Synchrophasor-Only Dynamic State Estimation & Data Conditioning

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Dissertation submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering

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May 3, 2013
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Keywords: Synchrophasor, State Estimation, Power System

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A phasor-only estimator carries with it intrinsic improvements over its SCADA analogue with respect to performance and reliability. However, insuring the quality of the data stream which leaves the linear estimator is crucial to establishing it as the front end of an EMS system and network applications which employ synchrophasor data. This can be accomplished using a two-fold solution: the pre-processing of phasor data before it arrives at the linear estimator and the by developing a synchrophasor-only dynamic state estimator as a mechanism for bad data detection and identification. In order to realize these algorithms, this dissertation develops a computationally simple model of the dynamics of the power system which fits neatly into the existing linear state estimation formulation. The algorithms are then tested on field data from PMUs installed on the Dominion Virginia Power EHV network.
Acknowledgments

When I look back over the years since leaving home I can’t help but see the countless times where my friends and mentors have made a significant impact not only on the work I have pursued but the person that I have become. Undoubtedly, without their guidance and influence, I would be on a much different path in my life. The same has also been true about my pursuit of the Ph. D. degree at Virginia Tech. There are many people which have played tremendous roles in my graduate school experience and while I can’t possibly mention them all, there are a few that I would like to thank specifically.

I wish to thank all of the professors in the Center for Power & Energy for creating such a positive learning environment for myself and the rest of my fellow graduate students. I want to thank Dr. Jaime De La Ree for encouraging me to pursue a Ph. D. without which almost every aspect of my life would certainly now be different. I would also like to thank the Bradley Foundation for naming me a Harry Lynde Bradley Fellow and providing me with three years of financial support to pursue my education in my own way. I want to thank Dr. Matthew Gardner for being a great friend and an even better mentor. I wish to acknowledge my dear friends Dr. Santosh Veda and Kyle Thomas for their companionship and positive influence. And, of course, many thanks to my loving wife, Tania, for sharing the last two years and the rest of our lives together.

Finally, I wish to extend my sincere gratitude to my advisor, Dr. James Thorp. I was told before coming to graduate school that the people that you work with will make or break your graduate school experience and Dr. Thorp has proven to me that, without a doubt, this is the truth. Dr. Thorp treated me with patience and respect far beyond my merit, which is truly a testament to what a wonderful human being he is. Because of him, I have left graduate school with much more than an education but rather a sense of leadership and a lifelong commitment to learning.
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Chapter 1

Introduction & History

At the heart of every Energy Management System (EMS) are the algorithms which process the raw power system data to provide an optimal picture of the state of the system. Today, non-linear state estimation techniques driven by SCADA measurements are used to paint such a picture. Built upon this are the many network applications of the EMS such as real-time contingency analysis, reliability studies, and system alarming. With continued proliferation of Phasor Measurement Unit (PMU) technology in the electric utility industry, the transition of state estimation from a traditional non-linear formulation to one which is purely phasor based is imminent. A soon to be completed project which involved a large scale deployment of PMUs across Dominion Virginia Power’s EHV network also aimed to develop an estimator of this type[1]. The completed installation of these PMUs will see the first three-phase linear tracking state estimator accompanied by several complimentary network applications. Work related to this project is presented in [2][3][4]. This project promises to be very exciting as these applications come online. It is expected that things which have not yet been seen in the power system will soon become visible. This dissertation investigates methods for creating a high confidence portrait of the system state using only synchrophasor data by building on this previous development and implementation.
1.1 Motivation & Objective

In the context of an EMS, the role of state estimation, regardless of whether it is driven by SCADA data or phasor data, is one of providing the array of network applications which make up the EMS with the best possible picture of the power system. This notion is critical to understanding the significance of a PMU only state estimator as it is widely agreed upon that synchrophasors will allow for the painting of much higher quality pictures. It is this high-speed, high-confidence portrait that will allow for the EMS to grow and improve as the needs of managing the power system grow evermore complex. In order to create the optimal high-confidence picture of the grid, improvements to increase the quality and reliability of the output of the estimator should be made. A phasor-only estimator carries with it intrinsic improvements in this respect, however, insuring the quality of the data stream which feeds the linear estimator is crucial to establishing it as the front end of an EMS system. This is accomplished using a two-fold solution: the pre-processing of phasor data before it arrives at the linear estimator and the addition of a mechanism for bad data detection and identification by developing a synchrophasor-only dynamic state estimator.

To begin with the latter, while the static PMU-only estimator is intrinsically an improvement from traditional estimators for many reasons [5], one simplification of the estimator is that it assumes each new set of measurements and corresponding state estimates are a new problem relative to the measurement sets and estimates adjacent to it in time. This simplification may be satisfactory for traditional estimators where the window of time between samples is greater than a few seconds, but with a sampling rate of 30 frames per second, this assumption no longer makes sense. As a side effect of this simplification, the synchrophasor-only estimator lacks a quality mechanism for bad data detection. Additionally, if there is knowledge of previous states, it is logical that this information could be used to help determine even better estimates [6]. Previous treatments of dynamic state estimation that have been applied to traditional power system state estimators have typically had a state transition matrix equal to identity [7]. This simplification is taken due to the complexity of the true dynamic model of the power system accompanied by the fact that the system is too large to observe all of the state variables. Despite the many formulations and applications of traditional non-linear dynamic state estimators, the term *dynamic* is a bit of a mis-nomenclature. It has been called *dynamic* because it includes information about previous states of the power system. However, traditional state estimators do not solve fast
enough to see the true dynamics of the power system. Although, with frame rates of 30 fps (and higher if desired), a synchrophasor-only state estimator should be able to see the dynamics of the power system and the assumption of a state transition of identity is no longer valid. A synchrophasor-only dynamic state estimator, given a suitable model of the power system dynamics, would provide not only the optimal estimate of the state of the system based on past history but would also provide a mechanism for bad data detection because of the ability to predict future states.

However, the dynamic linear estimator is still dependent on a consistent, reliable stream of phasor data. Due to the streaming nature of the data, downstream applications which leverage the data are vulnerable to network congestion, configuration errors, and loss of GPS synchronization, among other issues. Again, with a relationship between past and present data points, development of data conditioning algorithms using proven smoothing techniques [6] becomes a viable solution. This then enables preprocessing of synchrophasor data before it ever reaches the linear estimator, mitigating drop-outs, outliers, and other data quality issues that decrease the value of the data at a downstream location.

Therefore, to accomplish the evolution of a synchrophasor-only estimator into a proper front end for a full suite of EMS applications, a computationally simple and clean model of these dynamics which fits neatly into the existing formulation is then a prerequisite. It is then an underlying goal of this work to establish a model as a suitable representation of the dynamics of the power system and demonstrate its efficacy in data conditioning algorithms and in a synchrophasor-only dynamic state estimator. Because there are already enough PMUs installed on Dominion Virginia Power’s 500kV network to run a sufficiently interesting state estimator, many of the algorithms in this dissertation will have the advantage of being demonstrated on real synchrophasor data.

1.2 Enabling Technologies

This dissertation presents an application for power system operation and control which employs the exclusive use of synchrophasor data to feed a truly dynamic linear state estimator. Discussed in this section are several of the enabling technologies which, viewed with a wide angle lens, make an application of this type possible. At the top of the list are the GPS synchronized metering devices called PMUs which provide (amongst a great many things) a
data set that allows for a linear formulation of the power system state equation. Included in the discussion is the infrastructure required for an implementation such as this because it is quite different than current EMS technologies. Also included in this section is a discussion of the history of power system state estimation and filtering algorithms as they apply to dynamic state estimation.

1.2.1 PMUs & Synchrophasor Data

The technology which sets the topic of this dissertation apart from similar constructs is the phasor measurement unit (PMU). This is mostly because of three key characteristics of synchrophasor data: it’s fast, it’s time synchronized, and it can directly measure complex voltages and currents. The first PMU was developed at Virginia Polytechnic Institute & State University in the 1980s [8] and spawned innovation in research organizations, electric utilities, and many other entities across the industry. Applications which use only synchrophasor data will also rely on an infrastructure which is different than that of traditional EMS systems. PMUs installed in EHV substations will collect high resolution, time synchronized measurements which will be aggregated by one or more substation PDCs (Phasor Data Concentrators). These PDCs will communicate over a fiber-optic network with the control center where a super PDC will collect and time align the data in preparation for use in applications such as state estimation. In the case of the project in [2] the infrastructure will resemble that in Fig. 1.1.

![Figure 1.1: High Level Infrastructure](image)
Because the work in this dissertation is a continuation of work completed as part of the [1] many of the algorithms presented in this dissertation have the benefit of being demonstrated on actual synchrophasor data. However, before the data can be used it must be validated for quality. This is to establish the data set used herein as a satisfactory control for all testing and experimentation. All of the synchrophasor data that has been used in this dissertation has been validated for quality using a Matlab script based on several criteria.

**Time Stamp** - The time stamps are checked for bad, missing, or repeated time stamps.

**Status Word** - The status word of the C37.118 frame provides data quality information about the measurements contained in that frame. Each bit in the status word represents a different metric. For example, a bit in the status word indicates if time synchronization has been lost by the PMU which would effectively tag all of the data as bad data. There are also user configurable bits which can be used to communicate user defined metrics.

**Frequency** - The average frequency must be near 60Hz with frequency excursions of 0.1Hz as an indicator of a potential problem.

**Rate of Change of Frequency** - The average rate of change of frequency must be zero with excursions not beyond 0.03 in either direction.

**Voltage Magnitude** - The voltage magnitude is checked using several criteria: low signal or polarity issues, noisy data, repeated values, and outliers.

**Current Magnitude** - The current magnitude is checked similarly to the voltage magnitude.

**Voltage & Current** - The voltage and current angles are checked to make sure there are no sudden jumps in angle, that the angle is changing smoothly and predictably, and that there are no repeated values.

**Phase Sequence** - Complex voltage and current are plotted in the complex plane to check for correct phase sequencing.
1.2.2 State Estimation

Power system state estimation as it is presently implemented was first proposed by Schweppe [9, 10, 11] in 1970. Present state estimation techniques employ a measurement set which consists of voltage magnitudes, real & reactive power injections, current injections, and real & and reactive power flows, yielding a non-linear, iterative solution based on the Newton-Raphson power flow equations. Since its inception, power system state estimation has evolved to be the cornerstone of modern operation & control applications. And with its roots firmly planted in both power systems and statistics, there has been much research involved in bad data detection & identification, robustness techniques, and many other formulations of the static state estimator. However, the most widely used estimator for power system applications is the well-known weighted-least-squares estimator [12].

PMUs have brought an entirely new dimension to power system state estimation because of several innate characteristics. These include, but are not limited to, the ability of the PMU to directly measure the system state, the high time precision achieved by GPS synchronization, and given the previous item, the ability to compare two measurements in time which are separated by large distances (Wide Area Measurement). The inclusion of PMU measurements into traditional state estimators for increased performance [13][14] has been investigated. Other research topics in the area of state estimation using phasor measurements include some problems of observability and optimum placement[15] and bad data detection [16]. Although, because of the small number and wide variety of PMU installations across the country, it is believed that there is still much to be learned about how ‘bad’ synchrophasor data behaves.

A true application of synchrophasor technology to state estimation would be one in which the measurement set is comprised purely of synchrophasor data [17]. This formulation would be linear and therefore never chance divergence. It would be a fast calculation which could take advantage of the high time resolution and precision of PMU devices. A three-phase linear tracking state estimator is presented in [2][17]. While there are many advantages to an estimator such as this (some of which are yet unknown) it is a bit of a mis-nomenclature. While it can get away with being called a tracking estimator because of its speed, it considers each new frame as a separate problem. Without a relationship between adjacent frames there is not a good mechanism for bad data detection at these speeds. The next state needs to be predicted and compared to the new measurements.
Tracking and dynamic state estimators, unlike the aforementioned linear estimator and the traditional non-linear static state estimator, use previous knowledge of the system state along with new measurements to estimate and then correct the estimate of the state. This is entirely based on the ideas developed by Gauss with recursive versions of Gauss’ ideas developed by Kalman [6]. Estimators like these for electric power systems were proposed by Debs and Larson in 1970 [18]. Debs and Larson and Nishiya et. al. [19] have presented a formulation in which the state transition matrix is identity and the control input function is zero under the assumption of the quasi-steady state behavior of the power system. Since then, a wide variety of mathematical models for predicting the next state of the power system have been studied and presented including those which employ artificial neural networks (ANN) [20], regression analysis [21], and fuzzy logic techniques [22]. Applications of these types of dynamic power system state estimation include state and load forecasting [21], anomaly detection and identification [19], and improvements to operators security analysis [7].

It should be noted that there are practical issues with dynamic state estimators applied to power systems. First, no power system state estimator can possibly include every measurement in the system because the system is just too large. Therefore, no formulation of the dynamic state estimator would be entirely complete and a deterministic model for the state transition matrix cannot be realized. The models grow exceedingly complex which is why many authors have made major simplifications to the dynamic model used in the estimator, particularly the assumption of the quasi-steady state behavior of the power system.

A simple dynamic model is needed which can be applied to the pure synchrophasor state estimation problem. This dissertation proposes a simple model for predicting the next state from previous knowledge of the states of the pure synchrophasor estimation problem and presents a dynamic state estimator which uses a filtering algorithms with a linear state transition matrix and linear system model. Bad data detection and identification is addressed with basic statistical methods such as the chi-squared and largest normalized residual tests. Data conditioning algorithms are also developed using the same model and are demonstrated on real field data.
1.3 Organization of the Dissertation

This dissertation is organized in the following way:

Chapter 1: Introduction & History - This chapter begins by first discussing the enabling technologies involved with the proposed application in this dissertation. A case is made for an high-confidence picture of the power system to drive EMS applications by making improvements on a phasor based linear tracking state estimator and an objective for the work pertaining to this dissertation is presented. The chapter concludes by outlining the contents of each chapter.

Chapter 2: The Quadratic Model - Presented in this chapter is the observation that complex voltages in a power system follow a quadratic trajectory. Several examples illustrate the efficacy of the quadratic prediction model on simulated power system data. Several examples are shown on field-collected phasor data. Higher order prediction models are acknowledged as well.

Chapter 3: Synchrophasor Data Conditioning - The quadratic model is discussed and demonstrated in the context of a Kalman filter. The quadratic model is demonstrated for use in smoothing techniques for the conditioning of synchrophasor data. A data conditioning algorithm is defined and tested on real synchrophasor data.

Chapter 4: Synchrophasor-Only Dynamic State Estimation - A review of phasor based linear tracking state estimation is presented. This is followed by the development of the equations for applying the previously developed quadratic model to a phasor based dynamic state estimator. Results of synchrophasor-only dynamic state estimator applied to real synchrophasor data are presented. Bad data detection is addressed as one of the benefits of this estimator’s formulation. The chi-squared and largest normalized residual test is demonstrated on the dynamic estimator formulation and simulation results are presented.

