THE APPLICATION OF LOAD DISTRIBUTION TO TRUSSED BEATS

by

Joseph E. Spagnuolo

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Approved:

In charge of Thesis

Head of the Department

Dean of Engineering

Chairman, Graduate Committee

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J. E. S.
Introduction

Greater simplification in structural design has always been encouraged by the engineering profession. Much has been done in recent years towards rendering the analysis and design of ordinary structural members a relatively simple matter. Structural hand-books of all descriptions are available to the engineer; design tables of various kinds are widely used, while new short-cut methods make their appearance regularly in engineering literature.

Such time and labor saving information, of course, is invaluable to the engineer. Indeed, such information has always been welcomed by structural designers, even if it has meant, at times, sacrificing accuracy of analysis. Of this, however, the practical engineer is not particularly concerned for he is aware of the uselessness of the many naive and unnecessary refinements often indulged in by his more theoretically minded colleagues. It becomes, for instance, a sheer waste of time and effort to compute in great detail the stresses involved at a column and girder connection with the idea of determining the minimum number of rivets required, when a standard connection, having many more rivets than needed, is what the fabricators would use.

The tendency towards greater simplification in structural design is further evidenced by the fact that tables for structural members other than those treated in standard hand-books have, in recent years, made their appearance. These design tables cover such structural members as steel joists and lightweight roof trusses some of which are capable of spanning sixty feet. Not uncommonly too, one finds in the catalogues of various manufacturers, tables referring to ready made trusses suitable for almost any ordinary purpose, and capable of handling spans up to one hundred feet.
One needs only compute the roof loads and select from the tables the light-
est truss capable of carrying them! It is usually found too, that these stock trusses are more economical than those designed for the purpose.

Undoubtedly, more of the ready made or stock pattern trusses would be used were it not for certain limiting restrictions which, as yet, have not been overcome, such as the type of truss, span and loading, let alone the more or less indeterminate conditions met with when trusses are supported on slender columns as is usually the case for long spans and high ceilings. Trussed bents, as these frames are usually called, offer of course, serious difficulties as to their correct analysis. This one fact perhaps more than any other is the chief reason for the lack of a readily applicable method of design.

Simplified solutions based on arbitrary assumptions are often resorted to, but because these solutions give only a very crude approximation of the stresses involved, such simplified analysis should not be attempted unless the problem is of a very simple nature with stresses not likely to be of such magnitude as would not be adequately provided for by the minimum requirements. The attitude usually adopted in the case of approximate solutions is to "play safe" by making the members of the frame much heavier than is actually necessary. This, of course, is not only wasting material, but, as is often observed, placing it where it is needed the least. In other words the procedure is wholly an empirical one, its ultimate result being an increase in cost without any appreciable structural advantage.

The older or classical methods of analyzing trussed bents, though fundamentally simple and direct, become very cumbersome when the number of unknowns are many. And then too, no simplified all inclusive, and readily applicable
data could be prepared from these methods with a view of rendering the design of this particular type of frame an easy matter.

This thesis concerns itself with the application of moment distribution to trussed bents, to the end that any case, however complicated, can be readily analyzed with a substantial saving in time and effort. It is believed that moment distribution is particularly adapted for the purpose. As a method of analysis it is very simple, direct, and accurate. It is a method that has become very popular since it was first made known by Professor Hardy Cross in 1932 and has been successfully applied to almost every type of frame.

This method would require the standardization of truss shapes and sizes in very much the same manner as has been done for steel beams and girders, for in order to apply moment distribution, certain truss constants must be determined; a matter this, implying stock truss design. Such constants and pertinent data could be arranged in chart form and thus made available to designers. The design of a trussed bent would then be a very simple undertaking accompanied by the benefits of greater economy, a more rational design, and a substantial reduction in the time required for analysis.

The following pages show how moment distribution can be applied to trussed bents. It is also shown how the constants were obtained in the case of a four and eight-panel truss, and the manner in which the data can be compiled for general use. Much additional work is, of course, necessary before the scheme set forth in this thesis can be advantageously exploited. However, the fundamentals of the method have been established and any further research in this field promises interesting and worthwhile possibilities.
The Research

As is well known, the classical method of analysis of indeterminate frames is to cut the structure back to a statically determinate condition, and then to find the forces required at the cut sections in order to restore continuity. The method of least work is typical of the classical theories.

