A Variable Sampling Interval Chart For A Combined Statistic

by

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(ABSTRACT)

This thesis is an extension of the work on variable sampling charts (VSI) for monitoring a single parameter. An attempt is made to develop a chart which can simultaneously monitor both the process mean and process variance. The chart is based on a statistic which combines both mean and variance. After developing such a chart variable sampling intervals are introduced and it is evaluated against alternative methods of monitoring mean and variance with variable sampling intervals. The statistic chosen is an approximate statistic and simulation studies are performed for the evaluation. The results are at times counter-intuitive thus an analysis of the properties of the chart is made and explanations are provided.
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CHAPTER I INTRODUCTION

The twentieth century may well be termed the age of mass-production. Mass production has made the luxuries of the past available to today's common man. The need to provide a quality product at a moderate price has contributed to the rise of quality control as a discipline. Ensuring that a process is in control so that the desired product quality is obtained and doing this at a minimum cost is the main aim of quality control.

A process is considered to be in control if the process parameters lie within a prescribed set of limits. As some of the process parameters may not be known, it is necessary to examine the output of the process to obtain an estimate of the process parameters. Instead of examining every item produced, which is time-consuming as well as expensive, representative samples are taken.

Traditionally, $\bar{X}$ and $R$ charts, also known as Shewhart charts, are used to display the information obtained from the sampling (Grant, 1952). Shewhart charts check whether the sample point lies within $\pm K$ standard deviation units from the mean. An $\bar{X}$ chart indicates the variation of the sample mean and verifies the stability of the process mean. $R$ charts are used to give an indication of the variability of the process, i.e. the deviations within the sample itself. This variation can be attributed to two main causes:
1. Chance causes which are inherent in any process. The variation due to this type of causes display statistical regularity.

2. Assignable causes which can be pinpointed as a source of variation. It appears as a disturbance in the pattern of inherent variation.

A point outside the control limits in either chart is taken as an indication of the process being out of control. S charts can be used in place of R charts. S charts are charts which signal when the standard deviation falls out of a specified range.

Shewhart charts can be modified by introducing variable sampling intervals. A sample is taken after a short interval or a long interval, depending on the location of the sampling mean. This method is found to respond faster to shifts in process parameters than the conventional method.

Another method of determining changes in the process mean and variance is to use a combined statistic which will reflect the changes. This method too has given good results in certain cases.

Extending the concept of variable sampling intervals to separate $\bar{X}$ and $S$ charts results in a very complex analysis as it is very likely that there would be a situation where conflicting signals arise from the charts. One way of overcoming this difficulty is to use a variable sampling interval chart which is based on a statistic which combines both $\bar{X}$ and $S$.

The primary aim of this thesis is to evaluate a chart based on an approximate combined statistic, Repko's Chart, against alternative sampling procedures. The criteria used for evaluation would be the Average Number of Samples to Signal (ANSS) and the Average Time to Signal (ATS). From previous studies it was found that certain charts give better results under certain conditions, while they do not perform as well under other conditions. Identification of the situations where Repko's chart may be utilized is one of the goals of this thesis. In addition, the properties of Repko's chart will be determined.
CHAPTER II LITERATURE REVIEW

2.1 Introduction

This chapter provides a brief review of the literature relevant to analysis and the subsequent modification of the Repko Control Chart. This study includes the description of the various VSI Control Charts surveyed, the types of combined statistic charts which pertain to the research and some basic statistical concepts which give insights into the behaviour of the Repko Chart.

2.2 Combined Control Charts For Mean And Variance

When the objective is to control both $\mu$ and $\sigma^2$, the standard practice of having separate charts for mean and variance may not be the most efficient method available. Rather than treat the two problems as if they were unrelated, it may be advantageous to use a procedure to control both $\mu$ and $\sigma^2$ simultaneously.

Reynolds and Ghosh (1981) have discussed a number of methods for setting the control limits in such situations. Of specific interest to us is the sum procedure which is described below.
Assume samples of size \( n \) are taken from a population which is normally distributed. Let the mean and standard deviation of the \( i \)th sample be given by \( \bar{X}_i \) and \( S_i \) respectively. Define 

\[
U_i = n(\bar{X}_i - \mu_0)^2/\sigma^2 \quad \text{and} \quad V_i = (n - 1)S_i^2/\sigma^2.
\]

Note that \( U_i \) is distributed according to a Chi-squared distribution with 1 degree of freedom while as \( V_i \) is distributed as a Chi-squared variable with \( n - 1 \) degrees of freedom.

One method of combining \( U_i \) and \( V_i \) is to use the sum \( U_i + V_i \). Since the sum has a Chi-square distribution with \( n \) degrees of freedom when \( \mu = \mu_0 \) and \( \sigma = \sigma_0 \), a signal is produced if

\[
U_i + V_i \geq \chi^2_{1-\alpha}(n)
\]

where \( \alpha \) is the probability of Type I error. A chart based on this statistic shall henceforth be referred to as the \( \chi^2_n \) chart.

It was determined by Reynolds and Ghosh that the sum procedure performed better if the change in the process involved variance only or a combination of mean and variance. The separate chart procedure performed better if the change in the process involved primarily the mean.

A point worth noticing here is that with the \( S^2 \) chart the \( \pm 3\sigma \) limits are not justified as the distribution of \( S^2 \) is skewed. Thus, having symmetric limits results in a bias which causes samples with low standard deviations to be accepted. This problem does not arise if the control chart is based on the above statistic.

### 2.3 Variable Sampling Interval Charts

The conventional method of using a control chart to monitor a process involves taking sample of a fixed size at uniform time intervals. After each sample a particular attribute (or a number of attributes) are measured and a statistic (or statistics) are calculated. If the statistic falls in a specific re-
region then a signal is given. A signal is interpreted as a change in the process distribution and appropriate action is taken to rectify the situation. If the statistic does not fall in the specified region then no action is taken and the process is allowed to continue until the next sample is taken.

Although the usual practice is to have a fixed sampling interval between samples, a logical extension of this is to vary the interval depending on the location of the observation with respect to the signal boundaries.

A Markovian sampling policy is used in the sampling plans mentioned below wherein the length of the sampling interval depends only on the previous statistic. If the statistic is close to the region producing a signal then it seems reasonable to sample after a shorter interval of time. Along the same lines, if a statistic is close to the target value then a longer interval between samples would be in order.

There have been quite a few papers suggesting methods of dealing with variable sampling rates. Very few deal with the properties of such a procedure. A few are discussed below.

**Variable Sampling Interval \( \bar{X} \) Charts** Reynolds, Armin and Arnold (1988) deal with the properties of such a procedure. They have found that the average time to sample (ATS) and the average number of samples to signal (ANSS) are lower than the corresponding figures for the standard Shewhart charts. A brief discussion of their results is given below.

Let \( q \) be the probability of a signal occurring when a sample is taken. If \( N \) is the number of samples to signal, then \( N \) has a geometric distribution with parameter \( q \) when the process does not change. Thus, the ANSS is

\[
E(N) = \frac{1}{q}.
\]  

[2.3.1]

The variance is given by the following expression
\[ Var(N) = \frac{1 - q}{q^2} \] \hspace{1cm} \[2.3.2\]

Let the minimum possible interval length be \( t_1 \) and the maximum possible interval length be \( t_2 \).

Let there be a finite number of intervals \( d_1, d_2, \ldots, d_k \) where \( d_1 < d_2 < \cdots < d_k \) and \( t_1 \leq d_i \leq t_2 \). Define \( p_j \) as the probability of the interval length being of length \( d_j \) and \( q \) as the probability of a signal being produced i.e. a sample mean lying outside the control limits. Let \( R_i \) be the length of the \( i^{th} \) interval.

The distribution of \( R_i \) must be the conditional distribution of an interval of a particular length given that there is no signal. Thus, the following result is obtained.

\[ E(R_i) = \frac{\sum_{j=1}^{k} d_j p_j}{(1 - q)} \] \hspace{1cm} \[2.3.3\]

The ATS can be written as follows

\[ E(T) = E(N)E(R_i) \] \hspace{1cm} \[2.3.4\]

Substituting,

\[ E(T) = \frac{\sum_{j=1}^{k} d_j p_j}{q(1 - q)} \] \hspace{1cm} \[2.3.5\]

\[ VAR(T) = \frac{\sum_{j=1}^{k} d_j^2 p_j}{q(1 - q)} + (1 - 2q) \] \hspace{1cm} \[2.3.6\]

Reynolds and Arnold have demonstrated that a variable sampling chart with two intervals gives the best results.

