Three Essays in Applied Time Series Econometrics

Atanu Rakshit

Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

in

Economics, Science

Kwok Ping Tsang, Chair
Richard A. Ashley
Raman Kumar
Eric A. Bahel

May 31st, 2013
Blacksburg, Virginia

Keywords: Kalman Filter, Uncertainty, Exchange rates, VAR, Threshold, Deficit, Interest Rates.
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(Abstract)

This dissertation is comprised of four chapters. Chapter 1 provides an introduction to Economic application of time series analysis and discusses the topics covered in each of the following chapters along with some main results therein.

In Chapter 2, I construct a measure of information asymmetry in the financial markets in U.S., by estimating an index of agency cost pertaining to U.S. manufacturing firms. The cyclical behavior of the unobservable agency cost is derived by a novel application of the Kalman filter within a Bayesian framework, using firm level data from 1984-2006. The preliminary results provide support to the financial accelerator mechanism in the business cycle literature.

In Chapter 3, I show that people’s expectation of uncertainty in financial markets is a significant factor impacting short-term real exchange rate movements. Specifically, a sudden increase in expectation of stock market volatility in a low interest rate country tends to appreciate their currencies against high interest rate currencies. I construct a measure of conditional expected uncertainty from volatility of returns of the dominant portfolio (indices) of 7 industrialized countries. I identify uncertainty shocks and its impact on dollar real exchange rate, and explain my results in the context of currency carry trade.

Chapter 4 of my dissertation documents the presence of significant non-linearity in the deficit-interest rate relationship in the U.S. economy. Using an asymptotic threshold test as per Hansen (2000), I find strong evidence for threshold effects in the impact of expected deficit on future long-term interest rates. I find that a percentage point increase in expected deficit in a regime where the expected deficit/GDP ratio is above 1.8 percent (the estimated threshold value) increases future nominal long term interest rates by 29-30 basis point, and a “news shock” to expectation of future deficit increases future real long term interest rates by 12-18 basis points. When expected deficit/GDP ratio is below 1.8 percent, an increase in expected deficit has no impact on future long-term interest rates.
Acknowledgements

I would like to thank my advisor, Kwok Ping Tsang, for his guidance throughout my graduate work. I have looked up to him for inspiration both as a researcher and a teacher, and my dissertation would not have been possible without his constant support and unwavering patience. I would also like to thank other members of my committee: Richard Ashley, Raman Kumar and Eric Bahel for their invaluable feedback and support. I am especially indebted to Richard Ashley for his support as Graduate Program Director during my tenure at the department. I have always felt that he went beyond his official role to help and support me during some of the more difficult times at graduate school.

A significant part of my research was concluded at Washington and Lee University, Lexington, where I have been teaching for the last two years. I thank faculties and staff at W&L for the support to make this happen, especially Carl Kaiser. I would like to thank my friends and colleagues at Blacksburg and elsewhere who made my graduate studies so memorable. They have been pillars of support and I will forever cherish the shared moments with them after I leave Blacksburg.

Finally I would like to thank my family for their support and their encouragements. My wife Sai’s faith in me is something that stills puzzles me, and she is the reason I never lost faith in myself. She gave me the best gift possible, and our son Umang Joshua was born last year, bringing previously unknown joys and emotions with him, for which I am thankful beyond measure! We are also expecting another one very soon, and I am glad that he/she will not have a graduate student for a father. I would also like to thank my elder brother Arindam Rakshit and my younger brother Amitava Rakshit (the other economist in the family) for their love and support, and especially for taking care of our parents when I could not be around for them.

Above all, I am indebted to my Ma and Baba for making me who I am today, and I hope that I have been able to do some small measure of justice to the struggles and sacrifices that they made to raise me.
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Chapter 1: Introduction

Considerable development in time series analysis has occurred in the last decade or so in the field of social sciences in general. Time series analysis accounts for the fact that data points taken over time may have an internal structure (such as autocorrelation, trend or seasonal variation) that should be accounted for. The very nature of analysis thus makes the application of time series analysis in the field of finance ubiquitous, mainly because of the nature of financial data, which is numerous, both due to sophistication as well as the frequency of financial market transactions. Time series econometrics, i.e. application of time series analysis to economic (and especially macroeconomic) data, also constitutes a large body of literature, albeit often utilizing much smaller data points.

The underlying models that economists use to decompose a given time series are many, and an applied econometrician will often have to make a choice among many competing models. Whatever the true data generating process may be, the econometrician has to decide what statistical model is the best approximation of that.

The focus of this dissertation is the application of time series analysis to problems of economic importance. While the time series techniques themselves not novel, the specific applications are somewhat new, and generate important and non-trivial predictions of some economic importance.

Macroeconomic datasets, typically multivariate time series, are only of moderate size, since they usually involve monthly, quarterly or annual observations. The research challenge is on how to build models that are accommodating enough to be empirically relevant, capturing key characteristic of the data, but not so flexible as to be significantly over parameterized. One common method suggested in literature is shrinkage, which takes the form of imposition of restrictions on parameters or shrinking them towards zero.
Using prior information, as is common in Bayesian analysis, provides a logical and formally consistent way to introduce shrinkage and reducing over parameterization. The second chapter of my dissertation uses a method similar to the above, and does Bayesian inference via the Gibbs sampler to estimate a latent variable, which can be thought of as an index of agency cost faced by US manufacturing firms in the last three decades. This measure of agency cost is extracted using a technique from signal processing, the Kalman filter, but within a Bayesian framework. The magnitude and cyclical behavior of this measure of agency cost empirically corroborates the “financial amplification” role of capital market imperfections on business cycles (Bernanke et. al, (1996,1999)).

In the third chapter I try to include time varying uncertainty for a subset of OECD countries in a Vector Auto Regression specification. I document a strong co-movement between expected stock market volatility (“uncertainty”) of a country, and real exchange rate. I also find that the inclusion of uncertainty greatly improves the explanatory power of a VAR than just using the real and nominal variables that are usual in the literature.

The empirical innovation of this paper pertains to the construction of a forward looking measure of volatility, which I call expected uncertainty, in financial markets, and using it as a variable to predict exchange rate movements. Apart from the improvement in explanatory power over exchange rates, this strategy also reveals some important insight regarding the role of carry trader’s expectations about uncertainty and consequent shift in their trading strategy (see also Brunnermeier 2008). I believe the results and our conjecture about trading behavior could be an important motivation to guide theoretical work in this area.

Threshold models have a wide range of application in economics, one of them being applications of empirical sample splitting, when the sample split is based on a continuously distributed variable such as firm size. They can also be used as a parsimonious strategy for non-parametric function estimation. One example is the threshold auto-regressive model or TAR, popular in non-linear time series literature.
Despite the large number of applications, the statistical theory of threshold estimation was undeveloped till quite recently. Hansen (2000) was one of the first studies to develop a statistical theory of threshold estimation in a regression context. One of the many contributions of this paper was the development of a method to construct asymptotic confidence intervals of the threshold estimate, by inverting the likelihood ratio statistic. My third essay (chapter 4) demonstrates the empirical relevance of this theory by an application to a very important economic question of our times, the impact of high fiscal deficit on future economic growth. Specifically, I estimate a threshold in a deficit-interest rate relationship to test and quantify crowding out, and specify and estimate a threshold VAR model to estimate response of future long-term rates to structural shocks to deficit expectations. I find the presence of significant non-linearity (threshold effect) in deficit-interest rate relationship, and also discover some important dynamics about how the economy form expectations about future long term interest rates based on fiscal projections.
Chapter 2. The Magnitude and Cyclical Behavior of Agency Cost

2.1 Introduction

The influence of firm’s financing decision on the evolution of the real economy has been largely ignored in macroeconomic literature, often evoking the Modigliani-Miller (1958) theorem as its justification. But as demonstrated by Bernanke, Gertler and Gilchrist (1996), the magnitude and persistence of business cycle fluctuations can be significantly amplified by informational asymmetries in financial markets. Such asymmetries result in agency costs which in turn induce a wedge between the cost of external and internal funds for investing firms. The presence of asymmetric information between entrepreneurs and investors in capital markets provide a theoretical link between entrepreneurs ‘financial health and the amount of borrowing (vis-à-vis their own internal funds) and hence play a crucial role in the investment decisions of entrepreneurs who are not sufficiently self-financed. The agency cost associated with such financial market imperfections involves the mechanism of “external finance premium” (the difference between the opportunity cost of internal funds and cost of funds raised externally)\(^1\) and its inverse relationship with the net worth of the borrowing entrepreneur. This inverse relationship is a result of the fact that when entrepreneurs bring little internal funds or net wealth to project financing, the potential divergence of interest between the supplier of external funds and the borrower is greater, implying increase agency costs. In equilibrium, thus, borrowers have to pay a higher premium to compensate for the higher agency costs. Assuming that debt contracts are structured in a manner that tries to mitigate such agency costs, uncollateralized external finance will thus be more expensive than internal finance.

The magnitude of such agency costs would thus indicate the level of asymmetric information in, or the imperfection of, financial markets. Because an increase in financial market imperfections increase the real cost of extending credit and decrease the efficiency of matching borrowers and lenders, widespread real macroeconomic effects

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\(^1\) Bernanke et al. (1999), introduces the concept of external finance premium and theoretically derives the financial accelerator or the amplification mechanism in a business cycle framework.
are a natural consequence, vividly exhibited by the recent financial crises and quantified by Gilchrist et al. (2009).

At the micro or firm level, earlier empirical work has focused on classifying groups of firms in terms of various characteristics, using panel data to study the relationship between capital market imperfections and firm level investment. Firm cash flow is often chosen as a measure of agency costs in such models. A study by Hubbard (1998) shows how cash flow might include information about a firm’s future profitability, and a further influential study by Kaplan and Zingales (1997) concurs with the view that cash flow is likely to be a faulty and incomplete measure of agency costs.

The extent and dynamics of agency costs are not directly observable of course, although earlier macroeconomic empirical work has focused on suitable proxies for the “external finance premium” in the form of observable credit spreads. But whether such spreads are a good proxy for the external finance premium is obviously model dependent. Moreover, the differential impact that agency costs might have on a cross section of firms is not answered by studies that use such proxies. Also, little empirical work has been done that considers the changes in aggregate level as well as the volatility of agency costs over time, both of which is important for the analysis of the relationship between capital market imperfections and business cycles (Bernanke et. al, (1996,1999)).

This study tries to bridge the gap between the firm level perspectives on agency costs as well as the practice of using model dependent proxies as a measure of aggregate agency costs. This is done by composing an index of agency cost by taking the approach proposed by Stock and Watson (1991) for the construction of a coincident index of the business cycle. Several variables that exhibit co-movement due to agency costs are used to estimate the unobservable factor or the agency cost index using a Kalman Filter. We estimate this index for the period 1984-2006 with respect to four classes of U.S. manufacturing firms and try to analyze how such an index of agency cost would affect firms belonging to different classes as well as how it evolves in the real economy. For the purposes of estimation, we use Bayesian inference via Gibbs sampling, following Carter and Kohn (1994); Kim and Nelson (1998). To confirm whether our estimated index has

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2 Fazzari et al. (1988); Hoshi et al. (1991) constitute studies that are representative of this approach.
3 Gilchrist et al. (2009), Natalucci and Zakrajsek (2006).
any economic implications, we compare our index with the evolution of GDP during our sample period including the two recessions that the economy experienced during the sample period.

2.2 The Econometric Model of Agency Cost

The optimal debt contact theory of Townsend (1979); Gale and Helwig (1985); Bernanke et al. al., (1996, 1999) suggests that agency costs are reduced by entrepreneurs’ net worth. Previous empirical research on the external finance premium and agency costs has mostly analyzed the influence of cash flow and net worth on firm level investment spending but has provided no direct evidence on the magnitude or cyclical properties of financial market asymmetry. Accordingly, this study tries to model agency costs from economic variables that exhibit co-movements, and are all closely related to agency costs. Specifically, following Uchiyama (2006), we model investment and borrowing, since a reduction in agency costs increases both these variables though a decrease in external finance premium. In addition, we also model an interest rate spread between risky assets and risk free assets, which reflects agency costs as the difference between the entrepreneur’s cost of internal and external funds. Accordingly, we model agency costs from investment and borrowing equations of four classes of firms and the spread that is not class dependent. The firm classes are based on their asset sizes in each period, and can be thought of as a control for firm sizes on agency cost. The model for each of the 3 variables is as follows:

2.2.1 Derivation of models

Investment equation

Under the assumption of an increasing cost-adjustment function, Hayashi (1982) showed that the investment equation of a firm under complete capital markets can be written as function of Tobin’s marginal tax adjusted q and the capital stock, i.e., $I_t = f(q_t, K_{t-1})$.

Expressing this function in a linear form:

---

\[
\frac{l_t}{K_{t-1}} = \alpha + \beta q_t + \varepsilon_t
\]  

(1)

where \( l_t \) denotes real investment in period \( t \), \( K_{t-1} \) denotes the capital stock at the end of the period \( t-1 \), \( q_t \) denotes Tobin’s marginal tax adjusted \( q \) in period \( t \), and \( \varepsilon_t \) is the error term.

Fazzari et al. (1988) included a cash flow per nominal stock term in the above equation, as a proxy for firm’s net worth affecting agency costs, assuming they are present. Their version of the equation, which takes into account financial frictions in the capital markets, is as follows:

\[
\frac{l_t}{K_{t-1}} = \alpha + \beta q_t + \gamma \frac{CF_t}{pK_{t-1}} + \varepsilon_t
\]  

(2)

where \( CF_t \) represents cash flow in period \( t \) and \( pK_{t-1} \) denotes the capital stock at replacement cost at the end of period \( t-1 \). But according to Hubbard (1998), cash flow is not an appropriate proxy for net worth, because cash flow also contains information about firm’s future profits, as does Tobin’s \( q \). Also, agency costs will usually reflect other firm characteristics such as maturity and forms of ownership, which are not captured by cash flow measures. A more precise mechanism of capturing agency costs would be to include it explicitly in the investment equation, which we try to do in our paper by replacing cash flow with the index of capital market imperfection, as below:

\[
\frac{l_t}{K_{t-1}} = \alpha_1 + \beta_1 q_t + \gamma_1 AC_t + \varepsilon_1 t
\]  

(3)

where \( AC_t \) denotes the index of agency costs in period \( t \).