Chapter 5: Summary & Future Work - The contributions of the dissertation are summarized and a practical vision for a complementary EMS system which is driven by synchrophasor data is presented.
Chapter 2

The Quadratic Model

If enough information is known about the system then the transition from one state to another can be included in the problem to obtain an optimal estimate for a single signal or for an entire power system network. The trouble is the mathematical models that currently exist, particularly in the area of dynamic state estimation, are very complex and computationally expensive. This is one of the central reasons for many of the simplifications to the assumed relationship between adjacent states in traditional dynamic estimators. This is also partly due to the large time scale of traditional estimators. However, the situation is different for synchrophasors.

One of the key goals of this dissertation is to develop a computationally simple model to serve as a relationship between two adjacent states and/or state variables of the power system. This chapter develops that model by first making the observation that the complex voltage at any node in the power system follows a quadratic trajectory in the complex plane for changes in load with a constant power factor. Determining the inverse of a certain Vandermonde matrix for a 2\textsuperscript{nd} order polynomial yields a set of coefficients which can interpolate (or predict) the next data point from three previous data points [23]. This prediction polynomial is quadratic in nature and can be used to predict future complex voltages from previous complex voltages in a power system. This simple relationship is experimentally verified using simulated data from a load flow and several dynamic simulations. Higher order prediction models are also acknowledged.
2.1 Observation of the Complex Voltage Trajectory

As previously mentioned, the first step in developing the simple relationship between past and future states in the power system is to first make the observation that the complex voltage trajectory is quadratic for changes in load at a constant power factor. At first, this may seem somewhat binding as the power factor changes often during the various conditions the power system experiences throughout a single day. However, at the speed that synchrophasors are reported (typically 30 frames per second) it is perfectly reasonable to assume that adjacent states have the same power factor. Later it will be seen that the quadratic nature of the complex voltages even exists during complex phenomena such as loss of excitation conditions. Again this is valid because of the small timescale of the synchrophasor data. To begin, first consider the scenario where the load is increased at a constant power factor.

\[ S_D = V_2 I^* = (x + jy) \left( \frac{1 - x + jy}{-j\chi} \right) = (1 + j\beta)t \]  

(2.1)

Isolating the real part of the load voltage, \( x \), yields the following equation.

\[ x^2 - x + \chi^2 t^2 + \chi \beta t = 0 \]  

(2.2)

Then, the imaginary part of the load voltage is \( y = -\chi t \) and the load voltage, \( x + jy \), is approximately quadratic in \( t \).

\[ x = \frac{1 \pm \sqrt{1 - 4\chi^2 t^2 - 4\chi \beta t}}{2} \]  

(2.3)

If desired, better accuracy could be achieved by writing a Taylor series expansion. Using the knowledge that the complex voltage follows a quadratic trajectory in the complex plane.

Figure 2.1: Load at Constant Power Factor
under the above stated conditions, $x(t)$, the time series function of the complex voltage, can be written as a 2nd order polynomial sampled at time $t$.

$$x(t) = \alpha_2 t^2 + \alpha_1 t + \alpha_0$$

(2.4)

However, it would be preferred to have an equation which relates $x$ at time $t$ with previously known values of $x$. Because $x$ is quadratic, three previous data points spread equally in time would be required.

$$-\beta_t \hat{x}(t) = \beta_{t-1} x(t - 1) + \beta_{t-2} x(t - 2) + \beta_{t-3} x(t - 3)$$

(2.5)

The first row of the inverse of a particular Vandermonde matrix [23],

$$[\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
 x(t)
 x(t - 1)
 \vdots
 x(1)
 x(0)
 \end{array}
\end{array}
\end{array}
\end{array}
\end{array}
= \begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
1
 t
 t^2
 \vdots
 1
 2
 2^2
 \vdots
 1
 1
 1
 1
 \end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\alpha_0
 \alpha_1
 \vdots
 \alpha_{t-1}
 \alpha_t
 \end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
= V \alpha$$

(2.6)

yields the desired coefficients in Eq. 2.5.

$$\beta^T = \begin{bmatrix} \beta_t & \beta_{t-1} & \ldots & \beta_1 & \beta_0 \end{bmatrix}$$

(2.7)

Because $x$ is quadratic, $t = 3$ and the coefficients of the quadratic prediction polynomial are $\beta^T = \begin{bmatrix} -1 & 3 & -3 & 1 \end{bmatrix}$ and therefore,

$$x(t) = 3x(t - 1) - 3x(t - 2) + x(t - 3)$$

(2.8)

Likewise, if $t = 6$, a quintic polynomial prediction, the coefficients of the quintic polynomial will then become $\beta^T = \begin{bmatrix} -1 & 6 & -15 & 20 & -15 & 6 & 1 \end{bmatrix}$ and therefore,

$$x(t) = 6x(t - 1) - 15x(t - 2) + 20x(t - 3) - 15x(t - 4) + 6x(t - 5) - x(t - 6)$$

(2.9)

From the above pattern it can be observed that the absolute value of the coefficients, as the order of the polynomial increases, are the same as the elements of Pascal’s Triangle as the rows progress downward.
In summary, based on the knowledge of the quadratic nature of the complex voltage trajectory under constant power factor and the coefficients of a 2\textsuperscript{nd} order prediction polynomial obtained from the inverse of a particular Vandermonde matrix, an equation which relates the previous three voltage states with a future state is given. Considering that $x$ is only approximately quadratic in $t$ an error term should be included in the equation when knowingly dealing with complex voltages.

$$x_{n+1} = 3x_n - 3x_{n-1} + x_{n-2} + v_n$$  \hspace{1cm} (2.10)$$

In the following section, this quadratic prediction equation will be demonstrated on simulated power system data (data which is assumed to be the true value of the state of the system).

### 2.2 Quadratic Prediction on Simulated Data

This section presents a look at the quadratic polynomial of Eq. 2.10 applied to simulated scenarios in a power system. The aim is to demonstrate the efficacy of the quadratic prediction on the true values of complex voltages during different operating conditions and contingencies. There were no random electromagnetic or electromechanical fluctuations added to the complex voltages in question; there was also no noise added to the values. Therefore, the difference between the given value and the predicted value can serve as a metric for the performance of the quadratic prediction polynomial. The following figure is the first example of the prediction polynomial applied to power system data.
Fig. 2.3 shows the complex voltage trajectory of a node in a 9 bus power system during a morning load increase. The load was increased by 60% over a 1 hour period. The red circles represent the true value of the complex voltage while the blue stars represent the predicted values from the quadratic prediction. Visual inspection yields observation of a remarkable ability of the quadratic prediction to accurately and consistently predict the change in the complex voltage from one state to another. It goes without saying that this is a simple scenario as it is considered quasi-steady-state. A better test would be one which employs cases with a transient event where the dynamics of the power system are visible.
Figure 2.4: Bus 9 in IEEE 118 Bus System After a 450 MW Generator Loss at Bus 10
The next five plots, contained in Fig. 2.4 & 2.5 correspond to a 450 MW generator outage in the IEEE 118 bus system. Fig. 2.4 shows the top plot, the complex voltage trajectory during the contingency. Again the blue stars represent the prediction and the red circles represent the true (simulated) value. Similarly, the middle plot shows the magnitude of the same complex voltage and lower plot the angle.

Figure 2.5: Difference Including Contingency

The upper plot of Fig. 2.5 shows the difference between the simulated and predicted value for the duration of the simulation including the time of the contingency. Note the spike in the value of the difference at the time of the contingency marked with the yellow box. Of course, the quadratic prediction cannot predict an instantaneous change in voltage. The
lower plot shows the same difference values as the upper plot, however, only the time after the contingency is shown in order to better see the values without the large spike at the time of the contingency. The plot has also been zoomed in.

It should be noted in Fig. 2.5 that the particular shape of the plot comes from the numerical precision of the simulation software which created the simulated data. When changes in voltages between adjacent time steps are sufficiently small the signal will not appear perfectly smooth in some places. A good analogy is like trying to draw a circle using pixels on a computer screen. The edge of the circle is not smooth to the naked eye until the resolution of the screen is high enough to make the incremental changes sufficiently small. Therefore, the larger values that show up in the difference between the simulated and prediction values after the contingency are not the true effectiveness of the prediction polynomial. In actuality, the error is closer to being on the order of $10^{-7}$ rather than $10^{-5}$.

Similarly the next five plots, shown in Fig. 2.6 & 2.7 correspond to a 277 MW load loss at bus 54 in the IEEE 118 bus network. The upper plot in Fig. 2.6 shows the complex voltage of bus 38 during the contingency and again, the blue stars are the predicted quantity and the red circles represent the true values. The middle plot shows the magnitude of the complex voltage and the lower plot the angle.

The top plot in Fig. 2.7 shows the difference between the simulated value and the predicted value for the duration of the simulation with the time of the contingency marked by the yellow box. And finally, the bottom plot shows the difference for the time after the contingency for a clearer plot. The numerical precision of the simulation software is more apparent from this example if the plot of the voltage magnitude is inspected. Again, the true error appears to be on the order of $10^{-7}$ and even lower during the pre-contingency state.

The next two plots correspond to complex bus voltages of two large generators in the WECC during a loss of excitation contingency. Note that these are simulated examples. It should be clear from visual inspection of the plots that the quadratic polynomial is able to predict the trajectory of the complex voltage even during something horrendous such as this loss of excitation condition.
Figure 2.6: Bus 38 in IEEE 118 Bus System After a 277 MW Load Loss at Bus 54
To further investigate the effectiveness of the quadratic prediction model on the loss of excitation scenario consider Fig. 2.9 & 2.10. These two figures show the observation residual of the raw quadratic prediction similar to Fig. 2.5 & 2.7. In both of the figures there are three plots of the same information at different levels of zooming. Again, the numerical precision of the simulation can be observed by the scale of the observation residuals of $10^{-4}$. However, upon closer examination, the true error of the prediction in both cases is again on the order of $10^{-7}$.
Figure 2.8: 500kV Bus Voltage on High Side of GSU of 2400 MW Generators in WECC
Figure 2.9: 500kV Bus Voltage on High Side of GSU of 2400 MW Generators in WECC
Figure 2.10: 500kV Bus Voltage on High Side of GSU of 2400 MW Generators in WECC
This section has demonstrated the predictive capability of the quadratic polynomial on various scenarios including a morning load pickup (quasi-steady-state), a loss of a 450 MW generator on the IEEE 118 Bus System, the loss of a 277 MW load on the IEEE 118 Bus System, and a loss of excitation on two different 2400 MW generators in the WECC. It has been seen that the predictive capability of the quadratic polynomial is strong during many different scenarios and contingencies. In later chapters, the quadratic polynomial will be adapted so that it can serve real synchrophasor data as a tool for filtering and smoothing techniques which will ultimately yield a synchrophasor-only dynamic state estimator and data conditioning algorithms.

### 2.3 Varying Time Between Samples

The frame rate of the synchrophasor measurements in [2][17] is 30 frames per second. At this speed, the assumptions made in Section 2.1, of course, remain valid. If the frame rate of the measurements was increased to 60 frames per second or perhaps 120 frames per second the same would be true still. Additionally, if the frame rates that are supported are lower than 30, potentially 15, 10, 5, or maybe even 1 frame per second the assumption may still also hold true with an obvious practical limit as the frame rate slows. This limit may be different for different operating conditions of the power system. This section discusses the idea that the frame rate need not be fixed at 30 frames per second and that there are certain situations which may take advantage of this. With an on-line application, the rate of the filtering algorithm would most likely be the same as the reporting frame rate of the PMUs. However, there may exist a scenario where it may be beneficial to expand the amount of time between samples, even in the presence of higher resolution data.

![Figure 2.11: Illustration of State Vector Predicting the Next State](image)

Fig. 2.11 shows the prediction when the resolution of the state vector matches the
reporting frame rate of the PMU. In Fig. 2.12, the measurement vector contains three values but they are separated by the minimum amount of time (the reporting frame rate of the PMU). If there is interest in the motion of an object in two dimensions (the complex plane in the case of voltages) it is reasonable to assume that a better picture of the objects motion can be attained by taking samples which are spread farther apart in time. Then, the inferred motion of the object is less sensitive to small perturbations in the measurements of position. Imagine a scenario where the measurement vector of the quadratic model still contains three values yet the snapshots of the system come from every other sample instead of adjacent samples.

![Figure 2.12: State Vector with \( \frac{1}{2} \) of the Resolution of the Reporting Frame Rate](image)

In Fig. 2.13 as in Fig. 2.12, the light blue boxes represent spaces in time relative to the reporting frame rate which *have not been used* in the state vector or measurement vector. The dark blue boxes represent the spaces in time relative to the reporting frame rate with *are used* in the state vector and measurement vector. When the snapshots of the system are separated by more than the time between adjacent samples the measurement window covers a longer time history of the motion of the object. Because the object can move more between each snapshot of the system it should be less sensitive to perturbations movement of the object. However, because the size of the state vector and measurement vector do not change in size, there is no increase in computational burden for a formulation such as this. Additionally, there is no difference in the filtering algorithm. It can likewise be imagined that the time between snapshots of the system can be increased further such that there may be many data points between system snapshots which are ignored. This is demonstrated in Fig. 2.13.
Scenarios where decreasing the resolution of the data used in the algorithm include but are not limited to:

- When a slower frame rate or down-sampling is desired or is a design constraint.
- When a data point has been flagged as ‘bad’ and it is desired to still provide a full prediction for the proceeding state.
- If one or all of a set of data is lost for a particular time \( k \) and it is still desired to provide a prediction for the proceeding state.

