DESIGN AND ANALYSIS OF INTERCROPPING EXPERIMENTS

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(ABSTRACT)

The statistical problems of intercropping experiments (which involve the growing of two or more crops together) are investigated in this study.

Measures of combined yield are discussed; the Land Equivalent Ratio (LER) is shown to be the 'best' index for intercropping. Problems that arise in the standardization of LER are investigated, and use of a single pair of divisors is recommended.

The use of systematic designs are advocated for yield-density studies, to reduce the number of guard rows. A 3-way systematic design is proposed and methods of analysis are suggested. A regression model is employed for the combined yield data (LER), from which estimates of the optimum densities can be calculated.

The study also deals with varietal trials in intercropping. Methods for reducing the large number of possible varietal combinations to be tested in the field and ways of reducing the block size are given. The field layout is discussed, and illustrated by examples.

Stability measures that can be used in intercropping are derived and it is shown how they can be used in evaluating stable varietal

combinations. It is also shown how information about the contribution to stability of each crop can be obtained.

The best proportions of the component crops in the intercropping mixture is also investigated. Design and analysis for an experiment on proportions in conjunction with varying densities is given.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I. INTRODUCTION

1.1 Background

Intercropping, which involves the growing of two or more crops simultaneously on the same area of land, is widely prevalent in the developing tropics. It is now generally recognized that intercropping can often produce higher yields than sole crops. By mixing crops the farmer can have a longer cropping season, greater protection of the soil from sun, rain and weeds, and more constant use of labor. He can also enjoy increased stability of income.

For 'sole crops' procedures for design and analysis are well established. However, very little work on statistical aspects has yet been done where intercropping is concerned. With more than one crop, the plant population and spatial arrangement become more complex. A single plant of one crop is seldom directly comparable to a single plant of another crop. Thus assessing the degree of yield advantage of intercropping becomes problematic. Experimental design becomes more complex, for in addition to the intercrops, 'sole crops' are needed to provide a comparison for the intercropping yields. Land areas required for the experiments are larger, especially in the case of spacing experiments, where a large proportion of the experimental area must be used as guard areas. To use the experimental area more efficiently, many experimenters and some statisticians advocate the use of systematic designs.

In terms of analysis, there is no standard method for analyzing intercropping data. It is generally accepted that more than one analysis should be applied to intercropping data (Mead and Stern, 1979).

Many types of intercropping experiments are possible, each with a different goal and each demanding a different type of analysis. Three different intercropping situations can be distinguished (Willey, 1979). They are briefly:

(i) Intercropping must give a full yield of a main crop and some yield of a second crop. This situation arises where the primary requirement is for a full yield of some staple food crop. A yield advantage occurs if there is any yield of a second crop. The objective is therefore to maximize the yield of the second crop without reducing yield of the main crop.

(ii) The combined yield must exceed the highest 'sole crop' yield. This criterion is useful only when unit yield of each 'component crop' is equally acceptable, as in grassland mixtures.

(iii) The combined intercrop yield must exceed the combined 'sole crop' yields. This is the most common situation and the one that presents problems of aggregation of the two component yields. This situation will be the main focus of this study.

Another problem in intercropping is the measurement of the degree of stability. Much of the advantage of intercropping, in particular, the possible stability of yield, can be attributed to the use of different niches, either of time or environment of the two crops (Mead and Riley, 1981).

1.2 Terminology

1) Component crop: 'component crop' is used to refer to either

of the individual crops making up the intercropping mixture.

2) Sole crop: "Sole crop' refers to a component crop being grown alone at optimum density.

3) Intercrop yield: It is the yield of a component crop when grown in intercropping and expressed over the total intercropped area (i.e. area occupied by both crops.

4) Variety: A subdivision of a species.

5) Replacement series: This is a series of treatments which contains the pure stands of each species and some mixture treatments formed by replacing given proportions of one species with equivalent proportions of the other.

1.3 Indices of Combined Yield

Various indices of combined yield have been proposed; of these, the land equivalent ratio (LER) appears to be the most popular, since it can be applied to any intercropping situation. The most common indices are given below:

(i) Relative crowding coefficient (K): This index assumes that mixture treatments form a replacement series. For each species, a coefficient (k) can be calculated, which indicates whether that species has produced more or less yield than expected. For species A in mixture with B, it can be written as:

$$k_{ab} = \frac{(\text{mixture yield of A})(\%\ \text{B sown})}{(\text{pure stand yield of A} - \text{A's mixture yield})(\%\ \text{A sown})} \qquad (1.1)$$

and $K = k_{ab} \cdot k_{ba}$. If $K > 1$, this indicates a yield advantage for the combination of the species.

(ii) Aggressivity (A): This index was proposed by McGilchrist (1965). It also assumes that mixtures form a replacement series and it gives a simple measure of how much the relative yield increase in species A is greater than that for species B.

$$A_{ab} = \frac{\text{mixture yield of A}}{\text{expected yield of A}} - \frac{\text{mixture yield of B}}{\text{expected yield of A}} \qquad (1.2)$$

An aggressivity value of zero indicates that the component species are equally competitive. For any other situation, both species will have the same numerical value, but the sign of the dominant species will be positive and that of the dominated negative.

(iii) Land equivalent ratio (LER): The LER is defined as the relative land required as sole crops to produce the same yields as in intercropping (Mead and Willey, 1980).

$$LER = L_A + L_B = Y_a/S_a + Y_b/S_b \qquad (1.3)$$

where L_A and L_B are LERs for the component crops, Y_a and Y_b are the component crop yields and expressed over the total intercropped area, and S_a and S_b are the yields as sole crops of A and B respectively.

The calculation of the LER can be viewed essentially as a standardization of the two component crop yields. LER does not assume that mixture treatments form a replacement series and is therefore widely applicable.

1.4 Use of Systematic Designs

The chief advantage of using randomized designs is that they provide an unbiased estimate of the variance of treatment effects, as well as giving unbiased estimates of the treatment means. The device of randomization enables one to treat plot errors as if they were random, even though they may show correlation patterns in the field. In spacing experiments, however, randomization may have disadvantages. If spacing is being maintained in a square pattern one may either keep a constant number of plants per plot, in which case the plots are all of different sizes and awkward to fit together in a block, or keep all plots the same size, in which case the close spacings may have an unnecessarily large number of plants. In addition, efforts to keep block size small will lead to a large proportion of the plants being used as guards. On the other hand using systematic designs where the density is allowed to change gradually over the block will require guards only on the outside of the block. Since each row is surrounded by others at nearly the same density, the usual requirement for guard or discard area around each plot is avoided.

The main advantage of using systematic designs in intercropping experiments that concerns spacing, is that they greatly increase the proportion of the total area that is harvested. This advantage is more notable when a very wide range of crop densities is being investigated.

1.5 Yield Stability

One of the chief reasons for the predominance of intercropping in developing countries is that it can give greater stability of yields. The basis for this is that if one crop fails or grows poorly, the other component crop can compensate. Thus a major part of intercropping research concerns the selection of suitable varieties of each crop to be grown in a mixture. The appropriate varieties for intercropping need not be the same as those used in sole cropping. Preliminary trials will be necessary for the initial screening of a few varieties that can be used in conventional types of designs later.

The optimal proportions of the two crops A and B, to be used in intercropping is another aspect of stability studies. However, experimental work in this area is practically nil.

1.6 Scope of Present Study

In a very recent paper, Federer (1984) gives a review of the statistical problems of design, analysis and inference associated with experiments on intercropping. According to Federer, some specific questions that need answering are:

(i) What are the properties of various statistical methods for combining responses? What are their distributional properties?

(ii) How to study spatial arrangements and density relationships? The number of possible arrangements and relationships being quite large, how is the best way to reduce this number and obtain desired information?

(iii) How does one measure stability once one has defined it? What is the statistical distribution of the stability measure?

(iv) How does changing proportions of crops affect the response and competition models?

Although this study does not pretend to answer all these questions, attempts are made to answer certain questions. Problems involved in combining yields of the two crops, stability analysis, combined-yield density trials and varietal trials are discussed in the following chapters.

In Chapter 2, the common indices of combined yield are critically reviewed. It is argued that the LER is the most useful index of combined yield. Various methods of standardization of LER and the distributional properties associated with each method are discussed. A small simulation study is used to find out whether 'blockwise' standardization is better than single standardization. The final recommendation is for single standardization.

In intercropping, yield-density trials involve a large number of density combinations. Conventional blocked designs cannot be used because of the large area of land involved, part of which is wasted as guard or discard areas. In Chapter 3, a 3-way systematic design is proposed, in which intra-row spacing for each crop and the inter-row spacing can be varied independently.

Chapter 3 also gives a review of the yield-density functions used in intercropping. A combined yield-density model is developed which can be used to find estimates of the optimal density for each crop in

the intercropping mixture. Using this, a few density combinations can be chosen, on which a detailed study can be made in conventional block designs. Using the systematic design is to be thought of as a preliminary trial to reduce the number of density combinations to be used in future experiments.

In Chapter 4, a varietal trial with mixtures of two crops is considered. The model is written in terms of general and specific mixing effects. The estimates of the general and specific mixing effects are used in predicting which varieties will be useful for further study. For variety trials done in conjunction with different densities for each crop, a design is given which takes into account the practical aspects of field layout. In order to reduce block size, part of the specific mixing effects is confounded with blocks.

If the number of varieties for each crop is large, then the number of varietal combinations will be even larger. One way to reduce the size of the experiment is to consider only some of the varietal combinations, for the field experiment. Chapter 4 discusses an explicit procedure for selecting the combinations, using an incomplete block design. To estimate the general mixing effects (g · m · e), the specific mixing effects (s · m · e) are assumed to be random or negligible. Even after selecting only certain combinations for the experiment, we may still find that the block size is large. In the final Section therefore, a design is given where the varietal combinations are arranged in an incomplete block framework within the main plot (density), in a strip-plot design. The analysis of variance for the design is also given.

The concept of stability, and measures of stability used in monocrops are reviewed in Chapter 5. It is then shown how the method can be extended to measure stability in intercrops. A method for assessing the contribution of the individual varieties of each crop is developed.

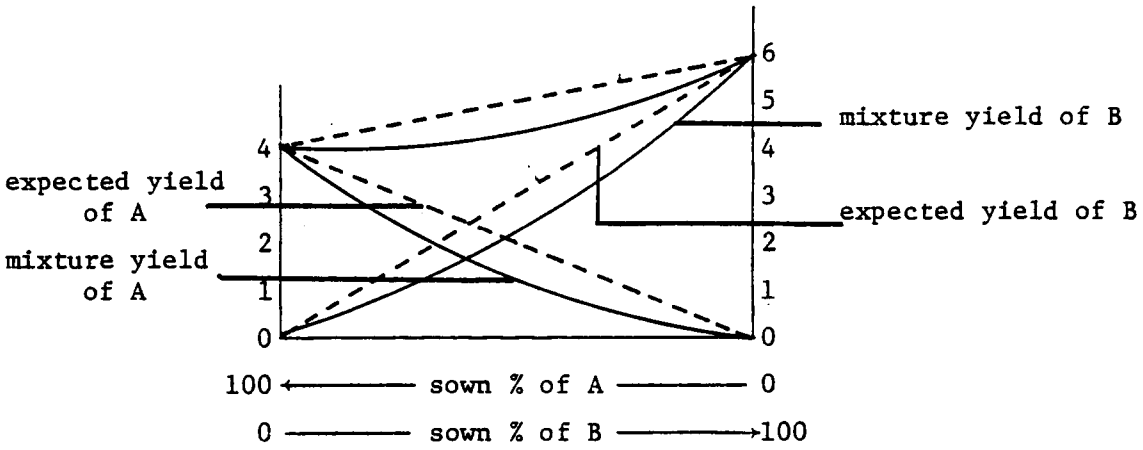
Chapter 6, deals with the question "what are the best proportions of the two crops in an intercropping mixture"? It is shown how the best proportion problem in intercropping can be put into the framework of the general mixture problem. Design and analysis of an experiment with different proportions at different density combinations are given.

II. COMBINING INTERCROPPING YIELDS

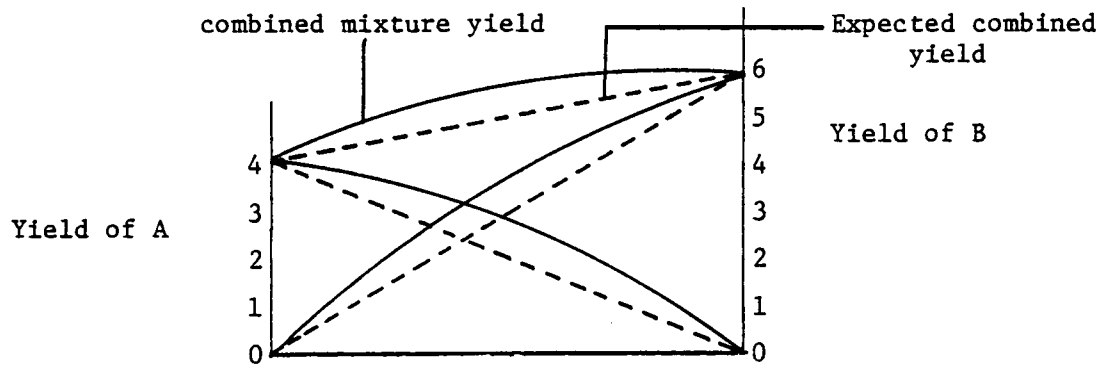
In the absence of a standard method of analyzing data from intercropping experiments, it has been suggested by Mead and Stern (1979), to have first separate analyses for the sole-plot yields of each crop, for the yields from both the sole and mixed plots of each crop, and possibly for only the mixed-plot yields of each crop. However, an analysis of combined yields of the crops has to be performed finally. Although combining crops on the basis of calorific value or income is helpful, there are certain objections to their usage. For instance monetary value is subject to market conditions, which are by no means constant. Using calorific value on the other hand may appeal to the dietician but does not enter into the decision making process of the farmer. Various indices of combined yield which do not take into account the price or the dietary value in their calculations have been proposed; they are critically reviewed in the following sections.

2.1 Indices of Combined Yield

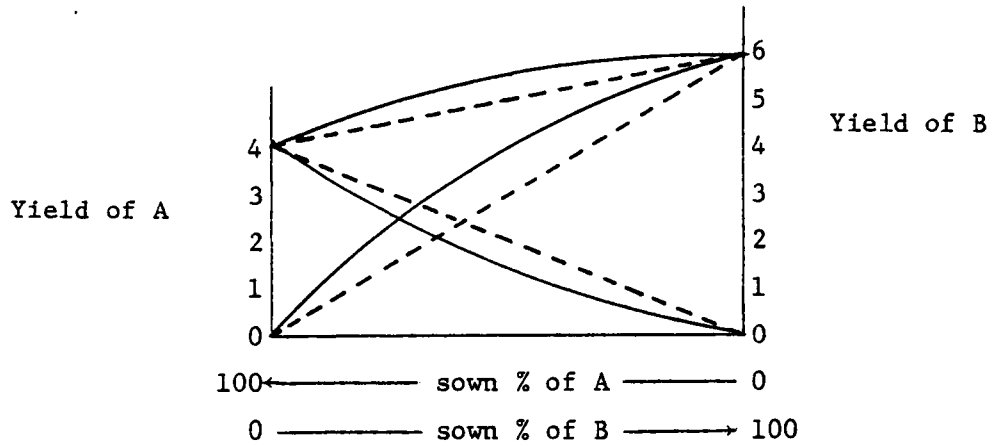
Most of the early indices of combined yield were used initially for investigations into competition between varieties of a particular species, rather than between intercropped species. Some of the indices as mentioned before are restricted to interpreting replacement series experiments. Willey (1979) described the possible patterns of results from a replacement series experiment as shown in Figure 2.1. Mutual inhibition (Fig. 2.1, a) occurs when the actual yield of each



(a) Mutual inhibition



(b) Mutual co-operation



(c) Compensation

Figure 2.1. Types of Competition Between Species. (- - - expected yields, ——— actual yield)

species is less than expected. Mutual co-operation (Fig. 2.1, b) exists when both species yield more than expected and compensation (Fig. 2.1, c) occurs when one species yield more than expected and the other less. It is clear that mutual inhibition cannot give a yield advantage by mixing, whereas the opposite is true of mutual co-operation. However where compensation occurs, the possible advantage of mixing is not clear.

A disadvantage of this widely used diagrammatic approach is that with compensation, the apparent overall advantage may be illusory. The expected yields are calculated assuming that the area is divided into two equal pure stand areas, whereas the actual yields of the 50:50 mixture could have been achieved by dividing the area into two pure stand areas in the proportion π and $1 - \pi$. Willey (1979) argues that the simple comparison of Figure (2.1, c) is biased towards intercropping if the dominant species gives the higher yield, and towards sole cropping otherwise.

Willey and Osiru (1972) avoided this bias by calculating the proportions of land required by sole cropping which would give the same final yield proportions as in intercropping. More recently a simpler procedure analogous to the first part of the calculation used by Willey and Osiru has become common practice in intercropping studies. This is the calculation of the land equivalent ratio (LER). The land equivalent ratio is defined as the relative land required as sole crops to produce the same yield as in intercropping.

$$LER = L_A + L_B = \frac{Y_a}{S_a} + \frac{Y_b}{S_b} \qquad (2.1)$$

when L_A and L_B are the LERs for the component crops A and B. Y_a and Y_b are the respective yields of the component crops A and B, expressed over total intercropped area, and S_a and S_b are the respective yields of A and B as sole crops.

Various other indices of combined yield have been suggested, all of which can be related to the LER components L_A and L_B (Mead and Riley, 1981). The relative crowding coefficient (K) suggested by de Witt (1960) can be expressed as

$$K = \frac{L_A L_B}{(1-L_A)(1-L_B)} \qquad (2.2)$$

The coefficient of Aggresivity (A) proposed by McGilchrist (1965) and discussed in the Introduction, can also be expressed in terms of L_A and L_B.

$$A = \frac{L_A}{\pi} - \frac{L_B}{1-\pi} \qquad (2.3)$$

where π = sown proportion of species A.

2.2 LER as a Measure of Combined Yield

The coefficient of aggresivity and the relative crowding coefficient, although useful for indicating which species is more competitive, can only be applied to replacement series experiments. The

LER however is more versatile as it can be applied to any intercropping situations and not just replacement series experiments. Even in replacement series experiments the LER is more advantageous to use for the following reasons:

(i) LER values show which combinations do or do not give a full yield advantage; the aggresivity values are not able to do this.

(ii) LER values indicate the actual magnitude of any yield advantage, while the crowding coefficient does not. For instance, a LER value of 1.2 means that, to produe the combined mixture yield by growing pure stands would require 20% more land, i.e. the mixture gave a 20% yield increase. A similar value for the crowding coefficient would simply mean that there is a yield advantage, but does not indicate how much of an advantage was achieved.

The LER puts different crops, whatever their type or level of yield on a relative and directly comparable basis. Combining crops on monetary values also achieves this. However as mentioned earlier monetary values are unreliable as they are subject to market conditions, which may fluctuate dramatically. Willey (1979) compared the coefficients K, A, and LER using experimental data. He came to the conclusion that LER was the most useful index, with the additional merit that it can be used in any intercropping situation.

