AN EXPERIMENTAL INVESTIGATION OF HEAT TRANSFER THROUGH INSULATION UNIFORMLY APPLIED TO A FLAT END CYLINDER

by

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I. INTRODUCTION

The determination of the heat transfer through a flat wall of which one surface is isothermal at a temperature of t_a and the other surface is isothermal at a temperature t_b is the simple problem of heat transfer. The equation:

$$Q = \frac{kA(t_a - t_b)}{L}$$

permits an easy solution of the problem where Q is the heat transfer, k is the thermal conductivity, A is the area through which the heat is transferred, and L is the distance between the two surfaces. The equation is only applicable where the area A is constant. This equation may be used without appreciable error for insulated enclosures such as furnaces where the insulation thickness is very small in comparison with the dimensions of the enclosure.

Shape factors have been applied to this basic equation so the equation may be used in the determination of heat transfer where the area A is not constant and the effect of corners can not be neglected. The equation then becomes

$$Q = \frac{fkA(t_a - t_b)}{L}$$

where f is the shape factor.

In 1947, T. S. Nickerson for a Master's thesis at V. P. I. determined the values of the shape factor where the above equation is applied to cylindrical enclosures having flat ends and relatively thick walls of uniform thickness. Mr. Nickerson solved this problem analytically by the relaxation method. His solution depended upon the inside and outside surfaces of the insulation about the enclosure being isothermal surfaces. The values were calculated for combinations of ratios of insulation thickness to length of enclosure and length of enclosure to diameter of enclosure.

This investigation is an experimental determination of these values using gypsum plaster cylinders of different combinations of ratios of length to diameter. However, before tests could be conducted on the cylindrical enclosures, the conductivity of gypsum plaster, the insulation about the cylindrical enclosure, had to be found. The method of determination of the conductivity and the values are given in Appendix A.

II. REVIEW OF LITERATURE

The first reported attempt to evaluate the heat transferred through the insulation surrounding a heated cylinder was made by Carl Hering[1] in 1908. Mr. Hering converted formulae for the electrical resistance to resistance to heat flow through insulation around a rectangular enclosure, a spherical enclosure, and a segment of an infinitely long cylindrical enclosure. Then the heat transferred from a cyclindrical enclosure was the sum of the heat flowing radially through the cylindrical segment and the heat flowing radially through semispherical ends. Mr. Hering admitted that this sum was only approximate.

These formulae developed by Mr. Hering as well as all others in this discussion were for insulation of uniform thickness having isothermal interior and exterior surface and under steady flow condition.

A few years later Langmum, Adams and Mickle[2] derived "shape" factors which could be applied to find the heat flow through the insulation of an electrically heated furnace. These shape factors were found by assuming a set of isotherms which gave less than the actual heat flow and then a set of isotherms which gave more than the actual heat flow. The factors so obtained proved to be within five per cent of the factors found by experiment for insulated rectangular furnaces.[3]

In 1940 R. B. Southwell published a book explaining the relaxation method and showing its application to the engineering sciences.[4] Three years later Mr. H. W. Emmons applied the relaxation method to two dimensional heatflow problems and Mr. Frank Lockhard applied the method to three dimensional problems.[5] These results were within four per cent of the experimental results for rectangular insulated enclosures.

Several graphical methods for solving complex heat problems have been presented.[6 & 7] The methods were tedious trial and error. One of these methods was to assume the location of isotherms and construct heat flow lines. The flow lines must cross the isotherms at right angles and an equal quantity of heat must flow through the path between any two adjacent flow lines. Also in order to obtain equal temperature difference between adjacent isotherms, the radio of the length to the width of the curvilinear rectangles bound by two adjacent isotherms and by two adjacent flow lines must be a constant for all rectangles. The total heat flow may be computed when these conditions are realized.

In 1945 G. M. Dusinberre gave briefly the numerical solution to heat flow problems for two and three dimensions.[8] This was followed by a book explaining fully the application of the relaxation method to heat flow problems.[9] This was the method employed by Mr. T. S. Nickerson in computing the heat flow through the insulation about a cylinder. This was a two dimensional problem because of the symmetry of the insulation in any plane passed through the longitudinal axis of the cylinder.

The heat transfer Q was computed for ratios of outside diameter of insulation to length of insulation and for ratios of insulation thickness to outside diameter of insulation. The factor f was calculated from the equation $Q = \frac{fAk\ (\Delta t)}{T}$ where Q is heat transfer, A is inside area of insulation, k is thermal conductivity of insulation, Δt is temperature difference between inner and outer surfaces of insulation and T is thickness of insulation. The factor f was plotted against the ratio of inside diameter to inside length of insulation for ratios of insulation thickness to inside diameter of insulation.

This investigation will be an experimental determination of the factor f and the results will be presented in the form of graph.

III. OBJECT OF INVESTIGATION

The object of this investigation is to obtain experimentally the curves which will facillitate the calculation of heat transfer under steady flow conditions through insulation uniformly applied to cylindrical enclosures with flat ends or hollow cylinders of insulation as these enclosures will be called.

IV. PROCEDURE

Casting of Cylinders

The hollow cylinders tested in this investigation were cast of gypsum plaster. It was necessary to cast these cylinders in two halves so the hollow inside would be accessible. A division of the cylinder in the plane normal to the longitudinal axis at the midpoint of that axis would interfere less with the heat transfer than any other division. This is because the heat flow at this plane of division is parallel to that plane whereas any other plane would cut the heat flow lines.

The cylinder halves were cast in molds as shown in Fig. 1b. This mold consists of a wooden center piece shown in Fig. 1a cut from a single piece of wood in a lathe. The surface of the wooden form which is in contact with the plaster was varnished to make a smooth surface from which the plaster casting would easily separate. Around this wooden form was fitted 16 B.W.G. gauge sheet metal rolled to the same diameter as the larger cylinder of the wooden form. This sheet metal was held securely to the wooden form by two lengths of wire wrapped around the metal where it covers the larger wooden cylinder. The circumference of the rolled metal was about 1/16 inch less than that of the wooden cylinder. The ends of the metal did not quite butt so that the metal would fit very snugly to the form. This 1/16-inch gap was sealed with scotch tape on the outside of the metal.

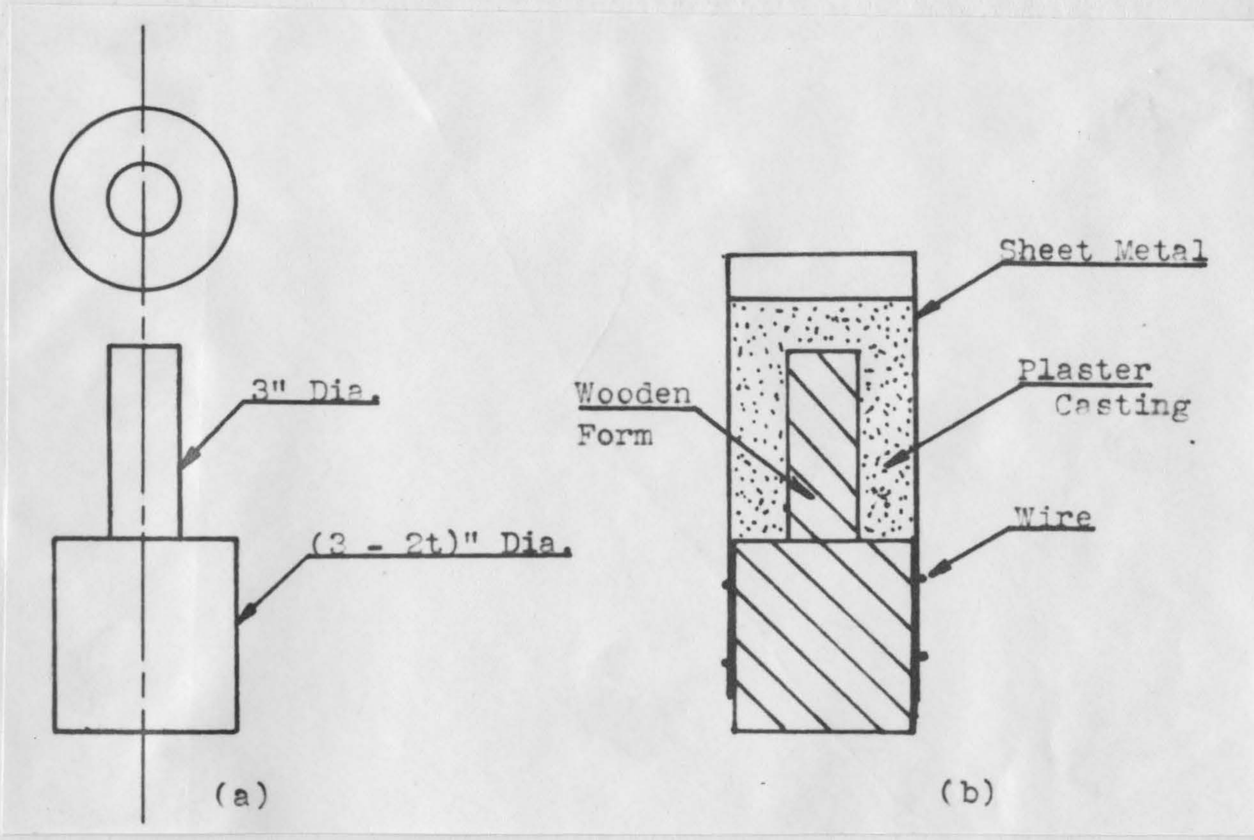


Fig. 1. Form Used to Cast Plaster Cylinders.

The resulting slight ridge on the casting was easily removed by scraping and sanding.

The metal and wooden surfaces which came in contact with the plaster-water mixture were covered with a very thick mixture of soap and water from which the water evaporated leaving a soap film which prevented the plaster from adhering to the wood or metal surfaces.

The mixture used in all castings was 70 parts water to 100 parts plaster by weight. The water and plaster were mixed until all plaster lumps had dissolved. The mixture was then poured into the form to a predetermined depth 1/16 inch more than the depth to give the required thickness to the closed end of the cylinder.

After the cylinder halves were removed from the molds, the ends were planed down with a straight edge so that the closed end had the required uniform thickness and the required length was obtained.

Heat Source

The heat source placed inside the cylinder was a nichrome resistance wire wound on a two-inch hollow refractory cylinder cover with one layer of asbestos paper to protect the inside cylinder wall from radiant energy. The elements were heated by D. C. power. The resistance of the element used in the hollow cylinders having a 12-inch internal length was 11.5 ohms. For the hollow cylinders having six inches internal length the resistance was 5.5 ohms, and

3.5 ohms for cylinders of three inches internal length. These heating elements were made one inch shorter than the internal length of the plaster cylinder in which they were placed. This allowed air to circulate freely around the ends of and through the element. The uninsulated copper leads to these elements were brought out through the opening between the cylinder halves. The heating element cylinder was placed coaxially with the plaster cylinder and supported by two 3/8-inch x 3/8-inch x 1-inch wooden blocks placed 90° apart on the circumference of the element at each end of the element. The one-inch length was placed parallel to the longitudinal axis of the element and in the same plane with the corresponding support on the other end. These wooden supports were necessary to prevent contact between the element and plaster and to center the element.

Measurement of Surface Temperature

The surface temperatures were measured in all cases by copper-constantan thermocouples (Leeds and Northrup, No. 22 BWG, #38 calibration). The hot junction of the thermocouples were glued into the surfaces whose temperatures they were measuring and the cold junctions were kept in a crushed ice bath at 32° F. In order to permit the use of one potentiometer to measure the e.m.f. generated in each thermocouple the copper leads from the hot junctions were connected to a common copper lead. This common copper lead was connected to the center tap of several single-pole, double-throw switches. The copper leads from the cold junction of the thermocouples were connected to the outside taps of these switches so that by closing one switch at a time each thermocouple circuit could be completed. The potentiometer was connected in this common lead and measured the e.m.f. of each circuit separately.

