A RELIABILITY BASED DESIGN PROCEDURE FOR WOOD PALLETS

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(ABSTRACT)

Pallets are widely used to efficiently store and handle goods and are often subjected to bending and impact loads. The consequences of structural failure of a loaded pallet can include loss of goods, increased labor and equipment costs, and possible severe or fatal injury to humans. The pallet industry, which annually consumes nearly 20% of all lumber manufactured in the United States, recognized a need for a rational design methodology, based upon engineering principles, to ensure consistent safety and economy in pallets of any geometry. To satisfy this need a cooperative research project between Virginia Tech, the U.S. Forest Service, and the National Wooden Pallet and Container Association was established. The objective of the project was to develop methods to design pallets for strength, stiffness, and durability. A primary expected benefit of the design methodology is to allow comparison of different pallet designs on a performance basis, without the need for extensive physical testing. This dissertation presents the results of this cooperative research project.

The developed methodology was computerized (Pallet Design System (PDS)) and is intended to allow pallet manufactures to obtain estimates of the maximum safe load capacity or the member dimensions required to resist known loads. Additionally, the program produces estimates of the durability and cost-per-use for pallets in specific service environments. PDS is limited in scope to pallets with up to four stringers and a maximum of 15 deckboards. Five different load types and four support modes can be analyzed. These include uniformly distributed and concentrated loads, and racked, stacked, and sling support modes. The techniques for estimating the strength and stiffness are based on matrix structural analysis and classical beam theory. The deckboard-stringer joints are modeled as spring elements, the stiffness of which are based upon characteristics of the fastener. Most fasteners commonly used in pallet construction (i.e. threaded nails or staples) can be analyzed. A probabilistic design technique based on mean value methods was applied in PDS to ensure safety in the resulting designs. The safety index was calibrated to pallet designs associated with warehouse load data. The physical properties of the material are estimated using either a modified clear-wood property approach (ASTM D-245 method), or in-graded testing of pallet lumber. The durability estimates are based upon studies of field data and economic analysis.

ACKNOWLEDGEMENTS

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1.0 INTRODUCTION

Since its inception during World War II, the wooden pallet has revolu-
tionized the manner in which materials, goods, and products are trans-
ported and stored. Today most products and goods are palletized, as unit
loads, for easy handling with a fork-truck. Use of the pallet and fork-
truck "system" has nearly eliminated the need for manual loading and un-
loading of transport carriers, such as trucks, thus eliminating an
expensive, labor-intensive, intermediate step between the manufacturer
and the consumer of goods. Product distribution warehouses commonly use
pallets as the foundation for efficient space utilization systems. Loaded
pallets can be stored in stacks, or multiple story racks, thus conserving
expensive warehouse floor space.

In the past three decades the manufacture of wooden pallets has increased
at an exponential rate. For example, in 1970 approximately 125 million
pallets were produced in the United States. In 1984, over 308 million
pallets were produced at a cost of approximately 2.5 billion dollars.
Tremendous volumes of wood (both hardwoods and softwoods) and fasteners
are consumed annually by the pallet industry. In 1984 nearly 7.5 billion
board feet of lumber, representing approximately 20% of all lumber cut
in the United states, was used for pallet construction, making this in-
dustry the second largest consumer of lumber in the USA (NWPCA, 1985).
("Only the housing industry consumes more lumber and fasteners",
(Stern,(1985)).

THE PROBLEM: Despite the fact that wooden pallets are used extensively by industry and consume a large percentage of the annual timber harvest, pallet design procedures have not been standardized. Some pallet designs are based upon tradition and intuition, backed up by occasional laboratory tests. Other designs are based upon the results of extensive laboratory and field tests and have been used to produce standard practice specifications. However, there is no universally accepted technique to account for the influence of design variables on pallet performance, in terms of strength, stiffness, or economic life. The design variables include those associated with material properties, fastener properties, overall pallet geometry, part geometry, load conditions, support conditions, service environment, economic life, and durability. Due to the complex relationships between these variables, wood pallet design has traditionally been based on "trial and error" and limited engineering analysis. The "design" process usually stopped upon discovery of an adequate structural configuration, and generally few attempts were made to improve the structural efficiency of the design. This system leads to inefficient utilization of timber resources, a potential for product damage, and even human injury by encouraging structures that are either under- or overdesigned for their intended use. This system also makes it difficult or impossible to compare pallet designs on a performance basis, unless the design is physically tested.

COOPERATIVE RESEARCH PROJECT: In response to the problem of establishing standard procedures for pallet design, a cooperative pallet research program (PRP) was initiated in 1980 by three cooperating agencies: 1)

Virginia Polytechnic Institute and State University, 2) the U.S. Forest Service (with scientists at the Forestry Sciences Laboratory at Princeton, West Virginia, and the U.S. Forest Products Laboratory at Madison Wisconsin), and 3) the National Wooden Pallet and Container Association. This dissertation presents the results of this research program.

OBJECTIVE: The overall objective of the PRP and this dissertation is to establish a rational methodology for wooden stringer-pallet design. The methodology allows the user to design pallets based on estimates of the required geometry of the wooden members. In addition the approach establishes fundamental techniques to produce estimates of the material properties and load effects, and defines the specific equations needed to perform the structural analysis. Furthermore, the methodology specifies techniques for ensuring safety in the resulting designs, and for predicting the economic life of a pallet designed for a specific service environment.

SCOPE: This dissertation discusses the development of the techniques used to establish the pallet design methodology, particularly the methods developed to predict the strength, stiffness, safety, and durability of pallets. The vehicle of the methodology is a computer program called the Pallet Design System (PDS). This program is written in the BASIC language for several brands of mini-computers and is intended to allow pallet manufacturers to optimize pallet designs for the requirements of specific

pallet users. The techniques used in PDS are also described in this thesis. The design methodology is limited in scope to the following:

1. Lumber pallets (stringer type), of commonly used North American species,

2. Notched or unnotched stringers of any rectangular geometry,

3. Pallets with two, three, and four stringers,

4. Deckboards of any rectangular geometry and a maximum of fifteen boards in each deck,

5. Deckboards ends: flush (with stringer edge), single winged, or double winged (i.e. winged top and bottom decks),

6. Commonly used fasteners, such as smooth or threaded nails, of hardened, stiff stock, or low carbon steel, or staples,

7. Five common load types: full and partial uniformly distributed, single, double, and triple concentrated line loads,

8. Four support conditions: racked across the stringers, racked across the deckboards, sling support (under the top deck wing), and stack or floor support mode.

The expected benefits of this design methodology are difficult to translate directly into dollars. The benefits of rationally designed pallets can be far reaching and include consistent safety, economy, and enhanced utilization of low grade wood, particularly the many hardwood species which generally have few uses of commerical importance other than pallet manufacture. For example, to maintain structural safety, PDS uses a reliability based design technique based on an exact formulation for com-

paring log-normal variates (similiar to the First-Order-Second-Moment (FOSM) method). This probabilistic design method rationally accounts for the variability of both the loads and the resistance and provides consistent safety in terms of probability of failure. The design procedure enables the user to determine the required minimum member dimensions that will safely carry the loads, thereby providing a means to select the most economical pallet in terms of strength and stiffness. From predicted durability and cost-per-use the user can rationally select from competing designs, one that is most economical in terms of expected service life. Since the design methodology can account for the variability of the material property distribution, pallets designed and constructed from lower grade wood will have approximately the same level of safety as those of higher quality wood. Also, since the mechanism for reliability based design is in place, the potential exists for efficient utilization of species that have traditionally been neglected or underutilized for uses other than pallets. This potential can be realized by the relatively simple task of developing, and incorporating in PDS, the material property data for such species. This data can then be used to design, specifically for such a species, pallets of general geometry. Another potential benefit of the design methodology was stated above, namely, conservation of timber resources. Because such a large volume of timber is annually manufactured into pallets, a small percentage reduction in the amount of material in a pallet may result in savings of timber resources especially if an industry wide shift is made from using the valuable, higher quality hardwoods or softwoods to the lower quality woods and the underutilized species. The pallet manufacturer can also benefit from a standardized

pallet design methodology. Rational selection of the optimum design for his customer's service environment ensures customer satisfaction and probably additional future pallet orders. In other words, PDS may become an important sales tool for pallet manufacturers.

The techniques developed to produce the pallet design procedure are presented in the following Chapters. First, some background information concerning pallet design is presented in Chapter 2. Then, Chapter 3 discusses some specific details concerning the scope of the Pallet Design System and the variables to be considered in pallet design. Chapters 4, 5, and 6 discuss the analysis techniques for racked and stacked pallets. Chapter 7 describes the procedures used to estimate material properties of pallet lumber (shook). Chapter 8 presents the probability based design method used to provide structural safety in the pallet designs resulting from use of PDS. The methods for estimating the durability and cost-per-use of a pallet are presented in Chapter 9. Lastly, Chapter 10 presents a summary of the pallet design procedure and describes some areas where data is lacking and may warrant further research.

2.0 LITERATURE REVIEW

The objective of this chapter is to provide a brief summary of some relevant background information. Some concepts of structural design methodology, particularly those used in the development of a reliability based pallet design procedure are presented first. Next, some results of previous pallet-related research are given, including topics such as the structural analysis of pallets, pallet shook properties, and pallet load types. Last, a brief discussion of pallet design for durability and life expectancy is presented.

2.1 STRUCTURAL DESIGN METHODOLOGY

To produce safe structures for the protection of life and property, a set of rules or procedures are established for society by the engineering profession. These rules form the basis of a design methodology or format. The goal of a design methodology is to establish systematic procedures for determining the structural geometry and materials that will produce "an economical structure with an acceptably low probability of failure" (Goodman et. al 1983). Additionally, the format is used to specify the requirements, or limit states, which a proposed design must satisfy. The design methodology includes instructions on how to translate loads into load effects (i.e. pounds into stress), how to compute the material resistance, how to account for the variabilities and interrelationships of

the material properties and the loads, and how to compute estimates of the life-expectancy or durability of the structure.

The underlying concept of any design format is to balance the material resistance and the load effects; this balance ensures that neither failure in a limit state, such as exceeding the strength or stiffness requirements, nor an uneconomical design results. This concept can be represented as:

$$R \geq S \qquad (2.1)$$

where:

R=design resistance

S=effect of design loads

To account for uncertainty due to variability of input quantities such as loads or material strength and stiffness, various design methods or formats, each having different levels of sophistication, have been developed by the engineering community. The traditional working-stress-design format and the target-probability-of-failure format represent the extreme levels of sophistication; other design formats rank between these two levels.

In the working-stress-design format, safety is achieved by using a high estimate of load to compute a load effect and a low estimate of the resistance to account for uncertainty and to satisfy equation (2.1). A

drawback to this approach is that the resulting designs will not be uni-
formly reliable and therefore will not be uniformly economical (Zahn
1977).

At the other extreme is the target-probability-of-failure format: Here,
safety is achieved by requiring that the probability of failure of a
particular design is less than or equal to a specified or target proba-
bility of failure:

$$P_{f_{target}} \geq P_f = P(R < S) = \int F_R(x) \, f_s(x) \, dx \qquad (2.2)$$

where:

$P_{f_{target}}$ = target probability of failure

P_f = computed probability of failure

$F_R(x)$ = cumulative distribution function for resistance

$f_s(x)$ = probability density function of load effects

The main disadvantage with the target-probability-of-failure format is
need for detailed information. The probability of failure is very sen-
sitive to the extreme tails of the distributions of R and S (Figure 1 on
page 10). Unfortunately, we usually have little information concerning
these tails. Also, the integration shown in equation 2.2 often must be
done numerically with a computer.
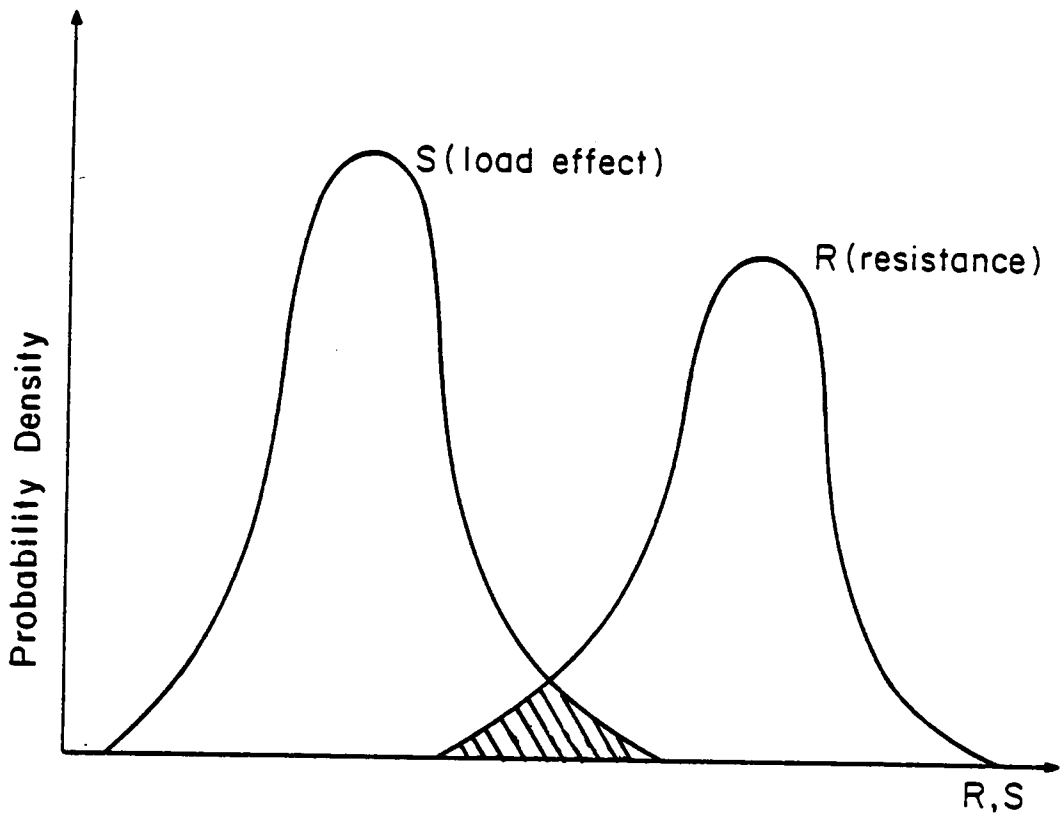
Figure 1 . Probability of failure.

2.1.1 FIRST-ORDER SECOND-MOMENT (FOSM) METHODS

An intermediate level design format based on first-order second-moment methods is used in the Pallet Design System (PDS). A brief discussion of the FOSM method is presented here, and a more detailed explanation is in a later section. FOSM methods can be used to provide more consistent levels of safety and economy than the traditional working-stress-design format but do not have the information demands of the target-probability-of-failure format. FOSM methods characterize the load effects and resistance distributions by their first two moments, namely, the mean and variance. Because the exact shapes of the distributions of either R or S are not used, the resulting designs are less sensitive to the tails of the distributions than are those of the target-probability-of-failure format.

The FOSM method achieves safety by use of a safety index called Beta. Given independent random variables, resistance (R), and load effects (S), failure occurs when:

$$R < S \quad \text{or} \quad \ln(\frac{R}{S}) \equiv u < 0 \qquad (2.3)$$

"The probability of failure is the area of the probability distribution curve of u in the tail where u < 0. This area is a function only of the number of standard deviations between u and 0. This number is by definition the safety index Beta" (Allen 1975). (See Figure 2 on page 12). High beta values result in high structural reliability and vice-versa (Zahn 1977). Beta can be computed as follows:
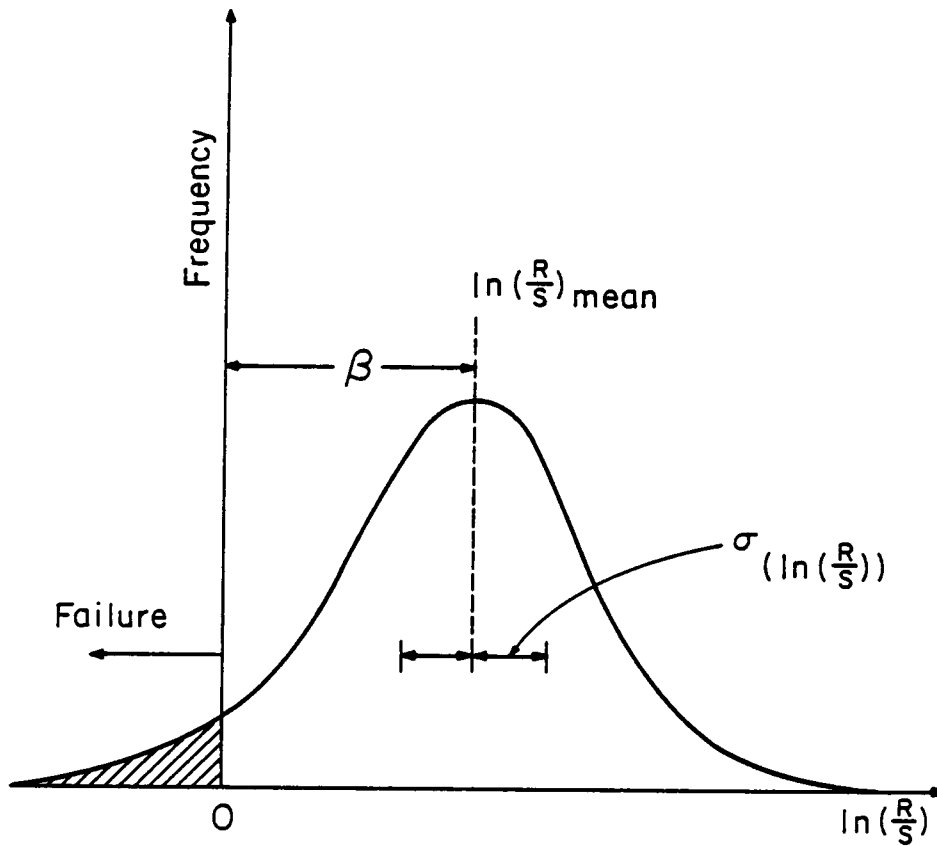
Figure 2. The safety index, Beta.

$$\beta = \frac{\ln\left(\frac{\bar{R}}{\bar{S}}\right)}{\sqrt{V_S^2 + V_R^2}} \tag{2.4}$$

where:

β =safety index

V_R and V_S =coefficient of variation of R and S respectively

Note that this formulation is an approximation for small variance situations (V_R and V_S must less than 0.30) (Ellingwood et al., 1980). For cases where the probability of failure is fairly high ($P_f > 0.001$), Beta, computed from equation (2.4), is related to P_f as (Zahn 1977):

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta) \tag{2.5}$$

where:

Φ =cumulative area under standard normal distribution

Where P_f is low, Beta can only be used as a relative measure of safety (Zahn 1977). If R and S are lognormal variates then an exact formulation of Beta can be written:

$$\beta = \frac{\ln\left[\frac{\bar{R}}{\bar{S}}\sqrt{\frac{1 + V_S^2}{1 + V_R^2}}\right]}{\sqrt{\ln\left[(1 + V_S^2)(1 + V_R^2)\right]}} \tag{2.6}$$

where:

$\bar{R}$=mean resistance (psi)

$\bar{S}$=mean load effects (psi)

β=safety index

V_S=coefficient of variation of S

V_R=coefficient of variation of R

This is applicable in large and small variance situations and P_f is evaluated exactly from equation (2.5) (Ellingwood et al. 1980). This formulation is used in PDS because the COV of the load distribution can be 0.45.

Numerical values for Beta are usually prescribed by a design code and are generally established by calibrating with satisfactory structures designed by a traditional method or a previous code. In this way the new process is made to mirror the safety implied by previous codes. [1]

2.2 ANALYSIS AND DESIGN PROCEDURES FOR WOODEN PALLETS

At first glance, the wooden pallet appears to be a rather simple structure composed of wood and nails. However, the structural action of a pallet is complex and involves composite action and load sharing among the wood

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[1] More details regarding the FOSM method and the calibration of Beta will be presented in Chapter 8.

members and possible non-linear behavior of the nail joints. In addition, the complex structural action of a pallet may be affected by many variables such as pallet geometry, species, shook quality, moisture content, fastener types, fastener patterns, loading characteristics, and service environment.

Several attempts have been made to produce a standardized design procedure for wooden pallets. However, no procedure has been accepted as a definitive design method. This section describes the techniques used in these early procedures. First, however a brief discussion of what is needed in a pallet design procedure is presented.

The ideal pallet design procedure should provide the user with a method to select the most economical pallet, based upon two main criteria: durability, and adequate strength and stiffness. The durability of a pallet is related to its ability to resist impact loads and survive in the handling environment. Impact loading of pallets due to lift-truck contact and general rough handling is often the most severe loading which a pallet will receive, hence, the static "load-carrying capacity is seldom the critical element of pallet design" (Protective Packaging Group, 1976). However, despite the fact that the pallet performance is more sensitive to impact loading, the ideal design methodology must include a static strength evaluation method to provide pallet producers with a strong, legally defendable case in product liability suits. Also, adequate pallet stiffness plays an important role in automated handling systems, where allowable deflection must be maintained within close tolerances. There-

fore, both static loading and durability of pallets should be considered in the pallet design procedure. (Design procedures to predict pallet response to dynamic loads may also be useful. However, dynamic loading is complex and such procedures would generally be difficult to apply.)

The ideal design method should also account for the influence on performance caused by the properties of the pallet components, such as lumber species, allowable lumber defects, shook moisture content, and fasteners (Heebink 1957).

Two categories of design procedures are found in the literature: theoretical and empirical. Each of these categories is discussed in the following sections.

2.2.1 THEORETICAL METHODS

Several theoretical design procedures for pallets are contained in the literature. The underlying concept of each procedure is to determine the relationship between load effects (stresses and deformations) and the applied loads.

Heebink (1957, 1959) developed a very simplified design procedure, based upon beam theory, which was used to calculate the load-carrying capacity of deckboards in the stack support mode. Using a simple statics and strength of materials approach, Heebink assumed that the load placed on a pallet could be modeled as either a point load or a uniform load on a

simply supported beam. For general design purposes, he concluded that a compromise between these two loading conditions would best approximate pallet use and behavior. Since the deckboards on a pallet are often continuous over two or more spans, Heebink's equations do not always provide accurate results.

To account for material defects in the deckboards, Heebink developed correction factors which reduce the effective cross-sectional area occupied by the deckboards. The allowable design bending stress was elucidated by applying correction factors to bending data of small, clear specimens. These factors account for the variability of test results within a species, duration of load, and a safety factor.

Wallin, Stern, and Johnson (1976) developed a procedure for designing and evaluating the performance of pallets and skids. This simplified procedure was computerized and used on a trial basis by several pallet manufacturers. Engineering principles developed for other types of structures were applied to pallets. The pallet parts were considered to act both individually and in combination as composite beams, depending upon the type of supports and the loading conditions.

The procedures developed by Wallin et. al. (1976) are based on the theory of elasticity as commonly applied to structures. Two load cases were considered, namely, a distributed load and a concentrated line load. Three support cases were considered:

1. full support of the pallet's bottom deck (stacked)

2. support along the stringers in a rack, causing both the top and bottom decks to be stressed as a composite beam (i.e. racked across the deckboards)

3. support along the ends of the stringers (i.e. racked across the stringers).

To predict pallet deflection and load capacity Wallin et. al.(1976) developed equations in which the joints were modeled as either fixed or pinned connections. Because the actual behavior of a semi-rigid pallet joint lies somewhere between these two theoretical end conditions Wallin, et al. (1976), conservatively recommended use of the pinned condition for safety until more joint fixity data becomes available.

