

**A STRUCTURAL SYSTEM FOR MULTI-STORY CONSTRUCTION  
IN PRESTRESSED-PRECAST CONCRETE**

by

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**Thesis submitted to the Graduate Faculty of the  
Virginia Polytechnic Institute  
in candidacy for the degree of  
MASTER OF SCIENCE**

in

**Architectural Engineering**

**May, 1962**

**Blacksburg, Virginia**

Abstract of Thesis

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It is the major purpose of this thesis to examine, through a practical engineering problem, the different aspects affecting the development of the structural concept. For this purpose, a prestressed-precast concrete structural system for multi-story construction is developed.

After the introductory sections, the development of the structural system is explained in the third section. The section includes an analysis of the system, as well as a description of the individual components of the structure and their interaction. The analysis provides a corroboration of the adequacy of the concepts of the system. Illustrations aid in the explanation of the problems involved.

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## I. INTRODUCTION

As much as the crisis of the functional relationship of architects and engineers involved in building design is generally recognized, there is very little of a concerted effort to achieve an adequate basis for such a relationship. It is the major purpose of this thesis to study the interrelation of the different aspects of the building design process, through the development of a structural system of prestressed-precast concrete for a specific building type : a multi-story office building.

Today, when specialization is an existing and necessary approach to all technical endeavors, there should be a clear understanding of the boundaries of each aspect and an integration of all of them to produce a unified whole. The structural solution of a building cannot, then, be achieved without consideration of all aspects of Architecture: structure, function, aesthetics and mechanical equipment, among others. Beside the practical and theoretical solutions inherent in structural design, the engineer must be continuously aware of the requirements of all these other aspects. The overall concept of the design should correlate all the factors involved. In this thesis the emphasis is on the structural aspects of the building design process. As far as it is possible, encroachment on the aspects not directly affecting the structural aspects are avoided. Certain decisions of purely aesthetic or non-technical character are made for the purpose of achieving an adequate study.

With automation and rapid technological advancements in evidence in most technical fields, the building industry seems to lag in wait of a revolution in the concepts of building technology. The building industry should develop techniques to use advantageously the concepts of automation and assembly line process, with a greater exploitation of the materials' capabilities. New possibilities must be examined, tried and developed, in order to free the industry from anachronistic methods. The development should be aimed at perfecting the new techniques without abandoning the basic structural principles of established concepts. The development of production methods has opened the way for the concept of mass produced structural components. This concept of construction by assembly of components, perhaps, best brings to light the necessity of a close interrelationship of the engineer, architect, manufacturer and builder. The cooperation of these - with an understanding of each others functions - should open the way for bold and imaginative articulate building design and construction.

In the process of the development of this thesis many possible structural systems that would apply to prestressed-precast concrete construction were considered. Some were discarded because of lack of the necessary knowledge to make an adequate structural analysis. The possibilities of a two-way structural system (rectangular diagrid) was extensively examined and abandoned for the reason stated above. Other systems were examined and abandoned for not satisfying other requirements such as flexibility, ease of construction and handling. The development of the structural forms is based not only on structural considerations, but also on considering them as an integral part in a

total architectural concept. The major purpose of this thesis is to examine these thoughts.

After the explanation of the development of the structural components, an analysis is undertaken to corroborate the design concept and its structural adequacy. For this purpose additional assumptions are made and are stated as used.

## II. PRESTRESSED-PRECAST CONCRETE CONSTRUCTION

As a matter of introductory reference a brief survey of the techniques of prestressed-precast concrete construction and its development is presented.

### A. Development of Prestressed-precast Concrete Construction

The principle of prestressed concrete is essentially the introduction of stresses to counteract stresses of superimposed loads. The stresses are introduced by tensioning of wires in the concrete section, producing bending moments inverse to those produced by external loads. The tensioning is usually done by either of two methods:

1. Pretensioning consists of manufacturing the member in prestressing beds. The wires are tensioned, before settling of the concrete, through anchorage at both ends.

2. Post-tensioning occurs when the member is cast without prestressing. After the concrete hardens to an adequate strength, post-tensioning wires or rods are pulled through ducts imbedded in the concrete member and stressed from one or both ends by use of hydraulic jacks acting against the end of the concrete section.

Although the concept of prestressing a material to obtain greater strength is relatively old, the development of the present concept of concrete prestressing is recent. The principle of prestressed concrete was first used around the year 1886 by P. H. Jackson, an American engineer. The practical advancements of the concept are credited, however, to the French engineer, E. Freyssinet. Around 1928 Freyssinet prestressed, by use of wires, concrete pieces together and used them in bridge structures. The concept of precasting was a

natural outcome of the prestressed concrete concept. Freyssinet's practical structures opened the way for a rapid development of the prestressed-precast concrete techniques. Many new systems appeared with innovations in the stressing and anchoring devices.

The potentialities of prestressed-precast concrete construction were first exploited in the field of bridge engineering, and later in the long span industrial buildings. Recently, however, the techniques have been applied to all building types, taking advantage of the structural efficiency and assembly by parts possibilities of the material. At present the use of prestressed-precast concrete buildings is increasing, with experimentation and new developments continuously bringing new improvements.

B. Review of Examples of Prestressed-precast Concrete Multi-story Construction

1. The Medical Research Building - Philadelphia

Louis Kahn, architect; A. Komendant, consulting engineer.

The structural system consists of four main girders supported on exterior columns (drawing 1a). The girders are of the Vierendeel type. Two of the girders are monolithic with pretensioned top and bottom chords; the other two consist of three sections and are post-tensioned. In each of the nine panels, formed by the girders, there are secondary members that aid in supporting the poured-in-place slab and the mechanical equipment. The columns are one-story pieces, precast and tied together with tendons running through the spandrel beam and the girders. The poured-in-place slab acts compositely with the girders and secondary members. The system, when finally prestressed,

is continuous.

The exterior wall is placed after the structural frame is completed. Lateral forces are taken by poured-in-place concrete service towers.

## 2. The Philadelphia Police Administration Building

Geddes, Brecher and Qualls, architects; A. Komendant, consulting engineer.

The building plan consists primarily of two circular towers connected by a curved segment. The exterior wall units provide the support for the radiating horizontal units, employing a registered European system called "Schokbeton" (drawing 1b). The core wall supports the other end of the horizontal units. Each horizontal unit consists of three joists with a thin slab filled in between joists. There is no continuity between the supporting wall and the horizontal units. The core wall provides all the resistance to lateral forces.

## 3. The Worthing Secondary School - England

The structural system is of the "intergrid" type. The main horizontal members are units tied together to form open trusses. The primary trusses are supported on boundary beams and column heads (drawing 1c). Secondary trusses span between the primary trusses, incorporating a two-way grid system after post-tensioning of the tendons running through the truss units. The tendons are passed along grooves in the outer edge of the lower flange of the secondary beam and through grooves in the primary beam. Stressing is done upon completion of each structural bay. Floor panels are placed in position within the square formed by the top flanges of the trusses, and after grouting act compositely with the trusses.

Cast-in-place staircase towers provide the resistance to lateral loads. Exterior wall panels are fitted into grooves in the column and fixed with reinforced concrete pins.

There are other buildings of prestressed-precast construction in existence in the United States and Europe; however, the ones discussed above are those that are most closely akin to the building system selected for examination.

### III. THE DEVELOPMENT OF THE SYSTEM

#### A. General

As in the economics of all industries, the economics of the building industry depend largely on the rapidity of completion of the product. A precise organization of the building process is essential. This organization, starting at the design stage and continuing through the fabrication and erection stages, is the determining factor in the time aspect of the building process. Throughout the development of the different structural components, the underlying total organization is kept in mind.

The building type limitations - a multi-story office building with maximum space arrangement flexibility - provide the point of departure of the units' development. The structural units are developed through consideration of all the aspects involved in the unified design of a building. The individual units are first discussed, and then the overall structural behaviour is explained.

#### B. The Units

##### 1. The Horizontal Unit (drawing 3)

The unit consists essentially of three joists connected transversally by stiffeners at the ends and the third points. The spacing of the joists is five feet. Each unit is to be placed parallel to another unit spaced about twenty inches apart, to provide for supply air ducts. The arrangement allows the ducts to run without interference, and permits a reduction of the story height with savings in wall material and decreased dead load. The unit is to be cast without the flooring material in order to reduce its weight during handling and erection.

The top flange of each joist is provided with a pocket to hold a rigid insulating board, that also serves as centering for the slab to be poured after erection of the unit. The poured-in-place slab, after setting, would act compositively with the joist along a portion of the span. Shear keys and stirrup extensions are to be provided to assure composite action. The slab also would serve the purpose of covering up the dimensional and constructional inconsistencies.

Each joist of the unit would have a perforation of about eight inches by twelve inches at mid-span to allow the branching of the air ducts and the electrical conduits to the middle area of the unit. The transverse stiffeners would have two functions: the stiffening of the unit against buckling, and stabilization of the unit during the erection stage before the poured-in-place slab sets.

The fabrication of the horizontal unit would be done either at the site of construction or at the manufacturing plant. Economics would determine this. The forty-five feet length of the unit would not make its transportation prohibitive. If site fabrication would be used, the section of the unit is simple enough to allow employment of wood forms or conventional collapsable steel forms. The repetitive nature of the structural system, would permit an economical reuse of forms. Metal conduits for the post-tensioning tendons, mild-steel reinforcement, bearing plates, lifting collars, stirrups and shear keys would all be cast with the unit. Provisions for the joists' perforation would be made by blackout of the required area by use of a sleeve in the forming.

The setting of the concrete would be controlled by use of admixtures, artificial cooling and steam curing. The steam curing process would allow a more complete crystallization and a more stable product, by preventing premature evaporation of water. Since no part of the unit would be exposed, special aggregates are unnecessary.

## 2. The Vertical Unit (drawing 2)

The vertical unit would serve as support for the horizontal unit and the girder, as well as space encloser. Only the three protruded portions of the unit would receive superimposed loads and would resist bending moments. The rest of the unit would serve as an enclosing and stiffening element. The unit would be of the same width as the horizontal unit in order that the joists of the horizontal unit would rest directly on the protruding portions.

The unit would be one-story high for the purpose of easing its handling erection. The protruded portions would provide the only contact with the unit above or below; the rest of the unit would slip past the similar portion of the other unit. The arrangement would allow for easier erection and alignment of the unit.

The unit would be notched and provided with brackets to seat the horizontal unit or the girder, and for covering of the anchorage devices. The exterior surfaces of the unit would be shaped so as to prevent excessive weathering by use of sloping surfaces to avoid the collection of water. This surfaces would be treated with silicon to aid against weathering. Drips and weep holes would also be provided. The interior surfaces would have recesses for positioning of thermal insulation and

finishing material. The exposed surfaces would have aggregates to achieve a desired appearance. Red granite, quartz aggregate and marble chips would be some of the possibilities. Acid-etching and bush hammering would be other possibilities in the treatment of the exposed surfaces to achieve different textures. The fenestration would be fitted into the openings in the unit, and made watertight by use of neoprene gaskets or similar devices.

For the fabrication of the unit, molds of either plaster, concrete or glass fiber reinforced plastic would be used, depending on the economics and the availability of the materials. Mild-steel reinforcement - to prevent cracking and to strengthen the unit during handling - together with bearing plates and lifting collars would be cast with the unit. The protruded portions would have recesses at top and bottom to accommodate leveling plates and anchorage devices.

### 3. The Main Girder (drawing 4)

The main girder would be of I-shape. This shape is adequate for members resisting loads that produce a dead load moment to live load moment ratio that is relatively large. Such would be the case on the girder. In expectancy of a large prestressing force, the girder would have an end block at each end. Brackets for support of the horizontal units, would be provided on one side and at the required spacing.

Considering the simplicity of girder section the fabricating could easily be done on the site of construction. If plant manufacturing were necessary, the girder would be easily transported since its length would permit it.

#### 4. The Core Element (drawing 5)

The core element would provide enclosure to the service facilities of the building - stairs, elevators, toilets and return air shaft. It would also serve as support to the horizontal members and as the lateral loads resisting element. The element would consist of: four corner precast columns of one-story height, directly supporting the main girders (drawing 6); and precast panels of load bearing capacity. The whole element would act monolithically by use of welded steel dowel connections, prestressing and grouting. Both the columns and the panels would be prestressed from the top by use of continuous steel rods. The panels would support the horizontal units and a poured-in-place slab, spanning the interior of the core element, at every story level. The precast concept of the core element is considered necessary in order to assure a simultaneous build up of the building. The "slip-form" construction technique - which is essentially a cast-in-place construction - would have required a complete pouring of the core element before commencing the erection of the precast units.

Some of the precast panels would be provided with openings for the passage of return air ducts leading to the shaft within the core.

