

CENTRAL BANK CREDIBILITY, ENDOGENOUS BELIEFS
AND SHORT-RUN PHILLIPS CURVES

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(ABSTRACT)

The effects of monetary policy on real economic variables have been debated for some time. This debate became more intense after the discovery of the Phillips curve which appeared to show a stable trade-off between inflation and unemployment. This curve in its original form has now been abandoned and debate has centered around the question of a short run trade-off. It is this question, do short-run trade-offs exist and if so, why, and what affects their length, that this dissertation addresses.

Chapter II explores this question in a model where the Federal Reserve does not have full credibility among all the agents in the economy and where beliefs are endogenous. It is shown that when the Federal Reserve announces a new monetary policy rule, temporary nonneutrality of money can result if some agents are skeptical of the Fed's intentions to follow the announced rule or if some agents merely believe some other agents are skeptical whether or not they truly are. The magnitude of the trade-off depends on the proportion of agents who are skeptical and how different the old and new rules are. The length of time the trade-off exists depends on how skeptical the agents are.

The more skeptical they are, the longer it takes the Fed to convince these agents that it is following and will continue to follow the announced rule.

Chapter III develops an empirical model to determine if the evidence supports the existence of short-run trade-offs in general and the credibility implications in particular. A Bayesian Vector Autoregression is used for the estimations. It is shown that short-run trade-offs do exist and do vary in length and magnitude. These variations are related to the implications of the credibility theory. It is found the degree of skepticism and the proportion of agents who are skeptical could have caused these short-run trade-offs to vary in length and magnitude.

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CHAPTER I

INTRODUCTION AND LITERATURE REVIEW

The effects of monetary policy on real economic variables have been debated for some time. Classicists saw the economy as being divided into a nominal sector and a real sector where the nominal is unable to affect the real. Thus, changes in the supply of money would not affect real variables such as unemployment and real output, but only nominal ones such as the price level. The Keynesian models, in contrast, did not dichotomize the economy in this way. This Keynesian viewpoint gained considerable support after the discovery of the Phillips curve. Since this curve appeared to show a stable trade-off between inflation and unemployment, it was in direct conflict with the classical dichotomy. But ideas about the Phillips curve changed quickly as it became evident that the trade-off was not in fact stable. Quite a few schools of thought were developed to explain this curve, or lack of one. The Phillips curve in its original form was abandoned and debate centered around the question of a short-run trade-off. It is this question, do short-run trade-offs exist and if so, why, and what affects their length, that this dissertation addresses.

This chapter is a discussion of some of the different views that have evolved about the Phillips curve and the theories behind them. The second chapter is a presentation of a theoretical model showing how money can temporarily affect real variables. This model provides

justification for the existence of a short-run Phillips curve. In chapter III, empirical evidence is presented which will support the existence of a temporary trade-off between inflation and unemployment.

Although the idea of an inflation-unemployment trade-off had existed long before 1958, it was not until then that modern Phillips curve analysis began. In that year, A.W. Phillips published an article in which he fitted a statistical equation relating percentage rates of change of money wages to the unemployment rate [Phillips 1958]. He interpreted the resulting curve as showing the response of wages to the excess demand for labor (which was proxied by the inverse of the unemployment rate). The important aspect of this curve was that it appeared to be stable. After transforming the curve from one showing a wage change relation to one showing a price change relation, it became a useful tool for policymakers. They could determine how much unemployment would be associated with any given rate of inflation, then choose which inflation-unemployment combination they preferred. However, since the relationship was an inverse one, policymakers believed they could not lower both inflation and unemployment at the same time, unless they could somehow shift the curve inward.

Around the mid-1960s it was found that the Phillips curve for the U.S. was not as stable as was the original one that had been identified for the United Kingdom. As a result, three major changes, which tried to explain this instability, were made in the Phillips curve equation in the early 1970s. Excess demand was redefined as the difference between the natural and actual rates of unemployment. Price expectations were included and a separate equation specified how these

expectations were formed. Generally, an adaptive expectations mechanism was used where price expectations are adjusted by some fraction of the error made in forecasting the current period's prices.

The adaptive expectations assumption was soon criticized because it allowed for the possibility of predictable errors being made. So this mechanism gave way to the rational expectations approach. This hypothesis, as advanced by Muth (1961), stated that the public's expectations of economic variables are essentially the same as the predictions of the relevant economic theory. This idea was very appealing since other theories about expectations formation implied that people make systematic errors when forecasting economic variables.

These developments in Phillips curve analysis were the base upon which the natural rate hypothesis was built. The natural rate theories postulate no permanent trade-offs between inflation and unemployment: the long-run Phillips curve is vertical. But, short-run trade-offs can exist if there is unexpected inflation. Obviously, there are different views on the likelihood of this occurring. Some of the more strict rational expectations models show the Phillips curve as being vertical even in the short run. Benjamin Friedman (1979) discusses the informational assumptions required for this result to hold and concludes that rational expectations is a long-run concept and is not useful for talking about short-run Phillips curves.

The dynamics of the natural rate hypothesis are explained in a discussion by Milton Friedman (1977). Briefly, he begins with an economy that has been stable but suddenly has an increase in nominal

aggregate demand. Each producer believes the increase is at least partially specific to him and so expects the relative price of his product to increase. He will want to produce more for which he needs more workers. To get them he will offer a higher nominal wage which he believes to be a lower real wage in terms of the price of his product. Workers on the other hand are not concerned with the real wage in terms of the good they produce but in terms of the general price level, which they do not realize has increased. Therefore, workers believe their real wage is higher and as a result are willing to work more. Thus, the increase in the rate of inflation has caused an increase in the rate of production and in the rate of employment. This situation is only temporary. Firms and workers will both realize that the rate of inflation has increased and readjust to their original pattern of behavior which causes the economy to return to the natural rate of unemployment.

Lucas (1972) formalized the natural rate hypotheses in an overlapping generations model with money. There are two markets between which no communication is possible. Fluctuations in relative prices can occur because the allocation of traders across the two markets in each period is in part stochastic. The economy can also experience nominal price fluctuations because of stochastic changes in the money supply. So the traders in this economy are not sure if the price changes they observe are due to relative or nominal shocks. They will generally guess that a particular price change is due partly to

both types of disturbances. This will result in temporary nonneutrality of money.

These developments had major implications for economic policy. The original Phillips curve was not valid and some rational expectationists were even questioning the existence of any exploitable short-run trade-offs. As Sargent and Wallace (1975) write, "there is no systematic rule that the authority can follow that permits it to affect the unexpected part of the price level." If an unpredictable random term is added to the money supply rule, there may be real effects but there is no way the authority could control it for purposes of countercyclical policy. So money may have an affect on real variables, but not in any systematic way controllable by the monetary authority.

Recently, some qualifications to the rational expectations approach have been suggested, which would imply that short-run trade-offs may last longer and occur more frequently than the rational expectations literature suggests. Whether or not these trade-offs are exploitable is still questionable. Most of this literature centers around explaining what happens after the Federal Reserve changes the money supply rule. Since agents know that average expectations affect economic variables, each agent must know average opinion in order to make rational forecasts. Beliefs about average expectations will in turn affect individual expectations, so beliefs are endogenous. The learning literature, the credibility literature and the sunspot literature all deal with this problem in some way.

Learning Literature

The learning literature tries to determine whether or not agents can learn a new monetary policy rule and eventually form expectations that are rational. Frydman (1982) considers the question of whether individuals can learn the parameters of the rational expectations equilibrium price distribution. He concludes that they cannot, even if they have the correct specification of the model, observations of market prices, and local information. This happens because it is not possible for agents to know average forecasts ex ante. To obtain convergence to rational expectations equilibrium, he must impose a consensus condition requiring all firms to form guesses on average opinion and make output decisions in the same way.

Townsend (1978) addresses the problem of each individual trying to guess what the others are expecting from the point of view of game theory. The problem is similar to an oligopolistic market where the actions of one firm influence the actions of the other. So, each firm bases his actions on his guess of what the other firms will do. In rational learning models each person tries to guess what average expectations will be. Townsend attempts to resolve the problem with a Nash equilibrium concept. He finds that convergence to a self-fulfilling equilibrium is possible even if the public is not very well informed. Also, even though the structural relations of the model cannot be identified, individuals can learn the reduced form price equation with a Bayesian update procedure.

Most of the learning literature simply sidesteps this interdependence of forecasts problem. These types of models begin with an ad hoc specification of the model used by the public to make forecasts. Some of the parameters are unknown and must be learned. The original model may not be consistent with the true model but through learning, the equations themselves may be changed. This approach seems to make convergence to rational expectations equilibrium more certain. For example, Cyert and DeGroot (1974) conclude that by using a Bayesian learning process, convergence to rational expectations equilibrium will occur. Turnovsky (1969), Taylor (1975) and Lewis (1981) also find convergence by using a Bayesian update process. Bray (1982) finds that convergence occurs if the stability parameter in her model has an appropriate value. However, it is possible for this parameter to take on other values which will cause the system to not converge to rational expectations.

Sunspot Literature

The sunspot literature attempts to determine if extrinsic uncertainty can be important in rational expectations models. In other words, can random phenomenon that does not affect tastes, endowments, or production possibilities have a real impact on the economy? This could happen because if agents believe prices depend on some extraneous phenomenon, (i.e. sunspots), prices actually will. If sunspot activity is believed to be important, its level will become a state variable which prices will depend upon. This can also be put into the context of the problem of forecasting average expectations. If for some reason

extrinsic to the economy, (i.e. sunspots), some or all of the agents believe that on average other agents are forecasting inflation, this agent will also forecast inflation. In this way, inflation will occur even if individual agents know there is no intrinsic reason why it should. It has been shown theoretically that sunspots can exist even in rational expectations models. See for example Cass and Shell (1980) and (1983).

There has also been empirical work done to determine if sunspots have mattered during certain historical periods. For example, Flood and Garber (1980) and Hamilton and Whiteman (1985) have tried to determine if speculative bubbles caused the German hyperinflation. They have concluded that this was not a factor. However, Hamilton and Whiteman also show that we really cannot empirically test whether prices are driven by completely extraneous factors. They write, "one can always relax restrictions on the dynamics of the fundamental driving variables so as to interpret what appears to be a speculative bubble as instead having arisen from rational agents responding solely to economic fundamentals not observed by the econometrician."

Credibility Literature

The credibility literature tries to determine what happens when the Federal Reserve announces a new monetary policy rule but the public is skeptical that the Fed will actually follow the new rule. The problem for agents is to learn whether the old rule or the new rule is being followed and which rule the other agents believe is being followed. So forecasting average expectations is once again a problem.

Credibility is generally defined as the extent to which the public believes that a shift in policy has actually taken place when such a shift has occurred. Credible policy must also be consistent with the public's information about the objectives and constraints facing the central bank. The public will not believe an announced policy if it knows the policy is incompatible with the current objectives of the policymakers. There are two types of approaches in the credibility literature to the Fed's motives. One type sees the central bank's objective function as a social welfare function. The other type sees the objectives of the central bank as being politically motivated. Here, the importance of preventing inflation relative to stimulating output depends on the current views of the central bank and the pressure placed on them by the government and the private sector.

Backus and Driffill (1985) take the first approach and view the credibility problem as a game between policymakers and the public. The policymaker may announce a disinflationary policy, but if the public is skeptical, they will continue to ask for wage increases. They may be skeptical because they know the government has an incentive to cause unexpected inflation in order to increase output. Thus, a recession will occur if the government sticks to its disinflationary policy.

The public updates their views about Fed credibility in a Bayesian fashion. If the Fed continues to stick to its announced rule, the public will eventually believe them and the recession will end as wages are adjusted accordingly. However, if the Fed inflates just once, its

reputation is ruined forever, obviously a definite drawback of this approach.

The second approach is taken by Cukierman and Meltzer (1986). They specify an objective function for the Fed in which its marginal preference for economic stimulation relative to inflation prevention shifts randomly through time. The marginal rate of substitution is a random variable which reflects the current compromise between backers of economic stimulation and those preferring price stability. Its value is not known by the public, but changes in its value can gradually be detected by observing changes in the growth rate of the money supply. So there can be temporary nonneutrality of money while agents determine what the Fed is doing.

The policymaker knows that current actions which raise future inflation expectations make it more costly (in terms of inflation) to further stimulate the economy in the future. More complications occur because the policymaker does not have perfect control of the money supply and does not know for certain what his future objectives will be. So the policy he forms today must also take into account these uncertainties. Cukierman and Meltzer show that agents adjust their expectations faster, so that the period of nonneutrality is shorter, the more precise monetary control is. This implies that if the Fed values stimulation, it will prefer imprecise control so that when the money supply is increased it will take agents longer to realize this, so the positive output effects will last longer. If the Fed is trying

to reduce inflation, it will want to have had more precise control so that the negative output effects will not last so long.

Marianne Baxter (1985) develops a theory where credibility is a function of both monetary and fiscal policies. She argues that a credible monetary reform depends on a compatible fiscal policy. Agents choose a value for the probability that the new policy is going to be continued and then estimate, in a Bayesian fashion, the parameters of the new rule. She applies this theory to Chile and Argentina and concludes that the reason reforms worked in Chile but not in Argentina may be that the Chilean reforms were credible while the Argentinian ones were not.

Di Tata (1983) adds a twist to the credibility problem and the problem of forecasting average expectations. In his paper, all of the agents actually believe the Fed, but individual agents do not realize that everyone else believes the Fed. So money is temporarily nonneutral again. This paper will be discussed in more detail in the next chapter since it is the foundation for the theoretical work presented there. Additional literature on forecasting average expectations can be found in the collection of papers edited by Frydman and Phelps (1983) of which Di Tata's article is a part.

This dissertation incorporates some aspects of each of the above strands of literature as it looks at what may happen when a new money supply rule is announced. Agents in the economy will face two decision problems. First, they must decide if the announcement is a credible one. Then the agents forecast average expectations, since they know it

is the average forecast that affects the economy. For example, if the Federal Reserve announces that it is going to increase the money supply by 10%, this agent, believing that money is neutral, knows that the price level should increase by 10%. However, if he believes there are others in the economy who act as if money is not neutral, he knows the price level will not increase by 10% and output will increase above the natural rate. So this agent will expect money to not be neutral, which results in these expectations being fulfilled. As time passes, agents will learn more about the motives of the Fed and the beliefs of the other agents. They will then update their own beliefs about the policy and about the beliefs of other agents in a Bayesian fashion.

CHAPTER II
THE THEORETICAL MODEL

Definition of the Problem

When the Federal Reserve announces a new monetary regime, we might expect that all agents in the economy would adjust their behavior accordingly so there would normally be no real effects on output. But, the problem of forecasting average expectations as discussed in Chapter I can cause other types of behavior. To analyze this, Di Tata (1983) sets up the following scenario. An economy exists where past governments have not been following their announcements about policy changes. Now, a new government with a different background takes over and announces a new, permanent change in policy. If each agent completely believes the new regime, is this enough to ensure that the change in policy will have no effect on output? Rational expectationist would say yes, because they implicitly assume that each agent presumes everyone else forms expectations by using the same model he is and that everyone assigns the same degree of credibility to the monetary rule that he does. However, Di Tata shows that the answer is no if an agent incorrectly believes that average opinion does not completely believe the new government. Because information about other agents' beliefs is costly to obtain, individual agents do not know that all other agents are following the new rule.

The Basic Model

Di Tata sets up an economy consisting of the following equations;

$$(1) \quad Y^s_t = a_0 + a_1(P_t - {}_{t-1}P_t) + e_t$$

and

$$(2) \quad Y^d_t = M_t - P_t + v_t$$

where Y_t , P_t , and M_t represent the natural logarithms of output, the price level, and the money supply, respectively. v_t and e_t are normally distributed, serially uncorrelated stochastic terms with mean zero and variances σ^2_v and σ^2_e . Shocks to velocity are represented by v_t and shocks to the aggregate supply schedule are represented by e_t . The shocks are distributed independently of each other. The variable ${}_{t-1}P_t$ represents the average psychological expectation, not the mathematical expectation, held by the public as of the end of period $t-1$. The natural level of output is represented by the constant a_0 ; a_1 is also a constant.

This economy then has a Lucas type supply function where output is a direct function of the difference between the current price level and peoples' prior expectations of the current price level. Unexpected increases in the price level lead to an increase in output because suppliers mistake the general price increase for a relative one, and thus increase their production. The aggregate demand function is a simple velocity equation.

The Federal Reserve tries to offset shocks to velocity so as to stabilize the economy. Thus the following reaction function is followed:

$$(3) \quad M_t = h_0 - k_0 v_{t-1}$$

where $h_0 > 0$ is the average money supply and $k_0 > 0$ is a constant representing the reaction coefficient. At the end of period t everyone can see v_t , so the monetary authorities are responding to disturbances that have already been seen by the public.

Assume that the rule described by (3) has been followed for a long time and the economy is at the natural rate of output. Now suppose that at the end of period t_{-1} the Fed announces that starting in period t_0 it will follow a new rule of the form

$$(4) \quad M_t = h_1 - k_1 v_{t-1}$$

where $h_1 \neq h_0$ and $k_1 \neq k_0$. Agents have this information at the end of period t_{-1} to use in their calculations of expectations of the price level that will exist in period t_0 .

Di Tata shows that under rational expectations, this shift in policy has no real effects. He then shows that temporary nonneutrality can result if, instead of assuming rational expectations, that is ${}_{t-1}P_t = E_{t-1}P_t$, assume that people form expectations according to the model they believe to be in existence at the time (equations 1,2 and 3 or equations 1,2 and 4), and also develop expectations about others' expectations, which may be different from their own. Di Tata refers to this as model interlinked expectations. To forecast average opinion, each agent assumes that the average agent is uncertain about the Fed's intentions and assigns probability θ to the new rule being followed and probability $(1-\theta)$ to the old rule continuing to be followed. Transitional nonneutrality results because of the incorrect perceptions

about average opinion as shown below in the final expression for output that Di Tata obtains;

$$(5) \quad Y_{t0+s} = a_0 + \frac{a_1^2(1-\theta)^{s+1}(h_1-h_0)}{(1+a_1)^2} + \frac{a_1 v_{t0+s} + e_{t0+s}}{1+a_1} + \frac{a_1^2(1-\theta)^{s+1}(k_0-k_1)v_{t0+s-1}}{(1+a_1)^2} \quad (s \geq 0).$$

The mean value of output is,

$$(6) \quad EY_{t0+s} = a_0 + \frac{a_1^2(1-\theta)^{s+1}(h_1-h_0)}{(1+a_1)^2} \quad (s \geq 0).$$

If h_1 is lower than h_0 , the mean value of output will be lower than the natural rate and vice versa, unless $\theta=1$. In this case, everyone believes that everyone else believes the Fed with certainty.