The hot junction of the thermocouples were glued in short grooves in the inside and outside surfaces. These grooves were just deep enough to allow the metal junction to be buried completely below the surface. The junction was then covered with a thin film of glue to prevent the junction being in contact with the air film on the surfaces. This contact would result in the thermocouple generating an e.m.f. partially due to the plaster surface temperature and partially due to the air film temperature which changes considerably in the few thousandths of an inch of thickness of film. The preliminary experiments to determine the conductivity of gypsum plaster demonstrated very clearly this effect of allowing air film and hot thermocouple junction contact by not setting the thermocouple below the surface. (See page 52)

The thermocouple leads from the inside surface were brought out through the opening between the halves. The thermocouples on the inside surface could be more easily and accurately glued in place before the cylinder was mounted for testing. This meant that the thermocouple leads had to be cut to get them out of the test chamber. When the cut leads were connected again, the ends were twisted together and clamped between two washers on small bolts which were tightened to insure electrical contact between the severed ends.

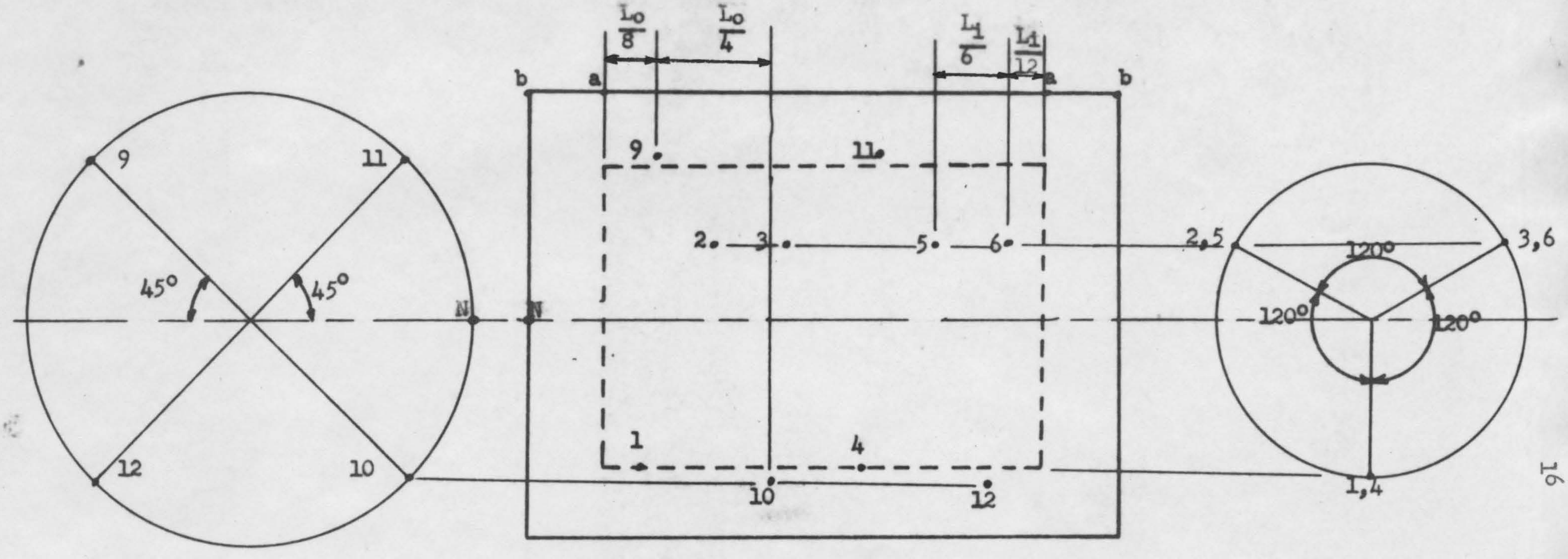
Six thermocouples were glued to the inside and five to the outside surface. The six thermocouples inside were glued to the surface within 1/2 inch of the six positions, 1 through 6, of Fig. 2. The five thermocouples on the outside were attached to the surface within 1/2 inch of the positions, 9, 10, 11, 12, and N shown in Fig. 2. Preliminary tests showed the greatest variation on the inside surface so more thermocouples were used on the inside than the outside surface to obtain a better average.

Mounting of Cylinders

The cylinder halves were mounted on six short wooden contact lines as shown in Fig. 3. Short contact lines were employed to obtain minimum interference with heat flow and air flow about the cylinder.

A 1/16-inch cotton felt gasket was placed between the two halves of the cylinders to minimize the air flow into and out of the hollow inside of the cylinders.

The cylinders were mounted in a 26-inch x 26-inch x 20-inch wooden box to obtain steady flow conditions. The heat transferred through the cylinder was removed from the box by two copper cooling plates nine inch in diameter (for description of these plates see page 45 of Appendix A and Fig. 6). One of the two plates was



L_o - Outside Length
L_i - Inside Length

Fig. 2 Outline of Plaster Cylinder Showing the Disposition of the Thermocouples on the Outside and Inside Surfaces

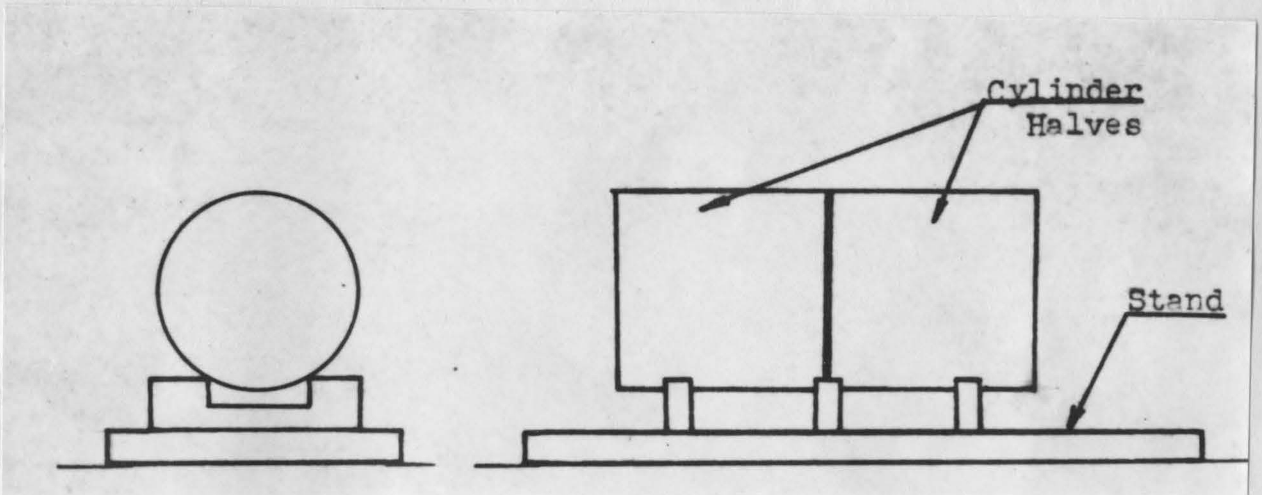


Fig. 3. Method of Mounting Cylinders for Testing.

suspended horizontally two inches below the center of the top of the test chamber. The other plate was placed in a vertical position two inches from the back wall of the test chamber. Both plates were shielded from radiant energy from the heated cylinder.

Cylinders Tested

Three pairs of cylinder halves were cast for testing. The internal diameter and internal length of the three hollow cylinders were three inches and 12 inches, respectively. The plaster thickness of each cylinder was varied, the three thicknesses being 3/4 inch, 1-1/2 inches, and 2-1/4 inches.

The cylinder of 2-1/4-inch thickness cracked due to thermal stresses set up by an 80° F. temperature difference between inside and outside surface temperatures. Because the walls cracked, the data of this test is not included in this thesis.

The 3/4-inch and 1-1/2-inch thick cylinders were tested. Then three inches was sawed off the open ends of the two halves to make cylinders having internal lengths of six inches. The cut ends were made smooth by scraping with a straight edge.

After the cylinders having an internal length of six inches were tested, 1-1/2 inches was cut from the open end of each half to make cylinders having internal lengths of three inches which were also tested.

All cylinders before testing were dried beside a hot radiator until three successive daily weighings were constant. The dry densities of the cylinders were determined from these dry weights and the calculated volumes of the cylinder.

V. RESULTS

Tables I through VIII inclusive are the data results of the tests.

The conductivity k of the plaster for each test was found from the equation $k = k_o \left[1 + .0015 \, (t - 148) \right]$, where k_o is the conductivity of plaster at a mean temperature of 148° F. found from Fig. 9, and t is the mean temperature of the plaster of the cylinder during the test. The determination of the graph of Fig. 9 and the derivation of the preceding formula for conductivity is given in Appendix A.

Fig. 4 is a reproduction of the analytical results of Mr. S. T. Nickerson's investigation of this problem. The experimental results have been plotted on the same graph for comparison. The results of the two tests on each cylinder at an L/D ratio of 4 were plotted to show the possible experimental error in testing these cylinders.

The L/D and T/D ratios are the ratios of internal length to internal length and thickness to internal diameter respectively.

The average outside temperature was taken as the average of the temperatures measured on the cylindrical outside surface which is equidistant from the inside surface. The outside temperature decreases 20 or 30° from point a to point b in Fig. 2 as indicated by the difference of the average of the temperatures measured by thermocouples 9, 10, 11, and 12, and that measured by thermocouple N at the outside corner. This average was used because the t so obtained corresponded to the cylinder wall thickness used in calculating the factor f. The horizontal broken line across each table is drawn in above the data used to calculate the results.

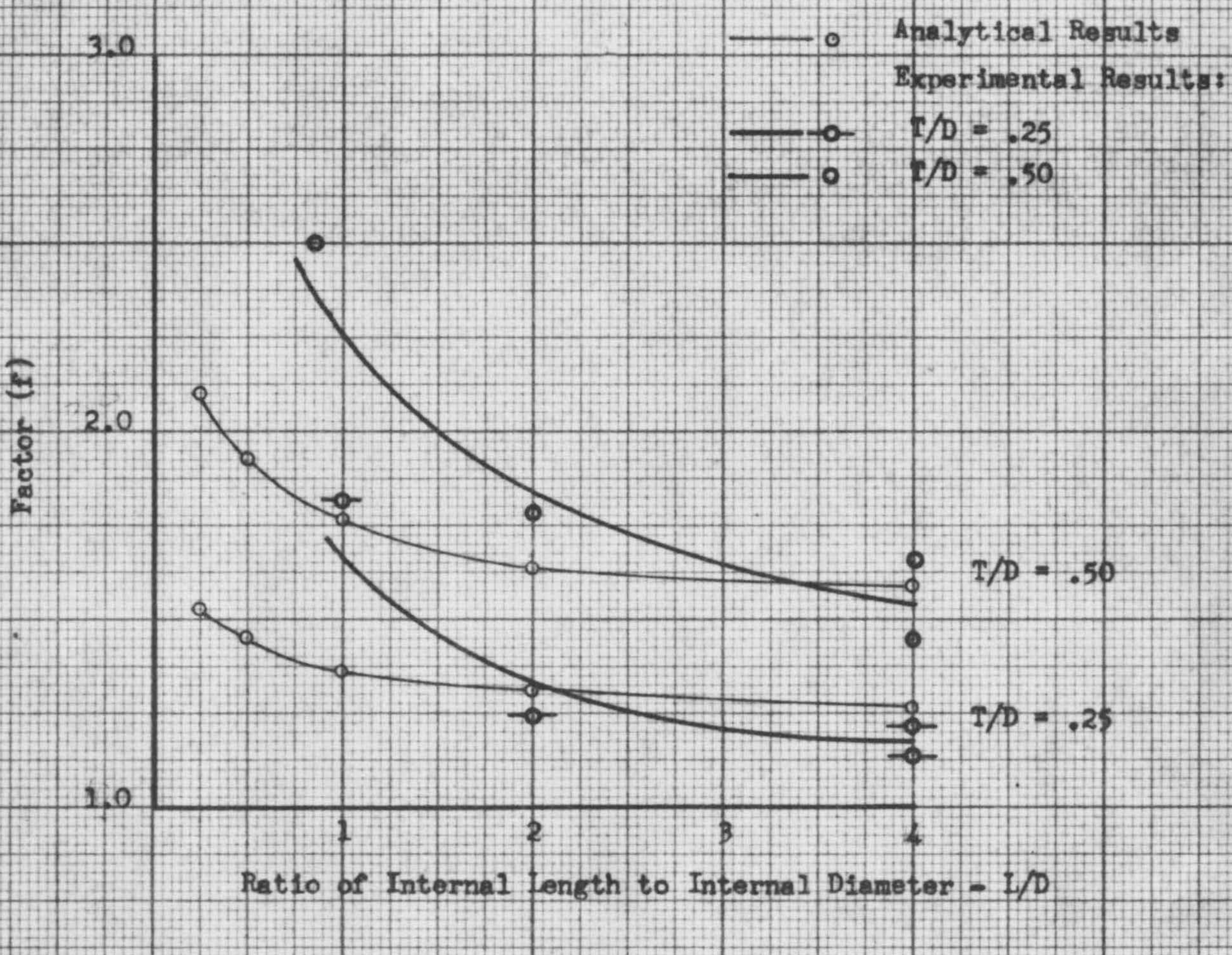


Fig. 4 Curves for Calculation of Heat Transfer Through Insulation Applied to Cylindrical Enclosures