CHAPTER II LITERATURE REVIEW
At this juncture it is important to note that the expressions stated above give the ATS for any $\mu_1$ under the simplifying assumption that this value of $\mu$ is the process mean from time zero onward. The more realistic situation would be the case where the process mean shifts at a random time in the future. The new measure of the chart performance would be the adjusted time to signal, $T'$. The adjusted ATS is given by

$$E(T') = E(Y) + E(N - 1)E(R)$$ \[2.3.7\]

where

$T'$ = time from the process shift until a signal
$Y$ = time from process shift until the next sample
$N$ = number of samples after shift until a signal
$R_i$ = length of the $i^{th}$ interval

A logical assumption would be that $Y$ is distributed uniformly over the particular interval in which the shift occurs. The final expression for expected value of $T'$ is

$$E(T') = \sum_{j=1}^{k} d_{N}^{2} p_{nj} + \frac{(1 - q_{1})}{q_{1}} \sum_{j=1}^{k} d_{N}^{p_{nj}}$$ \[2.3.8\]

**Cusum Charts with Variable Sampling Intervals** A standard cusum chart for controlling the process mean involves taking samples at fixed length sampling intervals and the use a control statistic based on a cumulative sum of differences between the sample mean and the target value. Such charts have been found to be much more efficient than the simpler Shewart $\bar{X}$ charts in detecting small and moderate shifts in the process mean.

**CHAPTER II LITERATURE REVIEW**
Consider the statistic \( C_j = \sqrt{n} (\overline{X}_j - \mu_0)/\sigma \) where \( \overline{X}_j \) is the sample mean. A cusum chart accumulates deviations of the sample mean which are more than \( k \) standard deviations from \( \mu_0 \). The parameter \( k \) is called the reference value of the cusum chart. In order to detect positive changes in \( \mu \) using a cusum chart, the statistic \( S_j \) is used where

\[
S_j = \max\{S_{j-1} + (C_j - k), 0\}
\]

[2.3.9]

The cusum chart signals when \( S_j \geq h \).

The parameters \( h \) and \( k \) are determined to achieve certain properties. The recommended value for \( k \) is \( \delta/2 \) where

\[
\delta = \frac{\sqrt{n} (\mu_1 - \mu_0)}{\sigma}
\]

[2.3.10]

In order to detect negative shifts in \( \mu \) similar statistics are used.

Amin, Reynolds and Arnold (1988) consider an extension of cusum charts where variable sampling intervals are introduced. In order to incorporate the VSI feature into the cusum chart the cusum statistic is modified as shown below

\[
S_j = \max\{S_{j-1}, 0\} + (C_j - k)
\]

[2.3.11]

Depending on whether the value of \( S_j \) is large or small, the sampling interval is short or long. As mentioned earlier, a Markovian sampling policy is followed here. Thus, it is possible to express a sequence of transitions as a Markov chain.

Suppose the interval \( (-\infty, h) \) is divided into \( r \) intervals, \( E_1, E_2, \ldots, E_r \), where each interval corresponds to a state of the Markov chain. There is an absorbing state \( A \) corresponding to \( S_j \) falling into the interval \([h, \infty)\)

The transition matrix \( P \) for the Markov chain can be written as

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\[ P = \begin{bmatrix} Q & (I - Q)1 \\ Q' & 1 \end{bmatrix} \]  

[2.3.12]

where \( Q \) is the submatrix of \( P \) corresponding to the \( r \) transient states. \( Q' \) is a 0 vector of dimension \((r \times 1)\). 1 is a unit vector of dimension \((r \times 1)\).

Define the matrix \( M \) as follows

\[ M = (I - Q)^{-1} = [M_{ij}] \]  

[2.3.13]

It can be shown that \( M_{ij} \) is the expected number of times the process is in transient state \( E_j \) before absorption into state \( A \), given that the process starts in state \( E_i \). Thus, the \( ANSS \) of the cusum chart, given that the process starts in state \( i \) is

\[ E(N_i) = \sum_{j=1}^{r} M_{ij} \]  

[2.3.14]

The vector of \( ANSS \) values \( E(\overline{N}) = M\overline{1} \) gives the expected number of signals corresponding to various initial states.

The \( ATS \) is the sum of time spent in each transient state prior to absorption. Following a procedure similar to the one shown above, the vector of \( ATS \) is given by

\[ E(\overline{T}) = M\overline{b} \]  

[2.3.15]

where \( \overline{b} \) is the vector of time intervals in each state.

The proof has been omitted for the following results

\[ Var(\overline{N}) = (2M - 1)E(\overline{N}) - (E(\overline{N}))^2 \]  

[2.3.16]

\[ Var(\overline{T}) = MB(2M - 1)\overline{b} - (M\overline{b})^2 \]  

[2.3.17]
Discussions on the adjusted time to signal are omitted and are discussed by Amin, Reynolds, and Arnold (1987).

**Variable Sampling Intervals for Multi-Parameter Charts** Chengalur, Arnold, and Reynolds (1988) consider the case where separate charts are used for monitoring mean and variance as well as the case where a single combined statistic is used. The separate charts used here are the VSI $\bar{X}$ and the VSI $S^2$ charts. The combined statistic mentioned here is the Reynolds-Ghosh statistic discussed earlier. A brief summary of the results obtained is given.

- Variable sampling with two widely spaced intervals is uniformly and substantially better than the fixed sampling interval procedures in case of both the combined statistic as well the separate charts.

- Both of the variable sampling procedures have a substantially smaller adjusted time to signal for small to moderate shifts in $\mu$ and $\sigma$. For very large shifts, the adjusted time to signal is smaller for the fixed interval than for variable sampling interval, but the difference is not very significant.

- Larger sampling interval widths give better results in terms of the $ATS$. However, when the adjusted time to signal is considered the fixed sampling procedures outperform the VSI procedures for large shifts in $\mu$ and $\sigma$.

- When comparing the two variable sampling interval procedures one chart does not clearly out-perform the other. The VSI procedure applied to separate charts performs well and is flexible. By varying the intervals on the individual charts it is possible to make a chart more sensitive to one statistic than the other.

- The VSI combined statistic chart has a smaller adjusted time to signal than the VSI separate chart procedure except when there is a shift in $\mu$ only.

CHAPTER II LITERATURE REVIEW
2.4 Process Capability Plot

A Process Capability Plot is a graphical method for studying process behaviour. There are many approaches to developing a Process Capability Chart. The Process Capability Plot developed by Repko (1986) is based on the distribution of ordered pairs \((S, \bar{X})\) obtained from random samples of the process. The plot can be used to evaluate the process stability and the process capability.

In this section only the principle behind Repko's chart will be discussed. A detailed description and analysis will follow later.

From the assumption of normality of the population, the following properties hold:

1. \(\bar{X}\) is normally distributed with mean \(\mu\) and variance \(\sigma^2/n\).

2. For large \(n\), \(S\) is approximately normal with mean \(\sigma\) and variance \(\sigma^2/2n\) (Duncan, 1974).

3. \(\bar{X}\) and \(S\) are independent random variables.

Standardizing by:

\[
U' = (\bar{X} - \mu)/\sigma/\sqrt{n} \quad \text{and} \quad V' = (S - \sigma)/\sigma/\sqrt{2n}.
\]

The ordered pair \((U', V')\) is distributed in a standard bi-normal distribution. Thus, \(R^2 = U'^2 + V'^2\) is approximately chi square distributed with 2 degrees of freedom. This is similar to the equation of a circle. Thus,

\[
P(U'^2 + V'^2 > R^2) = P(U'^2 + V'^2 > \chi^2_2)
\]  \[2.4.1\]

From the above equation it can be seen that the probability of a point falling within a circle of a specified radius is a function of the radius of the circle.

The above equation can be written as follows:

CHAPTER II LITERATURE REVIEW 11
If $\bar{X}$ was plotted against $S$ then from the above equation it can be shown the bounding contour is an ellipse. If the Process Capability Plot is constructed with a unit on the $S$ axis equal to $\sqrt{2}$ times the unit on the $\bar{X}$ axis, then the bounding contour is a circle with its center at $(\mu, \sigma/\sqrt{n})$.