2.3 Problems in Standardization

Two problems arise in the use of LER to compare different mixtures. The first problem is that the magnitude of the LER values

depend on both the intercrop and sole crop yields. The dependence on the success or failure of the sole crop makes the LER unreliable for comparing intercropping mixtures (Mead and Riley, 1981). However if S_A and S_B are regarded as standardizing factors (i.e. constants for a given experiment and obtained from prior experience) the comparisons between different mixtures only involve the mixture yields. Mead and Willey (1980) suggested it is best to define S_A and S_B as the maximum or average of the sole crop yields over the whole experiment.

The second problem with LER has to do with its distributional properties. Even if yields are assumed to be normally distributed, the distribution of the ratio of normal variables is not. The distribution of LER which is the sum of two such ratios would be even more complicated. The different methods of standardization used in calculating LER also affect its distributional properties. These ideas are explored further, later on.

The aims of the investigation should determine the method of standardization. The following examples show different situations that are possible and the appropriate standardization for each situation.

(i) In an intercropping experiment involving different plant densities, Huxley and Maingu (1978) are of the opinion that all intercrop yields should be compared with the sole crops grown at the sole crop optimum densities. The optimum density for intercropped component crops may however, be different from the optimum density for sole crops. In such a situation one could use either the average or

maximum sole crop yields over the entire experiment as standardization factors.

(ii) Consider an intercropping experiment involving fertilizer levels as the treatments. If we assume that fertilizer is constantly available to the farmer, then intercropping yields should be compared with the sole crops at the optimum fertilizer levels. In such as case the average or maximum sole crop yield at the optimum fertilizer level can be used as standardization factors for all mixture plot yields. However, in most developing countries fertilizer is not constantly available to the farmer and he would like to know the relative advantages of intercropping over sole-cropping at different fertilizer levels. In this situation standardization of intercrop yields at a particular fertilizer level should be done using sole crop values at the same fertilizer level.

(iii) An experiment with different varietal combinations of the two crops, present another situation where different divisors have to be used. If the goal is to determine the highest yielding intercrop combination, the intercrop yield should be compared with the highest yielding variety of each crop (Mead and Riley, 1981). However to determine the relative biological efficiency of a combination, the comparison should be with the sole crop varieties making up that particular combination. In the former case the yields of the highest yielding variety of each crop can be used as standardization factors, while in the latter case standardization of a particular combination should be done using the yields of the sole crops that make up that

combination.

In blocked experiments a further consideration is whether the standardization factors should be the same for the entire experiment or different for each block. Fisher (1977), is of the opinion that standardization within each block separately leads to reduced standard errors and skewness of the distribution of LERs. However Pantelides (1979) using data from two experiments involving different varietal combinations, calculated LER values with blockwise standardization and without blockwise standardization and came up with inconclusive results regarding Fisher's claim of reduced standard errors and skewness. Oyejola and Mead (1981) also working with experimental data, came to the conclusion that the more divisors required by standardization, the greater the doubts about the validity of the normality assumption and poorer the precision of treatment comparisons.

2.4 Distributional Properties of LER

The LER has been used in most intercropping situations as the measure of combined yields. Mean comparisons have also been made, using normal theory tests and the foremost question raised by many authors has been whether it is valid to do so. It would certainly be very desirable if LER values were approximately normal. Many tests based on the normal theory are not very sensitive to mild departures from normality. On the other hand if the exact distribution of LER (which is the sum of two ratios of normal random variables) was known parametric tests of hypotheses can be formulated.

It is known that the ratio of standard normal random variables is distributed according to the Cauchy distribution. However, it has been shown by Marsaglia (1965) that the distribution of the ratio of two non-negative normal random variables can take many different forms, ranging from unimodal symmetric curves to bimodal, positive skewed curves with extreme kurtosis.

Marsaglia considered the distribution of

$$w = \frac{a+x}{b+y} \tag{2.4}$$

where a, b, are non-negative constants and x, y are independent standard normal variables. He showed that the distribution of the ratio of two normal random variables, x_1/y_1 can be obtained from the distribution of w in (2.4) by translations and changes of scale.

The distribution of w,

$$F(t) = P\left[\frac{a+x}{b+y} < t\right] \tag{2.5}$$

can be expressed as

$$F(t) = \tfrac{1}{2} + \frac{1}{\pi}\tan^{-1}t + 2V\left(\frac{bt-a}{\sqrt{1+t^2}}, \frac{b+at}{\sqrt{1+t^2}}\right) - 2V(b, a) \tag{2.6}$$

where $V(h, q) = \int_0^h \int_0^{qx/h} \phi(x)\phi(y)\,dy\,dx$ and $\phi(.)$ is the standard normal density. By differentiating (2.6) and simplifying Marsaglia derived the density function to be,

$$f(t) = \frac{e^{-\frac{1}{2}(a^2+b^2)}}{\pi(1+t^2)} \left[1 + \frac{q}{\phi(q)} \int_0^q \phi(y)dy\right] \qquad (2.7)$$

where $q = \frac{b+at}{\sqrt{1+t^2}}$.

When $a > 2.257$ the density is bimodal. As the values of a and b get larger the density exhibits more and more positive skewness. The density given by (2.7) is very complicated and therefore the distribution of LER which is the sum of two ratios of the type given by (2.4) becomes intractable. Based on the fact that the density in (2.7) gives rise to many forms including curves with severe skewness and kurtosis, it is to be expected that the density of LER shows the same type of variation in form, to say the least.

One way to avoid this complicated situation with regard to the distribution of LER is to consider the divisors in the LER equation as standardizing constants (such as the average value over the entire experiment). This may not be entirely unreasonable since very often information about the yield of sole crops is readily available in the form of 'historical' data. In such a case:

$$\text{If} \quad Y_a \sim N(\mu_a, \sigma_a^2)$$

$$\text{Then} \quad \frac{Y_a}{S_a} \sim N\left(\frac{\mu_a}{S_a}, \frac{\sigma_a^2}{S_a^2}\right.$$

Similarly if

$$Y_b \sim N(\mu_b, \sigma_b^2)$$

Then

$$\frac{Y_b}{S_b} \sim N\left(\frac{\mu_b}{S_b}, \frac{\sigma_b^2}{S_b^2}\right)$$

Hence

$$\frac{Y_a}{S_a} + \frac{Y_b}{S_b} = LER \sim N\left(\left[\frac{\mu_a}{S_a} + \frac{\mu_b}{S_b}\right], \left[\frac{\sigma_a^2}{S_a^2} + \frac{\sigma_b^2}{S_b^2} + \frac{2}{S_a S_b} \operatorname{cov}(Y_a, Y_b)\right]\right)$$

If on the other hand standardization is done separately for each block (in a blocked experiment) based on the average or maximum value for the block, we get:

$$\frac{Y_{ai}}{S_{ai}} \sim N\left(\frac{\mu_a}{S_{ai}}, \frac{\sigma_a^2}{S_{ai}^2}\right)$$

and

$$\frac{Y_{bi}}{S_{bi}} \sim N\left(\frac{\mu_b}{S_{bi}}, \frac{\sigma_a^2}{S_{bi}^2}\right)$$

where $i = 1, 2, 3, \ldots, r$ and r = number of blocks. For block i, the LER values, denoted by $LER_i = \frac{Y_{ai}}{S_{ai}} + \frac{Y_{bi}}{S_{bi}}$ are normally distributed.

$$LER_i \sim N\left[\left(\frac{\mu_a}{S_{ai}} + \frac{\mu_b}{S_{bi}}\right), \frac{\sigma_a^2}{S_{ai}^2} + \frac{\sigma_b^2}{S_{bi}^2} + \frac{2}{S_{ai} S_{bi}} \operatorname{Cov}(Y_{ai}, Y_{bi})\right]$$

But in this case the LER values in different blocks will have a different variance. The effect of different variances on the analysis

will be discussed in the next section.

2.5 Assumptions in the Analysis of Variance

If the distribution of the data is known, exact tests of hypotheses can be formulated. However, the exact distribution is rarely known and in the majority of experiments the assumption that the data follows a normal distribution is made. At least in the field of biology, disturbances from normality are usually not sufficiently great to invalidate the technique of analysis of variance for yield data. For instance, crop yields are typically assumed to be normally distributed when using the analysis of variance technique.

The assumptions made in the analysis of variance are that treatment and 'environmental' effects are additive, and that the experimental errors are identically, independently, normally distributed. For distributions that are decidedly skewed or heavy tailed, using the ANOVA assumptions in the analysis may lead to erroneous conclusions.

In section (2.4) it was shown that the distribution of LER can be markedly skewed or possess excessive kurtosis or both. Thus, analyzing LER values, as if they were normally distributed can lead to erroneous conclusions. If the divisors used in the LER equations are constant for the whole experiment, it was shown that the LER values are normally distributed if yields can be assumed to be normally distributed. In such a case the LER values in addition will have a common variance and the ANOVA procedure can be applied. However, when using separate standardizations for each block, it was

shown that LER values have different variances in different blocks, thus violating the assumptions of identical distributions and common variance. Therefore the analysis of variance procedure for LER values obtained by blockwise standardization may be invalid.

The recommendations regarding the method of standardization are discussed at the end of this chapter.

2.6 Simulation Study with Different Methods of Standardization

To test Fisher's (1977) claim that blockwise standardization results in reduced positive skewness in the distribution of LER, a small simulation study was performed.

Yields of the two crops A and B denoted by y_a and y_b were generated from the normal distribution such that:

$$Y_a \sim N(\mu_a , \sigma_a^2)$$

and

$$Y_b \sim N(\mu_b , \sigma_b^2) .$$

The normal variates were generated using the Polar method (Rubinstein, 1981). The LER values were then calculated using single standardization as well as blockwise standardization. The algorithm for the generation of LER values is as follows:

(i) Generate 300 values of $z_1 \sim N(0, 1)$; $Y_a = \mu_a + \sigma_a z_1$.

(ii) Generate 300 values of $z_2 \sim N(0, 1)$; $Y_b = \mu_b + \sigma_b z_2$.

(iii) Calculate average values of Y_a and Y_b for the entire experiment and for each block separately, to be used as standardization values S_A and S_B.

(iv) Calculate LER_A and LER_B using constant values S_A and S_B;

$$LER = LER_A + LER_B$$

(v) Calculate LER_{A_i} and LER_{B_i} for each block separately.

$$LER_i = LER_{A_i} + LER_{B_i}; \; i = 1,2,\ldots,r; \; r = \text{number of blocks.}$$

Five values each of μ_a, μ_b, σ_a and σ_b were used and for each set of values (five hundred sets in all were used), the above algorithm was run. Ten replicate runs were made for each set of values of μ_a, μ_b, σ_a and σ_b using a different random seed each time, thereby giving a total of five thousand runs. The LER values calculated by using single and blockwise standardizations for each run, were then tested for normality, using the skewness and kurtosis measures. The results of the tests of normality on the LER values are shown in Table 2.1.

The results indicate that when using a single divisor for each crop, the LER values were, as expected, not significantly different from the normal distribution as judged by the skewness and kurtosis values. However using separate divisors for each block resulted in LER values, which were significantly different from the normal distribution in many of the runs. As the number of pairs of divisors were increased from two to three, the percentage of runs deviating from the normal distribution also increased. It was also noticed that as the number of pairs of divisors increased, the distribution tended to

Table 2.1. % of Runs Deviating from the Normal Distribution for Different Types of Standardization.

number of pairs of divisors used in the LER equation	% runs deviating from the normal distribution
single pair	0
2 pairs (2 blocks)	20
3 pairs (3 blocks)	38

become more and more positively skewed, thus contradicting Fisher's (1977) claim that skewness would decrease, with blockwise standardization.

It is therefore clear that using blockwise standardization leads not only to LER values that are distributed with different variances for the different blocks, but also to distributions that are positively skew. Blockwise standardization, therefore has to be avoided, unless there is some other strong reason for doing it. If the analysis of variance procedure is to be used on the LER values (which will now form the data), use of a single pair of divisors is therefore recommended. Since the LER values obtained using a single standardization, can be assumed to be normal, if crop yields are, procedures of analysis that can be used with crop yields can be therefore used also with the LER values. Tests of LER means at different levels of a factor can be carried out as usual. Unless otherwise mentioned, the combined measure of yield (from intercropping experiments), used in the succeeding chapters is the LER.

III. YIELD-DENSITY STUDIES IN INTERCROPPING

In order to model combined yield-density functions for intercropping, it is useful to first consider the various yield-density functions that have been used in monocropping (sole cropping).

3.1 Yield Density Functions in Monocropping

A variety of mathematical functions have been proposed to describe the relationship between yield of a crop and density of planting, measured as number of plants per unit area sown. Good reviews of the various models used are found in Willey and Heath (1969) and Mead (1979). There are essentially two types of relationships observed in practice, the 'asymptotic' and 'parabolic' yield-density relations. If x is the plant density (i.e. number of plants/unit area) and w is the yield per plant, then $y = xw$ is the total yield per unit area. It is to be noted that w itself can be a function of x. y is said to obey the asymptotic yield-density relation if it tends to approach an asymptote as plant density increases. On the other hand if y rises to a maximum and declines at higher densities it is said to obey the parabolic yield-density relation. The parabolic relation is common where reproductive yield is concerned (Holliday, 1960). The models commonly used in yield-density relationships employ w as the response variable rather than y. This practice can be attributed to competition experiments where the interest is in response functions for weight per plant, and has been simply extended also to reproductive yields. Two of the most commonly used models are:

$$w = (\alpha + \beta x)^{-1/\theta} + \varepsilon \tag{3.1}$$

$$w = (\alpha + \beta x + \gamma x^2)^{-1} + \varepsilon \tag{3.2}$$

Model (3.1) was proposed by Bleasdale and Nelder (1960), and model (3.2) by Holliday (1960). When $\theta = 1$ in model (3.1) or $\gamma = 0$ in model (3.2), both models reduce to,

$$w = (\alpha + \beta x)^{-1} + \varepsilon \tag{3.3}$$

Model (3.3) is commonly referred to as the asymptotic model or the reciprocal model. All three models considered above are non-linear models. To obtain least squares estimates one would have to go through an iterative procedure starting from some assumed values of the parameters. Using several sets of data in fitting the various models, Ratkowsky (1983) claimed that the Holliday model (3.2), was more suitable for general use in yield-density studies, based on the fact that the Holliday model behaved in a manner closely resembling the behavior of a linear model.

Modified versions of models (3.2) and (3.3) have been used by Willey and Rao (1981), Huxley and Maingu (1978) and Wahua and Miller (1978) to model component crop yields, ignoring the fact that they were in a mixture. The models used by these authors were:

$$\frac{1}{w} = \alpha + \beta x + \varepsilon \tag{3.4}$$

and

$$\frac{1}{w} = \alpha + \beta x + \gamma x^2 + \varepsilon \tag{3.5}$$

These models are similar to the ones discussed earlier, except that they are both linear models. Model (3.4) fits asymptotic relationships and (3.5) fits quadratic or parabolic relationships. The assumption made in these two models is that the variance of the reciprocal of w is constant throughout the range of the data. However, this may not be a reasonable assumption. The error variance structure for combined yield models will be discussed later on, in this chapter.

3.2 Hypothetical Response Curves for Component Crops

As mentioned previously quadratic or parabolic response is shown by crops whose yield is the reproductive yield. In intercropping one of the component crops grown is invariably a cereal such as sorghum while the other crop is usually a legume such as soybean. Thus in an intercropping mixture it is usual to find both crops exhibiting parabolic response. Figure 3.1 depicts a situation where both crops have distinctive parabolic yield-density response under sole cropping, but with different optimal densities.

Leaving aside the competition between the two crops, the optimum total plant density of the inter-crops will depend on the summation of the half values of the two sole crop curves for a 1:1 mixture. However, competition between the crops may reduce the yield of crop A even further as density is increased, since crop A is exhibiting more severe yield reduction as density is increased, than crop B.

A relatively few systematic spacing experiments conducted at the beginning of any series of intercropping investigations into particular

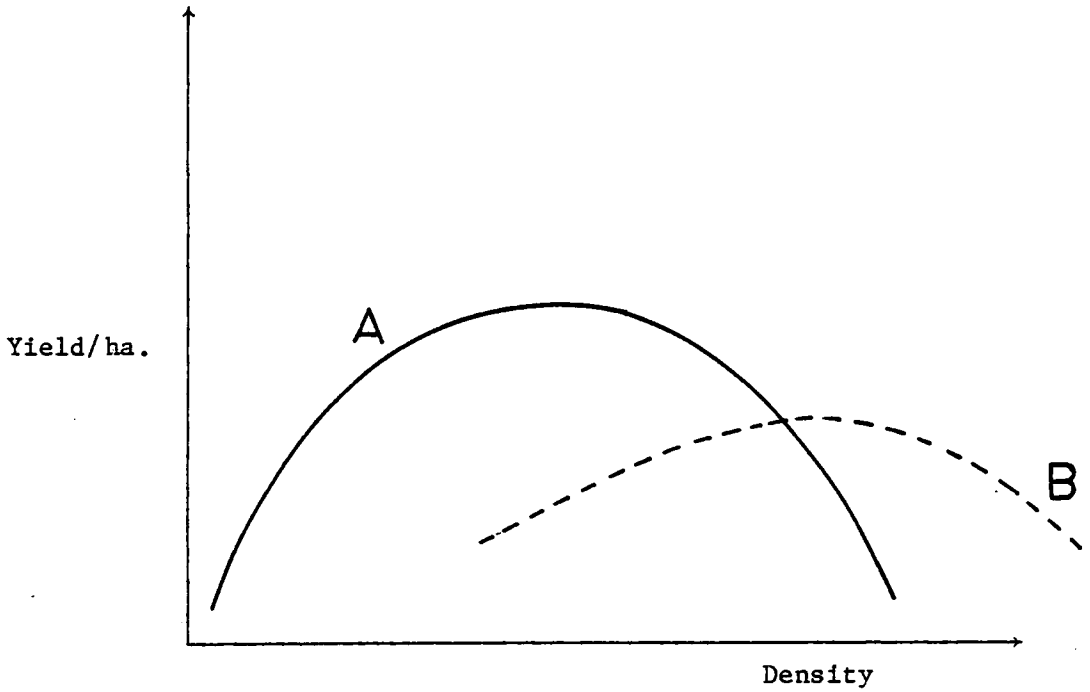


Figure 3.1. Yield-density Relations for Two Crops A and B, Both Showing Parabolic Response.

crop combinations, could offer good insights as to the most likely optimum combinations among the otherwise wide range of choices of densities for the two crops. Larger substantiating experiments of a conventional type can then be carried out based on a few density combinations.

3.3 Use of Systematic Designs in Intercropping

Investigating yield-density relationships in intercropping requires a wide range of plant densities to be used in the experiment, since optimum plant densities under intercropping may be different from the sole crop optimum, due to component crop competition. If conventional block designs are used a large proportion of the land will have to be devoted to guard or discard rows. Guard rows are needed as there may be some carryover of density effects from plots to neighboring plots (Kempthorne, 1979). Conventional block designs may have adjacent plots with widely different density combinations, thus necessitating the use of guard rows. On the other hand using systematic designs where density is allowed to change gradually over the block will require guard rows only on the outside of the block. As each row is surrounded by other plants at nearly the same density, the usual requirement for guard or discard areas around each plot is avoided.

