as yield prediction improved. It would appear that soybean hedging with futures could best be performed once yields are virtually known. It can be seen that superior yield prediction equations would greatly improve hedging with futures contracts. Late season hedging gave E-V efficient returns over cash.

Yield variability at Orange was less than Warsaw and subsequently the minimum-variance hedge levels were less. Minimum-variance hedge levels for corn at Orange were almost one-half those of Warsaw. First crop soybean hedge ratios were only slightly less for early season hedges but fell from 67.48 to 7.13 and 217.35 to 143.88 percent of expected yields for July and September hedges, respectively. The distribution of returns and E-V comparisons were almost the same.

Hedging levels at the Midwest location were less than either Warsaw and Orange for corn and significantly less for soybeans. Again, yield variability and relative price movements underly the difference in optimum hedge levels using the futures contracts. Average futures losses declined as yield predictability improved. Late season hedges for corn were E-V efficient over cash.

A comparison of hedging using options contracts at Warsaw resulted in option gains for early period corn hedging and option losses for late period hedging. Options gains were as high as $12.91/acre for hedges placed in April. It appears that increased price volatility during corn's critical growing period makes options in mid-summer an expensive means of hedging. It should also be noted that as the season progresses, prices adjust, and many pricing opportunities are lost. Returns to option hedging for corn are E-V efficient to futures hedges and cash positions. Minimum variance ratios are large ranging from 235.70 to 503.06 percent of expected yield. There ratios are high due to the absolute value of option gains and losses being far less than gain and losses to cash returns. The minimum-variance criteria forces greater amounts to be hedged in order to offset cash gains and losses. Variability of returns decreased from 25.83 to 39.60 percent and average returns increased up to 16.99 percent.

Table 10. Single Crop Futures Hedges For All Crops at All Three Locations, 1966-1980.

WARSAW

top PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN FUTURES RETURNS [2]	MEAN HEDGE RETURNS	STANDARD DEVIATION	R^2
CORN					
CASH	N/A	N/A	75.9957	71.3699	
1 [1]	1.0483	-14.2391	61.7567	52.0555	.47
2	1.1386	-15.9575	60.0384	47.2287	.56
3	1.3099	-19.4138	56.5819	43.5793	.63
4	1.3265	-10.1803	65.8155	50.3230	.46
5	1.8006	- 4.9194	71.0763	60.7346	.27
6	N/A				
SOYBEANS					
CASH	N/A	N/A	47.1977	37.8683	
1	1.0090	- 9.2354	37.9622	29.5701	.39
2	.9804	-10.0134	37.1843	28.3678	.44
3	1.0524	- 9.1147	38.0829	27.7588	.46
4	.8451	- 2.4873	44.7103	32.6810	.25
5	.6748	2.4280	49.6257	36.7936	.06
6	N/A				
7	2.1735	1.8220	49.0197	32.1897	.28
SECOND SOYBEANS					
CASH	N/A	N/A	25.0806	48.8147	
1	1.2393	-12.5476	12.5330	36.4446	.44
2	1.1635	-14.2812	10.7994	32.6564	.55
3	1.2948	-13.3949	11.6857	30.0537	.62
4	1.2418	- 5.2937	19.7869	36.0649	.45
5	1.6273	4.5637	29.6443	44.3250	.17
6	.7134	1.6310	26.7116	47.8846	.04
7	3.4899	- .9486	24.1319	37.8489	.40

[1] Variables identified as 1 through 7 are monthly hedging periods March through August for corn and March through September for soybeans.
[2] All returns and standard deviations are deflated.

ORANGE

PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN FUTURES RETURNS	MEAN HEDGE RETURNS	STANDARD DEVIATION	R^2
CORN					
CASH	N/A	N/A	52.2754	40.5078	
1	.4856	-4.9323	47.3431	36.5565	.18
2	.5302	-6.0710	46.2044	35.4264	.23
3	.6247	-7.6554	44.6200	34.5116	.27
4	.6519	-3.6678	48.6076	35.6848	.22
5	1.1277	-1.6341	50.6413	34.6104	.27
6	N/A				
SOYBEANS					
CASH	N/A	N/A	55.7121	35.9289	
1	.9845	-10.2925	45.4197	24.5042	.53
2	.9062	- 9.9895	45.7226	25.8263	.48
3	.8838	- 8.3492	47.3630	26.8679	.44
4	.4254	- 1.2197	54.4924	33.2436	.14
5	.0713	.2774	55.9896	35.9107	.00
6	N/A				
7	1.4388	.0545	55.7667	33.1832	.15
SECOND SOYBEANS					
CASH	N/A	N/A	30.0744	36.8579	
1	1.1726	- 9.9495	20.1248	26.2253	.49
2	1.2121	-10.1907	19.8837	25.8683	.53
3	1.2375	- 9.0193	21.0551	26.2640	.52
4	.8541	- 2.2330	27.8413	30.2642	.35
5	1.0333	2.5675	32.6416	34.5569	.13
6	.1122	.2758	30.3502	36.8772	.00
7	2.9455	- .0737	20.0007	38.9485	.37

MIDWEST

PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN FUTURES RETURNS	MEAN HEDGE RETURNS	STANDARD DEVIATION	R^2
CORN					
CASH	N/A	N/A	50.3977	38.0773	
1	.4747	-6.4763	43.9214	29.9685	.38
2	.4403	-6.4031	43.9947	30.6691	.35
3	.4853	-7.6622	42.7355	30.2087	.37
4	.5414	-3.8432	46.5545	32.9166	.25
5	.7856	.2172	50.6149	34.8174	.16
6	N/A				
SOYBEANS					
CASH	N/A	N/A	40.3706	20.8298	
1	.2419	-2.9312	37.4394	19.3847	.13
2	.1479	-1.9239	38.4467	20.2429	.06
3	.1094	-1.2496	39.1209	20.5083	.03
4	N/A				
5	N/A				
6	N/A				

Table 11. Single Crop Options Hedges For All Crops At All Three Locations, 1966 - 1980

WARSAW

PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN OPTIONS RETURNS [2]	MEAN HEDGE RETURNS	STANDARD DEVIATION	R^2
CORN					
CASH	N/A	N/A	75.9958	71.3699	
1 [1]	2.3570	7.7906	83.7864	43.1063	.63
2	2.7874	12.9137	88.9094	44.3401	.61
3	3.3580	8.7281	84.7238	45.9570	.58
4	2.7106	9.5455	85.5412	52.7531	.45
5	5.0306	- 6.5897	69.4060	52.9358	.44
6	N/A				
SOYBEANS					
CASH	N/A	N/A	47.1977	37.8683	
1	1.8499	- .4431	46.7545	33.2535	.23
2	1.5001	.2231	47.4208	34.8219	.15
3	1.4026	- .5630	46.6347	35.1330	.14
4	1.2903	.1550	47.3526	35.3370	.13
5	2.4181	.5079	47.7056	35.3081	.13
6	N/A				
7	4.6769	.7776	47.9753	32.1205	.28
SECOND SOYBEANS					
CASH	N/A	N/A	25.0806	48.8147	
1	2.7508	- .6878	24.3927	38.1625	.39
2	2.5478	- .0639	25.0166	39.3952	.35
3	2.3365	- 1.4503	23.6302	40.1470	.32
4	2.0672	- .1625	24.9180	42.6488	.24
5	4.1667	.3145	25.3951	43.9212	.19
6	.8779	.9805	26.0611	48.2897	.02
7	7.1261	- 1.3590	23.7215	40.2067	.32

[1] Variables identified as 1 through 7 are monthly hedging periods
March through August for corn and March through September for soybeans.
[2] All returns and standard deviations are deflated.

ORANGE

PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN OPTIONS RETURNS	MEAN HEDGE RETURNS	STANDARD DEVIATION	R^2
CORN					
CASH	N/A	N/A	52.2754	40.5078	
1	1.1263	3.4857	55.7611	34.8078	.26
2	1.3912	4.9655	57.2409	34.5484	.27
3	1.6688	3.1838	55.4592	34.9731	.25
4	1.1933	3.6037	55.8792	36.5364	.19
5	2.0692	- .8669	51.4085	35.2927	.24
6	N/A				
SOYBEANS					
CASH	N/A	N/A	55.7121	35.9289	
1	2.0036	- .6277	55.0844	28.5622	.37
2	1.5563	.3524	56.0645	31.6738	.22
3	1.5962	- .3657	55.3464	31.5269	.23
4	1.1655	.9900	56.7021	31.3734	.24
5	.8411	- .0351	55.6770	35.5219	.02
6	N/A				
7	4.6670	-2.4502	53.2619	30.4551	.28
SECOND SOYBEANS					
CASH	N/A	N/A	30.0744	36.8579	
1	2.1080	- .4652	29.6092	31.1819	.28
2	1.5614	.6617	30.7361	33.6542	.17
3	1.6816	- .1073	29.9670	33.4509	.18
4	1.2279	.9296	31.0012	33.6541	.17
5	2.9056	.5257	29.5487	33.7364	.16
6	N/A				
7	7.5661	-3.1695	26.9087	27.2185	.45

MIDWEST

PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN OPTIONS RETURNS	MEAN HEDGE RETURNS	STANDARD DEVIATION	R^2
CORN					
CASH	N/A	N/A	50.3977	38.0773	
1	1.0751	2.7190	53.1168	29.2733	.41
2	1.3451	4.4628	54.8605	29.0582	.42
3	1.7619	1.7986	52.1963	28.5008	.44
4	1.4275	1.5484	51.9462	30.8157	.34
5	1.9622	-2.6420	47.7558	32.5684	.27
6	N/A				
SOYBEANS					
CASH	N/A	N/A	40.3706	20.8298	
1	.5227	-.1522	40.2183	19.6223	.11
2	.3557	.1438	40.5144	20.2574	.05
3	.4778	-.1031	40.2675	19.7669	.10
4	.8167	-.3802	39.9904	18.7374	.19
5	N/A				
6	N/A				

Soybean hedging with options was mixed resulting in option gains and losses. The inability of options to offset income variability is evident in the low R^2s. The performance of options hedging with second crop soybeans was improved over first crop soybeans as evidenced by higher R^2s and lower levels of standard deviations relative to cash. Again, options appear preferred where yield uncertainty was a problem. Average income and income variability was higher with options than with futures contracts when hedging soybeans.

Hedging with options at Orange was similar to hedging with options at Warsaw. Corn returns were E-V efficient over cash and futures and soybean hedging was mixed. Overall hedge levels were generally less and were similar to futures hedging at Orange.

Returns to corn hedging with options at the Midwest location were also E-V efficient to cash and futures hedges. Return and risk reduction were best for early season hedging. Soybean hedging was mixed showing very little change in income and income variability when compared to cash. Again, returns to option hedging were lower as the season progressed.

Results from the minimum-variance analysis provide information which is not currently available in the literature. When yield variability is accurately addressed, E-V efficient alternatives to strict cash sales are available. These E-V efficient alternatives were obtained assuming efficient futures prices. Improved price prediction could improve producer returns even beyond these levels. Commodity options provide E-V efficiency for crops with highly variable and unpredictable yields and for hedges placed early in the season when yield capability is unknown. Futures contracts give E-V efficiency with improved yield prediction. It appears that hedging opportunities could be greatly enhanced with improved yield prediction.

The results also support the author's hypothesis that hedging is less desirable for grain producers located in major production areas. This, in part, explains why very few producers hedge. It also demonstrates the need to encourage producers in the fringe areas of production to hedge.

Through hedging, producers in peripheral areas of production could greatly reduce variation in income.

4.2 Optimum Allocation of Crop Hedges

Quadratic programming techniques were used to determine if an allocation of hedged positions throughout the growing season would be preferred to a single hedge decision. The analysis compared allocations of both futures and options contracts for various levels of risk aversion parameters and determined optimum hedge levels for option and futures combined. The Q.P. algorithm used the Wolfe reduced gradient method. Analysis was performed with the nonlinear programming package MINOS. A Fortran subroutine was added to allow changes in the risk aversion parameters. The program was also standardized to allow any size variance-covariance matrix for the nonlinear section.

Since hedging has traditionally been considered a risk reduction technique, only risk aversion parameters between zero and one were considered. The analysis did not seek solutions optimal to risk neutral or risk seeking individuals. Risk parameters used were consistent with estimates found by Knowles for commercial size farmers with incomes between $20,000 and $100,000. Their risk aversion coefficients fell within the range 0 to .0003. Several other studies including Wilson, and Lin, Dean, and Moore indicate that farmers exhibit risk averting, risk neutral, and risk preferring preferences. Risk aversion parameters of 1, .1, .01, .001 and .0001 were used in this study. They would represent risk aversion coefficients of .002[3] to .0000002 for average farm incomes ranging between $10,000 and $100,000 and .02 to .000002 for average incomes between $1,000 and $10,000, etc., for equivalent coefficients of variation.

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[3] Risk aversion coefficients are reduced by one-half as defined by the negative exponential utility function. (See Chapter 2).

The allocation of hedges across the growing season was analyzed for the Warsaw location. The results obtained from repeating the process at all locations did not appear to justify the data requirements needed to perform the analysis since optimum allocations of hedges were also determined in the crop diversification models which were performed at all locations.

The optimum hedge levels are equivalent to the percentage of the predicted yield that should be hedged at that particular hedging period. Since we are assuming that the future's price is the best predictor of future price, the optimum hedge levels would be placed regardless of the absolute level of price. For example, even if prices were at historically high levels, one could not say that the price could not go even higher. Therefore, the optimum hedge levels depend on historical price patterns within the hedging period relative to improvements in yield predictions. The optimum allocation of hedges should be viewed as the hedging plan prior to planting. Should price expectations be built into the model, the hedge ratios would change during the year and be higher or lower dependent on those price expectations.

Hedging in the Q.P framework was constrained to no more than 100 percent of the acreage allocated to each crop. Government program restrictions were not included in the model but the effects of past government programs are incorporated in the price. These results are recommended hedge levels given that past government programs were in existence.

An evaluation of the allocation of futures hedges at Warsaw reveals all crops were completely hedged with the exception of wheat (Table 12).[4] Optimal hedges involved both single hedges and allocations of hedges across the season. Hedging produced lower levels of income variation than cash at high levels of risk aversion and higher levels of income variation than cash at low levels of risk aversion. At low levels of risk aversion, the producers would in fact be speculating in short positions. This is evidenced by a higher level of standard deviation than a strict cash sale. The optimization procedure is selecting a short position which is risk increasing rather than risk reducing

[4] Table 12 gives the percentage of the total crop hedged in each period for risk parameters g 1 through g .0001.

which we normally associate with hedging. The reason the short position is selected is because it adds returns with small increases in variance.

Hedging with options contracts at Warsaw resulted in fully hedged positions but only two plans were allocations of hedges across the season (Table 13). Average returns and variance were generally higher than with futures. Higher levels of risk aversion resulted in hedges placed early in the season. Lower levels of risk aversion selected late season hedges.

Table 12. Optimum Allocation of Futures Hedges at Warsaw, 1966 - 1980.

RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN					
CASH	1.	1.	1.	1.	1.
CF1 [1]					
CF2			.1812		
CF3	1.	1.			
CF4			.8188		
CF5					
CF6				1.	1.
RETURN	61.17 [2]	61.17	67.17	79.81	79.81
STANDARD DEVIATION	45.58	45.58	50.51	55.08	55.08
CASH RETURN	76.00				
STANDARD DEVIATION	71.37				
SOYBEANS					
CASH	1.	1.	1.	1.	1.
SF1 [3]					
SF2					
SF3	.9132	.8179			
SF4					
SF5			.7938		
SF6				1.	1.
SF7	.0868	.1821.	.2062		
RETURN	39.36	40.27	50.23	50.79	51.00
STANDARD DEVIATION	27.75	27.84	36.21	37.04	41.80
CASH RETURN	47.20				
STANDARD DEVIATION	37.87				

RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
2nd CROP					
SOYBEANS					
CASH	1.	1.	1.	1.	1.
SF21 [4]					
SF22					
SF23	1.	1.	.14	.14	
SF24			.30	.30	
SF25			.56	.56	1.
SF26					
SF27					
RETURN	14.74	14.74	24.00	24.00	27.89
STANDARD DEVIATION	31.30	31.30	39.47	39.47	45.02
CASH RETURN	25.08				
STANDARD DEVIATION	48.81				
WHEAT					
CASH	1.	1.	1.	1.	1.
WF1 [5]					
WF2					
WF3					
WF4			.33	1.	1.
WF5					
WF6					
WF7					
WF8					
RETURN	21.66	21.66	22.49	24.18	24.18
STANDARD DEVIATION	14.48	14.48	16.44	23.09	23.09
CASH RETURN	21.66				
STANDARD DEVIATION	14.48				

[1]CF1 through CF6 represent corn hedging periods March through August.
[2]Returns and standard deviations of returns are deflated.
[3]SF1 and SF7 represent soybean hedging periods March through September.
[4]SF21 through SF27 represent second crop soybean hedging periods March through September.
[5]WF1 through WF8 represent wheat hedging periods October through May.

Table 13. Optimum Allocation of Options Hedges at Warsaw, 1966 - 1980.

RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN					
CASH	1.	1.	1.	1.	1.
CO1 [1]	1.	1.	1.		
CO2				1.	1.
CO3					
CO4					
CO5					
CO6					
RETURN	79.30	79.30	79.30	80.63	80.63
STANDARD DEVIATION	54.13	54.13	54.13	57.03	57.03
CASH RETURN [2]	76.00				
STANDARD DEVIATION	71.37				
SOYBEANS					
CASH	1.	1.	1.	1.	1.
SO1 [3]	1.	1.	.8507		
SO2					
SO3					
SO4					
SO5					
SO6					
SO7			.1493	1.	1.
RETURN	46.96	46.95	47.02	48.86	48.86
STANDARD DEVIATION	34.28	34.28	34.33	39.77	39.77
CASH RETURN	47.20				
STANDARD DEVIATION	37.87				

RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
2nd CROP					
SOYBEANS					
CASH	1.	1.	1.	1.	1.
SO21	1.	1.	.96		
SO22 [4]			.04		
SO23					
SO24					
SO25					
SO26				1.	1.
SO27					
RETURN	24.83	24.83	24.83	25.20	26.20
STANDARD DEVIATION	42.80	42.80	42.81	48.30	48.30
CASH RETURN	25.08				
STANDARD DEVIATION	48.81				
WHEAT					
CASH	1.	1.	1.	1.	1.
WO1 [5]					
WO2					
WO3					
WO4					
WO5		.01	1.	1.	1.
WO6					
WO7					
WO8					
RETURN	21.66	21.69	24.78	24.78	24.78
STANDARD DEVIATION	14.48	14.49	18.04	18.04	18.04
CASH RETURN	21.66				
STANDARD DEVIATION	14.48				

[1]CO1 through CO6 represent corn hedging periods March through August.
[2]Returns and standard deviations of returns are deflated.
[3]SO1 and SO7 represent soybean hedging periods March through September.
[4]SO21 through SO27 represent second crop soybean hedging periods March through September.
[5]WO1 through WO8 represent wheat hedging periods October through May.

Table 14 presents optimum hedge levels when options and futures were combined and hedging involved both hedging tools. Options were preferred for corn hedgers with low levels of risk aversion. Futures appear preferred for more risk averse corn producers and all risk averse soybean producers. Options entered into wheat hedging as a short speculative position as evidenced by higher levels of variation than the cash position. Again, all crops were fully hedged with an allocation of hedging across the season preferred at several levels of risk aversion. The results presented differ from the E-V comparisons of the minimum-variance hedge ratios. A high level of option hedging was needed to obtain minimum variance. The optimum allocation of hedges here was restricted to no more than 100 percent of the acreage in the Q.P. analysis.

Optimum allocations of hedges should be viewed as the optimum levels to hedge during the year regardless of price expectations because an efficient market is assumed. The levels to hedge, however, would change as yield expectations change. The allocation of hedges addresses within crop year hedging which, to the author's knowledge, has yet to be addressed in the literature. Berck suggested placing the study in an optimal control framework. The optimal hedge levels presented here could be adjusted for price expectations different from the futures if the decision making process were placed in an optimal control framework. This topic is discussed later in the area of further research.