The radius of the circle $R$ is given by:

$$R = \sqrt{\chi^2_2 \times \frac{\sigma}{\sqrt{n}}}$$  \hspace{1cm} [2.4.3]

In the Process Capability Plot the target value for the process mean is taken as $\mu$. $\sigma$ is usually taken to be tolerance width divided by 6. The rationale behind choosing such a value for $\sigma$ is that the $\pm 3\sigma$ limits will lie within the the specified tolerance range if the variance is not larger than the above value.

It is necessary to point out that Repko has assumed that $S$ behaves as a normal random variable while in reality $S$ approximates normal behaviour only if the sample sizes are large (more than 30). As such large sample sizes are unusual the validity of Repko's method will have to be checked for smaller sample sizes.

### 2.5 The Chi-Squared Distribution

An understanding of the Chi-squared distribution would be very useful in gaining insights to the problem of obtaining a combined statistic and the graphical representation of the statistic. Knowledge of the chi-squared distribution is essential in developing simplifying approximations.
A geometrical approach is used to obtain the density function and explain properties of the Chi-Square distribution (Kendall, 1963).

Consider the sum of squares of \( n \) independent \( N(0,1) \) variables. The joint distribution of the \( n \) variables is given by:

\[
dF = \frac{1}{2\pi^{n/2}} \exp\left( -\frac{1}{2} (x_1^2 + x_2^2 + \cdots + x_n^2) \right) dx_1 \cdots dx_n. \tag{2.5.1}\]

The distribution of \( z = x_1^2 + x_2^2 + \cdots + x_n^2 \) is desired.

From the above equation we see that the density is constant along the surface \( z = \text{constant} \) which is the equation of an \( n \)-dimensional hypersphere. The frequency function of \( z \) is then an integral of the constant density between the hyperspheres \( z \) and \( z + dz \), i.e., is proportional to \( e^{-\frac{z}{2}} \) times the volume of the hypersphere, which itself is proportional to the \( n \)th power of radius. Substituting \( z = P^2 \), we get

\[
dF = k \exp\left( -\frac{1}{2} P^2 \right) \frac{d}{dz} P^n dz. \tag{2.5.2}\]

On evaluation of the constant,

\[
dF = \frac{1}{2^{n/2}\Gamma\left(\frac{n}{2}\right)} e^{-1/2 z^{(n-2)/2}} dz. \tag{2.5.3}\]

Hald (1952) has given the following expression for the cumulative distribution function

\[
P(\chi^2) = \frac{1}{2^{n/2}\Gamma\left(\frac{n}{2}\right)} \int_0^{\chi^2} x^{(\frac{n}{2}-1)} e^{-x/2} dx \tag{2.5.4}\]
Suppose there were $p$ homogeneous restrictions of the type

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = 0. \quad [2.5.5]$$

The variables $x$ will be constrained to lie on $p$ hyperplanes. Thus the hypersphere of constant density will be reduced by $p$ dimensions to a hypersphere of $n - p$ dimensions. The distributions under these circumstances will be as before, but with $n - p$ replacing $n$.

### 2.6 The Non-Central Chi-Squared Distribution

When either the mean or the variance of the process changes, the statistics discussed earlier are no longer described by the central Chi-squared distribution. The non-central Chi-squared distribution is used to deal with cases where the process parameters have shifted.

If a variable $Z$ is normal $(c, 1)$, then $Z^2$ is said to be distributed as non-central $\chi^2$ with 1 degree of freedom and a non-centrality parameter $c^2$. Extending this definition to $n$ dimensions where $Z_1, Z_2, \ldots, Z_n$ are each distributed $(c_n, 1)$, we have $\sum Z_i^2$ distributed as a non-central $\chi^2$ with non-centrality parameter, $\lambda = \sum c_i^2$ and $n$ degrees of freedom. This can be written as follows,

$$\chi^2 = Z_1^2 + Z_2^2 + \cdots + Z_n^2 \quad [2.6.1]$$

A non-central $\chi^2$ with $n$ degrees of freedom can be represented as the sum of a non-central $\chi^2$ with one degree of freedom and the same parameter $\lambda$ and a central $\chi^2$ with $(n - 1)$ degrees of freedom, where the two variables are mutually independent. An intuitive explanation is attempted. A rigorous proof is given by Lancaster (1969).

The surface of constant density of a non-central chi-squared distribution is again an $n$ dimensional hypersphere but this time with its center at $(c_1, c_2, \ldots, c_n)$. We are only interested in the distribution of the sum of the squares. In other words, we are interested only in the probability mass contained
in a $n$ dimensional hypersphere located at the origin. It can be seen that the coordinates of the initial hypersphere does not affect the probability mass as long as the distance from the origin is constant. Thus, an orthogonal transformation can be performed such that the center of the sphere lies along one of the axes. This in turn is equivalent to having 1 non-central chi-squared variable and $n - 1$ central chi-squared variable.

Patnaik (1949) has given an expression for the density function,

$$g(w) = \frac{e^{-\frac{1}{2} w \sigma^2 - \frac{1}{2} w \frac{1}{2} (n-2)}}{2^{\frac{1}{2}} n \Gamma\left(\frac{1}{2} \cdot n\right)} \left\{1 + \frac{1}{n} \left(\frac{w\lambda}{2}\right) + \frac{1}{n(n+2)} \left(\frac{w\lambda}{2}\right) + \cdots \right\} \quad [2.6.2]$$

As this expression is difficult to integrate, a numerical integration is usually performed to obtain the cumulative probability distribution function.
3.1 Description Of The Process Capability Chart

In his paper, Repko has not discussed the principle behind the Process Capability Chart. An attempt is made to reason out the method. As stated earlier, the maximum standard deviation the process is permitted to have is determined by the tolerance range. In addition, the prediction circle is obtained by drawing a circle of a specified radius on a plot where a unit on the $S$ axis is equal to $\sqrt{2}$ units on the $\bar{X}$ axis. After the prediction circle is drawn, tangents to the prediction circle are drawn from the points indicating the tolerance limits, A and B (Fig. 1).

The geometry of the region can be explained as follows:

- If the sample has zero standard deviation and is at the upper or lower values of the tolerance specification then it lies on the upper or lower tolerance boundaries which is acceptable as there is no point in the sample which lies outside the tolerance range.
Figure 1. Process Capability Plot
• The boundary of the region intuitively seems to be concave. If the boundary of the region was convex, then a point with standard deviation greater than zero and a mean value outside the tolerance range would be acceptable. A conservative characterization of the boundary would be a straight line.

3.2 Density Function

As the sum of $U'$ and $V'$ is distributed as a chi-square with 2 degrees of freedom, the expression obtained earlier by Hald (1952) can be used.

$$P(\chi^2) = \frac{1}{2^\frac{n}{2}\Gamma\left(\frac{n}{2}\right)} \int_0^{\chi^2} x^\frac{n}{2} e^{-\frac{x}{2}}dx$$ \[3.2.1\]

Putting $n = 2$,

$$P(\chi^2) = \frac{1}{2\Gamma(1)} \int_0^{\chi^2} x^0 e^{-x/2}dx$$ \[3.2.2\]

$$P(\chi^2) = 1 - e^{-\chi^2}$$ \[3.2.3\]

This is not the exact probability mass contained in Fig.1. The correct figure would also reflect the probability mass contained outside the circle. The exact figure would be difficult to obtain. An approximation is given below.

Consider the plot shown in Figure 2. While most of the probability mass is within the circle, a significant fraction lies outside the circle boundary. A better approximation is the probability mass contained in the area indicated by the hatched lines, which is a sector of a circle of radius $R'$ equal
Due to the circular symmetry of the Chi-squared distribution the probability mass $P_s$ within the sector is proportional to the area of the sector. Thus,

$$P_s = \frac{2\theta}{360} (1 - e^{-R'^2})$$  \[3.2.4\]

$$\theta = \tan^{-1} 3$$  \[3.2.5\]

The fraction of the sector mass within the circle, $P_c$, is given by:

$$P_c = \frac{2\theta}{360} (1 - e^{-x^2})$$  \[3.2.6\]

Thus the probability mass lying outside the circle boundary is the difference between the two expressions. The following approximation can thus be made.

$$M' = M(1 + x)$$  \[3.2.7\]

$$x = .4 \left( \frac{1 - e^{-R'^2}}{M} - 1 \right)$$  \[3.2.8\]

where

- $M$ is the probability mass contained in the circle,
- $M'$ is the approximate mass contained in the region,
- $R' = \sigma$

In the event of a change in the process parameters then a non-central Chi-squared distribution has to be used. The region of intersection can be determined if the shift in the process mean and variance are known.
Figure 2. Geometrical Approximation Of Probability Mass
3.3 Modification of the Repko Chart

In order to use the Repko statistic for the purpose of variable interval sampling it was considered necessary to modify the chart as shown in the Fig 3. If a sample point lies outside the inner circle the next sample is taken after a shorter interval of time. In the present analysis only the circular portion is used. This modification proves to be beneficial as it improves the sensitivity of the chart. It may be argued that the number of false alarms is bound to increase due this modification. It turns out that this increase is negligibly small.