**Borrowing equation**

Uchiyama (2006) uses a borrowing equation to relate change in firm debt with agency cost and Tobin’s \( q \). The rationale for modeling a borrowing equation is that the magnitude of the difference between the costs of external and internal finance will affect
the level of firm borrowing. Thus when agency costs are low, firm has more incentive to raise debt from the market. Also, an increase in Tobin’s q, through its positive effect on investment, might tend to increase borrowing. Hence the ratio of borrowing to nominal capital stock is expressed as a function of Tobin’s q and the index of agency costs.

\[
\frac{\Delta B_t}{pK_{t-1}} = \alpha_2 + \beta_2 q_t + \gamma_2 AC_t + \varepsilon_{2t}
\]  

(4)

where \(\Delta B_t\) is the borrowing increment in period \(t\).

**Interest rate spread equation**

Corporate credit spreads—the difference in yields between corporate debt instruments and government securities of comparable maturity— is indicative of the default risk of firms. An increase in such spreads reflects worsening of the quality of firm’s balance sheet (as well as deterioration in the health of financial intermediaries that supply credit) and a subsequent increase in agency costs. Hence we model the interest rate spread as an increasing function of agency costs.

\[
spread_t = \alpha_3 + \gamma_3 AC_t + \varepsilon_{3t}
\]  

(5)

**2.2.2 State space representation**

We base our methodology on the approach proposed by Stock and Watson (1991) for the construction of a coincident index of business cycle. They used a Kalman filter algorithm to construct a high frequency business cycle index, by regarding this index as a common factor of four observable variables: industrial production, personal income, total manufacturing and trade sales, and employment (these classes of estimation strategies are hence also called factor models). All the variables in the Stock and Watson study are likely to be closely linked in both cause and effect with business cycles.

In this study, the unobservable variable is the agency cost, which we regard as the common factor of 3 variables: investment per net capital stock \(\frac{I_t}{K_{t-1}}\), borrowing per nominal capital \(\frac{\Delta B_t}{pK_{t-1}}\) and the interest rate spread. We divide the first two variables for four classes of firms, which effectively gives us nine variables and a greater degree of
freedom in our estimation. Since all the variables are closely linked with agency cost in both cause and effect, irregular movement of one variable in relation to the other is removed during estimation. The isolation of their variation due to a common factor (agency costs) is the primary feature of the model, an advantage that makes factor models superior to single proxy models.

Representing the three equations in our models in linear and Gaussian state-space form, we use the Kalman filter to construct the index of agency cost. The state space model consists of a measurement equation and a transition equation.

**Measurement equation**

\[
\begin{pmatrix}
\frac{I_{jt}}{K_{jt-1}} \\
\frac{ΔB_{jt}}{pK_{jt-1}} \\
\text{spread}_t
\end{pmatrix} = \begin{pmatrix}
Y_{j1} \\
Y_{j2} \\
Y_{j3}
\end{pmatrix} \cdot AC_t + \begin{pmatrix}
\beta_{j1} \\
\beta_{j2} \\
0
\end{pmatrix} \cdot q_t + \begin{pmatrix}
ε_{j1t} \\
ε_{j2t} \\
ε_{j3t}
\end{pmatrix} \quad j = 1, 2, 3, 4
\]

where \(q_t\) is the exogenous, observable variable and \(R\) is a diagonal variance-covariance matrix. The transition equation or the state equation for the factor \(AC_t\) is assumed in our study to follow an AR (4) process, since we are using quarterly data to derive the factor. Note that the index \(j\) in the above equation refers to firm size classes. Hence the above equation is illustrated with a slight abuse of notation, and the actual measurement equation should have a total of 9 variables i.e. along with the variable \(\text{spread}\), there are two variables (investment and borrowing) for each size class. Specifically, if we have just once size class, this is how the measurement equation should look like.

**Transition equation**

\[
AC_t = \varphi_1 AC_{t-1} + \varphi_2 AC_{t-2} + \varphi_3 AC_{t-3} + \varphi_4 AC_{t-4} + \theta_t, \quad \theta_t \sim iidN(0, 1), \quad E(e_t \theta_t^t) = 0 \quad (6.2)
\]

The regular condition for the above equation to be stationary is assumed to hold i.e. the roots of \(1 - \varphi_1 L - \varphi_2 L - \varphi_3 L - \varphi_4 L = 0\) must lie outside the unit circle. The variance of innovation \(\theta_t\), is normalized to unity to identify all the parameters in the model. The

---

5 Harvey (1989) and Hamilton (1994a) introduces and discusses state space representations as well the use of Kalman filter algorithm.
Kalman filter algorithm then estimates the time series of agency costs $AC_t$, as well as the vector of parameters $(\beta, \gamma, \varphi, R)$.

2.3 Econometric methods (Bayesian Inference and the Gibbs Sampler)

2.3.1 Rationale for use of the Bayesian Approach

Macroeconomic datasets, typically multivariate time series, are only of moderate size, since they usually involve monthly, quarterly or annual observations. The research challenge is on how to build models that are accommodating enough to be empirically relevant, capturing key characteristic of the data, but not so flexible as to be significantly over parameterized. One common method suggested in literature is shrinkage, which takes the form of imposition of restrictions on parameters or shrinking them towards zero. Using prior information, as is common in Bayesian analysis, provides a logical and formally consistent way to introduce shrinkage and reducing over parameterization.

As the sample size in our paper is small, Bayesian inference is appropriate for our model, given by equation (6.1) and (6.2), because the latter is a complicated form with many parameters and also involves the estimation of the stochastic unobservable variables, $AC_t$, for $t=1,2,3…T$. In this case, there is a significant chance that the classical maximum likelihood procedure might suffer from inefficiency and inference bias.

Bayesian inference often requires the solution of complicated integrals to compute posterior density functions, but the evolution of Markov Chain Monte Carlo (MCMC) simulation (footnote) and advancement in computing technology in the last decade has assisted in this type of inference. The flexibility to impose restrictions on the size and sign of estimated parameters is another advantage of Bayesian inference.

2.3.2 Bayesian Inference via the Gibbs Sampler

Whereas optimization problems are the main source of complexity in maximum likelihood methods (multiple modes, solution of likelihood equations, links between likelihood equations and global modes etc.), Bayesian analysis involves a different kind of difficulty, those of integration. In the Bayesian paradigm, information brought by the

---

6 Koops et al constitutes a textbook instance of Bayesian time series analysis. Chib and Greenberg offer a survey of Bayesian econometrics using MCMC.
data $x$, a realization of $X \sim f(x|\Theta)$, is combined with prior density $\Pi(\Theta)$ which incorporates prior beliefs about the parameters $\Theta$, to derive the marginal posterior distributions of the parameters of interest. Specifically, the posterior is derived from the joint distribution $f(x|\Theta) \Pi(\Theta)$, according to Bayes formula

$$
\Pi(\Theta|x) = \frac{f(x|\Theta)\Pi(\Theta)}{\int f(x|\Theta)\Pi(\Theta) d\Theta},
$$

where $m(x) = \int f(x|\Theta)\Pi(\Theta) d\Theta$ is the marginal density of $X$. In order to find the marginal posterior distributions from the given conditional posterior distributions, the Gibbs sampler\(^7\) is used. The procedure for the Gibbs sampler of the state space model in equation (6.1) and equation (6.2) can be divided into four steps and is described below, which essentially follows the algorithm of Carter and Kohn (1994)\(^8\) and as applied by Uchiyama (2006).

**Step 1**

Generate the unobservable variables $AC_t^{(j)}$ for $t=1,2,3...T$ from the normal distribution, conditional on the given parameters $\gamma^{(j-1)}$, $\beta^{(j-1)}$, $\phi^{(j-1)}$, $\sigma^2_{(j-1)}$, using the Kalman filter algorithm. A more detailed explanation is given in Appendix A.

**Step 2**

Generate coefficients for the measurement equation, $(\gamma, \beta)$ from a joint truncated normal distribution, conditional on the given parameters $\phi^{(j-1)}$ and $\sigma^2_{(j-1)}$, and on $AC_t^{(j)}$, as follows below.

$$(\gamma^{(j)}, \beta^{(j)}) | \phi^{(j-1)}, \sigma^2_{(j-1)}, AC_t^{(j)}, Y \sim N(\Gamma_1, \Sigma_1) I(\gamma, \beta),$$

$$\Sigma_1 = ((\Sigma_0^{-1} + \sigma^{-2}_{(j-1)}X'X)^{-1}$$

$$\Gamma_1 = \Sigma_1 (\Sigma_0^{-1} \Gamma_0 + \sigma^{-2}_{(j-1)}X'y)$$

where $\Gamma_1$ and $\Sigma_1$ are the posterior means and variances of the coefficient vector; $(\gamma^{(j)}, \beta^{(j)})$, $\Gamma_0$ and $\Sigma_0$ are the corresponding priors; $Y$ is the sample data; and $y$ and $X$ are the dependent variable and the explanatory repressors $AC_t$ and Tobin’s $q$, given in equation (6.1), respectively. Furthermore, $I(\gamma, \beta)$ is an indicator function that returns unity if the signs of $\gamma$ and $\beta$ are theoretically correct, and zero otherwise.

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\(^7\) See Casella and George (1992) for a comprehensive explanation of a Gibbs sampler.

\(^8\) They proposed a multi-move method for using the Kalman filter via the Gibbs sampler. This method is very efficient, and the speed of convergence is especially fast when compared with single move method.
Step 3

Generate coefficients for the transition equation, equation $B$, $\varphi$, from a truncated normal distribution, conditional on the given parameters $(\gamma^{(j)}, \beta^{(j)})$ and $\sigma^{2,(j-1)}$, and $AC_i^{(j)}$, as shown below.

$$
\varphi^{(j)} \mid (\gamma^{(j)}, \beta^{(j)}, \sigma^{2,(j-1)}, AC_i^{(j)}, Y) \sim N(\Phi_1, \Sigma_{\varphi,1})I(s(\varphi)),
\Sigma_{\varphi,1} = (\Sigma_{\varphi,0}^{-1} + Z'Z), \quad \Phi_1 = \Sigma_{\varphi,1}(\Sigma_{\varphi,0}^{-1} \Phi_0 + Z'z)
$$

where $\Phi_1$ and $\Sigma_{\varphi,1}$ are the posterior means and variances of the coefficients, $\Phi_0$ and $\Sigma_{\varphi,0}$ are the corresponding prior values, and $z$ and $Z$, respectively, the dependent variable and explanatory variables given in equation $B$. In addition, $I(s(\varphi))$ is an indicator function that returns unity if the roots are of $\Phi(L)=0$ lies outside the unit circle and zero otherwise.

Since we have just one factor (or one state variable in the state space notation) in the model, we assume it to follow an AR 4 process with innovation $\nu_t \sim iid N(0,1)$ where are variance of the innovation has been normalized to one. The algorithm can easily be extended to more than one factor (state variables) where the transition equation is in the form of a VAR in the factors. If then the assumption of a fixed and diagonal residual variance-covariance matrix is relaxed for the VAR, the natural conjugate prior for normal data (a usual assumption in vector auto regressions) is Normal-Wishart (Kadiyala and Karlsson, (1998)).

Step 4

Generate the variances for the measurement equation, equation $A$, $\sigma^2^{(j)}$ from an inverted gamma distribution, conditional on the given parameters $(\gamma^{(j)}, \beta^{(j)}, \varphi^{(j)})$, and on $AC_i^{(j)}$, as follows.

$$
\Sigma^2^{(j)} \mid (\gamma^{(j)}, \beta^{(j)}, \Phi^{(j)}, AC_i^{(j)}, Y) \sim IG \left(\frac{1}{2} \nu_1, \frac{1}{2} \delta_1\right),
\nu_1 = \nu_0 + T, \quad \delta_1 = \delta_0 + (y' - X\Gamma^{(j)})(y' - X\Gamma^{(j)})
$$

Where $\nu_1$ and $\delta_1$ are the posterior variances and degrees of freedom, $\nu_1$ and $\delta_1$ are priors, and $\Gamma$ denotes the vector of coefficients, $\gamma$ and $\beta$.

In the above steps, the $j$-th iteration is denoted by the $j$ superscript in parenthesis. To obtain the invariant marginal densities of the parameters, these four steps are reciprocally iterated until the full conditional densities of each one of the parameters converge to their marginal posterior densities. Statistical estimators like the mean, median, and standard deviations can then be computed from these posterior densities.
2.4 Data and estimation strategy

We use a sample of quarterly data on U.S. non-financial corporation reported in the S&P’s COMPUSTAT database, as well as a time series of standard Baa-Treasury credit spread. Specifically, our sample includes quarterly data from the first quarter of 1984 through the last quarter of 2006, for 471 publicly traded firms in the industry groups with U.S. Standard Industrial Classification (SIC) codes of 011 to 399, reported in the Center for Research in Security Prices (CRSP) and S&P’s COMPUSTAT. These firms include the industry groups of agriculture, forestry and fishing, mining, construction and manufacturing. The 471 firms are the only firms in their category who has survived for the full sample period of 1984 through 2006 and who does not have substantial missing or unreported data in the COMPUSTAT database. Moreover, our sample consists of firms with assets more than 10 million to get rid of small firm’s effect in our results.

The firms divided into four groups based on the size of their assets in each period, with the quartiles serving as the boundary of the groups. To mitigate the effects of outliers in the sample, the first and the last quartile category exclude firms whose asset size belongs to the first percentile and the ninety ninth percentile respectively. While determining the four classes of firms, we also exclude firms that exhibit borrowings and investment that lie in the ninety ninth and first percentile or their distribution.

Nominal investment, borrowing increments and capital stock are composed from the relevant balance sheet data in the COMPUSTAT Industrial Quarterly database for each firm-quarter. Real investment, debt and capital stock are deflated from their nominal value by the quarterly GDP deflator. Furthermore, the flow data, borrowing increments and real investment are seasonally adjusted via the U.S. Census Bureau’s X12 seasonal adjustment program. From these data sources, the ratios of investment to capital stock, \( \frac{I_t}{K_{t-1}} \), of borrowing to nominal capital stock, \( \frac{\Delta B_t}{pK_{t-1}} \), are derived. \( \frac{I_t}{K_{t-1}} \) and \( \frac{\Delta B_t}{K_{t-1}} \) are multiplied by 100 for expression as percentages and the average is taken for the number

---

9 This method modifies the X-11 variant of Census Method II by J. Shiskin A.H. Young and J.C. Musgrave of February, 1967 and the X-11-ARIMA program based on the methodological research developed by Estela Bee Dagum, Chief of the Seasonal Adjustment and Time Series Staff of Statistics Canada, September, 1979.
of firms in each group in each quarter. Being standardized ratios, this makes for a weighted average of investment and changes in borrowing, which takes into account firm size differences within groups. As is evident from Figure 1, firms belonging to Class 1 and Class 4 exhibit lesser variance in borrowing changes for most of the sample period than the other two firm classes. The level of borrowing changes is much higher in the case of firms belonging to Class 1, usually ranging within 7 percentages around zero, except during 2005 when it jumped up to around 30 percent. The investment per capital stock data indicates a gradual decline of investment for all class of firms during the early 1990s, followed by a gradual upswing, which again went down for all class of firms from 2000 onwards till 2002. Data on all the groups indicate a rising average investment from 2002 till the end of the sample period in 2006.