#### C. The Overall Behaviour

The structural arrangement of the elements is linear in concept (drawing 6). The four main girders would be supported by the corner columns of the core and by the column portions of two adjacent vertical units. The girders, in turn, would support the horizontal units on one side, transmitting the loads to the vertical units and the core columns. A total of four horizontal units would rest on each main girder. The

extreme joist of the outer most horizontal unit would rest on the vertical units that would not support any joist or girder (drawing 7a). These vertical units would only support the vertical units above and resist lateral forces, and would be kept in place by welding of steel plates on the vertical unit and the bottom of the extreme joist.

The girders would be post-tensioned and tied to the vertical units and the core columns by threading the tendons through these, producing a beam fixed at both ends. The large portion of the load that would be carried by the girder would necessitate the fixing of both ends.

The horizontal units that would be supported by the girder on one side would rest on brackets provided along that side (drawing 9a). The torsion due to the unsymmetrical placing of the horizontal units is probably negligible and is not considered. The expected large prestress force would be instrumental in the prevention of excessive deflection under working loads. The draping of the tendons would be such that it would roughly follow the expected moment diagram in reverse (figure 1).

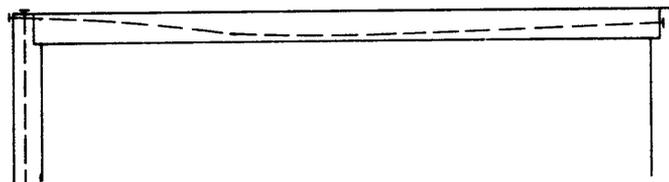
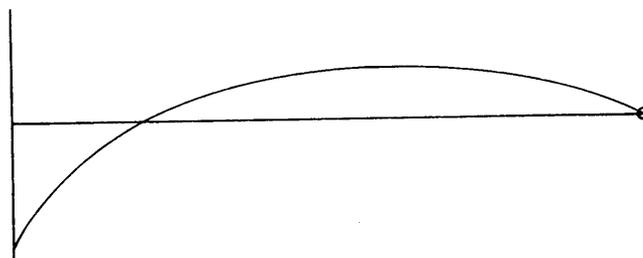


Figure 1.

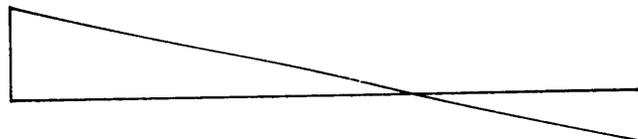
Although the use of the same number of tendons - determined by the end condition - throughout the member would tend to be wasteful of material, the practicality of such an arrangement for fabrication and erection purposes would determine its use for the system under consideration. The draped tendons would create secondary moments on the members

due to the restraint at the supports. These are dealt with in the analysis of the members (Part III-E). The shortening of the members due to the thrust of the prestress force and creep effects is normally neglected, and would be neglected in this case. Shims would be used to avoid excessive distortion of the frame. The post-tensioning system would not only serve to form a rigid frame connection, but also would provide a way of assembling the individual parts.

The horizontal units would rest either on the girders or the core wall at one end and on the vertical unit at the other. The spacing of the units would be such as to allow for the mechanical and electrical appurtenances (drawing 7b). The exterior end of the horizontal units would be continuous with the vertical unit in all cases; the interior end would be pinned connected. In this manner the largest bending moment would occur at the exterior end, avoiding a large bending moment at mid-span where the joists would be perforated for the passage of mechanical equipment and considerably reducing the reaction on the girder and core wall (figure 2).



Moment Diagram



Shear Diagram

Figure 2.

Also the girders' non-resistance to rotation would make the continuity with the horizontal units resting thereon impossible. The middle joist of the horizontal unit would obviously be carrying a heavier load than the extreme joists. The application of prestressing forces in proportion to the loads carried would permit the use of a uniform joist section.

The horizontal units would support rigid insulating boards of lightweight nature - such as "Cemesto" boards - that would serve as centering for the poured-in-place slab. After its setting, the poured-in-place slab would act compositely with the precast unit along the portion of member that would be resisting positive bending moment (see Part VI for sign convention). To assure composite action of the slab, shear keys and extension of the stirrups would be provided along the top of the joists. The horizontal units that would rest on the girders would be subjected to additional bending moments due to the deflections of the girders. However, these deflections are anticipated to be small, and the induced moment negligible.

The horizontal units would be post-tensioned from the exterior end and through the vertical units, where the anchoring devices would rest (figure 3). The interior end would have bearing plates to avoid large concentration of stresses.

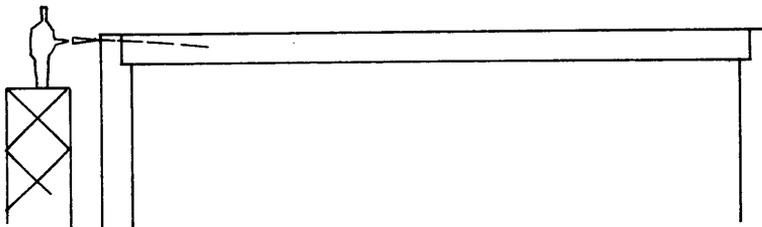


Figure 3.

Since some of the vertical units would be resisting bending moments and heavy axial loads, while others would resist no moment and carry light axial loads, varied prestress forces would be applied to the units through the tendons stressed from the top of each unit. The system would allow the use of the same basic vertical units throughout the building.

The unsupported length of the vertical unit would be small since the spandrel portion would serve as stiffener against lateral buckling of the column portion.

The core element, in addition to supporting the horizontal members, would resist all lateral forces transmitted to it by the horizontal units and the poured-in-place slab. The monolithic nature of the core element would permit an assumption of nearly infinite stiffness for it. The core element's columns and wall panels would be assembled similarly to the vertical units. Grooves on the columns would permit the threading of the girder tendons for post-tensioning and continuity. Lateral dowels in the panels and columns would overlap and be welded within a recess on the panel which is then grouted (drawing 5). Since the wall panels would have no continuity with the horizontal units the only bending moments to be resisted by the panels would be those induced by the floor slab within the core element.

#### D. The Assembly Procedure

The system of prestressing and anchoring that would be employed is, of course, a matter of choice. There are variations within each system to make it applicable to almost every situation. For the purpose of ease in making definite suggestions, the Freyssinet system is assumed

that would be employed. The system is the most commonly used system and its mechanics are very simple. Others equally applicable systems are the Magnel, Roebling, Prescon and variations of these. In the Freyssinet system the stressing is done by hydraulic jacking of the tendons, that upon reaching the desired stress are fixed by use of an anchorage device based on the wedge principle (drawing 8). After anchoring of the wires, grout can be forced into the conduits by application of pressure through the jack.

The horizontal members - girders and horizontal units - would be temporarily prestressed with an adequate force after their fabrication for purpose of their handling. This prestressing would be done after the element has reached a predetermined strength, and would be done on the site or at the plant depending on where the fabrication would take place. The order of prestressing the wires would be so as to avoid unsymmetrical stressing of the member, and their releasing would be gradual (figure 4).

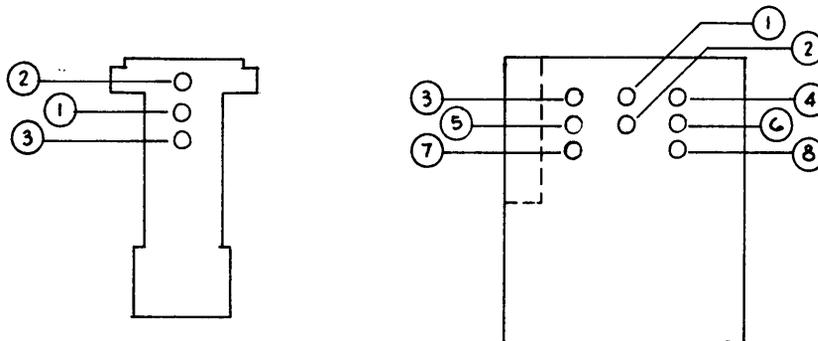


Figure 4.

The tendons that would be in the member during the handling stage would be anchored by use of releasable wedges until their final post-tensioning. The member, after being adequately prestressed, would be picked up through lifting collars by cranes, derricks or

davits and set in position.

The general erection procedure would be done by taking a quadrant of the structural frame at a time (drawing 6), with the core element completed first for each story.

For the purpose of outlining the erection procedure, an intermediate typical floor - where no abnormal conditions occur - is taken. An outline of general erection procedure is given below:

1. Placing of the core element
  - a. Alignment and stressing of corner columns.
  - b. Erection and jointing of the wall panels.
2. Placing of the exterior vertical that would support the girder
  - a. Leveling by use of sleeve nuts at the top of the unit below. Joints would be filled in with one quarter inch steel plates instead of grout so that the sections would be post-tensioned as soon as the girder is placed. Grouting would be done after tensioning of the section.
  - b. Alignment of the units by use of wood spacers and guy wires.
3. Placing of the girder on the seat provided by the vertical units, and on the core column (drawing 9b).
  - a. Threading of the tendons of the vertical units through the grooves in the girder; and the tendons of the girders through the grooves in the vertical units and core columns.
  - b. Leveling of the girder by use of shims. Temporary stability would be provided by bolted steel angles at the vertical supports.

4. Post-tensioning of the vertical units. Anchoring of the tendons and welding of their extensions.

5. Post-tensioning of the girder from both ends.

6. Placing of the rest of the vertical units on the quadrant.

See item 2.

7. Placing of the horizontal units (drawing 10a).

a. Placing of the units that would rest on the girder starting at the middle toward the ends, with proportional increases of the prestressing force on the girder (drawing 9a).

b. Placing of the units that would rest on the core wall, and welding of the bearing plates (drawing 10b).

8. Post-tensioning of the vertical unit. See item 4.

9. Post-tensioning of the horizontal units. The tensioning would be done from the exterior end and against the vertical unit. The interior end would have permanent anchorage devices.

10. Grouting of the joints and conduits.

The cast-in-place part of the structure would be done at any time after the erection of each floor frame.

#### E. Analysis

##### 1. General

As stated in the introduction the size characteristics of the building are predetermined for the purpose of allowing a definite analysis of the structure. The limitations are:

1. Four-story high office building.
2. The roof is unoccupied except for servicing purposes.
3. The core element is adequate in size to provide for

all service equipment.

The Elastic Theory is used throughout the analysis, except for the shear calculations in which ultimate strength criteria are used.

The usual Elastic Theory assumptions are made:

1. Elasticity-power of the body to recover from strain.
2. Isotropy-uniformity in all directions of the body.
3. Hooke's law of proportionality is applicable.
4. The Principle of Superposition can be applied.

Only an analysis of two typical frames is done. The core element is not analyzed considering it to be analyzable by conventional methods for similar shear wall structures.

## 2. Typical Joist and Column

The joist to be analyzed is the middle joist of the horizontal unit, with the connecting column (drawing 6). The resulting frame is shown in figure 5. The assumption that a fixed base is provided.

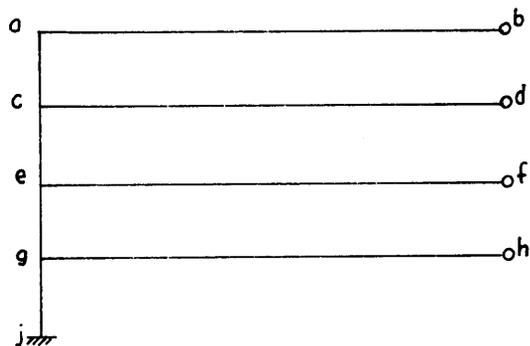


Figure 5.

Data and Specifications.

Spans:                   45'-0" joist span  
                           5'-0" typical spacing  
                           11'-0" column height

Loads: from the National Building Code.

floor LL = 80 psf

roof LL = 20 psf

partitions = 5 psf per sq. ft. of partition surface

assume wearing surface = 5 psf

Allowable stresses: (1)

	Temporary	Under working load
Concrete:	$f'_{ci} = 5,000 \text{ psi}$	$f'c = 5,000 \text{ psi}$
	$f_{ci} = 0.55 f'_{ci} = 2,750 \text{ psi}$	$f_c = 0.45 f'c = 2,250 \text{ psi}$
	$f_{ti} = 3\sqrt{f'_{ci}} = 212 \text{ psi}$	$f_t = 3\sqrt{f'c} = 212 \text{ psi}$
		$u = 0.08 f'c = 400 \text{ psi}$
Steel:	$f's = 240,000 \text{ psi}$	
	$f_{si} = 0.80 f's = 192,000 \text{ psi}$	
	$f_s = 0.70 f's = 168,000 \text{ psi}$	
	$f'v = 40,000 \text{ psi}$	
	$E_s = 29,000,000 \text{ psi}$	
	$E_c = 4,000,000 \text{ psi}$	
	$n = 7$	

Trial Sizing.

Total depth is tentatively established as 2'-0", including slab thickness.

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1. See sections 207.1 and 207.3 - "ACI-ASCE Recommendations for Prestressed Concrete."