4.3 Optimal Hedge Levels Under Crop Diversification

Hedging is only one means of reducing income variability. The degree of financial leverage, use of off-farm income, and crop diversification can also influence a producer's risk exposure. Crop diversification can reduce income variation if returns to two or more crops are less than perfectly positively correlated over time. Crop diversification may, in fact, be a more efficient means of reducing income variability than hedging and therefore reduce the overall need to hedge (Berck).

Table 14. Optimum Allocation of Options and Futures Hedges at Warsaw, 1966 -1980.

RISK PARAMETER	$\lambda = 1.$	$\lambda = .1.$	$\lambda = .01.$	$\lambda = .001.$	$\lambda = .0001.$
CORN					
CASH	1.	1.	1.	1.	1.
CF3 [1]	1.	1.			
CO1			1.		
CO2				1.	1.
RETURN [2]	61.17	61.17	79.30	80.63	80.63
STANDARD DEVIATION	45.58	45.58	54.13	57.03	57.03
CASH RETURN	76.00				
STANDARD DEVIATION	71.37				
SOYBEANS					
CASH	1.	1.	1.	1.	1.
SF3 [3]	.91	.82			
SF5			.79	1.	
SF6					1.
SF7	.09	.18	.21		
RETURN	39.36	40.27	50.23	50.80	51.00
STANDARD DEVIATION	27.74	27.83	36.21	37.04	41.79
CASH RETURN	47.20				
STANDARD DEVIATION	37.87				

RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
2nd CROP					
SOYBEANS					
CASH	1.	1.	1.	1.	1.
SF23 [4]	1.	1.	.14		
SF24			.30		
SF25			.56	1.	1.
RETURN	14.74	14.74	24.00	27.89	27.89
STANDARD DEVIATION	31.30	31.30	39.47	45.02	45.02
CASH RETURN	25.08				
STANDARD DEVIATION	48.81				
WHEAT					
CASH	1.	1.	1.	1.	1.
WO5 [5]		.01	1.	1.	1.
RETURN	21.66	21.69	24.78	24.78	24.78
STANDARD DEVIATION	14.48	14.49	18.04	18.04	18.04
CASH RETURN	21.66				
STANDARD DEVIATION	14.48				

[1] CO1 and CO2 represent corn options hedging in March and April, CF3 is futures hedging in May.
[2] Returns and standard deviation of returns are deflated.
[3] SF3, 5, 6, and 7 represent soybean futures hedging May, July, August, and September, respectively.
[4] SF24 and SF25 represent second crop soybean futures hedging in June and July.
[5] WO5 is wheat options hedging in February.

This section addresses the effects of crop diversification on hedging and optimum hedge levels. The analysis also determines optimal farm plans when decisions are made on acreage and hedging simultaneously. The analysis defines the effects of those decisions on overall farm level income risk exposure.

Table 15 presents optimum allocations of crops and crop hedges when diversification was considered. The percentage of acreage allocated to each crop is shown along with each crops hedges. The percentage of the acreage hedged could not exceed the percentage allocated to each crop. The analysis included all locations and five levels of risk aversion. Land was limited to 1 acre which represents allocations of each crop not to exceed 100% of the land available. Double crop soybeans had no direct land constraint but could not exceed the number of wheat acres. Other resource constraints such as labor, capital, and crop rotation were not considered. The allocation of crops and crop hedges should be viewed as an overall optimum plan. The total amount hedged would change as predicted yields changed. The percent to hedge would not change in this case because price expectations assume an efficient futures market.

In general, crop diversification reduced the overall level of income variability but did not reduce the need to hedge. All but one solution resulted in optimum organizations that were completely hedged. The use of hedging in addition to crop diversification reduced the standard deviation of income by almost one-half in some instances when comparing standard deviations of cash and hedged positions. This was particularly true for high levels of risk aversion. Standard deviation of income was higher for hedged positions than cash positions for some low levels of risk aversion. These optimum organizations included speculative short position.

Table 15. Optimum Allocation of Crops and Hedges Under Crop Diversification For All Locations, 1966 - 1980.

Warsaw

RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN				1	1
CO2 [1]				1	1
SOYBEAN	.3040	.3040	.3615		
SF3 [2]	.0303	.0303			
SF4	.1658	.1658	.1753		
SO6			.1226		
SO6	.1079	.1079	.0636		
WHEAT	.6960	.6960	.6385		
WO1 [3]			.4543		
WO2	.0359	.0359			
WO8	.6600	.6600	.1842		
RETURN [4]	27.90	27.90	29.79	80.63	80.63
STANDARD DEVIATION	7.95	7.95	8.58	57.02	57.02
CASH RETURN	29.42	29.42	30.89	76.00	76.00
STANDARD DEVIATION	14.26	14.26	22.73	71.37	71.37

Orange

RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN	.0509	.1318	.3954		
CF5	.0509	.1217			
CO2			.3954		
CO4		.0101			
SOYBEAN	.3423	.4308	.6046	1	1
SF3	.0662	.1874			
SF6				1	1
SO4			.6046		
SO5	.2761	.2434			
WHEAT	.6068	.4374			
WF1		.0703			
WF8	.6068	.3671			
RETURN	27.65	33.87	56.28	59.45	59.45
STANDARD DEVIATION	15.31	16.46	27.02	45.56	45.56
CASH RETURN	29.97	36.83	54.35	55.71	55.71
STANDARD DEVIATION	17.45	18.66	28.56	35.93	35.93

Midwest

RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN	.1464	.1910	.7672	1	1
CF6				.1092	1
CO2		.0336	.7672	.8908	
CO4	.1464	.1574			
SOYBEAN	.8357	.8090	.2328		
SO4	.8357	.8090	.2328		
RETURN	42.38	42.94	50.72	53.83	54.77
STANDARD DEVIATION	18.13	18.23	27.23	34.69	54.22
CASH RETURN	41.84	42.28	48.06	50.40	50.40
STANDARD DEVIATION	21.35	21.70	33.31	38.08	38.08

[1]CO1 through CO6 and CF1 through CF6 represent corn hedging periods March through August for options and futures, respectively.
[2]SF1 through SF7 and SO1 and SO7 represent soybean hedging periods March through September for futures and options, respectively.
[3]WO1 through WO8 and WF1 through WF8 represent wheat hedging periods October through May for options and futures, respectively.
[4]All returns and standard deviations of returns are deflated.

Optimum organizations resulted in a mix of crops and hedges for high levels of risk aversion. Primary crops included wheat and soybeans and included a mixture of futures and options hedges. Low levels of risk aversion generally resulted in one-crop one-hedge organizations. Hedges were mixed using both futures and options contracts. Corn was the crop selected at low levels of risk aversion at Warsaw and the Midwest. Single crop soybeans were selected at the Orange location. The results can be compared to those of Berck who indicated that crop diversification reduced the need to hedge. In this analysis diversification reduced hedge levels from the minimum-variance levels indicated earlier; but, hedging also reduced income variation beyond that of strict diversification. The results, then should be viewed from the fact that hedging reduces income variation beyond what is obtainable through crop diversification. In many cases the use of hedges produced an E-V efficient alternative to a strict cash position.

4.4 The Effects of Financial Leverage On Hedging Decisions

The Separation Theorem as developed by Sharpe and discussed in Chapter 2 describes how financial leverage can take portfolio selection off the efficient frontier. All efficient portfolios then become a linear combination of one efficient portfolio and various degrees of financial leverage. It becomes more efficient at some point to use financial risk as a replacement for operating risk. The degree of financial risk will depend on the cost of capital.

To determine optimal portfolios given financial leverage would involve a shift from using "average returns" and the "variance of average returns" to "returns on equity" and the "variance of returns on equity". Portfolio analysis in the financial management area uses the "state preference model" where various states of nature are compared in an E-V framework. Since the original model was not developed in returns on equity, optimization for returns on equity and variance of equity for various levels of risk aversion were not performed. A similar E-V analysis for selected strategies incorporating leverage was developed and E-V efficient sets determined.

The Q.P. model for Warsaw was adjusted to consider limited amounts of capital and the alternative of borrowing. Capital requirements for crops were set equal to the deflated total cost of production per acre. Debt-equity ratios of 0, .25, .50, and .75 were analyzed at all four levels of risk aversion. Results are presented in Table 16.

Twenty hedging portfolios were determined and their returns, standard deviations, returns on equity, and standard deviation equity ratios (SD/E) identified. They are labeled in Table 16 as A through T. There were fourteen different portfolios with return on equity and SD/E ratios ranging from .59 and .17 to 3.82 and 2.80, respectively. Only ten of the set were E-V efficient.

Financial leverage would allow a producer to take a low operating risk portfolio and use financial leverage to achieve E-V efficiency over a high operating risk portfolio. For example, portfolios P and Q are low operating risk portfolios combining soybeans and wheat. Leveraged at 75 percent these two portfolios are E-V efficient over portfolios C, D, E, H, I, and J which are corn and corn soybean combinations. More indepth study of hedging when financial leverage is considered is needed before specific recommendations of hedging levels can be made.

Table 16. The Effects of Financial Leverage on Optimum Hedge Levels and Hedge Selection, Warsaw Location, 1966 - 1980.

D/E=0					
PORTFOLIOS	A	B	C	D	E
RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN			.85	1	1
CO1 [1]			.85		
CO2				1	1
SOYBEAN	.30	.36	.15		
SF3 [2]	.03				
SF4	.17	.18	.15		
SF5		.12			
SO5	.10	.06			
WHEAT	.70	.64			
WO1 [3]		.45			
WO2	.04				
WO8	.66	.19			
CAPITAL RETURN [4]	27.90	29.79	74.09	80.63	80.63
STANDARD DEVIATION	7.95	8.58	48.56	57.03	57.03
ROE [5]	.59	.63	.97	.99	.99
SD/E [6]	.17	.18	.64	.70	.70

D/E=.25

Portfolios	F	G	H	I	J
RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN			.82	1	1
CO1			.82		
CO2				1	1
SOYBEAN	.30	.36	.18		
SF3	.03				
SF4	.17	.18	.18		
SF6		.12			
SO6	.10	.06			
WHEAT	.70	.64			
WO1		.45			
WO2	.04				
WO8	.66	.19			
CAPITAL			14.33	20.34	20.34
RETURN	27.90	29.79	72.20	79.64	79.64
STANDARD DEVIATION	7.95	8.58	47.34	57.03	57.03
ROE	.59	.63	1.18	1.31	1.31
SD/E	.17	.18	.77	.93	.93

D/E = .50					
Portfolios	K	L	M	N	O
RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN			.82	1	1
CO1			.82		
CO2				1	1
SOYBEAN	.30	.36	.18		
SF3	.03				
SF4	.17	.18	.18		
SF6		.12			
SO6	.10	.06			
WHEAT	.70	.64			
WO1		.45			
WO2	.04				
WO8	.66	.19			
CAPITAL	4.44	4.55	32.67	38.68	38.68
RETURN	27.68	29.57	71.30	78.74	78.74
STANDARD DEVIATION	7.94	8.58	47.34	57.03	57.03
ROE	.64	.69	1.67	1.85	1.85
SD/E	.19	.20	1.11	1.34	1.34

D/E=.75					
Portfolios	P	Q	R	S	T
RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN			.82	1	1
CO1			.82		
CO2				1	1
SOYBEAN	.30	.36	.18		
SF3	.03				
SF4	.17	.17	.18		
SF6		.12			
SO6	.10	.07			
WHEAT	.70	.64			
WO1		.45			
WO2	.04				
WO8		.19			
CAPITAL	26.77	26.88	55.00	61.01	61.01
RETURN	26.59	28.48	70.21	77.65	77.65
STANDARD DEVIATION	7.94	8.58	47.34	57.03	57.03
ROE	1.31	1.40	3.45	3.82	3.82
SD/E	.39	.42	2.33	2.80	2.80

[1] CO1 through CO6 and CF1 through CF6 represent corn hedging periods March through August for options and futures, respectively.
[2] SF1 through SF7 and SO1 through SO7 represent soybean hedging periods March through September for futures and options, respectively.
[3] WO1 through WO8 and WF1 through WF8 represent wheat hedging periods October through May for options and futures, respectively.
[4] Returns and standard deviations of returns are deflated.
[5] Return on equity.
[6] Standard deviation - equity ratio.

Chapter 5. The Out-of-Sample Test

The optimum hedge ratios developed and outlined in Chapter 4 were tested over a new data set. The purpose was to determine if the strategies would perform outside the original data set. All strategies with the exception of optimum allocation of single crop hedges at Warsaw were tested in the out-of-sample test.

The out-of-sample test included years 1981 through 1985. A simulation model similar to the previous simulation discussed in Chapter 3 was developed. The model included 1981-1985 yields, cash prices, futures prices, option premiums, and predicted yields. A summary of data used is presented in Appendices 9 through 12.

Yields were predicted using the polynomial specification models as presented in Chapter 3. The out-of-sample test included extreme weather conditions at all three locations. Many of the rainfall variables were out of the range of the original data set. Yield predictions were extremely high and extremely low in several cases. When lack of rainfall caused predicted yield to be negative, a zero prediction was substituted. Yields were not restricted on the up side.

The out-of-sample test was a relatively good data set from which to test the strategies. Both the Virginia and Midwest locations experienced drought and good growing conditions over the period 1981-85. The Midwestern drought in combination with the Payment in Kind (PIK) program in 1983 sent prices to comparable levels with 1974. In 1985, prices of commodities considered were at or near the loan rate. The Virginia locations again experienced years of low yields and low prices.

Results of the out-of-sample test were mixed. Futures and options hedges gave both higher and lower returns and higher and lower levels of variation when compared to each other and compared to cash. In general, hedging increased average returns over strict cash sales. This is due, in part, to a general decline in commodity prices over the period with the exception of 1983. Options performed significantly better than futures when hedging corn and somewhat better when hedging soybeans. It would appear that the optimum hedge ratios were performing as expected. They were forcing average returns and income variability back to the levels expressed in the original data set.

5.1 Futures Hedges in the Out-of-Sample Test

Futures hedges in the out-of-sample test are presented in Table 17. The hedge levels tested were minimum variance hedge ratios. Corn hedging with futures in period two increased average returns by 138 percent and standard deviations by 38 percent when compared to a strict cash position. Returns and standard deviation for all corn hedges increased. Overall returns, however, were less than in the original data set. Average returns for first crop soybeans increased slightly with futures hedges while returns to second crop soybean hedging actually declined. Post sample yields at Orange were less than during the previous period. Income variation increased slightly.

Corn and first crop soybean hedging at Orange Experiment Station actually increased returns and lowered income variability over strict cash sales. Income gains were not as great as seen at the Warsaw station. Second crop soybean hedging at Orange significantly increased returns but also increased the overall level of income variability.

Hedging at the Midwest location produced similar results. Futures positions for both corn and soybeans produced positive returns. The futures gains increased hedged returns for corn and

Table 17. Futures Hedging in the Out-Of-Sample Test, All Locations, 1981-1985.

Warsaw

PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN FUTURES RETURNS [1]	MEAN HEDGE RETURNS	STANDARD DEVIATION
CORN				
CASH	N/A	N/A	19.0719	48.1581
1 [2]	1.0483	17.5861	36.6580	59.1871
2	1.1386	26.2788	45.3507	62.4120
3	1.3099	22.2382	41.3108	64.7622
4	1.3265	17.4958	36.5677	70.8253
5	1.8006	15.6054	34.6773	73.4968
6	N/A			
SOYBEANS				
CASH	N/A	N/A	24.3908	12.2581
1 [3]	1.0090	5.0489	29.4397	19.9897
2	.9804	6.5618	30.9527	20.0506
3	1.0524	1.2602	25.6511	19.7677
4	.8451	.5849	24.9757	19.5808
5	.6748	-.2732	24.1176	14.2540
6	N/A			
7	2.1735	-.3813	24.0095	12.8131
SECOND CROP SOYBEAN				
CASH	N/A	N/A	12.2739	17.1584
1 [4]	1.2393	7.3965	19.6703	13.3922
2	1.1635	7.8677	20.1415	16.6962
3	1.2948	3.2023	15.4761	23.9668
4	1.2418	-2.1039	10.1699	26.4524
5	1.6273	-5.0255	7.2483	15.0422
6	.7134	2.8871	15.1609	18.4926
7	3.4899	-3.1850	9.0889	16.9392

Table 17. (continued)

Orange

PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN FUTURES RETURNS	MEAN HEDGE RETURNS	STANDARD DEVIATION
CORN				
CASH	N/A	N/A	74.5116	27.0536
1	.4856	4.6490	79.1606	22.9535
2	.5302	6.3467	80.8582	22.6437
3	.6247	5.3225	79.8341	21.8599
4	.6519	4.3344	78.8460	21.5191
5	1.1277	5.6874	80.1990	25.1858
6	N/A			
SOYBEANS				
CASH	N/A	N/A	44.2448	35.4768
1	.9845	4.7459	48.9908	19.5269
2	.9062	6.6100	50.8549	22.2159
3	.8838	5.6331	49.8779	22.5740
4	.4254	2.1136	46.3584	28.2130
5	.0713	.1833	44.4282	34.6535
6	N/A			
7	1.4388	1.1325	45.3773	34.2465
SECOND SOYBEANS				
CASH	N/A	N/A	-5.8887	3.3423
1	1.1726	5.7179	- .1708	24.2167
2	1.2121	10.2752	4.3864	21.1791
3	1.2375	8.6890	2.8002	20.7026
4	.8541	4.1908	-1.6980	16.9524
5	1.0333	2.9695	-2.9193	14.7286
6	.1122	.3081	-5.5806	3.2144
7	2.29455	2.3925	-3.4962	5.3507

Table 17. (continued)

Midwest

PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN FUTURES RETURNS	MEAN HEDGE RETURNS	STANDARD DEVIATION
CORN				
CASH	N/A	N/A	22.1491	16.5472
1	.4747	6.6834	28.8325	22.1375
2	.4403	8.2022	30.3513	21.5832
3	.4853	6.6491	28.7983	23.3628
4	.5414	5.7471	27.8962	27.0751
5	.7856	8.6420	30.7911	28.5398
6	N/A			
SOYBEANS				
CASH	N/A	N/A	13.0376	10.9497
1	.2419	1.3771	14.4147	9.4691
2	.1479	1.2164	14.2540	9.7386
3	.1094	.6443	13.6819	9.7833
4	N/A			
5	N/A			
6	N/A			

[1]Returns and standard deviation of returns are deflated.
[2]Periods 1 through 6 represent corn hedging periods March through September.
[3]Periods 1 through 7 represent soybean hedging periods March through September.
[4]Periods 1 through 7 represent second crop soybean hedging periods March through September.

also increased the levels of standard deviation. Hedge returns to corn were higher than for soybeans with the standard deviation of hedged returns of soybeans slightly less than cash.

5.2 Options Hedging in the Out-of-Sample Test

Corn hedging with options at Warsaw significantly increased average returns. Mean hedge returns increased up to 318 percent over cash and up to 101 percent over the same hedge with futures (See Table 18). First crop soybean hedging with options resulted in increased returns over cash and futures hedging. Standard deviation of returns increased slightly over cash and significantly less than futures hedging. Returns to second crop soybeans were mixed with both increases and decreases in average returns and standard deviations. Average returns for second crop soybeans were more than doubled for early season hedges when compared to cash.

Average returns to corn hedging with options at Orange produced somewhat higher average returns than cash and futures with income variability less than cash and essentially unchanged from futures hedging. Option hedging of first crop soybeans resulted in higher returns than futures hedging with slight increases in income variability. A comparison of options to futures hedging with second crop soybeans were similar with higher returns and slightly higher standard deviations.

Options hedging in the Midwest resulted in higher average returns and standard deviations than cash and futures hedging for both corn and soybeans. Average returns for some corn option hedging resulted in returns over twice the amount for cash sales. Options hedges again performed better for corn than for soybeans.

Table 18. Option Hedging in the Out-Of-Sample Test, All Locations, 1981-1985.