Tobin’s \( q \) in each quarter is calculated as the market value of assets divided by the book value of assets, where the market value of assets consists of book value of assets added to balance sheet deferred taxes and market value of common equity less book value of common equity\(^\text{10}\). The average level of Tobin’s \( q \) for each class of firms peaks around 1996-1998 and again around 2000, although for the two lowest percentiles of firms, they again start going up after a sharp fall around 2003 (Figure 2.1).

The spreads in corporate credit are assigned to the difference between the yield of an index of seasoned long-term Baa-rated corporate bonds and the yield on the constant maturity 10-year Treasury note. The sources for all Treasury yields and the yield on Baa-rated long-term corporate bonds are “Selected Interest Rates” (H.15) Federal Reserve Statistical release. The interest rate spread data are expressed as percentages. As is evident from Figure 2.2, the spread increased to its historical highest point (since the beginning of the sample period) to reach approximately 3.6 percent around 2002-2003, which coincides with the period when average investment for all class of firms fell quiet significantly, as did their Tobin’s \( q \).

\(^{10}\) Kaplan and Zingales (1997) discuss how to derive Tobin’s Q and Investment from COMPUSTAT balance sheet data.
Figure 1: Time Series of borrowing increment and investment for different size classes of firms

Notes: The Class number refers to the class of firms based on the size of their asset, with Class four being the biggest asset size class. All units are percentages (along the vertical axis) and the time period is 1984 Quarter 3 - 2006 Quarter 3, along the horizontal axis.
Notes: The graphs for Tobin’s Q are for the four classes of firms, with Q1 referring to the time series of average Tobin’s Q for Class1 and so on. The interest rate spread is the difference between the yield of an index of seasoned long-term Baa-rated corporate bonds and the yield on the constant maturity 10-year Treasury note and the unit is percentage per year. The sample period for all the graphs is 1984 Quarter 3-2006 Quarter 3.
2.5 Estimation of agency cost

The estimation of equation A and B is implemented for the time period 1984 Q3 to 2006 Q3 using the Gibbs sampler, as explained in the last section. Our prior specification is mostly natural conjugates of the joint likelihood of our empirical model. The prior distributions of the coefficients $\gamma, \beta$ and $\varphi$ are Normal, $N (0, 1)$, and that of the variances, $\sigma^2$ is the inverted Gamma distribution $IG (5, 3)$ with mean one and variance 2. The coefficients $\varphi_t$ of the transition equation are constrained to be stationary, as explained in section. The estimation results for the parameters are displayed in Table 1.

As is evident from the results in Table 1, all the parameters except the coefficients on Tobin’s q are estimated significantly, in that they lie within their Bayesian confidence interval at 95% level, with low standard deviations and median values of the draws matching the mean value of the parameter draws.

In this paper, the first 5000 iterations of Gibbs sampling are discarded for convergence; the posterior distribution is then computed from the means of 10 successive elements of the Markov chain of the next 95,000 iterations, thus generating 950 samples for each estimator. The reforming of the posterior with means of every nth draw ($n=10$ in our study) is standard in literature, and is done to reduce the autocorrelation that is a feature of Gibbs iteration.

Based on the figures in Table 1, the measurement and transition equation for two classes of firms are as below:

**Measurement Equation for Class 1 and Class 4 firms (respectively) and Transition equation for Agency cost (1984Q1-2006Q4)**

\[
\begin{align*}
\begin{pmatrix} I_{1,t} \\ K_{1,t-1} \\ \Delta B_{1,t} \\ pK_{1,t-1} \\ \text{spread}_t \end{pmatrix} &= \begin{pmatrix} -0.665 \\ -1.437 \\ 0.315 \end{pmatrix} AC_t + \begin{pmatrix} 0.048 \\ 0.021 \\ 0 \end{pmatrix} q_{1,t} + \begin{pmatrix} \varepsilon_{1,1t} \\ \varepsilon_{1,2t} \\ \varepsilon_{3t} \end{pmatrix} \\
\begin{pmatrix} I_{4,t} \\ K_{4,t-1} \\ \Delta B_{4,t} \\ pK_{4,t-1} \\ \text{spread}_t \end{pmatrix} &= \begin{pmatrix} -0.675 \\ -1.182 \\ 0.315 \end{pmatrix} AC_t + \begin{pmatrix} 0.015 \\ 0.042 \\ 0 \end{pmatrix} q_{4,t} + \begin{pmatrix} \varepsilon_{4,1t} \\ \varepsilon_{4,2t} \\ \varepsilon_{3t} \end{pmatrix} \\
\bar{AC}_t &= 0.928AC_{t-1} + 0.066AC_{t-2} + \theta_t, \quad \theta_t \sim iid \mathcal{N}(0,1), \quad E(\varepsilon_t \theta_t) = 0
\end{align*}
\]
Table 1: Results of the State Space AR (4) model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
</tr>
<tr>
<td>$\varphi_1$</td>
<td>0.928</td>
<td>0.297</td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td>0.066</td>
<td>0.421</td>
</tr>
<tr>
<td>$\varphi_3$</td>
<td>-0.033</td>
<td>0.407</td>
</tr>
<tr>
<td>$\varphi_4$</td>
<td>-0.111</td>
<td>0.283</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.665</td>
<td>0.145</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>-0.766</td>
<td>0.157</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>-0.782</td>
<td>0.160</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>-0.675</td>
<td>0.146</td>
</tr>
<tr>
<td>$\gamma_5$</td>
<td>-1.437</td>
<td>0.206</td>
</tr>
<tr>
<td>$\gamma_6$</td>
<td>-1.605</td>
<td>0.223</td>
</tr>
<tr>
<td>$\gamma_7$</td>
<td>-1.592</td>
<td>0.207</td>
</tr>
<tr>
<td>$\gamma_8$</td>
<td>-1.182</td>
<td>0.188</td>
</tr>
<tr>
<td>$\gamma_9$</td>
<td>0.315</td>
<td>0.124</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.021</td>
<td>0.072</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.011</td>
<td>0.065</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.004</td>
<td>0.055</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>0.042</td>
<td>0.055</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>0.048</td>
<td>0.072</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>-0.011</td>
<td>0.061</td>
</tr>
<tr>
<td>$\beta_7$</td>
<td>-0.001</td>
<td>0.055</td>
</tr>
<tr>
<td>$\beta_8$</td>
<td>0.015</td>
<td>0.059</td>
</tr>
<tr>
<td>$\sigma_1^2$</td>
<td>1.753</td>
<td>0.252</td>
</tr>
<tr>
<td>$\sigma_2^2$</td>
<td>2.470</td>
<td>0.410</td>
</tr>
<tr>
<td>$\sigma_3^2$</td>
<td>1.394</td>
<td>0.215</td>
</tr>
<tr>
<td>$\sigma_4^2$</td>
<td>1.540</td>
<td>0.224</td>
</tr>
<tr>
<td>$\sigma_5^2$</td>
<td>1.152</td>
<td>0.188</td>
</tr>
<tr>
<td>$\sigma_6^2$</td>
<td>0.584</td>
<td>0.112</td>
</tr>
<tr>
<td>$\sigma_7^2$</td>
<td>0.331</td>
<td>0.077</td>
</tr>
<tr>
<td>$\sigma_8^2$</td>
<td>0.424</td>
<td>0.081</td>
</tr>
<tr>
<td>$\sigma_9^2$</td>
<td>0.199</td>
<td>0.033</td>
</tr>
</tbody>
</table>

a. The notations of the parameters are based on Eqs. (6.1) and (6.2).
b. $\sigma_1^2$ represents the variance of $\varepsilon_{1,t}^1$, $\sigma_2^2$ is the variance of $\varepsilon_{1,t}^2$ and so on.
c. S.D. denotes the standard deviation.
d. 95 percent band means Bayesian confidence interval of 95% level
The variable *spread* is of course common between these two systems of equations, since both of them are actually part of one single measurement equation and the above demarcation is just for illustrative purposes. That the absolute value of the loadings for borrowings are greater than the absolute value of the loadings for investment shows that agency cost, being a financial factor, affects borrowing more than investment. Also, the variation in size of the loading parameter across firm size classes also indicate that smaller size firms are affected more by information asymmetry and the consequent agency costs, than their bigger counterparts. These results are consistent with the fact that information asymmetry does indeed exists in the capital markets for manufacturing firms. The transition equation parameters are only significant for the second order auto regressive term, and insignificant for the remaining, hence we retain agency cost as an AR (2) process because of this reason. The estimator of agency costs and its 95 percent band is depicted in Figure 3.

**Figure 4: Estimated time series of agency costs and its 95 percent band**
Figure 5: Estimated time series of agency costs with NBER recession dates

Notes: The shaded lines in Figure 4 represents NBER recession dates, which lasted from July 1990 (Q3) till March 1991(Q1) and from March 2001(Q1) till November 2001(Q4). The GDP is HP filtered log real GDP for the sample period. The agency cost has been multiplied by 100 in all time period to make the graph comparable.

The index lies within its upper and lower 95% band for the entire sample period. Figure 4 also compares the time series of agency cost with HP filtered log real GDP for the U.S. economy for the sample period. The shaded line represents the NBER recession dates during the sample period. Figure 4 gives some support to the hypothesis that agency cost is negatively correlated and amplifies business cycles. Although the evidence is weak for the recession starting 1990Q3 and ending 1991Q1, the agency cost exhibits a sharp rise during the recession starting 2001Q1 and ending 2001Q4, lagging the sharp drop in GDP by approximately a quarter.

2.6 Conclusion

To evaluate the index of agency cost constructed in Section 4, we compare the estimated index $\tilde{AC}_t$, to models of earlier studies, given by Eqs. (1) and Eqs. (2) described in Section 2. My results show that coefficients on Tobin’s Q in these equations (both of which describes complete capital market scenarios) are likely to overestimated because of omitted variable bias, caused by the omission of financial factors. Also, the
absolute value of the coefficient on cash flow measure is smaller than the agency cost index parameter, which can mean that cash flow under-estimates the effect of imperfections in capital market supply factors, as compared to the agency cost index.

The lack of data going back beyond 1984 does significantly limits the comparison that can be made between agency cost behavior and business cycles. Also, our data on both investment and borrowing is dependent on specific items of reported balance sheet data in the COMPUSTAT database. Although the former is a problem we deal with to some extent using Bayesian methods of inference, meaningful economic interpretation would still require a times series that goes back beyond 1984. An obvious extension is to incorporate all listed firms in any quarter in the sample, which is exactly what I have done for a second version of the paper. But since data on investment is often not recorded (or not recorded quarterly) in COMPUSTAT prior to the 1980s, even the inclusion of all firms does not solve the problem entirely. But I do indeed hope to get rid of survivor bias in the sample, thus getting more precise estimates of agency cost behavior.

Another, more important issue is whether the second moment of agency cost is more important than its mean level, and if it is, then in what specific way. In other word, whether the fluctuation of agency cost have been wider during (or before) a recession, as compared to other periods, and whether these swings have any amplification effects on business cycle peaks and troughs (the Bernanke et al. financial accelerator concept). My future version of the paper also considers this point through an appropriate Markov switching model and causality test. This paper is thus an initial attempt to marry a specific statistical methodology to the application of capital market imperfections in understanding business cycles.
Appendix : Algorithm and Gibbs sampler of Kalman filter

A.1 Kalman Filter
Kalman filter algorithm estimates unobservable variable, $\beta_t$, from a state space model which consists of the measurement equation and transition equation 6.1 and 6.2. In measurement equation, dependent variable, $y_t$, is explained by unobservable variable, $\beta_t$, and exogenous variable, $z_t$, while the transition equation is auto-regression of the unobserved variable.

Measurement equation
$$y_t = H\beta_t + Az_t + e_t$$

Transition equation
$$\beta_t = \mu + F\beta_{t-1} + v_t \quad e_t \sim iidN(0, R), \quad v_t \sim iidN(0, Q), \quad E(e_t v'_t) = 0$$

Where $y_t$ is n×1 vector of dependent variable at time $t$; $\beta_t$ is a k×1 vector of unobservable variable; $z_t$ is r×1 vector of exogenous variable; H is an n×k matrix of coefficients; A is n×r matrix; F is a k×k matrix of coefficients; $e_t$ and $v_t$ are the error terms in each equations; R is a n×n covariance matrix of $e_t$ and Q is a k×k covariance matrix of $v_t$.

This algorithm is composed of the prediction process and updating process as below. In prediction process, $\beta_t$ and its variance covariance matrix, $P_t$, are estimated from all information up to period $t-1$. In updating process, $\beta_t$ and $P_t$ are more precisely updated from predicted value at period $t-1$, using the prediction error of period $t$. These processes are recursively calculated from the first period 1 up to the end of period T.

Prediction Process
$$\beta_{t|t-1} = \mu + F\beta_{t-1|t-1}, \quad P_{t|t-1} = FP_{t-1|t-1}F' + Q,$$
$$\eta_{t|t-1} = y_t - y_{t|t-1} = y_t - H\beta_t - Az_t, \quad f_{t|t-1} = HP_{t|t-1}H' + R,$$

Updating process
$$\beta_t = \beta_{t|t-1} + K\eta_{t|t-1}, \quad P_t = P_{t|t-1} - K_tHP_{t|t-1},$$
where $K_t = P_{t|t-1}H'f_{t|t-1}^{-1}$ is the Kalman gain; $\eta_{t|t-1}$ is the prediction error; $\beta_{t|t}$ is expectation of $\beta_t$ at period $t$. From this algorithm, we get expectation of unobservable variable, $\beta_{t|t}$, in terms of period $t$. 
A.2 Gibbs sampler of the state space model

In section 2.3.2, we have already seen the inference method of parameters, $\Theta$, using the Gibbs sampler. Here, I explain the Bayesian inference of $\beta_t$. To find the estimator of the Kalman filter, the calculation of each conditional distribution, $\pi(\beta_t|\theta, y_t)$ and $\pi(\theta|\beta_t, y_t)$ are recursively iterated until the convergence of each distribution. This convergent distribution can be regarded as the invariant marginal target distribution. We pick up sample from this distribution and make estimators from this sample. Algorithm of this process is divided into two steps.

Step 5
Run the Kalman filter algorithm to calculate $\beta_{t|t}$ and $P_{t|t}$ for $t=1,2,...,T$ and save them.