Selection of the short and long intervals is very important. Amin et al (1988) mentioned that best results for the $VSI \bar{X}$ charts are obtained when the longest and shortest possible intervals are used. Table 1 gives the results obtained when different combinations of intervals are used. The sample size $n$ is 5. The number of samples is equal to 10,000. The $ATS$ and $ANSS$ are matched for the in-control case. It may be noted that with the Repko chart too the smallest and largest possible intervals give the best results. For any kind of deviation of either the mean or variance the chart with the smallest interval and largest interval is consistently better. It should be noted that the outer interval is the same for each chart so that the $ANSS$ is the same.

It may be recalled that separate $\bar{X}$ and $S$ charts provide greater flexibility than the conventional combined single statistic chart. This flexibility can be incorporated into the Repko chart by introducing inner limits as shown in Fig. 4. The inner limits were introduced on the premise that the $VSI$ Repko chart would signal faster in situations when there where shifts in mean only if it would sample more frequently. The new inner limits reduce the sensitivity of the chart in detecting shifts in the process variance. It may be noted that a sample point located at the position indicated by the cross would not result in a quicker signal by the 'hybrid' Repko chart.
Figure 3. Modification Of Repko Chart For Variable Interval Sampling
### Table 1. ATS For Repko Charts With Different Intervals

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>INTERVALS</th>
<th>$\sigma = 1.0$</th>
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<th>$\sigma = 1.3$</th>
<th>$\sigma = 1.5$</th>
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<tr>
<td>0</td>
<td>(.90, 1.25)</td>
<td>100.0</td>
<td>43.0</td>
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<td>4.7</td>
</tr>
<tr>
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<td>42.9</td>
<td>10.0</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>(.68, 1.25)</td>
<td>100.0</td>
<td>42.3</td>
<td>9.6</td>
<td>4.4</td>
</tr>
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<td>92.6</td>
<td>38.5</td>
<td>9.9</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>(.85, 1.25)</td>
<td>92.2</td>
<td>38.3</td>
<td>9.8</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>(.68, 1.25)</td>
<td>92.3</td>
<td>37.8</td>
<td>9.4</td>
<td>4.3</td>
</tr>
<tr>
<td>0.3</td>
<td>(.90, 1.25)</td>
<td>47.5</td>
<td>25.1</td>
<td>8.1</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>(.85, 1.25)</td>
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<td>24.9</td>
<td>8.0</td>
<td>4.1</td>
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<td>3.9</td>
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<tr>
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<td>3.6</td>
</tr>
<tr>
<td></td>
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<td>19.1</td>
<td>12.3</td>
<td>5.7</td>
<td>3.4</td>
</tr>
<tr>
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<td>(.90, 1.25)</td>
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<td>7.4</td>
<td>4.4</td>
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<td>(.85, 1.25)</td>
<td>9.4</td>
<td>7.2</td>
<td>4.3</td>
<td>2.9</td>
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<tr>
<td></td>
<td>(.68, 1.25)</td>
<td>8.7</td>
<td>6.7</td>
<td>4.1</td>
<td>2.8</td>
</tr>
<tr>
<td>1.0</td>
<td>(.90, 1.25)</td>
<td>3.9</td>
<td>3.5</td>
<td>2.8</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>(.85, 1.25)</td>
<td>3.8</td>
<td>3.4</td>
<td>2.8</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>(.68, 1.25)</td>
<td>3.5</td>
<td>3.1</td>
<td>2.6</td>
<td>2.2</td>
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<tr>
<td>1.5</td>
<td>(.90, 1.25)</td>
<td>1.7</td>
<td>1.7</td>
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<td>1.6</td>
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<tr>
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<td>1.7</td>
<td>1.7</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>(.68, 1.25)</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
</tr>
</tbody>
</table>
Figure 4. Modification of the Inner Limits of VSI Repko Charts
A similar strategy would have to be pursued if there was a need to detect shifts in variance quickly. In the following chapter comparisons between various sampling strategies will be made. The variable interval Repko chart and the hybrid Repko chart are also considered.
CHAPTER IV COMPARISON OF CHARTS

4.1 Introduction

Based on the discussion by Chengalur, Arnold, and Reynolds (1988), the following VSI charts are considered for comparison against the VSI Repko chart.

- Separate $\bar{X}$ and $S$ charts with variable sampling intervals.
- Combined statistic $VSI$ chart; more specifically, the $\chi^2$ chart with variable sampling intervals
- $VSI \bar{X}$ chart

In addition, the modified Repko chart discussed in the previous chapter is also included in the comparisons.

The Repko statistic being an approximate statistic, it is difficult to predict the exact behaviour of the chart. Hence, simulation was considered to be a better alternative to a mathematical analysis. In order to avoid a bias due the peculiarities of the data that was generated the entire comparison was based on simulation results for all the above mentioned charts. For analytical data on the
ATS (the average time to signal) the reader is referred to the papers by Amin et al (1988) and Chengalur et al (1988). The process is assumed to be in-control when the mean $\mu$ is 0 and the standard deviation $\sigma$ is 1. Unless otherwise mentioned, the sample size is assumed to be 5 and the number of samples as 10,000.

These simulations can be classified into two categories

1. Type 1 Simulation: Each of the principal statistics was set at a critical value with $\alpha = .0027$.

2. Type 2 Simulation: Matching of ATS and ANSS for the in-control case was done based on the simulation results. The chosen values were $ATS = 100$ and $ANSS = 100$.

4.2 Comparison Based On Type 1 Simulation

This comparison was made primarily to determine the behaviour of the charts and to get a better understanding of the relative sensitivity of the charts. The number of samples used for the simulation is 1000. The value of $\alpha$ chosen is .0027. The observations made below are based on the results shown in Table 2. As there is no matching of ANSS and ATS, it was decided to specify the number of signals and time taken to finish the 1,000 samples instead.

- For the in-control case, both the Repko and the $\chi^2$ chart give more signals than the $\bar{X}$ chart.

- For a moderate shift in the mean only, the Repko chart gives good results initially. However for larger shifts in the mean, the $\bar{X}$ chart significantly outperforms the Repko chart. A plausible explanation for this apparent high sensitivity at the beginning would be that there is a bias due to the large number of initial false alarms are repeated again. From the observations made it may be concluded that for situations where there would be primarily changes in mean, the
Table 2. Number of Signals And Time Taken By A Type I Simulation

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>CHART</th>
<th>$\sigma = 1$</th>
<th>$\sigma = 1.5$</th>
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<tbody>
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<td>0</td>
<td>X</td>
<td>(3,1695)</td>
<td>(41,1536)</td>
</tr>
<tr>
<td></td>
<td>REPKO</td>
<td>(7,1856)</td>
<td>(281,1584)</td>
</tr>
<tr>
<td></td>
<td>CHIN</td>
<td>(8,1855)</td>
<td>(296,1500)</td>
</tr>
<tr>
<td></td>
<td>HREPKO</td>
<td>(7,1698)</td>
<td>(281,1669)</td>
</tr>
<tr>
<td>0.7</td>
<td>X</td>
<td>(221,1325)</td>
<td>(312,1499)</td>
</tr>
<tr>
<td></td>
<td>REPKO</td>
<td>(153,1423)</td>
<td>(512,1662)</td>
</tr>
<tr>
<td></td>
<td>CHIN</td>
<td>(43,1529)</td>
<td>(479,1590)</td>
</tr>
<tr>
<td></td>
<td>HREPKO</td>
<td>(153,1256)</td>
<td>(512,1665)</td>
</tr>
<tr>
<td>1.5</td>
<td>X</td>
<td>(968,1968)</td>
<td>(881,1889)</td>
</tr>
<tr>
<td></td>
<td>REPKO</td>
<td>(933,1934)</td>
<td>(915,1922)</td>
</tr>
<tr>
<td></td>
<td>CHIN</td>
<td>(690,1702)</td>
<td>(862,1870)</td>
</tr>
<tr>
<td></td>
<td>HREPKO</td>
<td>(933,1933)</td>
<td>(915,1918)</td>
</tr>
</tbody>
</table>
best choice would the $\bar{X}$. It may be noted that this result is consistent with the observation made by Reynolds and Ghosh (1981).