TABLE I

Test of Hollow Cylinder 3/4" Thick and Having a 12" Internal Length

Time	Power		Thermocouples-mv										
			Inside Surface						Outside Surface				
	Amps.	Volts	1	2	3	4	5	6	9	10	11	12	N
7:30	3.0	29.0	3.62	4.22	4.00	4.05	3.55	4.18	2.10	1.80	1.80	1.92	1.42
7:40	2.6	27.0	3.92	4.42	4.12	4.12	3.72	4.38	2.42	2.10	2.10	2.22	1.60
7:50	2.4	26.0	4.02	4.52	4.30	4.30	3.92	4.50	2.65	2.30	2.28	2.42	1.72
8:00	2.3	25.0	4.02	4.60	4.38	4.38	3.90	4.50	2.70	2.31	2.30	2.48	1.75
8:10	2.3	25.0	4.06	4.60	4.38	4.37	4.00	4.48	2.71	2.33	2.33	2.50	1.83
8:20	2.3	25.0	4.10	4.62	4.38	4.40	4.01	4.48	2.70	2.40	2.40	2.50	1.85
8:30	2.3	25.0	4.10	4.61	4.40	4.41	4.00	4.50	2.70	2.36	2.37	2.52	1.85
8:40	2.3	25.0	4.10	4.64	4.41	4.44	4.02	4.52	2.72	2.40	2.40	2.53	1.84
8:50	2.3	25.0	4.11	4.64	4.42	4.43	4.04	4.51	2.72	2.42	2.40	2.50	1.86
9:00	2.3	25.0	4.13	4.65	4.41	4.44	4.04	4.51	2.73	2.41	2.41	2.50	1.86
Average	2.3	25.0	4.09	4.62	4.40	4.41	4.00	4.50	2.71	2.38	2.37	2.50	1.83

RESULTS OF DATA OF TABLE I

Internal diameter of cylinder	3"
Thickness of cylinder	3/4"
Inside length of cylinder	12"
Weight of "A" half of cylinder	1,100 gms.
Weight of "B" half of cylinder	1,115 gms.
Difference of densities of "A" and "B"	1.5%

$$\text{Average density} = \frac{(1110 + 1115)\ 2.20}{\dfrac{13.5 \times 2.25^2 \times 3.14}{1728} - \dfrac{12 \times 1.5^2 \times 3.14}{1728}} = 65.0\ \#/ft^3$$

Average inside temperature	214° F.
Range of inside temperatures	201° F. to 225° F.
Average outside temperature	141° F.
Range of outside temperatures	136° F. to 150° F.
Mean temperature of plaster	178° F.
Outside surface temperature at end of cylinder	112° F.

$K = 1.90 \quad 1 + .0015\ (178 - 148) \quad = \quad 1.98$ BTU-in./hr-ft^2-°F.

Corrected amperage	2.2 amps.
Corrected voltage	26.0 volts

$$f = \frac{2.2 \times 26.0 \times 3.413 \times .75}{\dfrac{12 \times 3 \times 3.14 + 15^2 \times 3.14}{144} + 1.98\ (214 - 141)} = 1.22$$

$$L/D = \frac{12"}{3"} = 4$$

$$T/D = \frac{.75"}{3"} = .25$$

TABLE II

Test of Hollow Cylinder 3/4" Thick and Having a 12" Internal Length

Time	Power		Thermocouple-mv										
			Inside Surface						Outside Surface				
	Amps.	Volts	1	2	3	4	5	6	9	10	11	12	N
2:55	2.6	29.4	5.31	4.90	5.30	5.28	4.92	5.42	2.99	2.62	2.65	2.80	2.10
3:05	2.6	29.0	5.40	4.90	5.33	5.30	4.91	5.43	2.98	2.66	2.70	2.81	2.10
3:15	2.5	27.7	5.42	4.90	5.36	5.35	4.96	5.50	2.99	2.62	2.72	2.78	2.10
3:25	2.5	27.8	5.28	4.66	5.30	5.30	4.76	5.30	2.90	2.60	2.66	2.73	2.10
3:35	2.5	27.9	5.30	4.67	5.24	5.21	4.80	5.38	2.88	2.60	2.67	2.72	2.04
3:45	2.5	27.9	5.38	4.64	5.24	5.26	4.80	5.42	2.89	2.61	2.61	2.70	2.10
3:55	2.5	27.9	5.42	4.64	5.28	5.28	4.80	5.44	2.88	2.56	2.61	2.70	2.04
4:05	2.5	27.9	5.40	4.63	5.28	5.27	4.85	5.43	2.88	2.55	2.62	2.73	2.04
4:15	2.5	27.9	5.42	4.64	5.30	5.27	4.85	5.44	2.88	2.55	2.64	2.73	2.06
4:25	2.5	27.9	5.43	4.66	5.32	5.29	4.86	5.44	2.89	2.57	2.64	2.72	2.06
4:35	2.5	27.9	5.41	4.65	5.31	5.30	4.86	5.46	2.91	2.56	2.63	2.70	2.05
Average	2.5	27.9	5.40	4.65	5.29	5.27	4.83	5.43	2.89	2.56	2.67	2.71	2.06

RESULTS OF DATA OF TABLE II

Same cylinder as used in Test of Table I

Density = 65.0 #/ft^3

Average inside temperature	243° F.
Range of inside temperatures	226° F. to 256° F.
Average outside temperature	150° F.
Range of outside temperature	144° F. to 157° F.
Mean temperature of plaster	197° F.
Outside surface temperature at end of cylinder	124° F.
K = 190 1 + .0015 (197 - 148) = 2.04 BTU-in/hr-ft^2-°F.	
Corrected amperage	2.4 amps.
Corrected voltage	29.0 volts

$$f = \frac{2.4 \times 29.0 \times 3.413 \times .75}{\frac{12 \times 3 \times 3.14 + 1.5^2 \times 3.14}{144} \times 2.04\,(243 - 150)} = 1.13$$

$$L/D = \frac{12''}{3''} = 4$$

$$T/D = \frac{.75''}{3''} = .25$$

TABLE III

Test of Hollow Cylinder 1-1/2" Thick and Having a 12" Internal Length

Time	Power		Thermocouple-mv										
			Inside Surface						Outside Surface				
	Amps.	Volts	1	2	3	4	5	6	9	10	11	12	N
2:50	2.4	26.7	4.50	4.72	4.60	4.63	4.71	4.28	2.64	2.45	2.40	2.40	2.10
3:00	2.3	26.0	4.70	4.98	4.80	4.90	5.02	4.53	2.64	2.49	2.42	2.42	2.10
3:10	2.3	25.0	4.82	5.02	4.90	4.99	5.18	4.68	2.71	2.53	2.48	2.48	2.10
3:20	2.1	23.3	4.77	4.99	4.85	4.95	5.12	4.67	2.78	2.58	2.52	2.52	2.12
3:45	1.56	16.0	3.85	3.98	3.84	4.01	4.18	3.85	2.70	2.58	2.50	2.50	2.15
3:55	1.54	16.4	3.83	3.92	3.84	3.94	4.12	3.80	2.79	2.53	2.42	2.42	2.16
4:05	1.57	16.7	3.80	3.90	3.83	3.90	4.10	3.75	2.62	2.52	2.40	2.40	2.10
4:15	1.57	16.7	3.78	3.89	3.79	3.90	4.08	3.70	2.62	2.48	2.40	2.38	2.10
4:25	1.58	16.8	3.78	3.88	3.78	3.88	4.06	3.70	2.60	4.47	2.38	2.38	2.10
4:35	1.58	16.8	3.77	3.88	3.79	3.88	4.04	3.70	2.60	2.48	2.36	2.37	2.10
4:45	1.58	16.8	3.78	3.86	3.78	3.88	4.05	3.68	2.59	2.48	2.36	2.37	2.10
4:55	1.58	16.8	3.78	3.87	3.78	3.87	4.04	3.69	2.59	2.47	2.37	2.37	2.10
Average	1.58	16.8	3.78	3.88	3.79	3.88	4.06	3.70	2.60	2.47	2.38	2.38	2.10

RESULTS OF DATA OF TABLE III

Internal diameter of cylinder	3"
Thickness of cylinder	1-1/2"
Inside length of cylinder	12"
Weight of "A" half of cylinder	2926 gms
Weight of "B" half of cylinder	2963 gms
Difference of densities of "A" and "B"	1.3%

$$\text{Average density} = \frac{(2926 + 2963)\ 2.20}{\frac{15 \times 3^2 \times 3.14}{1728} - \frac{12 \times 1.5^2 \times 3.14}{1728}} = 66.0\ \#/\text{ft}^3$$

Average inside temperature	196° F.
Range of inside temperature	193° F. to 204° F.
Average outside temperature	139° F.
Range of outside temperature	136° F. to 146° F.
Mean temperature of plaster	167° F.
Outside surface temperature at end of cylinder	125° F.

$$K = 2.01 \quad 1 + .0015\ (167 - 148) \quad = 2.07\ \text{BTU-in/hr-ft}^2\text{-}^\circ\text{F.}$$

Corrected amperage	1.56 amps.
Corrected voltage	18.0 volts

$$f = \frac{1.55 \times 18.0 \times 3.413 \times 1.5}{\frac{12 \times 3 \times 3.14 + 1.5^2 \times 3.14}{144} \times 2.07\ (196 - 139)} = 1.45$$

$$L/D = \frac{12''}{3''} = 4$$

$$T/D = \frac{1.5''}{3''} = .50$$

TABLE IV

Test of Hollow Cylinder 1-1/2" Thick and Having a 12" Internal Length

Time	Power		Thermocouple-mv										
			Inside Surface						Outside Surface				
	Amps.	Volts	1	2	3	4	5	6	9	10	11	12	N
9:25	2.5	27.4	4.00	4.38	4.22	4.12	4.18	3.80	2.30	2.28	2.17	2.12	1.90
9:35	2.4	26.5	4.25	4.62	4.50	4.42	4.52	4.11	2.40	2.32	2.23	2.20	1.90
9:45	2.0	23.0	4.22	4.52	4.40	4.40	4.58	4.11	2.46	2.43	2.32	2.24	1.91
9:55	2.0	23.0	4.28	4.55	4.40	4.42	4.60	4.16	2.55	2.51	2.38	2.30	1.97
10:05	2.0	22.5	4.30	4.55	4.40	4.44	4.68	4.20	2.60	2.52	2.40	2.32	1.97
10:15	2.0	22.5	4.30	4.56	4.40	4.50	4.68	4.20	2.60	2.60	2.42	2.32	2.00
10:25	1.95	22.0	4.32	4.58	4.44	4.55	4.70	4.22	2.62	2.60	2.47	2.35	2.01
10:35	1.95	21.9	4.35	4.58	4.44	4.54	4.70	4.28	2.62	2.60	2.47	2.38	2.03
10:45	1.95	21.7	4.37	4.58	4.42	4.53	4.70	4.28	2.70	2.62	2.48	2.39	2.02
10:55	1.95	21.7	4.35	4.59	4.42	4.53	4.70	4.28	2.70	2.63	2.48	2.39	2.02
11:05	1.95	21.7	4.35	4.60	4.42	4.54	4.70	4.27	2.70	2.63	2.47	2.40	2.02
11:15	1.95	21.8	4.35	4.59	4.42	4.55	4.70	4.27	2.69	2.63	2.47	2.40	2.03
11:25	1.95	21.8	4.38	4.61	4.43	4.55	4.70	4.28	2.69	2.64	2.47	2.39	2.03
Average	1.95	21.8	4.35	4.59	4.43	4.54	4.70	4.27	2.68	2.62	2.47	2.39	2.02

RESULTS OF DATA OF TABLE IV

Same cylinder as used in Test of Table III

Density = 66.0 $\#/ft^3$

Average inside temperature	220° F.
Range of inside temperature	212° F. to 228° F.
Average outside temperature	143° F.
Range of outside temperature	137° F. to 149° F.
Mean temperature of plaster	182° F.
Outside surface temperature at end of cylinder	122° F.