The last iteration of the Kalman filter provides us with $\beta_{T|T}$ and $P_{T|T}$ and these can be used to generate $\beta_T$ based on

$$\beta_T|\theta, y_T \sim N(\beta_{T|T}, P_{T|T})$$

Step 6
For $t=T-1$, T-2,.....1, given $\beta_{t|t}$ and $P_{t|t}$, we can derive updating equation as

$$\beta_{t|t, \beta_{t+1}} = \beta_{t|t} + P_{t|t}F'\left(FP_{t|t}F' + Q\right)^{-1}(\beta_{t+1} - \mu - FP_{t|t})$$

$$P_{t|t, \beta_{t+1}} = P_{t|t} - P_{t|t}F'\left(FP_{t|t}F' + Q\right)^{-1}FP_{t|t}$$

Given the above two equations, we generate $\beta_t$, $t=T-1,T-2,.....1$ based on

$$\beta_t|\theta, y_t, \beta_{t+1} \sim N(\beta_{t|t, \beta_{t+1}}, P_{t|t, \beta_{t+1}})$$
REFERENCES


Koop, G., 2003, *Bayesian Econometrics*, John Wiley and Sons Ltd.


Chapter 3. The Impact of Stock Market Uncertainty on Real Exchange Rates

3.1 Introduction

Since the breakdown of the Bretton Woods system of fixed exchange rates, the real exchange rates of the world's largest economies have been highly volatile. The vast empirical literature explaining the movement of nominal exchange rates between countries has mainly focused on the monetary policy channel (interest rate rules) of major industrialized countries\textsuperscript{11}. Empirical studies on real exchange rate movements, on the other hand, have ranged from impact of fiscal and current account shocks on real exchange rates to impact of money demand shocks on the real exchange rate\textsuperscript{12}. Other strands of recent empirical literature have looked at productivity changes as a determinant of real exchange rates, along the line of the classic Balassa-Samuelson hypothesis\textsuperscript{13}. Following the influential paper by Chari, Kehoe and McGrattan (2002), a vast body of literature has also focused on the response of the real exchange rate to monetary shocks. But the literature is still plagued by a number of puzzles and anomalies, one of them being the asymmetrical response real exchange rate of U.S. and other G-7 countries to similar interest rate movement. In other words, a contractionary shock to U.S. interest rate is associated with an impact appreciation of the U.S. dollar (Eichenbaum and Evans (1995) whereas similar shock to interest rate in other G-7 countries is frequently associated with an impact depreciation of their currency relative to U.S. dollar (Grilli and Roubini (1995); Sims(1992)).

In this paper, we investigate the impact of financial market uncertainty shocks to the U.S. dollar real exchange rates for seven industrial countries. We create a forward looking expected uncertainty measure based on realized volatility of the dominant indices for all the countries, and identify shocks to uncertainty using a structural VAR with short

\textsuperscript{12} Corsetti and Müller, (2006); Monacelli and Perotti, (2006); Kim and Roubini (2008); Clarida and Gali (1994).
\textsuperscript{13} Corsetti, Dedola and Leduc (2008); Alquist and Chinn (2002).
run restrictions. We then study the impact of U.S. and foreign country shocks on the dollar real exchange rate of each country. Swings in exchange rates in recent years have been attributed to carry trades in the foreign exchange markets, where investors exploit interest rate differentials between countries to earn excess return on currency holdings. Carry traders profit from the well-known anomaly in international financial markets: the violation of uncovered interest parity condition (the forward premium puzzle), which predicts the absence of any excess return in currency markets for trades based on interest differential between two countries. Some very recent papers have attempted to explain the excess return in currency markets with the inclusion of time varying uncertainty.

The inclusion of exogenous source of uncertainty or risk and its impact on exchange rate is not novel in theoretical exchange rate economics: early contributions by Frankel and Meese (1987) in a partial equilibrium setting, and Hodrick (1989) in general equilibrium, have pointed out the role of uncertainty in explaining exchange rate determination. A more recent macroeconomic literature has examined the role and the effects that risk or uncertainties have on macroeconomic variables, which gives support to the role of uncertainty as a factor in real and nominal exchange rate determination.

The literature on time varying uncertainty and its impact on exchange rates have mostly been theoretical in nature, with some exceptions. The empirical literature on exchange rates has mostly depended on using VAR specifications to identify real and nominal shocks that drive exchange rate movements. Benigno, Benigno and Nistico, 2011 is the only paper so far who has explicitly included time varying uncertainty in a simple VAR specification, focusing on identifying the effect of monetary policy shock on exchange rates in the presence of time varying uncertainty. Their empirical specification only accounts for an uncertainty shock to U.S. productivity, U.S. inflation targets and U.S. federal funds rate. In our paper, we try to include time varying uncertainty for all the

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14 Cavallo (2006); Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006)
15 Benigno, Benigno and Nistico (2011); Obstfeld and Rogoff (2002)
countries in a VAR specification. We document a strong co-movement between expected stock market volatility ("uncertainty") of a country, and real exchange rate. We also find that the inclusion of uncertainty greatly improves the explanatory power of a VAR than just using the real and nominal variables that are usual in the literature.

### 3.2 Empirical motivations

Papers by Hansen and Sargent (2003, 2005, 2007), Hansen, Sargent, Turmuhambetova, and Williams (2006), Chen and Epstein (2002), Maenhout (2004, 2006), among others, have shown how uncertainty affects asset prices and optimal financial decisions. Although mostly theoretical in nature, it is safe to conclude from all of these papers that, in principle, when agents are unsure of the correct probability laws governing the market return, they demand a higher premium in order to hold the market portfolio. Even with some knowledge of the correct distribution of outcomes, it seems that an exogenous shock to the market’s expectation of uncertainty usually leads to a sharp drop not only in the financial markets, but also in real activity (e.g. employment and industrial production). Also, such second-moment shocks usually generate transitory ups and downs in economic activity, which is very different from the much more persistent movement that is typically exhibited in response to the type of level (first moment) shock that is usually modeled in the literature (see Christiano, Eichenbaum, and Evans 2005 and references therein).

Bollerslev and Zhou (2007) and Bekaert, Engstrom and Xing (2008) provides evidence of short-run predictive ability (in financial markets) of the CBOE’s Volatility Index computed using the whole set of S&P 500 options maturing at a given date (based on recent work on model-free implied volatility). For the case of the real economy, Christiano, Motto and Rostagno (2008) find that shocks to idiosyncratic uncertainty, as well as signal (news) shocks generates a lot of the volatility of business cycles in U.S. and Eurozone. Bloom (2009) provided further evidence for the relationship between stock market volatility (as a proxy for uncertainty) and real activity using the VIX and U.S. manufacturing employment and industrial production. Given the facts, it would seem that a valid measure of conditional stock market volatility (or physical expected
variance) of financial markets should be a forward looking variable, partially predicting movement in financial returns as well as real variables. Since the real exchange rate between two open economies with a floating regime is also a forward-looking variable (being an asset price), we should thus expect uncertainty to explain movements in real exchange rates.

In this paper, we investigate whether such a relationship between some suitable uncertainties indexes (some alternative index to VIX for reasons we will specify later) and real activity holds for other country as well. We then try to empirically model uncertainty shocks in two-country VAR (U.S. and foreign) to see how real exchange rate movements respond to shock to uncertainty. To see whether uncertainty shocks have any significant impact on real outcomes, we start with conducting a VAR on our measure of the uncertainty index, described in the next section, along with inflation, short-term interest rate and unemployment for each country we are studying. Figure 1 displays the response of unemployment to a shock to domestic uncertainty and also compares it with a level shock to rate of interest, for each country. As is evident, uncertainty shock and interest rate shock affect unemployment is very different ways, a point also noted in Bloom 2009 for U.S. data. The impulse responses are done based on a Cholesky decomposition of a 4-lag VAR with the variables mentioned above, and our results are robust to different specifications of lag as well different ordering of the variables. Except for New Zealand, all countries exhibit a rise in unemployment in response to an uncertainty shock, and in most cases (except for Japan and Australia), the rise in unemployment is much less persistent than if an interest rate shock would have hit the economy. For Japan and Australia, unemployment reverts to mean at around the same time for both kind of shocks.

We expect that for countries where uncertainty has low predictive power over real activity, (or where they do not contain any extra information for predicting real variables), they poorly explain exchange rate movements. And for countries where uncertainty has significant predictive power over real variables like unemployment, the exclusion of uncertainty in an econometric model of real exchange rate movement will
lead to poor results as compared to when uncertainty is included. We test our hypothesis by conducting a structural VAR for country pairs, with U.S. always a member of each pair, in keeping with its relative economic importance among the countries. We use variables from both countries, U.S. and foreign, and also isolate the differing impact of uncertainty shock to U.S. as opposed to uncertainty shock in the foreign country on the real exchange rate.

3.3 Data

We use monthly data for four out of the seven G7 countries (U.S., U.K., Japan and Canada) and also Australia, New Zealand and Switzerland, with a sample period ranging from January 1992 till July 2007, and estimate a structural VAR using country-differentials of the variables (except of course for uncertainty) for each country pair (U.S. and foreign). Our indicator for uncertainty in each country is based on the most prominent market portfolio in that country (for example, the S&P 500 for the U.S. For a complete list, refer to Table 1). Following Bloom (2009), it has become the usual practice in current empirical literature to use the stock market option-based implied volatility, or the VIX index, as an approximate measure of market uncertainty. However, since data for the VIX are only available starting January 1990, and the fact that similar indices for other countries start even later, we use within month realized volatilities for the whole sample period. Specifically, we construct conditional volatilities by considering the fitted values of an AR (1) regression for the indicator in each country, following Bekaert and Engstrom’s (2009) application to U.S. data. In other words, for each country we create measures of the total realized variation of the market (using daily squared return), or realized variance, for the months in our sample. We then project monthly realized variances on the one past realized variance, and use the fitted value of this regression, which is primarily driven by the past realized variance, as the estimated physical expected variance. We call the logarithm of this estimate “uncertainty”. Our measure is highly correlated (.84) with the VIX for the duration of the VIX (1990-2011). Figure 2 displays the dynamic properties of this indicator for each country.

The graphs in Figure 2 clearly indicate the strong correlation between historical expectations of uncertainty in U.S., U.K., Japan, Canada and Switzerland, and between
Australia and New Zealand. Also, all countries exhibit similar expectation of uncertainty from 2007 onwards, during and after the financial crises.

Other than the uncertainty indicator for each country, the data vector for structural VAR for a country pair (U.S. and foreign) also consists of their CPI inflation ($\pi$ and $\pi^*$), unemployment ($U$ and $U^*$), a short-term interest rate ($i$ and $i^*$) (specifically, the yield on a 3 month Treasury bill of the country) and the real exchange rate (defined as $q = s + p^* - p$ where $s$ denotes the log nominal exchange rate, expressed in terms of units of USD needed to buy one unit of foreign currency and $p^*$ and $p$ are foreign and U.S. log price levels (CPI Index). As such, an increase in $q$ (or $s$) denotes U.S. Dollar real (nominal) depreciation. Data has been taken from International Financial Statistics of IMF, OECD Main Economic Indicators database and Thomson Reuters DataStream database. A more detailed definition of the data and their specific sources are provided in Table 1.

The first three variables are well known variables in monetary business cycle literature. Although it might seem that we should be introducing an additional variable to control for the component of U.S. monetary policy that is a reaction to foreign monetary policy shocks (Kim and Roubini 1995 suggests the Federal Funds rate for this purpose), our identification scheme and consideration of country pairs makes this redundant. Finally, the real exchange rate is introduced to consider the effects of identified stock market shocks and macroeconomic shocks to the value of the currencies.

3.4 Identification Scheme

3.4.1 Structural VAR modeling

We assume that the economy is described by a structural form equation

$$A(L) y_t = e_t$$

(1)

where $A(L)$ is a matrix polynomial in the lag operator $L$, $y_t$ is a $n \times 1$ data vector, and $e_t$ is a $n \times 1$ structural disturbances vector. The vector $e_t$ is serially uncorrelated and $\text{var} (e_t) = \Lambda$ and $\Lambda$ is a diagonal matrix where diagonal elements are the variances of
structural disturbances; therefore, structural disturbances are assumed to be mutually uncorrelated.

Our reduced form VAR thus takes the form

\[ y_t = B(L)y_t + u_t \]  \hspace{1cm} (2)

where \( B(L) \) is a matrix polynomial (without the constant term) in lag operator \( L \) and \( \text{var} \,(u_t) = \Sigma \).

Recovery of the structural parameters from estimated parameters in the reduced form equations can be done using several methods. A popular and convenient method is to utilize Cholesky decomposition of the reduced form error covariance matrix, which is just a method to orthogonalize reduced form disturbances (as in Sims (1980)). However, this approach to identification necessarily assumes a recursive structure for the variables or a Wold-causal chain. Blanchard and Watson (1986), Bernanke (1986), and Sims (1986) suggest a generalized method (structural VAR) in which non-recursive structures are allowed, while still restricting contemporaneous structural parameters.

Let \( A_0 \) be the coefficient matrix (non-singular) on \( L^0 \) in \( A \, (L) \), that is, the contemporaneous coefficient matrix in the structural form, and let \( A^0 \, (L) \) be the coefficient matrix in \( A \,(L) \) without contemporaneous coefficient \( A_0 \). That is

\[ A(L) = A_0 + A^0 \, (L) \]  \hspace{1cm} (3)

Then, the parameters in the structural form equation and those in the reduced form equation are related by

\[ B(L) = -A_0^{-1} A^0 \, (L) \]  \hspace{1cm} (4)

In addition, the structural disturbances and the reduced form residuals are related by

\[ e_t = A_0 u_t \], which implies

\[ \Sigma = A_0^{-1} AA_0^{-1} \]  \hspace{1cm} (5)
Maximum likelihood estimates of $A$ and $A_0$ can be obtained only through sample estimates of $\Sigma$. The right hand side of Eq. (5) has $n \times (n + 1)$ free parameters to be estimated. Since $\Sigma$ contains $n \times (n + 1)/2$ parameters, we need at least $n \times (n + 1)/2$ restrictions. By normalizing $n$ diagonal elements of $A_0$ to 1’s, we need at least $n \times (n - 1)/2$ restrictions on $A_0$ to achieve identification. In the VAR modeling with Cholesky decomposition, $A_0$ is assumed to be a triangular matrix (hence the implied recursive structure). However, in the structural VAR approach, $A_0$ can be any structure as long as it has enough restrictions.

