

Estimability of natural mortality within a statistical catch-at-age model: a framework and simulation study based on the Gulf of Mexico red snapper

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## Abstract

Estimation of natural mortality within statistical catch-at-age models has been relatively unsuccessful and is uncommon within stock assessments. The models I created estimated population-dynamics parameters, including natural mortality, through Metropolis-Hastings algorithms from Gulf of Mexico red-snapper *Lutjanus campechanus* data. I investigated the influences of assumptions regarding model configuration of natural mortality and selectivity-at-age parameters by comparing multiple models. The results of this study are preliminary due to parameter estimates being bounded by uniform priors and thus a potential lack of convergence to the posterior distribution. Estimation of a natural-mortality parameter at age 0 or a Lorenzen natural-mortality parameter could be confounded with selectivity-at-age-1 parameters for bycatch from the shrimp fisheries. The Lorenzen natural-mortality curve was calculated by dividing the parameter by red snapper length at age. An age-1 natural-mortality parameter might not be estimable with the currently available data. Values of the natural-mortality parameter for ages 2 and older appear to be slightly less influenced by assumptions regarding selectivity-at-age parameters. We conducted a simulation study to determine the accuracy and precision of natural-mortality estimation assuming the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries equaled 1.0 and a Lorenzen natural-mortality curve. The simulation study indicated that initial abundance-at-age parameters may be inestimable within the current model and may influence other parameter estimates. The preliminary simulation results showed that the

Lorenzen natural-mortality parameter was consistently slightly underestimated and apical-fishing-mortality parameters were considerably underestimated. The estimation of natural mortality within a statistical catch-at-age model for Gulf of Mexico red snapper has many caveats and requires additional investigation.

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## List of Abbreviations

M – Natural Mortality ( $\text{year}^{-1}$ )

M0 – Natural Mortality at age 0 ( $\text{year}^{-1}$ )

M1 – Natural Mortality at age 1 ( $\text{year}^{-1}$ )

F<sub>MSY</sub> – Fishing mortality at Maximum Sustainable Yield

SEAMAP – South East Area Monitoring and Assessment Program

NMFS – National Marine Fisheries Service

GOM – Gulf of Mexico

SEDAR – South East Data Assessment and Review

N0 – Abundance at age in the first year of the model

Z – Total Mortality ( $\text{year}^{-1}$ )

F – Fishing Mortality ( $\text{year}^{-1}$ )

DIC – Deviance Information Criterion

BIC – Bayesian Information Criterion

$\Delta$ DIC – Difference in Deviance Information Criterion values between two models

$\Delta$ BIC – Difference in Bayesian Information Criterion values between two models

CV – Coefficient of variation

REE – Relative Estimate Error

## **Chapter 1 – Introduction**

### **Review of Natural Mortality Estimation Methods**

The instantaneous rate of natural mortality is the rate at which a cohort's abundance exponentially declines over time due to natural causes such as predation, senescence, disease and malnutrition. This rate is extremely difficult to estimate because death of individuals and the quantity of death overtime in a population are usually unobservable in a marine environment (Xiao 2001). Natural mortality has been designated as the least known factor affecting stock variability (Quinn and Deriso 1999). This parameter is extremely important in determining the age composition of a population, which in turn affects the estimate of spawning capabilities of the stock and other population-dynamics parameters. Therefore, natural mortality is an extremely important population parameter to estimate for use in a statistical catch-at-age model. A statistical catch-at-age model incorporates catch data, age-composition data, and other sources to estimate population abundance and fishing mortality parameters for management use. Yet, to date, reliable methods for estimation of natural mortality within a statistical catch-at-age model have not been widely used in stock assessments.

Traditional stock assessments use methods outside of the statistical catch-at-age model to estimate natural mortality. These methods include mark-recapture, catch-curve analysis and empirical models that perform a regression on life-history parameters to natural mortality for a variety of species (Wang et al. 2007). The most common parameters used to predict natural mortality include von Bertalanffy growth coefficient, von Bertalanffy maximum length, maximum observed age, age at maturity, and temperature or some combination therein (Beverton and Holt 1959; Hoenig 1983; Pauly 1980; Ralston and Williams 1988). It has been hypothesized

that natural mortality can be estimated in some cases where age-structure data are available for a fishery prior to exploitation (Schnute and Richards 1995). Yet there are caveats associated with each of these methods.

Methods that estimate natural mortality outside of a statistical catch-at-age model are often problematic because of their strong assumptions that must not be violated and limited applicability to an entire stock. For instance, catch-curve analyses are only viable options when the fishing mortality rate is negligible because they provide a measure of total mortality of the population (Chapman and Robson 1960). However, the data required for a catch-curve analysis for most fisheries are obtained when fishing mortality is high. Methods that estimate natural mortality from life-history parameters are the result of meta-analyses that show natural mortality is positively correlated to growth parameters and negatively correlated to temperature and average reproductive output through a regression (Pascual and Iribarne 1993). However, correlation does not prove causation and the predictive ability of these regression equations is uncertain for stocks and species not part of the meta-analysis. Yet these methods are used in the literature and current stock assessments as if they give completely accurate predictions of natural mortality (Pascual and Iribarne 1993, SEDAR 31 2013). Limitations to estimating natural mortality from age-structure data are also present for currently available data. This form of natural mortality estimation for unexploited fisheries may not be feasible because populations that are underexploited or economically unimportant fisheries often have limited data. This limited data is due to lack of funding from management agencies to conduct the required sampling and the priority of economically important species. Likewise, age-structure data are usually only available for highly exploited populations, however these data are often only available for the past few decades. Additionally, these methods rarely provide an estimate of

uncertainty surrounding the natural mortality parameter. Therefore, the instantaneous rate of natural mortality is often treated as a known constant in assessment models, despite comprehensive documentation that shows using an incorrect value for the natural mortality parameter drastically changes the assumed fishing status of the stock within many different stock assessment methods (Sims 1984; Saila et al. 1985; Lapointe et al. 1989; Bax 1993; Schnute and Richards 1995; Xiao and Wang 2007).

Within a statistical catch-at-age analysis, incorrect specifying a value of natural mortality will cause improper estimation of key population dynamics parameters, such as recruitment and biological reference points used for management and determining fishery status, e.g.  $F_{MSY}$  and  $F/F_{MSY}$ . More specifically, if the estimate of natural mortality is higher than the true value, this will cause an underestimation of the abundance in older age-classes, an overestimation of the recruitment potential, and overestimation of  $F_{MSY}$  (Clark 1999). On the other hand, if the estimated natural mortality is less than the true value, the opposite results occur but the magnitude of error in the parameter estimates is not as large as when natural mortality is overestimated (Clarke 1999). Additionally, erroneous values for natural mortality when average fishing mortality ( $\bar{F}$ ) was low caused a greater impact on population size estimates compared to when  $\bar{F}$  was high (Clark 1999). Clark (1999) claimed that if  $\bar{F}$  was greater than or equal to 0.3 then error in natural mortality less than  $\pm 0.1$  would not be a serious concern. Despite research showing the negative results of assigning an incorrect value for natural mortality, attempts to estimate natural mortality within a population-dynamics model have been limited to recent studies. Therefore, additional investigation into the estimation of natural mortality within a statistical catch-at-age model is required to prevent misspecification of natural mortality by stock assessment scientists.

An *a priori* estimate of natural mortality has been a prerequisite in stock assessments since the first catch-at-age analyses of virtual population analysis (VPA) (Gulland 1965), and has been used since for other cohort analysis (Pope 1972; Pope and Sheperd 1982; Sheperd 1999) and early statistical catch-at-age models (Doubleday 1976; Deriso et al. 1985; Liu and Pitcher 1995). The sustained use of an *a priori* estimate of natural mortality may be due to early studies which found that natural mortality (M) and catchability coefficient (q) parameters were highly correlated and both parameters could not be simultaneously estimated reliably (Butler and McDonald 1979; Paloheimo 1980; Fournier and Archibald 1982). Similarly, recent studies have found that natural mortality estimates are highly correlated with other model parameters, such as selectivity at age (Thompson 1994; Schnute and Richards 1995; Fu and Quinn 2000; Lee et al. 2011). These correlations are commonly due to the model formulation in a statistical catch-at-age model along with the structure and informativeness of the data. However, these model formulations are currently unavoidable and these issues are not easily resolvable. Finally, many attempts to estimate natural mortality resulted in an estimate on the lower bound of the parameter, which suggests that this parameter may not be estimable within the model (Schnute and Richards 1995).

Natural mortality and fishing mortality are estimated from the decline in cohort abundance at older ages. The natural mortality and selectivity parameters are estimated from the observed age-composition data. Most often, age frequency comes from commercial catch which is affected by selectivity of the gear and the observed data do not give a full representation of the population present. One example the observed age-composition data providing a very poor representation of the population is when the selectivity at age has a “dome-shaped” curve, i.e. small proportions of the youngest and oldest of the population are collected. This causes

parameter estimates to be biased by the preponderance of data available for middle age-classes. As is often the case, the recorded landings are assumed to be the most certain data in stock assessments and therefore the most heavily weighted during the fitting process, which further compounds issues with potentially biased age-composition sampling. These are some of the many issues that cause problems when estimating natural mortality within a statistical catch-at-age model.

Recent research has shown, through simulation studies for various stock assessment methods, that natural mortality can, under certain circumstances, be estimated accurately within the model (Wang and Liu 2006; Wang et al. 2007, 2009; Lee et al. 2011). For a simulated species with one fishery, Wang and Liu (2006) discovered that natural mortality could be estimated from catch-at-age data and fishery-effort data within a statistical catch-at-age analysis. The estimability of natural mortality decreased as error in annual recruitment, catch-at-age data, and effort data increased (Wang and Liu 2006). Natural mortality estimation was found to be most sensitive to recruitment variation. They found that when the coefficient of variation (CV) ( $100 \cdot SD / \text{mean}$ ) for a lognormal distribution reached 10% on recruitment error, natural mortality estimates were unreliable ( $>10\%$  Relative Estimate Error). Error within the catch-at-age data had a higher impact upon the estimate of natural mortality than did error in effort data. Wang and Liu (2006) also showed that the type of fishing scenarios modeled affected the estimability of natural mortality and hypothesized that a high cumulative fishing mortality (F) in recent years would result in low error in estimates of natural mortality. This study is encouraging because it shows natural mortality can be estimated from within a statistical catch-at-age model, but there may be some limitations in its applicability to genuine fisheries data.

A study by Lee et al. (2011) found that natural mortality can be estimated within a statistical catch-at-age model using the program Stock Synthesis II. The analysis was performed for 12 groundfish stock assessments and was not restricted to the assumption of constant natural mortality across time, age, or sex. This study found that some unrealistic estimates of natural mortality may be a result of high correlation with parameters associated with productivity, e.g. growth and steepness in the spawner-recruit model, or those that describe the decline in observed proportions of older ages, e.g. selectivity at age (Lee et al. 2011). They concluded that unreasonable estimates of natural mortality were due to model misspecification rather than the inability to estimate the parameter. Yet methodology was not presented for determining when model misspecification was present or how to determine the correct model. However, this result may be extremely important to consider when making assumptions during model creation such as constant mortality across ages or through time (Jiao et al. 2012).

The identifiability of estimated parameters is a concern for complex models using Bayesian parameter estimation, but recognition of such problems and some solutions are presented by Rannala (2002) and are discussed below. Complex models are easy to overparameterize, especially when using Bayesian analysis due to the ease with which parameters can be added to models. Overparameterization occurs when a model obtains a sample configuration probability that is identical to another model that uses fewer parameters. Unidentifiable parameters are present in models that have been overparameterized. Extreme correlation between parameters in Bayesian estimation may be a sign that one of the parameters is unidentifiable, especially when parameters are independent under the prior distribution. However, correlation in the posterior distribution between parameters occurs for many models where all parameters are identifiable. Therefore, correlation can only be used as a general guide



for detecting overparameterized models and potentially unidentifiable parameters. Unidentifiable parameters may also be recognized by determining the robustness of the posterior distribution to the prior. A solution to unidentifiable parameters is to determine a function of the parameters that can be identified, and estimate values for this function instead. This solution is only possible for simple models where additional functions of parameters can be found, and is not likely in fisheries assessments. Another possible solution for Bayesian analysis is to apply an informative prior to the unidentifiable parameters. The use of informative priors in overparameterized models causes the parameter to be highly dependent upon the prior. The use of informative priors is not common practice in formal stock assessments, but the application of these priors might be superior to the point estimates currently used.

### **Review of the Red Snapper Fishery in the Gulf of Mexico**

Red snapper *Lutjanus campechanus* is considered to be the most economically important species in the snapper-grouper commercial fishery in the Gulf of Mexico (SEDAR 31 2013). The red snapper is a long-lived (approximately 60 years) marine teleost that occupies the continental shelf of the Gulf of Mexico (GOM) and Atlantic Ocean as far north as Cape Hatteras, NC. Coral reefs, rock outcrops and other three-dimensional structures are common habitat for red snapper (Moran 1988). Red snapper are carnivorous fish that feed primarily on crustaceans, squid and other fish. According to the most recent assessment of red snapper in the GOM there are two spawning stocks of the species that are separated by the Mississippi River (SEDAR 31 2013). Spawning occurs across the entire GOM shelf and upper slope, over firm sand bottoms with little relief (Moran 1988). The peak spawning months for red snapper in the GOM is from June through August, though spawning occurs from April through October (Fitzhugh et al. 2004).

Estimating the reproductive potential of red snapper continues to be a challenge due to the large variations in size at age and batch fecundity between individuals.

This species is managed under one management plan, despite the biological separation of the stocks. Currently, the Gulf of Mexico red snapper is classified as overfished and is subsequently managed through a rebuilding plan (SEDAR update assessment 2009). The overfished status is a result of the high fishing mortality from the directed fisheries and high bycatch mortality of juveniles from the GOM shrimp fisheries. Management of the species is enacted through size restrictions, bag limits, season closures, limited access to the fishery, and catch quotas. A list of historical and current restrictions on the season, and minimum size limits of harvest for both commercial and recreational fisheries, can be found in the most recent stock assessment of red snapper in the GOM (SEDAR 31 2013). The management of the red snapper fishery is a complex and contentious issue that is beyond the scope of this paper. Yet a clear understanding of the sources of fishing mortality involved, both directed and bycatch, is essential to modeling this species.

Gulf of Mexico red snapper are harvested by multiple directed fisheries: a commercial longline fishery, a commercial handline fishery, and a recreational fishery, which all operate in both the eastern and western GOM. The red snapper fishery is a very economically important fishery, not only commercially but also recreationally. Current regulations allocate 51% of the quota to the commercial fleet and 49% to the recreational sector. Due to management restrictions, there are limited red snapper seasons for both the commercial and recreational fisheries, resulting in fishery closures. During these seasonal closures red snapper are caught as bycatch by other commercial and recreational fisheries. Large quantities of juvenile red snapper are also killed as bycatch in the GOM shrimp fisheries. An estimate of the number of red snapper

killed due to bycatch mortality is required for each bycatch fishery to be used in a stock assessment.

Red snapper can be found at depths of 10 to 200 meters, with older fish often moving away from bottom structure to deeper areas (Galloway et al 2009). This puts the species at severe risk of experiencing barotrauma when they are rapidly brought to the surface by fishermen. Release mortality is a concern for directed fisheries during the open season because individuals smaller than the legal-harvest size limit make up a large proportion of the catch (Burns et al. 2002). Release mortality is estimated to range between 15% for the eastern GOM recreational fishery and 82% for commercial fisheries in the western GOM (Table 1.1). This difference between fisheries in the estimated value of release mortality is very controversial and can drastically impact the assessed status of the species.

Juvenile red snapper, age-0 and age-1 fish, are caught and killed in very high frequencies as bycatch in the GOM shrimp fisheries (SEDAR update assessment 2009). An estimated 25 - 45 million red snapper are annually returned to the sea as a result of bycatch from the shrimp fisheries, which correspond to roughly 2 - 5 million pounds. On the other hand, the directed commercial and recreational fisheries remove only 3 - 4 million individuals, which correspond to 9 million pounds of red snapper. The 2004 stock assessment predicted that the red snapper population would be unable to rebuild its stock size, even with a complete closure of the directed fishery, without a reduction in the amount bycatch by the shrimp fisheries (SEDAR 7 2005). The Gulf of Mexico Fishery Management Council therefore enacted both Shrimp Amendment 9 and 14 in response to the high juvenile red snapper mortality caused by shrimp trawling. Due to the complex interaction of multiple fisheries, the results stock assessments on red snapper are often highly controversial due to the economic impact they may impose.

Natural mortality is often a controversial topic during many stock assessments: red snapper is no exception. Controversies arise from the subjective decisions scientists make in assigning a value to natural mortality for use in a statistical catch-at-age model. For red snapper, controversy exists regarding the value of natural mortality for age-0 and age-1 red snapper. Recent studies presented in the 2009 stock assessment of red snapper (Gazey et al. 2009, SEDAR update assessment 2009) suggested that juvenile natural mortality may be highly density dependent, thus implying that the estimate of natural mortality used in SEDAR 7 (2005) for age-0 and age-1 red snapper was too low. Due to time constraints, no models for red snapper assuming density dependent natural mortality were created during the update assessment, instead the rate of natural mortality for age 0 and age 1 were changed to the values presented in Table 1.2. Estimated values of natural mortality for age 0 ranged from 0.98 to 3.7 and for age 1 from 0.6 to 1.4 (SEDAR update assessment 2009). Due to this large range of estimated values, two scenarios of natural mortality were used for age 0 and age 1 in the update assessment (Table 1.2). Assigning a value for natural mortality creates controversies, and an incorrect choice can drastically change the results of the assessment. Therefore, the estimation of natural mortality and the uncertainty surrounding this parameter within a statistical catch-at-age model for red snapper is prudent to resolve these issues.

The study by Wang and Liu (2006) show natural mortality is estimable for a simplistic fishery model. Nonetheless, caution must be taken to assure that variability in the data does not cause poor estimability of natural mortality. The estimation of natural mortality within a statistical catch-at-age model has been limited to stocks with simple fisheries and complete catch-at-age datasets. A general methodology to estimate natural mortality within a statistical

catch-at-age model that is applied to a species with multiple, complex fisheries must be tested and shown to be reliable to stimulate the use of such methodologies in future assessments.

### **Research Goals and Objectives**

This study examined the complex population dynamics of the Gulf of Mexico red snapper using a Bayesian approach and the estimation of natural mortality within the statistical catch-at-age models for the red snapper fisheries. Specifically, Chapter 2 explored the impact of assumptions regarding the shape of the curve for natural mortality at age and selectivity at age upon the estimates of natural mortality. A simulation study was performed in Chapter 3 for a model that assumes a Lorenzen natural mortality curve where the accuracy and precision of parameter estimates from simulated data are compared to the “true” parameter values. Chapter 4 comments upon the methodology used by Gazey et al. (2008) and disputes the assumptions made therein. Through these methods the estimability of natural mortality for red snapper in the Gulf of Mexico is assessed and points for future research possibilities are offered.

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Table 1.1. Estimated value of release mortality of red snapper for each fishery in the eastern and western Gulf of Mexico (GOM) obtained from the SEDAR update assessment of red snapper (2009).

<b>Fishery</b>	<b>Western GOM</b>	<b>Eastern GOM</b>
Recreational	40%	15%
Commercial (longline and handline)	82%	71%

Table 1.2. Rates of natural mortality of red snapper in the Gulf of Mexico assumed for ages 0, 1, and 2 and older (2+) assumed as the most representative on the population for SEDAR 31 (2013) and the SEDAR update assessment (2009).

<b>Age</b>	<b>SEDAR 31</b>	<b>SEDAR update assessment</b>
0	2	0.9833
1	1.2	0.59
2+	0.1	0.1

## **Chapter 2 - Natural mortality estimation within statistical catch-at-age models for Gulf of Mexico red snapper: A comparison of modeling assumptions**

### **Abstract**

Natural mortality estimation within catch-at-age models has been problematic since the use of virtual population analysis in the 1960s. Regression equations, of life-history parameters and natural mortality, are often used to estimate natural mortality, which are then inserted into an assessment model. Data from the red snapper SEDAR update assessment in 2009 were collected and used to create statistical catch-at-age models in which natural mortality and other parameters were estimated through a Bayesian Metropolis-Hastings algorithm. The estimation of natural mortality at age 0 was highly dependent upon the age-composition data for bycatch from the shrimp fisheries and the modeling assumptions for selectivity parameters for this fishery at age 0 and 1. Estimation of a separate parameter for natural mortality at age 1 does not seem feasible with the currently available data for Gulf of Mexico (GOM) red snapper and is heavily influenced by the prior on this parameter but is strongly negatively correlated with natural mortality at age 0. Estimated values of natural mortality for ages 2 and older were influenced by the estimated natural mortality values at age 0 and age 1, but were estimated to be around 0.1 for the majority of models. The estimation of all natural mortality parameters appears to be dependent upon the assumption regarding the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries being set to fully selected and may be problematic for inclusion in an assessment framework. Additionally, the estimation of natural mortality within a statistical

catch-at-age model is likely to be an artifact of the assumed shape of the selectivity curve for the fisheries in the model.

## **Introduction**

Red snapper *Lutjanus campechanus* is considered to be the most economically important species in the snapper-grouper commercial fishery in the Gulf of Mexico (SEDAR 31, 2013). Currently, Gulf of Mexico (GOM) red snapper is classified as overfished and is subsequently managed through a rebuilding plan (SEDAR 31, 2013). The overfished status is a result of the high fishing mortality from directed fisheries especially from the recreational fishery and high bycatch mortality of juveniles from the GOM shrimp fisheries. The stock has begun to recover from its lowest point in the 1990s through management regulations upon the directed fisheries and reductions in bycatch by the shrimp fisheries through reductions in shrimping effort (SEDAR31 2013). Due to the complex interaction of multiple fisheries, the results of red snapper assessments are often highly controversial due to the economic impact they might impose.

One contentious issue that has arisen during the previous assessments of red snapper is what values should be input into assessment models for natural mortality at age 0 ( $M_0$ ) and natural mortality at age 1 ( $M_1$ ) (SEDAR 7 2005; SEDAR update assessment 2009; SEDAR31 2013). This controversy arises from the decisions that scientists must make when assigning a value to natural mortality for use in a statistical catch-at-age model. The assumed values for  $M_0$  and  $M_1$  input into the assessment model have increased in the past three assessments of red snapper from 0.5 and 0.3, respectively (SEDAR 7 2005) to 1 and 0.6, respectively (SEDAR update 2009) and the current best estimates are assumed to be 2 and 1.2, respectively (SEDAR 31). During this time period, the assumed natural mortality of ages 2 and older ( $M_{2+}$ ) have remained consistently at 0.1, though a Lorenzen mortality curve was instituted in SEDAR 31

(2013). Since assigning a value for natural mortality creates controversies, and an incorrect choice can drastically change the results of an assessment (Sims 1984; Saila et al. 1985; Lapointe et al. 1989; Bax 1993; Schnute and Richards 1995; Mertz and Myers 1997; Clark 1999; Xiao and Wang 2007), the estimation of natural mortality and the uncertainty surrounding this parameter within a statistical catch-at-age model for red snapper seems prudent.

Recent studies suggest that the estimation of natural mortality within a statistical catch-at-age model is potentially possible. The study by Wang and Liu (2006) is a framework from which more-complicated statistical catch-at-age models can be built to estimate natural mortality. Similarly, Lee et al (2011) discovered that the inclusion of high quality length or age-composition data is necessary to reliably estimate natural mortality within an assessment model. Yet these data alone are not always sufficient to prevent high correlation with parameters in the model, which can then result in unrealistic estimates of  $M$  (Lee et al 2011). Parameters that are often highly correlated with  $M$  are those associated with production, such as growth and recruitment curve steepness, or those that can be used to describe the decline observed in proportion at age of older age-classes, such as selectivity. Lee et al. (2011) concluded that model misspecification causes unrealistic estimated values of  $M$  that are approximately the bounds of the parameter rather than an inability to estimate  $M$  from the data. Despite these successes in estimating natural mortality within a statistical catch-at-age model, the estimation of natural mortality within an assessment model is rare.

The goal of this study is to investigate the estimation of natural mortality within a statistical catch-at-age model for the complex population dynamics of the GOM red snapper using a Bayesian approach. Various assumptions about the shape of the curve for natural mortality at age were compared and estimated within the statistical catch-at-age model.

Additional assumptions were investigated regarding the selectivity at age for bycatch from the shrimp fisheries and the corresponding selectivity parameters for fisheries from eastern and western GOM. The results of this study provide some interesting insight into the estimation of natural mortality within statistical catch-at-age models for GOM red snapper.

## **Methodology**

Several models were fit to red snapper data to test a variety of assumptions relating to natural mortality and selectivity at age. Though data exist for commercial catch of red snapper as far back as 1872, the models described below are for the time period of 1987 to 2008 because data were not sufficient to reliably estimate annual recruitment before 1987. Additionally, the SEAMAP age-1 and age-0 survey data were hypothesized to be influential in the estimation of natural mortality; therefore, the start of the model coincides with the first available year of data for the SEAMAP age-1 survey (SEDAR update assessment 2009). In the following methodology, the data sources are identified, model calculations are shown, differences between model configurations are explained, the statistical estimators for the models are calculated, and the Bayesian methods employed are summarized. All data sources described below were obtained from the SEDAR update assessment (2009). Descriptions of each symbol used are presented in Table 2.1, the equations used in the models are presented in Table 2.2, and the equations used for the statistical estimator of the posterior distributions are presented in Table 2.3. The equations are referenced as Equation x.y where equation y is within Table 2.x.

## ***Landings Data***

Based upon the precedent set by SEDAR 7 (2005), the SEDAR update Assessment (2009) and SEDAR 31 (2013), the landings data by fisheries were separated into two fisheries by region, where one fishery operates in the eastern GOM and the other in the western GOM. There



are four commercial directed fisheries for which landings in weight of red snapper in the GOM were reported: handline east, handline west, longline east and longline west. The models created used the commercial landings data for these fisheries from 1987 to 2008 and separate landings data in number of fish for the eastern and western recreational fisheries for these years. The landed catch and amount of dead discards for fish larger than the fishery specific legal-harvest size limits are incorporated in the landings estimates for both commercial and recreational fisheries. Estimates of discards for fish smaller than the harvest size limit are calculated by the model and are described below. The numbers of fish caught by the NMFS bottom-longline survey for intermittent years from 1996 to 2008 were incorporated into the models as landings.

Two sources of bycatch estimates were input into the assessment model as landings in the update assessment: shrimp-trawl bycatch and closed-season bycatch. Bycatch estimates in number of red snapper caught by the shrimp fisheries from 1987 to 2008 for both regions of the GOM were used in the models. Red snapper closed-season bycatch estimates from 1991 to 2008 for the eastern and western GOM were incorporated into the models as number of fish caught by a hypothetical fishery, where the legal-harvest size limit equaled zero and all fish caught are “harvested”. Bycatch estimates from the shrimp trawl were treated as the total number of fish caught by a hypothetical shrimp fleet, where all fish caught are assumed to be “harvested” and the legal-harvest size limit was zero.

### ***Age-Composition Data***

Age-composition data for the eastern and western GOM were calculated from subsamples of the catch from the commercial longline fisheries, commercial handline fisheries, and recreational fisheries, and incorporated into the model for available years of data from 1991 to 2008. Estimated proportion of age-0, age-1, and age-2 red snapper data for bycatch from the

shrimp fisheries were incorporated into the model for the eastern and western GOM from 1992 to 2008. Age-composition data were estimated for the SEDAR update assessment (2009) for the bycatch of red snapper during the closed-season fishery in the eastern and western GOM. The final age-composition data were from the western GOM NMFS bottom-longline survey for years the survey was active. The precedent set in the SEDAR update assessment (2009) of red snapper modeled age-composition data for the NMFS bottom-longline as though it were from a fishery to connect the survey and the age composition together. Therefore, 11 fisheries were modeled to harvest red snapper in the GOM for these models presented (Table 2.4). The numbers of samples obtained for each year and fishery were used in the statistical estimator, but sample sizes greater than 200 were set equal to 200 to reduce the weight the age-composition data had upon model fits.

### ***Shrimp Fishing Effort Data***

Estimates of offshore shrimp fishing effort from 1987 to 2008 for the eastern and western GOM were used in the models. Fishing effort estimates were used together with estimates of bycatch harvest by the shrimp fisheries from 1987 to 2008 to improve the accuracy of the estimated number of bycatch red snapper from the shrimp fisheries. Both data sources were used to estimate a log-scale catchability coefficient for bycatch from the shrimp fisheries in the model to appropriately scale the fishing effort.

### ***Indices of Abundance***

Abundance indices from the SEDAR update assessment (2009) used in this study were from both fishery-dependent and fishery-independent sources. Table 2.4 presents the eight fishery-independent abundance indices used in the model and their respective survey number in the model. The model incorporated fishery-dependent abundance indices for the commercial

handline fisheries in the eastern and western GOM from 1996 to 2006, for the recreational fisheries in the eastern and western GOM from 1981 to 2008 and for the NMFS bottom-longline survey in the western GOM from 1996 to 2008, but missing data for 1998 and 2005. The SEAMAP reef fish video surveys from the eastern and western GOM were used for intermittently available years from 1993 to 2008. The SEAMAP larval or ichthyoplankton survey data, collected using bongo nets in the eastern and western GOM, were available from 1987 to 2007. The SEDAR update assessment (2009) found that the SEAMAP larval index was highly correlated to the number of adult red snapper and used it as an index of spawning stock. Similar formulation, as described below, was used to calculate the expected value of this survey in all the models. The SEAMAP bottom trawl survey abundance indices of age-0 red snapper for both the eastern and western GOM were used in the models for years 1987 to 2008. The SEAMAP bottom trawl survey abundance index of age-1 red snapper for the eastern GOM from 1987 to 2008 and for the western GOM from 1989 to 2008 were incorporated into the models. The SEAMAP bottom trawl survey data were used as one index in SEDAR 7 (2005), but were separated into distinct indices for age 0 and age 1 as were used in the SEDAR update assessment (2009). All indices were scaled to ensure catchability coefficients were close to but no more than 1.0 and the natural logarithm was applied, to allow for easier parameter estimation.

### ***Estimated Variability of Data***

For the landings data, abundance indices, and effort data, CVs for each year of available data were used as a measure of variance in the statistical estimator. The calculation of the variance for each data point is described below (Table 2.3). These estimates and the rationale behind the values can be found in the SEDAR update assessment (2009).

### ***Relative Fecundity***

The SEDAR update assessment (2009) of red snapper used a measure of relative fecundity, where age-30 red snapper have a relative fecundity equal to 1.0 and all younger ages are calculated from a logistic function that was adopted in the model of this study. These values were used in the model to weight the contribution of each age-class to the egg production estimated by the model for comparison to the SEAMAP larval survey data.

### ***All Model Calculations***

The model described by Porch (2007) was a basis from which all models were built utilizing the red snapper data described above. The model in this study spans the time period 1987 to 2008, with three seasons in each year and spawning is assumed to occur during the second season. Each age-class is assumed to progress to the next age-class after 3 full seasons, i.e. at the beginning of the second season as described in Equations 2.2a, 2.2b, and 2.2c. The model uses 16 age-classes starting at age 0 and assumes a plus group at age 15 to represent the abundance of fish 15 years old and older as described by Equations 2.3a, 2.3b, and 2.3c.

Despite the recent season closures, commercial and recreational fisheries were modeled as operating all year every year because season closures varied from year to year and did not always occur strictly within one or two seasons. The impacts of this assumption are assumed to be relatively small because season closures did not occur until 1999 and this assumption was used in the SEDAR update assessment (2009). To incorporate the red snapper killed as dead discards during the red snapper closed season, bycatch during the closed-season fishery was modeled as operating during the second season where all fish caught were “harvested” and the legal-harvest size limit was equal to zero. Similarly, the red snapper killed as bycatch in the shrimp fisheries were treated as “harvest” with the legal-harvest size limit equal to zero. All

models described assumed no processes error in the population and no aging error was present in the age-composition data. Differing from the methodology used in the SEDAR update assessment (2009) described by Porch (2007), recruitment in each year and the abundance in each age-class for the first year were estimated as free parameters (Equation 2.1a and 2.1b).

The process equations of the model are shown by equations 2.1a, 2.1b, 2.2a, 2.2b, 2.2c, 2.3a, 2.3b, and 2.3c. The initial age-class in season 1 of the first year and recruitment in season 2 of all subsequent years in number of fish are estimated parameters in the model and form the basis that the rest of the process equations are calculated from (Equations 2.1a and 2.1b). The calculations to produce abundance at age for ages that are less than the plus group (age 15) through seasons and years are described below. The transition from season 1 to season 2 in the process equation calculates an exponential decay due to total mortality ( $Z$ ) to a cohort which is then moved to the next age-class and the next season (Equation 2.2a). In season 2 a cohort is transitioned to season 3 after applying a negative exponential population function (Equation 2.2b). After applying an exponential decay to the number of fish in season 3, the cohort is moved to the first season of the next year, but kept in the same age-class (Equation 2.2c). These calculations are slightly different for the plus group at age 15. The transition from season 1 to season 2 is the sum of the two negative exponential population functions for age 14 and the plus group of age 15. The transition of season 2 to season 3 applies an exponential decline due to  $Z$  to the plus group in the same year and plus group. In season 3 an exponential population decline is applied to the plus group which is then moved to the next year but kept at age 15.

In order to apply the exponential decay due to total mortality ( $Z$ ), this parameter must first be calculated.  $Z$  is the sum of all fishing mortality for all fisheries that operate in the specific region, season and year plus the instantaneous rate of natural mortality ( $M$ ) divided by the

number of seasons (Equation 2.4). The calculation of fishing mortality (F) for all fisheries, except the bycatch from the shrimp fisheries, is the product of selectivity at age, apical fishing mortality, the probability that a fish dies once it is caught based upon the legal-harvest size limit for the fishery, an indicator variable that equals 1 if the fishery is operating during that season and equals 0 if the fishery is not operating during that season, and an indicator variable that equals 1 if the fishery is operating in a region and equals 0 if the fishery is not operating in that region, divided by the number of seasons that the fishery is active (Equation 2.5a). For the bycatch from the shrimp fisheries F is calculated by the product of selectivity at age, the shrimp fishing effort, an indicator variable that follows the rule if the fishery is operating during that season equals 1 otherwise equals 0, and an indicator variable that follows the rule if the fishery is operating in a region equals 1 otherwise equals 0, and then divided by the number of seasons that the fishery is active times the natural logarithm of the catchability coefficient (Equation 2.5b). The natural logarithm of the catchability coefficient was used because it showed better convergence in the Metropolis-Hastings algorithm to the posterior distribution of the parameters.

Selectivity at age was modeled the same as was done in the SEDAR update assessment (2009). For the commercial handline fishery in the eastern GOM, commercial handline fishery in the western GOM, bycatch during the closed-season fishery in the eastern GOM, and bycatch during the closed-season fishery in the western GOM, fisheries 1, 2, 7, and 8 respectively (Table 2.4), the selectivity-at-age-0 parameter was assumed to equal zero, the parameters for age 1 through 9 selectivity at age were estimated individually, and one selectivity-at-age parameter was assumed and estimated for age 10 through age 15. Commercial longline fishery in the east GOM and commercial longline fishery in the west GOM, respectively fisheries 3 and 4 (Table 2.4), modeled the selectivity-at-age parameters for age 0 and age 1 to equal zero, the selectivity-

at-age parameters for age 2 through 9 were estimated individually and one selectivity-at-age parameter for age 10 through age 15 was estimated. The selectivity-at-age-0 parameter was assumed to equal zero for the recreational fishery in the eastern GOM and recreational fishery in the western GOM, fisheries 5 and 6 respectively (Table 2.4). For these fisheries, selectivity-at-age parameters for age 1 through 8 were estimated as individual parameters and one selectivity-at-age parameter for age 9 through 15 was estimated. For the bycatch from the east and west shrimp fisheries, fisheries 9 and 10 respectively (Table 2.4), the selectivity-at-age parameters for age 3 and older were assumed to equal zero, selectivity-at-age parameters for age 0 and age 2 were estimated separately, and the selectivity-at-age-1 parameter was assumed to equal one for the majority of models. However, alternative assumptions were modeled and are described below for specific models. Selectivity at age for the NMFS bottom-longline survey, fishery 11 (Table 2.4), was assumed to follow a logistic function (Equation 2.6) where parameters  $\alpha$  and  $\beta$  were estimated by the model.

The probability that a fish would die once it was caught was modeled in the same fashion as the SEDAR update assessment (2009) and is based on the assumption that only fish that are less than the legal limit are discarded during the open season and all fish larger than the limit are harvested. Therefore,  $\zeta$  is calculated as 1 minus the product of the probability that a fish lives if it is caught (1 minus the discard mortality for the fishery) and the probability that a fish is less than the size limit ( $G_{L|a,s}$ ) (Equation 2.7). The values for the discard mortality rates were taken from the SEDAR update assessment (2009) of red snapper for each fishery. The discard mortality for the commercial handline and commercial longline fisheries was assumed to be 0.71 in the eastern GOM and 0.82 in the western GOM. For the recreational fisheries discard mortality was assumed to equal 0.15 in the eastern GOM and 0.40 in the western GOM. The differences in

discard mortality were due to differences in the assumed depths at which the fisheries caught red snapper and the assumed benefit of gas bladder venting required in the recreational fishery (SEDAR update assessment 2009). All other fisheries were modeled to harvest all fish and thus there were no discards. The probability that a fish is less than the size limit is calculated based off of the von Bertalanffy growth curve presented in Equation 2.8. The von Bertalanffy growth curve used in this study was not used in the assessment model of SEDAR 31 (2013) but was the most current growth model from the data workshop of SEDAR 31 at the time of this study. The probability that a fish is less than the size limit was calculated assuming a discretized gamma distribution scaled to a maximum of one following the methodology used in the update assessment CATCHEM model (Porch 2007).

