

UNIT COMMITMENT

FOR OPERATIONS

by

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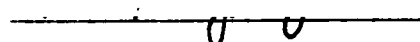
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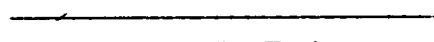
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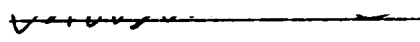
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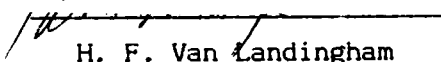
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UNIT COMMITMENT FOR OPERATIONS

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(ABSTRACT)

The topic of unit commitment has been and continues to be of interest to many researchers and is a primary operation for most utilities. Past research has utilized integer programming, dynamic programming, linear programming, gradient, and heuristic techniques. This research combines both linear programming and dynamic programming for unit commitment decisions within a weekly time frame. The result provides most of the advantages of linear programming and dynamic programming with less stringent requirements on the pre-solution information needed for unit transition sequences. Further, the research yields a new tool for the solution of the Transaction Evaluation problem.

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CHAPTER 1

INTRODUCTION

The subject of this dissertation is Unit Commitment for operations. Unit Commitment is the hourly scheduling of generating units for production within the next one-hundred-sixty-eight (168) hours. The scheduling of generating units is primarily the scheduling of the start-up and shut-down times for each generating unit while ensuring that all operating constraints are satisfied. Unit Commitment bridges the gap between medium-range planning (e.g., maintenance scheduling, hydro and/or pumped storage scheduling, and fuel contract scheduling) and hourly production and transportation (e.g., automatic generation control, economic dispatch, interchange scheduling).

The key question of this research is how to reduce the computation requirements to solve the unit commitment problem.

1.1 RESEARCH FOCUS

Unit Commitment has increased in importance due to the escalating costs of fuel, the decreasing generation and transmission capacity margins, and the increasing delays with the installation of new equipment. Additionally, the aging equipment and the new plant designs have increased the dynamic considerations of generating unit operation. Finally, the new generation designs for alternate fuels, the growing acceptance of co-generation, and the increasing variation between daily minimum and maximum load demands have forced the power system equipment to respond at their dynamic limits.

The actual operation of electric power systems has evolved from the immediate problem of coordinating the interconnection of two synchronous generators to the planning of energy resources for proper weekly operation. The temporal decomposition has evolved into the following functional groupings:

- o Automatic Generation Control (nearly instantaneous allocation of generation based upon dynamic response and upon economic considerations, primary decision is allocation of power) [15,64,120,184,194,196,219].
- o Scheduling of energy resources on hourly basis (primarily deals with short-term interchanges with neighboring utilities) [2,33,41,42,71,143,147,148,175,180,220].
- o Scheduling of energy resources on daily basis (primary decision is start-up or shut-down of cycling thermal units, pumping or generation with pumped storage units, and hydro storage schedules for shallow or run-of-river units) [4,14,99,114,139,157,170,212].
- o Scheduling of energy resources on weekly basis (primary decision is start-up and shut-down of base thermal units, pumping or generation with pumped storage units, and hydro storage schedules for medium-term reservoir units) [25,54,60,93,94,130,152,213].
- o Scheduling of energy resources on monthly basis (primary decision is maintenance of units and fuel management) [52,61,100,102,117,131,206,236].
- o Scheduling of energy resources on yearly basis (primary decision is allocation of long-term interchange contracts, medium-term fuel contracts, and seasonal hydro coordination) [35,36,69,132,186,228].
- o Scheduling of energy resources on multi-year or decade basis (primary decision is allocation of long-term fuel contracts and nuclear refueling) [34,37,86,90,182].

The last two temporal groups are normally part of system planning since the coordination of long-term fuel contracts has to account for

alternative expansion construction plans and for fuel inventory strategies.

Unit Commitment research has been intensive, especially within oil-dependent countries. This research has been focused upon finding a better Dynamic Programming algorithm to solve the most general allocation problem recognizing the above temporal decomposition [70]. The development has been directed to provide not only the most complete allocation algorithm but also the algorithm which would provide the most complete support function [181].

The remaining parts of this section outline the general formulation of the Unit Commitment problem, discuss the methods which have been applied to the Unit Commitment problem, discuss the interconnections with other temporal groups, and outlines this document.

1.2 GENERAL UNIT COMMITMENT PROBLEM FORMULATION

The general intent of Unit Commitment is to produce the hourly unit schedule and the hourly unit dispatch. The unit schedule is the start-up and shut-down times for each unit, the unit dispatch is the generation level for each unit. The main requirements which Unit Commitment must handle include:

- o Forecasted Load Demand
- o Scheduled Interchange
- o Equipment Operating Constraints
- o System Operating Constraints

The general Unit Commitment objective is to satisfy all of the requirements for each hour of the study period such that the total operating cost is minimized.

The benefits of Unit Commitment have been conservatively estimated and reported in the literature [65,74,153,196]. The reason for these conservative estimates is that the operating environment is not static but dynamic. The system operating conditions and the available fuel and plant resources change from one year to the next. Additionally, the methods used to determine the estimated benefits have been subjective in nature. It is hard to estimate what would have been done without the Unit Commitment program once the Unit Commitment program is available. Nevertheless, the following major benefits have been quoted within the technical literature:

- o Decreased Fuel Costs (between 1.5% and 8% per year)
- o Better Allocation of Reserve Margins
- o Better Control of Unit Cycling

These benefits are critically dependent upon the quality of the input data and upon the response capability of the program to find an alternate solution when unplanned events change the operating environment. The response capability of the program depends upon the time delay to prepare the input data for current conditions, the computer resources to solve the algorithm, and the time delay to either implement the new schedule or acquire alternate energy resources. The response time is minimum when the algorithm is implemented on-line as part of the power system control computer. These power system

control computers are presently called "Energy Management Systems." These systems enable power system dispatchers to remotely remove equipment from service, to automatically control generation to meet changes in demand and in interchange, and to analyze current operating conditions. The benefits of an on-line Unit Commitment program include:

- o Solution correctly includes real-time conditions
- o Alternate solutions can be more quickly determined in anticipation of changing conditions
- o Alternate resources (e.g., hydro energy) can be more accurately evaluated and can be correctly related to present and future unit schedules.
- o Schedules can be updated as future conditions change (e.g., if an interface exists with an on-line, Short-Term Load Forecast)

The goal of an on-line Unit Commitment program is to achieve the largest savings possible by the proper use of current, correct data.

Additionally, an on-line Unit Commitment program can be used to evaluate alternative interchange schedules. Both Economy A and Economy B schedules may be evaluated such that the maximum benefit of the energy resource is attained. Such studies are addressed in Section 6.2.

1.3 METHODS

The number of methods which have been tried for the Unit Commitment Problem is extensive [70,163]. The method selected by any single electric utility is often customized to those problems which the utility encounters on a daily basis. The amount of hydro generation, the type of hydro generation (e.g. run-of-river,

controllable storage, pumped hydro), the existence of "must-use" fuel constraints, of "take-or-pay" fuel contracts, and the existence of load shedding contracts or load management, determine the methodology which a utility may select. Generally, the number of constraints modeled within a particular Unit Commitment Problem is very high. This research focused on a generic set of constraints "typical" for a straightforward, thermal system. This research was additionally restricted to deterministic models since appropriate data and operating criteria for stochastic analysis are not generally available nor in general use.

The algorithms which have been successfully applied to the Unit Commitment problem include:

- o Heuristic
- o LaGrangian Relaxation
- o Mixed Integer Linear Programming
- o Generalized Benders Decomposition
- o Dynamic Programming

Heuristic algorithms have been the classical solution due to the operating simplicity of electric utilities in the past [84,101]. The most global heuristic algorithm is called "merit order commitment." This algorithm was implemented at a large number of electric utilities and is still used. This algorithm is an extension of "merit order loading" used for economic dispatch. The basic approach is to commit each unit in a predefined sequence to satisfy the system operating and the unit equipment constraints for the current hour and to satisfy the

unit transition limitations based upon the previous hour only. The status of each unit (committed or decommitted) is known at the start of the study period either from a previous study or from real-time conditions. This concept lead to the Priority List Dynamic Programming algorithm described below.

The LaGrangian technique (relaxation) is to decompose the Unit Commitment Problem into single-generator subproblems or into single time period subproblems. The fundamental approach of LaGrangian relaxation is to include constraints as part of the objective function with LaGrangian multipliers. This relaxed problem should be easier to solve than the original problem. Such an approach is based upon the dual of the original problem and a method to solve the dual problem. The relaxed problem solution provides a lower bound to the original problem and is a function of the LaGrangian multipliers. Once the relaxed subproblem is decomposed into time periods or single-generators, then each subproblem is solved individually (e.g. Dynamic Programming). The subproblems are related to the original problem as nodes of an enumeration tree for a branch and bound procedure. The solution at a node can be found by a minimum-path algorithm to connect each of the subproblems such that only valid transitions are generated between each time period for each generation unit. The quality of the feasible solution is based upon it's cost which is also an upper bound to the optimal cost. The drawback of this approach [24,27,137,138, 154,156,165] is the large number of LaGrangian multipliers which have to be included for each inequality constraint.

Another LaGrangian-related approach [27,29] approximates the dual problem with a twice-differentiable problem, which is then solved by Newton's method. Sufficient information is obtained by this alternative, such that only a single node of the branch and bound tree need be evaluated. This is a very complex approach which requires extensive code changes as constraints are added and which requires extensive computer resources. The minimum solution time is cited as ten minutes on a VAX-11/780 for 200 units over a 24 hour study period.

The Mixed Integer Linear Programming approach is similar to the last method since a branch and bound procedure is often used. The difference is that a convex-piece-wise-linear model of the operating costs is used. The status of a unit (committed or decommitted) is represented by 0-1 integer variables. The generation level of each unit is given by continuous variables. The drawback of this approach [38,55,63,81,82,88,92,141,153,173] is the high computational burden for realistically sized power systems.

The Generalized Bender's Decomposition method is based on a mixed-integer-linear model as above. The solution progresses in two steps. First, each time interval is solved for all feasible combinations of unit statuses. The optimal set is found by Bender's Decomposition Principle. Second, a coupled-optimal search is performed for all time intervals such that only valid transitions are allowed. Dynamic Programming is often used for such a search

process. The drawback of this approach is the large number of combinations which have to be optimized for each time interval [20].

The most widely accepted algorithm is Dynamic Programming. Due to the high dimensionality of the Unit Commitment Problem, Dynamic Programming is not a practical methodology for large or for medium-sized electric utilities as it was originally devised by Bellman. The many approximations which reduce the "curse of dimensionality" are discussed in Section 3. The most general method is the benchmark for the successive approximation technique advanced by this research [10,11,39,89,94,97,113,119,145,159,164,168,169,171,181,218,222,229].

1.4 INTERCONNECTION BETWEEN TEMPORAL GROUPS

The benefit of the temporal decomposition is that an algorithm for any temporal group has to interface only with the temporal group directly above and directly below. Figure 1-1 shows some of the variables which are exchanged between groups for one implementation. Note that the more conventional program names have been substituted for the group descriptions. The data interface is primarily limited to the minimum information needed to connect each subsequent layer. The data interface is dependent upon the operating constraints considered. The existence of take-or-pay fuel contracts would add data on the status of each contract and the allocation of the take-or-pay fuel for every time period simulated by each subsequent layer. The same is true for hydro generation which would add the allocated water usage for each time period simulated by each

- o Automatic Generation Control
 - Unit Status for next hour
 - Unit Desired Generation for next hour
 - Interchange Status for next hour
 - Interchange Value for next hour

- o Economy A
 - Unit Status for next 24 hours
 - Unit Desired Generation for next 24 hours
 - Interchange Status for next 24 hours
 - Interchange Values for next 24 hours
 - Interchange Availability for next 24 hours

- o Unit Commitment
 - Unit Status for next 168 hours
 - Unit Desired Generation for next 168 hours
 - Interchange Status for next 168 hours
 - Interchange Values for next 168 hours
 - Unit Availability Status for next 168 hours

- o Economy B
 - Unit Status for next 168 hours
 - Unit Desired Generation for next 168 hours
 - Interchange Status for next 168 hours
 - Interchange Values for next 168 hours
 - Unit Availability Status for next 168 hours
 - Interchange Availability for next 168 hours

- o Hydro Thermal Interchange Scheduling
 - Unit Status for next 52 weeks
 - Unit Desired Generation for next 52 weeks
 - Interchange Status for next 52 weeks
 - Interchange Values for next 52 weeks
 - Unit Availability Status for next 52 weeks

- o Maintenance Period Scheduling
 - Unit Status for next 104 weeks
 - Unit Desired Generation for next 104 weeks
 - Interchange Status for next 104 weeks
 - Interchange Values for next 104 weeks
 - Unit Availability Status for next 104 weeks

- o Fuel Cycle and Interchange Contract Analysis
 - Unit Status for next 10 years
 - Unit Desired Generation for next 10 years
 - Interchange Status for next 10 years
 - Interchange Values for next 10 years
 - Unit Availability Status for next 10 years

FIGURE 1-1. TEMPORAL GROUP INTERFACE

subsequent layer. The following discussion focuses the decomposition of operational planning from the longest time group to the shortest time group.

Fuel Cycle and Interchange Contract Analysis ranges from a few years to a decade for nuclear fuel cycling and interchange contracts. Note that interchange contracts must be "in place" before dispatchers may enter into interchange schedules. An interchange contract defines the types of interchange and the attributes of each type of interchange which may be implemented. The long-range scheduling of nuclear fuel cycling and interchange contracts involves multi-yearly planning of resources on a quarterly or monthly basis. The resultant schedules ensure that there will be adequate resources of energy to meet expected load trends at an acceptable cost. The objective is not purely minimum cost since most of the variables involve significant uncertainty. The techniques used for these studies are based on Load Duration Curve or Equivalent Load Duration Curve algorithms. These solutions are typified by the use of integer variables and are solved by Linear Programming and/or Dynamic Programming. The results of these studies are used to constrain the next shorter decision time frame. Additionally, sensitivities are often used to signal the need to reconsider the results of these long-range studies when conditions indicate that the long-term solutions are no longer valid.

Maintenance Period Scheduling ranges from a few weeks to a few years. The long-range scheduling of maintenance periods involves multi-year planning of resources on a monthly basis. The resultant

schedules ensure that there will be adequate resources of energy to meet expected load trends at an acceptable cost. The objective is not purely minimum cost, since most of the variables involve significant uncertainty. The techniques used for these studies are based on Load Duration Curve or Equivalent Load Duration Curve formulations. These solutions are typified by the use of integer variables and are solved by Linear Programming and/or Dynamic Programming. The results of these studies are used to constrain the next shorter decision time frame. Additionally, sensitivities are often used to signal the need to reconsider the results of these long-range studies when conditions indicate that these solutions are no longer valid.

The medium-term scheduling of interchange and hydro-thermal coordination involves yearly planning of resources on a weekly basis. This planning ensures that there will be adequate resources of energy to meet expected load levels at an acceptable cost. The objective is often purely minimum cost even though the actual load demands and the actual weather influence include large uncertainties. The main considerations are to schedule surplus interchange between companies, to provide for sufficient schedule flexibility for unscheduled maintenance, to provide for variations in the expected load demand, and to provide adequate regulation capability.

The short-term scheduling of interchange coordination with thermal units involves weekly planning of resources on an hourly basis. The objective is to ensure that there will be adequate resources of energy to meet expected load levels at an acceptable

cost. The objective is often purely minimum cost even though the actual load demands and the actual weather influence include large uncertainties. The main considerations are to schedule surplus interchange between companies, to provide for sufficient schedule flexibility for unscheduled maintenance, to provide for variations in the expected load demand, and to provide adequate regulation capability.

The real-time scheduling of thermal units and Economy A interchange involves hourly planning of resources on an hourly basis. The objective is to ensure that there will be adequate resources of energy to meet expected load levels at an acceptable cost. The objective is purely minimum cost even though the actual load demands and the actual weather influence include some uncertainty. The main considerations are to schedule surplus interchange between companies, to provide for sufficient schedule flexibility for forced outages, to provide for variations in the expected load demand, to provide adequate operating reserve capability, and to provide adequate regulation capability.

The real-time control of thermal units and interchange involves hourly allocation of resources on a four second basis. The objective is to ensure that there will be adequate resources of energy to meet actual load levels at an acceptable cost. The objective is minimum cost even though the actual load demands include significant deviations from the expected load demand. The main considerations are to minimize deviations of interchange schedules between companies, to

provide for variations in the expected load demand, to provide adequate operating reserve capability, and to maintain adequate regulation capability.

1.5 DISSERTATION OUTLINE

This dissertation is divided into six chapters. This first chapter introduces the subject matter and identifies the problem addressed by this research. The problem identification includes the coupling of Unit Commitment within an operational organization. The result of the first chapter is to review the highlights of the literature search which preceded this analysis and to define the type of problem considered.

The second chapter introduces the models used for the Unit Commitment problem. The modeling has often been a neglected subject since the modeling applicable for the time duration of the simulation does strongly influence the algorithms which may be applied.

The third chapter discusses the problem formulation and the Dynamic Programming algorithms which have been applied to the Unit Commitment program.

The fourth chapter outlines the algorithm considered by this research and its implementation as a computer program. Additionally, the "de facto" standard algorithm, which is Priority List Dynamic Programming, is also outlined in pseudo-code.

The fifth chapter presents one of the sample electric power systems used to evaluate the proposed algorithm and the results from the programs described in chapter four.

The sixth chapter discusses two extensions of the proposed algorithm to solve more complex problems involving interchange evaluation and hydro scheduling.

1.6 CONCLUSIONS

The proposed algorithm achieves a significant decrease of computational requirements compared to the industry standard algorithm. Additionally, when the resultant output of the proposed algorithm is compared to the standard algorithm, the proposed algorithm provides a surplus of information which may benefit operations when forced outages or other unexpected events occur. It is this resultant sensitivity data output which can give power system dispatchers the necessary information to quickly recover from unplanned events.

CHAPTER 2

POWER SYSTEM MODELS

The representations of thermal generating units for the Unit Commitment Problem can be separated into the following model categories:

- o Operating Costs
- o Unit Equipment Limitations
- o System Operating Requirements

Each category is discussed below, a more complete development can be found in the references [7,62,72,91,202,207,237,241].

2.1 OPERATING COSTS

The operating costs considered within the Unit Commitment problem include:

- o Unit Start-up
- o Unit Shut-down
- o Unit No-Load (idle, banking or standby)
- o Unit Production
- o System Losses

The costs associated with interchange is normally assumed fixed for the Unit Commitment problem. However, some implementations have the capability to model interchange contracts as equivalent thermal units. Another approach is to compare two Unit Commitment solutions to determine if a potential interchange schedule, a change in unit

availability, a change in unit capability, etc. would be beneficial. Such modifications have not been included within this research.

The unit start-up and shut-down costs are collectively referred to as Transition Costs. The unit start-up cost is a highly nonlinear function relating the latent heat of the boiler and the response time to bring the boiler to operating conditions. This nonlinear function is most often approximated by an exponential curve:

$$\text{UNIT_START_UP_COST} = \text{FUEL_COST} \times \text{START_UP_HEAT} \times (1.0 - \text{EXP}(\text{HOURS SINCE SHUT_DOWN} / \text{BOILER_TIME_CONSTANT}))$$

A more complex representation is to model the start-up sequence explicitly. Such a model is shown in Table 2-1. This model shows the gradual loading of a unit each hour and the fuel usage by fuel type for each hour. The operation of the unit for the sixth hour is governed only by the Unit Equipment Limitations. The normal duration for a start-up sequence is between two (2) and eight (8) hours.

The unit shut down costs are primarily the cost of the fuel used after the unit has been disconnected from the power grid and the cost of the plant crew who control the shut-down process. This cost is normally a constant:

$$\text{UNIT_SHUTDOWN_COST} = \text{UNIT_CONSTANT}$$

The unit shut-down costs do not include the possibility of "banking" a unit until it's generation is needed again. This condition occurs frequently for cycling thermal units. These unit no-load costs are primarily due to the fuel used to keep the unit

TABLE 2-1. THERMAL UNIT START-UP SEQUENCE

Hour	Generation (MW)	Fuel Mix	
		Primary (MBTU)	Secondary (MBTU)
1	20	20	80
2	80	20	80
3	110	20	80
4	140	40	60
5	150	70	30

"warm." The unit is kept "warm" such that all seals are properly lubricated and seated for the high pressures and high temperatures required for generation. Another component is the cost of the plant crew to control the boiler at such a low level. This cost is normally a constant:

$$\text{UNIT_NO_LOAD_COST} = \text{UNIT_BOILER_CONSTANT} \times \text{HOURS_IDLE}$$

A typical operating decision is whether to bank the unit at no-load cost until the unit's generation is needed or to shut-down the unit and to incur a start-up cost when the unit's generation is needed. When this decision is compounded by the unit's minimum down time constraint, which is included in all Unit Commitment programs, the decision is more complex than most dispatchers would consider. These constraints are discussed below.

The unit production cost is composed of two components: fuel cost and repair cost. The fuel cost is simply the fuel used to generate power. The repair cost is the incurred maintenance cost due to the amount of power generated or due to the crew cost to control the unit. The fuel cost is a highly non-linear time-variant function which relates the amount of fuel burnt in the boiler and the unit's generation. This function is called the unit Input/Output Curve. The derivative of this curve is actually used within Unit Commitment or Economic Dispatch programs. The derivative of the Input/Output Curve is the Incremental Heat-Rate Curve. The repair cost is typically represented as a linear function of unit generation.

However, at least one utility has used piece-wise linear maintenance curves [193].

The data used to construct the production cost curves are obtained from field tests on the unit while it operates at different levels of generation. Unfortunately, the gathering of this data is a costly and difficult task since current measurement techniques are highly inaccurate. A typical estimate of the error in estimating these curves is on the order of ten per-cent. This error magnitude allows wide variation for the selection of a mathematical formula to represent the function for Unit Commitment. Currently, quadratic curve(s) are used to represent the Unit Input/Output Curve. The more exact representations use piece-wise quadratic representations for the unit Input/Output Curve. This yields a piece-wise linear Incremental Heat Rate Curve as shown in Figure 2-1. Most research reports use a single segment linear Incremental Heat Rate Curve, the only difference is the simplicity of the Economic Dispatch solution. This research used such a simplistic approach, primarily to reduce the computer requirements. The differences in Economic Dispatch solutions is discussed below in Section 3.3, Economic Dispatch.