1. Slab. In order to estimate the thickness of the slab, a preliminary design is performed with the following specifications:

$$L = 5'-0''$$

$$f'_c = 3,000 \text{ psi}$$

$$f_c = 1,350 \text{ psi}$$

$$f_s = 20,000 \text{ psi}$$

$$n = 10$$

Assuming a 3" slab thickness:

$$W_{DL} = \frac{3}{12} \times 150 / 1 \times 5 = 37.5 \text{ plf}$$

$$W_{LL} = 80 \times 1 = \underline{80.0}$$

$$w = 117.5 \text{ plf.}$$

$$\text{Taking maximum } M = \frac{wL^2}{8}$$

$$= \frac{0.118(s)^2}{8} = 0.37 \text{ K-ft.}$$

From Table 2 - "Reinforced Concrete Design Handbook."

$$\text{minimum } d = 2''$$

∴ the 3" thickness is adequate.

## 2. Precast Section.

Assuming a 6" x 22" joist section:

$$W_{DL} = \frac{22 \times 6}{144} \times 150 = 138 \text{ plf}$$

$$W_{slab} = \frac{3 \times 60}{144} \times 150 / 5 \times 5 = 213$$

$$W_{LL} = 80 \times 5 = 400$$

$$\text{Assuming a 9' high partition} = 9 \times 5 = \underline{45}$$

$$W = 796 \text{ plf}$$

For a beam with one end pinned, the other one fixed and uniform load on, the moments are as shown in figure 6.

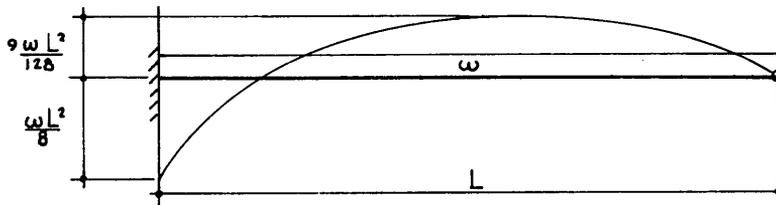


Figure 6.

Assuming the maximum moment to be reduced due to moment distribution, take

$$\begin{aligned}
 M_{\max} &= \frac{wL^2}{10} \\
 &= \frac{0.796(45)^2}{10} = 161 \text{ K-ft.} \\
 \text{Using } F &= \frac{M_{\max}}{0.65h} \quad (2) \\
 &= \frac{161 \times 12}{0.65 \times 24} = 124 \text{ K.}
 \end{aligned}$$

Assuming a prestress losses of 15%,

$$\begin{aligned}
 F_0 &= \frac{F}{0.85} \\
 &= \frac{124}{0.85} = 146 \text{ K.}
 \end{aligned}$$

The approximate concrete area required is:

$$\begin{aligned}
 A_c &= \frac{F}{0.50f_c} \quad (3) \\
 &= \frac{124}{0.50 \times 2.25} = 112 \text{ sq. in.}
 \end{aligned}$$

2. Formula (6-1-1) - T. Y. Lin, "Prestressed Concrete Structures"

3. Formula (6-1-3) - T. Y. Lin, "Prestressed Concrete Structures"

The minimum bearing can be approximated by

$$A_c = \frac{F_o}{f'ci} \quad (4)$$

$$= \frac{146,000}{5,000} = 29.2 \text{ sq. in.} \quad (3)$$

Based on the concrete areas determined above and on considerations of tendon coverage the tentative section is determined (figure 7).

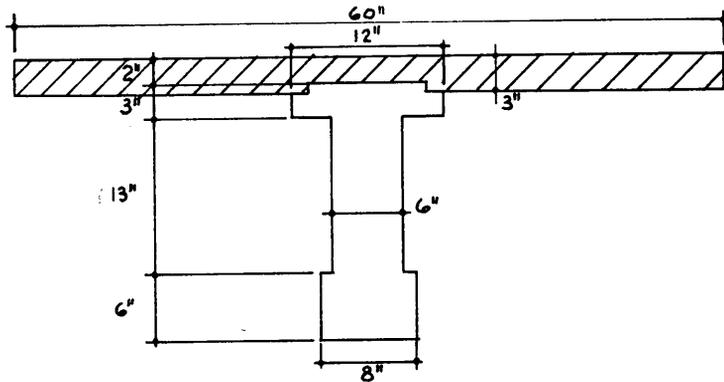


Figure 7.

### 3. Column.

Using  $P = N(1 + \frac{Be}{t}) \quad (5)$  (4)

Assuming the moment to be half of that on the joist:

$$M = \frac{161}{2} = 80.5 \text{ K-ft.}$$

and estimating  $N = 84.0 \text{ K.}$

$$e = \frac{M}{N} = 0.96 \text{ ft.}$$

Taking  $B = 3.0$  ;  $t = 26''$

Substituting in (4),

$$P = 198 \text{ K.}$$

4. See Section 207.35 - "ACI-ASCE Recommendations for Prestressed Concrete"

5. Formula (20) - "ACI Building Code" - (ACI 318-56)

Using  $P = 0.80 A_g (0.225 f'_c + f_{spg})^{(6)}$  (5)

with  $p_g = 0.04$

$A_g = 167 \text{ sq. in.}$

The dimensions are then determined (figure 8).

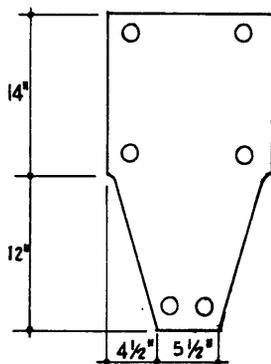


Figure 8.

### Section Properties.

#### 1. Precast Section:

$$A_c = 162 \text{ sq. in.}$$

$$c_b = 11.5''$$

$$I = 7,727 \text{ in.}^4$$

$$K = \frac{I}{L} = 14.2$$

#### 2. Composite Section (slab and precast section):

$$A_c' = 330 \text{ sq. in.}$$

$$c_b' = 17.0$$

$$I' = 17,980 \text{ in.}^4$$

$$K = \frac{I'}{L} = 33.0$$

---

6. See Section 1104-a- "ACI Building Code" (ACI 318-56)

## 3. Column:

$$A_c = 302 \text{ sq. in.}$$

$$cb = 14.4$$

$$I = 14,093 \text{ in}^4$$

$$K = \frac{I}{L} = 107$$

$$K = \frac{4EI}{L} = 107,000 \text{ K-ft.}$$

Uniform Loads for Moment Distribution.

## 1. Floor:

$$W_1 = \frac{162}{144} \times 150 = 168 \text{ plf}$$

$$W_2: \quad \text{slab} = 213 \text{ plf}$$

$$\text{joist} = \underline{168}$$

$$381 \text{ plf}$$

$$W_3: \quad \text{LL} = 400 \text{ plf}$$

$$\text{partitions} = \underline{45}$$

$$445 \text{ plf.}$$

## 2. Roof.

$$W_1 = 168 \text{ plf}$$

$$W_2 = 381 \text{ plf}$$

$$W_3: \quad \text{LL} = 100 \text{ plf}$$

$$\text{roofing} = \underline{25}$$

$$125 \text{ plf.}$$

Moment distribution can now be made for the different loading stages on the precast section.



Moments due to DL - precast section ( $M_2$ ).

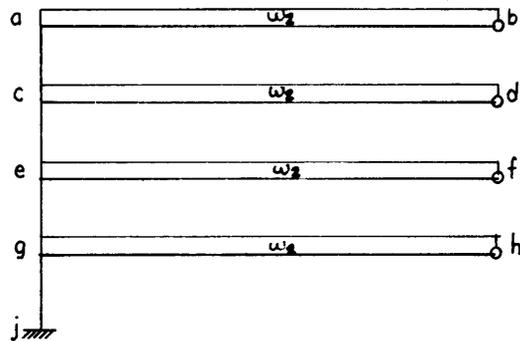


Figure 9.

The moments obtained after distribution are:

$$M_{ab} = / 87.7 \text{ K-ft.}$$

$$M_{cd} = / 93.4$$

$$M_{ef} = / 92.9$$

$$M_{gh} = / 91.9$$

$$M_{gi} = - 38.2$$

The poured-in-place slab does not act structurally at the beam portion where there is negative moment; however, at the beam portion where there is positive moment, the slab will act compositely with the precast portion. In order to approximate the point at which the slab ceases to act structurally, a distribution of fixed-end moments due to dead loads on the precast section and a distribution of fixed-end moments due to live loads on the composite section - with the slab acting throughout - is performed. The average distance of the points of counter flexure is taken as the cut-off point for the slab.

The cut-off point for the slab is found,

$$x = 10.75 \text{ ft. from end E.}$$

Since the member is no longer prismatic for structural purposes, carry-over factors, stiffnesses and fixed-end moments must be found for the member. To obtain these the following procedure is used:

1. Assuming the rotation ( $\phi$ ) at the left end to be one unit, an equation is written by use of the moment-area method (figure 10).
2. Assuming the deflection at the left end to be zero, another equation is written by use of the  $\frac{M}{EI}$  diagram.
3. Having the equations, the carry-over factor and stiffness for the left end is obtained.
4. The fixed-end moment is found by of the  $\frac{M}{EI}$  diagram, using integration for the parabolic areas.
5. The procedure is repeated for the right end.

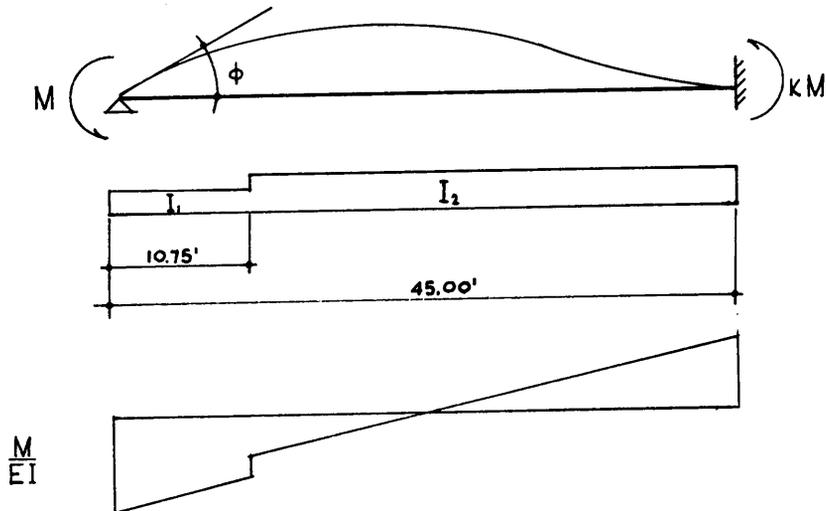


Figure 10.

The values obtained are:

- carry-over (left to right) = 0.77
- carry-over (right to left) = 0.20
- stiffness (left end) = 19,700 K-ft

stiffness (right end) = 88,000 K-ft

FEM (left end) = 0.101  $wL^2$

FEM (right end) = 0.180  $wL^2$

With this values the fixed-end moments due to live load are distributed throughout the frame.



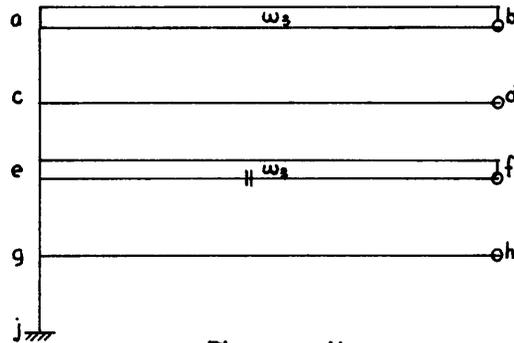
Maximum positive moment on ef-composite section (M<sub>4</sub>).

Figure 11.

The moments obtained after distribution are:

$$M_{ab} = / 28.3 \text{ K-ft.}$$

$$M_{cd} = / 3.5$$

$$M_{ef} = / 113.1$$

$$M_{gh} = / 2.4$$

$$M_{gj} = / 13.0$$

Due to the its eccentricity, the prestress force produces primary moments in the member which in turn produce secondary moments, caused by the restraint of the reaction at the support. The end result is that the center of the line of compressive force (C-line) no longer coincides with the c.g.s. line; thus, the effective  $e$  is changed.<sup>(7)</sup>

The cable profile is first established by use of the following:

$$e_{\min} = \frac{M_{\min}}{F}$$

---

7. See pp. 284 to 322 - T. Y. Lin, "Prestressed Concrete Structures."

$$e_g = \frac{M_1}{F_0}$$

from top kern line:

$$e_{max} = \frac{M_{max}}{F}$$

$$e_g = \frac{M_1}{F_0}$$

The distances obtained from the above relations are plotted - upward for negative moments and downward for positive moments - from the top or bottom curve.<sup>(8)</sup> The  $e$  distances are found for the random points along the member. Having plotted the distances, an area is determined within which the tendons may lie (drawing 11a). The profile of the tendon is then fitted within the boundaries, in order that no tension exists in the concrete. For reasons of adequate coverage and bearing considerations, the cable is taken as being three inches from the extreme fiber at end E and at mid-span. Since there is no moment at F, the c.g.s is taken at the c.g.c.