In contrast to this, the procedure in moment distribution is to restrain the structure by preventing the rotation and translation of certain joints so that the end moments and thrusts in the restrained structure are easily determined. Then by successive removal of these joint restraints, the structure is finally freed and the true moments are thus found.

However, the application of moment distribution to any indeterminate frame requires the following design constants:

1. Fixed end moments
2. Carry-over factor
3. Moment and stiffness factor
4. Thrust stiffness factor

These constants are readily calculated for prismatic beams having uniform section. For beams having variable moment of inertia, charts giving the constants for varying conditions have been prepared, and are available for general use.

Constants for trusses, however, are not available and are not very easily calculated. Following is the procedure adopted for obtaining the constants for an arbitrarily chosen example; a four panel parallel chord Pratt truss.

Fixed end moments in trusses.

The calculation of the fixed end moments in trusses rigidly anchored
to masonry walls was most conveniently done by the method of least work.

A four panel truss having parallel chords as shown in Fig. 1 was assumed for study. The cross sectional area of all members was arbitrarily taken as being the same throughout. This was done in order that the computations be simplified, since the example was to serve as an illustration of method rather than as a specific case.

If the ends of the truss are not permitted to rotate or translate, forces as shown by the arrows (Fig. 1) must be applied in order that this be accomplished. Finding the magnitude of these forces is, in effect, finding the end moments. But in order to find the forces sought it is first necessary to reduce the truss to a statically determinate condition by cutting the redundant members A-J and H-J and placing a roller, let us say, under the left end. (Fig. 2.)
The stresses in all members due to the imposed loads can be found by statics. These stresses are designated as $S_0$. (See Plate No. 1.)

Now if in the case of the redundant $A$-$J$, we apply a force $X = 1$ and determine the stress $S_x$ in all members due to this unit load, the stresses can be expressed in terms of the redundant load $X$, thus $X S_x = \text{Fig. 3a}$. (See also Plate 1)

![Diagram](a)

![Diagram](b)

![Diagram](c)

Fig. 3.
Similarly if a force $Y = 1$ is applied in the redundant $H-J$, and the stress $Sy$ determined in all members due to the unit load, the stresses can be expressed in terms of the redundant load $Y$, thus $YSy$ — Fig. 3b. (Plate I)

Under the imposed loads the roller will have displaced inward a certain amount, but since the ends of the truss are not permitted to translate, a force $Z$ is necessary to bring the roller back to its original position. Now if we apply a force $Z = 1$ as shown in Fig. 3c, and find the stresses $Sz$ due to this unit force, we can express the stresses in terms of the indeterminate force $Z$, thus $2Sz$. Obviously the total stress in each member of the truss is:

$$S = S_o + XSx + YSy + ZSz$$

Now the total internal work in each member is:

$$W = \int \frac{S^2}{AE}$$

And the partial derivatives of work in each member with respect to the redundants $X$, $Y$, $Z$ are:

$$\frac{\delta W}{\delta X} = 2 \int \frac{(S_0 + XSx + YSy + ZSz) SxL}{AE} = 0$$

(1)
Equations (1), (2), (3) become respectively:

\[
\frac{\delta W}{\delta Y} = \frac{2}{2} \int \left( S_{x} + X S_{x} + Y S_{y} + Z S_{y} \right) S_{y} L \bigg|_{\text{AE}} = 0
\]

(2)

\[
\frac{\delta W}{\delta Z} = \frac{2}{2} \int \left( S_{o} + X S_{x} + Y S_{y} + Z S_{z} \right) S_{z} L \bigg|_{\text{AE}} = 0
\]

(3)

\[
\frac{\delta W}{\delta Z} = \frac{2}{2} \int \left( S_{o} + X S_{x} + Y S_{y} + Z S_{z} \right) S_{z} L \bigg|_{\text{AE}} = 0
\]