- For detecting a shift in $\sigma$ only, the best results are given by the $\chi^2$ chart. However the Repko chart gives comparable results. In addition, for shifts in $\mu$ and $\sigma$ the Repko chart gives much better results than the $\chi^2$ chart. The $\bar{X}$ chart does not perform as well as the other charts.

- When the value of $\alpha$ is set at .84, the sensitivity of Repko’s chart appears to go down sharply. $\alpha$ was chosen to be .84 it corresponds to the $1\sigma$ limits. $\bar{X}$ charts were still the best. The $\chi^2$ chart performs marginally better than the Repko chart.

The low sensitivity of the Repko chart at $\alpha = .16$ prompted a study of the relative behaviour of the two combined statistics.

The Repko statistic is based on the premise that the standard deviation $s$ is distributed normally with mean $\sigma$ and standard deviation $\sigma/\sqrt{2n}$. Thus $s$ can be written as follows

$$\frac{s - \sigma}{\left(\frac{\sigma}{\sqrt{2n}}\right)} = Z$$  \[4.2.1\]

Putting $\sigma = 1$,

$$\sqrt{2n} (s - 1) = Z$$  \[4.2.2\]

On squaring both sides,

$$2n(s - 1)^2 = \chi^2_1$$  \[4.2.3\]

Taking the square root on both sides of the expression and rearranging,

$$s = \sqrt{\frac{\chi^2_{1,\alpha}}{2n}} + 1$$  \[4.2.4\]
Squaring the expression,

\[ s'^2 = (\sqrt{\frac{\chi^2_{n-1, \alpha}}{2n}} + 1)^2 \]  \[\text{[4.2.5]}\]

The \( \chi^2 \) statistic uses the fact that \((n - 1)s^2/\sigma\) is distributed according to a chi-squared distribution with \( n - 1 \) degrees of freedom.

\[(n - 1)(\frac{s^2}{\sigma})^2 = \chi^2_{n-1} \]  \[\text{[4.2.6]}\]

Putting \( \sigma = 1 \) and rearranging the terms,

\[ s^2 = \frac{\chi^2_{n-1}}{(n - 1)} \]  \[\text{[4.2.7]}\]

In order to distinguish between the \( s \) obtained from the Repko assumption and the \( s \) obtained above, the latter is designated as \( s' \). Thus

\[ s'^2 = \frac{\chi^2_{n-1, \alpha}}{(n - 1)} \]  \[\text{[4.2.8]}\]

Table 3 gives the value of \( s/s' \) for various values of \( \alpha \) for \( n = 5 \). The values of \( \alpha \) taken are not at equal intervals in order to highlight the differences between the two combined statistics at the upper and lower end of the scale.

At the lower end the ratio is less than 1, indicating that Repko’s statistic underestimates the lower limits. At the upper end of the scale the ratio is greater than 1, indicating that Repko overestimates the upper limits. On account of this behaviour of the Repko statistic, the values of \( s \) which would be considered beyond the limits indicated by the Reynolds-Ghosh statistic would be within the limits of the Repko statistic. This behaviour is probably the cause of the apparent high sensitivity for the out-of-control cases and the large number of false alarms for the in-control case. It may be noted that as \( \alpha \) tends to 1, the ratio of the two statistics tends to 1.
Table 3. The Repko Statistic Versus The Reynolds-Ghosh Statistic

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$s/s'$</th>
<th>$\alpha$</th>
<th>$s/s'$</th>
</tr>
</thead>
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<td>0.005</td>
<td>0.639</td>
<td>0.500</td>
<td>1.412</td>
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<tr>
<td>0.010</td>
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<td>1.417</td>
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<tr>
<td>0.015</td>
<td>0.803</td>
<td>0.600</td>
<td>1.420</td>
</tr>
<tr>
<td>0.020</td>
<td>0.852</td>
<td>0.650</td>
<td>1.421</td>
</tr>
<tr>
<td>0.025</td>
<td>0.892</td>
<td>0.700</td>
<td>1.421</td>
</tr>
<tr>
<td>0.030</td>
<td>0.927</td>
<td>0.750</td>
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</tr>
<tr>
<td>0.035</td>
<td>0.955</td>
<td>0.800</td>
<td>1.416</td>
</tr>
<tr>
<td>0.040</td>
<td>0.981</td>
<td>0.850</td>
<td>1.411</td>
</tr>
<tr>
<td>0.045</td>
<td>1.004</td>
<td>0.900</td>
<td>1.405</td>
</tr>
<tr>
<td>0.050</td>
<td>1.025</td>
<td>0.905</td>
<td>1.404</td>
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<tr>
<td>0.055</td>
<td>1.044</td>
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<td>1.403</td>
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<td>1.061</td>
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<td>0.950</td>
<td>1.395</td>
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<td>1.165</td>
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<td>0.450</td>
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CHAPTER IV COMPARISON OF CHARTS
4.3 Comparison Based On Type 2 Simulation

The Type 2 simulation involves simulation of the charts which have matching ANSS and ATS for the in-control case. Prior to discussing the results of the simulations, it would be useful to note that the ANSS is independent of the ATS but not vice versa. Thus matching of the ATS is done only after the matching of the ANSS. The ANSS are matched by the manipulation of the outer limits. The matching of ATS was then done by manipulation of the inner limits. The sampling intervals which are chosen are .9 and 1.25 time units. The adjusted ATS is calculated using the expression obtained in eqn. 2.3.8. The values for the probabilities are obtained from the results of simulation. For example, the probability of a signal is assumed to be the number of signals obtained during the simulation divided by the total number of samples taken which in this case happens to be 10,000. The program REPKO9 was used to carry out the simulation. REMINC was used to determine the inner and outer limits of the VSI Repko chart so that the specified ANSS and ATS are obtained.

Tables giving the result of the simulation for different combinations of shifts in mean and variance are given below. Tables 4, 5 and 6 give the ANSS, ATS and the adjusted ATS respectively. CXS stands for the separate $\bar{X}$ and $S$ charts. REPKO and HREPKO represent the Repko and hybrid Repko charts respectively. CHIN stands for the $\chi^2_1$ chart while X stands for the $\bar{X}$ chart.

1. For a shift in mean only, it is noticed that though the separate $\bar{X}$ and $s$ charts initially give the best results, the single $\bar{X}$ chart performs the best for larger shifts in the mean. It may be noted that at this stage the hybrid Repko chart does not prove to be any better than the VSI Repko chart. The $\chi^2_1$ chart does not prove to be very quick in picking up the shifts in mean.

2. For a change in variance only, the $\chi^2_1$ chart signals faster than any other chart. However, the $\chi^2_1$ chart gives only marginally better results than the Repko chart for moderate to large shifts in variance. The single $\bar{X}$ chart has the much worse results than corresponding results for other
### Table 4. ANSS for Type II Simulation (Sample Size = 5)

<table>
<thead>
<tr>
<th>$\mu$</th>
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<th>$\sigma = 1.3$</th>
<th>$\sigma = 1.5$</th>
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<td>REPKO</td>
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<td>CHIN</td>
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<td>HREPKO</td>
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<td>10.0</td>
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<tr>
<td></td>
<td>CHIN</td>
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</tr>
<tr>
<td></td>
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<td>38.8</td>
<td>10.0</td>
<td>4.5</td>
</tr>
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<td>X</td>
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<tr>
<td></td>
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CHAPTER IV COMPARISON OF CHARTS
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charts. This is to be expected as its purpose is to detect the changes in the mean only and not changes in variance.

3. For moderate to large shifts in mean and variance, the Repko chart gives figures comparable to the ones that were the best.

4. A direct comparison of the two combined statistics being considered shows that the Repko chart picks up shifts in mean faster than the $\chi^2$ chart. The $\chi^2$ chart picks up changes in variance marginally faster than the Repko chart.