From Drawing 11a, the angle ( $\beta$ ) indicating the change in slope between the tangents to the cable profile at each end.

Graphically

$$\beta = 7.5^\circ = 0.132 \text{ radians}$$

The total force ( $W$ ) due to prestress can be approximated by:

$$W = F\beta \quad (9) \quad (6)$$

---

8. See Section 10-5- T. Y. Lin, "Prestressed Concrete Structures."

9. See p. 300 - T. Y. Lin, "Prestressed Concrete Structures."

and  $W_4 = 0.52 \text{ Klf}$

With the equivalent uniform load obtained above, a distribution of the corresponding fixed-end moments. The distribution is done first for the precast section acting only; afterward, for the composite section. For each case the four-story frame is assumed to be stressed at all levels. (figure 12).

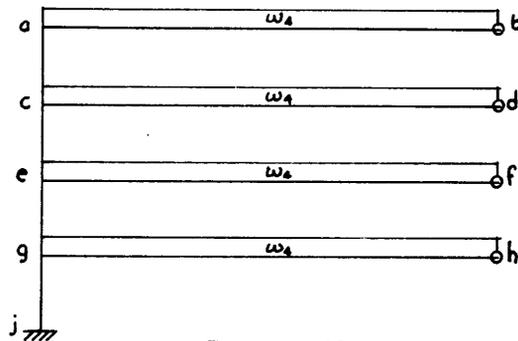


Figure 12.

The moments for end E are:

precast section acting,  $M_5 = -128.0 \text{ K-ft.}$

composite section acting,  $M_6 = -137.2 \text{ K-ft.}$

### Check of Sections.

#### 1. Joist.

At end E:  $C_b = 11.5 \text{ in.}$

$$C_t = 10.5$$

$$I = 7,727 \text{ in}^4.$$

$$A_c = 162 \text{ sq. in.}$$

$$r^2 = \frac{I}{A_c}$$

$$= 47.8$$

$$K_t = \frac{r^2}{C_b}$$

$$= 4.15 \text{ in.}$$

$$K_b = \frac{r^2}{C_t}$$

$$= 4.55 \text{ in.}$$

Taking the initial assumption of,

$$F = 177 \text{ K.}$$

$$F_o = 208 \text{ K.}$$

The location of the C-line for any particular condition is obtained by the algebraic summation of the eccentricities due to the prestress and the external loads present.

$$e = \frac{M}{F} \quad (7)$$

Bottom fiber: for the critical condition in the bottom fiber at E,  $M_2$  and  $M_3$  are the moments due to external loads.

$$\text{due to loads, } e_1 = \frac{M_2 + M_3}{F}$$

$$= 14.0 \text{ in. below c.g.c.}$$

$$\text{due to prestress, } e_2 = \frac{M_6}{F}$$

$$= 9.4 \text{ in. above c.g.c.}$$

$$\text{effective } e = 4.6 \text{ in. below c.g.c.}$$

using

$$f_b = \frac{F}{A_c} - \frac{F_e}{A_c K_t} \quad (10) \quad (8)$$

$$= 2.27 \text{ Ksi} \approx f_c, \text{ adequate.}$$

Top fiber: for the critical condition in the top fiber at E, only  $M_1$  participates with the prestress force; ( $M_5$ ).

---

10. See p. 173 - T. Y. Lin, "Prestressed Concrete Structures."

$$e_1 = \frac{M_1}{F_0}$$

$$= 2.4 \text{ in. below c.g.c.}$$

$$e_2 = \frac{M_5}{F_0}$$

$$= \underline{7.1} \text{ in. above c.g.c.}$$

$$\text{effective } e = 4.7 \text{ in. above c.g.c.}$$

$$f_t = \frac{F_0}{A_c} + \frac{F_0 e}{A_c K_b}$$

$$= 2.60 \text{ Ksi} < f_{ci}, \text{ adequate.}$$

Stress in the poured-in-place slab is of no concern at points where the moment is negative, since the slab does not act compositely with the beam.

At mid-span: the properties of the section are affected by the composite action of the poured-in-place slab and the perforation (figure 13).

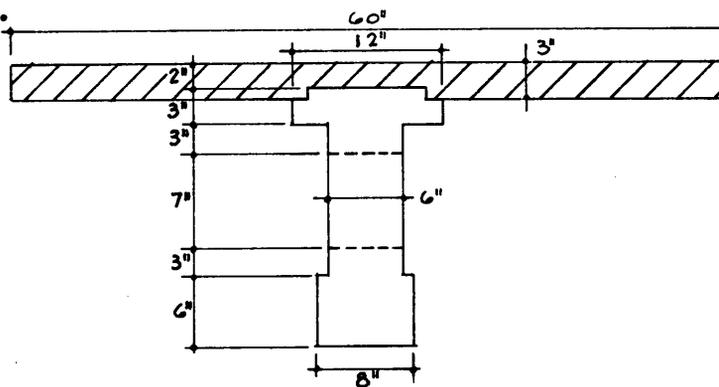


Figure 13.

The properties are:

$$\text{precast section, } A_c = 120 \text{ sq. in.}$$

$$I = 7,587 \text{ in.}^4$$

$$C_t = C_b = 11.0 \text{ in.}$$

$$r^2 = \frac{I}{A_c}$$

$$= 63.0$$

$$K_b = K_t = 5.7 \text{ in.}$$

$$\text{composite section, } A'_c = 288 \text{ sq. in.}$$

$$I' = 17,127 \text{ in.}^4$$

$$cb' = 16.8 \text{ in.}$$

$$ct' = 5.2 \text{ in.}$$

$$m_t = \frac{I/ct}{I'/ct'}$$

$$= 0.21$$

$$m_b = \frac{I/cb}{I'/cb'}$$

$$= 0.68$$

From the previous moment distributions, the moments at mid-span are obtained:

$$M_1 = + 22.0 \text{ K-ft.}$$

$$M_2 = + 49.0 \text{ K-ft.}$$

$$M_4 = + 57.9 \text{ K-ft.}$$

$$M_5 = - 60.9 \text{ K-ft.}$$

$$M_6 = - 62.8 \text{ K-ft.}$$

Bottom fiber: for the critical condition only  $M_1$  acts with  $M_5$  at mid-span.

$$e_1 = \frac{M_1}{F_o}$$

$$= 1.26 \text{ in. above c.g.c.}$$

$$e_2 = \frac{M_5}{F_o}$$

$$= \underline{3.50} \text{ in. below c.g.c.}$$

$$\text{effective } e = 2.24 \text{ in. below c.g.c.}$$

$$f_b = \frac{F_o}{A_c} + \frac{F_{oe}}{A_c K_t}$$

$$= 2.43 \text{ Ksi} < f_{ci} \text{ , adequate}$$

Top fiber: for the critical condition, a portion of  $M_4$ ,  $M_2$  and  $M_6$  act at mid-span.

$$e_1 = \frac{(M_4)_{mt}}{F} + \frac{M_2}{F}$$

$$= 4.41 \text{ in. above c.g.c.}$$

$$e_2 = \frac{M_6}{F}$$

$$= 4.26 \text{ in. below c.g.c.}$$

$$\text{effective } e = 0.12 \text{ in. below c.g.c.}$$

$$f_t = \frac{F}{A_c} + \frac{F_e}{A_c K_b}$$

$$= 1.44 \text{ Ksi} < f_c \text{ , adequate}$$

The top fiber of the poured-in-place top flange is determined

by

$$f = \frac{(M_4)_c}{I}$$

$$f = 0.29 \text{ Ksi}$$

Which is less than the allowable stress for conventional reinforced concrete ( $f_c = 1.35 \text{ Ksi}$ ).

To check the stresses during handling, a pick-up from the ends is assumed.

$$M = \frac{wL^2}{8}$$

$$= 42.5 \text{ K-ft.}$$

Assuming half of the prestressing force,

$$F_1 = 88.5 \text{ K.}$$

for c.g.s. at 3 in. from the bottom fiber,

$$e = c_b - 3 = 8.5 \text{ in.}$$

$$f_t = \frac{F_i}{A_c} - \frac{(F_i e - M)}{I}$$

$$= 0.40 \text{ Ksi}$$

and  $f_b = 0.88 \text{ Ksi} < f_{ci}$ , adequate.

Shear check:

The web reinforcement to resist shear is design by the ultimate method.

At end E,  $V = 23.3 \text{ k.}$ , from the moment distributions.

Taking a load factor of 2.

$$V' = 46.6 \text{ k}$$

Assuming a parabolic cable profile,

$$V's = 4 \frac{Fh'}{L} \quad (9)$$

$$\text{if } h = 15 \text{ in., } = 19.5 \text{ k}$$

$$V'c = V' - V's = 27.1 \text{ k}$$

$$s = \frac{hAvf'v}{V'c} \quad (10)$$

Using 3/8 in. diameter mild-steel U-stirrups

$$s = 7.80 \text{ in.}$$

Use 7.5 in. spacing

Maximum spacing  $= 3/4 h = 18 \text{ in.}$

In theory, no stirrups are required beyond 9-in., - assuming the concrete to resist shear - but it is recommend that they be used irrespective of theory. (12)

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11. Formula (7-4-2) - T. Y. Lin, "Prestressed Concrete Structures".

12. Section 210-2-3- "ACI-ASCE Recommendations for Prestressed Concrete".

Stirrups are used at throughout at a maximum spacing of 18 in. (drawing 11b).

Bond Stress:

$$A_s = \frac{F}{f_s}$$

$$= 1.05 \text{ sq. in.}$$

Use 3 Freyssinet cables of 12-0.20 in. diameter wires each.

Bond stress is check at end E for two conditions: (12)

between tendons and grout by,

$$u = \frac{v_{yn} D^{(13)}}{4I} \quad (11)$$

$$= 0.003 \text{ ksi}$$

and between conduit and concrete by,

$$u = \frac{nA_s y V^{(14)}}{\Sigma_o I} \quad (12)$$

$$= 0.037 \text{ ksi } 0.08 \text{ f'c, adequate.}$$

Prestress Losses:

The losses are determined by the following relations: (13)

$$\text{Losses} = 3,000 \text{ } \neq \text{ fcs } \neq 0.04 \text{ fsi} \quad (13)$$

$$\% \text{ friction loss} = \overline{WB} \neq \overline{KL} \quad (14)$$

and found to be 14.1% which compares adequately with the assumed 15%.

Deflection:

The deflection would differ at the different stages of loading. The maximum downward deflection would occur upon the

13. Section 7-5 - T. Y. Lin, "Prestressed Concrete Structures".

14. See Appendix D, p. 444 - T. Y. Lin, "Prestressed Concrete Structures."

application of all loads. The deflection at mid-span is found by use of the moment-area method, and is found to be:

$$\delta = 0.09 \text{ in. downward.}$$

## 2. Column.

Since the column is a prestressed compression member - with good capacity for flexure resistance, but not so for axial load - it is investigated for the condition of maximum axial load and corresponding bending moment. For this condition full live load is applied at all levels, and column g<sub>i</sub> investigated.

$$N = 87.9 \text{ k.}$$

$$M = 86.6 \text{ k-ft.}$$

$$\text{Using } f = \frac{F}{A_c} \pm \frac{Fec}{I} \pm \frac{N}{A_t} \pm \frac{BMc}{I} \quad (15)$$

Assuming an effective prestress in steel

$$= 125 \text{ ksi}$$

and using 6-½ in. diameter wires as shown in figure 14.

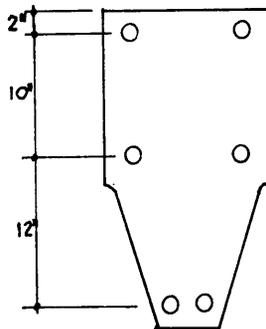


Figure 14.

---

15. Formula (12-4-1) - T. Y. Lin, "Prestressed Concrete Structures".

for the exterior fibers,

$$f = 0.210 \text{ ksi (tension)} < f_t, \text{ adequate}$$

for the interior fibers,

$$f = 1.59 \text{ ksi (compression)} < f_c, \text{ adequate}$$

The elevation of the final joist is shown in drawing 11b.

### 3. Girder and Column

The data and specifications are the same as in section E-2.

#### Trial Sizing.

Four of the typical joists - analyzed previously - and six of the less loaded joists rest on the girder (drawing 6).

The reaction of the typical joists can be obtained from the previous analysis. In order to obtain the reactions of the other joists, the loads on them are found and then the fixed-end moments distributed. In order to approximate the reaction on the girder it is assumed that the carry-over factors, stiffnesses and fixed-end moments for the joist are the same as in the previous analysis. A safe assumption since the moments of inertia of the section differ by less than five percent.