Equations (1), (2), (3) become respectively:

\[
\frac{S_{o} S_{x} L}{2 \text{AE}} + \gamma \int \frac{S_{x} S_{x} L}{2 \text{AE}} + \gamma \int \frac{S_{y} S_{x} L}{2 \text{AE}} + Z \int \frac{S_{z} S_{x} L}{2 \text{AE}} = 0
\]

(5)

\[
\frac{S_{o} S_{y} L}{2 \text{AE}} + \gamma \int \frac{S_{x} S_{y} L}{2 \text{AE}} + \gamma \int \frac{S_{y} S_{y} L}{2 \text{AE}} + Z \int \frac{S_{z} S_{y} L}{2 \text{AE}} = 0
\]

(6)

\[
\frac{S_{o} S_{z} L}{2 \text{AE}} + \gamma \int \frac{S_{x} S_{z} L}{2 \text{AE}} + \gamma \int \frac{S_{y} S_{z} L}{2 \text{AE}} + Z \int \frac{S_{z} S_{z} L}{2 \text{AE}} = 0
\]

(7)
The partial derivatives obtained as shown above are summed and set equal to zero. The three resulting simultaneous equations are solved for the values of the redundants $X$, $Y$, and $Z$. The true stresses in the members and consequently the fixed end moments are thus easily found.

Plate I shows in complete detail the operations involved in finding the fixed end moments for the truss as assumed above. The true forces of $X$, $Y$, and $Z$ appear in general terms of "P" and "L".

It will be noted that the true forces of $X$, $Y$, and $Z$ when applied to the truss, are not readily adaptable for moment distribution, but must be altered in the following manner.

![Diagram](image)

**Fig. 4.**

In Fig. 4, the forces $X$, $Y$, and $Z$ are shown applied on the truss. If $.50$ is subtracted from the force $Z = 1.75$ and then added as indicated, and similarly $.50$ added and subtracted to the force $X$ and $Y = .75$ there re-
results a moment of \(1.25 \pi \ell - 2.5\) and a pull of 1 as indicated in the figure.

With this slight transformation moment distribution is readily applicable.

The carry-over factor in trusses.

The carry-over factor in moment distribution is the second constant necessary in order that the process of balancing moments can be accomplished.

The carry-over factor is defined as the ratio of the moment at the fixed end of a beam to the moment producing rotation. Thus in Fig. 5 the ratio of \(M_2\) to \(M_1 = \text{carry-over factor.}\)

\[
\frac{M_2}{M_1} = r
\]

Fig. 5.

The carry-over factor represents the measure with which a beam is capable of transferring a moment from the end being rotated to the end being held fixed. In the case of prismatic beams of uniform section, the carry-over factor is, for instance, one-half.

In the case of the truss assumed (Fig. 1.) the carry-over factor was also obtained by the method of least work. (Fig. 6., and Plate II.)
After having reduced the truss to a statically determinate condition as was done for the F. E. M., a moment equal to unity was applied at the left end of the truss while the right end was kept rigidly anchored into the wall. The simple ratio of the moments thus produced gave the carry-over factor sought.

Plate II shows the operations involved. Here also, as in the case of the fixed end moments, the computations were made in terms of the variable "L" and "a".

The moment stiffness factor in trusses.

Moment stiffness may be thought of as the property of a beam to resist rotation, this property being directly proportional to the cross sectional area and depth of the beam and inversely proportional to the length of the beam. The moment stiffness factor, the third constant necessary in moment distribution, may be defined as the moment required to produce unit rotation at one end of the beam while the other end is being held fixed. Thus in
Fig. 7. The moment required to make $\phi = 1$ is the stiffness factor.

The final stresses in the members as computed in the case of the carry-over factor can be utilized in the calculation of the stiffness factor though here the method of the elastic weights must be resorted to.

The method of elastic weights is identical to that of area-moments as applied to beams except that in trusses, the angle changes $\frac{M}{E} \, \frac{dx}{I}$ are concentrated at the panel points instead of occurring in any small portion $dx$ as in the case with beams.