The data used to generate the maintenance cost curves is obtained from historical records of maintenance for each individual unit. The maintenance costs were modeled as a linear function of unit generation for this research:

$$\begin{aligned} \text{UNIT_MAINTENANCE_COST} &= \text{UNIT_MAINTENANCE_CONSTANT} \times \text{HOURS_ON-LINE} \\ &+ \text{UNIT_GENERATION} \times \text{UNIT_MAINTENANCE_VARIABLE_CONSTANT} \end{aligned}$$

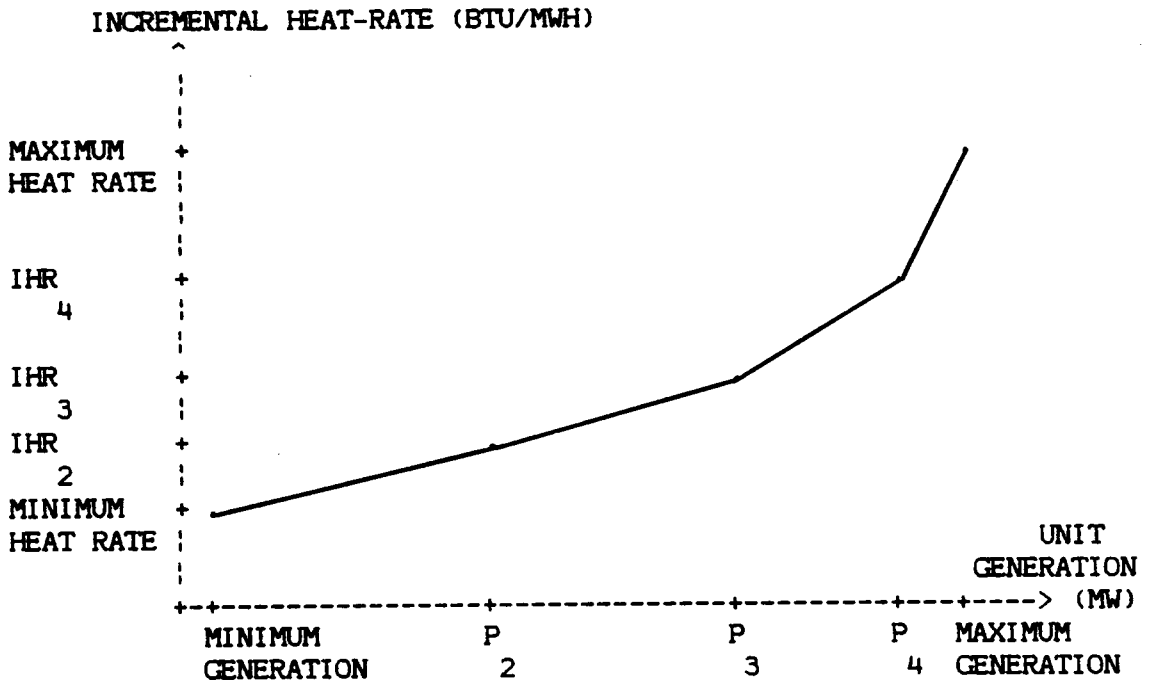


FIGURE 2-1. UNIT INCREMENTAL HEAT RATE CURVE

The cost of system losses is due to the resistance of the transmission system. The transmission system is obviously crucial to transporting the power from the generating units to the end user. Thus, it is necessary to include a model of the transmission system within the Unit Commitment problem. However, The model used for the transmission system can be very simplistic since the transmission system does not normally constrain generation at any unit or group of units. The model normally used is simply the "penalty factors" which reflect the incremental amount of power which will not reach the end user due to the resistance losses of the power system. The penalty factors do not provide the actual transmission line flows for any solution. The penalty factors are calculated by a network sensitivity program from on-line estimator adjusted and verified data. The sensitivities are calculated for every generating unit and every interchange company.

The current sensitivities are used by the Automatic Generation Control program for proper Economic Dispatch. The current sensitivities are also used by a loss model program to update the system loss model for subsequent Unit Commitment studies. The network sensitivities are defined by solving either the general transmission loss formula by power flow techniques or by calculating the gradients for the loss objective function. The sensitivities can be used to estimate the new system loss for changes in generation from the base transmission system loss:

$$\text{NEW_SYSTEM_ESTIMATED_LOSS} = \sum_{n=1}^N \left(\frac{dL}{dX_n} \right) X_n + \text{OLD_LOSS}$$

where: $\frac{dL}{dX_n}$ = the average sensitivity of the change in loss to a change in generation for unit n

X_n = the change in generation at unit n,

OLD_LOSS = the base transmission loss.

The penalty factors are more commonly used since only the effect of the system loss is needed for the Unit Commitment problem. The penalty factors are related to the network sensitivities by a simple conversion formula:

$$\text{PENALTY_FACTOR}_n = \left(\frac{1.0}{1.0 - \frac{dL}{dX_n}} \right)$$

As discussed in Section 3.2, Economic Dispatch, this representation is used to reflect the amount of generation which would actually be available for the end user.

The above representation is inadequate only when the transmission system limits a unit or a group of units as is common for power pool studies. A power pool is a group of utilities which have agreed to schedule interchange and/or generation for the benefit of the group as a whole. The transmission lines which connect utility companies are not as extensive as the transmission lines which connect the generation and the end user within a utility. The net result is often

the restriction of power transfer from one utility to another. The lack of data for these models have restricted this study from including such restrictions, often called inter-area flow constraints. However, the models are easily included in the Economic Dispatch algorithms as shown in Section 3.2.

2.2 UNIT EQUIPMENT LIMITATIONS

The Unit Equipment Limitations represent the capabilities of each unit to satisfy the System Operating Requirements. These limitations often include:

- o Pre-Schedules (Fixed, Must Run, Outaged, Available)
- o Nameplate Limits (Maximum and Minimum Capacity)
- o Capacity Derations (Partial forced outages)
- o Operating Limit Restriction
- o Economic Dispatch Restriction
- o Ramping Rate Limit Restriction
- o Spinning Reserve Restriction
- o Ready Reserve Restriction
- o Minimum Up-Time and Minimum Down-Time Restriction

The pre-schedule limitations are one technique to coordinate the Unit Commitment solution with the solutions of other temporal problems, such as Nuclear Fuel Scheduling. Pre-Scheduled units (Fixed or Must Run) were excluded from this research. Such units reduce the dimensionality of the Unit Commitment Problem and can be easily included. The capacity limitations and response restrictions are depicted in Figure 2-2. The nameplate limitation is simply the

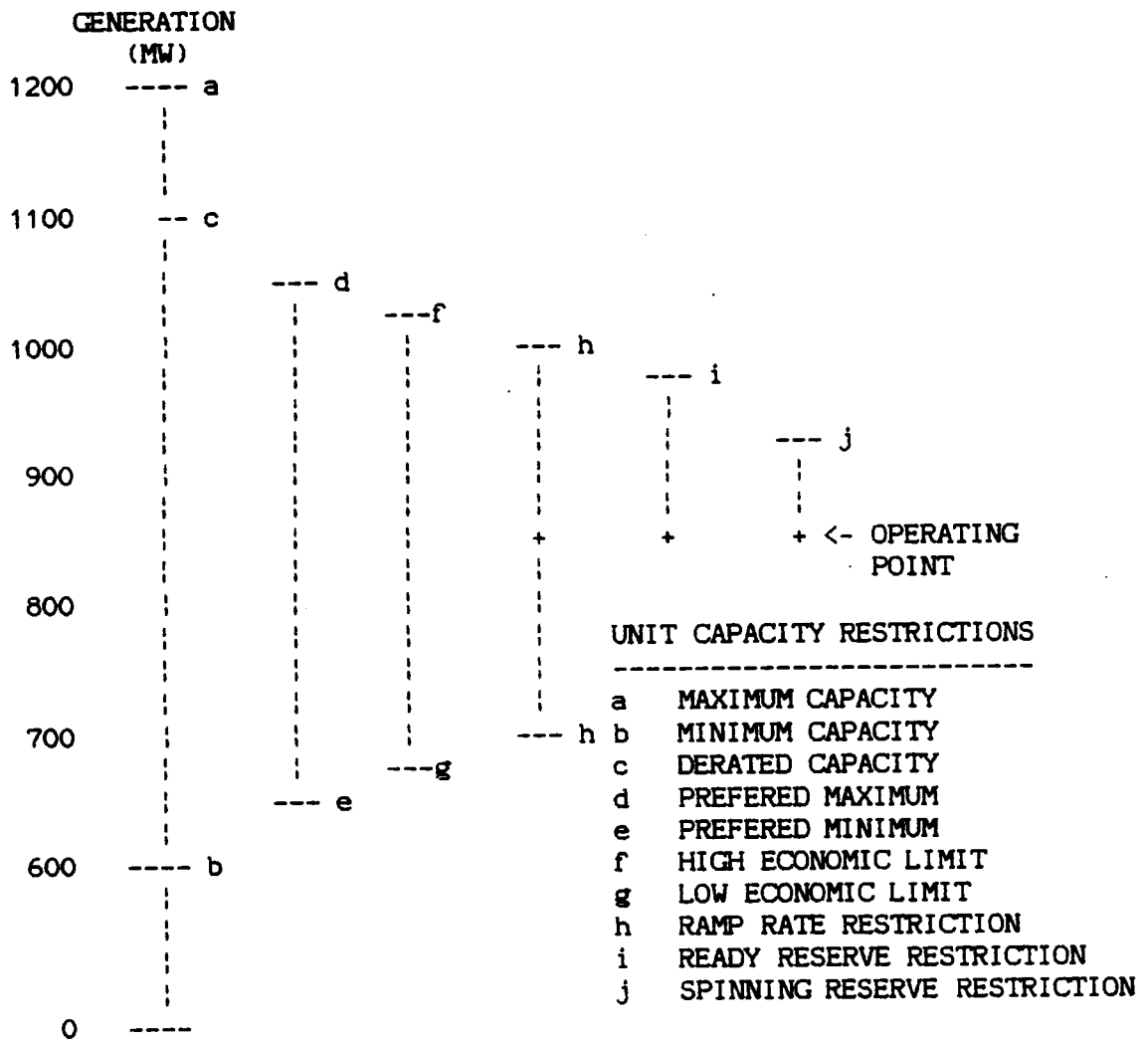


FIGURE 2-2. UNIT EQUIPMENT LIMITATIONS

designed range of the equipment. The capacity deration is one method of representing a limitation due to partial equipment failure (e.g., steam valve) or due to environmental restriction (e.g., thermal discharge). This type of constraint reduces the feasible solution space for the Economic Dispatch problem but does not alter the Unit Commitment problem. The Operating Limit Restrictions, which are annotated as the Preferred Maximum and Minimum, often define the controllable generation capability. The Economic Dispatch Restrictions, which are annotated as the High Economic Limit and the Low Economic Limit, often define the dispatchable generation capability. The rate restrictions are due to limitations of the boiler control system to follow the generator controls or due to equipment thermal stress limitations. The minimum up-time and down-time restrictions limit the number of transitions a unit would be subjected to such that the steam system will not be stressed. Note that the restrictions are based upon the unit's current generation value (i.e., operating point).

2.3 SYSTEM OPERATING REQUIREMENTS

The System Operating Requirements define the amount of generation needed to satisfy the current demand, the unexpected demand, the unexpected changes in unit status, and to satisfy the expected change in demand. The System Operating Requirements include:

- o Load
- o Spinning Reserve
- o Ready Reserve

- o Ramping Reserve
- o Instantaneous Regulation
- o Sustained Regulation

The Load restriction is simply that the generation must satisfy all demands plus system losses:

$$\text{TOTAL_GENERATION} + \text{NET_INTERCHANGE} = \text{SYSTEM_LOAD} + \text{SYSTEM_LOSSES}$$

The reserve restrictions maintain a response margin to ensure that the power system can survive simple forced outages. The methods to determine the required reserve values are primarily heuristic. The results of many planning studies, the cumulative operational experience over many years, and the results of negotiated support between neighboring utilities are used to set the required levels.

The type of reserves maintained vary between utilities. This research used the term spinning reserve as defined by the following generic System Reserve Constraint:

$$\text{TOTAL_CAPABILITY} - \text{TOTAL_GENERATION} \geq \text{REQUIRED_RESERVE}$$

This research also excluded Ramping Reserve since most utilities do not have unit hourly limitations. Ramping limitations are typical only during the start-up period. The best method of modeling such ramping restrictions to start a unit is to model the actual operating procedures. Note that a generic implementation of Ramping Reserve would apply to both increases and decreases in generation.

The Regulation Restrictions are new to the utility industry and are not included within any existing Unit Commitment programs. Since these restrictions are not generally included and since the necessary

data for these restrictions are not available, the Regulation Restrictions were not included within this research.

2.4 UNIT COMMITMENT PROBLEM FORMULATION

The sum total of the above models is summarized generically in Figures 2-3 and 2-4. The units of measurement are given in Sections 2.1, 2.2 and 2.3 above. The notation used [190] has been generally accepted as the basic Unit Commitment Problem formulation.

The first equation is the objective function. As defined within this report, the objective is to minimize the total operating cost of all committable units. The objective function includes terms for production cost and transition cost. The production cost is classically defined as the cost of fuel consumption. However, the accrued costs of maintenance have become a significant component and may be included without any change to this formulation. The transition cost is either the cost of starting a unit or stopping a unit. These latter costs may include labor as well as startup fuel consumption. The remaining equations define the constraints which restrict operation.

There are typically two levels of optimization applied to Unit Commitment: one for global and one for local constraints. The second equation shown in Figure 2-3 is a global constraint. A global constraint is a stage dependent restraint on the decision or control variables. The third equation shown is a local constraint. A local constraint restrains the decision and control variables within a stage

$$\begin{array}{l}
 \text{Minimize: } \quad \text{PI} = \text{SUM} \left(p_{t,t}(u_t) + s_{t,t}(u_t) \right) \\
 \text{Subject to: } \quad \text{SUM} h_{t,t}(u_t) \leq b \\
 \quad \quad \quad \quad g_{t,t}(u_t) \leq y_t \\
 \quad \quad \quad \quad u_t \in S_t
 \end{array}
 \quad \left. \begin{array}{l}
 \text{For all time} \\
 \text{intervals} \\
 t = 1, \dots, T
 \end{array} \right\}$$

Where:

- PI = objective function (total operating cost)
 $p_{t,t}(u_t)$ = production cost at stage t
 $s_{t,t}(u_t)$ = transition cost at stage t
 u_t = control variable (unit generation) at stage t
 T = schedule horizon (number of stages/hours)
 h_t = time dependent constraints
 g_t = time independent constraints
 b, y_t = constraint limits
 S_t = control variable limits
 N = number of variables (units),
 (implicit in the vector u_t above)

FIGURE 2-3. UNIT COMMITMENT - GENERAL MATHEMATICAL FORMULATION

$$\text{Minimize PI} = \sum_{n=1}^N p_n(u_n)$$

Subject to:

$$\sum_{n=1}^N u_n = \text{GTBD} = \text{BLD} + \text{LOSS}$$

Where:

1. Unit cost function:

$$p_n = f_n * e_n * F_n(u_n)$$

where: p_n = production cost

u_n = production power

f_n = fuel cost

e_n = efficiency

F_n = heat rate curve

For

all

units

$n=1, \dots, N$

2. Unit heat rate curve:

$$F_n(u_n) = a_n + b_n * u_n + c_n * u_n^2$$

where:

u_n = production power

a_n, b_n, c_n = curve constants

For all

units

$n=1, \dots, N$

FIGURE 2-4. ECONOMIC DISPATCH FORMULATION

3. Unit capacity limits:

$$u_n^{\max} \geq u_n \geq u_n^{\min} \quad \begin{array}{l} | \\ | \\ | \end{array} \begin{array}{l} \text{For all units} \\ n=1, \dots, N \end{array}$$

4. Power system model: $GTBD = BLD + LOSS$

where: $GTBD$ = the generation to be dispatched,
 BLD = the base load demand (system load),
 $LOSS$ = the transmission loss.

FIGURE 2-4. ECONOMIC DISPATCH FORMULATION (Continued)

and is, thus, stage independent. An equipment restriction is shown in the last equation. This restriction is upon the values to which a control variable may be assigned.

CHAPTER 3

SOLUTION ALGORITHMS

3.1 GENERAL FORMULATION

The Unit Commitment problem has been summarized in Section 2.4. The separation of constraints into a global time-dependent set and a local time-independent set is often used to apply multiple algorithms. Such a separation is useful for all of the approaches listed in Chapter 1, even if the separation is used only to manage main and bulk memory requirements. This separation is basic to this research.

The most widely accepted global algorithm is dynamic programming. The most widely accepted local algorithm (economic dispatch) is linear programming. Both are discussed below.

The units for each variable (e.g., MW for generation) are given in Sections 2.1, 2.2, and 2.3. The units are not listed in this section for presentation clarity.

3.2 DYNAMIC PROGRAMMING

The Unit Commitment problem has all of the characteristics of the classical Resource Allocation problem. A formulation [16,17,18,19] for the classical Resource Allocation problem is given in Figure 3-1. The objective function corresponds with the objective of the Unit Commitment problem except that the Unit Commitment problem does not clearly show the state variable (x).

The second equation is the state transition function which was not given for the Unit Commitment problem. This issue is addressed

$\min_{x_n, y_n} J(\{x_n, y_n\}) = \sum_{n=0}^N h_n(x_n, y_n)$ <p>s.t:</p> $x_{n+1} = g_n(x_n, y_n)$ $x_0 = c$ $x_n \in S_n(c)$ $y_n \in D_n(c)$ $\sum_{n=0}^N k_n(x_n, y_n) \leq a$ $y_n = f_n(x_n)$		<p>For all time</p> <p>Intervals</p> <p>$n = 0, \dots, N$</p>
---	--	--

recursive function:

$$f_n(c) = \min [h_n(c, y_n) + f_{n-1}(g_{n-1}(c, y_{n-1}))]$$

FIGURE 3-1. GENERAL DYNAMIC PROGRAMMING
RESOURCE ALLOCATION PROBLEM
FORMULATION AND SOLUTION

below. The control variable (y) corresponds directly with the unit generation (u).

The third equation defines the initial conditions. The initial conditions for the Unit Commitment problem is the present status of each unit. As discussed below, this may or may not correspond to the initial condition for the state variables.

The fourth equation defines the domain for the state variables. The fifth equation defines the domain for the control variables. The sixth equation corresponds with the global and the local constraints of the Unit Commitment problem. The last equation defines the relationship between the state and control variables and is not normally defined for the Unit Commitment problem explicitly. This relationship is the Economic Dispatch optimization for the Unit Commitment problem.

The main idea of Dynamic Programming is the recursive function shown at the bottom of Figure 3-1. The recursive function defines the relationship between stages which must be optimized if the overall problem is to be optimized. This equation is the same as the one used for the Unit Commitment problem.

The classical definition of the Unit Commitment problem defines the unit status as a component of the state variable [184]. Thus, the state variable can be any integer number between one and the maximum number of combinations. There are many classifications of unit status

but only the binary values (one, zero) are needed. The state transition function is simply a function of the present and past state variables and not the control variables. This definition is depicted in Figure 3-2. Note that the state variable does not explicitly give the status of each unit if the number of the state variable is saved. However, this is easily circumvented by using a binary code for each state variable. A computer word is defined large enough to contain one bit for each unit. Then the words are treated as an array (e.g., the sixth word entry would show the bit pattern for the set corresponding to X_6). This array of words is all that is needed to define the information passed between stages.

An alternative approach is to define the state variables as the required reserve margins and as the estimated load. The state variables would then be continuous variables which could range from the required margin to some reasonable upper limit. The load could range from the estimated load minus the estimation error to the estimated load plus the estimation error. This formulation is reserved for future research.

3.2.1 General Dynamic Programming

The solution algorithm for the full Dynamic Programming algorithm is outlined in Figure 3-3. The problem with applying Dynamic Programming to any problem is the "curse of dimensionality". This is not evident from the solution outline. This can only be seen if the number of states is explicitly shown as in Figure 3-4. An example of the solution is shown in Figure 3-5. This figure shows all of the

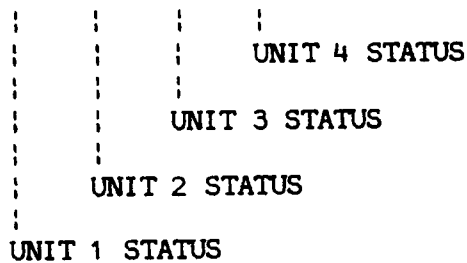
THE STATE IS THE SET X:

$$X = \{ X_1, X_2, X_3, \dots, X_i, \dots, X_N \}$$

THE COMBINATIONS MAY BE REPRESENTED AS A BINARY COUNTER:

STATE -----	SET ----			
X 1	1	0	0	0
X 2	0	1	0	0
X 3	1	1	0	0
X 4	0	0	1	0
X 5	1	0	1	0
X 6	0	1	1	0

ETC.



(ONE INDICATES THAT THE UNIT IS ON-LINE)
 (ZERO INDICATES THAT THE UNIT IS OFF-LINE)

FIGURE 3-2. UNIT COMMITMENT STATE

1. Forward Path
 - A. For each stage; $t = 1, \dots, T$
 - B. For each state; $i = 1, \dots, N$
 - i. For unit on, if feasible path from combination previous stage exists then continue else go to step vi.
 - ii. Economically Dispatch all combination units
 - iii. Production Cost all combination units
 - iv. Transition Cost combination units by calculating start-up or shut-down costs if transition dependent constraints are not violated.
 - v. Save transition of least total cost as the optimal path from previous stage to this stage as an optimal segment.
 - vi. For unit off, repeat steps ii. through vi. if all transition constraints are satisfied, else do next unit.
2. Trace Optimal Path
 - A. Find minimum total cost from last stage's set of optimal segments.
 - B. For each stage segment; $t = T, \dots, 1$
 - i. Find previous stage's state from optimal segment.
 - ii. Save state for each stage as global optimal path.
3. Cost Optimal Path
 - A. For each stage, $t = 1, \dots, T$
 - B. For optimal path segment
 - i. Determine status of all dispatchable units.
 - ii.--iv. Repeat steps ii. through iv. of Step 1. above.
 - v. Save transition and production costs.
 - C. Generate reports.