A systematic spacing design consists of a grid of points, each representing the position of a plant and has the property that area per plant and/or the rectangularity (ratio of inter- and intra-row distances) change is some consistent fashion over the different parts

of the grid. Since each density change is small any plant has neighbors in positions close to those that would occur in a strictly rectangular array. Two types of systematic designs commonly used in intercropping experiments are:

(i) Fan designs, due to Nelder (1962).

(ii) Row modification of the fan design called the 'parallel row' design due to Bleasdale (1967).

The grid for Nelder's fan design is formed by the radii and arcs of concentric circles. The fan design has been used in intercropping experiments designed to investigate yield-density responses by Wahua and Miller (1978) and Huxley and Maingu (1978). Figure 3.2 shows the systematic fan design used by Wahua and Miller in an experiment to determine the effect of different plant densities of sorghum and soybean on total grain yield of sorghum.

In the row modification of the fan design, the rows are laid out parallel to each other. This arrangement, called the parallel row design, has considerable potential in intercropping. A limitation of fan designs is that the harvest area (usually 1 or 2 arcs) tends to be small and this could be a particular problem in intercropping experiments where the yield of each component crop is estimated from part of this harvest area. This limitation can be overcome by using parallel row designs in which row length can be adjusted to give any required harvest area. This arrangement is more easily laid out than fan designs and will usually give more efficient use of the experimental area, because the experimental units fit together more conveniently;

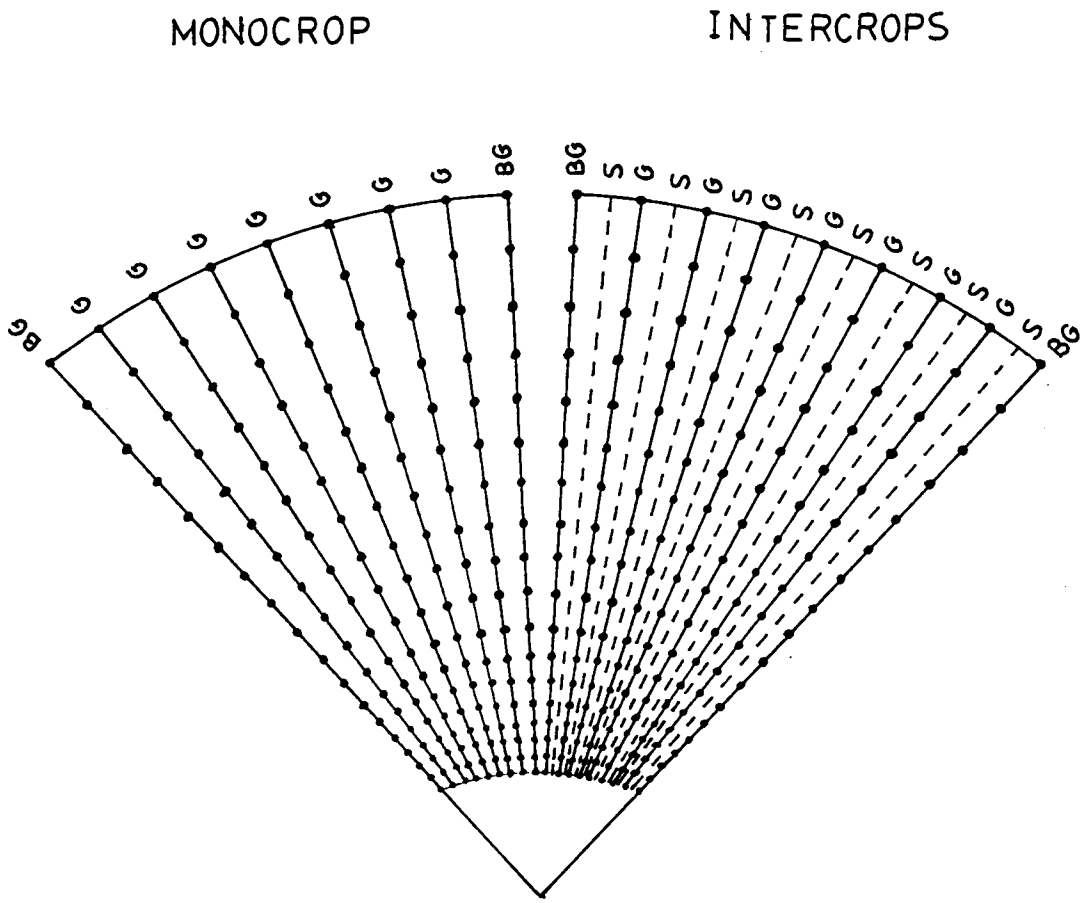


Figure 3.2. Crop Arrangement in a Single Fan. (S = soybean spoke, G = sorghum spoke, BG = border sorghum.)

parallel rows are also more related to normal cropping practice. Mead and Stern (1980) give an example of a 2-way parallel row design that allows the densities of the two component crops to be varied independently. This is shown in Figure 3.3.

3.4 Optimum Densities for Intercropping

In intercropping there are two ways to change the density of the crops. One is by changing the intra-row spacing and the other is by varying the inter-row spacing. Since research into intercropping is in its infancy hardly any research for optimal inter-row spacing has been done. Practically all experiments in intercropping achieve density changes by varying the intra-row spacing at a constant inter-row spacing. Due to practical reasons, inter-row spacing cannot be made too small. However, there is a need to experiment with different levels of inter-row spacing so as to come up with an optimum. In this section we will consider an experiment with different intra-row spacing for each of the two component crops together with varying levels of inter-row spacing.

In such an experiment we then have three factors to contend with, namely the intra-row spacings for crops A and B and the inter-row spacing. Due to the large number of possible treatment combinations, a 3-way systematic design will be appropriate, to save the large area of land that would otherwise be wasted as guard rows. Sole crops at the optimum pure stand density could be grown alongside the actual experiment for standardization purposes. It is not necessary to have

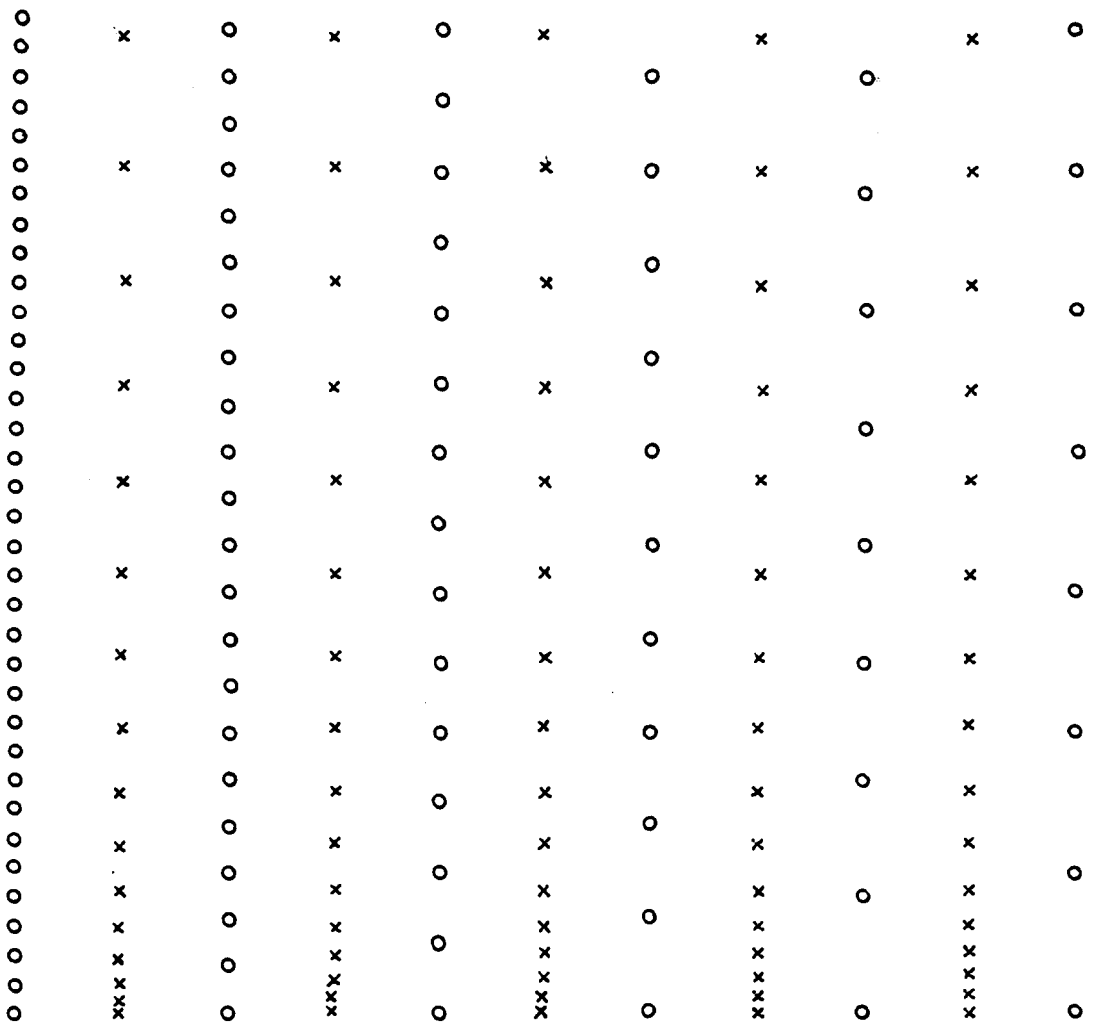


Figure 3.3. Parallel Row Design for 2 Crops (x and 0) with Densities Varying in Perpendicular Direction.

pure stands of the component crops at different densities, since to ascertain whether or not one is better off with a mixture or sole crop, the comparison of the mixture has to be performed with the pure stand yield at optimum density. Standardization will then involve using only a single pair of divisors in the LER equation.

Figure 3.4 shows a 3-way systematic design that is being proposed for an experiment with different intra-row spacings for each component crop together with varying inter-row spacing. The experimental area is divided into segments S_1, S_2, S_3, ... Within each segment the intra-row spacing for crop B (D_{B_1}, D_{B_2}, ...) is the same, while it increases vertically between segments in a systematic manner. The inter-row spacing (D_{I_1}, D_{I_2}, ...) increases systematically in the horizontal direction. The first two inter-row spacings are of the same length, the next two are slightly larger and so on. The intra-row spacing for crop A (D_{A_1}, D_{A_2}, ...) varies within each segment. The average intra-row spacing for crop A within a given harvest area can be considered as the level of intra-row spacing D_{A_i} ($i = 1, 2, 3, \ldots$) for crop A, whereas the inter-row spacing and intra-row spacing for crop B is constant for a given harvest area. In S_1 the intra-row spacing for crop A decreases from top to bottom. In S_2 it increases from top to bottom, while in S_3 it again decreases from top to bottom and so on, such that each succeeding segment is a mirror image of the preceding segment, where intra-row spacing of crop A is concerned. The harvest areas can be denoted by H_{ijk} (i = level of intra-row spacing for crop A, j = level of intra-row spacing for crop B, and k = level of inter-row spacing). For purposes of analysis four typical

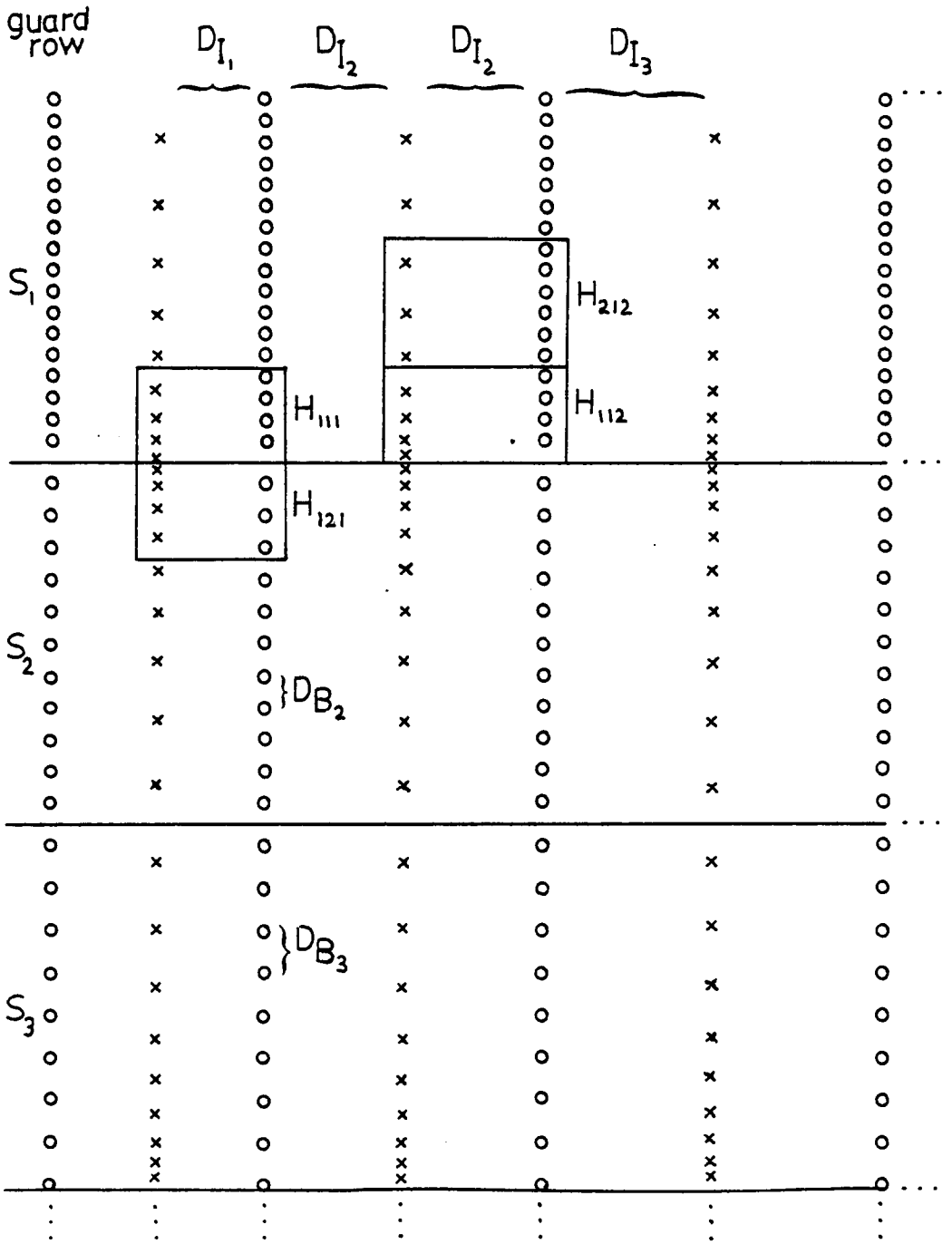


Figure 3.4. 3-Way Parallel Row Systematic Design for 2 Crops, Crop A = X, Crop B = O. [D_I = inter-row distance, D_B = intra-row distance for crop B and H = harvest area.]

harvest areas H_{111}, H_{121}, H_{112} and H_{212} are marked in Figure 3.4 The size of the harvest area is adjustable and will depend on the type of crops that are grown in the mixture.

In passing, it is to be noted that if there are a large number of segments in the experiment, we need to have some type of blocking. For instance, if there are ten levels of intra-row spacing for crop B, then we need ten segments in the experiment. This will involve a large area of land. Therefore we may consider blocking it so that block 1 will contain the spacing levels D_{B_1}, D_{B_2}, D_{B_3}, and D_{B_4} while block 2 will contain D_{B_4}, D_{B_5}, D_{B_6}, and D_{B_7} and block 3 will contain D_{B_7}, D_{B_8}, D_{B_9}, and $D_{B_{10}}$. This arrangement will also ensure that we have a connected design. However, this aspect will not be considered further here.

Suppose we have n_a levels of intra-row spacing for crop A, n_b levels of intra-row spacing for crop B and n_I levels of inter-row spacing. The total number of treatment combinations is then $N = n_a n_b n_I$. Although an analysis of variance is, strictly speaking not appropriate for a systematic design, we can nevertheless perform one to use it as a diagnostic tool for determining the importance of the factors and interactions among them and to make mean comparisons of the LER between different treatment combinations. This would give us an idea, as to which treatment combinations are most likely worthy of further investigation. The outline of the analysis of variance for the experiment is given in Table 3.1, assuming that the systematic pattern has been replicated r times (i.e. number of replicates for each treatment combination will be equal to r). However, it may not always be possible

Table 3.1. ANOVA Table for the 3-Way Systematic Spacing Experiment.

Source	d.f.	SS
Replicates	$r-1$	SS(rep)
Intra-row, Crop A	n_a-1	SS(A)
Intra-row, Crop B	n_b-1	SS(B)
Inter-row	n_I-1	SS(I)
A × B	$(n_a-1)(n_b-1)$	SS(AB)
A × I	$(n_a-1)(n_I-1)$	SS(AI)
B × I	$(n_b-1)(n_I-1)$	SS(BI)
A × B × I	$(n_a-1)(n_b-1)(n_I-1)$	SS(ABI)
Error	$(r-1)(n_a n_b n_I-1)$	SS(error)
Total	$n_a n_b n_I r-1$	SST

to have replications of the systematic pattern for want of land. In such a case the triple interaction A × B × I can be used as the error term, on the assumption that it is negligible (see later).

Harvest areas in the same column but in different segments, e.g. H_{111} and H_{121} in Figure 3.4 have the same inter-row spacing and the same intra-row spacing for crop A. Therefore a comparison between the mean LER of these two harvest areas is a comparison between two intra-row spacing levels of crop B. However, if the two harvest areas are in the same segment and in the same column, e.g. H_{112} and H_{212}), they differ only in the intra-row distances of crop A. A comparison between H_{112} and H_{212} is therefore a comparison of two intra-row spacing levels of crop A. To compare two inter-row distances, comparisons are based on harvest areas in the same segment but in different columns, e.g. H_{111} and H_{112}. The main reason for laying out a systematic design with several levels of intra- and inter-row spacing is, however, to fit a combined yield-density relationship and use the estimated equation, to determine the optimum levels of spacing.