The dead and live load reactions transmitted by the joists to the girder are found to be:

DL reaction	=	61.0 k
LL reaction	=	<u>66.0 k</u>
Total	=	127.0 k

The total depth for the precast section is tentatively determined as 2'-0". A slab thickness of three inches is used, as before.

Assuming a section of about 460 sq. in. the total uniform load on the girder is found

$$w = 3.752 \text{ plf}$$

Assuming the maximum moment to be

$$M_{\max} = \frac{wL^2}{12}$$

$$= 635 \text{ k-ft.}$$

by formula (1),  $F = \frac{M_{\max}}{0.65 h}$

$$= 490 \text{ k}$$

$$F_o = 575 \text{ k}$$

and by (2),  $A_c = \frac{F_o}{0.50f_c}$

$$= 435 \text{ sq. in.}$$

The section is then sized (figure 15).

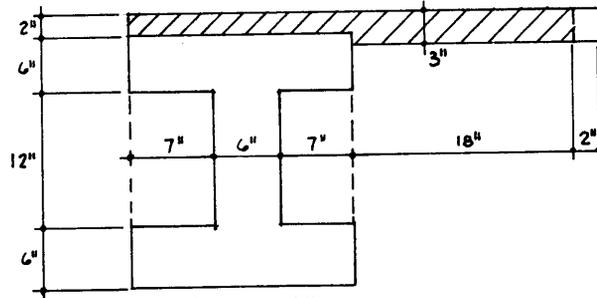


Figure 15.

The girder is to have end blocks of about five feet long at each end in anticipation of a large prestressing force and to provide the area required by formula (2).

Precast Section:

$$A_c = 312 \text{ sq. in.}$$

$$c_b = 12 \text{ in.}$$

$$I = 20.985 \text{ in.}^4$$

$$K = 39.0$$

at the ends,

$$A_c = 480 \text{ sq. in.}$$

$$I = 23,200 \text{ in.}^4$$

Composite Section: The maximum effective flange overhang is 18 inches. (16) The flange overhang only acts on one side of the girder since the rotation of the horizontal units would crack the slab making it ineffective on the side on which the units rest. The total effective slab is then,

$$\text{effective slab} = 18 / 20 = 38 \text{ in.}$$

$$A_c' = 406 \text{ sq. in.}$$

$$c_b' = 17.2 \text{ in.}$$

$$I' = 34,839 \text{ in.}^4$$

$$k = 64.5$$

Column: (figure 16).

$$A_c = 654 \text{ sq. in.}$$

$$c_b = 14.8 \text{ in.}$$

$$I = 52,070 \text{ in.}^4$$

$$k = 394.0$$

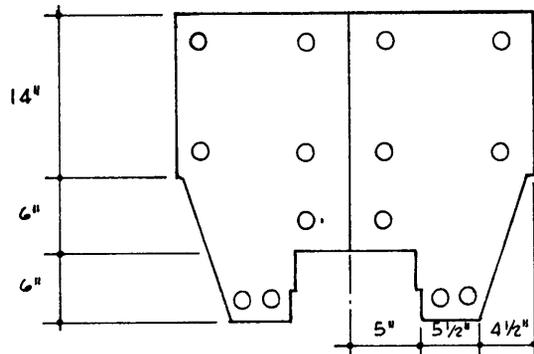


Figure 16.

Uniform Loads for Moment Distribution.

Floor:

$$w_5: \quad w_{DL} = \frac{350}{144} \times 0.150 = 0.360 \text{ klf}$$

$$w_{slab} = \frac{30 \times 3 \times 10 \times 2}{144} \times 0.150 = 0.100$$

$$w_{joists} = \frac{61.0}{45} = \underline{1.340}$$

$$w_6: \quad 1.800 \text{ klf.}$$

$$w_7: \quad w_{joist} = \frac{66.0}{45} = 1.460 \text{ klf}$$

$$w_{LL} = \frac{40}{12} \times 0.080 = 0.265$$

$$w_{part.} = \underline{0.045}$$

$$1.770 \text{ klf.}$$

Roof:

$$w_1 = 0.360 \text{ klf}$$

$$w_2 = 1.800 \text{ klf}$$

$$w_3: \quad w_{joist} = \frac{66.0}{45} \times \frac{20}{80} = 0.365 \text{ klf}$$

$$w_{LL} = \frac{40}{12} \times 0.020 = 0.067$$

$$w_{roofing} = \frac{40}{12} \times 0.005 = \underline{0.016}$$

$$0.448 \text{ klf}$$

Moments due to girder load - precast section (M7).

$DF_{ab} = 0.09$

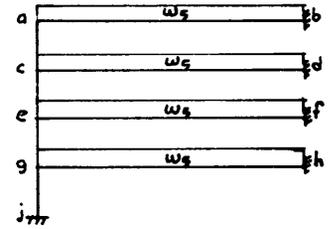
$DF_{cd} = 0.06$

$DF_{ac} = 0.91$

$DF_{cc} = 0.47$

$FEM_{ab} = \frac{wL^2}{12} = 61.5 \text{ k-ft.}$

$FEM_{cd} = 61.5 \text{ k-ft.}$



	0.09	61.5	5.5	0.0	1.3	0.0	0.9	56.4											
	0.91	0.0																	
		-56.0																	
		-14.5																	
		-36.7	13.2																
		-6.3	10.0																
		6.6	-9.1																
		20.0	56.4																
		-28.0																	
		-29.0	61.5	3.5	0.0	2.5	0.0	0.8	59.7										
	0.06	0.0																	
	0.47	0.47																	
		0.0																	
		-29.0																	
		-14.5																	
		-26.2	20.0																
		-6.3	6.8																
		10.0	-6.3																
		13.6	-23.0																
		-14.5																	
		-29.0	61.5	3.5	0.0	1.8	0.0	0.8	59.0										
	0.06	0.0																	
	0.47	0.47																	
		0.0																	
		-29.0																	
		-14.5																	
		-33.1	13.6																
		-3.2	3.4																
		6.8	-6.3																
		6.8	-32.8																
		-14.5																	
		-29.0	61.5	3.5	0.0	0.9	0.0	0.4	58.5										
	0.06	0.0																	
	0.47	0.47																	
		0.0																	
		-29.0																	
		0.0																	
		-11.1	6.8																
		0.0	0.0																
		3.4	-3.2																
		0.0	-25.4																
		-14.5																	
		0.0																	
		0.0																	
	0.00	0.00																	

	-63.6	0.0	0.7	0.0	1.8	0.0	61.5	0.00
	-62.0	0.0	1.3	0.0	1.8	0.0	61.5	0.00
	-62.4	0.0	0.9	0.0	1.8	0.0	61.5	0.00
	-61.8	0.0	0.5	0.0	1.8	0.0	61.5	0.00

Moments due Dead Loads - precast section ( $M_D$ ).

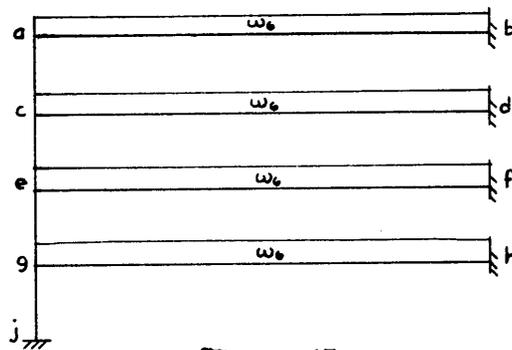


Figure 17.

$$M_{ab} = +283.4 \text{ k-ft.} \quad M_{ba} = -298.0 \text{ k-ft.}$$

$$M_{cd} = +288.0 \quad M_{dc} = -295.0$$

$$M_{ef} = +287.6 \quad M_{fe} = -295.2$$

$$M_{gh} = +286.5 \quad M_{hg} = -294.0$$

$$M_{gj} = -125.0$$

The cut-off points for the structurally acting poured-in-place slab are found using the procedure employed for the joist analysis (figure 18).

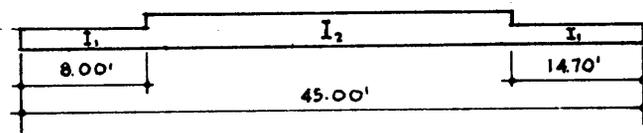
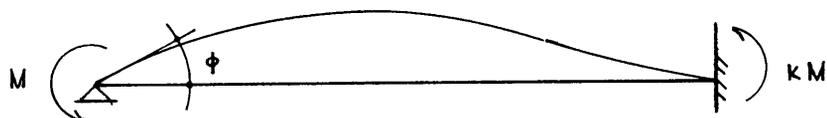


Figure 18.

To find the carry-over factors, stiffnesses and fixed-end moments for the girder, the same assumptions as for the joist previously analyzed are made and the procedure repeated (figure 19).



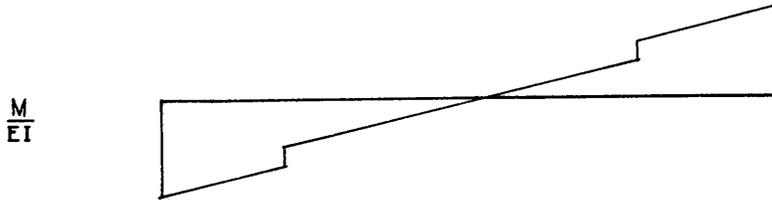


Figure 19.

carry-over factor (left-right) = 0.43

relative stiffness (left end) = 52.5

FEM (left end) =  $0.074 wL^2$

FEM (right end) =  $0.079 wL^2$

Maximum positive moment on member ef-composite section ( $M_{10}$ ).

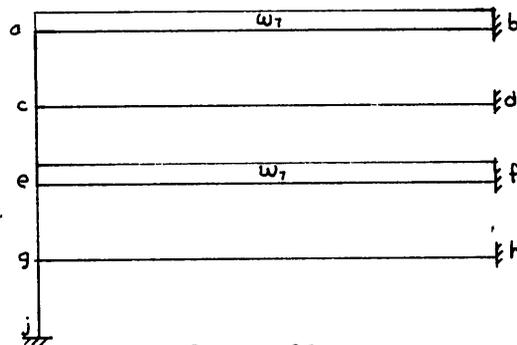


Figure 20.

$$M_{ab} = + 65.0 \text{ k-ft} \quad M_{ba} = -732 \text{ k-ft}$$

$$M_{cd} = + 24.7 \quad M_{cd} = -29.2$$

$$M_{ef} = + 240.3 \quad M_{fe} = -270.0$$

$$M_{gh} = + 247.0 \quad M_{hg} = -273.2$$

$$M_{gj} = -101.5$$

Using (1),

$$F = \frac{M}{0.65h}$$



$$= \frac{(M_8 \neq M_9) 12}{0.65 \times 24}$$

$$= 465 \text{ k}$$

assuming a 15% prestress loss

$$F_o = 545 \text{ k.}$$

Half of the tendons would be prestressed immediately after placing. The prestressing force would gradually be increased as the horizontal units are placed on the girder.

Cable Profile:

As in the joist's analysis - from the moments obtained -  $A_{min}$ ,  $A_{max}$  and  $A_g$  are obtained the boundaries of the area - within which the c.g.s. must lie in order not to produce tension - are found (drawing 11c).

Graphically

$$\beta' = 12.7^\circ = 0.222 \text{ radians}$$

using (6),

$$W = F$$

$$= 465 (0.222) = 103.0 \text{ k}$$

and

$$w_g = 2.3 \text{ klf}$$

As before, the moments due to the prestress eccentricity are found for the precast section and the composite section.

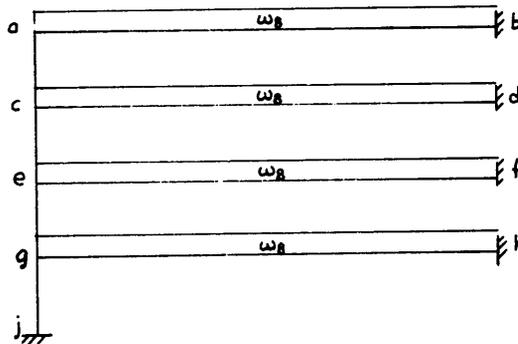


Figure 21.

The moments for member ef are:

$M_{11}$  precast section acting,  $ef = / 331.2$  k-ft.

$$f_e = - 370.0$$

$M_{12}$  precast section acting,  $ef = / 361.0$  k-ft.

$$f_e = -393.2$$

### Check of Sections.

1. Girder. The girder is checked at end F first, since there is the largest moment.

At end F:  $cb = ct = 12.0$  in.

$$I = 23,200 \text{ in.}^4$$

$$A_c = 480 \text{ sq. in.}$$

$$r^2 = \frac{I}{A_c}$$

$$= 48.5$$

$$K_b = k_t = \frac{r^2}{cb}$$

$$= 4.0 \text{ in.}$$

Using (7),  $e = \frac{M}{F}$

Bottom fiber:

due to loads,  $e_1 = \frac{M_8 / M_9}{F}$

$$= 15.0 \text{ in. below c.g.c.}$$

due to prestress,  $e_2 = \frac{M_{11}}{F}$

$$= \underline{10.0} \text{ above c.g.e}$$

effective  $e = 5.0$  below c.g.c.