Briefly the application of this method to our particular problem is as follows:

As implied in the definition of moment stiffness, the slope at the fixed end must be zero, while that at the rotated end must equal unity. Now according to the area-moment proposition that the true slope at any point in a beam is equal to the shear at the corresponding point in the conjugate beam as loaded with the $\frac{M}{EI}$ diagram, we need only determine the slope caused by a unit moment at the rotated end and then, by a simple proportion, obtain the end moment or stiffness required to make this slope equal to unity.

Plate III shows the operations involved for obtaining the moment stiff-
Thrust stiffness in trusses.

In addition to the three constants so far described, a fourth constant known as "thrust stiffness" must be obtained if moment distribution is to be applied to trussed bents.

Thrust stiffness may be defined as the thrust needed to produce a unit horizontal displacement without rotation at one end of the truss while the other end is being held fixed. With reference to the four panel truss assumed in Fig. 1, we obtained the thrust stiffness in the following manner:

\[
\sum \frac{SL}{AE} = \Delta; \quad \frac{t/2 \cdot L/4 \cdot \Delta}{AE} = 1 = \Delta
\]

\[
t = \frac{2AE}{L}
\]

Fig. 3.

In Fig. 3, a horizontal displacement \( \Delta = 1 \) is produced by a thrust \( t \).
Now the stress in the chords as far as any particular panel is concerned, is \( \frac{t}{2} \), and since deformation \( \Delta = \sum SL/AL \), it becomes only necessary to substitute in this relation and solve for \( t \).

Thrust stiffness is a very essential constant in the application of moment distribution to trussed bents, for it implies a distribution of thrusts not reckoned with in the case of prismatic beams. Trusses when loaded deform in a manner different from that of beams and therefore the process of balancing moments is further complicated in that a distribution of moments in a truss causes a thrust at the base of the supporting column. Likewise the balancing of a thrust causes a moment. (See Fig. 9.—a. and b.)

![Fig. 9.](image)

On the contrary as shown in Plate II and Fig. 8, it is to be noted that when applying a moment at one end of the truss (the other end being fixed) no thrust is produced, and vice versa, when applying a thrust to one end of the truss (the other end being fixed) no moment is produced.
Applying moment distribution to a trussed bent.

With the aid of the four constants: fixed end moments, carry-over factor, moment stiffness, and thrust stiffness as obtained in Plates I, II, and III the analysis of a simple bent was undertaken by moment distribution. But first, as a check against the answer given by moment distribution the bent was analyzed by the method of least work as shown in Plate IV.

The bent itself is made up of the same truss for which the constants were calculated and is supported on columns whose moment of inertia is $10a$. Dimensions assumed are also clearly indicated. Hinges were placed at the base of the columns with the view of simplifying the computations.

To reduce the bent to a statically determinate condition, a roller is placed under the left column and the stresses in the members due to the unit loads $P$ are found. These stresses might be called $S_0$. (See Plate IV.)

Now by applying a unit horizontal force $H = 1$ at the roller, the stress in all members of the truss due to this unit load can be found and expressed in terms of the redundant $h$, thus $hS_h$.

Obviously then the total stress in each member is:

$$S = S_0 + hS_h$$
$$M = M_0 + hM_h$$

And the total internal work in each member is:

$$W = \int \frac{S^2 L}{2AE} + \int \frac{h^2 dx}{2EI}$$
But:

\[ \frac{\delta W}{\delta h} = \int \frac{S_0 Sh L}{2AE} + h \int \frac{Sh^2 L}{2AE} + h \int \frac{M_0 Mhdx}{EI} + h \int \frac{M_h^2 dx}{EI} = 0 \]

\[ = \sum \frac{S_0 Sh L}{A} + h \sum \frac{Sh^2 L}{A} + \sum \frac{M_0 Mhdx}{EI} + h \sum \frac{M_h^2 dx}{EI} = 0 \]

Since \( M_0 = 0 \), the equation becomes:

\[ \sum \frac{S_0 Sh L}{A} + h \sum \frac{Sh^2 L}{A} + 0 + h \sum \frac{M_h^2 dx}{EI} = 0 \]

In Plate IV this equation is solved as there indicated, giving the

\( h = .30109 \) and therefore a moment of \( (.30109 \times 6) = 1.8065 \) at point "C" of

the bent.