In postulating a model for the combined yield-density relationship, it is necessary to consider the effect of the three variables, i.e. intra-row spacings for crops A and B, the inter-row spacing and the interaction effects on the yields of A and B. As intra-row spacing for each component crop is reduced (i.e. density increased) the yield per unit area of the component crop will first increase and then decline as intra-row spacing becomes even smaller. A similar behavior can be expected as inter-row spacing is progressively reduced. If we consider the component yield of, say, crop A, then a possible model

for component crop yield, in the absence of interaction effects is:

$$y_a = \beta_0^a + \beta_1^a D_A + \beta_2^a D_A^2 + \beta_3^a D_I + \beta_4^a D_I^2 + \varepsilon_a \qquad (3.6)$$

where y_a is the yield per unit area of crop A, D_A and D_I are the intra-row distances of crop A and the inter-row distances, respectively. The response variable used here is the yield per unit area rather than the yield per plant. One reason for doing this is that when inter-row distance is increased beyond a certain value, w, the yield per plant will not be affected, whereas y, the yield per unit area will decrease. Another reason is that our final aim is to model the yields of A and B together and in combining yields one uses the yield per unit area and not yield per plant. According to the parabolic relationship outlined in Section 3.1, the coefficients β_1^a and β_3^a are positive, while the coefficients β_2^a and β_4^a are negative

In model (3.6) we have assumed that the interaction between inter- and intra-row spacings is absent. However, in practice a combined reduction of inter- and intra-row spacings may lead to further reduction (or increase) in yield, i.e. yield reduction (increase) may not be additive. Hence we need to include a $D_A D_I$ term in the model. The coefficient for this term may be positive or negative. We may also need a $D_A D_B$ term in the model, reflecting the interaction between crops A and B. But the effect of this interaction may be very slight when compared to the interaction $D_A D_I$, as it is the inter-row distance that will affect the yield of A, rather than the intra-row spacing of crop B. Similarly the intra-row spacing of crop A may not affect the

yield of B. Therefore the 3 factor interaction A × B × I can be assumed to be negligible and is not included in the model. Using the interaction terms, the component yields of A and B can then be jointly modelled as follows:

$$\text{Crop A:}\quad y_a = \beta_0^a + \beta_1^a D_A + \beta_2^a D_A^2 + \beta_3^a D_I + \beta_4^a D_I^2 + \beta_{13}^a D_A D_I + \beta_{12}^a D_A D_B + \varepsilon_a \tag{3.7}$$

$$\text{Crop B:}\quad y_b = \beta_0^b + \beta_1^b D_B + \beta_2^b D_B^2 + \beta_3^b D_I + \beta_4^b D_I^2 + \beta_{13}^b D_B D_I + \beta_{12}^b D_A D_B + \varepsilon_b \tag{3.8}$$

To study the behavior of the intercropping system as a whole, we need to combine equations (3.7) and (3.8) to obtain an equation for combined yield. Let s_a and s_b represent the average yield of A and B grown as sole crops at the sole crop optimum densities. Dividing equation (3.7) by s_a and equation (3.8) by s_b, we obtain equations (3.9) and (3.10), respectively, for the component LER's:

$$LER_A = \gamma_0^a + \gamma_1^a D_A + \gamma_2^a D_A^2 + \gamma_3^a D_I + \gamma_4^a D_I^2 + \gamma_{12}^a D_A D_B + \gamma_{13}^a D_A D_I + \varepsilon_1 \tag{3.9}$$

$$LER_B = \gamma_0^b + \gamma_1^b D_B + \gamma_2^b D_B^2 + \gamma_3^b D_I + \gamma_4^b D_I^2 + \gamma_{12}^b D_A D_B + \gamma_{13}^b D_A D_I + \gamma_2 \tag{3.10}$$

where $LER_A = y_a/s_a$, $LER_B = y_b/s_b$, $\gamma_0^a = \beta_0^a/s_a$, $\gamma_0^b = \beta_0^b/s_b$, $\gamma_1^a = \beta_1^a/s_a$, $\gamma_1^b = \beta_1^b/s_b$, $\gamma_2^a = \beta_2^a/s_a$, $\gamma_2^b = \beta_2^b/s_b$, $\gamma_3^a = \beta_3^a/s_a$, $\gamma_3^b = \beta_3^b/s_b$, $\gamma_4^a = \beta_4^a/s_a$, $\gamma_4^b = \beta_4^b/s_b$, $\gamma_{12}^a = \beta_{12}^a/s_a$, $\gamma_{12}^b = \beta_{12}^b/s_b$, $\gamma_{13}^a = \beta_{13}^a/s_a$, and $\gamma_{13}^b = \beta_{13}^b/s_b$.

Combining equations (3.9) and (3.10), we finally obtain:

$$LER = \gamma_0 + \gamma_1^a D_A + \gamma_1^b D_B + \gamma_2^a D_A^2 + \gamma_2^b D_B^2 + \gamma_3 D_I + \gamma_4 D_I^2$$

$$+ \gamma_{12} D_A D_B + \gamma_{13}^a D_A D_I + \gamma_{13}^b D_B D_I + \varepsilon \qquad (3.11)$$

where $LER = LER_A + LER_B$, $\gamma_0 = \gamma_0^a + \gamma_0^b$, $\gamma_3 = \gamma_3^a + \gamma_3^b$, $\gamma_4 = \gamma_4^a + \gamma_4^b$, and $\gamma_{12} = \gamma_{12}^a + \gamma_{12}^b$.

In estimating the coefficients of equation (3.11) the use of ordinary least squares may be inappropriate as the variability of yields may be different at different densities. In fact empirical evidence indicates that variability of yields are higher as competition increases (Mead, 1967). In O·L·S we assume that the vector of error terms $\underline{\varepsilon}$ is distributed $N(\underline{\varepsilon}:\underline{0}, \sigma^2 I)$ i.e. homogenous error structure. In the case of density trials, it can be expected, that the combined yield is inversely related to the area occupied by the component crops A and B. However, reducing the area by a factor of f, may not increase the variability by as much as a factor f. We make the assumption that variability of yields is inversely proportional to the square root of the combined area occupied by a pair of plants, one each from crops A and B, i.e.

$$\text{variability} \propto \frac{1}{\sqrt{(D_A+D_B)D_I}}$$

In other words, the assumption we make regarding the error, is that $\underline{\varepsilon}$ is distributed $N(\underline{\varepsilon}:\underline{0}, \sigma^2 V)$ where V is a positive definite matrix given by:

$$V = \begin{bmatrix} \frac{1}{\sqrt{(D_{A_1} + D_{B_1})D_{I_1}}} & & & \\ & \frac{1}{\sqrt{(D_{A_2} + D_{B_1})D_{I_1}}} & & \underline{0} \\ & & \ddots & \\ \underline{0} & & & \frac{1}{\sqrt{(D_{A_{na}} + D_{B_{nb}})D_{I_{nI}}}} \end{bmatrix}$$

Writing model (3.11) in matrix form

$$\underline{L} = D\underline{\gamma} + \underline{\varepsilon} \qquad (3.12)$$

where $\underline{L}$ is an Nr × 1 vector of LER values

D is an Nr × p matrix of fixed observable values (the spacing levels) and

p = number of parameters in the model.

For the above model p = 10.

$\underline{\gamma}$ is a p × 1 vector of parameters.

$$\underline{\gamma} = \begin{pmatrix} \gamma_0 \\ \gamma_1^a \\ \gamma_1^b \\ \gamma_2^a \\ \gamma_2^b \\ \gamma_3 \\ \gamma_4 \\ \gamma_{12} \\ \gamma_{13}^a \\ \gamma_{13}^b \end{pmatrix}$$

and $\underline{\varepsilon}$ is an Nr × 1 unobservable random vector such that $E(\underline{\varepsilon}) = \underline{0}$ and $\text{cov}(\underline{\varepsilon}) = \sigma^2 V$.

The solution vector $\hat{\underline{\gamma}}$ is obtained by using generalized least squares:

$$\hat{\underline{\gamma}} = (D'V^{-1}D)^{-1} D'V^{-1}\underline{L} \tag{3.13}$$

The equation for estimating LER can be written as,

$$\hat{LER} = \hat{\gamma}_0 + \hat{\gamma}_1^a D_A + \hat{\gamma}_1^b D_B + \hat{\gamma}_2^a D_A^2 + \hat{\gamma}_2^b D_B^2 + \hat{\gamma}_3 D_I$$

$$+ \hat{\gamma}_4 D_I^2 + \hat{\gamma}_{12} D_A D_B + \hat{\gamma}_{13}^a D_A D_I + \hat{\gamma}_{13}^b D_B D_I \tag{3.14}$$

The outline of the analysis of variance for model (3.12) is given in Table 3.2; it is a further subdivision of the sources of variation in Table 3.1. The table shows how each source of variation given in Table 3.1 can be partitioned into a source of variation due to regression and a lack of fit component. If replication was not possible the A × B × I interaction, which was assumed to be negligible can be used as the error term. However, if replication is done, we can test whether the A × B × I interaction is needed or not.

To find the estimates of the optimum densities for the two crops in the mixture, we maximize $\hat{LER}$ in equation (3.14) with respect to D_A, D_B and D_I.

$$\frac{\partial \text{LER}}{\partial D_A} = \hat{\gamma}_1^a + 2\hat{\gamma}_2^a D_A + \hat{\gamma}_{12} D_B + \hat{\gamma}_{13}^a D_I = 0 \tag{3.15}$$

Table 3.2. ANOVA for Regression Model.

Source	d.f.
replicates	$r-1$
Spacing A	n_a-1
Regression A	2
Lack of fit	n_a-3
Spacing B	n_b-1
Regression B	2
L.O.F.	n_b-3
Inter-row spacing	n_I-1
Regression I	2
L.O.F.	n_I-3
A × B	$(n_a-1)(n_b-1)$
Regression A B	1
L.O.F.	$(n_a-1)(n_b-1)-1$
A × I	$(n_a-1)(n_I-1)$
Regression A × I	1
L.O.F.	$(n_a-1)(n_I-1)-1$
B × I	$(n_b-1)(n_I-1)$
Regression B × I	1
L.O.F.	$(n_b-1)(n_I-1)-1$
A × B × I	$(n_a-1)(n_b-1)(n_I-1)$
Error	$(r-1)(n_a n_b n_I-1)$
Total	$n_a n_b n_I r-1$

$$\frac{\partial \hat{LER}}{\partial D_B} = \hat{\gamma}_1^b + 2\hat{\gamma}_2^b D_B + \hat{\gamma}_{12} D_A + \hat{\gamma}_{13}^b D_I = 0 \qquad (3.16)$$

$$\frac{\partial \hat{LER}}{\partial D_I} = \hat{\gamma}_3 + 2\hat{\gamma}_4 D_I + \hat{\gamma}_{13}^a D_A + \hat{\gamma}_{13}^b D_B = 0 \qquad (3.17)$$

Solving equations (3.15), (3.16), and (3.17) simultaneously, we obtain the estimates of the optimum intra-row spacing for each component crop and the optimum inter-row spacing.

$$D_A^* = \frac{\hat{\gamma}_1^a(\hat{\gamma}_{13}^{b\,2} - 4\hat{\gamma}_2^b\hat{\gamma}_4) + \hat{\gamma}_1^b(2\hat{\gamma}_{12}\hat{\gamma}_4 - \hat{\gamma}_{13}^a\hat{\gamma}_{13}^b) + \hat{\gamma}_3(2\hat{\gamma}_{13}^a\hat{\gamma}_2^b - \hat{\gamma}_{12}\hat{\gamma}_{13}^b)}{\text{Det.}}$$

$$D_B^* = \frac{\hat{\gamma}_1^a(2\hat{\gamma}_{12}\hat{\gamma}_4 - \hat{\gamma}_{13}^a\hat{\gamma}_{13}^b) + \hat{\gamma}_1^b(\hat{\gamma}_{13}^{a\,2} - 4\hat{\gamma}_2^a\hat{\gamma}_4) + \hat{\gamma}_3(\hat{\gamma}_{12}\hat{\gamma}_{13}^a - 2\hat{\gamma}_2^a\hat{\gamma}_{13}^b)}{\text{Det.}}$$

$$D_I^* = \frac{\hat{\gamma}_1^a(2\hat{\gamma}_{13}^a\hat{\gamma}_2^b - \hat{\gamma}_{12}\hat{\gamma}_{13}^b) + \hat{\gamma}_1^b(\hat{\gamma}_{12}\hat{\gamma}_{13}^a - 2\hat{\gamma}_2^a\hat{\gamma}_{13}^b) + \hat{\gamma}_3(\hat{\gamma}_{12}^2 - 4\hat{\gamma}_2^a\hat{\gamma}_2^b)}{\text{Det.}}$$

where Det. $= 8\hat{\gamma}_2^a\hat{\gamma}_2^b\hat{\gamma}_4 - 2\hat{\gamma}_2^a(\hat{\gamma}_{13}^b)^2 - 2\hat{\gamma}_{12}^2\hat{\gamma}_4 - 2\hat{\gamma}_2^b(\hat{\gamma}_{13}^a)^2 + 2\hat{\gamma}_{12}\hat{\gamma}_{13}^a\hat{\gamma}_{13}^b$.
To achieve a maximum the matrix of the second derivatives,

$$\begin{bmatrix} 2\hat{\gamma}_2^a & \hat{\gamma}_{12} & \hat{\gamma}_{13}^a \\ \hat{\gamma}_{12} & 2\hat{\gamma}_2^b & \hat{\gamma}_{13}^b \\ \hat{\gamma}_{13}^a & \hat{\gamma}_{13}^b & 2\hat{\gamma}_4 \end{bmatrix}$$

must be negative definite.

IV. VARIETAL SELECTION

A major part of intercropping research centers around the selection of suitable varieties of the component crops. Varieties appropriate for intercropping need not be the same as those used in sole cropping. Suitable varieties for intercropping have to be determined through large scale experimentation. Since there is much similarity between selection for varietal mixtures and selection of hybrids using diallel cross experiments, we base the development of our model and terminology to run parallel with the ideas used in hybrid cross selection. It is to be noted here that the best parent lines (individuals) do not necessarily give the best hybrid cross, likewise the best sole crop varieties need not perform the best in an intercropping mixture.

4.1 Response Model for Varietal Mixtures

In Chapter 3 we assumed that we had information on the best pair of varieties and our aim was to get an idea of the optimum density for each crop when grown as a mixture. However, this is an unlikely case and in practice we may have to choose between v_a varieties of crop A and v_b varieties of crop B. Let us first consider the case where all possible pairs of combinations of the varieties are grown, assuming for the moment that optimum densities for the individual varieties are known. This assumption will be relaxed later.

Suppose all possible combinations of v_a varieties of A (A_1, A_2, ..., A_{v_a}) and v_b varieties of B (B_1, B_2, ..., B_{v_b}) are made. The response model for the LER values can be written as follows:

$$y_{ijr} = \mu + \alpha_i + \gamma_j + (\alpha\gamma)_{ij} + \varepsilon_{ijr} \qquad (4.1)$$

where y_{ijr} is the LER value for A_i in mixture with B_j in the r^{th} replicate; $i = 1, 2, \ldots, v_a$; $j = 1, 2, 3, \ldots, v_b$; $r = 1, 2, \ldots, n$; μ is the overall mean effect. α_i can be termed the general mixing effect (g · m · e) of the i^{th} variety, i.e. A_i, of crop A ($\sum_{i=1}^{v_a} \alpha_i = 0$). γ_j is the g · m · e of the j^{th} variety, i.e., B_j, of crop B ($\sum_j \gamma_j = 0$). $(\alpha\gamma)_{ij}$ is the specific mixing effect (s · m · e) of A_i when grown in mixture with B_j ($\sum_i (\alpha\gamma)_{ij} = 0$; $\sum_j (\alpha\gamma)_{ij} = 0$). ε_{ijr} is the random component of variation.

An assessment of the α_i's, γ_j's and $(\alpha\gamma)_{ij}$'s will provide a better understanding of how each variety is performing in a mixture. In most cases one is, obviously interested in the "best" mixture (A_i, B_j), i.e. a mixture with the highest expected LER given by

$$E(LER) = \mu + \alpha_i + \gamma_j + (\alpha\gamma)_{ij} \qquad (4.2)$$

This can be achieved in several ways which have different implications from a practical point of view. For example, the dominant term in (4.2) may be $(\alpha\gamma)_{ij}$, the specific mixing effect, in which case the particular mixture (A_i, B_j) is to be preferred. Another possibility is that one of the general mixing effects, say α_i, is very large, then mixtures involving A_i will generally be good, i.e. (A_i, B_ℓ) with $\ell = 1, 2, \ldots, v_b$ may be possible mixtures to be considered. In general, this assessment will be useful in selecting a few varieties of each crop for further experimentation such as stability analysis, which is discussed in Chapter 5.

4.2 Varietal Experiments in Conjunction with Varying Densities

As mentioned earlier, some of the varietal combinations may do better at a slightly higher density than others. Therefore varietal selection has to be done in conjunction with varying densities of each crop. The densities that are to be used in the experiment can however, be selected on the basis of the preliminary density trial discussed in Chapter 3.

Suppose in our experiment we have all possible combinations of v_a varieties A and v_b varieties of B, and we have selected P density combinations (see Chapter 3). Then the total number of "treatments" for the experiment is $T = P \cdot v_a \cdot v_b$. The problem then arises how these should be arranged in a field experiment. Due to practical considerations the density combinations have to be laid out as main plots (in a split plot framework). The $v_a v_b$ varietal combinations then occur within each density combination (main plot), and as a means of reducing the number of guards will be laid out as strips across the main plots. Up to this point the design resembles a typical strip plot design. However, as the number of varietal combinations in the experiment increases, the block size also increases. To keep the block size small, some type of confounding has to be performed. We may want to sacrifice information on the specific mixing effects to do this, in which case we confound the A × B interaction with blocks.

If v_a is not equal to v_b, we are dealing with a mixed factorial. Confounding in such a case is related to a certain type of PBIB design, namely the GD - PBIB(2) design. However, in practice v_a and v_b can be

made equal easily, by adding or deleting a variety. If $v_a = v_b = v$, and if v is a prime number, the systems of confounding the interaction A × B are AB, AB^2, ..., AB^{v-1}. For example, if we confound AB, the set of varietal combinations that constitute each block is determined by the following set of equations.

$$\begin{aligned} x_1 + x_2 &= 0 \\ &= 1 \\ &\vdots \\ &= v-1 \end{aligned} \qquad (4.3)$$

In this case (v-1)d.f. are confounded with the blocks. The number of blocks per replicate will be v. If we needed only s blocks we can combine the sets of varietal combinations to achieve this (see example 4.1).

On the other hand if v is a prime power, we can use the method of 'pseudofactors' in confounding. The basic idea is to treat the $(p^m)^n$ factorial as a p^{nm} factorial, i.e. as a factorial system with nm factors at p levels each, by introducing so called pseudofactors. For a discussion of the method see (Hinklemann, 1983). The method is illustrated in the following example.

Example 4.1

Suppose we had four varieties of each crop. This would be a 4^2 factorial for the varietal combinations. In using pseudofactors we would treat it as a $(2^2)^2$ factorial, by using the following

correspondence for the factors and levels:

	4^2 system	2^4 system
factors	A	x_1, x_2
	B	y_1, y_2
levels	0	0, 0
	1	1, 0
	2	0, 1
	3	1, 1

The A × B interaction would then correspond to x_1y_1, x_1y_2, x_2y_1, x_2y_2, $x_1y_1y_2$, $x_2y_1y_2$, $x_1x_2y_1$, $x_1x_2y_2$ and $x_1x_2y_1y_2$ in the 2^4 system. In this representation the 9 d.f. for interaction are expressed as single d.f. in the 2^4 system. Suppose we want the number of blocks per replicate, s = 2, then we need to confound only 1 d.f. If x_1y_1 was used for confounding, the set of varietal combinations satisfying $x_1 + y_1 = 0$ are (0, 0), (2, 0), (0, 2), (2, 2), (1, 1), (3, 1), (1, 3), and (3, 3). The remaining set (0, 1), (0, 3), (1, 0), (1, 2), (2, 1), (2, 3), (3, 0), and (3, 2) satisfy $x_1 + y_1 = 1$. The block size in this case will be eight. Figure 4.1 shows the layout of the design, for a single replicate.