Because this is independent of the algorithms discussed herein and is only dependent on the selection of the elements of the measurement vector then using a technique such as this for skipping over bad data is excellent as the resolution could be extended selectively for those signals which have data points flagged as errant. It should be noted that because the quadratic prediction model is a local phenomenon, despite the fact that separating samples farther apart in time increases the depth of the prediction it is entirely different than attempting the following where the error in the prediction is compounded as time progresses.

\[
\hat{x}(k + 2|k) = \Phi^2 \hat{x}(k|k) + \Phi \Gamma w_k
\]  

(2.11)

Before proceeding, an experimental verification of the quadratic model with larger time-steps is needed. Begin with the previously presented example of a simulated loss-of-excitation on a 2400MW generator in the WECC. The quadratic model was again applied to this data set four times each with a different resolution. Fig. 2.14 shows the magnitude of the residuals for each of these four iterations. Shown in red is the residuals for the highest resolution (30fps), in green (15fps), in blue (10 fps), and in cyan (7.5 fps).
Figure 2.14: Voltage Magnitude Residuals for Different Frame Rates For LOF Example

The upper and lower plot in the figure are the same thing but the lower plot is zoomed in to better see the residuals during the first half of the plot. Past the first half of the plot, the residuals begin to oscillate and increase in magnitude. However, this occurs in the simulation after a time when the generator would have tripped off-line so it is an operating condition that is of no interest. The residuals for the first half are on the order of $2 \times 10^{-4}$ and vary little between the different data resolutions. Statistics for the first half of the loss-of-excitation example are shown in Table 2.1. A comparable standard deviation among the different time resolution indicates that there is a good deal of flexibility for using different time resolutions with the same quadratic model.
Table 2.1: Mean and Standard Deviation of Residuals Using Different Frame Rates

<table>
<thead>
<tr>
<th>Loss of Excitation</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Every Sample</td>
<td>0.00000040487</td>
<td>0.00013577</td>
</tr>
<tr>
<td>Every Other Sample</td>
<td>0.000000050518</td>
<td>0.00013533</td>
</tr>
<tr>
<td>Every Third Sample</td>
<td>0.0000026097</td>
<td>0.00013203</td>
</tr>
<tr>
<td>Every Fourth Sample</td>
<td>0.0000017904</td>
<td>0.00012605</td>
</tr>
</tbody>
</table>

An observant individual would note that if something like this was used continuously that it would be effectively downsampling the data set. However, this can be avoided in the examples shown here. Maintaining a full resolution result while decreasing the resolution of the state and measurement vector can be accomplished by running several algorithms in parallel with each other. It can be noted that the purpose of the following discussion is simply one of acknowledgement as the practicality of the content is limited.

First, assume that there is a scenario such as that described in Fig. 2.13. In this case, the data at every fourth time step is used by the filter (while it may foreshadow later chapters to call it a filter, to do so clarifies the idea). Is it not possible to include the directly adjacent data in the same algorithm. Imagine now this same scenario where data from every fourth time step is used by one of four predictors. If a formulation is used where there is data between the snapshots used by the filter then another predictor can be run in parallel to take advantage of those data points which have been ignored. Fig. 2.15 demonstrates a scenario such as this where there are four predictors running in parallel. There are four which run every fourth frame which in turn yields an output at the same frame rate as the data is streamed in from the network. The computational burden for this formulation would not increase relative to a single algorithm running at full resolution.
2.4 Higher Order Prediction Models

The quadratic prediction model that has been discussed throughout this chapter is a 2\textsuperscript{nd} order polynomial and while it satisfies the desired criteria, polynomials of higher order will also serve the same purpose. It has been shown that the elements of Pascal’s triangle yield the absolute value of the coefficients of the quadratic polynomial. Moving down the rows of the triangle will yield the coefficients of higher order polynomials which function in the same way as the quadratic polynomial to predict the next data point given some number of previous data points.
Stepping down through the triangle one row yields the third order or cubic prediction polynomial.

\[ y_{n+1} = 4y_n - 6y_{n-1} + 4y_{n-2} - y_{n-3} \quad (2.12) \]

Stepping down again yields the fourth order, or quartic prediction polynomial.

\[ y_{n+1} = 5y_n - 10y_{n-1} + 10y_{n-2} - 5y_{n-3} + y_{n-4} \quad (2.13) \]

And continuing to step down yields the fifth and sixth order, or quintic and sextic prediction polynomials.

\[ y_{n+1} = 6y_n - 15y_{n-1} + 20y_{n-2} - 15y_{n-3} + 6y_{n-4} - y_{n-5} \quad (2.14) \]
\[ y_{n+1} = 7y_n - 21y_{n-1} + 35y_{n-2} - 35y_{n-3} + 21y_{n-4} - 7y_{n-5} + y_{n-6} \quad (2.15) \]

All of the principles discussed for the quadratic polynomial throughout this chapter apply to each of these polynomials as well. Given the assumption that higher order polynomial prediction models can be used in the same fashion as the quadratic prediction model, their performance should be compared to the quadratic prediction model relative to the baseline established using the simulated data in Section 2.2 where the performance of the quadratic model was demonstrated against four separate scenarios. This section aims to evaluate the performance of the higher order polynomials listed above against one of the same scenarios from Section 2.2 (obviously, there are an infinite number of these polynomials but it has been assumed impractical to continue beyond the list in the previous section).

Fig. 2.17 shows the absolute difference (linear distance in the complex plane) between the true (simulated) value and the predicted value for the case of the generator trip from the previous section. In order to get a better view, the above figure shows the difference after the trip and therefore does not include the time before the contingency or during the contingency. It is clear that while all of the models do perform objectively well, the quadratic model is the most effective at predicting the next data point in the series. The efficacy of the prediction model decreases faster than linearly with the increase in the order of the polynomial.
2.5 Summary

This chapter presented a simple model to be used to predict future data points from previous data points. The quadratic prediction model was demonstrated on simulated data in its raw form but will later be applied to a Kalman filter and other algorithms as a state transition for data conditioning and dynamic state estimation. Later sections demonstrated how the quadratic prediction model is still valid for different resolutions of synchrophasor and discussed why it may be desirable to do so. An idea for maintaining full resolution under this scenario was acknowledged. And finally, higher order prediction models were compared to the effectiveness of the quadratic model. In summary, the quadratic prediction model has been shown to be a satisfactory computational model for the dynamics of a power system when using synchrophasor data.
Chapter 3

Conditioning of Synchrophasor Data

Despite the many advantages of using synchrophasor data, it is still vulnerable to certain data quality issues. Loss of GPS synchronization, incorrect PMU configuration, and communication network congestion are a few examples of problems that can plague applications that digest the phasor data to produce meaningful information. This chapter addresses many of these issues by leveraging the quadratic prediction model developed in the previous chapter. The quadratic prediction model can be used in conjunction with filtering and smoothing algorithms to improve the data quality of the raw synchrophasor data received by the operations center. These algorithms will be shown to be able to fill in missing data, reduce gross errors in measurement, and provide an optimal value for data points even in the absence of larger issues. In the context of the larger goal, this chapter defines algorithms that can be used for the pre-processing of synchrophasor data before it is received by the linear state estimator. The output of the linear estimator is what will become the final stop for the phasors before they are made available to other network applications.

3.1 Plausibility Checks

A first pass for the detection and identification of bad data can be done with several plausibility/sanity checks. There are many common sense criteria that can be applied before the estimator even runs. It is becoming common practice to validate the PMU data after an installation before the stream is even connected to the operations center. Any issue resulting
from an incorrect configuration of the PMU, problems with the GPS clock, or an incorrectly connected signal wire will most likely be mitigated before the data even makes it into the state estimator. However, it is very simple to include some of these plausibility checks before performing the estimation. In addition to preventing unworthy data from being consumed by an application, an online algorithm for plausibility checks would also provide a mechanism for making operators and engineers aware of a data quality problem which will require manual mitigation. Measurement statistics generally calculated by the PDC will also be useful in identifying cases which require manual mitigation. Below is a non-exhaustive list of the types of plausibility checks that would cause measurements to be eliminated before state estimation. This mirrors the discussion of phasor data validation in the first chapter.

- Measurements who’s timestamps do not match the rest in the measurement set should be eliminated.
- Voltage magnitude measurements with zero or near zero readings should be eliminated.
- Voltage magnitude measurements with negative readings should be eliminated.
- Current magnitude measurements with zero or near zero readings should be eliminated.
- Current magnitude measurements with negative readings should be eliminated.
- Measurements who’s C37.118 status word shows the DataValid bit asserted should be eliminated.
- Measurements who’s C37.118 status word shows the PMUSync bit asserted should be eliminated.
- Measurements who’s C37.118 status word shows the PMUError bit asserted should be eliminated.
- Measurements that have other problems communicated via the C37.118 status word (i.e. A very common implementation uses digital relays as PMUs. When these dual-use devices are undergoing relay testing, there will be good signals measured by the PMU portion of the device. Since these are not really the measurement on the system and have no intrinsic characteristic which separates it from good data, the configurable bits in the C37.118 stream can be configured to communicate when the relay us undergoing maintenance.)
3.2 Evaluation of Signal Quality using SNR

There are simple ways to go beyond the common sense plausibility checks of the previous section for determining the quality of a stream of synchrophasor data. One example is to use the signal-to-noise ratio for evaluating the quality of each phasor. Reconstructing the original sinusoid and performing the signal-to-noise ratio calculations would be non-trivial. Therefore, it can be assumed that for most operating conditions, that the components of the phasor can be considered DC signals. Then, the calculation of SNR is simply the mean of the signal divided by the standard deviation of the signal taken over a moving window.

\[
SNR_{DC(\text{dB})} = 10 \log \frac{\mu}{\sigma} = 10 \log \frac{\text{mean}}{\text{std}}
\]  

(3.1)

It is clear that the phasor magnitude would be considered a DC signal but the angle is somewhat tricky. The raw phase angle rotates based on the frequency deviation from 60Hz and in order for the phase angles to be considered DC signals, they would need to be unwrapped and then referenced to a particular phase angle. Multiple references would be needed to detect signal quality issues in the reference signal itself. Signal-to-noise ratio can be an indication not only of a loose connection or potential hardware problem but could also help to diagnose certain equipment issues such as instrument transformer failure. Shown in Fig. 3.1 is data discovered post-mortem from the failure of a PT device in Dominion Virginia Power’s 500kV network.
While this was discovered after the device failed, signs of failure (because this and other types of devices often fail slowly) were evident up to three days in advance of the final failure. The data shown in the figure is from days prior to the event. Alarming actions based on the voltage level associated with the event would be difficult. However, using a simple signal-to-noise ratio calculation yields a diagnostic tool which could help spot issues like this ahead of time. Shown in Fig. 3.2 is the SNR for the magnitude and angle during the window of data captured here. Both the magnitude and the angle show a clear indication of a potential problem. While voltage is difficult to alarm on for this situation, decibels is
a metric that could be used to alarm on because it is a relative measure, not an absolute one. Additionally, in this case, because three phase data is used, an algorithm could quickly acknowledge that phase C was experiencing issues while the other were not. Having three phases of data is in and of itself a good mechanism for the detection of anomalies like this.

![Signal-to-Noise Ratio of PMU Data during C Phase PT Failure](image)

Figure 3.2: Signal-to-Noise Ratio of PMU Data during C Phase PT Failure

A key assumption for this quality criteria is the quasi-steady state operating condition of the power system. The size of the moving window used to calculate the mean and standard deviation components is critical for establishing a baseline criteria for alarming. Therefore, an intelligent alarming scheme would be necessary to prevent misinterpretation of the signal-
to-noise ratio during oscillation behavior and discrete changes on the network. The latter can be avoided by counting the number of time steps which the SNR is above a certain ratio over a certain period of time. This prevents ‘blips’ in the network such as faults on a line from actuating an alarm based on SNR. To alleviate the prior, a smaller moving window could be employed (but could cause other problems) or information from an oscillation detection algorithm could be used to block the alarm.

### 3.3 Quadratic Model on Predicting, Filtering, and Smoothing of Synchrophasor Data

Armed with the newly found knowledge that there is a ‘quadratic’ relationship between present, past, and future states in a power system, the synchrophasor-only estimation problem can now be realized without having to consider each iteration of the algorithm as a new problem. To be clear, in the context of this chapter, the ‘system’ refers to a single complex voltage or current signal and the ‘state’ of the system is just the value of that signal at each moment in time. In a dynamic system where the state is a function of time and there is a finite measurement set available for determining the state at any particular instant in time, the optimal estimate can be written in the following way.

\[
\hat{x}(k|j) = \Phi[z(i), i = 1, \ldots, j]
\]  

(3.2)

where \(\hat{x}(k|j)\) is the best estimate of \(x\) at some discrete time \(k\) given a finite measurement set of size \(j\), and where \(\Phi\) is a function of the measurement set \(z\). There are three scenarios for this problem. The first of which is for the case when \(k > j\); this is considered to be a prediction problem. The second is for the case when \(k = j\); this is considered a filtering problem. The final scenario is that when \(k < j\) and is a smoothing problem [6].

For the case of a stream of synchrophasor data, a practical solution (one which is not prohibitively large) is one where the data is processed more or less as it arrives [6]. To no surprise, this is how it is actually implemented. A previous implementation of a synchrophasor-only linear state estimator [2][17] are examples of the case where \(k = j\). Unfortunately, without knowledge of a relationship between prior and future states, this is effectively considering each new observation of the state at time \(k\) a different problem and the case becomes \(k = j = 0\).
Now that it is known that state variables in a power system (complex voltages) can be predicted from previous state variables, this knowledge comes together to form a system where the state is observed via direct measurement of the system state (because of synchrophasors) as well as knowledge of the system state at different times. To reiterate, for this chapter, the equations and examples will be presented with a system which is just a single complex voltage or current signal. This may be telling of topics to be covered in the next chapter, but these ideas can be easily expanded to a large power network. In real implementations, the state is redundantly observed with voltage phasors but also indirectly observed via current flow phasors which are linearly related to the system state. Finally, the relationship between the present and future state is derived from the quadratic model.

Following the desire to extend the synchrophasor-only estimation problem to include the relationship between states (allowing \( k \) and \( j \) to grow beyond 0) the practical solution is, again, one which processes data as it arrives. This will yield a process which in its general form is the following system [6].

\[
\begin{align*}
x(k+1) &= \Phi(k+1,k)x(k) + \Gamma(k+1,k)w(k) \quad (3.3) \\
z(k+1) &= H(k+1)x(k+1) + v(k+1) \quad (3.4)
\end{align*}
\]

where \( k \) is the discrete time index, \( x \) is the vector of state variables, \( \Phi \) is the state transition matrix derived from the quadratic model, \( \Gamma \) is the disturbance transition matrix, and \( w \) is the process noise of the state transition (basically, governed by the effectiveness of the quadratic prediction) and is Gaussian with zero mean. The measurement vector is \( z \) and in this section is a direct observation of a single signal. In this section, all observations of the state are direct. This dictates that the observation matrix, \( H \), will contain identity relationships between the measured values and the states. However, when current phasors are included in the context of a larger network, \( H \) will contain an upper and lower partition. The upper partition relates the direct observations to the state and the lower partition relates the indirect measurements (current phasors) to the state based on network impedances. The measurement noise, \( v \), is also considered Gaussian with zero mean. Essentially, the system as it existed in [17] is representative of only Eq. 3.4. The addition of the quadratic model enables the inclusion of Eq. 3.3. With this it will be shown in the following subsections that the ability now exists to perform predicting, filtering, and smoothing techniques on synchrophasor data. An example is given for the optimal filtering and optimal smoothing algorithms.
3.3.1 Predicting

The optimal predicted estimate (the case where \( k > j \)) can be applied to synchrophasor data when the quadratic model is included as the relationship between past and future states. Given the same system as Eq. 3.3 & 3.4, if the optimal filtered estimate, \( \hat{x}(j|j) \), and the error covariance matrix, \( P(j|j) \) are known then the optimal predicted estimate can be described in the following way, \[6\]

\[
\hat{x}(k + 1|k) = \Phi(k + 1, k)\hat{x}(k|k)
\] (3.5)

where the error covariance is governed by the following equation.

\[
P(k + 1|k) = \Phi P(k|k)\Phi^T + \Gamma Q \Gamma^T
\] (3.6)

Similar to the filtering algorithm, for complex voltage phasors as states of the system, the optimal prediction algorithm can be simplified by removing the time dependence notation from the affected parameters.

\[
\hat{x}(k + 1|k) = \Phi \hat{x}(k|k)
\] (3.7)

\[
P(k + 1|k) = \Phi P(k|k)\Phi^T + \Gamma Q \Gamma^T
\] (3.8)

It should be apparent that the optimal predicted estimate and the optimal filtered estimate are intimately related. They could have been defined in the opposite order since the optimal filtered estimate is considered to be an initial condition for the optimal predicted estimate.