As discussed in Chapter I, a second problem that can occur when a new monetary regime is announced is the lack of Federal Reserve credibility. This could occur if the Fed has not followed its announcements in the past or if some agents do not believe the new policy is feasible. Some agents could also believe that the Fed may follow the announced rule for a couple of periods but then revert back to the old rule. This could be planned by the Fed in advance or could result from political pressures aimed against the new policy. So even though the Fed may follow the announced rule for a couple of periods, everyone may not be immediately convinced that the Fed is credible.

This chapter studies the related problems of credibility and of forecasting average expectations. It will be assumed that the Fed does follow the new rule this time so that agents who believe the old rule is still being followed are wrong. However, their expectations will be updated in a Bayesian fashion until they converge to the true policy

rule. It is found that how long this takes will depend in part on how strongly skeptical the agents are. The more skeptical they are, the more weight the agent attaches to his prior belief. Thus, more observations are needed to convince the agents that the Fed will continue to follow the new rule. Obtaining more observations in this case of course means that more time periods must pass before agents are convinced. The agents who originally believed the Fed will continue to believe that this new rule is being followed. However, both types of agents will alter their expectations of average expectations as realizations of economic variables are observed.

The following model is an extension of Di Tata's model. In this extension, there is no assumption that all agents believe the announced policy. An explicit Bayesian update process is also added.

Extension of the Model

The economy is again described by equations (1) through (4), with one exception. It will be assumed that v_t follows a first order autoregressive process

$$(7) \quad v_t = \rho v_{t-1} + \psi_t, \quad 0 < |\rho| < 1,$$

where ψ_t is normally distributed and serially uncorrelated with mean zero and variance σ^2_ψ . This makes the rule which the Federal Reserve follows more plausible. If there is no persistence in v_t , the Fed has no reason to offset the past periods' velocity shocks. However, when there is persistence in the shocks, it makes sense for the Fed to act in this way as it will truly be following a stabilizing monetary rule.

To solve this model, impose market clearing by equating equations

(1) and (2) to obtain,

$$(8) \quad a_0 + a_1(P_t - {}_{t-1}P_t) + e_t = M_t - P_t + v_t .$$

Rearranging gives,

$$(9) \quad P_t = \frac{M_t + v_t - a_0 + a_1 {}_{t-1}P_t - e_t}{1 + a_1} .$$

Let $f^i_{t-1}P_t$ represent the forecast of P_t at $t-1$, held by agent i . Also assume that $f^i(ax) = af^i(x)$ and $f^i(x+y) = f^i x + f^i y$, so that the forecast operator is taken to be linear. Taking expectations of (9) we get

$$(10) \quad f^i_{t-1} P_t = \frac{a_1 f^i_{t-1}({}_{t-1}P_t) + f^i_{t-1}M_t - a_0}{1 + a_1} .$$

Let $f_{t-1}P_t$ represent average forecasts, where the average is taken over all n agents. Thus, $f_{t-1}P_t = 1/n \sum_{i=1}^n f^i_{t-1}P_t$. Let $f_{t-1}(f_{t-1}P_t)$ be represented by $F_{t-1}P_t$ so that $F_{t-1}P_t$ is the average forecast of the average forecast of the price level. Rewriting (10) in terms of averages gives,

$$(11) \quad f_{t-1}P_t = \frac{a_1 F_{t-1}P_t + f_{t-1}M_t - a_0}{1 + a_1} .$$

We could continue taking average expectations of average forecasts of the price level to higher degrees in the same way. To avoid an infinite regress problem, assume as does Di Tata, that $F_{t-1} P_t = F^2_{t-1} P_t$ where $F^2_{t-1} P_t$ represents the average expectation of the average opinion about the average forecast of the price level. Therefore,

$$(12) \quad F_{t-1}P_t = \frac{a_1 F_{t-1}P_t + F_{t-1}M_t - a_0}{1+a_1}$$

which implies,

$$(13) \quad F_{t-1}P_t = F_{t-1}M_t - a_0 .$$

Now solve for $f_{t-1}P_t$ as

$$(14) \quad f_{t-1}P_t = \frac{a_1 F_{t-1}M_t + f_{t-1}M_t}{1 + a_1} - a_0$$

and for P_t as

$$(15) \quad P_t = \frac{(1+a_1)M_t + a_1 f_{t-1}M_t + a_1^2 F_{t-1}M_t - (1+a_1)(a_0(1+a_1) + e_t - v_t)}{(1+a_1)^2}.$$

Substituting (14) and (15) into (1) we can solve for supply as

$$(16) \quad Y_t^s = a_0 + a_1 \left\{ \frac{(M_t - f_{t-1}M_t) + a_1 (M_t - F_{t-1}M_t)}{(1 + a_1)^2} \right\} + \frac{a_1 v_t + e_t}{1 + a_1}.$$

It can be seen from (16) that to obtain the usual rational expectations result, M_t must equal both $f_{t-1}M_t$ and $F_{t-1}M_t$. In that special case any deviations of Y_t from its natural rate would be due to purely random shocks. Now we must determine how $f_{t-1}M_t$ and $F_{t-1}M_t$ are formed in order to determine how likely it is for the neutrality result to be obtained and what may happen when it is not. First, it will be shown that before the new rule was announced, money was indeed neutral. Then we will determine what may happen after the new rule is announced.

It was assumed that the rule shown in equation (3) has been followed for a long time so that the economy is operating at the natural rate of output. Everyone has had sufficient time to learn that this rule is going to be followed and that everyone else believes this rule is the one being followed. This implies

$$(17) \quad f_{t-1}^i M_t = h_0 - k_0 v_{t-1} \quad \text{for all } i$$

and

$$(18) \quad F_{t-1}^i M_t = h_0 - k_0 v_{t-1} \quad \text{for all } i.$$

If we substitute (17) and (18) into (15) and (16), we can solve for P_t and Y_t as,

$$(19) \quad P_t = h_0 - k_0 v_{t-1} - \frac{a_0(1+a_1) + e_t - v_t}{(1+a_1)^2}$$

and

$$(20) \quad Y_t = a_0 + \frac{a_1 v_t + e_t}{1+a_1}$$

since $M_t = f_{t-1} M_t = F_{t-1} M_t$. Any changes in the money supply are fully absorbed into the price level with only the random component influencing output. Now consider the more interesting case of what happens when the new rule is announced. First, we will analyze the static case, but assume all n agents may be different.

Static Model with Different Beliefs

Let B^i be the probability agent i attaches to the new rule being followed. Therefore,

$$(21) \quad f_{t-1}^i M_t = B^i (h_1 - k_1 v_{t-1}) + (1-B^i) (h_0 - k_0 v_{t-1})$$

and

$$(22) \quad f_{t-1} M_t = \frac{1}{n} \sum_{i=1}^n [B^i (h_1 - k_1 v_{t-1}) + (1-B^i) (h_0 - k_0 v_{t-1})]$$

Similarly, let D^i be agents i 's forecast of the average forecast of the probability that the new rule is being followed. Thus,

$$(23) \quad F_{t-1}^i M_t = D^i (h_1 - k_1 v_{t-1}) + (1-D^i) (h_0 - k_0 v_{t-1})$$

and

$$(24) \quad F_{t-1} M_t = \frac{1}{n} \sum_{i=1}^n [D^i (h_1 - k_1 v_{t-1}) + (1-D^i) (h_0 - k_0 v_{t-1})]$$

Now we can substitute (22) and (24) into (15) and (16) to solve for P_t and Y_t . Also, let \bar{B} represent average B and \bar{D} represent average D .

Thus,

$$(25) \quad P_t = \frac{(1+a_1)(h_1-k_1v_{t-1})+a_1[\tilde{B}(h_1-k_1v_{t-1})+(1-\tilde{B})(h_0-k_0v_{t-1})]}{(1+a_1)^2} \\ + \frac{a_1^2[\tilde{D}(h_1-k_1v_{t-1})+(1-\tilde{D})(h_0-k_0v_{t-1})]}{(1+a_1)^2} - \frac{(e_t-v_t)}{1+a_1} - a_0$$

and

$$(26) \quad Y_t^s = a_0 + a_1 \frac{[h_1-k_1v_{t-1}-\tilde{B}(h_1-k_1v_{t-1})-(1-\tilde{B})(h_0-k_0v_{t-1})]}{(1+a_1)^2} \\ + a_1^2 \frac{[h_1-k_1v_{t-1}-\tilde{D}(h_1-k_1v_{t-1})-(1-\tilde{D})(h_0-k_0v_{t-1})]}{(1+a_1)^2} + \frac{a_1v_t + e_t}{1+a_1} .$$

What effect the change in the money supply rule will have on Y_t^s will depend on the values of \tilde{B} and \tilde{D} . Several general cases will now be analyzed.

Case 1: $\tilde{B} = \tilde{D} = 1$

In this case each individual completely believes that the Fed will follow the new rule that is announced and each person believes that everyone else also completely believes the Fed. Intuitively, it seems that the outcome should be the same as it would be under the rational expectations assumption. This is indeed what happens. We can easily see from looking at equation (26) that only random shocks and the natural rate will enter into the equation. Specifically,

$$(26') \quad Y_t^s = a_0 + \frac{a_1v_t + e_t}{1+a_1} .$$

So by making these specific assumptions, the same result is obtained as would be if we assumed rational expectations.

Case 2: $\tilde{B} = \tilde{D} = 0$

In this case the average forecast of average expectations is correct but average expectations themselves are not accurate. In other

words, on average, agents accurately predict what the other agents believe but, on average, they do not accurately predict what the Fed is going to do. In fact, no individual believes the Fed and no one believes that anyone else does either. In this case we get

$$(26'') \quad Y^S_t = a_0 + a_1 \left[\frac{h_1 - k_1 v_{t-1}}{1 + a_1} - \frac{(h_0 - k_0 v_{t-1})}{(1 + a_1)^2} \right] + \frac{a_1 v_t + e_t}{1 + a_1} .$$

This is just the opposite of Case 1. There we found the smallest output effect. Here, we have the largest effect, because everyone has been completely fooled. How large the actual output effect is, depends on how different the two rules are.

Case 3: $0 < \tilde{B} = \tilde{D} < 1$ or $0 < \tilde{B} < 1$ and $0 < \tilde{D} < 1$ and $\tilde{B} \neq \tilde{D}$

In this case, average expectations of the average forecast may not equal the actual average forecast. Also, agents may not accurately predict what the Fed is going to do. In this case, Y^S_t can be rewritten as follows;

$$(26^*) \quad Y^S_t = a_0 + a_1 \frac{\{ [h_1 - k_1 v_{t-1}] [(1 - \tilde{B}) + a_1 (1 - \tilde{D})] - [h_0 - k_0 v_{t-1}] [(1 - \tilde{B}) + a_1 (1 - \tilde{D})] \}}{(1 + a_1)^2} + \frac{a_1 v_t + e_t}{1 + a_1} .$$

We can see again that the larger are \tilde{B} and \tilde{D} , the smaller is the forecast error. The difference between the two does not matter. This is why there are not separate cases for $\tilde{B} = \tilde{D}$ and $\tilde{B} \neq \tilde{D}$. By looking at this, one may think it possible that agents have no incentive to accurately forecast average expectations. However, in this general setup it is difficult to talk about individual agents' motives since everything is expressed in terms of averages. The model will now be

arbitrarily restricted so that individual agents can be discussed more easily. The method by which expectations are updated each period after the original choice of prior parameters is also discussed.

Dynamic Model; Two Types of Agents

There will be two types of agents existing in this economy. The first type, Type I, hears the announcement and believes that the Fed will follow this new rule. The second type, Type II, also hears the announcement but they do not believe that the Fed will actually follow this rule. Instead, they believe the Fed will either continue following the old rule, or will soon revert back to it. The proportion of Type I vs. Type II will largely depend on how credible the Fed has been in the past and how believable the new rule is. The number of Type II agents will increase if Fed credibility decreases and/or if the new policy is not credible itself for some reason.

This division of agents, although arbitrary, can be justified in a couple of ways. First, we could assume that the Type I agents hear the announcement but Type II agents do not. So some agents are informed and others are uninformed. To retain the credibility problem however, we must assume that Type I agents do not completely believe the Fed. Otherwise, we just have an information problem. Alternatively, we could have Type II agents receiving a noisier signal about the new money rule than Type I agents do. This could cause Type II agents to be more uncertain about the motives of the Fed.

It is also not necessary to assume that Type I agents completely believe the Fed and Type II agents do not believe the Fed at all. The

update process will be the same if we allow them both to be somewhat skeptical of the Fed with the degree of skepticism differing between the two types. The priors will be different and Type Is will also have to update their beliefs. For simplicity, the original assumption of Type Is believing and Type IIs disbelieving will be retained.

Thus the agents are now divided into two groups, Type I and Type II, with the agents within a group being identical to the others in that group. Rewriting equation (10) under these assumptions gives

$$f_{t-1}^I P_t = \frac{a_1 f_{t-1}^I(r_{t-1}P_t) + f_{t-1}^I M_t - a_0}{1 + a_1} \quad (27)$$

$$f_{t-1}^{II} P_t = \frac{a_1 f_{t-1}^{II}(r_{t-1}P_t) + f_{t-1}^{II} M_t - a_0}{1 + a_1}$$

where I and II stand for Type I agents and Type II agents respectively. Now let $F_{t-1}^i P_t$ represent $f_{t-1}^i(r_{t-1}P_t)$ for $i=I,II$, where $F_{t-1}^i P_t$ is Type i 's average forecast of the average forecast of the price level. Thus (27) becomes

$$f_{t-1}^I P_t = \frac{a_1 F_{t-1}^I P_t + f_{t-1}^I M_t - a_0}{1 + a_1} \quad (28)$$

$$f_{t-1}^{II} P_t = \frac{a_1 F_{t-1}^{II} P_t + f_{t-1}^{II} M_t - a_0}{1 + a_1}$$

Therefore, we have

$$F_{t-1}^I P_t = \frac{a_1 F_{t-1}^I P_t + F_{t-1}^I M_t - a_0}{1 + a_1} \quad (29)$$

which implies,

$$F_{t-1}^I P_t = F_{t-1}^I M_t - a_0 \quad (30)$$

$$F_{t-1}^{II} P_t = F_{t-1}^{II} M_t - a_0$$

Substituting (30) into (28) gives

$$f_{t-1}^I P_t = \frac{a_1 F_{t-1}^I M_t + f_{t-1}^I M_t - a_0(1+a_1)}{1+a_1}$$

(31)

$$f_{t-1}^{II} P_t = \frac{a_1 F_{t-1}^{II} M_t + f_{t-1}^{II} M_t - a_0(1+a_1)}{1+a_1}$$

Now, to solve for the price level, take a weighted average of (31) to determine average expectations. Thus,

$$(32) \quad f_{t-1} P_t = \rho f_{t-1}^I P_t + (1-\rho) f_{t-1}^{II} P_t$$

where ρ is the proportion of Type 1 agents and $(1-\rho)$ is the proportion of Type 2 agents. Substituting (31) into (32) gives

$$(33) \quad f_{t-1} P_t = \frac{\rho[a_1 F_{t-1}^I M_t + f_{t-1}^I M_t]}{1+a_1} + \frac{(1-\rho)[a_1 F_{t-1}^{II} M_t + f_{t-1}^{II} M_t] - a_0(1+a_1)}{1+a_1}$$

Next, substitute (33) into (9) and solve for the price level as

$$(34) \quad P_t = \frac{(1+a_1)(M_t + v_t - e_t) + a_1\{\rho[a_1 F_{t-1}^I M_t + f_{t-1}^I M_t]\}}{(1+a_1)^2} + \frac{a_1\{(1-\rho)[a_1 F_{t-1}^{II} M_t + f_{t-1}^{II} M_t]\} - a_0(a_1^2 + 2a_1 + 1)}{(1+a_1)^2}$$

Substituting (34) and (33) into (1) we can solve for supply as

$$(35) \quad Y_t^s = a_0 + \frac{a_1\{M_t - (\rho f_{t-1}^I M_t + (1-\rho) f_{t-1}^{II} M_t)\}}{(1+a_1)^2} + \frac{a_1\{a_1[M_t - (\rho F_{t-1}^I M_t + (1-\rho) F_{t-1}^{II} M_t)]\}}{(1+a_1)^2} + \frac{a_1 v_t}{1+a_1} + \frac{e_t}{1+a_1} .$$

We now need to make more explicit assumptions about the behavior of each agent.

Recall that during the first period, $t-1$, Type I agents believe with certainty that the Fed is following the new rule and will have no

reason to alter this particular belief in the following periods of this regime. During the first period, Type II agents believe that the Fed is following the old rule. When the realizations of economic variables are different from what they expected, they will begin to attach less probability to the old rule being followed and more probability to the new rule. Thus, Type II agents are updating their beliefs each period.

A Bayesian update process will be used to show how each agent updates his beliefs about the Federal Reserve's policy. The type of prior that will be specified is referred to as a g-prior distribution which is a special form of the natural conjugate prior. More details and further references can be found in Judge, et. al., 1985.

The general class of natural conjugate priors are preferred because they combine easily with the likelihood function to give a posterior distribution of the same functional form. The g-prior in particular was chosen because it is more easily specified than the general natural conjugate prior. It also makes analyzing the results much easier. The equations for the point estimates and the updating formula explicitly show how the choice of the prior enters and how forecast errors effect the updating process. A general discussion of natural conjugate priors and the g-prior in particular, as well as the updating formulas is given in Appendix A.

Before incorporating this Bayesian process into the model, an error term needs to be added to the money supply equations so they are true regression equations. The Fed is assumed to have no control over

this term. Thus, the interpretations remain basically the same. The Fed tries to follow the rule as specified in equations (3) or (4). However, the actual money supply will be affected by random shocks. The specified model is,

$$(36) \quad M = \beta_0 + \beta_1 X + \tau$$

where M is a vector of observations on the money supply, X is a vector of observations on velocity and τ is a disturbance vector with the properties $\tau \sim N(0, \sigma^2 I)$. Agents will have to form prior beliefs about β_0 and β_1 as well as specifying how strongly they hold these beliefs, which is reflected in the choice of g_0 . Specifically, g_0 determines how much weight is given to the prior relative to sample information when determining the mean of the posterior distribution. Ordinarily, agents would also form beliefs about variances and covariances, but the choice of the g -prior for β and the diffuse prior for σ relieves them of this. (The diffuse prior assumes the agents have no information about the value of σ .)

At period t_{-1} when the announcement about the new rule is made, Type I agents will choose $\bar{\beta}_0 = h_1$ and $\bar{\beta}_1 = k_1$ while Type II agents will choose $\bar{\beta}_0 = h_0$ and $\bar{\beta}_1 = k_0$ as their priors for period t_0 . A single bar indicates this is a prior parameter. Later, a double bar will indicate a posterior parameter. When combining these priors with sample information (with the likelihood function) we only include the observations beginning with the first period under the new rule. Information from previous regimes is helpful only in determining

factors such as Federal Reserve credibility and these concerns are already reflected in the choice of the prior parameters.