FIGURE 3-3. GENERAL DYNAMIC PROGRAMMING
ALGORITHMIC SOLUTION OF THE
UNIT COMMITMENT PROBLEM

<u>Number of Units</u>	<u>Number of Combinations</u>
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
20	1.04 10**6
30	1.07 10**9
40	1.09 10**12
50	1.13 10**15

FIGURE 3-4. DYNAMIC PROGRAMMING
CURSE OF DIMENSIONALITY

(* denotes initial condition)
 (! denotes optimal end point)

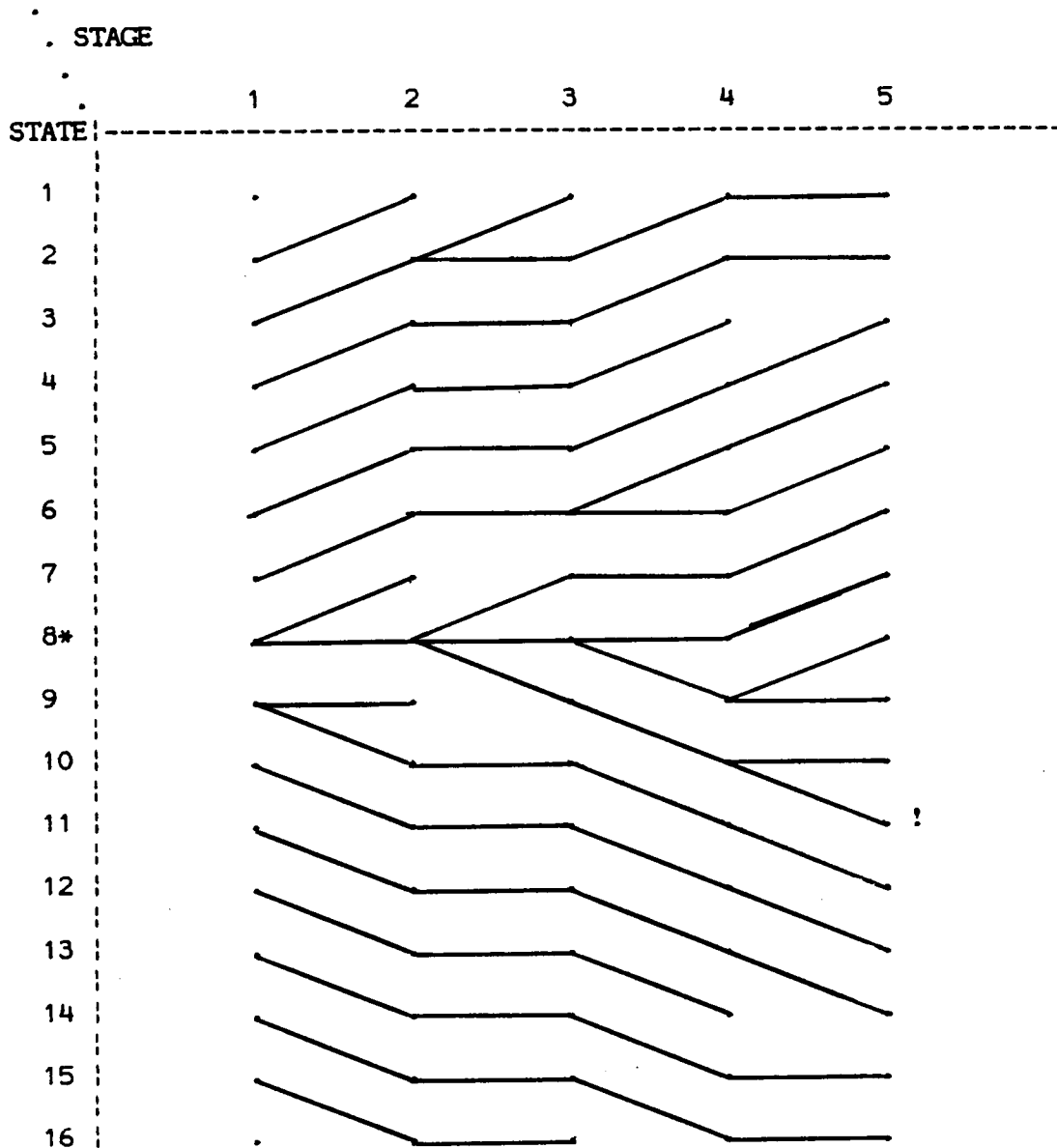


FIGURE 3-5. DYNAMIC PROGRAMMING
 FOUR UNITS AVAILABLE
 SOLUTION PATHS SHOWING OPTIMAL PATH

combinations and paths considered for a four (4) unit study lasting five (5) hours. Note that the initial conditions are not used to constrain the solution process. The combination which represents the initial conditions is marked with an asterisk. The optimal end point is marked with an exclamation point. The optimal path is the sequence of lines between these two points.

A practical algorithm would have selected only a fractional number of the total number of combinations considered. For example, if only twenty percent (20%) of all eighty (80) combinations were considered, then only sixteen (16) combinations would have been evaluated. If the neighborhood of the optimal path was known, then only the combinations within the neighborhood have to be evaluated. If the neighborhood was known to be two combinations above and below the optimal path, then only twenty-one (21) combinations have to be evaluated. Other research has evaluated such an approach with mixed results [39, 128].

There have been some utilities which use the full Dynamic Programming algorithm because the number of units is small (e.g., less than twenty) or because the computer capability is available. However, thirty (30) units would not be considered feasible because of the computer expense and because, in reality, only a few number of units need to be considered at each stage. Most units are committed because of long-term cost considerations such as nuclear units which are scheduled on a five-year basis and hydro units which are scheduled on a yearly basis because of hydraulic constraints.

There have been various methods of limiting the number of states considered each stage. Additionally, alternative Dynamic Programming algorithms have been used (e.g., successive approximations).

One of the more successful methods of limiting the number of combinations is to pre-define the combinations to be evaluated each stage [6,68,116,134]. This approach was implemented at Union Electric Company, in St. Louis, MO., and is still in use. This is a heuristic method which requires very careful analysis to detect whenever the combinations should be altered. The only way to guarantee that the optimum is found would be to analyze the choice of combinations whenever a significant change occurs with one of the following: the load patterns, the fuel costs, the units outaged for maintenance, etc. Clearly, this is not a trivial task for the general case. Since this approach is system dependent, it was not evaluated as part of this research.

3.2.2 Priority List Dynamic Programming

This is the most popular technique used in the electric power industry [75]. The method collapses the number of combinations evaluated by considering units only within a sequential priority list as shown in Figure 3-6. Thus, for five (5) units, as shown in the figure, only six (6) of the total number of combinations (32) are evaluated. Many implementations have cited a savings of one-half percent (.5%) to eight percent (8%) for this method over manual methods [168].

The actual units considered are reduced by an ordering algorithm similar to the optimal ordering techniques applied to the power flow

THE STATE IS THE SET X:

$$X = \{X_1, X_2, X_3, \dots, X_i, \dots, X_N\}$$

THE COMBINATIONS MAY BE REPRESENTED AS A BINARY COUNTER:

<u>STATE</u>	<u>SET</u>				
X 0	0	0	0	0	0
X 1	1	0	0	0	0
X 2	1	1	0	0	0
X 3	1	1	1	0	0
X 4	1	1	1	1	0
X 5	1	1	1	1	1
ETC.	:	:	:	:	:
	:	:	:	:	UNIT 5 STATUS
	:	:	:	:	UNIT 4 STATUS
	:	:	:	:	UNIT 3 STATUS
	:	:	:	:	UNIT 2 STATUS
	:	:	:	:	UNIT 1 STATUS

(ONE INDICATES THAT THE UNIT IS ON-LINE)
 (ZERO INDICATES THAT THE UNIT IS OFF-LINE)

FIGURE 3-6. UNIT COMMITMENT STATES FOR SEQUENTIAL PRIORITY LIST

and other network analysis algorithms. An example of this ordering is shown in Figure 3-7. The Fixed Schedule Units (Nuclear, Hydro, Take-or-Pay, etc.) are sorted to the bottom of the list because the decision to commit and to dispatch these units has been made by an analysis at a higher temporal level. The Must Run Units are committed due to exogenous considerations such as voltage control or stability. The Must Run Units are sorted to the second lowest position within the priority list. The Must Run Units may not be decommitted but they may be dispatched to higher levels of generation. The Outaged Units are sorted to the top of the list since they are removed for maintenance and may not be committed or dispatched. It is only the Available Units which are of any concern to the Unit Commitment problem. Thus, any evaluated combinations need only include these units. This report will ignore all other classifications of units except Available. However, an industrial grade algorithm would have to account for all of the above unit classifications.

Since many units are not part of the commit decision, this approximation is probably not excessive in general. However, it has been shown to be wrong for some utilities.

This method is the basis for all comparisons and has been coded as part of this research.

3.2.3 Truncated Priority List Dynamic Programming

This is a relatively new technique which has been implemented by this author for a major computer vendor. This program has been or is being delivered to four (4) major electric utilities. The algorithm

	/LAST	UNIT N
OUTAGED UNITS	/	
	\FIRST	UNIT M
	/HIGHEST	UNIT L
AVAILABLE UNITS	/ PRIORITY	
	/	
	\ LOWEST	
	\PRIORITY	UNIT K
	/HIGHEST	UNIT J
MUST RUN UNITS	/ PRIORITY	
	/	
	\ LOWEST	
	\PRIORITY	UNIT I
	/HIGHEST	UNIT H
FIXED SCHEDULE	/ PRIORITY	
UNITS	/	
	\ LOWEST	
	\PRIORITY	UNIT 1

FIGURE 3-7. SELECTION PROCESS--OPTIMAL ORDERING

was developed by Dr. C. Pang as part of his doctoral research at Purdue University [169]. This approach only changes the combinations evaluated at each stage as shown in Figure 3-8. The number of units eligible for combination are determined by the window length. This figure assumed a window length of four (4). The combinations evaluated are shown in Figure 3-9. Note that the fifteenth and sixteenth combinations are not valid since the Kth and K-1st units are assumed to be required to satisfy all constraints. When the Kth unit is the first available unit, then the sixteenth combination can be included in the evaluated combinations. A second special case is when the constraints are satisfied before the first available unit, then the fifteenth and sixteenth combination can be added to the evaluated combinations.

The programming required to handle all of these special cases is very complex. Thus, the above program was augmented to use only the priority list as an option to reduce the computational requirements.

This algorithm has not been coded as part of this research.

3.2.4 Successive Approximation in State Space

This is a very powerful technique when applied to continuous differential state transition functions [126,133,135,160]. However, the Unit Commitment problem is a zero-one state variable and not a continuous variable. An algorithm for implementing this technique is shown in Figure 3-10. Notice that the first step is to find an initial feasible solution. Once a feasible solution is found the successive

DYNAMIC PROGRAMMING

SELECTION PROCESS

1. DETERMINE NUMBER OF COMMITTED UNITS REQUIRED BASED UPON GENERATION CONSTRAINT AND PRIORITY LIST, DENOTE LAST UNIT COMMITTED AS UNIT K.
2. DETERMINE NUMBER OF AVAILABLE UNITS ELIGIBLE FOR COMBINATIONS. DENOTE THESE AS K-1, K, K+1, K+2.
3. GENERATE $2^M - 2$ COMBINATIONS, WHERE M IS THE NUMBER OF ELIGIBLE AVAILABLE UNITS.
4. DELETE THE COMBINATIONS WHICH VIOLATE CONSTRAINTS.

FIGURE 3-8. TRUNCATED PRIORITY LIST

UNIT COMBINATIONS	K-1	K	K+1	K+2	
1	0	1	0	0\ 0/	> 2 UNITS
2	1	1	0	0/	
3	0	0	1	0\ 0	> 3 UNITS
4	1	0	1	0	
5	0	1	1	0	
6	1	1	1	0/	
7	0	0	0	1\ 1	> 4 UNITS
8	1	0	0	1	
9	0	1	0	1	
10	1	1	0	1	
11	0	0	1	1	
12	1	0	1	1	
13	0	1	1	1	
14	1	1	1	1/ 0\ 0/	
15	0	0	0	0\ 0/	> SPECIAL CASES
16	1	0	0	0/	

FIGURE 3-9 TRUNCATED DYNAMIC PROGRAMMING
COMBINATIONS

1. Find feasible solution
2. For each state variable (unit); $i = 1, \dots, N$
 - a. For each stage (hour); $j = 1, \dots, T$
 - i. For the unit on ($X^{(i)}(j) = 1$) find $\{u^{(i)}(j)\}; PI_{opt}$
 - ii. For the unit off ($X^{(i)}(j) = 0$) find $\{u^{(i)}(j)\}; PI_{opt}$
 - iii. Find best transition, from $j-1$ to j for i. and ii.
 where $PI_j = PI_{tr} + PI_{opt}$
 - b. Find best sequence $\{X^{(i)}(j)\}$
3. Repeat Step 2 if any state sequence has changed.

FIGURE 3-10. SUCCESSIVE APPROXIMATION--STATE SPACE
UNIT COMMITMENT

evaluation of each state variable should find slight improvements until the optimal solution is found.

The Objective Function or Performance Index (PI) is found by an Economic Dispatch algorithm for the dispatchable units. Any of the Economic Dispatch algorithms discussed below could be used.

This algorithm would not find the optimum if the ordering of the variables changes the solution space characteristics. Thus, this algorithm has not been evaluated as part of this research.

3.2.5 Successive Approximation in Control Space

This algorithm has been used for the Unit Commitment problem [6,135,160]. The idea is the same as the previous algorithm but the implementation is more suited for hydro-thermal coordination or take-or-pay fuel contracts. This algorithm is shown in Figure 3-11. The best transition is evaluated by determining if a unit can be turned off for any sequence of hours beyond the unit's minimum down-time. The main difference is that the unit status is not considered explicitly.

This algorithm can be combined with the previous such that the Economic Dispatch is replaced with the successive approximation. Since the Economic Dispatch algorithms are so well behaved and economic, there are no apparent benefits to combine these algorithms.

This algorithm has not been evaluated as part of this research.

3.2.6 Successive Approximation in Demand Space

This technique is the classical method to find a closed form solution when there is one equality constraint. This technique has

1. Find feasible solution
2. For each control variable (unit); $i = 1, \dots, N$
 - a. For each stage (hour); $j = 1, \dots, T$
 - i. Determine coarse grid of control variable
 - ii. Economically Dispatch each point of grid
 - iii. Find best transition from stage $j-1$ to j
 where $PI = PI_j + PI_{tr\ opt}$
 - b. Find optimal path
3. Repeat Step 2 if grid mesh is not within solution tolerance and change in objective (PI) exceeds tolerance.

FIGURE 3-11. SUCCESSIVE APPROXIMATION--CONTROL SPACE
UNIT COMMITMENT

been successfully applied to the Unit Commitment problem [126,133,160]. The algorithm is outlined in Figure 3-12. The problem is that the number of evaluations are prohibitive when all of the normal constraints are added.

This algorithm was not evaluated as part of this research.

3.2.7 Successive Approximation in Solution Space

This algorithm is outlined in Figure 3-13. This approach is very similar to gradient optimization techniques since first the neighborhood is identified and then small changes are tried to determine if the minimum can be reduced while maintaining feasibility. Any heuristic method may be used to find the initial feasible solution. The results of the feasible solution are then used to define a coarse grid over the solution space. The previous optimal solution is no longer needed. Next the Economic Dispatch is calculated for each possible transition from a previous solution point. The best transition is found and saved for the next stage. After all stages have been evaluated, the optimal transition is traced from the final stage to the first stage. This process is repeated around the new optimal solution path until the change in objective function is within the desired tolerance or the grid size is within the desired tolerance.

The primary benefit of this approach is that a unit priority list to consider transitions is not needed. Additionally, the solutions for each successive grid would yield sensitivity information not previously available with any Dynamic Programming algorithm. This algorithm is believed to be an original approach to the Unit Commitment

1. Determine coarse grid of load, range from minimum capacity to maximum capacity of system
2. For each stage (hour); $j = 1, \dots, T$
3. For each unit; $n = 1, \dots, N$
 - a. Economically Dispatch each point of grid
 - b. Find best transition, from $j-1$ to j for each grid point
 - c. Find optimal path
4. Repeat Step 2 with reduced grid mesh if grid mesh is not within solution tolerance and change in objective (PI) exceeds tolerance.

FIGURE 3-12. SUCCESSIVE APPROXIMATION--REQUIREMENTS SPACE
UNIT COMMITMENT

1. Find feasible solution
2. For each stage (hour): $j = 1, \dots, T$
 - a. Determine coarse grid of solution space
 - b. Economically Dispatch each point of grid
 - c. Find best transition, from $j-1$ to j for each grid point
where $PI_j = PI_{tr} + PI_{opt}$
 - d. Find optimal path
3. Repeat Step 2 with reduced grid mesh if grid mesh is not within solution tolerance and change in objective (PI) exceeds tolerance.

FIGURE 3-13. SUCCESSIVE APPROXIMATION--SOLUTION SPACE
UNIT COMMITMENT

problem. This algorithm was evaluated for this research. A more complete description is given in Chapter 4.

3.3 ECONOMIC DISPATCH

One of the most thoroughly researched areas within power system analysis is Economic Dispatch. Economic Dispatch is the process of allocating the required load demand amongst generation units such that the cost of operation is minimum. This section will discuss the modeling and the algorithms which have been investigated as part of this research.

Economic Dispatch is the most intensive part of a unit commitment program. Approximately eighty (80) percent of the computer time of a unit commitment program is expended by the Economic Dispatch algorithm. Thus, the selection and implementation of an Economic Dispatch algorithm is a central issue of any unit commitment research.

The first part of this section discusses the models used and the resulting optimization formulation. Then each optimization algorithm, which has been researched for the Economic Dispatch Problem, is shown in detail. The optimization algorithms have been divided into the following groups:

- o Merit Order Loading
- o Range Elimination
- o Binary Section
- o Graphical/Table Look-Up
- o Linear Programming
- o Convex Simplex

- o Dantzig-Wolf Decomposition
- o Separable Convex Linear Programming
- o Reduced Gradient with Linear Constraints
- o Steepest Descent Gradient
- o First Order Gradient

The selection of the Economic Dispatch algorithm for this research was based upon the test programs written to evaluate each of the above algorithms. The factors included within this evaluation are presented after each of the above descriptions. The final topic of this section includes directions for additional research (e.g., model changes to increase the power system representation).

3.3.1 Economic Dispatch Models and Formulation

The general representation of the electric power system for the Economic Dispatch Problem was presented in Chapter 2. This section summarizes the models used for this research and the general mathematical statement which results.

The representation for the electric power system for Economic Dispatch consists of models for the generating units and of models for the transmission system. The models for the generating units are summarized in Figure 3-14. These models represent the cost of producing electricity, the generation capability, and the reserve capability of each unit. The only model change required for each of the above algorithms is the representation of the Input/Output Curve. The Input/Output Curve represents the conversion of energy from one form (e.g., coal) into electricity. The input may be tons of coal,

1. Unit cost function:

$$y_n = f_n * e_n * P_n(u_n) + M_n(u_n)$$

where: y_n = production cost

f_n = fuel cost

e_n = efficiency

P_n = energy conversion curve

u_n = production power

M_n = maintenance conversion curve

For all

units

$n=1, \dots, N$

2. Unit capacity limits:

$$u_n \geq U_{n \min}$$

$$u_n \leq U_{n \max}$$

For all
units

$n=1, \dots, N$

3. Unit reserve limits:

$$r_n \geq R_{n \min}$$

$$r_n \leq R_{n \max}$$

For all
units

$n=1, \dots, N$

FIGURE 3-14. GENERATING UNIT ECONOMIC DISPATCH MODELS

barrels of oil, gallons of diesel, cubic feet of water, or cost. The output is always level of generation. The Incremental Heat Rate Curve is the first derivative of the Input/Output Curve. Most Economic Dispatch algorithms use the Incremental Heat Rate Curve. The corresponding Incremental Heat Rate Curves for each of the Input/Output Curves, used to represent the energy conversion curves for this research, included:

- | | |
|------------------------------|-------------------------------------|
| o Polynomial Curve | Polynomial Curve of one less order; |
| o Cubic Curve | Quadratic Curve; |
| o Quadratic Curve | Linear Curve; |
| o Linear Curve | Constant; |
| o Piece-Wise Quadratic Curve | Piece-Wise Linear Curve. |

None of the other generating unit models have to be changed for any of the above energy conversion curves. However, the form of energy conversion curve used does impact the solution algorithm best suited to solve the resulting optimization formulation.

The models for the transmission system do not include the individual transmission lines, transformers, and bus loads. Instead, the incremental impact on the network is estimated from the network sensitivities, as calculated from real-time conditions either from an optimal power flow algorithm or from a Newton-Raphson power flow transpose solution.

The use of sensitivity factors is shown in Figure 3-15. The basic power system model is shown as the first model. This is the model used by many Automatic Generation Control programs which do not include Economic Dispatch. Simply stated, this model requires all generation to be equal to the demand and losses. The inclusion of the transmission system into the simple model requires some formulation for the dependencies of the control variable (generation and interchange) upon the system losses, as shown in the second model.

The second model is the most used representation within on-line Economic Dispatch programs which include the dependencies of the control variables to the system losses. This model assumes that any changes in generation and/or in interchange can be represented as a linear (differential) change in system losses. The sensitivity factors can be calculated by an on-line set of network analysis programs. The on-line set of programs which are required to calculate such sensitivity factors are: State Estimator, External Model Estimator, and Penalty Factors. The State Estimator uses the latest on-line telemetered values of power flows and of power injections to determine the current power flow state (voltages and angles). The External Model Estimator uses the results of the State Estimator and other information on neighboring companies to determine the current power flow state for the neighboring companies. The Penalty Factors program calculates the network sensitivities for the current power flow state, as calculated by the State Estimator and by the External Model Estimator. The flaw in this approach is that an Economic Dispatch program executes once

1. Power system model: $GTBD = (BLD + LOSS)$

where: $GTBD$ = the generation to be dispatched,
 BLD = the base load demand,
 $LOSS$ = the transmission loss.

2. Transmission Loss Equation:

$$LOSS = \sum_{n=1}^N \left(\frac{dL}{du_k} \right)_n * u_n + \sum_{m=1}^M \left(\frac{dL}{dI_k} \right)_m * I_m + L_k$$

where: $\left(\frac{dL}{du_k} \right)_n$ = the network sensitivity of the change in loss to a change in generation for a given range of load, interchange, topology, etc.

u_n = the unit generation,

$\left(\frac{dL}{dI_k} \right)_m$ = the network sensitivity of the change in loss to a change in interchange for a given range of load, interchange, topology, etc.

I_m = the interchange,

L_k = the reference transmission loss for the given range.

3. Transmission Adaptive Factors:

$$LOSS = \sum_{n=1}^N \left(\frac{dL}{du_k} \right)_n * (u_n - \hat{u}_n) + \sum_{m=1}^M \left(\frac{dL}{dI_k} \right)_m * (I_m - \hat{I}_m) + \hat{L}_k$$

FIGURE 3-15. TRANSMISSION SYSTEM ECONOMIC DISPATCH MODELS

where: $\hat{\frac{dL}{du_k}}$ = the average sensitivity of the change in loss to a change in generation for a given range of load, interchange, topology, etc.

u_n = the unit generation,

$\hat{\frac{dL}{dI_k}}$ = the average sensitivity of the change in loss to a change in interchange for a given range of load, interchange, topology, etc.

I_n = the interchange,

\hat{u}_n = the average unit generation for the given range,

\hat{I}_n = the average interchange for the given range,

\hat{L}_k = the reference average transmission loss for the given range.

FIGURE 3-15. TRANSMISSION SYSTEM ECONOMIC DISPATCH MODELS
(continued)

every one to five minutes while the network analysis programs can execute only once every ten to fifteen minutes. The discrepancy is due to the lack of computer resource to execute all of these programs coherently. This lack of computer resource is due to lack of justification to install sufficiently fast and large enough computer systems to supply the required computer resource.