3.3.2 Filtering

The optimal filtered estimate, \( \hat{x}(k + 1|k + 1) \) is the sum of the best estimate of \( x \) at time \( k \) given all prior information up to and including time \( k \) and projected into the future time of \( k + 1 \), plus the filter gain matrix multiplied by the observation residual. The observation residual is the difference between the projected values of the state at time \( k + 1 \) and the measured values from time \( k + 1 \) related to the state values via the measurement matrix or observation matrix. The initial condition for the state vector can be zero.

\[
\hat{x}(k + 1|k + 1) = \Phi(k + 1, k)\hat{x}(k|k) + K(k + 1) [z(k + 1) - H(k + 1)\Phi(k + 1, k)\hat{x}(k|k)]
\] (3.9)
The filter gain matrix can be recursively calculated from the following equations [6].

\[
K(k+1) = P(k+1|k)H^T(k+1)\left[H(k+1)P(k+1|k)H^T(k+1) + R(k+1)\right]^{-1} \\
P(k+1|k) = \Phi(k+1,k)P(k|k)\Phi^T(k+1,k) + \Gamma(k+1,k)Q(k|k)\Gamma^T(k+1,k) \\
P(k+1|k+1) = \left[I - K(k+1)H(k+1)\right]P(k+1|k)
\]

In the case of synchrophasors, the measurement noise covariance matrix, \( R \), the process noise covariance matrix, \( Q \), and the error covariance matrix, \( P \) are diagonal matrices (unless there is an error model like that described in the first section of Chapter 4). The initial condition for the error covariance matrix is the zero matrix.

There are some simplifications that can be made when the filtering algorithm is applied to synchrophasor data. First, the measurement matrix, \( H \), is not a function of time. This is true for single and multiple voltage & current phasors. Secondly, due to the nature of the quadratic prediction model, adjacent state vectors will share two of three state variables in common yielding an augmented state vector. The state vector can be thought of as a moving window (Fig. 3.3) containing three snapshots of the system which moves forward only one snapshot at a time. This is actually a providing smoothing to the oldest variable in the state vector in a single step. The section to follow on smoothing techniques will elaborate on this.

![Figure 3.3: Moving Window State Vector](image)

To some it may be more clear when written as an equation. The following equations are
the quadratic model equivalent of Eq. 3.3 & 3.4 when applied to a single phasor value.

\[
\begin{bmatrix}
\hat{x}_{n+1} \\
x_n \\
x_{n-1}
\end{bmatrix}
= \begin{bmatrix}
3 & -3 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_n \\
x_{n-1} \\
x_{n-2}
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

\[w_n\]  \hspace{1cm} (3.13)

\[
\hat{x}_{n+1} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_{n+1} \\
x_n \\
x_{n-1}
\end{bmatrix}
+ v_{n+1}
\]

\[v_{n+1}\]  \hspace{1cm} (3.14)

Additionally, the measurement noise covariance, process noise covariance, and disturbance transition matrices (as seen above) are not functions of time. This yields a simplified filtering algorithm and therefore the notation of time will be dropped from those affected matrices.

\[
K(k+1) = P(k+1|k)H^T[HP(k+1|k)H^T + R]^{-1}
\]

\[K(k+1)\]  \hspace{1cm} (3.15)

\[
P(k+1|k) = \Phi P(k|k)\Phi^T + \Gamma Q\Gamma^T
\]

\[P(k+1|k)\]  \hspace{1cm} (3.16)

\[
P(k+1|k+1) = [I - K(k+1)H]P(k+1|k)
\]

\[P(k+1|k+1)\]  \hspace{1cm} (3.17)

Fig. 3.4 shows the filtering algorithm presented in this subsection applied to a voltage phasor from an oscillation at a nuclear plant. It can be imagined that it would be very difficult to design a model for the relationship between states in situations such as this one. This demonstrates the power and flexibility of the quadratic prediction on actual synchrophasor data. Blue stars are the measured value (real synchrophasor data) and the red circles are the optimal filtered estimate.
Fig. 3.5 shows the observation residual of the filtering algorithm. The optimal prediction is closely related to the optimal filtered estimate. Because of this, the observation residual gives information not only about the effectiveness of the prediction but of the filtering algorithm on the whole. It can be seen from the plots that the results are on the order of $10^{-5}$. 

Figure 3.4: Filtering Example on Nuclear Oscillation Data
In the previous chapter, different time resolutions were explored using the quadratic prediction model. The same was done here, only in the context of a Kalman filter as described in Eq. 3.3 & 3.4. Two examples were performed: the first is a 500kV complex voltage from quasi-steady-state operating conditions and the second is a 230kV complex voltage from a 250 MW oscillation at a nuclear plant. Statistics for these two examples are given in Table 3.1. From the plots and the table it can be seen that decreasing the resolution of the filtering algorithm has different affects for the quasi-steady-state condition than for the low frequency oscillation. As expected given the loss-of-excitation example, the standard deviation of the different formulations are comparable during the quasi-steady-state scenario.
However, this is not the case with the oscillation example. There also appears to be a lag between the prediction and the measurement which grows as the resolution decreases just as the magnitude of the observation residual grows.

<table>
<thead>
<tr>
<th></th>
<th>Quasi-Steady-State</th>
<th>Low Hz Oscillation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td><strong>Every Sample</strong></td>
<td>0.00000026911</td>
<td>0.000018092</td>
</tr>
<tr>
<td><strong>Every Other Sample</strong></td>
<td>0.00000059645</td>
<td>0.000026074</td>
</tr>
<tr>
<td><strong>Every Third Sample</strong></td>
<td>0.00000199820</td>
<td>0.000034423</td>
</tr>
</tbody>
</table>

### 3.3.3 Smoothing

Smoothing techniques are also applicable with the addition of the quadratic prediction model as a state transition for states of a power system. Smoothing problems can generally be classified into three groups. These include the fixed-interval smoothing, the fixed-point smoothing, and the fixed-lag smoothing [6].

**Fixed Interval Smoothing**

A fixed-interval smoother is appropriate for off-line or post-experiment analysis and therefore may be uninteresting in the case where an online synchrophasor-only dynamic state estimator is desired. However, similar to several other ideas discussed in this dissertation, acknowledgement of a fixed-interval smoother using the quadratic prediction model is important. To give an example, consider the route of the raw synchrophasor data and the phasor data which will be considered at a high enough confidence to be fed to the network applications. The raw synchrophasor data will be saved to a historian, as would the output of the linear state estimator. While most, if not all, on-line network applications will leverage the ‘clean’ data from the linear estimator, it is difficult to say that off-line analysis might not find use for the raw data. implementing a fixed-interval smoother for the off-line back-office applications may be of value. Again, this section is purely one of acknowledgement of the compatibility of the quadratic prediction model for fixed-interval smoothing techniques.
Fixed Point Smoothing

The fixed-point smoother is also not appropriate for an on-line application as it is concerned with the optimal estimate of the state at a single instant of time. However, it provides the initial conditions to the fixed-lag smoother (the ideal algorithm for on-line smoothing) and must therefore be implemented as well. Removing time notation where it is unnecessary,

$$\hat{x}(k|j) = \hat{x}(k|j-1) + B(j) [\hat{x}(j|j) - \hat{x}(j|j-1)]$$  \hspace{1cm} (3.18)

where \( k \) represents some fixed point in time and \( j \) progresses through the data set. The optimal filtered estimate is an initial condition for this equation and is provided by the Kalman filter. Also,

$$B(j) = \prod_{i=k}^{j-1} A(i)A(i) = P(i,i)\Phi^T P^{-1}(i+1|i)$$  \hspace{1cm} (3.19)

and the process error covariance is computed with the following equation where the initial condition, \( P(k|k) \) is also provided by the Kalman filter.

$$P(k|j) = P(k|j-1) - B(j)K(j)HP(j|j-1)B^T(j)$$  \hspace{1cm} (3.20)

Fig. 3.6 shows an example of the quadratic model applied to the fixed point smoothing algorithm with the same oscillation example shown in the section on filtering.

![Figure 3.6: Fixed-Point Smoother Example with Nuclear Oscillation](image-url)
For demonstration purposes, the fixed-point smoothing algorithm was applied to each of the points in the series using a window of the same length for each, in this particular case, three. While this same approach could be employed for an online application which uses the fixed-point smoothing techniques, it is not computationally optimized. This is the reason for the fixed-lag smoother. Generally speaking, the fixed-lag smoother is the same as the fixed-point smoother except that it allows the point of interest to move forward in time at each iteration by recursively removing the oldest measurement and covariance information and including the latest. Fig. 3.7 shows the residuals of both the filtering algorithm and the fixed-point smoothing algorithm applied to the previous example.

![Observation Residuals for Fixed-Point Smoother Example](image)

Figure 3.7: Observation Residual for Fixed-Point Smoother Example with Nuclear Oscillation

**Fixed Lag Smoothing**

The fixed-lag smoother can also be used for off-line studies but is really suited for an on-line application where some delay can be tolerated. There are many types of fixed lag smoothing. There are two in particular which are the topic of this section. The fixed lag smoother which is presented in [6] and is a recursive version of the fixed-point smoother from the previous section is given first. Removing time notation where it is unnecessary yields the optimal fixed-lag smoothed estimate employing the quadratic prediction for synchrophasor data.
\[
\hat{x}(k+1|k+1+N) = \Phi_f \hat{x}(k|k+N) + \\
C(k+1+N)K(k+1+N)\hat{z}(k+1+N|k+N) + \\
U(k+1)[\hat{x}(k|k+N) - \hat{x}(k|k)]
\]

(3.21)

where the forward and backward gain matrices are given by

\[
C(k+1+N) = \prod_{i=k+1}^{k+N} A(i)
\]

(3.22)

\[
A(i) = P(i,i)\Phi_f^T P^{-1}(i+1|i)
\]

(3.23)

and by

\[
U(k+1) = \Gamma Q \Gamma^T \Phi_b^T P^{-1}(k|k)
\]

(3.24)

and the error covariance is given by the following equation.

\[
P(k+1|k+1+N) = P(k+1|k) - \\
C(k+1+N)K(k+1+N)HP(k+1+N|k+N)C^T(k+1+N) - \\
A^{-1}(k) [P(k|k) - P(k|k+N)] [A^T(k)]^{-1}
\]

(3.25)

However, a more interesting and perhaps relevant fixed-lag smoothing technique is one which is in the form of a Kalman filter. [24] presents a model for a fixed-lag smoother which in its general form has the following discrete time state equation with an augmented state vector and an associated augmented dynamical system.

\[
\begin{bmatrix}
\dot{x}_{k+1} \\
x_k \\
\dot{x}_{k-1} \\
\vdots \\
x_{k-N}
\end{bmatrix} = \begin{bmatrix}
A_k & 0 & 0 & \ldots & 0 \\
I & 0 & 0 & \ldots & 0 \\
0 & I & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & I & 0
\end{bmatrix} \begin{bmatrix}
x_k \\
x_{k-1} \\
x_{k-2} \\
\vdots \\
x_{k-N-1}
\end{bmatrix} + \begin{bmatrix}
G_k \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix} \nu_k
\]

(3.26)

and an augmented measurement equation of the following

\[
z_k = \begin{bmatrix}
H_k & 0 & 0 & \ldots & 0
\end{bmatrix} \hat{x}_k + \omega_k
\]

(3.27)
This should look strikingly familiar to the filter equations using the quadratic prediction model in the previous section. Here, the elements of the matrices have been defined as vector quantities but in the case of the quadratic prediction model applied to a single complex voltage or current signal they are defined as constant scalars. The only other difference is in the top row of what serves as the state transition matrix. The future state is a function of the three previous states and not just the directly preceding state. As stated previously, the filtering equations are actually smoothing the older data points in the augmented state vector. This algorithm executes in the same way as the filtering algorithm and knowing how to execute the filtering algorithm is enough to execute this algorithm. The time update, measurement update, and covariance update are all executed in the same way. The initial conditions for all of the estimates and covariances can be zero and not require an additional Kalman filter to provide this information as long as it is acknowledged that the values for these won’t be correct until after the smoothing window has been fully populated with data [6]. Due to it’s simplicity, this is the particular smoothing algorithm that will be used in the next section do create the data conditioning algorithm.

3.4 Data Conditioning Algorithm

The ultimate goal of this chapter is to use the knowledge of the quadratic prediction model and the filtering and smoothing techniques to create an algorithm that will improve the quality of the synchrophasor data stream provided to the linear state estimator. It is desirable to mitigate data problems such dropouts caused by network congestion (among other causes), randomly occurring outliers and other bad data, as well as any other intermittent data problems. A blanket filter or smoother would not be the optimal solution because while decreasing the effect of bad data on a stream of data, when there is a large enough error in the data (100% TVE in the case of a dropout) the quality of the stream local to that particular time would suffer. It is ideal, then, to replace measurements with gross errors with estimates generated from measurements taken closely in time which have been deemed ‘good’ quality. Therefore, the algorithm should be able to take action for each measurement at each time step based on the quality of the measurement. There are four scenarios which encompass all possibilities.

Scenario 1 - Under ‘blue sky’ conditions, the algorithm will receive a good quality measure-
ment at each time step for each signal in the stream. In this case, the data conditioning algorithm needs to take no action outside of its usual operation of predict, correct, and smooth.

**Scenario 2** - In the presence of a manageable amount of bad or missing data, the bad points will be replaced by the optimal predicted estimate and the algorithm will assume those values to be measurements and the algorithm continues as usual.

**Scenario 3** - In the case of a discrete change in the network such as a line outage or generator trip the algorithm should report the data points as measured with no conditioning for the duration of the smoothing window starting at the time step corresponding to the first data point after the contingency.

**Scenario 4** - For the case where there is not enough quality information for the algorithm to make a guess at the value of the measurement, no measurement should be reported. This is a last resort.