$K = 2.0 \quad 1 + .0015\,(182 - 148) \;= 2.11$ BTU-in/hr-ft^2-°F

Corrected amperage	1.94 amps.
Corrected voltage	22.6 volts

$$f = \frac{1.94 \times 22.6 \times 3.413 \times 1.5}{\dfrac{12 \times 3 \times 3.14 + 1.5^2 \times 3.14}{144} \times 2.11\,(220 - 143)} = 1.66$$

$$L/D = \frac{12''}{3''} = 4$$

$$T/D = \frac{1.5''}{3.0''} = .50$$

TABLE V

Test of Hollow Cylinder 3/4" Thick and Having a 6" Internal Length

Time	Power		Thermocouple-mv										
			Inside Surface						Outside Surface				
	Amps.	Volts	1	2	3	4	5	6	9	10	11	12	N
2:25	2.6	14.0	4.20	3.90	5.00	4.00	4.30	4.36	2.50	2.78	2.76	2.48	2.29
2:35	2.6	14.0	4.50	4.18	5.28	4.30	4.61	4.64	2.60	2.90	2.87	2.56	2.40
2:45	2.4	12.1	4.42	4.08	5.02	4.22	4.50	4.53	2.62	2.99	2.96	2.58	2.48
2:55	2.4	12.2	4.48	4.10	5.08	4.28	4.51	4.53	2.62	2.99	2.95	2.62	2.42
3:05	2.4	12.3	4.48	4.18	5.10	4.29	4.60	4.56	2.66	3.02	2.98	2.60	2.48
3:15	2.4	12.3	4.51	4.18	5.12	4.32	4.62	4.60	2.61	3.02	2.99	2.62	2.48
3:25	2.4	12.3	4.53	4.20	5.12	4.36	4.63	4.60	2.62	3.06	2.88	2.62	2.48
3:35	2.4	12.3	4.53	4.19	5.14	4.37	4.64	4.61	2.62	3.08	2.89	2.63	2.46
3:45	2.4	12.3	4.52	4.20	5.16	4.37	4.70	4.62	2.62	3.08	2.90	2.62	2.49
3:55	2.4	12.3	4.56	4.22	5.20	4.39	4.70	4.62	2.63	3.08	2.92	2.62	2.52
4:05	2.4	12.3	4.55	4.22	5.19	4.38	4.72	4.64	2.62	3.10	2.93	2.64	2.53
4:15	2.4	12.3	4.56	4.24	5.21	4.38	4.72	4.64	2.62	3.08	2.93	2.64	2.50
Average	2.4	12.3	4.54	4.21	5.16	4.37	4.78	4.62	2.62	3.07	2.92	2.63	2.49

RESULTS OF DATA OF TABLE V

Internal Diameter of cylinder	3"
Thickness of cylinder	3/4"
Inside length of cylinder	6"
Weight of "A" half of cylinder	660 gms
Weight of "B" half of cylinder	645 gms.
Percent difference of densities of "A" and "B"	2.5%

$$\text{Average density} = \frac{(660 + 645) \times 2.20}{\frac{7.5 \times 2.25^2 \times 3.14}{1728} - \frac{6 \times 1.5^2 \times 3.14}{1728}} = 64.4\#/ft^3$$

Average inside temperature	221° F.
Range of inside temperature	210° F to 231° F.
Average outside temperature	151° F.
Range of outside temperature	141° F to 164° F.
Mean temperature of plaster	186° F.
Outside surface temperature at end of cylinder	142° F.

$$K = 1.83 \quad 1 + .0015\,(186 - 148) \quad = 1.94 \text{ BTU-in/hr-ft}^2\text{-}^\circ\text{F.}$$

Corrected amperage	2.3 amps.
Corrected voltage	12.7 volts

$$f = \frac{2.3 \times 12.7 \times 3.413 \times .75}{\frac{6 \times 3 \times 3.14 + 1.5^2 \times 3.14}{144} \times 1.94\,(221 - 151)} = 1.25$$

$$L/D = \frac{6"}{3"} = 2$$

$$T/D = \frac{.75"}{3.0"} = .25$$

TABLE VI

Test of Hollow Cylinder 1-1/2" Thick and Having a 6" Internal Length

Time	Power		Thermocouples-mv										
			Inside Surface						Outside Surface				
	Amps.	Volts	1	2	3	4	5	6	9	10	11	12	N
10:15	2.7	14.1	4.68	3.90	4.40	3.69	4.17	3.76	2.08	2.11	2.22	2.14	1.98
10:25	2.4	13.9	4.68	3.99	4.42	3.75	4.22	3.88	2.10	2.18	2.30	2.20	2.00
10:35	2.2	11.8	4.52	3.90	4.30	3.70	4.20	3.82	2.12	2.21	2.42	2.25	2.02
10:45	2.3	12.0	4.56	3.93	4.38	3.75	4.20	3.88	2.13	2.21	2.41	2.27	2.05
10:55	2.3	12.0	4.62	4.00	4.42	3.86	4.28	3.92	2.20	2.30	2.44	2.32	2.10
11:05	2.2	11.9	4.68	4.05	4.52	3.90	4.32	4.00	2.20	2.30	2.42	2.32	2.08
- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
11:15	2.2	11.8	4.70	4.08	4.51	3.98	4.35	4.00	2.20	2.30	2.50	2.32	2.10
11:25	2.2	11.8	4.71	4.11	4.53	3.91	4.42	4.04	2.23	2.32	2.50	2.32	2.10
11:35	2.2	11.9	4.68	4.10	4.51	3.94	4.38	4.02	2.24	2.32	2.54	2.32	2.10
11:45	2.2	11.9	4.70	4.10	4.52	3.92	4.39	4.02	2.24	2.34	2.54	2.31	2.11
11:55	2.2	11.9	4.70	4.12	4.54	3.95	4.41	4.04	2.27	2.33	2.33	2.33	2.11
12:05	2.2	11.9	4.71	4.12	4.54	3.95	4.42	4.06	2.27	2.34	2.33	2.33	2.13
12:15	2.2	11.9	4.72	4.13	4.56	3.97	4.42	4.06	2.28	2.36	2.53	2.35	2.13
Average	2.2	11.9	4.70	4.11	4.53	3.95	4.40	4.03	2.25	2.33	2.52	2.33	2.11

RESULTS OF DATA OF TABLE VI

Internal diameter of cylinder	3"
Thickness of cylinder	1-1/2"
Internal length of cylinder	6"
Weight of "A" half of cylinder	1839 gms
Weight of "B" half of cylinder	1875 gms
Difference of densities of "A" and "B"	2.0%

$$\text{Average density} = \frac{(1839 + 1875)\ 2.20}{\frac{9 \times 3^2 \times 3.14}{1728} - \frac{6 \times 1.5^2 \times 3.14}{1728}} = 66.7\ \#/ft^3$$

Average inside temperature	212°F
Range of inside temperature	199° F to 228° F.
Average outside temperature	135° F
Range of outside temperature	131° F to 142° F.
Mean temperature of plaster	179° F
Outside surface temperature of cylinder	126° F

$$K = 2.08\ \ 1 + .0015\ (179 - 148) = 2.18\ \text{BTU-in/hr-ft}^2\text{-}^\circ\text{F}.$$

Corrected amperage	2.1 amps.
Corrected voltage	12.3 volts

$$f = \frac{2.1 \times 12.3 \times 3.413 \times 1.5}{\frac{6 \times 3 \times 3.14 + 1.5^2 \times 3.14}{144} \times 2.18\ (212 - 135)} = 1.79$$

$$L/D = \frac{6"}{3"} = 2$$

$$T/D = \frac{1.5"}{3"} = .50$$

TABLE VII

Test of Hollow Cylinder 3/4" Thick and Having a 3" Internal Length

Time	Power		Thermocouple - mv										
			Inside Surface						Outside Surface				
	Amps.	Volts	1	2	3	4	5	6	9	10	11	12	N
7:45	3.9	8.8	5.42	5.43	5.40	4.60	4.80	4.70	3.18	3.30	2.98	3.36	2.62
7:55	4.0	9.0	5.60	5.62	5.56	4.90	5.10	5.06	3.20	3.40	3.04	3.42	2.75
8:05	3.5	8.0	5.40	5.40	5.43	4.80	5.02	5.02	3.20	3.40	3.07	3.42	2.76
8:15	3.5	8.0	5.41	5.38	5.45	4.86	5.00	5.01	3.20	3.40	3.08	3.42	2.76
8:25	3.5	8.0	5.41	5.40	5.50	4.90	5.00	5.01	3.20	3.32	3.08	3.42	2.78
8:35	3.5	8.0	5.41	5.46	5.60	4.90	5.00	5.02	3.21	3.32	3.06	3.43	2.78
8:45	3.5	8.0	5.50	5.50	5.60	4.90	5.00	5.02	3.21	3.30	3.06	3.42	2.76
8:55	3.5	8.0	5.50	5.50	5.60	4.88	5.00	5.02	3.23	3.34	3.07	3.44	2.77
9:05	3.5	8.0	5.51	5.52	5.60	4.90	5.01	5.04	3.21	3.34	3.06	3.42	2.77
9:15	3.5	8.0	5.50	5.52	5.61	4.90	5.01	5.04	3.23	3.33	3.07	3.42	2.78
9:25	3.5	8.0	5.50	5.53	5.60	4.90	5.00	5.03	3.23	3.33	3.07	3.44	2.78
Average	3.5	8.0	5.47	5.49	5.59	4.90	5.00	5.03	3.22	3.32	3.07	3.43	2.77

RESULTS OF DATA OF TABLE VII

Internal diameter of cylinder	3"
Thickness of cylinder	3/4"
Internal length of cylinder	3"
Weight of "A" half of cylinder	421 gms
Weight of "B" half of cylinder	430 gms
Difference of densities of "A" and "B"	2.1%

$$\text{Average density} = \frac{(421 + 430)\ 2.20}{\dfrac{4.5 \times 2.25^2 \times 3.14 - 3 \times 1.5^2 \times 3.14}{1728}} = 64.3\ \#/ft^2$$

Average inside temperature	251° F.
Range of inside temperature	237° F to 265° F.
Average outside temperature	174° F
Mean temperature of plaster	216° F
Outside surface temperature at end of cylinder	153° F

$$K = 1.82\ \left[1 + .0015\ (216 - 148)\right] = 2.01\ \text{BTU-in/ft}^2\text{-hr-}^\circ\text{F}.$$

Corrected amperage	3.3 amps.
Corrected voltage	8.2 volts

$$f = \frac{3.3 \times 8.2 \times 3.4.3 \times .75}{\dfrac{3 \times 3 + 1.5^2}{144} \times 2.01\ (251 - 174)} - 1.81$$

$$L/D = \frac{3}{3} = 1$$

$$T/D = \frac{3/4}{3} = .25$$

TABLE VIII

Test of Hollow Cylinder 1-1/2" Thick and Having a 3" Internal Length

Time	Power		Thermocouples-mv										
			Inside Surface						Outside Surface				
	Amps.	Volts	1	2	3	4	5	6	9	10	11	12	N
7:25	4.6	10.7	5.28	6.30	6.76	5.99	5.20	6.51	2.50	2.50	2.50	2.50	2.20
7:35	4.6	10.7	5.60	6.70	7.50	6.20	5.45	6.78	2.40	2.60	2.63	2.65	2.24
7:45	3.5	8.1	5.18	6.01	6.80	5.70	4.99	6.30	2.43	2.63	2.70	2.62	2.30
7:55	3.5	8.1	5.09	6.00	6.78	5.60	4.89	6.29	2.43	2.73	2.70	2.60	2.30
8:05	3.5	8.1	5.06	6.01	6.79	5.60	4.88	6.30	2.43	2.72	2.72	2.61	2.30
8:15	3.5	8.1	5.06	6.01	6.79	5.60	4.88	6.30	2.43	2.72	2.72	2.61	2.30
8:25	3.5	8.1	5.05	6.02	6.79	5.62	4.89	6.30	2.45	2.72	2.74	2.61	2.30
8:35	3.5	8.1	5.05	6.00	6.78	5.62	4.89	6.32	2.46	2.74	2.73	2.63	2.31
8:45	3.5	8.1	5.06	6.01	6.78	5.61	4.91	6.32	2.46	2.75	2.74	2.63	2.31
8:55	3.5	8.1	5.05	6.00	6.80	5.62	4.91	6.33	2.45	2.75	2.74	2.62	2.31
Average	3.6	8.1	5.06	6.01	6.79	5.61	4.89	6.31	2.44	2.73	2.73	2.62	2.31