Once the abundance in each age-class during all seasons has been calculated by the process equations described above, the catch is calculated. The catch equation represented by Equation 2.9 is the Baranov catch equation divided by the probability that the fish dies once it is caught. This formulation was used because the probability that the fish dies cancels out in the fishing mortality, but is still accounted for in the total mortality. Therefore, the value calculated is the total number of fish that are caught including fish that are discarded. Since landings data are the amount of harvested fish and does not include discards, the expected landings calculated by the model are the number of fish caught by a fishery times the probability that a fish is over the size limit given its age, for that fishery dependent upon the legal size-limit that year (Equation 2.10a). The directed commercial fisheries harvest data are recorded in weight of fish and therefore the number of fish caught are converted into weight of fish harvested (Equation 2.10b). This calculation was performed by multiplying the fishery catch in numbers by the probability that a fish is over the size limit given its age and the weight at age. The weight at age



in a season is calculated by converting the length at age from the von Bertalanffy growth curve using the length to weight conversion function (Equation 2.11). The length to weight conversion used in this study was not used in the assessment model of SEDAR 31 (2013) but was the most current weight model from the data workshop at the time of this study. The number of red snapper harvested at each age for the fisheries were then summed over age and season to give the harvest within a year.

Though not of direct interest in this study, the number of discards less than the size limit were estimated by multiplying the probability that a fish is less than the size limit given its age times the catch (Equation 2.12). Additionally, the number of dead discards for each fishery can be calculated by multiplying the number of discards by the probability a fish dies given it is discarded. Likewise, the total number of fish killed is calculated as the sum of the dead discards and the harvest for each fishery.

The calculations to derive the expected values of the CPUE indices depended upon the type of index being modeled. The fishery-dependent indices of abundance for the commercial handline fisheries in the east and west GOM incorporate number of fish in the stock, the probability that the fish is larger than the size limit, the weight at age, the selectivity at age of the fishery, the number of seasons the fishery is active during a year and a catchability coefficient (Equation 2.13a). The abundance index for the recreational fishery for the east and west GOM uses the estimated population size in number of fish, the selectivity at age of the fishery, the number of seasons the fishery is active in one year and a catchability coefficient (Equation 2.13b). The abundance index for the NMFS bottom-longline survey was calculated similar to the commercial handline indices but did not include the weight at age of the fish (Equation 2.13c). Fishery-independent abundance indices, except for the larval survey in the east and west GOM,

were calculated with the estimated population size of red snapper in number of fish, the assigned selectivity at age for the survey and the estimated catchability coefficient (Equation 2.14a). The larval abundance indices were calculated in a similar fashion as the other fishery-independent abundance indices but incorporated the relative fecundity at age as was used in the SEDAR update assessment (2009; Equation 2.14b). All surveys were summed over age and season to give a yearly estimate of the abundance index.

The final calculation required, before the statistical estimator could be used to compare the estimated values from the parameter estimates to the data, was the predicted proportion of each age-class in the harvest within a year for each fishery. This was calculated by taking the expected number of fish harvested by a fishery in a year of each age-class summed over seasons and dividing by the total number of fish harvested for the fishery that year summed over seasons and ages (Equation 2.15). This calculation was performed for each age-class for all years and fisheries from which the expected age compositions were created.

### ***Statistical Estimator***

A Bayesian analysis was performed to estimate the parameters in the model using the data described above. Ability to use an informative prior, to incorporate outside knowledge about model parameters not present in the data, was the primary impetus for using Bayesian methodologies. Additionally, there was concern that the complexity of the model would create difficulties in the optimization of the statistical estimator using maximum likelihood techniques. This analysis uses Bayes' theorem, which can be simplified to show that the posterior distribution conditional on the data is proportional to the likelihood of the data given the parameters times the prior distributions of the parameters as presented in Equation 3.1. This theorem was expanded to incorporate the likelihoods of the numerous data sources. Log-

likelihood functions assuming lognormal error were used to compare the observed and predicted values for harvest (Equation 3.2), abundance indices (Equation 3.4) and shrimp fishing effort (Equation 3.5). The variance used in the likelihood functions was calculated for each data point using Equation 3.3, where  $\sigma_H$  could be replaced by either  $\sigma_I$  or  $\sigma_e$ , for abundance indices or shrimp fishing effort respectively. The models assumed  $\sigma_H$ ,  $\sigma_I$  and  $\sigma_e$  were constants as specified in Table 2.1. The other likelihood function incorporated into this theorem assumed a multinomial distribution to compare the age-composition data to the calculated proportion at age from the parameter estimates (Equation 3.6). To prevent extreme variation in estimates of vulnerability (selectivity at age) between age-classes within a fishery, a curvature penalty was added to the likelihood functions (Equation 3.7). All log-likelihood functions were then summed together to give the total log-likelihood of the data conditional on the parameters (Equation 3.8).

### ***Priors***

To estimate the posterior distribution of the parameters in the model, a prior for each parameter was specified. All priors on parameters were uniform distributions. The upper and lower bounds corresponding to the parameters estimated are presented in Table 2.7. The lower bounds on the uniform priors for the parameters  $N_0$  were calculated in the following manner. The minimum number of red snapper caught at each age within a region as predicted by the SEDAR update assessment (2009) for the time period 1987 to 2008 were obtained. A fishing mortality of 5.0 and a natural mortality of 0.04 were assumed and the minimum catch-at-age values were incorporated into the Baranov catch equation (Equation 2.9 excluding  $\xi$ ). This equation was then solved for  $N$  to give an estimate of the minimum stock size required to support those catches. The upper bounds on the uniform priors for the parameters  $N_0$  were calculated in the following manner. The maximum number of red snapper caught for each age within a region

as predicted by the SEDAR update assessment (2009) of red snapper for the time period 1987 to 2008 were obtained. A fishing mortality of 0.05 and a natural mortality of 0.08 were assumed and the maximum catch-at-age values were integrated into the Baranov catch equation (Equation 2.9 if excluding  $\zeta$ ). The equation was solved for N to give an estimate of the maximum stock size that would create those catches. The natural logarithms of the priors were then added to the log-likelihood of the data conditional on the parameters to get the posterior distribution.

An additional set of priors that were created to be more informative to the models were tested to determine the influence of priors on the parameter estimates. The uniform distribution priors used for all parameters is labeled “Wide priors”, and those with normal distributions on natural mortality and tighter constraints on the uniform priors is labeled “Tight priors” in Table 2.7. The tight priors were constructed based upon the results of the SEDAR update assessment (2009) to give realistic bounds on the recruitment and fishing mortality. The bounds on the initial abundance at age ( $N_{0a}$ ) were assumed to be informative in both scenarios to assist with convergence and were the same between the two prior scenarios.

### ***Metropolis-Hastings Algorithm***

A Metropolis-Hastings algorithm was used in conjunction with the statistical estimator described above to approximate the posterior distribution of the parameters using a Markov Chain Monte Carlo (MCMC). A MCMC is an algorithm for sampling from a posterior distribution by taking successive samples from a target distribution. Each sample is dependent upon the previous sample, which is one of the properties of a Markov chain. A Metropolis-Hastings algorithm uses a proposal function that proposes the next value of the MCMC based upon the current value and is then either accepted or rejected based upon its likelihood relative to the current value. The difference between the likelihood at the proposed value and the likelihood

at the current value was calculated. A random number between 0 and 1 was generated and the log of the value was taken. The difference in likelihoods was compared to the log random number. If the value of the difference in likelihoods was greater than the random number then the proposed value was accepted as the next value in the MCMC. On the other hand, if the value of the difference in likelihoods was less than the random number then the proposed value was rejected and the previous value in the MCMC was used as the next value in the MCMC. A proposal function involving all parameters resulted in an extremely low acceptance rate of proposed values and therefore was not used. Likewise, using a proposal for all parameters separately greatly increased the computation time for samples. Therefore, nine proposal functions were used which grouped similar parameters together. A normally distributed random number was added to the current value of the respective parameters within the proposal function, while all other parameters were kept at the initial value. The variance of the normal distribution was calculated as the percentage of the current parameter value, where the percentage was tuned for each proposal distribution to give an average acceptance rate between approximately 20% and 40%.

The specific parameters within each proposal function in the Metropolis-Hastings algorithm are described below. The first proposal function included all natural mortality parameters used by the model. The second proposal function was composed of recruitment ( $R$ ) for each year and the initial abundance ( $N_0$ ) for each age-class in both regions of the GOM. Apical fishing mortality and selectivity-at-age parameters for the commercial handline fisheries in the east and west GOM were associated with the third proposal function. The fourth proposal function included the apical fishing mortality parameters and selectivity-at-age parameters for the commercial longline fisheries in the east and west GOM. Parameters of apical fishing

mortality and selectivity at age for the recreational fisheries, bycatch during the closed-season fisheries, bycatch from the shrimp fisheries, and NMFS bottom-longline fishery were associated with fifth through eighth proposal functions, respectively. The seventh proposal function also included the catchability coefficients for bycatch from the shrimp fisheries. The last proposal function included the catchability coefficients for the abundance indices. These proposal functions were used to estimate the posterior distributions of each parameter through Metropolis-Hastings methods.

Three chains were used to assist with visually testing if the posterior distribution had been converged upon through sampling. One of the three chains started at the values estimated in the SEDAR update assessment (2009), while the other two chains were started by adding a random variable from a normal distribution with a standard deviation of ten percent of the parameter estimates to the aforementioned estimates. All models were tested for 500,000 samples before the first sample was saved (burnin), the first of every 25 samples was saved (thinning interval) and at least 10,000 samples were saved. Visual diagnostics of the trace plots for saved posterior values were conducted for each model to determine if convergence to the posterior distribution were possible. Additionally, Gelman and Rubin diagnostic test were performed on the 3 chains of each parameter to determine if convergence to the posterior distributions were likely.

### ***Model Descriptions***

The model equations described above explain the methodology similar between all models used to estimate parameters to fit to the data. The following section describes the differences between the models investigated. Table 2.6 indicates the corresponding mathematical

representations of the modeling assumptions made by each model, where the row at the top indicates the number of the equation found in Table 2.5.

To determine if natural mortality could be estimated in any capacity for GOM red snapper data, a simple model was created assuming natural mortality was constant over all ages (Equation 5.1). This model also assumed that selectivity-at-age parameters for similar fisheries were the same for eastern and western GOM fisheries of the same type (Equation 5.5), e.g. recreational fishery in the east GOM and recreational fishery in the west GOM were the same. The third constraint of the first model investigated was that selectivity-at-age-1 parameter for bycatch from the shrimp fisheries was fixed at one (Equation 5.7). This model will later be referred to as the following: the model that estimated constant mortality over all ages.

The SEDAR update assessment (2009) and SEDAR 31 (2013) had identified the rate of natural mortality for age-0 and age-1 red snapper as parameter in the model that strongly affected the results of the assessments. Therefore, the second model assumed three separate natural mortality values (Equation 5.2): natural mortality at age 0 ( $M_0$ ), natural mortality at age 1 ( $M_1$ ), and natural mortality at age 2 and older ( $M_{2+}$ ). The second model also assumed the selectivity-at-age parameters were the same for similar type fisheries in the eastern and western GOM (Equation 5.5), and the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries was fixed at unity (Equation 5.7). The parameters of this model were estimated using four different prior distributions on the three  $M$  parameters. The first prior tested assumed uniform distribution with wide bounds on the natural mortality parameter and other parameters (Table 2.7). This model formulation will be referred to as: the model that assumed three  $M$  parameters. The second prior tested once again modeled uniform distributions but the lower bound of the  $M_1$  parameter was assumed to equal 0.5 and assumed the wide priors on all other parameters. This

model formulation will be referred to as: the model that assumed three M parameters with the lower bound of M1 prior equal to 0.5. The third prior tested incorporated normal distributions centered at the high values of natural mortality assumed in SEDAR 31 (2013), but assumed the tight uniform priors for other distributions, and will be referred to as: the model that assumed three M parameters with normal priors at high values. The last prior tested for this model created the prior as normal distribution with a mean at the low values of natural mortality assumed in the SEDAR update assessment (2009), but assumed the tight uniform priors on all other parameters, and will be referred to as: the model that assumed three M parameters with normal priors at low values.

A Lorenzen natural mortality curve was then assumed in an attempt to create a model that assumed a higher natural mortality for age 0 and age 1, and was estimable by the model. The Lorenzen M function (Equation 5.3) was created by dividing the estimated parameter by the length at age for red snapper calculated by Equation 2.8. The weight at age for red snapper is approximately the length at age cubed and the estimated exponent of the weight used in the function proposed by Lorenzen (1996) is approximately equal to one third. Therefore, the estimated parameter divided by the length at age was used instead of the function proposed by Lorenzen (1996) to create a Lorenzen style natural mortality curve. This model also assumed the selectivity-at-age parameters were the same for similar eastern and western GOM fisheries (Equation 5.5), and the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries equaled one (Equation 5.7). This model will be referred to as: the model that assumed a Lorenzen M.

Additional complexity was added to the Lorenzen M model through the estimation of separate selectivity-at-age parameters for fisheries in the eastern and western GOM that had



previously been mirrored. The model that assumed separate selectivity-at-age parameters between eastern and western GOM fisheries integrated the same available data but estimated 40 more parameters compared to the model that assumed a Lorenzen M. This model assumed a Lorenzen M and estimated separate selectivity-at-age parameters for eastern and western GOM fisheries (Equation 5.6) but still assumed the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries equaled one (Equation 5.7). This model will be referred to as: the model that estimated separate selectivity-at-age parameters.

An additional model was created that assumed natural mortality was different between the eastern and western GOM and estimated separate selectivity-at-age parameters for eastern and western GO fisheries. This model estimated a separate Lorenzen M curve for each region, (Equation 5.4), and assumed the selectivity-at-age parameters were estimated separately for eastern and western GOM fisheries (Equation 5.6), and the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries equaled 1.0 (Equation 5.7). This model will be referred to as: the model that estimated separate selectivity-at-age parameters and separate Lorenzen M parameters.

A model was created that assumed separate Lorenzen M parameters for each region of the GOM, but assumed the selectivity-at-age parameters were mirrored across eastern and western GOM fisheries. This model assumed different Lorenzen natural mortality curves for the eastern and western GOM (Equation 5.4), but assumed selectivity was mirrored for similar fisheries (Equation 5.4), and the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries were fixed to equal one (Equation 5.7). This model will be referred to as: the model that estimated separate Lorenzen M parameters.

In the assessment model of SEDAR 31 (2013) the selectivity-at-age parameters for bycatch from the shrimp fisheries were estimated for all age-classes. This was very different from the previous assessment which had assumed that age 1 was fully selected by the gear based upon cod-end trawl selectivity experiments. Additionally, it was hypothesized that the selectivity-at-age parameters for bycatch from the shrimp fisheries may be correlated with the estimation of  $M_0$  and  $M_1$  because the only age-composition data available for age 0 and age 1 were from these fisheries. To investigate these concerns a model was created by setting the upper bound of the priors for the selectivity-at-age parameters for bycatch from the shrimp fisheries equal to two to allow the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries to be scaled to less than one. This model, like the others before it assumed the selectivity-at-age-1 parameter equaled one for bycatch from the shrimp fisheries; however this value was scaled to less than one if any other selectivity-at-age parameters were estimated to be greater than 1. The following calculation was performed for all iterations of the MCMC chain: if either values of the selectivity-at-age parameters for age 0 or age 2 were greater than one, then all selectivity values, including the age 1 value equal to one, were scaled by the maximum proposed selectivity value from the proposal function. However, if neither proposed value for the selectivity-at-age parameters for age 0 or age 2 were greater than one, then the values were not scaled and the selectivity-at-age-1 parameter equaled one. This model also assumed a Lorenzen  $M$  (Equation 5.3) and the selectivity-at-age parameters were the same for eastern and western GOM fisheries (Equation 5.5). The set of informative priors was not tested on this model because this was an intermediate model in the investigation of this study between the model that assumed a Lorenzen  $M$  and the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen  $M$  parameter. Additionally, it was believe that the model was unlikely

to be used in an assessment. This model will be referred to as: the model that assumed the upper bound of the priors on the selectivity-at-age parameters equaled two for bycatch from the shrimp fishery.

A new model was created to get a clearer picture of how the estimation of the selectivity-at-age parameter for bycatch from the shrimp fisheries at age 1 and natural mortality estimation interact within the model. This model estimated a selectivity-at-age-1 parameter for bycatch from the shrimp fisheries as an independent parameter (Equation 5.8). The model also assumed one Lorenzen M curve (Equation 5.3) and the selectivity-at-age parameters were the same for eastern and western GOM fisheries (Equation 5.5). This model will be referred to as: the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen M parameter.

To determine if the estimation of natural mortality at specific ages was influenced by estimating the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries, another model was created with a different assumption of natural mortality. The model assumed an M0, an M1 and M2+ parameters (Equation 5.2), the selectivity-at-age parameters were the same for the east and west GOM fisheries (Equation 5.5), and estimated a selectivity-at-age-1 parameter for bycatch from the shrimp fisheries (Equation 5.8). This model will be referred to as: the model that estimated selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and three M parameters.

Two models were created to compare the results of the other model to situations when the natural mortality values were fixed. Both models assumed the selectivity-at-age parameters for similar fisheries in the eastern and western GOM were mirrored and the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries was fixed to equal one. One model assumed the

high rates of natural mortality for age 0 and age 1 from the SEDAR 31 (2013), where  $M_0$  equaled 2.0,  $M_1$  equaled 1.2 and  $M_{2+}$  equaled 0.1. This model will be referred to as: the model with fixed  $M$  parameters at high values. The other model created assumed the lower rates of natural mortality from the SEDAR update assessment (2009), where  $M_0$  equaled 1.0,  $M_1$  equaled 0.6 and  $M_{2+}$  equaled 0.1 and will be referred to as: the model with fixed  $M$  parameters at low values. All models estimated the posterior distributions of their respective parameters using the equations, statistical operators, and Metropolis-Hastings methodology described above.

## **Results**

### ***Uniform Priors Constraining Parameter Estimates***

The estimated values of some parameters for multiple models were nearly equal to the bounds of the uniform priors. The estimates for the number of fish in each age-class for the first year for all models were constrained by either the upper or lower bounds of the prior, depending upon the specific parameter. The estimation of all initial abundance-at-age ( $N_0$ ) parameters for all models was problematic using the Bayesian methods described. None of the  $N_0$  parameters could safely be assumed to have converged to their true posteriors for various reasons. Either the trace plot of the parameter showed an increasing trend or decreasing trend or both, or the trace plot showed the parameter estimate to lie on one of the bounds of the prior (Figure 2.2; Appendix A). Broader bounds on the priors were tested but usually resulted in the trace plots of parameters with a highly variable random walk between the bounds. Increasing the number of samples removed before the first saved value (burnin) and sample size of the saved values were tested to see if convergence to the posterior were likely but resulted in similar trace plots. An alternative model was created that started in 1991 coinciding with the start of available age-composition data, because it was assumed that these data would provide more information to the model to

estimate the  $N_0$  parameters. This model also had difficulty estimating the initial abundance at age of the population for both stocks. In general, the estimation of the  $N_0$  parameters was problematic.

The  $N_0$  parameters were not the only values which were close to the bounds of the priors in some models. The models that estimated  $M_1$  as an independent parameter resulted in mean posterior values of this parameter approximately equal to the lower bound of the uniform prior distribution (Table 2.8 and 2.9; Figure 2.3 and 2.13). The model that estimated a selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen  $M$  with wide priors, estimated the natural mortality parameter only slightly larger than the lower bound, due to a peak in this parameter's trace plot raising the average value slightly above the lower bound of the prior (Figure 2.1 and 2.3; Table 2.8). The model that estimated selectivity-at-age-1 parameter for bycatch from the shrimp fishery and three  $M$  parameters with both wide and tight priors had a parameter estimate for  $M_0$  that approximated the lower bound of the prior (Table 2.8 and 2.9).

Trends were not observed in the trace plots for most parameters in the models and good mixing between the three chains were observed for most parameters. The trace plots of the deviance calculated for each sample were also monitored for trends in the likelihood and for good mixing between chains. However, the lack of convergence of the  $N_0$  parameters in all models invokes doubt that the posterior of the model has been reached in any of the models.

### ***Gelman Rubin Diagnostics***

The results of the Gelman Rubin diagnostic in general were less than 1.1, with the exception of a few initial  $N_0$  parameters in nearly all models. Gelman Rubin test statistics for models that estimated parameters with tight priors were all less than 1.2 with the exception of the model that estimated separate selectivity-at-age parameters and separate Lorenzen  $M$  parameters.

In this model for parameters NOE and selectivity at age for fishery 5 the Gelman Rubin test statistics were greater than 1.2. All models with tight priors had Gelman Rubin test statistics greater than 1.1 for at least one  $N_0$  parameter either in the eastern or western GOM. The two models with fixed Ms also had Gelman Rubin test statistics greater than 1.1 for catchability coefficients of surveys.

On the other hand, there were a few models with wide priors that had Gelman Rubin test statistics greater than 1.1 for parameters other than NOE and NOW. The Gelman Rubin diagnostic for the model that estimated separate selectivity-at-age parameters with wide priors had a value greater than 1.1 for many parameters including recruitment in the western GOM, apical fishing mortality for fisheries 1, 2, 3, 4, 6, 8, and 11, catchability coefficient for bycatch from the shrimp fisheries in the western GOM and the selectivity-at-age parameter for the oldest age-class for fisheries 1 and 3. However, no values of the Gelman Rubin diagnostic were greater than 1.2 for the model that estimated separate selectivity-at-age parameters.

The model that estimated separate selectivity-at-age parameters and separate Lorenzen M parameters with wide priors had values of Gelman Rubin test statistic greater than 1.1 for parameters of  $N_0$  in the east and west GOM, recruitment in the eastern (RE) GOM, apical fishing mortality for fisheries 1, 3, 5, and 7, selectivity-at-age parameters for fisheries 3 and 7, and the catchability coefficient for bycatch from the shrimp fisheries in the eastern GOM. Gelman Rubin test statistics were greater than 1.2 only for one parameter for NOE and two selectivity-at-age parameters for fishery 7. In general it appears that the model that estimated separate selectivity-at-age parameters and separate Lorenzen M parameters had difficulty estimating parameters from the eastern GOM for this model.

The model that assumed three M parameters with the lower bound of M1 prior equal to 0.5 also appeared to have some difficulty estimating parameters for the eastern GOM including NOE, RE, apical fishing mortality for fisheries 1, 3, 5 and 7 and selectivity for the oldest ages for fisheries 3 and 4 combined with Gelman Rubin test statistics greater than 1.1. However for this model only one NOE parameter had Gelman Rubin test statistic greater than 1.2. The MCMC chains were continued using the Metropolis-Hastings algorithm and additional samples were added to the original samples to assist convergence but similar results were observed.

The model that assumed the upper bound of the priors on the selectivity-at-age parameters equaled two for bycatch from the shrimp fisheries had Gelman Rubin test statistics greater than 1.1 for NOE, NOW, RE, apical fishing mortality for fisheries 1, 3, 5, and 7 and catchability coefficient for bycatch from the shrimp fishery in the eastern GOM. Unlike other models this model had Gelman Rubin test statistics greater than 1.2 for many parameters of N0, RE and apical fishing mortality for fisheries 1 and 3. Convergence of this model was not pursued more because it served as an intermediate model in the investigation of the influence of the selectivity-at-age parameters for bycatch from the shrimp fisheries on the estimation of natural mortality. With the exception of a few N0 parameters in each model, all other models with wide priors had Gelman Rubin test statistics that were less than 1.1 for most parameters,

### ***Natural Mortality Estimated Values***

The estimated values of natural mortality parameters varied greatly between models (Table 2.8 and 2.9). The model that estimated constant mortality over all ages with wide priors gave promising results that natural mortality could be estimated for GOM red snapper because the estimated M was close to the assumed value of 0.10 in the update assessment for ages 2 and older and the estimate was not influenced by the bounds on the uniform prior. The estimate of

natural mortality for the model that estimated constant mortality over all ages with the tight priors had a fairly similar estimate of natural mortality with a difference between the two estimates of 0.0237.

The model that assumed three M parameters with wide priors had difficulties estimating M1 and the trace plot for this parameter lay directly upon the lower bound of 0.01. The posterior samples for this model showed very little correlation between M0, M1, selectivity-at-age-0 parameter for bycatch from the shrimp fisheries, and catchability coefficients for bycatch from the shrimp fisheries. However, M0 was moderately correlated with the selectivity-at-age-2 parameter for bycatch from the shrimp fisheries (0.5984).

The model that assumed three M parameters with normal priors at high values estimated the values of natural mortality to be very different. The value of M0 was less than when run with wide priors, M1 was a more reasonable estimated value of 0.7688 and M2+ was nearly doubled the value estimated by the wide prior (Table 2.8). The values of the posterior distribution for M0 and M1 were found to be strongly negatively correlated to one another (Table 2.10). The M0 parameter was also strongly correlated to other parameters, namely negatively correlated with the selectivity-at-age-0 parameter for bycatch from the shrimp fisheries (Table 2.10) and positively correlated with catchability coefficients for bycatch from the shrimp fisheries in the east and west GOM (0.6335 and 0.6681, respectively). The M1 parameter was found to be positively correlated with the selectivity-at-age-0 parameter for bycatch from the shrimp fisheries and negatively correlated with catchability coefficients for bycatch from the shrimp fisheries in the east and west GOM (Table 2.10). Interestingly, the estimate of M2+ was found to be positively correlated to 3 years of the apical fishing mortality estimates for the NMFS bottom-longline survey (approximately 0.5).



The model that assumed three M parameters with normal priors at high values resulted in interesting results compared to the model that assumed three M parameters with normal priors at low values. The estimated value of M0 was higher for when the prior on the parameter was lower than when ran with a higher prior, contrary to what would be expected. The estimated value of M1 for the model that assumed three M parameters with normal priors at low values was slightly lower than the model that assumed three M parameters with normal priors at high values. However, the estimates of M2+ between the two models were very similar. Similar correlation of parameter estimates observed for the model that assumed three M parameters with normal priors at high values was observed for the model that assumed three M parameters with normal priors at low values.

The estimate of M1 for the model that assumed three M parameters with the lower bound of M1 prior equal to 0.5 resulted in a trace plot on the new lower bound of the prior and thus an estimate close to 0.5. Other models, not presented here were created, with a lower bound of 0.0001 and 1, respectively, to see if M1 would be estimated by the value of the lower bound of the prior. Though convergence of these test models was not assured due to trends in the trace plots of other parameters, the trace plots of M1 were all directly upon the lower bound of the prior.

The model that assumed a Lorenzen M with both wide and tight priors predicted a value for M that did not interact with the bounds on the uniform prior and gave fairly realistic estimates of natural mortality for various ages (Table 2.8 and 2.9; Figure 2.3). The estimate of M0 in the first season based upon the Lorenzen M curve is consistent with the assumed value for natural mortality of 2 at age 0 in SEDAR 31 (2013; Table 2.8 and 2.9). The model also predicted M for the plus group close to the assumed value of 0.10 that was used in the SEDAR update

assessment (2009; Table 2.8 and 2.9). On the other hand, the model that assumed a Lorenzen M with wide priors estimated M1 to equal 0.4698, which is lower than the SEDAR 31 (2013) assumed value for M1 equal to 1.2. The model that assumed a Lorenzen M with tight priors estimated the Lorenzen M value similar to the same model with wide priors but increased by 0.2389 (Table 2.9).

The Lorenzen M value for the model that estimated separate selectivity-at-age parameters with tight priors was very slightly higher than the estimate for when it was run with wide priors (Table 2.8 and 2.9). Additionally, these two models had similar estimated values compared to the model that assumed a Lorenzen M with tight priors. Natural mortality in the eastern GOM was estimated to be much lower for the model that estimated separate selectivity-at-age parameters and separate Lorenzen M parameters with tight priors compared to the same model with wide priors. Additionally, the model that estimated separate selectivity-at-age parameters and separate Lorenzen M parameters with tight priors estimated natural mortality in the western GOM slightly lower compared to the same model with wide priors (Table 2.8 and 2.9). The parameter estimates of the posterior showed correlation of the Lorenzen M value for each respective region to be highly positively correlated with recruitment and moderately correlated with the catchability coefficients for bycatch from the shrimp fisheries. Additionally, the yearly estimates of recruitment in the eastern and western GOM showed strong positive correlation with other recruitment parameters in the same region as well as the other region.

Models that assumed a Lorenzen natural mortality curve generally had estimates that are close to each other (Table 2.8 and 2.9). However, when the selectivity-at-age-1 parameter was estimated as a free parameter for bycatch from the shrimp fisheries the estimated value of the Lorenzen M changed drastically (Figure 2.3 and 2.13). The model that estimated the selectivity-

at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen M parameter with wide priors resulted in an estimate of the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries of approximately 0.33 (Figure 2.4) and a Lorenzen M trace plot that mainly resided upon the lower bound of the prior, though a few peaks were observed (Figure 2.1). These peaks bring doubt as to whether the posterior distribution has actually been reached for the Lorenzen M curve parameter, but are likely due to the constraining nature of the bounds of the prior.

The influence of the prior on estimation of natural mortality for the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen M parameter is clear when comparing the estimate for the model with tight priors and the model with wide priors (Table 2.8 and 2.9; Figure 2.3 and 2.13). The Lorenzen M value of the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen M parameter with tight priors resulted in a value that is very similar to the estimate from the model that assumed a Lorenzen M with tight priors. On the other hand, the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen M parameter with wide priors estimated the Lorenzen M value that was approximately equal to the lower bounds of the prior. Correlation of posterior samples between the Lorenzen M parameter and catchability coefficients for bycatch from the shrimp fisheries were relatively low (-0.3618 and -0.3389, respectively), between Lorenzen M and yearly recruitment parameters in the east and west was moderate (0.56 to 0.78) and Lorenzen M was moderately negatively correlated with the selectivity-at-age-0 parameter for bycatch from the shrimp fisheries (-0.4899).

Like previous models that attempted to estimate natural mortality at age 1 as a separate parameter, the model that estimated selectivity-at-age-1 parameter for bycatch from the shrimp

fisheries and three M parameters had difficulty estimating this parameter and resulted in an estimate close to the lower bound of the prior for the wide priors (Table 2.8). However, this model with tight priors on M1 resulted in a very high value of 1.85 that was strongly influenced by the prior (Table 2.9; Figure 2.13). The estimated value of natural mortality at age 0 was approximately equal to the lower bound of the prior for the wide and tight priors (Table 2.8; Table 2.9). This result is similar to the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen M parameter estimated the Lorenzen natural mortality parameter to be approximately equal to the lower bound of the prior. However, natural mortality for ages 2 and older was estimated to equal 0.0545 for the model that estimated selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and three M parameters. This value of M2+ is nearly half of the estimated values from other models (Table 2.8 and 2.9). For the model that estimated selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and three M parameters with tight priors, the posterior samples of the M1 parameters were negatively correlated with the catchability coefficients for bycatch from the shrimp fisheries in the eastern and western GOM (-0.5810 and -0.06105, respectively). However, the posterior estimates of natural mortality for the model that estimated selectivity-at-age-1 parameter for the bycatch from the shrimp fisheries and three M parameters and the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen M parameter were not strongly correlated with selectivity-at-age parameters for the bycatch from the shrimp fisheries, unlike the other models.

### ***Selectivity at Age for Bycatch from the Shrimp Fisheries***

The estimated value of selectivity-at-age parameters for age 0 and age 2 for bycatch from the shrimp fisheries were fairly consistent between models when age 1 was assumed to be fully

selected (Figure 2.4 and 2.14). However, when a selectivity-at-age-1 parameter was estimated the shape of the selectivity curve changed drastically (Figure 2.4 and 2.14). Additionally, the estimation of the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries greatly changed the estimates of natural mortality within the models. The model that assumed the upper bound of the priors on the selectivity-at-age parameters equaled two for bycatch from the shrimp fisheries resulted in the trace plot for the selectivity-at-age-0 parameter for bycatch from the shrimp fisheries to reside on the upper bound of the prior equal to two; therefore the selectivity-at-age-1 parameter was estimated to be near 0.5 (Figure 2.4). With this large decrease in the value of the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries for the model that assumed the upper bound of the priors on the selectivity-at-age parameters equaled two for bycatch from the shrimp fisheries, there was also a large decrease in the estimated Lorenzen M parameter value (Table 2.8; Figure 2.3).

A change in the shape of the selectivity curve for the bycatch from the shrimp fisheries compared to other models was noted for the models that fixed M parameters at high and low values. The model that fixed M parameters at high values had the lowest estimated value for the selectivity-at-age-0 parameter for the bycatch from the shrimp fisheries. Additionally, the estimated value for the selectivity-at-age-2 parameter for the bycatch from the shrimp fisheries was higher compared to all other models. For the model that fixed M parameters at low values, the estimated selectivity-at-age-0 parameter for the bycatch from the shrimp fisheries was approximately 0.85, which was less than the estimated values by other models. The estimated value of selectivity-at-age parameters for the bycatch from the shrimp fisheries changed with changes in the estimated values of natural mortality parameters.

An interesting phenomenon was observed in the estimation of the selectivity-at-age-2 parameter for bycatch from the shrimp fisheries between the models. The combined value estimated in the model that assumed a Lorenzen M equaled 0.0256 but when estimated separately for the eastern and western fisheries resulted in values of 0.0003 and 0.0109, respectively, for the model that estimated separate selectivity-at-age parameters. The model that estimated separate selectivity-at-age parameters and separate Lorenzen M parameters estimated the selectivity-at-age-2 parameter for bycatch from the shrimp fisheries for the eastern and western GOM to equal 0.070 and 0.0001, respectively. The estimate of the selectivity-at-age-2 parameter for bycatch from the shrimp fishery was influenced by the lower bound for the eastern GOM fishery in the model that estimated separate selectivity-at-age parameters. Conversely, the model that estimated separate selectivity-at-age parameters and separate Lorenzen M parameters had a selectivity-at-age-2 parameter for bycatch from the shrimp fishery that was influenced by the lower bound for the eastern GOM fishery.

### ***Comparison of Abundance for Ages 2 and Older***

The estimated abundance of red snapper for ages 2 and older were summed together and plotted for each model and compared to the estimate from the SEDAR update assessment (2009). The parameter estimates for the models with the wide priors resulted in estimated abundances what were two orders of magnitude larger than the estimated values by the SEDAR update assessment (2009; Figure 2.9 and 2.10). The estimated abundances by models ran with tight priors compared to the SEDAR update assessment (2009) values were much more reasonable (Figure 2.11 and 2.12). However, the estimated value of many of the abundance indices for these models showed there was a tendency to overestimate the abundance compared to the observed values. Additionally, the catchability coefficients for some models interacted with the upper

bound of the prior when run with tight priors, specifically the age-1 and age-0 surveys in the eastern GOM. The estimates of catchability coefficients for the MRFSS survey from the recreational fishery in the eastern and western GOM were close to the bounds of the prior but did not appear to be strongly constrained by the upper bound. The scale of the abundance and thus fishing mortality appears to be difficult to estimate using these models.

## **Discussion**

### ***Inestimable Nature of Initial Abundance-at-Age Parameters***

The objective of this project was to determine the estimability of natural mortality within a statistical catch-at-age model, but estimability of other parameters is also important. All models investigated had difficulty estimating the initial abundance at age (N0) parameters and estimates were often approximately equal to the bounds of the specified prior. The N0 parameters appear to be inestimable based upon the current configuration of the model. There are many possibilities that could be leading to this issue. First, the age-composition data for red snapper in the Gulf of Mexico do not begin until 1991, four years after the start of the model. However, even when the model was started in 1991 the estimability of the parameters did not improve. Second, the complexity of the model may be causing the inestimable nature of these parameters, especially with ages 2 and older where four fisheries are interacting on the same age-classes each with different age-composition data. Third, the complexity of the model may be creating complications. Specifically, incorporating three seasons into each year with spawning occurring in the second season is likely having an effect on the estimability of the initial abundance at age parameters. For the first season of the first year in the model, the abundance at age parameter is constructed to be in the third season of life. Therefore, during the second season of the model the red snapper would then be transition to the next age-class. For example, the  $N0E_1$  parameter is

the abundance of one year olds in their third season of life and thus would become age 2 in the second season of the model. Thus the initial abundance at age is experiencing the fishing mortality at age 1 and age 2 during the first year of the model and may be causing the problems with estimating the  $N_0$  parameters. This methodology is consistent with the model used in the SEDAR update assessment (2009), but was likely not an issue there because the recruitment and initial abundance of the model in 1872 was fixed at 70 million recruits for the initial 100 years. It appears that the overall complexity of the model created a scenario in which the data are not able to provide information to estimate these parameters in a reliable fashion. Finally, the initial abundance-at-age parameters may also have difficulty converging because of the model's overall difficulty in estimating the scale of abundance and recruitment compared to the SEDAR update assessment (2009).

However, the constraining influence of the priors on the initial abundance-at-age parameters elicits suspicion that convergence to the posterior distribution was not obtained for the models presented. Therefore, the models cannot be assumed to give unaffected parameter estimates and comparison between models is likely not valid. Additionally, conclusions made regarding these models are not straightforward. Consequently the results of this study provide only preliminary deductions that could potentially change with future research and additional model evaluation.

### ***Catchability Coefficient Estimation***

All models presented had difficulty in estimating the relative scale of the abundance of the population and thus influenced the estimates of fishing mortality. The SEDAR update assessment (2009) used indices of abundance that were scaled so that the estimated value of the catchability coefficient would be close to 1.0, to aid in the estimability of these parameters. Yet



the catchability coefficients were estimated to be no more than 0.6 by the model. The models presented had difficulty estimating the catchability coefficients in this manner, and therefore the lognormal abundance indices were modeled. The upper bound on the catchability coefficients of -0.01 (corresponding to a value of 0.99 on normal scale) was thought by the authors to be sufficiently high to accommodate the survey estimates. However, based upon the poor fit to the survey data and the parameter estimates being nearly equal to the upper bounds of the priors for the age-1 and age-0 trawl survey, the catchability coefficient on a normal scale may have been greater than 1.0 for these surveys. It appears that the log-transformations of these data and the catchability parameters are causing the difficulty in estimating the scale of abundance. To fit the surveys with catchability coefficients constrained by the bounds of the priors, namely the age-0 and age-1 surveys, the scale of recruitment was driven higher. As a result of these higher recruitment values, the estimates of fishing mortality were reduced to lower values than expected to fit to the catch data. Despite these changes in fishing mortality parameters, a drastic change in the estimates of other parameters were not noted, specifically natural mortality. Additionally, the fit to the catch data was approximately the same between the models with wide and tight priors, though the scale of recruitment and fishing mortality were largely different (Figure 2.5 and 2.15). Additionally, a better fit to the survey data can be seen when comparing the fits with the wide priors on the recruitment parameters compared to the tight priors (Figure 2.8 and 2.17). Often the catchability coefficients are referred to as nuisance parameters and are not deemed very important to the model. However, in this scenario the catchability coefficients caused large changes in the estimates due to the priors on the parameters and modeling the abundance indices and catchability coefficients on the log-scale.