### 3.4.1 Leveraging the Optimal Prediction & Intrinsic Smoothing Properties of the Dynamic State Equation

As discussed in a previous section, the because of the nature of the augmented dynamical system with the augmented state and measurement vector, the filtering algorithm also smooths the older state variables in the state vector. Additionally, under normal operating conditions the optimal prediction and subsequent observation are used to calculate an observation residual. That residual is used to determine whether the data was expected or not. When the data received is ‘expected’, i.e. there is no bad data, repeating data, or dropouts detected, the algorithm simply carries on 'business as usual' by predicting, correct, and simultaneously smoothing the oldest states.
Essentially, there are two possibilities when the algorithm receives unexpected data: either the data is bad or a discrete change on the network has just occurred. A simple residual calculation cannot determine the difference alone so this idea will be revisited shortly. For now, assume that it is the case in which the data is bad or missing. The large observation residual will indicate to the algorithm that the measurement should be replaced with the optimal predicted estimate calculated by the Kalman filter. This measurement will then be smoothed as it progresses naturally through the smoothing window. Theoretically, the algorithm should be able to provide reliable estimates in the presence of bad or missing data as long as the algorithm receives good data intermittently. The minimum required time between good data points would be determined by the size of the smoothing window; if enough subsequent data points are not reported or are ‘bad’, then the algorithm cannot make accurate predictions and resort to some default operation such as resetting itself or simply reporting the raw data, potentially flagged with an ‘unprocessed’ or ‘untrustworthy’ flag because the quality cannot be determined. As will be seen in this section, it would require a substantial amount of poor quality data before this happens.

Several examples will follow where this algorithm has been subjected to varying levels of drop-outs or unreported data. This is considered the worst case scenario because it is effectively 100% total vector error (TVE) and no contextual information can be extracted from it. A quasi-steady state operating condition was chosen as the candidate scenario for analysis in this section, however, the power oscillation at the nuclear plant is included as an aside. To begin, there are seven scenarios with, as mentioned, various levels of data drop-outs: 1%, 2%, 5%, 10%, 20%, 30%, & 40%. The drop-outs are simulated by removing data points from the stream probabilistically. For example, for a 10% drop-out rate, the likelihood
that a data point will be reported is 90%. A uniformly distributed random number generator between 0 and 1 is used to accomplish this. The results of each of these tests can be viewed in Fig. 3.8, 3.9, 3.10, 3.11, 3.12, 3.13, 3.14, & 3.15, respectively. The results show that the algorithm can provide performance quality on par with receiving the data as if nothing had ever happened up to and including the 20% range. As the drop-out rate increases, the likelihood that adjacent frames are dropped increases exponentially. And as previously discussed, if enough adjacent frames are missing, the algorithm will lose its ability to make an accurate prediction. Although operating at a 30-40% drop rate is evidently not the end of the world, the effects of sustained drop-outs clearly are sufficiently large enough that they could be misinterpreted by downstream applications as something gone wrong.

It is worth mentioning, however, that discussion of 40%, 30%, 20%, or even 10% data drop rates is somewhat irrelevant because higher data quality will be demanded by other entities such as Regional Transmission Organizations (RTOs). For example, the Pennsylvania - Jersey - Maryland Power Pool (PJM) requires drop-out rates of members to be below 5%. If data quality falls below this level, then manual mitigation must take place. This is the purpose of the plausibility checks providing a mechanism to make engineers and operators aware when there is a problem that cannot and should not be solved with data conditioning algorithms.

The plots showing the results on the following pages are grouped into sets of three. The top plot on each of the pages shows the phasor magnitude of the raw data, optimal filtered estimate, and optimal smoothed estimate. The red dotted lines were left in the plots to better visualize the occurrence of a dropped frame; each vertical red line corresponds to a time when data is not reported. Some of the plots of the phasor magnitude have been zoomed in on the x-axis to better visualize the effectiveness of the smoothing aspect of the algorithm even in the presence of bad or missing data points. The latter two plots on each page represent the observation residual of the optimal filtered estimate and the optimal smoothed estimate with the measured values. However, this calculation/comparison was made with the phasor data as if it had not been removed. While the actual stream of data being passed through the data conditioning algorithm contained the drop-outs, the data used to make the observation residual comparison was the original good data. This way, the performance of the algorithms can be compared to what would have happened if the data had actually been received by the algorithms rather than suffering a drop-out. The nuclear oscillation scenario is also presented at the 30-40% drop-out range with similar results.
Figure 3.9: Results of Data Conditioning Algorithm with 1% Dropouts
Figure 3.10: Results of Data Conditioning Algorithm with 2% Dropouts
Figure 3.11: Results of Data Conditioning Algorithm with 5% Dropouts
Figure 3.12: Results of Data Conditioning Algorithm with 10% Dropouts
Figure 3.13: Results of Data Conditioning Algorithm with 20% Dropouts
Figure 3.14: Results of Data Conditioning Algorithm with 30% Dropouts
Figure 3.15: Results of Data Conditioning Algorithm with 40% Dropouts
Figure 3.16: Results of Data Conditioning Algorithm with 40% Dropouts (Oscillation Example)
On this and the following page, two plots (Fig. 3.17 & 3.18) are shown for the nuclear oscillation scenario with a 30% drop-out rate. There are no additional observations that are made using these plots than what has been previous made. However, these plots were graphed in the complex plane with the red dashed lines and the optimal filtered estimate removed. The only data in the plots are the raw data points (from the drop-out set - i.e. the missing data is visually reflected in this plot by its absence) and the optimal smoothed estimate. Viewing the results for this particular scenario in this way yields a very impressive understanding of the effectiveness of the algorithm to condition the data in the presence of such poor data quality.

Figure 3.17: Complex Voltage Results of Algorithm with 30% Dropouts (Oscillation Example)

Another type of data quality issue that shares similar characteristics with drop-outs due to network congestion is the case of repeated values. Again, data-validation procedures
actuated before the stream is connected to the operations center can alleviate many of the causes of data problems such as repeated data values. However, as with the drop-outs, repeated values can occur intermittently and should be addressed. The difference between missing data and repeated data is clearly the TVE; a missing data point has a total vector error of 100% whereas the total vector error of a repeated data point is very small. At first, it may appear that this data quality problem is not necessarily a problem because the values do not change as much. This might be the case if the raw phase angle values were somehow already unwrapped and referenced. The reality is that the phase angle is changing at each new frame based on the deviation from nominal frequency. Therefore, repeated values have the ability to cause problems with the quality of the phase angles. Fig. 3.19 shows an example where (probabilistically) 20% of the values are repeated values. The performance characteristics of the algorithm in the presence of the repeated data points is very similar to the characteristics in the presence of data drop-outs. Again, the performance will suffer as the probability of adjacent data points being repeated values increases.
Figure 3.18: Complex Voltage Results of Algorithm with 30% Dropouts (Oscillation Example)
Figure 3.19: Results of Data Conditioning Algorithm with 10% Repeated Values
3.4.2 Knowing When to Reset the Algorithm

As was seen in the previous subsection, as the probability of adjacent data points increases, the data conditioning algorithm will lose the ability to make accurate predictions. This is critical for the overall performance of the algorithm. While the content of the previous subsection addresses Scenario 1 & 2 of the algorithms functionality, this section addresses what happens for Scenario 3 & 4 when the algorithm is not presented with enough quality data to perform its operation reliably.

To begin, Fig. 3.20 shows the phasor magnitude for a scenario where there is a 50% likelihood that each data point will be lost. While the algorithm is able to perform for some time, it does not take long until enough adjacent data points are lost that the algorithm is unable to make accurate predictions and diverges. This is because if the observation residual is high enough the raw measurement is replaced by the optimal predicted estimate. Therefore, even when the good data returns, the error in the optimal prediction has compounded enough times that it is not able to track the raw measurements anymore.

![Figure 3.20: Results of Data Conditioning Algorithm with 50% Dropouts (Divergence)](image)

In order to prevent the algorithm from becoming numerically unstable and diverging, it should have the ability to reset itself in the event that it loses the ability to make accurate predictions. If the smoothing window contains enough measurements which are not raw data but rather pseudo-measurements generated from optimal predictions, then the quality of the
next prediction is severely degraded and will continue to degrade as time progresses. For example, after the smoothing window has filled with sub-optimal data, the algorithm should be reset.

![Figure 3.21: Timeline for Resetting the Data Conditioning Algorithm](image)

If the need arises for the algorithm to reset itself it will need to fill the smoothing window with data before it can begin operation. Therefore, there will be at least a delay of three (for the algorithm described herein) frames before the algorithm can continue operation as normal. Additionally, proper selection of the initial conditions can help the algorithm begin to track the synchrophasor stream more quickly. For example, the error covariance matrix and the Kalman filter reach ‘steady-state’ during normal operation. These values can be saved and used to re-initialize the algorithm when required. *Fig. 3.22* demonstrates the effectiveness of this when beginning to track a stream of data.
It is acknowledged that for error rates approaching the 40% - 50% mark, the data conditioning algorithm won’t be able to do much for the improvement of the raw measurements because of the lack of 'good' adjacent measurements. However, incorporating an automatic reset into the algorithm will prevent it from diverging in the presence of overwhelming data loss. The pseudocode for the algorithms reset function is shown below.

\[
\text{if } \text{numberOfSubOptimalDataPoints} \geq \text{smoothingWindowLength} \text{ then}
\]
\[
\text{Algorithm} \leftarrow \text{Reset}
\]

\[
\text{else}
\]
\[
\text{if } \text{BadOrMissingDataDetected} \text{ then}
\]
\[
\text{Measurement} \leftarrow \text{OptimalPredictedEstimate}
\]
\[
\text{numberOfSubOptimalDataPoints} \leftarrow \text{numberOfSubOptimalDataPoints} + 1
\]

\[
\text{else}
\]
\[
\text{numberOfSubOptimalDataPoints} \leftarrow 0
\]
\[
\text{end if}
\]

\[
\text{end if}
\]

The example from Fig. 3.20 is repeated only this time the algorithm has the ability to reset itself under the previously described conditions. Fig. 3.23 shows the resulting performance. As can be seen in the plot the majority of the output will simply be reporting the
raw data because there are not enough adjacent data points to support a quality prediction. This is expected as a side effect of resetting and was described in the functional specifications in the beginning of the section.

![Figure 3.23: Results of Data Conditioning Algorithm with 50% Dropouts (No Divergence)](image)

Figure 3.23: Results of Data Conditioning Algorithm with 50% Dropouts (No Divergence)

Another side effect of adding reset functionality to the algorithm is that in combination with the innate lag time of the algorithm, contingencies or discrete changes in the system can be properly conditioned. Recall in Chapter 2 the behavior of the quadratic prediction at the time of a contingency. The step changes are not accounted for with the quadratic prediction model and therefore cannot provide accurate predictions at the time of the contingency. It takes several samples until the window moves past the step change before it can properly track the stream again. *Fig. 3.24* shows a contingency from the IEEE 118 bus system (one of the cases from Chapter 2) and demonstrates the effectiveness of the algorithms reset functionality in the presence of a discrete network change such as a loss of generation.
Figure 3.24: Effect of Algorithm Reset During True Contingency
3.5 Summary

This chapter has developed and demonstrated a data conditioning algorithm for synchrophasor data based on the quadratic prediction model from Chapter 2. It included discussions on the applicability of the quadratic model in several filtering and smoothing techniques and provided numerical examples using real synchrophasor data under several conditions. Techniques were chosen to serve in the data conditioning algorithm and functional specifications were made. These included operating with a lag to provide an optimal smoothed estimate during ‘blue sky’ conditions, using the optimal predicted estimate to replace a bad or missing measurement, and incorporating an automatic reset functionality to prevent divergence in the presence of high volumes of bad/missing data and to improve the quality of the smoothed estimate when discrete changes (contingencies) in the network occur. The algorithm has been shown to mitigate data loss at levels of 30% - 40% while remaining stable for even more severe conditions. As 5% is considered the threshold for normal data loss by PJM, Dominion Virginia Power’s RTO, the algorithm gives an ample cushion for constantly providing a clean data stream to the linear estimator and network applications. Plausibility checks and the signal-to-noise ratio of the magnitude and phase of the synchrophasor were discussed and demonstrated as mechanisms for both preventing poor quality data from propagating downstream and alerting technicians of problems which will require manual mitigation.
Chapter 4

Synchrophasor-Only Dynamic State Estimation

With previous synchrophasor-based state estimators, each iteration of the algorithm is treated as a new problem and historical information about the state of the system was not included. This chapter represents the culmination of the development of the quadratic prediction model as a relationship between present and future states of a power system by developing equations for the first synchrophasor-only dynamic state estimator. While the quadratic prediction model is known to be an approximation, it is a very useful one given the time scales of adjacent measurement sets as it has been seen to have the ability to track system dynamics of all kinds.

The chapter begins with a discussion of the correlation between real and imaginary components of phasor measurements and how the measurement covariance matrix will then be a function of the angles of the measurement vector. A brief presentation of the static phasor-based estimation analogue is also discussed. The dynamic algorithm is developed as an optimal filter and its parameters defined. Different variations of the dynamic algorithm are discussed; this includes different three phase equations and separating the real and imaginary components. The algorithm is tested with data from a subsection of Dominion Virginia Power’s EHV network for several scenarios and bad data detection and identification were presented and demonstrated as applications of the dynamic formulation.
4.1 Complex Error Model

For the purpose of accurate modeling, it is important to discuss the error model of the phasor measurement. It should be noted that this is different from CT and PT errors which can be considered constant for similar loading conditions. Previous work on linear tracking state estimation assumed an error model where the real and imaginary errors are uncorrelated. This will appear as a circle around the true value in the complex plane.