Warsaw

PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN OPTIONS RETURNS [1]	MEAN HEDGE RETURNS	STANDARD DEVIATION
CORN				
CASH	N/A	N/A	19.0719	48.1581
1 [2]	2.3570	38.8584	57.9303	56.2150
2	2.7874	62.8597	81.9316	77.1003
3	3.3580	64.1659	83.2378	76.6019
4	2.7106	45.3521	64.4240	66.7807
5	5.0306	41.2716	60.3434	99.8709
6	N/A			
SOYBEANS				
CASH	N/A	N/A	24.3908	12.2581
1 [3]	1.8499	13.3867	37.7776	13.7856
2	1.5001	12.2366	36.6275	14.7345
3	1.4026	7.0796	31.4704	13.7954
4	1.2903	6.8913	31.2822	13.7814
5	2.4181	2.4319	26.8228	18.9156
6	N/A			
7	4.6769	- 5.5848	18.8060	12.6326
SECOND SOYBEANS				
CASH	N/A	N/A	12.2738	17.1584
1 [4]	2.7508	23.8219	36.0958	4.6614
2	2.5478	24.5608	36.8346	12.6741
3	2.3365	19.7265	32.0000	18.3649
4	2.0672	12.4483	24.7222	13.1574
5	4.1667	- 3.3999	8.8739	21.9628
6	.8779	.4231	38.6806	50.8699
7	7.1261	-13.4656	-1.1918	14.5826

Table 18. (continued)

Orange

PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN OPTIONS RETURNS	MEAN HEDGE RETURNS	STANDARD DEVIATION
CORN				
CASH	N/A	N/A	71.5116	27.0539
1	1.1263	10.5007	85.0123	19.0983
2	1.3912	16.7993	91.3108	22.2704
3	1.6688	17.0113	91.5228	22.8800
4	1.1933	11.0891	85.6006	23.0744
5	2.0692	11.7086	86.2201	27.5407
6	N/A			
SOYBEANS				
CASH	N/A	N/A	44.2448	35.4768
1	2.0036	13.8344	58.0793	24.8665
2	1.5563	12.3876	56.6324	25.9851
3	1.5962	12.6504	56.8952	26.0820
4	1.1655	8.1630	52.4078	30.2846
5	.8411	1.9498	46.1947	33.0113
6	N/A			
7	4.6769	- 1.4836	42.7612	28.9137
SECOND SOYBEANS				
CASH	N/A	N/A	-5.8887	3.3423
1	2.1080	14.8873	8.9986	14.5851
2	1.5614	13.1753	7.2865	13.3693
3	1.6816	13.7514	7.8626	12.5058
4	1.2279	8.0901	2.2014	9.1769
5	2.9056	7.1161	1.2273	19.8307
6	.8779	.5150	-5.3737	3.1452
7	7.5661	- 2.1537	-8.0425	12.4068

Table 18. (continued)

Midwest

PERIOD	MINIMUM VARIANCE HEDGE RATIO	MEAN OPTIONS RETURNS	MEAN HEDGE RETURNS	STANDARD DEVIATION
CORN				
CASH	N/A	N/A	22.1491	16.5472
1	1.0751	14.5568	36.7060	23.1142
2	1.3451	11.4822	33.6314	22.7268
3	1.7619	27.6907	49.8399	32.9002
4	1.4275	20.6957	42.8449	26.6012
5	1.9622	20.4923	42.6414	32.4201
6	N/A			
SOYBEANS				
CASH	N/A	N/A	13.0374	10.9497
1	.5227	4.3319	17.3695	9.5440
2	.3557	3.3340	16.3716	9.5142
3	.4778	4.4476	17.4853	9.0175
4	.8167	5.5844	18.6221	9.2315
5	N/A			
6	N/A			

[1]Returns and standard deviation of returns are deflated.
[2]Periods 1 through 6 represent corn hedging periods March through August.
[3]Periods 1 through 7 represent soybean hedging periods March through September.
[4]Periods 1 through 7 represent second crop soybean hedging periods March through September.

5.3 Crop Diversification and the Out-of-Sample Test

The results of optimal allocation of crops and corresponding crop hedges were tested in the out-of-sample test. (See Table 19) Average returns and standard deviations were similar to those obtained in the original data set. Returns and standard deviations are very similar for optimum organizations which are highly diversified.

The stability of income demonstrated here is interesting in light of a worsening economic condition for many producers over the period 1981-85. It was demonstrated in the financial leverage section that the highly diversified low risk portfolios combining wheat and soybeans could be leveraged to produced comparable returns and lower variances when compared to higher risk organizations. Less risk averse producers could have leveraged these low risk portfolios and would have experienced relatively stable incomes over the period 1981-85. Outside the work of McKinnon, very few hedging studies have concentrated on long-term income stability. McKinnon's work concentrated on long-term hedging and storage and concluded that combined use could result in long-term income stability. It appears here that diversification of crops and crop hedges are also effective in stabilizing income over time

Table 19. Optimum Hedges and Crop Diversification, Test on the New Data Set, All Locations, 1981 -1985.

Warsaw

RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN				1	1
CO2 [1]				1	1
SOYBEAN	.3040	.3040	.3615		
SF3 [2]	.0303	.0303			
SF4	.1658	.1658	.1753		
SF6			.1226		
SO6	.1079	.1079	.0636		
WHEAT	.6960	.6960	.6385		
WO1 [3]			.4543		
WO2	.0359	.0359			
WO8	.6600	.6600	.1842		
RETURN [4]	25.6625	25.6625	25.6625	28.1027	28.1027
STANDARD DEVIATION	10.9215	10.9215	10.9215	13.1336	13.1336

Table 19. (continued)

Orange

RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN	.0509	.1318	.3954		
CF5	.0509	.1217			
CO2					
CO4		.0101	.3954		
SOYBEAN	.3423	.4308	.6046	1	1
SF3	.0662	.1874			
SF6					
SO4					
SO5	.2761	.2434			
WHEAT	.6068	.4374	.6046		
WF1		.0703			
WF8	.6068	.3671			
RETURN	47.5117	57.5295	63.9908	46.8917	46.8917
STANDARD DEVIATION	2.1175	3.2700	19.2394	36.5091	36.5091

Table 19. (continued)

Midwest

RISK PARAMETER	λ = 1.	λ = .1.	λ = .01.	λ = .001.	λ = .0001.
CORN	.1464	.1910	.7672	1	1
CF6				.1092	1
CO2		.0336	.7672	.8908	
CO4	.1464	.1574			
SOYBEAN	.8536	.8090	.2328		
SO4	.8357	.8090	.2328		
RETURN	22.2084	22.8785	28.1689	30.3907	27.9859
STANDARD DEVIATION	10.1502	9.9341	13.2730	16.4828	18.0577

[1]CO1 through CO6 and CF1 through CF6 are corn hedging periods March through August for options and futures, respectively.
[2]SF1 through SF7 and SO1 through SO7 are soybean hedging periods March through September for futures and options, respectively.
[3]WO1 through WO8 and WF1 through WF8 are Wheat hedging periods October through May for options and futures, respectively.
[4]Returns and standard deviation of returns are deflated.

Chapter 6. Summary and Conclusions

The purpose of this study was to compare commodity options and futures contracts as hedging tools and determine optimum levels of hedging. The comparison objective has been thoroughly addressed and commodity options appear to be preferred to futures for hedging crops under most scenarios. The optimum levels to hedge, however, are very sensitive to type of crop, location, crop diversification, and degree of financial leverage. Optimum levels of hedging are in fact an individual farmer decision. This is particularly true in light of producer price expectations which may be different from the futures market. The approach presented here could be adapted to determine farm level hedge and crop alternatives given individual farm constraints and farmer price expectations.

The results are too diverse and varied to be able to make specific recommendations on hedge levels that would encompass all grain producers. There are, however, several general conclusions which have surfaced. A list of results and general observations are as follows:

Results and General Observation

1. Hedging grain with commodity options contracts is a viable alternative for reducing long run income variation.

2. Commodity options contracts are superior to futures contacts as hedging tools for early season hedges.

3. Hedging with options appear preferred to hedging with futures for crops with highly variable and uncertain yields.

4. An allocation of hedges across the growing season seems preferred to single hedges for most ranges of risk aversion.

5. Crop diversification, in general, reduced income variability but did not reduce overall need to hedge.

6. Crop and hedge diversification in combination with financial leverage is an effective means of stabilizing producer income over time.

7. The degree of risk aversion substantially alters the optimum allocation of crops and corresponding crop hedges.

8. Combinations of hedges using both commodity options and futures contracts are preferred for many levels of risk aversion.

9. Optimum hedge levels vary depending upon farm location and alternative farm enterprises.

10. A strict cash position for wheat offers more income stability than a hedged position.

The study presented here is unique in a number of ways. First, very little if any published work is available on hedging grains with commodity options contracts. Second, the study addresses hedging strategies under the realm of production uncertainty. Third, the study demonstrates that there are E-V efficient alternatives to strict cash sales. And fourth, crop and hedge diversification in combination with financial leverage is an effective means of obtaining long term income stability.

This study addresses a wide range of topics which may influence hedging decisions and introduces a number of new concepts. The study, however, is far from complete. An indept study of yield estimation in cooperation with agronomist might produce better yield forecasting techniques which may influence the results. On farm data of commercial grain operations would provide yield information superior to those taken from variety tests at the various Agricultural Experiment Stations. The model would also need to be reformulated so that optimization for various levels of risk aversion would incorporate financial leverage and optimal solutions determined rather than just producing E-V frontiers.

Recommendations for further study include refinement of the current model to incorporate realistic assumptions surrounding price expectations. Economic, time series, and technical trading systems could produce price expectations different from the market. These price expectation can be expressed as profit expectations and incorporated into the model. Again, the model should be reformulated so that optimization would incorporate financial leverage as a relevant variable. Lastly, the analysis should be performed at numerous locations so that regional differences could be compared.

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Appendix

Appendix Table 1. Summary of Average Crop Yield at All Locations, 1966 - 1980. [1]

	Warsaw				Orange				Midwest	
Year	Corn	SB	SB2	Wheat	Corn	SB	SB2	Wheat	Corn	SB
1966	93	20	16**	60	70*	29	21**	41	120	38
1967	131	37	30**	64	147	39	28**	43	129	42
1968	127	33	27**	59	131	33	24**	46	117	40
1969	113	37	30**	67	123	51	37**	51	133	44
1970	117	28	23**	70	94	36	26**	42	114	38
1971	102	41	33**	71*	110	41	30**	61*	145	44
1972	128	40	19	59*	116	34	25**	52*	128	42
1973	130	37	31	44	113	44	32**	60	130	39
1974	160	41	43	49	95	36	36	43	76	20
1975	175	39	31	36	153	42	30	60	156	45
1976	162	32	28	49	77	38	33	27	148	43
1977	43	36	11	42	83	35	31	63	131	42
1978	113	44	51	52	124	50	50	60	139	45
1979	168	38	35	61	129	43	27	53*	144	44
1980	91	22	14	58	76	25	27	46	87	40

* Yields extrapolated from country averages.
** Yields were extrapolated from the full season crop.

[1] Crops analyzed included corn, soybeans (SB), second crop soybeans (SB2), and wheat.

Appendix Table 2. Summary of Cash Prices [1] at All Locations, 1966 - 1980.

	Warsaw			Orange			Midwest	
Year	Corn	Soybeans	Wheat	Corn	Soybeans	Wheat	Corn	Soybeans
1966	1.45*	2.97	1.35*	1.45*	2.97*	1.35*	1.30	2.91
1967	1.19*	2.58	1.50*	1.19*	2.58*	1.50*	1.00	2.53
1968	1.04*	2.45	1.25*	1.04*	2.45*	1.25*	.96	2.42
1969	1.16	2.28	1.26*	1.19	2.27	1.26*	1.10	2.32
1970	1.41	2.83	1.15	1.46	2.79	1.15	1.38	2.93
1971	1.09	2.99	1.26	1.15	2.90	1.25	.99	3.01
1972	1.24	3.16	1.35	1.27	3.18	1.36	1.24	3.29
1973	2.12	5.95	1.27	2.10	6.03	1.31	2.29	5.67
1974	3.27	7.90	2.35	3.27	7.90	2.41	3.69	7.48
1975	2.60	4.93	3.68	2.54	4.86	3.56	2.60	5.01
1976	2.53	6.15	3.27	2.54	6.10	3.25	2.38	6.17
1977	1.72	5.00	3.21	1.65	4.91	3.17	1.74	5.02
1978	1.98	6.46	1.99	2.00	6.37	1.84	2.08	6.69
1979	2.64	6.13	3.05	2.53	6.00	2.92	2.55	6.53
1980	3.45	8.25	4.36	3.44	8.13	4.28	3.35	8.32

* State average price as reported by Virginia Agricultural Statistics

[1] Local cash prices are on the 15th or closest day to the 15th of the harvest month. Harvest months are September for Virginia corn, October for Midwest corn, October for all soybeans, and June for wheat.

Appendix Table 3. Summary of Commodity Futures Prices [1] For All Crops, 1966 - 1980.

December Corn

	Mar [2]	Apr	May	Ju	Jl	Aug	Sep	Oct
1966	1.18	1.21	1.21	1.32	1.47	1.53	1.43	1.34
1967	1.39	1.40	1.36	1.31	1.24	1.17	1.16	1.14
1968	1.29	1.27	1.20	1.19	1.14	1.03	1.02	1.08
1969	1.15	1.17	1.24	1.23	1.26	1.15	1.17	1.20
1970	1.17	1.19	1.22	1.26	1.30	1.49	1.54	1.47
1971	1.47	1.44	1.39	1.59	1.43	1.22	1.14	1.16
1972	1.27	1.29	1.27	1.22	1.26	1.27	1.43	1.33
1973	1.47	1.58	1.71	2.01	2.36	3.03	2.50	2.54
1974	2.76	2.47	2.47	2.67	3.20	3.62	3.51	3.80
1975	2.66	2.58	2.43	2.54	2.55	3.22	3.01	2.92
1976	2.66	2.67	2.65	2.78	2.90	2.78	2.89	2.66
1977	2.71	2.65	2.49	2.38	2.16	1.94	1.99	2.13
1978	2.49	2.57	2.59	2.56	2.39	2.26	2.21	2.34
1979	2.57	2.63	2.70	3.17	3.15	2.84	2.83	2.78
1980	3.03	2.95	2.99	2.92	3.37	3.43	3.52	3.57

November Soybeans

	Mar	Apr	May	Ju	Jl	Aug	Sep	Oct
1966	2.70	2.69	2.80	2.90	3.30	3.25	3.15	2.90
1967	2.82	2.78	2.77	2.77	2.70	2.68	2.67	2.63
1968	2.71	2.64	2.62	2.60	2.57	2.52	2.53	2.52
1969	2.36	2.35	2.38	2.36	2.39	2.35	2.39	2.43
1970	2.52	2.58	2.62	2.72	2.97	2.84	2.85	2.95
1971	2.87	2.82	2.98	3.19	3.40	3.25	3.08	3.24
1972	3.14	3.23	3.24	3.23	3.20	3.31	3.37	3.32
1973	4.08	4.49	4.76	6.65	7.58	8.39	6.39	6.29
1974	6.18	5.51	5.54	5.62	7.43	7.70	7.58	8.49
1975	5.68	5.59	4.96	5.14	5.37	6.23	5.93	5.28
1976	5.01	5.08	5.46	6.35	7.27	6.59	6.85	6.31
1977	7.15	7.34	7.37	6.90	5.92	5.23	5.14	5.14
1978	6.22	6.13	6.31	6.15	6.02	6.37	6.52	6.88
1979	7.04	6.98	7.24	8.13	7.60	7.23	7.26	6.45
1980	6.91	6.54	6.53	6.62	8.26	7.74	8.54	8.49

[1]Commodity futures prices are closings on the 15th of each hedging month or the first trading day after the 15th.
[2]Column headings are abbreviations for the months January through October.

July Wheat

	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Ju
1966	1.48	1.43	1.60	1.63	1.59	1.56	1.57	1.60	1.71
1967	1.68	1.74	1.80	1.67	1.65	1.78	1.68	1.67	1.67
1968	1.59	1.54	1.54	1.51	1.51	1.57	1.41	1.40	1.30
1969	1.36	1.40	1.34	1.35	1.34	1.29	1.30	1.32	1.26
1970	1.37	1.37	1.36	1.37	1.38	1.37	1.39	1.36	1.37
1971	1.59	1.62	1.58	1.63	1.56	1.54	1.59	1.50	1.73
1972	1.39	1.40	1.48	1.45	1.41	1.48	1.48	1.46	1.43
1973	1.92	2.11	2.45	2.42	2.11	2.12	2.28	2.44	3.00
1974	3.85	3.48	4.63	5.03	5.33	4.66	4.27	3.68	3.88
1975	3.54	3.97	3.78	3.98	3.73	3.53	3.36	3.15	3.17
1976	4.20	3.69	3.41	3.66	4.00	3.77	3.65	3.56	3.60
1977	3.12	2.74	2.78	2.90	2.95	2.84	2.79	2.52	2.48
1978	2.72	2.94	2.74	2.68	2.66	2.92	3.30	3.21	3.10
1979	3.34	3.22	3.21	3.21	3.22	3.23	3.24	3.72	4.22
1980	4.51	4.53	4.52	4.50	4.66	4.33	4.00	4.24	4.02

Appendix Table 4. Midmonth Total Rainfall [1] for All Locations, 1966 -1980.

Warsaw

	Jan [2]	Feb	Mar	Apr	May	Ju	Jl	Aug	Sep	Oct
1966	1.31	4.40	2.22	1.93	3.51	2.55	3.53	3.50	4.63	7.85
1967	3.07	1.98	2.25	1.84	3.56	1.64	4.62	5.54	3.84	1.88
1968	4.65	0.48	3.31	2.65	3.08	6.18	3.41	5.42	1.99	0.87
1969	1.21	4.26	3.53	2.39	2.55	6.29	3.02	7.26	9.45	2.96
1970	3.75	1.75	1.46	4.09	2.02	3.73	6.52	2.36	1.11	0.51
1971	4.16	3.36	1.38	2.91	3.30	7.84	3.47	3.38	3.76	4.21
1972	2.62	2.49	3.23	3.21	5.14	6.83	10.65	3.57	3.26	2.67
1973	3.23	3.91	1.94	3.46	2.97	2.66	3.32	3.53	3.87	3.60
1974	2.00	2.11	1.06	5.77	3.13	2.34	2.57	9.42	5.62	0.65
1975	5.40	3.14	2.65	4.64	3.99	4.92	9.54	3.31	5.49	9.24
1976	3.97	4.12	1.47	2.01	3.41	2.45	3.87	2.43	3.61	10.35
1977	2.40	0.36	1.41	2.90	4.57	4.89	2.36	1.07	6.59	2.31
1978	8.17	4.29	2.76	2.08	8.77	6.26	2.08	4.65	2.33	0.44
1979	3.76	3.87	6.22	3.29	1.77	8.42	3.66	7.02	12.79	6.43
1980	1.77	2.08	2.23	4.20	2.19	2.92	1.96	3.86	1.34	3.47

Orange

	Jan	Feb	Mar	Apr	May	Ju	Jl	Aug	Sep	Oct
1966	1.24	2.73	2.63	1.54	4.49	4.97	1.75	3.70	2.04	9.98
1967	1.82	2.10	2.88	2.75	2.81	1.36	2.98	8.45	5.12	2.68
1968	4.59	0.18	2.26	1.65	1.68	6.20	4.18	4.64	4.08	1.35
1969	0.67	3.27	2.60	1.58	1.69	2.86	4.30	6.13	2.10	5.34
1970	3.87	2.86	1.09	4.49	3.74	3.41	3.55	3.29	1.96	0.42
1971	3.58	3.71	2.54	4.27	2.92	9.14	4.04	3.26	5.97	6.38
1972	1.61	3.73	1.80	2.42	5.03	3.70	13.34	3.55	0.83	7.93
1973	4.18	4.35	1.90	5.75	3.90	3.16	2.55	3.05	7.32	4.64
1974	5.16	2.28	1.79	3.77	3.31	2.64	2.62	5.89	7.71	0.04
1975	2.92	3.18	2.61	5.01	3.16	6.02	8.31	4.26	3.45	8.27
1976	3.50	3.45	2.89	3.13	2.09	3.69	3.65	1.84	1.61	9.32
1977	1.42	0.48	1.84	2.84	2.20	0.89	1.80	6.50	1.22	1.65
1978	6.54	5.03	1.35	2.35	6.59	2.18	4.25	5.11	6.47	1.43
1979	4.09	4.68	5.26	4.80	2.33	8.77	1.65	4.14	8.27	10.49
1980	1.56	2.57	2.55	4.14	1.79	3.91	4.12	3.08	1.48	1.28

[1] Midmonth total rainfall is accumulated rainfall from the 15th of the previous month to the 14th of the current month.
[2] Column headings are abbreviations for the months January through October.