### ***Apical Fishing Mortality Parameter Estimation***

Natural mortality and fishing mortality are often assumed to be negatively correlated. However, none of the posterior estimates of fishing mortality or natural mortality parameters were strongly negatively correlated. On the other hand, the apical fishing mortality parameters for the NMFS bottom-longline survey were positively correlated to some Lorenzen  $M$  values or  $M_{2+}$  parameters. Even with the drastic change in fishing mortality estimated when using different priors in the same model, the estimates of natural mortality were relatively stable for the model that estimated constant mortality over all ages, the model that assumed a Lorenzen  $M$ , the model that estimated separate selectivity-at-age parameters, and the model that estimated separate selectivity-at-age parameters and separate Lorenzen  $M$  parameters. It appears that natural mortality is not as correlated to the fishing mortality as might be expected. However, the estimation of natural mortality is greatly influenced by the selectivity-at-age parameters; in these models specifically, the estimation of natural mortality and selectivity-at-age parameters for bycatch from the shrimp fisheries are strongly interrelated.

### ***Lorenzen $M$ Parameter Estimation***

Since the Lorenzen curve resulted in natural mortality estimates near those assumed in SEDAR 31 (2013) for both age 0 and ages 2 and older, it is difficult to determine which data are contributing to the estimation of the Lorenzen mortality parameter without additional analysis. However, the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen  $M$  parameter and the model that estimated selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and three  $M$  parameters gave some interesting insight into the estimation of the parameter in the Lorenzen curve. The results of the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen  $M$  parameter showed that a Lorenzen curve natural mortality parameter was

constrained by the lower bound of the prior. On the other hand, the model that estimated selectivity-at-age-1 parameter for the bycatch from the shrimp fisheries and three M parameters shows that age-0 and age-1 natural mortality parameter estimates would reside on the lower bounds of the priors but natural mortality for age 2 and older would still provide an estimate different from the bounds of the priors. Thus it appears that the Lorenzen curve parameter may be influenced most strongly by the age-composition data for bycatch from the shrimp fisheries compared to the other fisheries. On the other hand, the peak observed in the trace plot of natural mortality for this parameter in the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen M parameter (Figure 2.1) and the potential for lack of convergence to the posterior may be due to the influence of the other age-composition data sources. Alternatively, some data in the abundance indices may be supporting the estimation of natural mortality, but the weight given to these data sources is minimal due to the assumed high variance and relative weight of the age-composition data. Therefore, the abundance indices data are not sufficient to change the estimate of natural mortality compared to the relative weight imposed by the age-composition data and the fit of the selectivity parameters. Investigation into changing the assumed variance of the abundance indices and the sample size input of the age composition and assumed multinomial distribution is recommended in the future. An additional concern raised by the results of some models suggests that the multinomial distribution used to fit the age composition may be sensitive to the age composition data for bycatch from the shrimp fisheries. The sensitivity to a few extreme data points with a very large change in the estimated parameters of the model invokes questions as to whether the multinomial distribution assumed in the models was suitable. Investigation into more robust multinomial distributions is recommended for comparing fit to the data between models and could potentially change the

estimate of some model parameters. Yet it is apparent that the estimation of the Lorenzen curve parameter is most strongly influenced by the estimation of natural mortality at age 0 as opposed to the other ages and data sources. Therefore, the estimation of a Lorenzen-curve natural mortality should be used with caution and the influence of each source of age-composition data should be specifically tested within each model.

The model that estimated separate selectivity-at-age parameters and separate Lorenzen M parameters with wide priors estimated two separate Lorenzen M values that indicated the eastern GOM may be experiencing a higher rate of natural mortality than the western GOM. Yet when an informative prior with a mean of 3.0 was used in both models, the estimates of the Lorenzen M for the western GOM were lower. This phenomenon is very surprising and is currently unexplained. The age-composition data for bycatch from the shrimp fisheries influences the estimation of a Lorenzen M function. Therefore, the estimation of two separate Lorenzen M curves between the two regions is driven by the separate age-composition data on bycatch from the shrimp fisheries. Therefore, it is likely that the estimated difference in natural mortality is due to the slight difference in the value of the selectivity-at-age parameter for bycatch from the shrimp fisheries in the east and west (Figure 2.4 and 2.14). Additionally, the difference in M estimates could be attributed to differences in the sampling of the age-composition data on bycatch from the shrimp fisheries. However, the possibility that the rate of natural mortality between the eastern and western GOM is different should be considered in future modeling of these stocks of red snapper.

### ***M0 Parameter Estimation***

The estimation of natural mortality at age 0 is determined by the age-composition data of bycatch from the shrimp fisheries. Approximately 60% to 80% of the observed bycatch

proportion-at-age data from the shrimp fisheries are observed to be age-0 red snapper, while 2% or less are age 2 and the remaining proportion is age 1. This large decrease in proportion at age from age 0 to age 1 can be explained in two very different fashions depending upon the modeling assumptions used. If the parameter for the bycatch from the shrimp fisheries is assumed to be fully selected for age-1 red snapper, and selectivity-at-age parameters for age 0 and age 2 are estimated, then this decline is accounted for by a high value for  $M_0$ . The model estimates the selectivity-at-age-0 parameter for bycatch from the shrimp fisheries to be near a value of 1.0, i.e., close to fully selected for the fisheries, so that the observed decline in the proportion at age is similar to the decline in proportion at age in the whole population. If the model were to predict that the age-0 red snapper were much less than fully selected to the gear, this then would indicate that there is a much larger proportion of age-0 red snapper than that observed in the catch. If the model were to indicate there were more age-0 red snapper than observed, then the decline observed in the proportion-at-age data was only a fraction of the total decline and thus would estimate a much greater value for  $M_0$ . The alternative modeling assumption that can account for this decline in observed proportion at age is to allow the selectivity-at-age parameters for ages 0, 1 and 2 to be estimated for bycatch from the shrimp fisheries. When these three selectivity-at-age parameters are estimated for bycatch from the shrimp fisheries, the model assumes that the decline in observed abundance from age 0 to age 1 is due entirely to a change in vulnerability of the fish to the gear. Thus the model estimated the natural mortality at age 0 for red snapper to be negligible and constrained by the assumed lower bound of the prior. Therefore, the estimation of selectivity-at-age parameters for bycatch from the shrimp fisheries at age 0 and age 1 is confounding with the estimation of the  $M_0$  parameter. Thus the claim by Clarke (1999) that the estimation of natural mortality is an artifact of the assumed shape of the selectivity curve maybe

true for the estimation of natural mortality within a statistical catch-at-age model. Additionally, the estimation of  $M_0$  was strongly influenced by the prior on  $M_1$ . This is due to a strong negative correlation between  $M_0$  and  $M_1$  when informative priors were used on the models. The negative correlation between the parameters was not present when uniform priors were used, but the value of  $M_1$  was estimated to be approximately equal to the lower bound of the prior. Since the model with fixed  $M$  parameters at high values, and the model with fixed  $M$  parameters at low values did not have any variation in the  $M_0$  parameter, it is not possible to determine a correlation between this parameter and the selectivity-at-age parameters for bycatch from the shrimp fisheries. However, since these two models had different values of  $M_0$  and there is a very significant change in the estimated values of the selectivity-at-age parameters for bycatch from the shrimp fisheries, it can be assumed that the two are very closely interrelated. In the model that estimated selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and three  $M$  parameters the estimation of  $M_0$  with an informative prior still resulted in an unreasonably low estimated value. The prior on  $M_1$  and the ability to estimate a selectivity-at-age-1 parameter for bycatch from the shrimp fisheries appears to be overpowering the influence of the prior on  $M_0$ . The interaction of these parameters makes this a very complex model and estimability of these parameters very difficult. The modeling component wherein age is modeled to change partway through the year might be influencing or causing the correlation between  $M_0$  and  $M_1$  to a greater degree than if age were modeled to change at the start of the year. This hypothesis should be investigated further as this factor also may be influencing the estimability of the initial abundance at age.

The only data source that includes a substantial proportion of age-0 red snapper is the age-composition data from bycatch by the shrimp fisheries and therefore is the main source to contribute to the estimation of  $M_0$ . The SEAMAP age-0 and age-1 surveys may provide some

information that assist with the estimation of  $M_0$  but are not sufficient to estimate natural mortality when the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries is estimated separately. Traditionally, if natural mortality is confounding with selectivity-at-age parameters in an assessment model, then natural mortality is estimated outside the assessment model and inserted as a known constant into the model, while the selectivity-at-age parameters are estimated by the model. An alternative option that may be applicable to red snapper in the GOM would be to input the values for the selectivity-at-age parameters of bycatch from the shrimp and estimate a natural mortality parameter at age 0. This method would require an experimental estimation of the selectivity at age for the shrimp trawl nets, which may be a difficult task. One potential experimental solution to determining the selectivity at age for a shrimp trawl may be to conduct a video survey of an area to determine the estimated proportion at length of the fish in a specific area shortly before conducting an experimental trawl. Then a comparison of the observed proportion at length in the video survey can be compared to the observed proportion at length in the trawl from which a selectivity-at-age curve may be calculated. An additional way to estimate the selectivity at age of the shrimp trawl is to perform an experimental shrimp trawl with a smaller mesh size outside of the cod end of the net, or twin-trawl nets with different mesh sizes. Estimating the selectivity at age during different seasons of the year to account for spawning, emigration from trawlable habitat, and the high rate of growth during age 0 and age 1 would increase the accuracy of the assessment model and potentially increase the ability to estimate natural mortality at age 0. This methodology is not certain to produce accurate estimates of natural mortality and potentially could produce inflated estimates of natural mortality. Therefore, additional investigation is required into the estimation of natural mortality at age 0 within the assessment model through the assumption of input values for

selectivity-at-age parameters in bycatch from the shrimp fisheries. Additionally, investigation into various assumptions of the shape of the selectivity curve for GOM red snapper bycatch from the shrimp fisheries and all other fisheries may provide more insight into whether the estimation of natural mortality within a statistical catch-at-age model is truly an artifact of these assumptions.

### ***M1 Parameter Estimation***

The estimation of natural mortality at age 1 is problematic within the assessment model for GOM red snapper based upon the available data. Natural mortality estimation within the assessment model appears to be driven primarily by the age-composition data of the fisheries with some additional input from the surveys. The estimation of natural mortality at age 1 requires a change in proportion at age in the observed data that is not accounted for by the model as a change in selectivity at age for the fishery. The majority of fisheries are not fully selected for red snapper until they are at least age 2. Therefore the model observes an increase in the proportion at age from age 1 to age 2 in the age-composition data and accounts for this change through a difference in selectivity at age for the fishery, and the estimate of natural mortality is approximately zero. The age-composition data from bycatch in the shrimp fisheries shows a decline in proportion at age for age 1 to age 2, but this decline is attributed solely to a change in selectivity at age for the fishery over those ages. When an informative prior on M1 is used, the estimated value is relatively close to the mean of the prior. However, the negative correlation between M1 and M0 can lower the estimate of M1 away from the mean of the prior if the mean is set to a high value, as is the case for the model that assumed three M parameters. Correlation between M0 and M1, in the posterior samples of the model that assumed three M parameters and the model that assumed three M parameters with normal priors at low values, may suggest that



one or both of the parameters may be inestimable in this model. Additionally, M1 was found to be strongly negatively correlated with the catchability coefficient for bycatch from the shrimp fisheries and positively correlated with the selectivity-at-age-0 parameter for bycatch from the shrimp fisheries. These results suggest that M1 is strongly confounding with other parameters in the model and can potentially drive the estimation of other parameters. Assuming input values for the selectivity-at-age parameter of bycatch from the shrimp fisheries at age 1 and age 2 may produce an estimate of natural mortality at age 1, but has some caveats that must be considered. A transition from the low-profile oyster shell habitat to the high-profile coral habitat occurs during the first to second year in the red snapper life cycle (Gallaway et al. 2009). Therefore, the observed decline in the proportion at age in bycatch from the shrimp fisheries may be due primarily to age-2 red snapper no longer residing in habitat that undergoes shrimp trawling, as would be indicated by a decrease in the vulnerability to the gear. If the selectivity-at-age parameters for age 1 and age 2 were set to be fully selected based upon experimentation, then the model would predict that the decline in observed proportion at age would be due to natural mortality and would not account for emigration from the area. This overestimation of natural mortality is inadvisable as it would impact the estimation of biological reference points used for management purposes, such as  $F_{MSY}$  and others (Mertz and Meyers 1997, Clarke 1999, Williams 2002). Therefore, the estimation of red snapper natural mortality at age 1 for in the GOM within an assessment model does not appear to be feasible based upon the currently available data sources. However, informative priors can be used, but will likely influence the estimation of natural mortality at age 0 and/or the selectivity parameters of bycatch from the shrimp fisheries.

### ***M2+ Parameter Estimation***

The influence of the age-composition data from bycatch in the shrimp fisheries upon the estimation of natural mortality for ages 2 and older in the model that estimated selectivity-at-age-1 parameter for the bycatch from the shrimp fisheries and three M parameters is surprising. The decline in proportion at age in the age-composition data would be expected to be modeled sufficiently by the fixed value of 0 for ages 3 and older. However, the influence of estimating a selectivity-at-age-1 parameter for bycatch from the shrimp fisheries may change the estimated recruitment enough to drastically change the estimated population dynamics of the stocks and thus the natural mortality estimate. On the other hand, the estimated value of natural mortality for ages 2 and older might be an artifact of how the selectivity-at-age parameters are modeled for fisheries other than the bycatch from the shrimp fisheries. The model assumes that all the selectivity-at-age parameter for ages 10 through the plus group at age 15 have the same value for the commercial handline, commercial longline, recreational, and bycatch during the closed-season fisheries, for each fishery separately. This assumption was made because the stock assessment scientist from the SEDAR update assessment (2009) believed that there would not be sufficient information in the age-composition data to estimate the selectivity for these age-classes separately because these ages are encountered infrequently by most fisheries. The estimation of natural mortality for ages 2 and older is likely due to this assumption because any decline observed in the proportion at age as age increases would be attributed to natural mortality. However, if each age class were estimated separately the decline in proportion at age would be accounted for by the selectivity and natural mortality may be estimated to be approximately zero. Therefore, the estimation of separate selectivity-at-age parameters for ages 10 through 15 should be attempted to determine if the estimation of natural mortality for ages 2 and older is an artifact of the assumption made in this model. Additionally, the NMFS bottom-longline survey assumes

a logistic function that is very nearly fully selected by age 9 in all the models and could also be a contributing factor to the estimation of natural mortality for ages 2 and older. The correlation of  $M_{2+}$  and the apical fishing effort for the NMFS bottom-longline survey arouses concerns. It is possible that the age-composition data for this fishery is driving this parameter estimation and the logistic functional curve for the selectivity-at-age parameters may be influencing or driving the estimate. Comparisons between the resulting estimates of natural mortality for various assumptions on the shape of the selectivity-at-age curve for these fisheries must be conducted to determine if natural mortality estimation for ages 2 and older in GOM red snapper is a residual of the logistic shape of the selectivity function for the NMFS bottom-longline survey. Additional investigation on the influence of the assumed shape of the selectivity-at-age curve upon the estimation of natural mortality at ages 2 and older is required.

### ***Influence of Selectivity on M Estimation***

A model was created that estimated selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and three M parameters to determine if a biologically reasonable value of natural mortality could be estimated concurrently with a selectivity-at-age-1 parameter for bycatch from the shrimp fisheries. The assumptions regarding the selectivity-at-age parameters for bycatch from the shrimp fisheries can influence the estimation of all natural mortality parameters. Therefore, doubt is invoked on the estimability of any natural mortality parameter within a statistical catch-at-age model. This result is also surprising because the age-composition data from the bycatch by the shrimp fisheries do not inherently contain any information for the decline in abundance after age 2. However, the estimation of all natural mortality parameters for GOM red snapper appears to be dependent upon the selectivity parameters assumptions made regarding the bycatch from the shrimp fisheries. This evidence suggests that estimation of natural

mortality in juvenile red snapper maybe extremely influenced by the estimation of the selectivity-at-age parameters for bycatch from the shrimp fisheries. This result is likely true for other studies that attempt to estimate juvenile natural mortality, including studies that rely upon fisheries length-at-age abundance data (Gazey et al. 2008). The informative priors on  $M_0$  and  $M_1$  caused a very large change in the estimated values of  $M_1$  and the selectivity-at-age parameters for bycatch from the shrimp fisheries in the modeled compared to when run with wide uniform priors. These parameters appear to be extremely cross-correlated and are likely confounding as suggested in other models (Paloheimo et al. 1982; Thompson 1994; Schnute and Richards 1995; Clark; 1999; Wang 1999; Fu and Quinn 2000; Magnusson and Hilborn 2007).

If natural mortality is specified and selectivity-at-age parameters are estimated, then it is likely that the estimate of selectivity at age for bycatch from the shrimp fisheries will be highly influenced by the value of natural mortality used. A larger assumed rate of red snapper natural mortality for age 0 in the GOM would result in an estimated value of the selectivity-at-age parameter closer to 1.0 for age-1 red snapper bycatch from the shrimp fisheries, assuming the parameter for age 0 is estimated to be fully selected. This difference in the value of selectivity-at-age parameters may appear to be trivial, but potentially could result in a very different estimate of fishing mortality in bycatch from the shrimp fisheries and even the assessed status of the stock. Therefore, within a statistical catch-at-age model for red snapper in the GOM given the currently available data, stock assessment scientist must decide whether an assumption of values for natural mortality at various ages or an assumption of values of selectivity at age for bycatch from the shrimp fisheries is more reasonable within the model. Two models should be tested and compared to one another within a GOM red snapper assessment. The first model will assume selectivity-at-age function parameter values or values at age for bycatch from the shrimp

fisheries and natural mortality will be estimated. The second model will assume values for natural mortality and selectivity-at-age parameters for bycatch from the shrimp fisheries will be estimated. The comparison of these two models will aid in the determination of the implications these assumptions regarding selectivity at age and natural mortality have on the estimated status of the stock.

### ***Future Research***

The models presented were based upon the configurations and data sources from the SEDAR update assessment (2009). To include as much realism as possible for Gulf of Mexico red snapper the models incorporated the 11 fisheries, 13 surveys, 3 seasons per year, and the variable legal-harvest size limit for each fishery. The purpose of incorporating these factors was to test the applicability of estimating natural mortality within a statistical catch-at-age model to real data, with the added complexity of multiple fisheries interacting with different portions of the population. The complexity of this model appears to have led to the inestimable nature of the initial abundance-at-age parameters and thus clouded the certainty of any deductions that might be drawn from the results of this study. Therefore, additional investigation must be conducted on these models to decisively conclude the estimability of natural mortality within a statistical catch-at-age model. The first action would be to resolve the difficulty with estimating the scale of recruitment by allowing the catchability coefficients to be greater than or equal to 0 or rescaling the indices of abundance. If this action is unable to resolve the inestimable nature of the initial abundance-at-age parameters, then priors with diffuse normal distributions will be employed within the model. Additional sensitivity analysis will be performed to determine the influence of different priors upon the estimated value of parameter within the model. When convergence to the true posterior distribution is no longer questioned within the models, a

comparison of the models presented will be conducted to ascertain the validity of the deductions made herein.

Additional investigation on the use of informative priors on the selectivity at age for the shrimp fishery is recommended. A possible avenue of further research is the use of two selectivity-at-age curves for the shrimp fishery. The first selectivity curve will be assumed values that describe the “contact vulnerability” of red snapper that encounter the shrimp trawl. The second selectivity curve will be estimated values that describe the gear selectivity to the population. This may help to explain the two contradictory effects of the increase in contact vulnerability with increasing age but decrease in gear selectivity to the population with increasing age as older fish leaving the trawlable habitat. However it is possible that the age-composition data do not contain enough information to estimate such values concurrently with natural mortality. The implication of the estimated values using two selectivity curves must be carefully considered before use within an assessment. Many questions currently remain regarding natural mortality estimation that must be answered before natural mortality estimation within a statistical catch-at-age model is performed for Gulf of Mexico red snapper.

One concern, as a result of this study, was that the estimated value of natural mortality might not truly reflect the population dynamics of the fish but instead are an artifact of the assumptions regarding selectivity-at-age in the fisheries. To address this question a simulation study could be conducted. The use of a hypothetical fish population might be suitable for this simulation but could assume that multiple fisheries impact the population dynamics. These fisheries could have different selectivity curves, which could be similar to those that harvest red snapper in the Gulf of Mexico. The simulation study would focus primarily upon the impact of assuming a logistic function versus a double-logistic function for the selectivity-at-age for

fisheries that harvest adult fish. To test this premise, two types of simulation datasets will be generated: one that models all fisheries to have a double-logistic selectivity function and another that assumes one fishery has a logistic selectivity function. Two models will estimate population dynamics parameters including natural mortality for each dataset. The first model will assume all fisheries have a double-logistic selectivity function. The second model will be assumed to correctly identify the fishery that has a logistic function and will model it as such, while all others will have double logistic functions. Alternative models could also be test, such as a model that assumes a logistic function for the incorrect fishery or models that estimate selectivity-at-age parameters for each age. Additionally, the influence of different priors upon the estimation of natural mortality could be investigate through a simulation study, but would require a different design. The considerable potential remains for future research on the estimability of natural mortality within a statistical catch-at-age model.

### ***Conclusions***

The results of this study are preliminary and clear conclusions cannot yet be made. However, these results suggest that natural mortality estimation within a statistical catch-at-age model for GOM red snapper has many caveats. First, the estimation of natural mortality at age 1 as an independent parameter does not appear to be currently viable based upon the available data from the SEDAR update assessment (2009). Yet, natural mortality at age 1 can be strongly influenced by a prior and a value within the range of biologically reasonable values can be obtained. However, the M1 parameter is strongly negatively correlated with M0 and thus will influence the results of the model greatly. Estimation of natural mortality at age 0 is dependent upon the assumptions about the selectivity-at-age parameters for bycatch from the shrimp fisheries. A confounding nature between these two parameters was identified and estimation of

the two concurrently might not be possible. Lorenzen natural mortality curves are driven by the age-composition data from the bycatch in the shrimp fisheries and may also be confounding with the selectivity-at-age parameters for these fisheries. However, when an informative prior is placed upon the Lorenzen  $M$  parameter a reasonable value of natural mortality is estimated and the selectivity-at-age-1 parameter is estimated to be approximately 1.0. The values of natural mortality parameters for ages 2 and older between models were also shown to change depending upon the assumptions made about selectivity-at-age parameters for bycatch from the shrimp fisheries. However, the alterations in values of this parameter between models may be due to the changes in the values of natural mortality for age 0 and age 1, which change drastically under the various assumptions within the models. Additional investigation is recommended on the influence of modeling assumptions of the selectivity at age for other fisheries, especially the logistic function of the NMFS bottom-longline survey. Estimation of any natural mortality parameter within a statistical catch-at-age model may inherently be a consequence of the assumed shape of the selectivity-at-age curve. Therefore, multiple selectivity-at-age functions or parameterization assumptions should be compared when estimating natural mortality. It may be possible to estimate a Lorenzen curve natural mortality parameter or a natural mortality parameter at age 0 if informative priors are placed on the selectivity-at-age parameters of the fisheries, specifically the bycatch from the shrimp fisheries. However, these priors may yield an estimated value of natural mortality in the model that appears to be biologically reasonable, but may not represent the true population dynamics for Gulf of Mexico red snapper. A simulation study could be configured to potential answer this concern. Consequently, natural mortality estimation within statistical catch-at-age models should still be investigated in the hopes of a breakthrough, but should be approached with caution.



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Table 2.1. List of symbols and variables with their associated descriptions that are used in the equations presented in Table 2.2 to model the population dynamics of red snapper in the Gulf of Mexico.

Symbol	Description of variable and use in equations
$a$	Subscript representing the age of the fish
$A$	The maximum age and plus group used in the model (age 15 and older)
$i$	Subscript indicating the fishery (1 to 11)
$j$	Subscripts indicating the survey (1 to 8)
$k$	Subscript representing the region in the GOM (1= eastern, 2 = western)
$t$	Subscript representing the year from 1987 to 2008
$s$	Subscript representing the season in a year with a total of 3 seasons per year
$N$	The abundance of red snapper in number of fish for respective subscripts
$R$	Recruitment in number of fish recruiting in season 2
$N_0$	Number of fish in an age-class at the start of the model
$F$	Instantaneous rate of fishing mortality rate for respective subscripts
$Z$	Instantaneous rate of total mortality for respective subscripts
$M$	Instantaneous rate of natural mortality for respective subscripts
$v$	Relative selectivity at age or vulnerability of fish at age to the gear maximum value is scaled to 1
$f$	Apical fishing mortality for respective subscripts
$\xi$	Probability that a fish dies once it is caught based upon the legal size limit for the corresponding subscripts
$\delta$	Indicator variable whether a fishery is active during a season (value = 1 or =0)
$\gamma$	Indicator variable whether a fishery is active in a region (value = 1 or =0)
$n_i(s)$	Number of seasons fishery $i$ is active during one year
$\alpha$	Parameter used in logistic equation to model selectivity at age in NMFS bottom-longline fishery
$\beta$	Parameter used in logistic equation to model selectivity at age in NMFS bottom-longline fishery
$Q$	Log scale catchability coefficient to scale shrimp fishing effort

$e$	Shrimp fishing effort estimated by the model
$d$	Probability that a fish dies if it is released for the corresponding subscripts
$G_{L a,s}$	Probability that a fish is less than the harvest size limit given its age in seasons for corresponding subscripts
$L_{a,s}$	Length at age calculated for each season in inches
$C$	Total number of fish caught
$H$	Total number or weight of fish harvest depending upon the fishery
$w_{a,s}$	Weight at age calculated for each season
$D$	Total number of discards for corresponding subscripts
$I$	Log scale Abundance Index for corresponding subscripts
$q$	Log scale catchability coefficient to scale population abundance for corresponding subscripts
$r_a$	Relative fecundity at age
$P$	Proportion at age of harvested fish for corresponding subscripts
$n$	Number of age composition samples taken for corresponding subscripts
$\sigma$	Standard deviation calculated for the statistical estimator for corresponding subscripts
$\lambda$	CV assigned to weight data point for corresponding subscripts
$E()$	Expected value of corresponding value as calculated by the model
$p(\theta D)$	Posterior distribution of parameter $\theta$ conditional on the data $D$
$LH(D \theta)$	Likelihood of the data $D$ given parameter $\theta$
$\pi(\theta)$	Prior distribution for parameter $\theta$
$\sigma_o$	CV of overall model set at 0.01
$\sigma_H$	CV of harvest data set at 0.01
$\sigma_I$	CV of abundance index data set at 0.01
$\sigma_e$	CV of shrimp fishing effort data set at 0.1

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Table 2.2. Mathematical equations used to calculate the abundance at age, expected harvest, expected value of abundance indices, and expected proportion at age within the harvest from estimated parameters for Gulf of Mexico red snapper.

Equation
(2.1a) $N_{k,0,t,2} = R_{k,t}$
(2.1b) $N_{k,a,1,1} = N0_{k,a}$
(2.2a) For $a < A$ and $s = 1$ : $N_{k,a+1,t,s+1} = N_{k,a,t,s} e^{-Z_{k,a,t,s}}$
(2.2b) For $a < A$ and $s = 2$ : $N_{k,a,t,s+1} = N_{k,a,t,s} e^{-Z_{k,a,t,s}}$
(2.2c) For $a < A$ and $s = 3$ : $N_{k,a,t+1,s=1} = N_{k,a,t,s} e^{-Z_{k,a,t,s}}$
(2.3a) For $a = A$ and $s = 1$ : $N_{k,A,t,s+1} = N_{k,A-1,t,s} e^{-Z_{k,A-1,t,s}} + N_{k,A,t,s} e^{-Z_{k,A,t,s}}$
(2.3b) For $a = A$ and $s = 2$ : $N_{k,A,t,s+1} = N_{k,A,t,s} e^{-Z_{k,A,t,s}}$
(2.3c) For $a = A$ and $s = 3$ : $N_{k,A,t+1,s=1} = N_{k,A,t,s} e^{-Z_{k,A,t,s}}$
(2.4) $Z_{k,a,t,s} = \sum_i F_{k,i,a,t,s} + \frac{M_{k,a,s}}{3}$
(2.5a) For $i \neq 9,10$ $F_{k,i,a,t,s} = v_{k,i,a} f_{k,i,t} \xi_{k,i,a,s,t} \delta_{k,i,s} \gamma_{k,i} / n_i(s)$
(2.5b) For $i = 9,10$ $F_{k,i,a,t,s} = v_{k,i,a} e_{k,i,t} \delta_{k,i,s} \gamma_{k,i} \log_e(Q_{k,i}) / n_i(s)$
(2.6) $v_{i=11,a,s} = 1/1 + \exp\left(-\frac{3*a+s-.5-\beta}{\alpha}\right)$
(2.7) $\xi_{k,i,a,s,t} = 1 - (1 - d_{k,i}) G_{L a,s k,i,t}$
(2.8) $L_{a,s} = 33.4047 \left(1 - e^{-0.2188 \left( \left(a + \frac{s-1}{3}\right) + 0.0611 \right)}\right)$
(2.9) $C_{k,i,a,t,s} = \frac{F_{k,i,a,t,s}}{\xi_{k,i,a,t} Z_{k,a,t,s}} N_{k,a,t,s} (1 - e^{-Z_{k,a,t,s}})$
(2.10a) For $i > 4$ : $E(H_{k,i,t}) = \sum_a \sum_s (1 - G_{L a,s k,i,t}) C_{k,i,a,t,s}$
(2.10b) For $i = 1,2,3,4$ : $E(H_{k,i,t}) = \sum_a \sum_s H_{k,i,a,t,s} * w_{a,s}$
(2.11) $w_{a,s} = 0.000447 (L_{a,s})^{2.994}$
(2.12) $D_{k,i,a,t,s} = G_{L a,s k,i,t} C_{k,i,a,t,s}$

$$(2.13a) \text{ For } i = 1, 2 : E(\log_e I_{k,i,t}) = \log_e(\sum_s \sum_a N_{k,a,t,s} * (1 - G_{L|a,s k,i,t}) * w_{a,s} * v_{k,i,a}/n_i(s)) + \log_e(q_{k,i})$$

$$(2.13b) \text{ For } i = 5, 6 : E(\log_e I_{k,i,t}) = \log_e(\sum_s \sum_a N_{k,a,t,s} * v_{k,i,a}/n_i(s)) + \log_e(q_{k,i})$$

$$(2.13c) \text{ For } i = 11 : E(\log_e I_{k,i,t}) = \log_e(\sum_a (N_{k,a,t,s=3} * (1 - G_{L|a,s k,i,t}) * v_{k,i,a}/n_i(s)))$$

$$(2.14a) \text{ For } j \neq 3, 4 : E(\log_e I_{k,j,t}) = \log_e(\sum_a N_{k,a,t,s=2} * v_{k,j,a}) + \log_e(q_{k,i})$$

$$(2.14b) \text{ For } j = 3, 4 : E(\log_e I_{k,j,t}) = \log_e(\sum_a N_{k,a,t,s=2} * r_a * v_{k,j,a}) + \log_e(q_{k,j})$$

$$(2.15) E(P_{k,i,a,t}) = \sum_s H_{k,i,a,t,s} / \sum_s \sum_a H_{k,i,a,t,s}$$


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Table 2.3. Log-likelihood equations used to fit the estimated parameters by comparing the expected values from the model to the observed data for Gulf of Mexico red snapper. The statistical estimator used assumed log-scale likelihoods and priors.

Equation
(3.1) $p(\theta D) \propto LH(D \theta)\pi(\theta)$
(3.2) $l_H = -\sum_k \sum_i \sum_t \frac{(\log_e(H_{k,i,t}) - \log_e(E(H_{k,i,t})))^2}{2\sigma_{H_{k,i,t}}^2} - \log_e\left(H_{k,i,t} * \sqrt{2\pi\sigma_{H_{k,i,t}}^2}\right)$
(3.3) $\sigma_{H_{k,i,t}}^2 = \log_e((\sigma_H * \sigma_O)^2 + \lambda_{k,i,t} + 1)$ ,
(3.4) $l_I = -\sum_k \sum_j \sum_t \frac{(\log_e(I_{k,j,t}) - \log_e(E(I_{k,j,t})))^2}{2\sigma_{I_{k,j,t}}^2} - \log_e\left(I_{k,j,t} * \sqrt{2\pi\sigma_{I_{k,j,t}}^2}\right)$
(3.5) $l_S = -\sum_k \sum_i \sum_t \frac{(\log_e(e_{k,i,t}) - \log_e(E(e_{k,i,t})))^2}{2\sigma_{e_{k,i,t}}^2} - \log_e\left(e_{k,i,t} * \sqrt{2\pi\sigma_{e_{k,i,t}}^2}\right)$
(3.6) $l_P = \sum_k \sum_i \sum_a \sum_t \log_e(n_{k,i,t}!) - \log_e(n_{k,i,t} * P_{k,i,a,t}!) + n_{k,i,t} * P_{k,i,a,t} \log_e(E(P_{k,i,a,t}))$
(3.7) $v_{penalty} = \sum_k \sum_j \sum_a \frac{1.5(v_{k,j,a+1} - v_{k,j,a})^2}{0.3e^{-8(a+\frac{1}{6})}}$
(3.8) $-\log_e(LH(D \theta)) = l_H + l_I + l_S + l_P + v_{penalty}$



Table 2.4. Fisheries that harvest Gulf of Mexico red snapper and surveys that provide an index of abundance for each stock and their associated number for modeling purposes are presented in the table below. East refers to fisheries and surveys that operate east of the Mississippi River and West refers to fisheries and surveys that operate west of the Mississippi River.

<i>i</i>	Fishery	<i>j</i>	Survey
1	Commercial Handline East	1	Video East
2	Commercial Handline West	2	Video West
3	Commercial Longline East	3	Larval East
4	Commercial Longline West	4	Larval West
5	Recreational East	5	Age-1 East
6	Recreational West	6	Age-1 West
7	Bycatch during the Closed-Season East	7	Age-0 East
8	Bycatch during the Closed-Season West	8	Age-0 West
9	Bycatch from the Shrimp East		
10	Bycatch from the Shrimp West		
11	NMFS Bottom Longline Survey West		

Table 2.5. Equations used within the various models that describe the assumptions concerning natural mortality ( $M$ ) and selectivity-at-age ( $v$ ) parameters. In these equations  $k$  represents stock,  $a$  represents age with max age is represented by  $A$ ,  $i$  represents the fishery and  $s$  represents season with 3 in each year. The bycatch from the shrimp fisheries in the eastern and western Gulf of Mexico are represented by fisheries number 9 and 10 respectively.

Equation
(5.1) For all $k$ , $a$ , and $s$ : $M_{k,a,s} = M_{k,a,s}$
(5.2) For all $k$ : $M_{k,a=0} \neq M_{k,a=1} \neq M_{k,a=2}$ and $M_{k,a=2} = M_{k,a=3} = \dots = M_{k,a=A}$
(5.3) $M_{k,a,s} = g/L_{a,s}$ and $M_{k=1,a,s} = M_{k=2,a,s}$ , where $g$ is an estimated parameter
(5.4) $M_{k=1,a,s} = g/L_{a,s}$ , $M_{k=2,a,s} = h/L_{a,s}$ and $M_{k=1,a,s} \neq M_{k=2,a,s}$ , where $g$ and $h$ are estimated parameters
(5.5) $v_{k=1,i,a} = v_{k=2,i,a}$ , where $i$ corresponds to the same type of fishery, i.e. recreational east $i=5$ and recreational west $i=6$
(5.6) For all $i$ and $a$ : $v_{k=1,i,a} \neq v_{k=2,i,a}$
(5.7) $v_{k=1,i=9,a=1} = 1$ and $v_{k=2,i=10,a=1} = 1$
(5.8) $v_{k=1,i=9,a=1} = v_{k=2,i=10,a=1} = g$ , where $g$ is an estimated parameter

Table 2.6. Comparison of modeling assumptions used by each model where the number in the top row represents the equation used to represent the assumption in Table 2.5. The \* symbol in the row for ShrVsUB2 indicates that the selectivity-at-age-1 parameter was scaled by the maximum selectivity-at-age parameter for bycatch from the shrimp fisheries for each saved sample of this model. ConstM represents the model that estimated constant mortality over all ages; 3Ms represents the model that assumed three M parameters; 3MsM1LB0.5 represents the model that assumed three M parameters with the lower bound of M1 prior equal to 0.5; LorM represents the model that assumed a Lorenzen M curve; SepVsLorM represents the model that estimated separate selectivity-at-age parameters for east and west GOM; SepVsSepMs represents the model that estimated separate selectivity-at-age parameters and separate Lorenzen M parameters for east and west GOM; SepLorMs represents the model that estimated separate Lorenzen M parameters for east and west GOM; ShrVsUB2 represents the model that assumed the upper bound of the priors on the selectivity-at-age parameters equaled two for bycatch from the shrimp fisheries; ShrV1LorM represents the model that estimated the selectivity-at-age-1 parameter for the shrimp fishery and a Lorenzen M parameter; ShrV13Ms represents the model that estimated selectivity-at-age-1 parameter for bycatch from the shrimp fishery and three M parameters; FixedMsHigh represents the model with fixed M parameters at values from SEDAR 31 (2013); FixedMsLow represents the model with fixed M parameters at values from SEDAR update assessment (2009).

Model	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8
ConstM	X				X		X	
3Ms		X			X		X	
3MsM1LB0.5		X			X		X	
LorM			X		X		X	
SepVsLorM			X			X	X	
SepVsSepMs				X		X	X	
SepLorMs				X	X		X	
ShrVsUB2			X		X		X*	
ShrV1LorM			X		X			X
ShrV13Ms		X			X			X
FixedMsHigh		X			X		X	
FixedMsLow		X			X		X	

Table 2.7. Two sets of priors on the parameters used in the models to estimate population dynamics of red snapper in the Gulf of Mexico. The first and second values for uniform distributions are the lower and upper bounds of the prior, respectively. The first value for the normal distribution is the mean and the second value is the standard deviation. The prior for natural mortality varied by model and 3MsM1LB0.5 represents the model that assumed three M parameters with the lower bound of M1 prior equal to 0.5 and corresponds to the priors in the “Wide Priors” column. The “Tight Priors” column and the corresponding rows to the 3MsM1LB0.5 labels are the priors for the model that assumed three M parameters with normal priors at the values from the SEDAR update assessment (2009). The meaning of other parameters are listed in Table 2.1.