Mack (1975) developed a theoretical procedure for analyzing pallets racked across the deckboards. The procedure was used to calculate the deflection, due to a central, concentrated load, of a pallet section racked across the deckboards. The method is based upon the theory of elasticity, and it accounts for the material properties of both the wood members and the joints. The joints are treated as semi-rigid connections whose restraining force depends on the rotation modulus of the joint and a function of the applied load. The modulus of elasticity and the moment of inertia of both the top and bottom deckboards, the span between outside stringers, stringer thickness, and the number of fasteners per joint are incorporated into the deflection calculation. Mack's procedure recognizes the contribution of both the bottom and top decks in resisting the

applied load, but it is only applicable to pallets loaded with a central concentrated point load.

Kyokong (1979) applied the method of matrix structural analysis to the analysis of pallets. He devised a computer program in the FORTRAN language which analyzes the pallet as a plane framework of elements loaded normal to its plane (i.e., a grid). The analysis assumes that a joint will not deflect in the plane of the grid nor rotate about an axis normal to the plane. Nail joints were modeled as pinned connections constrained by a rotational spring. A method for predicting the strength and stiffness of a notched stringer was also used.

Kyokong included a modification, based on Mack's (1975) work, to allow for the analysis of pallets racked across the deckboards. Kyokong expanded on Mack's derivation to account for the case where the ends of the bottom deckboards are unsupported,(i.e. four corner support). Kyokong (1979) also included an additional modification to Mack's work to account for the radial compression which develops at the inside edges of the outer stringers. The procedures developed by both Mack and Kyokong only provide analysis techniques for pallets and do not consider techniques to provide safety in the resulting designs.

Mulheren (1982) developed a three dimensional structural analysis computer program, called SPACEPAL [2], based on the matrix displacement method.

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[2] SPACEPAL is an acronym for SPACE frame analysis of wood PALlets.

SPACEPAL can be used to analyze any linear, three dimensional, framed structure with either rigid or semi-rigid joints. The semi-rigid nail joints are modeled as zero-length spring elements, the stiffness of which are found through testing of actual joints. SPACEPAL was used to develop and verify the generalized design equations which form the basis for P.D.S.--the new pallet design procedure.

2.2.2 EMPIRICAL METHODS

Several researchers have devoted considerable effort to establish pallet durability design methods based largely upon empirical procedures (Wallin, Stern, Whitenack, Strobel, etc.). This section describes some of these studies.

Wallin and Whitenack (1974) collected data over a four-year period related to the performance of 22 different pallet designs; this study was called the Pallet Exchange Program (PEP). The purpose of the PEP study was to develop a method to insure uniform in-service pallet performance irrespective of the materials used for pallet construction. "Performance of the pallets (was) measured in terms of maximum allowable loads, deflections, and structural integrity, and performance (was) evaluated by the cost of using the pallet (i.e. cost per use) " (Wallin, Stern, and Strobel 1975). To evaluate the influence of factors such as species, defects, or environmental conditions on performance, Wallin and Whitenack released 2,075 pallets into commercial shipping operations and collected data on each use of individual pallets. The recorded data included the

amount of use, number of pallet damages by part, severity of the pallet

damage, and damage to the palletized product. For simplicity of analysis,

damage was measured in terms of costs of replacement or repair of either

the pallet or the palletized product. Pallet damage was related by eco-

nomic analysis and regression techniques to both the number of uses and

the design of the pallets. The economic life and the minimum average cost

of use were calculated for each of the various designs, species, shook

qualities, shook-grade-placements, and nail types. The contribution of

each of the above factors to the performance of the pallet was then as-

sessed (Wallin and Whitenack 1979, 1981,1984).

A computer model based on these results was developed and can be used to

compute estimates of the life expectancy, cost-per-use, durability [3],

strength, and stiffness of a pallet design. The strength and stiffness

computations which are used in the program were based on the procedures

described by Wallin, Stern, and Johnson (1976). The life expectancy and

cost-per-use are based on empirical relations obtained from the PEP study.

The Protective Packaging group of the Eastern Forest Products Laboratory

published a comprehensive report dealing with the selection and proper

design of wood pallets (1976). The report summarized the findings of many

past studies dealing with the performance and durability of pallets from

a non-engineering standpoint. The Protective Packaging Group (1976)

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[3]   Portions of this program were incorporated in P.D.S. and form the
    basis of the procedure for predicting the durability and economic life
    of pallets. More details of are given in Chapter 9.

stated "Pallet design is controlled by the handling environment, the expected frequency of pallet loss and the type of unit load." They recommended that pallets intended for use primarily by non-automated handling systems be reinforced around the periphery with end deckboards and outer stringers that have medium to high density. Pallets intended for use primarily by automated handling systems need not be designed to resist the impact loads of fork truck contact but should be designed to meet the required stiffness of the handling system. The frequency of pallet loss can be used to determine the required quality and durability of new pallets. If unavoidable pallet loss is a frequent occurrence, then lower quality pallets should be used to minimize the cost of loss, while high quality, durable pallets can be justified if pallet loss is rare. The type of unit load can also influence the severity of pallet damage. Delicate high value products are often handled under strict supervision, resulting in low incidence of pallet damage. Less delicate goods are often handled more roughly by warehouse personnel, leading to increased pallet damage.

2.3 PALLET LOADS AND SUPPORT CONDITIONS

To establish an acceptable pallet design procedure, the general types of load and loading conditions as well as supporting conditions must be known. Literature dealing with pallet loads and loading conditions is scanty, but some studies have been reported and are reviewed in this section.

The load configuration placed on pallets is generally called the unit load. Tanchoco and Agee (1980) investigated the relationship between load and pallet geometry and optimum warehouse space utilization. They stated:

> "The unit load is composed of one or more bulk items or bulk material arranged on a pallet or other base which can be picked up by handling equipment."

They classified unit loads into three categories based upon the strength and form of the products making up the palletized load:

1. materials which are strong enough to withstand crushing and are of a shape which permits direct construction of a unit load, such as lumber or bricks

2. strong materials of irregular shape requiring intermediate cartons or boxes to facilitate stacking, such as canned goods or grocery items, or electronic equipment

3. bagged materials capable of compressing into a relatively flat surface such as grains or cement

Past research suggests that the pallet size and unit load configurations be designed as functions of carrier size, product size, warehouse shape and size, and rack or storage bay size (Tanchoco and Agee 1980; Goehring and Wallin 1981). Goehring and Wallin (1981) outlined a non-engineering procedure which can be used to design the unit load and choose an appropriate pallet size which will conform to warehouse conditions. Other researchers have produced similar procedures, but these are beyond the scope of this paper. In summary, the geometry of a pallet is generally

determined by its expected function rather than by strength and stiffness consideration.

To characterize the actual loading and support conditions of typical in-service pallets, Goehring and Wallin conducted an on-site survey of 88 materials handling environments. They found that the static loading of pallets can be grouped into three classes:

- uniformly distributed loads covering the entire deck
- partially concentrated or uniform loads covering only a portion of the deck
- concentrated line or point loads

They also classified the support conditions into three groups:

1. pallets loaded and dead-piled into stacks, resulting in the top and bottom deckboards being stressed as simple or continuous beams (69%)
2. loaded pallets supported along the outside stringers in racks (racked across the deckboards) (10%)
3. loaded pallets supported under ends of the stringers in drive-in racks (racked across the stringers) (21%)

Goehring and Wallin observed that some unit load types shifted from the theoretical load model to some other load condition due to interactions with the handling environment. For example, bagged goods tended to slump inward, causing a non-uniformly distributed load. Also, boxed goods

tended to bridge and transfer the load in an uncertain manner to the pallet. No attempt was made to quantify the difference between the assumed model loads and actual pallet loads. Goehring and Wallin found that the pallet loads in the survey varied from 19,000 pounds to 350 pounds with a median load of 1800 pounds.

The data gathered by Goehring and Wallin was used to calibrate the safety index, Beta, in the new pallet design procedure. The details of this step are in Chapter 8.

2.3.1 LOAD-BRIDGING

Load-bridging is a phenomenon that occurs when the unit load is stiff in relation to the pallet. During loading the pallet's deflection causes the load to "bridge" between the supports as shown in Figure 3 on page 26. Products that are intrinsically rigid such as stiff boxes or machine parts may cause load-bridging. In such cases the assumption of a uniformly distributed load may be unrealistic, resulting in erroneous predictions of pallet deflection and load capacity. Due to sparse information on load-bridging and its effects on pallet performance, two projects were initiated within the scope of the cooperative pallet research program.

The first study, conducted by G. B. Fagan (1983), had objectives as follows: 1) to determine if load bridging of package components has an effect on pallet performance, 2) to develop and verify structural models of load and support conditions of pallets as found in common usage, and
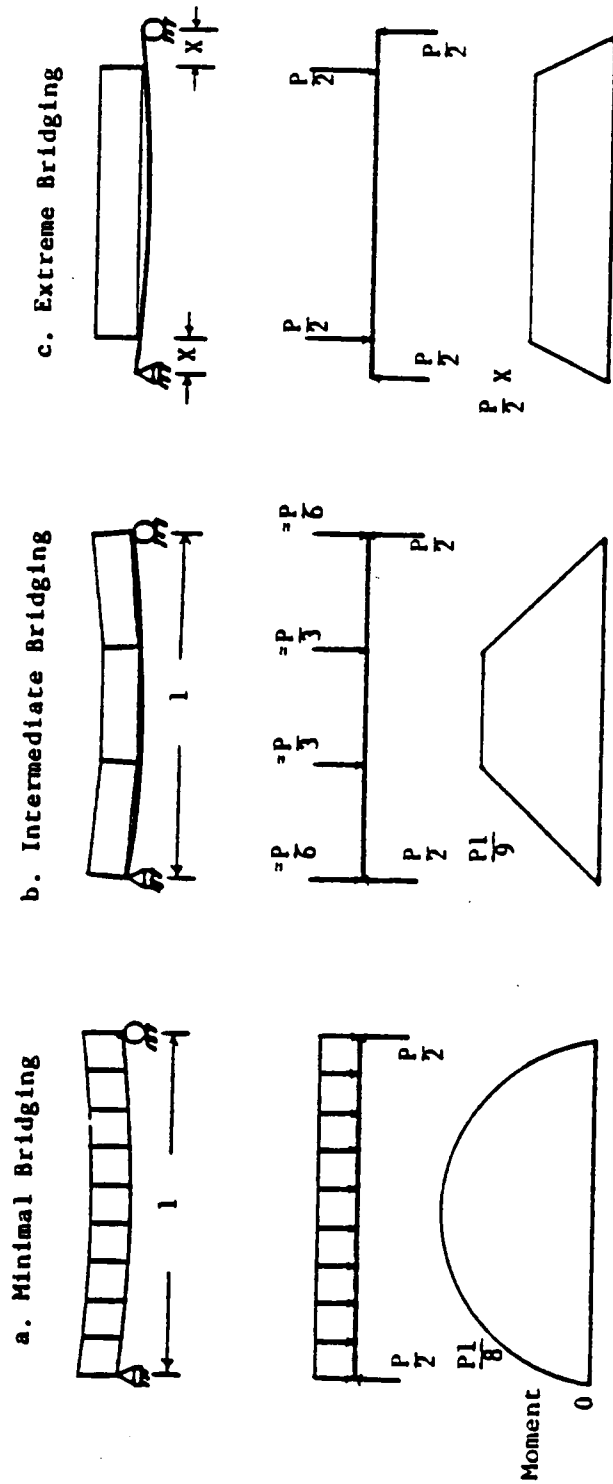
Figure 3.   Analogs illustrating three levels of bridging of a total load P.

(Collie, 1984)

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3) to design and build a pallet testing machine which allows for the simulation of in-service load and support conditions of pallets. The load conditions he investigated included uniform or concentrated loads applied by either an airbag or boxed goods, and the support types were racked across the stringers or deckboards. Fagan found that load bridging of package components was inversely related to pallet stiffness, but that quantitative prediction of the effect of bridging on pallet response was difficult. To satisfy his second objective, Fagan found that for RAS pallets the clear span is the best estimate of the effective span. In other words, within the limits of his investigation, the rack bearing width had minimal influence on the response of the pallet. He also identified the need for a new structural analysis model for pallets racked across the deckboards. He concluded that an accurate model should allow for a change of joint rigidity for bottom joints located near the support.

The second project, conducted by S. T. Collie (1984), had three objectives: 1) to characterize the load distribution of pallets in the stacked support mode, 2) to further investigate the effects of load bridging on pallet performance, and 3) to provide experimental data for verification of the new pallet design procedure.

For bagged or boxed goods Collie found that the percentage of total stack load distributed to the top deckboards of the bottom pallet was related to the number of stacked pallets, and neither the pallet stiffness nor the load type or configuration significantly affected the load distribution. The proportion of load distributed to the top deck of pallets

stacked 1, 2, or 3 high was 100%, 80% and 66% respectively. The remaining load is transferred through the stringers directly to the floor and therefore does not contribute to the bending stress of the top deckboards of the bottom pallet in a stack. This conclusion differs from the assumed load distribution used for previous stacked analysis developed by Wallin and is used to modify PDS output for specific conditions as detailed in Chapter 5. Collie also found that, in either the RAS or RAD support modes, pallets of low stiffness will experience significant load bridging and their behavior will not follow that of a true uniform load. However, he cautioned against recognizing this phenomenon in general design situations because it is very difficult to quantify. Ignoring load bridging may result in slightly conservative designs. In the design verification phase of Collie's work, he tested 125 pallets of twelve different pallet designs in three support conditions, RAD, RAS, and Stacked. These results are presented in later sections and are used to evaluate the accuracy of the pallet design system.

2.4 PALLET-SHOOK MECHANICAL PROPERTIES

A keystone in the rational design of pallets is knowledge of the mechanical properties of the lumber used in pallet manufacture, particularly the modulus of elasticity, the maximum bending stress, and the variability of both. Such lumber, generally called 'shook', is often produced from lower quality logs of both hardwood and softwood species, or the cull material from products which require high quality wood such as furniture.

The traditional method of assigning allowable strength properties to structural lumber, as detailed in ASTM Standard D-245, if used directly, may not provide the accuracy needed for estimating the strength of pallet shook obtained from low quality logs (Walters, et. al. 1971). Briefly, the ASTM procedure is executed by first establishing clear wood strength values for the desired species. The clear wood strength values are obtained by extensive testing of small, clear specimens according to ASTM Standard D-143, or by previously established property-specific gravity relationships. The clear wood properties are then adjusted to allowable design properties for full size lumber by applying various correction factors (depending on the property) as detailed in ASTM D-245 [4].

Using these procedures with modifications for cases not covered by the standards, Wallin et. al. (1976) developed estimates of design values for pallet shook. To account for the effect of strength reducing defects, Wallin recommended the use of five visually graded classes. The strength and stiffness values for the five grades were established by testing a random sample of material from each grade and evaluating the percentage of strength reduction [5] due to the grade limiting defect.

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[4] P. D. S. uses a modified form of the ASTM method to assign design values to pallet shook. This work was done by McLeod (1985). More details are presented in chapter 9.

[5] The strength reduction of a piece containing defects is relative to the strength of a similar piece which contains no defects (i.e. clear wood).

To establish the allowable bending stress and stiffness for a sample of pallet shook composed of a grade-mix, Wallin, et. al. (1976) suggested the use of grade-mix factors. The grade-mix factor is found by computing the percentage of strength reduction, based on the percentages of the grades mixed in the pallet and the percentage of strength reduction for each grade [6]. To calculate the average allowable bending stress for a pallet, the uncorrected bending stress was adjusted proportionately to the grade factor. The modulus of elasticity was adjusted in proportion to the square root of the grade factor.

The uncorrected allowable stress for both the west coast woods and the southern pines were taken from the National Design Specifications For Stress-Graded Lumber (1973). Wallin, et. al., (1976) adjusted the table values for a two month load duration. A grade factor of 0.83 for west coast woods and a factor of 0.74 for southern pines is applied to the allowable stress.

The allowable stresses for hardwoods were derived from the average stress values for the mix of species that were currently used in pallet construction. The basic bending stresses were related to the density and the geographic region from which the shook came. The unadjusted stresses were obtained from the Wood Handbook (1974). The basic stress was adjusted similarly to the softwoods.

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[6]   This is similar to a weighted average based on the percent of strength reduction for each grade in the mix.

2.4.1 PHYSICAL TESTING

"Because MOE and allowable bending stress depend on the wood species, defects, and variability, information on the clearwood strength and stiffness as given in the Wood Handbook (1974) is not sufficient to estimate the MOE and the allowable bending stress for a specific pallet material" (Polensek, 1979). Therefore, experimental data for commercially important pallet species must be collected from actual pallet material. This data can be used to verify the property estimates obtained by applying the standard methods. Unfortunately this data, for many species, is scanty or nonexistent in the literature. This section describes the results of some studies that investigated the mechanical properties of selected species commonly used in pallet construction.

Holland (1980) investigated the mechanical properties of yellow-poplar pallet material. The main objectives of his study were:

1. to determine the strength and stiffness of yellow-poplar pallet shook obtained from a random sample in the principle growth range,

2. to determine the suitability of the N.W.P.C.A. grades for segregating pallet shook by strength and stiffness.

Holland tested 450 stringers and 480 deckboards and found that the N.W.P.C.A. (1962) grading rules produced a reasonable classification of the relative strength and stiffness of the stringers. However, the grading rules were not effective for identifying the relative strength

and stiffness of the deckboards. Holland's data was used to form part of the data base for estimating the properties of pallet shook described in chapter 9. (The specific results of this study were incorporated in PDS and can used by selecting species class 21.)

Bastendorf and Polensek (1984) evaluated the MOR and MOE of red alder and bigleaf maple pallet materials in both the green and dry conditions. This study included two sizes of deckboards (1 x 4 and 1 x 6 inches) and both notched and unnotched stringers. Two sampling methods were used for red alder: random samples and serially selected samples. The serially se- lected samples were included to simulate the board selection sequence in commercial pallet assembly. The average MOR's of deckboards for both species were about 75% of the clear wood values. The average MOE's were found to be approximately the same as those for the clear wood values. The MOE of notched stringers was 20% lower and the MOR was 43% lower than those of unnotched stringers.

Wallin (1981) reported on the results of four research projects which were initiated to establish the working stress for pallet shook. The results of these projects are used to supplement the data base for estimating properties of pallet shook described in chapter 9.

Spurlock (1982) investigated the mechanical properties of mixed oak pallet shook. The shook was sampled from 33 mills in 16 eastern states in proportion to the amount of oak grown in each of the states. Approx- imately 3000 boards were sampled. The effect of defects on the strength

and stiffness of the shook and the accuracy of a proposed visual grading scheme (Wallin 1979) were also evaluated [7] .

McLeod (1985) has developed and modified standard techniques to produce estimates of the strength and stiffness of pallet shook based on visual grading criteria. These procedures are used in P.D.S. and are described in detail in Chapter 7.

2.5 PALLET JOINTS

The service life of a pallet has been shown to be highly influenced by the type and quality of the fasteners (Wallin and Stern, 1974). When a pallet is subjected to impact loads it must be capable of absorbing and distributing the shock-energy throughout the structure. "Rigid joints which cannot absorb shock without failure are undesirable" (Wallin and Stern, 1974). Instead, the joints should be flexible to allow stressing without failure but stiff enough to resist bending stresses up to the crushing strength of the wood. Wallin (1981) describes the general performance requirements for pallet nails as follows:

> "Nails must be employed in numbers and sizes sufficient to provide
> the maximum shear resistance in the joints; they must be embedded
> in the wood members to a sufficient depth to resist separation
> forces sufficient to pull the head through the board members; they
> must be able to retain withdrawal resistance after the wood members
> dry to equilibrium moisture content--this requires that they be
> threaded."

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[7] This study is part of the Cooperative Pallet Research Program. The results were incorporated into the material property files of P.D.S. and can be used by selecting species class 29.

The two most common types of nails used in pallet construction are stiff-stock nails and hardened steel nails. (Wallin 1981, Eichler 1976, Wallin and Stern 1974c). Wallin and Stern (1974c) stated:

> "Stiff-stock nails are non-hardened, medium or medium-high carbon-steel nails and provide greater stiffness at high flexural loads than bright low-carbon steel nails of the same wire diameter. Hardened steel nails are heat treated and tempered, medium to medium-high carbon steel nails, providing at least the stiffness of low-carbon steel nails of one gauge larger diameter at high flexural loads.

To determine the relative hardness of the steel, nails are often subjected to the MIBANT test (ASTM standard F680- Testing nails) (Stern 1970). The test involves dropping a standard weight onto a clamped nail and measuring the resulting angle formed between the bent shank and the unbent shank portion. Hardened steel nails must have a MIBANT angle of 28 degrees or less, while stiffstock nails may vary from MIBANT angles of 29 to 46 degrees. (Wallin and Stern 1974c). Pallet nails should have a helical thread with a minimum of 4 flutes and a thread angle of 60 degrees (+ or - 5 degrees) (Wallin and Stern 1974c). The threaded nail greatly improves the withdrawal resistance of the fastener.

Wallin and Stern (1974) provided equations, based on empirical data, which can be used to calculate the allowable lateral load and the allowable static withdrawal loads for either stiff-stock or hardened steel nail joints in the side-grain of lumber. The lateral load equation is a modified form of an equation found in the Wood Handbook (1974). The following parameters are used to compute the allowable lateral load: nail diameter, wood specific gravity, a load-displacement function based on

Mack's (1966) work, moisture content factor, number of nails, nail-type factor, and species factor (for species with high splitting resistance). The withdrawal load equation is based on a function of: nail diameter, length, wood specific gravity, nail-type factor (thread vs. no thread), thread angle factor, species factor (for splitting resistance), moisture content factor, and a wood-seasoning factor. These equations are discussed further in chapters 5 and 9.

In addition to the load carrying capacity of joints, three other joint parameters are necessary for modeling pallet structural behavior. These are translational stiffness (slip modulus), separation or withdrawal modulus, and the rotational stiffness (rotation modulus).

The translational stiffness is a measure of joint stiffness in lateral loading. Empirical values for the translational stiffness can be obtained from load-slip curves of nail joints. Also, several theoretical procedures have been developed to predict the joint stiffness based on properties of the connected materials (Wilkinson 1971, McLain 1976). Mack (1975) demonstrated that the lateral slip exhibited by the deckboard-stringer joint in a pallet under static load is small and can be ignored in a simple analysis.

The rotation modulus and the separation modulus are "constants describing the degree of fixity of a nailed joint under moment and axial force respectively. The separation modulus is defined as the ratio of the applied withdrawal force to the corresponding separation; whereas the rotation

modulus is the ratio of the applied moment to the angular rotation "(Kyokong 1979). Kyokong (1979) developed an equation which relates the separation modulus to the rotation modulus. Therefore, the fixity of a pallet joint can be modeled by one factor, either the rotation or separation modulus.

Wilkinson (1983) investigated the effect of the material properties of the stringers, deckboards and fastener types on the rotation modulus, and developed an empirical relation between rotation modulus and material properties. The results of this project have been incorporated into P.D.S. and are described in detail in chapter 3 [*] .

2.6 SUMMARY OF THE LITERATURE

The wooden pallet is a deceptively complex framework whose structural performance involves load sharing, composite action, and possible non-linear behavior. The structural action and performance of pallets may be affected by many factors. The important variables which influence pallet performance have been reported in literature and were briefly presented in the previous pages. This information, together with the results of concurrent research projects, were used to form the foundation for a rational design procedure for wooden pallets. The remainder of this dissertation contains the specific techniques used to develop this procedure.

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[*]   This study is part of the Cooperative Pallet Research Project.

3.0 GENERAL PALLET DESIGN

The objective of this chapter is to introduce some fundamental concepts used in the development of a design procedure for wooden pallets. Some basic terms, limitations, and geometries are defined here for use throughout this thesis. This chapter also provides the reader with a global view of the relationship between the various elements involved in the design process. In subsequent chapters the techniques developed for design of pallets in specific load and support conditions are detailed.