Using formula (8),

$$f_b = 2.17 \text{ ksi} < f_c, \text{ adequate.}$$

Top fiber:

$$e_1 = \frac{M_7}{F_0}$$

$$= 1.4 \text{ in. below c.g.c.}$$

$$e_2 = \frac{M_{12}}{F_0}$$

$$= \underline{8.6} \text{ in. above c.g.c.}$$

effective  $e = 7.2 \text{ in. above c.g.c.}$

$$f_t = 3.1 \text{ ksi}$$

The stress is higher than the allowable, but the full  $F_0$  has been taken for the calculations when actually only half of it would act when there is no load on the girder, and would be increased as the dead loads increase.

At mid span: the properties are

precast section,  $A_c = 312 \text{ sq. in.}$

$$C_b = C_t = 12.0 \text{ in.}$$

$$I = 20.985 \text{ in.}^4$$

$$r^2 = 67.0$$

$$k_b = k_t = 5.6 \text{ in.}$$

composite section,

$$A'_c = 406 \text{ sq. in.}$$

$$c_{b'} = 17.2 \text{ in.}$$

$$c'_{t'} = 6.8 \text{ in.}$$

$$m_t = \frac{I/c_t}{I'/c'_{t'}} = 0.34$$

$$m_b = \frac{I/c_b}{I'/c'_{b'}} = 0.86$$

From the previous moment distributions, the moments at mid-span are obtained:

$$M_7 = + 30.6 \text{ k-ft.}$$

$$M_8 = + 164.6 \text{ k-ft.}$$

$$M_{10} = + 194.8 \text{ k-ft.}$$

$$M_{11} = - 234.4 \text{ k-ft.}$$

$$M_{12} = - 241.8$$

Top fiber:

$$e_1 = \frac{M_{10} \text{ mt } + M_8}{F}$$

$$= 6.0 \text{ in. above c.g.c.}$$

$$e_2 = \frac{M_{12}}{F}$$

$$= \underline{6.0} \text{ in. below c.g.c.}$$

$$\text{effective } e = 0$$

Using formula (8)

$$f_t = 1.50 \text{ ksi} < f_c, \text{ adequate}$$

Bottom fiber:

$$e_1 = \frac{M_7}{F_o}$$

$$= 0.7 \text{ in. above c.g.c.}$$

$$e_2 = \frac{M_8}{F_o}$$

$$= \underline{5.1} \text{ in. below c.g.c.}$$

$$\text{effective } e = 4.4 \text{ in. below c.g.c.}$$

$$f_b = 2.8 \text{ ksi} \approx f_c, \text{ adequate.}$$

The stress on the top fiber of the slab is checked by

$$f = \frac{M_c}{I}$$

$$= 0.40 \text{ ksi, adequate.}$$

The stresses during handling are checked assuming,

$$M = \frac{WL^2}{8}$$

and  $F = 100 \text{ k}$

$f_b = 0.18 \text{ ksi}$

$f_t = 0.46 \text{ ksi} < f_c$ , adequate.

Shear Check:

At end F,  $V = 81.2 \text{ k}$

Taking a load factor of 2,

$V' = 162.4 \text{ k}$

$$V's = \frac{4Fh'}{L} \quad (9)$$

if  $h = 18''$   $= 61.5 \text{ k}$

$V'c = V' - V's = 100.9 \text{ k}$

by formula (10),

$s = 2.3''$  for  $3/8$  in. diameter

U-stirrups. Use 2.25 in. spacing.

Maximum spacing  $= 3/4 h = 18 \text{ in.}$

Bond Stress:

Using 8 Freyssinet cables of 12-wires of 0.196 in. diameter each, the bond stress is checked by formulas (11) and (12):

Maximum  $u = 0.24 \text{ ksi}$   $0.08 f'c$ , adequate.

Prestress Losses:

By use of formulas (13) and (14), the prestress losses are found to be 14.6%, which compares adequately with the assumed 15%.

Deflection:

The deflection at mid-span is found to be:

$\delta = 0.10 \text{ in.}$ , downward.

2. Column. For full live load on the frame, the axial load and bending moment on column g<sub>j</sub> are:

$$N = 292.2 \text{ k}$$

$$M = 237.8 \text{ k-ft.}$$

Using 14-1/2 in. diameter wires (figure 16), and assuming an effective prestress of 125,000 psi, the stresses are found by formula (15).

for the exterior fibers,

$$f = 0.235 \text{ ksi (compression)}$$

for the interior fibers,

$$f = 1.736 \text{ ksi (compression)}$$

$< f_c$ , adequate.

The elevation of the final girder is shown in drawing 11d.

## IV. CONCLUSIONS

The structural system suggested provides a basis for an integrated building, in which there is an approach to the problem of correlating of the structural elements of the system with the other elements pertinent to Architecture.

The system indicates that a prestressed-precast concrete construction does not necessarily causes the possibility of monolithic action to vanish. The system shows what is possible in a relatively new type of construction, where members are tied together by post-tensioning. The analysis of the system shows the difficulties that are encountered due to the large number of loading and stressing conditions.

This thesis, above all, brings to light the inseparability of the different aspects of the building design process. Improvements on the system developed in this thesis are, of course, possible; however, the scope of this thesis would have been surpassed if all possibilities would have been examined. A system that would approach the best use of the construction concept of prestressed-precast concrete and that would be analyzable without much intricacies, was the necessary compromise.

## V. ACKNOWLEDGEMENT

Appreciation is expressed to Professor John F. Poulton, Department of Architecture, Virginia Polytechnic Institute, for suggestions and guidance not only during the development of this thesis, but also throughout the graduate year.

## VI. NOTATIONS

- $f$  Positive Moment, top fiber in compression.
- $A_b$  bearing area of anchor plate.
- $A_{cb}$  maximum area of portion of the member that is geometrically similar to and concentric with  $A_b$ .
- $A_c$  net cross-sectional area of concrete.
- $A'_c$  net cross-sectional area of composite section.
- $A_t$  gross cross sectional area of concrete, including steel transformed by ratio  $n$ .
- $A_v$  cross-sectional area of one set of steel stirrups.
- $a_1, a_2, a...$  lever arm between the centers of compression and tension in a beam section.
- $\beta$  change in angle of tendons.
- $b'$  width of web of member.
- $C$  center of compressive force.
- $c$  distance from c.g.c. to extreme fiber.
- $c_b, c_t$   $c$  to bottom (top) fibers;  $c_b'$ ,  $c_t'$  for composite section.
- c.g.c. center of gravity of concrete section.
- c.g.s. center of gravity of steel area.
- $D$  diameter of bars or wires.
- DF distribution factor.
- DL dead load.
- $\delta$  deflection.
- $e$  eccentricity in general.
- $e_1, e_2$  eccentricity under various conditions.
- $E$  modulus of elasticity in general.

$F$	total effective prestress after deducting losses.
$F_i$	total initial prestress - for handling
$F_o$	total prestress, just after transfer
FEM	fixed-end moment.
$f$	unit stress in general.
$f_c$	unit stress in concrete.
$f'_c$	ultimate unit stress in concrete.
$f'_{ci}$	ultimate unit stress in concrete, at time of transfer.
$f_{ci}$	allowable unit compressive stress in concrete, at transfer.
$f_{cp}$	allowable unit compressive stress on concrete bearing area.
$f_{cs}$	average concrete stress along the c.g.s. line.
$f_s$	unit stress in steel.
$f'_s$	ultimate unit stress in steel.
$f_{si}$	initial stress in prestressing steel after seating of anchorage.
$f_t$	allowable unit tensile stress in concrete.
$f'_v$	ultimate unit stress in steel stirrups.
$h$	overall depth of member
$h'$	rise of parabolic cable
$I$	moment of inertia of section; $I'$ for composite section.
$k$	relative stiffness.
$K$	absolute stiffness.
$\bar{K}$	coefficient for wobble effect of tendons.
$k_t, k_b$	Kern distances from c.g.c for top (bottom).
$L$	length of member.
LL	live load.

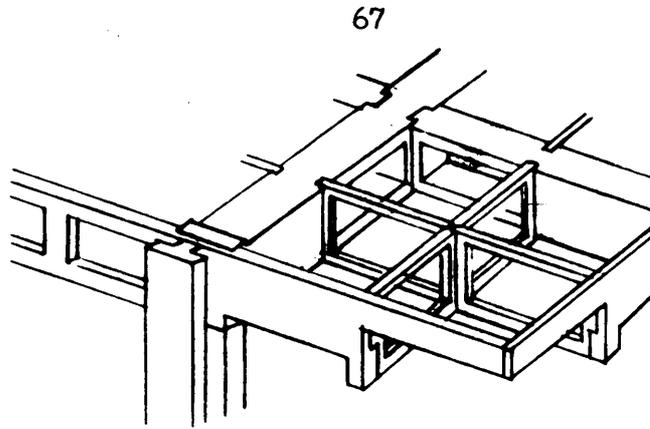
M	bending moment in general.
M <sub>1</sub> , M <sub>2</sub> , ...	bending moment under various conditions.
M <sub>max</sub>	algebraically largest moment at a section.
M <sub>min</sub>	algebraically smallest moment at a section.
m <sub>b</sub> , m <sub>t</sub>	ratio of section moduli of precast section to composite section for bottom (top).
N	axial load.
n	modular ratio E <sub>s</sub> /E <sub>c</sub> .
r	radius of gyration.
s	stirrup spacing.
u	unit bond stress.
V	total shear in member.
V'	total shear at ultimate load.
V' <sub>c</sub>	shear carried by concrete at ultimate load.
V' <sub>s</sub>	shear carried by steel at ultimate load.
W	total weight.
w	load or weight per length.
$\bar{w}$	friction coefficient.
$\Sigma o$	sum of bar perimeter areas.

VII. BIBLIOGRAPHY

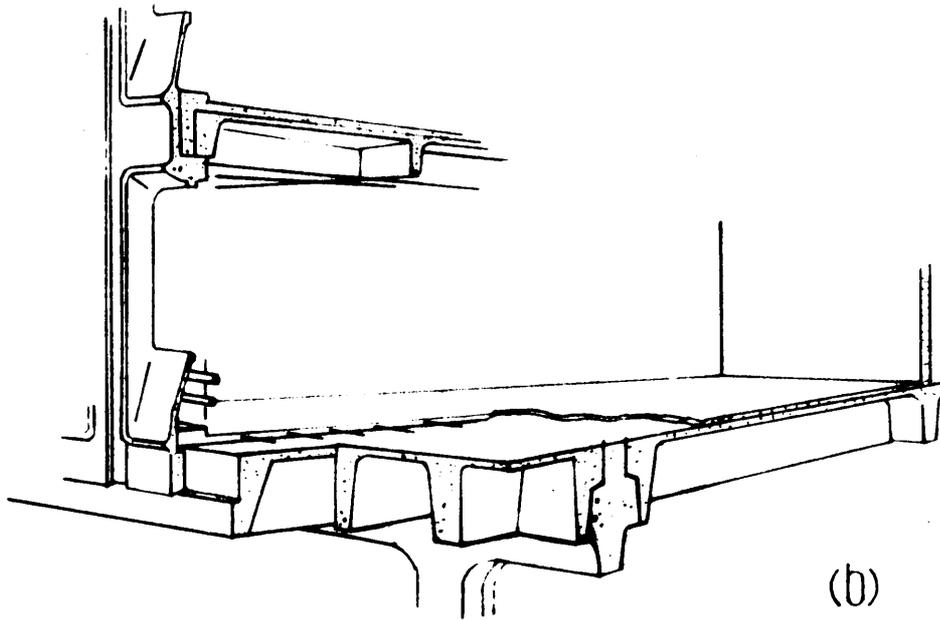
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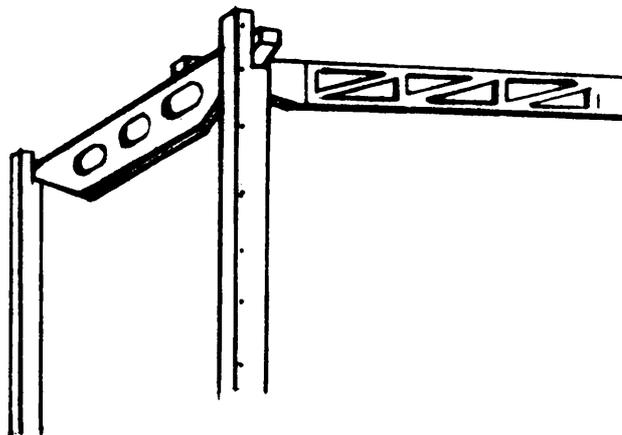
**IX. DRAWINGS**