Midwest

	May	June	July	August	September
1966	3.91	1.86	1.63	2.32	4.29
1967	4.57	3.59	0.85	3.55	2.48
1968	2.58	7.79	4.25	2.68	0.57
1969	2.38	0.53	3.08	2.22	0.79
1970	4.19	3.49	3.91	2.54	4.17
1971	2.19	1.63	9.27	4.90	2.25
1972	5.60	1.58	1.70	4.58	3.89
1973	4.78	6.20	8.43	4.16	1.44
1974	4.58	7.63	6.16	6.29	2.82
1975	3.46	4.96	3.65	5.41	4.03
1976	1.40	2.60	3.07	5.02	2.30
1977	5.22	1.32	4.45	9.68	4.02
1978	5.56	0.90	4.88	5.19	2.34
1979	2.31	0.83	3.88	7.33	1.59
1980	1.51	4.99	1.82	2.94	5.81

Appendix Table 5. Summary of Annual Total Cost of Production for All Crops at All Locations, 1966-1980.[1]

	Warsaw and Orange				Midwest		
	Corn	First Crop Soybeans	Second Crop Soybeans	Wheat	Corn	Soybeans	PPI
1966	82.89	57.79	61.47	52.59	101.04	77.37	99.8
1967	83.06	57.91	61.59	52.69	101.24	77.52	100.0
1968	85.14	59.36	63.13	54.01	103.77	79.46	102.5
1969	88.46	61.67	65.59	56.11	107.82	82.56	106.5
1970	91.70	63.93	68.00	58.17	111.77	85.58	110.4
1971	94.69	66.02	70.21	60.07	115.41	88.37	114.0
1972	98.92	68.97	73.35	67.75	120.58	92.33	119.1
1973	111.88	78.00	82.96	70.97	136.37	104.42	134.7
1974	132.98	92.71	98.61	84.36	162.09	124.11	160.1
1975	145.27	101.29	107.72	92.16	177.07	135.58	174.9
1976	152.00	105.98	112.71	96.42	185.27	141.86	183.0
1977	161.30	112.46	119.61	102.32	196.61	150.54	194.2
1978	173.85	121.21	128.91	110.28	211.89	162.25	209.3
1979	195.69	136.44	145.11	124.14	238.52	182.64	235.6
1980	223.27	155.66	165.55	141.63	272.13	208.37	268.8

[1] 1980 is the base year for cost.

Appendix Table 6. Summary of Commodity Option Premiums [1] For All Crops at All Locations, 1966-1980.

CORN

Mar[2]	Apr	May	Ju	Jl	Aug	SMar[3]	SApr	SMay	SJu	SJl	SAug
.0413	.0345	.0343	.0439	.0980	.1236	.0075	.0110	.0110	.0347	.1028	.1419
.0390	.0710	.0662	.0565	.0390	.0250	.2300	.2400	.2000	.1500	.0800	.0158
.0173	.0264	.0303	.0249	.0251	.0572	.2700	.2500	.1800	.1700	.1200	.0146
.0303	.0211	.0503	.0702	.0346	.0427	.0058	.0136	.0700	.0600	.0900	.0058
.0296	.0167	.0348	.0265	.0654	.0614	.0012	.0017	.0028	.0051	.0088	.0567
.0344	.0629	.0581	.0958	.1112	.0597	.3300	.3000	.2500	.4500	.2900	.0871
.0349	.0349	.0280	.0260	.0410	.0376	.0036	.0054	.0036	.0011	.0028	.0036
.1163	.0969	.1623	.2693	.2854	.2437	.0005	.0014	.0045	.0306	.1293	.5702
.3678	.2651	.1955	.1892	.2692	.2973	.0071	.0007	.0007	.0038	.0689	.2528
.3115	.2268	.1639	.1325	.1806	.2327	.0336	.0210	.0074	.0162	.0173	.2849
.1082	.0734	.0597	.1411	.1991	.1308	.0274	.0295	.0255	.0606	.1136	.0606
.1052	.1037	.1217	.1085	.1483	.0960	.7200	.6600	.5000	.3900	.1801	.0386
.0999	.1660	.1389	.1323	.0841	.0606	.2800	.3600	.3800	.3500	.1825	.0786
.0620	.0591	.0957	.1319	.2459	.1867	.0271	.0404	.0011	.3588	.3388	.0216
.1748	.0960	.1235	.0867	.1454	.1870	.0014	.0005	.0009	.0003	.0341	.0503

SOYBEANS

Mar[4]	Apr	May	Ju	Jl	Aug	Sep	OMar[5]	OApr	OMay	OJu
.0385	.0369	.1133	.0445	.1533	.1250	.0970	.0000	.0000	.0002	.0183
.0327	.0408	.0282	.0908	.0209	.0254	.0321	.1900	.1500	.1400	.1400
.0245	.0377	.0222	.0181	.0318	.0126	.0154	.1900	.1200	.1000	.0800
.0335	.0202	.0160	.0212	.0217	.0218	.0138	.0000	.0000	.0000	.0000
.0381	.0369	.0444	.0699	.1360	.0722	.0646	.0000	.0000	.0000	.0000
.0415	.0526	.0595	.1004	.1679	.0833	.0675	.0000	.0000	.0000	.0420
.0996	.0879	.0989	.0751	.0868	.0774	.0505	.0001	.0031	.0042	.0031
.4894	.2951	.4414	.9934	1.0425	.7881	.3564	.0000	.0000	.0000	.3988
.5407	.4945	.3881	.2279	.5410	.5038	.3278	.0000	.0000	.0000	.0000
.5360	.4696	.2640	.2654	.3437	.4210	.4831	.4000	.3103	.0002	.0086
.1651	.1684	.1613	.3771	.5229	.3720	.2915	.0000	.0000	.0000	.1163
.3840	.3974	.6223	.4645	.6273	.3377	.1426	2.0100	2.2000	2.2300	1.7600
.4184	.3035	.3790	.3457	.2301	.1894	.1541	.0002	.0000	.0006	.0001
.3278	.1891	.1805	.2996	.5493	.2927	.2436	.5900	.5300	.7900	1.6800
.2150	.2724	.3045	.1500	.4189	.4569	.2481	.0000	.0000	.0000	.0000

OJl	OAug	OSep
.4000	.3500	.2500
.0700	.0501	.0403
.0501	.0096	.0154
.0001	.0000	.0001
.0294	.0001	.0002
.2400	.0918	.0025
.0011	.0231	.0598
1.2900	2.1000	.2101
.0000	.0000	.0000
.1086	.9500	.6500
.9600	.2947	.5408
.7800	.1498	.0925
.0000	.0018	.0071
1.1500	.7800	.8100
.0201	.0000	.1218

WHEAT

Jan[6]	Feb	Mar	Apr	May	JJan[7]	JFeb	JMar	JApr	JMay
.0395	.0445	.0385	.0259	.0212	.0001	.0000	.0000	.0000	.0000
.0705	.0600	.0627	.0813	.0422	.0142	.0063	.1100	.0198	.0142
.0361	.0388	.0323	.0353	.0216	.2100	.2100	.2700	.1100	.1000
.0329	.0324	.0204	.0234	.0175	.0314	.0230	.0013	.0029	.0029
.0406	.0221	.0316	.0134	.0229	.0060	.0123	.0060	.0206	.0022
.0547	.0438	.0252	.0431	.0233	.0001	.0000	.0000	.0000	.0000
.0565	.0477	.0434	.0276	.0164	.0213	.0012	.0500	.0500	.0304
.2751	.1896	.1824	.2067	.2227	.0001	.0000	.0000	.0000	.0001
.5949	.4911	.3920	.3301	.1788	1.1500	1.4500	.7800	.3949	.0233
.2825	.3048	.2122	.1346	.0814	.8100	.5600	.3600	.1902	.0228
.2064	.2679	.2694	.1160	.0830	.0763	.4000	.1718	.0692	.0220
.1077	.0706	.1155	.0672	.0699	.4200	.4700	.3600	.3100	.0424
.1042	.0715	.1227	.1809	.1103	.0000	.0000	.0005	.2002	.1145
.0929	.0781	.0743	.0451	.0942	.0000	.0000	.0000	.0000	.0002
.3058	.2359	.1421	.2066	.1714	.4800	.6400	.3100	.0393	.2217

Oct	Nov	Dec	JOct	JNov	JDec
.0353	.0580	.0612	.0000	.0000	.0000
.0881	.0931	.0874	.0198	.0703	.1300
.0538	.0555	.0513	.2900	.2400	.2400
.0406	.0588	.0532	.0406	.0800	.0230
.0667	.0546	.0238	.0060	.0060	.0022
.0515	.0434	.0475	.0000	.0000	.0000
.0947	.0640	.0819	.0001	.0004	.0500
.4790	.2934	.1768	.0000	.0000	.0001
.4184	.2155	.6104	.0779	.0030	.7500
.3042	.2766	.3440	.3700	.8000	.6100
.6453	.5889	.3059	.6000	.0997	.0008
.4803	.1120	.1230	.6400	.2600	.3000
.1062	.1509	.1697	.0000	.0010	.0000
.2504	.2083	.1010	.0000	.0000	.0000
.2652	.2241	.1320	.4900	.5100	.5000

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[1] Commodity option premiums were dtermined using the "Black-Scholes" commodity option pricing model.
[2] Column headings are abbreviations for corn hedging months March through August.
[3] Column headings are abbreviations for September corn harvest month premiums for hedges placed March through August.
[4] Column headings are abbreviations for soybean hedging months March through September
[5] Column headings are abbreviations for October soybean harvest month premiums for hedges placed March through September.
[6] Column headings are abbreviations for wheat hedging months January through May.
[7] Column headings are abbreviations for June wheat harvest month premiums for hedges placed October through May.

Option Values For Midwest October Corn Harvest

	O Mar [8]	O Apr	O May	O Ju	O Jl	O Aug
1966	.0070	.0019	.0019	.0246	.1322	.1900
1967	.2500	.2600	.2200	.1700	.1000	.0354
1968	.2100	.1900	.1200	.1100	.0600	.0000
1969	.0046	.0092	.0468	.0394	.0632	.0046
1970	.0000	.0000	.0000	.0002	.0010	.0468
1971	.3100	.2800	.2300	.4300	.2700	.0601
1972	.0123	.0181	.0123	.0038	.0099	.0123
1973	.0001	.0000	.0001	.0051	.0604	.5032
1974	.0000	.0000	.0000	.0000	.0056	.0724
1975	.0329	.0190	.0055	.0140	.0152	.3316
1976	.0779	.0831	.0729	.1532	.2510	.1532
1977	.5800	.5200	.3600	.2519	.0742	.0054
1978	.1551	.2300	.2500	.0760	.0760	.0167
1979	.0088	.0180	.0363	.3900	.3700	.1041
1980	.0000	.0000	.0000	.0000	.0101	.0201

[8] Column headings are abbreviations for October midwest corn harvest month premiums for hedges placed March through August.

Appendix Table 7. Summary of Predicted Yields For All Crops at All Locations. 1966-1980. [1]

WARSAW

CORN

Year	March	April	May	June	July	August
1966	114.493	118.643	118.446	122.895	101.656	88.800
1967	119.710	124.138	124.016	129.579	136.750	151.249
1968	124.927	126.858	126.008	121.190	96.268	109.275
1969	130.144	132.877	131.222	126.002	88.353	122.647
1970	135.361	132.855	130.396	132.679	167.665	137.689
1971	140.578	141.708	141.192	129.563	106.497	91.900
1972	145.795	146.001	148.278	140.984	141.277	129.429
1973	151.013	150.448	149.431	153.713	125.957	113.534
1974	156.230	148.546	147.772	152.521	98.742	153.656
1975	161.447	157.245	157.777	157.049	179.475	163.856
1976	166.664	170.567	170.218	174.814	163.411	134.523
1977	171.881	173.042	174.453	173.812	111.975	61.040
1978	177.098	180.786	188.572	183.462	110.358	113.441
1979	182.315	182.274	179.436	165.047	147.673	179.385
1980	187.532	184.687	182.486	186.344	108.228	100.505

SOYBEANS

Year	March	April	May	June	July	August	Sept
1966	33.6935	32.1907	31.8021	31.7557	29.9580	29.8060	29.7977
1967	33.8506	32.2378	31.9987	32.9609	33.5539	34.2330	33.8968
1968	34.0077	33.3846	31.7107	33.8052	31.6913	32.2715	31.1677
1969	34.1648	33.2240	29.9658	32.2791	29.0649	31.8477	33.8395
1970	34.3218	35.4581	30.6157	30.1838	32.8689	33.0389	31.5699
1971	34.4789	34.1735	33.1572	39.5155	37.5610	37.4163	37.0470
1972	34.6360	34.6971	39.1808	42.6993	40.8350	40.6816	40.1048
1973	34.7931	35.1596	33.1569	33.0305	30.6726	30.5197	30.1960
1974	34.9502	38.1390	36.6146	36.7459	32.1239	39.3668	39.7693
1975	35.1073	36.9155	37.9617	38.1961	38.7790	38.6415	38.9900
1976	35.2644	33.8594	33.1719	33.2059	32.2472	32.3811	31.9496
1977	35.4215	35.1038	37.8837	38.0883	32.7587	33.9731	34.7781
1978	35.5786	34.2591	49.5932	51.8459	45.5228	45.6172	44.6545
1979	35.7357	35.8945	30.3048	38.6388	37.1719	39.5841	42.9619
1980	35.8928	37.1634	32.8292	32.5498	25.7833	25.6467	24.2732

[1] See Appendix Table 1 for a comparison to actual yields.

SECOND CROP SOYBEANS

Year	March	April	May	June	July	August	Sept
1966	34.6858	29.4945	28.7141	26.6283	22.0337	18.2771	18.2882
1967	35.2086	29.6374	29.1572	26.3541	27.6970	31.7498	32.1983
1968	35.7313	33.5788	30.2168	32.0844	26.6250	30.2712	31.7438
1969	36.2541	33.0043	26.4603	28.4749	19.9162	29.0755	26.4180
1970	36.7768	40.7019	30.9761	29.9839	33.4200	24.4708	26.4307
1971	37.2996	36.2444	34.2032	38.4604	33.4386	29.1635	29.6562
1972	37.8223	38.0334	47.0388	49.7986	21.5897	18.1326	18.9021
1973	38.3451	39.6113	35.5888	33.5972	27.4619	23.8340	24.2658
1974	38.8678	49.8835	46.8216	44.5606	31.8763	45.5329	44.9959
1975	39.3906	45.6370	47.7383	48.0358	33.2040	28.6232	28.1582
1976	39.9134	35.0597	33.6789	31.5090	29.1370	20.5238	21.0996
1977	40.4361	39.3388	44.9221	45.1848	30.3738	14.8324	13.7583
1978	40.9589	36.4007	67.1992	69.1736	51.3276	52.2073	53.4917
1979	41.4816	42.0303	30.8035	35.9817	32.2766	40.8045	36.2979
1980	42.0044	46.3937	37.6885	35.9258	16.7093	14.4691	16.3016

WHEAT

Year	December	January	February	March	April	May
1966	58.4411	62.6900	61.4039	62.1618	67.7436	68.1861
1967	57.2458	57.9673	58.6763	59.3428	65.3911	65.4518
1968	56.0504	53.6053	55.5509	53.0658	54.8333	59.3944
1969	54.8551	59.3045	58.1338	55.0133	57.5651	67.6851
1970	53.6598	53.0185	53.9171	57.0299	54.6687	70.5273
1971	52.4645	51.0014	50.5727	53.9379	55.4098	57.3861
1972	51.2692	52.8926	53.1811	50.9286	51.8347	44.3466
1973	50.0739	50.4747	49.5926	51.2091	50.6183	55.7530
1974	48.8785	51.7446	52.3464	56.7296	53.2563	56.6968
1975	47.6832	43.7349	43.4876	42.9472	40.1208	37.3074
1976	46.4879	45.4056	44.3504	47.4317	52.7594	53.8248
1977	45.2926	47.3569	49.4015	52.6720	54.8412	48.6496
1978	44.0973	34.5973	33.4019	32.5333	41.5140	48.2903
1979	42.9020	42.2406	41.3914	31.0244	28.2386	49.9031
1980	41.7067	45.0337	45.6602	46.3876	43.6664	57.5473

ORANGE

CORN

Year	March	April	May	June	July	August
1966	117.489	111.989	112.903	116.730	84.287	83.784
1967	117.218	115.355	115.096	95.149	87.736	141.352
1968	116.947	111.778	110.732	119.120	119.228	120.842
1969	116.676	111.297	110.257	102.070	95.848	123.525
1970	116.405	119.770	120.161	115.615	102.080	98.145
1971	116.134	118.838	118.657	130.660	136.567	117.447
1972	115.863	113.008	114.298	111.528	126.452	129.052
1973	115.592	122.743	123.246	117.089	113.339	87.415
1974	115.321	116.523	116.613	106.868	105.322	108.835
1975	115.049	119.978	119.964	127.797	150.838	150.677
1976	114.778	114.057	113.297	110.467	96.576	83.218
1977	114.507	112.915	112.231	88.048	53.142	89.365
1978	114.236	111.171	113.550	100.362	109.974	108.151
1979	113.965	118.263	117.670	129.783	119.339	99.993
1980	113.694	116.008	115.039	113.492	103.397	99.458

SOYBEANS

Year	March	April	May	June	July	August	Sept
1966	45.1884	46.1588	44.5044	40.5263	26.9929	27.2511	22.5587
1967	44.5637	44.8925	45.5059	70.0872	64.0800	39.4261	42.1835
1968	43.9391	44.8511	46.9899	39.2050	38.9646	38.8277	39.0696
1969	43.3144	44.2636	46.3889	55.9540	56.2047	50.8568	46.3095
1970	42.6898	42.0959	41.4539	46.6350	43.5611	43.2908	38.4049
1971	42.0651	41.5879	42.0528	37.3682	36.5352	36.2095	41.0228
1972	41.4405	41.9442	39.5609	42.6729	35.1710	35.2852	27.6662
1973	40.8158	39.5538	38.6958	45.7951	37.3426	36.5654	44.6440
1974	40.1912	39.9791	39.9175	51.4048	43.3640	39.2354	48.2573
1975	39.5665	38.6969	38.8378	31.4220	39.1336	39.4258	38.1439
1976	38.9419	39.0691	40.6545	43.8350	41.2395	35.6850	29.9526
1977	38.3172	38.5983	40.0351	70.2429	57.0473	49.5340	42.8582
1978	37.6926	38.2334	33.7443	49.5619	49.6099	48.4362	54.4589
1979	37.0679	36.3096	37.5710	31.5500	17.3328	17.6846	28.0611
1980	36.4433	36.0350	38.0253	39.7433	39.2516	38.5457	32.4988