The partial ANOVA for this design is given in Table 4.1, where c = number of d.f. confounded with blocks and r = number of replicates.

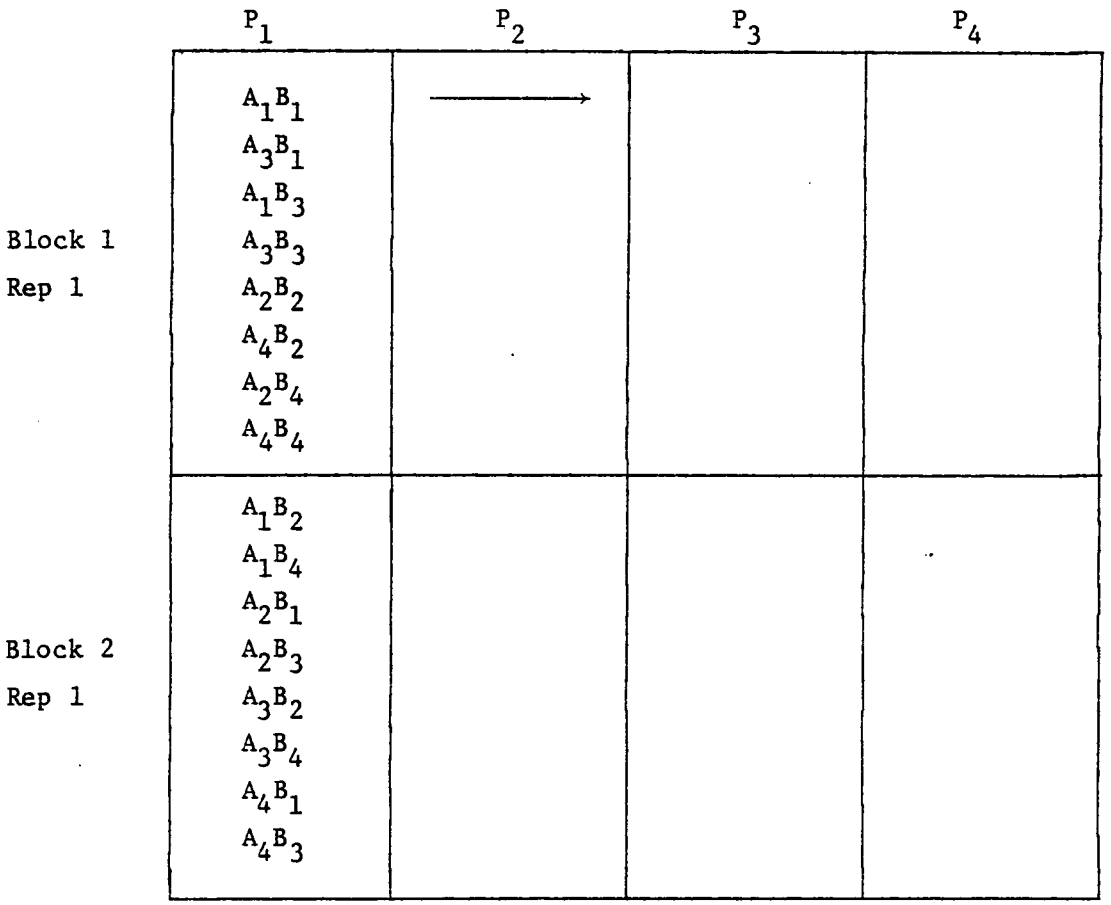


Figure 4.1. Modified Split Plot Design (P_1, P_2, P_3, P_4 density combinations in main plot; (A_iB_j) varietal combinations in strip plots).

Table 4.1. ANOVA for Modified Split Plot Design (with number of reps r = 2).

Source	d.f.	d.f. for example
Reps	(r-1)	1
Densities	P-1	3
Blocks/rep	r(s-1)	2
Error(a)	(P-1)(r-1)	3
Density × Block/rep	r(P-1)(s-1)	6
Varietal Combinations	$v_a v_b - c - 1$	14
A	$v_a - 1$	3
B	$v_b - 1$	3
A × B	$(v_a-1)(v_b-1)-c$	8
Error(b)	$(v_a v_b - c - 1)(r-1)$	14
Density × Varietal Combinations	$(P-1)(v_a v_b - c - 1)$	42
Error(ab)	$(P-1)(v_a v_b - c - 1)(r-1)$	42
Total	$v_a v_b Pr - 1$	127

4.3 Choosing a Subset of Varietal Combinations for Experimentation

The number of varietal combinations even with relatively few varieties of each crop will, in general be quite large, for a conventional block design. Even using balanced incomplete blocks in this situation will result in an extremely large number of blocks. One way to reduce the size of the experiment would be to reduce for a given number of varieties for crops A and B, the number of varietal combinations that are to be tested.. A strategy to accomplish this and its implications are discussed below.

Let us consider each variety of A in combination with only r_a varieties of crop B ($r_a < v_b$), and each variety of B in combination with r_b varieties of A ($r_b < v_a$), such that the total number of varietal combinations is $N = v_a r_a = v_b r_b$. Although there is no restriction with regard to what combinations are selected for experimentation, it is desirable to have an explicit procedure for performing such a selection. Hinkelmann (1975) gave such a method, although in a different context, namely for incomplete or partial diallel mating designs of type I. The procedure is as follows.

Consider an incomplete block design with parameters (t, r, b, k). By identifying crop A varieties with the treatment and crop B varieties with the blocks (or vice versa) and letting $t = v_a$, $r = r_a$, $b = v_b$ and $k = r_b$, we say that a varietal combination (A_i, B_j) is to be included in the experiment, if and only if treatment i occurs in block j in the incomplete block design. The types of incomplete blocks that are particularly useful for this type of selection are BIB designs and

2-associate class of PBIB designs (Hinkelmann, 1975).

Example 4.2

To illustrate the procedure of selecting varietal combinations that can be used in the subsequent experiment, consider four varieties each of crops A and B. Suppose we want to select only 12 out of the 16 possible combinations for the field experiment, such that each variety of A is in combination with 3 varieties of B. The varietal combinations can be selected by considering a BIB design (e.g. Cochran and Cox, 1957) with parameters ($t = b = 4$, $k = r = 3$). The selected combinations are marked with an x, as shown in Figure 4.2.

4.4 Design of Field Experiment with Subset of Varietal Combinations and Densities

The procedure discussed in Section 4.3, will reduce the number of varietal combinations that are to be tested in the field. However, the subset of the varietal combinations, may still be too large for a complete block design. Confounding the A × B interaction, as done for the complete set of varietal combinations (Section 4.2) cannot be used for the subset of varietal combinations, as some of the varietal combinations are absent. An alternative is then, to use an incomplete block for the subset of varietal combinations, and to assume that the $s \cdot m \cdot e$ are either negligible or random. Here again we are sacrificing information on the $s \cdot m \cdot e$, to obtain estimates for the $g \cdot m \cdot e$. If P is the number of density combinations in the experiment then the total number of 'treatments' for the experiment is $T = NP = v_a r_a P = v_b r_b P$. As

crop B varieties \ crop A varieties	1	2	3	4
1	x	x		x
2		x	x	x
3	x		x	x
4	x	x	x	

Figure 4.2. Scheme for Selecting Varietal Combinations (x denote combination selected).

done previously, the density combinations are laid out as main plots in the field. The N varietal combinations are laid out as strips across the main plots in incomplete blocks (see example 4.3).

The model for the experiment is

$$y_{mqij\ell s} = \mu + \rho_q + \lambda_m + w_{qm} + \beta_\ell + \alpha_i + \gamma_j + \varepsilon_{ij\ell s} + (\lambda\alpha\gamma)_{m(ij)} + \delta_{mqij\ell s} \qquad (4.4)$$

where $y_{mqij\ell s}$ is the LER for the plot containing the i^{th} variety of crop A in mixture with the j^{th} variety of crop B, at the density m, in the ℓ^{th} block of the q^{th} replicate (of the basic pattern).

$i = 1, 2, \ldots, v_a$; $j = 1, 2, \ldots, v_b$; $\ell = 1, 2, \ldots, b/R$.

b = total number of blocks for the whole experiment.

s = 0, 1 is the number of times a given combination occurs in the incomplete block ℓ.

$m = 1, 2, \ldots, P$;

$q = 1, 2, \ldots, R$; R is the number of times the basic pattern is replicated.

w_{qm} = main plot error; $\varepsilon_{ij\ell s}$ = variety plot error; $\delta_{mqij\ell s}$ = density × super variety plot error.

Since all the density combinations (main plots) are found in the basic pattern, the estimates for the density effects $\hat{\lambda}_m$ and the s.s. for density is found by the usual methods for split plot designs. However this is not so for estimates and s.s. of the g · m · e's $\hat{\alpha}_i$ and $\hat{\gamma}_j$. To obtain estimates of α_i and γ_j we consider, that part of the

model that pertains to the varietal combination plots (since g · m · e and density effects are orthogonal to each other).

$$y_{ij\ell s} = \mu + \alpha_i + \gamma_j + \beta_k + \varepsilon_{ij\ell s} \tag{4.5}$$

Assume that the incomplete block part of the design is equireplicate and proper, i.e. block size = k and number of replicates of each varietal combination within the basic pattern = r.

In matrix notation the model is:

$$\underset{nN\ 1}{\underline{y}} = \mu \underset{nN\times 1}{\underline{I}} + \underset{Nn\times v_a}{x_\alpha}\ \underset{v_a\times 1}{\underline{\alpha}} + \underset{Nn\times v_b}{x_\gamma}\ \underset{v_b\times 1}{\underline{\gamma}} + \underset{Nn\times b}{x_\beta}\ \underset{b\times 1}{\underline{\beta}} + \underline{\varepsilon} \tag{4.6}$$

where

n = total number of replicates of each varietal combination in the whole experiment and b = total number of blocks.

$\underline{\alpha}' = (\alpha_1, \alpha_2, \ldots, \alpha_{v_a})$; $\underline{\gamma}' = (\gamma_1, \gamma_2, \ldots, \gamma_{v_b})$;

$x_\alpha = (\underline{x}_{\alpha_1}, \underline{x}_{\alpha_2}, \ldots, \underline{x}_{\alpha_{v_a}})$, each vector $\underline{x}_{\alpha_i}$ contains $r_a \cdot n$ unit elements and $n(N - r_a)$ zero elements.

$x_\gamma = (\underline{x}_{\gamma_1}, \underline{x}_{\gamma_2}, \ldots, \underline{x}_{\gamma_{v_b}})$, each vector $\underline{x}_{\gamma_j}$ contains $r_b \cdot n$ unit elements and the rest zeros.

x_β is Nn × b matrix with the 1^{st} k elements = 1 in column one and the rest zeros, the 2^{nd} set of k elements = 1 in column two and the rest zeros and so on. (The observations are assumed to be arranged according to the order of the blocks.)

The normal equations for model (4.6) are written as

$$\begin{pmatrix} \underline{I}' \\ x_\alpha' \\ x_\gamma' \\ x_\beta' \end{pmatrix} (\underline{I} \quad x_\alpha \quad x_\gamma \quad x_\beta) \begin{pmatrix} \mu \\ \underline{\alpha} \\ \underline{\gamma} \\ \underline{\beta} \end{pmatrix} = \begin{pmatrix} \underline{I}' \\ x_\alpha' \\ x_\gamma' \\ x_\beta' \end{pmatrix} \underline{y} \qquad (4.7)$$

i.e.

$$\begin{bmatrix} \underline{I}'\underline{I} & \underline{I}'x_\alpha & \underline{I}'x_\gamma & \underline{I}'x_\beta \\ x_\alpha'\underline{I} & x_\alpha'x_\alpha & x_\alpha'x_\gamma & x_\alpha'x_\beta \\ x_\gamma'\underline{I} & x_\gamma'x_\alpha & x_\gamma'x_\gamma & x_\gamma'x_\beta \\ x_\beta'\underline{I} & x_\beta'x_\alpha & x_\beta'x_\gamma & x_\beta'x_\beta \end{bmatrix} \begin{bmatrix} \mu \\ \underline{\alpha} \\ \underline{\gamma} \\ \underline{\beta} \end{bmatrix} = \begin{bmatrix} \underline{I}'\underline{y} \\ x_\alpha'\underline{y} \\ x_\gamma'\underline{y} \\ x_\beta'\underline{y} \end{bmatrix} \qquad \begin{matrix} (4.8.1) \\ (4.8.2) \\ (4.8.3) \\ (4.8.4) \end{matrix}$$

i.e.

$$\begin{bmatrix} nN & r_a n\underline{I}' & r_b n\underline{I}' & k\underline{I}'_b \\ r_a n\underline{I}_{v_a} & r_a nI & S & N_\alpha \\ v_b n\underline{I}_{v_b} & S' & r_b nI & N_\gamma \\ k\underline{I}_b & N_\alpha' & N_\gamma' & kI_b \end{bmatrix} \begin{bmatrix} \mu \\ \underline{\alpha} \\ \underline{\gamma} \\ \underline{\beta} \end{bmatrix} = \begin{bmatrix} G \\ \underline{T}_\alpha \\ \underline{T}_\gamma \\ \underline{B} \end{bmatrix} \qquad \begin{matrix} (4.9.1) \\ (4.9.2) \\ (4.9.3) \\ (4.9.4) \end{matrix}$$

where

N_α is the incidence matrix for varieties of crop A and has dimensions $v_a \times b$.

N_γ is the incidence matrix for varieties of crop B and has dimensions $v_b \times b$.

S is a $v_a \times v_b$ matrix with an entry n in the i^{th} row, j^{th} column, if the combination (A_i, B_j) is in the experiment and zero otherwise.

$\underline{I}$ is a column vector of unit elements.

G is the grand total, $\underline{T}_\alpha$ is the vector of variety A totals, $\underline{T}_\gamma$ is the vector of variety B totals and $\underline{B}$ is the vector of block totals.

Using equation (4.9.4) we see that

$$k\underline{I}_b\mu + N_\alpha'\underline{\alpha} + N_\gamma'\underline{\gamma} + k\underline{\beta} = \underline{B}$$

i.e.

$$k(\underline{I}_b\mu + \underline{\beta}) = \underline{B} - N_\alpha'\underline{\alpha} - N_\gamma'\underline{\gamma}$$

i.e.

$$\underline{I}_b\mu + \underline{\beta} = \frac{1}{k}(\underline{B} - N_\alpha'\underline{\alpha} - N_\gamma'\underline{\gamma}) \tag{4.10}$$

Equation (4.9.2) can be written as

$$r_a n\underline{I}_{v_a}\mu + r_a n\underline{\alpha} + S\underline{\gamma} + N_\alpha\underline{\beta} = \underline{T}_\alpha \tag{4.11}$$

Since $r_a n\underline{I}_{v_a} = N_\alpha\underline{I}_b$ we can rewrite (4.11) as

$$N_\alpha\underline{I}_b\mu + N_\alpha\underline{\beta} + r_a n\underline{\alpha} + S\underline{\gamma} = \underline{T}_\alpha$$

i.e.

$$N_\alpha(\underline{I}_b\mu + \underline{\beta}) + r_a n\underline{\alpha} + S\underline{\gamma} = \underline{T}_\alpha \tag{4.12}$$

Substituting from (4.10), we write (4.12) as

$$\frac{1}{k}N_\alpha(\underline{B} - N_\alpha'\underline{\alpha} - N_\gamma'\underline{\gamma}) + r_a n\underline{\alpha} + S\underline{\gamma} = \underline{T}_\alpha$$

i.e.

$$(r_a nI - \frac{1}{k}N_\alpha N_\alpha')\underline{\alpha} + (S - \frac{1}{k}N_\alpha N_\gamma')\underline{\gamma} = I_\alpha - \frac{1}{k}N_\alpha\underline{B}$$

i.e.

$$C_A\underline{\alpha} + C_{AB}\underline{\gamma} = \underline{Q}_\alpha \tag{4.13}$$

where $C_A = (r_a nI - \frac{1}{k} N_\alpha N_\alpha')$; $C_{AB} = (S - \frac{1}{k} N_\alpha N_\gamma')$ and $\underline{Q}_\alpha = \underline{T}_\alpha - \frac{1}{k} N_\alpha \underline{B}$. Similarly using equation (4.9.3) and substituting for $\underline{I}_b\mu + \underline{\beta}$ from (4.10) we obtain

$$(r_b nI - \frac{1}{k} N_\gamma N_\gamma')\underline{\gamma} + (S' - \frac{1}{k} N_\gamma N_\alpha')\underline{\alpha} = \underline{T}_\gamma - \frac{1}{k} N_\gamma \underline{B}$$

i.e.

$$C_B\underline{\gamma} + C'_{AB}\underline{\alpha} = \underline{Q}_\gamma \tag{4.14}$$

where $C_B = (r_b nI - \frac{1}{k} N_\gamma N_\gamma')$ and $\underline{Q}_\gamma = \underline{T}_\gamma - \frac{1}{k} N_\gamma \underline{B}$. From (4.13) $C_A\underline{\alpha} = \underline{Q}_\alpha - C_{AB}\underline{\gamma}$. A solution for $\underline{\alpha}$ is

$$\underline{\hat{\alpha}} = C_A^{-}(\underline{Q}_\alpha - C_{AB}\underline{\gamma}) \tag{4.15}$$

Substituting $\underline{\hat{\alpha}}$ for $\underline{\alpha}$ in equation (4.13) we obtain

$$C_B\underline{\gamma} + C'_{AB}C_A^{-}(\underline{Q}_\alpha - C_{AB}\underline{\gamma}) = \underline{Q}_\alpha$$

i.e.

$$C_B\underline{\gamma} - C'_{AB}C_A^{-}C_{AB}\underline{\gamma} = \underline{Q}_\gamma - C'_{AB}C_A^{-}\underline{Q}_\alpha$$

i.e.

$$(C_B - C'_{AB}C_A^{-}C_{AB})\underline{\gamma} = \underline{Q}_\gamma - C'_{AB}C_A^{-}\underline{Q}_\alpha$$

∴ A solution for $\underline{\gamma}$ is

$$\underline{\hat{\gamma}} = (C_B - C_{AB}C_A^{-}C_{AB})^{-}(\underline{Q}_\gamma - C_{AB}C_A^{-}\underline{Q}_\alpha)$$

i.e.

$$\underline{\hat{\gamma}} = C_B^{*-} \underline{Q}_\gamma^{*} \tag{4.16}$$

where $C^{*} = C_B - C'_{AB}C_A^{-}C_{AB}$ and $\underline{Q}_\gamma^{*} = \underline{Q}_\gamma - C'_{AB}C_A^{-}\underline{Q}_\alpha$. Similarly a solution for $\underline{\alpha}$ is

$$\underline{\hat{\alpha}} = (C_A - C_{AB}C_B^{-}C'_{AB})^{-}(\underline{Q}_\alpha - C_{AB}C_B^{-}\underline{Q}_\gamma)$$

i.e.

$$\underline{\hat{\alpha}} = C_A^{*-}\underline{Q}_\alpha^{*} \tag{4.17}$$

where $C_A^{*} = C_A - C_{AB}C_B^{-}C'_{AB}$ and $\underline{Q}_\alpha^{*} = \underline{Q}_\alpha - C_{AB}C_B^{-}\underline{Q}_\gamma$.