When the Fed announces the new policy, the agents choose their priors for β as just described as well as a value for g_0 . If an agent is confident in his prior, he will choose a large value for g_0 so the prior is weighted more heavily than sample information when forming the posterior distribution. The generic forecasting equation is given as follows,

$$(37) \quad F_{t+1} M_{t+1} = \beta_0 M_{t+1} + \beta_1 M_{t+1} v_{t+1}$$

where

$$\beta_{t+1} = \beta_{t+1} + \frac{[(X'X)_{t+1} - 2(g_0+1)]^{-1} x_{t+1} (M_{t+1} - x'_{t+1} \beta_{t+1})}{1 + x'_{t+1} [(X'X)_{t+1} - 2(g_0+1)]^{-1} x_{t+1}}$$

and

$$[(X'X)_{t+1} - 2(g_0+1)]^{-1} = [(X'X)_{t+1} - 2(g_0+1)]^{-1} -$$

$$\frac{[(X'X)_{t+1} - 2(g_0+1)]^{-1} x_{t+1} x'_{t+1} [(X'X)_{t+1} - 2(g_0+1)]^{-1}}{1 + x'_{t+1} [(X'X)_{t+1} - 2(g_0+1)]^{-1} x_{t+1}} .$$

Also,

$$\beta_{t+1} = (h_1 \ k_1) \text{ for Type I agents}$$

and

$$\beta_{t+1} = (h_0 \ k_0) \text{ for Type II agents.}$$

This equation is referred to as generic because it applies to both Type I agents and Type II agents. The two types of agents both update in the same manner, they just start out with different priors. This is what causes the posterior mean (and distribution) for Type I agents to be different from that of Type II agents until the Type II's have learned the correct values for β as given in the model.

We can see from equation (37), that in general the larger the forecast error, the larger is the change in β . However, the amount by which β is updated also depends on the value of g_0 . If g_0 is large, then the amount of the update is less than it would be if g_0 was small for a given error. So, even though the error may be large, if this agent is very confident in his prior, he will not change his beliefs a great deal in any one period.

Now we must specify how agents update their beliefs about others' beliefs. Here, agents are only concerned with estimating average expectations because they know that it is the average that affects output. They must form beliefs about how many agents are Type I and how many are Type II as well as the beliefs about the Fed that each type holds.

Let Type I agents believe that θ is the proportion of other agents who are also Type I and $(1-\theta)$ is the proportion of other agents that are Type II. Type II agents believe γ is the proportion of agents who are Type I, so that $(1-\gamma)$ is the proportion believed to be Type II. With this information we can determine $F^I_{M_t}$ and $F^{II}_{M_t}$. For Type I agents we have,

$$F^I_{t-1}M_{t0} = \theta(h_1 - k_1 v_{t0-1}) + (1-\theta)(h_0 - k_0 v_{t0-1})$$

$$F^I_{t0}M_{t0+1} = \theta(h_1 - k_1 v_{t0}) + (1-\theta)[\theta(h_1 - k_1 v_{t0}) + (1-\theta)(h_0 - k_0 v_{t0})]$$

and

$$(38) F^I_{t0+s-1}M_{t0+s} = \theta \sum_{i=0}^s (1-\theta)^i (h_1 - k_1 v_{t0+s-1}) + (1-\theta)^{s+1} (h_0 - k_0 v_{t0+s-1})$$

For Type II agents,

$$F^{II}_{t-1}M_{t0} = \gamma(h_1 - k_1 v_{t0-1}) + (1-\gamma)(h_0 - k_0 v_{t0-1})$$

$$F^{II}_{t0} M_{t0+1} = \gamma(h_1 - k_1 v_{t0}) + (1-\gamma)[\gamma(h_1 - k_1 v_{t0}) + (1-\gamma)(h_0 - k_0 v_{t0})]$$

and

$$(39) F^{II}_{t0+s-1} M_{t0+s} = \gamma \sum_{i=0}^s (1-\gamma)^i (h_1 - k_1 v_{t0+s-1}) + (1-\gamma)^{s+1} (h_0 - k_0 v_{t0+s-1}).$$

We see that both types of agents update their forecasts each period by a constant proportion which depends on their initial choice for θ and γ . Type I agents believe that Type II agents begin to attach a higher probability to the new rule each period because they assume Type IIs are becoming more convinced the new policy is being and will continue to be followed. Similarly, as Type II agents change their own beliefs about policy, they assume other Type II agents are doing the same.

Now, substitute equations 4 and 37-39 into 35 to solve for supply as,

$$(40) Y^s_{t0+s} = a_0 + \frac{a_1}{(1+a_1)^2} \{h_1 - k_1 v_{t0+s-1} + \tau_{t0+s} - [\rho(\beta^I_{0t0+s-1} - \beta^I_{1t0+s-1} v_{t0+s-1})]\} \\ - \frac{a_1(1-\rho)(\beta^{II}_{0t0+s-1} - \beta^{II}_{1t0+s-1} v_{t0+s-1}) + a_1^2 [h_1 - k_1 v_{t0+s-1} + \tau_{t0+s}]}{(1+a_1)^2} \\ - \frac{a_1^2 \rho \{ \theta \sum_{i=0}^s (1-\theta)^i (h_1 - k_1 v_{t0+s-1}) + (1-\theta)^{s+1} (h_0 - k_0 v_{t0+s-1}) \}}{(1+a_1)^2} \\ - \frac{a_1^2 (1-\rho) \{ \gamma \sum_{i=0}^s (1-\gamma)^i (h_1 - k_1 v_{t0+s-1}) + (1-\gamma)^{s+1} (h_0 - k_0 v_{t0+s-1}) \}}{(1+a_1)^2} \\ + \frac{a_1}{1+a_1} v_{t0+s} + \frac{e_{t0+s}}{1+a_1}.$$

where β^I indicates the posterior mean for Type I agents and β^{II} indicates the posterior mean for Type II agents. Since it is assumed Type I agents immediately believe the new rule, they have no updating process to go through. Therefore, $\beta^I_{0t0+s-1}$ will be substituted for h_1 and $\beta^I_{1t0+s-1}$ for k_1 in the following case studies.

Some observations can be made from equation (40). First, recall that β^I_{t0+s-1} will be the vector $(h_1 \ k_1)$. This corresponds to the

actual rule that the Fed is following. Therefore, the higher is ρ , the closer we are to the rational expectations world where everyone knows and believes the new rule. Also, the larger are the values of θ and γ the closer we are to rational expectations. Possible outcomes of the model will now be discussed.

Case 1: $\rho=\theta=\gamma=1$

In this case, we would expect the rational expectations outcome since everyone believes the Fed's announcement and everyone believes everyone else believes the Fed. This is similar to the Case 1 in the earlier version of the model. In this case output can be written as follows,

$$(40') \quad Y^s_{t0+s} = a_0 + \frac{a_1(v_{t0+s} + \tau_{t0+s})}{1 + a_1} + \frac{e_{t0+s}}{1 + a_1} .$$

This is the outcome that would occur if we assumed rational expectations. Supply is just equal to the natural rate plus random shocks. So the Fed cannot influence output.

Case 2: $\rho=\gamma=\theta=0$

In this case, no one believes the Fed and no one believes that anyone else does either. Intuitively, this case should give the largest output effect possible. This is similar to the previous case 2 where the Fed did in fact have a very large output effect. Equation (40) can be written as follows,

$$(40'') \quad Y^s_{t0+s} = a_0 + a_1 \frac{\{(h_1 - k_1 v_{t0+s-1} + \tau_{t0+s}) - (\beta^{II} \pi_{t0+s-1} - \beta^{II} \pi_{t0+s-1} - v_{t0+s-1})\}}{(1 + a_1)^2} \\ + a_1^2 \frac{\{(h_1 - k_1 v_{t0+s-1} + \tau_{t0+s}) - (h_0 - k_0 v_{t0+s-1})\}}{(1 + a_1)^2} + \frac{a_1 v_{t0+s}}{1 + a_1} + \frac{e_{t0+s}}{1 + a_1} .$$

In the first period, the output effect depends on how different the two rules are since in the first period $\bar{\beta}_{t_0+s-1}$ is the vector $(h_0 \ k_0)$. In subsequent periods, the output effect will depend partly on how fast agents learn that the new rule is going to be stuck to. This will largely depend on their earlier choice of g_0 which, for Type IIs, measured how strongly these agents felt that the Fed was trying to fool them. Note here that expectations of average expectations are not being updated. The forecast itself is being updated, but since γ and θ started out as zero, they dropped out of the equation completely. Thus, money will remain nonneutral. Intuitively, this may happen because agents initially see that the economy behaves as if money is nonneutral, so even though each agent updates his own beliefs, he believes some other agents are not. As long as this continues, money will indeed be nonneutral and beliefs will be selffulfilling. See Appendix B for an explicitly Bayesian method that allows the updating of beliefs about average beliefs to occur, but requires more restrictions on the model.

Case 3: $0 < \rho < 1, 0 < \theta < 1, 0 < \gamma < 1$

Since this is the intermediate case, we would expect that the larger are the above parameters the smaller will be the output effect since we are getting closer to the rational expectations result. The interesting question to consider here is, given ρ , will the choice of $\theta = \gamma = \rho$ be optimal for the agents. The best way to determine this is with a numerical example. Let $\rho = \theta = \gamma = .5$. In this case (38) becomes,

(40''')

$$\begin{aligned}
Y^s_{t0+s} = & a_0 + a_1 \left\{ \frac{.5(h_1 - k_1 v_{t0+s-1}) + \tau_{t0+s} - .5(\beta^{II}_{0t0+s-1} - \beta^{II}_{1t0+s-1} v_{t0+s-1})}{(1 + a_1)^2} \right\} \\
& + a_1^2 \frac{[h_1 - k_1 v_{t0+s-1} + \tau_{t0+s} - .25 \sum_{i=0}^s .5^i (h_1 - k_1 v_{t0+s-1}) - (.5) \cdot 5^{s+1} (h_0 - k_0 v_{t0+s-1})]}{(1 + a_1)^2} \\
& - \{ a_1^2 \cdot 25 \sum_{i=0}^s .5^i (h_1 - k_1 v_{t0+s-1}) - (.5) \cdot 5^{s+1} (h_0 - k_0 v_{t0+s-1}) \} \frac{1}{(1 + a_1)^2} + \frac{a_1 v_{t0+s} + e_{t0+s}}{1 + a_1}
\end{aligned}$$

If $\rho = .5$ and $\theta = \gamma = 1$ we have

(39*)

$$\begin{aligned}
Y^s_{t0+s} = & a_0 + a_1 \left\{ \frac{.5(h_1 - k_1 v_{t0+s-1}) + \tau_{t0+s} + .5(\beta^{II}_{0t0+s-1} - \beta^{II}_{1t0+s-1} v_{t0+s-1})}{(1 + a_1)^2} \right\} \\
& + \frac{a_1^2 (h_1 - k_1 v_{t0+s-1} + \tau_{t0+s})}{(1 + a_1)^2} + \frac{a_1 v_{t0+s}}{1 + a_1} + \frac{e_{t0+s}}{1 + a_1} .
\end{aligned}$$

We can see that the total effect on output is less when $\rho = \theta = \gamma$. So agents have an incentive to accurately predict what average expectations are, instead of predicting what they would be if everyone believed the Fed. The better their predictions, the less the real effect from a change in Fed policy is.

Chapter Summary and Conclusions

The analysis in this chapter shows that the length and degree of nonneutrality of money largely depends on the proportion of people who are skeptical and the degree to which they are skeptical. First, for a given error, if there are many people who are only slightly skeptical, (a large number of Type IIs choosing a small value for g_0), there will be a large output effect which will die quickly. Since these agents do not have a high degree of confidence in their prior, they will become

convinced early on that the Fed is following the new rule and that other agents are also becoming convinced of this.

Second, if there are only a few agents who are very skeptical, (a small number of Type IIs who choose a large value for g_0), the initial output effect will be smaller than in the previous case, but will last longer. More observations will be required to convince these agents that the Fed is going to stick to the new rule.

Third, if there are a lot of agents who are very skeptical, (a large number of Type IIs who choose a large g_0), there will be a large output effect that will continue for a relatively long time.

The fourth possible outcome is the rational expectations one. Here, all agents completely believe the Fed so there are no Type II agents around. In this case, we should see a very small output effect, if any, which will die out very quickly.

Empirically, these results imply that after determining when a regime shift has occurred, we must determine, at least in general terms, how believable the new policy was to the public. If it is found that the policy was believed, we should see very little nonneutrality. However, if the public was skeptical of the Fed's actions or of the effectiveness of those actions, we should see some temporary nonneutrality of money.

There exists a suggestion of a way to quantify credibility, see Cukierman (1981). This work was not looked into, but should be considered in future extensions of the research at hand. No quantifiable measure of credibility has been used here. Instead, the

mood of the country is largely gauged from the Annual Reports of the Federal Reserve Board, which often summarize results of surveys conducted to determine inflationary expectations as well as other inflationary expectations indicators such as major wage settlements. If the Fed has announced an anti-inflationary policy, but the public continues expecting inflation, we can infer that the Fed's credibility is low.

Other factors may also influence the amount of nonneutrality observed. As we saw from equation (37), a large forecasting error will cause a large change between the prior and posterior means. But, since a large update is needed to get closer to the true mean, we cannot say whether large forecasting errors are more quickly alleviated than are smaller ones. But, there is an interesting aspect of this problem that was hinted at earlier. This is that large forecast errors can result from large unanticipated shocks. We are assuming the agents try to fit a statistical model to the data they observe, keeping in mind their priors. The model will not exactly fit the data because of things such as random disturbances in the economy or measurement error. So even though the agents may have estimated the coefficients of the money supply rule correctly, their predictions will often be incorrect because of the error term. Usually, this is too small to concern the agents. However, if there is some large disturbance in the economy, the error term will be larger. This may cause agents to believe they have not in fact estimated the coefficients of the regression equation correctly. Note that this implies Type I agents may also update their

expectations, which is something that was not allowed in this model. So agents may have estimated the coefficients of the money supply rule correctly, but not realizing this, may alter their estimates unnecessarily. Thus, large unanticipated shocks could cause nonneutrality of money to last longer than it otherwise would. So, when interpreting the data, we must keep in mind shocks such as the OPEC oil shocks or perhaps major agricultural shocks. These supply shocks cause inflation to increase and most likely cause the public to be more skeptical of the Fed's fight against inflation.

I have also shown that convergence to a world where money is neutral is not certain. In the main text, beliefs about others' beliefs are updated automatically. This artificially makes convergence more certain since when each agent believes that everyone believes the new rule, he should also believe the rule. (The exception was for Case 3 where no updating occurred.) However, when beliefs about others' beliefs are updated in a Bayesian fashion as shown in Appendix B, we may never reach this point. In the Bayesian world, the update process depends on the choice of prior and on the data observed. If an agent believes that other agents act as if money is not neutral he will act this way also. So money will not be neutral. When the agent observes this, his beliefs about others will be reinforced. So an equilibrium outcome where money is not neutral is possible in this model. This is similar to the result of Margaret Bray. Recall from the previous chapter that she found the economy may not always converge to a rational expectations world where money is neutral. Whether or not

this occurs is an empirical problem. How important credibility, or lack of it, is in Federal Reserve policy making is also an empirical problem which will now be discussed in the following chapter.

We should find that periods when Federal Reserve credibility was low, there was a large output response to a shock in the money supply. We would also expect to find that periods of significant change would give a large output response, at least initially, unless the Fed was completely believed immediately by everyone and all agents knew this. Some sort of legally binding action taken by the administration to support the Fed's policy could possibly cause this high level of credibility.

CHAPTER III
THE EMPIRICAL EVIDENCE

Introduction

This chapter empirically studies transitional nonneutrality of money to determine whether this transition process is significant enough to be included in rational expectations models. If short-run trade-offs occur very rarely or disappear very quickly, these models may not need to be complicated by including this process. However, if short-run trade-offs seem to be important, not including the transition process would mean that rational expectations models may not be appropriate for analyzing short-term monetary policy. In this case, a model such as the one presented in the previous chapter may be more appropriate.

The transition process is studied by first determining when regime shifts have occurred, then determining the time required for the transition to a new regime to be completed. The Federal Reserve is assumed to have been following a given policy rule long enough for the economy to be operating at the natural rate of output. This implies that the rule is known by agents, they have complete confidence that the rule is a credible one, and they believe that all other agents believe the rule is credible. If a new rule is then announced, agents will know the rule, but will not know if the Fed is being truthful in its announcement. Only credible announcements, (actual regime shifts),

are considered since this is the way the theoretical model is structured. An interesting extension would be to analyze periods when the Federal Reserve made an announcement that was not credible.

There is some debate about whether or not true regime shifts occur very often. Sims (1982) says that by definition they cannot, since if permanent changes in policy occurred very often they would not be permanent. A somewhat different view is taken here by loosely defining a regime shift as a change in policy that cannot be anticipated by agents. This definition is more inclusive than that of Sims'. For example, if there are two views on monetary policy, A and B, the actual outcome is usually some combination of these two polar views. If the policy outcomes have ranged over the entire scale from A to B, agents can see the entire policy rule and will be able to predict the polar outcomes as well as the intermediate ones. However, if the policy outcomes have all been close to A, agents will have no experience with which to predict the B result. So a jump from the A outcome to the B outcome will appear to agents as if a regime shift has occurred, although actually it is just a different outcome of the same policy.

Determining Regime Shifts

Numerous methods of determining when regime shifts occurred were used in the hope that the results of the different methods would reinforce each other. If they do not, the decision as to which result is the more accurate one will be based on statistical concepts such as goodness of fit. The first method was to divide the monthly M-1 series from January 1948 to March 1986 into subperiods corresponding to

different money growth rate variances. Several subperiods were chosen rather arbitrarily, then the Goldfeld-Quant test for heteroscedasticity was applied to the money growth rate series. Twelve observations were omitted between each subperiod, then an F-statistic was formed by taking the ratio of the sample variance of the second subperiod to the first and of the third subperiod to the second. Specifically, I test

$$H_0: \sigma_1^2 = \sigma_2^2 \quad H_0: \sigma_2^2 = \sigma_3^2$$

and

$$H_a: \sigma_1^2 < \sigma_2^2 \quad H_a: \sigma_2^2 < \sigma_3^2$$

where σ_i for $i=1$ to 3 is the variance of the corresponding subperiod. The results of three of the many different splits tried are reported below.