The above discussion was based on comparing the $ATS$ only. Comparison based on the $ANSS$ yields the same type of results as seen above. This is to be expected as if the $ANSS$ is small then it is logical to expect the $ATS$ to be small unless of course the sampling intervals are far apart.

In situations where the intervals are far apart a better criterion for comparison is the adjusted $ATS$. In the simulation the sampling intervals being fairly close the adjusted $ATS$ is not very different from the $ATS$, so the observations made above are still valid if the comparison was based on the adjusted $ATS$. It is interesting to note that though there is not a great deal of difference between results for the the Repko chart and the hybrid Repko chart, the difference between the adjusted $ATS$ and the $ATS$ is consistently smaller for the hybrid Repko chart. A plausible explanation for that sampling at shorter intervals is probably more frequent with the hybrid Repko chart.

**Effect of Sample Size** The results of a simulation where the sample size is increased to 10 is shown on Tables 7, 8, and 9. Matching of the $ANSS$ and $ATS$ was not done. The $\alpha$ values chosen were the same as were used in the earlier simulation.

A summary of the results is given below.

1. The separate $\bar{X}$ and $S$ charts give a large number of false alarms for the in-control case. This large number of false alarms can be traced down to the $S$ charts which are still using the same
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Table 8. ATS for Type II Simulation (Sample Size = 10)

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Table 9. Adjusted ATS for Type II Simulation (Sample Size = 10)

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</table>
values of $K$ (refer page 1) as were used in the earlier simulation. As the comparison in this part
deals with the effect of sample size on individual charts there is no effort made to find a new
way to deal with the false alarm problem.

2. The number of false alarms drops in all the other charts.

3. The $\bar{X}$ chart does not appear to behave in the same manner as the combined statistics. For
a change in variance only, there is a slight increase in $ATS$ of the $\bar{X}$ charts with an increase in
sample size. For the combined statistics, the $ATS$ falls with an increase in sample size. For
shifts in the mean only, the increase in sample size causes the $ATS$ of the $\bar{X}$ chart to fall. With
the combined statistics, the $ATS$ increases marginally at first and then falls.

This difference in behaviour between the $\bar{X}$ and the combined statistic charts can be attributed to
the lowering of the number of signals for the in-control case and the relative sensitivity of the charts
to changes in process parameters. In other words, the $ATS$ falls faster for a chart when there is a
change in the dominant variable of the governing statistic.

CHAPTER IV COMPARISON OF CHARTS
An attempt has been made to compare various VSI charts. The analysis was aimed at determining whether the VSI Repko chart displays sufficiently good behaviour to consider it as an alternative to the other VSI charts. Since the Repko statistic is an approximate statistic, comparison based on data obtained from simulation was considered a better alternative to an analytical comparison. The results of the comparison indicate that the Repko chart is more sensitive to changes in variance than the $\bar{X}$ chart and is comparable to the $\chi^2$ chart. For shifts in the mean, the Repko chart does not perform as well as the $\bar{X}$ chart but performs better than the $\chi^2$ chart. It would appear that the combined statistic chart is a useful alternative to the VSI $\bar{X}$ and $S$ charts as well as the VSI $\chi^2$ chart.

For situations where it is important to detect changes in mean and variance equally fast with a fixed probability of Type I error, the $\bar{X}$ chart can not be used alone as it is not very sensitive to changes in the variance. The Shewart $S$ chart too is not the best choice when one takes into account the fact that the limits are not logical and probably reduce its sensitivity. As the VSI Repko chart picks up changes in variance almost as fast as any other chart and can detect shifts in the process mean very quickly, it could be a considered a viable alternative to the VSI $\bar{X}$ and $S$ charts.

CHAPTER V CONCLUSION AND EXTENSIONS
A certain amount of flexibility is present as was seen by introducing the new inner limits. By manipulating the inner limits it is possible to signal faster based on the location of the sample point with respect to the target mean or the target variance.

The fact that very large changes in the mean and variance are not considered here may appear to make this study incomplete. There were two main reasons for this omission. Firstly, most changes in real-life situations are of a small to moderate level. Secondly, it is observed that the $ANSS$ is very small for extreme changes in the mean and variance. Thus, considering larger changes in the mean or variance will not produce significant changes in either $ANSS$ or $ATS$.

Direct extensions of this work could be the following:

- A study on the manipulation of the inner bounds of $VSI$ combined statistic charts to get optimal or near optimal behaviour.

- A cost analysis of the $VSI$ Repko chart.

- A weighted combined statistic chart which is based on an exact statistic. An exact statistic is preferred as it allows for a better study of the properties through analytical methods instead of by means of simulation studies.
BIBLIOGRAPHY


Appendix A. SIMULATION PROGRAM

Given below is the listing of the program 'REPKO9' that was used to simulate the behavior of the different charts.

REAL LBD, UBD, LBD1, UBD1
REAL LBX, UBX, LBS, UBS, LBX1, UBX1, LBS1, UBS1
REAL NCT, NCT5, NCT6, NCT4, NCT3
DIMENSION XCH(100000), XM(10000), SM(10000), A(7), B(4)
INTEGER L

C******************************************************************************
C ARRAY A CONTAINS THE VARIOUS VALUES OF THE SHIFT IN THE MEAN
C ARRAY B CONTAINS THE VARIOUS VALUES OF THE STANDARD DEVIATION
C******************************************************************************

B(1) = 1.0
B(2) = 1.1
B(3) = 1.3
B(4) = 1.5
A(1) = 0.0
A(2) = 0.1
A(3) = 0.3
A(4) = 0.5
A(5) = 0.7
A(6) = 1.0
A(7) = 1.5

SAMN = 10.

C**.......................... .......................................................... ..........................................................

C THE VARIOUS INNER AND OUTER BOUNDS HAVE BEEN CALCULATED TO GET
C AN ANSS OF 100 AND AN ATS OF 100.

C**.......................... .......................................................... ..........................................................

LBD1 = -.352/(SAMN**.5)
UBD1 = .352/(SAMN**.5)
LBD = -2.62/(SAMN**.5)
UBD = 2.62/(SAMN**.5)
LBX1 = -0.903/(SAMN**.5)
UBX1 = 0.903/(SAMN**.5)
LBX = -2.786/(SAMN**.5)
UBX = 2.786/(SAMN**.5)

C4 = .9727
LBS = 1. - (2.578/C4)*(1. - C4*C4)**.5
UBS = 1. + (2.578/C4)*(1. - C4*C4)**.5
LBS1 = 1. - (.573/C4)*(1. - C4*C4)**.5
UBS1 = 1. + (.573/C4)*(1. - C4*C4)**.5

CALL GGNML ( 1.D0,100000,XCH)
CALL MDCHI ( .3089,2.,CHIR,IER)
R1 = CHIR / SAMN
CALL MDCHI ( .9923,2.,CHIR,IER)

Appendix A. SIMULATION PROGRAM
R2 = CHIR / SAMN
CALL MDCHI (.2770,SAMN,CHIR,IER)
RN1 = CHIR / SAMN
CALL MDCHI (.99010,SAMN,CHIR,IER)
RN2 = CHIR / SAMN
DO 50 L2 = 1,4,1
  SGM = B(L2)
DO 40 L1 = 1,7,1
  SFT = A(L1)
  L = 1
  SXM = 0.0
  SSM = 0.0

C*****************************************************************************
C CALCULATION OF MEAN AND STANDARD DEVIATION FOR EACH SAMPLE
C ARRAY XM CONTAINS THE SAMPLE MEANS
C ARRAY SM CONTAINS THE SAMPLE STANDARD DEVIATIONS
C*****************************************************************************

     DO 10 I = 1,10000,1
       SX = 0.0
       K = I * INT(SAMN)
       DO 20 J = L,K,1
         SX = SX + XCH(J)*SGM + SFT
     20 CONTINUE

     XM(I) = SX / SAMN
     SS = 0.0
     DO 30 J = L,K,1
       SS = ((XCH(J)*SGM + SFT-XM(I)**2) + SS
     30 CONTINUE

Appendix A. SIMULATION PROGRAM 46
SM(I) = (SS/(SAMN - 1.))**.5
L = L + INT(SAMN)
SXM = SXM + XM(I)
SSM = SSM + SM(I)

10 CONTINUE
XMM = SXM / 10000.
SMM = SSM / 10000.
NSIG = 0
NSIG1 = 0
NSIG2 = 0
NSIG3 = 0
NSIG4 = 0
NSIG5 = 0
NSIG6 = 0
ISIG = 0
ISIG1 = 0
ISIG2 = 0
ISIG3 = 0
ISIG4 = 0
ISIG5 = 0
ISIG6 = 0
NCT = 0.
NCT3 = 0.
NCT4 = 0.
NCT5 = 0.
NCT6 = 0.