3.4.2 Identifying uncertainty shocks

The identification scheme allows for identifying the impact of level shocks i.e. impact of structural shocks to interest rate and inflation disturbances, on real exchange rate, which provides us with an empirical benchmark, somewhat along the spirit of Eichenbaum and Evans (1995). Apart from the real variables for each U.S. - foreign country pair, the two uncertainty variables are included in the VAR specifications to isolate and identify the impact of a structural shock to volatility (second order effect) on the levels of real exchange rate as well as the other real variables.

Before we explore the structural identification, we would like to briefly comment on some reduced form VAR statistics. We use a VAR with 2 lags for each variable in our analysis, based on lag-selection criteria like Akaike (AIC), Hannan-Quinn (HQIC) and Schwarz (SBIC). While the Schwarz criterion selects a VAR with 3 lags for all the countries, the AIC and HQIC criteria both select a VAR with two lags for all the countries. We focus the remainder of the paper on VAR with two lags.

For the restrictions on the contemporaneous structural parameters $A_0$, we follow the general idea of Sims and Zha (1995), and Kim and Roubini (2000), but modify it substantially. The following equations summarize our identification scheme based on Eq. 5.

\[ e_t = A_0 u_t. \]
where $e_{(\pi^*-\pi)}$, $e_{e^*}$, $e_{(U^*-U)}$, $e_{(l^*-l)}$ are the structural disturbances or exogenous shocks to relative foreign inflation (relative to U.S.), foreign uncertainty, domestic (U.S.) uncertainty, relative foreign unemployment rate (relative to U.S.) and short-term interest differential respectively (i.e. notations with * superscript signify variables of the foreign country). The structural shock to the real exchange rate is denoted by $e_{RER}$. The residuals in the reduced form equations, representing unexpected movements (given information in the system) of each VAR variable are $u_{(\pi^*-\pi)}$, $u_{e^*}$, $u_{(U^*-U)}$, $u_{(l^*-l)}$. The real exchange rate residual is denoted as $u_{RER}$.

It is worth noting that the restrictions indicated in Eq. 6 are contemporaneous restrictions on the structural parameters of $A_0$ without further restrictions on the lagged structural parameters. The inflation equation assumes that inflation in each country, relative to U.S. inflation, reacts to the expectation of uncertainty in that country and in U.S. within that month, as well as relative unemployment. While any exchange rate movements will eventually feed through to the domestic CPI, the pass through effect is not instantaneous and will likely to vary relatively slowly (Goldberg and Knetter, (1996)). Hence we excluded any contemporaneous impact of exchange rate on inflation. Any other exogenous supply shock will likely to increase uncertainty in the market, and hence should already be captured in the inflation equation in its present form. The uncertainty variable, being a beginning of month variable incorporating expectations of volatility for the coming month, is not affected contemporaneously by any of the endogenous variables in the VAR. The real exchange rate, which is a forward-looking asset price, is contemporaneously affected by relative inflation, relative unemployment and interest differential between the country and U.S. It is also contemporaneously affected by uncertainty in both countries.
The unemployment variable for a country, being a sluggish real variable in almost all industrialized countries, is affected only by domestic uncertainty within that period and nothing else. We assume that one month is too small a time period for unemployment to react to changes in domestic inflation, real exchange rate or the domestic rate of interest. Hence relative unemployment should react to uncertainty shocks to both foreign and U.S. stock markets. Finally, the short-term interest rate in our model reacts to domestic inflation and unemployment; following the lines of a Taylor rule type Fed reaction function. In addition, we include our short-term rate of interest to also have a same period relationship with uncertainty, in order to isolate the component of monetary policy that is a reaction to market’s expectation of uncertainty. Hence interest differential is affected by all the above. Figure 3 illustrates the historical relative (to U.S.) levels of short-term interest rate for all the countries. Other than Japan and Switzerland, all the other countries have consistently experienced a higher level of short-term interest rate relative to U.S. for the sample period in our study (except for two short intervals of time when the Canadian interest rate drops relative to U.S.). The Swiss interest rate rises relative to U.S. during the period of 1990 to 1995, and remains consistently below the U.S. interest rate otherwise, converging only recently. Japan interest rate remains consistently at a lower level relative to U.S. rate.

In Table 2 we report the estimated coefficients from our VAR. The numbers in the brackets are the standard errors. Data are monthly and the estimation period is from 1992 January-2007 June. For Australia the estimation period is shorter, from 1995:7 – 2007:6, because of limitation of getting data for constructing the uncertainty index. We use log of uncertainty and real exchange rates throughout, and use unemployment rate less its past 12 month average value.

3.5 The effects of Uncertainty Shocks

We couch our main results in the form of impulse-response functions (IRFs henceforth), estimated in the usual way. We compute 90% bootstrapped confidence
intervals based on 1000 replications, and focus our discussion on significant responses. We report the resulting structural impulse-response functions in Figure 4.

Before we report the empirical results from our model, we discuss the expected movements of the macro variables (mainly the real exchange rate) given available theories about the effects of time varying uncertainty. The literature on time varying volatility and its effect on exchange rates are relatively new and scarce, although some notable early papers confronting the issue are those of Hodrick (1989) and Obstfeld and Rogoff (2001). But those papers relate the nominal exchange rate to monetary uncertainty through alternative specifications of money demand, and do not really tackle the issue of exogenous risk factors and its impact on exchange rates. Some of the very recent empirical papers that include exogenous time varying uncertainty and its role in exchange rate behavior are Engel (2011) and Benigno et al (2011), but they rely on theories of exchange rate determination based on interest rate rules. In other words, time varying uncertainty affect exchange rates through the monetary policy channel. Our purpose, on the other hand, is to show that our measure of expected uncertainty is a forward looking variable, and although it is correlated to monetary policy rules, contains information about real exchange rate movements that is not completely captured by monetary policy stance.

Because of currency carry trades in the countries in our sample, we expect to see different set of results for relatively high interest rate currencies as opposed to relatively low interest rate currencies in our sample. In other words, we expect different results from VAR where the interest differential is positive as compared to those where it is negative. A shock to expected home (U.S.) uncertainty, as is evident from Figure 4, leads to an impact rise in real exchange rate (U.S. dollar weakens with respect to Yen and British Pound in real terms) for Japan and U.K., and the rise is persistent. In the case of Australia, New Zealand, Switzerland and Canada, the immediate impact of a shock to uncertainty in U.S. stock markets is to strengthen the U.S. dollar. For all these countries, the impact appreciation of U.S. dollar is persistent and it remains at the higher level.
(compared to its long run mean) long after any effect of uncertainty shock to real variables (unemployment, industrial production) has died out.

For shocks to expectations of uncertainty in a foreign country, the real exchange movement is more ambiguous when compared to home (U.S.) shocks. For Japan, an uncertainty shock to the Japanese stock market has the same effect on dollar-yen exchange rates as in the case of a U.S. shock, i.e. the dollar weakens with respect to the yen, but to a greater extent than before, but similarly persistent (Figure 4.a). For a domestic shock to uncertainty in Canada, Australia, and Switzerland the dollar moves in the opposite direction to when the shock was to U.S. markets i.e. it weakens relative to these three countries currency. We find that a shock to both U.K. and New Zealand stock markets tends to strengthen the dollar relative to their currencies.

Our impulse responses are consistent with the dynamics of currency carry trade. Currency carry traders borrow low interest currency, convert them into the high interest currency, and lends out at the higher interest rate. According to economic theory, the exchange rate risk involved in such a transaction should result in zero predictable profits. This is because the uncovered interest parity condition in international financial markets states that the interest differential between the high and low interest countries simply reflect how much investors expect the low interest currency to appreciate against the high interest currency. So any attempt at arbitrage would lead to an appreciation of the borrowing currency, thereby making returns worth less until any return from lending equals the borrowing cost. But investors do tend to invest in and make profits in carry trade. This is because excess returns in currency carry trade are generated by exploiting the anomaly that currencies with relatively low interest rates (or those that can be sold at a forward premium) tend, on an average, to depreciate (also known in literature as the forward premium puzzle), rather than appreciate. The opportunity to exploit this violation of the uncovered interest parity condition (and even cause or/and amplify it) thus prompts significant changes in supply and demand for currencies. Specifically, carry trades based on interest rate differentials and forward premiums affect the balance of supply and demand for borrowing and target currencies in foreign exchange markets, resulting in sizeable and persistent exchange rate movements. We believe that any exogenous shock
to the financial market’s expectation of uncertainty will significantly affect the risks of carry trade, thereby influencing the demand and supply of currencies and their exchange rates.

A shock to uncertainty in the stock markets of a low interest country should reduce borrowing (and subsequent selling) of that country’s currency (either due to increased liquidity frictions or increased risk aversion), thereby appreciating it against the high interest rate country’s currency. Uncertainty shock to U.S. reduces carry trade i.e. reduces the sale of U.S. dollars for the purpose of converting them into Australian, New Zealand or Canadian dollar or the U.K. pound. This should result in an impact appreciation of the U.S. dollars against these currencies, which is exactly what we see in our impulse responses (except for U.K). The size of impact appreciation of the U.S. dollar is biggest against Australia, the country with the relatively higher level of interest rates on an average during the sample period. This is in keeping with our carry trade hypothesis, and supported as well by evidence from a Bank of International Settlements (BIS) study which found a high degree of foreign exchange market turnover (size of carry trade) between U.S. and Australia during 2001-2004. Uncertainty shock to Japan reduces carry trade i.e. reduces the sale of Japanese yen for the purpose of buying U.S. dollars, thereby appreciating the yen (or depreciating the USD), which is again consistent with our result. We get the same result for the U.S. dollar Swiss franc exchange rate, with the Swiss franc appreciating to an uncertainty shock to the Swiss stock market, being a relatively (relative to U.S.) low interest rate country. Brunnermeier et.al. (2008) provides empirical evidence of the fact that any shock to risk-appetite, or to expected uncertainty, dampens carry traders willingness to speculate. In the face of shocks, including those to risk preferences, market liquidity will go down, traders will unwind their positions, and this can cause large swings in exchange rates. Such liquidity frictions (Brunnermeier and Pedersen (2008)) can also help explain why shocks to uncertainty in U.S. markets and Japanese markets dominate in their impact on exchange rates. Any increase in liquidity friction there (say a tightening of interbank liquidity) will affect the borrowing of the dollar and yen more than new or existing positions in the high yield currency.

18 Galati, and Melvin (2004);
In case of an uncertainty shock to markets in countries characterized by relatively high rate of interest, a rational investment strategy would be to reduce exposure to the market or exit. That would entail re-conversion of the high interest currency to the borrowing/funding currency, resulting in excess supply and consequent downward pressure on the high interest currency. This is consistent with what we find for the case of New Zealand and U.K. But our model predicts opposite movement of real exchange rate in the case of Australia and Canada i.e. the Canadian and Australian dollar both appreciate in response to uncertainty shocks to their markets.

We also report the results regarding the sources of real exchange rate fluctuation. In Table 3, we report the forecast error variance decompositions of real exchange rate due to shocks in foreign and U.S. uncertainty, shocks to relative inflation, shocks to interest differential, and shocks to relative unemployment rate. We highlight the effect of uncertainty shocks on real exchange rate fluctuations for each foreign-U.S. country pair at a horizon of 24 months, and all intermediate periods are also reported. For most countries, the uncertainty shocks’ contribution in explaining exchange rate fluctuations is very high, exceeding 25 percent in most cases at its peak as well as at 24-month horizon. Only for the case of U.K. and New Zealand are real exchange rate fluctuations explained less by uncertainty shocks, as compared with other shocks. Even then, the U.K-U.S. exchange rate fluctuation is 16% explained by uncertainty shocks at a 6-month horizon (its peak) and never drops below 13% after that. The dominant source of fluctuation in the NZ dollar –U.S. dollar exchange rate is definitely not uncertainty shocks, since they never manage to explain more than 8% of the fluctuations in the real exchange rate.

3.6 Robustness check

We conducted robustness checks for different identifications schemes as well as implementing VAR in levels rather than differences in variables of country pairs and then using different identification schemes. Our results are robust to such alternative specifications. In Figure 5 we report the impulse responses of dollar real exchange rates for our VAR for the sample period 2007 M6 to 2011 M9. The purpose of the exercise was to check whether our empirical specification holds up to the highly volatile data that
is a feature of the recent financial crises and economic downturn. We find that the effect of uncertainty shock on real exchange rate is very transitory during this period as well estimated imprecisely, given the short sample period. What is more important is the fact that for some of the country pairs, the impact switch signs, which could mean several things. It could be because of the fact during the crises, monetary policy responses has been highly unconventional among most of the industrialized world, as well as the fact that some of the major economies actively intervened in the foreign exchange market, thereby dampening some of the pure market signals that could have been captured by the VAR. It could also mean that carry trade speculative behavior during volatile periods is not adequately captured by a VAR specification like ours.

3.7 Conclusions

In this paper we developed an open economy structural VAR model to address the effects of uncertainty shocks in open economies on their real exchange rates. We created a forward looking measure of uncertainty using stock market volatility data and looked at the effect of structural uncorrelated shocks to U.S. and foreign uncertainty. The identification scheme used in the paper appears to be successful in identifying uncertainty shocks and significantly explaining fluctuations as and movements in real exchange rate as a result of an impact of those uncertainty shocks. It will be premature to claim that our identification scheme generates ‘sensible’ uncertainty policy shocks, mainly due to the lack of priors on dynamic response functions of real exchange rates (to uncertainty shocks) in the literature. But the impact response of exchange rates to other macro-shocks in our model (short term interest rate and inflation shock) leads us to believe that the identification scheme is not grossly at odds with sensible economic priors.

The impact of a shock to expected volatility of U.S. stock markets seems to appreciate the U.S. dollar against all relatively high interest rate countries (except U.K.). The impact of uncertainty shock to Japan and Switzerland tends to appreciate their currency against the relatively high yield U.S. dollars. We believe that this is consistent with the currency trade explanation of movements in short-term exchange rates. A shock to low interest currency (the funding currency in carry trade) should reduce speculative
activity and selling, leading to an appreciation of the funding currency and depreciation of the high interest currency.

Hence volatility shock, both in U.S. and foreign markets, does have a distinct and direct effect on dollar exchange rates. This result complements the well-known empirical literature on UIP regressions and exchange rate movements, and illustrate the necessity of including financial market based uncertainty, both U.S. and foreign, in any empirical and theoretical models of exchange rates.
Appendix: Tables and Figures

Table 2: Data and Variable Description

Data used for computation of stock market uncertainty for each country

<table>
<thead>
<tr>
<th>Country</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.A.</td>
<td>Historical daily return on the S&amp;P 500 index</td>
</tr>
<tr>
<td>U.K.</td>
<td>Historical daily return on the FTSE 100 index</td>
</tr>
<tr>
<td>Japan</td>
<td>Historical daily return on the Nikkei 250 index</td>
</tr>
<tr>
<td>Canada</td>
<td>Historical daily return on the S&amp;P/TSX 300 and S&amp;P TSX Composite index</td>
</tr>
<tr>
<td>Australia</td>
<td>Historical daily return on the S&amp;P/ASX 200 index</td>
</tr>
<tr>
<td>New Zealand</td>
<td>Historical daily return on the NZSE 40 index and NZSX 50 index</td>
</tr>
<tr>
<td>Switzerland</td>
<td>Historical daily return on the SMI (Swiss Market Index)</td>
</tr>
</tbody>
</table>

Source: Thomson Reuters DataStream.