SECOND CROP SOYBEANS

Year	March	April	May	June	July	August	Sept
1966	30.8634	35.1364	35.1466	31.8472	20.4117	21.3218	17.2254
1967	31.1367	32.5384	32.6246	51.8551	46.8758	27.4718	29.8789
1968	31.4099	35.4218	35.5590	28.8210	28.7152	28.3676	28.5788
1969	31.6831	35.8612	35.9979	43.6139	43.9181	38.7841	34.8144
1970	31.9564	29.2293	29.2735	33.4335	30.9446	31.9765	27.7112
1971	32.2296	30.0246	30.1058	23.8759	23.2741	24.3047	28.5067
1972	32.5029	34.6877	34.6736	37.1847	27.8922	28.8770	22.2256
1973	32.7761	27.0592	27.0961	32.7737	25.7055	26.6883	33.7408
1974	33.0494	32.0308	32.0943	41.2134	34.4975	30.3660	38.2420
1975	33.3226	29.3616	29.4320	23.0627	28.9694	29.2944	28.1753
1976	33.5958	34.0959	34.2146	36.7806	34.6959	34.0780	29.0737
1977	33.8691	35.0573	35.1710	58.6998	47.5555	40.7022	34.8744
1978	34.1423	36.4933	36.4087	48.8930	49.0280	47.5404	52.7982
1979	34.4156	30.9529	31.0607	24.0900	12.0645	12.5557	21.6142
1980	34.6888	32.7923	32.9245	34.3164	34.0003	34.9941	29.7153

WHEAT

Year	December	January	February	March	April	May
1966	41.8970	47.3507	48.3974	48.2086	40.4621	48.0930
1967	41.8362	45.4071	49.2015	48.8655	45.1133	45.0705
1968	41.7755	36.3543	48.5225	48.6624	41.0976	46.6313
1969	41.7147	49.0187	47.7103	47.5433	39.8586	45.3051
1970	41.6540	38.5700	39.0498	41.1016	50.7883	52.6347
1971	41.5932	39.4507	36.2233	36.1023	43.5885	43.4641
1972	41.5324	45.7850	42.4704	43.2039	37.9270	52.0785
1973	41.4717	37.3814	31.3627	31.9498	57.0350	59.7962
1974	41.4109	34.1393	37.1487	37.8974	40.9222	41.1680
1975	41.3501	41.3501	40.4343	40.2598	55.7246	55.7063
1976	41.2894	39.4066	37.3131	36.9726	35.3805	37.8926
1977	41.2286	46.0980	56.9577	57.6316	54.3518	56.2466
1978	41.1679	29.4165	20.4321	21.9445	16.3861	60.4534
1979	41.1071	37.3090	29.8511	31.1441	44.1779	45.4487
1980	41.0463	45.4612	47.2058	47.0768	53.3310	57.9424

MIDWEST

CORN

Year	March	April	May	June	July	August
1966	163.539	163.539	164.200	146.857	145.453	131.508
1967	162.631	162.631	164.796	166.218	143.678	136.917
1968	161.722	161.722	159.351	114.970	94.126	82.673
1969	160.813	160.813	157.986	111.287	119.878	106.364
1970	159.904	159.904	161.203	162.139	156.871	144.935
1971	158.995	158.995	155.735	134.243	139.629	139.751
1972	158.086	158.086	162.599	140.154	129.351	131.481
1973	157.177	157.177	159.821	148.003	108.398	104.782
1974	156.268	156.268	158.457	118.191	79.444	88.104
1975	155.359	155.359	154.995	155.699	153.496	157.398
1976	154.451	154.451	149.390	142.761	134.301	135.043
1977	153.542	153.542	157.189	129.491	137.478	141.033
1978	152.633	152.633	157.055	119.825	130.674	133.668
1979	151.724	151.724	148.738	109.794	133.937	138.930
1980	150.815	150.815	146.005	146.539	133.127	125.103

SOYBEANS

Year	March	April	May	June	July	August
1966	40.1239	40.1239	40.6296	33.2818	39.5797	44.6526
1967	40.9370	40.9370	42.5935	43.3703	43.3246	43.8383
1968	41.7502	41.7502	39.9367	31.4063	21.3776	25.1644
1969	42.5633	42.5633	40.4011	22.0633	28.1481	34.7780
1970	43.3764	43.3764	44.3703	44.8675	42.6622	47.4114
1971	44.1896	44.1896	41.6961	32.7427	41.6493	40.0255
1972	45.0027	45.0027	48.4552	39.1364	40.7406	42.9760
1973	45.8158	45.8158	47.8385	47.6689	32.5025	31.5816
1974	46.6290	46.6290	48.3029	40.8805	21.8169	23.7295
1975	47.4421	47.4421	47.1631	49.4233	49.1380	48.9718
1976	48.2552	48.2552	44.3842	41.3656	39.4845	37.8750
1977	49.0684	49.0684	51.8583	40.5456	43.7808	45.9115
1978	49.8815	49.8815	53.2643	38.3979	42.5820	42.4540
1979	50.6946	50.6946	48.4104	32.9117	44.5443	41.6462
1980	51.5078	51.5078	47.8286	50.0723	49.9724	54.6751

Appendix Table 8. Skewness and Test of Normality Statistics for Crop Yields, Prices, and Returns at Warsaw.

Warsaw

Corn	Skewness	W	Prob < W	Soybean	Skewness	W	Prob < W
Price	.65	.89	.08	Price	.39	.89	.07
Yield	-.40	.96	.59	Yield	-1.13	.88	.06
Cash	.90	.94	.40	Cash	1.40	.88	.04
1F [1]	-.39	.96	.71	1F [2]	-.45	.94	.42
2F	-.41	.94	.41	2F	-.68	.93	.33
3F	-.46	.94	.45	3F	-.58	.90	.13
4F	-.27	.96	.71	4F	-1.26	.85	.02
5F	-.81	.93	.35	5F	-.11	.96	.63
6F	1.70	.86	.02	6F	1.59	.87	.03
1O	.63	.94	.39	7F	-.81	.91	.21
2O	.81	.92	.23	1O	2.04	.78	.01
3O	.56	.91	.20	2O	2.69	.70	.01
4O	.36	.97	.80	3O	2.42	.75	.01
5O	.00	.97	.77	4O	1.63	.78	.01
6O	2.42	.73	.01	5O	-.01	.97	.80
				6O	1.83	.81	.01
				7O	.60	.94	.40

Second Crop Soybeans	Skewness	W	Prob < W	Wheat	Skewness	W	Prob < W
Price	.39	.89	.07	Price	.81	.83	.01
Yield	.35	.96	.67	Yield	-.39	.96	.64
Cash	1.43	.87	.04	Cash	-.83	.92	.27
1F [3]	-.40	.94	.44	1F [4]	-1.27	.90	.10
2F	-1.06	.90	.11	2F	-.64	.97	.77
3F	-.85	.87	.04	3F	-.11	.98	.91
4F	-1.45	.83	.01	4F	.65	.95	.50
5F	-.39	.92	.29	5F	.61	.92	.26
6F	.95	.89	.08	6F	-.68	.94	.46
7F	-1.59	.85	.02	7F	-1.44	.88	.04
1O	2.12	.77	.01	8F	-1.39	.87	.03
2O	2.53	.72	.01	1O	-.24	.92	.27
3O	2.47	.74	.01	2O	.60	.94	.45
4O	1.72	.80	.01	3O	.37	.96	.62
5O	-.28	.98	.94	4O	.95	.87	.04
6O	1.17	.83	.01	5O	2.19	.77	.01
7O	-.02	.92	.28	6O	1.05	.88	.06
				7O	.00	.97	.90
				8O	-.95	.86	.03

[1]Variable names 1F through 6F and 1O through 6O represent monthly corn hedging periods March through August for commodity futures and options, respectively.
[2]Variable names 1F through 7F and 1O through 7O represent monthly soybean hedging periods March through September for commodity futures and options, respectively.
[3]Variable names 1F through 7F and 1O through 7O represent monthly second crop soybean hedging periods March through September for commodity futures and options, respectively.
[4]Variable names 1F through 8F and 1O through 8O represent monthly wheat hedging periods October through May for commodity futures and options, respectively.

Orange

Corn	Skewness	W	Prob < W
Price	.716	.89	.07
Yield	.010	.95	.57
Cash	.715	.94	.45
1F	-.413	.96	.60
2F	-.527	.93	.36
3F	-.556	.92	.23
4F	.071	.96	.67
5F	-.226	.97	.88
6F	1.21	.89	.08
1O	.547	.91	.14
2O	.607	.90	.13
3O	.403	.90	.10
4O	1.00	.93	.30
5O	.509	.94	.45
6O	2.51	.73	.01

Soybean	Skewness	W	Prob < W
Price	.398	.88	.06
Yield	.081	.98	.94
Cash	1.32	.85	.02
1F	-.606	.93	.33
2F	-.579	.93	.33
3F	-.649	.91	.20
4F	-1.04	.83	.01
5F	-.094	.96	.61
6F	2.07	.80	.01
7F	-1.60	.88	.03
1O	1.86	.81	.01
2O	2.69	.69	.01
3O	2.29	.77	.01
4O	2.23	.74	.01
5O	-.587	.96	.63
6O	2.31	.76	.01
7O	.398	.97	.75

Second Crop Soybeans	Skewness	W	Prob < W
Price	.398	.88	.06
Yield	.081	.98	.94
Cash	1.44	.78	.01
1F	-.456	.94	.41
2F	-2.96	.94	.41
3F	-.447	.91	.20
4F	-.881	.83	.01
5F	-.340	.93	.34
6F	1.83	.83	.01
7F	-1.56	.86	.02
1O	2.08	.78	.01
2O	2.91	.66	.01
3O	2.51	.73	.01
4O	2.43	.72	.01
5O	-.532	.97	.89
6O	2.18	.79	.01
7O	-.374	.96	.70

Wheat	Skewness	W	Prob < W
Price	.825	.83	.01
Yield	-.580	.92	.26
Cash	1.90	.78	.01
1F	-1.21	.91	.15
2F	-.686	.96	.73
3F	-.175	.97	.88
4F	.479	.95	.49
5F	.111	.97	.79
6F	-.531	.98	.94
7F	-1.02	.90	.10
8F	-2.03	.79	.01
1O	-.441	.93	.37
2O	.532	.96	.65
3O	.226	.95	.56
4O	.879	.86	.03
5O	1.66	.82	.01
6O	1.06	.87	.04
7O	-.428	.93	.35
8O	-1.77	.80	.01

Midwest

Corn	Skewness	W	Prob < W	Soybean	Skewness	W	Prob < W
Price	.535	.89	.08	Price	.340	.90	.09
Yield	-1.15	.90	.10	Yield	-2.93	.65	.01
Cash	.956	.91	.22	Cash	.820	.93	.36
1F	.961	.90	.12	1F	-.375	.94	.44
2F	-1.13	.88	.05	2F	-.392	.94	.45
3F	-1.30	.87	.04	3F	-.506	.91	.18
4F	.647	.95	.54	4F	-1.43	.86	.02
5F	-.245	.96	.70	5F	.225	.96	.65
6F	1.11	.88	.05	6F	1.55	.87	.03
1O	.413	.89	.07	1O	2.16	.77	.01
2O	.500	.89	.06	2O	2.83	.67	.01
3O	.367	.93	.38	3O	2.40	.75	.01
4O	.671	.93	.35	4O	1.13	.83	.01
5O	.154	.98	.91	5O	.299	.93	.37
6O	1.25	.90	.12	6O	1.76	.83	.01

Appendix Table 9. Range of Hedge Returns Using Commodity Options and Futures, All Crops at All Locations, 1966-1980.

Future Hedges

Warsaw

Crop	Period	Hedge Ratio	Range Futures Returns		Range Hedge Returns	
Corn						
	Cash	N/A	N/A		-40.54	243.74
	1	1.0483	-14.24	48.82	-3.38	167.01
	2	1.1386	-15.96	53.51	12.07	134.04
	3	1.3099	-19.41	56.52	-14.68	118.00
	4	1.3265	-10.18	50.61	-22.72	137.58
	5	1.8006	-4.92	37.48	-22.90	209.31
	6	N/A				
Soybeans						
	Cash	N/A	N/A		1.61	144.40
	1	1.0090	-57.60	36.99	-11.68	93.52
	2	.9804	-69.60	38.99	16.82	74.80
	3	1.0524	-73.41	45.78	-15.58	80.56
	4	.8451	-55.67	29.17	-9.52	112.99
	5	.6748	-14.35	19.82	8.12	130.05
	6	N/A				
	7	2.1735	-49.13	32.10	7.65	110.40

Appendix Table 9. (continued)

Crop	Period	Hedge Ratio	Range Futures Returns		Range Hedge Returns	
Second Crop Soybeans						
	Cash	N/A	N/A		-33.27	150.59
	1	1.2393	-77.97	51.87	-49.22	81.09
	2	1.1635	-108.04	51.85	-57.78	80.64
	3	1.2948	-115.49	66.79	-54.21	72.12
	4	1.2418	-99.20	50.85	-49.66	86.49
	5	1.6273	-34.34	42.80	-20.95	118.14
	6	.7134	-16.03	26.51	-32.78	134.56
	7	3.4899	-89.26	43.55	33.27	81.63
Orange						
Corn						
	Cash	N/A	N/A		-12.54	139.14
	1	.4856	-42.92	20.61	3.11	127.96
	2	.5302	-44.45	20.35	.01	123.49
	3	.6247	-47.32	18.05	-1.30	114.28
	4	.6519	-36.55	33.62	-3.69	116.75
	5	1.1277	-44.74	39.18	-7.29	99.79
	6	N/A				

Appendix Table 9. (continued)

Crop	Period	Hedge Ratio	Range Futures Returns		Range Hedge Returns	
Soybeans						
	Cash	N/A	N/A		17.70	139.06
	1	.9845	-65.93	39.04	-3.38	82.56
	2	.9062	-67.43	39.62	-5.98	91.16
	3	.8838	-67.21	40.63	-6.80	100.21
	4	.4254	-39.20	27.08	5.94	144.27
	5	.0713	-2.05	2.55	17.46	141.61
	6	N/A				
	7	1.4388	-39.46	20.40	18.57	143.83
Second Crop Soybeans						
	Cash	N/A	N/A		-4.22	116.05
	1	1.1726	-63.06	41.10	-10.48	77.96
	2	1.2121	-72.27	48.14	-8.76	74.73
	3	1.2375	-75.66	49.98	-9.64	78.31
	4	.8541	-63.10	45.44	-2.30	89.14
	5	1.0333	-23.60	25.44	-2.78	107.10
	6	.1122	-1.68	4.67	-4.22	114.37
	7	2.9455	-64.02	30.84	-3.40	89.04

Appendix Table 9. (continued)

Midwest

Crop	Period	Hedge Ratio	Range Futures Returns		Range Hedge Returns	
Corn						
	Cash	N/A	N/A		6.80	130.52
	1	.4747	-59.27	21.76	-7.57	119.52
	2	.4403	-57.16	18.62	-8.50	117.22
	3	.4853	-63.88	17.59	-8.48	109.44
	4	.5414	-45.16	27.41	-12.37	112.21
	5	.7856	-25.51	25.98	-.97	102.39
	6	N/A				
Soybeans						
	Cash	N/A	N/A		14.52	86.64
	1	.2419	-18.18	12.28	-1.75	68.46
	2	.1479	-12.84	8.22	1.68	77.59
	3	.1094	-10.07	6.51	4.45	80.70
	4	N/A				
	5	N/A				
	6	N/A				

Options

Warsaw

Crop	Period	Hedge Ratio	Range Options Returns		Range Hedge Returns	
Corn						
	Cash	N/A	N/A		-40.55	243.93
	1	2.3570	-82.96	128.25	3.91	160.77
	2	2.7874	-68.38	138.17	14.14	175.36
	3	3.3580	-60.38	114.11	4.46	183.36
	4	2.7106	-73.83	109.12	16.24	195.86
	5	5.0306	-84.30	84.03	-35.32	181.59
	6	N/A				
Soybeans						
	Cash	N/A	N/A		1.61	144.40
	1	1.8499	-23.38	54.86	-.79	122.57
	2	1.5001	-17.67	48.88	-.18	126.73
	3	1.4026	-15.24	43.99	-3.44	131.95
	4	1.2903	-18.81	32.78	.53	137.65
	5	2.4181	-26.25	22.92	.36	119.16
	6	N/A				
	7	4.6769	-38.08	48.30	4.28	106.32

Appendix Table 9. (continued)

Crop	Period	Hedge Ratio	Range Options Returns		Range Hedge Returns	
Second Crop Soybeans						
	Cash	N/A	N/A		-33.27	150.59
	1	2.7508	-38.32	93.13	-27.86	114.48
	2	2.5478	-39.26	93.03	-30.60	111.33
	3	2.3365	-28.39	86.89	-28.60	124.07
	4	2.0672	-30.66	62.31	-22.76	137.48
	5	4.1667	-44.88	34.29	-28.95	105.71
	6	.8779	-12.58	20.38	-34.53	138.01
	7	7.1261	-65.65	62.18	-35.80	91.66
Orange						
Corn						
	Cash	N/A	N/A		-12.54	139.14
	1	1.1263	-29.26	40.83	5.94	118.55
	2	1.3912	-26.77	45.00	8.47	119.49
	3	1.6688	-24.09	36.48	5.46	121.22
	4	1.1933	-24.76	48.44	2.69	128.99
	5	2.0692	-29.14	44.32	-10.74	109.99
	6	N/A				

Appendix Table 9. (continued)

Crop	Period	Hedge Ratio	Range Options Returns		Range Hedge Returns	
Soybeans						
	Cash	N/A	N/A		17.90	139.06
	1	2.0036	-29.71	64.28	11.86	109.35
	2	1.5563	-19.22	55.76	12.02	125.57
	3	1.5962	-20.24	52.90	10.83	118.82
	4	1.1655	-23.56	54.61	15.12	115.50
	5	.8411	-12.32	8.28	12.81	144.83
	6	N/A				
	7	4.6670	-46.11	31.48	10.58	116.43
Corn						
	Cash	N/A	N/A		-4.22	116.05
	1	2.1080	-25.10	59.78	-1.60	92.52
	2	1.5614	-15.45	50.81	-1.12	100.60
	3	1.6816	-14.93	48.96	-5.79	102.97
	4	1.2279	-17.76	48.08	-2.08	108.85
	5	2.9056	-33.87	24.08	-4.56	95.38
	6	N/A	-9.56	25.99		
	7	7.5661	-59.24	39.31	-8.11	78.38

Appendix Table 9. (continued)

Midwest

Crop	Period	Hedge Ratio	Range Options Returns		Range Hedge Returns	
Corn						
	Cash	N/A	N/A		6.81	130.52
	1	1.0751	-38.60	41.32	-3.73	105.19
	2	1.3451	-34.81	44.27	-.43	105.69
	3	1.7619	-34.09	44.66	-5.07	105.78
	4	1.4275	-41.44	56.18	.06	115.46
	5	1.9622	-35.53	38.16	-6.37	102.03
	6	N/A				
Soybeans						
	Cash	N/A	N/A		14.52	86.64
	1	.5227	-8.70	21.47	6.29	77.94
	2	.3557	-5.12	16.20	9.40	83.07
	3	.4778	-7.49	20.51	8.92	79.15
	4	.8167	-17.18	22.09	9.77	69.46
	5	N/A				
	6	N/A				

Appendix Table 10. Yields and Cash Prices for the Out of Sample Test, All Locations, 1981 - 1985.

	Warsaw						
	Yields				Prices		
Year	Corn	Soybean	2nd Soybean	Wheat	Corn	Soybeans	Wheat
1981	156	42	27	76	2.62	6.32	3.26
1982	127	40	34	76	2.08	5.07	3.00
1983	79	31	36	66	3.57	8.26	3.20
1984	162	39	36	67	3.01	6.14	3.30
1985	48	38	38	65	2.21	4.89	2.98
	Orange						
1981	196	43	23	75	2.63	6.27	3.20
1982	170	41	33	56	2.12	5.10	3.02
1983	152	56	17	61	3.75	8.50	3.25
1984	172	44	26	76	2.95	6.19	3.36
1985	197	47	32	51	2.22	4.78	2.87
	Midwest						
1981	150	45			2.57	6.31	
1982	145	41			2.12	5.30	
1983	97	36			3.55	8.50	
1984	160	44			2.83	6.28	
1985	170	57			2.24	5.06	

Appendix Table 11. Predicted Yields of All Crops at All Locations, Out of Sample Test, 1981-1985.