Subperiod	Variance	F-statistic	Significance Level
1: 1948,2-1960,12	.000007892137		
2: 1962,1-1975,12	.00001157705	1.47	.001
3: 1977,1-1986,3	.00002906107	2.5	.001

1: 1948,2-1966,12	.000008138948		
2: 1968,1-1978,12	.00001091220	1.34	.001
3: 1980,1-1986,3	.00003773095	3.46	.001

1: 1948,2-1960,12	.000007892137		
2: 1962,1-1978,12	.00001156537	1.47	.001
3: 1980,1-1986,3	.00003773095	3.26	.001

As can be seen, the F-statistics are highly significant in these three splits indicating that we can reject both null hypotheses and accept both alternative hypotheses, for each split. The implication is that the variance of the money growth rate is significantly different in each period.

Using this crude division as a guideline for detecting policy changes, finer divisions are searched for using Chow tests. This test will determine whether dividing the sample into two parts and performing regressions over each, gives a significantly better fit than would be obtained by fitting one regression over the entire sample.

First, changes in the way the money supply has depended on its own past are considered. The model used is

$$(41) \quad M_t = \beta_1 M_{t-1} + \beta_2 M_{t-2} + \varepsilon_t$$

where M is the rate of growth of the $M-1$ measure of money supply and the error term possesses the classical properties. (The variance of the error term can be considered constant within subperiods although it may change over different periods.) Using ordinary least squares (OLS), the model is estimated over two different subperiods that I believe may be separated by a regime shift. These two subperiods are then combined and the model is reestimated. Then, the appropriate F-statistic is formed for the Chow test. Although many of these Chow tests were performed, during this initial search, only dates after 1960 were tested. Later, shifts in the 1950s are also considered. Those dates where the Chow test revealed the most significant split are reported below.

Subperiods	F-statistic	Significance Level
1948,5-1967,2	4.41	.005
1967,2-1970,1		

1967,2-1970,1	0.07	.97
1970,1-1980,1		

1970,1-1980,1	2.13	.01
1980,1-1986,3		

Since not all of these F-statistics are highly significant, the results are not completely persuasive. The results for the 1970 shift in particular are not good. (The statistics are included here for comparison with the next test which does give a significant result for a 1970 shift.) In an attempt to get better results or to strengthen these findings, a policy response function is estimated over the same subperiods and more Chow tests are performed. The response function used was motivated by a similar one in Gordon and King (1982). The following equation is estimated using OLS;

$$(42) \quad M_t = \sum_{i=1}^{10} A_i M_{t-i} + \sum_{i=1}^{10} B_i P_{t-i} + \sum_{i=1}^{10} C_i Y_{t-i} + \varepsilon_t$$

where M is again the growth rate of M-1, P is the rate of inflation as measured by the rate of change in the Consumer Price Index (CPI) and Y is the rate of change of output as measured by the rate of change of the Industrial Production Index (IPI). The error term is assumed to possess the classical properties. All data are seasonally adjusted. The results of these tests are reported below.

Subperiods	F-statistic	Significance Level
1949,6-1967,2	18.28	.193E-09
1967,3-1970,1		
1967,3-1970,1	8.10	.751E-04
1970,2-1980,1		
1970,2-1980,1	16.28	.466E-07
1980,2-1985,2		

The F-statistics here are highly significant indicating that there were regime shifts in February 1967, January 1970 and in January 1980.

The final test for regime shifts was performed by including one more variable on the right hand side of the money supply equation. The interest rate, as measured by the 3-month Treasury Bill rate was added in light of the fact that the Fed at times put more emphasis on targeting the interest rate than on monetary aggregates. The new feedback rule is as follows;

$$(43) \quad M_t = \sum_{i=0}^{10} \beta_i M_{t-i} + \sum_{i=0}^{10} \nu_i Y_{t-i} + \sum_{i=0}^{10} \eta_i P_{t-i} + \sum_{i=0}^{10} \alpha_i B_{t-i} + \epsilon_i$$

where B is the 3-month Treasury Bill rate and the other variables are as defined before. This final equation fit the data better in terms of the \bar{R}^2 statistic and gave results that could be fit with policy announcements more easily. These results are reported below.

<u>Subperiods</u>	<u>F-Statistic</u>	<u>Significance Level</u>
1949,1 - 1957,1	19.29	.183 D-06
1957,2 - 1966,7		

1957,2 - 1966,7	47.11	.861 D-08
1966,8 - 1971,8		

1966,8 - 1971,8	23.48	.440 D-07
1971,9 - 1980,2		

1971,9 - 1980,2	29.96	.276 D-07
1980,3 - 1985,2		

As can be seen, the results here are slightly different than the results from the previous models. Since this final model fit the data better, its results should be more reliable than those of the previous ones. Thus, the shifts indicated above will be used.

Determining regime shifts by analyzing the data is preferred to the method of looking at policy announcements since I only want to analyze regime shifts that actually occurred. This also seems to be the usual method followed among researchers and my findings are similar to those of others. Carlson and Parkin (1975) found the inflation rate increased significantly in 1967. Gordon and King also find a shift occurring around early 1967. Geweke (1986) finds shifts in 1966 and 1970. (The fact that he finds a shift in 1970 instead of 1971 is not too troublesome since he used annual instead of monthly data, so in this sense our results are different by only one data point.)

After having identified regime shifts, policy announcements of the Fed were matched to these time periods. Some of the matches were quite

close, while others did not seem quite as obvious. Early 1957 was characterized by a general tightening of the money supply which had begun to some extent in 1955, to curb the inflation that had begun then and which accelerated quickly in 1956. Four times in 1955, the discount rate was increased for a total increase of 1%. The discount rate was increased twice in 1956 for a total of 1/2%. From August of 1956 to August of 1957, the discount rate remained at 3%. However, there were other measures taken around January 1957 to affect the money supply. First, in a meeting on Dec. 3, 1956, the Federal Reserve Board increased the maximum rates of interest allowable under Regulation Q, effective January 1, 1957. This was designed to prevent money being taken out of depository institutions and put into instruments earning higher rates of interest. The Fed also sold \$1.8 billion of U.S. securities from January to June. During this time, member bank borrowing increased from an average of \$400 million in Jan. to \$1 billion in June. This tightening that began in 1955 seems to have become quite a bit tighter in early 1957 which could account for the January, 1957 shift that was identified in the data.

Mid-1966 seemed to be characterized much the same as the mid-1950s. An expansion had gathered momentum in mid-1965 and accelerated more rapidly into 1966. The discount rate had been increased in December of 1965 and in February, 1966, the Fed decreased the rate at which it supplied reserves to member banks. In June, the reserve requirement was increased 1%, effective July 14 and 21. In July, the Fed granted temporary authority to the Federal Reserve banks

to provide emergency credit to nonmember depository-type institutions as much money was being withdrawn from these institutions to be put into instruments earning higher rates of interest. In August, the reserve requirement was increased again by 1%. So this regime change in July, 1966 also seemed to start earlier than was picked up in the data but, as before, the tightening seemed to become more acute around the time the shift was detected as being statistically significant.

The 1971 shift is probably the most obvious one to discuss. There was again an expansion taking place in early 1971 and inflation was on the rise. Surveys taken in the first half of the year indicated consumers were concerned about rising prices and fears of unemployment. The U.S. balance of payments situation had gotten worse which further increased concern about the effectiveness of policies being pursued to stop inflation. Price increases continued and large wage settlements were frequent. The discount rate was increased by 1/4% on July 15. Then, in the middle of August, President Nixon announced a 90-day freeze on prices and wages which was to be followed by a second phase characterized by more flexibility. Also, convertibility of the dollar into gold or other exchange reserve assets was suspended. A temporary surtax of up to 10% on dutiable imports was imposed but was rescinded on Dec. 18. There were also tax incentives created to stimulate spending. The interest rate plunged as inflationary expectations declined and fears subsided that the worsening balance of payments situation might lead to even tighter money in the U.S. So, this shift in August, 1971 seems to be quite obvious and much easier to detect.

The shift of early 1980 was also characterized by a tightening of the money supply after a period of high inflation. The tightening began to some extent in late 1978 and early 1979 with several increases in the discount rate. In February 1980, the discount rate was again increased by a full percentage point. Then, on March 14, President Carter announced an anti-inflationary program. The Federal Reserve Board was authorized to use temporary measures to restrain the growth of credit. Banks were asked to curtail their lending for speculative or purely financial purposes and the reserve requirement was increased. There was also a surcharge of 3% on frequent borrowings by institutions with deposits of \$500 million or more imposed. So, again the general tightening that had begun earlier was increased greatly at the time of the data detected shift in February, 1980.

The exchange rate was also put into the money supply equation to check the last two shifts that had previously been detected. These shifts were reinforced by doing this. The exchange rate was not included in the equation previously because the series did not begin until 1967 so earlier shifts could not be tested for. The results for the 1971 and 1980 tests were not reported since they merely reinforce the previous results.

Estimation Methodology

Having determined when regime shifts occurred, the transition process that takes place after each shift can be investigated. To do this, a relatively new statistical technique called Bayesian vector autoregression (BVAR) is used to estimate a system of equations. This

technique has been used by others in different applications. Litterman (1979), and Doan, Litterman and Sims (1983) use this technique for forecasting purposes. Their methodology is followed closely here although it is applied to a different problem. Since the BVAR method is a new one, the methodology will be discussed before turning to the particular estimation. Vector autoregressions (VARs) that do not include a prior will be discussed first.

VARs summarize the relationships between a specified set of variables at various lags. The current value of each variable is regressed on lagged values of all the variables in the system. There is one equation for each variable. So each equation has a current variable on the left hand side and lagged values of all variables on the right hand side. The length of the lags is the same for each variable. Although lag lengths are often chosen arbitrarily, a ratio test can be used to determine the best lag length. This test simply reveals whether a longer lag gives a significantly better fit than a shorter one would.

The vector autoregressive specification is capable of modeling a very wide class of covariance stationary stochastic processes. A stochastic process Y_t with mean μ_t and covariance $\sigma_{t,s}$ is covariance stationary if μ_t is independent of t and if $\sigma_{t,s}$ depends only on $t-s$. Wold's decomposition theorem shows that any covariance stationary stochastic process with mean zero can be written as follows,

$$(44) \quad X_t = \sum_{j=0}^{\infty} d_j \varepsilon_{t-j} + \eta_t$$

where the ε_t are the one period ahead prediction errors for X_t given all past observations, $E\varepsilon_t=0$, $E\varepsilon_t\varepsilon_s=0$ for $t\neq s$, $E\varepsilon_t^2=\sigma^2>0$ and $E\eta_t\varepsilon_s=0$ for all t and s ; and η_t is the deterministic part of X_t which can be predicted arbitrarily well by a linear function of only past values of X_t .

If the deterministic part of the X_t process is removed and if we normalize so that $d_0=1$, the moving average representation of X_t can be written as follows,

$$(45) \quad X_t = \sum_{s=0}^{\infty} d_s \varepsilon_{t-s} = \varepsilon_t + d_1 \varepsilon_{t-1} + d_2 \varepsilon_{t-2} + \dots$$

or

$$(46) \quad X_t = d(L)\varepsilon_t \quad \text{where} \quad d(L) = \sum_{j=0}^{\infty} d_j L^j$$

and L is the lag operator defined by $L^n X_t = X_{t-n}$ for $n = \dots, -2, -1, 0, 1, 2, \dots$ and $d(L)$ is a polynomial in the lag operator defined as above.

If $d(L)$ has an inverse that is one-sided in non-negative powers of L , an autoregressive representation can also be derived. A necessary and sufficient condition for this to hold is that the roots μ of $\sum_{j=0}^n d_j \mu^j = 0$ all lie outside the unit circle. Define $a(L)$ to be this inverse of $d(L)$. Operating on both sides of the above equation with $a(L)$ we get,

$$(47) \quad a(L)X_t = \varepsilon_t \quad \text{where} \quad a(L) = a_0 - \sum_{j=1}^{\infty} a_j L^j$$

Since we normalized to make $d_0=1$, a_0 will also equal 1. Thus, we have the autoregressive representation of X_t as,

$$(48) \quad X_t = a_1 X_{t-1} + a_2 X_{t-2} + \dots + \varepsilon_t.$$

This analysis of univariate stochastic processes can be generalized to vector stochastic processes. In the moving average representation, redefine X_t to be an $(m \times 1)$ vector of observations on m variables at time t and ε_t to be an $(m \times 1)$ vector of one step ahead prediction errors for X_t given $(X_{t-1}, X_{t-2}, \dots)$. Also, $E(\varepsilon_t) = 0$, $E(\varepsilon_t \varepsilon_s) = 0$ for $t \neq s$, and $\text{Cov}(\varepsilon_t) = M$. (This is the covariance of ε between equations at time t .) Also, redefine D_s to be an $(m \times n)$ matrix of moving average coefficients. As before, this moving average representation can also be represented as an autoregression if $D(L)$ is invertible.

In practice, vector autoregressive estimation works in the reverse order of the theory just discussed. First we assume an autoregressive representation exists. Then, the moving average coefficients are derived. These coefficients are important because they may be used to form impulse response functions. These functions show how a particular variable will respond in the future to a current unit shock in any variable in the system including itself.

The vector autoregressions without priors are not new and are attractive because they are inexpensive and easy to use. However, there is a major drawback, referred to as overparameterization. In an unrestricted VAR, the number of parameters is often quite large. Thus, there often is not enough data to obtain good estimates. The equations may appear to fit the data very well but will have high variances. Obtaining very long data series may not solve the problem either since data in the remote past may not be relevant for the current estimation. This problem can be partially overcome by using Bayesian methods to

place restrictions on the model. If a researcher desires to avoid overparameterization by non-Bayesian methods he must either shorten the lag length, completely remove some variables from the model, or both. From a Bayesian point of view, this is equivalent to saying he is certain the coefficient for that variable is zero.

The Bayesian method is well suited for studying the transition process, as it seems logical that expectations actually are updated in this fashion. Theoretical models dealing with the updating of expectations have often included a Bayesian learning mechanism. (For example, Lewis 1981, Turnovsky 1969, Taylor 1975, and Baxter 1985). People are assumed to have some prior beliefs about the variables they are trying to forecast. As the values of economic variables are revealed, people may wish to revise their previous beliefs. Bayes' rule describes how these priors are combined with the data to form new beliefs (called posterior beliefs). Bayes' rule may be written as,

$$(49) \quad f(\theta | X) = \frac{f(X|\theta)f(\theta)}{f(X)}$$

where $f(\theta | X)$ is the posterior density function for θ (the unknown parameters) given the observed data X . $f(X|\theta)$ is the density function for X given θ , referred to as the likelihood function for X . $f(\theta)$ is the prior density function for θ and $f(X)$ is the density function for X . This Bayesian approach is just providing a structure within which specification searches can be undertaken. In classical estimation, different models are estimated. The specification that is determined to be the best is the one that is reported. No mention is made of the previous models. By explicitly using Bayesian methods, the researcher

has a structure within which to make his specification searches. When the Bayesian approach is combined with the VAR approach the result is a very general and minimally restricted method of determining the relationships between economic variables.

Estimation Procedure

The model that was estimated is given by the following equations

$$(50) \quad M_t = \sum_{i=1}^{15} \xi_i M_{t-i} + \sum_{i=1}^{15} \delta_i P_{t-i} + \sum_{i=1}^{15} \upsilon_i B_{t-i} + \sum_{i=1}^{15} \omega_i Y_{t-i} + C_1$$

$$(51) \quad P_t = \sum_{i=1}^{15} \tau_i M_{t-i} + \sum_{i=1}^{15} \beta_i P_{t-i} + \sum_{i=1}^{15} \kappa_i B_{t-i} + \sum_{i=1}^{15} \alpha_i Y_{t-i} + C_2$$

$$(52) \quad B_t = \sum_{i=1}^{15} \eta_i M_{t-i} + \sum_{i=1}^{15} \phi_i P_{t-i} + \sum_{i=1}^{15} \sigma_i B_{t-i} + \sum_{i=1}^{15} \psi_i Y_{t-i} + C_3$$

$$(53) \quad Y_t = \sum_{i=1}^{15} \gamma_i M_{t-i} + \sum_{i=1}^{15} \tau_i P_{t-i} + \sum_{i=1}^{15} \rho_i B_{t-i} + \sum_{i=1}^{15} \theta_i Y_{t-i} + C_4$$

where P is again the rate of inflation as measured by the percentage rate of change of the CPI, Y is the percentage rate of change of output as measured by the percentage rate of change of the IPI, M is the percentage rate of change in the money supply as measured by the percentage rate of change in M-1, B is the percentage rate of change in the interest rate as measured by the percentage rate of change in the 3-month Treasury bill rate, and the C_i for $i=1$ to 4 are constants. The lag length was chosen after performing several ratio tests involving both longer and shorter lags.

The next step was to impose a prior on this system. The prior used specifies that the coefficients on all variables except the constant have independent normal distributions. A non-informative prior is placed on the constant, since there is no information as to its value. The mean of each of these normal distributions is zero except for the mean of the first own lag in each equation. This form is chosen because as pointed out in Doan, Litterman and Sims (1983), many economic variables seem to approximately follow a random walk or random walk with drift process. However, the means of the first own lags are not chosen to be one. Instead, an unrestricted univariate autoregression was estimated for all the parameters of the system. The coefficient on the first lag in these estimations was specified as the prior of the first own lag in the BVAR. This coefficient for the inflation rate was .36, for output it is .45, for the interest rate it is .43 and for the money supply it is .30 (all are for percentage rates of change). After these means are specified, the standard deviations of all the normally distributed variables must be specified. The standard deviations follow the formula

$$(54) \quad S(i,j,k) = \gamma f(i,j) g(k) s_j / s_i$$

where i is the equation, j is the variable and k is the lag. s_i is the standard error of a univariate autoregression on equation i . s_j then is the standard error of variable j . The standard errors are basically scaling factors to adjust for relative sizes of the variables.

The function $f(i,j)$ gives the tightness of the prior for variable j in equation i relative to the tightness on the own lags. (Obviously,

$f(i,i)$ must equal one.) In other words, this determines how influential the inflation rate is allowed to be in the output equation for instance and symmetrically for the other equations and parameters. The value is $f(i,j)=.9$ for $i \neq j$. This means that the output growth rate is given 1/10 as much weight in the inflation equation as inflation itself and symmetrically for the other equations. A smaller number would increase the tightness of the prior about the mean of zero for the lags of variable j in equation i . So in the output equation the coefficients on the inflation rate would be more tightly constrained to be close to zero. Since Doan, et. al. point out that many economic variables follow univariate autoregressions, a number smaller than .9 would seem to be more appropriate. However, for this study .9 is preferred, since we do not want to restrict the parameters from influencing each other during the transition period when we are no longer as sure of the relationships between the parameters. To determine how sensitive the results are to the specification of the parameter, the estimation was performed using several different values. The results were not very different for any of these values. However, using .9 gave a slightly better fit in terms of the \bar{R}^2 statistic.