Plate V shows the analysis of the same bent by moment distribution. Two
sets of calculations are shown, one for the distribution of thrusts, the other
for the distribution of moments. However, before the distribution of either
the moments or thrusts can be performed, it is first necessary to find the
constants for the column. This is readily done as shown in Fig.10. and Fig.
11.
Moment stiffness in columns.

Since the moment stiffness in the column is the moment required to produce unit rotation, from Fig. 10, we derive:

\[ P_1 \text{ SLOPE} = \Theta_1 = \left[ \frac{1.6h}{2EI} \right] \frac{4}{7} = \frac{72h}{7EI} \]

Hence, \[ h = \frac{7}{72} \cdot EI \]

The moment stiffness, then:

\[ M = \frac{7}{72} \cdot EI = \frac{49}{72} \cdot EI \]
AND IF $I = 10a$

$$M = \frac{490}{72} \Delta E = 6.81 \Delta E$$

It is to be noted that $h$ is the thrust accompanying the moment stiffness required in produce $\Theta = 1$ and is equal to:

$$\frac{7}{72} \frac{E I}{h} = 70 \Delta E$$

**Thrust stiffness in columns**

![Diagram](image)

**Fig. 11.**

From Fig. 11, the thrust required to produce $\Delta = 1$ is:

$$\Delta = 1 = \left[ \frac{1}{2} \cdot \frac{6h}{EI} \right] 4 \ 	ext{(4)}$$

$$1 = \frac{72h}{EI}$$
It is to be noted here that the moment accompanying the thrust to produce $\Delta = 1$ is:

\[
\frac{10}{72} \cdot aE = 0.139 aE
\]

Referring again to Plate V the constants are arranged in the following manner. The fixed end moments of 2.5 are immediately made use of by placing them as indicated under "moment distribution". The carry-over factor for the truss $R = 0.1167$ is also set down for ready use. The values shown in the rectangular boxes represent the proportions with which the moments are to be distributed; the value 0.0767 pertains to the truss and 0.9233 to the column. These values are obtained from the moment stiffness factors of the truss and column as shown in Fig. 12.
M. STIFFNESS IN COL. (Fig. 10) STIFFNESS IN TRUSS (PLATE III)

\[
\begin{align*}
S_1 &= \frac{490 \Delta}{72} = 6.81 \\
S_2 &= \frac{24L}{14.14^2} = \frac{8}{14.14^2} = 0.565 \\
\frac{S_1 + S_2}{S_1 + S_2} &= \frac{6.81 + 0.565}{6.81 + 0.565} = 0.9233 \\
\frac{S_2}{S_1 + S_2} &= \frac{0.565}{6.81 + 0.565} = 0.0767
\end{align*}
\]

Fig. 12.

Under "thrust distribution" the constants are arranged in a similar manner. The end thrust \( P = 1 \) as obtained in Plate I is placed as indicated. The carry-over factor \( R = 1 \) is also set down for ready use. The values in the rectangular boxes represent the values according to which the thrusts are distributed; the value 0.641 pertaining to the truss, and 0.359 to the column. These values are obtained from the thrust stiffness of both the truss and the column as indicated in Fig. 13.

T. STIFFNESS IN COL. (Fig. 11) THRUST STIFFNESS IN TRUSS (Fig. 8)

\[
\begin{align*}
h &= \frac{EI}{72} \\
t &= \frac{2AE}{L} = \frac{2}{21} = \frac{1}{4} \\
\frac{t_1}{t_1 + t} &= \frac{0.139}{0.139 + 0.25} = 0.359 \\
\frac{t}{t_1 + t} &= \frac{0.25}{0.139 + 0.25} = 0.641
\end{align*}
\]

Fig. 13.
The thrust and moment at the base of the column and point "c" of the bent, as obtained by moment distribution, check very closely with the same values as obtained by the least work. It must be noted however, that what little discrepancy exists between the values as gotten by the two methods, is due to the fact that the portion of the column connected to the truss was considered infinitely stiff, in the case of moment distribution, a fact, of course, not strictly true. Should this restriction be considered when analyzing the bent by least work, the results would check exactly.