Parameter	Wide Priors	Tight Priors
$M_0$	Uniform (0.01, 6.00)	Normal (2, 0.5)
$M_1$	Uniform (0.01, 3.00)	Normal (1.2, 0.3)
$M_2 +$	Uniform (0.01, 0.75)	Normal (0.1, 0.027)
<i>Lorenzen M</i>	Uniform (0.01, 6.00)	Normal (3, 0.75)
$M_0$ for 3MsM1LB0.5	Uniform (0.01, 6.00)	Normal (1, 0.3)
$M_1$ for 3MsM1LB0.5	Uniform (0.5, 3.00)	Normal (0.6, 0.2)
$M_2 +$ for 3MsM1LB0.5	Uniform (0.01, 0.75)	Normal (0.1, 0.027)
<i>R East</i>	Uniform (1e6, 1e10)	Uniform (1e7, 1e8)
<i>R West</i>	Uniform (1e6, 1e10)	Uniform (1e7, 4e8)
$f_{k=1,i=1,t}$	Uniform (0.0001, 5)	Uniform (0.01, 1)
$f_{k=2,i=2,t}$	Uniform (0.0001, 5)	Uniform (0.075, 0.75)
$f_{k=1,i=3,t}$	Uniform (0.000001, 5)	Uniform (0.001, 0.2)
$f_{k=2,i=4,t}$	Uniform (0.000001, 5)	Uniform (0.0005, 0.1)
$f_{k=1,i=5,t}$	Uniform (0.0001, 5)	Uniform (0.5, 2.5)
$f_{k=2,i=6,t}$	Uniform (0.0001, 5)	Uniform (0.075, 1.5)
$f_{k=1,i=7,t}$	Uniform (0.000001, 5)	Uniform (0.01, 0.5)
$f_{k=2,i=8,t}$	Uniform (0.0001, 5)	Uniform (0.005, 0.1)
$f_{k=1,i=9,t}$	Uniform (0.0001, 5)	Uniform (0.05, 2.0)
$f_{k=2,i=10,t}$	Uniform (0.0001, 5)	Uniform (0.05, 2.0)
$f_{k=2,i=11,t}$	Uniform (10e-10, 5)	Uniform (5e-7, 1e-4)
$v$	Uniform (0.0001, 1.00)	Uniform (0.0001, 10)
$\alpha$	Uniform (0.0001, 10)	Uniform (0.0001, 10)
$\beta$	Uniform (0.0001, 10)	Uniform (0.0001, 10)
$Q$	Uniform (-0.0001, -10)	Uniform (-0.0001, -10)
$q$	Uniform (-0.01, -10)	Uniform (-0.01, -10)
$N_0_{k=1,a=0}$	Uniform (1.63e5, 1.03e8)	Uniform (1.63e5, 1.03e8)
$N_0_{k=1,a=1}$	Uniform (9.14e4, 2.47e7)	Uniform (9.14e4, 2.47e7)
$N_0_{k=1,a=2}$	Uniform (2.60e5, 1.62e7)	Uniform (2.60e5, 1.62e7)
$N_0_{k=1,a=3}$	Uniform (1.18e5, 1.34e7)	Uniform (1.18e5, 1.34e7)
$N_0_{k=1,a=4}$	Uniform (4.36e4, 6.37e6)	Uniform (4.36e4, 6.37e6)
$N_0_{k=1,a=5}$	Uniform (1.83e4, 2.12e6)	Uniform (1.83e4, 2.12e6)
$N_0_{k=1,a=6}$	Uniform (1.03e4, 1.33e6)	Uniform (1.03e4, 1.33e6)

$NO_{k=1,a=7}$	Uniform (1.31e3, 6.20e5)	Uniform (1.31e3, 6.20e5)
$NO_{k=1,a=8}$	Uniform (714, 3.09e5)	Uniform (714, 3.09e5)
$NO_{k=1,a=9}$	Uniform (692, 1.76e5)	Uniform (692, 1.76e5)
$NO_{k=1,a=10}$	Uniform (476, 1.02e5)	Uniform (476, 1.02e5)
$NO_{k=1,a=11}$	Uniform (311, 6.77e4)	Uniform (311, 6.77e4)
$NO_{k=1,a=12}$	Uniform (255, 5.05e4)	Uniform (255, 5.05e4)
$NO_{k=1,a=13}$	Uniform (233, 3.81e4)	Uniform (233, 3.81e4)
$NO_{k=1,a=14}$	Uniform (219, 2.76e4)	Uniform (219, 2.76e4)
$NO_{k=1,a=15}$	Uniform (575, 8.76e4)	Uniform (575, 8.76e4)
$NO_{k=1,a=0}$	Uniform (3.41e6, 1.80e9)	Uniform (3.41e6, 1.80e9)
$NO_{k=1,a=1}$	Uniform (1.46e6, 4.82e8)	Uniform (1.46e6, 4.82e8)
$NO_{k=1,a=2}$	Uniform (2.91e5, 6.12e8)	Uniform (2.91e5, 6.12e8)
$NO_{k=1,a=3}$	Uniform (2.06e5, 5.34e7)	Uniform (2.06e5, 5.34e7)
$NO_{k=1,a=4}$	Uniform (8.37e4, 3.14e7)	Uniform (8.37e4, 3.14e7)
$NO_{k=1,a=5}$	Uniform (2.84e4, 9.75e6)	Uniform (2.84e4, 9.75e6)
$NO_{k=1,a=6}$	Uniform (1.72e4, 1.52e6)	Uniform (1.72e4, 1.52e6)
$NO_{k=1,a=7}$	Uniform (9.85e3, 8.45e5)	Uniform (9.85e3, 8.45e5)
$NO_{k=1,a=8}$	Uniform (5.26e3, 3.60e5)	Uniform (5.26e3, 3.60e5)
$NO_{k=1,a=9}$	Uniform (3.15e3, 2.75e5)	Uniform (3.15e3, 2.75e5)
$NO_{k=1,a=10}$	Uniform (1.72e3, 1.94e5)	Uniform (1.72e3, 1.94e5)
$NO_{k=1,a=11}$	Uniform (1.71e3, 1.74e5)	Uniform (1.71e3, 1.74e5)
$NO_{k=1,a=12}$	Uniform (748, 1.38e5)	Uniform (748, 1.38e5)
$NO_{k=1,a=13}$	Uniform (701, 1.09e5)	Uniform (701, 1.09e5)
$NO_{k=1,a=14}$	Uniform (674, 9.75e4)	Uniform (674, 9.75e4)
$NO_{k=1,a=15}$	Uniform (4.65e3, 1.50e6)	Uniform (4.65e3, 1.50e6)

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Table 2.8. Estimated value of natural mortality by age-class for each model with wide priors where \* denotes the value of natural mortality listed is calculated by dividing the estimate by the length-at-age for the first season of the age-class and † denotes parameter estimates that were approximately equal to the lower bound of the prior and ‡ denotes parameter values that are fixed for the model. Model abbreviations are described in the caption of Table 2.6.

Model	Age-0 M value	Age-1 M value	Age- 2 M value	Plus group M value	Lorenzen M
ConstM	0.1567	0.1567	0.1567	0.1567	N/A
3Ms	1.4601	0.0169†	0.0735	0.0735	N/A
3MsM1LB0.5	1.2316	0.5238†	0.0683	0.0683	N/A
LorM	2.2764*	0.4698*	0.2868*	0.1151*	3.6967
SepVsLorM	2.3713*	0.4894*	0.2988*	0.1199*	3.8509
SepVsSepMs	East 3.0775* West 1.9602*	East 0.6351* West 0.4045*	East 0.3878* West 0.2470*	East 0.1556* West 0.0991*	East 4.9976 West 3.1833
SepLorMs	East 2.8214* West 1.8030*	East 0.5822* West 0.3721*	East 0.3555* West 0.2272*	East 0.1424* West 0.0909*	East 4.5817 West 2.9279
ShrVsUB2	0.9329*	0.1925*	0.1176*	0.0472*	1.5150
ShrV1LorM	0.0160*	0.0033*	0.0020*	0.0008*	0.0261†
ShrV13Ms	0.0182†	0.0138†	0.0545	0.0545	N/A
FixedMsHigh	2.0‡	1.2‡	0.01‡	0.01‡	N/A
FixedMsLow	1.0‡	0.6‡	0.01‡	0.01‡	N/A

Table 2.9. Estimated value of natural mortality by age-class for each model with tight priors where \* denotes the value of natural mortality listed is calculated by dividing the estimate by the length-at-age for the first season of the age-class and ‡ denotes parameter values that are fixed for the model. 3MsHalf represents the model that assumed three M parameters with normal priors at values from SEDAR update assessment (2009). Other model abbreviations are described in the caption of Table 2.6.

Model	Age-0 M value	Age-1 M value	Age- 2 M value	Plus group M value	Lorenzen M
ConstM	0.1804	0.1804	0.1804	0.1804	N/A
3Ms	1.0428	0.7688	0.1496	0.1496	N/A
3MsHalf	1.1038	0.6600	0.1493	0.1493	N/A
LorM	2.4235*	0.5001*	0.3054*	0.1222*	3.9356
SepVsLorM	2.4859*	0.5130*	0.3132*	0.1253*	4.0399
SepVsSepMs	East 3.0084* West 1.2282*	East 0.6208* West 0.2534*	East 0.3791* West 0.1548*	East 0.1517* West 0.0619*	East 4.8854 West 1.9945
SepLorMs	East 2.7934* West 1.3560*	East 0.5764* West 0.2798*	East 0.3520* West 0.1709*	East 0.1408* West 0.0684	East 4.5363 West 2.2020
ShrV1LorM	2.4067*	0.4967*	0.3033*	0.1213*	3.9084
ShrV13Ms	0.0250	1.8457	0.1511	0.1511	N/A
FixedMsHigh	2.0‡	1.2‡	0.01‡	0.01‡	N/A
FixedMsLow	1.0‡	0.6‡	0.01‡	0.01‡	N/A

Table 2.10. The ten highest magnitude correlation coefficients are presented in the far right column for posterior samples of the two variables listed, middle columns, from various statistical catch-at-age models assuming tight priors, left column, for Gulf of Mexico red snapper. The definitions of each variable are presented in Table 2.1. 3MsHalf represents the model that assumed three M parameters with normal priors at values from SEDAR update assessment (2009). Other model abbreviations are described in the caption of Table 2.6.

Model	Parameter 1	Parameter 2	Correlation Coefficient
ConstM	$NO_{k=1,a=7}$	$NO_{k=1,a=10}$	0.8380
ConstM	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=3}$	0.8097
ConstM	$NO_{k=1,a=0}$	$NO_{k=1,a=1}$	-0.7841
ConstM	$0_{k=1,a=0}NO_{k=1,a=0}$	$f_{k=1,i=5,t=1}$	0.7811
ConstM	$NO_{k=2,a=3}$	$NO_{k=2,a=4}$	-0.7673
ConstM	$NO_{k=1,a=10}$	$0_{k=2,a=13}NO_{k=2,a=13}$	0.7599
ConstM	$NO_{k=1,a=1}$	$f_{k=1,i=5,t=1}$	-0.7463
ConstM	$v_{k=1\&2,i=5\&6,a=3}$	$v_{k=1\&2,i=5\&6,a=4}$	0.7438
ConstM	$NO_{k=1,a=7}$	$0_{k=2,a=6}NO_{k=2,a=6}$	0.7400
ConstM	$\alpha$	$\beta$	0.7379
3Ms	$M0$	$M1$	-0.8956
3Ms	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=3}$	0.8911
3Ms	$v_{k=1\&2,i=5\&6,a=3}$	$v_{k=1\&2,i=5\&6,a=4}$	0.8462
3Ms	$M0$	$v_{k=1\&2,i=9\&10,a=0}$	-0.8294
3Ms	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=4}$	0.8165
3Ms	$M1$	$Q_{k=2,i=10}$	-0.7765
3Ms	$M1$	$Q_{k=1,i=9}$	-0.7663
3Ms	$R_{k=2,t=21}$	$R_{k=2,t=22}$	0.7223
3Ms	$v_{k=1\&2,i=9\&10,a=0}$	$Q_{k=2,i=10}$	-0.7068
3Ms	$M1$	$v_{k=1\&2,i=9\&10,a=0}$	0.7043
3MsHalf	$M0$	$M1$	-0.9065
3MsHalf	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=3}$	0.8853
3MsHalf	$M0$	$v_{k=1\&2,i=9\&10,a=0}$	-0.8755
3MsHalf	$v_{k=1\&2,i=5\&6,a=3}$	$v_{k=1\&2,i=5\&6,a=4}$	0.8305
3MsHalf	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=4}$	0.8020
3MsHalf	$M1$	$Q_{k=2,i=10}$	-0.7941
3MsHalf	$M1$	$Q_{k=1,i=9}$	-0.7703
3MsHalf	$v_{k=1\&2,i=9\&10,a=0}$	$Q_{k=2,i=10}$	-0.7489
3MsHalf	$M1$	$v_{k=1\&2,i=9\&10,a=0}$	0.7453
3MsHalf	$NO_{k=2,a=4}$	$NO_{k=2,a=5}$	-0.7346
LorM	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=3}$	0.8737
LorM	$v_{k=1\&2,i=5\&6,a=3}$	$v_{k=1\&2,i=5\&6,a=4}$	0.8232
LorM	$NO_{k=1,a=10}$	$NO_{k=2,a=14}$	0.7884
LorM	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=4}$	0.7841
LorM	$M$	$R_{k=1,t=14}$	0.7821



LorM	$M$	$R_{k=1,t=13}$	0.7721
LorM	$M$	$R_{k=1,t=16}$	0.7719
LorM	$R_{k=2,t=21}$	$R_{k=2,t=22}$	0.7619
LorM	$NO_{k=1,a=0}$	$f_{k=1,i=5,t=1}$	0.7463
LorM	$NO_{k=1,a=0}$	$NO_{k=1,a=1}$	-0.7391
SepVsLorM	$v_{k=1,i=5,a=2}$	$v_{k=1,i=5,a=3}$	0.9589
SepVsLorM	$v_{k=1,i=5,a=3}$	$v_{k=1,i=5,a=4}$	0.9251
SepVsLorM	$v_{k=1,i=5,a=2}$	$v_{k=1,i=5,a=4}$	0.9086
SepVsLorM	$v_{k=1,i=5,a=1}$	$v_{k=1,i=5,a=2}$	0.8857
SepVsLorM	$v_{k=1,i=5,a=1}$	$v_{k=1,i=5,a=3}$	0.8686
SepVsLorM	$NO_{k=2,a=10}$	$NO_{k=2,a=15}$	-0.8568
SepVsLorM	$v_{k=2,i=6,a=2}$	$v_{k=2,i=6,a=3}$	0.8387
SepVsLorM	$v_{k=1,i=5,a=1}$	$v_{k=1,i=5,a=4}$	0.8278
SepVsLorM	$f_{k=1,i=5,t=11}$	$v_{k=1,i=5,a=2}$	-0.8183
SepVsLorM	$f_{k=1,i=5,t=8}$	$v_{k=1,i=5,a=2}$	-0.8105
SepVsSepMs	$M$ East	$R_{k=1,t=14}$	0.9053
SepVsSepMs	$M$ East	$R_{k=1,t=13}$	0.9014
SepVsSepMs	$M$ East	$R_{k=1,t=16}$	0.8945
SepVsSepMs	$M$ East	$R_{k=1,t=9}$	0.8873
SepVsSepMs	$R_{k=1,t=14}$	$R_{k=1,t=16}$	0.8715
SepVsSepMs	$M$ East	$R_{k=1,t=3}$	0.8631
SepVsSepMs	$M$ East	$R_{k=1,t=7}$	0.8613
SepVsSepMs	$M$ East	$R_{k=1,t=15}$	0.8581
SepVsSepMs	$M$ East	$R_{k=1,t=11}$	0.8564
SepVsSepMs	$R_{k=1,t=16}$	$R_{k=1,t=18}$	0.8550
SepLorMs	$M$ East	$R_{k=1,t=16}$	0.8809
SepLorMs	$M$ East	$R_{k=1,t=14}$	0.8789
SepLorMs	$M$ East	$R_{k=1,t=13}$	0.8717
SepLorMs	$M$ East	$R_{k=1,t=9}$	0.8494
SepLorMs	$M$ East	$R_{k=1,t=18}$	0.8464
SepLorMs	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=3}$	0.8428
SepLorMs	$R_{k=1,t=14}$	$R_{k=1,t=16}$	0.8266
SepLorMs	$M$ East	$R_{k=1,t=3}$	0.8194
SepLorMs	$M$ East	$R_{k=1,t=7}$	0.8184
SepLorMs	$M$ East	$R_{k=1,t=17}$	0.8105
ShrV1LorM	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=3}$	0.8625
ShrV1LorM	$v_{k=1\&2,i=5\&6,a=3}$	$v_{k=1\&2,i=5\&6,a=4}$	0.8082
ShrV1LorM	$M$	$R_{k=1,t=14}$	0.7849
ShrV1LorM	$M$	$R_{k=1,t=16}$	0.7781
ShrV1LorM	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=4}$	0.7736
ShrV1LorM	$M$	$R_{k=1,t=13}$	0.7724
ShrV1LorM	$R_{k=2,t=21}$	$R_{k=2,t=22}$	0.7619
ShrV1LorM	$NO_{k=1,a=0}$	$NO_{k=1,a=1}$	-0.7519
ShrV1LorM	$NO_{k=1,a=0}$	$f_{k=1,i=5,t=1}$	0.7472

ShrV1LorM	$M$	$R_{k=1,t=9}$	0.7404
ShrV13Ms	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=3}$	0.8501
ShrV13Ms	$v_{k=1\&2,i=5\&6,a=3}$	$v_{k=1\&2,i=5\&6,a=4}$	0.7615
ShrV13Ms	$f_{k=1,i=3,t=1}$	$f_{k=1,i=3,t=2}$	0.7511
ShrV13Ms	$NO_{k=1,a=0}$	$NO_{k=1,a=1}$	-0.7302
ShrV13Ms	$f_{k=1,i=3,t=2}$	$f_{k=1,i=3,t=3}$	0.7294
ShrV13Ms	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=4}$	0.7240
ShrV13Ms	$NO_{k=2,a=4}$	$NO_{k=2,a=5}$	-0.7101
ShrV13Ms	$M1$	$R_{k=1,t=16}$	0.7089
ShrV13Ms	$M1$	$R_{k=1,t=14}$	0.7033
ShrV13Ms	$NO_{k=1,a=0}$	$f_{k=1,i=5,t=1}$	0.7016
FixedMsHigh	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=3}$	0.8733
FixedMsHigh	$v_{k=1\&2,i=5\&6,a=3}$	$v_{k=1\&2,i=5\&6,a=4}$	0.8151
FixedMsHigh	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=4}$	0.7749
FixedMsHigh	$NO_{k=1,a=0}$	$f_{k=1,i=5,t=1}$	0.7643
FixedMsHigh	$NO_{k=1,a=0}$	$NO_{k=1,a=1}$	-0.7602
FixedMsHigh	$R_{k=2,t=21}$	$R_{k=2,t=22}$	0.7216
FixedMsHigh	$NO_{k=1,a=1}$	$f_{k=1,i=5,t=1}$	-0.6851
FixedMsHigh	$NO_{k=1,a=1}$	$NO_{k=1,a=2}$	-0.6738
FixedMsHigh	$v_{k=1\&2,i=5\&6,a=4}$	$v_{k=1\&2,i=5\&6,a=5}$	0.6662
FixedMsHigh	$NO_{k=2,a=8}$	$NO_{k=2,a=14}$	0.6614
FixedMsLow	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=3}$	0.8403
FixedMsLow	$v_{k=1\&2,i=5\&6,a=3}$	$v_{k=1\&2,i=5\&6,a=4}$	0.7823
FixedMsLow	$v_{k=1\&2,i=5\&6,a=2}$	$v_{k=1\&2,i=5\&6,a=4}$	0.7436
FixedMsLow	$R_{k=2,t=21}$	$R_{k=2,t=22}$	0.7195
FixedMsLow	$f_{k=1,i=3,t=1}$	$f_{k=1,i=3,t=2}$	0.6916
FixedMsLow	$\alpha$	$\beta$	0.6837
FixedMsLow	$NO_{k=2,a=4}$	$NO_{k=2,a=5}$	-0.6654
FixedMsLow	$f_{k=1,i=3,t=2}$	$f_{k=1,i=3,t=3}$	0.6614
FixedMsLow	$NO_{k=1,a=7}$	$NO_{k=2,a=9}$	0.6612
FixedMsLow	$f_{k=1,i=3,t=1}$	$f_{k=1,i=3,t=3}$	0.6514

Figure 2.1. Trace plot for three chains of saved values, where each chain is a different color, for Lorenzen curve natural mortality parameter is shown in the top portion from the model where a selectivity-at-age-1 parameter for bycatch from the shrimp fisheries is estimated, fisheries are assumed to have mirrored selectivity parameters across east and west Gulf of Mexico and natural mortality is modeled as a Lorenzen curve. The bottom portion of the figure shows the trace plot of the deviance calculated at each saved set of parameters for the three chains in this model.

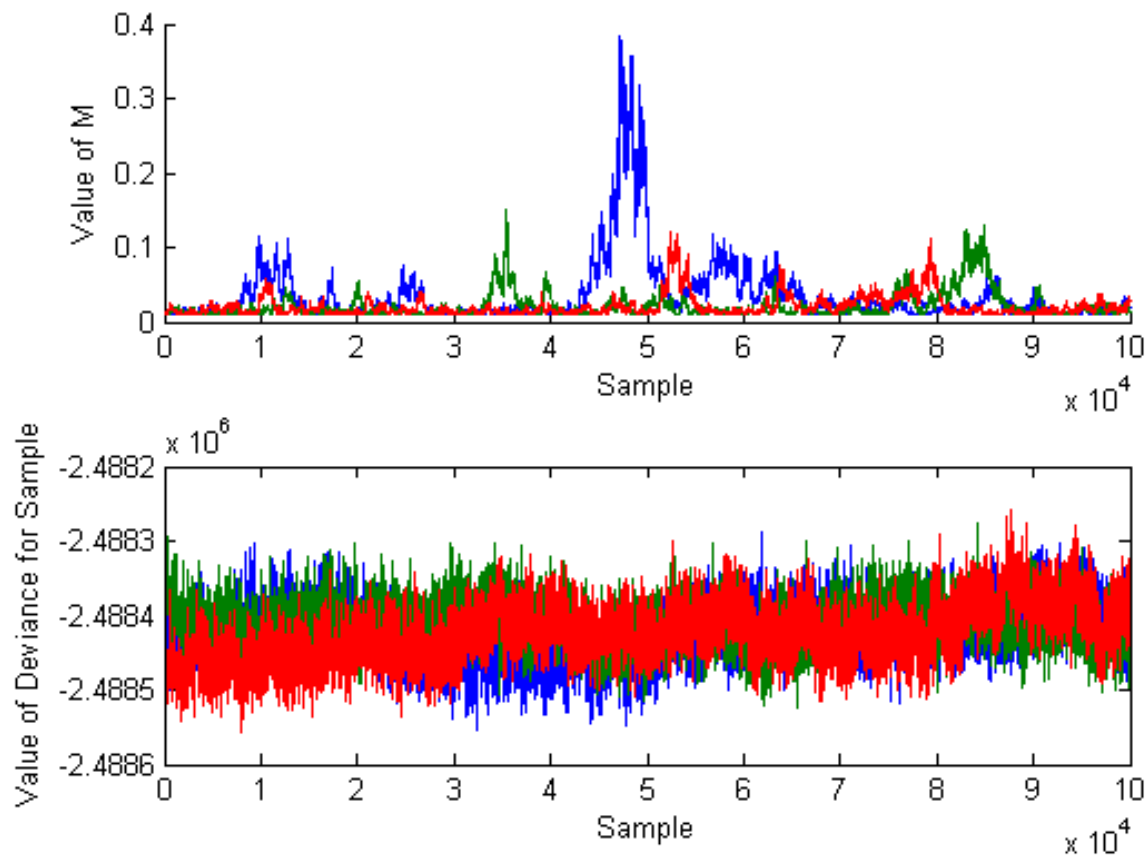


Figure 2.2. An example trace plot of poor convergence of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen M parameter with tight priors, where each color represents a different chain of saved values.

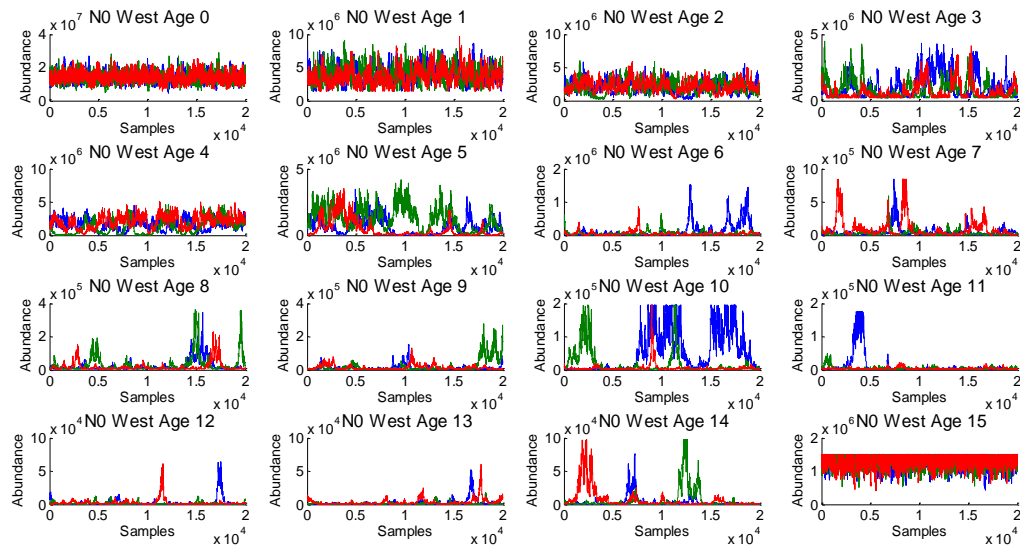


Figure 2.3. The estimated rates of natural mortality at age are depicted for model configurations with wide priors, where the title for each graph corresponds to the modeling assumptions. In the figure two separate Lorenzen curve functions were estimated for the models SepVsSepMs and SepLorMs. Model abbreviations are described in the caption of Table 2.6.

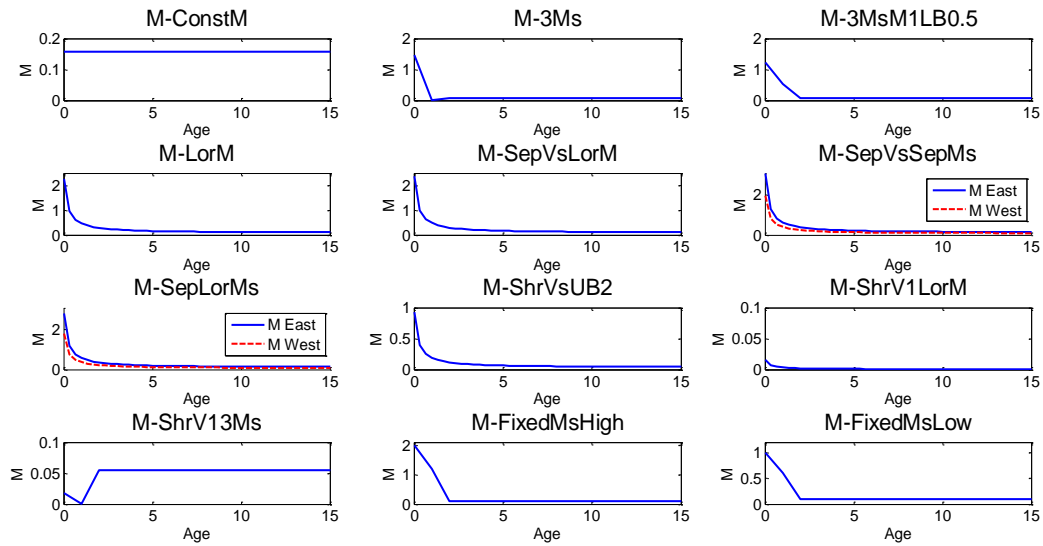


Figure 2.4. Selectivity-at-age curve for ages zero through three of bycatch from the shrimp fisheries for each of the model configurations with wide priors, as the title of the graph. Graphs that have only one line assumed selectivity was mirrored across east and west Gulf of Mexico fisheries, whereas graphs with two lines estimated separate parameters. Model abbreviations are described in the caption of Table 2.6.

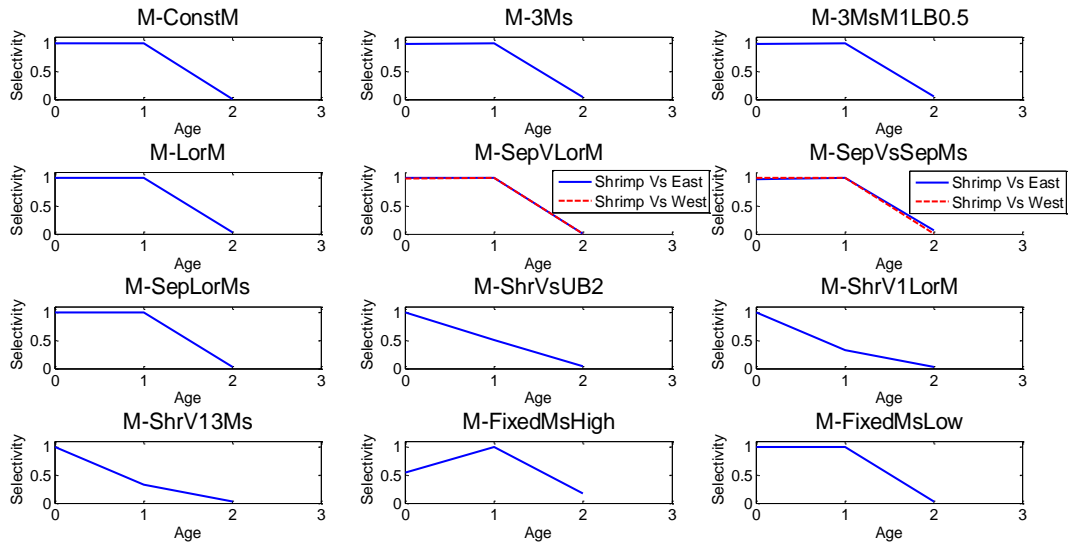


Figure 2.5. The harvest data for Gulf of Mexico red snapper from each fishery from the SEDAR update assessment (2009) as the points and the estimated value at the mean of the posterior distributions for all parameters for each model with wide priors as a line. CHLE and CHLW represents the commercial handline fishery in the east and west Gulf of Mexico (GOM), CLLE and CLLW is the commercial longline fishery in the east and west GOM, RE and RW represent the recreation fisheries in the east and west GOM, CLOSEDE and CLOSEDW represent the bycatch from the closed-season fisheries in the east and west GOM, SHRE and SHRW represent the bycatch from the shrimp fisheries in the east and west GOM, and NMFS BLL represents the NMFS bottom-longline survey. Original Data is the values from the SEDAR update assessment (2009) to which each of the models were fit. Model abbreviations are described in the caption of Table 2.6.

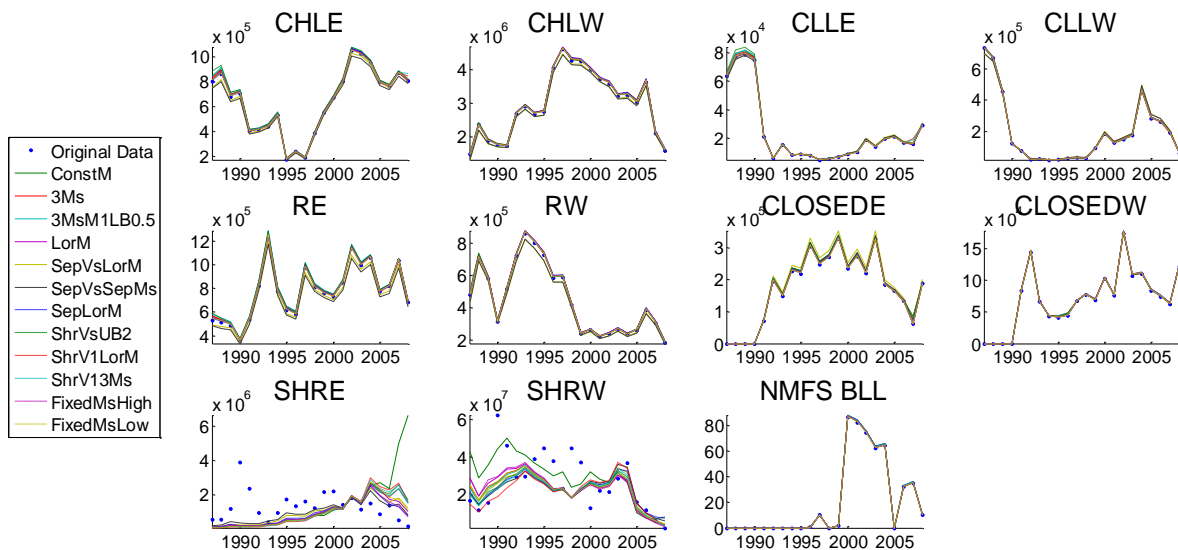


Figure 2.6. The mean of the posterior distribution for the selectivity-at-age parameter for each fishery estimated by the models indicated with wide priors. CHLE and CHLW represents the commercial handline fishery in the east and west Gulf of Mexico (GOM), CLLE and CLLW is the commercial longline fishery in the east and west GOM, RE and RW represent the recreation fisheries in the east and west GOM, CLOSEDE and CLOSEDW represent the bycatch from the closed-season fisheries in the east and west GOM, SHRE and SHRW represent the bycatch from the shrimp fisheries in the east and west GOM, and NMFS BLL represents the NMFS bottom-longline survey. Model abbreviations are described in the caption of Table 2.6.

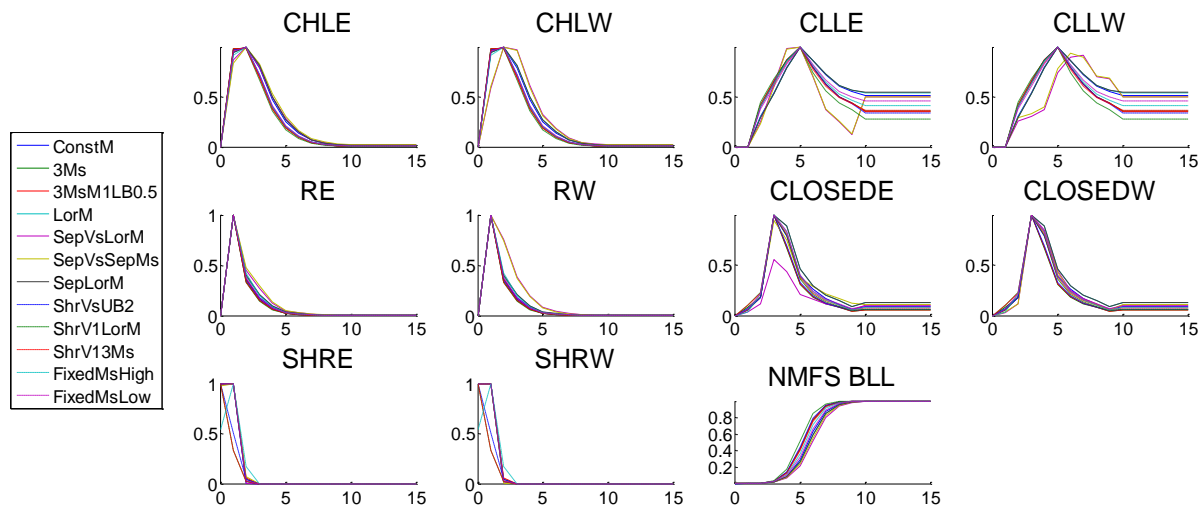




Figure 2.7. Shrimp fishing effort from the SEDAR update assessment (2009) plotted as points (Original Data) versus the estimated value for each model with wide priors as the mean of the posterior distribution. Model abbreviations are described in the caption of Table 2.6.

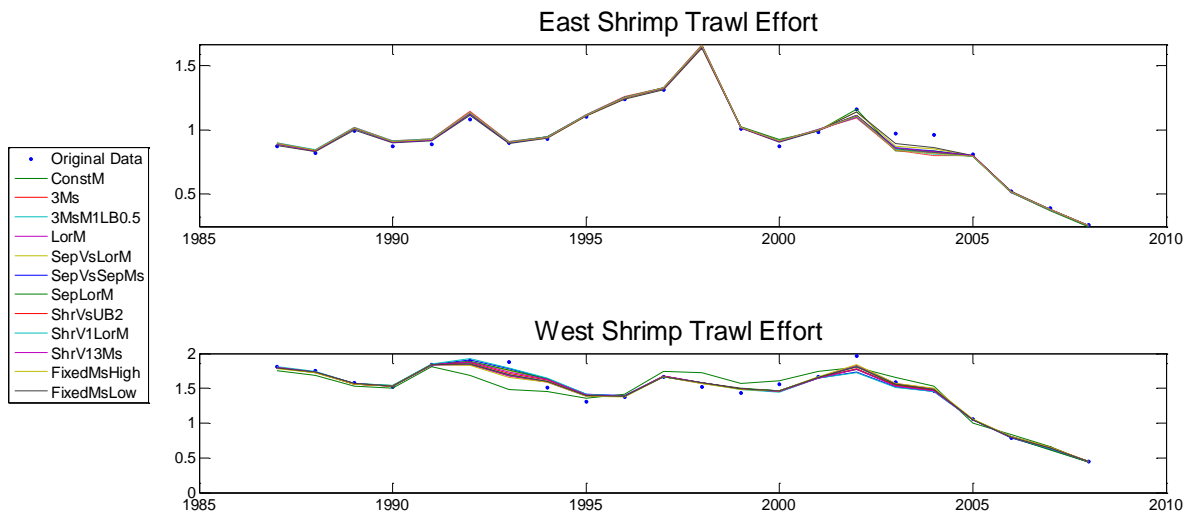


Figure 2.8. Log-scale abundance indices from surveys for Gulf of Mexico red snapper extracted from SEDAR update assessment (2009) plotted as points (Original Data) versus the expected value from each model with wide priors at the mean of the posterior for the parameter estimates. CHLE Index and CHLW Index are the catch per unit effort for the commercial handline fishery in the east and west Gulf of Mexico (GOM), MRFSS E and MRFSS W are the catch per unit effort from the recreation fisheries in the east and west GOM, Video E and Video W are the SEAMAP reef fish video survey in the east and west GOM, Larval E and Larval W are the SEAMAP ichthyoplankton survey in the east and west GOM, Trawl 1 E, Trawl 1 W, Trawl 0 E, and Trawl 0 W are the SEAMAP bottom trawl survey for age-0 and age-1 red snapper in the east and west GOM respectively, and NMFS BLL is NMFS bottom-longline survey in the western GOM. Model abbreviations are described in the caption of Table 2.6.