3.1 PALLET DESIGN--A GLOBAL PERSPECTIVE

As described in chapter 2, a design methodology based on probabilistic concepts was developed for use with wood pallets. Traditional design methodology associates design uncertainties with either the load or the resistance side of equation (2.1) and treats the load and resistance as if they were independent. However, in the design process "four major sources of uncertainty and variability can be recognized, and there can be appreciable interaction among most or all possible pairs of these four sources" (Criswell, 1979). The major sources of variability are the material resistance, the applied loads, the analysis methodology, and the actual service life. Figure 4 on page 38 schematically shows these sources and the possible interactions among them.
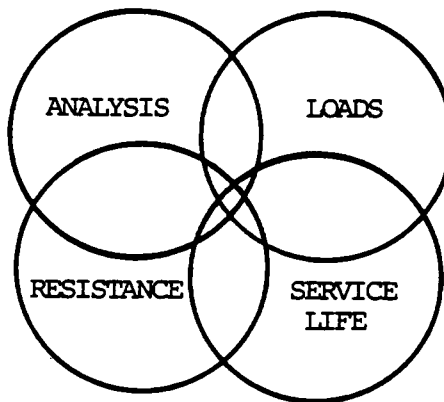
Figure 4 . Diagram of interrelationship
among design variables.

Each of the four sources of variability can be defined specifically for wood pallets. The _resistance_ is defined as the property of the material (wood, nails, etc.) that is associated with resisting the effects of applied loads. For pallets the most important material properties that provide load resistance are the strength and stiffness of the wood members (measured by the modulus of rupture (MOR) and the modulus of elasticity (MOE), respectively). Resistance variability arises from several sources including "material properties, dimensions, workmanship, and construction processes" (Criswell, 1979).

The _applied loads_ are future events and depend upon the use of the structure. For most structures the loads are truly random events and are often dictated by nature (i.e snow or wind loads on buildings). For pallets, the static loads are often accurately known. For example, in a warehouse catering to a single product, all pallets might carry the same magnitude of unit load. However, in other warehouses the coefficient of variation of the load distribution may exceed 50% thus increasing the probability of observing loads that will exceed the load carrying capacity of the structure (i.e increased probability of failure). Additionally, the form or type of the load is variable, and may be uniformly distributed or concentrated as point loads.

The _analysis_ is a series of procedures or formulas that translate the loads into load effects (stress and deflection). This translation process is necessary to allow comparison of the load effects and material resistance on an equal basis (i.e. in the same units). The analysis also

General Pallet Design   39

evaluates the resistance provided by the geometry and properties of the material. The variability that is associated with the analysis step can arise from several sources such as simplifying assumptions, idealization of the load and support conditions, and errors and approximations in the analysis calculation.

"The actual service life that the structure will experience is unknown at the time of design" (Criswell, 1979). The variability associated with the service life is primarily related to interactions with both the resistance and the applied loads. For example, the material properties can change drastically with time, joints may weaken from repeated loading, and wood can become decayed and lose much of it's strength. Also the probability of experiencing extreme loads increases with increased service life.

Other interrelationships between these four parameters exist. For example, "the analysis method must consider that the characteristics of the loads, and the calculated load effects depends upon both the loads and the analysis. Also, the loads and the loading history may influence the resistance. Such is the case when fatigue, creep, or some other form of accumulated damage occurs. Additionally, if the resistances change appreciably with time or such time dependent items as general moisture conditions, then the service life and resistance are not independent" (Criswell, 1979).

The overall goal of the Pallet Research Project was to develop and apply (in a computerized form) rational techniques to account for all these variabilities and therefore to improve upon traditional design methodology.   To satisfy this this goal, the variables and problems associated with each circle in Figure 4 on page 38 were addressed. These define the general format of this dissertation. Specifically, the techniques developed for the analysis of wooden pallets in the various support modes and load conditions are described in chapters 4, 5, and 6. The methods developed to estimate the resistance of wood pallet materials are described in chapter 7. Methodology developed to account for the variability of the loads and to achieve safety is described in chapter 8. The techniques used to estimate the service life of a pallet in a specific environment and the cost associated with its use are presented in chapter 9. Finally, chapter 10 summarizes the PRP project and identifies areas where data or other information is lacking and may warrant further research.

Before discussing the specifics of the design procedures it is necessary to define some basic terms, assumptions, and limitations which consititute the scope of this proposed design procedure.

3.2 THE PALLET DESIGN SYSTEM (PDS)

The PALLET DESIGN SYSTEM is a set of procedures developed to provide pallet manufacturers with tools for designing pallets to meet various performance and serviceability criteria, such as strength and stiffness

in specific support modes, life-expectancy, and minimum cost-per-use. For expediency, the system was computerized for two commonly available mini-computers namely, the Apple II and IBM-PC mini-computers. (PDS has also been translated for the TRS-80 and the Wang machines by individual users.) The program is "user friendly" and requires minimal user knowledge of computers and engineering concepts. The PDS program is written in the BASIC language and is executed as a series of subroutines driven by a main program. The user is required to provide a specific description of the pallet and component geometry, species, fastener characteristics, support conditions, and load type. The program automatically requests this information from the user and has built-in provisions for modifying any input parameter. This feature allows the user to optimize the structure for strength or durability by modifying the geometry of the structural elements, or to correct erroneous (mistyped) input. For simplicity, the program is menu driven and has a separate menu screen for each pallet part type (i.e stringers, top deckboards etc.), load and support modes, durability parameters, etc.

When using PDS the user has several options. After describing the pallet and load and support conditions the user can select any of the following: a schematic diagram of the pallet for visual verification; a specification sheet summarizing the parts, fasteners, and overall pallet geometry can also be examined; store the pallet description in a file on a mini-diskette for use in a subsequent design session; analyze the pallet for strength and stiffness in several support modes, lateral collapse poten-

tial, and, durability; or have all analysis results and a complete pallet description sent to an on-line printer.

3.3 SCOPE OF THE PALLET DESIGN PROCEDURE AND PDS

GENERAL GEOMETRY AND MATERIALS: The PALLET DESIGN SYSTEM can analyze lumber pallets having 2, 3, or 4 stringers (notched or unnotched) and a maximum of 15 boards on a deck (top or bottom). The pallet decks can be reversible (i.e identical top and bottom decks) or nonreversible, single winged (i.e top deckboards extending past outer stringer edge forming an overhang), double winged, or flush (i.e deckboard ends flush with stringer edge). The parts can be of any geometry (width, thickness, and length), and commonly used timber species (defined in detail in Material Resistance chapter). Most commonly used fasteners such as, staples, threaded (helical or annular) or smooth shank nails of hardened, stiffstock, or low carbon steel can be analyzed.

SUPPORT MODES: Four principal pallet support modes may be analyzed by PDS: racked across the stringers (RAS), racked across the deckboards (RAD), stacked, and sling supported. These modes are schematically shown in Figure 5 on page 44. (These represent the vast majority of generic support conditions found in a field study by Gohering and Wallin). The RAS mode causes the stringers to be stressed as parallel beams. The RAD mode causes both top and bottom decks to be stressed as a composite structure. The stack mode causes the top deck of the bottom pallet and bottom deck of second pallet in a stack to be stressed independently as continuous

a) Racked across deckboards (RAD)
   (end view)

b) Racked across stringers (RAS)
   (side view)

c) Stacked (end view)

d) Supported under top deck (sling
   support (end view)
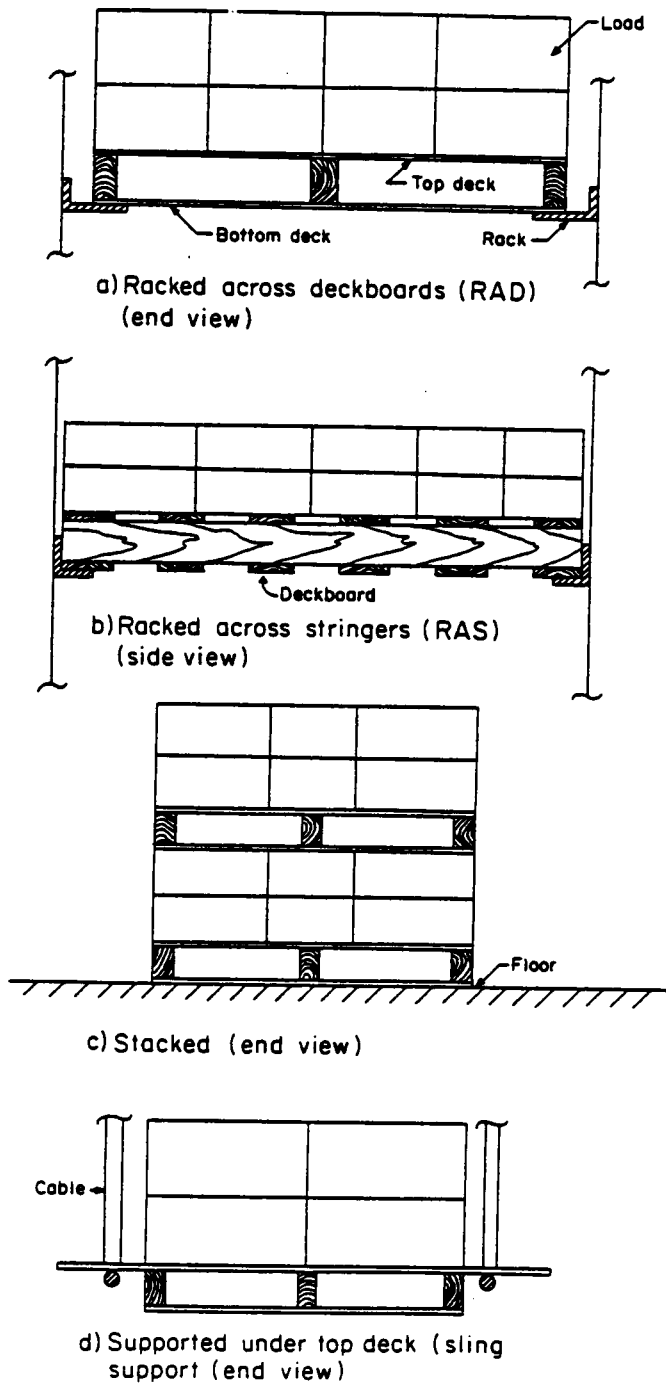
Figure 5. Common support types.

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beams.   The sling support mode causes the top and bottom decks to be
stressed similarly to the RAD mode except that load is transferred through
the top deck wing to the support.   The detailed explanation of the anal-
ysis techniques for pallets supported in these modes is presented in
chapters 4, 5, and 6.

LOAD CONDITIONS: In PDS, five potential load conditions are assumed for
each support mode as follows (see Figure 6 on page 46):

1.   Uniform load--This load type is typical of products such as boxed or
     bagged goods, covering the entire top deck and producing a uniformly
     distributed load.

2.   Partial coverage uniform load--Caused by a unit load that is smaller
     than one or both dimensions of the pallet.

3.   One centerline-line load--Produced by a product such as a horizon-
     tally positioned barrel located directly over the pallet centerline.

4.   Two symmetrically placed line loads.

5.   Three line loads (cases 3 and 4 acting simultaneously).

Some specific limitations regarding these load types follow: a) Since
line loads are assumed to be intrinsically rigid, this load type is not
analyzed in the stack mode if the loads are perpendicular to the stringers
or if the loads are located directly over and parallel to a stringer.
In other words, the floor supported stringers carry the entire load and
the deckboards are not stressed.   The total load is therefore governed
by the stringer's compression perpendicular to grain strength.   Likewise

LOAD TYPES
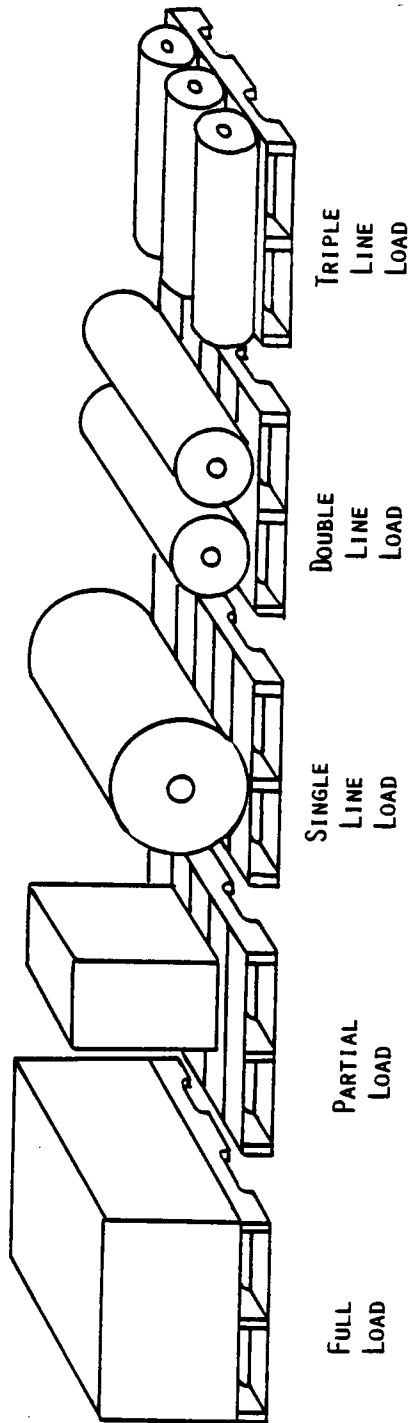
Figure 6 . Load types analyzed by PDS.

SINGLE LINE LOAD

DOUBLE LINE LOAD

TRIPLE LINE LOAD

PARTIAL LOAD

FULL LOAD

in the racked modes, no analysis is performed if the line loads are parallel to the free span. In this case it is assumed that the load itself bridges the free span. For example, if the line load is defined as being parallel to the deckboards, only the RAS mode is analyzed. b) Due to simplifying assumptions made in the RAD analysis, the minimum length of partial uniform loads are limited to two inches for two and three stringer pallets. For four stringer pallets, the minimum length is equal to the spacing between the centerlines of the inner stringers. For either case, symmetry about the pallet centerlines is assumed. The reason for these restrictions is described in detail in chapter five.

Specific details regarding the analysis of pallets loaded with any of these load types are presented in chapters 4, 5, and 6.

An additional load condition is allowed for the stack support mode namely, lateral loading resulting from horizontal forces similar to those that might be generated by fork truck impact. For this load condition, PDS produces an estimate of the lateral collapse potential (high, medium or low). Lateral collapse occurs when the horizontal shear capacity between the top deck and stringers is exceeded, thus the stringers become unstable and rotate causing the top deck to collapse to the floor (Figure 7 on page 48). Specific details are in Arritt's thesis (1985).

RACK SPANS: A racked pallet is assumed to pivot about the inside edges of the support; therefore, the effective span for any racked pallet is the distance between those inside edges. The rack-bearing width is not
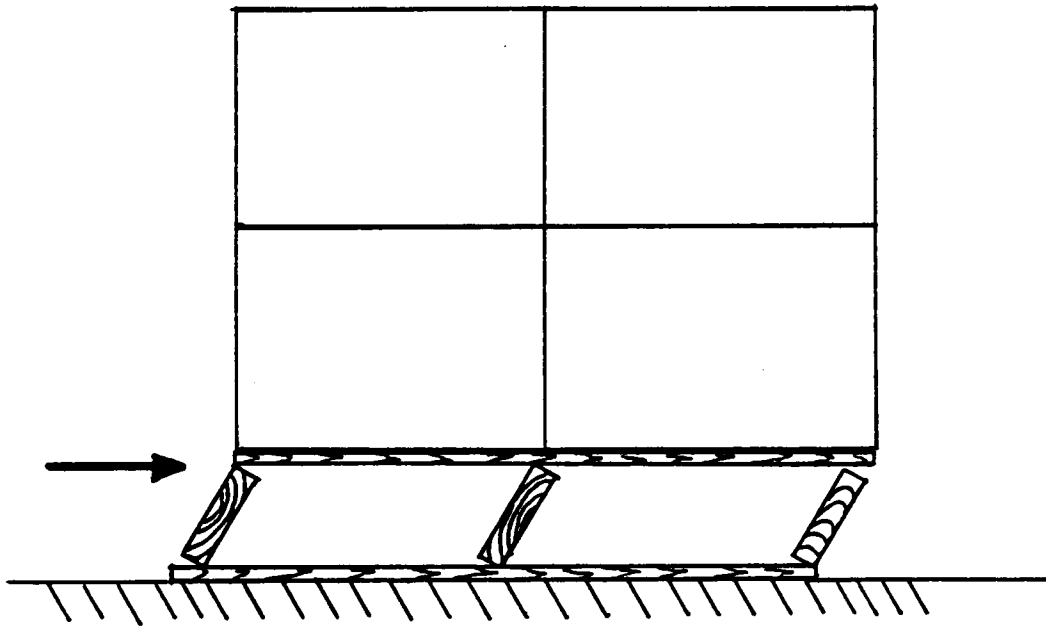
Figure 7 . Lateral collapse of a pallet.

LOAD

rack

O
overhang
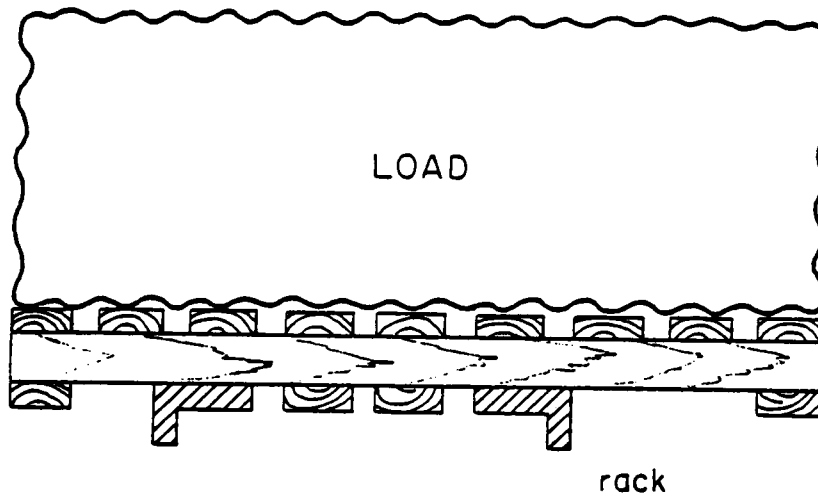
span
S

$O \leq 0.35\,S$

Figure 8. Limit for rack support placement.

directly considered in the analysis of racked pallets.: To simplify the analysis of uniformly loaded structures, the overhanging end is limited to 35 percent of the span (Figure 8 on page 49). Exceeding this limit causes the maximum bending moment to occur in the overhang between the support and the beam end (TCM 1974) thus requiring development of additional equations to compute the overhang moment. Since pallets are rarely racked in this fashion, limiting the overhang to 35 percent of the span is justified. Additionally, for notched stringer pallets, the equations for computing the stress at the notch are valid only for cases where the support is located between the beam end and the notch.

OPTIONS: For any support mode two options can be used: a) DESIGN option, or b) ANALYSIS option. The DESIGN option is selected when the user wishes to optimize the design of the pallet by balancing safety and economy; in other words, to determine the minimum amount of material which will safely carry the loads. For this option the user knows the magnitude of the loads being placed on the structure and has selected a trial pallet geometry and material (i.e., species). The computerized version of the design option checks the design and decides if the required strength and deflection limits are satisfied. The program then produces estimates for optimizing the structure by either increasing or decreasing the width and thickness of the critical structural elements.

The ANALYSIS option is used to compute the maximum load which can safely be placed on the structure. The maximum load capacity is based on either the strength of the critical structural elements or a deflection limit

input by the user. (A deflection limit is specified in cases where the maximum deflection of the pallet must be limited. Such limits are usually dictated by the physical limitations of automatic pallet handling equipment.)

The ANALYSIS and DESIGN options are described in more detail in chapters 4, 5, 6, and 8.

SUMMARY OF STEPS IN PALLET DESIGN PROCEDURE:

The general scheme used in the proposed pallet design procedure is shown in Figure 9 on page 52. This shows the interrelationships between the input parameters used to define the structure, loading, and support conditions. The analysis translates the loads into the load-effects. A separate analysis is conducted for each support mode. The safety requirements are used to ensure that the structure will perform satisfactorily in terms of strength and stiffness. The load-effects and resistance comparison produces either an estimate of maximum safe load (ANALYSIS option), or an estimate of the optimum critical member dimensions (DESIGN option). The estimated service life (and economic analysis) is made using the general pallet description and resistance estimate, and the characteristics of the use environment. The results of the strength and stiffness analysis and the durability estimate are reported to the user. These steps are discussed in detail in the remainder of this dissertation.
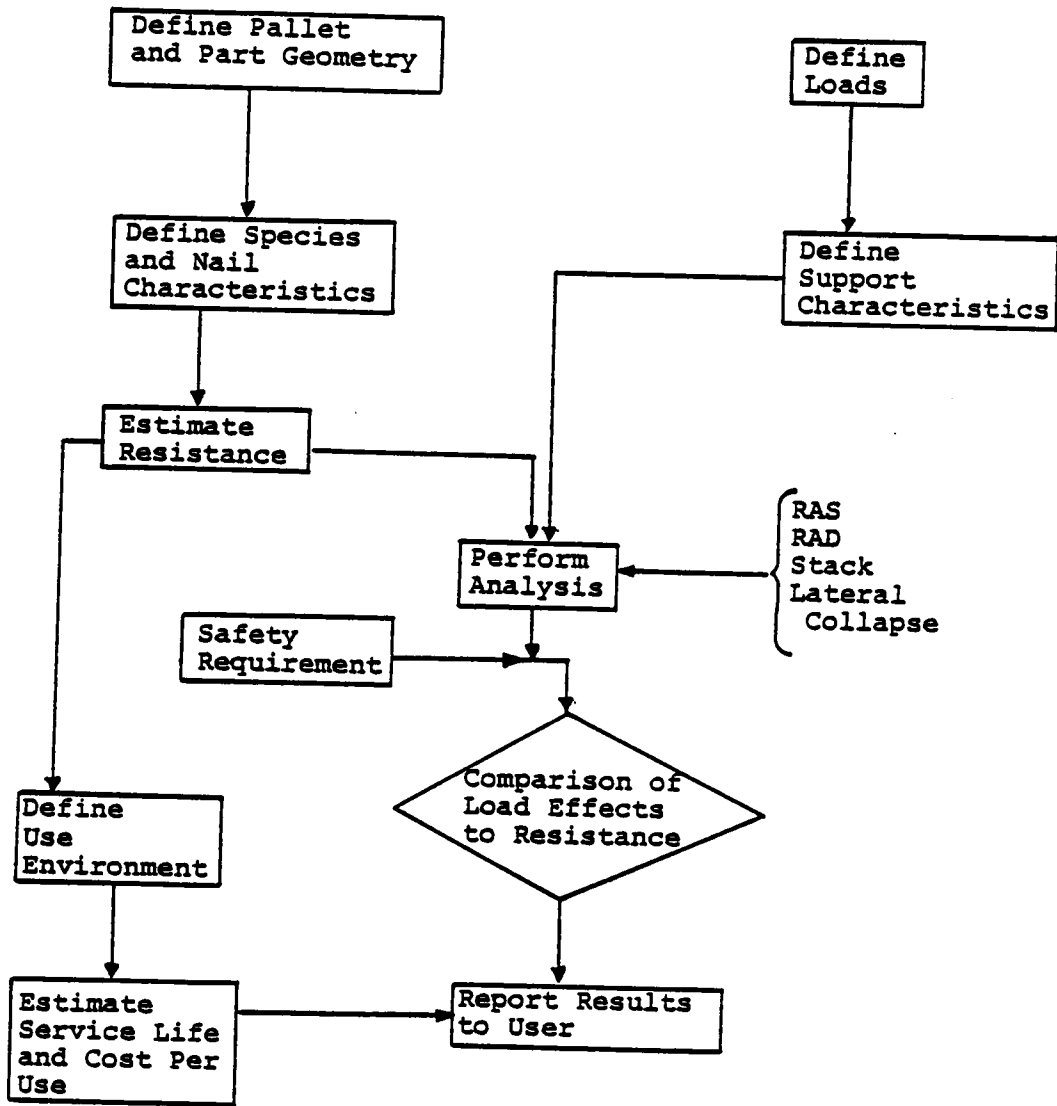
Figure 9. General flow chart of pallet design
         used in PDS.