The lack of justification is due to the uncertain solution quality of State Estimation and of External Model Estimation. The External Model Estimation solution quality is the most questionable, since most of the external solution is based upon heuristic techniques. These heuristic techniques are used to predict other power systems conditions based upon the known conditions. Until the external systems can be modeled to a higher level of accuracy, the total solution will still be relatively inaccurate. The methods of increasing this external information is to exchange data between power control centers. The present efforts of many utility organizations (e.g., Western Systems Coordinating Council) are directed to the exchange of such information. Until these exchange techniques are properly implemented, other approximations have to be made. The third model shown is one such approximation.

The third model has been used to bridge this discrepancy by categorizing a separate linear model for different operating ranges. The operating ranges are often characterized by net load level, by net interchange level, and by network topology. This approach is also used for on-line Unit Commitment studies to estimate what the system losses

will be in future hours. A typical categorization is shown in Table 3-1. There is a vector of penalty factor information saved for each category. The vector of penalty factor information includes the average system loss, the average penalty factor for each unit, the corresponding average generation for each unit, the average company penalty factor, and the corresponding average company net tie-line flow. The company values are calculated as weighted averages for the tie-lines between each company.

The research programs represented the transmission losses with the second model. Note that this does not preclude the use of the third model, since the second model is just a simplification of the third model.

The resulting Economic Dispatch formulation is summarized in Figure 3-16. The heat rate curve is selected from Table 3-2 depending upon the algorithm selected and the energy conversion model appropriate for the type of unit. All of the discussions within the remaining sections of this report assume that the formulation in Figure 3-16 and a curve from Table 3-2 is being referenced.

3.3.2 Economic Dispatch Algorithms

The Economic Dispatch algorithm uses the most computer time within the Unit Commitment solution process. Economic Dispatch was investigated to determine if any improvement could be made with the Unit Commitment problem by a more appropriate choice of Economic Dispatch algorithm. The algorithms which were evaluated for the Economic Dispatch problem are presented in order of complexity [1,5,9,57,58,105,146,192,200,201].

TABLE 3-1. PENALTY FACTOR MATRIX

NETWORK TOPOLOGY: 1.		ALL LINES IN SERVICE			
GENERATION RANGE (MW):		500-600	600-760	760-925	925-1225
INTERCHANGE RANGE (MW):	*				
-200,-125	PFIV #1	PFIV #2	PFIV #3	PFIV #4	
-125,-50	PFIV #5	PFIV #6	PFIV #7	PFIV #8	
-50,50	PFIV #9	PFIV #10	PFIV #11	PFIV #12	
50,125	PFIV #13	PFIV #14	PFIV #15	PFIV #16	
125-200	PFIV #17	PFIV #18	PFIV #19	PFIV #20	
NETWORK TOPOLOGY: 2.		765 KV LINE 1024-E OUT OF SERVICE			
GENERATION RANGE (MW):		500-575	575-660	660-875	875-1225
INTERCHANGE RANGE (MW):					
-200,-175	PFIV #1	PFIV #2	PFIV #3	PFIV #4	
-175,-100	PFIV #5	PFIV #6	PFIV #7	PFIV #8	
-100,0	PFIV #9	PFIV #10	PFIV #11	PFIV #12	
0,95	PFIV #13	PFIV #14	PFIV #15	PFIV #16	
95,200	PFIV #17	PFIV #18	PFIV #19	PFIV #20	
NETWORK TOPOLOGY: 3.		345 KV LINE 1036-N OUT OF SERVICE			
GENERATION RANGE (MW):		500-625	625-695	695-880	880-1225
INTERCHANGE RANGE (MW):					
-200,-100	PFIV #1	PFIV #2	PFIV #3	PFIV #4	
-100,0	PFIV #5	PFIV #6	PFIV #7	PFIV #8	
0,105	PFIV #9	PFIV #10	PFIV #11	PFIV #12	
105,165	PFIV #13	PFIV #14	PFIV #15	PFIV #16	
165,200	PFIV #17	PFIV #18	PFIV #19	PFIV #20	

*(PFIV = PENALTY FACTOR INFORMATION VECTOR)

$$\text{Minimize PI} = \sum_{n=1}^N y_n(u_n)$$

Subject to:

$$\sum_{n=1}^N u_n * PF_n = \text{BLD} + \text{LOSS}$$

Where:

1. Unit cost function:

$$y_n = f_n * e_n * F_n(u_n)$$

where: y_n = production cost

u_n = production power

f_n = fuel cost

e_n = efficiency

F_n = heat rate curve

For

all

units

$n=1, \dots, N$

2. Unit capacity limits:

$$U_{n \max} \geq u_n \geq U_{n \min} \quad \text{For all units } n=1, \dots, N$$

3. Network Loss Model:

PF_n = penalty factor For all units $n=1, \dots, N$

4. Power system model: BLD + LOSS

where: BLD = the base load demand (system load),

LOSS = the transmission loss.

FIGURE 3-16. ECONOMIC DISPATCH FORMULATION

TABLE 3-2. ENERGY CONVERSION CURVE MODELS

Input/Output Curve Function (F (u)) n n	Incremental Heat Rate Curve Function (f (u)) n n
1. Polynomial Curve;	
$F(u) = a * u^{**4} + b * u^{**3} + c * u^{**2} + d * u + e$ n n n n n n n n n n n n	
$f(u) = 4 * a * u^{**3} + 3 * b * u^{**2} + 2 * c * u + d$ n n n n n n n n n n	
2. Cubic Curve/Quadratic Curve;	
$F(u) = a * u^{**3} + b * u^{**2} + c * u + d$ n n n n n n n n n	
$f(u) = 3 * a * u^{**2} + 2 * b * u + c$ n n n n n n n	
3. Quadratic Curve/Linear Curve;	
$F(u) = a * u^{**2} + b * u + c$ n n n n n n n	
$f(u) = 2 * a * u + b$ n n n n n	
4. Linear Curve/Constant;	
$F(u) = a * u + b$ n n n n n	
$f(u) = a$ n n n	

TABLE 3-2. ENERGY CONVERSION CURVE MODELS (Continued)

Input/Output Curve Function (F (u)) n n	Incremental Heat Rate Curve Function (f (u)) n n
5. Piece-Wise Quadratic Curve/Piece-Wise Linear Curve	
For each range:	
$F(u) = a * u^{**2} + b * u + c$ n n n n n n n	$ U \geq u \geq U$ $ 1 n \min$ $ F() \geq F() \leq F()$ $ 1 n \min$
$f(u) = 2 * a * u + b$ n n n n n	or $ f() \geq f() \leq f()$ $ 1 n \min$
$F(u) = a * u^{**2} + b * u + c$ n n n n n n n	$ U \geq u \geq U$ $ 2 n 1$ $ F() \geq F() \leq F()$ $ 2 n 1$
$f(u) = 2 * a * u + b$ n n n n n	or $ f() \geq f() \leq f()$ $ 2 n 1$
$F(u) = a * u^{**2} + b * u + c$ n n n n n n n	$ U \geq u \geq U$ $ \max n 2$ $ F() \geq F() \leq F()$ $ \max n 2$
$f(u) = 2 * a * u + b$ n n n n n	or $ f() \geq f() \leq f()$ $ \max n 2$
6. Piece-Wise Linear Curve/Piece-Wise Constant Curve	
For each range:	
$F(u) = a * u + b$ n n n n n	$ U \geq u \geq U$ $ 1 n \min$ $ F() \geq F() \leq F()$ $ 1 n \min$
$f(u) = a$ n n n	or $ f() \geq f() \leq f()$ $ 1 n \min$
$F(u) = a * u + b$ n n n n n	$ U \geq u \geq U$ $ \max n 2$ $ F() \geq F() \leq F()$ $ \max n 2$
$f(u) = a$ n n n	or $ f() \geq f() \leq f()$ $ \max n 2$

The basic optimization formulation is shown in Figure 3-17. The objective function is augmented with the constraint, the first derivatives are found and set equal to zero. Mathematically, this satisfies the necessary condition but not the sufficient condition. The second order derivatives would have to be taken to satisfy the sufficient condition. Most techniques rely only on the necessary condition to find the optimal solution. If the energy conversion curve has favorable characteristics (as defined below) then the sufficient condition may be satisfied simultaneously with the necessary condition. The inclusion of system losses simply requires the addition of the penalty factors, as shown in Figure 3-18. The penalty factor (PF), the incremental transmission loss factor (ITL), and the network sensitivity factor are all related, as shown in Figure 3-18. Any technique which produces one of the three may be used [72,91,120,208,237]. This research assumes that the loss model parameters (PF, ITL, or NS) are available and included as part of the input data.

The first algorithm considered is a range elimination method. The general range elimination method is shown in Figure 3-19. Any of the above energy conversion curve models may be used with these algorithms. These methods are used in roughly one half of all Energy Management Systems for Economic Dispatch within Automatic Generation Control. The strengths of these approaches include: simplicity, easy to model unique unit constraints, uses mostly integer arithmetic, and will converge if the set of energy conversion curves are monotonically

1. Augment objective function with constraint

$$\text{MIN}_{u_n} \text{PI} = \sum_{n=1}^N F(u_n) - \text{LAMBDA} \left(\sum_{n=1}^N u_n - \text{BLD} \right)$$

2. Find all partial derivatives and equate to zero

$$\frac{d\text{PI}}{du_n} = \frac{dF}{du_n} - \text{LAMBDA} * (1) = 0 \quad \left. \begin{array}{l} | \\ | \\ | \\ | \end{array} \right\} n=1, \dots, N$$

$$\frac{d\text{PI}}{d\text{LAMBDA}} = - \left(\sum_{n=1}^N u_n - \text{BLD} \right) = 0$$

FIGURE 3-17. ECONOMIC DISPATCH SOLUTION TECHNIQUE
LAGRANGIAN MULTIPLIER
(CLASSICAL SOLUTION)

1. Augment objective function with constraint

$$\text{MIN}_{u_n} \text{PI} = \sum_{n=1}^N F(u_n) - \text{LAMBDA} \left(\sum_{n=1}^N u_n - \text{BLD} - \text{LOSS} \right)$$

2. Find all partial derivatives and equate to zero

$$\frac{d\text{PI}}{du_n} = \frac{dF}{du_n} - \text{LAMBDA} * \left(1 - \frac{d\text{LOSS}}{du_n} \right) = 0 \quad \begin{matrix} \vdots \\ n=1, \dots, N \\ \vdots \end{matrix}$$

$$\frac{d\text{PI}}{d\text{LAMBDA}} = - \left(\sum_{n=1}^N u_n - \text{BLD} - \text{LOSS} \right) = 0$$

Note: the loss coefficient is often rewritten into an equivalent form:

$$\left[1 - \frac{d\text{LOSS}}{du_n} \right] = \left[1 - \text{ITL}_n \right] = 1 / \text{PF}_n \quad (\text{Penalty Factor})$$

FIGURE 3-18. ECONOMIC DISPATCH SOLUTION TECHNIQUE
LA GRANGIAN MULTIPLIER WITH TRANSMISSION LOSSES
(CLASSICAL SOLUTION)

1. Determine minimum and maximum system lambda.
2. Guess new system lambda such that total generation will match the generation to be dispatched (GTBD).
3. Sum the generation of each unit for the new system lambda.
4. If the total generation is greater than GTBD, Then goto step 5, Else goto step 6.
5. Replace the maximum system lambda with the new system lambda, goto step 7.
6. Replace the minimum system lambda with the new system lambda.
7. If the total generation is within a tolerance of the GTBD Then goto step 10.
8. Update the iteration count, If the maximum iteration limit is exceeded, Then goto step 9, Else goto step 2.
9. Flag inability to converge within required number of iterations, return.
10. Save the new system lambda as the optimal solution, return.

FIGURE 3-19. ECONOMIC DISPATCH ALGORITHM
RANGE ELIMINATION METHOD(S)

increasing and sufficiently smooth. The weaknesses of these approaches include: slow convergence and lack of sensitivity information. This type of algorithm is the technique used for many commercial Unit Commitment algorithms.

The particular range elimination algorithm is the Binary Section method, as shown in Figure 3-20. The minimum and maximum values for the LaGrangian multiplier (LAMBDA) are calculated from the minimum and the difference between the upper LAMBDA and the lower LAMBDA. The generation for the estimated LAMBDA is found for each unit through the energy conversion curve and totaled. Next, the total generation is compared with the demand, if the total is too high, then the upper range can be discarded.

In either case, either the upper LAMBDA or the lower LAMBDA is replaced by the estimated LAMBDA, and the process is repeated. The process is repeated until too many iterations have occurred or the total generation is within solution tolerance. A quadratic curve model was assumed for the energy conversion curve.

The second algorithm considered is a graphical solution to the Economic Dispatch problem. The solution process is pictured in Figure 3-21. The process is to construct a system incremental cost curve which relates the total generation with the LaGrangian multiplier (LAMBDA). After the system curve is constructed, any solution can be found. The construction of the system incremental cost curve is built by summing all generation for different values of LAMBDA. Normally, the process starts at the lowest value of LAMBDA and continually

$$1. \quad a. \quad LMIN = \min_{n=1}^N \{ f_n * e_n (b_n + 2 * c_n * u_n) \}$$

N

$$b. \quad LMAX = \max_{n=1}^N \{ f_n * e_n (b_n + 2 * c_n * u_n) \}$$

$$2. \quad LAMBDA = (LMAX + LMIN) / 2; \quad PTOT = 0.0$$

3. For each unit:

$$\hat{u}_n = \frac{1}{2 * c_n} [LAMBDA / (PF_n * f_n * e_n)] - b_n$$

$$u_n = \begin{cases} u_n^{\min} & \text{if } \hat{u}_n < u_n^{\min} \\ \hat{u}_n & \text{otherwise} \\ u_n^{\max} & \text{if } \hat{u}_n > u_n^{\max} \end{cases}$$

$$PTOT = PTOT + u_n$$

4. If $PTOT > GTBD$ Then goto step 5, Else goto step 6.

5. $LMAX = LAMBDA$, goto step 7.

6. $LMIN = LAMBDA$.

7. If $ABS(GTBD - PTOT) \leq EPSILON$, Then goto step 10, Else continue.

8. $IT = IT + 1$; If $IT > ITMAX$ Then goto step 9, Else goto step 2.

9. Flag non-convergence and return.

10. Flag convergence, save optimum solution and return.

FIGURE 3-20. ECONOMIC DISPATCH ALGORITHM
BINARY SECTION/GOLDEN SECTION

A. Generate the System Incremental Cost Curve:

1. Find the minimum value of LAMBDA.
2. Sum the generation available for this value of LAMBDA, store this as the next point of the system incremental cost curve.
3. Increment the value of LAMBDA for the next curve point.
4. If this value of LAMBDA is greater than the maximum LAMBDA, Then goto step 5, Else goto step 2.
5. Set the value of LAMBDA equal to the maximum value of LAMBDA, sum the generation available, store this as the maximum point of the system incremental cost curve.

B. Find the Optimal Solution for a Given Demand:

1. Set PTOT = GTBD and the search index (l) to 1.
2. If PTOT is within the breakpoints (l,l+1), Then goto step 5, Else continue.
3. Increment the search index by one (l=l+1).
4. If the search index exceeds the number of breakpoints, Then goto step 8, Else goto step 2.
5. Find the optimal system incremental cost by extrapolation:

$$\text{LAMBDA} = \frac{\text{PTOT} - \text{P}(l)}{\text{P}(l+1) - \text{P}(l)} * (\text{LAMBDA}(l+1) - \text{LAMBDA}(l)) + \text{LAMBDA}(l)$$

6. For each unit (n=1,...,N):
 - a. Set the unit search index (j) to 1.
 - b. If LAMBDA is below the first unit breakpoint, Then goto step 6.g, Else continue.
 - c. If LAMBDA is within the unit breakpoints (j,j+1) Then goto step 6.f, Else continue.

FIGURE 3-21. ECONOMIC DISPATCH ALGORITHM
SYSTEM INCREMENTAL COST CURVE

- d. Increment the unit search index by one ($j=j+1$).
 - e. If the unit search index exceeds the number of breakpoints, Then goto step 6.h, Else goto step 6.c.
 - f. Find the optimal unit generation by extrapolation:

$$P(n) = \frac{\text{LAMBDA}-\text{LAMBDA}(j)}{\text{LAMBDA}(j+1)-\text{LAMBDA}(j)} * (P(j+1)-P(j)) + P(j)$$
 and Goto step 6.a.
 - g. Set the unit generation to P(min) and Goto step 6.a.
 - h. Set the unit generation to P(max) and Goto step 6.a.
- 7. Flag convergence, save optimum solution and return.
 - 8. Flag non-convergence and return.

FIGURE 3-21. ECONOMIC DISPATCH ALGORITHM--
SYSTEM INCREMENTAL COST CURVE (Continued)

increments LAMBDA until the maximum value of LAMBDA is attained. A solution is found by finding the system incremental cost for a given demand through the unit incremental cost curves. These unit generation values are the optimal solution. The strengths of this algorithm include: solution speed, solution accuracy, ease of modeling unique unit operating constraints, and wealth of sensitivity information. The weaknesses of this approach include: overhead to recreate system incremental cost curve, storage requirement for the system incremental cost curve, and inaccuracies of curve representation between successive values of LAMBDA. Note that the system incremental cost curve has to be regenerated whenever a unit status changes, the system loss model changes, fuel cost changes, fuel type changes, etc. If the piece-wise linear incremental heat rate curve is the selected model for the units, then the system incremental cost curve is also piece-wise linear. The values for the system incremental cost should be the breakpoints for each unit's incremental cost curve to eliminate all of the inaccuracies of curve representation. Figure 3-22 depicts a piece-wise linear system incremental cost curve. If the demand was 700 MW, then the system incremental cost at the optimal solution is 0.5. This value for the system incremental cost is then used for each unit incremental cost curve to find the unit generations (500.0,194.4).

The third algorithm considered is a piece-wise search of the graphical solution [32]. This algorithm is shown in Figure 3-23. The basis of this algorithm is the concept of unit-segment. A unit-segment is a section of the unit piece-wise linear incremental heat rate

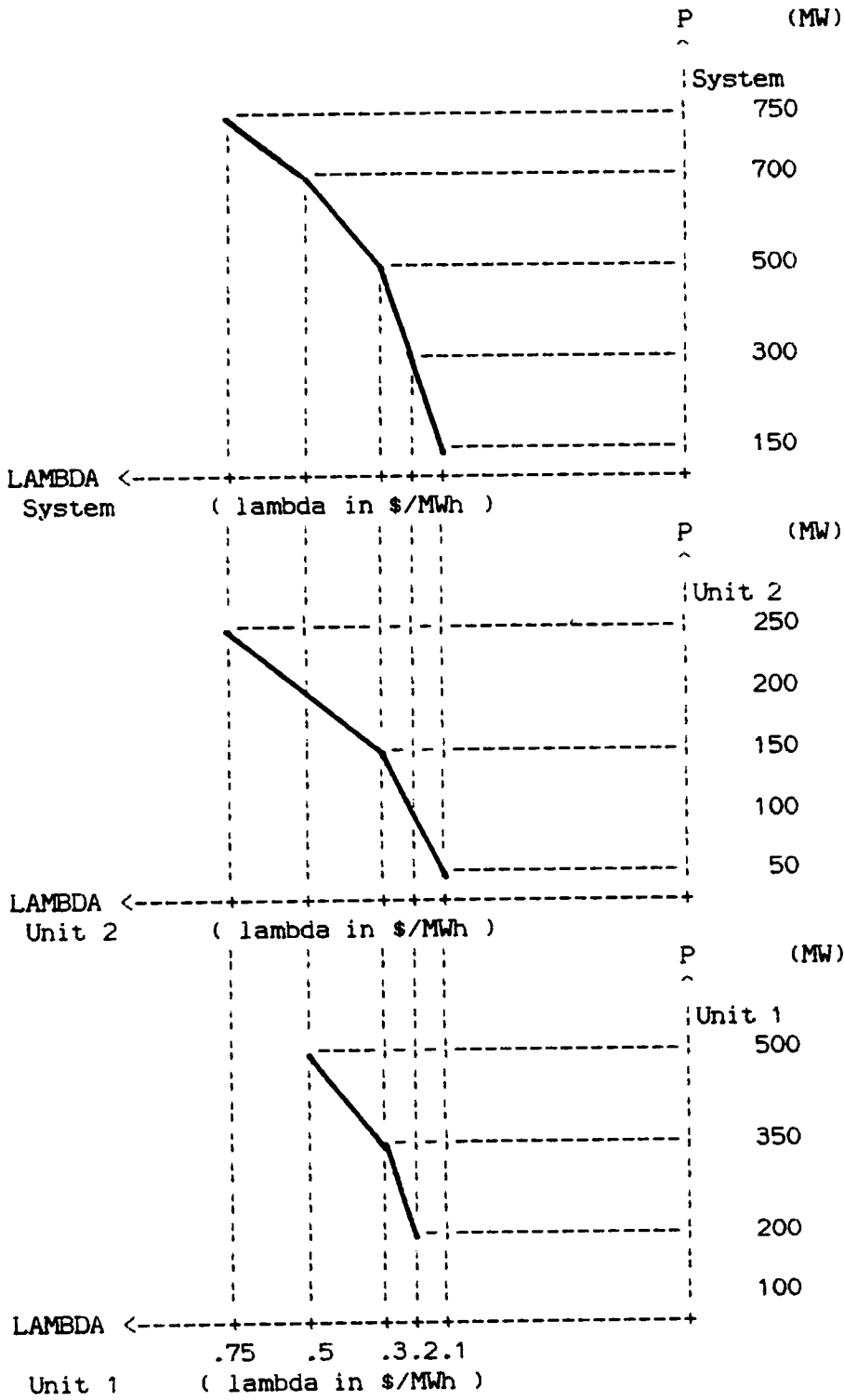


FIGURE 3-22. ECONOMIC DISPATCH ALGORITHM--
SYSTEM INCREMENTAL COST CURVE (TABLE LOOKUP)

- A. Generate the Unit-segment cost curve in ascending order:
1. Generate the incremental unit-segments (us) for all units ($n = 1, \dots, M*N$), where M is the number of segments per unit, and sum the minimum generation for each unit.
 2. SORT the unit-segments ($n = 1, \dots, M*N$) into ascending cost.
- B. Find the near-optimal solution for the given demand:
1. Set the unit-segment index to one ($n = 1$) and the total generation to the summation of the minimum generations.
 2. Dispatch the unit-segment by adding the incremental unit-segment generation into the total generation:

$$PTOT = PTOT + us \max_n$$
 3. If $PTOT > GIBD$, Then goto step 5, Else continue.
 4. Increment the unit-segment index ($n = n + 1$);
If there are no more unit-segments ($n > N*M$)
Then goto step 9, Else goto step 2.
 5. Calculate the amount of the generation needed from the last unit-segment for the total generation to equal the demand:

$$PART = PTOT - us \max_n - GIBD$$
 6. If the partial generation is negative, Then goto step 10, Else continue.
 7. Sum the unit-segments dispatched ($n = 1, \dots, n^*$) for each unit and add the partial dispatch (PART) from the last unit-segment.
 8. Flag "convergence", save optimum solution and return.
 9. Flag too little generation and return.
 10. Flag too much generation and return.