The function $g(k)$ specifies the lag pattern. Specifically it specifies the amount of shrinkage of the standard deviation as lag length increases. A geometric decay pattern with the decay parameter equal to .9 was specified. Thus, $g(k)=.9^{k-1}$. As k gets larger, $g(k)$ will get smaller. So as lag length increases, the standard deviations get smaller. In other words, the longer the lag length, the less its

coefficient is allowed to vary from zero. This reflects the idea that variables in the remote past are less important for determining current and future variables than are variables in the recent past.

γ is the overall tightness of the prior. Specifically, it is the standard deviation of the prior on the first own lag in each equation. This can be seen by observing that the f and g functions are both equal to one for the first own lag in each equation. If we are confident about the specifications for the prior, we will want this number to be small so that the data will not be highly influential in determining the estimates. However, in this study a loose prior is desired because it is not known how the system will change during the transition. Therefore, $\gamma=1.0$ is chosen. A strong prior would mean choosing γ to be around .01 or smaller. Choosing this large value for γ causes the standard deviations of the other coefficients to be higher than seems to be implied by the f and g functions. So the sensitivity of the system to this parameter is also tested. Smaller values of γ were tried with no significant difference resulting. So the prior specifications that were chosen do not seem to be overly influential on the system.

Using a BVAR with the prior just described, the model is estimated with the "RATS" computer package. The method used is least squares equation by equation. This method is efficient for VARs without a prior since the same variables appear on the right hand side of each equation. However, in systems with a prior, the variables are not all the same. Seemingly unrelated regression techniques could improve the

estimation. This has not been tried here but may be worth doing at a later time.

Using the least squares method, the system is initially estimated for the period 1949,5 until 1956,10. (The series that were used begin in 1947 and end in 1985,2.) It is then reestimated after adding three more observations. The updating process continues until the transition is complete in a way to be made more clear below. The same process is then followed for the other periods. In this estimation process the data for the previous regime is not used to estimate the system for the current regime and the initial prior is imposed at the beginning of each new regime.

The coefficients obtained from the estimations are not easily interpreted. This is partly due to the large number of coefficients that must be looked at and partly because the coefficients generally vary from positive to negative. If the moving average representation of the system is derived, results can be obtained that are easier to interpret. Recall that impulse response functions may be derived from the moving average representation by shocking the error terms in the equations. These shocks will affect the dependent variables in future periods, with the responses generally decreasing over time as the effects dissipate. The impulse response functions needed to study credibility are the ones showing the response of output to a shock in the money supply. These are derived for all estimations for all four periods. By comparing the magnitude of the responses and the time it takes for them to die out, it can be determined how long the transition

process is taking and in some sense, how inaccurate expectations were. The assumption here being the larger the output response the more inaccurate the forecasts were. So, if the magnitude of the response begins decreasing over time and the response functions begin dying out (converging to zero) faster, it may be assumed that people are learning about the new policy rule and the expectations of others.

The above ideas will be clarified momentarily: there is a complication with this process that must be dealt with at this time. There may exist contemporaneous correlations among the error terms of different equations. Therefore, a shock to one error term may cause contemporaneous shocks to the error terms of the other equations. This implies that the response we observe may not be solely due to the shock we imposed. The usual solution to this problem is to use orthogonalized innovations. The method suggested by Sims (1980), and the one used here, is to triangularize the system. So a shock to the first equation in the system will affect all others, a shock to the second equation will affect all others except the first and so on. In cases where the correlations among residuals are large, the impulse response functions will be sensitive to the ordering of the system. However, if the correlations are small, we need not worry about the ordering.

To determine how sensitive the response functions derived here may be to a different ordering, the covariance matrix for the residuals of the four equations is derived. In all cases the covariances were very small. As a further check, the order of the equations is changed and

the system is reestimated. The results were not different from the previous estimations. Therefore, the shock to the money supply can be assumed to be the sole cause of the observed response in output. The impulse response functions are graphed in Appendix C. These graphs show the responses for 50 periods (50 months). There are two graphs for each shift. The first graph only shows selected response functions to highlight the differences between individual functions. The second graph includes all seven response functions and should be referred to when we perform the detailed analysis.

The coefficients of the different estimations are not shown because of the previously mentioned difficulty in interpreting them. Interest here centers on the change in the way the money supply influences real output. This requires determining how the money supply coefficients change with each estimation. Since there are fifteen of these coefficients in each equation and seven different estimations for each subperiod, this is a huge task. A simpler and equivalent method of determining coefficient change is by looking at the impulse response functions. If the graph shows the second response function to be closer to zero than the first, then the money supply coefficients are smaller in the second estimation than in the first. If the initial response of the second function is large but approaches zero quickly, then the coefficients on the recent money supply are large but decrease very quickly as lag length increases. So the behavior of the response functions illustrates the behavior of the estimated coefficients.

Each of the graphs shows that a shock to the money supply will cause an output response. However, the response differs among the four periods. How these responses differ should give some indication of how Federal Reserve credibility influences the effectiveness of a change in policy. The first set of graphs in Appendix C shows the impulse response functions for the January 1957 shift. The range of these responses is quite large compared to the responses to the other shifts. (The range is next to the largest behind the February, 1980 shift.) However, the responses for three and six months afterwards have settled down quite a bit indicating that perhaps the Fed has achieved more credibility very quickly. (Settling down means that the response functions have decreased in magnitude and are becoming closer together.) Nonneutrality seems to occur again from October 1957 through April 1958. Since this is at least ten months after the original shift, and after an initial settling down, it most likely occurs because of later shocks. So, in this case, the Fed seems to have achieved a great deal of credibility within three to six months.

This suggests that this shift may have been characterized by many people who are only slightly skeptical. This was the first result discussed in the conclusions to the previous chapter. Recall that this means there are a large number of Type II agents who choose a small value for g_0 . The large number of skeptical agents would cause the large output effect. The lack of strong skeptics causes the output effect to die quickly.

The second set of graphs show the response functions for the July 1966 shift. In this case, the range of the responses is not quite as large as before. However, six to nine months pass before the response functions begin settling down. The responses begin increasing in magnitude again around July, 1967 and October, 1967. Once again, this seems to be due to another alteration of policy which occurred later. So in this case, it took the Fed longer to achieve credibility, but the output response was not as drastic as before.

This suggests that this shift may have been characterized by a few agents who are very skeptical. Recall that this means there are a small number of Type II agents who choose a large value for g_0 . This implies a small output effect, due to the small number of Type IIs.

The third set of graphs give the responses for the August, 1971 shift. The range of the responses is relatively small and they begin settling down in about three to six months after the shift. As in the previous cases, the response functions begin to indicate an increased output response in May 1972 and continuing through November, 1972. This again will be considered as responses to another unanticipated shock. Thus, here, as in the 1957 shift, the Fed seems to achieve credibility in three to six months.

Here, the result seems to be close to that of what the rational expectations result would be. In this case, there are no Type II agents and everyone knows that there are not any. This implies the output effect will be either extremely small or nonexistent and will die very quickly.

The fourth set of graphs show responses for the February, 1980 shift. The range of the output responses is very large in this case and the response functions begin to settle down in about six months. The magnitude of the responses increases again in November of 1980 and February of 1981, which is again attributed to a later policy change.

This outcome suggests that this shift was characterized by many agents who are very skeptical. This means there are a lot of Type II agents who choose a large value for g_0 . The large number of Type IIs would cause the large output effect and the high level of skepticism would cause the effect to not die as quickly as in the first and third shifts.

The response functions for the four periods differ both in magnitude of response and in the length of time until the responses begin to settle down. Before analyzing these differences further, recall from Chapter II what influences the amount of nonneutrality. The influence that has been discussed was the number of people who were skeptical and the degree to which they were skeptical. Another was the difference in the old rule and the new rule. We can determine the relative difference in the two rules by looking at the significance levels of the F-tests that were performed to find the regime shifts. The causes of the different degrees of skepticism will also be discussed, but only very generally.

Of the four shifts detected, two of them, 1971,8 and 1980,2, involved Presidential announcements about the new policy. This would seem to give the policy change of the Fed more credibility. This

should be especially true in August, 1971 because of the price and wage freeze and the suspension of convertibility of the dollar. The March 1980 announcement was not as stringent as it just gave the Fed more power to restrict the growth of money and credit. Thus, we would expect to find convergence (settling down of the response functions) occurring quicker in these two cases than in the other two and faster for 1971 than for 1980. This is what was found with one qualification. In 1971, the output responses begin settling down in three months, whereas a full six months were required in 1980. The magnitude of the response in 1980 was larger than that of 1971 reflecting the more significant shift of 1980.

The shifts of 1957 and 1966 are somewhat more difficult to analyze. Since there was no help from the White House in either of these years, we would expect that the Fed would have a more difficult time achieving credibility. This is true in the 1966 case, where convergence did not occur until six to nine months after the shift was detected. However, the 1957 policy change seems to be believed after only three to six months. To determine the cause of this, one needs to go back some time before the data determined shift occurred to see what the Fed had been doing.

The 1966 shift was the most significant one found. The shift from a policy of relatively easy money to one of tight money began seven months prior to the time it was detected in the data. Thus, the change had begun seven months before it reached its highest significance level. In 1957, the shift had begun nearly 2 years prior

to the data detected shift. This shift was also the least significant one found. From the very beginning of the change in regime until convergence is obtained, the 1966 shift converged faster than did the 1957 one. The range of output responses was also less for 1966 than for 1957. This would seem to imply that a large, significant shift is preferable when the Fed is decreasing the money supply than is a shift that begins more subtly and does not become significant for a couple of years. If, however, the Fed is increasing the money supply, it may want to be more subtle about the policy change so as to get a larger output response. This may actually be what has happened in the past. All of the regime shifts that were detected in the data occurred when money was becoming tighter and not easier. But, this could be unintentional on the Fed's part. The public may well believe the Fed when it says it will increase the money supply but needs some convincing when the Fed tries to slow the money supply growth. Thus, less drastic measures are required for trying to stimulate the economy than for trying to slow it. If this is true, unexpectedly stimulating the economy to achieve a temporary increase in real output has extremely high costs later on. More inflationary expectations are built into the economy and even more drastic measures are required to slow inflation.

The data determined shifts do not coincide with the beginning of the change in policy. Instead, they coincide with the period when that policy was the most significantly different from the previous policy. So, to more completely analyze these shifts, impulse response functions

were also derived for two years before the data determined shifts occurred. These graphs are shown in Appendix D.

The responses prior to the 1957 shift are not as large in magnitude as they are for the shift period. This is most likely because the old policy and the new are not greatly different at that point. Since the responses continue getting larger as policy gets tighter, it seems the public is remaining skeptical. To gain credibility, the Fed had to continue tightening the money supply over a two year period.

The same general scenario holds for the periods prior to the 1966 shift and the 1980 shift. However, the period prior to the 1971 shift is different. In this case, the range of responses is larger for the period prior to the shift than it is for the actual shift periods. The largest output response occurred in February, 1971. This could be because of the Fed's rather odd actions taken at this time. Surveys early in the year indicated that the public was concerned about rising prices as well as the effectiveness of policies designed to curb inflation. In this climate, the public would be expecting the Fed to tighten the money supply to try to reduce inflation. Instead, the discount rate was decreased by 1/2% in January and 1/4% in February. This may account for the large output effect seen in February. In July, the discount rate was raised again and in August the wage-price freeze took effect. If we discount the February 1971 response, we get the same type of scenario as for the previous regime shifts. The response functions gradually increase in magnitude after the new policy

is begun until a significant move is made which gives the Fed some credibility.

Chapter Summary and Conclusions

In this chapter, it has been shown, using Bayesian Vector Autoregressive techniques, that Federal Reserve credibility can influence the effectiveness of policy. If the Fed is not believed by the public, there is temporary nonneutrality of money. The length of this period depends on how bad the Fed's credibility is. However, in all the cases analyzed, at least three months passed before the economy came close to returning to the neutrality result. Even in the 1971 shift, which was close to the rational expectations result, there was an output effect for three months. So, it seems that we should include a transition period in rational expectations models. Otherwise, these models must be considered long-run models which are not useful for making short-run policy analysis.

In addition, it was found that it is quite difficult for the Fed to obtain credibility on its own very quickly. The two shifts that occurred without White House aid seemed to be the least believed. The Fed required more time to convince the public of its intentions than it did when the White House intervened. This would suggest that when the Fed wants to tighten the money supply, an announcement of an anti-inflation policy by the president may help. If this causes the public to have more faith in the Fed's intentions, it will mean the transition period (a recession in this case) will be shorter. However, there is a limit to what the administration can do. The wage-price

freeze of the Nixon era helped for a while but inflation became worse later on as the freeze was lifted and the public began expecting inflation again. So it seems that perhaps an ongoing, reliable policy by both the Federal Reserve and the administration is needed to control inflationary expectations and for the Fed to regain credibility.

CHAPTER IV

SUMMARY AND CONCLUSIONS

This dissertation has studied Central Bank credibility and its relation to short-run Phillips curves. A theoretical model was developed that showed lack of Federal Reserve credibility could cause a temporary trade-off between inflation and unemployment. Intuitively, this trade-off occurs because if agents are skeptical of the Fed's intentions to fight inflation, they will continue asking for money wage increases so that their real wage will not be eroded by the anticipated inflation. Employers will not be willing to give these increases and still retain all their workers for two possible reasons. First, if they believe inflation will continue, they will interpret the decrease in demand for their product that results from the decrease in the growth rate of the money supply, as a relative one and will want to decrease production. This means decreasing employment and decreasing, or at least not increasing money wages. So some employees will be laid off and some may quit, expecting to find better wages elsewhere. Second, if employers believe the Fed will successfully bring down inflation, they will not want to grant money wage increases because they believe this is also a real wage increase. So, again some employees will leave, expecting higher wages elsewhere.

The magnitude of the trade-off depends on the proportion of skeptical agents. The more skeptical agents there are, the more will

leave their current job, looking for better wages elsewhere. There will also be more workers laid off since more employers will be skeptical as well. The significance of the change in policy also affects the magnitude of the trade-off. The greater the decrease in the growth rate of the money supply for example, the greater the effect will be on employers and employees. How long this trade-off continues depends on how skeptical the agents are. The more skeptical they are, the longer it takes for them to be convinced that the Fed is serious about decreasing the rate of inflation.

It was also found that since beliefs are endogenous, the economy could reach an equilibrium where money remains nonneutral. This occurs if individuals believe other agents believe money is nonneutral. This individual then believes money is not neutral although he knows it should be. His beliefs are fulfilled which reinforces his beliefs about other agents next period.

An empirical model was then developed that showed lack of Federal Reserve credibility could cause these short-run trade-offs to occur. The length and magnitude of these trade-offs differed for the different periods studied. These differences were attributed to a different proportion of skeptical agents in each period and different degrees of skepticism they held each period. However, no evidence of an equilibrium where money is nonneutral was found. Although money was found to be nonneutral quite often, the economy seemed to return to neutrality, at least temporarily. The erratic nature of Federal Reserve policy during many of the periods studied caused difficulties

when trying to determine how likely it would be for nonneutrality to exist for a long period of time under a stable monetary policy.

Credibility, or lack of it, of the Federal Reserve seems to be related to the significance of the Fed's actions and to the actions of the U.S. president. If the Fed's actions are very drastic, credibility seems to be greater and to occur more quickly. If the president publicly supports the Fed's policy and implements policy of its own to achieve the same objective, credibility also seems to be higher. This may occur because the public feels that even if the Fed was not serious about decreasing inflation before, it will now be under more pressure from the administration to do this. So agents are more likely to have faith in the Fed's change of policy.

If the Fed wants to decrease inflation without large and prolonged negative output effects, it should take very strong actions immediately to gain credibility. The administration could help by announcing its own anti-inflationary measures and conduct its fiscal policy in such a manner as to reinforce these measures. It was also suggested that the costs of expansionary policy seem to be greater than the temporary benefits gained, because inflationary expectations become more deeply embedded causing the public to be more skeptical of the Fed's anti-inflationary measures later on.

The importance of Central Bank credibility in influencing the effectiveness of monetary policy is a subject for continuing research. A quantifiable measure of credibility may help in future studies of this topic. We also need to further investigate the costs and benefits

of expansionary policy. Another area of interest is whether the Fed can effectively control its credibility. If so, they may be able to reduce the negative output effects that can occur when contractionary monetary policy is used. Of related interest is if the Fed can create uncertainty about its objectives. If so, is there an optimal level of uncertainty that will give the desired output effect when the Fed is using expansionary policy? Can the Fed actually exploit any of these short-run trade-offs in any way?

Although there are many questions that have not be answered here, this dissertation has shown that temporary trade-offs do occur frequently and can continue for fairly long periods. This suggests that these trade-offs should be taken into account when analyzing the short-run affects of monetary policy.

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APPENDIX A

DISCUSSION OF NATURAL CONJUGATE PRIORS

The general form of the natural conjugate prior for the general linear model $Y=X\beta+e$ where y is a $(Tx1)$ vector of observations on a dependent variable, X is a (TxK) matrix of observations on K explanatory variables, e is a $(Tx1)$ disturbance vector with the properties $e \sim N(0, \sigma^2 I)$, and β and σ are unknown parameters about which we seek information, is as follows;

$$(1A) \quad g(\beta|\sigma) = (2\pi)^{-K/2} \sigma^{-K} |A|^{1/2} \exp[-1/2 \sigma^2 (\beta - \bar{\beta})' A (\beta - \bar{\beta})]$$

and

$$(2A) \quad g(\sigma) = \frac{2}{\Gamma(\bar{v}/2)} \left(\frac{\bar{v}s^2}{2}\right)^{\bar{v}/2} \frac{1}{\sigma^{\bar{v}+1}} \exp\left(-\frac{\bar{v}s^2}{2\sigma^2}\right)$$

where the parameters $\bar{\beta}$, A , \bar{v} , and \bar{s}^2 depend on prior information. In particular,

$$\begin{aligned} E[\beta|\sigma] &= E[\beta] = \bar{\beta} & \text{cov}[\beta|\sigma] &= \sigma^2 A^{-1} \\ E[\sigma] &= \frac{\Gamma[(\bar{v}-1)/2]}{\Gamma(\bar{v}/2)} \left(\frac{\bar{v}}{2}\right)^{1/2} \frac{1}{\bar{s}} & E[\sigma^2] &= \frac{\bar{v}s^2}{\bar{v}-2} \end{aligned}$$

and the prior mode for σ is

$$\text{mode}(\sigma) = \left(\frac{\bar{v}}{\bar{v}+1}\right)^{1/2} \frac{1}{\bar{s}}$$

A single bar over a symbol means it is a prior parameter. Later, a double bar will mean the symbol is a posterior parameter.