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4.0 DESIGN FOR RACKED ACROSS STRINGERS SUPPORT CONDITION

The previous chapter described the general scheme used in the Pallet Design System. This chapter describes the analysis techniques developed for computing the load effects of pallets racked across the stringers (RAS). The following chapter describes the techniques developed to analyze pallets racked across the deckboards.

Multiple story rack systems allow for efficient space usage in warehouses, where access to individual unit loads is necessary. The rack typically supports the pallet along two opposite edges, thus requiring the pallet to act as a bridge connecting the free span between its supports. The general geometry of RAS pallets (notched and unnotched) is as shown in Figure 10 on page 54.

The pallet must have sufficient strength and stiffness to successfully carry the load. The consequences of insufficient strength can include a cascading failure started by a pallet located high in a rack. If an over-stressed pallet fails, its load falls and can cause the next lower pallet to fail or become unstable. This can continue until many pallets in the rack have failed. The severity of failure increases if the load contains a high valued delicate product or caustic materials; in many cases, such failures can be life threatening to warehouse personnel.

Racked Across Stringers (RAS)

SPAN

SPAN
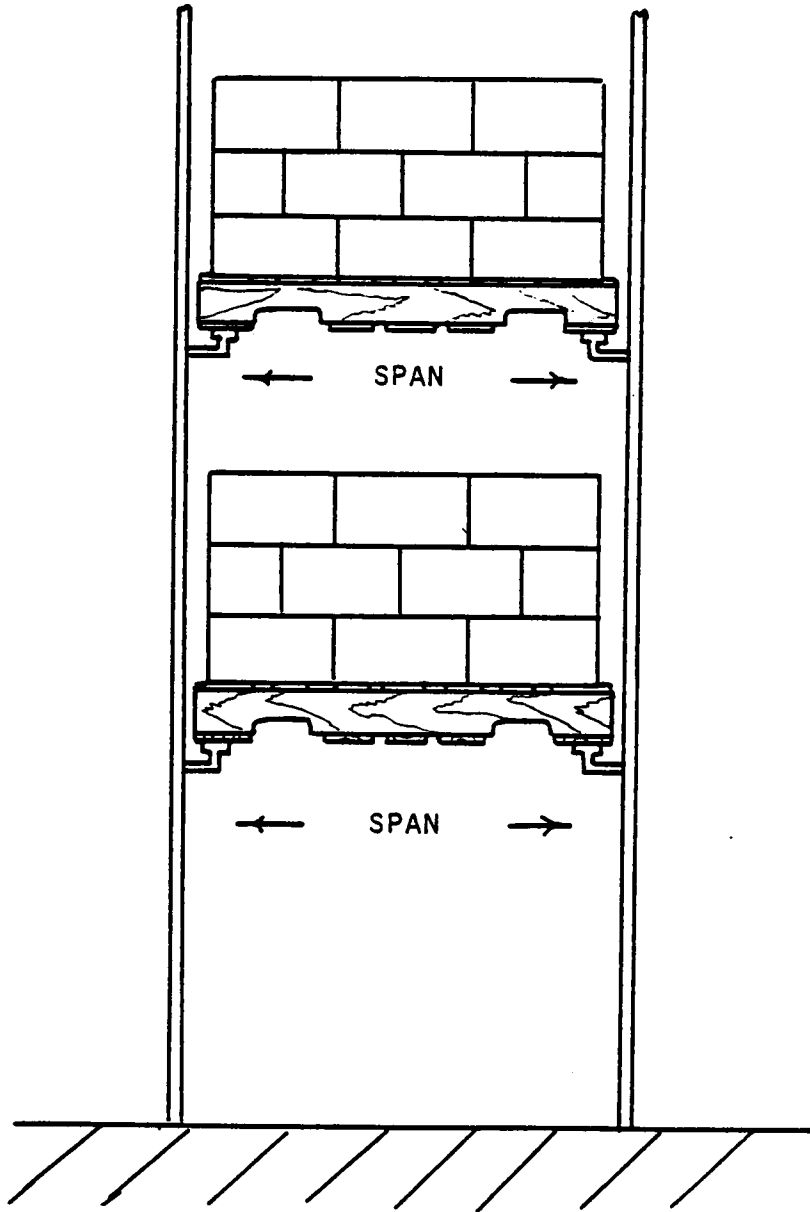
Figure 10. Pallets supported in racked across stringers (RAS) mode.

Occasionally, deflection limits are specified in addition to strength requirements. The maximum allowable deflection is usually limited by automatic pallet-handling equipment. If the loaded pallet deflects excessively, the machine might be unable to adjust its fork position in relation to the fork openings in the pallet, thereby making the pallet inaccessible to that machine. There is also an inherent "psychological deflection limit": an excessively deflected pallet looks unsafe even though it can successfully carry its load.

4.1 GENERAL LOAD AND SUPPORT CONDITIONS

The pallet design procedure considers two principle racking modes: 1) racked across the stringers (RAS), and 2) racked across the deckboards (RAD). (This chapter only deals with the RAS mode.) The RAS mode causes the stringers to be stressed as multiple parallel beams as shown in Figure 10 on page 54.

The objective of the analysis is to translate the applied load into the load effects. For pallets, the important load effects are the stress and deflection of the most highly stressed members. These critical members govern the maximum allowable pallet load. For the RAS mode the critical members are the stringers, which include either of the outer stringers in a two stringer pallet, the center stringer in a three stringer pallet, and either of the inner stringers in a four stringer pallet. In a typical analysis involving a pallet which has equal sized stringers, only the critical stringers are checked, because these have the larger tributary

area and therefore higher stress and deflection than the outer stringers. However, in pallets that have unequal sized stringers, the outer stringers may attract more load and thus experience higher stress and deflection than the inner stringers. Therefore, if the stringers are unequal sizes, each stringer must be checked to find maximum stress and deflection of the pallet.

RAS LOAD ANALOG: In PDS, five load conditions are allowed as described in chapter 3. These load types are idealized as shown in Figure 11 on page 57. Distributed loads are considered to act uniformly along the length of the deckboards, and are transferred from deckboards to stringers, as concentrated point loads, only at the deckboard-stringer joints or intersections. The resulting loads on the stringer are assumed to be located at each deckboard centerline for fully loaded deckboards or at the center of the load for partially loaded deckboards (Figure 12 on page 58).

Line loads are considered to act as concentrated point loads on stringers. The location of line loads are defined by the user.

SPANS: As described in chapter 3, the the effective span for any racked pallet is the distance between inside edges of the rack support.: The remainder of this chapter describes the analysis techniques developed to compute the load effects of RAS pallets for specific load conditions. To determine if the design is acceptable, the load effects are compared to the resistance using the FOSM methods described in Chapter 8. The
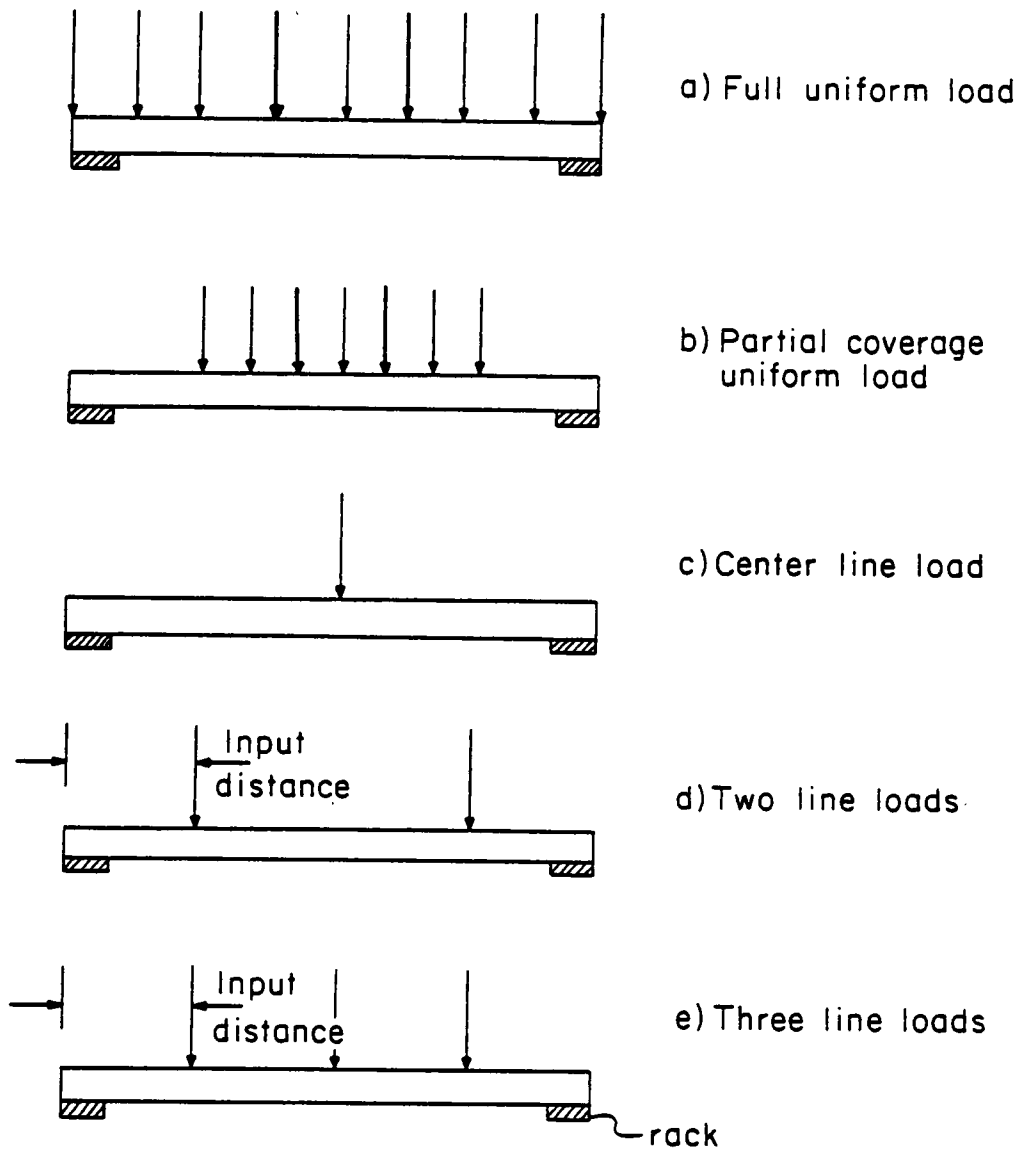
Figure 11. Analog load types analyzed by PDS.
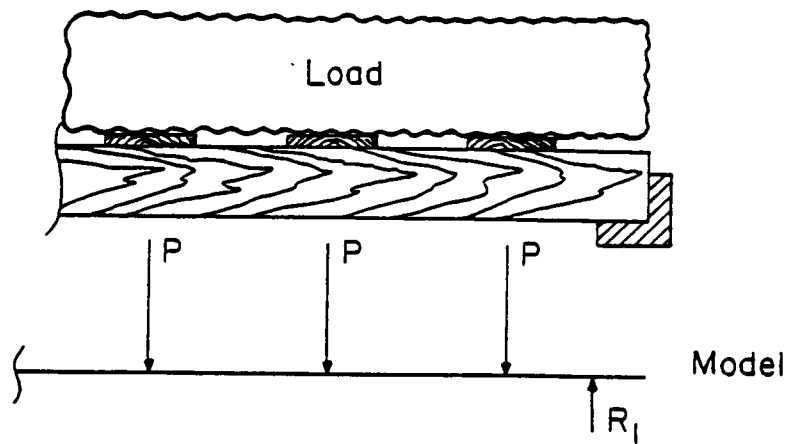
a) Fully loaded deckboards
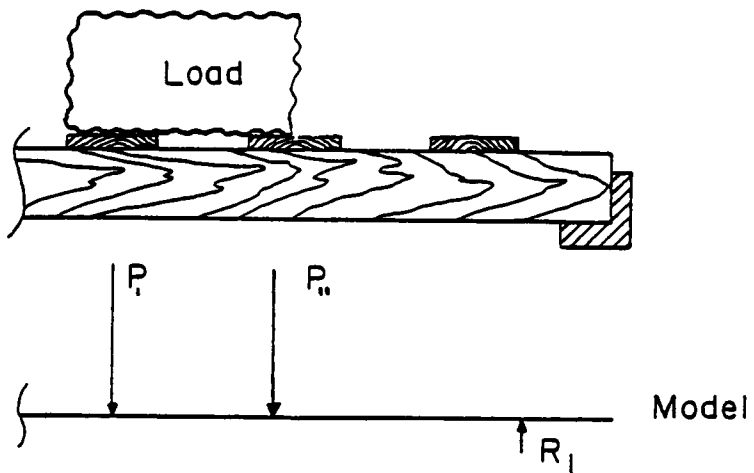
b) Partially loaded deckboards

Figure 12 . Analog models of a) fully and b) partially loaded deckboards.

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techniques used to analyze the five load types and to apply the ANALYSIS or DESIGN options are presented concurrently. The effectiveness of the analysis procedures are evaluated in the last section by comparison of predicted to measured pallet response.

4.2 ANALYSIS METHODOLOGY

Two different techniques were developed for use in computing the load effects (i.e. deflection and stress) of pallets supported in the RAS mode: 1) a simple strength of materials approach and 2) a matrix displacement solution. For all pallets, except those having unequal sized stringers, loaded with either a uniform or a partial uniform load, the strength of materials approach is used because it requires only a few seconds for a microcomputer to compute the solution and has relatively minimal computer memory requirements. The matrix approach, although it is more versatile and accurate, requires greater computer memory and may take considerable time for computation. Hence, it is only used for those cases where the analysis is too complex for the simpler strength of materials approach. The increased complexity is partly caused by the manner in which loads are distributed to the stringers, a phenomenon called load sharing. For structures having equal sized stringers, load sharing can be characterized by a fairly simple model described below. For structures having unequal sized stringers, prediction of load sharing effects becomes quite difficult, and, for some structures, unreliable answers may result if the simpler procedure is applied. Both methods were developed since a project

requirement was to produce solutions for a wide range of computer capabilities.

The following section discusses the methodology used to design or analyze RAS pallets by the strength of materials approach. Then the matrix displacement procedure is presented.

4.2.1 EQUAL SIZED STRINGERS LOADED WITH UNIFORM OR PARTIAL UNIFORM LOADS

GENERAL:

The structural response of uniformly loaded three and four stringer pallets is influenced by the lateral (in-plane) stiffness of the deckboards, and to a lesser extent, the lateral stiffness of the nail joints. This influence, known as differential deflection, is described as follows: In loaded pallets the center stringer (or stringers) usually has the largest tributary area and therefore, higher stress and deflection than the outer stringers. According to beam theory, as a consequence of the increased deflection of the inner stringer(s), the top and bottom fibers of the center stringer will experience greater horizontal motion (i.e., rigid body rotation about the neutral axis) than the corresponding fibers of the outer stringers (see Figure 13 on page 61). For example, consider points a and b in Figure 13 on page 61 which represent the intersection of the centerline of a deckboard with three stringers of an unloaded pallet. After loading, these points translate horizontally to a' and b'. Because of the increased bending of the center stringer the

a) Initial and deflected states showing differential stringer deflection
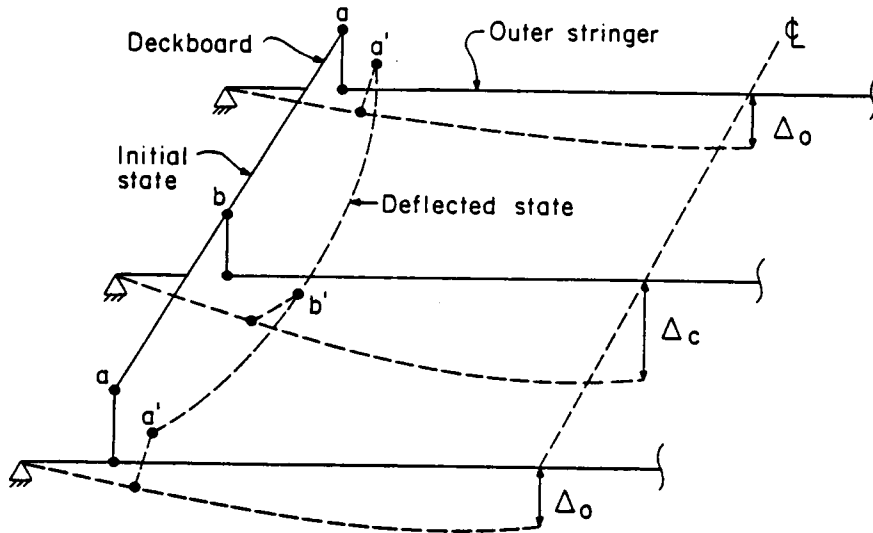
b) Schematic of center stringer
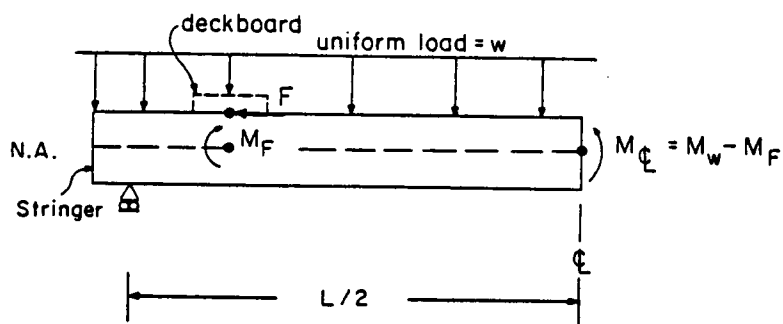
F = load caused by deckboard-stringer shear,
M_F = moment resisting F,
M_w = moment resisting vertical load w,
$M_{\text{¢}}$ = moment at span centerline.

Figure 13. Schematic of RAS pallet showing differential stringer deflection.

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horizontal distance between b and b' will be greater than that of a and a' depending on the inplane flexibility of the deckboard.

Because the deckboards are fastened to the stringers, in-plane bending forces are introduced in the deckboards due to differential deflection. The horizontal deckboard reactions appear on the stringer as horizontal forces (i.e. shear forces) located at the outer edges (i.e. top or bottom face). These reactions are greatest for deckboards at the ends of the stringer and decrease to zero for those in the center of the span. Because the reactions are displaced from the neutral axis by a distance equal to half the stringer height, moments are introduced in the stringer which reduce the maximum bending-load induced moment at centerline. Ignoring these moments can result in large errors (up to 30%) in the prediction of stress or deflection. For pallets having stringers of equal size and stiffness, the magnitude of the influence of the moments caused by differential stringer deflection can be computed as described below, thereby enabling the application of the simplified strength of materials approach. However, for unequal sized stringer pallets the complexity increases and the simplified procedure is abandoned for a numerical solution.

PREDICTION OF LOAD SHARING: For simplified design the question to be answered for RAS pallets is: "What percentage of the total load is carried by each stringer"? The distribution of load among the stringers defines the stress and deflection of each member in the pallet. Therefore, to compute the maximum load effects of the structure, an estimate of the

percentage of total pallet load carried by a critical stringer (PLOAD) is required. From this estimate, the stress and deflection of the critical stringer are computed using principles of statics and strength of materials.

For accurate results in a simplified procedure, a parameter in addition to PLOAD is needed to account for the effect of differential stringer deflection (PERROR). This subsection describes the development of equations for predicting PLOAD and PERROR based on the properties and geometry of a pallet.

Results of computer simulations of RAS pallets, using a modified version of SPACEPAL, provided a data base for developing regression equations to predict the percent of total load (PLOAD) on the critical stringer and the influence of differential stringer deflection (Perror). Simulations were conducted for pallets having two, three and, four stringers. (The two stringer pallets were included to verify the assumed load distribution of fifty percent in each stringer.) The input parameters to SPACEPAL included the number of structures simulated, the specific geometry of the pallet and span, an arbitrary uniform load (total magnitude of 2000 pounds), and the three parameters for the Weibull cumulative distribution function of the modulus of elasticity for both deckboards and stringers of eastern oak pallet shook (collected by Spurlock). The modified version of SPACEPAL used 'Monte Carlo' techniques, commonly described in literature (Woeste, Haan), to assign MOE values to each member in the pallet. The MOE values were randomly sampled from the Weibull cumulative dis-

tribution function. (Uniform random numbers were generated from the International Mathematical and Statistical Library (IMSL) subroutine called GGUBS.)

A wide variety of pallet geometries were simulated. The selected designs are representative of commonly manufactured pallets and include deckboard coverages from 29 to 95 percent and pallet dimensions of 46 by 19 inches to 75 by 60 inches. (See Appendix A for descriptions of these pallets). For the four stringer pallets the ratio of spacing between center and outer stringers was varied between 0.062 and 0.3. To produce a wide range of stringer-to-deckboard stiffness ratios the deck thickness was varied between 3/8 and 1 inch. Three-dimensional pallet models were used for all simulations. A typical model structure is shown in Figure 14 on page 65.

One hundred simulated structures of each geometry were generated from the input parameters. A total of 2 two-stringer, 80 three-stringer, and 40 four-stringer pallet geometries were simulated.

The joints were modeled as semirigid connections having finite lateral and rotational stiffness. The values for both rotation and lateral stiffness were selected from test curves of pallet joints obtained from the archives of the Sardo Pallet and container Laboratory at Virginia Tech (see below for more details). The data generated from each simulated structure included the maximum moment, stress, deflection, percentage of load carried by the critical stringer and, generated MOE values for
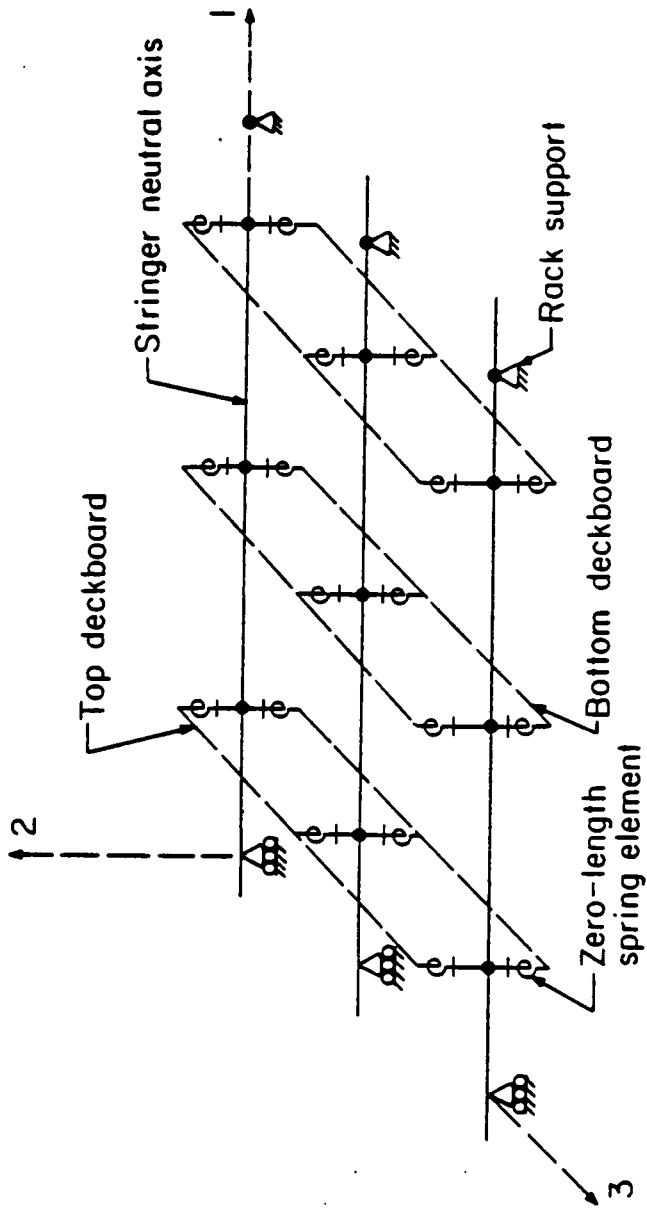
Figure 14. Three dimensional model for SPACEPAL of 3 stringer pallet with 3 top and bottom deckboards.

deckboards and stringers. The magnitude of the influence of differential stringer deflection (PERROR), was determined by computing the percent difference between the moment computed directly from the load distributed on the critical stringer (i.e. considering only the vertical loads applied by each deckboard and ignoring the effect of differential deflection) and the moment predicted by SPACEPAL.