FIGURE 3-23. ECONOMIC DISPATCH ALGORITHM--
MERIT ORDER LOADING

curve. A unit-segment could be the linear segment (.1,100) to (.2,200) for the first unit shown in Figure 3-22. The first step is to sort the unit segments into ascending cost. The cost used could be the average incremental cost for the segment, the minimum incremental cost for the segment, or the maximum incremental cost for the segment. Sum the unit-segments in ascending order until the total generation is greater than the given demand (GTBD). Delete the last unit-segment added and use extrapolation to find the generation needed from the last unit-segment for the total generation to equal the demand. The strengths of this algorithm include: solution speed, solution accuracy, ease of modeling unique unit operating constraints, and some sensitivity information. The weaknesses of this approach include: overhead to recalculate unit-segments, storage requirement for the unit-segment cost curves, and inaccuracies of curve representation within each unit-segment. Note that the unit-segments have to be reordered whenever a unit status changes, and regenerated whenever the system loss model changes, fuel cost changes, fuel type changes, etc. If the piece-wise linear incremental heat rate curve is the selected model for the units, then each unit-segment cost curve is also piece-wise linear. The end-points for each unit-segment cost curve should at least be the breakpoints for each unit's incremental cost curve to eliminate all of the inaccuracies of curve representation. The end-points could be determined by the amount of generation to be allocated to each unit for an incremental change in demand. This could require more computer memory than available, due to the increase in the

number of unit-segments. The optimal solution may not be found by this algorithm if any unit-segment is larger than the desired solution tolerance. Any energy conversion curve may be used for this algorithm as long as the curve is monotonically increasing.

The next type of algorithms considered are based upon directed search techniques. Directed search techniques use information available from the objective function to improve the solution without forcing the solution to be an infeasible solution along the way. The fourth algorithm considered was the steepest descent gradient search depicted in Figure 3-24. This algorithm starts from a known feasible solution and experimentally searches for the best direction to improve each variable (unit generation). Some of the more natural solutions to start from include: initial conditions, the optimal solution for the previous stage (hour), or the optimal solution for the previous combination. Any of the above energy conversion curve models may be used as long as the first derivative exists. The strength of this approach includes: simplicity, easy to model unique unit operating constraints, will converge if the set of energy conversion curves is monotonically increasing and sufficiently smooth, solution accuracy, and some sensitivity information. The weakness of this approach includes: slow convergence, inaccuracies of curve representation for non-polynomial models, the models have to be regenerated whenever a unit status changes (e.g., the system loss model changes, fuel cost changes, fuel type changes) for piecewise-linear models.

1. Find starting point, u
2. Compute gradient, $GRAD(u) = \sum_{n=1}^N \frac{df}{du} e_i$
3. Update generation of each unit ($n=1, \dots, N$):

$$\hat{u}_n = u_n + s * GRAD(u)_n$$

$$\hat{u}_n = \begin{cases} u_n & \text{if } u_n < u_{n, \min} \\ u_{n, \min} & \text{if } u_n < u_{n, \min} \\ u_n & \text{if } u_n > u_{n, \max} \\ u_{n, \max} & \text{if } u_n > u_{n, \max} \end{cases}$$

$$\hat{PTOT} = PTOT + \hat{u}_n$$

4. If $PTOT > GTBD$ then goto step 5, else goto step 6.
5. $s = s / 2.$, goto step 3
6. Update u and $PTOT$.
7. If $ABS(GTBD - PTOT) \leq EPSILON$, then goto step 10.
8. $IT = IT + 1$; If $IT > ITMAX$ then goto step 9, else goto step 2.
9. Flag non-convergence and return.
10. Flag convergence, save optimum solution and return.

FIGURE 3-24. STEEPEST DESCENT GRADIENT SEARCH

The next algorithm considered was the first order gradient method shown in Figure 3-25. This algorithm can be obtained from the Taylor series expansion, as shown in the text by Wood and Wollenberg [232]. This method starts from any feasible solution. The generating units, which will change the objective function the most, are selected and the unit generations changed accordingly. This method is obviously very similar to the previous method except that feasibility is never lost. Any of the above energy conversion curve models may be used as long as the first derivative exists. The strength of this approach includes: simplicity, easy to model unique unit operating constraints, will converge if the set of energy conversion curves is monotonically increasing and sufficiently smooth, solution accuracy, and some sensitivity information. The weakness of this approach includes: slow convergence, inaccuracies of curve representation for non-polynomial models, the models have to be regenerated whenever a unit status changes (e.g., the system loss model changes, fuel cost changes, fuel type changes) for piecewise-linear models.

The next logical algorithm for consideration would be the Second-Order Gradient Method [232]. The second order gradient method allows all generation to change simultaneously. This method was not considered for this research due to the process of inverting the required Hessian matrix even though there are not any mixed, second order derivatives. The Hessian matrix is a function of the present solution point and the active constraints. The computer resources necessary for such computations are just as extensive as the computer

1. Find feasible starting point, u

2. Compute gradient, $GRAD(u) = \sum_{i=1}^N \frac{df}{du_i} e_i$

3. Check for convergence at a stationary point (local optimum or saddle point):

If $\|GRAD(u)\| < \epsilon$ for all $i = 1, \dots, N$

then goto 12.

4. Select: $P_u = \text{MAX}_{n=1}^N \{ GRAD(u)_n \}$

5. Calculate relative costs: $c_i = \frac{df}{dp_i} - \frac{df}{dp_u}$ For all $i=1, \dots, N$ and $i \neq u$.

6. Select $P_l = \text{MAX}_{n=1}^N (c_n)$ $n \neq u$

7. Calculate generation change of each unit (l and u):

$$DELTA(P)_u = \begin{cases} P_u - P_u^{min} & \text{if } \frac{df}{dp_u} < 0 \\ P_u^{max} - P_u & \text{if } \frac{df}{dp_u} > 0 \end{cases}$$

8. Calculate maximum step size:

$$DELTA = \text{MIN} \{ DELTA(P)_u, DELTA(P)_l \}$$

FIGURE 3-25. FIRST ORDER GRADIENT SEARCH

9. Update solution:

$$P_u^{i+1} = P_u^i + \text{DELTA} \quad ; \quad P_l^{i+1} = P_l^i + \text{DELTA}$$

10. If $\text{ABS}(\text{DELTA}_i - \text{DELTA}_{i+1}) \leq 0$, then goto step 12.

11. $IT = IT + 1$;
 If $IT > ITMAX$ Then flag non-convergence and return
 Else goto step 2.

12. Flag convergence and return.

FIGURE 3-25. FIRST ORDER GRADIENT SEARCH (CON'T.)

resources to solve the Unit Commitment problem with a simple Economic Dispatch. Clearly, the selection of such a technique would only compound our problem unless only a few Economic Dispatches were needed for each stage.

The next algorithm selected for consideration was Linear Programming, shown in Figure 3-26 [58, pp 323-325]. This algorithm was the basis of previous research by this author to evaluate the cost of regulation. Linear Programming is the most widely used optimization algorithm. When applied to the Economic Dispatch problem and when appropriately modified for upper bounding, this algorithm converges very quickly. The algorithm implemented for this research follows the development in the text by Cooper and Steinberg. This algorithm uses only the original variables and a table indicating if a non-basis variable is at its lower or upper limit. The implemented algorithm is similar to the method in the text by Wood and Wollenberg [232] except that a heuristic technique is used to find an initial feasible solution. The choice of this algorithm was based upon the energy conversion curves supplied with the test data sets.

The choice of energy conversion curve can force additional refinements of the Linear Programming algorithm. The most general refinement, separable programming, is required if a high order polynomial curve is selected. This refinement turns the Linear Programming algorithm into a Reduced Gradient algorithm. If the constraints were not assumed to be linear, then the Generalized Reduced Gradient algorithm would be the most general solution

1. Find feasible starting point, u , used as basis,
 $-B$

$$\text{where: } u = B^{-1} * b \quad \text{and} \quad y = B^{-1} * a$$

$$-B \quad - \quad -j \quad -j$$

2. Calculate reduced costs:

$$c * y - c_j = z_j - c_j \quad \text{for all } j \in NB$$

$$-B \quad -j \quad j \quad j \quad j$$

3. Find best variable to adjust(entry):

$$a. \quad d_k = \text{MAX}_{n=1}^{NB} \left\{ \begin{array}{l} z_n - c_n \text{ if } u_n = 0 \\ c_n - z_n \text{ if } u_n = U_{n \max} \end{array} \right\}$$

- b. if $d_k \leq 0$ then optimal, goto step 8, else continue,
 k

4. Calculate optimal stepsize and leaving variable (1)

a. $u_k = 0$

$$i. \quad \text{DELTA}_1 = \frac{u_{bl} - 1}{y_{lk}} = \text{MIN} \left(\frac{u_{Bi}}{y_{ik}} \mid y_{ik} > 0 \right)$$

$$ii. \quad \text{DELTA}_2 = \frac{u_{bl} - u_{l1}}{y_{lk}} = \text{MIN} \left(\frac{u_{Bi} - u_{li}}{y_{ik}} \mid y_{ik} < 0 \right)$$

$$iii. \quad \text{DELTA}_k = \text{MIN} \{ u_k, \text{DELTA}_1, \text{DELTA}_2 \}$$

iv. if $\text{DELTA}_k = u_k$ then $u_k = \text{DELTA}_k$ and goto step 6,

else goto step 5.

FIGURE 3-26. LINEAR PROGRAMMING WITH UPPER BOUNDING

$$b. \quad u_k = U_{\max k}$$

$$i. \quad \text{DELTA}_1 = -\frac{u_{B1}}{y_{1k}} + u_k = \text{MAX}_i \left(-\frac{u_{Bi}}{y_{ik}} \mid y_{ik} < 0 \right) + u_k$$

$$ii. \quad \text{DELTA}_2 = \frac{u_{B1} - u_{l1}}{y_{1k}} + u_k = \text{MAX}_i \left(\frac{u_{Bi} - u_{li}}{y_{ik}} \mid y_{ik} > 0 \right) + u_k$$

$$iii. \quad \text{DELTA} = \text{MAX} \{ 0, \text{DELTA}_1, \text{DELTA}_2 \}$$

iv. if DELTA = 0 then $u_k = 0$ and goto step 6, else continue

5. Simplex pivot:

$$a. \quad \hat{y}_{ij} = y_{ij} - \frac{y_{lj} * y_{ie}}{y_{le}} \quad \text{for all } e \neq l$$

$$b. \quad \hat{y}_{lj} = \frac{y_{lj}}{y_{le}}$$

FIGURE 3-26. LINEAR PROGRAMMING WITH UPPER BOUNDING
(CONTINUED)

6. Update solution:

$$a. \quad \hat{z}_j - c_j = z_j - c_j - \frac{y_{lj}}{y_{le}} * (z_e - c_e)$$

$$b. \quad \hat{u}_{Bi} = u_{Bi} - u_{Bl} * \frac{y_{ie}}{y_{le}} ; \quad \hat{u}_{Bl} = \frac{u_{Bl}}{y_{le}}$$

7. Goto step 3.

8. Flag convergence and return.

FIGURE 3-26. LINEAR PROGRAMMING
WITH UPPER BOUNDING (cont)

algorithm. The Reduced Gradient algorithm is discussed below. If the objective function (performance index) is linear, then the Reduced Gradient algorithm can be transformed into the Simplex Linear Programming algorithm. If the constraints were removed, then the Reduced Gradient algorithm can be transformed into the Steepest Descent algorithm [9, pg. 473].

It is generally assumed that the energy conversion curve is convex to force a globally optimal solution. (Global for all values of generation, not for all values of time). A general algorithm for convex simplex programming is shown in Figure 3-27 [200, pp 297-305]. Any of the above energy conversion curve models may be used with this algorithm.

The strengths of this approach include: easy to model unique unit operating constraints, will converge if the set of energy conversion curves is monotonically increasing and sufficiently smooth, solution accuracy, and the availability of some sensitivity information at the optimal solution. The weakness of this approach includes: slow convergence, overhead to recalculate matrices and derivatives, storage requirement for the matrices and derivatives.

If the objective function is additively separable, then each function can be linearized separately by a set of points for each variable [58, pg. 249]. An algorithm for a piece-wise linear conversion curve is shown in Figure 3-28. If the objective functions are linear functions, then this grid linearization process can be transformed into the Dantzig-Wolfe decomposition algorithm shown in

1. Find feasible starting point, u , used as basis.

$$\begin{bmatrix} b \\ -0 \end{bmatrix} = B \begin{bmatrix} b \\ - \end{bmatrix}; u = b - \sum_{j \in NB} (u_j * y_j); y = B^{-1} a$$

2. Calculate reduced costs:

$$\frac{dz}{du_j} = \frac{df}{du_j} - \text{GRAD}(u_j) * y_j \quad ; \text{ For all } j \in NB$$

3. Find best variable to adjust (entry-nonbasic):

a. Compute :

$$\frac{dz}{du_p} = \text{MAX}_{j \in NB} \left\{ \frac{dz}{du_j} \right\}$$

$$\frac{dz}{du_q} * u_q = \text{MIN}_{j \in NB} \left(\frac{dz}{du_j} * u_j \right)$$

- b. Select variable:

i. increase u_p if $\frac{dz}{du_p} > 0$ and $\frac{dz}{du_q} * u_q \Rightarrow 0$

or

$\left| \frac{dz}{du_p} \right| \Rightarrow \left| \frac{dz}{du_q} * u_q \right|$, goto step 4a,

ii. decrease u_q otherwise, goto step 4b.

iii. if $\frac{dz}{du_p} \leq 0$ and $\frac{dz}{du_q} * u_q \Rightarrow 0$, goto step 12.

FIGURE 3-27. CONVEX SIMPLEX PROGRAMMING

4. Calculate stepsize limits:

a. u is to be increased:
 k

i. if at least one component of $y = B^{-1} * a > 0$
 $-k$ $-k$

$$\text{then DELTA} = \frac{u}{y_{lk}} = \text{MIN} \left(\frac{u}{y_{ik}} \mid y_{ik} > 0 \right)$$

set case = 1 and goto step 5.

ii. if no component of $y = B^{-1} * a > 0$
 $-k$ $-k$

then DELTA = +M (some very large number),

set case = 2 and goto step 5.

b. u is to be decreased: set case = 3
 k

$$\text{DELTA}' = \frac{u}{y_{lk}} = \text{MAX} \left(\frac{u}{y_{ik}} \mid y_{ik} < 0 \right)$$

If $y_{ik} \geq 0$ for all i , then DELTA' = -M

$$\text{DELTA} = \text{MIN} \left(-\text{DELTA}', u \right)$$

FIGURE 3-27. CONVEX SIMPLEX PROGRAMMING (cont)

5. Solve for optimum step size (θ) :

$$\text{MAX } f(\hat{u}) = \begin{cases} f(u + \theta * s) & \text{for case = 1 or 2} \\ f(u - \theta * s) & \text{for case = 3} \end{cases}$$

such that $0 \leq \theta \leq \text{DELTA}$

$$\text{and } s_j = \begin{cases} s_k = 1 & \\ s_j = 0 & \text{if } j < k \text{ and } j \in \text{NB} \\ s_j = -y_{jk} & \text{if } u_j = u_{B1} \text{ and } j \in B \end{cases}$$

6. If case = 1 goto step 7, else if case = 2 goto step 8, else if case = 3 goto step 9.

7. If $\theta < \text{DELTA}$ then goto step 10, else
opt

Simplex pivot: u_k replaces u_{B1} in basis, goto step 10.

8. If $\theta = +M$ (unbounded), then goto step 13, else goto step 10.
opt

9. If $\theta = -\text{DELTA}' < u_k$ then continue, else goto step 10
opt

Simplex pivot: u_k replaces u_{B1} in basis.

10. Update solution:

$$\hat{u}_{\text{opt}} = \hat{u} + \theta * s$$

11. Goto step 2.
12. Flag convergence and return.
13. Flag nonconvergence and return.

FIGURE 3-27. CONVEX SIMPLEX PROGRAMMING (cont)

Original problem:
$$\text{MIN } z = \sum_{n=1}^N f_n(u_n)$$

such that:
$$a_m * u_m = b_m \quad | \text{ For all } m=1, \dots, M$$

$$u_n \geq 0 \quad | \text{ For all } n=1, \dots, N$$

Piece-wise linear problem:
$$\text{MIN } \sum_{n=1}^N \sum_{p=1}^P (f_{np} * l_{np})$$

such that:
$$\sum_{n=1}^N \sum_{p=1}^P (a_{mnp} * l_{np}) = b_m \quad | \text{ For all } m=1, \dots, M$$

$$\sum_{p=1}^P (l_{np}) = 1 \quad | \text{ For all } n=1, \dots, N$$

$$l_{np} \geq 0 \quad | \text{ For all } n \text{ and } p$$

and each subproblem:
$$\text{MIN } z_i = (c_i - p_i * A_i) * u_i$$

such that:
$$B_i * u_i = b_i \quad | \text{ For all } i = 1, \dots, N$$

$$u_i \geq 0$$

FIGURE 3-28. SEPARABLE CONVEX LINEAR PROGRAMMING

Figure 3-29. The Dantzig and Wolfe decomposition principle [136] is considered to be the most efficient solution algorithm for convex simplex problems of linear objective functions with linear constraints. The methods to achieve such efficiencies are listed in the text by Lasdon [136]. The approach is to form an equivalent "master program" which has just a few rows but many columns. The technique to achieve efficiency is to generate the columns only when needed by the simplex algorithm.

The main Linear Programming application, which has been most successful, is the application of the Dantzig-Wolfe Decomposition to the Economic Dispatch algorithm. This is due partially to the predominant use of piece-wise linear curves. Another reason is due to the convexity of the energy conversion curves even though this convexity is normally forced since the raw data used to generate the curves are normally generated by a step function and even though there is considerable measurement error. The convexity is often required for the real-time control algorithms. This algorithm was tested for this research.

All of the energy conversion curve models may be used, but any Linear Programming based technique will work best with curves which are nearly linear. Linear Programming methods are used extensively in the general field of optimization. Linear Programming based algorithms are the techniques used for most commercial optimization packages. The literature abounds with information from special coding techniques to special solution algorithms for specially structured problems (e.g.

$$\begin{array}{l}
 \text{Original problem: } \text{MIN } z = \sum_{n=1}^N c_n * u_n \\
 \text{such that: } \sum_{n=1}^N A_n * u_n = b_0 \\
 B_n * u_n = b_n \quad \left\{ \begin{array}{l} \text{For all} \\ n=1, \dots, N \end{array} \right. \\
 u_n \geq 0 \quad \left\{ \begin{array}{l} \\ N \geq 1 \end{array} \right.
 \end{array}$$

$$\begin{array}{l}
 \text{Master problem: } \text{MIN } \sum_{j \in B} (f_j * l_j) \\
 \text{such that: } \sum_{j \in B} (p_j * l_j) = b_0 \\
 \sum_{j \in B} (l_j) = 1 \\
 l_j \geq 0 \quad \left\{ \begin{array}{l} \text{For all} \\ j \in B \end{array} \right.
 \end{array}$$

$$\begin{array}{l}
 \text{and each subproblem: } \text{MIN } z_i = (c_i - p * A_i) * u_i \\
 \text{such that: } B_i * u_i = b_i \quad \left\{ \begin{array}{l} \text{For all} \\ i = 1, \dots, N \end{array} \right. \\
 u_i \geq 0
 \end{array}$$

FIGURE 3-29. DANTZIG-WOLFE DECOMPOSITION ALGORITHM

network flows). The strengths of any Linear Programming based algorithms include: simplicity, easy to add models for unique unit operating constraints, convergence is easily detected, convergence to a global optimum is guaranteed if the set of energy conversion curves is monotonically increasing (convex), solution speed, solution accuracy if the curves are nearly linear, and a wealth of sensitivity information. The weakness of these algorithms is predominantly due to the energy conversion curve representation. These weaknesses may include: slow convergence if the curves are highly non-linear, storage requirement for the linearized models, inaccuracies of curve representation, the models have to be regenerated whenever a unit status changes (e.g., the system loss model changes, fuel cost changes, fuel type changes).

The last algorithm considered was the Reduced Gradient algorithm referenced above. This method is similar to the Convex Simplex Linear Programming algorithm above. The main difference is the step direction found at each iteration. The Reduced Gradient algorithm allows all variables to change just as the Generalized Reduced Gradient algorithm changes all variables simultaneously. The Convex Simplex Linear Programming algorithm changes just one variable at a time, just as Linear Programming changes only one variable at a time. This increase in freedom to select better directions, which are combinations of the non-basic variables, requires more complex computations. Since most Economic Dispatch solutions involve only a small number of active constraints at the optimum, the Reduced Gradient algorithm should prove faster than the Convex Simplex Linear Programming algorithm. The main difference between the Generalized Reduced Gradient and Linear

Programming, for the completely non-linear case, is that the Linear Programming solution has to occur at a boundary where constraints are active. The Generalized Reduced Gradient algorithm does not restrict the solution to a boundary but allows the solution to be any interior point.

There are many counter-examples to the above conclusion that the Reduced Gradient algorithm is better. These examples are primarily based upon the main weakness of all these algorithms: the derivatives must exist for all intermediate solution points and the energy conversion curve must be a smooth, monotonically increasing function. The main concern should be that degeneracy can always occur with linearly constrained problems even when Linear Programming is not used. The definition of degeneracy has to be based upon the linear dependency of the gradients of the active constraints [200].

The Reduced Gradient algorithm evaluated by this research is shown in Figure 3-30 [200, pp. 312-317]. This definition uses the familiar notation of Linear Programming. However, it should be noted that the value of the non-basic variables need not be either the lower (zero) or the upper bound. The most interesting result of verifying the solution process of the Reduced Gradient algorithm was the close similarity with the Merit Order Loading algorithm. If simple accounting procedures and backtracking logic are added to the Merit Order Loading algorithm, the Merit Order Loading algorithm can find the same solution sequence as any Linear Programming or Reduced Gradient algorithm. This would be beneficial if fast solutions would be needed for simply constrained problems without incurring the overhead needed for matrix generation or solution. This comparison has been left for future research.