RESULTS OF DATA OF TABLE VIII

Internal diameter of cylinder	3"
Thickness of cylinder	1.5"
Internal length of cylinder	2-9/16"
Weight of "A" half of cylinder	1175 gms
Weight of "B" half of cylinder	1196 gms
Difference of densities of "A" and "B"	1.8%

$$\text{Average density} = \frac{(1175 + 1196)\ 2.20}{\dfrac{5\text{-}9/16 \times 3^2 \times 3.14 - 2\text{-}9/16 \times 1.5^2 \times 3.14}{1728}} = 65.0\ \#/ft^3$$

Average inside temperature	266° F
Range of inside temperature	235° F to 305° F.
Average outside temperature	147° F
Range of inside temperature	139° F to 151° F.
Mean temperature of plaster	207° F
Outside surface temperature at end of cylinder	134° F

$$K = 1.90 \quad 1 + .0015\ (207 - 148) \quad = 2.07\ \text{BTU-in/hr-ft}^2\text{-°F.}$$

Corrected amperage	3.3 amps.
Corrected voltage	8.3 volts

$$f = \frac{3.3 \times 8.3 \times 3.413 \times 1.5}{\dfrac{2\text{-}9/16 \times 3 \times 3.14 \times 1.5^2 \times 3.14}{144} \times 2.07\ (266 - 147)} = 2.5$$

$$L/D = \frac{2\text{-}9/16}{3} = .855$$

$$T/D = \frac{1.5}{3} = .50$$

VI. DISCUSSION OF RESULTS

For values of T/D greater than two, the analytical and experimental results differ by not more than eight per cent. Mr. Nickerson stated in his thesis that he "feels certain that the results (of his thesis) are accurate to within ten percent for the conditions assumed".[15] Where the end effects are not pronounced his opinion is justified.

The large difference between experimental and analytical results occurs for values of T/D less than two. This is to be expected since a larger part of heat transfer passes through the ends which are not isothermal as Mr. Nickerson assumed.

However, there are two possible reasons that the experimental values differ so widely from the analytical.

One is that heat leakage through the joint is not negligible for the short cylinders. This possibility was discarded for the following reason. Immediately after each test was completed, the front cover of the test chamber was removed and the joint was examined. No difference temperature of the plaster on the surface right at the joint and the outside surface temperature could be felt by hand. Also, no hot air was flowing out through the joint that could be detected by hand.

The other reason is that the average surface temperatures are not representative. The inside surface temperatures varied widely, particularly for the short cylinders. On the other hand, if the average surface temperatures were not representative then it is unlikely that the results for the two cylinders (T/D = .25 and T/D = .50) would have plotted so symmetrically as they did in Fig. 4.

The variation of conductivity of the plaster in the ends of the cylinder at a different mean temperature will change less than 0.5% and has been neglected in the calculations.

When drawing in the curve the maximum deviation from the mean was taken as eight per cent from the results of the two tests on the cylinder having L/D and T/D ratios of four and 0.50 respectively.

Tests could not be run as hoped on cylinders having T/D ratios of 0.75 and 1.0 because thermal stresses caused the relatively thick walls to crack. However, the position of the experimental curves relative to the analytical curves can be assumed because of the similarity of the two sets of curves.

It should be pointed out that these curves applied to any size cylinder whose T/D and L/D ratios fall within the range of those of Fig. 4. Also these curves can be applied to the situation where the heat is flowing into hollow cylinders.

VII. CONCLUSIONS

1. The value of the factor f takes into account the variation of the outside surface temperatures at the ends of the hollow cylinders.

2. The correct solution to heat transfer problem by the relaxation method must be worked under known conditions and not assumptions.

3. The curves of Fig. 4 offer a convenient method of calculating the heat transfer through insulation uniformly applied to cylindrical tanks having flat ends.

VIII. RECOMMENDATIONS

The obvious recommendation is that tests be conducted to determine more points of the experimental curves of Fig. 4.

The interesting question is why the factor for experimental results for L/D ratios of less than 2 are greater than the analytically values. The author feels that the explanation of this can be given after a more thorough investigation of the inside and outside surface temperatures particularly at the corners.

More uniform inside surface temperature could be obtained by casting the plaster cylinders around a metal cylinder in which steam is condensed as a source of heat. If copper cylinders were used and constantan leads were soldered to the outside surface of the cylinder, the thermocouple hot junctions so formed could be used to measure the temperature of the inside surface of the insulation.

APPENDIX A

DETERMINATION OF CONDUCTIVITY OF GYPSUM PLASTER

INTRODUCTION

The alundum guarded ring method was used to determine the conductivity of gypsum plaster. This apparatus was assembled at V. P. I. as a master's thesis in 1939 and is practically identical with that used by the U. S. Bureau of Standards for many years.*

Essentially the apparatus consists of one hot plate or heat source and two hollow cooling plates through which cooling water is circulated. The test specimens are discs about 9-1/2 inches in diameter and 1/2 inch in height. The accuracy of test results depend upon using two specimens of equal height and density for each test. The apparatus and specimens are assembled in the following manner. One of the cooling plates is supported on the bottom surface in a horizontal position. A test specimen is placed on the top surface of this cooling plate, the hot plate is placed on this specimen, the second specimen is placed on the hot plate and the second cooling plate is placed on the second specimen (see Fig. 7). All plates and specimens are placed concentrically on each other and the weight of the top cooling plate presses the contact surfaces together to insure good thermal contact.

D. C. power is allowed to flow through the three parallel resistance circuits in the hot plate. One of these circuits, the main

---

* For a more complete description of this apparatus see <u>The Assemblage and Calibration of Apparatus for the Determination of Thermal Conductivities of Insulating Materials</u> by R. M. Johnston, V. P. I. Library, 1939.

heating element, extends over the inside area of the plate and is enclosed by the other two circuits which are known as guard rings. Variable resistors in series with each circuit permits the power input to each circuit to be varied. Thermocouples in the surface of the cooling plates, which are in contact with the test specimen, and in the surfaces of the hot plate are used to determine the temperature of those surfaces. By adjusting the variable resistors, the heat released in each of the three circuits is varied until the surface temperatures of the hot plate as measured by the thermocouples are equal. The water in the cooling plates maintains the temperature uniform at all points in both cooling surfaces.

When this equilibrium condition is obtained, the heat flowing from the main heating element is flowing normal to the hot plate surfaces through the two specimens to the cooling surfaces. The heat leakage across the exposed vertical surfaces of the plates and specimens is heat released by the guard ring. So the measured input to the main heating element is the transferred heat which is used to calculate the conductivity of the specimen as will be explained in detail in the following sections.

APPARATUS

The alundum hot plate used in this investigation was purchased from the Mellon Institute, Pittsburgh, Pennsylvania. The following is a description of the plate by Heilman[10] and is illustrated by Fig. 5.

> It consists essentially .... of a circular heating plate, 3/8" thick and 9-1/2 in. in diameter. .... The plate has a spiral groove cut in its face to a depth of slightly more than 3/16 in. The groove has a width of approximately 1/16 in. and starting from the center of the plate continues to the outer edge at the rate of 10 turns per inch. The heating elements consist of No. 22 (B & S) gage Nichrome III resistance wire. The center heating element extends over the center 7 in. of the plate.
>
> Two guard ring heating elements each 1/2 in. wide cover the remainder of the plate. The heating elements are cemented into the center of the heating plate with alundum cement.
>
> Embedded in one surface of the plate are 4 Chromel-Alumel thermocouples 1.75 in., 3.25 in., 3.75 in., and 4.25 in., respectively, from the center of the plate........

The three heating elements are connected in parallel across a D. C. potential. Variable resistors in series with each heating element permits the current and therefore the power in each element to be varied.

The two hollow cooling plates were built in the V. P. I. machine shop according to specifications of Fig. 6. The two holes tapped for 1/8 inch standard pipe are the cooling water inlet and outlet. The four 1/32 inch x 1/32 inch milled slots accommodate the copper-constantan used to measure the surface temperature of the