CORN

Year	March	April	May	June	July	August
1981	192.749	193.047	192.516	192.463	202.416	211.883
1982	197.966	203.380	201.513	199.804	98.235	137.010
1983	203.183	191.679	193.894	190.100	119.869	98.165
1984	208.401	189.530	189.667	186.214	150.261	161.635
1985	213.618	220.326	215.697	214.049	57.499	38.042

SOYBEANS

Year	March	April	May	June	July	August	Sept
1981	36.0498	36.0743	35.0281	35.0428	35.8630	36.2368	34.8052
1982	36.2069	34.2032	30.5266	31.1334	22.2829	25.7922	24.3855
1983	36.3640	41.0678	45.4320	46.9931	40.9237	40.8709	40.4642
1984	36.5211	44.1450	4.4140	45.8058	42.7388	43.2190	41.7210
1985	36.6782	34.1614	25.0446	25.6264	11.8451	11.7528	14.1223

SECOND CROP SOYBEANS

Year	March	April	May	June	July	August	Sept
1981	42.5271	42.6115	40.5103	40.5326	42.353	45.025	46.935
1982	43.0499	36.1282	28.7437	29.4321	3.662	13.887	15.763
1983	43.5726	59.8217	68.5870	70.0746	53.009	46.593	47.136
1984	44.0954	70.4316	70.9719	72.3311	64.194	67.392	69.391
1985	44.6181	35.9238	17.6128	18.2771	0.0	0.0	0.0

WHEAT

Year	January	February	March	April	May
1981	40.5113	46.0430	46.8427	49.3908	49.8711
1982	39.3160	41.3002	41.8113	37.9199	44.5802
1983	38.1207	40.8464	41.0690	42.7164	41.4321
1984	36.9254	35.3220	35.3797	30.4198	35.7118
1985	35.7301	38.2554	37.7937	43.1437	52.7978

Appendix Table 10. (continued)

ORANGE

CORN

Year	March	April	May	June	July	August
1981	113.423	108.315	107.903	100.813	103.666	115.095
1982	113.152	112.461	112.078	121.754	85.354	110.046
1983	112.881	122.618	123.894	120.825	111.489	109.482
1984	112.610	122.226	124.123	105.387	80.314	185.429
1985	112.339	106.029	104.815	116.838	109.856	82.556

SOYBEANS

Year	March	April	May	June	July	August	Sept
1981	35.8186	36.7201	37.6305	45.8581	47.7051	45.8874	44.968
1982	35.1940	35.3159	36.1723	40.3317	26.3208	23.2236	16.499
1983	34.5693	32.8513	30.4949	33.9517	31.2138	31.5695	28.981
1984	33.9447	32.2478	28.6901	51.6805	45.0667	0.00	0.00
1985	33.3200	34.4336	36.8963	32.0541	24.8208	20.7491	25.272

SECOND CROP SOYBEANS

Year	March	April	May	June	July	August	Sept
1981	34.9621	38.9265	39.0226	45.5888	47.2168	45.1097	44.3073
1982	35.2353	35.7116	35.8059	34.9781	23.1306	19.8762	14.0061
1983	35.5085	27.7510	27.7377	30.5246	28.3197	29.0946	26.8352
1984	35.7818	28.1191	28.0657	46.0767	40.5802	4.7055	0.00
1985	36.0550	40.9687	41.1167	34.7973	28.7719	28.7432	32.6920

WHEAT

Year	January	February	March	April	May
1981	40.9856	48.8739	51.4907	51.9811	44.4532
1982	40.9248	42.6129	37.7717	37.3867	35.8573
1983	40.8640	48.4278	48.2097	47.8932	86.2217
1984	40.8033	40.9331	39.1014	38.7327	76.3960
1985	40.7425	42.3007	40.3817	44.0118	35.9731

Appendix Table 10. (continued)

MIDWEST

CORN

Year	March	April	May	June	July	August
1981	163.539	163.539	174.777	177.555	193.297	197.481
1982	162.631	162.631	157.616	155.764	173.492	159.228
1983	161.722	161.722	172.230	174.200	156.415	136.113
1984	160.813	160.813	160.813	160.817	164.572	153.115
1985	159.904	159.904	157.237	159.443	148.920	148.168

SOYBEANS

Year	March	April	May	June	July	August
1981	40.1239	40.1239	48.7203	50.5899	57.7227	55.3865
1982	40.9370	40.9370	37.1009	39.0113	47.4013	54.6326
1983	41.7502	41.7502	49.7886	52.1014	44.3556	51.7185
1984	42.5633	42.5633	42.5633	42.5646	45.3830	47.9515
1985	43.3764	43.3764	41.3363	42.6244	39.2030	40.9348

Appendix Table 12. Futures Prices For the Out of Sample Test, All Crops, All Locations, 1981-1985.

	CORN							SOYBEANS					
Year	Mar	[1]Apr	May	Ju	Jl	Aug	Sep	Mar	Apr	May	Ju	Jl	Aug
1981	3.69	3.83	3.68	3.65	3.62	3.16	2.96	8.08	8.32	7.79	7.69	7.66	7.00
1982	2.84	2.97	2.85	2.76	2.64	2.28	2.24	6.43	6.68	6.77	6.44	6.23	5.54
1983	2.92	2.99	2.89	2.74	2.97	3.66	3.48	6.19	6.61	6.57	6.11	6.74	9.05
1984	2.93	2.98	2.94	3.04	2.87	2.86	2.87	7.24	7.15	7.31	7.13	6.34	6.58
1985	2.62	2.65	2.62	2.51	2.45	2.27	2.19	5.96	6.06	5.87	5.64	5.78	5.25

		WHEAT								
Sep	Oct	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Ju
6.79	6.64	5.35	5.30	4.97	5.03	4.69	4.34	6.36	4.17	3.99
5.56	5.30	4.65	4.65	4.02	4.07	3.87	3.65	3.87	3.62	3.38
8.57	8.66	3.41	3.56	3.41	3.56	3.66	3.41	3.59	3.53	3.52
5.92	6.23	3.68	3.48	3.38	3.41	3.26	3.47	3.50	3.43	3.51
5.13	5.00	3.39	3.41	3.30	3.35	3.32	3.32	3.33	3.20	3.30

[1]Column abbreviations represent hedging months January through December.

Appendix Table 13. Commodity Option Premiums [1] For the Out of Sample Test, All Crops at All Locations, 1981-1985.

CORN

Year	CoMar[2]	CoApr	CoMay	CoJu	CoJl	CoAug	CosMar[3]	CosApr	CosMay	CosJu	CosJl	CosAug
1981	.2071	.1925	.1305	.1405	.1752	.1133	.7300	.8700	.7200	.6900	.6600	.2319
1982	.1142	.0777	.0686	.0840	.0650	.0846	.6000	.7300	.6100	.5200	.4000	.1047
1983	.2046	.1476	.1077	.0677	.0996	.1625	.0097	.0155	.0078	.0023	.0136	.2639
1984	.0835	.0836	.1201	.1029	.1600	.1173	.1212	.1536	.1274	.1972	.0878	.0828
1985	.0472	.0685	.0527	.0755	.0733	.0579	.4300	.4600	.4300	.3200	.2600	.1058

SOYBEANS

Year	SoMar[4]	SoApr	SoMay	SoJu	SoJl	SoAug	SoSep	SoOMar[5]	SoOApr	SoOMay	SoOJu	SoOJl	SoOAug	SoOSep
1981	.5265	.5653	.2784	.2957	.3596	.2602	.1414	1.440	1.680	1.150	1.050	1.020	.3600	.1530
1982	.2603	.2540	.2445	.2821	.2023	.1532	.1198	1.130	1.380	1.470	1.140	.9300	.2452	.2638
1983	.2927	.3607	.2954	.2167	.3631	.4592	.4212	.0000	.0000	.0000	.0000	.0000	.4325	.1264
1984	.2905	.2153	.1500	.3046	.5090	.3103	.2597	1.010	.9200	1.080	.9000	.1149	.3500	.0000
1985	.1618	.2397	.1996	.2373	.2927	.1443	.0991	.9600	1.060	.8700	.6400	.7800	.2500	.1310

WHEAT

Year	WoOct[6]	WoNov	WoDec	WoJan	WoFeb	WoMar	WoApr	WoMay	WoJOct[7]	WoJNov	WoJDec	WoJJan	WoJFeb	WoJMar	WoJApr	WoJMay
1981	.3135	.2588	.2371	.2605	.2492	.1906	.1462	.0879	1.360	1.310	.9800	1.040	.7000	.3500	.3700	.1800
1982	.1449	.1006	.1965	.2700	.2724	.0949	.0995	.0649	1.270	1.270	.6400	.6900	.4900	.2700	.4900	.2400
1983	.1534	.1383	.1451	.1197	.1099	.1549	.1171	.0656	.0000	.0436	.0000	.0436	.1400	.0000	.0707	.0219
1984	.1712	.1406	.1031	.1073	.0792	.0911	.0938	.0551	.1712	.0242	.0033	.0066	.0001	.0206	.0326	.0100
1985	.0676	.0816	.0616	.0669	.0745	.1477	.0490	.0475	.0900	.1100	.0080	.1500	.0217	.0217	.0306	.0000

1Commodity option premiums were determined using the "Black-Scholes" commodity option pricing model.

2Column headings are abbreviations for corn hedging months Marhc through August.

3Column headings are abbreviations for September corn harvest month premiums for hedges placed March through August.

4Column headings are abbreviations for soybean hedging months March through September.

5Column headings are abbreviations for October soybean harvest month premiums for hedges placed March through September.

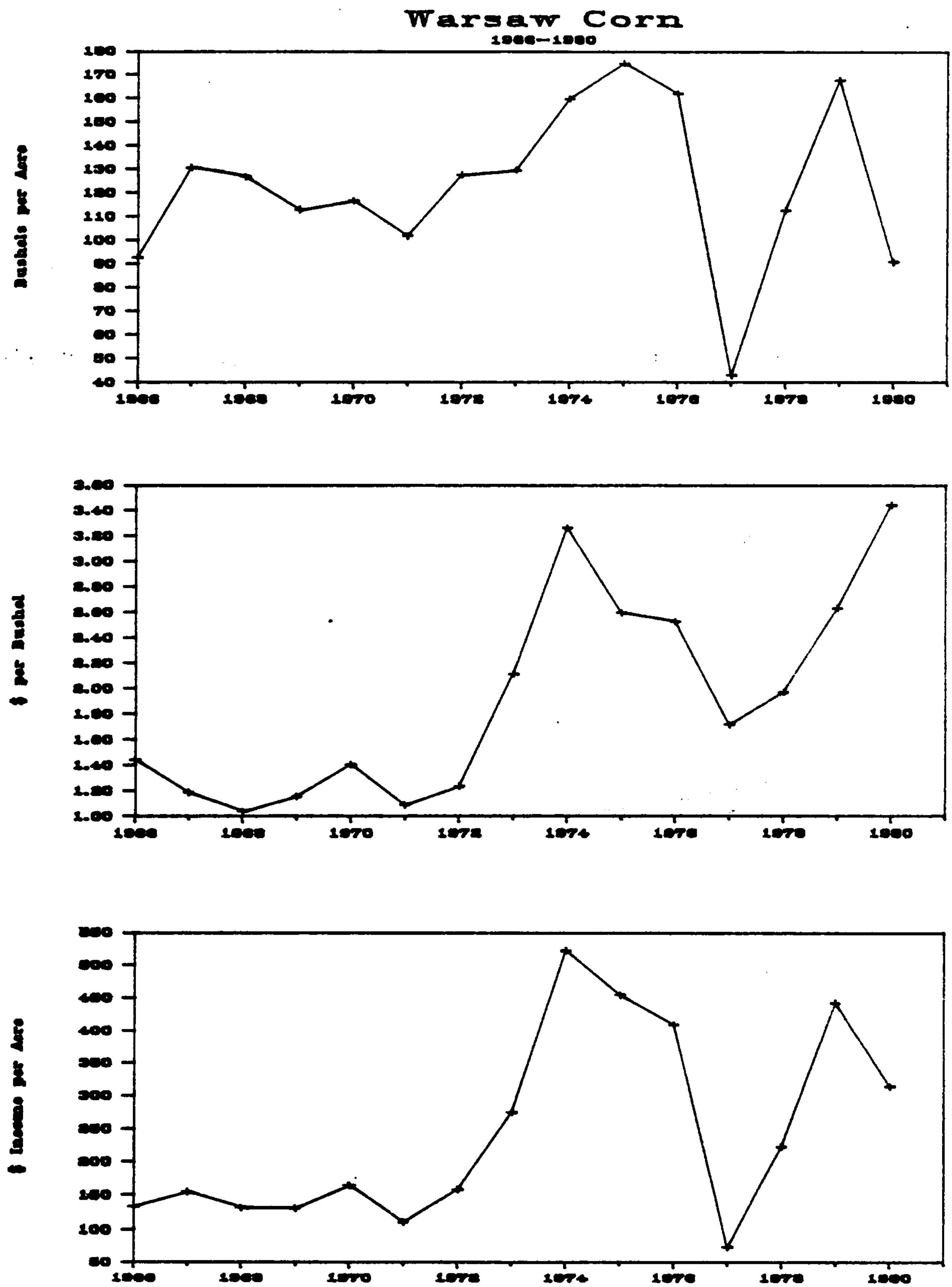
6Column headings are abbreviations for wheat hedging months January May.

7Column headings are abbreviations for June wheat harvest month premiums for hedges placed October through May.

Additional Midwest Option Prices

Year	COOMAR [8]	CCOAPR	COOJU	COOJ1	COOAUG	
1981	.7500	.0160	.7500	.7100	.6800	.2204
1982	.6200	.7500	.6300	.5400	.4200	.0927
1983	.0013	.0028	.0009	.0001	.0023	.1969
1984	.1224	.1700	.1314	.2300	.0726	.0725
1985	.4100	.5500	.4100	.3000	.2400	.0838

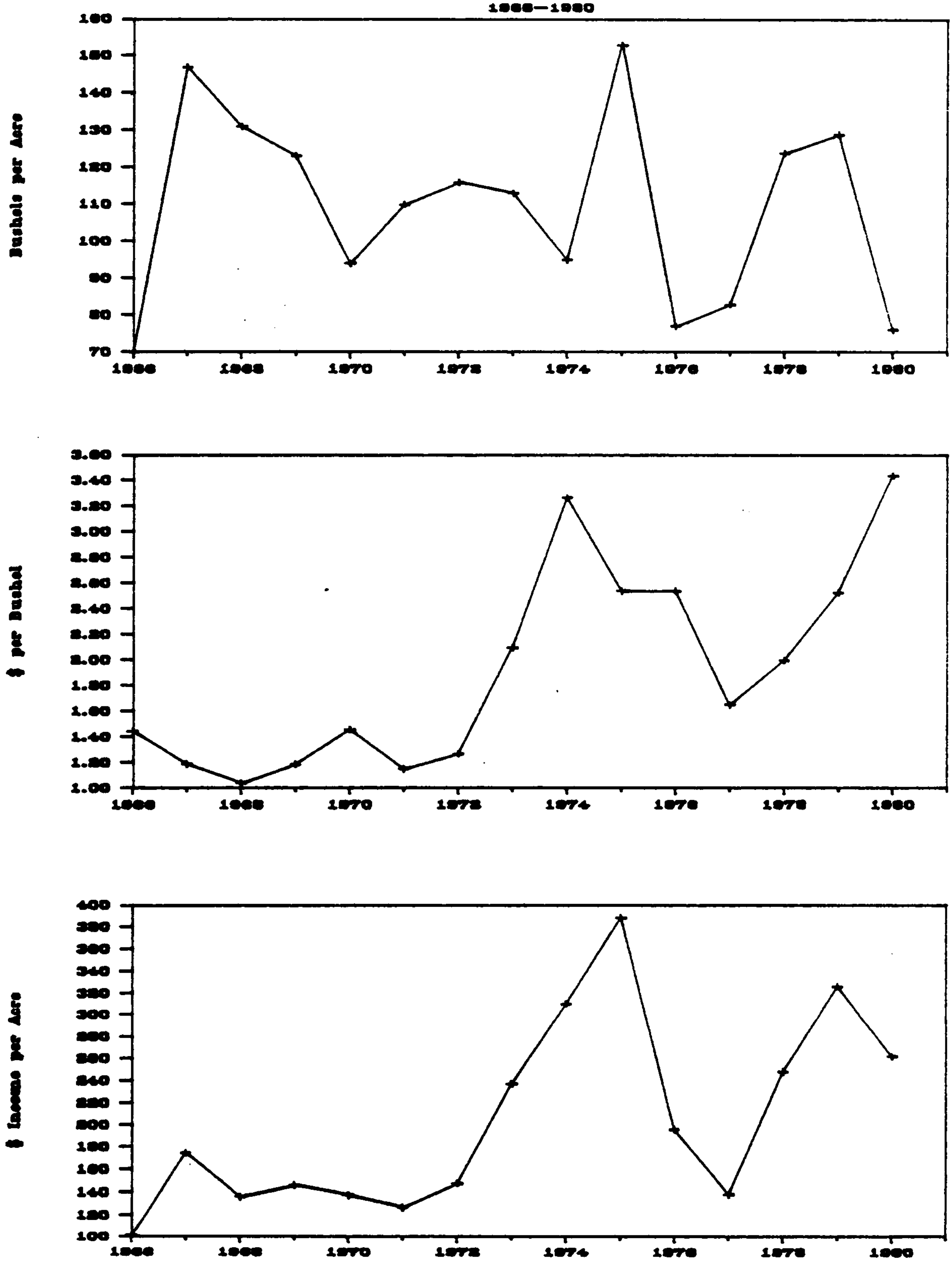
[8]Column hedgings are abbreviations for October midwest corn harvest months premiums for hedges placed March through August



Appendix Figure 1.

Orange Corn

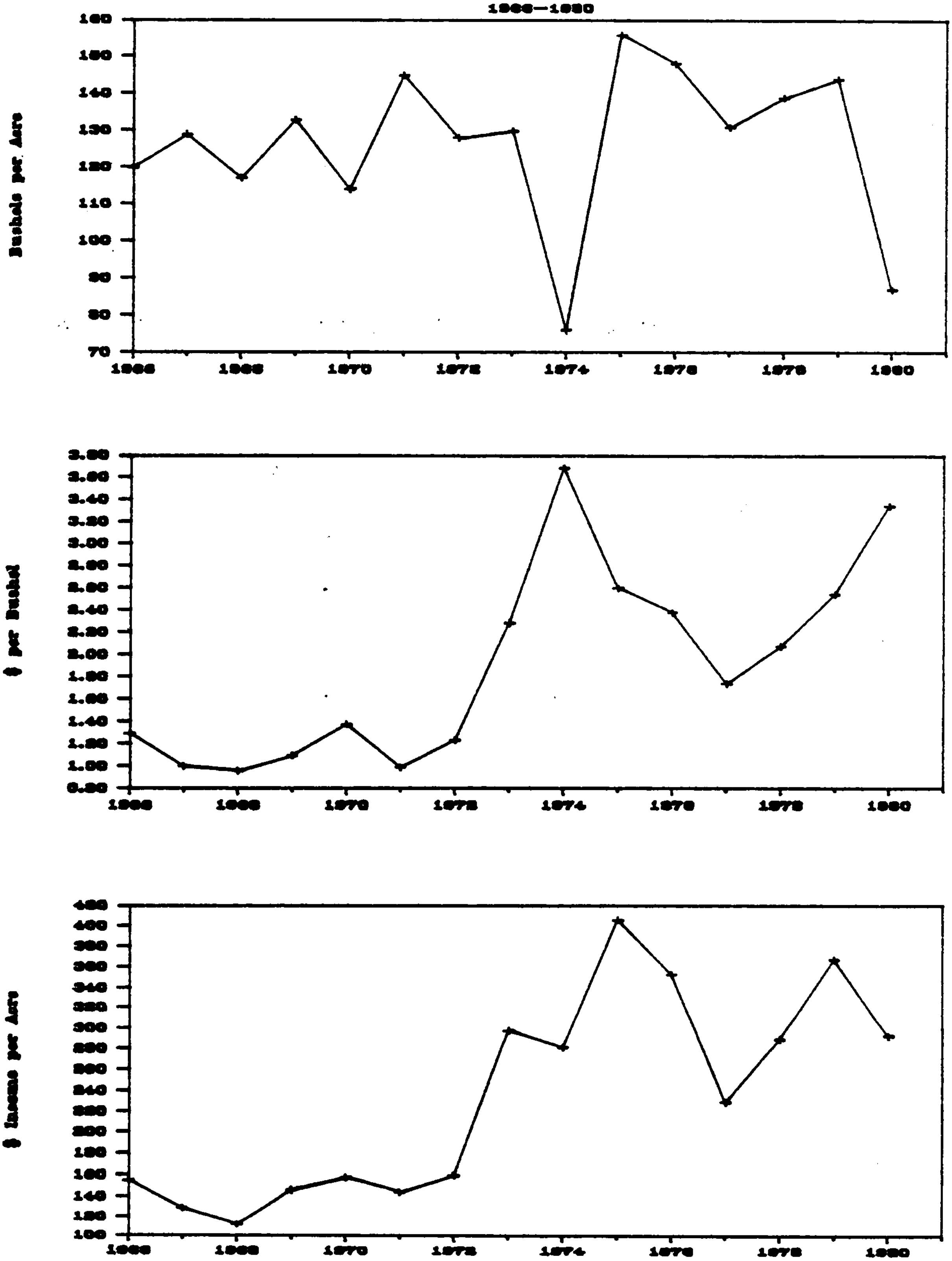
1966–1980



Appendix Figure 1 (cont.).