1. Find feasible starting point, u , used as basis.
 $-B$

2. Calculate reduced costs:

$$\frac{dz}{du_j} = \frac{df}{du_j} - \text{GRAD}(u)^T * y_j = \quad ; \text{ For all } j \in \text{NB}$$

3. Find the reduced gradient:

$$w_j = \begin{cases} 0 & \text{if } \frac{dz}{du_j} \leq 0 \text{ and } u_j = 0 \\ \frac{dz}{du_j} & \text{otherwise} \end{cases}$$

4. If $\|w\|_1 \leq 0$ for all j , then goto 12.

5. Calculate basic variable change:

$$v_i = -\sum_{j \in \text{NB}} w_j * y_{ij}$$

6. Calculate optimal stepsize:

a. Calculate stepsize to force nonbasic variable to zero

$$i. \quad \text{DELTA} = \max_{j \in \text{NB}} \left(\frac{u_j}{w_j} ; w_j < 0 \right)$$

ii. if no component of $w_j < 0$ then $\text{DELTA} = -M$

FIGURE 3-30. REDUCED GRADIENT WITH LINEAR CONSTRAINTS

b. Calculate stepsize to force basic variable to zero:

$$i. \quad \Delta = \max_{j \in NB} \left(\frac{-B_j}{A_{ij}} \mid v_j < 0 \right)$$

$u_j = u_j$

ii. if no component of $v_j < 0$ then $\Delta = +M$

$$c. \quad \Delta = \min(\Delta_1, \Delta_2)$$

7. Solve for optimum step size (θ):

$$\max_{\theta} f(\underline{u}) = f(\underline{u} + \theta * \underline{s})$$

such that: $0 \leq \theta \leq \Delta$

where: $s_j = w_j$ if $j \in NB$

$s_j = v_j$ if $j \in B$ and $u_j = u_{Bi}$

8. If $\theta_{opt} = \Delta = +M$, then unbounded, goto step 11.

9. Update solution:

$$a. \quad \underline{u}^{i+1} = \underline{u}^i + \theta_{opt} * \underline{s}$$

b. If $\theta_{opt} = -\Delta$ then Simplex pivot:

variable to enter basis found by: $\max_{j \in NB} \{ y_{rj} \}$

10. Goto step 2.

11. Flag nonconvergence and return.

12. Flag convergence and return.

FIGURE 3-30. REDUCED GRADIENT WITH LINEAR CONSTRAINTS (cont.)

CHAPTER 4

SUCCESSIVE APPROXIMATION METHODOLOGY

The first major subsection below defines the models implemented in the programs developed for this research. The second major subsection develops the Successive Approximation in Solution Space (SASS) algorithm. The third major subsection outlines the programs developed to evaluate the proposed algorithm. This algorithm is believed to be original at least for the Unit Commitment problem. This algorithm was evaluated for this research.

4.1 POWER SYSTEM MODELS

4.1.1 Unit Commitment Formulation

The general Unit Commitment formulation was discussed in Chapter 2. This section will define the models included within the programs developed to evaluate the SASS algorithm.

The first equation, shown in Figure 4-1, is the classical objective function which includes terms for production costs and transition costs. The production cost is the cost of fuel consumption. The transition cost is either the cost of starting a unit or stopping a unit. The start-up and shut-down costs are constants.

The second equation, shown in Figure 4-1, is the statement that the demand must equal the generation adjusted for the power system losses.

Operational Cost :

$$DC_t = \sum_{n=1}^N \{ F(u_n) + s_{n,t}(u_{n,t}) \}$$

Generation Requirement:

$$L_t = \sum_{n=1}^N (p_{n,t} * u_{n,t})$$

Reserve Constraint:

$$B_t \geq \sum_{n=1}^N (r_{n,t})$$

Up/Down Time Duration:

$$\bar{T}_n \geq g(u_{n,t}) \leq T_{-n}$$

Unit capability:

$$\bar{U}_n \geq u_{n,t} \geq U_{-n}$$

Reserve capability:

$$\bar{R}_n \geq r_{n,t} = (\bar{U}_n - u_{n,t})$$

For
All
Stages
t = 1, ..., T

For
All
Units
n = 1, ...
..., N

FIGURE 4-1. UNIT COMMITMENT PROBLEM FORMULATION

Where:

- L = system load capability (MW)
 DC_t = allowable cost of production for stage t (\$)
 p_n = penalty factor for unit n
 $F(u)_n$ = production cost for unit n (\$)
 $s_t(u)_n$ = transition cost at stage t for unit n (\$)
 $u_{n,t}$ = generation for unit n at stage t (MW)
 $r_{n,t}$ = reserve capability for unit n at stage t (MW)
 T = schedule horizon (number of stages)
 B_t = required spinning reserve for stage t (MW)
 g_n = unit up/down time transform
 \bar{T}_n, T_n = unit up/down time limits (stages)
 \bar{U}_n, U_{-n} = unit maximum/minimum generation limits (MW)
 \bar{R}_n = unit maximum reserve contribution limit (MW)
 N = number of units

FIGURE 4-1. UNIT COMMITMENT PROBLEM FORMULATION (Con't.)

The remaining equations define the constraints which restrict operation. These constraints are (listed in the order on the figure):

- o Required reserve for state t
- o Unit up and down time duration
- o Unit capability limits

The required reserve for each stage (hour) was assumed to be a function of the largest unit on-line and of the system load demand. The reserve contribution from each unit was constrained to a constant upper limit. The up and down time duration function (g) transforms the status of a unit for each stage into a duration value which can be compared with the unit's minimum up-time and down-time. If the minimum up-time is not satisfied then the unit has to be forced into more continuous unit operation. If the unit's minimum down-time is not satisfied then the unit has to be forced into a more continuous unit shut-down or the unit must be put on idle stand-by. The constant unit capability limits constrain the unit generation each hour.

4.1.2 Economic Dispatch Models

The general representation of the electric power system for the Economic Dispatch Problem was presented in Chapter 2. This section summarizes the models used by the programs developed for this research. The representation for the electric power system for Economic Dispatch consists of models for the generating units and of a model for the transmission system. The transmission system model was assumed as constant penalty factors within these programs. The values for penalty factors have always been neglected from scheduling test

cases. Consequently, this research did not include transmission loss values, but the programs have the capability of including penalty factors for each unit.

The models for the generating units are summarized in Figure 4-2. These models are a subset of the models in Figure 4-1 since each applies for the stage (hour) under simulation. These models represent the cost of producing electricity, the generation capability, and the reserve capability of each unit. The function used to represent the energy conversion curve for this research was a Linear Curve for the Input/Output Curve which results in a Constant Value for the Incremental Heat Rate Curve. The error introduced by this simplification was calculated from the quadratic curve parameters as part of input data processing.

The classical objective function (PI) for Economic Dispatch is shown as a function of only the production costs. The generation requirement shows that the total generation must be equal to the system load demand after adjustment for transmission losses. The reserve requirement shows that the generation allocation must be spread across the units and must be at least equal to the required value. The change in notation is shown to clarify the programming design given below. The primary change is the deletion of the transition costs and the deletion of the subscript "t".

Note that only the variable costs are considered by any Economic Dispatch algorithm. The programs developed for this research used a single reserve constraint.

1. Unit cost function:

$$y_n = f_n (a_n * u_n + b_n)$$

| For all
| units
| n=1,...,N

2. Unit capacity limits:

$$\bar{U}_n \geq u_n \geq U_{-n}$$

| For all
| units
| n=1,...,N

3. Unit reserve limits:

$$r_n = (\bar{U}_n - u_n) \leq \bar{R}_n$$

| For all
| units
| n=1,...,N

4. Operational Cost:

$$PI = \sum_{n=1}^N \{ F_n(u_n) \}$$

5. Generation Requirements:

$$L = \sum_{n=1}^N (P_n * u_n)$$

6. Reserve Constraint:

$$B \geq \sum_{n=1}^N (r_n)$$

FIGURE 4-2. GENERATING UNIT ECONOMIC DISPATCH MODELS

Where: $F_n(u_n)$ = production cost for unit n (\$)

f_n = fuel cost (\$/MBTU)

a_n = energy conversion variable
heat rate constant (MBTU/MW)

b_n = energy conversion heat rate
constant (MBTU/MW)

u_n = unit generation (MW)

L = system load capability (MW)

p_n = penalty factor for unit n

B = required reserve capability (MW)

r_n = reserve capability for unit n (MW)

\bar{U}_n, U_{-n} = unit maximum/minimum generation limits (MW)

\bar{R}_n = unit maximum reserve contribution limit (MW)

N = number of units

FIGURE 4-2. GENERATING UNIT ECONOMIC DISPATCH MODELS (Con't.)

4.2 PROPOSED METHODOLOGY

The following sections define the algorithm developed as part of this research. The idea originated from a previous study on the cost of regulation [226,227]. The cost of regulation study used Linear Programming for the Economic Dispatch within a Sequential Priority List Dynamic Programming algorithm. The first subsection outlines the basis of the idea. The second subsection outlines an explanation based upon Decision Analysis [40,48,49,50,68,106,107,115,144,149,180,214] and a justification for convergence based upon the Maximum Principle [8,68,97,98,105]. The third subsection defines the Successive Approximation in Solution Space (SASS) algorithm in pseudo-code. The last subsection compares the SASS algorithm with the Truncated Priority List Dynamic Programming algorithm.

4.2.1 Duality and Successive Approximations

The idea for this method originated from the Binary Section Economic Dispatch (BSED) algorithm (Section 3.3.2). The BSED algorithm samples the objective function for various values of the costs. Based upon the samples, a new value is generated from the assumed topology of the solution surface. Essentially, the solution space is searched with a range elimination method. The Successive Approximation in Solution Space (SASS) algorithm attempts to search the Unit Commitment Solution Space. While the BSED algorithm is only a single dimension search, the SASS algorithm is a two-dimensional search of time and of unit combination versus cost.

Consider the simplified Economic Dispatch problem in Figure 4-3. This is a two unit example with only the load constraint and no limits

Primal problem:

$$\begin{aligned} \text{Min } z &= a * u_1 + b * u_2 \\ \text{subject to:} \quad & 1 * u_1 + 1 * u_2 = 1 \\ & u_1, u_2 \geq 0 \end{aligned}$$

Dual problem:

$$\begin{aligned} \text{Max } z' &= 1 * x_1 \\ \text{subject to:} \quad & 1 * x_1 \leq a \\ & 1 * x_1 \leq b \end{aligned}$$

FIGURE 4-3. DUALITY EXAMPLE IN LINEAR PROGRAMMING

on each unit's maximum contribution. The primal cost coefficients (a,b) would vary depending upon the segment presently active for the Input/Output energy conversion curve. The method for adjusting the equations for piece-wise linear curves is shown below in Figure 4-17. The following discussion compares the solution sequences for the BSED algorithm, Primal Linear Programming and Dual Linear Programming.

The BSED algorithm would sequentially guess a value for the dual variable "X" (Lambda) and for the primal value "u". Then the BSED algorithm determines in which range the resulting value of the constraint function is contained. The range found from the last evaluation is segmented and a new dual or primal variable estimated. One such sequence is shown in Figure 4-4. The BSED algorithm would start with the two extreme points (a,f) based upon the units' maximum and minimum capability. The first guess is assumed to be at point "b," and the first region excluded for future searches would be the range from point "a" to point "b." The next guess is assumed to be point "g," and the second region excluded for future searches would be the range from point "f" to point "g." The next guess is assumed to be point "c," and the third region excluded for future searches would be the range from point "b" to point "c." The next guess is assumed to be point "d," and the last range from point "c" to point "g" is assumed to be less than the solution tolerance. Note that these guesses were based upon the break-points of the Input/Output Curve to enable comparison with the Linear Programming solutions.

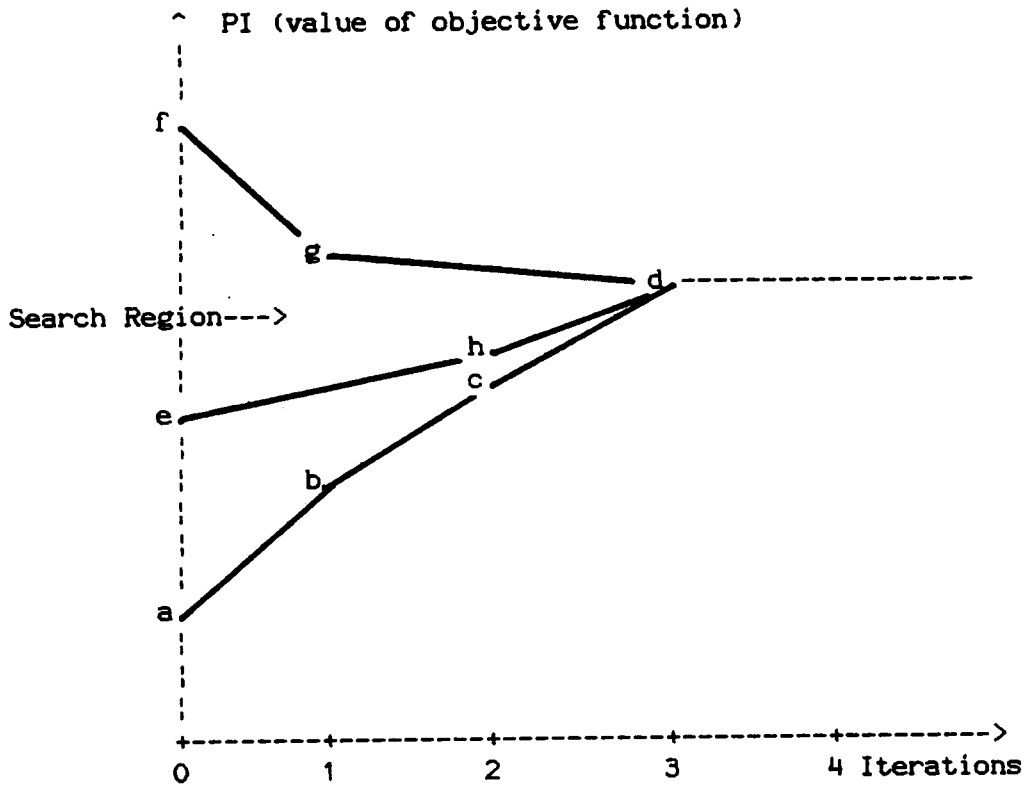


FIGURE 4-4. VALUE OF OBJECTIVE FUNCTIONS.

The Primal Linear Programming algorithm would first find a feasible solution (point "e") based upon some "crashing" algorithm. A crashing algorithm is a heuristic technique of selecting legitimate non-basic variables to replace artificial variables. The remaining steps would load each unit segment according to relative cost going to point "h" and finally to point "d." The Dual Linear Programming algorithm could start from the maximum capability of each unit segment (point "f"). The remaining steps would unload each unit segment according to relative cost going to point "g" and finally to point "d."

The BSED algorithm can now be compared with both Linear Programming algorithms. The BSED algorithm can be viewed as a sequence of guesses, half of which solve the dual Economic Dispatch problem. The process is to keep guessing a solution until the constraint(s) is (are) the tightest (satisfied).

It is assumed that the problems of degeneracy and redundant constraints are not being considered. Degeneracy and redundancy need not be considered for the Economic Dispatch formulation. Degeneracy would occur only if two units have identical energy conversion curves and reserve contribution functions. Such units need not be explicitly considered separately but they may be combined into equivalent units. This is a common method of reducing the curse of dimensionality since the equivalent unit can be transformed into the real units after the optimum solution is found. Redundancy would occur only if the generation constraint and one of the reserve constraints or if two or more of the reserve constraints are linearly dependent. Since the unit

contribution capabilities for each type of reserve are unique (Figure 2-2), this should not occur. The upper limits on generation have been neglected for the above discussion since the resulting dual problem would be more complex than necessary for this discussion.

4.2.2 Decision Analysis

The Successive Approximation in Solution Space (SASS) algorithm can also be outlined in terms of Decision Analysis and optimal paths. The problem is analogous to searching a known path through a "valley" for alternate paths which require less total expenditures than the known path. The first step is to segment the "valley", as shown in Figure 4-5. The next step is to determine the best path from each grid point of the current stage to the next grid point of the next stage. The Unit Commitment problem for daily operations always starts from real-time, which are the initial conditions. This step is shown graphically in Figure 4-6. Note that each of the grid points for a stage defines a state. The combination of units, which determine the optimal path, are shown as "C" with subscripts to note the combination number. The best combination is that combination which least exceeds the required load capability for the monetary expenditure determined from the state. An example of the total process is shown in Figure 4-7. The optimal path coordinates are given in Table 4-1. It is assumed that the fourth (4th) state of the last stage is the least accumulated cost path. The optimal path is found by simply following the pointers to the initial conditions.

Initial Conditions:					+
Stage:	-20%	-10%	0%	10%	20%
1			5000		
2	-20%	-10%	8000	10%	20%
3	-20%	-10%	8600	10%	20%
4	-20%	-10%	12400	10%	20%
5	-20%	-10%	16800	10%	20%
6	-20%	-10%	14500	10%	20%
7	-20%	-10%	14000	10%	20%
8	-20%	-10%	9600	10%	20%
9	-20%	-10%	8200	10%	20%
10	-20%	-10%	7350	10%	20%

FIGURE 4-5. SUCCESSIVE APPROXIMATION IN SOLUTION SPACE
GRID GENERATION

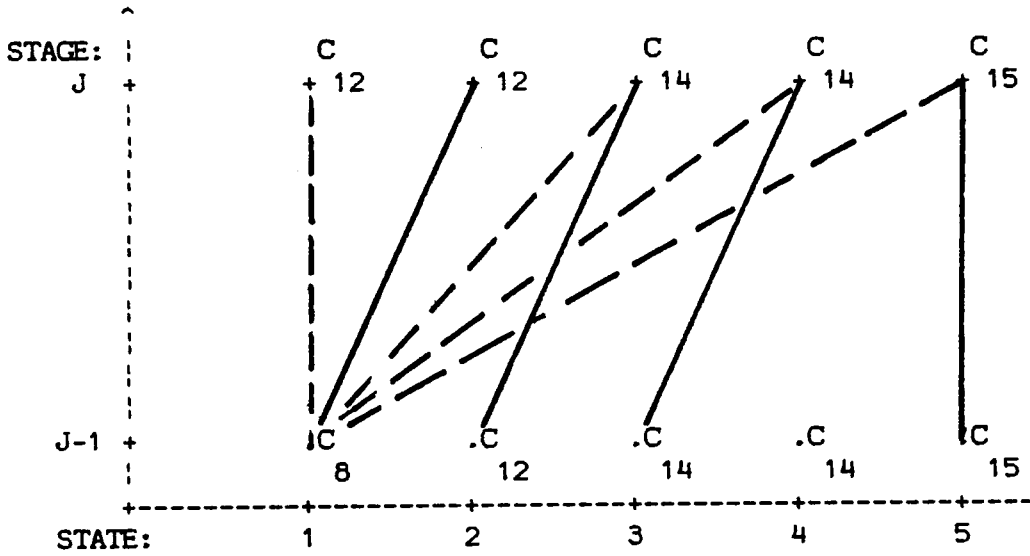


FIGURE 4-6. EXAMPLE COMBINATION EVALUATION FOR THE SUCCESSIVE APPROXIMATION IN SOLUTION SPACE ALGORITHM

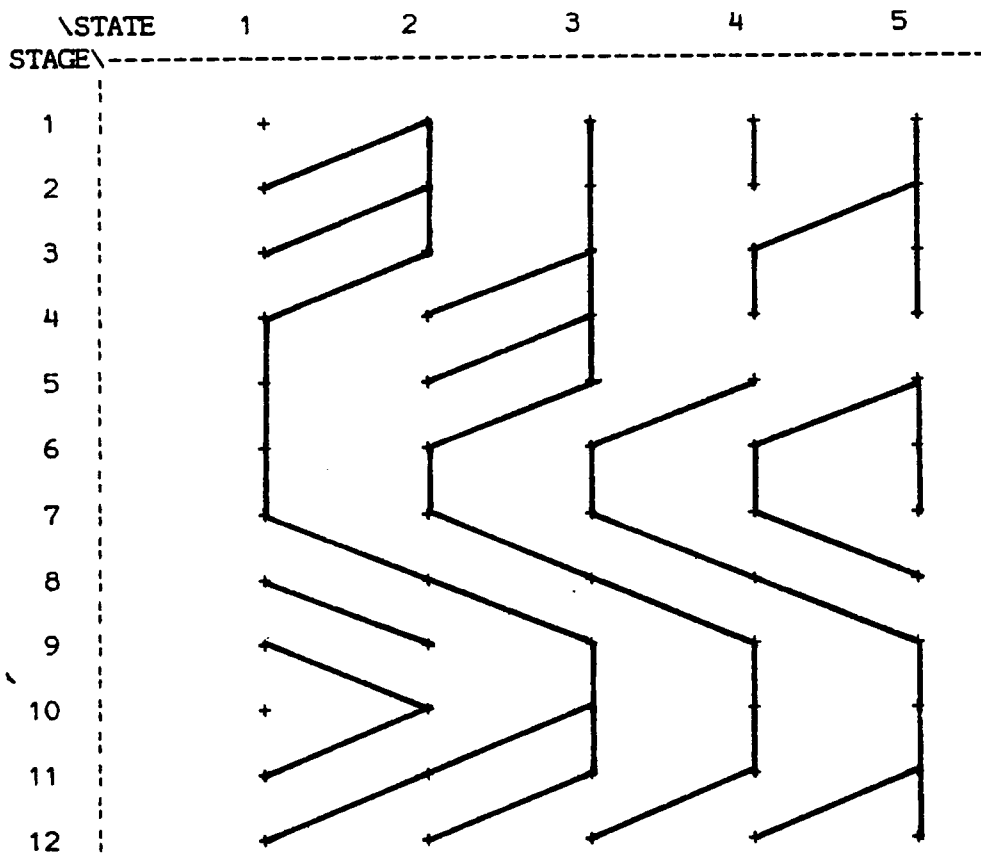


FIGURE 4-7. EXAMPLE SUCCESSIVE APPROXIMATION GRID OF FIVE

TABLE 4-1. OPTIMAL PATH COORDINATES

<u>STAGE</u>	<u>STATE</u>
1	3
2	3
3	3
4	3
5	3
6	2
7	2
8	3
9	4
10	4
11	4
12	3

The next step is to refine the grid around the previous optimal path found above. This step is shown in Figure 4-8 where it is assumed that the optimal path did not change. The above process is then repeated. The reason for refining the grid is to avoid any narrow valleys caused by the integer nature of the problem.