ALUNDUM GUARDED

HOT PLATE

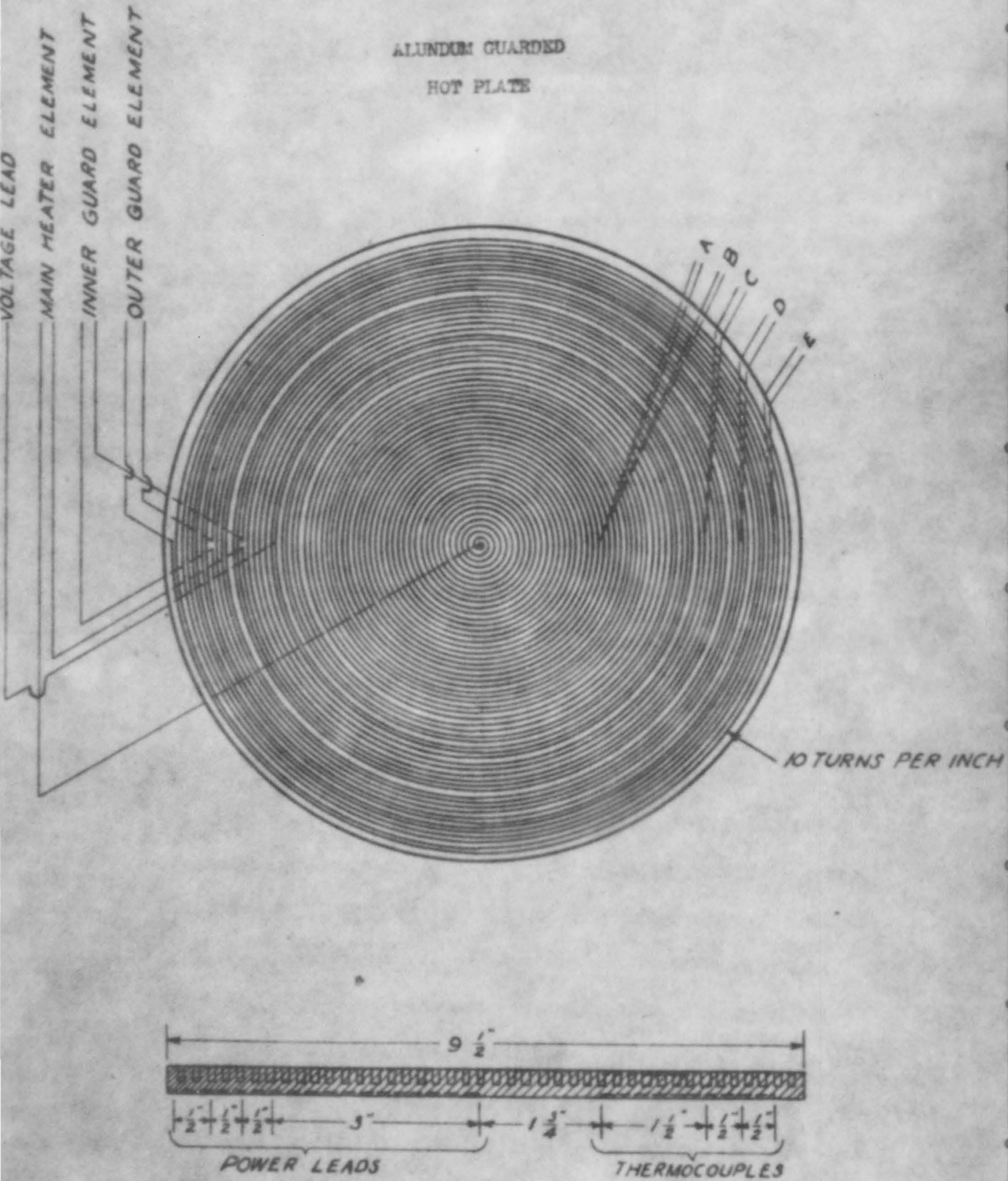


FIG. 5

ALUNDUM GUARDED HOT PLATE

COOLING PLATE

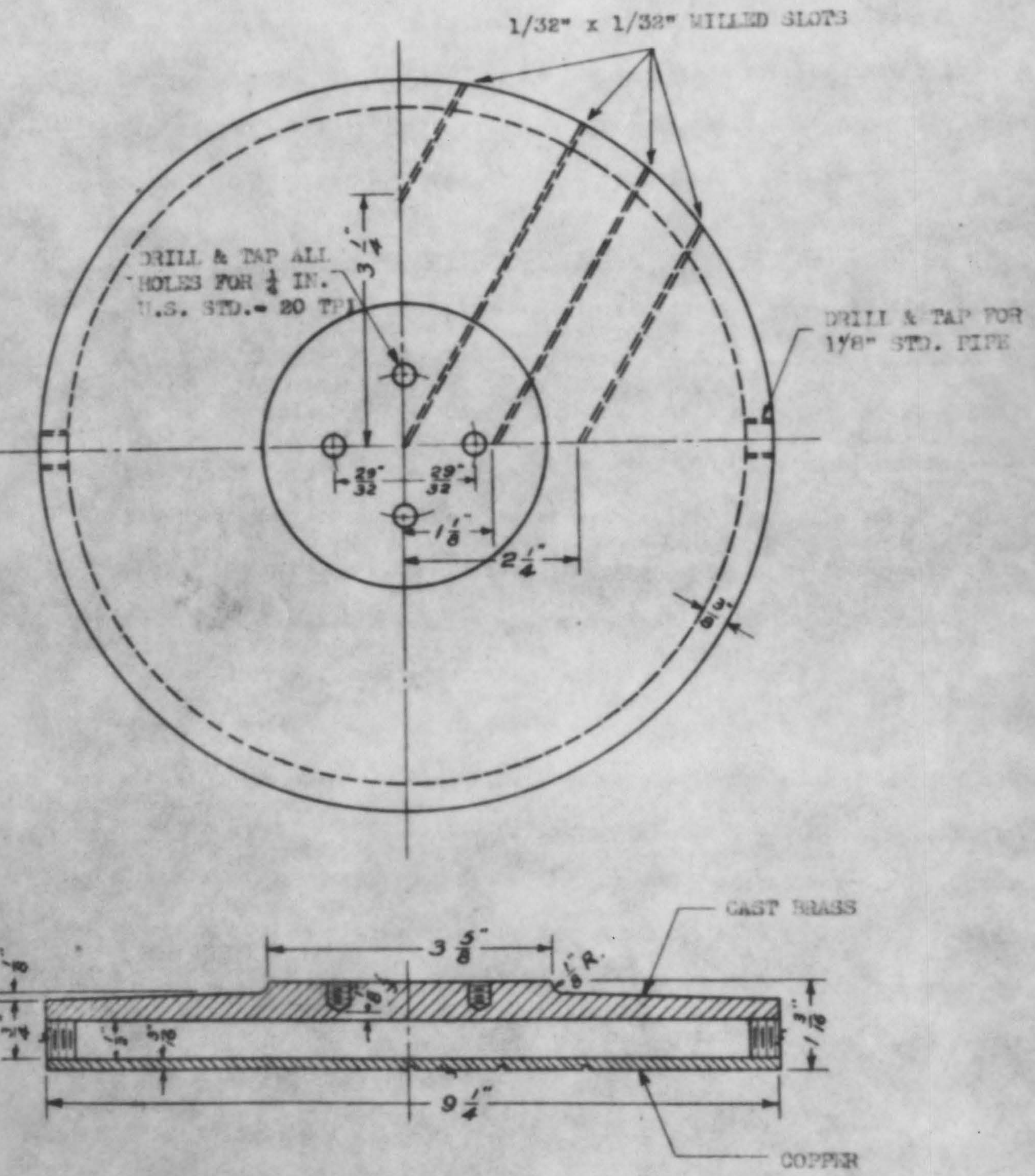


FIG. 6

COOLING PLATE

cooling plate.

Two specimens of the same thickness or height are placed as indicated in Fig. 7 between the heating plate and cooling plates. D. C. power is allowed to flow to the parallel circuits. The surface temperatures of the heating and cooling plates are measured by the thermocouples in the surfaces. Adjustments are made on the three variable resistors until the temperatures across the hot plate surfaces are equal. The temperatures of both the cooling surfaces are maintained uniform and equal by the flow of cooling water through the hollow plates. Where this condition is obtained the heat lost through the sides is heat lost from the two guard rings as indicated by wavy heat flow arrows in Fig. 7. The power input to the heating elements is measured as the current in the main heating element and the voltage drop across the element from the center out to a three-inch radius. The voltage lead at the three-inch radius is so labeled in Fig. 5. So the area of measured heat transfer is a circle six inches in diameter.

If the conductivity of the specimen is less than 1 Btu-in/ft^2-$^{\circ}$F-hr, then the surface temperatures of the cooling and heating plates is taken as the cold and hot surface temperatures of the specimen. If the conductivity is greater than 1, then thermocouples must be placed in the surfaces of the specimen to measure those surface temperatures.

The conductivity K of the two specimens is found from the equation $Q = \frac{KA \Delta t}{L}$, where Q is the electrical power input in Btu to

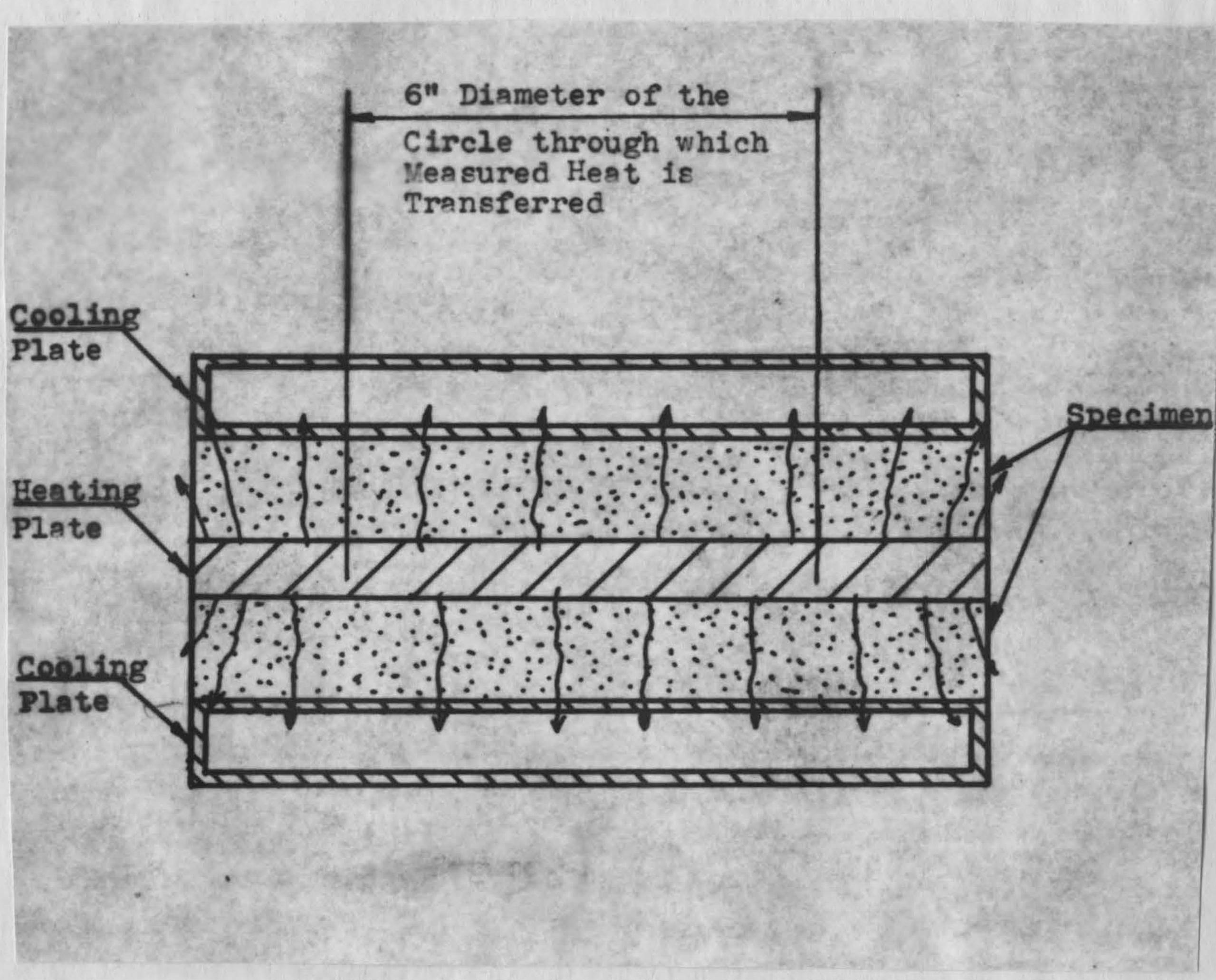


Fig. 7. Cross Section of Apparatus Assembled for Testing.

the center six-inch circle of heating element, Δt is the difference in degrees Farenheit in surface temperatures, L is the thickness in inches of the samples, and A is the sum of the two 6-inch circular areas in square feet through which the heat is transferred from the heating plate to the specimens.

PROCEDURE

Preparation of Specimens

In order to obtain the same density in each member of the pairs of specimens, each pair was cast from the same plaster-water mixture. The specimens were cast in circular molds 9.1 inches in diameter and varying in height from 1/2 inch to 3/4 inch. It was necessary to plane down the top and bottom surfaces of each specimen with a straight edge to make smooth surfaces and get uniform thickness in both specimens of each pair.

The plaster used in all tests was "Pottery Plaster" a form of gypsum plaster sold by the United States Gypsum Company.

The thickness of the specimens were measured with outside calipers and reported to the nearest 1/64 inch.

Mounting of Specimens

Because of warping of the heating and cooling plate surfaces of the testing apparatus, it was impossible to get complete thermal contact between the surfaces of the plate and of the specimens. Cotton felt 1/16 inch thick was placed between these surfaces to minimize the effects of air films due to surface irregularities.

Measurement of Surface Temperatures

Because the conductivity of gypsum plaster is more than 1 Btu-in/ft^2-hr-$^\circ$F and felt had been placed between the surfaces as explained above, the surface temperatures of the specimens must be measured. This was accomplished by making small grooves just large enough to accommodate the thermocouple wire across the whole length of the surfaces of the specimens, laying the thermocouple wire in the groove and glueing the bimetal junction to the plaster to insure thermal contact between the plaster and junction.

The conductivity of the acetate glue used at the thermocouple joints is 1 - 2 Btu-in/ft^2-hr-$^{\circ}$F[16] which is very nearly that of gypsum plaster. Because the conductivities of the glue and plaster are practical equal, the isotherms in the glue were assumed to be coplanar with those in the plaster. A thermocouple measured the temperature of the isotherm A, Fig. 8, passing through the midplane of the thermocouple junction. This isotherm was taken as .015 inch below the surface for reasons given in the following paragraph.