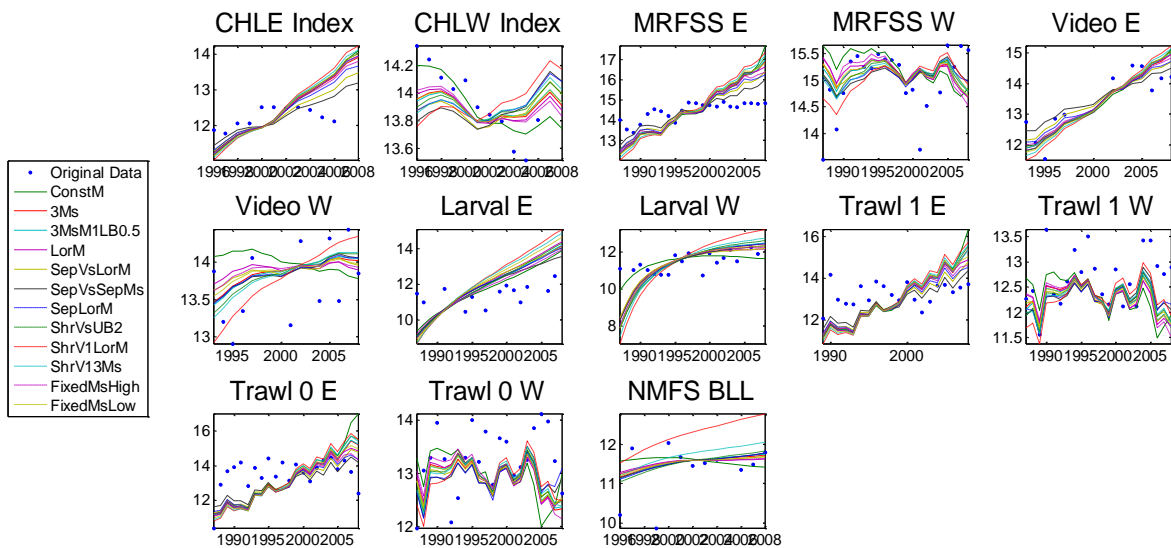


Figure 2.9. Number of red snapper age two and older in the eastern GOM estimated by the respective models with wide priors and the estimates from the SEDAR update assessment (2009) labeled as Catchem. Model abbreviations are described in the caption of Table 2.6.

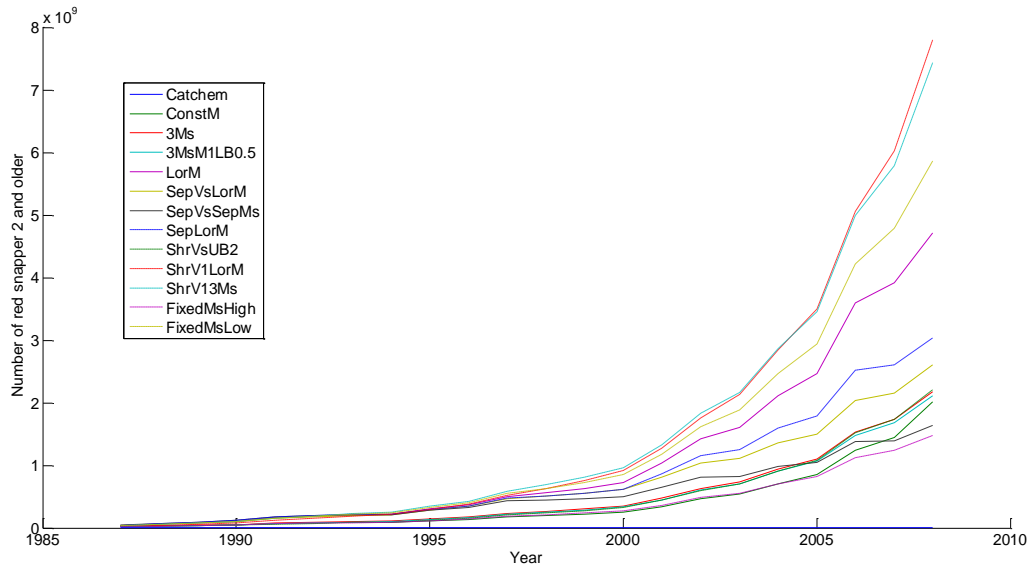




Figure 2.11. Number of red snapper age two and older in the eastern GOM estimated by the respective models with tight priors and the estimates from the SEDAR update assessment (2009) labeled as Catchem. 3MsHalf represents the model that assumed three M parameters with normal priors at values from SEDAR update assessment (2009). Other model abbreviations are described in the caption of Table 2.6.

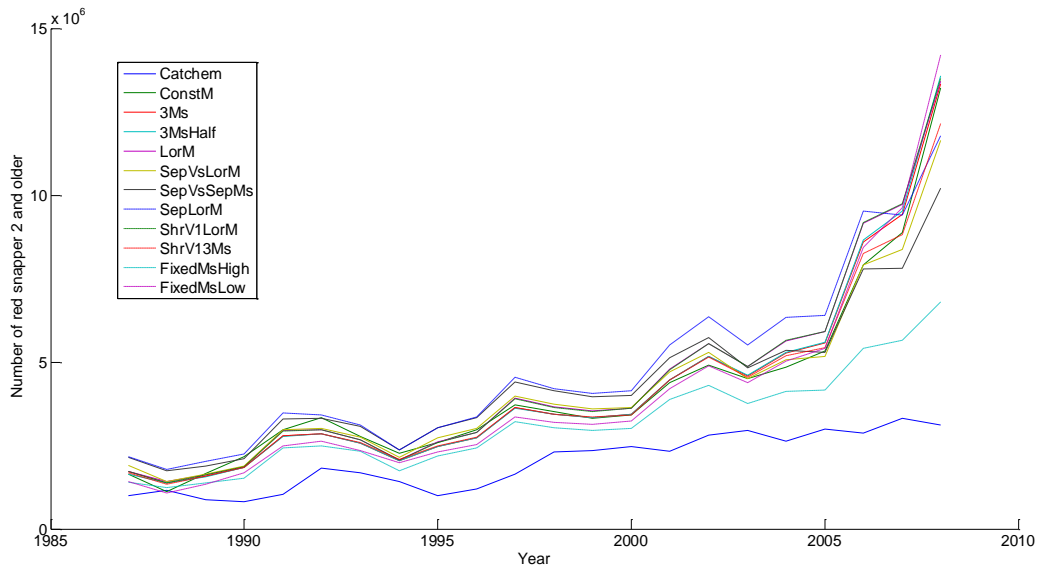


Figure 2.12. Number of red snapper age two and older in the western GOM estimated by the respective models with tight priors and the estimates from the SEDAR update assessment (2009) labeled as Catchem. 3MsHalf represents the model that assumed three M parameters with normal priors at values from SEDAR update assessment (2009). Other model abbreviations are described in the caption of Table 2.6.

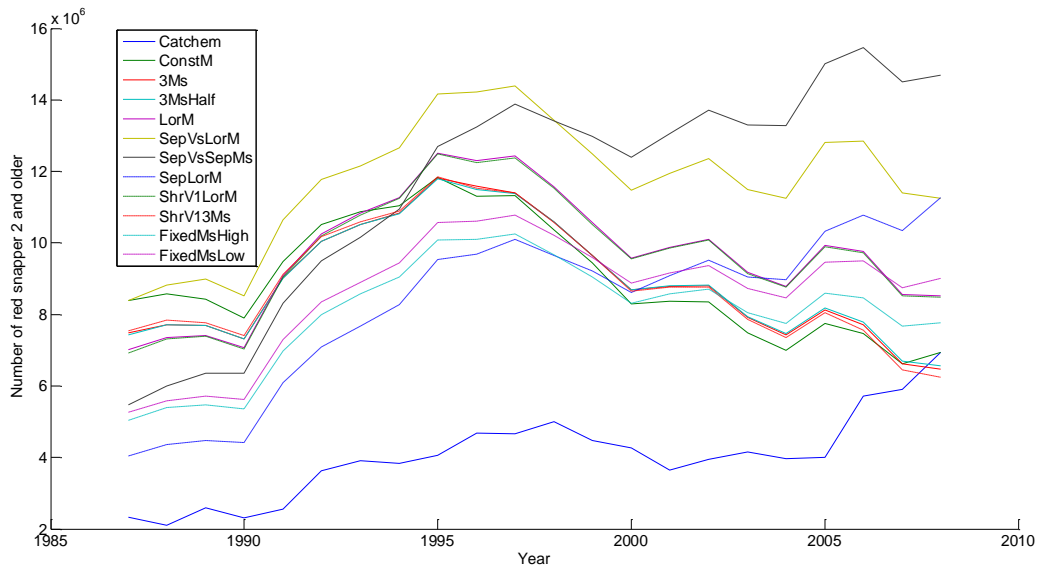


Figure 2.13. The estimated rates of natural mortality at age are depicted for model configurations with tight priors, where the title for each graph corresponds to the modeling assumptions. In the figure two separate Lorenzen curve functions were estimated for the models SepVsSepMs and SepLorMs. 3MsHalf represents the model that assumed three M parameters with normal priors at values from SEDAR update assessment (2009). Other model abbreviations are described in the caption of Table 2.6.

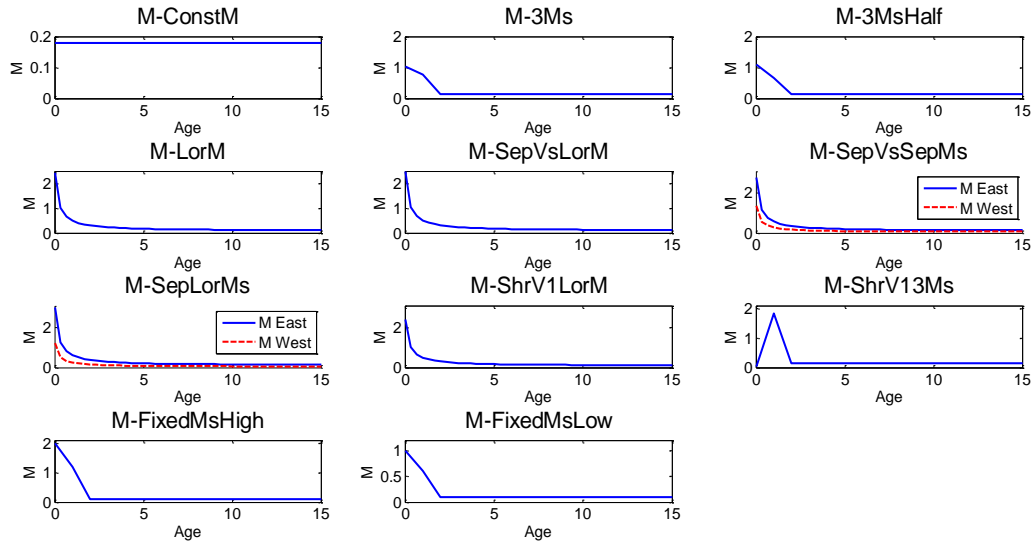


Figure 2.14. Selectivity-at-age curve for ages zero through three of bycatch from the shrimp fisheries for each of the model configurations with tight priors, as the title of the graph. Graphs that have only one line assumed selectivity was mirrored across east and west Gulf of Mexico fisheries, whereas graphs with two lines estimated separate parameters. 3MsHalf represents the model that assumed three M parameters with normal priors at values from SEDAR update assessment (2009). Other model abbreviations are described in the caption of Table 2.6.

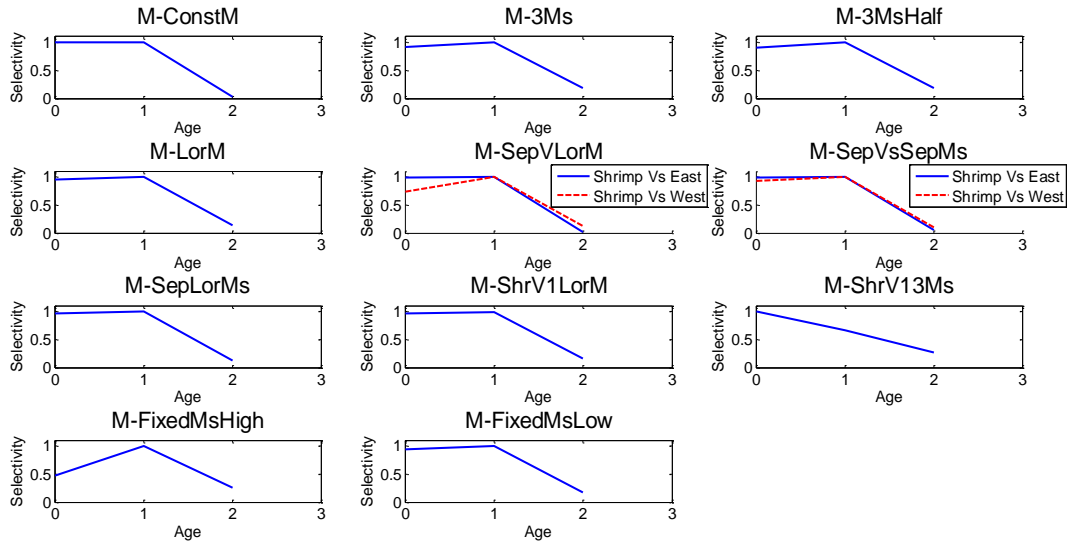




Figure 2.15. The harvest data for Gulf of Mexico red snapper from each fishery from the SEDAR update assessment (2009) as the points and the estimated value at the mean of the posterior distributions for all parameters for each model with tight priors as a line. CHLE and CHLW represents the commercial handline fishery in the east and west Gulf of Mexico (GOM), CLLE and CLLW is the commercial longline fishery in the east and west GOM, RE and RW represent the recreation fisheries in the east and west GOM, CLOSEDE and CLOSEDW represent the bycatch from the closed-season fisheries in the east and west GOM, SHRE and SHRW represent the bycatch from the shrimp fisheries in the east and west GOM, and NMFS BLL represents the NMFS bottom-longline survey. Original Data is the values from the SEDAR update assessment (2009) to which each of the models were fit. 3MsHalf represents the model that assumed three M parameters with normal priors at values from SEDAR update assessment (2009). Other model abbreviations are described in the caption of Table 2.6.

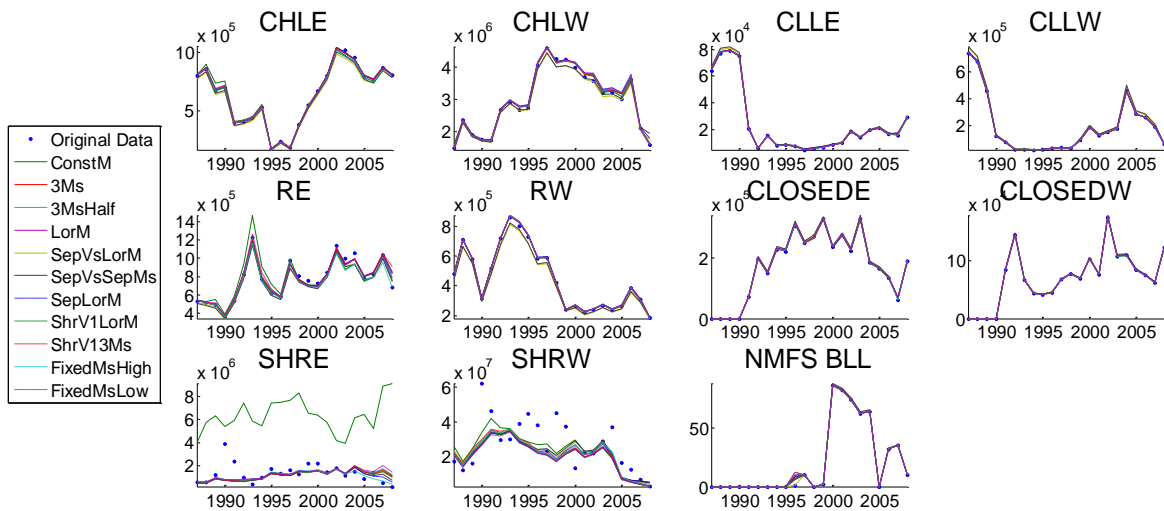


Figure 2.16. Shrimp fishing effort from the SEDAR update assessment (2009) plotted as points (Original Data) versus the estimated value for each model with tight priors as the mean of the posterior distribution. 3MsHalf represents the model that assumed three M parameters with normal priors at values from SEDAR update assessment (2009). Other model abbreviations are described in the caption of Table 2.6.

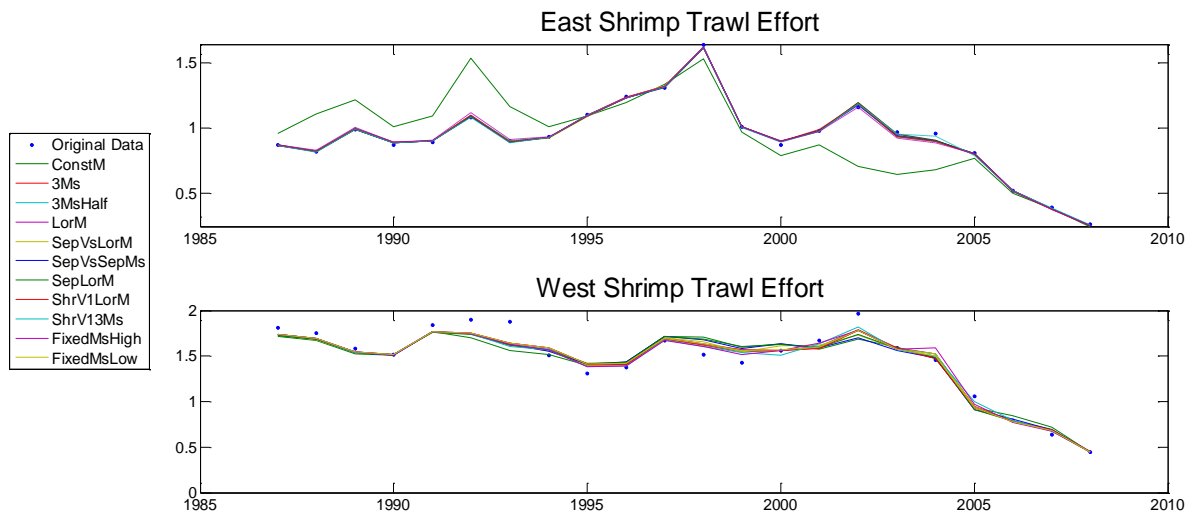
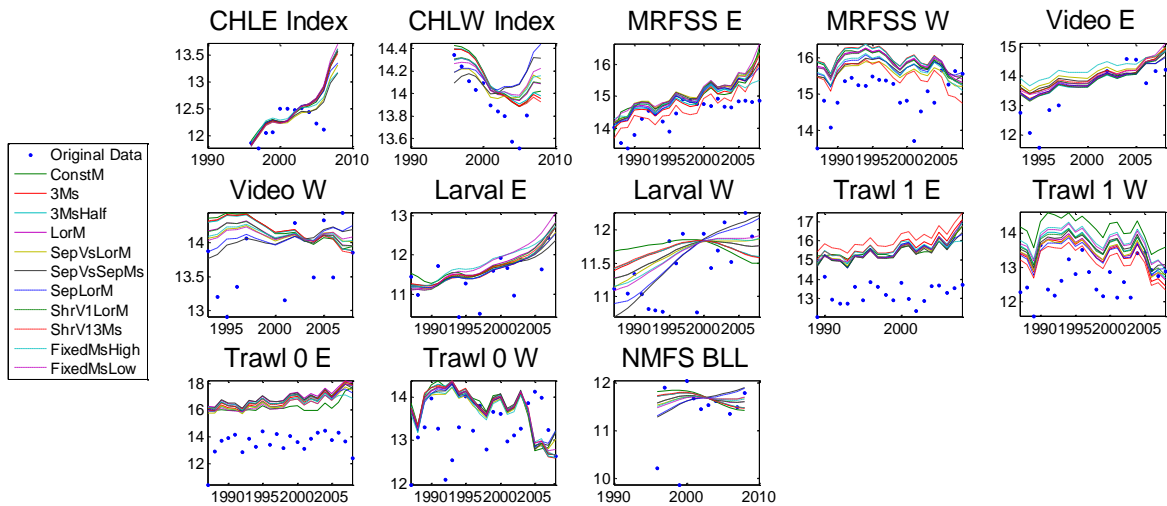


Figure 2.17. Log-scale abundance indices from surveys for Gulf of Mexico red snapper extracted from SEDAR update assessment (2009) plotted as points (Original Data) versus the expected value from each model with tight priors at the mean of the posterior for the parameter estimates. CHLE Index and CHLW Index are the catch per unit effort for the commercial handline fishery in the east and west Gulf of Mexico (GOM), MRFSS E and MRFSS W are the catch per unit effort from the recreation fisheries in the east and west GOM, Video E and Video W are the SEAMAP reef fish video survey in the east and west GOM, Larval E and Larval W are the SEAMAP ichthyoplankton survey in the east and west GOM, Trawl 1 E, Trawl 1 W, Trawl 0 E, and Trawl 0 W are the SEAMAP bottom trawl survey for age-0 and age-1 red snapper in the east and west GOM respectively, and NMFS BLL is NMFS bottom-longline survey in the western GOM. 3MsHalf represents the model that assumed three M parameters with normal priors at values from SEDAR update assessment (2009). Other model abbreviations are described in the caption of Table 2.6.



## **Chapter 3 – Estimability of natural mortality within a statistical catch-at-age model: A simulation study based on the Gulf of Mexico red snapper fishery**

### **Abstract**

Methodology for the reliable estimation of natural mortality within a statistical catch-at-age model has not yet been widely used in fisheries assessments. A simulation study of the complex fisheries dynamics of the Gulf of Mexico red snapper was conducted to investigate the precision and accuracy of the estimation of a Lorenzen natural mortality curve and other model parameters. The Lorenzen natural mortality curve was calculated by dividing an estimated parameter by the length at age in a season from a von Bertalanffy growth equation. All relevant data were collected and a set of “true” population dynamics parameters were estimated, from which new datasets were simulated. A total of 250 datasets were simulated, from which simulation estimates were derived and compared against the “true” parameter estimates. The initial abundance-at-age parameters in the simulation estimates were often approximately equal to one of the bounds of the prior. Thus the posterior distributions of the simulation models were likely not reached and the results of this study are preliminary. Additionally, the simulation results suggest that the initial abundance-at-age parameters are inestimable. The Lorenzen natural mortality parameter showed a slight underestimation in all simulation estimates with an average underestimation of 5%. Recruitment parameters were highly variable but not biased in any direction, and appear to estimate the trend of recruitment over time well. However, apical fishing mortality parameters were biased toward underestimation by approximately 25%. The estimation of a Lorenzen natural mortality curve within a statistical catch-at-age model is likely to remain a controversial topic due to the slight bias towards underestimation of natural mortality and bias to underestimate fishing mortality parameters.

## Introduction

Natural mortality is an extremely important population parameter, but reliable estimation of this parameter within statistical catch-at-age models has been limited. Historically, stock assessments use methods outside of the statistical catch-at-age model to estimate natural mortality. These methods have many limitations, caveats, and assumptions associated with them, which makes their use less than ideal (Pascual and Iribarne 1992; Brodziak et al 2009). Additionally, incorrect assignment of natural mortality within a statistical catch-at-age analysis can cause improper evaluation of key population dynamics parameters and biological reference points used for management (Sims 1984; Saila et al. 1985; Lapointe et al. 1989; Bax 1993; Mertz and Myers 1997; Clark 1999; Xiao and Wang 2007). However, estimation of natural mortality within statistical catch-at-age models has proven to be difficult, even with sufficient data sources (Butler and McDonald 1979; Paloheimo 1980; Paloheimo et al. 1982; Thompson 1994; Fu and Quinn 2000).

Recent studies suggest that the estimation of natural mortality within a statistical catch-at-age model is possible. The study by Wang and Liu (2006) is a basis from which more complicated statistical catch-at-age models can be built to estimate natural mortality. Similarly, Lee et al (2011) discovered that the inclusion of high-quality length- or age-composition data is necessary to reliably estimate  $M$  within an assessment model. Yet these data alone are not always sufficient to prevent high correlation with parameters in the model, resulting in unrealistic estimates of  $M$  (Paloheimo et al. 1982; Thompson 1994; Schnute and Richards 1995; Clarke 1999; Fu and Quinn 2000; Lee et al 2011). Other parameters that are often highly correlated with  $M$  are those associated with production, such as growth parameters and recruitment curve steepness parameters, or those that describe the decline observed in proportions at age of older

age-classes, such as selectivity. Therefore, Lee et al. (2011) concluded that model misspecification is the cause for unrealistic estimates of  $M$  that are approximately equal to the bounds of the parameter rather than an inability to estimate  $M$  from the data. On the other hand, Clark (1999) postulated that the estimation of natural mortality within a statistical catch-at-age model may merely be the result of the assumed shape of the selectivity-at-age curve. Despite these studies, the estimation of natural mortality within assessments used to advise management for harvested species is rare, as uncertainty and questions remain.

The goal of this study was to determine the estimability of natural mortality within a statistical catch-at-age model that incorporates multiple fisheries, multiple surveys, landings data, effort data and age-composition data of the landed fish, using the Gulf of Mexico (GOM) red snapper as the model data source. This goal was pursued through a simulation study in which the available data for GOM red snapper were used to estimate a set of “true” parameters for the model. From these estimated values, datasets of the same type and availability as the real data were simulated. The simulated data were used to estimate the parameters for the simulation model, which were then compared against the “true” parameter values. This comparison identifies the precision and accuracy with which the parameters in the model are estimated.

## **Methodology**

A simulation study procedure was conducted to evaluate the estimability of natural mortality for GOM red snapper within a statistical catch-at-age model. This methodology estimated a set of “true” parameters from available data using Bayesian techniques. A data-generating model used the “true” parameters to create data of the same form as the available data for red snapper. The generated data were then used in an estimation model to estimate the population dynamic parameters. Performance of the model to estimate parameters was evaluated.

All data sources used to estimate the “true” model parameters described below were obtained from the SEDAR update assessment (2009). Descriptions of each symbol used are presented in Table 3.1, the equations used in the models are presented in Table 3.2, and the equations used for the estimation models are presented in Table 3.3. The equations are referenced as Equation x.y where equation y is within Table 3.x.

### ***“True” Parameter Estimation Model***

#### *Landings Data*

Based upon the precedent set by SEDAR 7 (2005), the red snapper SEDAR update assessment (2009) and SEDAR 31 (2013), the landings data by fisheries were separated into two fisheries by region with one fishery operating in the eastern GOM and the other in the western GOM. There are four directed commercial fisheries for which landings in weight of red snapper in the GOM were reported: handline east, handline west, longline east and longline west. The model used data on the commercial landings for these fisheries from 1987 to 2008. The year 1987 was selected as the starting date of the model to coincide with the first year of available data for the SEAMAP age-1 survey because it was hypothesized that the SEAMAP age-1 and SEAMAP age-0 surveys would be influential in the estimation of natural mortality. Additionally, starting in this year significantly reduced the number of recruitment parameters required to be estimated compared to starting in 1872. Landings data in number of fish for the recreational fisheries separated between eastern and western GOM were used for all years in the model. The landed catch and dead discards of fish larger than the fishery specific legal-harvest size limit are incorporated in the landings estimates for both commercial and recreational fisheries. Estimates of discards for fish smaller than the harvest size limit are calculated by the model and are

described below. The numbers of fish caught by the NMFS bottom-longline survey for intermittent years from 1996 to 2008 were incorporated into the models as landings.

Two sources of bycatch estimates from the update assessment were used in the model and treated as the landings of a fishery with no discards: bycatch in the shrimp trawl fisheries and bycatch during the closed-season fisheries. Bycatch estimates in number of red snapper caught by the shrimp trawl fishery from 1987 to 2008 for both regions of the GOM were used in the models. Closed-season bycatch estimates from 1991 to 2008 for the eastern and western GOM were incorporated into the models as number of fish caught by a hypothetical fishery, where the legal-harvest size limit is zero and all fish caught are “harvested”. Bycatch estimates from the shrimp fisheries were treated as the total number of fish caught by a shrimp trawl fishery, where all fish caught are assumed to be “harvested” and the legal-harvest size limit was zero.

#### *Age-Composition Data*

Age-composition data for the eastern and western GOM were calculated from subsamples of the catch from the commercial longline fishery, commercial handline fishery, and recreational fishery, and incorporated into the model for available years of data from 1991 to 2008. Proportions of age-0, age-1, and age-2 red snapper bycatch from the shrimp fisheries were used in the model for the eastern and western GOM from 1992 to 2008. The age-composition data were calculated in the SEDAR update assessment (2009) for the bycatch of red snapper during the closed-season fisheries in the eastern and western GOM and are available from 1991 to 2003. The final age-composition data are from the western GOM NMFS bottom-longline survey for 2000 through 2008, but not 2005. These 11 age-composition data sources were represented as coming from separate fisheries by the model, which correspond to specific fishery numbers that were used during modeling are presented in Table 3.4. Though the NMFS bottom-



longline survey does not operate like a commercial fishery, this data source was modeled as a fishery following the precedent of the SEDAR update assessment (2009). Numbers of samples obtained for each year and fishery were used in the statistical estimator, but sample sizes greater than 200 were capped at 200 to reduce the weight the age-composition data had upon model fits in the estimation models. The uncapped numbers of samples taken for each fishery by year were used to simulate the age-composition data in the data-generation model.

#### *Shrimp Fishing Effort Data*

Estimates of offshore shrimp fishing effort from 1987 to 2008 for the eastern and western GOM were used in the model. Fishing effort estimates were used together with estimates of bycatch harvest by the shrimp fisheries from 1987 to 2008 to improve the accuracy of the bycatch estimates from the shrimp fisheries. Both data sources were used to estimate a catchability coefficient for bycatch from the shrimp fisheries in the model to appropriately scale the fishing effort. Estimates of variability given as CVs for each year of available data were used as a measure of variance in the statistical estimator. These estimates and the rationale behind the values can be found in the SEDAR update assessment (2009).

#### *Indices of Abundance Data*

Relative abundance indices from SEDAR update assessment (2009) used in this study were from both fishery-dependent and fishery-independent sources. The eight fishery-independent abundance indices used in the model are presented in Table 3.4 and their associated number for modeling purposes. The model incorporated abundance indices that were calculated from fishery-dependent sources for the commercial handline fisheries in the eastern and western GOM from 1996 to 2006, for the recreational fisheries in the eastern and western GOM from 1981 to 2008, and for the NMFS bottom-longline survey from 1996 to 2008, excluding 1998 and

2005. The SEAMAP reef fish video surveys from the eastern and western GOM were used for intermittently available years from 1993 to 2008. The SEAMAP larval or ichthyoplankton survey data, collected using bongo nets in the eastern and western GOM, were available from 1987 to 2007. The SEDAR update assessment (2009) found that the SEAMAP larval index was highly correlated to the number of adult red snapper and used it as an index of spawning stock. Similar formulation, as described below, was used to calculate the expected value of this survey in all the models. The SEAMAP bottom trawl survey abundance indices of age-0 red snapper for both the eastern and western GOM were used in the models for years 1987 to 2008. The SEAMAP bottom trawl survey abundance indices of age-1 red snapper for the eastern GOM from 1987 to 2008 and for the western GOM from 1989 to 2008 were incorporated into the models. The SEAMAP bottom trawl survey data were used as one index in SEDAR 7 (2005), but were separated into distinct indices for age 0 and age 1 as were used in the SEDAR update assessment (2009). All indices were scaled to ensure catchability coefficients were close to but no more than 1.0 and then the natural logarithm was applied, to allow for easier parameter estimation.

#### *Estimated Variability of Data*

For the landings data, abundance indices, and effort data, CVs for each year of available data were used as a measure of variance in the statistical estimator. The calculation of the variance for each data point is described below (Table 3.3). These estimates and the rationale behind the values can be found in the SEDAR update assessment (2009).

#### *Relative Fecundity Data*

The update assessment of red snapper used a measure of relative fecundity, where age-30 red snapper have a relative fecundity equal to 1.0 and all younger ages are calculated from a

logistic function adopted in SEDAR 7 (2005). These values were used in the model to weight the contribution of each age-class in the spawning stock to the egg production estimated by the model for comparison to the SEAMAP larval survey data.

#### *“True” Model Calculations*

The model described by Porch (2007) was a basis from which the model was built utilizing the red snapper data described above. The model spans the time period 1987 to 2008, with three seasons in each year and spawning is assumed to occur during the second season. Each age-class is assumed to progress to the next age-class after 3 full seasons, i.e. at the transition from the first season to the second season (Equations 3.2a, 3.2b, and 3.2c). The model uses 16 age-classes, starting at age 0 and progressing to a plus group at age 15, which is assumed to represent the abundance of fish 15 years old and older as described by Equations 3.3a, 3.3b, and 3.3c.

Despite the recent season closures, commercial and recreational fisheries were modeled as operating all year every year because season closures varied from year to year and did not always occur strictly within seasons. The impacts of this assumption are assumed to be relatively small because season closures did not occur until 1999 and this assumption was used in the SEDAR update assessment (2009). To incorporate the red snapper killed as dead discards during the red snapper closed-season, bycatch during the closed-season fisheries were modeled as operating during the second season where all fish caught were “harvested” and the legal-harvest size limit was equal to zero. Similarly, the red snapper killed as bycatch in the shrimp fisheries were treated as “harvest” with the legal-harvest size limit equal to zero. The model assumed no aging error was present in the age-composition data and did not model process error. Differing from the methodology used in the SEDAR update assessment (2009), recruitment parameters in

each year and parameters for the abundance in each age-class for the first year of the model were estimated as free parameters (Equation 3.1a and 3.1b).

The process equations of the model are shown by equations 3.1a, 3.1b, 3.2a, 3.2b, 3.2c, 3.3a, 3.3b, and 3.3c. The initial age-class in season 1 and recruitment in season 2 in number of fish are parameters estimated by the model and form the basis from which the rest of the process equations are calculated (Equations 3.1a and 3.1b). The calculations to produce abundance at age for ages that are younger than the plus group (age 15) through seasons and years are described below. The transition from season 1 to season 2 in the process equation calculates an exponential decay due to total mortality ( $Z$ ) applied to a cohort which is then moved to the next age-class and the next season (Equation 3.2a). In season 2 a cohort is transitioned to season 3 after applying a negative exponential population function (Equation 3.2b). After applying an exponential decay to the number of fish in season 3, the cohort is moved to the first season of the next year, but kept in the same age-class (Equation 3.2c). These calculations are slightly different for the plus group at age 15. The transition from season 1 to season 2 is the sum of the two negative exponential population functions for age 14 and the plus group of age 15. The transition of season 2 to season 3 applies an exponential decline due to  $Z$  to the plus group, while remaining in the same year and plus group. In season 3 an exponential population decline is applied to the plus group which is then moved to the next year but kept at age 15.

In order to apply the exponential decay due to total mortality ( $Z$ ), this parameter must first be calculated.  $Z$  is the sum of fishing mortality ( $F$ ) for all fisheries that operate in the specific region, season and year plus the instantaneous rate of natural mortality ( $M$ ) divided by the number of seasons (Equation 2.4). The natural mortality for each age and season is calculated as the estimated parameter  $LM$  divided by the calculated length-at-age and season using the von

Bertalanffy growth curve in Equation 3.8. The Lorenzen natural mortality curve was calculated in this fashion because the exponent in the length to weight conversion (Equation 3.11) is close to 3 and the exponent estimated by Lorenzen (1996) is approximately one third and would thus cancel out. Therefore, a different natural mortality rate is assumed to impact a cohort each season and younger ages experience a higher rate of natural mortality than older age-classes (Equation 3.12). The calculation of fishing mortality for all fisheries is calculated by a fully separable fishing mortality model (Equation 3.5a and 3.5b). Fishing mortality calculation, for fisheries except the bycatch from the shrimp fisheries, incorporates an age effect through vulnerability or selectivity at age, a year effect through an annual apical fishing mortality, an annual harvest size limit calculates the probability that a fish dies once it is caught based upon the limit for the fishery given the age of the fish during a specific year, an indicator variable for whether the fishery is operating during that season, and an indicator variable for if the fishery is operating in a region, divided by the number of seasons that the fishery is active (Equation 3.5a). For the bycatch from the shrimp fisheries  $F$  is calculated by the product of an age effect through selectivity at age, a year effect from the shrimp fishing effort parameter, an indicator variable for if the fishery is operating during that season, and an indicator variable for whether the fishery is operating in a region divided by the number of seasons that the fishery is active times the natural logarithm of the catchability coefficient divided by the number of seasons the fishery is active (Equation 3.5 b). The natural logarithm of the catchability coefficient was used because it made for better convergence of the Metropolis-Hastings algorithm to the posterior distribution of the parameter.

The selectivity-at-age parameters were modeled in the same manner as the SEDAR update assessment (2009) of red snapper. In general for the directed commercial fisheries,

recreational fisheries, and bycatch during the closed-season fisheries, the selectivity-at-age-0 parameter was assumed to equal zero, ages 1 through 9 were estimated as separate parameters and ages 10 through 15 were assumed to equal one estimated parameter, though some slight differences between fisheries were modeled and are explained in the SEDAR update assessment (2009). The selectivity-at-age parameters for age 3 and older for bycatch from the shrimp fisheries were assumed to equal zero, selectivity parameters for ages 0 and 2 were estimated, and the selectivity-at-age-1 parameter was assumed to equal one. The age-composition data from the NMFS bottom-longline survey was assumed to have too few samples to estimate selectivity-at-age parameters for all ages separately, so a logistic function was used, where  $\alpha$  and  $\beta$  were estimated by the model (Equation 3.6). Fisheries of the same type, i.e. recreational fishery in the eastern GOM and recreational fishery in the western GOM, were assumed to have the same selectivity to reduce the number of estimated parameters.