C******************************************************************************

C CHECKING THE LOCATION OF EACH SAMPLE ON ALL THE CHARTS

Appendix A. SIMULATION PROGRAM
C NSIG - NUMBER OF SIGNALS GIVEN BY THE SEPARATE SAMPLING
C PROCEDURE
C NCT - COUNTER KEEPING TRACK OF THE TIME AT WHICH EACH SAMPLE
C IS TAKEN.
C SAMPLES ARE TAKEN EITHER AT EVERY 1.25 OR 0.9 TIME UNITS
C

DO 90 I = 1, 10000, 1
   IF ( XM(I).GT.UBX.OR.XM(I).LT.LBX) THEN
      NSIG1 = NSIG1 + 1
      NSIG = NSIG + 1
      NCT = NCT + 1.25
   ELSE
      IF ( SM(I).GT.UBS.OR.SM(I).LT.LBS ) THEN
         NSIG = NSIG + 1
         NSIG2 = NSIG2 + 1
         NCT = NCT + 1.25
      ELSE
         IF ( XM(I).GT.UBX1.OR.XM(I).LT.LBX1) THEN
            NCT = NCT + .90
            ISIG = ISIG + 1
            ISIG1 = ISIG1 + 1
         ELSE
            IF ( SM(I).GT.UBS1.OR.SM(I).LT.LBS1) THEN
               NCT = NCT + .90
               ISIG = ISIG + 1
               ISIG2 = ISIG2 + 1
            ELSE
               NCT = NCT + 1.25
   ELSE
      IF ( X
ENDIF
ENDIF
ENDIF
ENDIF

IF ( XM(I).GT.UBD.OR.XM(I).LT.LBD ) THEN
  NSIG3 = NSIG3 + 1
  NCT3 = NCT3 + 1.25
ELSE
  IF ( XM(I).GT.UBD1.OR.XM(I).LT.LBD1) THEN
    NCT3 = NCT3 + .9
    ISIG3 = ISIG3 + 1
  ELSE
    NCT3 = NCT3 + 1.25
  ENDIF
ENDIF

D = XM(I)**2 + 2*(SM(I)-1)**2

IF ( D.GT.R2) THEN
  NSIG4 = NSIG4 + 1
  NCT4 = NCT4 + 1.25
ELSE
  IF ( D.GT.R1) THEN
    NCT4 = NCT4 + .9
    ISIG4 = ISIG4 + 1
  ELSE
    NCT4 = NCT4 + 1.25
  ENDIF
ENDIF

DR = XM(I)**2 + (SM(I)**2)*(SAMN -1.)/SAMN
IF ( DR.GT.RN2) THEN
  NSIG5 = NSIG5 + 1
  NCT5 = NCT5 + 1.25
ELSE
  IF ( DR.GT.RN1) THEN
    NCT5 = NCT5 + .9
    ISIG5 = ISIG5 + 1
  ELSE
    NCT5 = NCT5 + 1.25
  ENDIF
ENDIF

IF ( D.GT.R2) THEN
  NSIG6 = NSIG6 + 1
  NCT6 = NCT6 + 1.25
ELSE
  IF ( XM(I).GT.UBD1.OR.XM(I).LT.LBD1) THEN
    NCT6 = NCT6 + .9
    ISIG6 = ISIG6 + 1
  ELSE
    NCT6 = NCT6 + 1.25
  ENDIF
ENDIF

90 CONTINUE

FNUM = (.9)*(.9)*(.7143) + (1.25)*(1.25)*(.2757)
FDEN = 1.8
FEXP = FNUM/FDEN
SIG = REAL(NSIG)
SIG3 = REAL(NSIG3)
SIG4 = REAL(NSIG4)
SIG5 = REAL(NSIG5)
SIG6 = REAL(NSIG6)
SSIG = REAL(ISIG)
SSIG3 = REAL(ISIG3)
SSIG4 = REAL(ISIG4)
SSIG5 = REAL(ISIG5)
SSIG6 = REAL(ISIG6)
SF = (10000. - SIG)/SIG
SS = ((.9* SSIG) + 1.25*(10000. -SSIG - SIG))/10000.
AATS = FEXP + SF/SS
SF3 = (10000. - SIG3)/SIG3
SS3 = ((.9* SSIG3) + 1.25*(10000. -SSIG3 - SIG3))/10000.
AATS3 = FEXP + SF3/SS3
SF4 = (10000. - SIG4)/SIG4
SS4 = ((.9* SSIG4) + 1.25*(10000. -SSIG4 - SIG4))/10000.
AATS4 = FEXP + SF4/SS4
SF5 = (10000. - SIG5)/SIG5
SS5 = ((.9* SSIG5) + 1.25*(10000. -SSIG5 - SIG5))/10000.
AATS5 = FEXP + SF5/SS5
SF6 = (10000. - SIG6)/SIG6
SS6 = ((.9* SSIG6) + 1.25*(10000. -SSIG6 - SIG6))/10000.
AATS6 = FEXP + SF6/SS6
RATIO = NCT / SIG
RATIO3 = NCT3 / SIG3
RATIO4 = NCT4 / SIG4
RATIO5 = NCT5 / SIG5
RATIO6 = NCT6 / SIG6

Appendix A. SIMULATION PROGRAM
ANSIG = 10000. / SIG  
ANSIG3 = 10000. / SIG3  
ANSIG4 = 10000. / SIG4  
ANSIG5 = 10000. / SIG5  
ANSIG6 = 10000. / SIG6  
WRITE (25, *) ‘########################################################’  
WRITE (25, *) XMM, SMM  
WRITE (25,111) NSIG, NCT, ANSIG, RATIO, AATS  
WRITE (25,112) NSIG1, NSIG2  
WRITE (25,111) NSIG3, NCT3, ANSIG3, RATIO3, AATS3  
WRITE (25,111) NSIG4, NCT4, ANSIG4, RATIO4, AATS4  
WRITE (25,111) NSIG5, NCT5, ANSIG5, RATIO5, AATS5  
WRITE (25,111) NSIG6, NCT6, ANSIG6, RATIO6, AATS6  
WRITE (25,113) LBX, LBX1, UBX1, UBX, LBS, LBS1, UBS1, UBS  
111 FORMAT (2X, I4, 2X, F10.1, 2X, F9.3, 2X, F10.4, 2X, F10.3)  
112 FORMAT (2X, I5, 2X, I5)  
113 FORMAT (8(2X, F6.3))  
40 CONTINUE  
50 CONTINUE  
STOP  
END
Appendix B. PROGRAM FOR ITERATIVELY COMPUTING LIMITS OF REPKO CHART

Given below is the listing of the program 'REMINC' which calculates the limits of the Repko Chart for a specified value of ATS and ANSS.