We can combine equations (1A) and (2A) and omit the constants of proportionality to get the normal-gamma joint prior density

$$(3A) \quad g(\beta, \sigma) \propto \sigma^{-K-\bar{v}-1} \exp\{-1/2 \sigma^2 [\bar{v}s^2 + (\bar{\beta} - \beta)' A (\bar{\beta} - \beta)]\}$$

where the symbol \propto means proportional to. If we combine (3A) with the likelihood function, we get a posterior distribution which is also of the normal-gamma type. Instead of writing out the distribution, I

will just show the posterior mean, standard deviation, and variance since these are the statistics we are most interested in. The posterior mean is

$$(4A) \quad \bar{\beta} = (A + X'X)^{-1}(A\bar{\beta} + X'Xb)$$

where b is the least squares estimate of β .

For σ and σ^2 , the posterior means are as follows,

$$(5A) \quad E[\sigma|y] = \frac{\Gamma[(\bar{v}-1)/2]}{\Gamma(\bar{v}/2)} \left(\frac{\bar{v}}{2}\right)^{1/2} \bar{s}$$

and

$$(6A) \quad E[\sigma^2|y] = \frac{\bar{v}\bar{s}^2}{\bar{v}-2}$$

The recursive updating formula shows how to update the posterior distributions after more sample information becomes available. This is given by,

$$(7A) \quad \bar{\beta}_{T+1} = \bar{\beta}_T + \frac{S_T x_{T+1} (y_{T+1} - x'_{T+1} \bar{\beta}_T)}{1 + x'_{T+1} S_T x_{T+1}}$$

and

$$(8A) \quad S_{T+1} = S_T - \frac{S_T x_{T+1} x'_{T+1} S_T}{1 + x'_{T+1} S_T x_{T+1}}$$

where $\bar{\beta}_T$ is the posterior mean for β calculated from T observations, and S_T is the matrix $(W'W)^{-1}$ based on T observations. The additional observation is given by (y_{T+1}, x'_{T+1}) . The $(W'W)$ matrix is defined as follows,

$$W = \begin{matrix} A^{1/2} \\ X \end{matrix}$$

where $A^{1/2}$ is the symmetric matrix such that $A = A^{1/2} A^{1/2}$.

The g -prior distribution makes an explicit assumption about the A matrix which simplifies the choice of prior parameters. In particular $A = g_0 X'X$ where g_0 is a scalar to be specified as a prior parameter. If we use a diffuse prior for σ so that $g(\sigma) \propto \sigma^{-1}$ we get a posterior density as follows,

$$(9A) \quad g(\beta, \sigma | y) \propto \sigma^{-K} \exp\left[-\frac{1+g_0}{2\sigma^2} (\beta - \bar{\beta})' X'X (\beta - \bar{\beta})\right] \cdot \sigma^{-(v+1)} \exp\left(-\frac{vs^2}{2\sigma^2}\right)$$

where $v = T$

$$v s^2 = (Y - X\bar{\beta})'(Y - X\bar{\beta}) + g_0 (\bar{\beta} - \bar{\beta})' X'X (\bar{\beta} - \bar{\beta})$$

$$\text{and } \bar{\beta} = \frac{g_0 \bar{\beta} + b}{1 + g_0} .$$

The posterior mean then is just a simple weighted average of the prior mean $\bar{\beta}$ and the least squares estimate b , with the relative weights determined by g_0 .

The recursive updating formula for the g -prior can be obtained by substituting in $g_0 X'X$ for A in the W matrix. The result is,

$$(10A) \quad \bar{\beta}_{T+1} = \bar{\beta}_T + \frac{[(X'X)_T(g_0+1)]^{-1} x_{T+1} (y_{T+1} - x'_{T+1} \bar{\beta}_T)}{1 + x'_{T+1} [(X'X)_T(g_0+1)]^{-1} x_{T+1}}$$

and

$$(11A) \quad S_{T+1} \equiv [X'X_{T+1}(g_0+1)]^{-1} = [X'X_T(g_0+1)]^{-1} - \frac{[(X'X)_T(g_0+1)]^{-1} x_{T+1} x'_{T+1} [(X'X)_T(g_0+1)]^{-1}}{1 + x'_{T+1} [(X'X)_T(g_0+1)]^{-1} x_{T+1}} .$$

APPENDIX B

BAYESIAN UPDATE OF EXPECTATIONS ABOUT AVERAGE EXPECTATIONS

Agents can update their beliefs about average expectations by using Bayesian methods. Agents can observe supply and can write down equation (35) of the main text.

Rewrite (35) in the following way,

$$(1B) \quad Y^s_t = a_0 + \frac{(a_1 + a_1^2)M_t}{(1+a_1)^2} - \frac{\rho a_1 f^I_{t-1} M_t}{(1+a_1)^2} + \frac{(1-\rho)a_1 f^{II}_{t-1} M_t}{(1+a_1)^2} \\ - \frac{a_1^2(\rho f^I_{t-1} M_t + (1-\rho)f^{II}_{t-1} M_t)}{(1+a_1)^2} + \frac{a_1 v_t + e_t}{1+a_1}$$

where all of the variables are as defined previously.

Since the economy had been operating at the natural rate of output before the new money supply rule was announced, it will be assumed that agents were able to determine the value of a_0 (which corresponds to the natural rate of output). It will also be assumed they have been able to determine the value of a_1 . From (20) (in the main text) it can be seen that although agents cannot see a_1 directly they can see a combination of a_1 and the error terms. So although the assumption is somewhat bothersome, it seems safe to assume that agents have at least some idea of what a_1 is. We will also let agents see ρ , the proportion of Type I agents in the economy. $f^I_{t-1} M_t$ is also observable, since all agents know that Type I agents initially believe the Fed. All agents observe that the Fed is following the new rule, so they will have no reason to believe that Type I's change their beliefs about the money supply.

Now, divide the average belief about average expectations terms into their observable and unobservable components. Since agents know ρ , the following equations hold,

$$(2B) \quad F^I_{t-1}M_t = \rho(f^I_{t-1}M_t) + (1-\rho)(f^{II}_{t-1}M_t)$$

and

$$(3B) \quad F^{II}_{t-1}M_t = \rho(f^I_{t-1}M_t) + (1-\rho)(f^{II}_{t-1}M_t).$$

The first term in both of these equations is known but the second must be estimated. (Actually, $f^{II}_{t-1}M_t$ in equation (3B) is known. Since all Type II agents are the same, Type IIs will know what other Type IIs are doing. However, for now put this term in with the unknowns. But, by separating the equations for F^I and F^{II} , we implicitly let the beliefs Type Is hold about Type IIs be different from the beliefs Type IIs hold about other Type IIs.)

Rewrite equation (1B) using equations (2B) and (3B) to get,

$$(4B) \quad Y^s_t = Z_0 + Z_1X + \delta$$

where

$$Z_0 \equiv a_0 + \frac{(a_1+a_1^2)M_t}{(1+a_1)^2} - \frac{F^I_{t-1}M_t(\rho a_1+a_1^2\rho^2+(1-\rho)\rho)}{(1+a_1)^2}$$

$$Z_1 \equiv F^{II}_{t-1}M_t$$

$$X \equiv \frac{a_1(1-\rho)[1-a_1\rho-a_1(1-\rho)]}{(1+a_1)^2}$$

and

$$\delta \equiv \frac{a_1 v_t + e_t}{1 + a_1}.$$

Y^s_t is a vector of observations on output, X is a vector of observations on the ρ s and a_1 s and δ is a disturbance vector with the properties $\delta \sim N(0, \lambda^2)$. Since Z_0 is completely known, its prior will be the definition of Z_0 as given above. The prior on Z_1 for period t_0 will be $M_t = h_0 - k_0 v_{t-1}$ for both types of agents. After the initial period, the new priors (posteriors) may differ because the agents have

different ideas about the speed with which Type IIs are updating their beliefs. We will again use a g-prior distribution for Z and a diffuse prior for λ .

The forecasting equation is as follows,

$$(5B) \quad F_{t0+s-1} Y^s_{t0+s} = \bar{Z}_{0t0+s-1} + \bar{Z}_{1t0+s-1} X$$

where

$$\bar{Z}_{t0+s-1} = \bar{Z}_{t0+s-2} + \frac{[(X'X)_{t0+s-2}(g_0'+1)]^{-1} x_{t0+s-1} (Y^s_{t0+s-1} - x'_{t0+s-1} \bar{Z}_{t0+s-2})}{1 + x'_{t0+s-1} [(X'X)_{t0+s-2}(g_0'+1)]^{-1} x_{t0+s-1}}$$

and

$$[(X'X)_{t0+s-1}(g_0'+1)]^{-1} = [(X'X)_{t0+s-2}(g_0'+1)]^{-1} - \frac{[(X'X)_{t0+s-2}(g_0'+1)]^{-1} x_{t0+s-1} x'_{t0+s-1} [(X'X)_{t0+s-2}(g_0'+1)]^{-1}}{1 + x'_{t0+s-1} [(X'X)_{t0+s-2}(g_0'+1)]^{-1} x_{t0+s-1}} .$$

Once again, the g_0' term is a measure of how confident the agent is in his prior relative to sample information. (The prime on g_0' is used to distinguish this g_0' from the g_0 agents chose when forecasting the money supply rule earlier.) If an agent strongly believes that Type II agents will update their beliefs slowly, he will choose a large value for g_0' . In this case, even if the observations on supply indicate that Type 2s are updating quickly, this agent will not change his forecast as readily as he would if he had chosen a lower value for g_0' .

We can now rework Case 2 using equation (5B).

Case 2B: $\rho=0$

Here, no agent believes the Fed and since ρ is known, all agents know that no one believes the Fed. However, updating will still occur

as the agents become convinced the Fed will stick to its new policy. As this occurs, expectations about average expectations will also be updated. The new equation for output becomes,

(37B)

$$\begin{aligned}
 Y^s_{t0+s} = & a_0 + \frac{a_1}{(1+a_1)^2} \{m_1 - g_1 v_{t0+s-1} + \tau_{t0+s} - a_1 (\beta^{II}_{0t0+s-1} - \beta^{II}_{1t0+s-1} v_{t0+s-1})\} \\
 & + \frac{a_1^2}{(1+a_1)^2} \{m_1 - g_1 v_{t0+s-1} + \tau_{t0+s} - (Z^{II}_{0t0+s-1} + Z^{II}_{1t0+s-1} x_{t0+s-1})\} \\
 & + \frac{a_1}{1+a_1} v_{t0+s} + \frac{e_{t0+s}}{1+a_1} .
 \end{aligned}$$

Money will eventually become neutral if β converges to the true money supply rule and if Z converges to the true value of average expectations. The time this takes will depend on the choice of g_0 and g_0' as well as how different the two rules are.