Using the Statistical Analysis System (SAS), regression equations based on the results of the simulations were developed to predict PLOAD and K for both three and four stringer pallets. It was hypothesized that the percent of total load carried by the center stringer(s) should be a function of the relative stiffness of the stringers and deckboards. For example, if the deckboards are extremely thick and stiff (i.e. rigid bars) they would tend to cause the stringers to deflect equally, leading to equal load sharing among the stingers. If the deckboards are extremely flexible in relation to the stringers, and the joints have zero stiffness (i.e deckboards are not fastened to stringers) the load is distributed among the stringers in proportion to the reactions of a continuous beam (deckboards) over multiple supports (stringers). This load distribution causes the center stringer(s) to carry more load than the outer stringers. Hence, the load carried by the center stringer of a three stringer pallet should vary between 33% and 62.5% of the total pallet load. Likewise, in a 4 stringer pallet with equal stringer spacing, the load carried by each center stringer should vary between 25% and 36.6% of the total load. However for pallets, the stiffness of the deckboards is typically between these extreme cases, also, a real deckboard to stringer joint has finite

stiffness (i.e. greater than zero). The stiffness of these components cause the central stringers to carry less total load percentage than the theoretical maximum of 62.5% for a three stringer or 36.6% for a four stringer pallet. To simplify matters, sensitivity studies were used to identify the variables that significantly influence RAS pallet response. The sensitivity studies showed that in the stiffness range exhibited by test joints (15000 to 60000 pounds per inch), lateral stiffness has little influence on pallet response in the RAS mode. However, assuming zero or very high stiffness, predictions of pallet response can be affected by 10 percent (Mulheren 1982). Consequently, the lateral joint stiffness for all simulated pallets was held constant and was equal to 30,000 in-pound per nail. Rotational stiffness, in the range exhibited by test joints (i.e. 2000 to 15000 inch-pounds per radian), was also found to have little influence on predicted RAS pallet response and for the development of the simplified RAS equations was held constant at 10000 in-pound per radian per joint. The sensitivity studies showed that for flexible deckboards (less than 3/8 inch thick) the difference in the percentage of total pallet load carried by the center stringer of a three stringer pallet changed from 61% to 53% assuming rotational stiffness of zero or 10000 inch-pounds per radian respectively. However, for stiffer deckboards (3/4 inch thick), the difference is less than 3% of the total pallet load (i.e. 55.3% versus 52.3% for rotational stiffness of zero or 10000 inch-pounds per radian respectively). Assuming deterministic values for joint stiffness greatly reduces the complexity of predicting the percentage of total pallet load carried by a stringer while sacrificing a only small amount of accuracy. Therefore, the independent variables

that were investigated for predicting the percent stringer load were related to the ratio of the stiffness of the stringers to the deckboards.

The relationship between the percent load (PLOAD) and the ratio of stringer to deckboard stiffness (R) for three stringer pallets is shown in Figure 15 on page 69. Figure 16 on page 70 shows a plot of the error in predicting the stringer moment (PERROR), (i.e. ignoring the effect of differential stringer deflection) versus the stiffness ratio. Several multivariate regression models were investigated for predicting PLOAD and PERROR. The best independent variable for predicting PLOAD and PERROR in three or four stringer pallets is:

$$R = \left[\frac{(EI/L^3)_{stringer}}{\sum_{i=1}^{n} (EI/\ell^3)_n + \sum_{i=1}^{nb} (EI/\ell^3)_{nb}} \right] \qquad (4.1)$$

where:

R=ratio of stringer to deckboard stiffness,

E=modulus of elasticity of stringer or deckboards (psi)

I=moment of inertia of critical stringer or accumulated deckboards

L=stringer span   (inch)

ℓ =deckboard length for three stringer pallets or distance between inner and outer stringer of four stringer pallets (inch)

n=number of top deckboards,

nb=number of bottom deckboards.
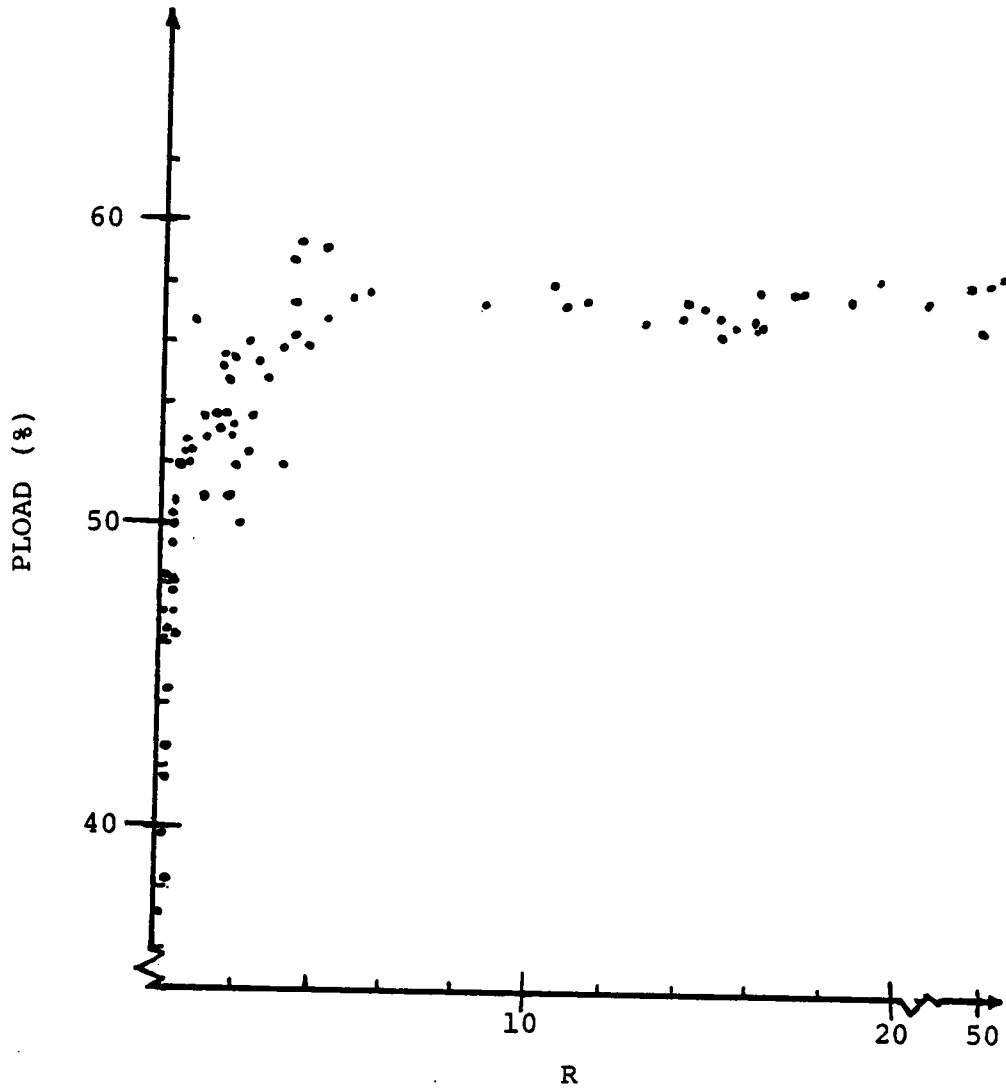
Where:

    R = stiffness ratio of stringer to
         deckboard defined in text.

Figure 15. Plot of data for three stringer pallets
            used to develop regression equation for
            predicting PLOAD.
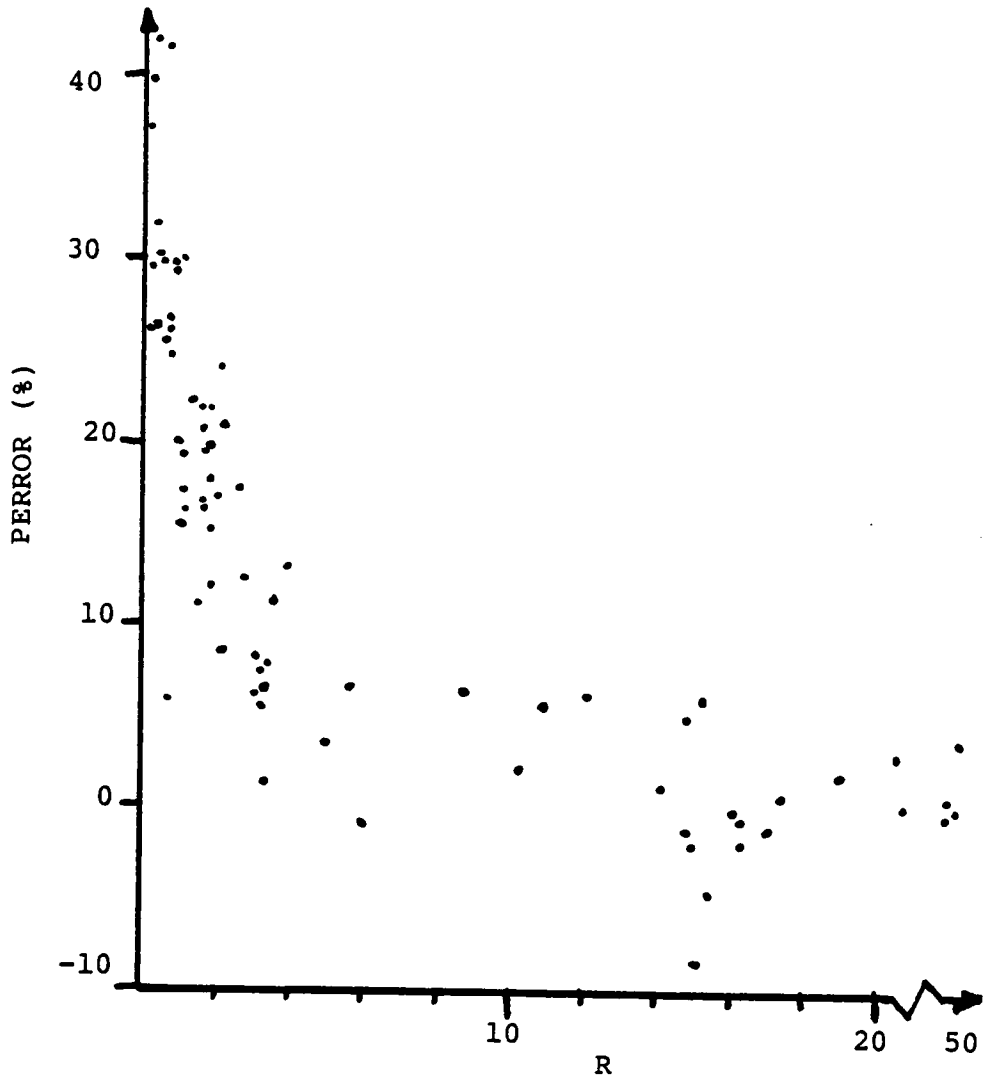
Where: R = Stringer to deckboard stiffness ratio.
          (Defined in text.)

Figure 16.   Data for three stringer pallets used to
            develop regression equation for predicting
            PERROR.
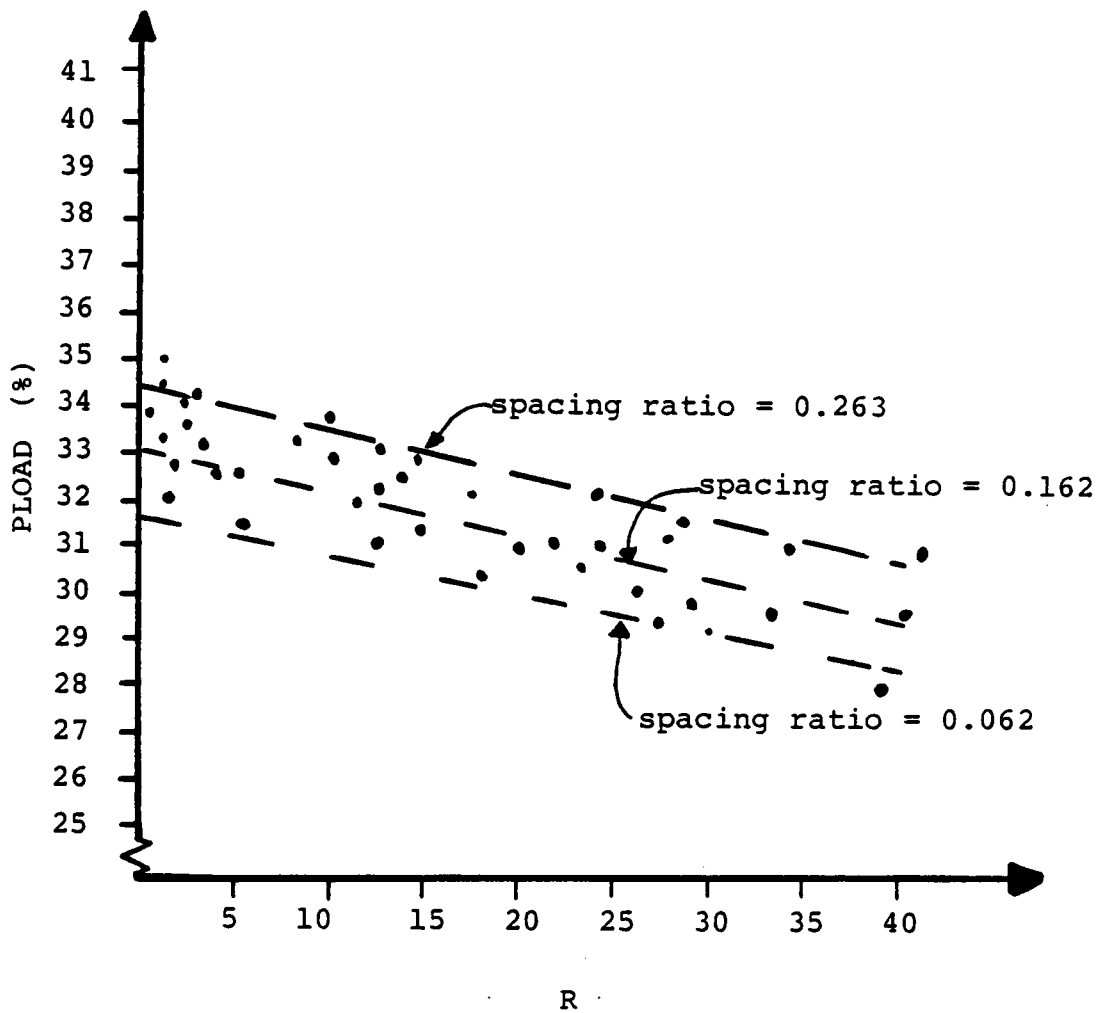
Figure 17. Four stringer pallet stiffness ratio vs PLOAD showing regression equation from Table 4.1 solved at three values of inner to outer spacing ratio.
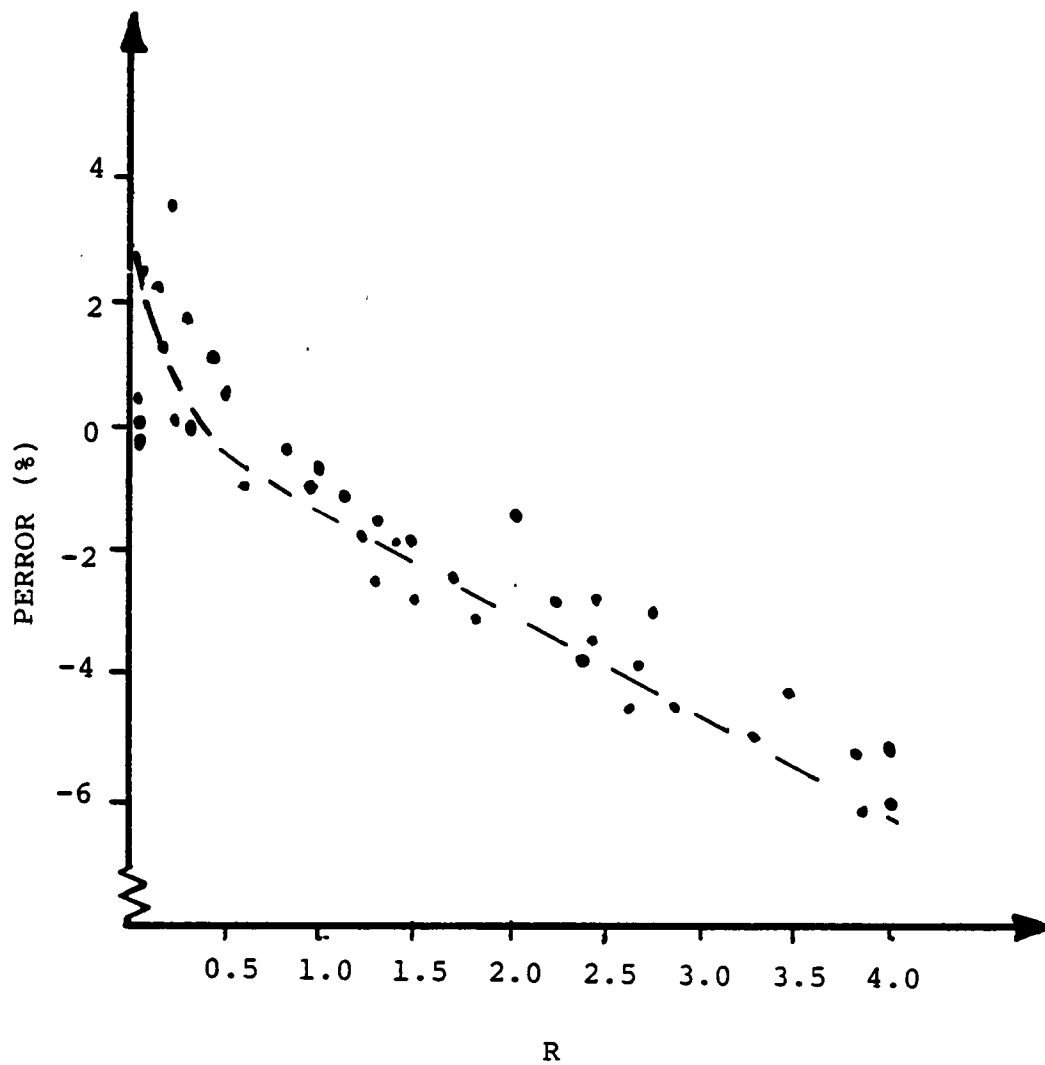
Figure 18. Four stringer pallet stiffness ratio vs PERROR with regression equation from Table 4.1 super-imposed.
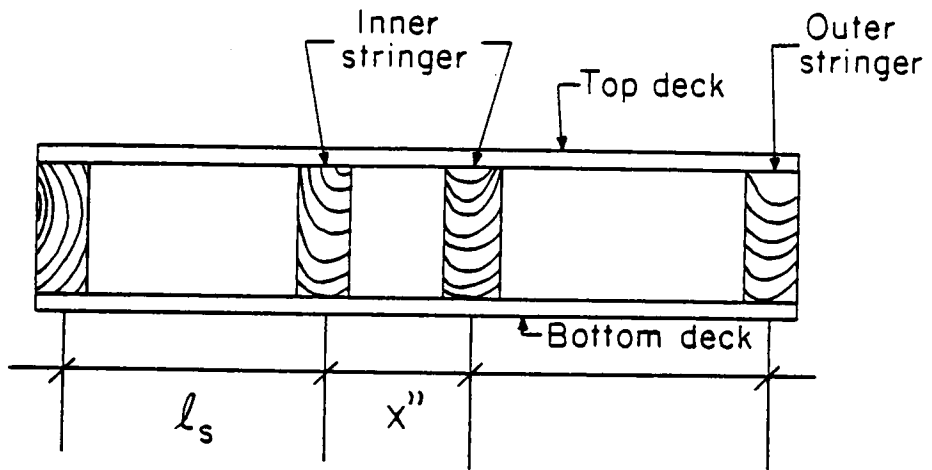
Figure 19. Pallet cross section showing spacing ratio for four stringer pallet.

For four stinger pallets the spacing of the inner stringers also influences the load sharing. By increasing or decreasing the inner stringer spacing the effective stiffness of the deckboards (i.e deckboard span). is changed. Therefore, for four stringer pallets the spacing was also included as an independent variable. This parameter was made dimensionless by taking the ratio of the distance between centerlines of the inner stringers to the distance between centerlines of an outer and inner stringer as shown in Figure 19 on page 73. A plot of PLOAD versus R for four stringer pallets is shown in Figure 17 on page 71. Figure 18 on page 72 shows the relationship between the percent load and the spacing ratio.

The best regression equations for predicting PLOAD and PERROR for either three or four stringer pallets are shown in Table 4.1. These equations are applied in the following subsection to compute stress and deflection of the critical stringers. Table 4.2 shows the verification of these equations by comparison of moment and deflection computed using the PLOAD and PERROR equations to those computed by SPACEPAL for structures that were not used to develop the regression equations. The table shows that the simplified approach provides reasonable accuracy for predicting the load effects of uniformly loaded RAS pallets.

4.2.2 MAXIMUM STRESS AND DEFLECTION-UNNOTCHED STRINGERS, UNIFORM LOADS

The following scheme was used to compute the maximum stress and deflection of unnotched pallets racked across the stringers.

Table 4.1. Computation of percent of total load carried by critical stringer (PLOAD) and effect of differential deflection PERROR.

No. of Stringers	PLOAD	PERROR
2	50%	0
3	$53.19 + 6.09(\log R) - 0.15(R)$	$15.57 + (R) \ 0.239 - 14.87(\log R)$
4	$30.87 - 0.79(R) + 13.99(\text{Spacing})$	$-0.123 - 1.531(R) + 0.16\left(\frac{1}{R}\right)$

where:

$$R = \frac{(EI/L^3) \text{ stringer}}{\left(\frac{\Sigma EI}{\ell^3}\right)_{\substack{\text{top} \\ \text{deck}}} + \left(\frac{\Sigma EI}{\ell^3}\right)_{\substack{\text{bottom} \\ \text{deck}}}}$$

spacing = defined in Figure 19 .

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Table 4.2. Comparison of simplified procedure for computing moment and deflection in pallets to SPACEPAL results for structures not used to develop the simplified procedure.