The Dynamic Programming definitions are listed in Figure 4-9. The recursion equation for this approach is simply a serial multistage decision system with additive returns [160]:

$$\begin{array}{l}
 \text{PI}_n(X) = \min_D \{ Q_n(X, D) \} \quad | \quad n=1, \dots, N \\
 \text{Q}_n(X, D) = r_n(X, D) \quad | \quad n=1 \\
 \text{Q}_n(X, D) = r_n(X, D) + \text{PI}_{n-1}(t_n(X, D)) \quad | \quad n=2, \dots, N
 \end{array}$$

The resulting paths can be evaluated by direct application of the Principle of Optimality, as defined in Figure 4-10. As stated by Bellman [18]:

"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

The policy is the sequence of unit transitions (e.g., start-ups and shut-downs) for each path. Since the policy is a highly non-linear function of the path, optimality cannot be guaranteed. The application

Initial Conditions:						
		+				
		5000				
Stage:	-10%	-5%	0%	5%	10%	
1						
		8000				
2	-10%	-5%	0%	5%	10%	
		8600				
3	-10%	-5%	0%	5%	10%	
		12400				
4	-10%	-5%	0%	5%	10%	
		16800				
5	-10%	-5%	0%	5%	10%	
		14500				
6	-10%	-5%	0%	5%	10%	
		14000				
7	-10%	-5%	0%	5%	10%	
		9600				
8	-10%	-5%	0%	5%	10%	
		8200				
9	-10%	-5%	0%	5%	10%	
		7350				
10	-10%	-5%	0%	5%	10%	

FIGURE 4-8. SUCCESSIVE APPROXIMATION IN SOLUTION SPACE
GRID REFINEMENT

THE STAGE IS EACH HOUR OF THE STUDY.

THE STATE IS DEFINED BY THE GRID POINTS OF EXPENDABLE DOLLARS

THE CONTROL VARIABLES ARE THE UNIT GENERATIONS DEFINED BY THE SET OF UNIT STATUSES:

$$C = \{C_1, C_2, C_3, \dots, C_i, \dots, C_N\}$$

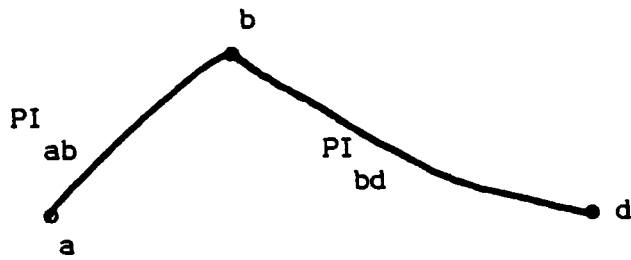
THE UNIT COMBINATIONS ARE REPRESENTED AS A BINARY COUNTER:

<u>COMBINATION</u>	<u>SET</u>			
C 1	1	0	0	0
C 2	0	1	0	0
C 3	1	1	0	0
.
.
.
C 13	1	1	0	1
C 14	1	1	1	0
C 15	1	1	1	1
				UNIT 4 STATUS
				UNIT 3 STATUS
				UNIT 2 STATUS
				UNIT 1 STATUS

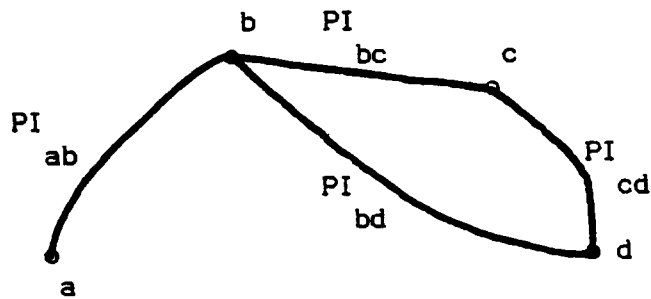
(ONE INDICATES THAT THE UNIT IS ON-LINE)
 (ZERO INDICATES THAT THE UNIT IS OFF-LINE)

FIGURE 4-9. DYNAMIC PROGRAMMING DEFINITIONS FOR SUCCESSIVE APPROXIMATION IN SOLUTION SPACE

Optimal Path



Alternate Optimal Path



Optimal Path

$$PI_{ad}^* = PI_{ab} + PI_{bd}$$

Alternate Path

$$PI_{ad}' = PI_{ab} + PI_{bc} + PI_{cd}$$

Principle

$$PI_{ad}' \geq PI_{ad}^* \text{ if } PI_{ad}^* \text{ is optimal}$$

FIGURE 4-10. DYNAMIC PROGRAMMING PRINCIPLE OF OPTIMALITY

of the Principle of Optimality to Decision-Making is shown in Figure 4-11 [121]. The Principle of Optimality implies that if path "a" to "b" is the initial segment of the optimal path from "a" to "z", then "b" to "z" is the terminal segment of this optimal path. Similarly, paths "a" to "c" and "a" to "d" indicate that the paths shown at the bottom of the figure are the only candidates for the optimal trajectory from "a" to "z." The optimal trajectory is found first by computing all feasible paths:

$$\begin{array}{c} * \\ \text{PI} \\ \text{abz} \end{array} = \begin{array}{c} * \\ \text{PI} \\ \text{ab} \end{array} + \begin{array}{c} * \\ \text{PI} \\ \text{bz} \end{array}$$

$$\begin{array}{c} * \\ \text{PI} \\ \text{acz} \end{array} = \begin{array}{c} * \\ \text{PI} \\ \text{ac} \end{array} + \begin{array}{c} * \\ \text{PI} \\ \text{cz} \end{array}$$

$$\begin{array}{c} * \\ \text{PI} \\ \text{adz} \end{array} = \begin{array}{c} * \\ \text{PI} \\ \text{ad} \end{array} + \begin{array}{c} * \\ \text{PI} \\ \text{dz} \end{array}$$

Then by comparing all candidate optimal paths:

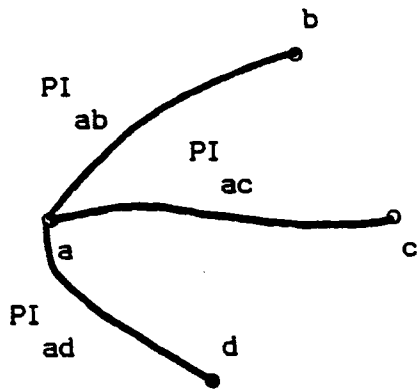
$$\text{Min} \left\{ \begin{array}{c} * \\ \text{PI} \\ \text{abz} \end{array}, \begin{array}{c} * \\ \text{PI} \\ \text{acz} \end{array}, \begin{array}{c} * \\ \text{PI} \\ \text{adz} \end{array} \right\} = \begin{array}{c} * \\ \text{PI} \\ \text{az} \end{array}$$

The optimal decision at "a" is the decision sequence which yields the minimum performance index.

4.2.3 Successive Approximations in Solution Space

The SASS algorithm is shown in Figure 4-12. Any "crashing" method may be used to find the initial feasible solution, as shown in Figure 4-13. A preferred method is the Priority List Economic Dispatch Unit Commitment algorithm which is an abbreviated Priority List Dynamic Programming algorithm. The Priority List Dynamic Programming algorithm was used for this research. The results of the feasible solution is

Paths from all allowable decisions at "a":



Optimal Paths to Stage "z":

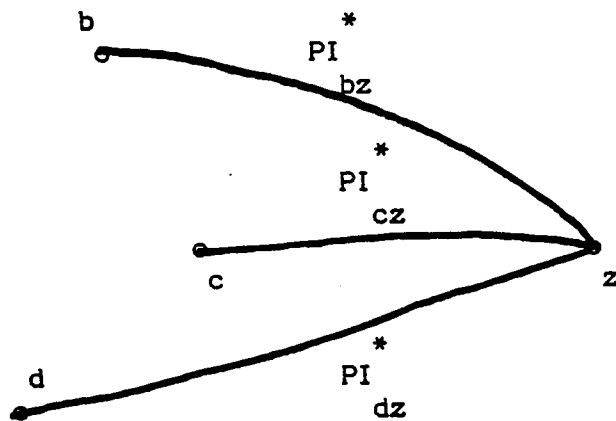


FIGURE 4-11. DYNAMIC PROGRAMMING - APPLICATION OF THE PRINCIPLE OF OPTIMALITY TO DECISION-MAKING

Candidate Optimal Paths:

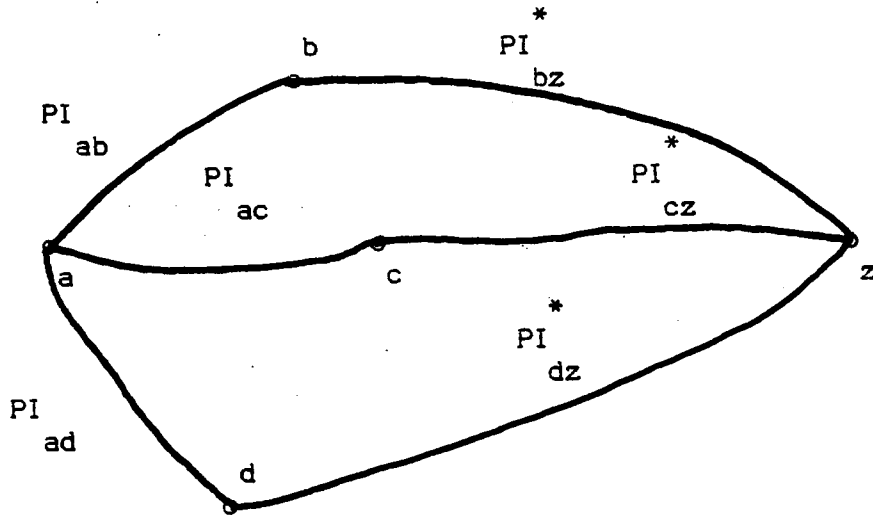


FIGURE 4-11. DYNAMIC PROGRAMMING - APPLICATION OF THE PRINCIPLE OF OPTIMALITY TO DECISION-MAKING (Continued)

1. Find feasible solution and retain the cost (for each stage).
2. Generate coarse grid of solution space by allocating DC for M steps above and N steps below optimal path with step size of SS.
3. For each stage: $j = 1, \dots, T$
 For each present state (grid point): $i = 1, \dots, M + N$
 For each previous state (grid point): $k = 1, \dots, K$
 For each valid unit combination: $l = 1, \dots, L$
 - a. Generate unit combination which is a valid transition from the previous state
 - b. Dispatch the unit combination to find the maximum load attainable for the given financial resource,
 - c. Save the transition from $k-1$ to k for each state such that the generation requirement is satisfied at lowest cost.

Next combination: l

Next previous state: k

Next present state: i

Next stage: j
4. Find optimal path from the sequence of states with the available load capability nearest the system load demand.
5. Repeat Steps 2 through 4 with reduced step size if the step size is not within solution tolerance or if the change in objective exceeds the desired solution tolerance.
6. Cost the Optimal Path.

FIGURE 4-12. SUCCESSIVE APPROXIMATION IN SOLUTION SPACE

UNIT COMMITMENT

Hour/Stage	Unit Status				Stage Cost
	Unit 1	Unit 2	Unit 3	Unit 4	
1	1	0	0	0	5000
2	1	1	0	0	8000
3	1	1	0	0	8600
4	1	1	1	0	12400
5	1	1	1	1	16800
6	1	1	1	0	14500
7	1	1	1	0	14000
8	1	1	0	0	9600
9	1	1	0	0	8200
10	1	0	0	0	7350
					etc.

(ONE INDICATES THAT THE UNIT IS ON-LINE)
(ZERO INDICATES THAT THE UNIT IS OFF-LINE)

FIGURE 4-13. SUCCESSIVE APPROXIMATION IN SOLUTION SPACE
PRIORITY LIST DYNAMIC PROGRAMMING CRASHING ALGORITHM

used to define a coarse grid over the solution space. Next, the Load Dispatch problem is solved for each possible transition from a previous solution point. The best transition is found and saved for the next stage. The best transition is defined to be the path from the last combination which is closest to the actual load demand for the current stage for the allowed cost. After all stages have been evaluated, the optimal transition is traced from the final stage to the first stage. This process is repeated around the new optimal solution path until the grid size is within the desired solution tolerance.

This approach is very similar to gradient optimization techniques since first the neighborhood is identified and then small changes are tried to determine if the minimum can be reduced while maintaining feasibility. The primary benefit of this approach is that a unit priority list to consider transitions is not needed. Additionally, the solutions for the last grid would yield sensitivity information not previously available with any of the previously described Dynamic Programming algorithms.

The Dynamic Programming algorithm can be graphically shown by the application of the grid search technique to the solution space for the previous Unit Commitment problem. The candidate paths are shown in Figure 4-6. The gradient search is conducted between the grid points based upon cost minimization between each iteration of the method. The search between grid points can be very fast since simplex based Linear Programming techniques allow rapid evaluation of alternate combinations. Only when a combination not considered in the Sequential

Priority List Dynamic Programming algorithm will the optimal path deviate from the original optimal solution.

Various optimal gradient search techniques are discussed in Simmons [200, pp 162-166] and in Luenberger [146, pp 133-163]. The primary difference between these classical techniques and this search algorithm is that feasible possibilities are examined at each stage (hour) for different number of units in combination. The change in cost is allocated (split) as an economic resource between unit transition costs and unit production costs for the given generation requirement.

Figure 4-6 shows the result of this algorithm for one iteration with the optimal path marked as a solid line and all other rejected paths marked with dashed lines. The nodes of this diagram are the combinations considered for one specific grid point of Figure 4-5.

The final question at each grid point is if there is sufficient financial resource to still satisfy the load requirement. This is called a Load Dispatch problem. A Linear Programming with Upper Bounding (LPUB) algorithm was used for this research. The LPUB algorithm found the amount of load which could be served for the expendable dollar amount minus any transition costs.

The steps to apply LPUB to the Load Dispatch problem are identical to those needed for the Economic Dispatch problem. Consider the Economic Dispatch problem shown in Figure 4-14. The problem has to be transformed by shifting the control variables such that zero is the minimum value. All right hand sides are adjusted by the constant

$$\text{Minimize PI} = f_1 * (a_1 * u_1 + b_1) + f_2 * (a_2 * u_2 + b_2)$$

Subject to:

1. Generation Requirement:

$$\text{LOAD} + \text{LOSS} = (p_1 * u_1) + (p_2 * u_2)$$

2. Unit capacity limits:

$$u_1 \leq \bar{U}_1$$

$$u_1 \geq U_{-1}$$

$$u_2 \leq \bar{U}_2$$

$$u_2 \geq U_{-2}$$

3. Reserve Constraint:

$$B \leq r_1 + r_2 - y_1 - y_2$$

4. Unit reserve limits:

$$r_1 + u_1 = \bar{U}_1$$

$$r_2 + u_2 = \bar{U}_2$$

$$r_1 + y_1 \leq R_1$$

$$r_2 + y_2 \leq R_2$$

FIGURE 4-14. ECONOMIC DISPATCH FORMULATION

values of each coefficient times the equation parameter as shown in Figure 4-15. The constant part of the objective function (PI) can be dropped for the LPUB algorithm. The Load Dispatch problem is shown in Figure 4-16. The corresponding Primal Load Dispatch problem is shown in Figure 4-17. The Load Dispatch formulation forces the generating units to pick-up the maximum load without violating the reserve constraints or without expending the total amount of financial resources allocated at the present grid point.

4.2.4 Comparison of Truncated Priority List Dynamic Programming to Successive Approximation in Solution Space

The Truncated Priority List Dynamic Programming (TPLDP) algorithm, presented in Section 3.2.3, is similar in approach to the Successive Approximations in Solution Space algorithm. Both techniques require a "crashing" method to position the search range close to the optimal path. The TPLDP algorithm uses the Sequential Priority List Dynamic Programming (SPLDP) algorithm to position the truncation window, as shown in Figure 4-18. The SPLDP algorithm commits enough units ($M+1$) to meet all constraints, as shown in this figure, for each stage. Then the TPLDP algorithm backtracks one unit to apply a window "K" units wide. A window "K" units wide may include 2^K combinations for evaluation by Economic Dispatch Production Costing and Transition Costing. The window for this example was assumed to be three units wide. The windows in Figure 4-18 assume that only four valid combinations are found at each stage. All of the possible combinations

$$\text{Minimize PI} = a'_1 * u_1 + a'_2 * u_2$$

Subject to:

1. Generation Requirement:

$$\text{LOAD} + \text{LOSSES} - \sum_{n=1}^N (p_n * U_n) = (p_1 * u'_1) + (p_2 * u'_2)$$

2. Unit capacity limits:

$$-u'_1 \geq U_1 - \bar{U}_1 = c'$$

$$-u'_2 \geq U_2 - \bar{U}_2 = d'$$

3. Reserve Constraint:

$$B \leq r_1 + r_2 - y_1 - y_2$$

4. Unit reserve limits:

$$r_1 + u'_1 = \bar{U}_1 - U_1 = e'$$

$$r_2 + u'_2 = \bar{U}_2 - U_2 = f'$$

$$r_1 + y_1 \leq R_1$$

$$r_2 + y_2 \leq R_2$$

$$u'_1, u'_2 \geq 0; r_1, r_2 \geq 0; y_1, y_2 \text{ unrestricted}$$

FIGURE 4-15. PRIMAL ECONOMIC DISPATCH FORMULATION

$$\text{Maximize PI} = (p_1 * u'_1) + (p_2 * u'_2)$$

Subject to:

1. Available resource:

$$DC = \sum_{n=1}^N f_n * (a_n * u_n + b_n)$$

2. Unit capacity limits:

$$\bar{U}_n \geq u_n \geq U_{-n} \quad \begin{array}{l} | \text{ For all} \\ | \text{ units} \\ | n=1, \dots, N \end{array}$$

3. Unit reserve limits:

$$r_n = (\bar{U}_n - u_n) \leq \bar{R}_n \quad \begin{array}{l} | \text{ For all} \\ | \text{ units} \\ | n=1, \dots, N \end{array}$$

4. Reserve Constraint:

$$B \geq \sum_{n=1}^N (r_n)$$

FIGURE 4-16. LOAD DISPATCH FORMULATION

$$\text{Maximize } PI' = p'_1 * u'_1 + p'_2 * u'_2$$

Subject to:

1. Available resource:

$$DC - \sum_{n=1}^N (f_n * b_n) = \sum_{n=1}^N f_n * a_n * u'_n$$

2. Unit capacity limits:

$$u'_1 \leq \bar{U}_1 - U_{-1}$$

$$u'_2 \leq \bar{U}_2 - U_{-2}$$

3. Reserve Constraint:

$$B \leq r_1 + r_2 - y_1 - y_2$$

4. Unit reserve limits:

$$r_1 + u'_1 = \bar{U}_1 - U_{-1}$$

$$r_2 + u'_2 = \bar{U}_2 - U_{-2}$$

$$r_1 + y_1 \leq R_1$$

$$r_2 + y_2 \leq R_{-2}$$

$$u'_1, u'_2 \geq 0; \quad r_1, r_2 \geq 0; \quad y_1, y_2 \text{ unrestricted}$$

FIGURE 4-17. PRIMAL LOAD DISPATCH FORMULATION

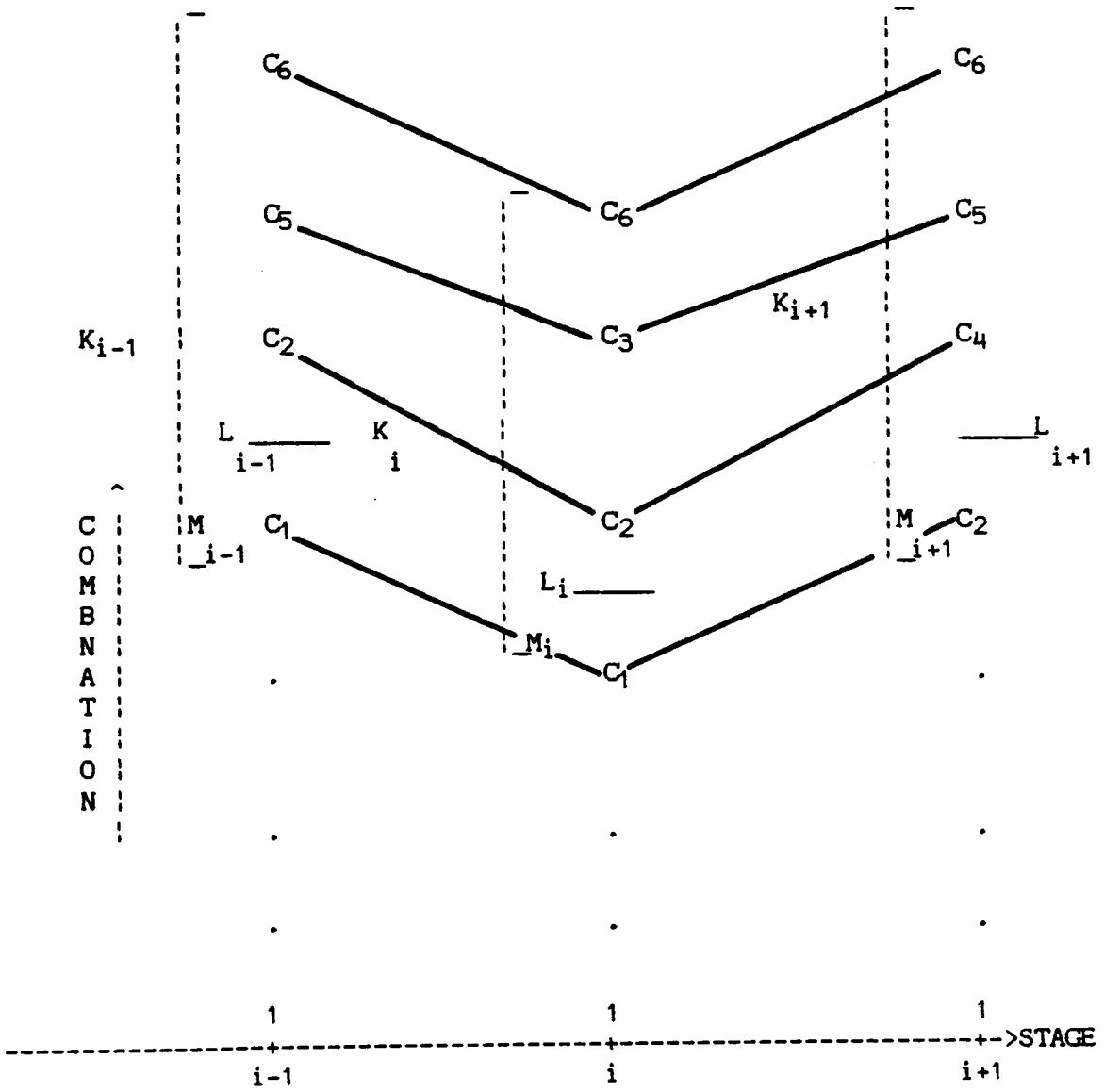


FIGURE 4-18. TRUNCATED DYNAMIC PROGRAMMING

for a three unit window are shown in Figure 3-9. The resulting solution for these windows is shown in Figure 4-19. Note that the accumulated cost is needed to find the optimal path for Dynamic Programming but only the differential costs need to be shown for each stage.