APPENDIX C
IMPULSE RESPONSE FUNCTIONS

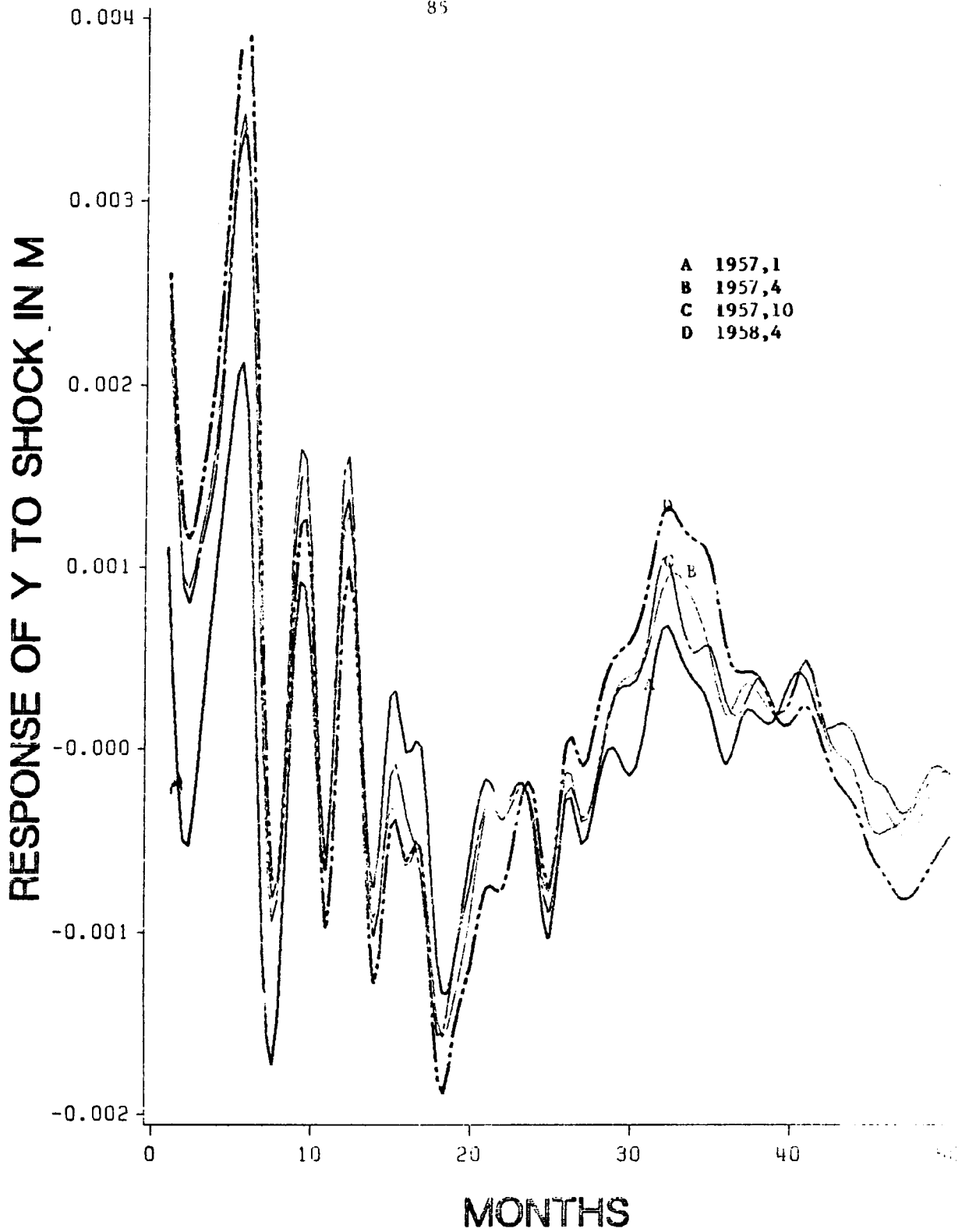


FIGURE 1

J.A.N. 1957 SHIFT

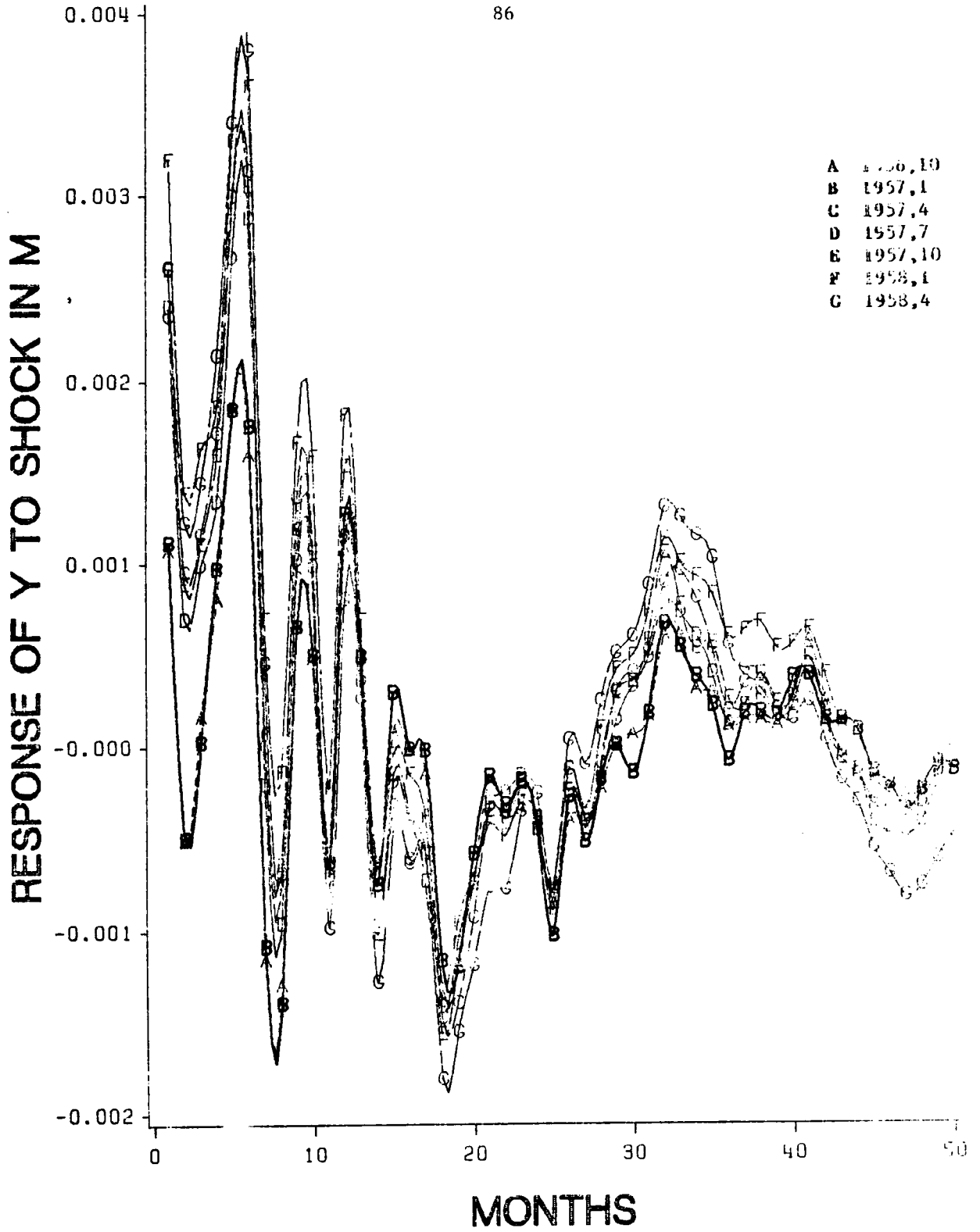


FIGURE 2

JAN. 1957 SHIFT

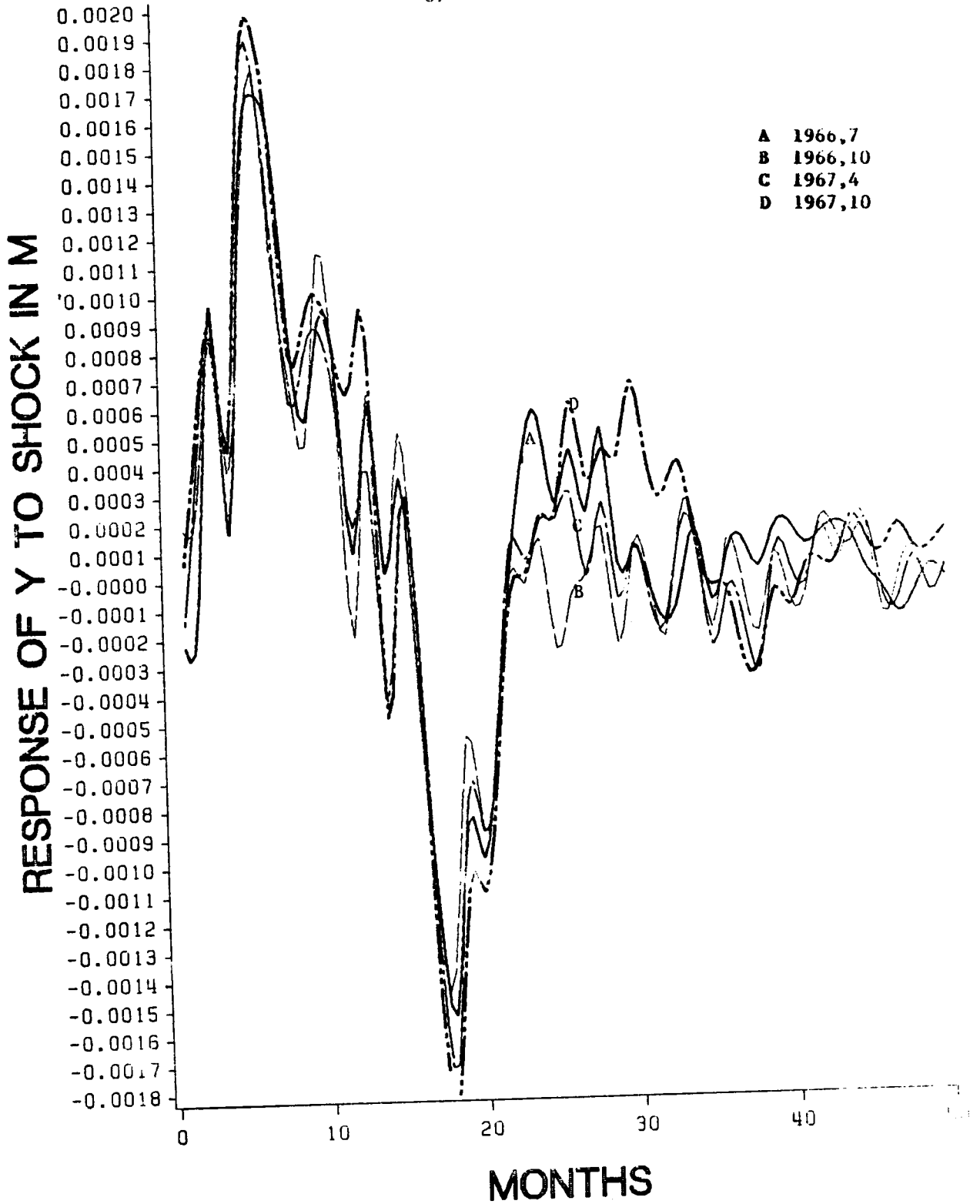


FIGURE 3

JULY 1966 SHIFT

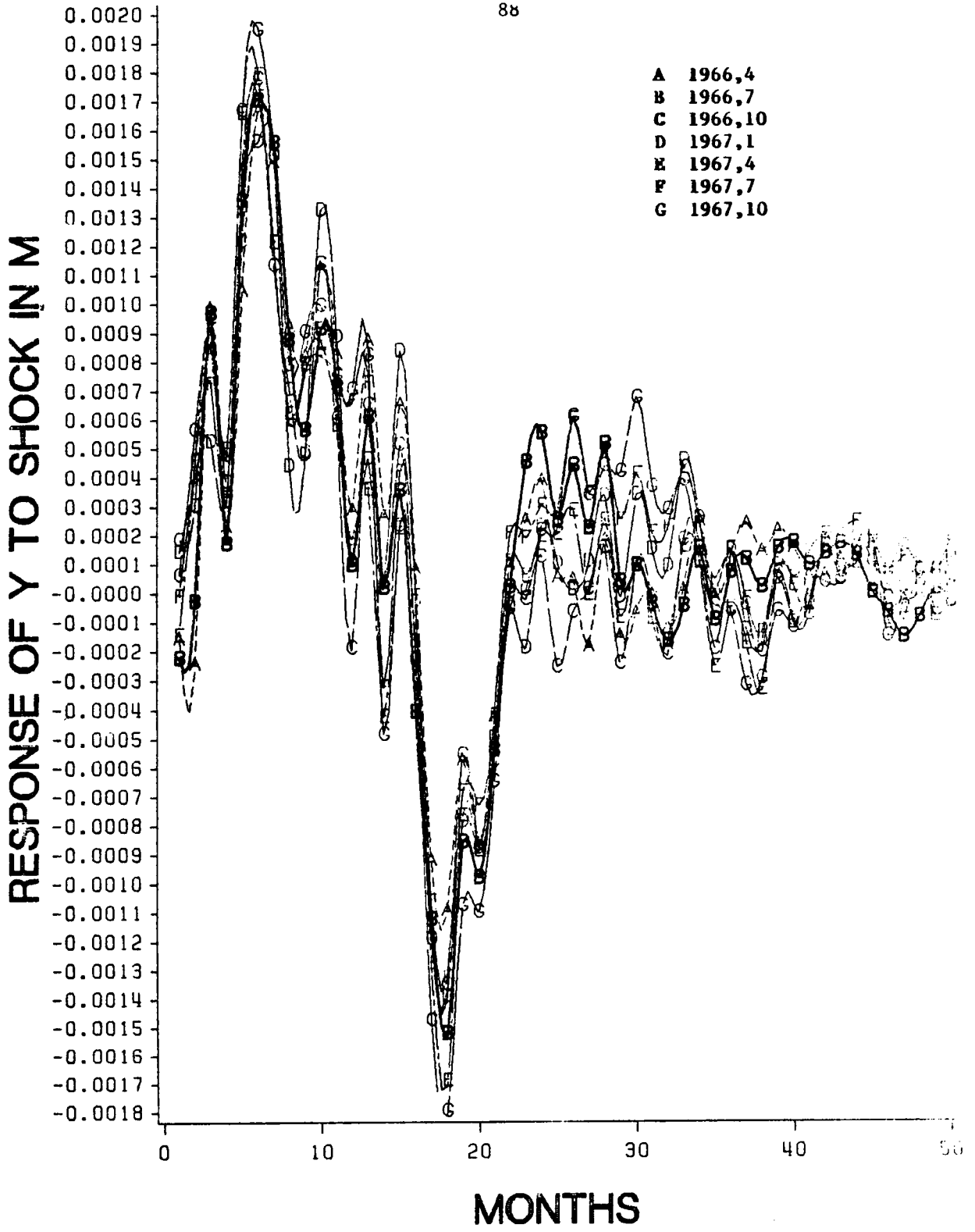


FIGURE 4

JULY 1966 SHIFT

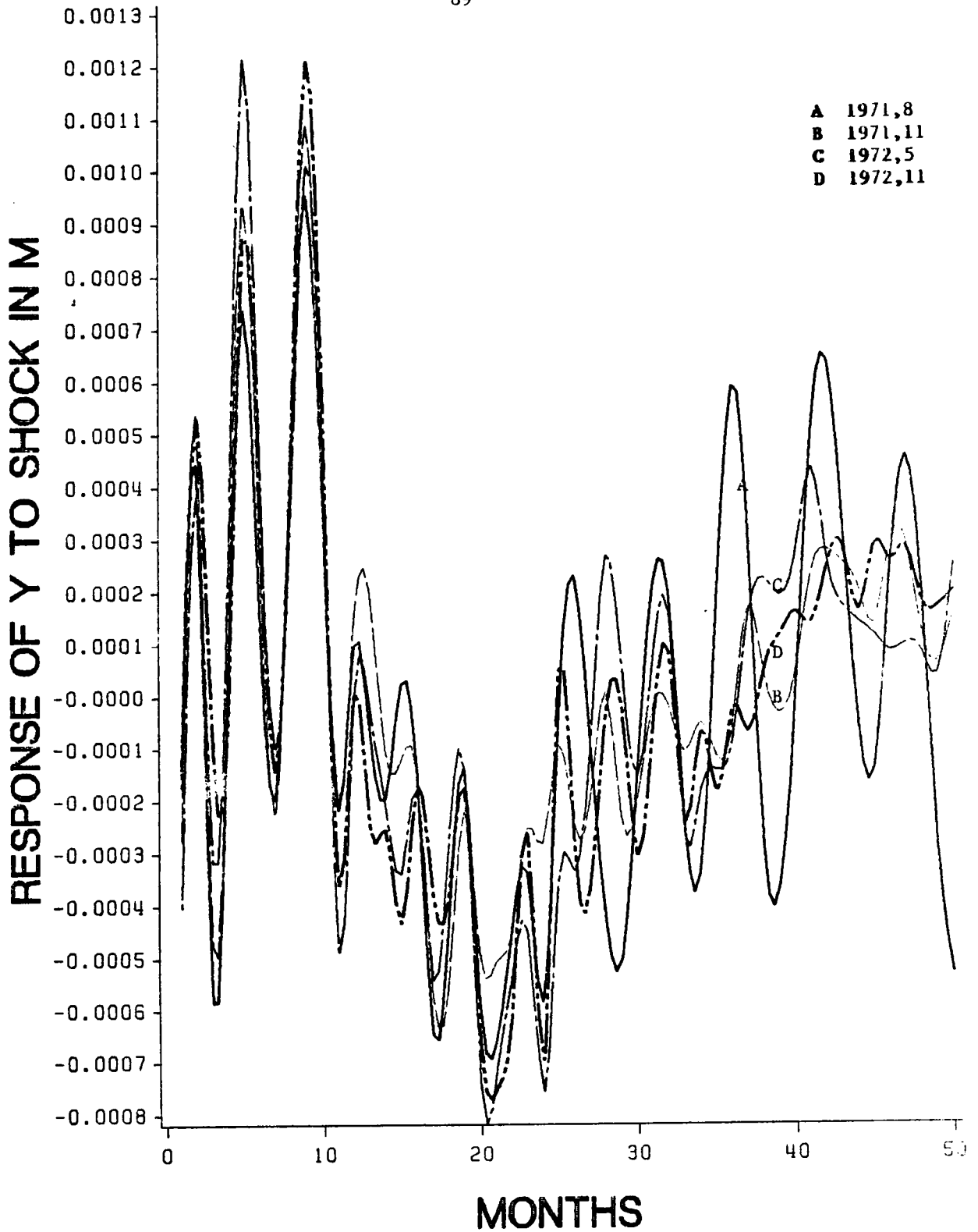


FIGURE 5

AUG. 1971 SHIFT

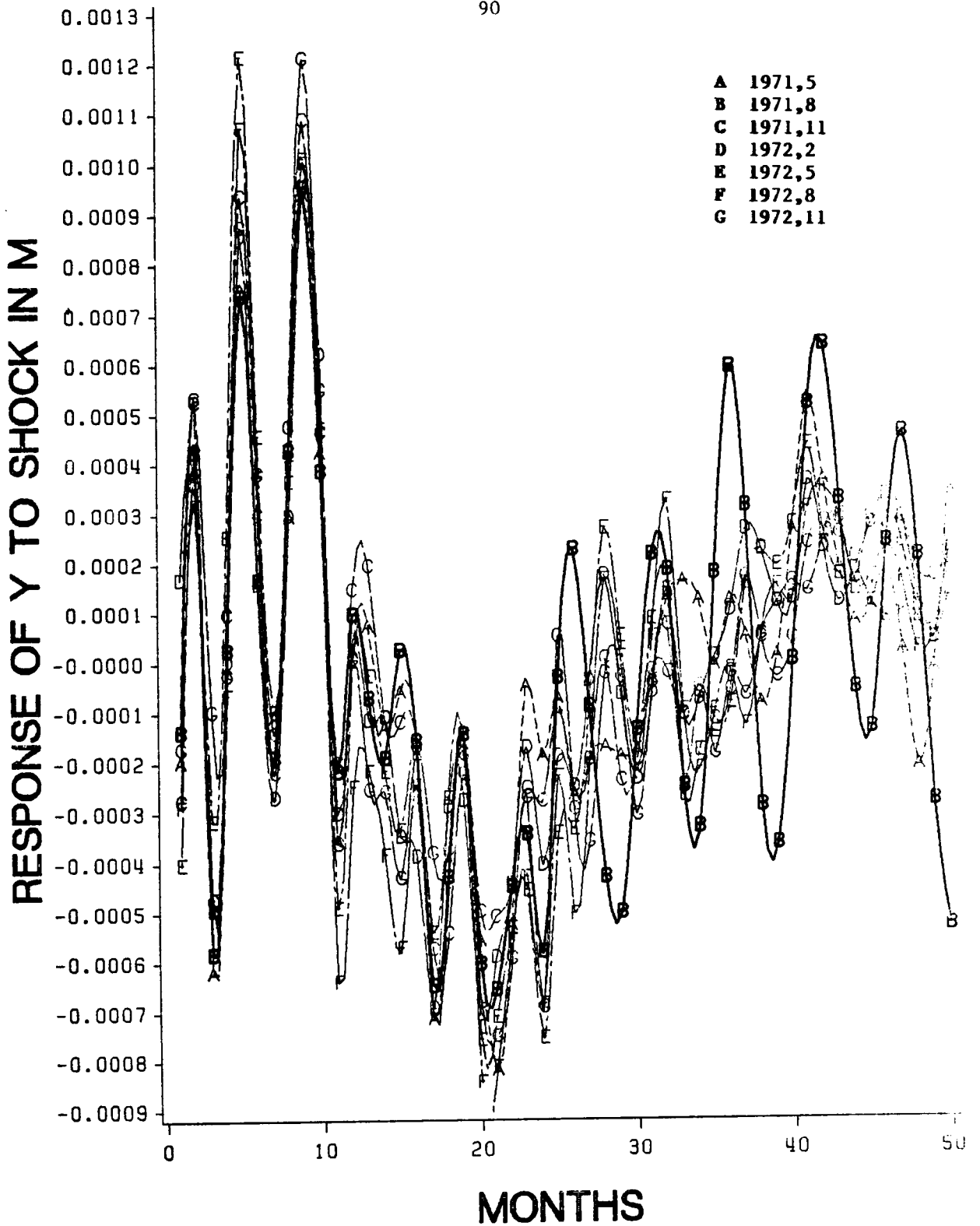


FIGURE 6

AUG. 1971 SHIFT

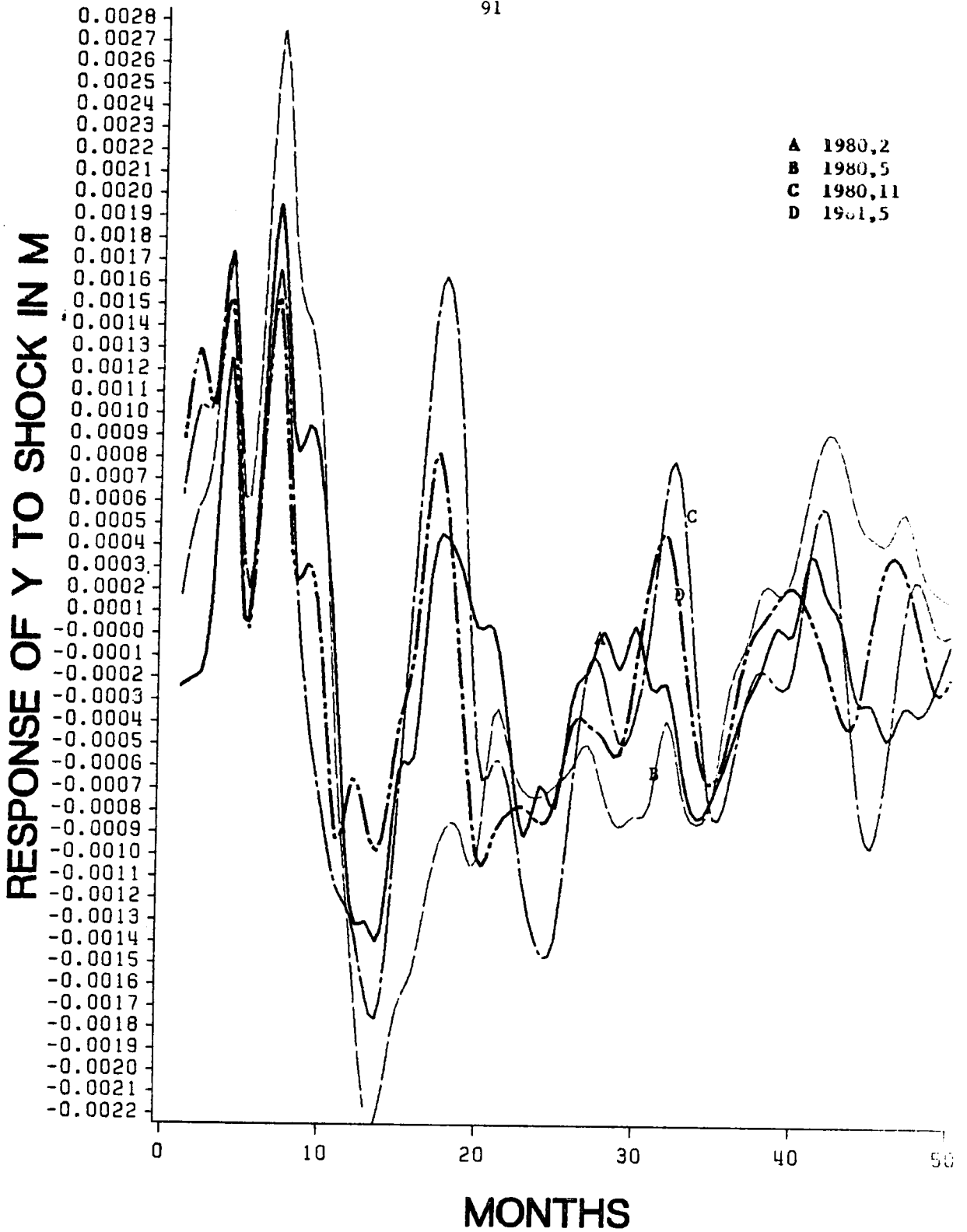


FIGURE 7

FEB. 1980 SHIFT