For a connected design all linear combinations of $\sum_i \lambda_i\alpha_i$ and $\sum_j \lambda_j\gamma_j$ with $\sum_i \lambda_i = 0$ and $\sum_j \lambda_j = 0$ are estimable functions. The estimate of any estimable function is invariant with respect to the solution of the reduced normal equations.

Special Case

Consider a BIB design, $N_\alpha N_\alpha'$ is the concordance matrix for varieties of crop A and will be of the form

$$\begin{bmatrix} m_\alpha & & & \underline{\lambda}_\alpha \\ & m_\alpha & & \\ & & \ddots & \\ \underline{\lambda}_\alpha & & & m_\alpha \end{bmatrix}$$

where m_α is a constant and is related to the number of times a particular variety of A occurs in the experiment, while λ_α is the number of times two varieties of A occur together in the same block (this symmetry is obtained only because we are using a BIB in the selection process as well as in the field experiment). C_A, now can be written as

$$C_A = r_a n I_k - \frac{1}{k}\begin{bmatrix} m_\alpha & m_\alpha & \cdots & \lambda_\alpha \\ \vdots & \ddots & \ddots & \vdots \\ \lambda_\alpha & \cdots & \cdots & m_\alpha \end{bmatrix}$$

$$= r_a n I - [\frac{1}{k}(m_\alpha - \lambda_\alpha)I + \frac{1}{k}\lambda_\alpha \underline{I}\underline{I}']$$

$$\therefore \quad C_A = (r_a n - \frac{m_\alpha - \lambda_\alpha}{k})I - \frac{1}{k}\lambda_\alpha \underline{I}\underline{I}'$$

If we take

$$\tilde{C}_A = C_A + \frac{1}{k}\lambda_\alpha \underline{I}\underline{I}'$$

Then

$$\tilde{C}_A^{-1} = C_A^{-} = \frac{1}{(r_a n - \frac{m_\alpha - \lambda_\alpha}{k})} I \tag{4.18}$$

Similarly

$$C_B^{-} = \frac{1}{(r_b n - \frac{m_\gamma - \lambda_\gamma}{k})} I \tag{4.19}$$

The partial ANOVA table for model (4.4) is shown in Table 4.2. The calculation of s.s. for density and "super" varieties follow the usual methods for strip plot and BIB designs respectively. The s.s. for varieties of crop A eliminating blocks and varieties of B is given by

$$SS(A) = \frac{1}{P}\,\underline{\hat{\alpha}}'\underline{Q}_{\alpha}^{*}$$

where P = number of density combinations, and the s.s. for varieties of crop B eliminating blocks and varieties of A is given by

$$SS(B) = \frac{1}{P}\,\underline{\hat{\gamma}}'\underline{Q}_{\gamma}^{*} \ .$$

In Table (4.2), k refers to the block size, b is the number of blocks in the whole experiment, n is the number of replicates of each variety combination in the whole experiment, and R is the number of replicates of the basic pattern.

Example 4.3

Consider an experiment with three varieties each of crops A and B. Using the selection procedure (Section 4.3) say we have selected the varietal combinations A_1B_1, A_1B_2, A_2B_2, A_2B_3, A_3B_1 and A_3B_3 for field experimentation, i.e. $v_a = v_b = 3$; $r_a = r_b = 2$; $N = r_a v_a = r_b v_b = 6$. If these combinations are labelled as 'super' varieties 1, 2, ... , 6; we can set up a correspondence between the 'super' varieties and varietal combinations as follows:

Table 4.2. Partial ANOVA for Model (4.4).

Source	d.f.	
Replicates	R-1	
Densities	P-1	
Rep × densities (Error(a))	(R-1)(P-1)	
Blocks/Reps.	b-R	
Densities × Block/Reps.	(P-1)(b-R)	
'Super' varieties	N-1	
A eliminating blocks and B		v_a-1
B eliminating blocks and A		v_b-1
residual		$(N-1)-(v_a-1)-(v_b-1)$
'Super' variety plot error (error(b))	b(k-1)-(N-1)	
'Super' variety × density	(N-1)(P-1)	
Error(ab)	b(k-1)(P-1)-(N-1)(P-1)	
Total	PNn-1	

Super variety	Varietal combination
1	A_1B_1
2	A_1B_2
3	A_2B_2
4	A_2B_3
5	A_3B_1
6	A_3B_3

Using a catalog for BIB designs (e.g. Cochran and Cox, 1957) we see that for $t = 6$, a BIB design exists with the following parameters.

$$BIB(t = 6,\ b = 10,\ k = 3,\ \lambda = 2)$$

The arrangement of the 'super' varieties within each block of the BIB design and the corresponding arrangement of the varietal combinations are shown in Figure 4.3.

The incidence matrices for varieties of crop A and B are respectively,

$$N_\alpha = \begin{bmatrix} 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 2 & 2 \end{bmatrix}$$

$$N_\gamma = \begin{bmatrix} 2 & 1 & 1 & 1 & 2 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 2 & 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 1 & 2 \end{bmatrix}$$

Hence

$$N_\alpha N_\alpha' = \begin{bmatrix} 14 & 8 & 8 \\ 8 & 14 & 8 \\ 8 & 8 & 14 \end{bmatrix}$$

	'super' varieties			varietal combinations		
Blocks 1	1	2	5	A_1B_1	A_1B_2	A_3B_1
2	1	2	6	A_1B_1	A_1B_2	A_3B_3
3	1	3	4	A_1B_1	A_2B_2	A_2B_3
4	1	3	6	A_1B_1	A_2B_2	A_3B_3
5	1	4	5	A_1B_1	A_2B_3	A_3B_1
6	2	3	4	A_1B_2	A_2B_2	A_2B_3
7	2	3	5	A_1B_2	A_2B_2	A_3B_1
8	2	4	6	A_1B_2	A_2B_3	A_3B_3
9	3	5	6	A_2B_2	A_3B_1	A_3B_3
10	4	5	6	A_2B_3	A_3B_1	A_3B_3
	(a)			(b)		

Figure 4.3. Arrangement of (a) super varieties (b) varietal combinations within each block of a BIB design.

and

$$N_\gamma N_\gamma' = \begin{bmatrix} 14 & 8 & 8 \\ 8 & 14 & 8 \\ 8 & 8 & 14 \end{bmatrix}$$

$$S = \begin{bmatrix} 5 & 5 & 0 \\ 0 & 5 & 5 \\ 5 & 0 & 5 \end{bmatrix}$$

Note that in the matrix S, zeros appear in positions (1, 3), (2, 1), and (3, 2) representing the missing combinations and the rest of the entries are equal to five, which is the number of times each varietal combination is replicated.

$$N_\alpha N_\gamma' = \begin{bmatrix} 11 & 11 & 8 \\ 8 & 11 & 11 \\ 11 & 8 & 11 \end{bmatrix} \qquad N_\gamma N_\alpha' = \begin{bmatrix} 11 & 8 & 11 \\ 11 & 11 & 8 \\ 8 & 11 & 11 \end{bmatrix}$$

$$C_A = C_B = 8\, I_{v_a} - \frac{8}{3}\, \underline{I}\, \underline{I}'_{v_a}$$

$$\therefore\ \tilde{C}_A = \tilde{C}_B = 8\, I_{v_a}$$

$$\therefore\ C_A^{-} = C_B^{-} = \frac{1}{8}\, I_{v_a} = \begin{bmatrix} \frac{1}{8} & 0 & 0 \\ 0 & \frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{8} \end{bmatrix}$$

The reason for C_A being equal to C_B is a result of considering equal number of varieties for each crop, and should be kept in mind by the

researcher, since it is easy to make the number of varieties of each crop equal, by adding or deleting a variety. The symmetry in the concordance matrices $N_\alpha N_\alpha'$ and $N_\gamma N_\gamma'$ is due to the use of a BIB design for the field experiment as well as for the selection procedure.

In model (4.4) we assumed that the s · m · e were either negligible or random. Therefore we do not have estimates of the s · m · e. However, based on the g · m · e of each variety we may judge which varieties are useful to be included in a mixture. For instance, if the g · m · e of the h^{th} variety of crop A and the g^{th} variety of crop B are very high, we use the combination (A_h, B_g) in future experimentation, even though it is not used in the present experiment.

4.5 Outline of Strategy in a Series of Intercropping Experiments

If a researcher in intercropping is confronted with the problem of no knowledge of the varietal combinations and densities to be used in an experiment, then the following strategy can be adopted.

(i) Use a particular pair of varieties (the ones commonly used by farmers as intercrops) and perform the preliminary density trial discussed in Chapter 3.

(ii) Choose a few density combinations from the results of the preliminary density trial and at each density combination, include the varietal combinations that are needed to be tested, as discussed in this chapter.

(iii) From (i) and (ii) we can get an idea as to which varietal combinations will perform well in a mixture and at what density. These

combinations can then be used in conventional block designs to study for example yield stability, which is discussed in Chapter 5.

V. YIELD STABILITY

One of the major reasons why intercropping is becoming an important practice in many developing areas of the world, is the improved stability of yields that farmers achieve. The basis of this is that if one crop fails or grows poorly, the other crop to some extent may compensate for that (Rao and Willey, 1980).

5.1 Definition of Stability

Stability of performance in a crop is the ability of the crop to show a minimum of interaction with the environment, i.e. less variability of yields over different seasons and/or locations.

In monocropping, the agronomist is interested in selecting for varieties that are stable. In intercropping, selection of suitable varieties of the two crops A and B is not simply a case of selecting the best variety of each crop. As pointed out earlier the varietal combination that does best in intercropping need not consist of those used in monocropping. Thus selecting varieties for intercropping involves a new round of experimentation. In trying to derive stability parameters for intercropping, it is first necessary to look at stability measures that have been used for comparing varieties in monocropping.

5.2 Stability Parameters in Monocropping

Finlay and Wilkinson (1963) computed a linear regression coefficient of mean individual yield of each variety on the mean yield of all

varieties for each site and season. The regression coefficient was then used as a measure of stability. Eberhart and Russel (1966) developed a more elaborate method based on the regression technique. They used the regression coefficient of a variety on the environmental index (calculated as the mean of all varieties at a particular environment minus the grand mean of all environments), and the deviation from regression as parameters for measuring stability of a variety. The mean yield of all varieties at each site and season provides a numerical grading of sites and seasons, and is used to describe a complex natural environment without the complexities of defining or analyzing the interacting edaphic and seasonal factors (Finlay and Wilkinson, 1963).

A more formal statement of this procedure is as follows. In regional variety trials with v varieties and ℓ environments (where environment can represent location and/or time) the ANOVA model is:

$$y_{ikr} = \mu + \gamma_i + \eta_k + (\gamma\eta)_{ik} + \varepsilon_{ikr} \qquad (5.1)$$

where y_{ikr} is the yield of the i^{th} variety in the r^{th} replicate of the k^{th} environment ($i = 1, 2, \ldots, v$; $k = 1, 2, \ldots, \ell$; and $r = 1, 2, \ldots, n$).

γ_i = effect of i^{th} variety.

η_k = effect of k^{th} environment.

$(\gamma\eta)_{ik}$ = interaction effect between i^{th} variety and k^{th} environment.

ε_{ikr} = random error component.

For stability analysis the interaction term is partitioned into two components, namely the linear response to environment and the

deviation from linear response. Let β_i' be the coefficient of linear response and δ_{ik} ($k = 1, 2, \ldots, \ell$) the deviation from linear response of the i^{th} variety; then

$$(\gamma\eta)_{ik} = \beta_i'\eta_k + \delta_{ik}$$

Model (5.1) can then be rewritten as

$$y_{ikr} = \mu + \gamma_i + \eta_k + \beta_i'\eta_k + \delta_{ik} + \varepsilon_{ikr} \tag{5.2}$$

or equivalently as

$$y_{ikr} = \mu + \gamma_i + \beta_i\eta_k + \delta_{ik} + \varepsilon_{ikr} \tag{5.3}$$

where $\beta_i = \beta_i' + 1$.

If in model (5.3) η_k is replaced by I_k, where I_k is the environmental index and given by

$$I_k = \bar{y} \cdot k \cdot - \bar{y} \ldots = \hat{\eta}_k$$

Model (5.3) becomes

$$y_{ikr} = \mu + \gamma_i + \beta_i I_k + \delta_{ik} + \varepsilon_{ikr} \tag{5.4}$$

An estimator for β_i is taken to be

$$b_i = \sum_k \bar{y}_{ik\cdot} I_k / \sum_k I_k^2 \tag{5.5}$$

with

$$\bar{b} = \sum_i b_i / v = 1$$

The first stability measure is the estimate of the regression coefficient β_i. A second stability measure can be obtained by considering δ_{ik} which represents the deviation from linear response. An obvious estimator for δ_{ik} is

$$\hat{\delta}_{ik} = (\bar{y}_{ik\cdot} - \bar{y}_{i\cdot\cdot}) - b_i I_k \tag{5.6}$$

$$\therefore \sum_k \hat{\delta}_{ik}^2 = \sum_k (\bar{y}_{ik\cdot} - \bar{y}_{i\cdot\cdot})^2 - b_i^2 \sum_k I_k^2$$

If we define

$$S_{d_i}^2 = \sum_k \hat{\delta}_{ik}^2/(\ell-2) \ , \tag{5.7}$$

then

$$\lambda_i = \frac{S_{d_i}^2}{MS(error)/n} \tag{5.8}$$

is considered as another stability measure, the meaning of which is explained in Section 5.3.

5.3 Interpretation of the Stability Measures

A stable variety is a variety that performs relatively the same under a wide range of environments. This means that a stable variety will have a regression coefficient $b_i = 0$ (this is equivalent to stating that the environment has no effect on that variety). However, this would mean that the stable variety performs relatively better under adverse

conditions and not so well in favorable environments. However, the breeder usually wants a variety that does above average in all environments, i.e. one with $b_i = 1.0$ and the deviations from regression as small as possible (i.e. $\lambda_i = 1$), since $\lambda_i = 1$ implies that $\delta_{i1} = \delta_{i2} = \ldots = \delta_{ik} = 0$ and hence a perfect fit to the linear regression. A variety having the values ($b_i = 1$, $\lambda_i = 1$) will be referred to as having average stability, while one with ($b_i = 0$, $\lambda_i = 1$) will be called a perfectly stable variety. Of course in addition to ($b_i = 1$, $\lambda_i = 1$) the breeder also wants the variety to be a high yielder. Finlay and Wilkinson (1963) categorized varieties on the basis of their mean yield and regression coefficients (b_i's). While a regression coefficient $b_i = 1$ is taken to be indicative of average stability, $b_i < 1$ is regarded as above average stability and $b_i > 1$ as below average stability. A high mean yield in addition to $b_i = 1$ indicates a variety having general adaptability; when associated with a low mean yield, varieties are regarded as poorly adapted to all environments. Figure 5.1 provides a schematic version of the above categorization, based on a survey of a large number of varieties. Figure 5.1 shows that varieties that give a high yield, do not possess above average stability. It also reveals that variability in the regression coefficients is inversely proportional to the mean yield, as indicated by the triangular configuration in Figure 5.1.

5.4 Derivation of Stability Measures in Intercropping

To derive stability measures for intercropping, consider an intercropping experiment with two crops A and B, A having v_a varieties

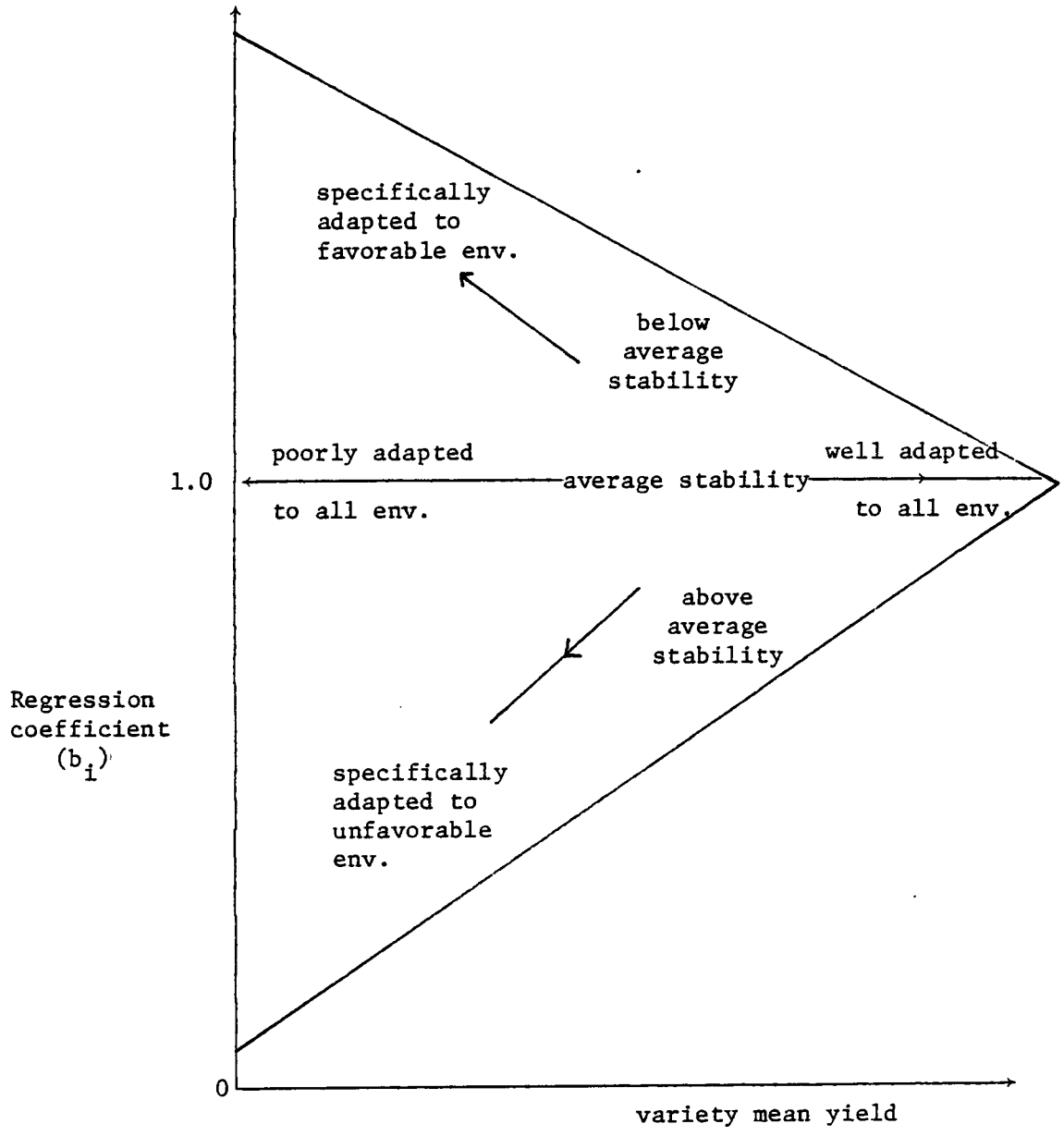


Figure 5.1. Categorization of Varieties According to Mean Yield and Regression Coefficient (b_i).