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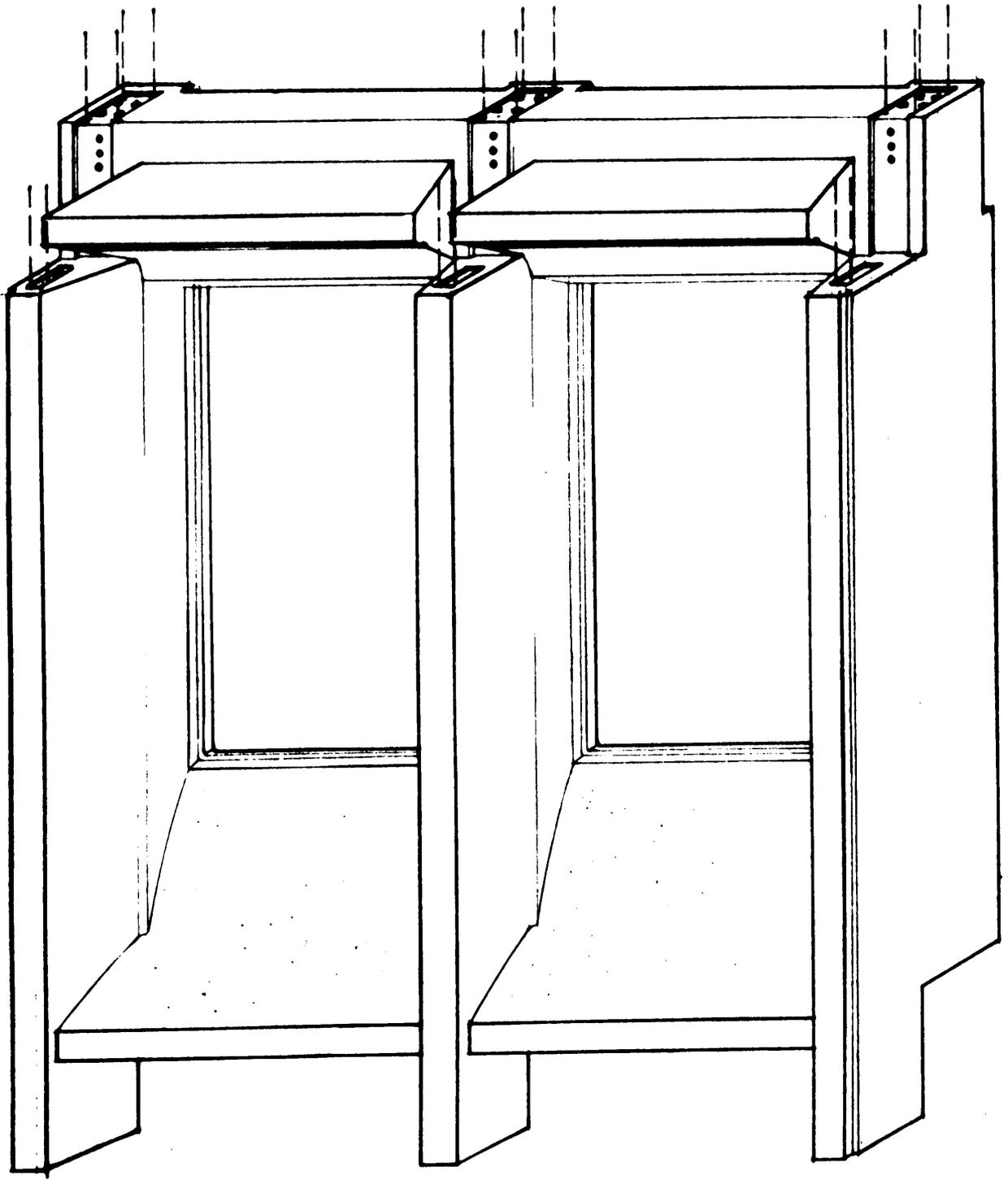


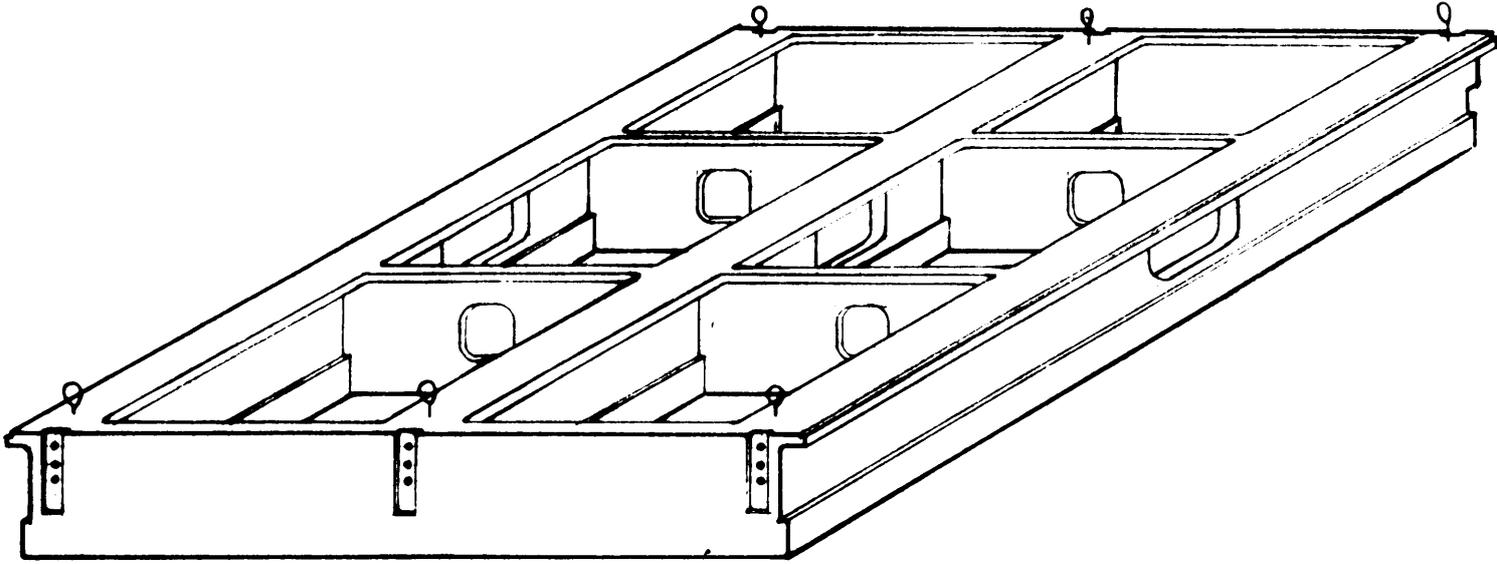
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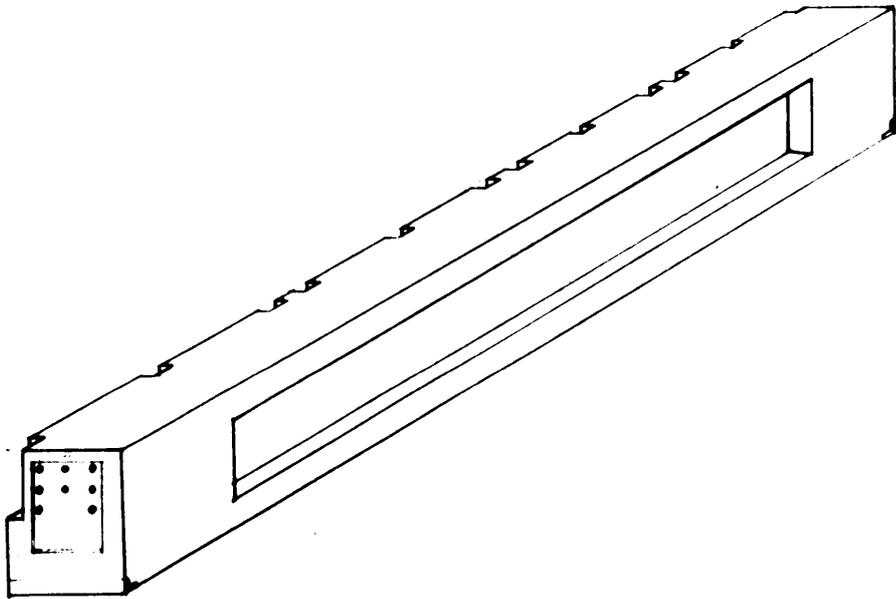
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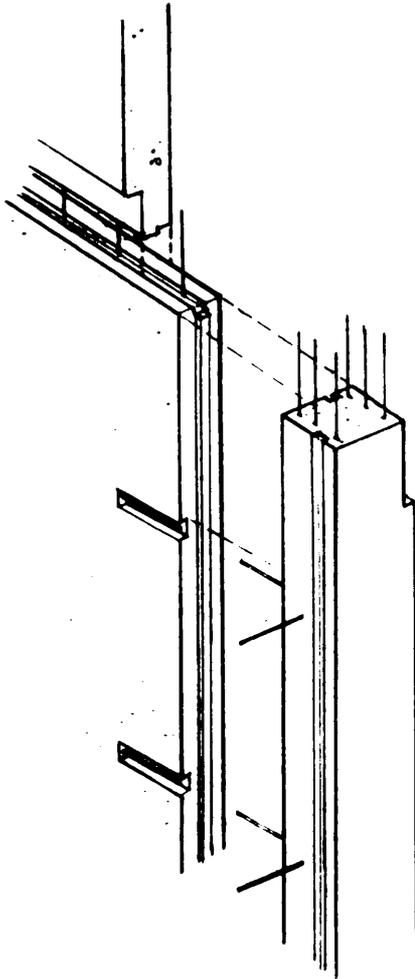


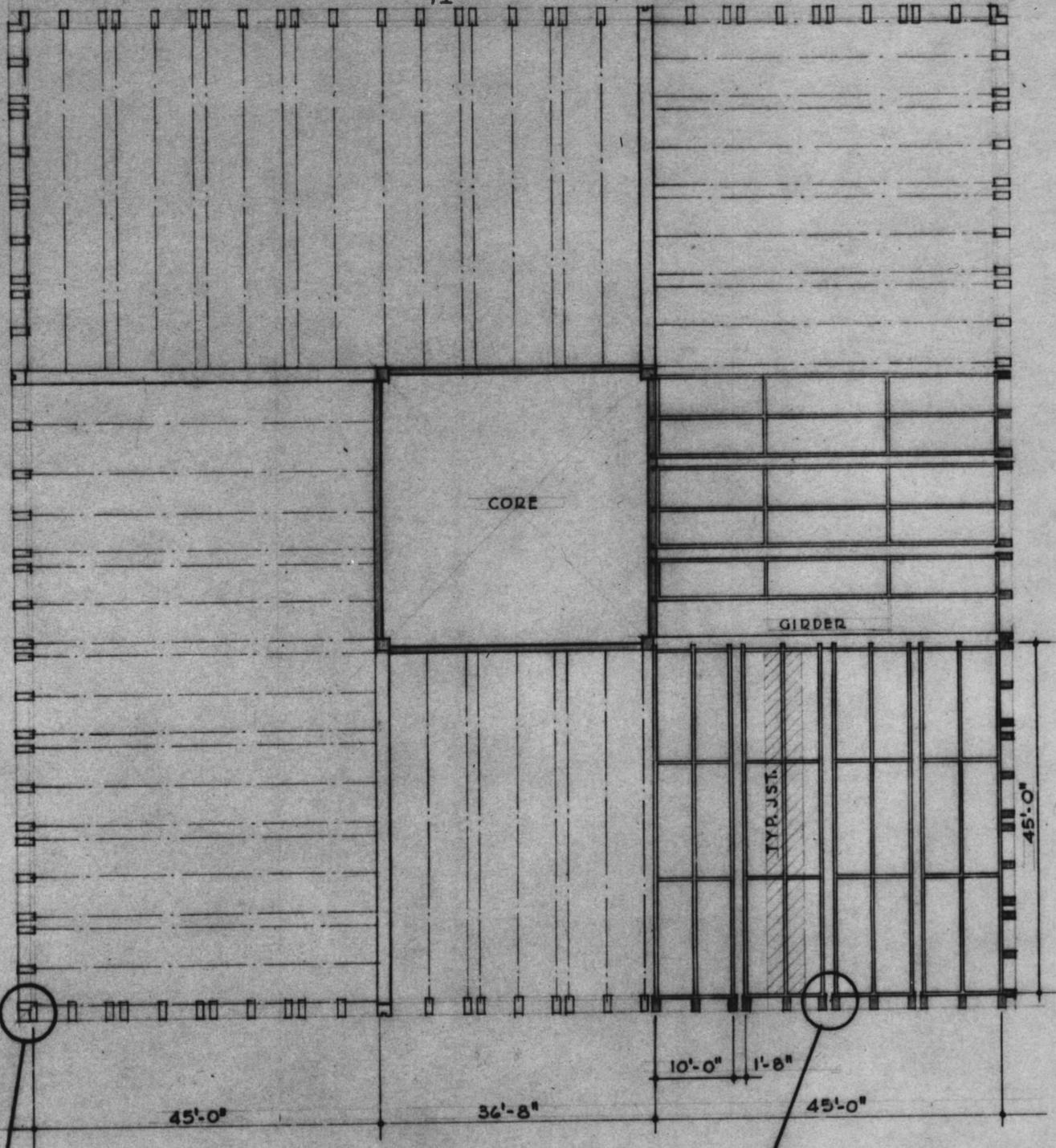


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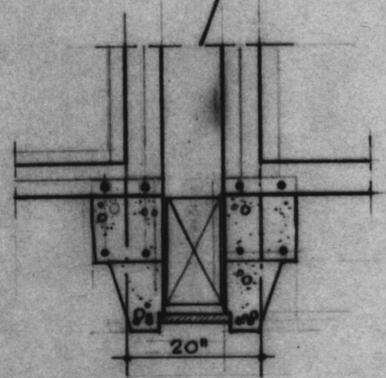
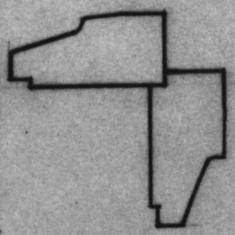