Percent Deck Coverage On Stringers		SPACEPAL		Simplified Prediction		% Difference		
Top	Bottom	Moment (in-lb)	Δ (in.)	Moment (in-lb)	Δ (in.)	Moment	Δ	
100	20	4846	0.213	4789	0.220	1.1	3.2	Three Stringer
100	100	4589	0.102	4936	0.113	7.5	10.7	
36	22	4970	0.109	5022	0.109	1.0	0	
62	50	4654	0.106	4786	0.108	2.8	2.0	
81	53	4719	0.109	4788	0.110	1.4	0.9	
82	82	4483	0.104	4590	0.105	2.3	1.0	
95	95	4767	0.109	4887	0.111	2.5	1.8	
81	00	5184	0.119	4926	0.113	4.9	5.0	
82	82	4610	0.106	4590	0.105	0.4	0.9	
95	95	4884	0.111	4885	0.111	0	0	
58	0	6340	0.221	5987	0.207	5.0	6.3	
90	90	3405	0.110	3428	0.102	0.6	7.0	Four Stringer
90	90	3376	0.108	3363	0.105	0.3	2.7	
90	90	3347	0.106	3260	0.104	2.5	3.7	
35	25	3316	0.110	3267	0.106	1.4	4.0	
35	25	3223	0.104	3133	0.102	2.7	1.9	
35	25	3170	0.101	2990	0.098	5.5	2.9	
60	50	3344	0.086	3306	0.080	1.1	6.0	
60	50	3199	0.080	3142	0.079	1.7	1.2	
60	50	3106	0.100	3103	0.101	2.9	1.0	

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1. DESIGN Option--unnotched stringers:

    a. The percentage of load (PLOAD) that is transmitted to the critical stringer(s) (i.e., the center stringer of three or four stringer pallets or either stringer for two-stringered pallets) is computed from the equations shown in table 4.1. PLOAD defines the load sharing among the stringers.

    b. Compute the tributary load carried by the critical stringer:

$$Q = \text{TLOAD} \left(\frac{\text{PLOAD}}{100} \right) \qquad (4.2)$$

    where:

    Q=tributary load on critical stringer (pounds)

    TLOAD=Total pallet load input by the user (pounds)

    c. Assume that the tributary load is distributed along the critical stringer as a series of point loads, P acting at the centerline of each deckboard as shown in Figure 20 on page 78. The magnitude of each point load is proportional to the surface area of the deckboard on which it acts.

$$P_i = Q \left[\frac{W_i}{\sum\limits_{i=1}^{nl} W_i} \right] \left[1 - \frac{\text{PERROR}}{100} \right] \qquad (4.3)$$

    where:

    P_i = load acting on critical stringer caused by deckboard i (pound)

    W_i = width of deckboard i (inch)

    ΣW_i = sum of the widths of all loaded deckboards (inch)

a) Actual loading

b) Assumed analog of load distribution

Note $\sum\limits_{i=1}^{7} P_i = w\ell$

$M_{\mathbb{C}} = -\dfrac{Q}{2}\dfrac{\ell}{2} + P_1 D_1 + P_2 D_2 + P_3 D_3$

c) Centerline moment
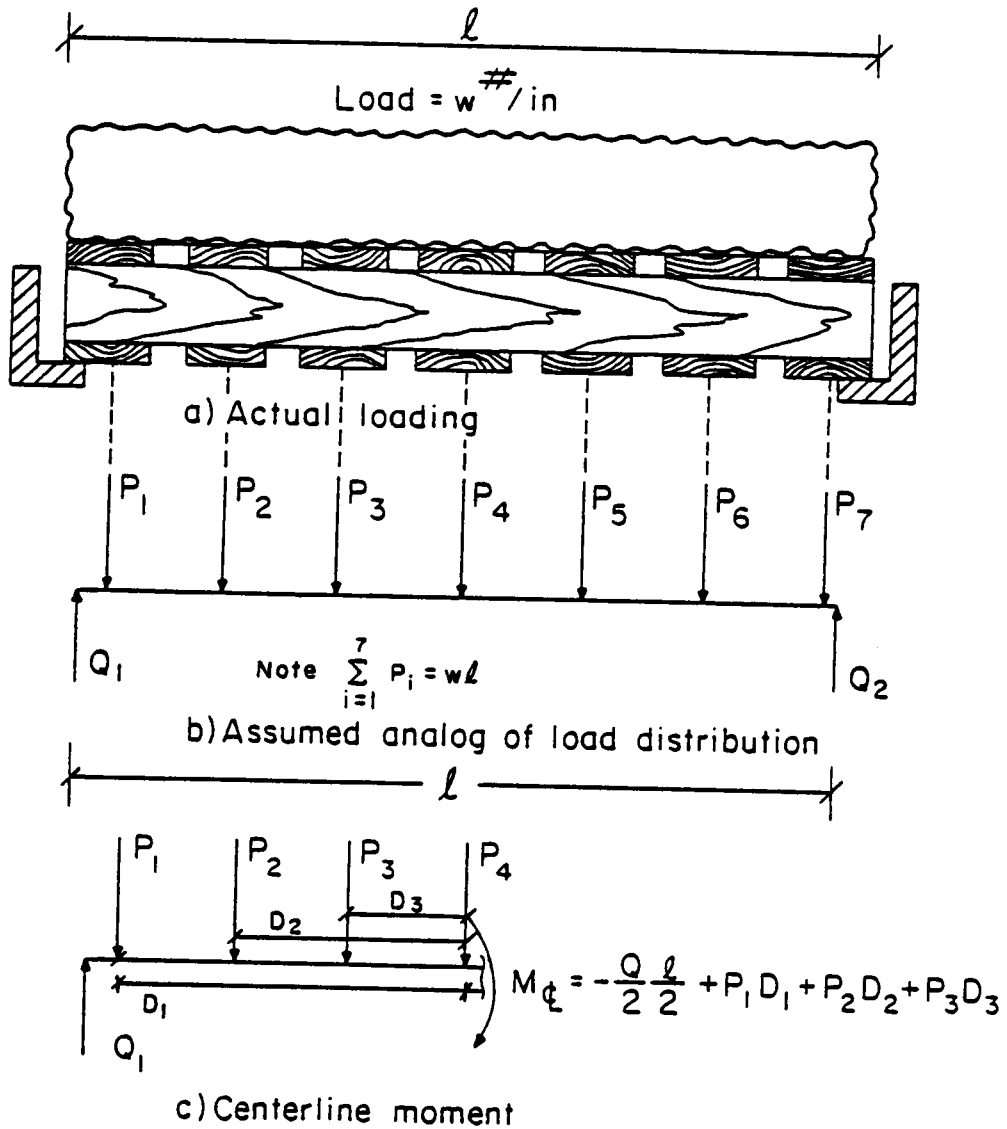
Figure 20. Load diagram for RAS uniform load.

PERROR=error due to differential deflection computed from equation in Table 4.1.

nl=number of loaded deckboards.

d. The moment at centerline is computed (see Figure 20 on page 78):

$$M_{\text{¢}} = \left[(-\frac{Q}{2})(\frac{L}{2})(1-\frac{PERROR}{100})\right]+P_1D_1+P_2D_2 \cdots P_{\text{int}(\frac{n}{2})}D_{\text{int}(\frac{n}{2})} \qquad (4.4)$$

where:

L=span between rack supports (inch)

$M_{\text{¢}}$ =moment at centerline of span (inch-pound)

D_i =distance from load point to centerline of span (inch)

int(n/2)= integer value of n divided by 2

n=number of deckboards

e. Compute maximum stress at centerline:

$$\sigma_{\text{max}} = \frac{6M_{\text{¢}}}{b(d^2)} \qquad (4.5)$$

where:

σ_{max} =maximum stress (psi)

$M_{\text{¢}}$ =moment at centerline (inch-pounds)

b=thickness of a stringer (inch)

d=height of a stringer (inch)

The maximum stress is the load effect and is used in the FOSM method to determine if the design is acceptable (as described in chapter 8).
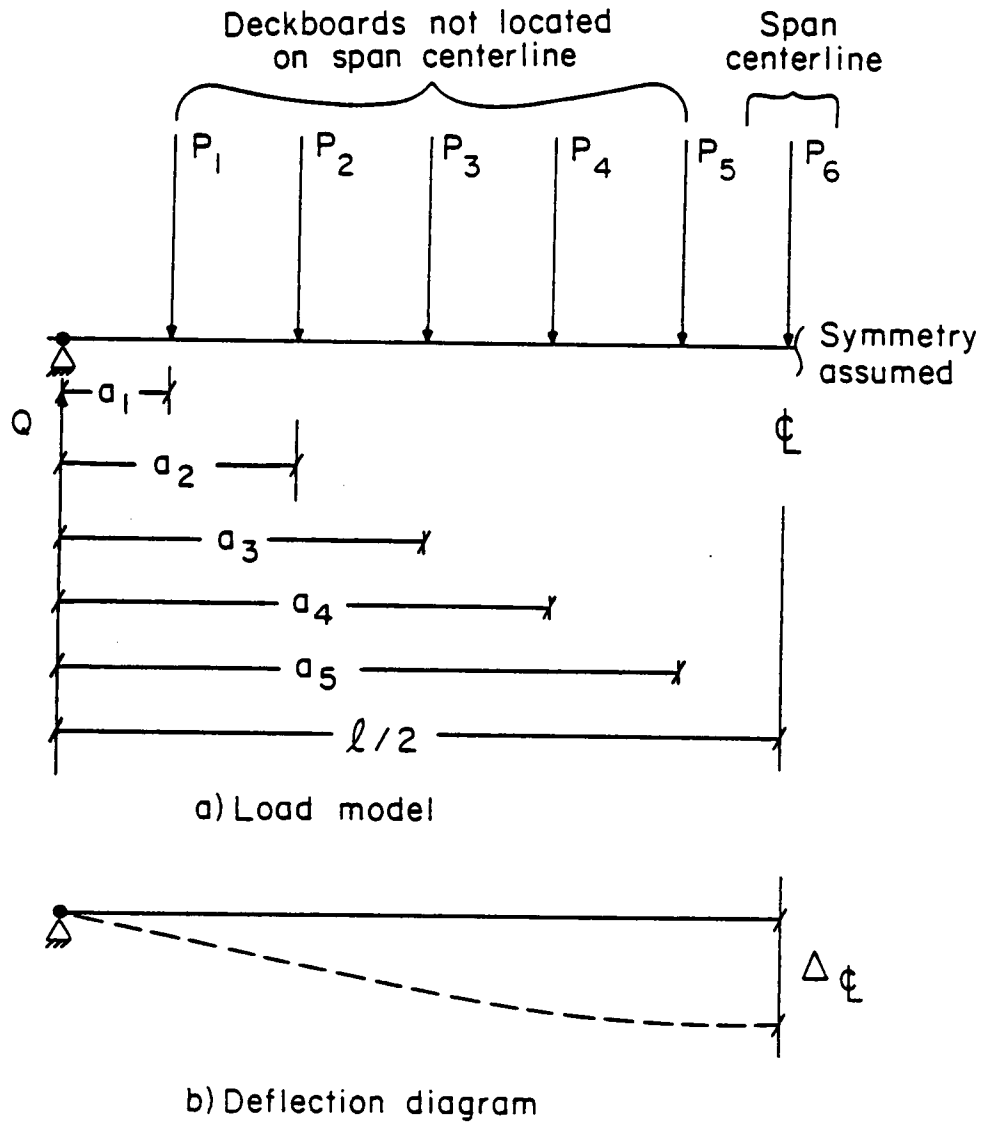
Figure 21. Diagram of symmetrically loaded stringer for computing stringer deflection RAS.

f. Compute the deflection at centerline using the principle of superposition. Symmetry about centerline is assumed (see Figure 21 on page 80). Two deflection equations are applied and are selected based upon the location of the deckboards that are transmitting the load to the stringer (Timber Construction Manual (TCM) 1974).

1) Deckboards not located on span centerline:

$$\Delta_i = \frac{aP_i}{24EI} \; [3L^2 - 4a^2] \qquad (4.6)$$

where:

Δ_i = deflection at centerline caused by symmetric point loads P_i (inch)

a=distance load point to support (inch)

E=elastic modulus (MOE) of stringer (psi)

I = moment of inertia of stringer

L=stringer span (inch)

2) Deckboard located on stringer centerline:

$$\Delta_c = \frac{P_c L^3}{48EI} \qquad (4.7)$$

where:

Δ_c = deflection at centerline caused by the center deckboard (inch)

P_c = point load of centerline deckboard (pound)

Find total deflection at centerline :

$$\Delta_T = \left[\sum_{i=1}^{int(\frac{n}{2})} \Delta_i \right] + \Delta_c \qquad (4.8)$$

where:

Δ_T = total deflection at centerline

2. ANALYSIS option:

a. Compute the percentage of load (PLOAD) that is carried by the critical stringer using the equations in Table 4.1. (This is the same as step a of the DESIGN option.)

b. Determine the MOR for the material and correct it for safety using the First-Order-Second-Moment (FOSM) equation. (The details of this step are presented in chapter 8). The resulting value, called SBAR, is the mean load effect, or stress, which can be safely resisted by the material.

The objective of the next two steps is to find the total pallet load which causes the mean load effect in the critical stringer. This load is assumed to be the maximum load which can be safely placed on the pallet without exceeding the strength of the critical stringer.

c. Compute the maximum load which can be safely placed on the critical stringer:

$$Q = \frac{SBAR}{\left[1- \frac{PERROR}{100}\right]\left[- \frac{L}{4} + \frac{1}{W}\left(\sum_{i=1}^{int(n/2)} W_i \, D_i\right)\right]} \qquad (4.9)$$

where:

Q=maximum load on critical stringer (pounds),

PERROR=parameter from Table 4.1 to account for effect of differential stringer deflection,

W_i = width of deckboard i (inch),

D_i = distance of load i from support (inch),

W=total accumulated width of loaded top deckboards (inch).

d. Translate the critical stringer load into the total pallet load:

$$TLOAD = Q \left(\frac{PLOAD}{100}\right) \qquad (4.10)$$

where :

   TLOAD=total pallet load for strength criteria (pounds)

e. Compute the magnitude of each point load for use in the deflection calculation (as in step c of the DESIGN option).

f. Compute the deflection at the maximum load (as in step f of DESIGN option).

g. Compute the maximum load for a critical stringer based on a deflection limit: The allowable mean deflection limit (DBAR) is computed from the user-input limit. The First-Order-Second-Moment equation presented in Chapter 8 is used for this calculation. The maximum allowable load for a critical stinger is:

$$Q_{\Delta Lim} = \frac{Q(DBAR)}{\Delta_{total}} \qquad (4.11)$$

where:

   $Q_{\Delta Lim}$ =maximum load on the critical stringer at deflection limit (pounds)

   Q=max. load on critical stringer at strength limit (pounds)

   DBAR= user input deflection limit adjusted for safety by FOSM (inch)

   Δ_{total}= deflection at load Q (inch)

h. Compute the maximum pallet load: This step translates the maximum load for a critical stringer into the maximum allowable load for the pallet based on the allowable deflection limit:

$$P_{max} = Q_{\Delta Lim} \left[\frac{PLOAD}{100}\right] \qquad (4.12)$$

where:

P_{max} = maximum pallet load for a deflection limit (pounds)

4.2.3 LINE LOADS

Some load types such as horizontally positioned barrels have the form of line loads as shown in Figure 22 on page 85. Loads of this type are assumed to be intrinsically rigid leading to equal load sharing among the stringers. Therefore, all the stringers in the pallet are assumed to deflect approximately equally and the moments induced from differential stringer deflection are assumed to equal zero. Consequently, the same procedure was used for pallets with both equal or unequal sized stringers.

Three line load cases can be analyzed by PDS:

1. A single central load (CL)

2. Two off center loads of equal magnitude and symmetrically placed (SL)

3. One center load and two off center line loads (CL and SL).

a) Real pallet

b) 2-D Load analog
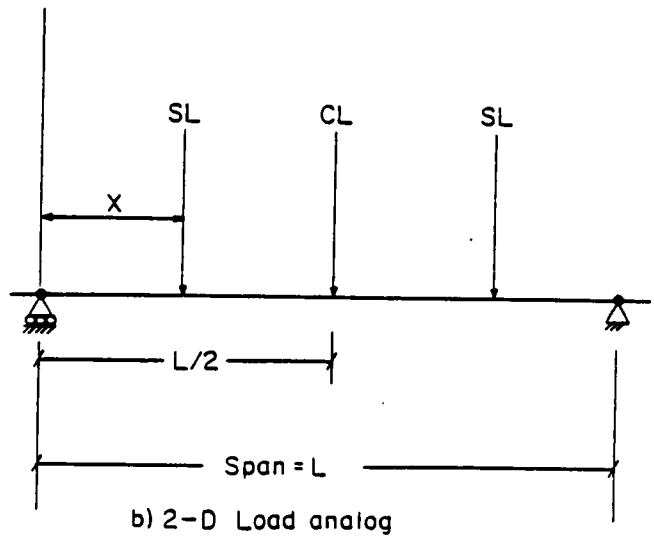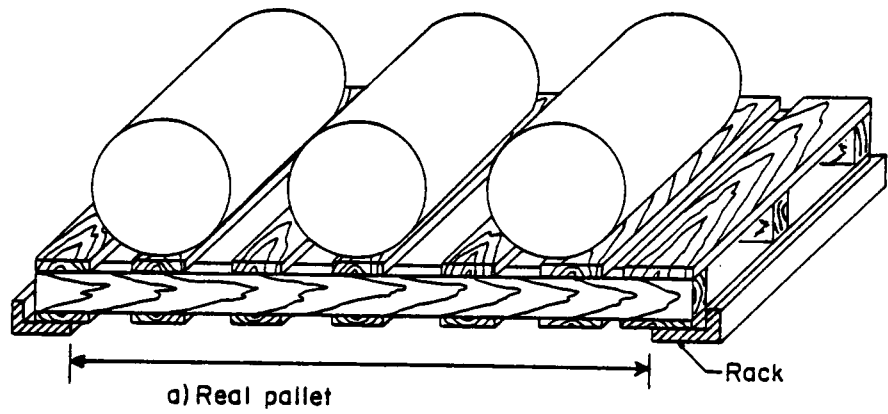
Figure 22.   Analog model of RAS pallet loaded with
line loads.

The following procedures are used to compute the response of pallets loaded with line loads.

1. DESIGN option for line loads: Assume (require) that the symmetrically placed loads are of equal magnitude. The center load can be of any magnitude (i.e. equal to or not equal to the side loads).

   a. Compute the moment in one stringer

   $$M_{max} = \frac{SL(X)}{Nst} + \frac{CL}{Nst} \left[\frac{L}{4}\right] \qquad (4.13)$$

   where:

   M_{max} = moment at centerline (inch-pound)

   SL = magnitude of one off centerline load (side load) (pound)

   CL = magnitude of centerline load (pound)

   Nst = Number of stringers

   L = span (inch)

   X = distance between support and one off center load (inch)

   b. Compute the load effect (stress):

   $$\sigma = \frac{6M_{max}}{bd^2} \qquad (4.14)$$

   where:

   σ = stress (psi)

   b = thickness of thinnest stringer (inch)

   d = height of stringer (inch)

Determine if the design is adequate by comparing the load effect to the MOR corrected for safety by the FOSM method (described in chapter 8).

c. Compute the maximum deflection from superposition of the loads.
(These equations are from TCM (1974)).

$$\Delta = \frac{SL(x)}{24EI}\left[3L^2 + 4x^2\right] + \frac{CL(L^3)}{48EI} \tag{4.15}$$

where:

Δ = maximum centerline deflection (inch)

E=elastic modulus of stringer (psi)

I=second moment of inertia

L=span (inch)

x=distance from support to load point (inch)

2. ANALYSIS option: Assume (require) that the line loads are equal in
magnitude.

a. Compute the maximum allowable moment at centerline for one
stringer:

$$M_{allow} = \frac{SBAR\ d^2 b}{6} \tag{4.16}$$

where:

M_{allow}=maximum allowable centerline moment (inch-pound)

SBAR = mean load effects (MOR corrected for safety) (psi)

b. Compute the maximum magnitude of one line load:

$$P_1 = \frac{M_{allow}}{DD} \tag{4.17}$$

where:

P_1= magnitude of one line load (pound)

DD=parameter dependent on load type:

= L/4 if single line load

Design for Racked Across Stringers Support Condition   87

= x if two line loads

= x+L/4 if three line loads

x= distance between support and one off center load

L=stringer span (inch)

c. Compute the maximum pallet load:

$$P_{max} = P_1 (Nst)(Nline) \qquad (4.18)$$

where:

P_{max}=maximum pallet load (pound)

Nst=number of stringers

Nline = number of line loads

d. Compute deflection at centerline for maximum pallet load (TCM 1974):

$$\Delta_{max} = P_{max} (AA) \qquad (4.19)$$

where:

AA=parameter that depends on number of loads as follows:

$= \dfrac{L^3}{48EI}$ if one line load,

$= \dfrac{x}{24EI} [3L^2 - 4x^2]$ if two line loads,

$= \dfrac{x}{24EI} [3L^2 - 4x^2] + \dfrac{L^3}{48EI}$ if three line loads.

4.2.4 NOTCHED STRINGERS

The procedure used to DESIGN or ANALYZE notched stringer pallets having any load type is similar to that used for the corresponding unnotched

Realbeam

w

$M_X = M_\mathbb{C} * 4X' * \dfrac{[\ell - x]}{(\ell_2)}$
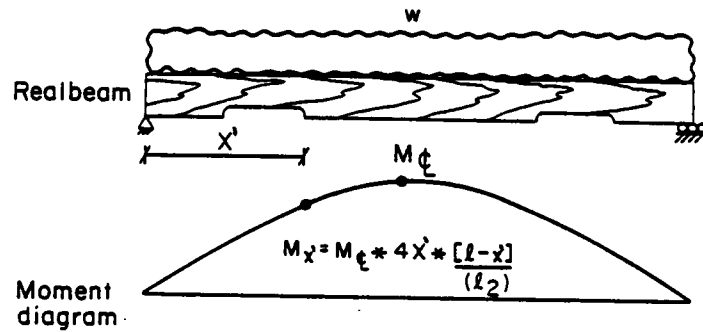
Moment
diagram

a) Uniform load – moment at notch

CL

Realbeam

$M_X = M_\mathbb{C} (\dfrac{2x}{\ell})$

Moment
diagram

b) Center line load

SL   SL

Realbeam

$M_{X'}$ $M_\mathbb{C}$
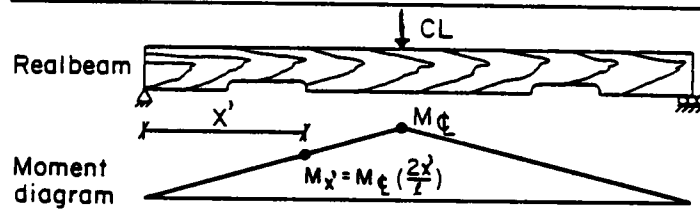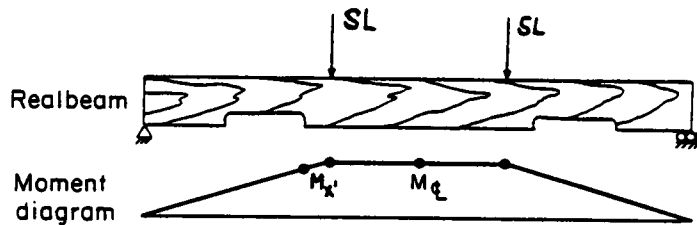
Moment
diagram

c) Two off center loads

Figure 23.   Computation of the moment at the notch in
              RAS mode for uniform loads and line loads.

pallet except that the critical bending stress is computed at the inside corner of the notch instead of at the center line of the span. The stress at the notch is then compared to a critical-stress. The critical-stress is the stress which causes an unstable crack to propagate from the corner of the notch nearest the center of the span. Exceeding this stress will cause failure in the stringer. The critical-stress is a material property and, for notches commonly found in pallets, is typically in the range of 40 to 60 percent of the MOR of an unnotched stringer. (Chapter 7 contains more details concerning the critical-stress.) The stress at the corner of the notch is computed using relations developed by Gerhardt (1984) (see Appendix). The equations are based on finite element analysis of notched stringers, and account for the geometry of the notch and stringer. These relations produce a parameter which is similar to and can be used as a stress concentration factor. The factor reflects the increase in stress intensity caused by the discontinuity in the material around the notch. In other words the notch is treated as 'a stress raiser'. The factor is used to multiply the stress of an unnotched stringer computed at a point corresponding to the inner notch corner (Figure 23 on page 89).