$A_1, A_2, \ldots, A_{v_a}$ and B having v_b varieties $B_1, B_2, \ldots, B_{v_a}$. The treatment combinations consist of the $v_a \times v_b$ varietal combinations (A_i, B_j). The experiment is to be repeated over ℓ environments. Within an environment, each varietal combination is replicated n times. For the sake of simplicity a completely randomized design is considered.

If each varietal combination is considered as a 'super variety', then a linear model for this experiment can be written as:

$$y_{ijkr} = \mu + \tau_{ij} + \eta_k + (\tau\eta)_{ijk} + \varepsilon_{ijkr} \qquad (5.9)$$

where y_{ijkr} is the combined intercrop yield (LER) of the plot containing the combination (A_i, B_j) in the r^{th} replicate of the k^{th} environment $(i = 1, 2, \ldots, v_a; j = 1, 2, \ldots, v_b; k = 1, 2, \ldots, \ell; r = 1, 2, \ldots, n)$.

The interaction term is partitioned into the two components, i.e. linear response to environment and the deviation from linear response, as a means of obtaining stability parameters.

$$(\tau\eta)_{ijk} = B^*_{ij}\eta_k + \delta_{ijk}.$$

Model (5.9) can then be written as

$$y_{ijkr} = \mu + \tau_{ij} + B_{ij}\eta_k + \delta_{ijk} + \varepsilon_{ijkr} \qquad (5.10)$$

where $B_{ij} = B^*_{ij} + 1$.

Model (5.10) resembles model (5.3) used for sole crops, except that instead of a single subscript i used for sole crops, we now have two subscripts ij to denote the varietal combination. Therefore formulas for b_{ij} and λ_{ij} will be similar in form to b_i and λ_i.

The regression coefficient b_{ij} is a measure of overall stability of the varietal combination (A_i, B_j). If $b_{ij} \approx 1$, the varietal combination can be referred to as having average stability, while $b_{ij} < 1$ is indicative of above average stability and $b_{ij} > 1$ is indicative of below average stability.

5.5 Partitioning the S.S.

Consider the identity,

$$(y_{ijkr} - \bar{y}....) = (\bar{y}_{ij}.. - \bar{y}....) + (\bar{y}..{}_k. - \bar{y}....) + (b_{ij} - 1)(\bar{y}..{}_k. - \bar{y}....)$$

$$+ [(\bar{y}_{ijk}. - \bar{y}_{ij}..) - b_{ij}(\bar{y}..{}_k. - \bar{y}....) + (y_{ijkr} - \bar{y}_{ijk}.)]$$

i.e.

$$(y_{ijkr} - \bar{y}....) = \hat{\tau}_{ij} + I_k + (b_{ij} - 1)I_k + \hat{\delta}_{ijk} + e_{ijkr} \qquad (5.11)$$

Squaring both sides of (5.11) and summing over all subscripts gives rise to

$$\sum_{ijkr} (y_{ijkr} - \bar{y}....)^2 = \sum_{ijkr} \hat{\tau}_{ij}^2 + \sum_{ijkr} I_k^2 + n \sum_{ij} (b_{ij} - 1)^2 \sum_{k} I_k^2$$

$$+ \sum_{ijkr} \hat{\delta}_{ijk} + \sum_{ijkr} e_{ijkr}^2 \qquad (5.12)$$

which represents a partition of the total sum of squares.

Simplifying (5.12), we obtain

$$\sum_{ijkr} (y_{ijkr} - \bar{y}....)^2 = \ell n \sum_{ij} \hat{\tau}_{ij}^2 + v_a v_b n \sum_{k} I_k^2$$

$$+ n \sum_{ij} (b_{ij}-1)^2 \sum_{k} I_k^2 + n \sum_{ijk} \hat{\delta}_{ijk}^2 + \sum_{ijkr} e_{ijkr}^2 \qquad (5.13)$$

Table 5.1 shows the breakdown of d.f. and s.s. for model (5.9).

To test the hypothesis

$$H_0: \beta_{11} = \beta_{12} = ... = \beta_{v_a v_b} = 1 ,$$

we can use

$$F = \frac{MS(slopes)}{[ss(deviations)+ss(error)]/[(v_a v_b-1)(\ell-2) + v_a v_b \ell(n-1)]} \qquad (5.14)$$

and to test H_0: all δ_{ijk}'s are zero, we can use

$$F = \frac{MS(deviations)}{MS(error)}$$

obtained from Table 5.1.

5.6 Subdivision of Slope (SS)

In evaluating the best varietal combination it is also necessary to look at the mean performance of that combination in addition to using the stability measures b_{ij} and λ_{ij}. It should be noted however, that b_{ij} is a measure of overall stability of the intercrop mixture, and does not give any information on the contribution of the individual varieties of each crop to the stability. For this we propose to take into

Table 5.1. Breakdown of s.s. for model (5.9).

Source	d.f.	s.s.
Varietal combinations	$v_a v_b - 1$	$\ell n \sum_{ij} \hat{\tau}_{ij}^2$
Environments	$\ell - 1$	$v_a v_b n \sum_{k} I_k^2$
Var. combinations × Env.	$(v_a v_b - 1)(\ell - 1)$	$n \sum_{ijk} (\hat{\tau\eta})_{ijk}^2$
slopes	$v_a v_b - 1$	$n \sum_{ij} (b_{ij} - 1)^2 \Sigma I_k^2$
deviations	$(v_a v_b - 1)(\ell - 2)$	$n \sum_{ijk} \hat{\delta}_{ijk}^2$
Error	$v_a v_b \ell(n-1)$	$\sum_{ijkr} e_{ijkr}^2$
Total	$v_a v_b \ell n - 1$	$\sum_{ijkr} (y_{ijkr} - \bar{y}_{....})^2$

account the factorial structure of the experiment. We now write τ_{ij} in model (5.10) as

$$\tau_{ij} = \alpha_i + \gamma_j + (\alpha\gamma)_{ij} \tag{5.15}$$

where

α_i is the effect of the i^{th} variety of A,

γ_j is the effect of the j^{th} variety of B, and

$(\alpha\gamma)_{ij}$ is the interaction effect due to the i^{th} variety of A and j^{th} variety of B ($i = 1, 2, \ldots, v_a$; $j = 1, 2, \ldots, v_b$). Model (5.10) can be restructured and written as

$$y_{ijkr} = \mu + \alpha_i + \gamma_j + (\alpha\gamma)_{ij} + \eta_k + (\alpha\eta)_{ik} + (\gamma\eta)_{jk} + (\alpha\gamma\eta)_{ijk} + \varepsilon_{ijkr} \tag{5.16}$$

We now partition each interaction term into two components, the linear response to environment and the deviation from linear response, i.e.

$$(\alpha\eta)_{ik} = \beta_i'\eta_k + \delta'_{ik} \tag{5.17}$$

$$(\gamma\eta)_{jk} = \theta_j'\eta_k + \delta'_{jk} \tag{5.18}$$

and

$$(\alpha\gamma\eta)_{ijk} = \beta_{ij}'\eta_k + \delta'_{ijk} \tag{5.19}$$

Combining (5.17), (5.18), and (5.19) we have

$$(\alpha\eta)_{ik} + (\gamma\eta)_{jk} + (\alpha\gamma\eta)_{ijk} = \beta^*_{ij}\eta_k + \delta_{ijk} \qquad (5.20)$$

where $\beta^*_{ij} = \beta_i' + \theta_j' + \beta_{ij}'$ and $\delta_{ijk} = \delta_{ik}' + \delta_{jk}' + \delta_{ijk}'$. To obtain estimates of β_i', θ_j' and β_{ij}' and also to partition ss(slopes) into the three different components, ss(slopes) due to A, B, and the A × B interaction, we proceed as follows.

Assuming a fixed effects model, and averaging over crop B, model (5.16) reduces to

$$\bar{y}_{i\cdot kr} = \mu + \alpha_i + \eta_k + (\alpha\eta)_{ik} + \bar{\varepsilon}_{i\cdot kr} \qquad (5.21)$$

Using the environmental index $I_k = \bar{y}_{\cdot\cdot k\cdot} - \bar{y}_{\cdots\cdot}$, instead of η_k and using relation (5.17), model (5.21) can be rewritten as

$$\bar{y}_{i\cdot kr} = \mu + \alpha_i + \beta_i I_k + \delta_{ik}' + \bar{\varepsilon}_{i\cdot kr} \qquad (5.22)$$

where $\beta_i = \beta_i' + 1$.

An estimator for β_i is $b_i = \sum_k \bar{y}_{i\cdot k\cdot} I_k / \sum_k I_k^2$. Similarly averaging yields over crop A (subscript i) gives rise to the model

$$\bar{y}_{\cdot jkr} = \mu + \gamma_j + \theta_j I_k + \delta_{jk}' + \bar{\varepsilon}_{\cdot jkr} \qquad (5.23)$$

An estimator for θ_j is $t_j = \sum_k \bar{y}_{\cdot jk\cdot} I_k / \sum_k I_k^2$. Finally, β_{ij}' can be estimated by $b_{ij}' = (b_{ij}-1) - (b_i-1) - (t_j-1)$.

Table 5.2 shows in detail how each interaction term is partitioned into the slopes and deviation s.s. components. SS(slopes) in Table 5.1, is partitioned into $ss(slopes)_A$, $ss(slopes)_B$ and $ss(slopes)_{AB}$. The deviations (s.s.) is also partitioned into three components, namely

Table 5.2. Partition of ss(slopes).

Source	d.f.	s.s
A × Envir.	$(v_a-1)(\ell-1)$	$v_b n \sum_{ik} (\hat{\alpha\eta})_{ik}^2$
slopes(A)	(v_a-1)	$v_b n \sum_i (b_i-1)^2 \sum_k I_k^2$
deviations(A)	$(v_a-1)(\ell-2)$	$v_b n \sum_{ik} (\hat{\delta}'_{ik})^2$
B × Envir.	$(v_b-1)(\ell-1)$	$v_a n \sum_{jk} (\hat{\alpha\eta})_{jk}^2$
slopes(B)	(v_b-1)	$v_a n \sum_j (t_j-1)^2 \sum_k I_k^2$
deviations(B)	$(v_b-1)(\ell-2)$	$v_a n \sum_{jk} (\hat{\delta}'_{jk})^2$
A × B × Envir.	$(v_a-1)(v_b-1)(\ell-1)$	$n \sum_{ijk} (\hat{\alpha\gamma\eta})_{ijk}^2$
slopes(A×B)	$(v_a-1)(v_b-1)$	by subtraction
deviations(A×B)	$(v_a-1)(v_b-1)(\ell-2)$	by subtraction

deviations from linear regression of A, B, and A × B. Notice that estimators of δ_{ik}' and δ_{jk}' are given by

$$\hat{\delta}_{ik}' = (\bar{y}_{i\cdot k\cdot} - \bar{y}_{i\ldots}) - b_i(\bar{y}_{\cdot\cdot k\cdot} - \bar{y}_{\ldots\cdot})$$

and

$$\hat{\delta}_{jk}' = (\bar{y}_{\cdot jk\cdot} - \bar{y}_{\cdot j\cdot\cdot}) - t_j(\bar{y}_{\cdot\cdot k\cdot} - \bar{y}_{\ldots\cdot}) \qquad (5.25)$$

respectively.

In Table 5.2, slopes(A × B) and deviations(A × B) are obtained by using the relationships given below.

$$\text{ss(slopes) in Table 5.1} = \text{ss(slopes)}_A + \text{ss(slopes)}_B + \text{ss(slopes)}_{A\times B} \qquad (5.26)$$

and

$$\text{ss(deviations) in Table 5.1} = \text{ss(dev.)}_A + \text{ss(dev.)}_B + \text{ss(dev.)}_{A\times B} \qquad (5.27)$$

Using equation (5.26), we can find out which crop is contributing most, to the ss(slopes). Examination of the estimates b_i, t_j and b_{ij}' also allows us to draw conclusions about which crop is contributing to the stability of the intercropping mixture. It may happen that the contribution of each crop to the stability is of no consequence, but it is the interaction of the crops that leads to the stability of the intercropping mixture.

For example, let $b_{ij} = 1.0$, $b_i = .6$, $t_j = 1.4$ and $(b_{ij}' + 1) = 1.0$, for a particular varietal combination (A_i, B_j). Note that $b_{ij} = (b_i - 1) + (t_j - 1) + b_{ij}' + 1$. The overall stability measure $b_{ij} = 1$ tells us that the varietal combination possesses average stability. However, the variety B_j of crop B, has less than average stability ($t_j > 1$) and variety A_i of crop A, has more than average stability ($b_i < 1$), while

the interaction of A_i and B_j is associated with an average stability value ($b_{ij}' + 1 = 1$). In this case, while the two crops have opposing stability indices, the interaction of the two varieties A_i and B_j of crops A and B respectively, lead to a mixture possessing average stability.

In passing it must be mentioned that since we utilized the varietal selection procedure of Chapter 4, in selecting a few varieties of each crop for stability analysis, we can use a randomized complete block design at each environment. Since the environment is to be measured as the location mean minus the overall mean, it is necessary to have as many locations as possible, to capture the variability of the environment. Since seasonal variability is a principle factor of the environment it will also be useful to repeat the experiment in time.

VI. MIXTURE PROPORTIONS IN INTERCROPPING

Choosing optimal proportions for the crops to be grown in a mixture, is an important aspect of intercropping research. However, very little work on designing experiments to find the optimal proportions has been done. Pearce and Edmondson (1982) have examined historical data on yields of the individual species that enter into an intercropping system, and thus determined the best proportions for each crop. But, it is not clear what would happen if they were to be grown together as intercrops.

There is a large body of literature on mixture problems in general. An excellent review is given by Cornell (1981). The optimum proportion problem in intercropping can also be put into the framework of the general mixture problem with very few modifications.

6.1 The General Mixture Problem

In the general mixture problem, the response that is measured is a function only of the proportions of ingredients (or components) present in the mixture and is not a function of the amount of the mixture. If we let q represent the number of components in the system and if we denote the proportion of the i^{th} component in the mixture by Z_i, then

$$Z_i \geq 0 \quad i = 1, 2, \ldots, q \tag{6.1}$$

and

$$\sum_{i=1}^{q} Z_i = 1.0 \ . \qquad (6,2)$$

For the mixture proportion problem in intercropping, if we consider two crops A and B, the number of components, q, is two. The proportion of crop A in the mixture can be represented by Z_1 and the proportion of crop B in the mixture by Z_2. In the intercropping context, the proportion of a crop, Z_i, in the mixture, can be defined as

$$Z_i = \frac{\text{number of rows occupied by crop } i}{\text{total number of rows}}$$

and $\sum_{i=1}^{2} Z_i = 1.0$, satisfying the restriction in (6.2). Thus a ½:½ proportion of crop A:B will be an experiment with every row of crop A followed by a row of crop B. A 0:1 mixture will be the pure stand of crop B and a 1:0 will be the pure stand of crop A.

One complication that arises in intercropping is that the combined yield does not only depend on the proportions of the component crops, but also on the intra- and inter-row spacings of both crops (see chapter 3). To circumvent this problem, we assume for the present that the crops are grown at their optimum density combination throughout the entire experiment.

6.2 The Simplex Lattice Design

An important design that can be used for investigation into the mixture problem is the simplex lattice design, which consists of points whose coordinates are defined by the following combinations of the

component proportions; the proportions assumed by each component will take on the m + 1 equally spaced values from zero to one given below, and m is the degree of the polynomial we want to fit.

$$Z_i = 0, \frac{1}{m}, \frac{2}{m}, \ldots, 1 \qquad (6.3)$$

The {q, m} simplex lattice consists of all possible combinations of the components where the proportions given by (6.3) are used, such that the restriction given in (6.2) is satisfied.

For the intercropping problem, say we want to fit a polynomial of degree two, i.e. m = 2. Then for each of the crops A and B, the proportions are: $Z_i = 0, \frac{1}{2}, 1$; $i = 1, 2$; $(q = 2; m = 2)$. The possible combinations (Z_1, Z_2) satisfying the restriction (6.2) are given by:

$$(Z_1, Z_2) = (0, 1), (\tfrac{1}{2}, \tfrac{1}{2}), \text{ and } (1, 0) .$$

The (1, 0) and (0, 1) represent the pure stands of crops A and B respectively and $(\frac{1}{2}, \frac{1}{2})$ the intercropping mixture with equal proportions of each crop.

6.3 Canonical Polynomials

For the second degree polynomial the response η can be written as

$$\eta = \beta_0 + \beta_1 Z_1 + \beta_2 Z_2 + \beta_{12} Z_1 Z_2 + \beta_{11} Z_1^2 + \beta_{22} Z_2^2 \qquad (6.4)$$

or equivalently as

$$\eta = \beta_0 + \sum_{i=1}^{2} \beta_i Z_i + \sum_{i=1}^{2} \beta_{ii} Z_i^2 + \beta_{12} Z_1 Z_2 \qquad (6.5)$$

Multiplying the β_0 term by $\Sigma Z_i = 1$ does not change β_0. Using also $Z_i^2 = Z_i(1 - \sum_{j \neq i} Z_j)$ in model (6.5) results in,

$$Z = \beta_0(\sum_i Z_i) + \sum_i \beta_i Z_i + \sum_i \beta_{ii} Z_i(1 - \sum_{j \neq i} Z_j) + \beta_{12} Z_1 Z_2$$

$$= \sum_{i=1}^{2} \beta_i^* Z_i - \sum_{i=1}^{2} \beta_{ii} Z_i \sum_{j \neq i} Z_j + \beta_{12} Z_1 Z_2$$

$$= \sum_i \beta_i^* Z_1 - \beta_{11} Z_1 Z_2 - \beta_{22} Z_2 Z_1 + \beta_{12} Z_1 Z_2 \quad .$$

$$= \beta_1^* Z_1 + \beta_2^* Z_2 + \beta_{12}^* Z_1 Z_2 \qquad (6.6)$$

where $\beta_1^* = \beta_0 + \beta_1 + \beta_{11}$; $\beta_2^* = \beta_0 + \beta_2 + \beta_{22}$ and $\beta_{12}^* = \beta_{12} - \beta_{11} - \beta_{22}$. Dropping the * notation model (6.6) can be written as

$$\eta = \beta_1 Z_1 + \beta_2 Z_2 + \beta_{12} Z_1 Z_2 \qquad (6.7)$$

Model (6.7) is known as the canonical polynomial form and it is in this form the model is used in estimating the β terms. In model (6.4) there are clear dependencies among the Z_i terms, since the restriction $\Sigma Z_i = 1$, is not used in that model.