The assembly of the constants in chart form.

Plate VI shows the manner in which the four constants and other pertinent data can be arranged for ready reference. In the lower part of the chart is shown the type of truss dealt with, in this case the truss being an eight panel parallel chord of the Pratt variety.

Adjacent to the truss are shown the fixed end moments, 5.25 PL/8 and the thrust or pull 2P; also the thrust stiffness. An interesting result of this investigation reveals the fact that regardless of the size of the web members, the fixed end moments and thrust stiffness factors do not change. On the chart are also shown the carry-over and the moment stiffness curves plotted against the ratio of the area of the web members to the area of the chords.

A characteristic of the carry-over curve is that it drops to a minus one value when Aw/AC ratio is zero. Both curves rise rapidly but soon flatten out to almost constant values.

The carry-over and stiffness curves for the truss pictured in Plate VI were plotted by calculating the constants for the following Aw/AC ratio --
The calculations for the constants obtained from the above ratios, for the sake of brevity, are not attached to this thesis. The procedure involved, however, is identical to that adopted for finding the constants for the experimental case of the four panel truss.

Conclusions.

From the foregoing investigation the following conclusions seem justifiable:

1. That moment distribution can be successfully applied to trussed bents, with comparatively little effort once the constants are available; and this with an accompanying saving of time and effort, as compared to the analysis of trussed bents by least work or any other classical method.

2. That the results obtained by moment distribution check closely with those obtained by the method of least work, and for all practical purposes are therefore acceptable.

3. That the compilation of constants in chart form, covering all types of trusses, is a perfectly feasible undertaking and promises to be the basis on which a practical, simple, and straightforward method of analysis will eventually be established.

4. That the adoption of moment distribution for the analysis of trussed bents together with the charting of the constants will aid in bringing about a standardization of truss construction heretofore unrealized.
### Fixed End Moments

#### Four Panel Truss

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<th>MEN</th>
<th>L/4a</th>
<th>Sx</th>
<th>Sy</th>
<th>Sz</th>
<th>Sx,Sz L/A</th>
<th>Sx² L/A</th>
<th>Sy² L/A</th>
<th>Sz² L/A</th>
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<tr>
<td>EB</td>
<td>L/4a</td>
<td>-1.5P</td>
<td>-1.4</td>
<td>+1.4</td>
<td>+1</td>
<td>-1.5P /L + 1.4 /L</td>
<td>-1.4 /L</td>
<td>+1 /L</td>
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<tr>
<td>ED</td>
<td>L/4a</td>
<td>-2.5P</td>
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<td>+1.2</td>
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<td>+1</td>
<td>-1.5P /L + 1.4 /L</td>
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<td>L/4a</td>
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<td>-1.4</td>
<td>+1.4</td>
<td>+1</td>
<td>-1.5P /L + 1.4 /L</td>
<td>-1.4 /L</td>
<td>+1 /L</td>
</tr>
</tbody>
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### Stress Diagrams

- **Stresses, Sx**:
  - ![Stress Diagram Sx](image)

- **Stresses, Sy**:
  - ![Stress Diagram Sy](image)

### Equations

- **Equation 1**: 
  
  $$[1.8945L / \Delta x] - [3.955L / \Delta y] - [0.5L / \Delta z] + 5 PL / a = 0$$

- **Equation 2**: 
  
  $$[-2 \cdot 1.8945L / \Delta x] + [1.8945L / \Delta y] + [0.5L / \Delta z] - 1.25 PL / a = 0$$

- **Equation 3**: 
  
  $$[-3 \cdot 1.8945L / \Delta x] + [1.8945L / \Delta y] + [0.5L / \Delta z] - 1.75 PL / a = 0$$