![Circular Error Model](image)

However, this may not be the case. Consider first the high level accuracy of the GPS clock which is inherently responsible for determining the angle of the phasor. The GPS clock signal is known to give the angle of the phasor measurement an accuracy of $0.02^\circ$. If the voltage magnitude is then assumed to have a 1% accuracy then the error model will no longer appear as a circle in the complex plane. It will become an ellipse oriented in the direction of the true angle of the phasor.
In a situation like this, there will be a strong correlation (large covariance) between the real and imaginary parts of each phasor for most locations in the complex plane. If the phasor is located directly on one of the axis then there will be no correlation between the real and imaginary parts. However, if the phasor sits directly between two axis at 45° then the correlation between the real and imaginary components of the phasor (the off-diagonal covariance) is at its peak. Consider the state variable of a single phasor broken into its magnitude & angle components.

\[ W_{\text{polar}} = \begin{bmatrix} \sigma_M & 0 \\ 0 & \sigma_A \end{bmatrix} \]  \hspace{1cm} (4.1)

Because of the linear formulation of the phasor-only estimator, the covariance needs to be in terms of real and imaginary. This can be calculated by performing a rotation of the polar covariances where \( \phi \) is the true angle of the phasor in question.

\[ W(\phi)_{\text{rect}} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \sigma_M & 0 \\ 0 & \sigma_A \end{bmatrix} \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \]  \hspace{1cm} (4.2)
This evaluates to the following.

\[ W(\phi)_{\text{rect}} = \begin{bmatrix} \cos \phi^2 \sigma_M^2 + \sin \phi^2 \sigma_A^2 & \cos \phi \sin \phi (\sigma_M^2 - \sigma_A^2) \\ \cos \phi \sin \phi (\sigma_M^2 - \sigma_A^2) & \cos \phi^2 \sigma_A^2 + \sin \phi^2 \sigma_M^2 \end{bmatrix} \] (4.3)

For a \( \sigma_M = 0.01 \text{ p.u.} \) and a \( \sigma_A = 0.000349 \text{ radians} \) or \( 0.02^\circ \) the covariance matrix for a phasor sitting at \( 0^\circ \) yield zero correlation between real and imaginary components.

\[ W(0)_{\text{rect}} = \begin{bmatrix} 1 \times 10^{-4} & 0 \\ 0 & 1.218 \times 10^{-7} \end{bmatrix} \] (4.4)

And as stated previously, there will be a high correlation between real and imaginary components of the phasor when the phasor is situated directly between the two axis.

\[ W(\pi/4)_{\text{rect}} = \begin{bmatrix} 5.0061 \times 10^{-5} & 4.9939 \times 10^{-5} \\ 4.9939 \times 10^{-5} & 5.0061 \times 10^{-5} \end{bmatrix} \] (4.5)

A covariance matrix for the linear estimator would no longer be a diagonal matrix but contain two bands running parallel to the diagonal which represent the correlation between the real and imaginary parts of individual phasors. It is still assumed that there is no correlation between any two phasors causing the rest of the elements of the covariance matrix to be zero.

### 4.2 Linear Tracking State Estimator

[2] describes the formulation of a phasor-based linear tracking state estimator in both positive sequence and three phases. It is important to understand the functionality of this linear tracking estimator before the formulation of a phasor-based dynamic estimator can be presented. All of the individual matrices developed as part of the tracking state estimator will be employed in the dynamic version. Finally, the solution to the static synchrophaser-only estimation problem exists under several different assumptions than that of its dynamic companion.

Begin by defining a state vector of complex voltages which represent all observable substations in the monitored network.

\[ x = \begin{bmatrix} v_1 & v_2 & \ldots & v_m \end{bmatrix}^T \] (4.6)
Then, the measurement vector will be the vertical concatenation of the complex voltage measurements and the complex current measurements from across the network.

\[ z = \begin{bmatrix} v_1 & v_2 & \ldots & v_m & i_1 & i_2 & \ldots & i_n \end{bmatrix}^T \]  

(4.7)

The measurement vector can be related to the state vector by the vertical concatenation of two matrices. The upper matrix, \( \mathbf{II} \), defined as the \textit{voltage measurement-bus incidence matrix}, represents the identity relationship between the voltage measurements and the complex state vector. Due to the implementation of dual-use line relays as PMUs in the substations there are direct, redundant observations of the system state. This, coupled with the fact that not every node in the network need be measured, the voltage measurement-bus incidence matrix will typically be taller than it is wide.

The lower matrix can be understood as a product of three other matrices that when combined yield the relationship between the current phasor measurements and the complex state vector. The \textit{current measurement-bus incidence matrix}, \( \mathbf{A} \), represents the relationship of each current measurement to its location in the monitored network. In the \textit{series admittance matrix}, \( \mathbf{Y} \), each diagonal element represents the series admittance of the transmission line being measured by that respective measurement. In the \textit{shunt susceptance matrix}, \( \mathbf{Y}_s \), each element represents the shunt susceptance of each line being measured by the current measurement.

\[ z = \begin{bmatrix} \mathbf{II} \\ \mathbf{YA} + \mathbf{Y}_s \end{bmatrix} \mathbf{x} + \epsilon \]  

(4.8)

A more detailed description including explicit rules on the population of these matrices can be found in [2]. The solution to this over-defined set is achieved by performing the pseudo-inverse of the state matrix and multiplying by the measurement vector.

\[ \hat{\mathbf{x}} = (\mathbf{H}^T \mathbf{W}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{W}^{-1} z \]  

(4.9)

where \( \mathbf{W} \) is the covariance matrix of the measurement normally distributed measurement errors. It is most important to note that this particular estimator considers each new frame (measurement set) to be a separate estimation problem than the previous and future measurement sets. Because of this, and the fact that the measurements are so close together in time (30 frames per second or more), that the same mechanisms used for bad data detection in traditional non-linear estimators could not be used with this estimator. Traditional non-linear estimators have solutions which are separated in time by as little as just one or
two seconds and as much as 15 seconds. This is enough time to assume no relation between adjacent solutions and the measurement residuals can be calculated from the estimated state vector. Because of the high time resolution of these phasor measurements this assumption no longer holds and in order to properly calculate a measurement residual it is necessary to have the ability to predict the next measurement set from previous data and compare that to the new measurement set as it comes in.

4.3 Linear Dynamic State Estimator

In order to form the equations for the synchrophasor-only dynamic state estimator two relationships are required. The first is the observation equation which relates measurements at time $k$ to the state at time $k$. The second is the state equation which relates the state at time $k$ to the state at time $k+1$. As with the development of the static version of the linear state estimation equations, it is convenient to begin with the simple two-port $\pi$-model. The assumption of a PMU at each end of the transmission line remains.

\[ \mathbf{x} = \begin{bmatrix} v_i \\ v_j \end{bmatrix}^T \]  \hspace{1cm} (4.10)

\[ \mathbf{z} = \begin{bmatrix} v_i & v_j & i_{ij} & i_{ji} \end{bmatrix}^T \]  \hspace{1cm} (4.11)

The two-port $\pi$-model example will be referred to in the following subsections when describing the state & measurement vectors and the measurement equations.
4.3.1 The State Equation

Ideally, in the static state estimation sense, the state of the system at time $k$ is the set of voltages in the power system which, if known, contain all of the information about the current operating conditions of the network at time $k$. It should be noted that practically speaking, no estimator actually includes all of the voltages and, therefore, the ‘state’ of the system will actually be a subset of all of the voltages. Continuing on, in order to maintain a consistent idea of a state transition, the state of the system needs to be slightly redefined. Because of the way the state transition is formed using the quadratic model, a single snapshot no longer suffices for the state of the system. The state of the system, or the state vector, must contain some past history of each of the state variables. Since the previously presented quadratic model requires three data points to predict the fourth, so should the state vector contain three state vectors which represent three adjacent snapshots of the system. Eq. 4.12 is now the state of the system at time $k$ for the two-port $\pi$-model example. It includes the snapshot of the system at time $k$ along with the snapshots of the system at the times $k-1$ and $k-2$.

\[
\mathbf{x}_k = \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}(k-1) \\ \mathbf{x}(k-2) \end{bmatrix} = \begin{bmatrix} v_i(k) \\ v_j(k) \\ v_i(k-1) \\ v_j(k-1) \\ v_i(k-2) \\ v_j(k-2) \end{bmatrix}
\] (4.12)

Now, this augmented state vector at time $k+1$ includes the snapshot of the system at time $k+1$ in addition to the snapshots at time $k$ and $k-1$. It may seem unintuitive at first that these adjacent state vectors share common state variables but it is required for a consistent idea of the system state and should become clearer. The state vector in the dynamic case is of length $3n$ where $n$ is the number of complex state variables in a single snapshot of the network.

\[
\mathbf{x}_{k+1} = \begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{x}(k) \\ \mathbf{x}(k-1) \end{bmatrix} = \begin{bmatrix} v_i(k+1) \\ v_j(k+1) \\ v_i(k) \\ v_j(k) \\ v_i(k-1) \\ v_j(k-1) \end{bmatrix}
\] (4.13)
The state transition equation contains the relationship between the present and future state vectors where \( x_k \) is the state vector at time \( k \), \( \Phi \) is the state transition matrix from the state at \( k \) to the state at \( k+1 \) (but is not a function of time), and the disturbance transition matrix, \( \Gamma \) (also not a function of time), \( w_k \) is the associated white noise with known covariance and mean zero.

\[
x_{k+1} = \Phi x_k + \Gamma w_k
\]

(4.14)

The state transition matrix, \( \Phi \), is the Kronecker product of the matrix form of the quadratic model with an \( n \) by \( n \) identity matrix where \( n \) is the number of complex state variables in one snapshot of the system. In the two-port \( \pi \)-model example, \( n \) will be equal to 2. Note that this is different than the number of state variables in the state vector.

\[
\Phi = \begin{bmatrix}
3 & -3 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix} \otimes I = \begin{bmatrix}
3I & -3I & I \\
I & 0 & 0 \\
0 & I & 0
\end{bmatrix}
\]

(4.15)

Also, the disturbance transition matrix is the Kronecker product of \( \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \) and the row vector which is the diagonal of the same identity matrix.

\[
\Gamma = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \otimes \text{diag} \{ I \}^T
\]

(4.16)

Then, the state equation takes the following form.

\[
\begin{bmatrix}
x(k+1) \\
x(k) \\
x(k-1)
\end{bmatrix} = \begin{bmatrix}
3I & -3I & I \\
I & 0 & 0 \\
0 & I & 0
\end{bmatrix} \begin{bmatrix}
x(k) \\
x(k-1) \\
x(k-2)
\end{bmatrix} + \begin{bmatrix}
\text{diag} \{ I \}^T & 0 & 0
\end{bmatrix} w_k
\]

(4.17)

### 4.3.2 The Measurement Equation

For the measurement equation, it is convenient to begin with the same measurement equation as its static counterpart. Again, the relationship between the state vector and the measurement vector contains an upper partition which identically relates the state to the voltage measurements and a lower partition which relates the state vector to the current measurements by the admittances of the line.

\[
z = \begin{bmatrix}
I \\
YA + Y_s
\end{bmatrix} x = Bx
\]

(4.18)
The observation matrix, $B$, for the static linear estimator has dimensions $m$ by $n$ where $m$ is the number of complex phasor measurements and $n$ is the number of complex state variables. Similar to the dynamic version of the state vector, $x_k$, the measurement vector, $z_k$, will be a concatenation of a series of observations of the system. The measurement vector will be of length $3m$ or 3 times the number of complex phasor measurements in the network. Therefore, the measurement vector must also be similarly redefined. The measurement vector for the dynamic formulation must contain the present measurement snapshot of the system as well as previous measurement snapshots of the system. In the present case where three snapshots predict the fourth, the measurement vector contains the present measurement snapshot and the two previous snapshots. Eq. 4.19 shows the measurement vector for the two-port π-model example.

$$z_k = \begin{bmatrix} z(k) \\ z(k-1) \\ z(k-2) \end{bmatrix} = \begin{bmatrix} v_i(k) \\ v_j(k) \\ i_{ij}(k) \\ i_{ji}(k) \\ v_i(k-1) \\ v_j(k-1) \\ i_{ij}(k-1) \\ i_{ji}(k-1) \\ v_i(k-2) \\ v_j(k-2) \\ i_{ij}(k-2) \\ i_{ji}(k-2) \end{bmatrix}$$

The measurement equation contains the relationship between the observations of the system state (voltage & current phasors) and the system state where $z_k$ is the measurement vector at time $k$, $H$ is the noiseless connection between the observation and the state, $x_k$ is the state vector at time $k$ and $v_k$ is the associated Gaussian measurement noise with mean zero and known covariance at time $k$.

$$z_k = Hx_k + v_k$$

The noiseless connection between the state vector, $x_k$, and the measurement vector, $z_k$, is simply a block diagonal matrix where each diagonal block is the state matrix from the static linear state estimation equation. Each diagonal is comprised of four separate matrices relating the measurement snapshot to the state snapshot. These include the current
measurement-bus incidence matrix, the voltage measurement-bus incidence matrix, the series admittance matrix, and the shunt susceptance matrix as defined in the previous section. The observation matrix for the dynamic case will have dimensions of $3m \times 3n$ where $m$ is the number of complex phasor measurements in the network and $n$ is the number of complex state variables in any snapshot of the network (for a single time, $k$).

$$
H = \begin{bmatrix}
B & 0 & 0 \\
0 & B & 0 \\
0 & 0 & B \\
\end{bmatrix}
$$

(4.21)

This yields the measurement equation for the dynamic state estimation problem.

$$
\begin{bmatrix}
z(k) \\
z(k-1) \\
z(k-2)
\end{bmatrix} =
\begin{bmatrix}
B & 0 & 0 \\
0 & B & 0 \\
0 & 0 & B \\
\end{bmatrix}
\begin{bmatrix}
x(k) \\
x(k-1) \\
x(k-2)
\end{bmatrix} + v_k
$$

(4.22)

4.3.3 The Filtering Algorithm

With the state equation and observation equation defined in the previous subsections, the filtering algorithm can be written for the formulation of the synchrophasor-only dynamic state estimator. As given by Eq. 3.9 and later simplified, the optimal filtered estimate can be written as

$$
\hat{x}(k+1|k+1) = \Phi \hat{x}(k|k) + K(k+1) [z(k+1) - H\Phi \hat{x}(k|k)]
$$

(4.23)

where the state transition matrix, $\Phi$, for all $k$ is given by

$$
\Phi =
\begin{bmatrix}
3I & -3I & I \\
I & 0 & 0 \\
0 & I & 0 \\
\end{bmatrix}
$$

(4.24)

and the observation matrix, $H$, for all $k$ is given by

$$
H = \begin{bmatrix}
B & 0 & 0 \\
0 & B & 0 \\
0 & 0 & B \\
\end{bmatrix}
$$

(4.25)

and the filter gain matrix can be recursively calculated using the following set of equations,

$$
K(k+1) = P(k+1|k)H^T [HP(k+1|k)H^T + R]^{-1}
$$

(4.26)

$$
P(k+1|k) = \Phi P(k|k) \Phi^T + \Gamma Q \Gamma^T
$$

(4.27)

$$
P(k+1|k+1) = [I - K(k+1)H] P(k+1|k)
$$

(4.28)
4.3.4 Three Phase Formulation

The three phase formulation of the synchrophasor-only dynamic state estimator is essentially the same as the positive sequence version. The two main differences are first, the representation of branch impedances as 3 by 3 impedance matrices and second, the size of all of the matrices and vectors used in the algorithm will triple in both dimensions. For simplification of the notation, the positive sequence version of the algorithm has been used throughout this dissertation. [2] describes, at a higher granularity, the details of the three phase formulation versus the positive sequence. Additionally, all of the examples in the remainder of this dissertation are using a three phase system with real three phase synchrophasor data.