When $Z_1 = 1$ in model (6.7) $Z_2 = 0$ and $\eta = \beta_1$. Therefore, β_1 represents the sole crop yield of crop A. Similarly β_2 represents the sole crop yield of B, while $\beta_{12} Z_1 Z_2$ represents the excess of model (6.7) over the following model;

$$\eta = \beta_1 Z_1 + \beta_2 Z_2 \qquad (6.8)$$

i.e. $\beta_{12}Z_1Z_2$ represents the advantage (or disadvantage) of intercropping over sole cropping.

The least squares estimates of β_1, β_2 and β_{12} are given by:

$$\hat{\beta}_1 = \bar{y}_1,\ \hat{\beta}_2 = \bar{y}_2 \text{ and } \hat{\beta}_{12} = 4\bar{y}_{12} - 2(\bar{y}_1 + \bar{y}_2) \tag{6.9}$$

where y_1 is the observed combined yield (LER) at $Z_1 = 1$ and $\bar{y}_1$ is the mean over the replicates, y_2 is the observed LER at $Z_2 = 1$ and y_{12} is the observed LER at $Z_1 = \frac{1}{2}$, $Z_2 = \frac{1}{2}$.

6.4 Estimating Optimal Intercrop Proportions

Using the Lagrangean function

$$L = \hat{\beta}_1 Z_1 + \hat{\beta}_2 Z_2 + \hat{\beta}_{12} Z_1 Z_2 + \lambda(Z_1 + Z_2 - 1)$$

and maximizing with respect to Z_1, Z_2 and λ yields,

$$\frac{\partial L}{\partial Z_1} = \hat{\beta}_1 + \hat{\beta}_{12} Z_2 + \lambda = 0 \tag{6.10}$$

$$\frac{\partial L}{\partial Z_2} = \hat{\beta}_2 + \hat{\beta}_{12} Z_1 + \lambda = 0 \tag{6.11}$$

$$\frac{\partial L}{\partial \lambda} = Z_1 + Z_2 - 1 = 0 \tag{6.12}$$

Solving the above equations, simultaneously, gives estimates of the optimal proportions Z_1^* and Z_2^*.

$$Z_1^* = \frac{\hat{\beta}_1 - \hat{\beta}_2 - \hat{\beta}_{12}}{2\hat{\beta}_{12}} \quad \text{and} \quad Z_2^* = 1 - Z_1^* .$$

Since the two crops in intercropping are grown in separate rows the values that Z_i can take are 1, 1/2, 1/3, etc., the estimates Z_i^* have to be rounded to the closest fractions for practical use. If $Z_i^* = .37$ for instance, we will use $Z_1 = 1/3$ and $Z_2 = 2/3$, i.e. 1 row of crop A alternating with 2 rows of crop B.

In the preceding sections we have assumed that the degree of the model was two (i.e. m = 2). However, if we want to test the lack of fit of the 2^{nd} degree model or to fit a 3^{rd} degree model, we need to include at least one more design point. As an additional point we can choose a point such as $(Z_1, Z_2) = (1/3, 2/3)$ or $(2/3, 1/3)$. To test the degree of the model, we can use the F test:

$$F_{r,n-p} = \frac{(SSE_{reduced} - SSE_{full\ model})/r}{SSE_{full}/n-p} \qquad (6.13)$$

where the full model refers to the higher degree polynomial equation and reduced refers to the polynomial equation that is one degree less than the full model, and r = reduction in the number of parameters; p = number of parameters in full model and n = total number of observations. Equation (6.13) can also be used to test whether we need the β_{12} term, i.e. whether the first degree polynomial $\eta = \beta_1 Z_1 + \beta_2 Z_2$ fits the data as well as the second degree model.

6.5 Studying Proportions in the Presence of Varying Densities

The optimal proportions of the crops A and B, may be different at different densities of the component crops. Studies on mixture proportions therefore, has to be performed in the presence of varying densities. The density combinations can be selected based on the preliminary trial discussed in Chapter 3. However, another complication arises when we consider mixture proportions that are not 1:1, because now we have two different types of inter-row spacings. One inter-row spacing between two rows of the same crop and another inter-row spacing between two rows of different crops. This aspect is not investigated further here. For the present study we assume that the inter-row spacing between two rows of different crops are selected on the basis of the density trial of Chapter 3, and the inter-row spacing between two rows of the same crop is selected from monocrop studies.

Consider an experiment where we want to experiment with crop B at two different densities X_1 and X_2 (number of plants/unit area), while crop A is grown at a constant density throughout the entire experiment. At each level of proportion (Z_1, Z_2), we must therefore include the two densities of crop B in the design. We cannot however, use a simplex lattice design, since at the proportion level $(Z_1, Z_2) = (1, 0)$, crop B is not present and combination of (1, 0) with different densities of crop B does not make sense. The proportion levels must contains crop B. Instead of the proportions (1, 0) and (0, 1) we can have (1/4, 3/4) and (3/4, 1/4), so that the mixture proportions used at each density of crop B will be,

$$(Z_1, Z_2) = (1/4, 3/4), (1/2, 1/2) \text{ and } (3/4, 1/4) .$$

Coded densities w = 1 for higher and w = -1 for lower density can be used in the model, instead of X_1 and X_2.

For practical reasons the field layout of the experiment needs to be of the split plot type. Proportions are used as the main plot and has to be arranged systematically, as also the densities of crop B (subplots), to avoid as much as possible the use of guard rows. The field layout is shown in Figure 6.1.

The model for the experiment is written in terms of the observed LER values (y), as

$$y = \gamma_1^0 Z_1 + \gamma_2^0 Z_2 + \gamma_{12}^0 Z_1 Z_2 + \nu + \gamma_1^1 Z_1 w + \gamma_{12}^1 Z_1 Z_2 w + \varepsilon \qquad (6.14)$$

where $\nu \sim N(0, \sigma_\nu^2)$ and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. This model is different from the types of models considered for density in Chapter 3. Here density merely appears as a type of dummy variable, as we are interested only in finding whether the optimal proportions are different at the two densities of crop B.

The split plot design is helpful in reducing the number of guards, since they are needed only at the boundaries between the different densities.

The O · L · S estimates of the parameters in model (6.14) are given below.

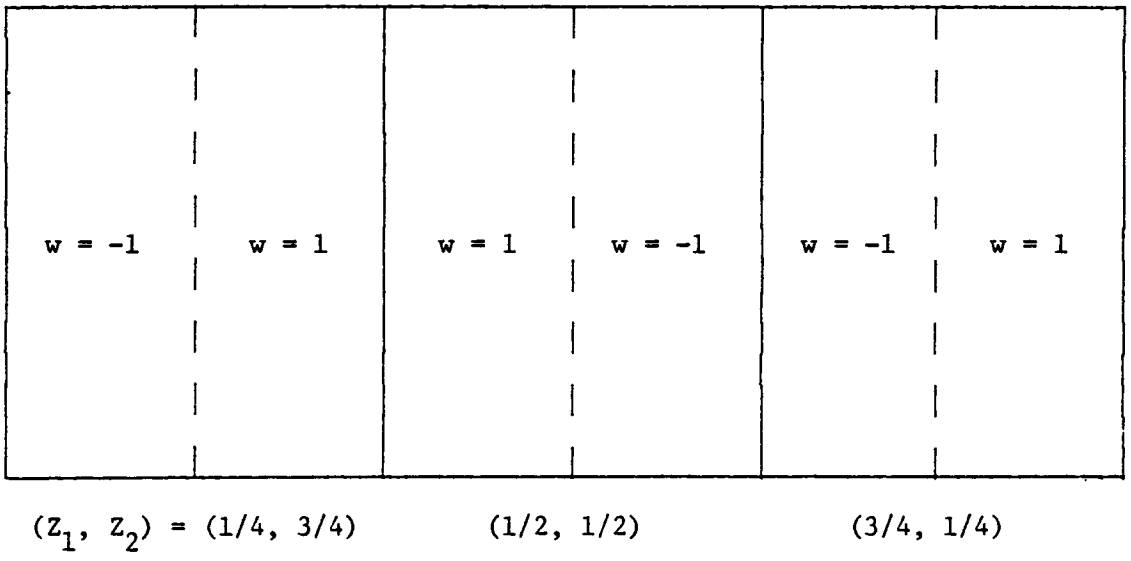


Figure 6.1. Design for Proportion Experiment, at Two Densities (w = 1 and -1) of Crop B, Shows One Replicate.

$$\hat{\gamma}_1^0 = \bar{y}_{1..} - 3(\bar{y}_{3..} - \bar{y}_{2..})$$

$$\hat{\gamma}_2^0 = \bar{y}_{2..} - 3(\bar{y}_{3..} - \bar{y}_{1..})$$

$$\hat{\gamma}_{12}^0 = 16\bar{y}_{3..} - 12(\bar{y}_{1..} + \bar{y}_{2..})$$

$$\hat{\gamma}_1^1 = 1/2(\bar{y}_{11.} - \bar{y}_{12.}) + 3/2(\bar{y}_{21.} + \bar{y}_{32.} - \bar{y}_{22.} - \bar{y}_{31.})$$

$$\hat{\gamma}_2^1 = 1/2(\bar{y}_{21.} - \bar{y}_{22.}) + 3/2(\bar{y}_{11.} + \bar{y}_{32.} - \bar{y}_{12.} - \bar{y}_{31.})$$

$$\hat{\gamma}_{12}^1 = 8(\bar{y}_{31.} - \bar{y}_{32.}) + 4(\bar{y}_{12.} + \bar{y}_{22.} - \bar{y}_{11.} - \bar{y}_{21.})$$

where y_{ijk} is the observed LER at the i^{th} proportion level, j^{th} density of crop B in the k^{th} replicate ($i = 1, 2, 3$; $j = 1, 2$; and $k = 1, 2, \ldots, r$).

We can test the hypotheses; $\gamma_1^1 = 0$ and $\gamma_{12}^1 = 0$ in the usual way to see if proportion levels are different at the two densities of crop B. If each crop is grown at two different densities, the density combinations can be arranged in a systematic manner with the main plots (proportion levels) in order to avoid the use of too many guard rows, as shown in Figure 6.2.

The model for the experiment is written in terms of the observed LER values as

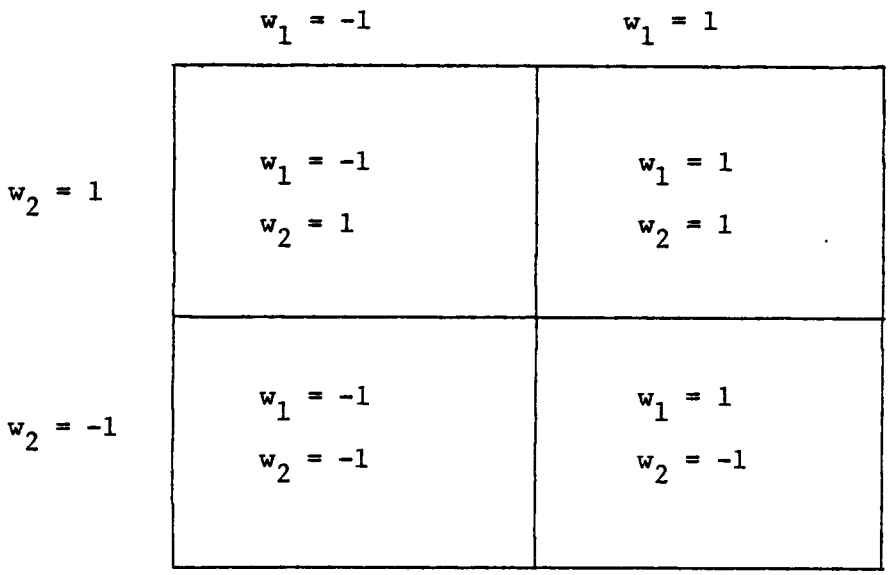


Figure 6.2. Systematic Arrangement of Densities Within a Main Plot (Proportion Level), [w_1 denote density of A, w_2 density of B].

$$y = \gamma_1^0 Z_1 + \gamma_2^0 Z_2 + \gamma_{12}^0 Z_1 Z_2 + \nu + \gamma_1^1 Z_1 w_1 + \gamma_2^1 Z_2 w_1$$

$$+ \gamma_{12}^1 Z_1 Z_2 w_1 + \gamma_1^2 Z_1 w_2 + \gamma_2^2 Z_2 w_2 + \gamma_{12}^2 Z_1 Z_2 w_2$$

$$+ \gamma_{12}^{12} Z_1 Z_2 w_1 w_2 + \varepsilon \qquad (6.15)$$

where ν and ε are as defined for model (6.14).

The O · L · S technique can be used to obtain the estimated equation. Using the Lagrangean technique outlined earlier, the best proportions of Z_1 and Z_2 can be found at each density combination.

The purpose of this chapter was merely to show how the proportion problem can be put into the general mixture framework, and to point out some of the difficulties encountered, e.g. different inter-row spacings (when neighboring rows are of the same crop and when they are of different crops). There is much scope for further research along these lines.

VII. SUMMARY AND FUTURE RESEARCH

7.1 Summary of Results

Methods of design and analysis of experiments in sole crops are well established. However, in intercropping which involves the growing of two or more crops together, research is still in its infancy. Therefore any investigation in intercropping will involve several aspects or factors each with a large number of levels. For example density trials with p levels of density for each of the two crops will result in p^2 'treatments'. Similarly a varietal trial with v varieties for each of the two crops, will involve v^2 varietal combinations. If these two factors were to be investigated together we end up with $p^2 \cdot v^2$ 'treatment' combinations. Matters will get worse if we were to consider more than two crops. The present study deals exclusively with a two-crop intercropping mixture. However, most of what is covered can be extended to more than two crops, but it may be impractical to consider more than three crops in the mixture.

The present study deals with designs in intercropping mainly from a practical point of view. Although it is theoretically possible to design experiments with many factors at different levels, it will be practically impossible to lay it out in the field, due to the large number of treatment combinations. The strategy that we use, is to first carry out preliminary trials separately for each factor or certain factor combinations involving an adequate number of levels. Using the results of the preliminary trials we isolate a few levels for

each factor, which can then be used in factorial combinations in future experimentation.

The factors considered in this study are

1. Densities:
   (i) Intra-row densities for crops A and B
   (ii) Inter-row densities
2. Varieties
3. Environments
4. Mixture proportions.

As mentioned earlier, to investigate all factors simultaneously, although statistically desirable, is practically impossible. Even the investigation of just one factor at a time may lead to large experiments, i.e. requiring large areas of land. One of the objectives of this study has therefore been to suggest reasonable reductions in the size of the experiments.

The only way that density trials can be carried out is in the form of systematic designs. To study intra-row and inter-row densities we develop a new 3-way systematic design (Chapter 3). Using different harvest areas in the design, we are able to make comparisons between different intra-row densities of each crop and also between different inter-row densities.

In Chapter 4, we outline a strategy for carrying out a series of experiments. Therein we consider density trials first, and suggest that

a few density combinations (chosen from the results of the density trial) be used in conjunction with the varietal screening procedure. In the final stage a few selected varieties can be grown at their optimum densities for stability analysis. This sequence of experimentation can be changed by the investigator depending on his preferences. For instance he may want to do the varietal screening procedure first, and then study the density variables using a few varieties.

For varietal trials (Chapter 4), the model is written in terms of general and specific mixing effects. Design of experiments with varieties and a few density combinations, takes into account the practical aspects of field layout. In order to reduce the size of the experiment, some varietal combinations were omitted from the experiment. An explicit method based on an analogy with BIB and PBIB designs, for choosing the varietal combinations to be tested is discussed. In this case, in order to estimate the general mixing effects ($g \cdot m \cdot e$), the specific mixing effects ($s \cdot m \cdot e$) are assumed to be either negligible or random. The selection of varieties for further study is based on their $g \cdot m \cdot e$'s, the idea being that varieties with good (i.e. large) $g \cdot m \cdot e$ will give rise to reasonable i.e. good yielding combinations. One must realize, however, that due to the nature of the design one may miss extremely good combinations. An incomplete strip plot design given at the end of Chapter 4, is useful in keeping the block size small.

Investigation of the factor "environment" is not an investigation about which environments to use but rather an investigation of the stability of varietal combinations i.e. intercropping systems. Not only

is it important to assess the stability of the combination as such, but we need also to learn about the contribution of the crop A and crop B varieties, singly and combined. We extend (Chapter 5) the regression approach for monocrops to intercropping systems and develop various stability measures useful for explaining overall stability. A method for assessing the contribution of the individual crops to the overall stability of the mixture is also developed.

Design of experiments that involve different proportions of the two crops is discussed in Chapter 6. It is shown how the best proportion problem in intercropping can be put into the framework of the general mixture problem. Design of experiments with different proportions at different densities and the field layout for such an experiment is given.

For all the above experiments the measure of combined yield that we use is the Land equivalent ratio (LER). In Chapter 2, we discuss the various indices of combined yield, and argue that the LER is the most useful index of combined yield. Problems of standardization of the LER are considered and it is shown that LER based on a single standardization (i.e. single pair of divisors for the LER equation) can be analyzed using normal theory methods. It was not possible to obtain an explicit expression for the density of LER, when random divisors are used.

In the foregoing chapters, we use different types of models, suited for finding answers to the different problems in intercropping. For yield density trials we use the regression model for purposes of

finding the optimal intra-row and inter-row spacings. In variety trials we consider the usual ANOVA model and for stability studies the usual ANOVA model is exploited (by partitioning the interaction terms into a linear component and deviations from the linear component) to obtain stability measures. For studies on mixture proportions, canonical polynomials are made use of due to the dependence of the proportions (i.e. since $Z_1 + Z_2 = 1$). Other types of models are possible, but they have to be structured to answer specific questions.

7.2 Areas for Future Research

The methods of design and analysis considered here were for two-crop mixtures. Future research will involve extension of these methods for a general n-crop mixture. This will necessitate having large experiments. It may not be even practical to consider more than three crops in the mixture. A yield density trial for a three-crop mixture will involve five spacing variables, namely two inter-row spacing variables and three intra-row spacing variables. Can we then come up with a systematic design in which all these five factors vary independently? The yield density function will also be more complicated.

Another interesting area with three-crop mixtures will be in the selection of varietal combinations to be tested. Can we find an explicit method for doing such a selection? Once the combinations are selected how does one lay them out in the field? Answering these questions offer a considerable challenge to statisticians both to show how the advances in statistical knowledge of experimental design and analysis

can be used to improve the efficiency of intercropping experiments and to develop new methods for the special requirements of experiments with more than one crop.

Even for two-crop mixtures there are further questions that need to be answered. Can we find an explicit expression for the distribution of LER, when the divisors in the LER equation are random? Are there other approaches to measure stability in intercrops? One possible approach is to consider minimizing risk to the farmer (in monetary terms). However, the detailed definition of risk for multiple crops needs much careful thought.

In Chapter 3, the use of systematic designs were discussed and for more than two crops, it seems that they will become increasingly useful in reducing the area wasted to discard rows. Therefore, an assessment of the practical and theoretical advantages of systematic designs is needed in the future. More work is also needed in the area of yield-density functions for multiple crops.

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