The probability that a fish would die once it was caught was modeled in the same fashion as the SEDAR update assessment (2009) and is based on the assumption that only fish that are less than the annual harvest size limit are discarded during the open season and all fish larger than the limit are harvested. Therefore,  $\zeta$  is calculated as 1 minus the product of the probability that a fish lives if it is caught (1 minus the discard mortality for the fishery) and the probability that a fish is less than the size limit ( $G_{L|a,s}$ ) (Equation 3.7). The values for the discard mortality rates were taken from the SEDAR update assessment (2009) of red snapper for each fishery. The discard mortality for the commercial handline and commercial longline fisheries was assumed to be 0.71 in the eastern GOM and 0.82 in the western GOM. For the recreational fisheries discard mortality was assumed to equal 0.15 in the eastern GOM and 0.40 in the western GOM. The differences in discard mortality were due to differences in the assumed depths at which the

fisheries caught red snapper and the assumed benefit of gas bladder venting required in the recreational fishery (SEDAR update assessment 2009). All other fisheries were modeled to harvest all fish and thus there were no discards. The probability that a fish is less than the size limit is calculated based on the von Bertalanffy growth curve presented in Equation 3.8. The von Bertalanffy growth curve used in this study was not used in the assessment model of SEDAR 31 (2013) or in the SEDAR update assessment (2009) but was the most current growth model from the data workshop of SEDAR 31 at the time of this study. The probability that a fish is less than the size limit was calculated assuming a discretized gamma distribution scaled to a maximum of one following the methodology used in the model described by Porch (2007).

Once the abundance in each age-class during all seasons was calculated by the process equations described above, the catch was calculated. The catch equation represented by Equation 3.9 is the Baranov catch equation divided by the probability that the fish dies once it is caught. This formulation was used because the probability that the fish dies cancels out in the fishing mortality, but is still accounted for in the total mortality and therefore calculates the total number of fish that are caught. Since landings data is the actual harvest of fish and does not include discards, the expected number of fish harvested is calculated as the number caught by a fishery times the probability that a fish is over the size limit given its age dependent upon the legal-harvest size limit for that fishery in that year (Equation 3.10a). The directed commercial fisheries harvest data are recorded in weight of fish and therefore the number of fish caught is converted into weight of fish harvested (Equation 3.10b). This calculation was performed by multiplying the fishery catch in numbers by the probability that a fish is over the size limit given its age and the weight at age. The weight at age in a season is calculated by converting the length at age in a season from the von Bertalanffy growth curve using the length to weight conversion function

(Equation 3.11). The length to weight conversion used in this study was the most current model from the SEDAR 31 data workshop at the time of this study.

The calculations to derive the expected values of the CPUE indices depended upon the type of index being modeled. The fishery-dependent indices of abundance for the commercial handline fisheries incorporate number of fish estimated in the stock, the probability that the fish is larger than the size limit, the weight at age, the selectivity at age of the fishery, the number of seasons the fishery is active and a log scale catchability coefficient (Equation 3.13a). The abundance index for the recreational fisheries uses the estimated population size in number of fish, the selectivity at age of the fishery, the number of seasons the fishery is active and a catchability coefficient (3.13b). The abundance index for the NMFS bottom-longline survey was calculated similar to the commercial handline indices but did not include the weight at age of the fish (Equation 3.13c). Fishery-independent abundance indices, except for the larval survey in the east and west GOM, were calculated with the estimated population size of red snapper in number of fish, the assigned selectivity at age for the survey and the estimated catchability coefficient (Equation 3.14a). The larval abundance indices were calculated in the same fashion as the other fishery-independent abundance indices but incorporated the relative fecundity at age as was used in the SEDAR update assessment (2009; Equation 3.14b). All surveys were summed over age and season to give a yearly estimate of the abundance index.

The final calculation required, before the statistical estimator could be used to compare the estimated values from the parameter estimates to the data, was the predicted proportion of each age-class within the harvest for every year and fishery. This was calculated by taking the expected number of fish harvested by a fishery in a year of each age-class summed over seasons and dividing by the total number of fish harvested for the fishery that year summed over seasons



and ages (Equation 3.15). This calculation was performed for each age-class for all years and fisheries to create the expected age compositions.

### *Statistical Estimator*

A Bayesian analysis was performed to estimate the parameters in the model using the data described above. Ability to use an informative prior, to incorporate outside knowledge about model parameters not present in the data, was the primary impetus for using Bayesian methodologies. Additionally, there was concern that the complexity of the model would create difficulties in the optimization of the statistical estimator using maximum likelihood techniques. This analysis uses Bayes' theorem, which can be simplified to show that the posterior distribution conditional on the data is proportional to the likelihood of the data given the parameters times the prior distributions of the parameters as presented in Equation 3.1. This theorem was expanded to incorporate the likelihoods of the numerous data sources. Log-likelihood functions assuming lognormal error were used to compare the observed and predicted values for harvest (Equation 3.2), abundance indices (Equation 3.4) and shrimp fishing effort (Equation 3.5). The variances used in the likelihood functions were calculated for each data point using Equation 3.3, where  $\sigma_H$  could be replaced by either  $\sigma_I$  or  $\sigma_e$ , for abundance indices or shrimp fishing effort respectively. The models assumed that  $\sigma_H$ ,  $\sigma_I$  and  $\sigma_e$  were constants as specified in Table 3.1 and the individual modifier to the variance were the CVs assumed in the SEDAR update assessment (2009). The other likelihood function incorporated into this theorem assumed a multinomial distribution to compare the observed age-composition data to the calculated proportion at age from the parameter estimates (Equation 3.6). To prevent extreme variation in estimates of vulnerability between age-classes within a fishery, a curvature penalty was added to the likelihood functions (Equation 3.7). All log-likelihood functions were then

summed together to give the total log-likelihood of the data conditional on the parameters (Equation 3.8).

### *Priors*

To estimate the posterior distribution of the parameters in the model a prior for each parameter was specified. All priors on parameters were uninformative uniform distributions. The upper and lower bounds corresponding to the parameters estimated are presented in Table 3.5. The lower bounds on the uniform priors for the parameters  $N_0$  were calculated in the following manner. The minimum number of red snapper caught for each age within a region as predicted by the SEDAR update assessment (2009) for the time period 1987 to 2008 were obtained from the update assessment. A high rate of fishing mortality of 5.0 and a low rate for natural mortality of 0.04 were assumed and the minimum catch-at-age values were incorporated into the Baranov catch equation (Equation 3.9 excluding  $\zeta$ ). The equation was then solved for  $N$  to give an estimate of the minimum stock size required to support those catches. The upper bounds on the uniform priors for the  $N_0$  parameters were calculated in a similar manner. The maximum number of red snapper caught for each age within a region as predicted by the SEDAR update assessment (2009) for the time period 1987 to 2008 were obtained from the update assessment. A low fishing mortality of 0.05 and a natural mortality of 0.08 were assumed and the maximum catch-at-age values were incorporated into the Baranov catch equation (Equation 3.9 if excluding  $\zeta$ ). This equation was then solved for  $N$  to give an estimate of the maximum stock size that would likely create those catches. The natural logarithms of the priors were then added to the log-likelihood of the data conditional on the parameters, which is assumed to be proportional to the distribution of the parameters conditional on the data (Equation 3.1).

*Metropolis-Hasting algorithm*

To approximate the posterior distribution of the parameters a Metropolis-Hastings algorithm was used with the statistical estimator described above using a Markov Chain Monte Carlo (MCMC). A MCMC is an algorithm for sampling from a posterior distribution by taking successive samples from a target distribution. Each sample is dependent upon the previous sample, which is one of the properties of a Markov chain. A Metropolis-Hastings algorithm uses a proposal function that proposes the next value of the MCMC based upon the current value and is then either accepted or rejected based upon its likelihood relative to the current value. The difference between the likelihood at the proposed value and the likelihood at the current value was calculated. A random number between 0 and 1 was generated and the log of the value was taken. The difference in likelihoods was compared to the log random number. If the value of the difference in likelihoods was greater than the random number then the proposed value was accepted as the next value in the MCMC. On the other hand, if the value of the difference in likelihoods was less than the random number then the proposed value was rejected and the previous value in the MCMC was used as the next value in the MCMC. A proposal function involving all parameters resulted in an extremely low acceptance rate of proposed values and therefore was not used. Likewise, using a proposal function for all parameters separately greatly increased the computation time for samples. Therefore, nine proposal functions were used which grouped similar parameters together. All proposal functions took the current value of the parameters and added a normally distributed random number to the respective parameters within the proposal function while all other parameters were kept at the initial value. The variance of the normal distribution was calculated as a percentage of the current parameter value where the

percentage was tuned for each proposal distribution to give an average acceptance rate between 20% and 40%.

The specific parameters within each proposal function in the Metropolis-Hastings algorithm are described below. The first proposal function included the natural mortality parameter used by the model. The second proposal function was composed of recruitment (R) for each year and the initial abundance (N0) for each age-class in both regions of the GOM. Apical fishing mortality and selectivity-at-age parameters for the commercial handline fisheries in the east and west were the parameters associated with the third proposal function. The fourth proposal function included the apical fishing mortality parameters and selectivity-at-age parameters for the commercial longline fisheries in the east and west GOM. Parameters of apical fishing mortality and selectivity-at-age parameters for the recreational fisheries, bycatch during the closed-season fisheries, bycatch in the shrimp fisheries, and NMFS bottom-longline fishery were associated with the fifth through eighth proposal functions, respectively. The seventh proposal function also included the catchability coefficients for bycatch from the shrimp fisheries. The last proposal function was the catchability coefficients for the abundance indices. These proposal functions were used to estimate the posterior distributions of each parameter through Metropolis-Hastings Markov Chain Monte Carlo methods.

Three chains were used to assist with visually testing if the posterior distribution had been converged upon through sampling. One of the three chains started at the values estimated in the SEDAR update assessment (2009), while the other two chains were started by adding a random variable from a normal distribution with a standard deviation of ten percent of the parameter estimates to the aforementioned estimates. The model was sampled for a burnin of 500,000 samples, a thinning interval of 5 was used and 200,000 samples were saved. Visual

diagnostics of the saved posterior values were conducted for the model to determine if convergence to the posterior distribution was possible.

### ***Data-Generating Model***

The “true” parameter estimates for recruitment and initial age-class abundance were used to initiate the calculations for the population dynamics of the generated data (Equation 3.1a and 3.1b). Total mortality was calculated based upon the fishery and natural mortality “true” parameters estimates (Equation 3.4, 3.5a and 3.5b) and abundance at age were calculated using the same equations as the estimation model (Equation 3.2a, 3.2b, 3.2c, 3.3a, 3.3b, and 3.3c). Apical fishing mortality, selectivity at age, and harvest size limit were used in conjuncture with the calculated population dynamics to calculate landings (Equation 3.9, 3.10a and 3.10b). Indices of abundance were created using the calculated abundance at age and catchability coefficients and the known values of selectivity at age for the specific survey. Catch data, effort data, and survey data were created by applying a multiplicative random lognormal error to the calculated values. The variance of the error applied to the calculations was the same values as that used in the estimation model and varied by data point. Years during which data were not available in the real dataset were removed from the simulated data. The available age-composition data were generated using a multinomial distribution. The underlying proportions at age for the distributions were the predicted proportion at age in the catch and were calculated from the parameter estimates (Equation 3.15). The number of samples used to generate the age-composition data was the total number of otoliths aged in each year for each fishery. These four data sources were the only portions of the model where error was incorporated. The legal-harvest size limit, von Bertalanffy growth curve, length to weight conversion, probability that a fish was less than the harvestable size limit, discard mortality and other fishery operation assumptions

were all assumed to have no error associated with them and were the same as the estimation model. In total 250 data sets were generated for this simulation study.

### ***Simulation Estimation Model***

The same model as described in the “true” parameter estimation section was used to estimate the parameters from the generated data. Three chains were used to estimate the posterior distributions of the parameters. One chain was started at the parameter estimates, another chain had a random error of 20% of the parameter estimates added to each value and the final chain had a random error of 20% of the parameter estimates subtracted from each value. Chains were assessed for convergence based upon visual inspection of the trace plots of the three chains. After extensive testing, it was assumed that a sample size of 15,000 with a burnin of 125,000 and a thinning interval of five for each chain would be sufficient for the model to approximate the posterior distributions. These samples were then saved for each simulation and parameter estimates were calculated as the mean of the saved samples.

### ***Simulation Estimation Model Evaluation***

To determine the estimability of natural mortality for red snapper in the GOM the parameter estimates from the simulation model were compared to the “true” parameter estimates, which were used to simulate the data. The mean of the saved samples for each simulation was calculated and used as the point estimate of the simulation run for comparison to the “true” parameter value. Relative Estimate Error (REE) was calculated by  $REE = \frac{\hat{X} - X}{X} * 100$ , where  $\hat{X}$  is the parameter of interest from the estimation model and  $X$  is the “true” parameter value. These REE percentages were calculated for all simulation runs that were assumed to converge and plotted as boxplots. The center of the box is marked by the median, the edges of the box are the 25<sup>th</sup> and 75<sup>th</sup> percentiles and the whiskers extend to the most extreme data point that are not

considered outliers and outliers are plotted individually. Points are considered outliers if they are larger than  $q_3 + 1.5 \cdot \text{IQR}$  or smaller than  $q_1 - 1.5 \cdot \text{IQR}$ , where  $q_1$  and  $q_3$  are the 25<sup>th</sup> and 75<sup>th</sup> percentiles respectively and IQR is calculated by  $\text{IQR} = q_3 - q_1$ . The boxplots were examined for systematic bias in estimates and overall variability of estimates.

## Results

### *“True” Parameter Estimation*

The “true” set of parameter estimates from the real dataset was the average values from the Metropolis-Hastings Markov Chain Monte Carlo saved values. The majority of parameters appeared to have converged to their posterior distributions through investigation of the trace plots of the parameters and the trace plot of the likelihood at the saved sample. However, the estimation of the abundance at age (N0) parameters during the initial year of the model had difficulty converging to a clear posterior distribution. The majority of parameter estimates were approximately equal to either the upper bound or the lower bound of the priors. The bounds of the priors were calculated to give realistic bounds on the N0 parameters in 1987 given the predicted number of red snapper for each age-class from the SEDAR update assessment (2009). When wider bounds on the priors for the initial abundance at age were used, the trace plots of the parameters showed a random walk around the parameter space and did not show signs of convergence to the posterior distribution. Additionally, with the wider bounds of the priors some parameters were still constrained by the bounds of the priors but gave unreasonable estimates of abundance at age. One example of an unreasonable parameter estimate was when the abundance at age was less than the predicted catch-at-age values and resulted in a negative abundance calculated within the model. In an attempt to converge to the posterior, the number of samples removed as burnin before saving the parameter estimates was increased but similar

results were observed regardless of the amount of burnin. The lack of estimability of these parameters was thought to be due to a lack of age-composition data during the first four years of the model. An alternative model was created that started in 1991, the first year of available age-composition data, but was also unable to successfully estimate all the initial abundance parameters. Since the priors for these parameters were realistic ranges, it was believed that the estimation of these parameters would not strongly influence the results of the simulation and the mean values of the saved samples for these parameters were used as the “true” values.

### ***Estimation Model Evaluation***

Datasets for landings, abundance indices, shrimp effort, and catch at age were simulated 250 times. Two simulations were removed after examining the trace plots from the estimation model due to poor mixing between chains. All other simulations were assumed to have converged to the posterior distributions of the parameters due to good mixing between chains and no trends in the trace plots. Statistical tests were not performed because visual inspections of trace plots were thought to be sufficient due to the number of parameters estimated and the number of simulations conducted.

Natural mortality was underestimated by the estimation model in all simulations (Figure 3.1). Of the 248 simulation studies assumed to converge, only 44 of the simulation studies included the “true” value of natural mortality within the 95% credible interval for the saved samples. All estimates of natural mortality were underestimated by no more than 20% of the “true” parameter, though the majority of the simulations underestimated the “true” value by approximately 7% (Figure 3.1). The majority of catchability coefficients corresponding to the surveys were underestimated slightly but by no more than 10% of the “true” parameter. On the other hand, the catchability coefficients corresponding to the age-0 and age-1 surveys in both the



east and west GOM were both overestimated and underestimated by as much as 10%, but no bias in either direction was observed except for a very slight bias in age-1 survey in the west (Figure 3.1). The  $N_0$  parameters in both the east and west GOM had the highest REE in the simulations. Some of the parameters for the initial abundance at age were estimated very well by the model, while other parameters were overestimated by as much as 8000% of the “true” parameter in one simulation (Figure 3.2). The estimated abundance for age 0 through age 6 were estimated fairly close to the parameter values, but ages 7 and older showed more variability in simulation estimates. The recruitment estimates for all years were found to have REE as large as 50% in both overestimating and underestimating the parameters but no substantial biases were observed (Figure 3.2).

REE of the selectivity-at-age parameters in the simulations varied by age and fishery, but in general the estimated values were relatively close to the “true” parameters (Figure 3.4). For all fisheries the age that was estimated to be fully selected in the “true” parameter set resulted in the smallest value of REE in the simulation estimates. Age-classes younger than the age at full selectivity overestimated the value of the selectivity-at-age parameter for the majority of simulation. The age-classes older than the fully selected age did not show any systematic bias for most fisheries, but the recreational fishery showed a bias toward overestimating these parameters. The REE associated with selectivity-at-age parameters appeared to increase the farther the parameter was from the age that was fully selected to the fishery. For the bycatch during the closed-season, the commercial handline, and the commercial longline fisheries the selectivity-at-age-10 parameter showed less variability in estimates than the selectivity-at-age-9 parameter. The logistic curve parameters for the NMFS bottom-longline survey did not show strong bias in either direction, though the parameter in the denominator showed greater

variability in estimates than the parameter in the numerator (Figure 3.1). The estimation of the selectivity-at-age-0 parameter for the shrimp fisheries was very accurate, but the selectivity-at-age-2 parameter for this fishery was underestimated by the model by 100% (Figure 3.1). Various selectivity-at-age parameters in the model resulted in some of the most accurate estimates, while others showed some of the worst systematic biases of underestimation in the model simulations.

The majority of apical fishing mortality rates were underestimated by approximately 25% REE (Figure 3.5, 3.6, and 3.7). Relative estimates error for some fisheries displayed an interesting phenomenon where the underestimation of the parameters increased during the first three years of the model and then remained fairly consistent for the remainder of the model. The phenomenon was not observed in the bycatch during the closed-season fisheries (Figure 3.6) or the bycatch from the shrimp fisheries (Figure 3.7) REE boxplots. Additionally, the simulation estimates for bycatch from the shrimp fisheries effort were extremely close to the “true” parameters, but the catchability coefficient for these fisheries were slightly underestimated on average.

## **Discussion**

### ***Inestimable Nature of Initial Abundance-at-Age Parameters***

The “true” parameter estimation model was successful in estimating the majority of parameters relatively easily. However, the estimation of the initial abundance-at-age parameters for the two regions was problematic. Most of the parameter estimates for abundance at age in the first year of the model (N0) resulted in values that were approximately equal to the bounds of the priors. The authors believed that the realistic constrains of the priors on the initial abundance-at-age parameters would aid in the estimation of these parameters during the simulations. However, this was not the case and large REE values observed for the N0 parameters (Figure 3.2) is

indicative of the difficulty with which the model estimated them. The results of the simulation study suggest that the  $N_0$  parameters might be inestimable based upon the current configuration of the model. The causes of these inestimable natures of these parameters are unclear but a few hypotheses have been developed by the authors as possible explanation. First, the first year of available age-composition data from Gulf of Mexico red snapper is 1991, which is four years after the start of the model. However, some test models were created that started in 1991 and the estimability of the  $N_0$  parameters did not improve. A second hypothesis is that incorporating age-composition data for four fisheries that all harvest red snapper at ages 2 and older is problematic. Each age composition has an implied age structure for the first year of the model, which may be conflicting and causing the inestimable nature of the  $N_0$  parameters. A third hypothesis is that the age-composition data do not contain enough information to inform the model on the scale of the  $N_0$  parameters and additional assumptions or constraints may be required. Finally, incorporating three seasons into each year with spawning occurring in the second season is likely having an effect on the estimability of the  $N_0$  parameters. The abundance at age during the first season of the first year of the model is modeled to be in the third season of life. The abundance is then transitioned to the next age-class during the second season of the model. For example  $NOE_1$  is the abundance of one year olds and in their third season of life and thus would become age 2 in the second season of the model. Therefore, the  $NOE_1$  parameter is experiencing the fishing mortality at age 1 and age 2 during the first year of the model and may be causing the problems with estimating the  $N_0$  parameters. The SEDAR update assessment (2009) modeled the age progression in this fashion, but was likely not an issue there because the recruitment and initial abundance of the model in 1872 was fixed at 70 million recruits for the initial 100 years. The simulation study shows that the initial abundance-at-age parameters for this

model configuration are likely inestimable with the currently available data. To estimate these parameters additional constraints or assumptions are likely required.

### *Lorenzen MREE*

The intention of this study was to determine if a Lorenzen curve natural mortality parameter could be successfully estimated within a statistical catch-at-age model for data simulated based upon real data from GOM red snapper. The simulation model estimated the natural mortality with a slight negative bias in the REE indicating that the parameter was underestimated by the model. The most extreme REE for natural mortality corresponds to a parameter estimate for the Lorenzen curve natural mortality of 3.0832 whereas the “true” parameter estimate was 3.6874. This corresponds to values of natural mortality during the first season of age 0 for the most extreme estimate to equal 1.8986 and the “true” parameter value to equal 2.2753. Likewise, natural mortality for the plus group equals 0.0952 for the most extreme estimate and equals 0.1141 for the “true” parameter value. The difference between the most extreme estimate and the “true” value was less than 0.1 for all ages older than age 0. If the REE in natural mortality were of this magnitude in both overestimation and underestimation, then the estimability of natural mortality within statistical catch-at-age model for the GOM red snapper would readily be used in stock assessments. However, the simulation study showed a slight negative REE bias and elicits some reservations about the reliability of its estimation. One potential explanation for this underestimation of natural mortality is the bias observed in the logarithmic scale catchability coefficients for the fishery dependent surveys, video surveys, and larval trawl surveys. Since the catchability coefficients of the “true” values and simulation estimates are negative, a positive REE indicates that the value is underestimated by the model and a negative value indicates that the parameter is overestimated by the model. The catchability

coefficient parameters for surveys that represent age-classes two and older showed a bias toward underestimating these parameters on approximately the same scale as natural mortality. A slight underestimation of natural mortality, if fishing mortality and recruitment remained the same, would result in a larger abundance in older age-classes. Therefore, it appears that the model estimated the catchability coefficient to be slightly less to account for the underestimation of natural mortality. However, whether the estimation of natural mortality or the estimation of the catchability coefficients is driving the other to behave in this fashion is difficult to determine and requires additional investigation. The catchability coefficients that correspond to the age-0 and age-1 surveys did not show this systematic bias but instead resulted in more variable estimates of these parameters in both directions. On the other hand, there is a very slight bias in the age-1 survey in the eastern GOM to underestimate the catchability coefficient parameter.

### ***Recruitment REE***

The estimation of recruitment parameters for each year by the simulation models was unbiased for both the east and west GOM in the simulation study. However, the REEs for these parameters were highly variable with a slightly higher variability in the western GOM. Yet a plot of the recruitment estimates for all 248 simulations shows that the trends in recruitment are similar to those of the “true” parameter values for both the east and west GOM (Figure 3.3). Therefore, the estimation of recruitment by the simulation estimation model does not appear to significantly destabilize the performance of the model estimation. Instead the high level of REE in the recruitment parameters illustrates the difficulty of estimating the true value of recruitment for red snapper in the GOM, though the trend over time is well estimated. The “true” parameter model was created such that individual recruitment parameters for each year were estimated because the steepness parameter of the recruitment curves for GOM red snapper stocks appears

to be inestimable within the assessment model based upon SEDAR 31 (2013) and SEDAR update assessment (2009). The estimation of annual recruitment within the assessment model and then fitting a stock –recruit curve to the estimated annual recruitment parameters might be prudent to reduce the number of assumptions required to obtain reasonable parameter estimates, and the estimated stock recruitment relationship can be used for projection simulations.

Additionally, this simulation study shows the high range of error in the estimation of annual recruitment parameters for GOM red snapper, which should be considered when setting management goals.

### ***Initial Abundance-at-age REE***

Estimated values of the initial abundance-at-age parameters within the simulation estimation model were approximately equal to the bounds of the prior. Since the “true” parameters were often estimated very close to the upper bounds of the priors and the data were simulated based on these values, it is slightly surprising that the REE of the simulation estimates for some of these parameters is the largest of all parameters in the simulation model. Those parameters that were estimated by the “true” parameter model to be close to the lower bound of the prior, but were estimated to be close to the upper bound of the prior in the simulation model, showed the largest overestimation in the simulation estimates. The lack of informative data for these parameters and a general incapability to estimate these parameters in this model is one potential explanation for the large REE values of these parameters. The underestimation of natural mortality by the model is another potential factor in the change of the parameter estimates from the lower bound of the prior in the “true” parameter estimates to the upper bound of the prior in the simulation parameter estimates. When the estimate of natural mortality is less than the “true” parameter, the model calculates the abundance at age of older age-classes to be

greater (Clark 1999). Therefore, the data may provide some information for estimation of abundance in the initial year for the older age-classes, but is not informative enough for the model to converge to a clear posterior distribution not influenced by or approximately equal to the bounds of the prior. The bounds of the priors for the “true” parameter estimates of the  $N_0$  parameters not only constrained the estimates of these parameters in the simulation model but also influenced the apical fishing mortality estimates.

### ***Apical Fishing Mortality REE***

The apical fishing mortality estimates for the majority for fisheries showed a decreasing trend in the negative REE of the estimates from one year to the next for the first three years. The increase in underestimation of the apical fishing mortality parameters after the first three years was not observed in the bycatch during the closed-season fisheries apical fishing parameters, because these fisheries were modeled to begin in the fourth year of the model. It is hypothesized that the lowest REE in the first years is due to the estimates of the  $N_0$  parameters. The trend in the REE for the apical fishing mortality appears to be more prominent in the western GOM fisheries estimates than in the eastern fisheries estimates. The REE of the initial abundance at age for the eastern GOM was greater than those observed for the western GOM. Additionally, more initial-abundance-at-age parameters were approximately equal to the upper bound of the prior in the “true” estimates for the eastern GOM initial abundance parameters than the western GOM. Therefore, it appears that the priors on the  $N_0$  parameters and thus the estimated values for these parameters are the potential cause for these trends in the boxplots of REEs. Upon closer inspection of the trend in REE of the simulation estimates, a difference between fisheries can be seen in the year of the model after which the REE boxplot are similar for parameters representing later years, i.e. the year in which the decline in REE stops. The age that is fully selected to the

fishery appears to be a driving factor for when the trend is no longer observed. The year in which the first estimated value of recruitment reaches the age at full selectivity to the fishery appears to be the date after which the boxplots of the REE estimates are similar to one another. Therefore, the decreasing trend in REE during the first few years of the model for apical fishing mortality parameters is likely due to the influence of the bounds of the priors for the  $N_0$  parameters.

The bias of underestimating the “true” parameter by approximately 25% for most fisheries is perplexing. The apical fishing mortality estimates for bycatch from the shrimp fisheries are not biased like the other fisheries. Yet the REEs for the catchability coefficients for these fisheries are slightly positively biased. Since values of these “true” parameters are negative, a positive REE indicates that the parameter was underestimated. Therefore, the bycatch from the shrimp fisheries was also slightly underestimated the fishing mortality in the simulation estimates though the magnitude was less than the other fisheries. Though the influence of the quasi-informative uniform priors explains the trend of increasing underestimation for the apical fishing mortality over time, the cause of the underestimation in general is unclear. An explanation for this biased result would be expected to be seen by a strong bias in other parameters to balance the estimates of the model, such as a bias to overestimate the recruitment might be expected. However, the underestimation of natural mortality would be expected to be matched by an overestimation of fishing mortality to result in similar total mortality. Yet the model underestimated fishing mortality and an explanation of this bias is not evident because an underestimation of total mortality would likely be observed alongside an increase in estimated abundance. Though the cause of this systematic bias in apical fishing mortality estimation is not clear, this bias may potentially lead to spurious results for assessment purposes in providing management advice.



### *Selectivity-at-Age REE*

Selectivity parameters estimated in the model did not show bias except for age 2 in bycatch from the shrimp fisheries and ages 2 and older for the recreational fisheries. The age at full selectivity for each fishery was correctly identified in the simulation model and resulted in the lowest range of REE of all parameters. The increase in range of REE for selectivity-at-age parameters representing the older age-classes is likely due to the “true” parameter estimates being relatively small numbers less than 1. Thus the absolute difference between the “true” parameter and the simulation estimates maybe similar for ages close to the age of full selectivity with a larger estimated value and those much older than the fully selected age with a smaller estimated value, but the REE would appear to be greater for the later. The oldest age-class for the majority of fisheries had a slightly smaller range of REE than the age-class one year younger. This is hypothesized to be due to the modeling assumption of the oldest age presented in the boxplot have the same selectivity for all older age-classes. This increase in number of bins in the age-composition data used to estimate the parameter is thought to be the cause of the decrease in range of the REE of the simulation estimates though the “true” parameter values were smaller than the next oldest age-class. The parameters for the logistic function used for the selectivity at age for the NMFS bottom-longline survey were accurately estimated by the simulation models. The bottom parameter has a higher variability in REE compared to the top parameter, which is likely a result of the “true” value of the former parameter being smaller and similar absolute differences between the “true” parameter and the estimates for both parameters. The bias in the selectivity-at-age-2 parameter for bycatch from the shrimp fisheries appears to be due to the data

created by the data-generating model. The multinomial distribution used often resulted in a simulated proportion at age 2 equal to zero. This was different from the original dataset and is likely due to the use of a sample size of 200 for most years to simulate the data. Thus the estimated values of the simulation for the selectivity-at-age-2 parameter for bycatch from the shrimp fisheries were very close to the lower bound of the prior equal to 0.01. Therefore, this estimation error is not due to poor model estimation but instead is a result of poor data simulation that could be improved by increasing the sample size of the simulated datasets. On the other hand, the overestimation of the selectivity-at-age parameters for the recreational fisheries for ages 3 and older appears to be due to the inability of the model to estimate these parameters accurately. The cause of this poor model performance for these fishery selectivity-at-age parameters is unclear and requires additional investigation.

### ***Simulation Assumptions***

This simulation study provided a “best case” scenario under which natural mortality estimation was examined. This simulation study was a “best case” because of the overall lack of variability in the system and the knowledge of some “true” parameters. Observation error was the only source of variability incorporated into the simulation study and was not of large magnitude. The population dynamics equations used to calculate the “true” population did not incorporate any variability and only used the “true” estimated value of recruitment to simulate data. The shape of the Lorenzen curve natural mortality was exactly the same in the “true” parameter estimation mode, the data-generating model, and the simulation estimation model, though the magnitude could be estimated differently. Additionally, the assumption that the selectivity-at-age-1 parameter equaled 1.0 for bycatch from the shrimp fisheries was assumed in all models. The estimation of a Lorenzen curve natural mortality within a statistical catch-at-age

model has been shown to be sensitive to this assumption for GOM red snapper, though can be ameliorated through the use of a prior (Chapter 2). Additionally, the growth curve parameters, discard mortality rates, the probability a fish is less than the legal-harvest size limit, length to weight conversion parameters, assumption of mirrored selectivity for eastern and western GOM fisheries, and bounds on the priors of the  $N_0$  parameters that were used in the data-generating model were also used in the simulation estimation model. Despite these optimal conditions, the estimation of natural mortality within a statistical catch-at-age model for GOM red snapper within the simulation study showed a slight negative bias.

### ***Conclusions***

Conclusions on the estimability natural mortality within a statistical catch-at-age model are not straightforward. The inability to consistently estimate the initial abundance-at-age parameters within the model obscures direct inference of results. The priors directly constraining the estimated values of the initial abundance-at-age parameters likely influenced the estimation of other parameters in the model. Therefore, the models may not have converged to the posterior distributions of the parameters for the simulation estimates and these results should be treated as preliminary. Though the estimate of natural mortality was only slightly underestimated, the bias of underestimating fishing mortality for all fisheries could impact the estimated biological reference points used for management decision-making. Investigation of the cause of this systematic bias in the simulation estimation model of the apical fishing mortality is recommended. Additionally, the inability to estimate the abundance at age during the first year influenced the behavior of the REE of simulation estimates. Investigation of the impact of other priors for the initial abundance at age is required. Finally, additional caution should be used when interpreting these results because the data-generating model only incorporated observation

error into the simulated data, and does not account for process error. Furthermore, the assumptions made in the data-generating model were exactly the same as those in the simulation estimation models. Some examples of these known assumptions include the true shape of the Lorenzen natural mortality curve and how selectivity at age was modeled and the values for other parameters were known exactly such as discard mortality and growth. The true shape of the Lorenzen natural mortality curve and selectivity at age model specification for the fisheries are not possible to know exactly within a stock assessment for Gulf of Mexico red snapper. Therefore, sensitivity of the parameter estimates to various modeling assumptions of natural mortality at age or temporal variability and selectivity-at-age curves for all the fisheries should also be investigated before natural mortality estimation within a statistical catch-at-age model is unquestionably accepted. Therefore, the estimation of a Lorenzen natural mortality curve within a statistical catch-at-age model for GOM red snapper should be approached with caution.

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Table 3.1. List of symbols and variables with their associated descriptions that are used in the equations presented in Table 3.2 to model the population dynamics of red snapper in the Gulf of Mexico.

Symbol	Description
$a$	Subscript representing the age of the fish
$A$	The maximum age and plus group used in the model (age-15 and older)
$i$	Subscript indicating the fishery (1 to 11)
$j$	Subscripts indicating the survey (1 to 8)
$k$	Subscript representing the region in the GOM (1= eastern, 2 = western)
$t$	Subscript representing the year
$s$	Subscript representing the season in a year with a total of 3 seasons per year
$N$	The abundance of red snapper in number of fish for respective subscripts
$R$	Recruitment in number of fish recruiting in season 2
$N_0$	Number of fish in age-class at the start of the model
$F$	Instantaneous rate of Fishing mortality rate for respective subscripts
$Z$	Instantaneous rate of Total Mortality for respective subscripts
$M$	Instantaneous rate of Natural Mortality for respective subscripts
$v$	Relative selectivity at age or vulnerability of fish at age to the gear maximum value is scaled to 1
$f$	Apical fishing mortality for respective subscripts
$\xi$	Probability that a fish dies once it is caught based upon the legal size limit for the corresponding subscripts
$\delta$	Indicator variable whether a fishery is active during a season (value = 1 or =0)
$\gamma$	Indicator variable whether a fishery is active in a region (value = 1 or =0)
$n_i(s)$	Number of seasons fishery $i$ is active during one year
$\alpha$	Parameter used in logistic equation to model selectivity at age in NMFS bottom-longline fishery
$\beta$	Parameter used in logistic equation to model selectivity at age in NMFS bottom-longline fishery
$Q$	Log scale catchability coefficient to scale shrimp fishing effort
$e$	Shrimp fishing effort estimated by the model
$d$	Probability that a fish dies if it is released for the corresponding subscripts
$G_{L a,s}$	Probability that a fish is less than the harvest size limit given its age in seasons for corresponding subscripts
$L_{a,s}$	Length at age calculated for each season in inches
$C$	Total number of fish caught
$H$	Total number or weight of fish harvest depending upon the fishery
$w_{a,s}$	Weight at age calculated for each season
$D$	Total number of discards for corresponding subscripts
$I$	Log scale Abundance Index for corresponding subscripts



$q$	Log scale catchability coefficient to scale population abundance for corresponding subscripts
$r_a$	Relative fecundity at age
$P$	Proportion at age of harvested fish for corresponding subscripts
$n$	Number of age composition samples taken for corresponding subscripts
$\sigma$	Standard deviation calculated for the statistical estimator for corresponding subscripts
$\lambda$	CV assigned to weight data point for corresponding subscripts
$E()$	Expected value of corresponding value as calculated by the model
$p(\theta D)$	Posterior distribution of parameter $\theta$ conditional on the data $D$
$LH(D \theta)$	Likelihood of the data $D$ given parameter $\theta$
$\pi(\theta)$	Prior distribution for parameter $\theta$
$\sigma_o$	Overall CV of the model set at 0.01
$\sigma_H$	CV of harvest data set at 0.01
$\sigma_I$	CV of abundance index data set at 0.01
$\sigma_e$	CV of shrimp fishing effort data set at 0.1

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Table 3.2. Mathematical equations used to calculate the abundance at age, expected harvest, expected value of abundance indices, and expected proportion at age within the harvest from estimated parameters for Gulf of Mexico red snapper.