The deflection of a pallet having notched stringers is computed by modifying the center deflection of a corresponding unnotched pallet. The deflection modification was also developed by Gerhardt and is based on the geometry of both the stringer and the notch.

To compute the moment at the notch location, two approaches were used:
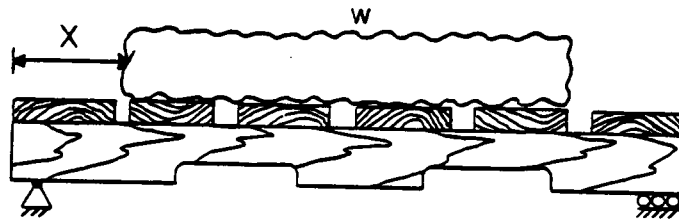
a) Load diagram
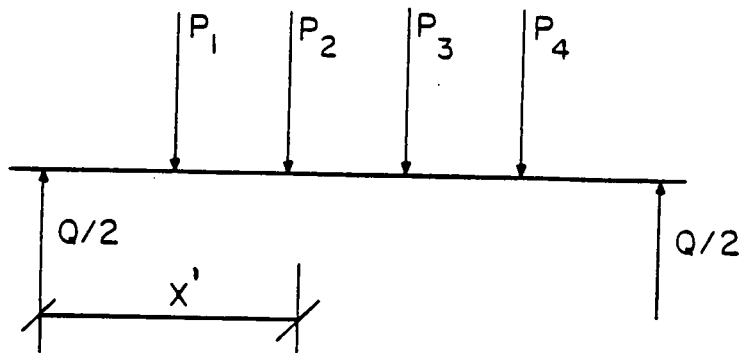
b) Load analog

c) Moment at notch

$$M_{x'} = -\frac{Q(x')}{2} + P_1(d_1)$$
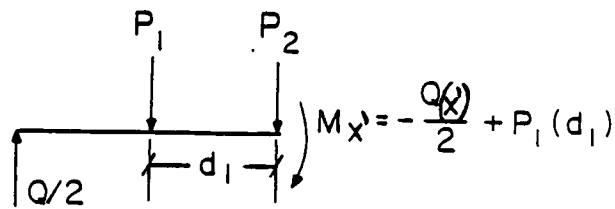
Figure 24. Computation of the moment at the notch for partial uniform loads--RAS.

1. For the full uniform and line load cases, the moment at the notch was found directly from the moment diagram as shown in Figure 23 on page 89. For these load types the peak of the moment diagram is defined by computing the centerline moment. The moment at the notch is then found by computing the value of the moment at a location corresponding to the inside corner of the notch.

2. For the partial uniform load case the moment at the notch is found by summing moments at a position corresponding to the notch location. This technique was used because it easily accounts for the location of the supports in relation to the load and the notch. The moment at the notch is computed from (see Figure 24 on page 91):

$$M_{x'} = -\frac{Qx}{2} + \sum_{i=1}^{nn} (P_i d_i) \qquad (4.20)$$

where:

$M_{x'}$=moment at the notch

x=distance from support to notch (inch)

Q=load on the critical stringer (pound)

P_i=equivalent point caused by a loaded deckboard (pound)

d_i=distance between centerline of loaded deckboard and notch (inch)

nn=number of loaded deckboards between end of beam and notch corner.

The moment computed by either method 1 or 2 is translated into stress at the notch by dividing by the stringer section modulus and then multiplying

by Gerhardt's stress concentration factor. The resulting stress is a load effect and can be compared to the critical-stress of a notched stringer.

4.3 MATRIX APPROACH TO PALLET STRUCTURAL ANALYSIS

GENERAL

The equations which were developed to predict both the load sharing and the influence of differential stringer deflection on the moment at centerline can produce erroneous results when applied to pallets which have stringers of unequal sizes. In addition, there is always some regression error. Therefore, to correctly analyze these pallets a solution was developed based on matrix structural analysis (stiffness method). The technique utilizes a grid model having quarter symmetry about the centerline to represent the pallet as shown in Figure 25 on page 94 and Figure 26 on page 95. To ensure that the quarter symmetric model behaves identically to a full model, shear releases are used to represent the cut ends of the members. The shear releases allow bending moments to be transmitted into the support while allowing the member end to translate vertically.

The advantage of using the reduced model lies in the fact that it has fewer members and joints than the full model. This leads to less com-

a) Two stringer pallet (top view)
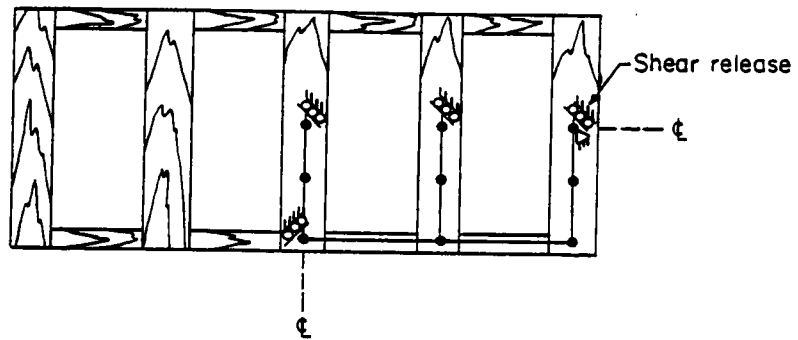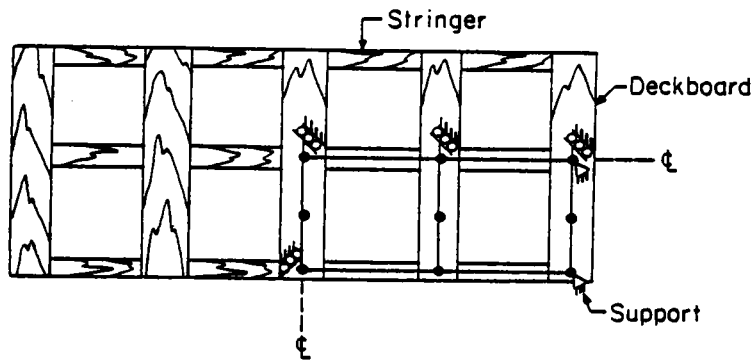
b) Three stringer pallet (top view)
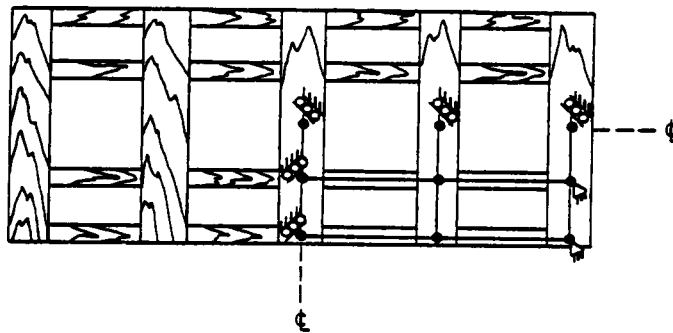
c) Four stringer pallet (top view)

Figure 25 . Example of grid model representing 2, 3 and 4 stringer pallets with 1/4 symmetry (other elements in grid model are "dummied out" by assigning zero properties).
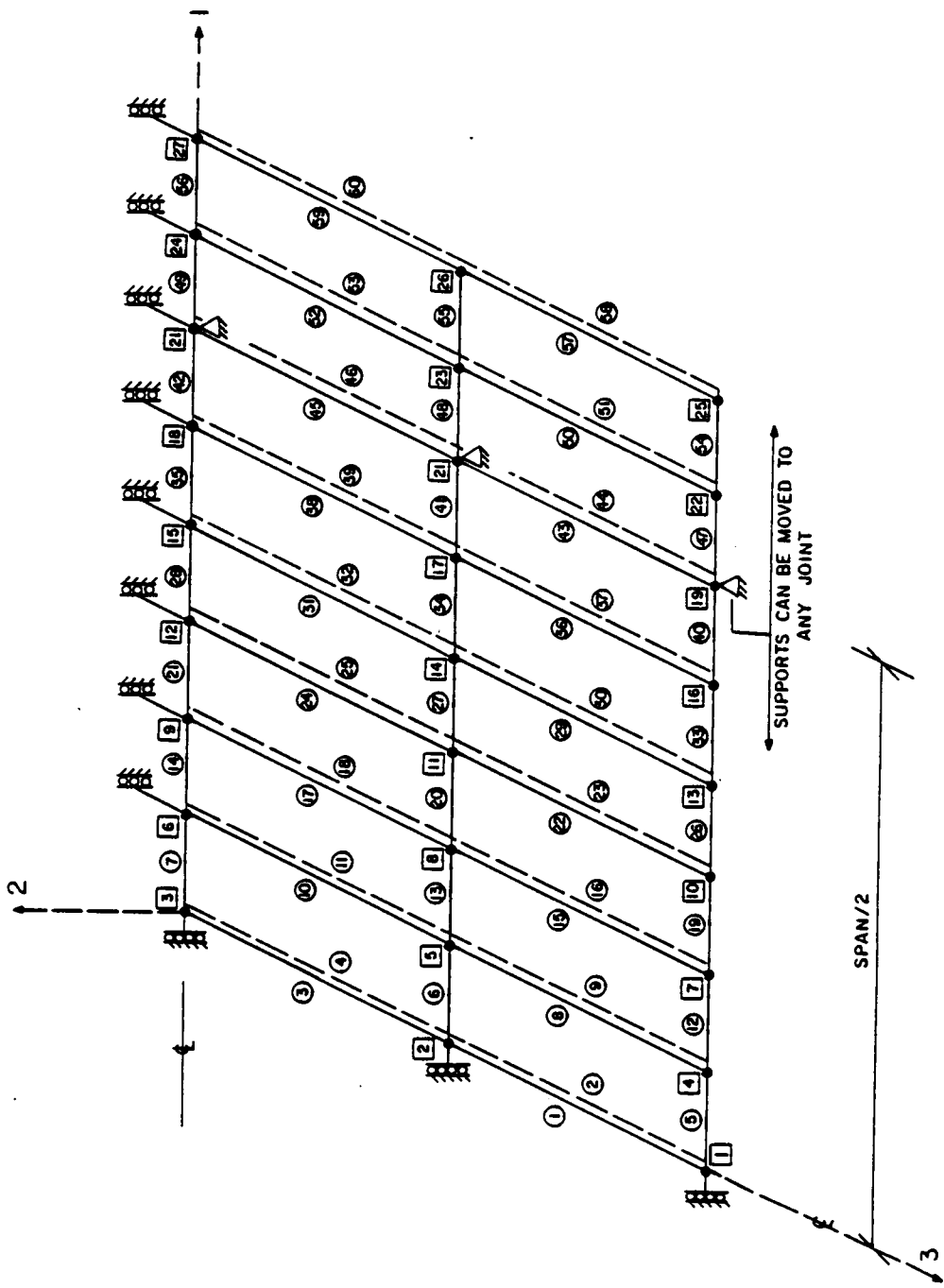
Figure 26. Matrix grid model for stringers RAS.

plication in the automatic assembly process and less time to compute a solution.

In the model shown in Figure 26 on page 95 elements representing stringers are oriented parallel to the 1 axis, and elements representing deckboards are parallel to the 2 axis. The complete model has 63 degrees of freedom. Details of the computerized assembly and analysis of this model are in this section, but first a brief disscusion of the concepts used in the analysis of any structure by matrix methods is presented [9]:

1. The first step is to produce a model which has members, joints, and constraints or supports placed in such a way that the model behaves in the same manner as the real structure. The members and joints in the model are assigned stiffness properties and geometries identical to those of the real structure.

2. The next step is to identify all possible joint displacements in the model. In a two dimensional model any unconstrained joint is free to move in three possible ways: horizontal and vertical translation and rotation. These motions are called degrees of freedom and the sum of all possible motions in the model is called the degree of freedom for the structure. The degrees of freedom for the structure are numbered in sequence and stored in the displacement vector, {D}. The matrix solution will define the magnitude of each possible joint

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[9] This method is described by Holzer (1982).

motion caused by an applied load to the structure. In other words, the solution will define the displaced configuration of the structure. From the displaced configuration the forces and stress in any member can be computed (assuming elastic response).

3. A system stiffness matrix, [K] is computed. This matrix defines in a compact form the interaction between all members in the structure. It is the stiffness matrix which provides the link between the known applied forces and the unknown joint displacements. The system stiffness matrix is assembled by transferring the values of individual element stiffness matrices into the proper cells of the system matrix.

4. The applied force vector, {F}, is defined. This vector contains, in a numbered sequence similar to the displacement vector, the magnitude of all applied (or equivalent) joint forces.

The matrix equation can now be written:

$$\{F\}=[K]\{D\} \tag{4.21}$$

The solution of this equation can be found by any of several techniques, however, the gauss elimination method was used for the computerized Pallet Design System (PDS). For structures with more than a few degrees of freedom, as are used in PDS, this equation cannot easily be solved by hand computation and a computer must be used.

5. From the joint displacements the stress in any member can be computed.

DETAILS OF RAS MATRIX SOLUTION:: The preceding section contained a brief explanation of the scheme used to analyze any structure by the matrix displacement method. This section describes in detail the procedure developed to automatically analyze pallets racked across the stringers by the matrix method.

1. Definition of the structure: The objective of this step is to define all member lengths and properties for both deckboards and stringers and to determine the location of supported joints in the model. Because of symmetry, the complete model shown in Figure 26 on page 95 represents a pallet having a maximum of 15 top and bottom deckboards, and 5 stringers.

    To represent pallets having less elements than the complete model, a special joint and element numbering scheme was used. This scheme was adopted for several reasons:
    *   To minimize the half band width of the system stiffness matrix thereby utilizing the symmetrical property of the matrix and reduce the time required to solve the equation,
    *   To allow for a variable number of elements and joints,
    *   To allow for easy placement of the supports based on the specific pallet and rack geometry.

    The numbering scheme allows for removal from the model any joints or elements which are unneeded in the analysis of a specific pallet.

The specific steps needed to define the structure follow:

a. The total number of joints in the model (NJ) is determined: The number of joints is based on the number of top deckboards. An array is defined which contains the number of joints in the model corresponding to the cell number in the array (JN). By addressing the array cell number which is equal to the number of top deckboards, the number of joints needed in the model to represent the actual deckboard-stringer joints is found. Three joints are added to the number of deckboard-stringer joints: these joints will be constrained from vertical displacement and will represent the rack support.

b. The number of elements in the model is determined:

$$NE = 2NJ + (\frac{NJ}{3} - 3)$$ (4.22)

where:

NE = number of elements,

NJ = number of joints.

c. Determine the joint coordinates in the direction of the global 1 axis: Since the origin is located at the pallet centerline, most global 1 joint coordinates are equal to the distance between the pallet centerline and the deckboard centerline. The coordinates of the supported joints are equal to half the span length. The joint coordinates are assigned in sequence starting at the pallet centerline and ending with the centerline location of the endboards. The number of the supported joint is defined by its coordinate and is saved for use in a later step. This procedure

allows for the placement of the support at any joint in the model. All member lengths in the global 1 direction are computed from the joint coordinates. The lengths are used in the stiffness matrix.

d. Determine the joint coordinates along the global 3 axis: The coordinates in the global 3 direction correspond to the length of the deckboard elements and are defined relative to the half width of the pallet. The coordinates of the joints along the outer stringer are equal to the distance between the outer stringer-centerline and the origin. If the pallet has four stringers the coordinates of the middle stringer elements are equal to the distance from the stringer-centerline to the origin. If the pallet has less than four stringers the coordinates of the middle elements are not critical and are set equal to half the distance between the outer elements and the origin. All member lengths in the global 3 direction can now be computed from the joint coordinates.

e. Define member properties and geometries: In this phase the properties of all elements in the model are computed. Most elements will be assigned properties and geometry corresponding to either the stringers or the deckboards which they represent in the real pallet. Some elements are given zero properties to correspond to elements contained in the model which are not in the real pallet. For example, zero properties are assigned to the elements representing the center stringer if a four stringer pallet is being analyzed. Also, zero properties are assigned to

some elements representing bottom deckboards if there are less bottom boards than top boards.

The parameters which are required for each member are the modulus of rigidity, the modulus of elasticity and the moment of inertia. These parameters are stored in vectors whose cell numbers correspond to the element numbers. To conform to rules for analyzing symmetric structures, the width of any member whose longitudinal axis is located on the line of symmetry in the model is reduced to half that of the real element. This reduced width is used to compute the moment of inertia and other geometric parameters.

2. Define joint constraints and determine the number of degrees of freedom for the structure: Because this is a grid model, only three possible joint actions, (or degrees of freedom), are allowed: rotation about the global 1 and 3 axes and translation in the global 2 direction. The joints which represent the supports must be constrained from motion in the proper directions. A special array, called JCODE, is used to identify the constrained directions for all joints. The JCODE array has three columns which represent the fixity of motion in the global 1,2, and 3 directions respectively. The number of rows in the array is equal to the number of joints in the model. JCODE is initially filled entirely with "ones" representing free joints. The array is then modified to account for the constrained joints. This is done by changing the value of the array cell which corresponds to the constrained direction. To represent a constraint in a given direction, the value of the cell is changed from

a) Unconstrained Beam

2

JCODE

	Direction 1	2	3
1	1	1	1
2	1	1	1
3	1	1	1
4	1	1	1

Joint Number
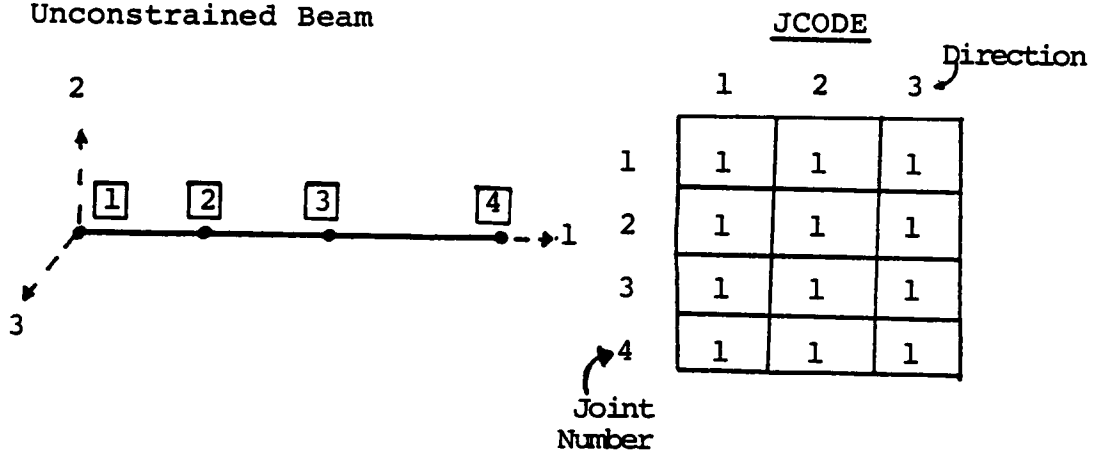
b) Constrained with Pin and
    Roller

JCODE

	1	2	3
1	.0	0	1
2	1	1	1
3	1	1	1
4	1	0	1
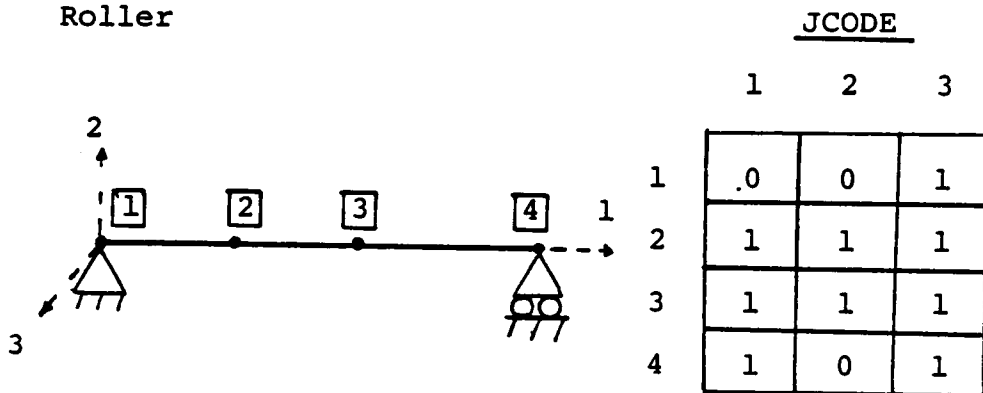
Figure 27. Example of use of JCODE to define
            constraints in a simple beam.

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a one to a zero.   For example, the JCODE for the simple beam shown in Figure 27 on page 102 is initially filled with "ones" to represent an unconstrained structure.   After defining the supported joints (joints 1 and 4) and the support type (pinned) the JCODE is modified to reflect the constraint in the horizontal and vertical directions by replacing the "1" with a "0" in the cells associated with the horizontal and vertical directions of joints 1 and 4.

All joints located on the axis of symmetry in the model are assumed to act as shear releases: They can transmit moments in either the global 1 or 3 directions but are free to translate vertically (Global 2 direction).   Therefore the JCODE values in the first and third columns are changed from "1" to "0" for these joints.

After defining the shear releases, the constraints representing the rack supports are defined and the appropriate cells of JCODE are modified.   The rack supports are assumed to act as pinned joints and restrain motion in the global 2 direction but allow rotation about both the 1 and 3 axes.   Therefore, the second column of JCODE is changed from a "1" to a "0" for each of these joints.

After modifying JCODE to account for the placement of the supports, numbers are assigned in sequence to each non-zero JCODE element.   The numbering sequence progresses row by row and, starts at one and ends with the total number of degrees of freedom for the structure.   Any supported joint contains a zero in the direction of the constraint.

The degree of freedom associated with the actions of each element are found next and stored in the MCODE array. MCODE is used to assemble the system stiffness matrix by identifying the members that influence the response of the structure in a given direction. The MCODE array has six columns: each represents the degree of freedom in each of the global directions for the joints connected to right and left ends of the member. The row numbers in MCODE correspond to the member number in the model. For example, the 15'th row of MCODE contains the degree of freedom numbers of the right and left ends of member number 15 from the model. Definition of the element actions is done by determining the joint number at each end of an element and transferring the corresponding degree of freedom numbers for the joint, from JCODE, into MCODE.

3. Assemble the Stiffness Matrix: The element stiffness matrix is first defined and then transferred into the proper locations of the system stiffness matrix. Since the model has elements oriented parallel to either the global 1 or 3 axes, two element stiffness matrices were defined, one to represent stringer elements (parallel to the global 1 axis) and the other to represent deckboard elements (parallel to the global 3 axis). A new array (INDEX), is also defined and is used to cross reference the position number of each cell in the element stiffness matrix with the computed numerical value of the cell. The INDEX array is used to save computer memory by utilizing the symmetric nature of the element stiffness matrix: of the possible 36 locations only 8 are unique as shown in Figure 28 on page 105. Use of the INDEX array therefore eliminates redundant calculations.