### Notes

- Area Web Men = 2
- Area Chords = 2
PLATE II

CARRY-OVER FACTOR

FOUR PANEL TRUSS

<table>
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<th>Mem.</th>
<th>L/A</th>
<th>S₀</th>
<th>Sₓ</th>
<th>S₀Sₓ/a</th>
<th>Sₓ²L/a</th>
<th>FINAL STRESSES</th>
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<td>2B</td>
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<td>-1/4</td>
<td>-1/4</td>
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<td>+L/64a</td>
<td>2.8832/L</td>
</tr>
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<td>4/16a</td>
<td>-4/16a</td>
<td>+L/64a</td>
<td>1.7664/L</td>
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<tr>
<td>2E</td>
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<td>4/16a</td>
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</tr>
<tr>
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<td>-3/16a</td>
<td>+9/64a</td>
<td>0.6497/L</td>
</tr>
<tr>
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<td>1</td>
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<td>0</td>
<td>16/4a</td>
<td>0.4671/L</td>
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<tr>
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<td>3/4</td>
<td>-3/16a</td>
<td>-3/16a</td>
<td>+9/64a</td>
<td>-0.6497/L</td>
</tr>
<tr>
<td>JC</td>
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<td>1/4</td>
<td>-3/16a</td>
<td>-3/16a</td>
<td>+L/64a</td>
<td>-2.8832/L</td>
</tr>
<tr>
<td>JA</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>-4.0000/L</td>
</tr>
<tr>
<td>AB</td>
<td>1/2</td>
<td>3/4</td>
<td>2.8/16a</td>
<td>2.8/16a</td>
<td>+L/22.6a</td>
<td>1.5793/L</td>
</tr>
<tr>
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<td>1/4</td>
<td>1/16a</td>
<td>1/16a</td>
<td>-L/22.6a</td>
<td>-1.1168/L</td>
</tr>
<tr>
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<td>1/2</td>
<td>3/4</td>
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<td>2.8/16a</td>
<td>+L/22.6a</td>
<td>1.5793/L</td>
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<td>1/4</td>
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<td>2.8/16a</td>
<td>+L/22.6a</td>
<td>-1.5793/L</td>
</tr>
<tr>
<td>FG</td>
<td>1/2</td>
<td>1/4</td>
<td>1/16a</td>
<td>1/16a</td>
<td>+L/22.6a</td>
<td>1.1168/L</td>
</tr>
<tr>
<td>GH</td>
<td>1/2</td>
<td>1/4</td>
<td>2.8/16a</td>
<td>2.8/16a</td>
<td>+L/22.6a</td>
<td>-1.5793/L</td>
</tr>
</tbody>
</table>