Data used for computation of variables in VAR

| Variable            | Description                                                                 \n|---------------------|-----------------------------------------------------------------------------|
| Inflation           | Annualized inflation calculated from monthly CPI data for each country based on the formula $\left(\frac{P_{t+12}}{P_t} - 1\right) \times 100$. Source: OECD (2010), "Main Economic Indicators - complete database", Main Economic Indicators, http://dx.doi.org/10.1787/data-00052-en (Accessed on 01/01/2011) |
| Short term interest rate | Yield on 3-month Treasury bills of respective countries. Source: International Financial Statistics, International Monetary Fund |
Table 3: The A matrix of structural parameters

\[
\begin{bmatrix}
1 & a_{12} & a_{13} & 0 & a_{15} & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
a_{41} & a_{42} & a_{43} & 1 & a_{45} & a_{46} \\
0 & a_{52} & a_{53} & 0 & 1 & 0 \\
a_{61} & a_{62} & a_{63} & 0 & a_{65} & 1
\end{bmatrix}
\]

Table 4: Contemporaneous coefficients in the structural model

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>Japan</th>
<th>U.K.</th>
<th>Australia</th>
<th>New Zealand</th>
<th>Switzerland</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_{41})</td>
<td>-0.004</td>
<td>0.000</td>
<td>0.004</td>
<td>-0.004</td>
<td>-0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>(a_{61})</td>
<td>-0.049</td>
<td>-0.004</td>
<td>0.042</td>
<td>-0.038</td>
<td>-0.104</td>
<td>0.022</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.055)</td>
<td>(0.028)</td>
<td>(0.033)</td>
<td>(0.020)</td>
<td>(0.037)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>(a_{12})</td>
<td>-0.220</td>
<td>-0.153</td>
<td>-0.073</td>
<td>-0.655</td>
<td>-0.432</td>
<td>-0.140</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.065)</td>
<td>(0.108)</td>
<td>(0.078)</td>
<td>(0.197)</td>
<td>(0.192)</td>
<td>(0.143)</td>
</tr>
<tr>
<td>(a_{42})</td>
<td>0.009</td>
<td>0.007</td>
<td>0.009</td>
<td>-0.005</td>
<td>0.013</td>
<td>-0.003</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.003)</td>
<td>(0.006)</td>
<td>(0.003)</td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>(a_{52})</td>
<td>0.059</td>
<td>-0.005</td>
<td>0.085</td>
<td>0.029</td>
<td>0.025</td>
<td>-0.016</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.033)</td>
<td>(0.039)</td>
<td>(0.026)</td>
<td>(0.044)</td>
<td>(0.043)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>(a_{62})</td>
<td>-0.079</td>
<td>-0.105</td>
<td>-0.019</td>
<td>-0.072</td>
<td>-0.024</td>
<td>-0.155</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.052)</td>
<td>(0.044)</td>
<td>(0.037)</td>
<td>(0.055)</td>
<td>(0.104)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>(a_{13})</td>
<td>0.115</td>
<td>-0.149</td>
<td>-0.073</td>
<td>0.323</td>
<td>-0.195</td>
<td>0.057</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.065)</td>
<td>(0.089)</td>
<td>(0.079)</td>
<td>(0.170)</td>
<td>(0.133)</td>
<td>(0.145)</td>
</tr>
<tr>
<td>(a_{43})</td>
<td>0.009</td>
<td>-0.013</td>
<td>-0.002</td>
<td>0.026</td>
<td>0.010</td>
<td>0.000</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.003)</td>
<td>(0.006)</td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>(a_{53})</td>
<td>0.049</td>
<td>0.005</td>
<td>-0.083</td>
<td>0.046</td>
<td>-0.007</td>
<td>-0.021</td>
</tr>
<tr>
<td>s.d.</td>
<td>(0.034)</td>
<td>(0.032)</td>
<td>(0.026)</td>
<td>(0.038)</td>
<td>(0.030)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>(a_{63})</td>
<td>-0.104</td>
<td>-0.078</td>
<td>0.044</td>
<td>0.009</td>
<td>0.056</td>
<td>-0.044</td>
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Table 5: Variance Decomposition of Log Real Exchange Rate

5a Variance Decomposition of Log Real Exchange Rate (US- Australia)

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5b Variance Decomposition of Log Real Exchange Rate (US- Canada)

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### 5c Variance Decomposition of Log Real Exchange Rate (US - Japan)

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### Variance Decomposition of Log Real Exchange Rate (US- UK)

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Figure 6: Response of Unemployment to Cholesky one S.D. innovations to *domestic* Short Term Interest Rate and *domestic* Uncertainty.

6 a Australia

6 b Canada
6c Japan

6d New Zealand
6 e Switzerland

6 f U.S.A.
**Notes:** The impact is of one standard deviation shock to uncertainty and short-term rate of interest. For all the countries, a shock to uncertainty exhibits a hump shaped response, i.e. unemployment goes up by around 2% to 4% (except for New Zealand, where unemployment is unresponsive) a month or two after impact of uncertainty shock and can be persistent. Also, a shock to interest differential generates either a similar or more persistent response of unemployment, but the dynamics are different. Unemployment, being a sluggish variable, responds to interest rate shock with a substantial lag, whereas uncertainty seems to affect unemployment almost immediately.
Figure 7: Graph of Expected Stock Market Uncertainty for Each Country

7 a Australia

7 b Canada

7 c Japan
7 d New Zealand

7 e Switzerland

7 f U.S.A.
Notes: The variable is the log of our measure of conditional physical variance computed from index daily returns data. The scale on the vertical axis has been forced to zero which gives a comparative idea of the both the magnitude as well as the relative fluctuations of uncertainty for the seven countries.
Figure 8: Interest Rate Differential

Notes: The above graph illustrated the difference between each country’s 3-month Treasury yield and the U.S. 3-month Treasury yields. Hence curves above the zero line are for countries and/or periods when the 3-month Treasury yield is lower than the U.S. counterpart. For example, Japan consistently stays above the zero line for most of the sample period, having lower short-term interest rate than U.S.
Figure 9: Impulse Responses from Structural VAR (1992 January-2007 June)

9 a. Response of Exchange Rate (U.S.–Japan) to Uncertainty Shock in Japan (top) and U.S. (bottom)

9 b. Response of Exchange Rate (U.S.–Australia) to Uncertainty Shock in Australia (top) and U.S. (bottom)
9 c. Response of Exchange Rate (U.S.–Canada) to Uncertainty Shock in Canada (top) and U.S. (bottom)

9 d. Response of Exchange Rate (U.S.–U.K.) to Uncertainty Shock in U.K. (top) and U.S. (bottom)
9. e Response of Exchange Rate (U.S. – New Zealand) to Uncertainty Shock in New Zealand (top) and U.S. (bottom)

9 f Response of Exchange Rate (U.S. – Switzerland) to Uncertainty Shock in Switzerland (top) and U.S. (bottom)

Notes: The black line going up to .05 on the vertical axis means that the USD has depreciated in real terms by 5%.
Figure 10: Robustness Check: Recent crisis period (2007 June-2011 September)

10 a. Response of Exchange Rate (U.S.–Canada) to Uncertainty Shock in Canada (top) and U.S. (bottom)

10 b. Response of Exchange Rate (U.S.–Japan) to Uncertainty Shock in Japan (top) and U.S. (bottom)
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10 c. Response of Exchange Rate (U.S.–Switzerland) to Uncertainty Shock in Switzerland (top) and U.S. (bottom)

10 f. Response of Exchange Rate (U.S.–U.K.) to Uncertainty Shock in U.K. (top) and U.S. (bottom)
REFERENCES


Chapter 4. Non-Linearity in Deficit-Interest Relationship: A Threshold Analysis

4.1 Introduction

Economic theory posits that the effect of deficit on long term interest rates is an important channel through which deficit affect the real sector of the economy. For instance, fluctuations in long-term rates may impact interest-sensitive components of private spending such as housing and business fixed investment. Hence, understanding how deficits impact long-term rates is of policy interest. However, this relationship can be confounded by the fact that different policy measures, monetary and fiscal, may impact deficit and interest rates contemporaneously. For example, if long-term interest rates fall due to monetary easing during recessions, while automatic stabilizers raise the deficit, deficits and interest rates may be negatively correlated even if the partial effect of deficits on interest rates - controlling for all other influences- is positive. The dynamics of the relationship between deficit and long-term interest rates is thus essentially an empirical question, which we try to address in this paper.

In the literature, the relationship between (deficit) and long-term interest rates has been extensively studied. In recent years, an important issue that one must address in estimating such a relationship is to account for the role of expectations about debt and deficit (see Gale and Orszag (2002, 2003), and Laubach (2009)). In this paper, we argue that it is also important to account for the possible nonlinearities in the relationship between long-term interest rates and the level of expected deficit.

In this paper we extend the Laubach (2009) single-equation framework by incorporating threshold effects in expected deficits. Specifically, we estimate a threshold model where the level of threshold is unknown and is estimated along with other parameters of the linear regression model (Hansen (2000)). Using the estimated threshold level for expected deficit from this model, we specify and estimate a threshold VAR
model to estimate response of future long-term rates to structural shocks to deficit expectations, and allow this response to differ across the threshold levels.

There are several findings of interest. First, similar to Laubach (2009), we find that expected deficit increases the future long-term rate\(^{19}\). However, we also find that there is a significant threshold effect in this relationship. Specifically, the relationship between expected deficit and interest rate is only significant during the regime of high expected deficit. Our estimated threshold level for the projected-deficit GDP ratio is 1.8. We find that for values of projected-deficit GDP ratio higher than this threshold level, a percentage point increase in the projected deficit GDP ratio increases future long term nominal interest rate by 30 basis points.

Using our estimated threshold level from the single-equation framework, we also estimated a Threshold VAR with a known threshold. We find that a one standard deviation shock to projected deficits increases the real future long term interest rate by 16 basis points in a ‘high’ deficit regime for almost a period of 12 months before returning to the mean. In contrast, in the ‘low’ deficit regime the impact is negligible. This means that the impact of an innovation to projected deficit acts not only through the expected inflation channel, but also directly on the economy’s real rate of return.

One way to interpret the results of our Threshold VAR is to consider an innovation to the projected deficit variable as a revision of market’s expectation of future deficit. In a regime where economic agents already expect deficit to be high in the future, any revision of such projections will likely have large and significant impact on future interest rates (agents expect future interest rates to be high). This will not be the case when economic agents expect future deficit to be below a certain threshold value.

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\(^{19}\) We measure expected deficit as CBO 5 year ahead projected deficit as a ratio of the potential GDP. We measure future long-term rate as 5-10 year ahead forward rates.
4.2 Related Literature

Although several different surveys over the past twenty years have evaluated the literature on the relationship between federal government debt/deficit and interest rates\(^{20}\), the empirical evidence on the relationship between deficit and interest rates is at best mixed.

A survey of the literature by Seater (1993) finds support for the Ricardian equivalence hypothesis, which implies that federal government debt has no effect on interest rates. Barro (1989) takes a similar position as Seater, concluding: "Overall, the empirical results on interest rates support the Ricardian view. Given these findings, it is remarkable that most macroeconomists remain confident that budget deficits raise interest rates."

Bernheim (1989) on the other hand finds little evidence for the neutrality of government debt. Elmendorf and Mankiw (1999) summarize this literature by stating that: “Our view is that this literature, like the literature regarding the effect of fiscal policy on consumption, is ultimately not very informative. Examined carefully, the results are simply too hard to swallow ".

A more recent survey paper by Gale and Orszag (2004) emphasizes that empirical study that (properly) incorporates deficit and debt expectations in addition to current deficit tends to find economically and statistically significant connections between anticipated deficits and current long-term interest rates. A recent paper by Laubach (2009) does just that. He explicitly focuses on long horizon forecasts of fiscal variables and interest rates. Laubach incorporates CBO deficit projections in his analysis as a proxy for expectations, and looks at the impact of the projections on future short and long interest rates, and finds significant positive relationship between these two variables.

The literature on non-linearity in fiscal policy (mainly dealing with correctly estimating the size of government spending or tax multiplier during a recessionary regime

\(^{20}\) (Bernheim (1987, 1989); Barro (1989); Seater (1993); Elmendorf and Mankiw (1999); and Gale and Orszag (2002, 2003)
and expansionary regime) is relatively new, with the most current and influential paper being Auerbach and Gorodnichenko (2012). In their paper, the authors incorporate regime in GDP growth in keeping with their research question, and endogenously determine regime switches and estimate regime dependent multipliers in a VAR specification.

Another paper that uses a more conventional Threshold VAR to incorporate non-linearity in fiscal policy is Candelon and Lieb (2011). Although structural VAR specifications have been the workhorse for finding out the impact of fiscal variables on monetary and vice versa (Mountford and Uhlig (2000), Perotti (2002), Engen and Hubbard (2004)), the existence or estimation of non-linearity in their relationship is rare in the literature (one reason could be the small sample size of most fiscal data and the potential empirical issues that researchers might have to encounter with the large number of parameters generated in any non-linear model).

The specific form of non-linearity that we are looking at is novel in two ways. Firstly, non-linearity in the impact of expected deficit on interest rates has not been addressed in the literature to our knowledge. Secondly, although it is fairly straightforward to do a Threshold VAR (along the lines of Tsay, 1998) where non-linearity in one of the variables (expected deficit in our case) can be endogenously determined (thereby generating regime dependent impulse responses), given the small sample size of government deficit data, it is near to impossible to get any meaningful results in a non-linear VAR with its numerous parameters. Hence we test for non-linearity in expected deficit and estimate its threshold value in a simpler single equation framework using Hansen (2000) asymptotic method. We then use this known threshold value to create a dummy that we interact with the projected deficit variable in our VAR to corroborate our results from the single equation framework.