4.3.5 Separating Real & Imaginary

In order to properly take into consideration the correlation of the real and imaginary errors of synchrophasor measurements as discussed in Section 3.1, the complex state & measurement equations and subsequently the filtering algorithm need to have their real and imaginary parts separated. Additionally, the state & measurement vectors must also be restated. Each of the parameters with separate real and imaginary parts will be denoted with the addition of the $s$ superscript.

The state vector can simply be a vertical concatenation of its real and imaginary parts.

\[
x_k^s = \begin{bmatrix}
    \text{Real}\{x(k)\} \\
    \text{Imag}\{x(k)\} \\
    \text{Real}\{x(k-1)\} \\
    \text{Imag}\{x(k-1)\} \\
    \text{Real}\{x(k-2)\} \\
    \text{Imag}\{x(k-2)\}
\end{bmatrix}
\]  

The same can be done for the measurement vector.

\[
z_k^s = \begin{bmatrix}
    \text{Real}\{z(k)\} \\
    \text{Imag}\{z(k)\} \\
    \text{Real}\{z(k-1)\} \\
    \text{Imag}\{z(k-1)\} \\
    \text{Real}\{z(k-2)\} \\
    \text{Imag}\{z(k-2)\}
\end{bmatrix}
\]
The observation matrix will also have separate real and imaginary parts.

\[
H^* = \begin{bmatrix}
B^* & 0 & 0 \\
0 & B^* & 0 \\
0 & 0 & B^*
\end{bmatrix}
\]  

(4.31)

where \(B^*\) is given by the following equation.

\[
B^* = \begin{bmatrix}
\text{Real}\{B\} & -\text{Imag}\{B\} \\
\text{Imag}\{B\} & \text{Real}\{B\}
\end{bmatrix} = \begin{bmatrix}
\Pi \\
gA + g_s \\
0 \\
bA + b_s
\end{bmatrix} \begin{bmatrix}
0 \\
-bA - b_s \\
0 \\
gA + g_s
\end{bmatrix}
\]  

(4.32)

The state transition matrix will follow similarly.

\[
\Phi^* = \begin{bmatrix}
\text{Real}\{\Phi\} & -\text{Imag}\{\Phi\} \\
\text{Imag}\{\Phi\} & \text{Real}\{\Phi\}
\end{bmatrix} = \begin{bmatrix}
3I & -3I & I \\
I & 0 & 0 \\
0 & I & 0 \\
0 & 3I & -3I & I \\
0 & I & 0 \\
0 & 0 & I & 0
\end{bmatrix}
\]  

(4.33)

The dimensions of the disturbance transition matrix will double as the number of measurements has doubled but will retain the same structure. The dimensions of all of the remaining parameters will also double in both dimensions. The measurement noise covariance matrix will become a tri-diagonal matrix which is a function of the angle of each of the entries of the measurement vector. The process noise covariance matrix will still maintain a single diagonal structure as well as the error covariance matrix. Despite all of these changes the filtering algorithm will be written in the same fashion by replacing each of the parameters with the separate real and imaginary parts. The same operations can be achieved for a three phase system as well.

### 4.4 Simulations Using Real Phasor Data

Now that the dynamic state estimator algorithm for a synchrophasor-based estimator has been presented it should be tested on a real system using real synchrophasor data. While
the experiments herein may be referred to as simulations, the data that is used is in no way simulated data. The data was captured from PMUs on Dominion Virginia Power’s EHV network. Substations and transmission lines have been renamed and renumbered in order to protect Critical Energy Infrastructure Information (CEII). Two examples are presented in this section. The first is a five minute data set from several substations in Dominion’s EHV network during a quasi-steady-state (blue sky) condition. The second example is a shorter data set from an oscillation at a nuclear power plant. The purpose of the two chosen examples is to demonstrate the efficacy and flexibility of the synchrophasor-only dynamic state estimator under different system operating conditions. Chapter 5 presents a discussion of the implementation that was used in order to perform these simulations.

4.4.1 Quasi-Steady State Data

The first example in this section is a quasi-steady-state operating condition from 3:30pm on August, 23, 2012 on a subsection of Dominion Virginia Power’s EHV Network. Fig. 4.4 shows the one-line diagram for the particular network subsection in question. The diagram is presented with substation level detail in order to help visualize the measurement redundancy in the substations. The sections of the bus which are colored represent sections which are monitored by phasor measurements. Sections that are left black are sections which are not monitored. However, reflected in the voltage measurement-bus incidence matrix is the fact that each substation is a single complex state variable. Three phase data is used for this and the next simulation and has been pre-validated for quality as discussed in the first chapter of the dissertation. Therefore, it can be observed from Fig. 4.4 that there are 24 (8∗3) state variables with 36 (12∗3) current phasor measurements and 45 (15∗3). This will dictate the size of the matrices used in the algorithm.

The results of this example are portrayed in Fig. 4.5, 4.6 & 4.7. Fig. 4.5 shows a plot of a small portion of the simulation for greater detail. The top plot shows the measured values and the optimal filtered estimate of the phase A voltage magnitude for Substation 1; in the bottom plot there is the phase A voltage angle. In both plots, the signals displayed in greyscale are the measured values and the signal displayed in bright green is the optimal filtered estimate. Visual inspection of the plots shows that the optimal filtered estimate is what intuition indicates. However, it should be noted that the phasor measurements are not calibrated so the estimate will fall somewhere in between the mean of the different voltage
magnitude signals. This plot demonstrates the ability of the dynamic estimator to accurately track the state of the power system during quasi-steady-state conditions even with small, random fluctuations in the operating condition. The plot of the angle is also an excellent reflection of the ability of the dynamic estimator follow the state of the system as it moves. Do note that the angle is changing so rapidly because the synchrophasor measurements have not been referenced to a particular location; they are in their raw form. The algorithms presented herein would work with both referenced and unreferenced phasor data, but for any synchrophasor estimator there is no reason to use referenced data.

Figure 4.4: Subsection of Dominion’s EHV Network
Figure 4.5: Substation 1 Phase A Voltage - Measured & Estimated Values
Figure 4.6: Voltage & Current Magnitude Observation Residuals
Figure 4.7: Voltage & Current Angle Observation Residuals
Fig. 4.6 & 4.7 show the observation residuals for the magnitude of the voltage & current measurements and the angle of the voltage & current measurements, respectively. The observation residuals are a key metric for the performance of the dynamic state estimator. In both cases, the plots demonstrate excellent performance of the algorithm in the presence of real synchrophasor data. If the phasors had been calibrated then the mean of each residual would be near or equal to zero. Because the phasors are uncalibrated (CT & PT errors are present), the average values are non-zero. These can also be associated with discrepancies in the true value of the impedances of the transmission lines and the value given in the model. However, the standard deviation of each residual is still a reflection of the performance of the algorithm and are on the order of $10^{-5}$ p.u.. The same principle applies to the residuals of the angle measurements. The mean of the residuals over the course of the simulation reflect the uncalibrated nature of the measurements. The key performance parameter is truly the standard deviation of the angle measurements. This too demonstrates very good performance.

4.4.2 Oscillation Data

This section presents the results for a second example on a more dynamic operation condition: that of an oscillation of a nuclear generating unit and an adjacent substation. Fig. 4.8 shows the one-line diagram for this example. Similar to the previous example, the substation level detail is shown in the one-line diagram is to demonstrate measurement redundancy. The elements that are colored in blue are measured by phasors and the elements that are colored in black are not measured. A single voltage signal from this example was used in the previous chapter for demonstrating the filtering algorithm on a single voltage signal. In this example there are 6 $(2 \times 3)$ complex state variables and 12 $(4 \times 3)$ complex voltage measurements and 3 $(1 \times 3)$ complex current measurements. Similar results are experienced with this simulation but it should be noted that there is no decrease in performance of the algorithm due to the dynamic nature of the operating condition in the example. Fig. 4.9 shows that the algorithm follows the dynamics of the oscillation very well and there is no lag in the filtered estimate relative to the measured values. Again, due to the uncalibrated nature of the phasor measurements, the peak values of the oscillation are different from one another. However, because the measurements are in phase with each other, this results in an observation residual that appears to have oscillatory properties itself. This is true in
both the voltage magnitude and the voltage angle. The observation residuals for the current measurements in this scenario have been omitted because there was only a single set of three phase current measurements. That particular set of current measurements provided observation of Substation 10 from Substation 9 and there were no other observations of Substation 10. Therefore, the current measurement residuals would carry little meaning here. Similarly, in the previous example, the residuals of the current measurements which observed the edges of the system (places where there is only the single, indirect observation) the residuals would practically zero always.

![Figure 4.8: Subsection of Dominion’s HV Network with Nuclear Unit](image)

This section has presented the results for two examples or real synchrophasor data for the implementation of the synchrophasor-only dynamic state estimator algorithm. The results demonstrated the performance that was expected considering the results presented in Chapter 2.
Substation 9 Phase A Voltage Magnitude

Substation 9 Phase A Voltage Angle

Figure 4.9: Phase A Magnitude Measured & Estimated Values
Figure 4.10: Voltage Magnitude Observation Residuals
4.5  Bad Data Detection & Identification

A clear addition to synchrophasor-only state estimation using the dynamic formulation is the ability of the estimator to find bad data. Because the dynamic state estimator forms a prediction of the next state, when the next set of measurements arrives, the estimator has a mechanism for comparison. There are many robust techniques for the detection and identification in the presence of bad data and many of them are required for dealing with multiple bad data especially in the case where there is interacting and conforming bad data. However, the purpose of this dissertation is not a rigorous study of robust estimation or bad data detection & identification. The purpose of this dissertation is the development and testing of a dynamic model and algorithms for the implementation of a synchrophasor-only dynamic state estimator and to demonstrate the propensity of the estimator to find bad data on a basic level. It is believed that future work should investigate the presence of more complex types of bad data as it deserves an entire dissertation study of its own.

4.5.1  Bad Data Detection using the $\text{Chi}^2$ Test

For this dissertation, the $\text{Chi}^2$ Test will be implemented for the detection of bad data and the largest normalized residual test will be used for identifying the bad data.

$$J(\hat{x}) = \sum_{i=1}^{m} \frac{(z_i - h_i(\hat{x}))^2}{\sigma_i^2}$$  \hspace{1cm} (4.34)

where $\hat{x}$ is the estimate of the state vector, $z_i$ is measurement $i$, $h_i(\hat{x})$ is the measurement $i$ as a function of the estimated state vector, $\sigma_i^2$ is the variance of measurement $i$, and $m$ is the number of measurements. While the above equation is the objective function for a weighted-least-squares estimator, because the synchrophasor-only estimator is linear, it can’t really be thought of as having an objective function. The formula can be slightly re-imagined to fit the equations from the dynamic state estimator in the previous chapter with the same effect of using the properties of the $\text{Chi}^2$ Distribution to test for bad data.

$$J(\hat{x}) = [z(k+1) - H\Phi\hat{x}(k|k)]^T R^{-1} [z(k+1) - H\Phi\hat{x}(k|k)]$$  \hspace{1cm} (4.35)

where $z(k+1)$ is the most recent measurement observation, $H$ is the noiseless connection
between the measurement vector and the state vector, $\Phi$ is the state transition matrix based on the quadratic model, and $\hat{x}(k|k)$ is the previous state vector corrected by the Kalman Gain. This becomes the observation residuals, or the difference between the predicted values and the observed values. $R$ is a diagonal matrix of measurement covariances for a complex formulation of the state estimator and is a block diagonal matrix of measurement covariances when the real and imaginary parts of the measurements are considered as separate measurements in the estimator.

After the value has been calculated, the $\chi^2$ test is performed by comparing it to the value for the $\chi^2$ distribution for the desired confidence level $p$ and $(m - n)$ degrees of freedom where $m$ is the number of measurements or observations of the state, and $n$ is the number of state variables. If the value calculated is greater than the value from the $\chi^2$ distribution then bad data is suspected to be present.

### 4.5.2 Identification using Largest Normalized Residual Test

The Largest Normalized Residual Test can be used to iteratively identify single bad data by comparing the largest residual (normalized by its covariance) to the other residuals. The first step is to compute the residuals. In the case of the dynamic state estimator the observation residuals will be used.

$$r = z(k+1) - H\Phi \hat{x}(k|k)$$  \hspace{1cm} (4.36)

Then normalize the residuals by dividing the absolute value of the residual by the corresponding element in the covariance matrix. The following is for a complex formulation where the covariance matrix is a diagonal matrix.

$$r_i^N = \frac{|r_i|}{R_{ii}}$$  \hspace{1cm} (4.37)

Functionally, the largest normalized residual test will be enacted upon the detection of bad data using the $\chi^2$ test and the measurement which corresponds to the largest normalized residual will be removed and the state recalculated. This will be repeated until the estimate passes the $\chi^2$ test.
4.5.3 Demonstration on Real Phasor Data

This section outlines results from a simulation using real synchrophasor data. Using the quasi-steady state case from the previous sections of this dissertation, simulations were ran using the base case data (assumed good quality and validated as described in Chapter 1), a case where two voltage measurements had 10% and 15% TVE added to their values, respectively, and the final case where removal of the bad data was facilitated by the $\chi^2$ and Largest Normalized Residual tests. Fig. 4.11 shows the location of the bad data on the one line diagram.

Figure 4.11: One-Line Diagram Showing Location of Bad Data
Table 4.1: Voltage Measurement Residuals (TVE) and Costs

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Case 1</th>
<th></th>
<th>Case 2</th>
<th></th>
<th>Case 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TVE</td>
<td>Cost</td>
<td>TVE</td>
<td>Cost</td>
<td>TVE</td>
<td>Cost</td>
</tr>
<tr>
<td>Sub 1 KV @ Line 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>6.17E-04</td>
<td>3.81E-03</td>
<td>1.70E-02</td>
<td>2.89E+00</td>
<td>5.96E-05</td>
<td>3.55E-05</td>
</tr>
<tr>
<td>B</td>
<td>8.93E-04</td>
<td>7.98E-03</td>
<td>8.59E-04</td>
<td>7.37E-03</td>
<td>9.02E-04</td>
<td>8.14E-03</td>
</tr>
<tr>
<td>C</td>
<td>8.10E-04</td>
<td>6.56E-03</td>
<td>8.83E-04</td>
<td>7.79E-03</td>
<td>7.95E-04</td>
<td>6.33E-03</td>
</tr>
<tr>
<td>Sub 1 KV @ Line 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1.33E-03</td>
<td>1.77E-02</td>
<td>1.64E-02</td>
<td>2.69E+00</td>
<td>7.75E-04</td>
<td>6.00E-03</td>
</tr>
<tr>
<td>B</td>
<td>1.51E-03</td>
<td>2.29E-02</td>
<td>1.51E-03</td>
<td>2.27E-02</td>
<td>1.50E-03</td>
<td>2.26E-02</td>
</tr>
<tr>
<td>C</td>
<td>2.28E-03</td>
<td>5.21E-02</td>
<td>2.42E-03</td>
<td>5.85E-02</td>
<td>2.31E-03</td>
<td>5.34E-02</td>
</tr>
<tr>
<td>Sub 1 KV @ Line 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>3.15E-03</td>
<td>9.92E-02</td>
<td>1.45E-02</td>
<td>2.10E+00</td>
<td>2.58E-03</td>
<td>6.66E-02</td>
</tr>
<tr>
<td>B</td>
<td>2.41E-04</td>
<td>5.82E-04</td>
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Table 4.3: Current Measurement Residuals (TVE) and costs

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### Table 4.4: Current Measurement Residuals (TVE) and costs (cont.)