Σ -4.1792a + 8.945L/a

EQ. 0.6945L/a - 179/2 = 0

CARRY-OVER FACTOR = (-4671/L - L²a) = 0.1167
### PLATE III

#### STIFFNESS FACTOR

- **FOUR-PANEL TRUSS**

<table>
<thead>
<tr>
<th>Mem</th>
<th>Stresses-S</th>
<th>∆ = SL/A</th>
<th>RAD(1)</th>
<th>ELASTIC W.G.T. AT JOINT</th>
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</thead>
<tbody>
<tr>
<td>RB</td>
<td>2.8832/L 2</td>
<td>L/4</td>
<td>+0.7208/a</td>
<td>L/4</td>
</tr>
<tr>
<td>BD</td>
<td>1.7664/L 2</td>
<td>L/4</td>
<td>+0.4416/a</td>
<td>L/4</td>
</tr>
<tr>
<td>DE</td>
<td>1.7664/L 2</td>
<td>L/4</td>
<td>+0.4416/a</td>
<td>L/4</td>
</tr>
<tr>
<td>RG</td>
<td>0.6497/L 2</td>
<td>L/4</td>
<td>+0.1624/a</td>
<td>L/4</td>
</tr>
<tr>
<td>JH</td>
<td>0.4671/L 2</td>
<td>L/4</td>
<td>+0.1167/a</td>
<td>L/4</td>
</tr>
<tr>
<td>JF</td>
<td>0.6497/L 2</td>
<td>L/4</td>
<td>-0.1624/a</td>
<td>L/4</td>
</tr>
<tr>
<td>JC</td>
<td>2.8832/L 2</td>
<td>L/4</td>
<td>-0.7208/a</td>
<td>L/4</td>
</tr>
<tr>
<td>JA</td>
<td>4.0000/L 2</td>
<td>L/4</td>
<td>-1.0000/a</td>
<td>L/4</td>
</tr>
<tr>
<td>AB</td>
<td>1.5793/L 2</td>
<td>L/2828</td>
<td>+0.5583,a</td>
<td>L/5.656</td>
</tr>
<tr>
<td>BC</td>
<td>1.1168/L 2</td>
<td>L/4</td>
<td>-0.2792/a</td>
<td>L/4</td>
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<tr>
<td>CD</td>
<td>1.5793/L 2</td>
<td>L/2828</td>
<td>+0.5583,a</td>
<td>L/5.656</td>
</tr>
<tr>
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<td>0</td>
<td>L/4</td>
<td>0</td>
<td>L/4</td>
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<tr>
<td>EF</td>
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<td>L/2828</td>
<td>-0.5583,a</td>
<td>L/5.656</td>
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<td>FG</td>
<td>1.1168/L 2</td>
<td>L/4</td>
<td>+0.2792/a</td>
<td>L/4</td>
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<tr>
<td>GH</td>
<td>1.5793/L 2</td>
<td>L/2828</td>
<td>-0.5583,a</td>
<td>L/5.656</td>
</tr>
</tbody>
</table>

\[ \sum = +7.158/aL + 6.8832/aL + 3.5328/aL + 0.1826/aL - 3.625/aL \]

#### Conjugate Beam

- **Angle Increase**
  - \( R = 14.142G \)
  - \( R_1 = -14.142G \)
  - \( R_2 = 0 \)

- **Angle Decrease**
  - \( \theta = 1 \)

By proportion:

\[ M_A : \theta = 1 \]

\[ M_A = 2L \times 14.142G = \text{STIFFNESS} \]
PLATE IV

ANALYSIS OF TRUSSED BENT
METHOD OF LEAST WORK

MEM | L/A | S0 | Sh | S0Sh L/A | Sh² L/A
---|-----|----|----|--------|--------
RB | L/4 | -1.5P + 3 | -4.5PL²/4 + 9L 4A
RD | -2.0P + 3 | -6PL²/4 + 9L 4A
RE | -2.0P + 3 | -6PL²/4 + 9L 4A
RG | -1.5P + 3 | -4.5PL²/4 + 9L 4A
JH | 0 | -4 | 0 + 16L 4A
JF | +1.5P | -4 | -6PL²/4 + 16L 4A
JC | +1.5P | -4 | -6PL²/4 + 16L 4A
JA | 0 | -4 | 0 + 16L 4A
AB | L²/288 + 2,1213P | 0 | 0 | 0
BC | L²/48 - 1.5P | 0 | 0 | 0
CD | L²/288 + .7071P | 0 | 0 | 0
DE | L²/48 - P | 0 | 0 | 0
EF | L²/288 + .7071P | 0 | 0 | 0
FG | L²/48 - 1.5P | 0 | 0 | 0
GH | L²/288 + 2,1213P | 0 | 0 | 0

Σ = -3.25PL²/4 + 25L 4A

EQUATION: [25L 4A - 8.25PL²/4] + 192, EI ∙ δ = 0
LET I/A = 10
THEN [25L 4A + 192/10] - 8.25PL²/4 = 0

NOTE:

∆w/Ac = 1

M(θ) = 0.30109 ∙ G = 1.8045
VALUES OF CARRY OVER FACTOR R

VALUES OF CARRY OVER STIFFNESS IN TERMS OF

VALUES OF AW/AC

NOTE:
FIXED END MOMENTS REMAIN CONSTANT
IRRESPECTIVE OF AW/AC RATIO

THRUST STIFFNESS \[ t = \frac{2AE}{L} \]

CARRY OVER STIFFNESS & F.E.M.-EIGHT PANEL TRUSS
BIBLIOGRAPHY

"Beam Constants for Continuous Trusses and Beams" A. S. C. E. Proceedings February, March, April, and October. 1940.