4.3 Specifications

In the absence of nominal rigidities in the economy, the real interest rate is equal to the “natural” rate of interest $r_t^*$ and only deviates from it in the presence of nominal
rigidities, where the short term rate takes the form \( r_t = i_t - E_t \pi_{t+1} \). Laubach (2009) is motivated by this theory to specify regressions for the real rate of the general form \( r_t^* = \alpha + \beta f_t + u_t \) where \( f_t \) is the given measure of fiscal policy, and \( u_t \) denotes other factors affecting the natural rate (specifically, growth rate of per capita consumption, the intertemporal elasticity of substitution, and household’s rate of time preference). The observed real short-term interest rate can then be written as \( r_t = \alpha + \beta f_t + u_t + (r_t - r_t^*) \) where the real-rate gap \( (r_t - r_t^*) \) is unobserved, and is subsumed in the residual of a regression of the current interest rate on fiscal factors. But this gives rise to a potential endogeneity problem in a setting where the real rate gap varies over time due to, for example, countercyclical monetary policy, and while at the same time automatic stabilizers induce cyclical variation in the fiscal variable \( f_t \). However, since nominal rigidities are only temporary, for sufficiently long horizons \( k \) the real rate gap should vanish in expectation, that is \( E_{t-k}(r_t - r_t^*) = 0 \). Laubach (2009) addressed the endogeneity problem by focusing on expectations of interest rates and fiscal variables sufficiently far into the future. Our test for the existence of threshold and its estimation is thus run on a variant of Laubach’s regression, which takes the form:

\[
E_t(i_{t+k}) = \beta_0 + \beta_1 E_t(\pi_{t+k}) + \beta_2 E_t(f_{t+k}) + \beta_3 E_t(u_{t+k}) + \varepsilon_t
\]

(1)

where the dependent variable is the long-term nominal interest rate expected to prevail \( k \) periods ahead, the coefficient \( \beta_1 \) on expected inflation can be different from 1, and \( u_t \) denotes additional regressors. The main interest is in the magnitude and statistical significance of \( \beta_2 \) and especially whether it takes significantly different values in different regimes of the fiscal variable (in the case where regimes are present).

The effects of fiscal variables like debt and deficit, and more specifically expected debt and deficit, on real and monetary variables, as well as their impact on future long-term rates remain unresolved in theoretical literature. Until we have more specific stylized facts about the interaction between interest rates and expected debt/deficit, such issues are bound to remain unresolved.

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\(^{21}\) Woodford (2003) gives detailed exposition on this.
In Section 3.2 we intend to specify one such stylized fact about the link between future long term rates and expected deficit/debt, mainly that there is an unambiguous threshold effect of expected deficit on future long term interest rate. In Section 3.3 we estimate the threshold and its confidence interval in a single equation regression framework of the type (1), and test whether there is a threshold effect of expected deficit/debt on future long rates. In Section 3.4, we estimate a Threshold VAR using current and projected fiscal variables and future interest rates. Our goal is to characterize the empirical relationship between future long-term interest rates, expected debt and deficit, expected inflation and expected GDP growth rates, while placing as few theoretical restrictions on the system dynamics as possible. Before we look into the nature and results of threshold tests and the Threshold VAR, we present the data in the next section.

4.3.1 Data and their Properties

Following related literature, we use Congressional Budget Office (CBO) published projections of debt, deficit \((E_Td_{t+k})\) and GDP growth rates \((E_Ty_{t+k})\) as the fiscal variables, with the first two expressed as percentages of projected GNP or GDP, as a proxy for expectations of future fiscal policy. The forecast horizon is 5 years in the future, which is the longest horizon for which a reasonably long time series of projections is available. Consistent with the use of 5-year-ahead projections of fiscal variables by the CBO, the analysis focuses on forward rates 5 years ahead embedded in the term structure of interest rates. In other words, we use 5 year ahead 5-year forward rate (and 5 year ahead 10-year forward rate) \(E_Ti_{t+k}\), calculated from zero coupon yield curve dataset of Gurkanayak, Sack and Wright (2006), details of which are in Appendix A. A significant component of any movement in future long term rates would be that of inflation expectations (of matching maturity), and we use a measure of long horizon inflation expectations \(E_T\pi_{t+k}\) composed of a spliced series from both survey data and models of professional forecasters over the time period of our sample (details in Appendix A).

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22 Laubach (2009), Wachtel and Young (1987), Cohen and Garnier (1991), Elmendorf (1993). For a brief review of these papers, see Laubach (2009). None of these papers deal with non-linear effects of debt on interest rates. We are grateful to Thomas Laubach for making his updated dataset on projected deficit and GDP growth rates available to us. For details regarding construction of the data and their stationary properties, see Laubach (2009).
series of interest rates and inflation expectations are shown in Figure 1. The unified budget deficit, debt and GDP projections used are at semi-annual frequency from 1985, rather than the annual frequency which goes back till 1976, since that way we get more observations in our sample. Also, we wanted to avoid the significantly higher levels (as also volatility) of projected deficit and expected inflation pre-1985 to drive our results.

In Figure 2, we show the CBO data on projected deficit, projected debt and projected GDP growth. For details on the forecast errors of the projections data and stationarity properties, refer to Laubach (2009) who gives a detailed exposition on the issue. The projections are shown for the fiscal year in which they are made. Table 4.1 and Table 4.2 provide some descriptive statistics of projected deficit/GDP ratio and projected GDP growth rates.

4.3.2 Threshold Estimation

The test for non-linear effects of expected debt on future interest rates based on an asymptotic distribution theory of threshold estimates in a regression developed by Hansen (2000). In a routine analysis of sub-sample stability of parameters of a regression of the form \( y_t = \beta' x_t + e_t \), either a threshold value to split the sample is specified, or in the case where such specification is lacking, some method is employed in its selection. Such practices can be formally treated as a special case of the threshold regression model. These take the form:

\[
\begin{align*}
    y_t &= \theta_1' x_t + e_t, & q_t \leq \gamma \\
    y_t &= \theta_2' x_t + e_t, & q_t > \gamma
\end{align*}
\]

Where \( q_t \) may be called the threshold variable, and is used to split the sample into two groups, which may be called “classes” or “regimes” depending on the context.

Hansen (2000) develops an asymptotic approximation to the distribution of the least squares estimate \( \hat{\gamma} \) of the threshold parameter \( \gamma \). The specification that we use to test for a threshold, as well as to search a threshold from information contained within the regression framework, is:
\[ i_t = \beta'X_t + e_t \]  

Where \( i_t \) is the 5-year-ahead 5 year and 10 year interest rates (more easily visualized as \( E_Ti_{t+k} \) where \( k = 5 \) in our case) and \( X_t \) is a \( t \times l \), \( (l = 4) \) vector of explanatory variables \([d_t, y_t, \pi_t, e_p] \). Although all the explanatory variables are denoted as current period time series for ease of notation, they should be more appropriately interpreted as expectational terms in the same vein as \( i_t \) i.e. \( d_t \) denotes \( E_Td_{t+k} \), \( y_t \) denotes \( E_Ty_{t+k} \), \( \pi_t \) denotes \( E_T\pi_{t+k} \). As a proxy for expected debt, the series of CBO semi-annual 5-year ahead deficit projections constitute \( d_t \), the series of CBO projections of GDP growth rates is what constitutes \( y_t \), and the series of long term inflation expectations are captured by the variable \( \pi_t \). The imposition of the variable of \( e_p \) in the regression (and in the VAR) is due to a different restriction. It denotes a time varying measure of relative risk aversion for the economy as a whole, constructed (following Laubach (2009)) from dividend yield, defined as the dividend component of national income divided by the market value of corporate equities held (directly and indirectly) by households as reported in the Federal Reserve’s Flow of Funds data (Appendix A).

To write Eq. 4 as a threshold regression of the type in Eq. 2 and Eq. 3, while still keeping the single equation framework, we define the dummy variable \( d_t(\gamma) = \{q_t \leq \gamma\} \) where \( \{.\} \) is the indicator function and \( q \) can be any element of the vector \( X \) (in our case, its projected deficit \( d_t \)). Setting \( X_t(\gamma) = X_t d_t(\gamma) \), we now have a threshold variant of Eq. 4.

\[ i_t = \beta_1'X_t + \beta_2'X_t(\gamma) + e_t \]  

where \( \beta_1' \) is the equivalent of \( \theta_2' \) in Eq. 1-2. The regression parameters are then \((\beta_1', \beta_2', \gamma)\) and the natural estimator is the Least Squares (LS). Hansen’s asymptotic method then not only tests for the presence of threshold, but also estimates the confidence interval around the “true” threshold. Figure 3 and Figure 4 shows the F test for linearity in projected deficit in the regression framework specified above, and it shows that the null of no-threshold is rejected at 5% for 5-year ahead 10 year interest rate and at 10% for 5-year-ahead 5 year interest rate.
4.3.3 Results of Threshold Estimation

A common method to form confidence intervals for parameters is through the inversion of Wald or the t-statistics. Given that the sampling distribution of threshold parameter depends on unknown parameters since the threshold itself is not identified in some parameter space, the Wald statistic will have very poor finite sample behavior\textsuperscript{23}. Hansen (2000) showed that under such circumstances, precise confidence interval can indeed be constructed using the likelihood ratio statistic $LR_n(\gamma)$ where $\gamma$ is the threshold parameter.

Figure 5 and Figure 6 displays the graph of the normalized (scaled to account for heteroskedasticity\textsuperscript{24}) likelihood ratio sequence $LR^*_n(\gamma)$ as a function of the threshold in projected deficit. The LS estimate of $\gamma$ is the value that minimizes these graphs, which occurs at $\hat{\gamma} = 1.800$ for both the cases. Table 1 elaborates the result of the threshold estimation and confidence interval construction. The 95\% critical value is also plotted (the dotted line) so we can read off the asymptotic 95\% confidence set from the graph where the $LR^*_n(\gamma)$ crosses the dotted line. These results show that there is reasonable evidence of two regime specification.

The impact of projected deficit on 5-year-ahead 5 year and 5-year-ahead 10 year interest rates, based on the regression in Equation 4 is shown in Table 2. In the full sample, a percentage point increase in projected deficit GDP ratio increases 5 year-ahead 5 year interest rates by 16 basis points. This is close to what Laubach (2009) derives, which is typically a 22 basis point increase for a percentage point increase in projected deficit. Similarly, in the full sample, a percentage point increase in projected deficit GDP ratio increases 5-year-ahead 10 year interest rates by 18 basis points.

Once we fix $\gamma$ at the LS estimate of 1.8 and split the sample in two based on this value of projected deficit, our sample approximately splits in half. Running the same regression for the two subsamples divided on the basis of the threshold, we find some

\textsuperscript{23} See Dufour (1997).
\textsuperscript{24} See Hansen (2000) for details.
very interesting results. Table 3.1 and 3.2 present the results of the regression (Equation 5). When projected deficit is below the threshold value of $\gamma = 1.8$ (we call this Regime 1), the impact of projected deficit becomes insignificant in both cases. In Regime 2, when projected deficit takes value higher than the threshold, a percentage point increase in projected deficit significantly raises 5-year-ahead 5 year interest rates by 30 basis points and 5-year-ahead 10 year interest rates by 29 basis points. The magnitude of the effect is thus more than double of what we get when we use the full sample (i.e. when we use a model in which we do not specify the presence of a threshold).

In other words, in a regime of high expected deficit, the market’s reaction to increased future deficit will markedly differ from when expected deficit is below a certain threshold. The economy’s expectation of future long term interest rates under the former scenario (being greater in magnitude and statistically significant) is thus different than under the later. This important distinction contains a new and important piece of information for the policy process. It is that the impact of expected deficit on future interest rates, and thus national savings and GDP, might be small or ambiguous during a period of low and low expected deficit (where outlay plans plus net interest payments does not exceed planned tax revenues by a certain (threshold) amount), but is clear and significantly large in a period of high or high expected deficit (where government outlay including net interest payments exceed tax revenue over a threshold value). To investigate the nature of the impact on future interest rates of expected deficit, we also carry out a Threshold VAR, so that that we can introduce richer dynamics among the variables. The specification of the Threshold VAR and impulse responses are discussed in the next section.

### 4.4 Threshold VAR

Threshold VARs are piecewise linear models with different autoregressive matrices in each regime. The regimes are determined by a transition variable, which is either one of the endogenous variables or an exogenous variable (Hansen 1996, 1997, Tsay 1998). In general it is possible to obtain more than one critical threshold value and therefore
more than two regimes, but since Hansen (2000) test generates one threshold value for our data on deficit, we will work with only two regimes.

Let a set of \( k \) stationary endogenous variables with \( x_t = (x_{1t}, \ldots, x_{kt})' \) and \( T \) observations describe a VAR of finite order \( p \)

\[
x_t = \Gamma_0 + \Gamma_1 x_{t-1} + \cdots + \Gamma_p x_{t-p} + u_t
\]

where \( \Gamma_0 \) is a \( k \)-dimensional vector containing deterministic terms such as a constant, a linear time trend or dummy variables. \( \Gamma_i \) with \( i = 1, \ldots, p \) are squared coefficient matrices of order \( k \), and \( u_t \) is a sequence of serially uncorrelated random vectors with mean zero and covariance matrix \( \text{Cov}(u_t) = \Sigma u \). We can rewrite equation (6) in the compact form

\[
x_t = \Gamma X_t + u_t
\]

With \( \Gamma = (\Gamma_0, \Gamma_1, \ldots, \Gamma_p) \) and \( X_t = (1, x_{t-1}, \ldots, x_{t-p})' \). Following this notation, a threshold VAR is represented by

\[
x_t = \Gamma_1 X_t + \Gamma_2 X_t I[z_{t-d} \geq z^*] + u_t
\]

\( z_{t-d} \) is the threshold variable determining the prevailing regime of the system, with a possible lag \( d \). \( I[\cdot] \) is an indicator function that equals 1 if the threshold variable \( z_{t-d} \) is above the threshold value \( z^* \) and 0 otherwise. The coefficient matrices \( \Gamma_1 \) and \( \Gamma_2 \), as well as the contemporaneous error matrix \( u_t \) are allowed to vary across regimes.