Midwest Corn

1966–1980



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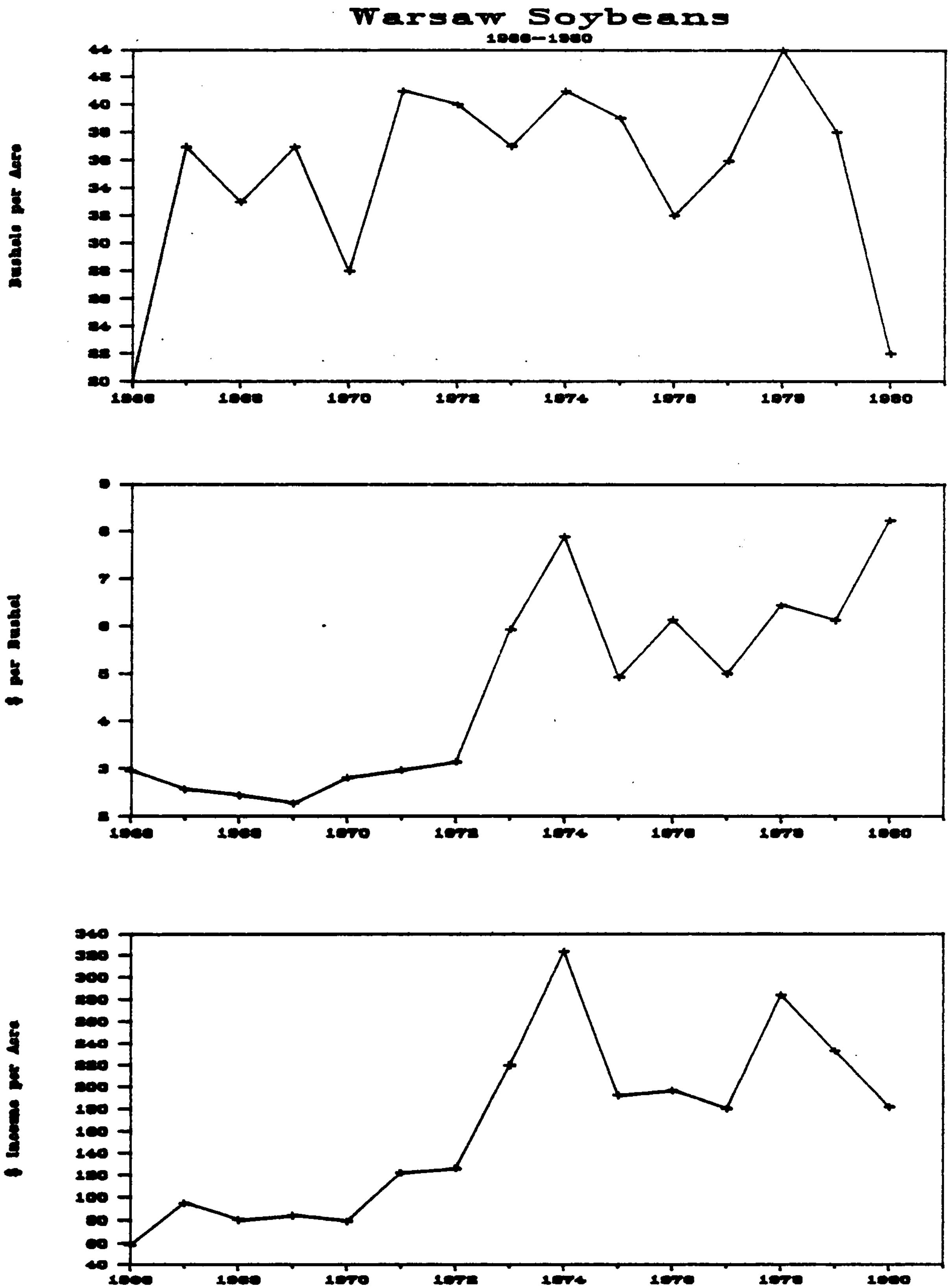
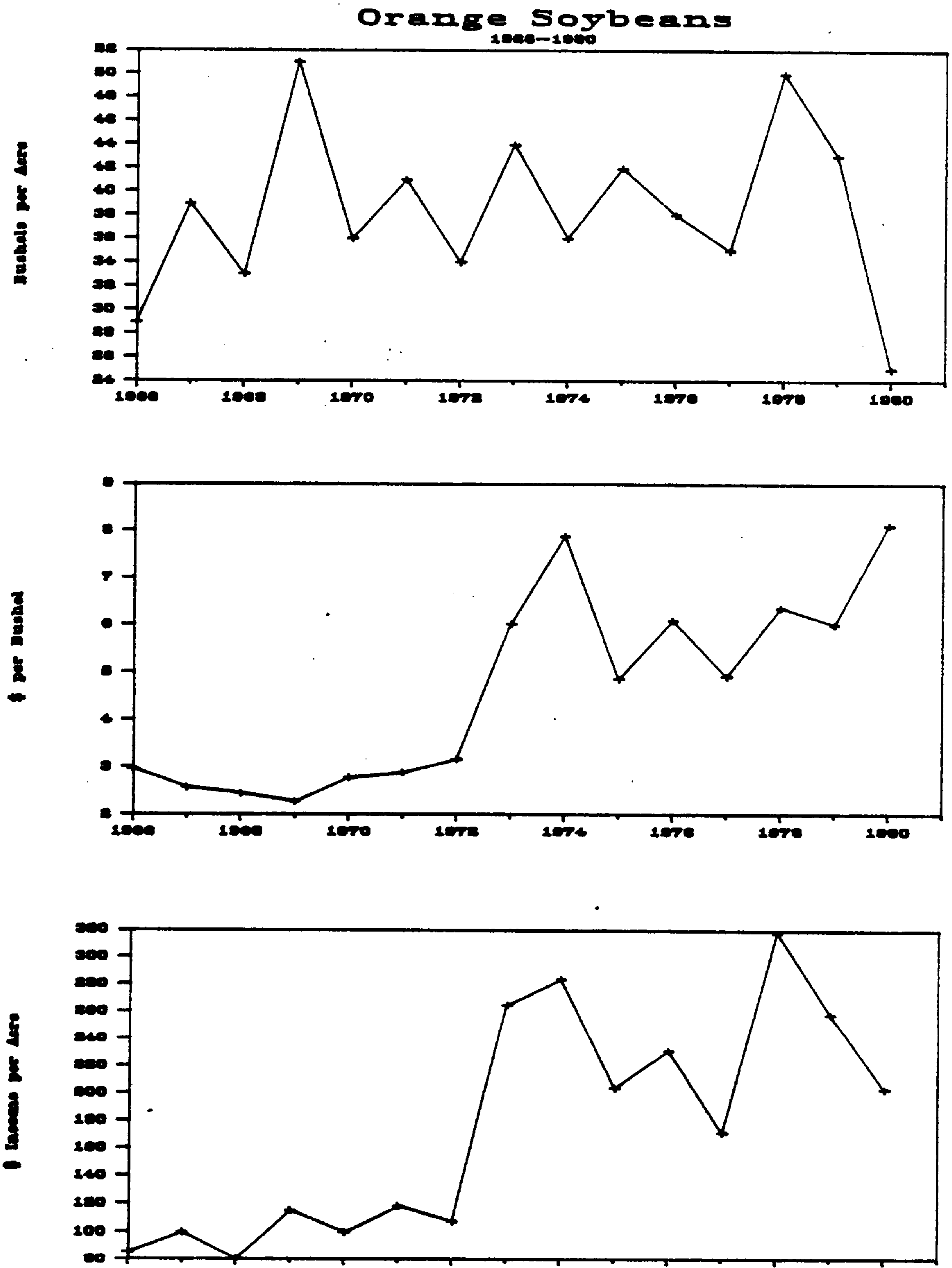


Appendix Figure 1 (cont.).



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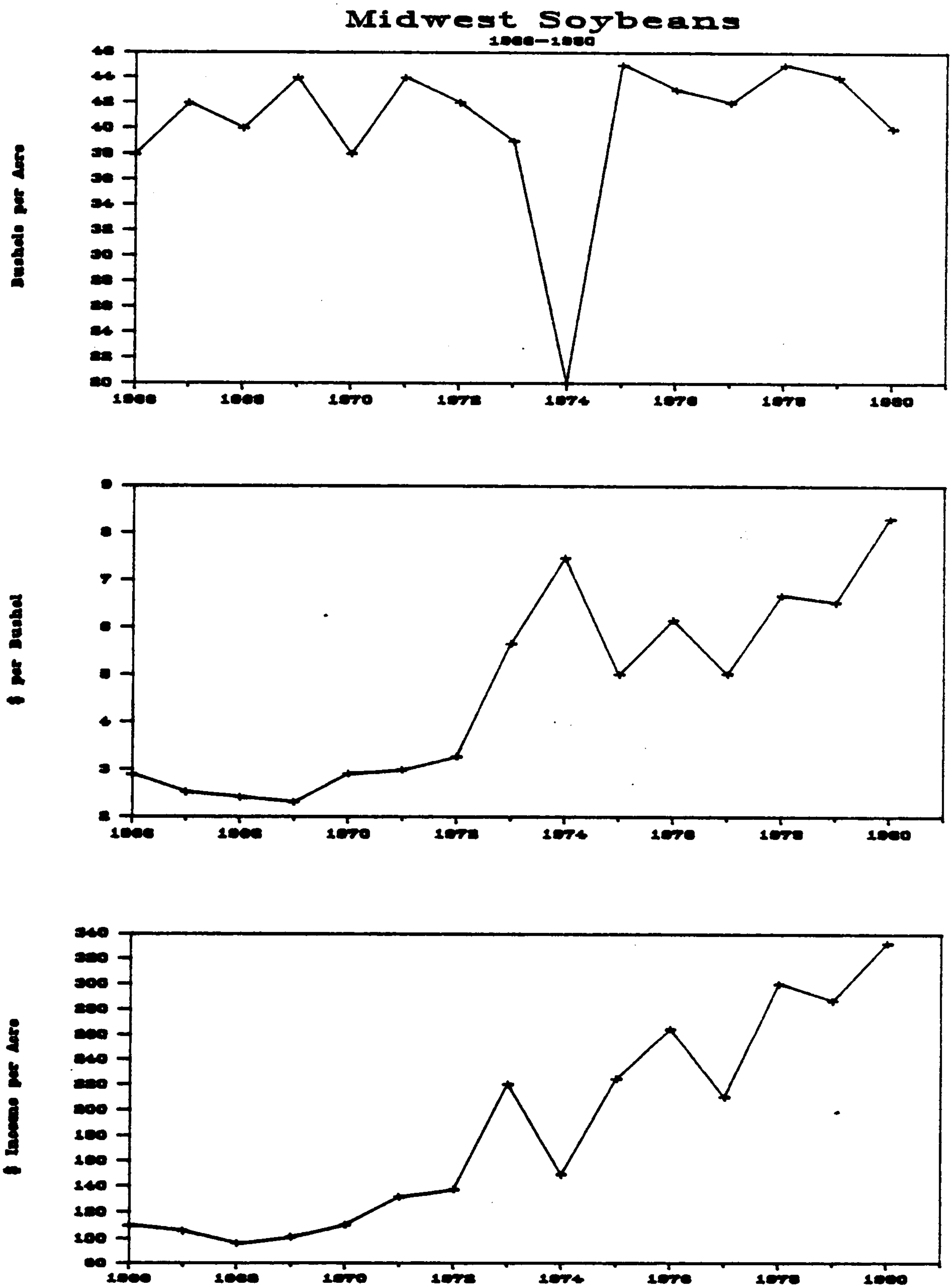

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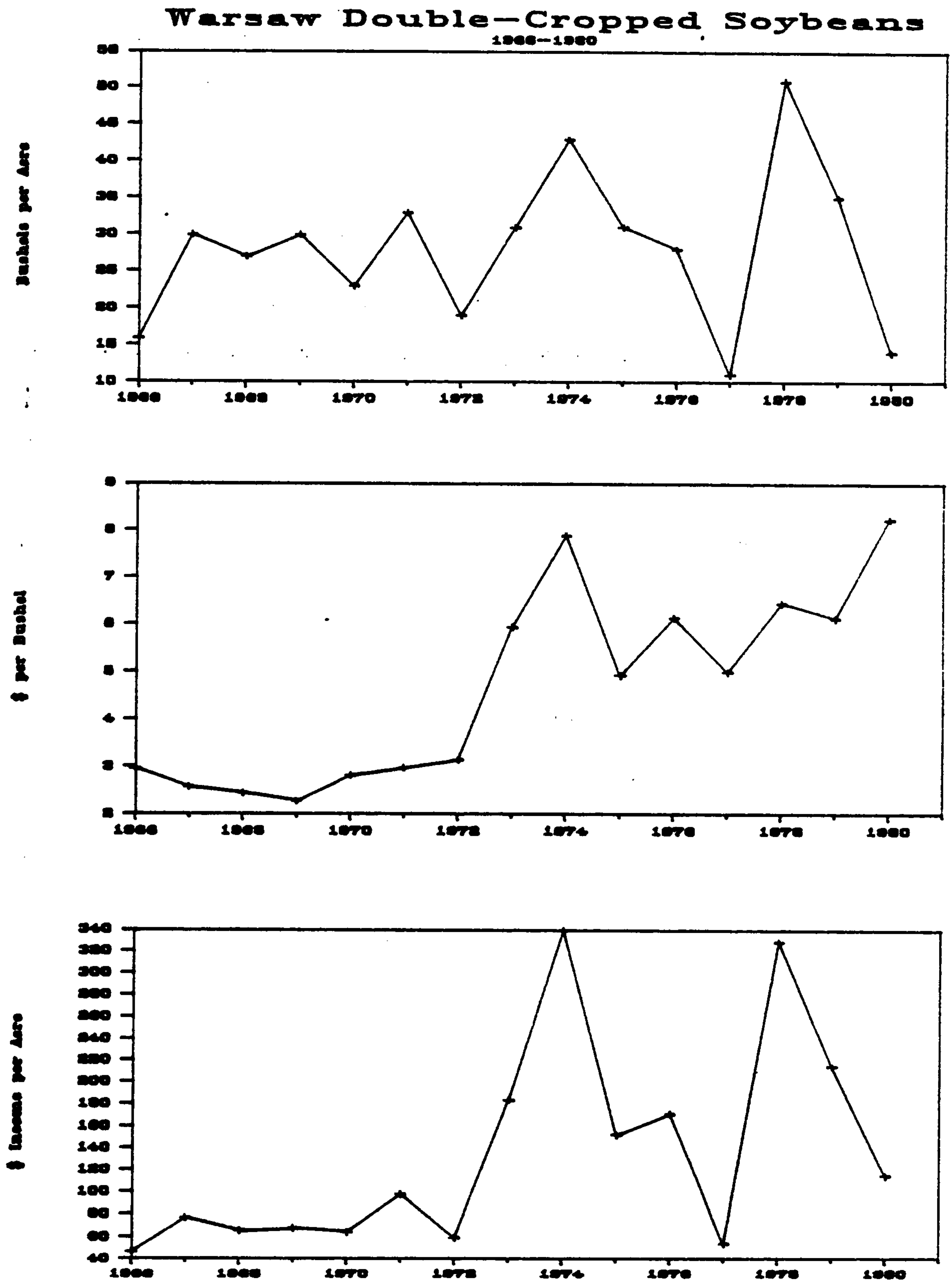
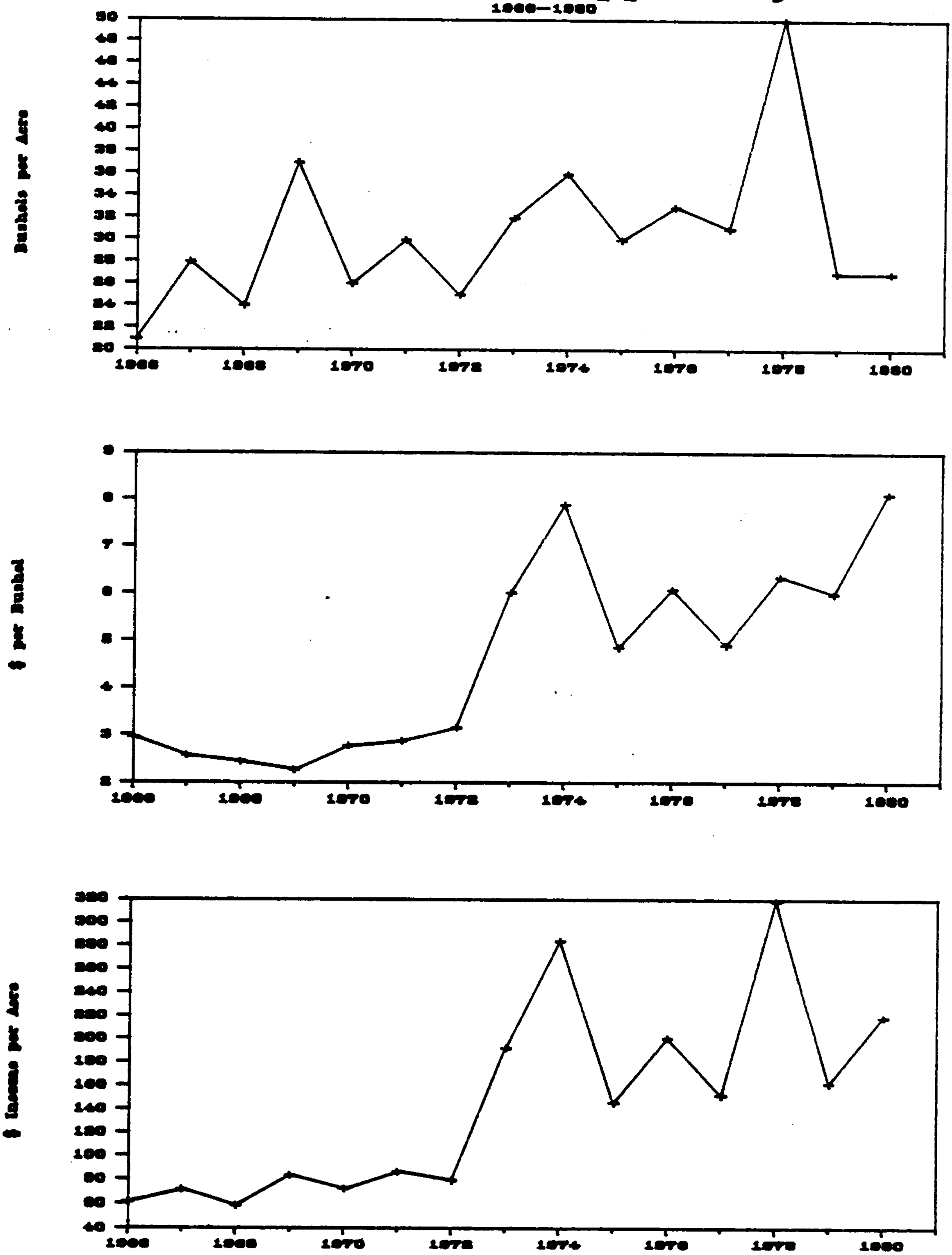


Appendix Figure 1 (cont.).