The SASS algorithm is depicted in Figure 4-20 for three stages and a window size of five. Note that the resulting solution spaces for both algorithms is conceptually similar. The savings between the two approaches is that the SASS algorithm would evaluate only fifteen Economic Dispatches while the TPLDP algorithm would evaluate eighteen Economic Dispatches. Since commercial grade TPLDP programs use a window size of six to ten units, the TPLDP algorithm would evaluate between sixty-four to one thousand twenty-four economic dispatches for each stage. The SASS algorithm would evaluate fifteen economic dispatches for each stage for a three pass iterative approximation and a window of five solution values. Additional algorithmic enhancements, such as parametric analysis, could improve this margin.

4.3 DEVELOPED PROGRAMS

Two major programs were developed for this research. The first program implemented the Sequential Priority List Dynamic Programming algorithm with Linear Programming for the Economic Dispatch. This pseudo-code description is given in Figure 4-21. The second program implemented the Successive Approximations in Solution Space Dynamic Programming algorithm with Linear Programming for the Load Dispatch.

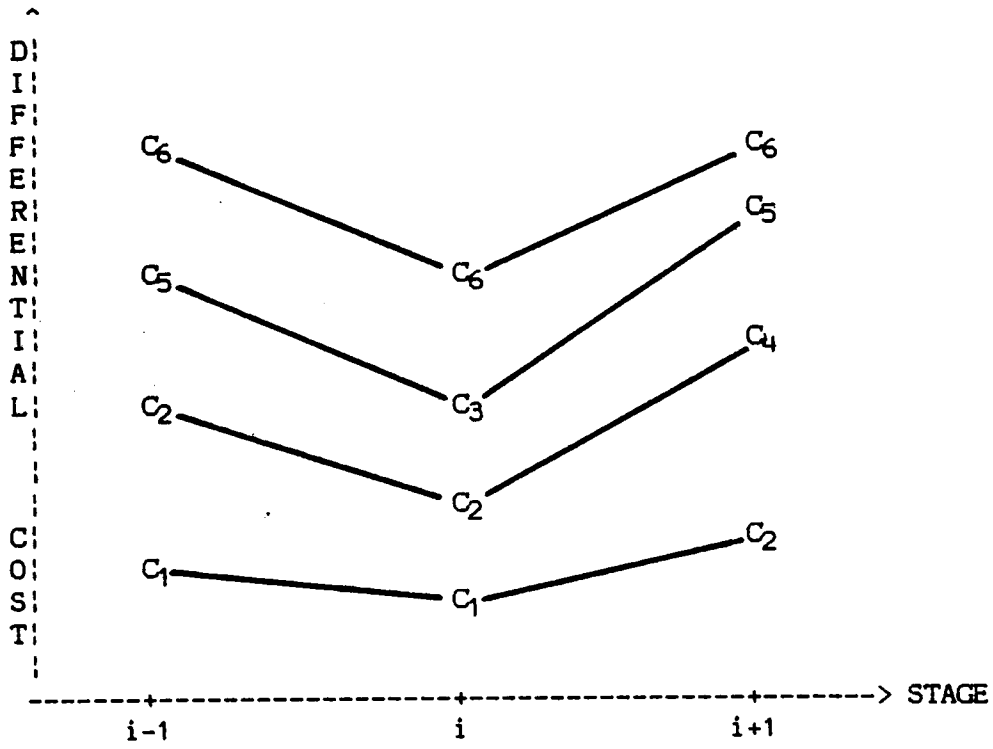


FIGURE 4-19. TRUNCATED DYNAMIC PROGRAMMING SOLUTION SPACE

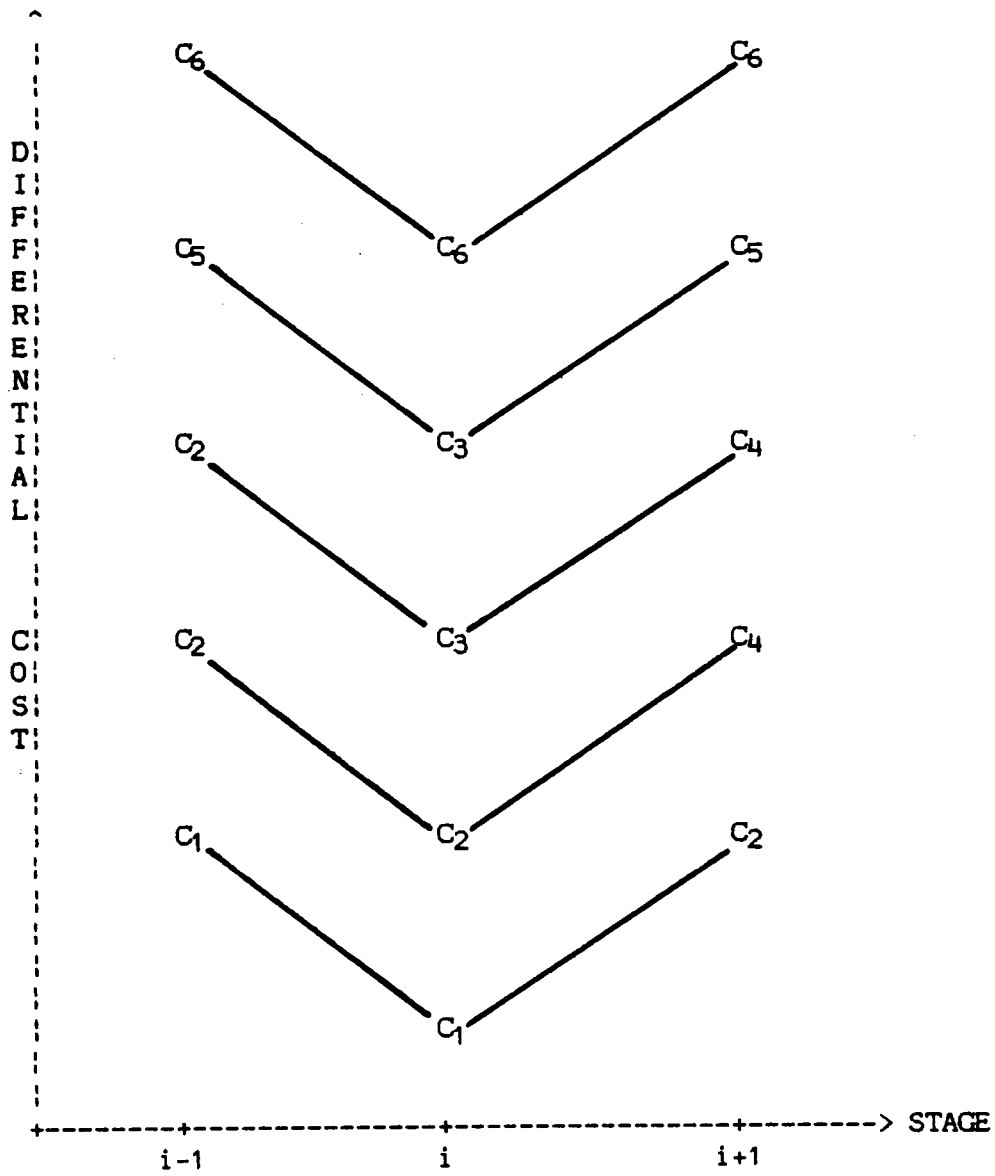


FIGURE 4-20. SASS SOLUTION SPACE

MAIN PROGRAM: SQPLLP

Code Sequence

1. CALL DBIN
2. Initializes DP Arrays
3. FOR EACH STAGE ($t = 1, \dots, T$)
 - a. CALL FEASBL
 - b. FOR EACH STATE - COMBINATION ($n=1, \dots, N$)
 - i. Turn on next unit in priority list
 - ii. CALL PROCOST
 - iii. Reject invalid combination
 - iv. CALL PTRCST
 - v. Calculate total cost for combination
 - vi. Save combination if it is the first or the least expensive
 - vii. Next state
 - c. Next stage
4. Find optimal solution by backtracking though paths generated above from final stage (least accumulated cost) to the first stage.
5. FOR EACH STAGE - Recalculate optimal path solution:
 - a. FOR EACH OPTIMAL STATE - COMBINATION
 - i. Turn on units in priority list
 - ii. CALL PROCOST
 - iii. CALL PTRCST
 - iv. Calculate total cost for combination
 - v. Next state
 - b. Next stage
 - c. Print results

FIGURE 4-21. SEQUENTIAL PRIORITY LIST DYNAMIC PROGRAMMING

SUBROUTINES:

DBIN - reads study data as card input

Data Input Sequence

1. Number of thermal units
2. First Title (for comments on unit data)
3. Thermal unit data:
 - Name
 - Unit Type
 - Reserve Flag
 - Minimum Capacity
 - Maximum Capacity
 - Variable Operation and Maintenance Cost
 - Startup Cost
 - Shutdown Cost
 - Quadratic I/O Curve
 - Fuel Cost
 - Penalty Factor
4. Second Title (for comments on load curve)
5. Number of Load Curve Points, Peak Load
6. Load Curve (input as per unit of peak load)
7. Security Constraints (reserve margin)
8. Initial Conditions (number of units committed)
9. Solution Control
 - Minimum and Initial Mesh Sizes
 - Priority Bias
 - Maximum Number of Iterations (DP)
 - Solution Tolerances
(absolute and percent of change)
 - Economic Dispatch Solution Tolerance
 - Maximum Number of Iterations (ED)

FIGURE 4-21. SEQUENTIAL PRIORITY LIST DYNAMIC PROGRAMMING
(Con't)

SUBROUTINES:

FEASBL - finds first feasible combination

Code Sequence

1. Calculate required reserve
2. FOR EACH UNIT IN PRIORITY SEQUENCE
(n = 1, ..., N)
 - a. Add maximum generation to total generation
 - b. Calculate reserve contribution
 - c. Limit reserve contribution if beyond unit capability
 - d. If sufficient reserve then goto 4, else continue
3. Flag insufficient units for feasible solution, and return
4. Flag sufficient units for feasible solution, and return the number of units committed

PRCOST - calculates economic dispatch and production costs

Code Sequence

1. Initialize LP variable and arrays.
2. CALL SETUP
3. CALL SIMPLX
4. Calculate total production cost

FIGURE 4-21. SEQUENTIAL PRIORITY LIST DYNAMIC PROGRAMMING
(Con't)

SUBROUTINES:

SETUP - sets up LP tableau and finds feasible solution

Code Sequence

1. Transform equations into standard LP notation (Figure 4-10)
2. Sequentially load units in order of ascending cost until load is satisfied
3. Initialize LP Tableau for feasible solution
4. Return

SIMPLX - solves LP tableau by algorithm shown in Figure 3-26

PTRCST - calculates transition costs

Code Sequence

1. FOR EACH UNIT COMMITTED
add start-up cost to total
2. FOR EACH UNIT DECOMMITTED
add shut-down cost to total

**FIGURE 4-21. SEQUENTIAL PRIORITY LIST DYNAMIC PROGRAMMING
(Con't)**

This pseudo-code description is given in Figure 4-22. The subroutines for each program were shared when possible (e.g., there is only one input routine - DBIN). The common areas were shared for all data except for the main Dynamic Programming solution arrays.

MAIN PROGRAM: SASSLP

Code Sequence

1. CALL DBIN
2. CALL CRASH
3. Initializes DP Arrays
4. FOR EACH STAGE ($t = 1, \dots, T$)
 - a. FOR EACH STATE ($m = 1, \dots, M$)
 - i. FOR EACH COMBINATION ($n=1, \dots, N$)
 - CALL COMBIN
 - CALL TRCOST
 - CALL OPLOAD
 - Reject invalid combination
 - Save combination if it is the first or the least expensive
 - Next combination
 - ii. Next state
 - b. Next stage
5. Find optimal solution by backtracking through paths generated above from final stage (least accumulated cost) to the first stage.
6. Adjust mesh for next iteration, if mesh size is less than tolerance or if maximum iterations have been exceeded, then goto 7, else goto 4
7. FOR EACH STAGE - Recalculate optimal path solution:
 - a. FOR EACH OPTIMAL STATE - COMBINATION
 - i. CALL COMBIN
 - ii. Turn on unit(s)
 - iii. CALL TRCOST
 - iv. CALL OPLOAD
 - v. Next state
 - b. Next stage
 - c. Print results

FIGURE 4-22. SUCCESSIVE APPROXIMATION IN SOLUTION SPACE

SUBROUTINES:

DBIN - reads study data as card input, same routine as for SQPLL

CRASH - generates initial feasible solution, reads in SWPLL solution

COMBIN - finds next feasible combination

Code Sequence

1. Calculate required reserve
2. FOR EACH UNIT IN SEQUENCE
(n = 1, ..., N)
 - a. Add maximum generation to total generation
 - b. Calculate reserve contribution
 - c. Limit reserve contribution if beyond unit capability
 - d. If insufficient reserve then goto f, else continue
 - e. If up/down time constraints are not violated, then goto 4,
 - f. Next unit
3. Flag insufficient units for another feasible combination and return
4. Flag sufficient units for another feasible combination and return the units committed

TRCOST - calculates transition costs

Code Sequence

1. FOR ANY UNIT COMMITTED
add start-up cost to total
2. FOR ANY UNIT DECOMMITTED
add shut-down cost to total

FIGURE 4-22. SUCCESSIVE APPROXIMATION IN SOLUTION SPACE
(Con't)

SUBROUTINES:

OPLOAD - calculates pseudo-dual economic dispatch and load capability

Code Sequence

1. Initialize LP variables and arrays.
2. CALL SETUPP
3. CALL SIMPLX
4. Calculate total load capability

SETUPP - sets up LP tableau and finds feasible solution

Code Sequence

1. Transform equations into standard LP notation (Figure 4-13)
2. Sequentially load units to maximum in order of ascending unit number until cost is expended
3. Initialize LP Tableau for feasible solution
4. Return

SIMPLX - solves LP tableau by algorithm shown in Figure 3-26

**FIGURE 4-22. SUCCESSIVE APPROXIMATION IN SOLUTION SPACE
(Con't)**

CHAPTER 5

TEST CASE AND RESULTS

The sample system contained in the Wood and Wollenberg text [232] was the main test case for all programs. This section presents the sample system data, the solution found by the Sequential Priority List (SQPL) Dynamic Programming algorithm, and the solution found by the proposed Successive Approximation in Solution Space (SASS) Dynamic Programming algorithm. Finally, the resulting solutions are compared.

5.1 EXAMPLE SYSTEM

The example system is shown in Figure 5-1 (reprinted with permission from the Wood and Wollenberg text). This system is not typical since there are no shut-down costs, a gas turbine is treated as a normal unit, and since the energy conversion curve is a single linear segment. Normally, gas turbines are excluded from the priority list and dispatched/committed each hour if the total cost is reduced. Note that fictional, nameless monetary unit (R) is used instead of dollars (\$) [232, pg. 3].

5.2 SEQUENTIAL PRIORITY LIST WITH LINEAR PROGRAMMING RESULTS (SQPLLP)

As discussed in Section 3.2.2 above, this is the industry wide accepted technique. This algorithm defines the search range for dynamic programming by specifying a fixed priority list order for the units to be started or stopped. The solution is shown in Figure 5-2. This solution was verified with the solution given in the text by Wood and Wollenberg.

THERMAL UNIT DATA

UNIT NUMBER	UNIT TYPE	PRIORITY ORDER	MINIMUM CAPACITY (MW)	MAXIMUM CAPACITY (MW)	COLD STARTUP (R)	HOT STARTUP (R)	SHUT DOWN (R)
1	1	3	25.0	80.0	350.0	150.0	0.0
2	2	2	60.0	250.0	400.0	170.0	0.0
3	2	1	75.0	300.0	1100.0	500.0	0.0
4	4	4	20.0	60.0	.02	0.0	0.0

UNIT NUMBER	INCREMENTAL HEAT RATE (BTU/kWh)	NO-LOAD COST (R/h)	FUEL COST (r/MBtu)	EFFICIENCY FACTOR	PENALTY FACTOR	VARIABLE MAINTENANCE (R/h)
1	10.44	213.00	2.0	1.000	1.000	0.0
2	9.00	585.62	2.0	1.000	1.000	0.0
3	8.73	684.74	2.0	1.000	1.000	0.0
4	11.9	252.0	2.0	1.000	1.000	0.0

UNIT NUMBER	MINIMUM UP TIME (h)	MINIMUM DOWN TIME (h)	STARTING HOURS (h)	INITIAL CONDITIONS (h)
1	4	2	4	-5
2	5	3	5	8
3	5	4	5	8
4	1	1	0	-6

LOAD PATTERN

HOUR	LOAD (MW)
1	450
2	530
3	600
4	540
5	400
6	280
7	290
8	500

FIGURE 5-1. CASE STUDY FROM WOOD AND WOLLENBERG

\ UNIT	1	2	3	4	COMBINATION NUMBER
1	0	1	1	0	12
2	0	1	1	0	12
3	1	1	1	0	14
4	0	1	1	0	12
5	0	1	1	0	12
6	0	0	1	0	5
7	0	0	1	0	5
8	0	1	1	0	12

FIGURE 5-2. SEQUENTIAL PRIORITY LIST SOLUTION

The grid shows the combination being considered next to each grid point (sequence in priority order). The optimum solution is marked with asterisks. The required generation is listed to the right of each stage (hour).

5.3 SUCCESSIVE APPROXIMATION IN SOLUTION SPACE RESULTS (SASS)

The proposed algorithm was used to generate the result shown in Figure 5-3. The SQPLLP solution was used to start the search. Note that most paths terminate immediately. It is expected that paths for a more complex system would terminate after a longer sequence. If any of these paths had returned to the optimal solution path, then the solution would have been degenerate. That is, this algorithm would have found two solutions. The result agrees with the full Dynamic Programming solution contained in the Wood and Wollenberg text.

The grid shows the combination being considered next to each grid point (sequence in priority order). The optimum solution is marked with asterisks. The required generation is listed to the right of each stage (hour).

5.4 DISCUSSION OF RESULTS

The proposed solution algorithm found an optimal path if the cost grid was small enough to force combination 13 on instead of combination 14. The main reason for the SQPLLP not finding the optimal path is that the priority list is not in proper order. The total cost for the SQPLLP algorithm is R 73,439. and the total cost for the SASS algorithm is R 73,274. This is the same solution as found by exhaustive enumeration or full Dynamic Programming.

STAGE \ STATE	1	2	3	4	5	LOAD
1	12	14	15			
1	*	+	+	+	+	450
2	12	14	15			
2	*	+	+	+	+	530
3	14	15				
3	*	+	+	+	+	600
4	12	14	15			
4	*	+	+	+	+	540
5	12	14	15			
5	*	+	+	+	+	400
6	5	12	14	15		
6	*	+	+	+	+	280
7	5	12	14	15		
7	*	+	+	+	+	290
8	12	14	15			
8	*	+	+	+	+	500

FIGURE 5-3. SEQUENTIAL PRIORITY LIST SOLUTION SPACE SUMMARY

\ UNIT	1	2	3	4	COMBINATION NUMBER
1	0	1	1	0	12
2	0	1	1	0	12
3	0	1	1	0	13
4	0	1	1	0	12
5	0	1	1	0	12
6	0	1	1	0	5
7	0	1	1	0	5
8	0	1	1	0	12

FIGURE 5-4. SUCCESSIVE APPROXIMATION IN SOLUTION SPACE UNIT
SOLUTION SUMMARY

\STATE	1	2	3	4	5	LOAD
STAGE\	0	0	12	13	15	
1	+	+	*	+	+	450
	0	0	12	13	15	
2	+	+	*	+	+	530
	0	0	13	14	15	
3	+	+	*	+	+	600
	0	0	12	13	15	
4	+	+	*	+	+	540
	0	0	12	13	14	
5	+	+	*	+	+	400
	0	0	5	8	9	
6	+	+	*	+	+	280
	0	0	5	8	9	
7	+	+	*	+	+	290
	0	0	12	13	15	
8	+	+	*	+	+	500

FIGURE 5-5. SUCCESSIVE APPROXIMATION IN SOLUTION SPACE SOLUTION SPACE SUMMARY

CHAPTER 6

CONCLUSIONS AND FUTURE RESEARCH

The algorithm developed from this research has been shown to have the potential to overcome the major objection, the use of a priority list, to the use of Dynamic Programming for the Unit Commitment problem. Additionally, the algorithm has been shown to have the potential to drastically reduce the computer resources necessary for accurate scheduling.

The results cited above do not conclusively demonstrate the worthiness of the proposed algorithm. The first subsection below defines the steps which should be taken to fully implement this algorithm. The second through fifth subsections outline future research which would justify development of a production grade program based upon this algorithm.

6.1 PRACTICAL IMPLEMENTATION

The implemented algorithm does not include many of the features which are available with modern scheduling packages. A practical implementation would have to include detailed modeling of the start-up cycle, unit equipment limitations, and of system operational constraints [109,111,195]. Additionally, actual data has to be obtained and results from a production grade unit commitment package has to be used for comparison.

6.2 SENSITIVITY/PARAMETRIC GRID GENERATION

I feel that the grid could be more effectively generated by a parametric analysis to determine the cost which would force the next most economic unit or units on-line. The segmenting of the grid by fixed percentages is arbitrary since it is the commitment or the decommitment of a unit which is at question. The sensitivity analysis algorithms of Linear Programming would be most appropriate for this task [58,165,174,203]. Additionally, the stopping criterion could be based upon the number of policy changes for the last iteration and not the grid size. Specifically, if there were no changes to the units schedules for the last iteration, then no additional iterations are needed. This would be a better stopping criterion than to prespecify a solution tolerance based upon the grid size.

6.3 INTERCHANGE EVALUATION

One of the benefits of this algorithm is the availability of sensitivity information and the possibility of quick resolutions [21,44,45,66,108,140,155]. This would be very beneficial to on-line operations for interchange contract evaluation and negotiation. The sensitivity information is available from the Linear Programming Tableaus for each hour (stage) and the load capability and cost information available from the grid. It would be most beneficial if the information for all of the evaluated grids were kept for further dispatcher analysis.

6.4 HYDRO-THERMAL COORDINATION

The availability of sensitivity information from the evaluated grids and the quick resolutions for small changes in system load demand would be beneficial for hydro-thermal coordination. All of the hydro-thermal coordination algorithms require economic information on incremental changes in thermal generation requirements [13,22,23,43,53,56,59,67,75,80,95,112,191,198]. Previous implementations have had problems with sensitivity information when alternate unit policies would be justified. This information is contained within the evaluated grid.

6.5 FUEL ALLOCATION

The unit commitment problem is hardest to solve when the fuel resources are constrained. Fuel resources can be constrained by contract, by transportation problems, and by environmental restrictions [85,126,160,223]. Contract constraints include both take-or-pay contracts, where the cost is fixed even if the fuel is not used, and rate limited use due to pipe-line pressure restrictions. Transportation problems include use of the same fuel at more than one unit, as for natural gas pipe-lines and intra-site storage restrictions. Environmental restrictions include limits on the type of fuel burnt and the mix of fuel burnt for emission restrictions.

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