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Tables on this and previous pages show the measurement residuals, costs, and total cost (Chi\(^2\) Test Results) for the three cases. The residual reflects total vector error between measured and estimated values. The Chi\(^2\) threshold for 99% confidence with 68 degrees of freedom is 138.

### Table 4.5: Chi\(^2\) Test Results - Threshold = 138

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Fig. 4.12 shows the measurement residuals for each of the three cases for all of the voltage measurements. The blue columns represent the base case where the original data was used. The red columns represent the case where a 10% and 15% TVE were added to phase A of two voltage measurements, respectively. Finally, the green columns represent the same case (bad data present) but with bad data detection and identification facilitating the removal of the bad measurements. Inferring from the plot, the statistical performance of the estimator between the first and third case are comparable because the bad data has been removed. It is clear not only that the red case has bad data present but which data is causing the most harm. The plots have also been grouped into A, B, & C groups and are distinguished by the alternating colored boxes. Notice the presence of bad data on A phase does contributes significantly to bias on A phase across the network. Fig. 4.13 shows the same information for the current measurements. Notice that the difference between the measured flows and estimated flows is only minimally affected by the presence and subsequent removal of bad voltage measurements.
4.6 Summary

This chapter has presented the quadratic prediction model applied to the synchrophasor-only dynamic state estimation problem. The state equation, measurement equation, and the filtering equations for a dynamic state estimator algorithm were developed. A discussion of a three phase formulation is given as the examples later in the chapter and dissertation are tested on three phase synchrophasor data. Also, the equations were rewritten to separate the complex state and measurement equations into their components so that the complex error model described in the first section of this chapter can be used. Two scenarios were used: the first was a quasi-steady-state (blue sky) operating condition and second, a low frequency oscillation at a nuclear plant. These two examples were chosen in order to demonstrate the flexibility of the dynamic state estimator under different operating conditions and the results of the experiments yielded very good performance. Finally, bad data detection and identification were presented and demonstrated as applications of the dynamic formulation.
Synchronized phasor measurements offer a paradigm shift in the way that transmission owners and operators monitor the electric grid. Many entities’ commitment to synchrophasor deployment has allowed them to step out of the realm of research and development and into one where they question how to extract value from their investment in synchrophasor technology. It is now being realized that not only is there value in engineering analysis and diagnostics but in real-time operations. With the hope of continued innovation in this direction, the work contained in this dissertation has laid the groundwork for the front end of an Energy Management System which is driven purely by synchrophasor data. Chapter 3 presented an effective data conditioning algorithm for improving the quality of the streams of PMU data before they reach a downstream linear estimator. Chapter 4 develops a synchrophasor-only dynamic state estimator which allows for a proper mechanism for bad data detection and identification at the state estimation level. The dissertation will conclude with a presentation of ideas on how to use this technology to continue to improve the status quo of real-time operations and prepare for a future where our Energy Management Systems are driven less and less by data from SCADA systems.

5.1 Investing in the Future of the Grid

It is important to not only address the benefits of network applications which are driven by a purely synchrophasor measurement base but to address how such a transition could
practically take place. Inciting change in an industry such as the electric utility industry requires much more than just technological developments and achievements. In many ways, even the market (the main mechanism for change in a capitalist society) for many of the products used by the electric power industry cannot drive the innovation required to keep up with the growing and ever-changing demands of the electric grid; there are simply too few producers and too few consumers to make innovation profitable. Vendors are simply locked in positions where they cannot afford to take risks for the betterment of the nation’s critical energy infrastructure. When change does happen it is often mandated by regulations of governing entities, however, it comes at a price. The strain that the overhead of regulation puts on utilities is resource intensive and can result in more time being dedicated to documentation than to innovation. That is why fundamental shifts or step changes in the way things such as real-time operation are executed are so slow to change and why a promise of value in the future is not enough to warrant investment presently.

The way to make the changes necessary to carry the electric infrastructure reliably into the future is to strategically leverage technologies such as those discussed in this dissertation to demonstrate added value in the short, middle, and long-term. Investments in technologies such as synchrophasors and synchrophasor based applications cannot gain the traction required to carry them into the future (where they will truly shine) without the ability to add value today. In a perfect world, the critical measurements (voltages and currents) provided to the EMS should all be synchrophasors, the state estimator should solve at the reporting frame rate of the PMUs and never diverge or crash, and downstream network applications capture the full potential of high resolution, time synchronized observations of the electric grid. The following sections outline a vision for how to achieve this by presenting ideas for added value not only in the distant future but for today, tomorrow, and the next day. This can be accomplished with a staged solution that first establishes a foundation or platform which creates a high confidence, high resolution, and highly available picture of the system (monitored by synchrophasors) and then use it to complement and augment traditional EMS functionality. Then, as the synchrophasor footprint and system demands grow, build on top of the foundation a suite of network applications to provide information to real-time operations that is not available with current technologies. Then, conclude by adding applications which can fully, yet more accurately and completely, perform the same duties as the current EMS network applications. Therefore, no step changes in technology or policy ever have to be made. Innovation can become organic because it will quickly add value at each step along
the way, all the while building confidence in the technology. And in the end, the systems can then be leveraged at the same time or either could serve as an operation backup.

5.1.1 Short-Term - A Redundant & Complementary Backup EMS

All EMS systems must be able to operate in a state of high availability. This means that redundant systems must be in place, ready to take over at a moments notice in the case of a failure of one or all parts of the production system. The backup system is typically identical to the production system and its only purpose is to function when the other system is down. Despite this redundancy, there are still certain types of failures which traditional EMS algorithms are vulnerable to. These parallel directly with many of the vulnerabilities of a non-linear state estimator as the state estimator is at the heart of the EMS. With a sufficient synchrophasor footprint, an additional redundant system can be built to provide backup in the case of these certain types of failures in the production EMS. However, the backup systems that are currently in place are fundamentally different than one which is driven by phasor measurements. This provides many advantages to this because it allows for more separate/parallel redundancy.

To establish context outside of the power system, consider a situation where a military soldier and his team are deployed for a night mission. Vision is critical for the success of the mission but the nature of the mission prevents humans from innately being able to see what is happening. Therefore, technology has been used to help augment soldiers’ capabilities in the field. This parallels the operation of the electric grid in that human visibility alone cannot properly monitor the electric grid. Where a soldier and his team may use night-vision technology, the electric grid operators use SCADA and EMS technologies. However, soldiers are often equipped with backup systems which are not simply duplicates of systems they already carry. In addition to night-vision, they may also be equipped with technologies which visualize heat signatures and radar tracking. These fundamentally different ways to monitor the soldiers surroundings can replace each other in certain scenarios but their true value is in complementing each other. Similarly, the true value of painting an optimal picture of the state of the system with synchrophasors versus painting a similar picture with SCADA data is not just one of redundancy but one of complementing an augmenting. This is similar to the reason that protection engineers will specify that backup relays be from a different manufacturers than the primary relays: it provides a different perspective and
further mitigates single-point-of-failure in the overall system.

A synchrophasor-only solution and a SCADA-based solution will typically only share two things in common. That is, they are both measuring a signal from the same instrument transformers and they will share the same fiber network (at the physical layer) to connect to the central operations. The remainder of the infrastructure and algorithms are not just parallel but also fundamentally different from each other. This includes, but is not limited to:

**The algorithms which measure the raw data at the substation level** - High resolution, time synchronized voltage and current measurements versus asynchronous real and reactive power flows.

**The device which concentrates the data at the substation level** - PDC versus RTU.

**The physical layer of communication to central operations** - Not all SCADA traffic is over fiber but all PMU traffic is.

**The communications protocol used** - IEEE C37.118 vs SCADA protocols.

**The central data concentration** - Super PDC versus Central RTU.

**The data conditioning and processing algorithms** - Linear state estimation which cannot diverge versus traditional non-linear techniques which can and do diverge. Linear state estimation can also provide a new state estimate as the same frame rate that the phasors are reported.

Therefore, an initial implementation of the algorithms described in this dissertation can provide immediate value to real-time operations by providing a complementary backup system to the production EMS state estimation application. This way, operators do not lose observability of critical parts of the network in the event of an EMS failure or state estimator crash. In the case of the deployment at Dominion Virginia Power, the entire EHV network and some of the lower 230kV and 115kV network are included in the redundancy.
5.1.2 Mid-Term - A Platform for Analytical Network Applications

Continuing to build upon the idea of complementing and augmenting the functionality of the EMS system, the next step is to leverage the high confidence picture of the system provided by the synchrophasors and aforementioned algorithms and use it as a platform for network application development. Establishing the synchrophasor-only state estimator as a platform for performing analytics which downstream applications can leverage is key for building the future Energy Management Systems. This has been the overarching goal of the work in this dissertation.

In many ways, the idea of an application platform reflects the ideas in [1] as several applications were built upon the static three phase linear state estimator. While the previous section focuses on the complementary characteristics of synchrophasor-only EMS front-end, this section will focus on how an implementation described in this dissertation can also augment traditional EMS systems by increasing their functionality and improving the quality of the state estimate. Additionally, the unique characteristics of a phasor-only data set can be leveraged to develop network applications which cannot be duplicated with current EMS technologies.

Several network applications that were developed as part of a DOE Demonstration project [1] will be the first to use the output of the linear state estimator to provide new, valuable functionality to real-time operations. For three phase implementations, instrument transformer calibration application [4] can be used to derive the ratio correction factor (RCF) and phase angle correction factor (PACF) of the instrument transformers. This information can not only be used to improve the quality of the output of the linear state estimator but can also be used to improve the quality of the raw SCADA data and therefore the traditional state estimator. Additionally, a three-phase implementation can yield a view of the system that current EMS systems are not capable of; this is the sequence components of the complex voltages and currents across the network. Unbalanced conditions in the network is a power quality issue that is of great value to be able to diagnose and troubleshoot if it is present. Additionally, real-time operations can make use of this because continued presence of negative sequence current can damage company assets such as the heating of generator rotors.

Another way to augment the functionality of current EMS implementations with the high confidence results of the PMU-only EMS front-end is to map the results of the linear state
estimator into the traditional EMS system. Once there, two things are possible. First, the values of the PMU-only estimator can be displayed next to the raw SCADA measurements and non-linear state estimation results. This will simultaneously build operator confidence in the phasor data (which is critical to the future) and help to troubleshoot discrepancies between SCADA measurements and traditional state estimator results. The second possibility is for the results of the linear state estimator to be included as measurements in the traditional state estimator. A high level of confidence can be assigned to these values with the intent of improving the quality and reliability of the output of the traditional state estimator. Additionally, improvements to the network model used by the traditional EMS can be improved by empirically determining the impedance of transmission lines using the high confidence picture provided by the linear state estimator. This is also the optimal time for introducing oscillation detection and monitoring into real-time operations.

They central idea to adding value in the mid-term is that whether the goal is to augment the capabilities of the current EMS or to build entirely new functionality to yield new information and perspective, the data source will be the output of the linear estimator. At this point there will be a linear relationship to additional synchrophasor deployments and the value that can be extracted from them; as the synchrophasor footprint grows, the information that can be taken from the applications will grow as well.

5.1.3 Long-Term - Adopting the Full Capabilities of the EMS

Ideally, the synchrophasor EMS front-end and suite of applications will have, while still serving as a complementary backup and augmentation of the traditional EMS, evolved into a stand-alone entity with its own intrinsic values and performance. Since it will have been established as an asset to real-time operations it then makes sense to take whatever functionality remains in the traditional EMS and duplicate it on the synchrophasor side. Again, looking for the short-term value add, any functionality that is migrated from traditional EMS implementations must be improved upon using the innate advantages of PMU measurements. Simply duplicating functionality is never valuable or desired.

One of the most valuable network applications used in traditional EMS systems is the real-time contingency analysis. Real-time contingency analysis is a steady state analysis tool to determine the security vulnerabilities of the network. However, it is a very CPU intensive process because of the number of simulations that it will have to run (one for each element in
the model). It is considered real-time because it runs at the same speed (or lower) than the state estimator solves and does so automatically. However, this speed is typically on the order of minutes and 30 seconds is an optimal performance. As computing power increases and becomes cheaper, super computers with hundred or even thousands of cores will be readily available and can be used to implement this type of steady state analysis at much higher speeds. This will also be made possible by the high time resolution of the synchrophasor data. The contingency analysis will always have the most up-to-the-millisecond accurate portrait of the system provided to it by the linear state estimator. With the ability to perform \((N - 1)\) analysis at these high speeds an \(((N - 1) - 1)\) or \((N - 2)\) analysis can be performed in the same time that an \((N - 1)\) analysis is currently performed. This clearly represents a step forward in terms of operating securely and situationally aware.

The traditional EMS will most likely never be fully phased out as it offers (much in the same way that synchrophasors do) a different perspective on the state of the system. Additionally, there will most likely be opportunities to fully integrate the two solutions. While this may appear to be the optimal decision, there is a tremendous amount of operational value in keeping them separate; the two systems in tandem provide much more value than either one separately or fully together. Even if the synchrophasor system is used to operate, the traditional EMS protects utilities against vulnerabilities such as a mass loss of GPS synchronization. Additionally, verification of contingency analysis results between the two systems will be invaluable for operating securely and troubleshooting each of the systems while in production.

5.2 Conclusion

This dissertation has attempted to develop and implement algorithms to serve as the front-end of an EMS system with is driven purely with synchrophasor data. This was accomplished with a two-fold solution: first, data conditioning algorithms were developed to improve the quality and reliability or the phasor streams coming in from the substation before they ever make it to the linear state estimator. Then, a synchrophasor-only dynamic state estimator which can truly track the dynamics of the state of the power system and provide a proper mechanism for bad data detection and identification at the state estimation level was developed and implemented. The resulting combination yields a high confidence highly available
portrait of the electric grid under all operating conditions. This was accomplished by ob-
serving that the state of the power system changes quadratically for changes in loading with
constant power factor. This assumption is perfectly suited for format and the fast reporting
rates of synchrophasors. Finally, in an attempt to drive usage of these technologies forward,
a vision was presented for how to use them to add value not only in the ideal, long-term
scenario, but in the short and near term. This allows for continued innovation by staging
investments with quick returns all while working towards the ultimate goal of operating the
electric grid to the highest standards possible.
Bibliography