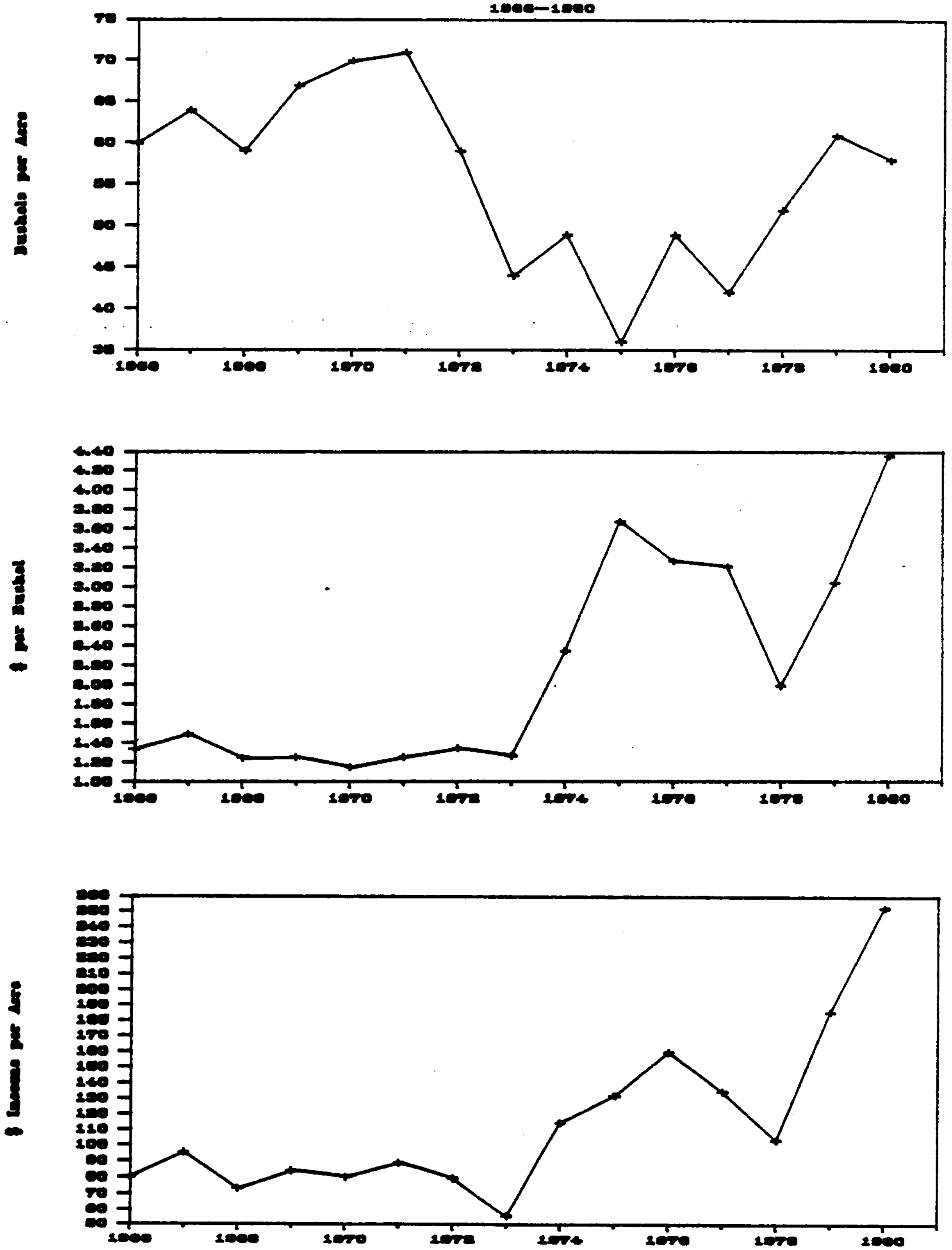
Orange Double-Cropped Soybeans

1966–1980



Appendix Figure 1 (cont.).

Warsaw Wheat
1966–1980



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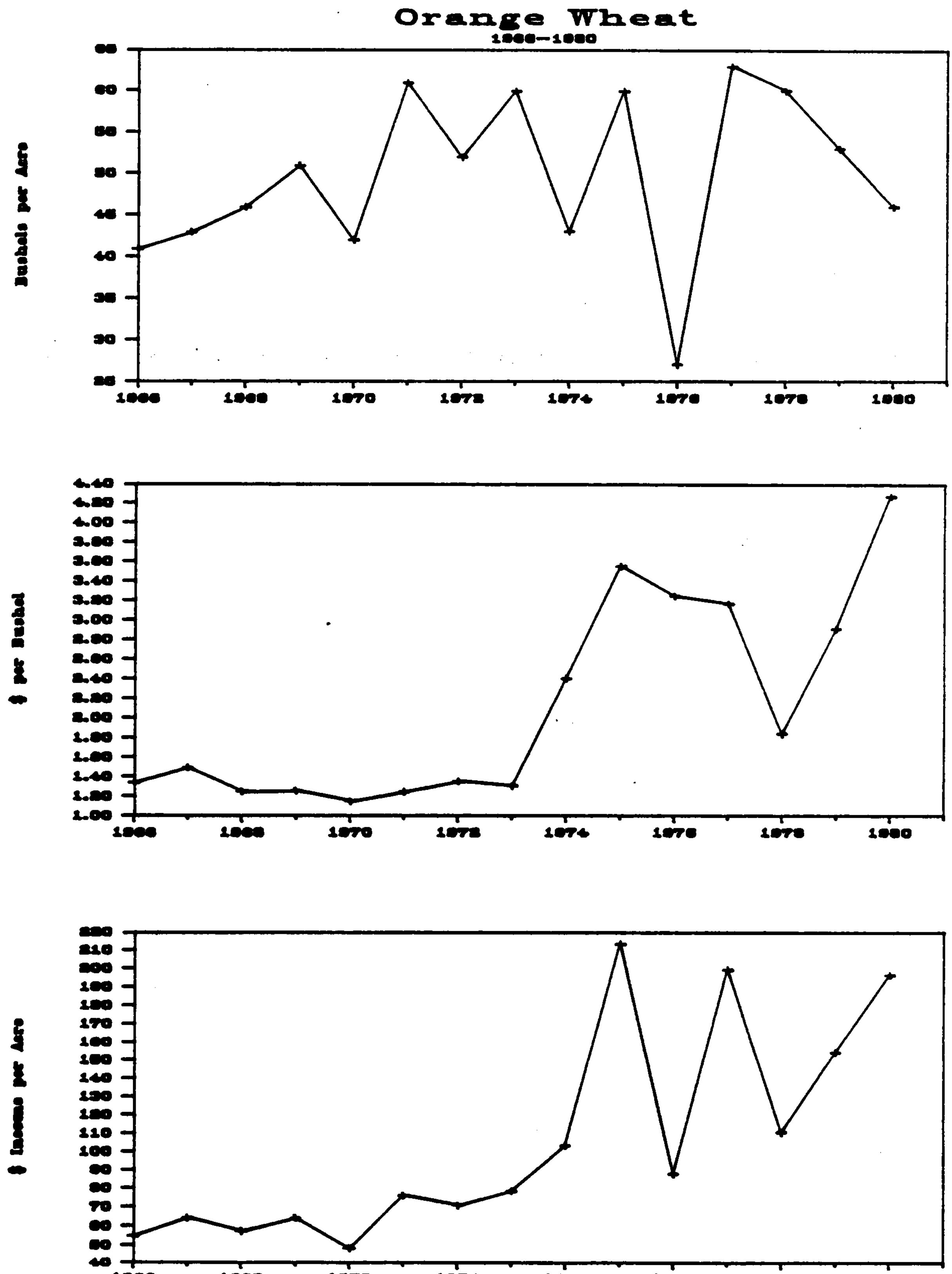
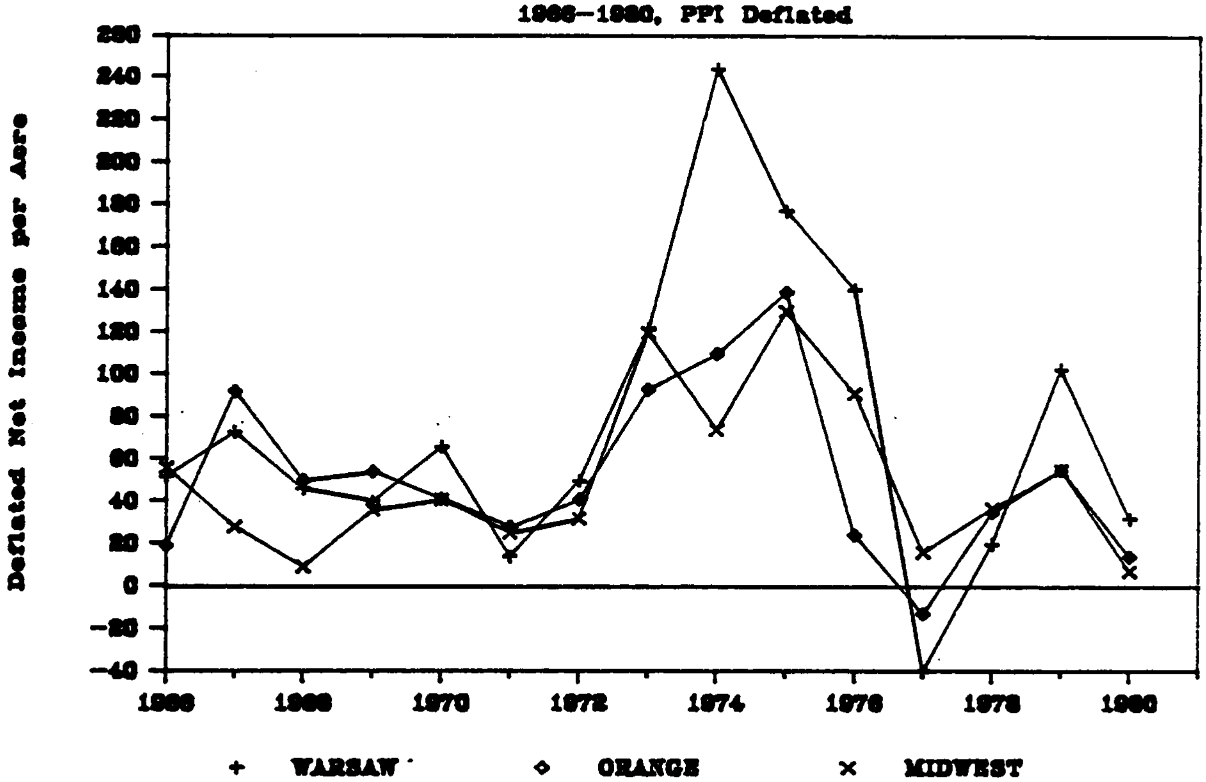


Appendix Figure 1 (cont.).

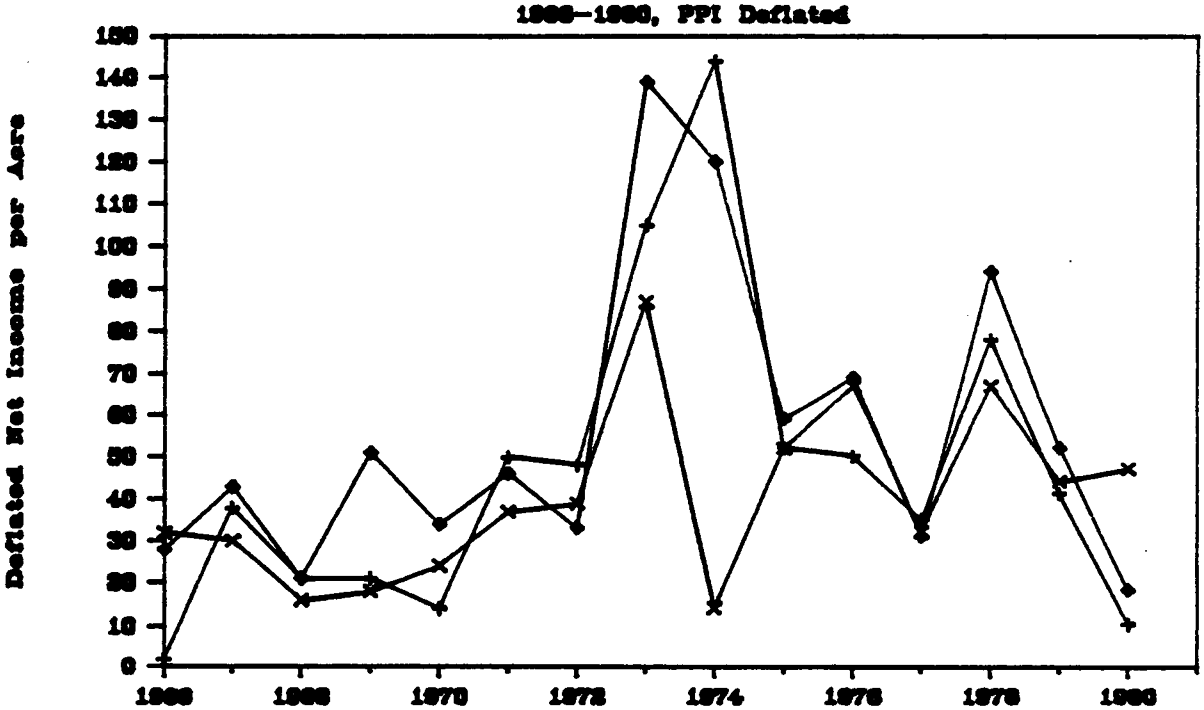
CORN

1966–1980, PPI Deflated



SOYBEANS

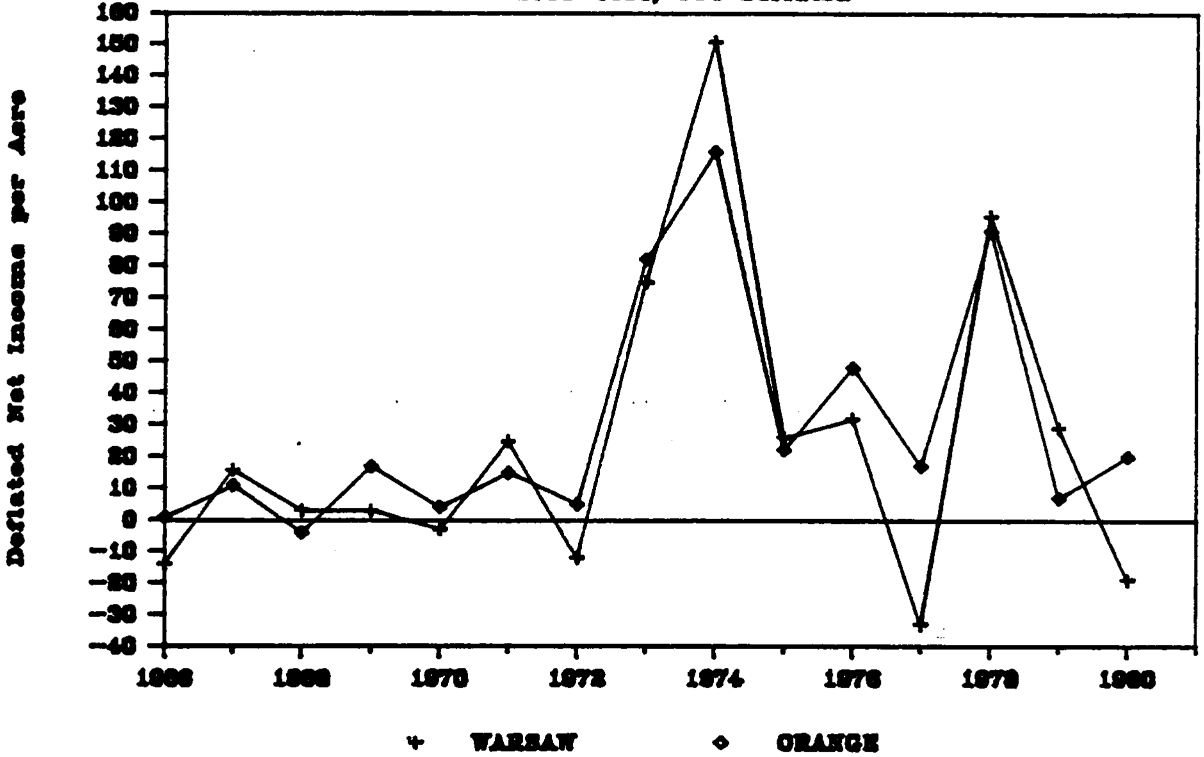
1966–1980, PPI Deflated



Appendix Figure 1 (cont.).

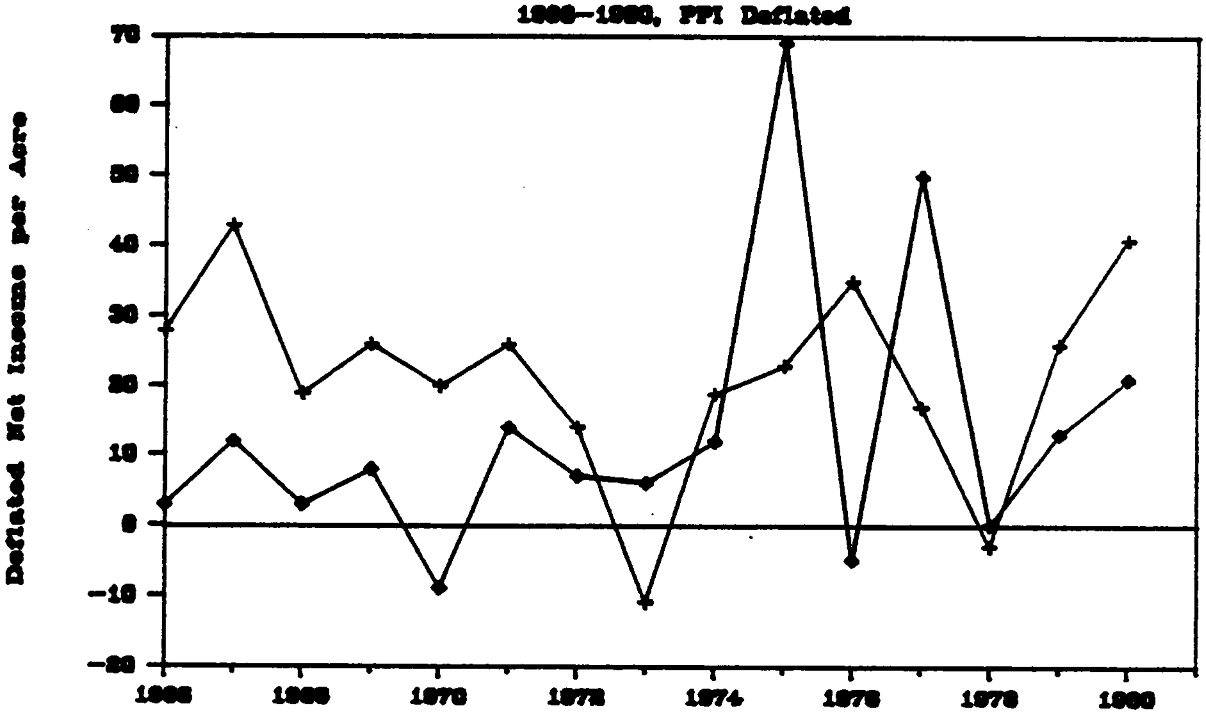
DOUBLE-CROPPED SOYBEANS

1966-1980, PPI Deflated



WHEAT

1966-1980, PPI Deflated



Appendix Figure 1 (cont.).

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