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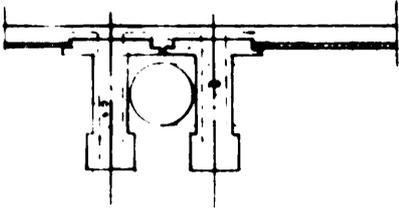




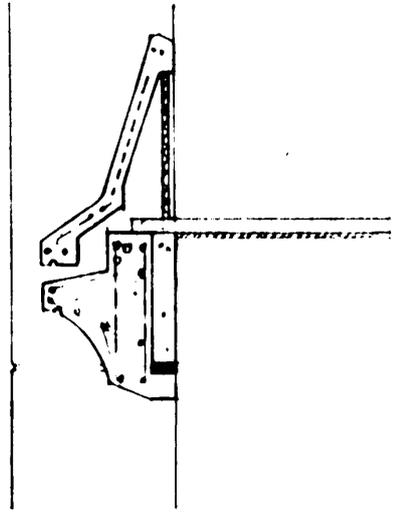
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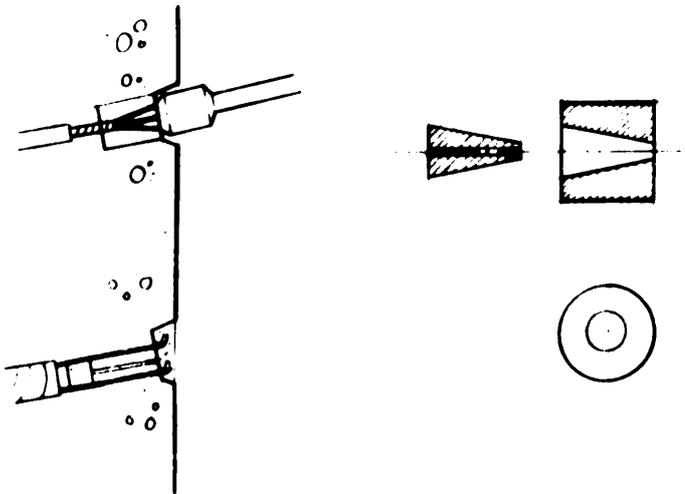


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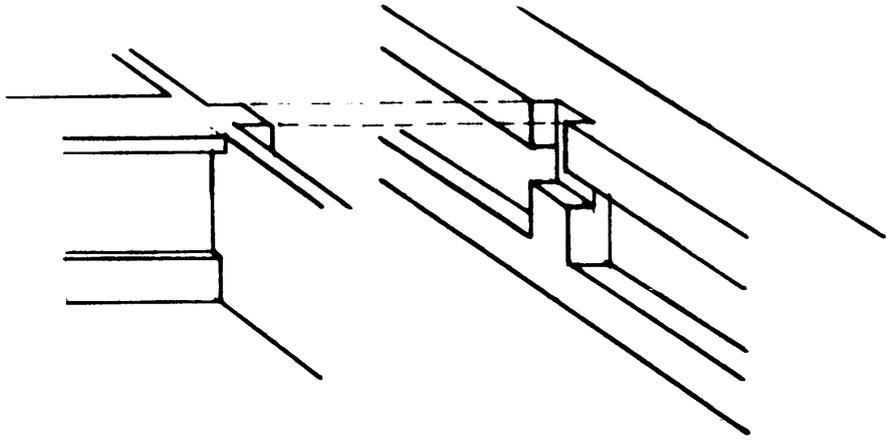


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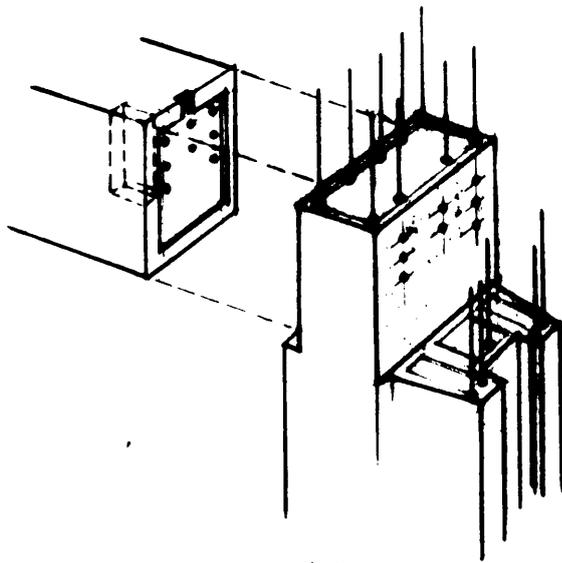
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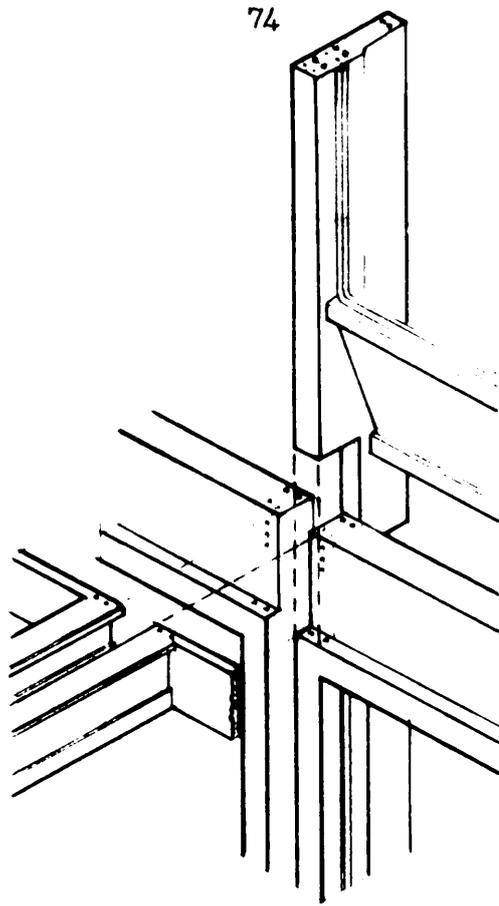
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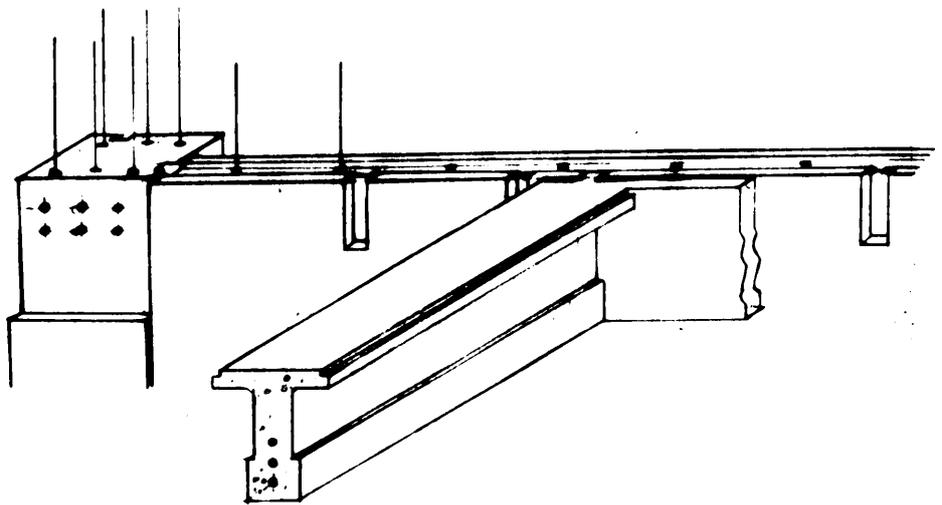
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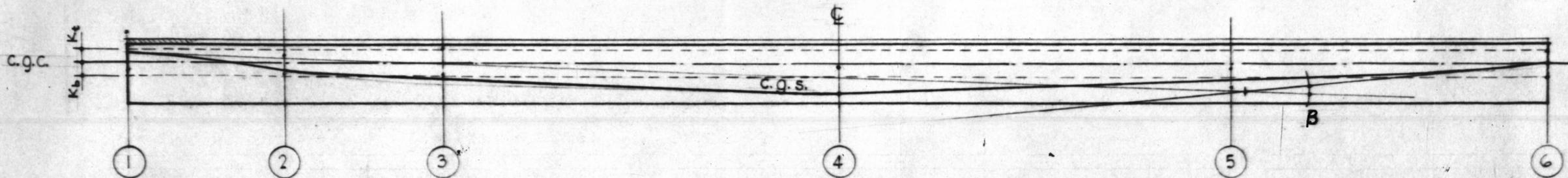
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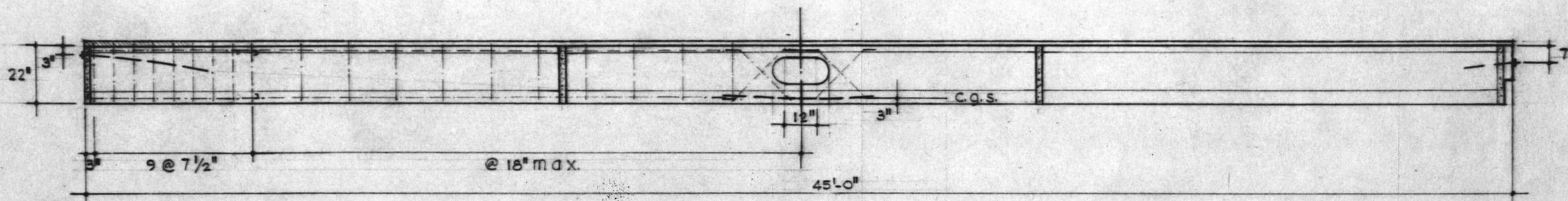
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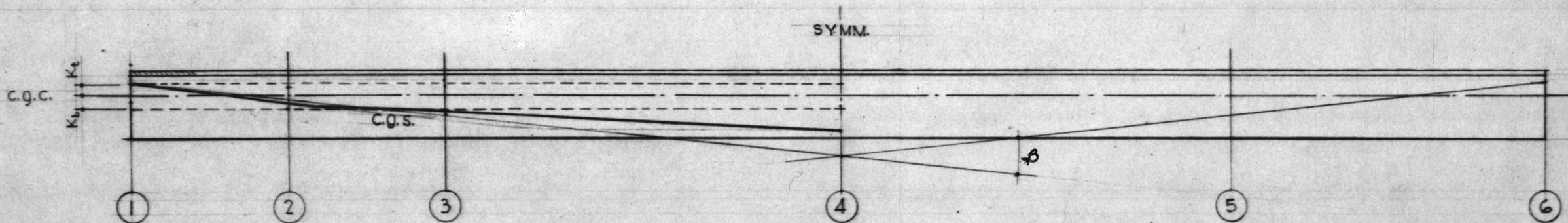
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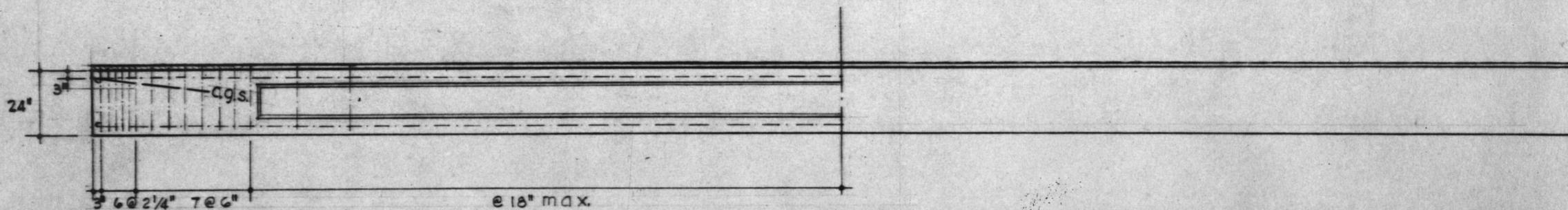
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