REAL ANR

DIMENSION ANR(50000), ASM(10000), ASX(10000)

C******************************************************************************
C THE PURPOSE OF THIS PROGRAM IS TO PROVIDE A MORE ACCURATE
C APPROXIMATION FOR THE RADIUS OF THE REPKO CIRCLE THAN THE ONE
C OBTAINED WITH THE MATHEMATICAL FORMULA.
C THE PROGRAM CHECKS THE APPROPRIATENESS OF THE RADII BY CHECKING
C THE NUMBER OF SIGNALS OBTAINED FROM A SIMULATION RUN AGAINST THE
C THEORETICAL NUMBER OF SIGNALS REQUIRED FOR A DESIRED SET OF ANSS
C AND ATS. IF THE RADII ARE FOUND TO BE INAPPROPRIATE THEN THE
C RADII ARE CHANGED UNTIL A SUITABLE PAIR ARE FOUND.
C
C THE INPUTS REQUIRED ARE:

Appendix B. PROGRAM FOR ITERATIVELY COMPUTING LIMITS OF REPKO CHART 53
C SAMN - SAMPLE SIZE
C ANSS - AVERAGE NO. OF SAMPLES TO SIGNAL
C ATS - AVERAGE TIME TO SIGNAL
C RSI - LONGEST POSSIBLE INTERVAL BETWEEN SAMPLES
C QSI - SMALLEST POSSIBLE INTERVAL BETWEEN SAMPLES
C***********************************************************************
READ(11,*) SAMN, ANSS, ATS, RSI, QSI
WRITE(21,110) SAMN, ANSS, ATS, RSI, QSI
ATBS = ATS/ANSS
110 FORMAT(5(2X,F9.3))
C***********************************************************************
C IT IS IMPORTANT TO REALIZE THAT THERE ARE A FEW SITUATIONS WHERE
C IT IS NOT POSSIBLE TO MEET THE SPECIFICATIONS SUCH AS
C (1) AVERAGE TIME BETWEEN SAMPLES CANNOT BE GREATER THAN THE
C    LARGEST INTERVAL NOR SMALLER THAN THE SMALLEST INTERVAL.
C (2) THE AVERAGE NO. OF SAMPLES CANNOT BE LESS THAN 1.
C***********************************************************************
IF ( ATBS.LE.QSI.OR.ATBS.GE.RSI.OR.ANSS.LE.1 ) THEN
   WRITE(21,111)
111 FORMAT(2X,'THESE SPECIFICATIONS ARE NOT POSSIBLE TO SATISFY.')
ELSE
   CALL GGNML ( 1.D0,50000,ANR)
C***********************************************************************
C THE PROBABILITY OF A SIGNAL DETERMINES THE ANSS.
C THE PROBABILITY OF A SAMPLE POINT LYING BETWEEN THE INNER AND OUTER
C CONTROL LIMITS DETERMINES THE ATS
C NS - THE NUMBER OF SIGNALS LYING OUTSIDE CONTROL LIMITS
C NIS - THE NUMBER OF SIGNALS LYING IN BETWEEN INNER AND OUTER

Appendix B. PROGRAM FOR ITERATIVELY COMPUTING LIMITS OF REPKO CHART
CONTROL LIMITS

P1 - THE PROBABILITY OF A SAMPLE LYING WITHIN OUTER CONTROL LIMITS

P2 - THE PROBABILITY OF A SAMPLE LYING WITHIN INNER CONTROL LIMITS

RNS = 10000./ANSS
RNIS = (10000.*RSI - 10000.)/(RSI - QSI)
NS = INT(RNS)
NIS = INT(RNIS)
P1 = 1. - (1/ANSS)
P2 = 1. - (RNS + RNIS)/10000.
IT = 0
IT1 = 0

FOR THE FIRST ITERATION THE RADIUS IS DETERMINED BY USING THE CHI-SQUARED CDF WITH TWO DEGREES OF FREEDOM

51 CALL MDCHI (P1,2.,CHIR,IER)
R1 = CHIR/SAMN
CALL MDCHI (P2,2.,CHIR,IER)
R2 = CHIR/SAMN
WRITE(21,119)P1,P2
WRITE(21,119)R1,R2
119 FORMAT(2(2X,F9.4))
NSIG = 0
NSIG1 = 0
CNT = 0.
L = 1
WRITE(21,* ) NS, NIS, P1, P2
IF (IT.LT. 10) THEN
DO 10 I = 1,10000,1
   L1 = L - 1 + INT(SAMN)
   SS = 0.
   SX = 0.
C******************************************************************************
C CALCULATION OF MEAN AND STANDARD DEVIATION FOR EACH SAMPLE
C******************************************************************************
   DO 11 J = L,L1,1
      SX = SX + ANR(J)
  11 CONTINUE
   XM = SX / SAMN
   DO 12 J = L,L1,1
      SS = SS + (ANR(J) - XM)**2
  12 CONTINUE
   SD = (SS / (SAMN - 1.)) **.5
   DC = (XM**2) + 2.*((SD -1.)**2)
C******************************************************************************
C CHECKING NUMBER OF SAMPLES LYING OUTSIDE THE OUTER CONTROL LIMITS
C******************************************************************************
   IF (DC.GT.R1) THEN
      NSIG = NSIG + 1
      CNT = CNT + RSI
   ELSE
C******************************************************************************
C CHECKING NUMBER OF SAMPLES LYING IN BETWEEN OUTER AND INNER
C CONTROL LIMITS
C******************************************************************************

Appendix B. PROGRAM FOR ITERATIVELY COMPUTING LIMITS OF REPKO CHART
IF (DC.GT.R2) THEN
    CNT = CNT + QSI
    NSIG1 = NSIG1 + 1
ELSE
    CNT = CNT + RSI
ENDIF
ENDIF
L = L1 + 1
10 CONTINUE
WRITE (21,*) NSIG, NSIG1
WRITE (21,*) CNT
C++++++++++++++++++++••••••••••••••••••••••+++++••••••••••••••••••••••
C SANSS ·A VERA
C NUMBER OF SAMPLES TO SIGNAL WITH CURRENT OUTER
C LIMITS
C SATS ·AVERAGE TIME TO SIGNAL WITH CURRENT INNER LIMITS
C++++++++++++++++++++++••••••••••••••••••••••••••••••••••••••••••••••••••••
SANSS = 10000./REAL(NSIG)
SATS = CNT/SANSS
WRITE (21,117) SANSS
WRITE (21,117) SATS
117 FORMAT(2X,F11.4)
C++++++++++++++++++++++••••••••••••••••••••••••••••••••••••••••••••••••••••
C THIS PART OF THE PROGRAM ADJUSTS THE VALUE OF P1 SUCH THAT THE
C SANSS WOULD BE CLOSER TO THE SPECIFIED VALUE OF ANSS
C++++++++++++++++++++++••••••••••••••••••••••••••••••••••••••••••••••••••••
IF (SANSS.LT.ANSS) THEN
    WRITE (21,113)
113 FORMAT(2X,'ENLARGE BOUNDARIES')
\[ P_1 = P_1 + \frac{(\text{ANSS} - \text{SANSS})}{10000}. \]

31 IF (\(P_1 > 1\)) THEN
   \[ P_1 = P_1 - (P_1 - 1) \cdot 2. \]
   IF (\(P_1 < P_2\)) THEN
      \[ P_2 = P_2 - \frac{(\text{SANSS} - \text{ANSS})}{10000}. \]
   ENDIF
   GO TO 31
ENDIF
IT = IT + 1
GO TO 51
ELSE
   IF (\(\text{SANSS} > \text{ANSS}\)) THEN
      WRITE (21, 114)
      114 FORMAT(2X, 'REDUCE BOUNDARIES')
      \[ P_1 = P_1 - \frac{(\text{SANSS} - \text{ANSS})}{10000}. \]
   32 IF (\(P_1 < P_2\)) THEN
      \[ P_2 = P_2 - \frac{(\text{SANSS} - \text{ANSS})}{10000}. \]
   GO TO 32
ENDIF
IT = IT + 1
GO TO 51
ELSE
   WRITE (21, 118)
   118 FORMAT(2X, 'SATISFACTORY MATCHING')
ENDIF
ENDIF
IF (IT1 .LE. 10) THEN
C

Appendix B. PROGRAM FOR ITERATIVELY COMPUTING LIMITS OF REPKO CHART
C THIS PART OF THE PROGRAM ADJUSTS THE VALUE OF P2 SUCH THAT THE C SATS WOULD BE CLOSER TO THE VALUE OF THE ATS
C

IF (NIS.GT. NSIG1) THEN
  WRITE (21,115)
115 FORMAT(2X,'REDUCE INNER BOUNDARIES')
  P2 = P2 - (NIS - NSIG1)/10000.
  IF (P2.LE.0) THEN
    P2 = -P2
  ENDIF
  IT1 = IT1 + 1
  GO TO 51
ELSE
  IF (NIS.LT.NSIG1) THEN
    WRITE (21,116)
116 FORMAT(2X,'ENLARGE INNER BOUNDARIES')
    P2 = P2 + (NSIG1 - NIS)/10000.
    IF (P1.LT.P2) THEN
      P2 = P2 - (P2 - P1)*2.
    ENDIF
    IT1 = IT1 + 1
    GO TO 51
  ELSE
    WRITE (21,118)
118 FORMAT(2X, 'CONTINUE')
  ENDIF
ENDIF
ENDIF
ENDIF
ENDIF
STOP
END
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