The thermocouples were carefully placed below the surface far enough that the exposed side of the thermocouples could be covered by a thin layer of glue so the air films above the surfaces were not in contact with the thermocouple junction. Preliminary tests in which the thermocouple junctions were partially embedded in plaster so that part of the metal was exposed to the air film yielded values of conductivities of 1 - 1.5 Btu-in/hr-ft^2-$^{\circ}$F. The same specimen tested with the thermocouples completely embedded were found to have conductivities of 2.0 - 2.9 which more nearly agrees with reference values of 3.0.[11] Several of the thermocouple junctions with the hardened glue surrounding them were pulled away from the specimens

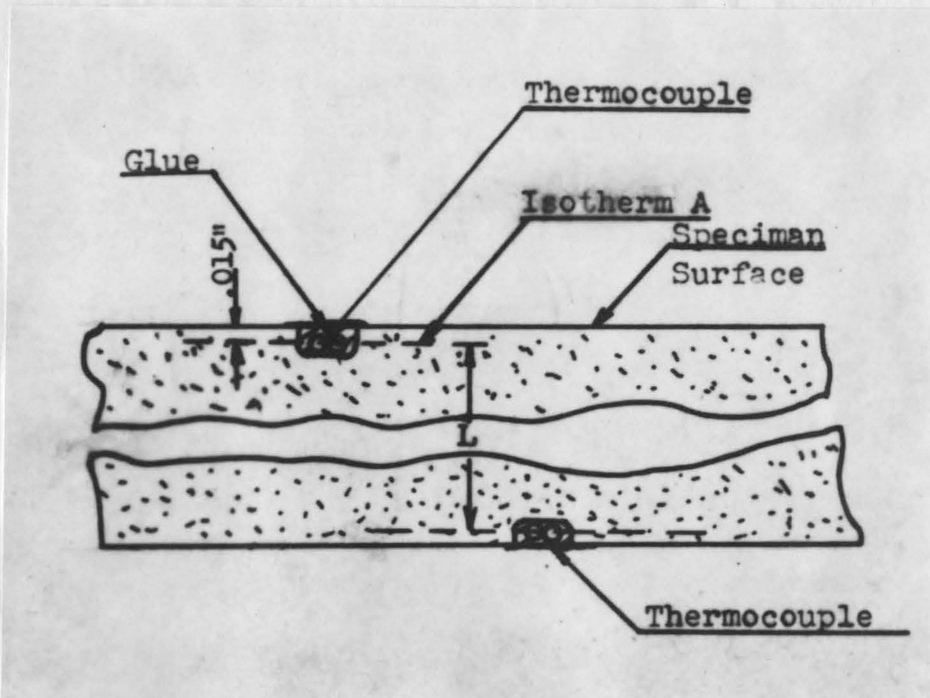


Fig. 8. Cross Section of Test Specimen Showing the Position of the Thermocouples.

after the specimens had been tested. The average thickness of the glue and joint measured with a micrometer was .025 inch. The measured average thickness of the soldered junction was .02 inch. Therefore, the average distance from the middle of the junction to the surface of the glue exposed to the air film is .015 inch. So the thickness L in the formula $Q = \frac{kA \, \Delta t}{L}$ was taken as the thickness of one specimen minus twice .015 inch or .03 inch.

Densities of Specimens

The free moisture was removed from all specimens by heat from the radiator in the room in which the testing equipment was set up. The specimens were placed on a table so that they were within one foot of the hot steam radiator and left for several days. Each day the specimens were weighed and when the weight was constant for three successive days, the specimens were assumed to be dry. The densities were calculated from these dry weights.

RESULTS

The results of tests conducted as described above are given in Tables 1 through 5, inclusive. Graphically the results of the tests are shown in Fig. 9 which gives the variation of conductivity of the plaster with density where the conductivities are determined at a mean temperature of 148^{o} F $\pm 8^{o}$.

In the following tables, the thermocouples B, C, D and E are those in the hot plate surfaces and thermocouples 2 and 3 and 6 and 7 are in the top and bottom cooling surfaces, respectively. The horizontal broken line across each table is drawn in above the data used to calculate the results. The data above the line was taken while the test was coming to equilibrium.

It was hoped that the plaster of the cylinders tested as described in the main body of this thesis could be kept at a mean temperature of 148^{o} F plus or minus several degrees. The mean temperatures of the cylinder walls actually fell in a range of from 175^{o} F to 200^{o} F. So it became necessary to find some method for finding the conductivity of gypsum plaster for mean temperatures other than 148^{o} F.

For small temperature ranges and constant density the conductivities of most materials vary with temperature according to the straight line relation $k_t = k_o \left[1 + a (t - t_o)\right]$ where k_t is the conductivity at temperature t, k_o is the conductivity at some datum temperature to 12. Time did not permit finding by the hot plate method values of the conductivity of gypsum plaster at mean temperatures other than 148° F. However, two values of conductivity for gypsum plaster, molded and dry, were obtained from published data and employed to find a in the above equation. These two values are: $k = 1.41$ Btu-in/hr-ft^2-$^\circ$F at a mean temperature of 70° F and a density of 62.8 lb/ft^3 [13] and k equals 3.0 at a mean temperature of 68° F and a density of 78 lb/ft^3.[12] Substituting in the equation $k_t = k_o\left[1 + a\,(t - t_o)\right]$, $k_t = 1.41$ at 70° F and density of 62.8 lb/ft^3

$$1.41 = 1.64\left[1 + a\,(70 - 148)\right]$$

where $k_o = 1.64$ from Fig. 9, the conductivity of gypsum plaster at a mean temperature of 148° F and density of 62.8 lb/ft^3.

Solving this equation,

$$a = .0018$$

Also from Fig. 9, $k = 3.3$ for a density of 78 lb/ft^3 at a mean temperature of 148° F and according to published data $k = 3.0$ at this same density but for a mean temperature of 68° F. Substituting the values in the above equation

$$3.0 = 3.3\left[1 + a\,(68 - 148)\right]$$

and solving,

$$a = .0012$$

The difference between the values is due to experimental inaccuracies and the difficulty of reading the graph of Fig. 9 accurately to the third place. The value of a substituted in the equation is the average

$$a = \frac{.0018 + .0012}{2} = .0015$$

and the equation becomes

$$k_t = k_o\left[1 + .0015\ (t - t_o)\right]$$

and using this equation with Fig. 9

$$k_t = k_o\left[1 + .0015\ (t - 148)\right]$$

This equation can be used for any density of plaster which lies between the values 62.8 and 78 lbs/ft^3.

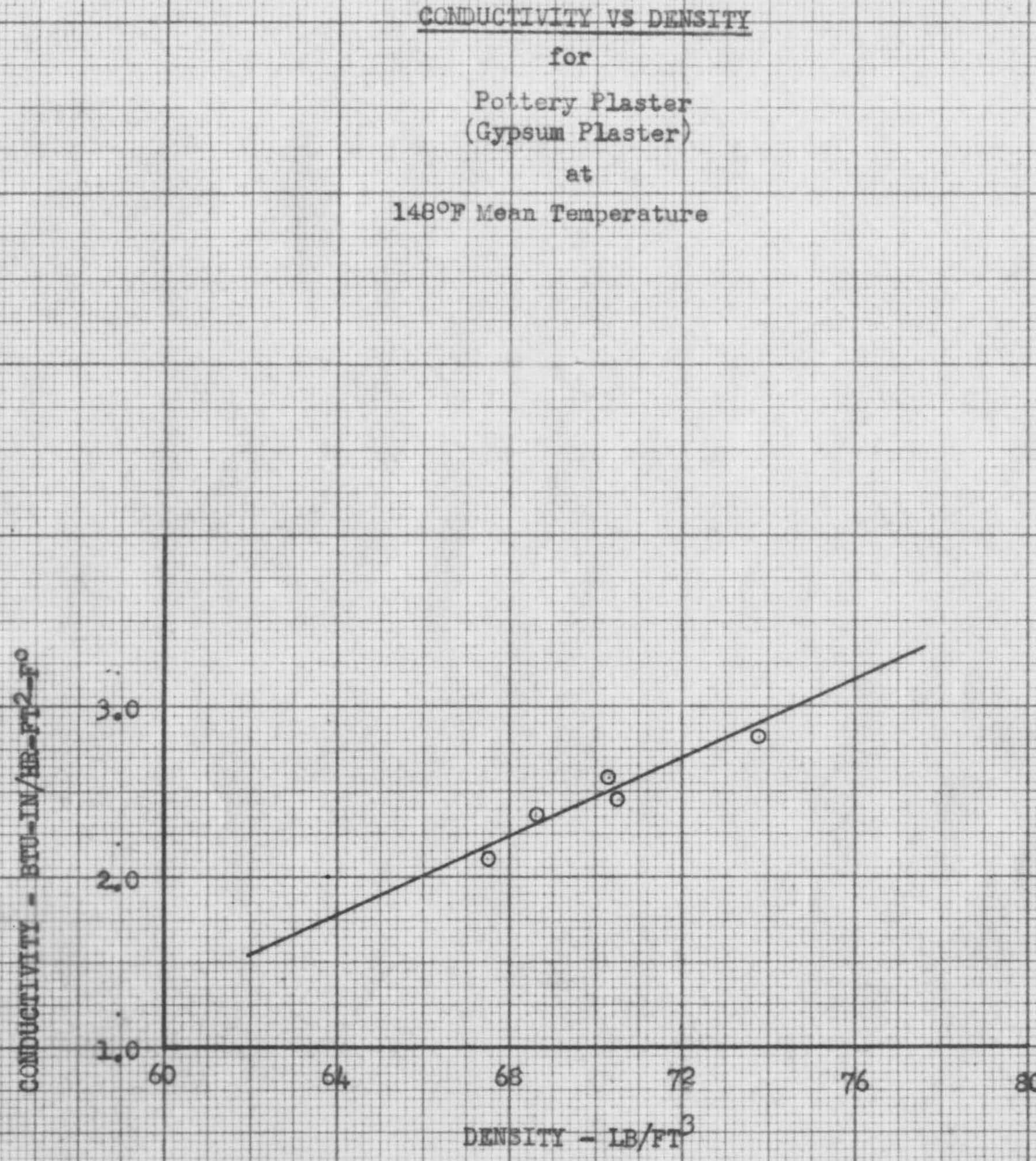


Fig. 9. Graph Showing Variation of Conductivity with Density for Gypsum Plaster

TABLE 1

TEST OF 37/64" SPECIMEN

Time	Power		Thermocouples-mv							
			Controls							
	Amp.	Volts	B	C	D	E	2	3	6	7
9:30	1.52	26.2	4.99	4.90	4.95	4.95	.61	.60	.62	.62
9:45	1.52	26.1	4.96	4.90	4.90	4.90	.61	.60	.63	.63
10:00	1.51	26.1	4.91	4.89	4.90	4.90	.61	.61	.61	.61
10:15	1.51	26.1	4.90	4.89	4.89	4.89	.60	.60	.60	.60
10:30	1.51	26.1	4.89	4.85	4.85	4.85	.60	.60	.61	.61
10:45	1.51	26.1	4.87	4.85	4.85	4.84	.61	.61	.61	.61
11:00	1.51	26.1	4.85	4.85	4.84	4.85	.61	.61	.62	.62
11:15	1.51	26.1	4.85	4.84	4.84	4.83	.61	.61	.61	.61
Average	1.51	26.1								

Weight of specimen = 1.465 lb.
Diameter of specimen = 9.1"
Thickness of specimen = 37/64"

$$\text{Density} = \frac{1.465}{\dfrac{37/64 \times 3.14 \times 9.1^2}{1728 \times 4}} = 67.5 \text{ lb/ft}^3$$

TABLE 1 (Continued)

Time	Thermocouples --mv							
	Hot Surfaces				Cold Surfaces			
	M	10	11	12	L	13	N	9
9:30	3.91	3.80	4.05	4.10	1.80	1.72	1.76	1.71
9:45	3.89	3.80	4.02	4.07	1.80	1.70	1.78	1.60
10:00	3.88	3.76	4.01	4.05	1.78	1.69	1.70	1.58
10:15	3.82	3.73	3.98	4.02	1.74	1.67	1.70	1.52
10:30	3.82	3.72	3.98	4.02	1.72	1.61	1.69	1.50
10:45	3.81	3.72	3.98	4.01	1.71	1.60	1.66	1.50
11:00	3.81	3.72	3.99	4.01	1.71	1.60	1.66	1.51
11:15	3.81	3.72	3.98	4.02	1.72	1.60	1.66	1.51
Average	3.81	3.72	3.98	4.01	1.72	1.60	1.67	1.51

Average hot surface temperature = 196.8° F
Average cold surface temperature = 104.8° F
Mean temperature = 150.8° F
Corrected amperage = 1.49 amps
Corrected voltage = 27.0 volts

$$k = \frac{1.49 \times 27.0 \times 3.413\ (37/64 - .03)}{.393\ (196.8 - 104.8)} = 2.10 \text{ Btu in/ft}^2\text{-hr-}^\circ\text{F}$$

Thermocouples on bottom specimen 11, 12, 13 and L
Thermocouples on top specimen M, 10, N and 9

TABLE II

TEST OF 3/4" SPECIMEN

Time	Power		Thermocouples - mv							
			Controls							
	Amp.	Volts	B	C	D	E	2	3	6	7
10:00	1.50	26.0	4.90	4.90	4.90	4.90	.54	.54	.54	.54
10:15	1.50	26.0	4.86	4.86	4.90	4.90	.53	.53	.53	.53
10:30	1.50	26.0	4.85	4.85	4.80	4.75	.53	.53	.53	.53
10:45	1.50	26.0	4.80	4.80	4.75	4.75	.54	.54	.54	.54
11:00	1.50	26.0	4.80	4.80	4.78	4.78	.54	.54	.53	.53
11:15	1.50	26.0	4.80	4.80	4.78	4.78	.53	.53	.53	.53
11:30	1.50	26.0	4.80	4.80	4.80	4.80	.53	.53	.53	.53
11:45	1.50	26.0	4.80	4.80	4.78	4.78	.53	.53	.53	.53
12:00	1.50	26.0	4.80	4.80	4.80	4.80	.53	.53	.53	.53
Average	1.50	26.0								