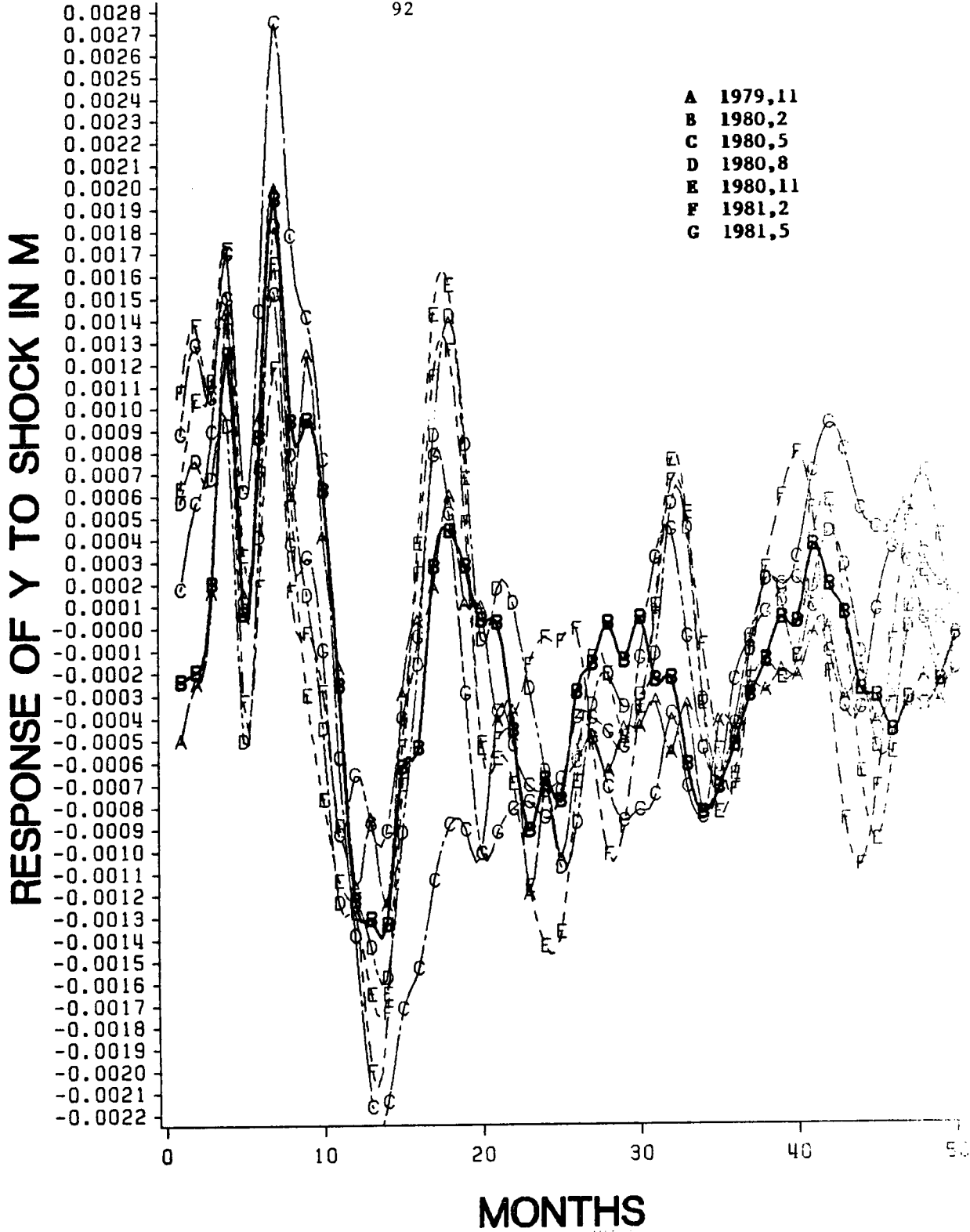


FIGURE 8

FEB. 1980 SHIFT

APPENDIX D

IMPULSE RESPONSE FUNCTIONS FOR PERIOD PRIOR TO SHIFTS

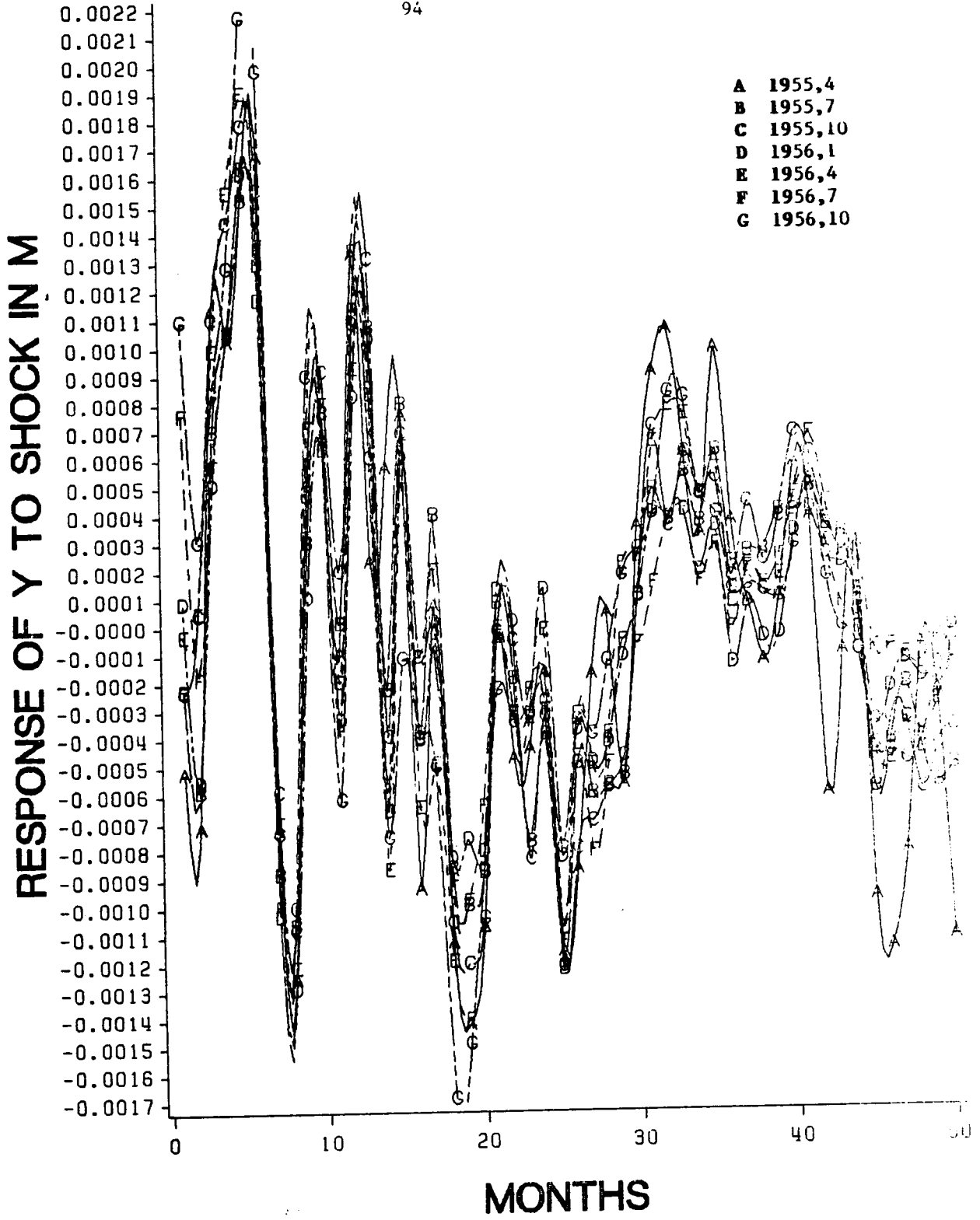


FIGURE 9 RESPONSES PRIOR TO JAN. 1957 SHIFT

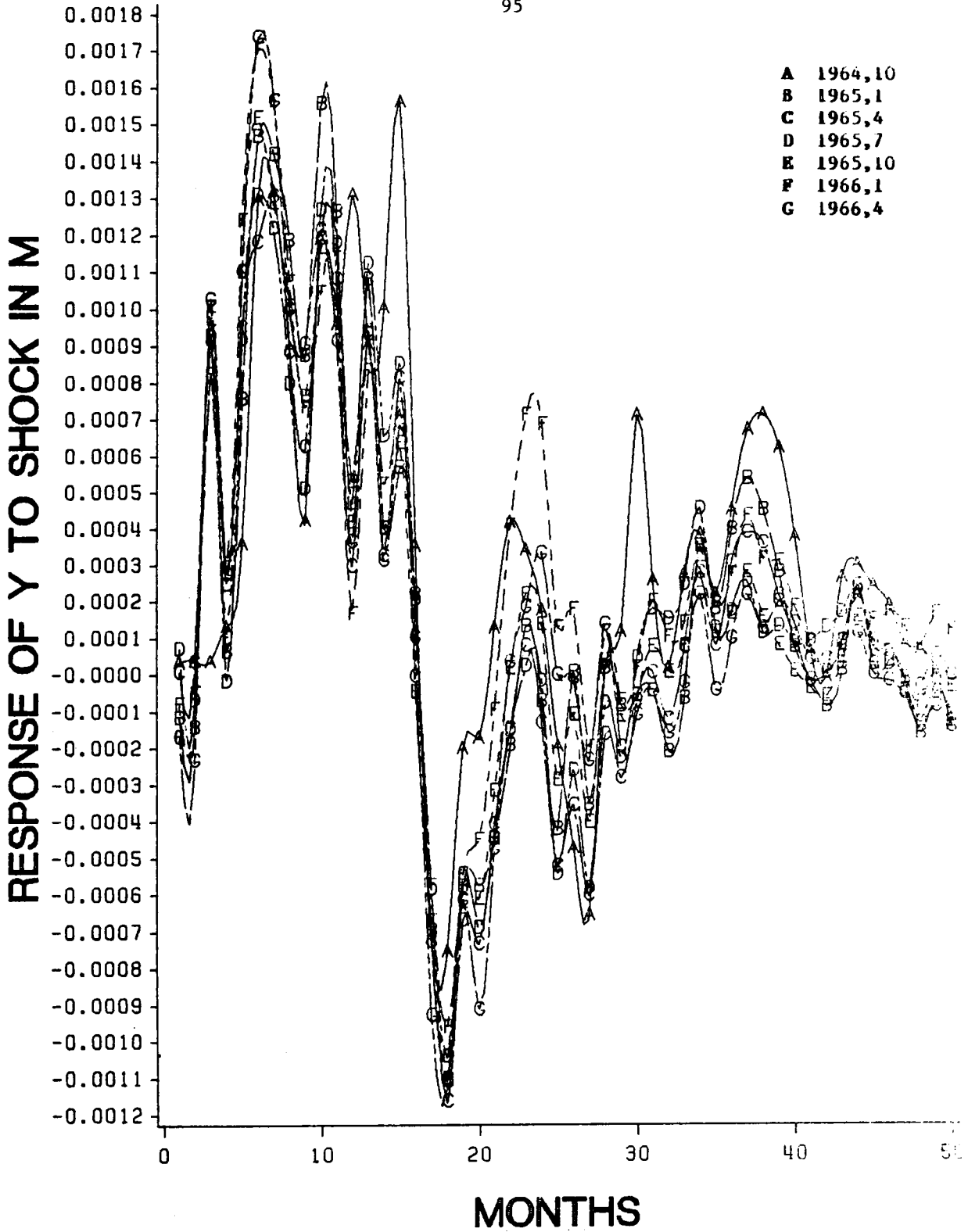


FIGURE 10 RESPONSES PRIOR TO JULY 1966 SHIFT

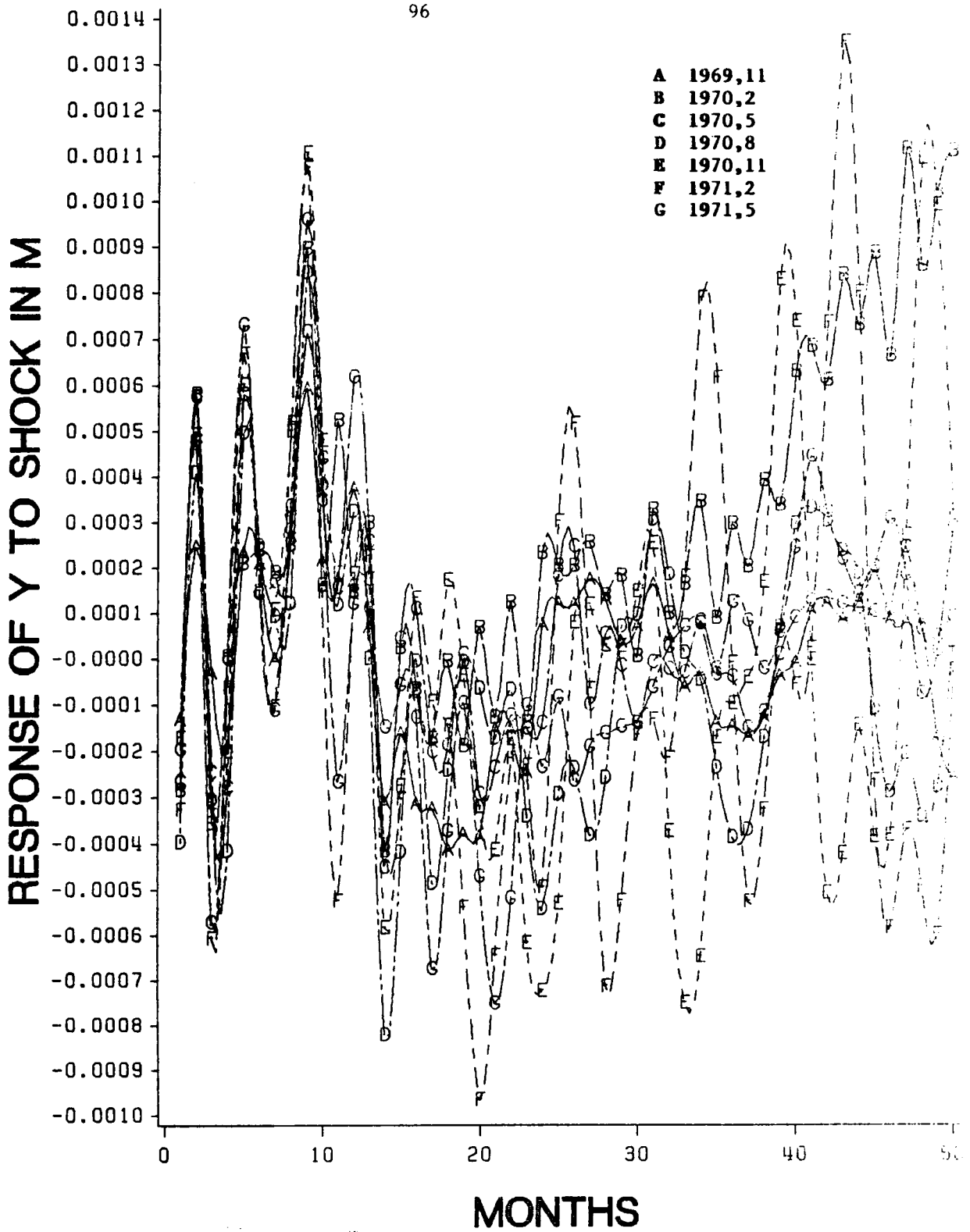


FIGURE 11

RESPONSES PRIOR TO AUG. 1971 SHIFT

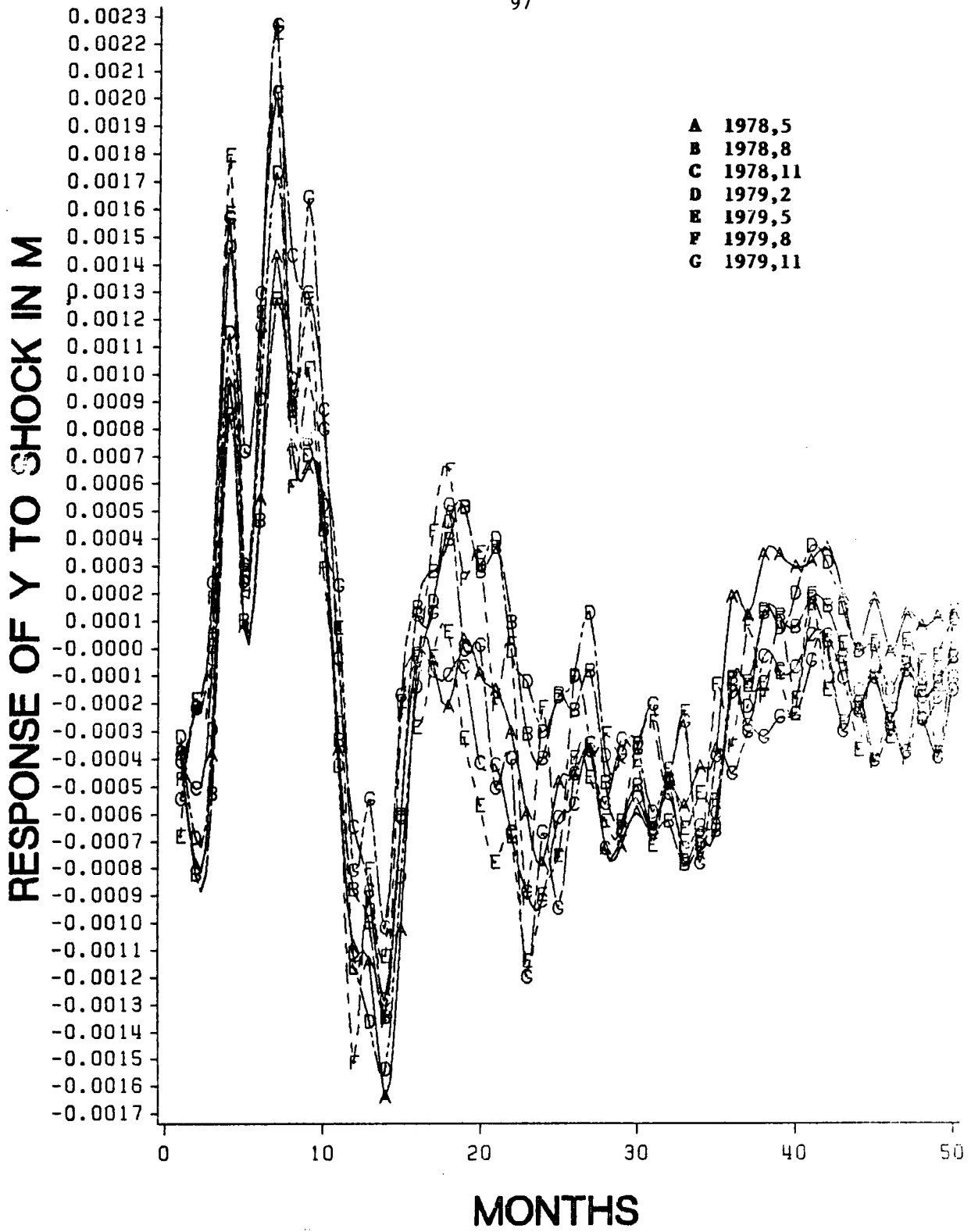


FIGURE 12

RESPONSES PRIOR TO FEB. 1980 SHIFT

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