Equation
(3.1a) $N_{k,0,t,2} = R_{k,t}$
(3.1b) $N_{k,a,1,1} = N0_{k,a}$
(3.2a) For $a < A$ and $s = 1$ : $N_{k,a+1,t,s+1} = N_{k,a,t,s} e^{-Z_{k,a,t,s}}$
(3.2b) For $a < A$ and $s = 2$ : $N_{k,a,t,s+1} = N_{k,a,t,s} e^{-Z_{k,a,t,s}}$
(3.2c) For $a < A$ and $s = 3$ : $N_{k,a,t+1,s=1} = N_{k,a,t,s} e^{-Z_{k,a,t,s}}$
(3.3a) For $a = A$ and $s = 1$ : $N_{k,A,t,s+1} = N_{k,A-1,t,s} e^{-Z_{k,A-1,t,s}} + N_{k,A,t,s} e^{-Z_{k,A,t,s}}$
(3.3b) For $a = A$ and $s = 2$ : $N_{k,A,t,s+1} = N_{k,A,t,s} e^{-Z_{k,A,t,s}}$
(3.3c) For $a = A$ and $s = 3$ : $N_{k,A,t+1,s=1} = N_{k,A,t,s} e^{-Z_{k,A,t,s}}$
(3.4) $Z_{k,a,t,s} = \sum_i F_{k,i,a,t,s} + \frac{M_{a,s}}{3}$
(3.5a) For $i \neq 9,10$ $F_{k,i,a,t,s} = v_{k,i,a} f_{k,i,t} \xi_{k,i,a,s,t} \delta_{k,i,s} \gamma_{k,i} / n_i(s)$
(3.5b) For $i = 9,10$ $F_{k,i,a,t,s} = v_{k,i,a} e_{k,i,t} \delta_{k,i,s} \gamma_{k,i} \log_e(Q_{k,i}) / n_i(s)$
(3.6) $v_{i=11,a,s} = 1/1 + \exp\left(-\frac{3*a+s-.5-\beta}{\alpha}\right)$
(3.7) $\xi_{k,i,a,s,t} = 1 - (1 - d_{k,i}) G_{L a,s k,i,t}$
(3.8) $L_{a,s} = 33.4047 \left(1 - e^{-0.2188 \left(\left(a + \frac{s-1}{3}\right) + 0.0611\right)}\right)$
(3.9) $C_{k,i,a,t,s} = \frac{F_{k,i,a,t,s}}{\xi_{k,i,a,t} Z_{k,a,t,s}} N_{k,a,t,s} (1 - e^{-Z_{k,a,t,s}})$
(3.10a) For $i > 4$ : $E(H_{k,i,t}) = \sum_a \sum_s (1 - G_{L a,s k,i,t}) C_{k,i,a,t,s}$
(3.10b) For $i = 1,2,3,4$ : $E(H_{k,i,t}) = \sum_a \sum_s H_{k,i,a,t,s} * w_{a,s}$
(3.11) $w_{a,s} = 0.000447 (L_{a,s})^{2.994}$
(3.12) $M_{a,s} = LM / L_{a,s}$
(3.13a) For $i = 1,2$ : $E(\log_e I_{k,i,t}) = \log_e (\sum_s \sum_a N_{k,a,t,s} * (1 - G_{L a,s k,i,t}) * w_{a,s} * v_{k,i,a} / n_i(s)) + \log_e(q_{k,i})$
(3.13b) For $i = 5,6$ : $E(\log_e I_{k,i,t}) = \log_e (\sum_s \sum_a N_{k,a,t,s} * v_{k,i,a} / n_i(s)) + \log_e(q_{k,i})$
(3.13c) For $i = 11$ : $E(\log_e I_{k,i,t}) = \log_e (\sum_a (N_{k,a,t,s=3} * (1 - G_{L a,s k,i,t}) * v_{k,i,a} / n_i(s)))$
(3.14a) For $j \neq 3,4$ : $E(\log_e I_{k,j,t}) = \log_e (\sum_a N_{k,a,t,s=2} * v_{k,j,a}) + \log_e(q_{k,i})$
(3.14b) For $j = 3,4$ : $E(\log_e I_{k,j,t}) = \log_e (\sum_a N_{k,a,t,s=2} * r_a * v_{k,j,a}) + \log_e(q_{k,j})$
(3.15) $E(P_{k,i,a,t}) = \sum_s H_{k,i,a,t,s} / \sum_s \sum_a H_{k,i,a,t,s}$

Table 3.3. Log-likelihood equations used to fit the estimated parameters by comparing the expected values from the model to the observed data for Gulf of Mexico red snapper. The statistical estimator used assumed log-scale likelihoods and priors.

Equation
(3.1) $p(\theta D) \propto LH(D \theta)\pi(\theta)$
(3.2) $l_H = -\sum_k \sum_i \sum_t \frac{(\log_e(H_{k,i,t}) - \log_e(E(H_{k,i,t})))^2}{2\sigma_{H_{k,i,t}}^2} - \log_e\left(H_{k,i,t} * \sqrt{2\pi\sigma_{H_{k,i,t}}^2}\right)$
(3.3) $\sigma_{H_{k,i,t}}^2 = \log_e((\sigma_H * \sigma_O)^2 + \lambda_{k,i,t} + 1)$ ,
(3.4) $l_I = -\sum_k \sum_j \sum_t \frac{(\log_e(I_{k,j,t}) - \log_e(E(I_{k,j,t})))^2}{2\sigma_{I_{k,j,t}}^2} - \log_e\left(I_{k,j,t} * \sqrt{2\pi\sigma_{I_{k,j,t}}^2}\right)$
(3.5) $l_S = -\sum_k \sum_i \sum_t \frac{(\log_e(e_{k,i,t}) - \log_e(E(e_{k,i,t})))^2}{2\sigma_{e_{k,i,t}}^2} - \log_e\left(e_{k,i,t} * \sqrt{2\pi\sigma_{e_{k,i,t}}^2}\right)$
(3.6) $l_P = \sum_k \sum_i \sum_a \sum_t \log_e(n_{k,i,t}!) - \log_e(n_{k,i,t} * P_{k,i,a,t}!) + n_{k,i,t} * P_{k,i,a,t} \log_e(E(P_{k,i,a,t}))$
(3.7) $v_{penalty} = \sum_k \sum_j \sum_a \frac{1.5(v_{k,j,a+1} - v_{k,j,a})^2}{0.3e^{.8(a+\frac{1}{6})}}$
(3.8) $-\log_e(LH(D \theta)) = l_N + l_{I_f} + l_I + l_H + l_P + l_S + v_{penalty}$

Table 3.4. Fisheries that harvest Gulf of Mexico red snapper and surveys that provide an index of abundance for each stock and their associated number for modeling purposes are presented in the table below. East refers to fisheries and surveys that operate east of the Mississippi River and West refers to fisheries and surveys that operate west of the Mississippi River.

<i>i</i>	Fishery	<i>j</i>	Survey
1	Commercial Handline East	1	Video East
2	Commercial Handline West	2	Video West
3	Commercial Longline East	3	Larval East
4	Commercial Longline West	4	Larval West
5	Recreational East	5	Age-1 East
6	Recreational West	6	Age-1 West
7	Bycatch during the Closed Season East	7	Age-0 East
8	Bycatch during the Closed Season West	8	Age-0 West
9	Bycatch from the Shrimp East		
10	Bycatch from the Shrimp West		
11	NMFS Bottom Longline Survey West		

Table 3.5. Priors on the parameters used in the models to estimate population dynamics of red snapper in the Gulf of Mexico. The first and second values for uniform distributions are the lower and upper bounds of the prior, respectively. The first value for the normal distribution is the mean and the second value is the standard deviation. The Lorenzen  $M$  stands for a natural mortality parameter that was divided by the length at age within a season.  $R$  stand for recruitment,  $f$  stands for apical fishing mortality,  $v$  is selectivity at age,  $\alpha$  and  $\beta$  are selectivity parameters for the NMFS bottom-longline survey,  $Q$  and  $q$  are catchability coefficients for the bycatch from the shrimp fisheries and the surveys respectively, and  $N0$  is the initial abundance at age of red snapper in the Gulf of Mexico where  $E$  and  $W$  represent the region and the subscript is the age.

Parameter	Wide Priors
<i>Lorenzen M</i>	Uniform (0.01, 6.00)
<i>R East</i>	Uniform (1e6, 1e10)
<i>R West</i>	Uniform (1e6, 1e10)
$f_{k=1,i=1,t}$	Uniform (0.0001, 5)
$f_{k=2,i=2,t}$	Uniform (0.0001, 5)
$f_{k=1,i=3,t}$	Uniform (0.000001, 5)
$f_{k=2,i=4,t}$	Uniform (0.000001, 5)
$f_{k=1,i=5,t}$	Uniform (0.0001, 5)
$f_{k=2,i=6,t}$	Uniform (0.0001, 5)
$f_{k=1,i=7,t}$	Uniform (0.000001, 5)
$f_{k=2,i=8,t}$	Uniform (0.0001, 5)
$f_{k=1,i=9,t}$	Uniform (0.0001, 5)
$f_{k=2,i=10,t}$	Uniform (0.0001, 5)
$f_{k=2,i=11,t}$	Uniform (10e-10, 5)
$v$	Uniform (0.0001, 1.00)
$\alpha$	Uniform (0.0001,10)
$\beta$	Uniform (0.0001,10)
$Q$	Uniform (-0.0001, -10)
$q$	Uniform (-0.01, -10)
$N0_{k=1,a=0}$	Uniform (1.63e5, 1.03e8)
$N0_{k=1,a=1}$	Uniform (9.14e4, 2.47e7)
$N0_{k=1,a=2}$	Uniform (2.60e5, 1.62e7)
$N0_{k=1,a=3}$	Uniform (1.18e5, 1.34e7)
$N0_{k=1,a=4}$	Uniform (4.36e4, 6.37e6)
$N0_{k=1,a=5}$	Uniform (1.83e4, 2.12e6)
$N0_{k=1,a=6}$	Uniform (1.03e4, 1.33e6)
$N0_{k=1,a=7}$	Uniform (1.31e3, 6.20e5)
$N0_{k=1,a=8}$	Uniform (714, 3.09e5)
$N0_{k=1,a=9}$	Uniform (692, 1.76e5)
$N0_{k=1,a=10}$	Uniform (476, 1.02e5)
$N0_{k=1,a=11}$	Uniform (311, 6.77e4)

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$NO_{k=1,a=12}$	Uniform (255, 5.05e4)
$NO_{k=1,a=13}$	Uniform (233, 3.81e4)
$NO_{k=1,a=14}$	Uniform (219, 2.76e4)
$NO_{k=1,a=15}$	Uniform (575, 8.76e4)
$NO_{k=1,a=0}$	Uniform (3.41e6, 1.80e9)
$NO_{k=1,a=1}$	Uniform (1.46e6, 4.82e8)
$NO_{k=1,a=2}$	Uniform (2.91e5, 6.12e8)
$NO_{k=1,a=3}$	Uniform (2.06e5, 5.34e7)
$NO_{k=1,a=4}$	Uniform (8.37e4, 3.14e7)
$NO_{k=1,a=5}$	Uniform (2.84e4, 9.75e6)
$NO_{k=1,a=6}$	Uniform (1.72e4, 1.52e6)
$NO_{k=1,a=7}$	Uniform (9.85e3, 8.45e5)
$NO_{k=1,a=8}$	Uniform (5.26e3, 3.60e5)
$NO_{k=1,a=9}$	Uniform (3.15e3, 2.75e5)
$NO_{k=1,a=10}$	Uniform (1.72e3, 1.94e5)
$NO_{k=1,a=11}$	Uniform (1.71e3, 1.74e5)
$NO_{k=1,a=12}$	Uniform (748, 1.38e5)
$NO_{k=1,a=13}$	Uniform (701, 1.09e5)
$NO_{k=1,a=14}$	Uniform (674, 9.75e4)
$NO_{k=1,a=15}$	Uniform (4.65e3, 1.50e6)

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Figure 3.1. Boxplot representation of the relative estimate error (REE) in percentage difference from “true” parameters for the 248 simulation estimates for various parameters. The top portion of the figure shows the REE for natural mortality and the catchability coefficient for the surveys, while the bottom section shows the REE of the selectivity parameters for bycatch from the shrimp fisheries and the NMFS bottom-longline survey and the catchability coefficient for bycatch from the shrimp fisheries. The catchability coefficient parameters for the surveys and bycatch from the shrimp fisheries are all negative; therefore, a negative REE represents an overestimation of the parameter and a positive REE represents an underestimation of the parameter.

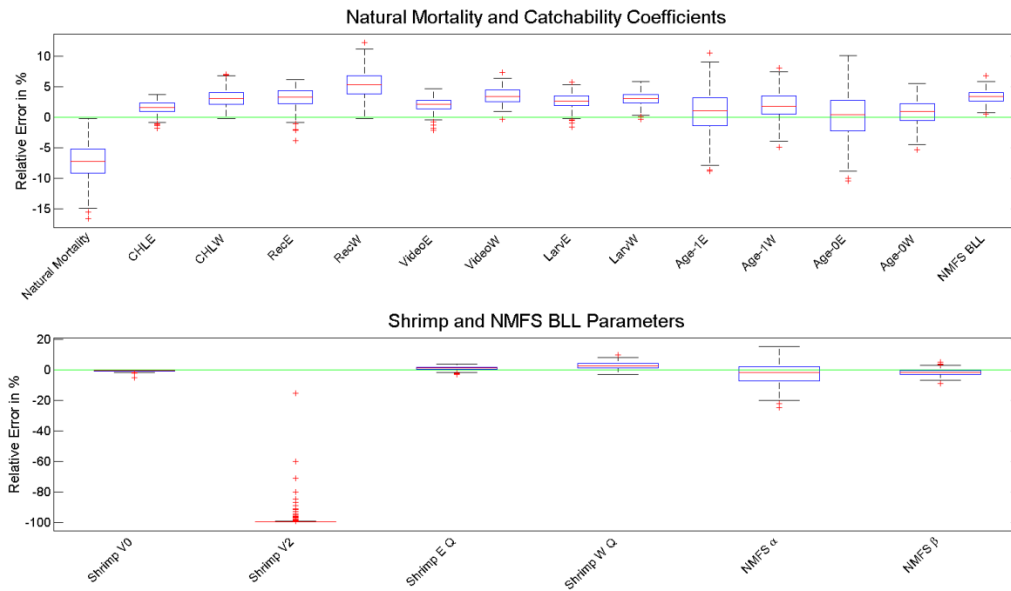


Figure 3.2. Boxplot representation of the relative estimate error (REE) in percentage difference from “true” parameters for the 248 simulation estimates for parameters of abundance at age in 1987 and recruitment by year. The x-axis of the top two graphs is the age-class of the abundance at age in the first year of the model and for the bottom two graphs is the year of recruitment. The y-axis for all graphs is the percentage difference in relative estimate error from the “true” parameter estimates.

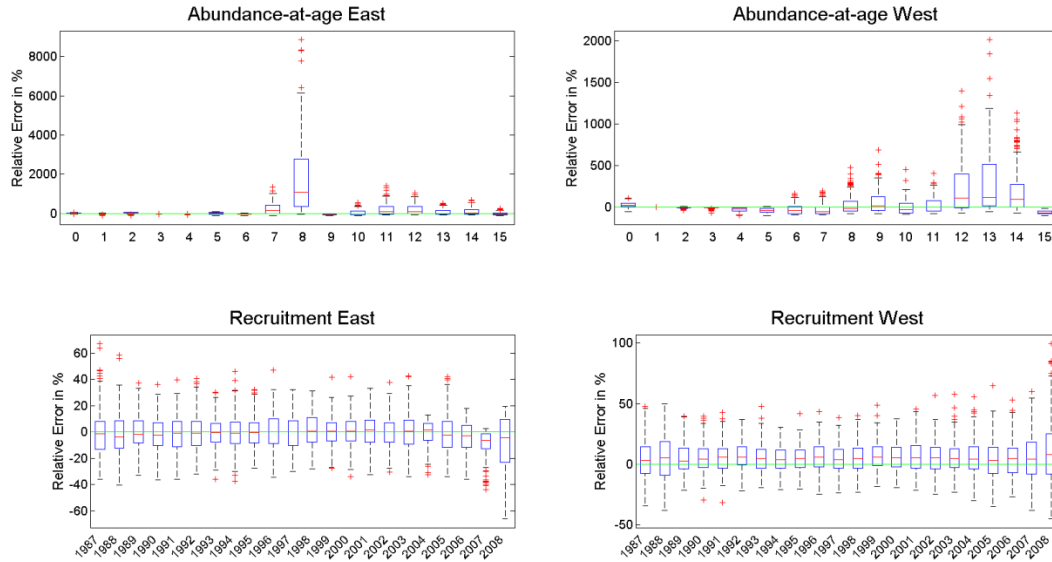




Figure 3.3. The recruitment estimates calculated as the mean of the posterior distribution for each year in the model for the 248 simulations for age-0 red snapper where recruitment in the eastern Gulf of Mexico is in the top portion of the figure and recruitment in the western Gulf of Mexico is in the bottom portion of the figure.

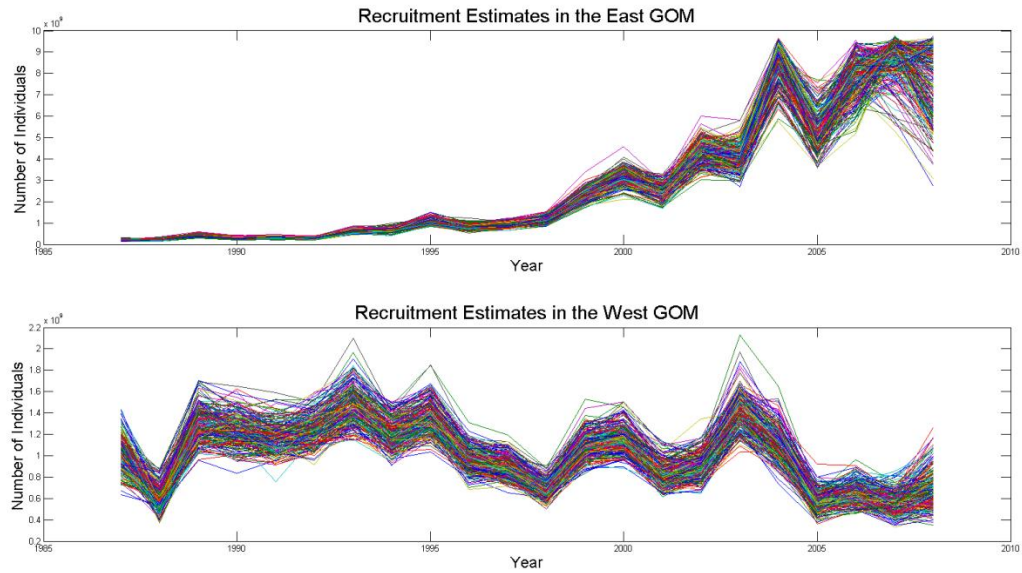


Figure 3.4. Boxplot representation of the relative estimate error in percentage difference from “true” parameters for the 248 simulation estimates for selectivity-at-age parameters for the commercial handline, commercial longline, recreational, and bycatch during the closed-season fisheries. The x-axis graph is the age-class of the selectivity-at-age for the fisheries. The y-axis for all graphs is the percentage difference in relative estimate error from the “true” parameter estimates.

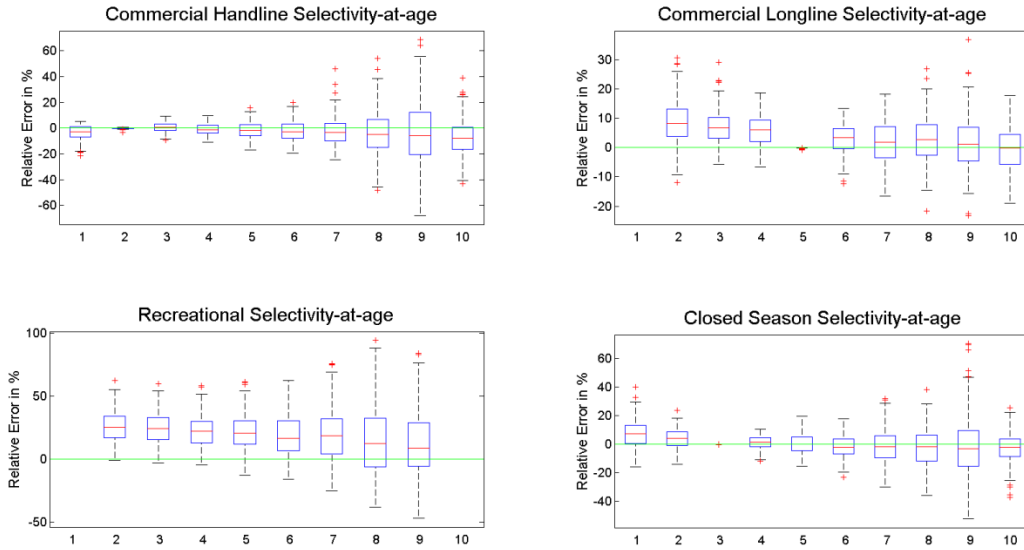


Figure 3.5. Boxplot representation of the relative estimate error (REE) in percentage difference from “true” parameters for the 248 simulation estimates for parameters of apical fishing mortality for the commercial handline fishery in the eastern GOM, commercial handline fishery in the western GOM, commercial longline fishery in the eastern GOM and commercial longline fishery in the western GOM. The x-axis of graphs is the year of the parameter in the model. The y-axis for all graphs is the percentage difference in relative estimate error from the “true” parameter estimates.

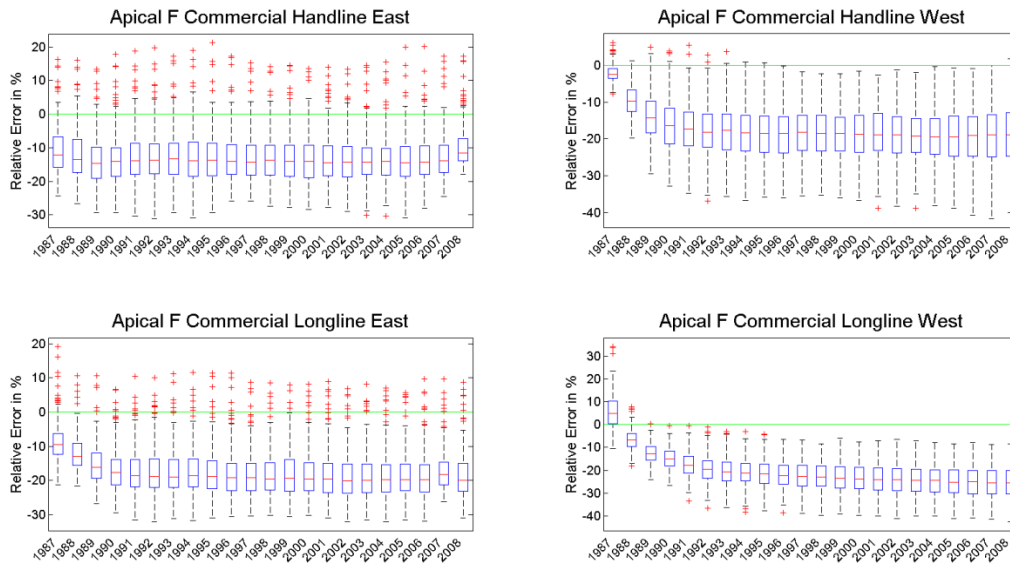


Figure 3.6. Boxplot representation of the relative estimate error (REE) in percentage difference from “true” parameters for the 248 simulation estimates for parameters of apical fishing mortality for the recreational fishery in the eastern GOM , recreational fishery in the western GOM , bycatch during the closed-season fishery in the eastern GOM and bycatch during the closed-season fishery in the western GOM . The x-axis of graphs is the year of the parameter in the model. The y-axis for all graphs is the percentage difference in relative estimate error from the “true” parameter estimates.

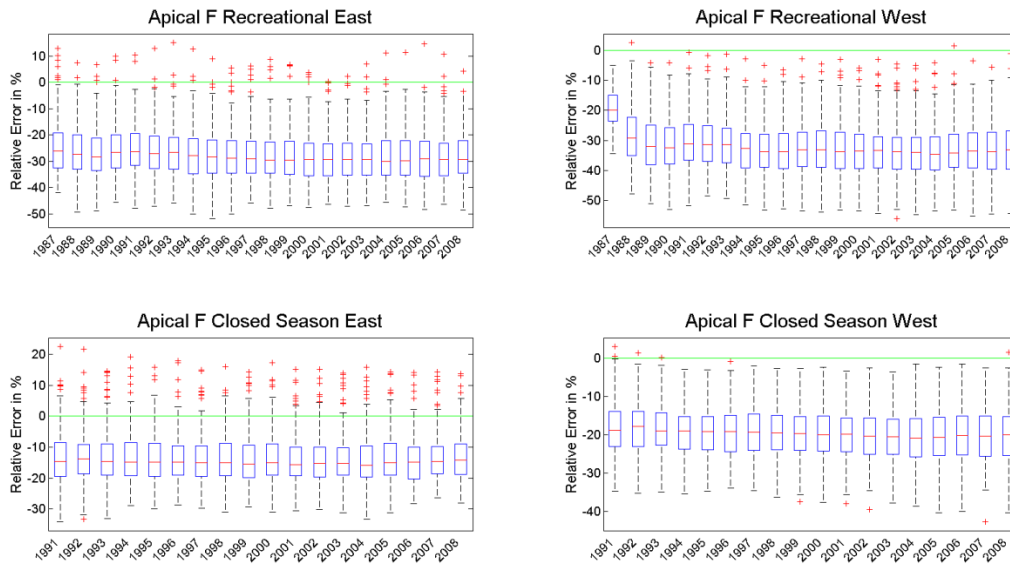
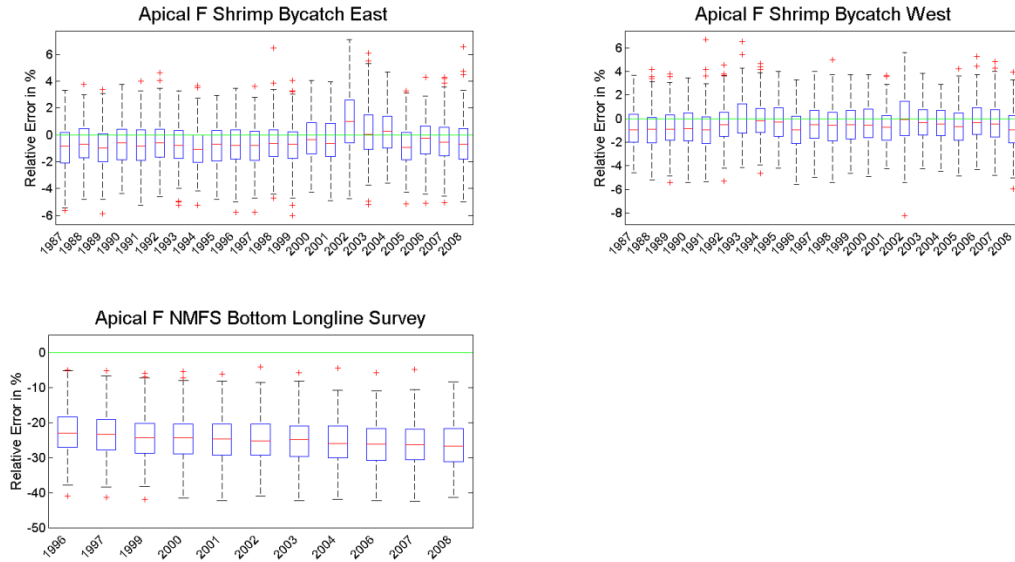


Figure 3.7. Boxplot representation of the relative estimate error (REE) in percentage difference from “true” parameters for the 248 simulation estimates for parameters of apical fishing mortality for bycatch from the shrimp fishery in the eastern GOM, bycatch from the shrimp fishery in the western GOM, and NMFS bottom-longline survey fishery in the western GOM. The x-axis of graphs is the year of the parameter in the model. The y-axis for all graphs is the percentage difference in relative estimate error from the “true” parameter estimates.



## **Chapter 4 – Accounting for fishing mortality when comparing density-dependent to density-independent mortality for the Gulf of Mexico red snapper**

**This Chapter has been accepted for publication in the North American Journal of Fisheries Management**

### **Comment**

Gazey et al. (2008) investigated the potential for density-dependent total mortality acting upon the juvenile red snapper *Lutjanus campechanus* population in the western Gulf of Mexico. The analysis compares two density-dependent total-mortality models and a density-independent total-mortality model using length-frequency data and standardized catch-per-unit-effort data of red snapper bycatch from the shrimp fishery from July 1999 – February 2007. The models were compared using Akaike Information Criterion (AIC) and Bayes posterior factor for their respective fits to the data through the model's estimation of total mortality parameters for young-of-year and age-1 red snapper ( $Z_0$  and  $Z_1$ ) and other parameters. The authors note the primary modeling assumption as, "total mortality is either a function of recruitment (density-dependent) or constant over the study." However, the use of a temporally constant total mortality for the density-independent model that does not account for known changes in fishing mortality is a strict assumption that creates complications when comparing to the density-dependent model.

The assumption of constant total mortality requires the density-independent mortality model to fit the data by averaging the changes in total mortality of red snapper over time into one value. During the study period, the relative shrimp fishing effort for the western Gulf of Mexico

varied between 0.79 – 1.97 (Figure 4.1; SEDAR update 2009). Because not all easily identifiable factors (i.e., fishing mortality) are accounted for in the model, the constraint of constant total mortality for the density-independent mortality model is comparable to fitting an intercept-only regression to data that shows a linear trend. The density-dependent mortality model may have a better fit compared to the density-independent mortality models because the former model has the ability to vary total mortality over time through changes in the estimated recruitment. The trend in recruitment estimated by Gazey et al. (2008) exhibits an opposite pattern of the recruitment estimates from the SEDAR update red snapper assessment (2009) rescaled with mean of 1.0 for 1999 to 2006 (Figure 4.2). These contrary patterns prompt questions about the driving factors for the estimation of recruitment deviations of the density-dependent models in Gazey et al. (2008) and indicate the parameters might not be estimated as would be expected. One speculative explanation for this discrepancy is that the estimation of recruitment is responding to the actual total mortality of the system because the model estimates one value for total mortality that is then modified by changes in recruitment. Thus, the density-dependent total mortality model presented by Gazey et al. (2008) might be forcing the estimates of recruitment to change in response to changes in total mortality, but do not accurately represent changes in recruitment. Therefore, these two models are not comparable and the ability to draw conclusions about the mechanisms of red snapper population dynamics is not valid. Thus, the statistical support for density-dependent natural mortality is equivocal.

Gazey et al. (2008) calls for an increase in natural mortality by 0.7 for age-0 red snapper by comparing the average estimate of total mortality from 2001 to 2003 of their model to the average of values estimated in SEDAR 7 (2005). The value of total mortality for age-0 red snapper in 2003 estimated by Gazey et al. (0.686; 2008) is well below the value assumed in the

assessment (0.983; SEDAR 7 2005) and is actually lower than the suggested amount of increase in age-0 natural mortality by Gazey et al. (0.700; 2008). If the estimate of fishing mortality from the update assessment for 2003 on age-0 fish (0.298; SEDAR update 2009) is assumed correct and the total mortality for 2003 from Gazey et al. (0.686; 2008) is used, the value calculated for natural mortality rate would be 0.388, also much lower than the value used in the assessment (0.983; SEDAR 7 2005). This one point is much different from what would be expected from a three consecutive year average, but displays the variability of the estimates from the model. Though not as convincing as the proposed point estimate of 0.7 increase in natural mortality, a range of possible values as estimated by the model would provide a more objective view of the natural mortality estimates and uncertainty therein for recommendation purposes to incorporate into the assessment.

If two models that account for changes in fishing mortality were constructed, a density-dependent natural mortality model which assumed natural mortality varied with recruitment and a density-independent natural mortality model which assumed constant natural mortality, then a comparison of the two models would be valid despite the difference in assumptions. However, none of the models presented in Gazey et al. (2008) adequately account for changes in fishing mortality. Even if a strong density-dependent natural mortality were present in the population, it would not be expected that all of the fish taken as bycatch by the shrimp fishery would have died due to natural causes in the same time period, though some undoubtedly might. Therefore, for an adequate comparison of density-dependent natural mortality to density-independent natural mortality the models must include an estimation of fishing mortality at age each year. To aid the estimation of fishing mortality, the models should incorporate data on relative shrimp fishing effort for the time series. This time series could be incorporated into the model in a similar to the



update assessment model (SEDAR update 2009), with the following added to the log-likelihood function for optimization  $LL_E = \sum_{i=1}^T F_i - qE_i$  where  $F_i$  is the estimated fishing mortality for time period  $i$ ,  $q$  is a catchability coefficient to be estimated for bycatch from the shrimp fishery and  $E_i$  is the effort data for time period  $i$ . The fishing mortality parameters do not necessarily need to be estimated on a monthly basis but instead could be configured into a yearly fishing mortality. This form of estimation may need additional total bycatch estimates from the shrimp fishery, but may increase the complexity of the model significantly and require an estimate of absolute abundance rather than relative deviations as implemented by Gazey et al. (2008). Additionally, a fishery-independent index of larval recruitment would provide a supplementary source of data to aid the estimation of recruitment and potentially assist the estimation of natural mortality. Some example abundance indices that could be incorporated include the Southeast Area Monitoring and Assessment Program (SEAMAP) groundfish trawl survey separated into an age-0 and an age-1 index in the recent update red snapper assessment or the SEAMAP larval trawl. Incorporating these recommendations into the framework provided by Gazey et al. (2008) would provide a convincing comparison of density-dependent against density-independent natural mortality in juvenile red snapper, from which a convincing argument for the use of density-dependent natural mortality could be made.

Density-dependent mortality may influence the population dynamics of juvenile red snapper in the Gulf of Mexico, but the inclusion of density-dependent natural mortality within an assessment based solely upon the results presented in Gazey et al. (2008) is not recommended. Additionally, an increase in the assumed value for natural mortality of age-0 red snapper within an assessment should be considered carefully, given that slight changes in natural mortality can have large ramifications upon the estimation of stock status. A comparison of many sources of

estimates of natural mortality and hypotheses of temporal variation in natural mortality should be conducted before any values are assumed or methodologies are instituted in an assessment.

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<http://www.sefsc.noaa.gov/sedar/download/Red%20Snapper%20Update%202009%205.0.pdf?id=DOCUMENT> (February 2013).

Figure 4.1. Observed shrimp effort values used in the stock assessment for years 1999 to 2007 (copied from Table 29 of SEDAR update 2009). Values are expressed as proportion relative to the 2001-2003 effort average.

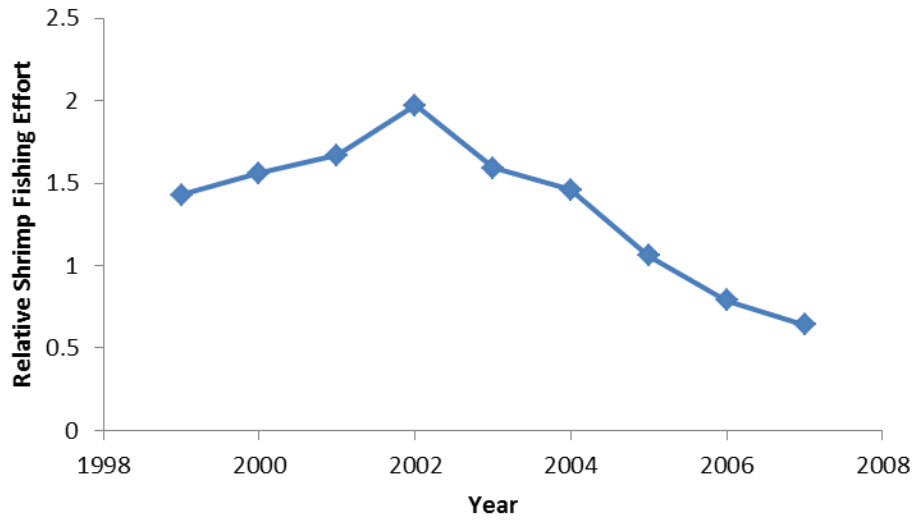
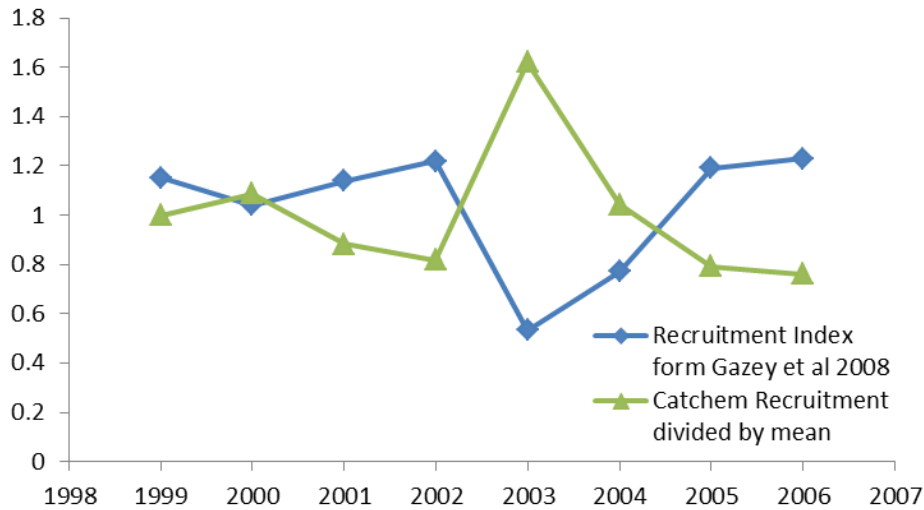


Figure 4.2. Estimate of red snapper Recruitment Index in the Western Gulf of Mexico from Table 7 in Gazey et al. (2008) represented by the solid line for 1999 to 2007. The dashed line represents estimates from the red snapper update stock assessment (SEDAR update 2009) divided by the mean recruitment for 1999 to 2007.



## Appendix A – Trace plots of Initial Abundance-at-Age Parameters

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Figure A.21. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model with fixed M parameters at high values from SEDAR update assessment (2009) with tight priors, where each color represents a different chain of saved values. .... 187

Figure A.22. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model with fixed M parameters at high values from SEDAR update assessment (2009) with tight priors, where each color represents a different chain of saved values. .... 188



Figure A.1. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model that estimated constant mortality over all ages with tight priors, where each color represents a different chain of saved values.

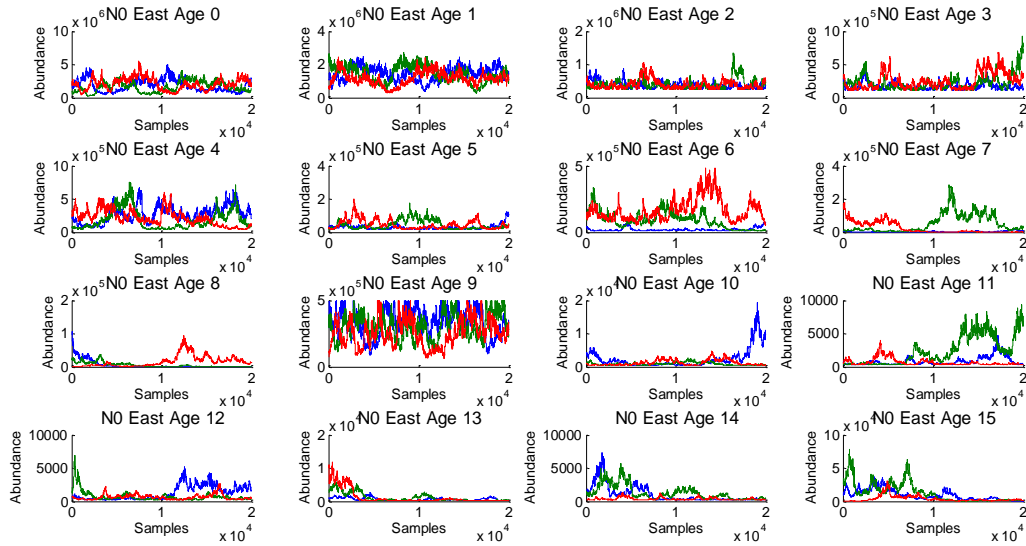


Figure A.2. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model that estimated constant mortality over all ages with tight priors, where each color represents a different chain of saved values.