Weight of specimen = 2.08 lb.
Diameter of specimen = 9.1 "
Thickness of specimen = 3/4"

$$\text{Density} = \frac{2.08}{\frac{3/4 \times 3.14 \times 9.1^2}{1728 \times 4}} = 73.8 \text{ lb/ft}^3$$

TABLE 2 (Continued)

Time	Thermocouples mv							
	Hot Surfaces				Cold Surfaces			
	M	10	11	12	L	13	N	9
10:00	3.58	3.50	3.90	3.70	1.62	1.62	1.44	1.30
10:15	3.58	3.48	3.82	3.70	1.60	1.60	1.42	1.25
10:30	3.55	3.48	3.83	3.65	1.60	1.58	1.44	1.25
10:45	3.52	3.45	3.80	3.62	1.58	1.58	1.40	1.28
11:00	3.50	3.44	3.80	3.62	1.57	1.57	1.40	1.25
11:15	3.50	3.42	3.80	3.62	1.56	1.56	1.40	1.25
11:30	3.52	3.42	3.80	3.62	1.58	1.58	1.40	1.25
11:45	3.52	3.42	3.81	3.62	1.56	1.56	1.40	1.23
12:00	3.52	3.42	3.80	3.62	1.54	1.54	1.40	1.25
Average	3.51	3.42	3.80	3.62	1.56	1.56	1.40	1.25

Average hot surface temperature = 185.2°F
Average cold surface temperature = 96.8°F
Mean temperature = 140.5°F
Corrected amperage = 1.48 amps
Corrected voltage = 26.9 volts

$$k = \frac{1.48 \times 26.9 \times 3.413\ (3/4 - .03)}{.393\ (185.2 - 96.8)} = 2.82\ \text{Btu-in/hr-ft}^2\text{-}^\circ\text{F}$$

Thermocouples on bottom specimen = 13, L, M and N
Thermocouples on top specimen = 9, 10, 11 and 12

TABLE 3

TEST OF 37/64" SPECIMEN

Time	Power		Thermocouples-mv							
			Controls							
	Amps.	Volts	B	C	D	E	2	3	6	7
9:20	1.51	26.0	4.86	4.86	4.86	4.89	.60	.60	.60	.60
9:35	1.51	26.0	4.85	4.82	4.80	4.88	.60	.60	.60	.60
9:50	1.51	26.0	4.80	4.80	4.80	4.82	.60	.56	.60	.60
10:05	1.50	26.0	4.80	4.80	4.80	4.82	.60	.55	.60	.60
10:20	1.50	26.0	4.80	4.80	4.80	4.81	.60	.56	.60	.60
10:35	1.50	26.0	4.80	4.80	4.80	4.81	.60	.54	.60	.60
10:50	1.50	26.0	4.80	4.78	4.80	4.80	.60	.55	.60	.60
11:05	1.50	26.0	4.80	4.78	4.80	4.80	.60	.56	.60	.60
Average	1.50	26.0								

Weight of specimen = 1.51 lb.
Diameter of specimen = 9.1"
Thickness of specimen = 37/64"

$$\text{Density} = \frac{1.51}{\dfrac{37/64 \times 3.14 \times 9.1^2}{1728 \times 4}} = 68.6\ \text{lb/ft}^3$$

TABLE 3 (Continued)

Time	Thermocouples-mv							
	Hot Surfaces				Cold Surfaces			
	M	10	11	12	L	13	N	9
9:20	3.78	3.70	3.82	3.75	1.81	1.78	1.78	1.80
9:35	3.70	3.65	3.78	3.74	1.78	1.78	1.78	1.80
9:50	3.70	3.64	3.72	3.70	1.71	1.72	1.72	1.78
10:05	3.68	3.60	3.70	3.69	1.68	1.72	1.72	1.75
10:20	3.65	3.56	3.69	3.67	1.67	1.71	1.70	1.72
10:35	3.61	3.55	3.69	3.67	1.61	1.68	1.70	1.72
10:50	3.60	3.55	3.68	3.67	1.60	1.70	1.70	1.72
11:05	3.60	3.56	3.68	3.66	1.60	1.69	1.69	1.70
Average	3.62	3.55	3.69	3.67	1.62	1.69	1.70	1.71

Average hot surface temperature = 186.8° F
Average cold surface temperature = 107.0° F
Mean temperature = 146.9° F
Corrected amperage = 1.48 amps
Corrected voltage = 26.9 volts

$$k = \frac{1.48 \times 26.9 \times 3.413 \times (37/64 - .03)}{.393\ (186.8 - 107.0)} = 2.36 \text{ Btu-in/hr-ft}^2\text{-}^\circ\text{F}$$

Thermocouples on bottom specimen = 10, 11, 13, and L
Thermocouples on top specimen = 9, 12, M and N

TABLE 4

TEST OF 3/4" SPECIMEN

Time	Power		Thermocouples-mv							
			Controls							
	Amps.	Volts	B	C	D	E	2	3	6	7
11:10	1.51	26.0	5.30	5.22	5.28	5.30	.64	.64	.64	.64
11:25	1.50	25.9	5.27	5.21	5.21	5.26	.65	.65	.65	.65
11:40	1.50	25.9	5.22	5.16	5.20	5.20	.66	.66	.66	.66
11:55	1.49	25.8	5.11	5.11	5.13	5.18	.66	.66	.66	.66
12:10	1.49	25.9	5.13	5.10	5.13	5.16	.68	.68	.68	.68
12:25	1.49	25.9	5.10	5.10	5.10	5.04	.68	.68	.68	.68
12:40	1.49	25.9	5.10	5.10	5.11	5.13	.67	.67	.67	.67
12:55	1.49	25.9	5.10	5.10	5.12	5.12	.67	.67	.67	.67
Average	1.49	25.9								

Weight of specimen = 1.99 lb.
Diameter of specimen = 9.1 "
Thickness of specimen = 3/4"

$$\text{Density} = \frac{1.99}{\dfrac{3/4 \times 3.14 \times 9.1^2}{1728 \times 4}} = 70.5 \text{ lb/ft}^3$$

TABLE 4 (Continued)

Time	Thermocouples-mv							
	Hot Surfaces				Cold Surfaces			
	M	10	11	12	L	13	N	9
11:10	4.16	4.26	4.29	4.29	1.80	1.57	1.80	1.82
11:25	4.11	4.20	4.28	4.20	1.73	1.52	1.78	1.80
11:40	4.10	4.17	4.20	4.20	1.70	1.45	1.70	1.75
11:55	4.02	4.11	4.18	4.11	1.67	1.42	1.68	1.73
12:10	4.00	4.03	4.12	4.11	1.61	1.42	1.60	1.68
12:25	3.99	4.04	4.10	4.10	1.60	1.42	1.58	1.62
12:40	3.96	4.04	4.10	4.03	1.60	1.40	1.48	1.62
12:55	3.98	4.02	4.10	4.03	1.59	1.40	1.48	1.62
Average	3.98	4.03	4.11	4.07	1.60	1.41*	1.56	1.64

Average hot surface temperature = 203.2° F
Average cold surface temperature = 103.2° F
Mean temperature = 153.2° F
Corrected amperage = 1.46 amps
Corrected voltage = 26.9 volts

$$k = \frac{1.46 \times 26.9 \times 3.413\ (3/4 - .03)}{.393\ (203.2 - 103.2)} = 2.46 \text{ Btu-in/hr-ft}^2\text{-}^\circ\text{F}$$

Thermocouples on bottom specimen = 9, 10, 11, and 12
Thermocouples on top specimen = 13, L, M. and N

* After the test this thermocouple was found to be exposed to air film on surface of specimen. Results were neglected.

TABLE 5

TEST OF 46/64" SPECIMEN

Time	Power		Thermocouples-mv							
			Controls							
	Amps.	Volts	B	C	D	E	2	3	6	7
10:05	1.51	26.0	5.10	5.04	5.11	5.11	.62	.68	.58	.62
10:20	1.51	26.0	5.10	5.10	5.10	5.10	.62	.67	.68	.68
10:35	1.51	26.0	5.10	5.10	5.10	5.10	.68	.64	.67	.67
10:50	1.51	26.0	5.10	5.10	5.10	5.10	.67	.64	.64	.64
11:05	1.51	26.0	5.04	5.04	5.10	5.10	.67	.66	.64	.64
11:20	1.51	26.0	5.08	5.08	5.10	5.10	.64	.65	.66	.66
11:35	1.51	26.0	5.10	5.10	5.10	5.10	.66	.66	.65	.65
11:50	1.51	26.0	5.08	5.08	5.10	5.10	.65	.65	.65	.65
12:05	1.51	26.0	5.10	5.08	5.10	5.10	.65	.65	.65	.65
Average	1.51	26.0								

Weight of specimen = 1.94 lb.
Diameter of specimen = 9.1"
Thickness of specimen = 47/64"

$$\text{Density} = \frac{1.94}{\dfrac{47/64 \times 314 \times 9.1^2}{1728 \times 4}} = 70.3 \text{ lb/ft}^3$$

TABLE 5 (Continued)

Time	Thermocouples-mv							
	Hot Surfaces				Cold Surfaces			
	M	10	11	12	L	13	N	9
10:05	4.20	4.08	3.82	4.20	1.62	1.80	1.80	1.80
10:20	4.20	4.15	3.78	4.20	1.70	1.80	1.79	1.79
10:35	4.20	4.11	3.72	4.11	1.65	1.78	1.78	1.78
10:50	4.20	4.10	3.73	4.10	1.65	1.77	1.70	1.70
11:05	4.20	4.10	3.75	4.10	1.60	1.78	1.70	1.70
11:20	4.20	4.10	3.75	4.10	1.60	1.78	1.70	1.70
11:35	4.20	4.10	3.75	4.10	1.60	1.78	1.69	1.70
11:50	4.20	4.10	3.75	4.10	1.60	1.78	1.69	1.68
12:05	4.20	4.10	3.75	4.10	1.60	1.60	1.70	1.72
Average	4.20	4.10	3.75*	4.10	1.60	1.78	1.70	1.70

Average hot surface temperature = 206.4° F
Average cold surface temperature = 108.0° F
Mean temperature = 157.2° F
Corrected amperage = 1.49 amps
Corrected voltage = 26.9 volts

$$k = \frac{1.49 \times 26.9 \times 3.413\ (47/64 - .03)}{.393\ (2064 - 108.0)} = 2.57 \text{ Btu in/hr-ft}^2\text{-}^\circ\text{F}$$

Thermocouples on bottom specimen = 9, 10, 11, and 12
Thermocouples on top specimen = 13, L, M, and N

* Thermocouple was embedded too deeply in plaster.
Results neglected.

DISCUSSION OF RESULTS

A comparison of results with published data is impossible because adequate data is not available other than the two references given in the section RESULTS.

There being no indication to the contrary, gypsum plaster was assumed to be one of the materials whose conductivity varies directly with temperature. The final criterion, of course, is an experimental determination of this relation. However, the conductivity of abestos varies 1.3 per cent as the mean temperature changes from 100° F to 200° F, that of loose infusorial earth varies 1.1 per cent for the same temperature difference, and for pulverized cork the variation is 2.2 per cent. The conductivity of gypsum plaster changes 1.5 per cent in 100° temperature difference which is within the limits of the three materials listed above.

Compared to the published data cited in the section RESULTS, the results plotted in Fig. 9 are reasonable. The crucial measurements employed in calculating the conductivity are thickness of specimen and the temperature difference between surfaces of the specimens.

The only available check on surface temperature measurements is to plot the results as in Fig. 9. Large deviations of the measured average temperature difference from the actual average temperature difference would result in wide spread points. The results plotted on Fig. 9 are within three per cent of the mean.

"The calculation of the temperature gradients ..... must be left to the judgment of the operator" in the case in which the thermocouples are attached to the surface of the specimen.[14] The method used by the operator in conducting these tests is that described under Measurement of Surface Temperatures.

The effect of using an erroneous method of determining this gradient is an error in one direction only and results in moving the mean curve of Fig. 9 either up or down on the ordinate depending on the error.

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