Although conventional Threshold VARs, following Tsay’s (1998) influential paper on the topic, treat the delay lag \( d \) and critical threshold value \( z^* \) as unknown parameters to be estimated, we treat them differently. Given our small sample size and the multitude of parameters that a VAR would estimate (more so in a Threshold VAR), we treat our threshold value as known. Specifically, we use the threshold value of deficit that we estimated using the Hansen (2000) methodology. Not only is Hansen’s method more robust to small sample size, the asymptotic theory developed by Hansen aids in the construction of confidence interval around the estimated threshold value, which Tsay’s
test does not allow for. The choice of $d$ in Threshold VAR usually relies on economic reasoning. We estimate the VAR using semi-annual data with a known value of $z^* = \gamma = 1.8$ and restrict $d$ to be zero i.e. Eq. 8 can be rewritten as

$$x_t = \Gamma_1 X_t + \Gamma_2 X_t I[z_{t-d} \geq \gamma] + u_t \quad (9)$$

where $\gamma = 1.8$. Conventional non-linear specification of VAR often includes an additional variable to distinguish between the regimes of interest (example: some transformation of output gap to indicate “good” and “bad” times in fiscal VAR literature\textsuperscript{25}). An important distinction between our specifications from those used in the literature is that we do not introduce such additional variable, instead restricting one of the endogenous variable of the VAR (projected deficit) to be the threshold variable. Although introducing a measure of output gap as a threshold variable would be fairly straightforward, the nature of the question that we are asking makes such a measure problematic. Government budget deficit, both current and projected, is counter-cyclical and would generally have relatively higher values during recessions. If we use output gap as a threshold variable as is the practice in non-linear fiscal VAR literature, we would be creating an endogeneity problem. Also, when we ran the Hansen (2000) asymptotic test in the single equation framework of the type in Eq.5, we found no non-linear effect of expected output gap on future long term interest rates.

In our version of threshold VAR, $x_t = [Rr_t, d_t, y_t]$, where all the variables are those that were used in the single equation regression, except $Rr_t$, which is the real 5 year ahead 5 year interest rate, which is obtained by subtracting the expected long term inflation from the nominal forward rates i.e. $Rr_t = i_t − \pi_t$, where the expectational nature of the variables are implicit in the notation, as explained in the Section 3.2.

We couch our main results in the form of impulse-response functions (IRFs henceforth), estimated in the usual way. We compute 90% bootstrapped confidence

\textsuperscript{25} See Auerbach and Gorodnichenko 2012a, 2012b.
intervals based on 1000 replications, and focus our discussion on significant responses. We report the resulting structural impulse-response functions in Figure 7-10.

4.4.1 Results of Threshold VAR

Using a known threshold value of 1.8 for the projected deficit-GDP ratio, the Threshold VAR corroborates our findings from the single equation framework and provides a new insight. The impulse responses show that a Cholesky one standard deviation shock to projected deficit increases the real future long term interest rate by 12 basis points in a ‘high’ deficit regime (projected deficit-GDP ratio greater than 1.8). This increase is persistent for almost a year and half before reverting back to its mean. In the “low” deficit regime, the impact is a small 3 basis point drop in the real rate, which reverts back to its pre-innovation level in 6 months’ time.

The impact of an innovation to projected deficit on real rates indicates that the deficit-interest dynamics acts not only through the expected inflation channel, but also directly on the economy’s real rate of return. One way to interpret the results of our Threshold VAR is to consider an innovation to the projected deficit variable as a revision of market’s expectation of future deficit that is independent of the CBO’s forecast. The source of the shock can also be thought of as some information private to CBO which becomes public. In a regime where economic agents already expect deficit to be high in the future, any revision of such projections will likely have large and significant impact on future real interest rates (agents expect future interest rates to be high). This will not be the case when economic agents expect future deficit to be below a certain threshold value. The VAR structure, while imposing the minimum if theoretical restrictions on the dynamics, controls for the fact that some part of projected deficit will have to incorporate future interest payments and GDP growth rates. It also enables us to look at the impact of an innovation to expected deficit on future real rates, which a single equation set up does not.
4.4.2 Robustness Checks

To see how robust our results are to the nature of data as well as specifications of our single equation regression and Threshold VAR, we carry out our entire exercise using an alternative measure of equity premium and using additional data on government spending projections and projections error in our VAR. The results of such exercises are qualitatively and quantitatively similar to what we present in our main results, both in the single equation Threshold estimation model and in the VAR. We also test the sensitivity of our results in the single equation to the threshold value of deficit/GDP that is estimated. Specifically, we carry out our threshold regression and our VAR using values of threshold around the neighborhood of 1.8, and find our results robust to such specification. Our VAR impulse responses are also robust to different ordering of the variables in the Cholesky ordering as long as we order slower moving variables last26.

4.5 Conclusion

In this paper we document empirical evidence in favor of significant non-linearity in the deficit-interest rate relationship. Building on Laubach (2009) single equation framework, we first test for a threshold effect in the impact of deficit expectations on the long term forward rates. We find that there is indeed a significant threshold that can be used to explore the deficit-interest rate relationship in two regimes. In the “low-regime”, projected deficit-GDP ratio has a negligible impact on long term rates. However, this effect is positive and significant (both economically and statistically) in the “high-regime” (30 basis points for one percentage point increase in expected deficit/GDP ratio).

We then use our estimated threshold from this single-equation framework and estimate a threshold VAR model with a known threshold. An advantage of this framework is that it allows us to explore the dynamics of the impact of a structural shock to deficit expectations on long term rates and how this dynamics differs across the threshold level of the deficit expectations. A structural shock to deficit expectations can be interpreted as a revision to CBO (hence the market’s) forecast of future revenue and spending scenario (the timing of taxes or the path of government spending). The result of this paper

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26 Results are available on request
suggests that any such revision will have significant impact on future interest rates (hence future investment, consumption and savings) in a scenario where expectations of deficit are already high.

This evidence should be not only interesting to policymakers in designing debt stabilization strategies but it can also help reconcile conflicting predictions about the effects of deficit on interest rates across different types of macroeconomic models. A novel implication of our result is that it might be worthwhile for policymakers to incorporate management of deficit-expectations in their policymaking framework. For instance, it may be beneficial to announce the path of government spending and taxes with some regularity and make this announcement reasonably visible for agents in the economy. This will be especially effective in a scenario of high deficit expectations, where an upward revision (“bad news”) of deficit projections have the potential to increase future interest rates. In so far as high deficit and debt dampen the Fed’s monetary policy effectiveness, this might be a prudent course of action for fiscal policymakers. Finally, we believe that our empirical result can be used as a motivation for future theoretical work to develop realistic DSGE that account for the non-linearity between deficit and interest rate to better understand the forces driving differences in the impact of deficit expectations on future interest rates during different regimes, the exact dynamics and consequences of such impact and policy implications of managing expectations of deficit.
Appendix: Tables, Figures and Data Description

Data Description:

Expected Inflation: the series is spliced using data from four sources:

- January 1960 – September 1980: the inflation endpoint constructed by Kozicki and Tinsley (JME 2001);
- October 1980 – October 1991: linearly interpolated data from Hoey is used after subtracting 0.55 percentage point (annual rate) to transform the data from a CPI inflation basis to a PCE inflation basis;
- November 1991 – 2005: linearly interpolated SPF survey data is used after subtracting 0.55 percentage point (annual rate) to transform the data from a CPI inflation basis to a PCE inflation basis.

5 year-ahead 5 year interest rate:
Average of one-year forward rates 5-9 years ahead, calculated from the zero coupon yield curve, sampled on the last trading day of the month of the CBO release.

5 year-ahead 10 year interest rate:
Average of one-year forward rates 5-14 years ahead, calculated from the zero coupon yield curve, sampled on the last trading day of the month of the CBO release.

Equity Premium: Calculated as the dividend component of national income, expressed as percent of the market value of corporate equity held (directly or indirectly) by households
Source: Flow of Funds account in FRED database.
Figure 11: Interest Rates and Long Horizon Inflation Expectations

Figure 12: Projected Deficit and Projected GDP Growth Rates (year of projection)
Figure 13: 90% F Test for presence of Threshold effect of expected deficit on 5-year-ahead 5 year interest rates.

Note: Linearity is rejected if F sequence becomes greater than or equal to the 90% critical value. For 5-year-ahead 5 year interest rates, the bootstrapped P-value for F sequence greater than 95% critical is 0.052; hence the line just about touches the 95% critical, but crosses the 90% critical line.
Figure 14: 95% F Test for presence of Threshold effect of expected deficit on 5-year-ahead 10 year interest rates:

*Note:* Linearity is rejected if F sequence becomes greater than or equal to the 95% critical value.
Figure 15: Confidence interval construction for threshold in Deficit/GDP ratio while regressing on 5-year-ahead 5 year interest rate.
Figure 16: Confidence interval construction for threshold in Deficit/GDP ratio while regressing on 5-year-ahead 10 year interest rate.
Figure 17: Response of 5-year-ahead 5 year Interest Rate to One S.D. Innovation to Projected Deficit-GDP ratio in Regime 1 ("regime of low expected deficit")

Figure 18: Response of 5-year-ahead 5 year Interest Rate to One S.D. Innovation to Projected Deficit-GDP ratio in Regime 2 ("regime of high expected deficit")
Figure 19: Response of 5-year-ahead 10 year Interest Rate to One S.D. Innovation to Projected Deficit-GDP ratio in Regime 1 ("regime of low expected deficit")

Figure 20: Response of 5-year-ahead 10 year Interest Rate to One S.D. Innovation to Projected Deficit-GDP ratio in Regime 2 ("regime of high expected deficit")
Table 6: Threshold Estimation in regression using 5-year-ahead 5 year interest rate and 5-year-ahead 10 year interest rate as the dependent variable

<table>
<thead>
<tr>
<th>Threshold Variable :</th>
<th>$d_t$</th>
<th>$d_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable :</td>
<td>5-year-ahead 5 year interest rate</td>
<td>5-year-ahead 10 year interest rate</td>
</tr>
<tr>
<td>Threshold Estimate:</td>
<td>1.800</td>
<td>1.800</td>
</tr>
<tr>
<td>0.95 Confidence Interval:</td>
<td>[-0.319 , 2.691]</td>
<td>[-0.319 , 2.691]</td>
</tr>
<tr>
<td>Sum of Squared Errors:</td>
<td>20.670</td>
<td>18.45</td>
</tr>
<tr>
<td>Residual Variance:</td>
<td>0.439</td>
<td>0.392</td>
</tr>
<tr>
<td>Joint R-squared:</td>
<td>0.885</td>
<td>0.887</td>
</tr>
<tr>
<td>Heteroskedasticity Test (P-Value)</td>
<td>0.657</td>
<td>0.704</td>
</tr>
</tbody>
</table>

Note: $d_t$ is projected deficit-GDP ratio.

Table 7: Regression of future interest rate on projected deficit and other regressors: Full Sample

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>3.226 (0.869)</td>
<td>3.339 (0.888)</td>
</tr>
<tr>
<td>$\pi_t$ (projected inflation)</td>
<td>1.619 (0.124)</td>
<td>1.521 (0.116)</td>
</tr>
<tr>
<td>$\gamma_t$ (projected GDP growth rate)</td>
<td>-0.592 (0.331)</td>
<td>-0.456 (0.331)</td>
</tr>
<tr>
<td>$d_t$ (projected deficit-GDP ratio)</td>
<td><strong>0.163 (0.052)</strong></td>
<td><strong>0.181 (0.051)</strong></td>
</tr>
<tr>
<td>Sum of Squared Errors:</td>
<td>28.915</td>
<td>26.752</td>
</tr>
<tr>
<td>Residual Variance</td>
<td>0.566</td>
<td>0.525</td>
</tr>
<tr>
<td>R-squared:</td>
<td>0.83</td>
<td>0.84</td>
</tr>
<tr>
<td>Heteroskedasticity Test (P-Value)</td>
<td>0.035</td>
<td>0.008</td>
</tr>
</tbody>
</table>
Table 8: Regression of future interest rates on projected deficit and other regressors: Split Sample (Regime 1)

<table>
<thead>
<tr>
<th>Regime 1:</th>
<th>( d_t \leq 1.8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable</td>
<td>5-year-ahead 5-year-ahead</td>
</tr>
<tr>
<td></td>
<td>5 year interest rate 10 year interest rate</td>
</tr>
<tr>
<td>Parameter Estimates</td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>Estimate</td>
</tr>
<tr>
<td>Constant</td>
<td>3.56 (0.89)</td>
</tr>
<tr>
<td>( \pi_t ) (projected inflation)</td>
<td>1.35(0.12)</td>
</tr>
<tr>
<td>( y_t ) (projected GDP growth rate)</td>
<td>-0.49(0.31)</td>
</tr>
<tr>
<td>( d_t ) (projected deficit-GDP ratio)</td>
<td>-0.18(0.21)</td>
</tr>
<tr>
<td>Observations:</td>
<td>30</td>
</tr>
<tr>
<td>Sum of Squared Errors:</td>
<td>11.37</td>
</tr>
<tr>
<td>Residual Variance</td>
<td>0.45</td>
</tr>
<tr>
<td>R-squared:</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 9: Regression of future interest rates on projected deficit and other regressors: Split Sample (Regime 2)

<table>
<thead>
<tr>
<th>Regime 2:</th>
<th>( d_t &gt; 1.8 )</th>
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</thead>
<tbody>
<tr>
<td>Dependent Variable</td>
<td>5-year-ahead 5-year-ahead</td>
</tr>
<tr>
<td></td>
<td>5 year interest rate 10 year interest rate</td>
</tr>
<tr>
<td>Parameter Estimates</td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>Estimate</td>
</tr>
<tr>
<td>Constant</td>
<td>3.05(0.92)</td>
</tr>
<tr>
<td>( \pi_t ) (projected inflation)</td>
<td>1.92(0.15)</td>
</tr>
<tr>
<td>( y_t ) (projected GDP growth rate)</td>
<td>-0.95(0.39)</td>
</tr>
<tr>
<td>( d_t ) (projected deficit-GDP ratio)</td>
<td>0.30(0.13)</td>
</tr>
<tr>
<td>Observations:</td>
<td>25</td>
</tr>
<tr>
<td>Sum of Squared Errors:</td>
<td>8.12</td>
</tr>
<tr>
<td>Residual Variance</td>
<td>0.38</td>
</tr>
<tr>
<td>R-squared:</td>
<td>0.90</td>
</tr>
</tbody>
</table>

*Note: Boot-strapped standard errors in parenthesis*
Table 10: Summary Statistics of Projected Deficit/GDP Ratio

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Mean</th>
<th>Median</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projected Deficit GDP Ratio</td>
<td>1.184733</td>
<td>1.719861</td>
<td>5.288041</td>
<td>-3.802997</td>
<td>1.931046</td>
<td>-0.645207</td>
<td>3.086528</td>
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<tr>
<td>Probability</td>
<td>0.147109</td>
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</tbody>
</table>

Table 11: Descriptive Statistics of Projected GDP/GNP Growth Rates

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Mean</th>
<th>Median</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projected GDP/GNP Growth Rates</td>
<td>2.621818</td>
<td>2.500000</td>
<td>3.500000</td>
<td>2.100000</td>
<td>0.367015</td>
<td>0.674924</td>
<td>2.500700</td>
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<tr>
<td>Probability</td>
<td>0.093157</td>
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<td></td>
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</tr>
</tbody>
</table>
REFERENCES