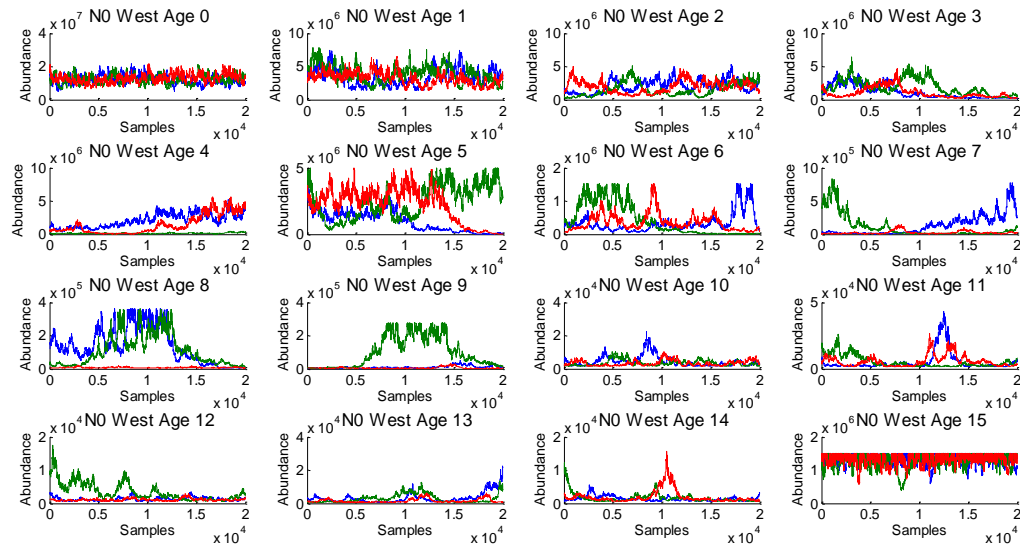


Figure A.3. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model that assumed three M parameters with normal priors at high values from SEDAR 31 (2013), where each color represents a different chain of saved values.

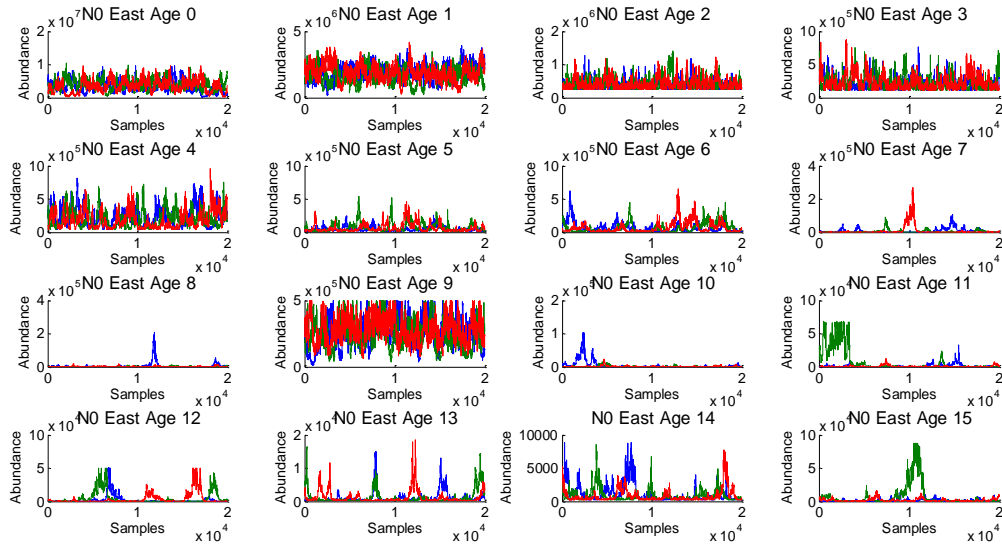


Figure A.4 Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model that assumed three M parameters with normal priors at high values from SEDAR 31 (2013), where each color represents a different chain of saved values.

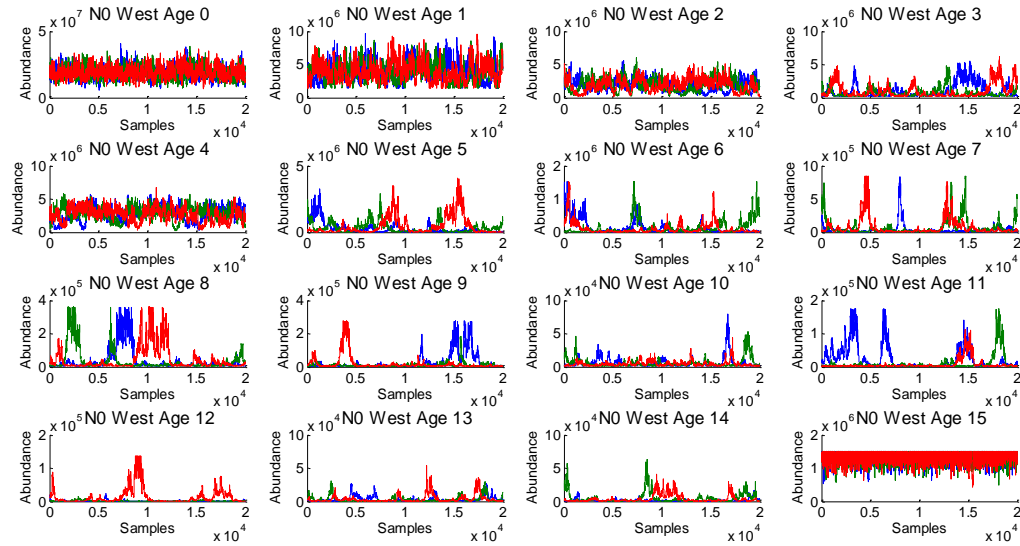


Figure A.5. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model that assumed three M parameters with normal priors at low values from SEDAR update assessment (2009), where each color represents a different chain of saved values.

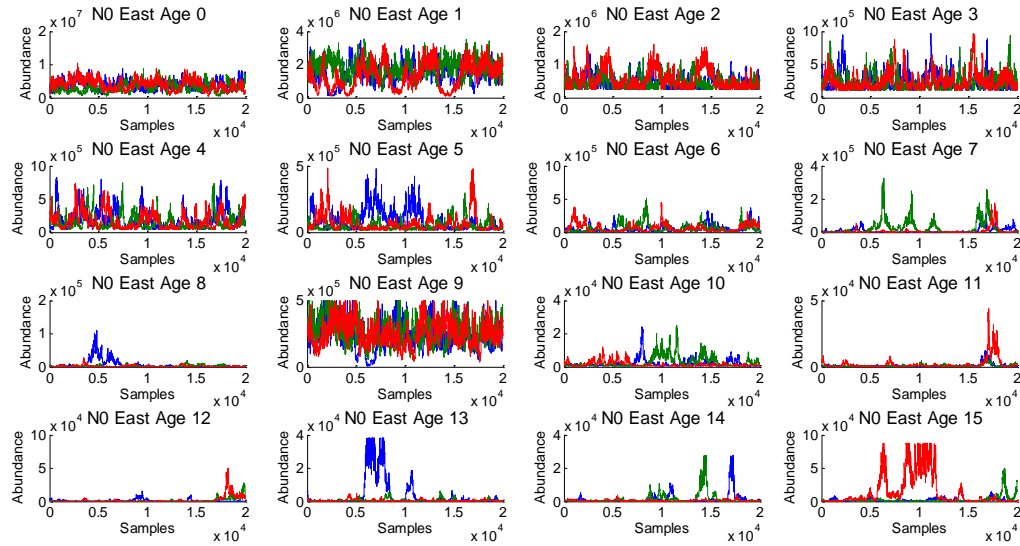


Figure A.6. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model that assumed three M parameters with normal priors at low values from SEDAR update assessment (2009), where each color represents a different chain of saved values.

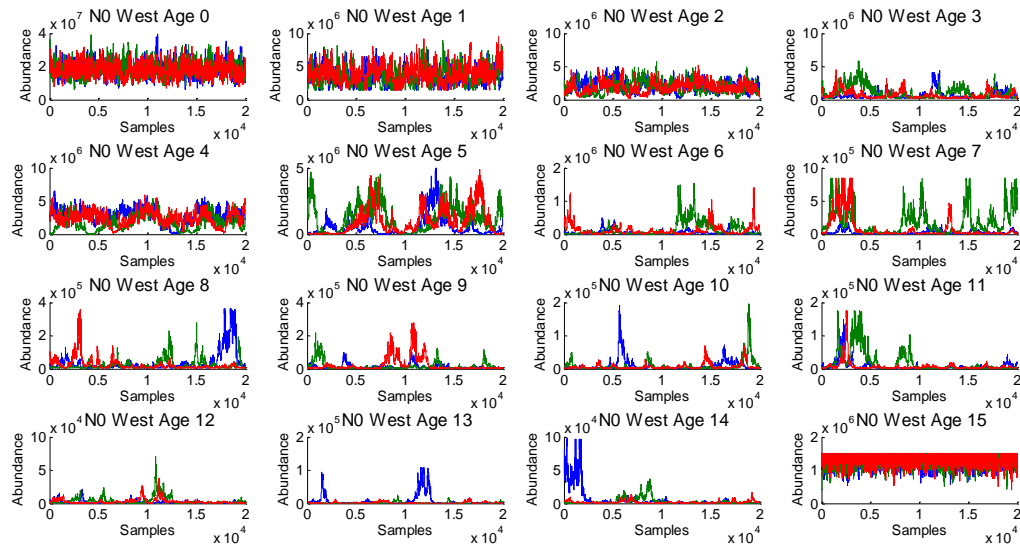


Figure A.7. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model that assumed a Lorenzen M with tight priors, where each color represents a different chain of saved values.

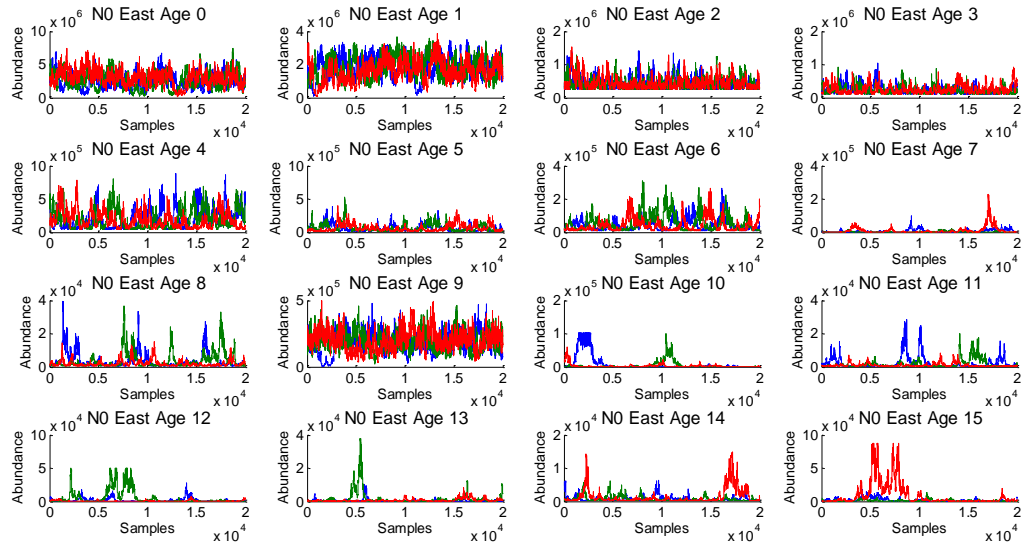


Figure A.8. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model that assumed a Lorenzen M with tight priors, where each color represents a different chain of saved values.

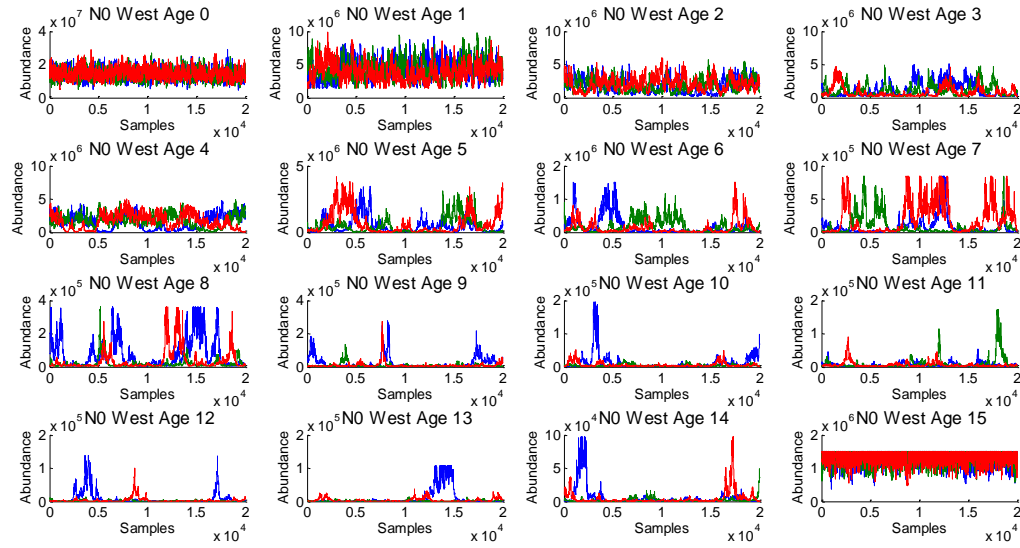




Figure A.9. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model that estimated separate selectivity-at-age parameters with tight priors, where each color represents a different chain of saved values.

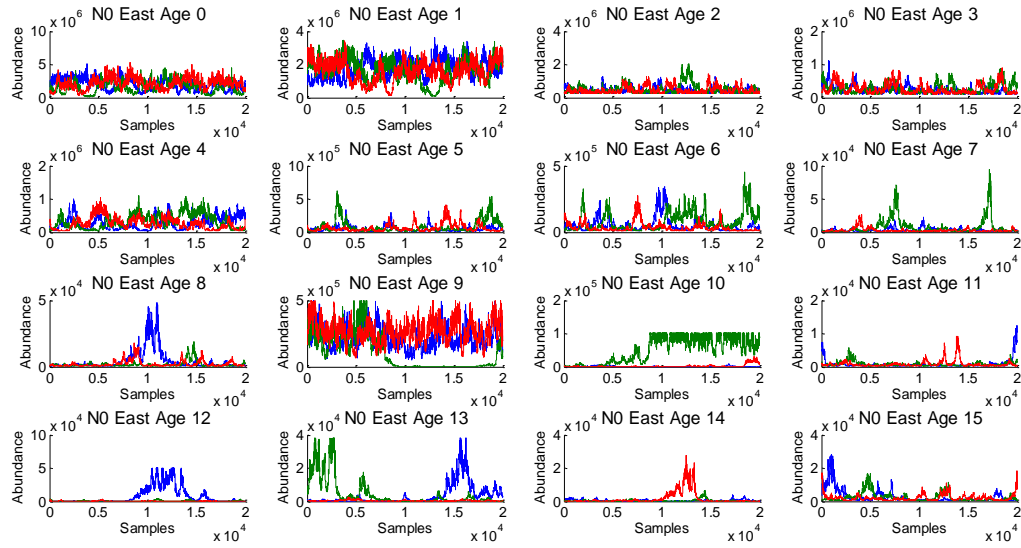


Figure A.10. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model that estimated separate selectivity-at-age parameters with tight priors, where each color represents a different chain of saved values.

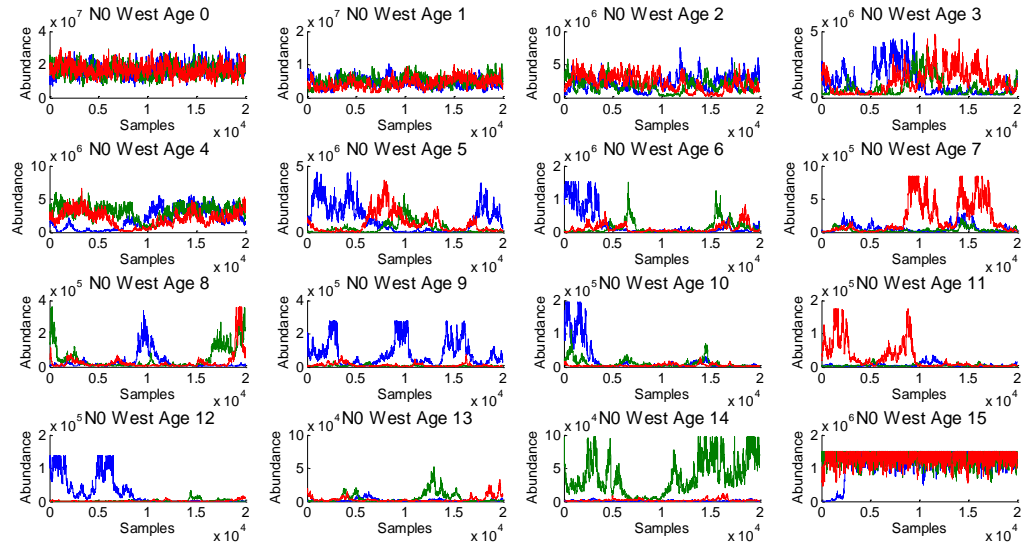


Figure A.11. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model that estimated separate selectivity-at-age parameters and separate Lorenzen  $M$  parameters with tight priors, where each color represents a different chain of saved values.

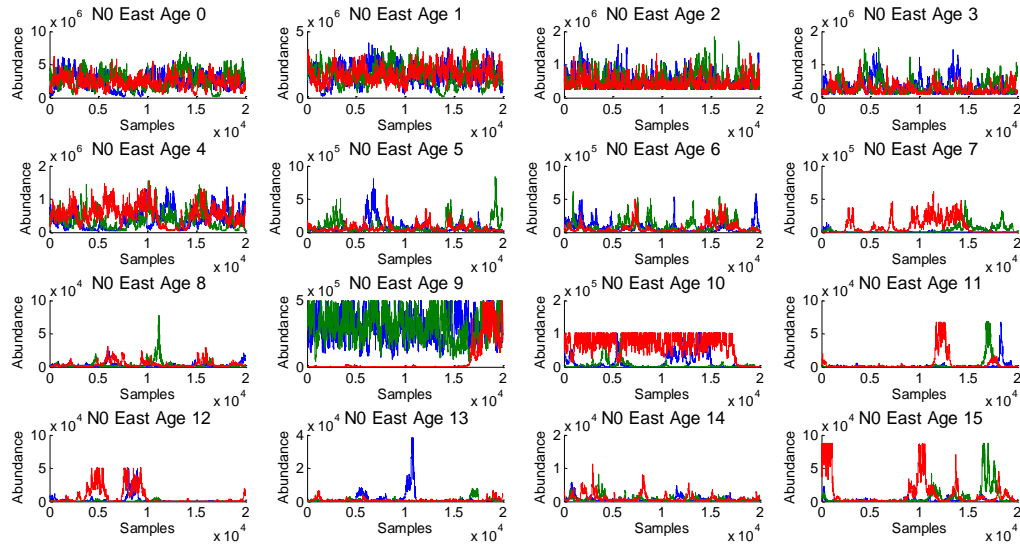


Figure A.12. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model that estimated separate selectivity-at-age parameters and separate Lorenzen  $M$  parameters with tight priors, where each color represents a different chain of saved values.

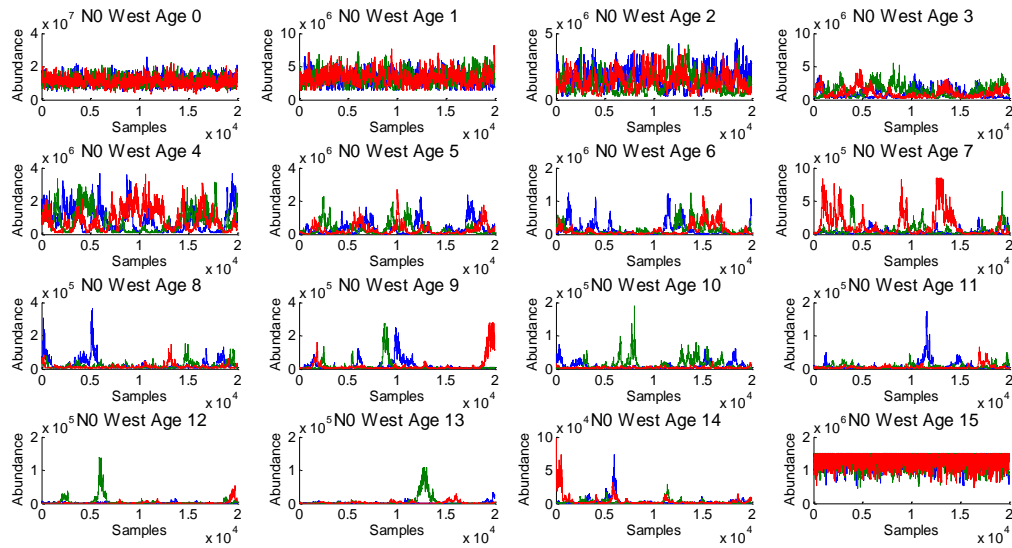


Figure A.13. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model that estimated separate Lorenzen M parameters with tight priors, where each color represents a different chain of saved values.

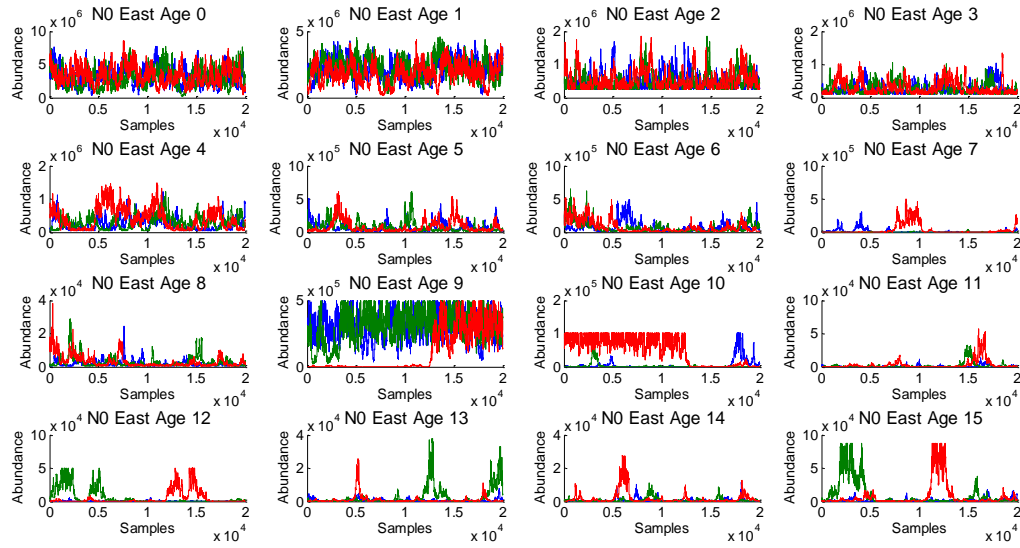


Figure A.14. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model that estimated separate Lorenzen M parameters with tight priors, where each color represents a different chain of saved values.

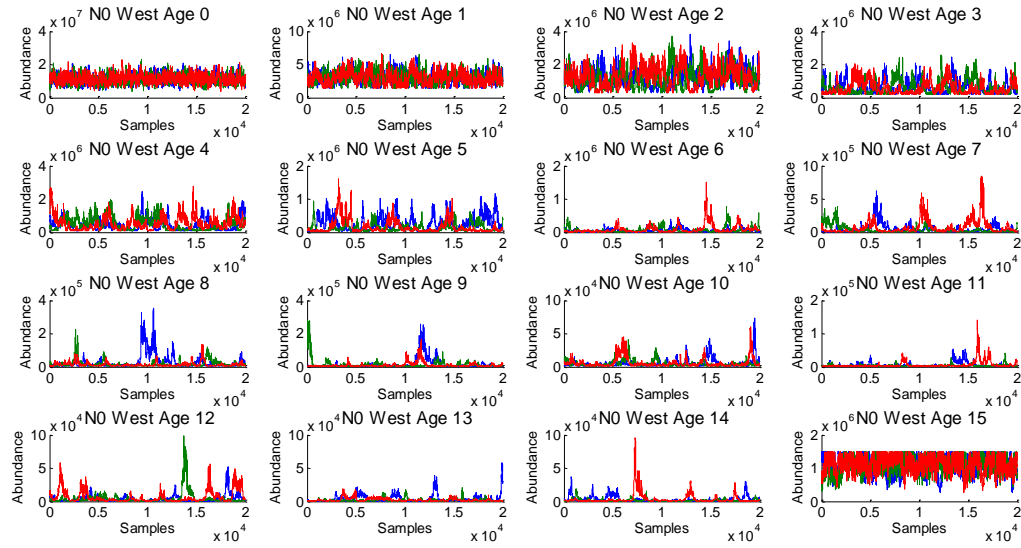


Figure A.15. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen M parameter with tight priors, where each color represents a different chain of saved values.

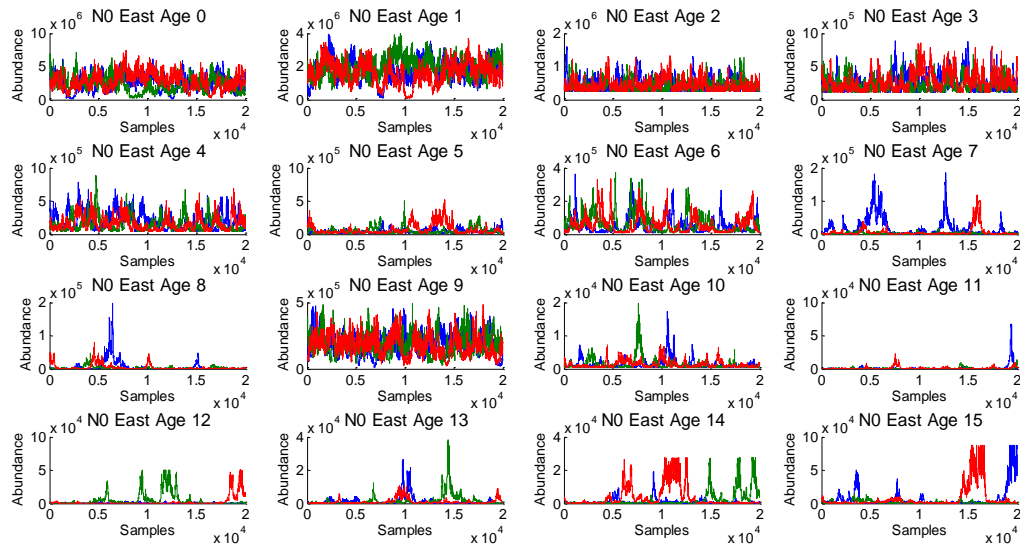


Figure A.16. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and a Lorenzen M parameter with tight priors, where each color represents a different chain of saved values.

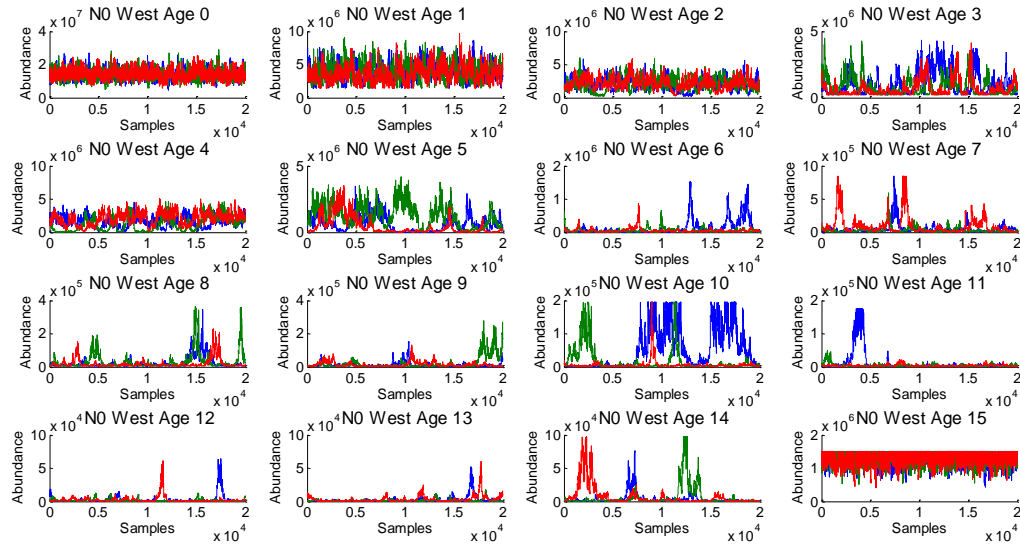




Figure A.17. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and three M parameters with tight priors, where each color represents a different chain of saved values.

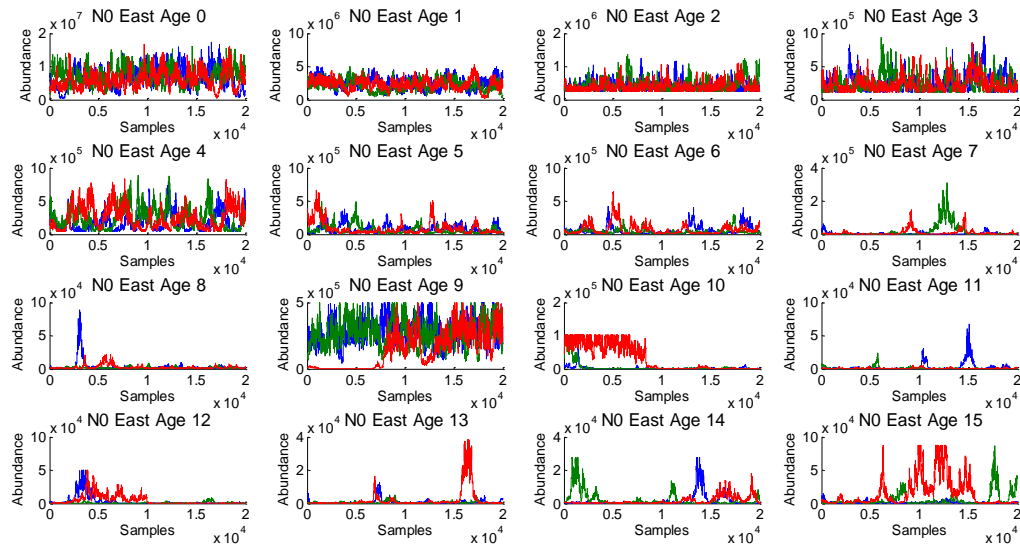


Figure A.18. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model that estimated the selectivity-at-age-1 parameter for bycatch from the shrimp fisheries and three M parameters with tight priors, where each color represents a different chain of saved values.

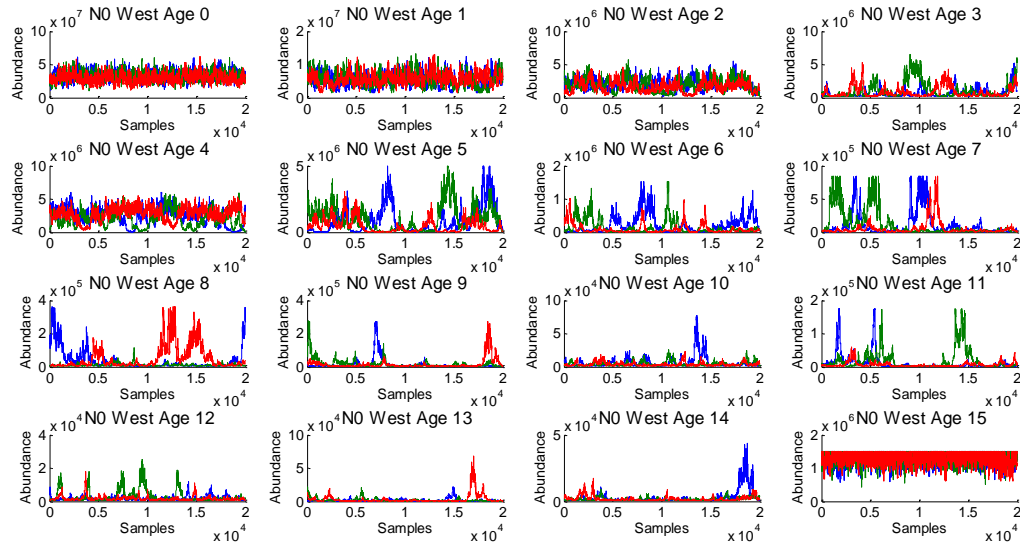


Figure A.19. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model with fixed M parameters at high values from SEDAR 31 (2013) with tight priors, where each color represents a different chain of saved values.

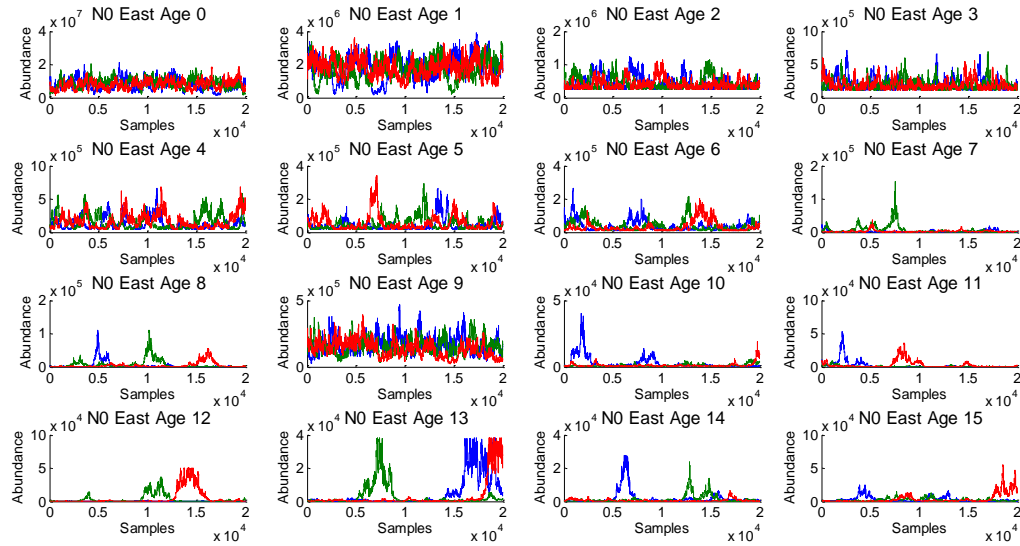


Figure A.20. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model with fixed M parameters at high values from SEDAR 31 (2013) with tight priors, where each color represents a different chain of saved values.

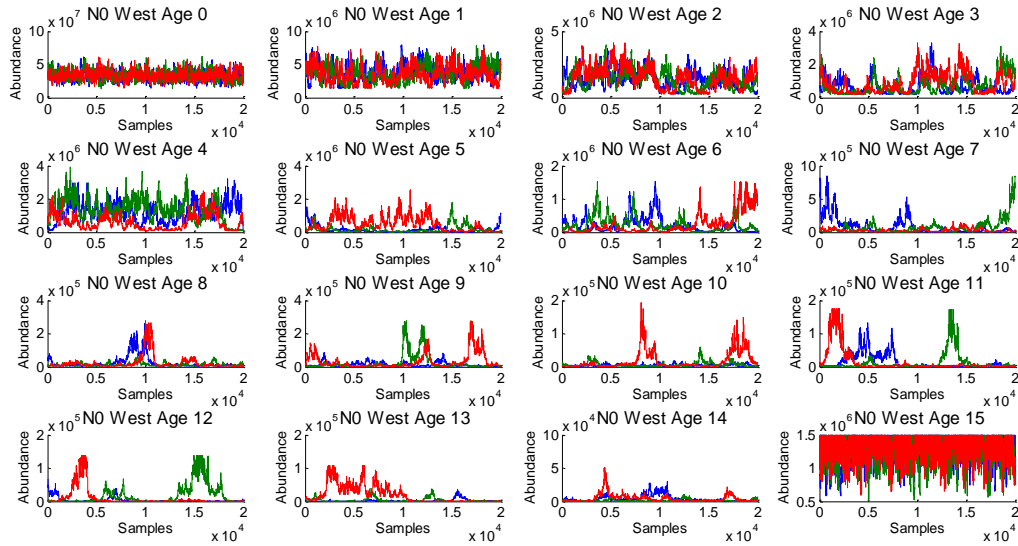


Figure A.21. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the eastern Gulf of Mexico from the model with fixed M parameters at high values from SEDAR update assessment (2009) with tight priors, where each color represents a different chain of saved values.

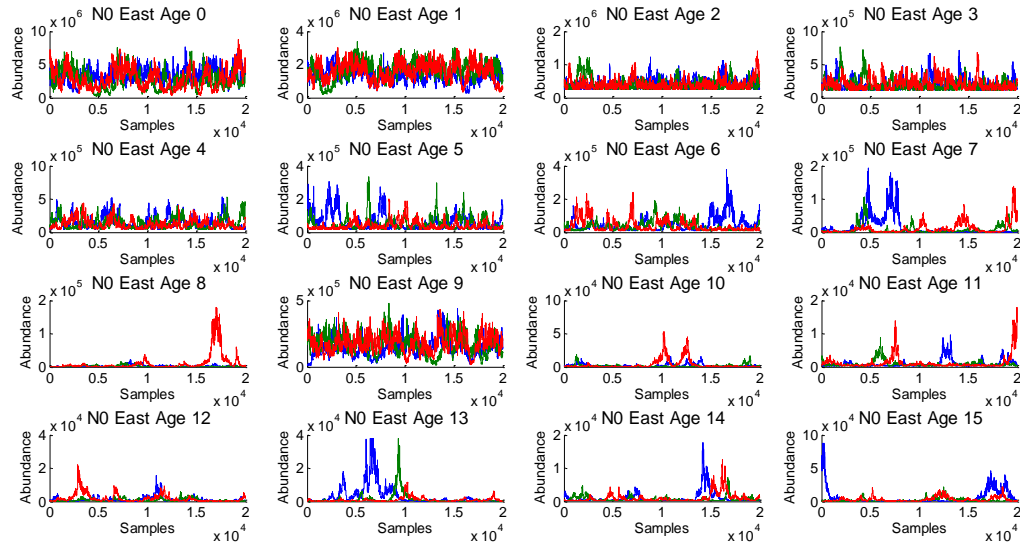


Figure A.22. Trace plot of the saved samples for three chains of initial abundance-at-age parameters for the western Gulf of Mexico from the model with fixed M parameters at high values from SEDAR update assessment (2009) with tight priors, where each color represents a different chain of saved values.