Index

	1	2	3	4	5	6
	G_1	G_2	G_4	G_8	$-G_2$	G_4
		G_3	G_5	G_2	$-G_3$	G_5
			G_6	$-G_4$	$-G_5$	G_7
Symmetric				G_1	$-G_2$	$-G_4$
					G_3	$-G_5$
						G_6

Where:

Stringer	Deckboard
$G_1 = \delta$	$G_1 = 4L^2\alpha$
$G_2 = 0$	$G_2 = 6L\alpha$
$G_3 = 12\alpha$	$G_3 = 12\alpha$
$G_4 = 0$	$G_4 = 0$
$G_5 = 6L\alpha$	$G_5 = 0$
$G_6 = 4L^2\alpha$	$G_6 = \delta$
$G_7 = 2L^2\alpha$	$G_7 = -\delta$
$G_8 = -\delta$	$G_8 = 2L^2\alpha$

$$\alpha = \frac{EI}{L^3} \; ; \; \delta = \frac{GJ}{L}$$

E = elastic modulus (psi),
I = moment of inertia (in.^{4}),
L = length (in.),
G = modulus of rigidity (E/16 assumed) psi,
J = polar moment of inertia (in.^{4}).

Figure 28. Element stiffness matrix index array used to reference deckboard or stringer stiffness coefficients.
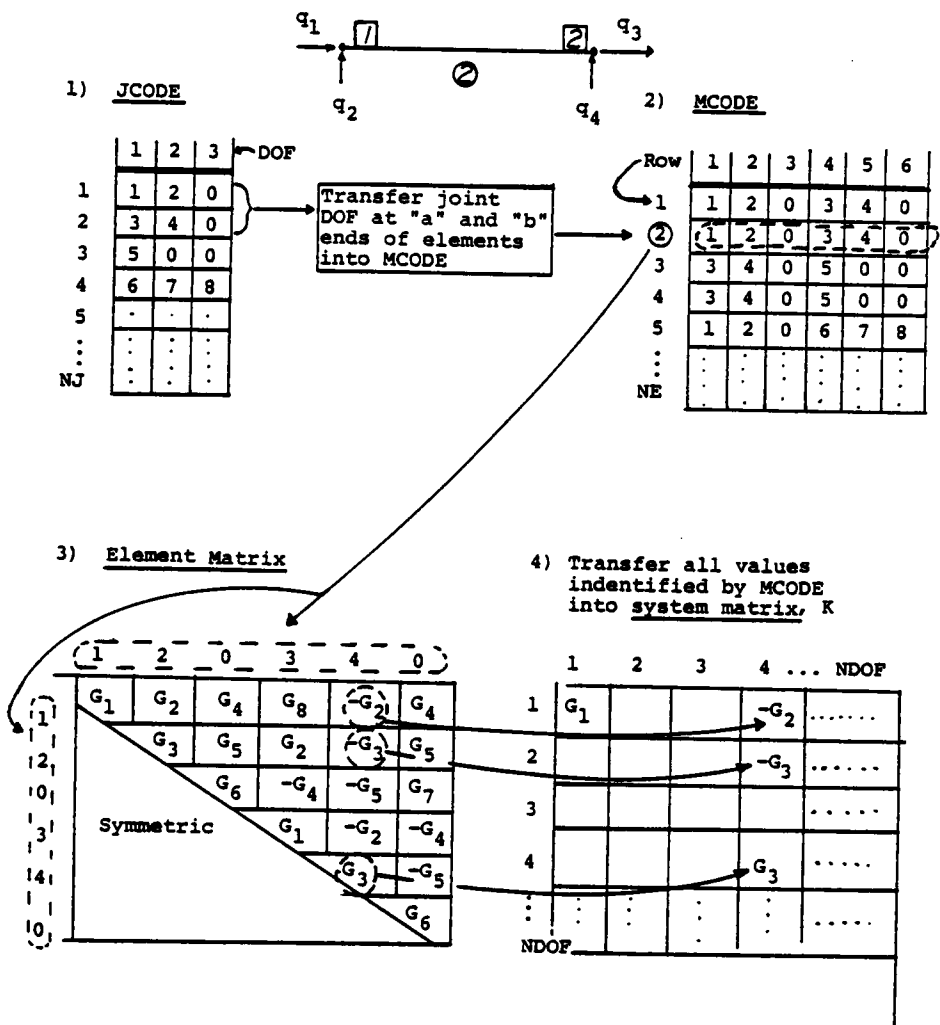
Figure 29.   Relationship between arrays used to assemble the
             system stiffness matrix (K). Example shows partial
             assembly for element 2.

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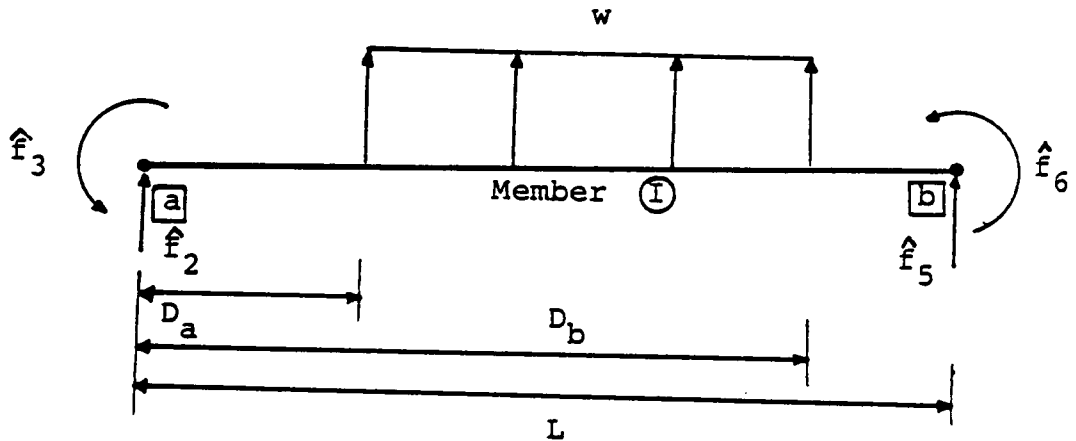
$$\hat{f}(2,I) = \frac{-W}{2L^2} [2L^3(D_b-D_a)-2L(D_b^3-D_a^3)+(D_b^4-D_a^4)]$$

$$\hat{f}(3,I) = \frac{-W}{12L^2} [6L^2(D_b^2-D_a^2)-8L(D_b^3-D_a^3)+3(D_b^4-D_a^4)]$$

$$\hat{f}(5,I) = \frac{-W}{2L^3} [2L(D_b^3-D_a^3)-(D_b^4-D_a^4)]$$

$$\hat{f}(6,I) = \frac{+W}{12L^2} [4L(D_b^3-D_a^3)-3(D_b^4-D_a^4)$$

Figure 30. Equations to compute the fixed end forces for deckboards loaded with full or partial uniform loads in the RAS model.

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The system stiffness matrix is assembled by referencing the MCODE to determine the cell coordinate in the system matrix, and the index array, to determine the value to be placed into previously identified cell. This referencing is done for all elements and is shown schematically in Figure 29 on page 106.

4. Compute the equivalent joint loads from the applied member loads: To conduct the analysis the uniform member loads must be translated into equivalent joint loads. In other words, joint loads which produce the same actions in the member as the uniform member load. Only those elements in the model which represent top deckboards are allowed to carry member loads. The scheme is to compute the fixed end forces (the shear and moments at the member ends) by using the equations in Figure 30 on page 107 .

The magnitude of the uniform load on an individual deckboard is found by dividing the total pallet load by the total loaded surface area and then multiplying by the width of an individual deckboard. If using the ANALYSIS option the input load is assumed to be an arbitrary 2000 pounds. The maximum load capacity is found from the ratio of allowable stress (MOR corrected for safety) divided by the computed stress (i.e. stress caused by the input load) multiplied by input load. If using the DESIGN option the input load is defined by the user.

The local fixed end forces for each member are transformed into the global fixed end forces, FF, by multiplying by a transformation matrix
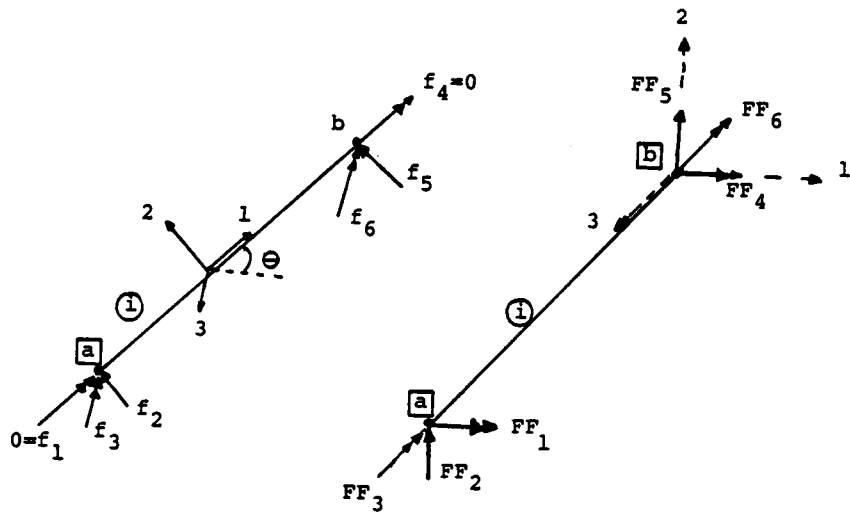
a) local reference
   system

b) global reference
   system

$$[\text{FF}] = [\lambda^T][f]$$

"a" end $\text{FF}_a = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} F_3 \\ F_2 \\ 0 \end{bmatrix}$

"b" end $\text{FF}_b = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ f_5 \\ f_6 \end{bmatrix} = \begin{bmatrix} F_6 \\ F_5 \\ 0 \end{bmatrix}$

Where:

λ = direction cosines = $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

FF_i = force vector in global coordinate system
       for i end of element,

f_i = force in local i direction,

F_i = force in global i direction.

Figure 31. Transformation of forces from local to
          global reference.

which contains the direction cosines for the element (Figure 31 on page 109). Finally the load vector, F, is assembled, element by element, using the MCODE to identify the degree of freedom number associated with each equivalent joint load.

5. Solution: At this point all the required matrices are computed and the system equation can be solved:

$$\{F\}=[K]\{D\} \qquad\qquad (4.23)$$

This relationship represents a set of equations having the number of unknowns equal to the number of degrees of freedom for the structure. Therefore the set of equations must be solved simultaneously. Several algorithms are available for this solution and a Gauss elimination method is used here.

6. Compute element stress: The element stresses are computed from the displacements found in the solution step. Only the stringer elements are checked because these are the critical elements in the RAS load case. Since the stringer elements do not carry member loads[10], maximum stress will always occur at the ends of the members, therefore, the stress is found at each end of every stringer element using the equations shown in Figure 32 on page 111. For the DESIGN option, the maximum stress for the structure is compared to the MOR, and the

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[10] Loads are only applied to the ends of the elements representing the stringers, and not to the elements themselves.
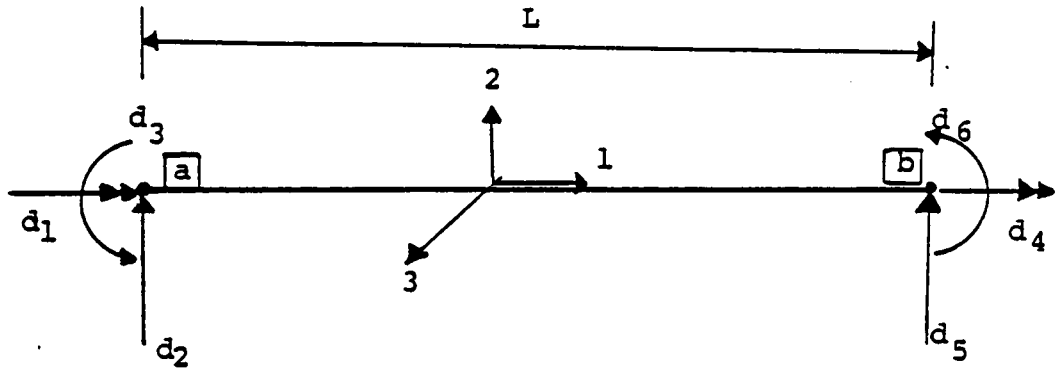
$$f_1 = \gamma(d_1 - d_4)$$

$$f_2 = 12\,\alpha(d_2 - d_5) + 6L\,\alpha(d_3 + d_6)$$

$$f_3 = 6L\,\alpha(d_2 - d_5) + 2L^2\,\alpha(2d_3 + d_6)$$

$$f_4 = -f_1$$

$$f_5 = -f_2$$

$$f_6 = f_2 L - f_3$$

Where:

$$\gamma = \frac{EA}{L} \quad ; \quad \alpha = \frac{EI}{L^3}$$

$$\sigma_a = \left|\frac{f_1}{A}\right| + \left|\frac{f_3}{S}\right| \qquad\qquad \sigma_b = \left|\frac{f_1}{A}\right| + \left|\frac{f_6}{S}\right|$$

Figure 32.   General equations to compute
stress from displacements--RAS.

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maximum deflection is found directly from the solution. To find the maximum load for the ANALYSIS option the maximum stress is ratioed with the MOR and the arbitrary input load. The deflection at the maximum load is then computed from the ratio of the input load and deflection multiplied by the maximum load.

7. Pallets with Notched Stringers: If the stringers are notched the moment at the notch location for each stringer is computed. This is done by assuming linear moment distribution and interpolating the moment between the two ends of the member that contains the notch. For the DESIGN option the maximum moment at the notch is then compared to the allowable notch moment (found from Gerhardt's' equations). For the ANALYSIS option the maximum load is found from the ratio of the input load divided by the notch moment multiplied by the allowable moment. The centerline deflection for the notched stringer pallet is found by multiplying the maximum deflection by the notch correction factor as described in the section for notched pallet which have equal sized stringers.

4.4 EXPERIMENTAL VERIFICATION

The previous sections described techniques developed to produce estimates of the load effects of RAS pallets. To reduce the complexity of the analysis some simplifing assumptions were made, such as estimation of the effective span, idealization of the load, and, the mechanism of load transfer. Also, to minimize computational time, simplified structural models were developed in two rather than three dimensions. These sim-

plifications were intended to simulate, as closely as possible, the structural action of RAS pallets while maintaining computational efficiency. The use of simplified procedures and assumptions may introduce error into the prediction of load effects. Evaluation of the adequacy of PDS in predicting the load effects requires comparison between predicted and experimental response of tested pallets. This section describes such a comparison for RAS pallets.

The experimental methods that were used to verify PDS were primarily conducted by Collie (1984). All tests were conducted in a similar manner and are briefly outlined here, but a detailed description is in Collie's thesis. Twelve pallet designs with 5 replications of each design were constructed, and tested destructively in the RAS mode. The pallets were constructed of green oak that was randomly sampled from the inventory of a southern Virginia pallet mill. The pallet types selected for testing included a wide variety of common commercial designs having either three or four stringers. The percentage of combined top and bottom deckboard coverage ranged from 58 to 165% The pallets were assembled with pneumatically or hand driven threaded pallet nails. After assembly the pallets were tested to failure (or to the testing machine capacity) with a uniformly distributed load applied by a constrained air-bag. (An additional test result was obtained from the archives of the Sardo Pallet Laboratory at Virginia Tech.) The load was measured by load cells located under each of the four pallet corners and the deflection was measured at three places: the center of the pallet and, the center of each unsupported edge. A Waters Longfellow potentiometer measured the deflection to an

accuracy of 0.004 inch. During testing a TRS-80 microcomputer automatically recorded the load and deflection measured by each transducer. After testing, the load and deflection data was transferred to an IBM 3084 mainframe computer for analysis.

The verification of PDS was accomplished by comparing the average observed strength and stiffness of each class of test pallet to predicted values obtained from PDS. Pallet stiffness was computed by dividing the load at a point in the linear region of a test curve, by the corresponding deflection. Unfortunately, nondestructive tests were not preformed on individual parts of the test pallets. Therefore, the exact properties of the members in each pallet are unknown and must be estimated. Because the MOE and MOR of pallet material is highly variable, this estimation can lead to large errors if the verification is performed on the basis of individual pallets. Consequently, the average stiffness for each type of test pallet was computed from a least squares fit to the linear portion of the experimental curves. Since PDS is a linear model, the stiffness predictions can be compared directly to the average stiffness of each type of test pallet. Similarly, the average maximum load for each class of test pallets was used to reduce sample-to-sample variation.

The predicted values were generated from PDS by setting the safety index equal to zero (i.e. predictions are uncorrected for safety) and conducting, for each pallet type, three separate analyses using different estimates of the material properties: the mean MOE and MOR, one standard deviation below the mean MOE and MOR, and, one standard deviation above
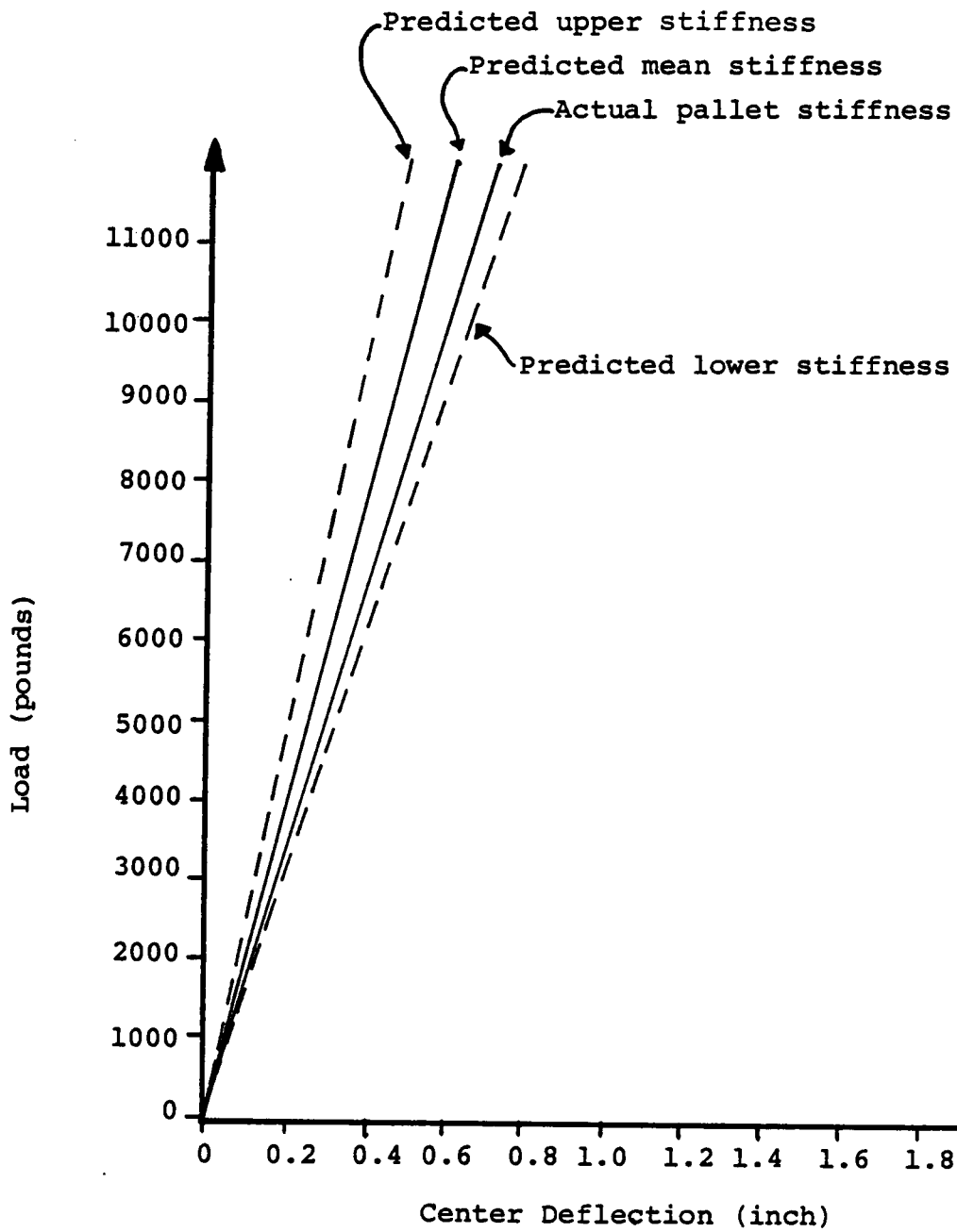
Figure 33. Typical curve showing average measured stiffness and predicted band one standard deviation above and below mean properties.

Table 4.3. Percent error and actual error in predicted stiffness for RAS tests. (Collie, 1984)

Design No.	Measured Stiffness (lbs/in.)	Percent Error In Predicted Stiffness			Actual Error In Stiffness (lbs/in.)		
		$\bar{X}-\sigma$	$\bar{X}$	$\bar{X}+\sigma$	$\bar{X}-\sigma$	$\bar{X}$	$\bar{X}+\sigma$
1	10764	9.2	-16.0	-42.0	1038	-1744	-4538
2	10256	9.1	-22.0	-51.0	388	-2438	-5304
3	16937	6.5	-17.3	-42.0	1112	-2934	-7268
4	6134	26.0	9.2	-11.7	1634	562	- 720
5	14497	-4.7	-44.0	-64.0	-688	-6416	-9314
6	17880	15.0	- 8.6	-31.0	2702	-1553	-5546
7	19753	4.0	-20.0	-49.0	822	-4025	-9783
8	16544	-2.0	-28.0	-53.0	-432	-4704	-8803
9	29244	9.0	-15.1	-40.1	2642	-4408	-11728
10	33293	18.1	- 3.6	-26.2	6009	-1228	-8743
11	9687	0.9	-23.1	-47.0	124	-2255	-4557
12	9310	18.1	13.0	- 3.9	2755	1247	- 368
VPI #1	25222		0			0	
Numerical Average		10.4	-15.9	-46.0	1810	-2717	-7667.2
Absolute Average		11.7	19.9	46.0	2034	3046	-7667.2

Note: Negative errors indicate over predicting stiffness.
$\bar{X}$ = mean value of material properties.
σ = standard deviation of material properties.

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the mean MOE and MOR. The resulting estimates from PDS, represent a predicted response region. This band is centered on the mean predicted strength and stiffness (see Figure 33 on page 116) and is expected to bound the average strength and stiffness values exhibited by the corresponding test pallet. The material property values (MOE, MOR) that were used as input parameters to PDS were obtained from bending tests of surplus material collected by Collie.

Due to the difficulty of estimating the exact material properties, the criteria for evaluating the effectiveness of PDS at predicting pallet behavior is based upon several factors: A) low bias-- the model should not consistently over- or under- predict the actual response, B)percent error--an adequate model should produce estimates that minimize the percent difference between the experimental and predicted pallet behavior, C) the measured response should fall within the predicted response region established by using property estimates of plus and minus of standard deviation about the mean.

The predicted and measured maximum loads and stiffnesses are shown in table 4.3. This table shows that, for the mean property estimates, the average absolute error in predicting the stiffness was 19.1%, and the difference in predicted stiffness was 3046 lbs/in. The main reason for these differences is probably due to errors in estimating the properties of the stringers. Since the material properties were not measured for each stringer in the pallets, some error in the predicted response is

expected. Establishing the properties of the center stringer is extremely important in the RAS mode because the response of the entire pallet is largely determined by this member's strength and stiffness. Table 4.1 also shows that for all but two designs the measured stiffness was bounded by the predictions.

In light of the highly variable properties it seems that PDS is able to produce predictions of RAS pallet response that generally meet the criteria stated above.

5.0 DESIGN FOR RACKED ACROSS THE DECKBOARDS SUPPORT MODE (RAD)

The previous Chapter presented the ANALYSIS and DESIGN techniques developed for pallets in the racked across the stringer mode. The objective of this Chapter is to describe similar methodology for pallets racked across the deckboards.

As described for the RAS mode, the analysis process translates the applied load into the load effects for subsequent comparison to the resistance, using the FOSM method described in chapter 8. The member exhibiting the maximum load effect governs the allowable pallet load and is a critical member. In the RAD mode the possible critical members are either the top or bottom deckboards. For design purposes, the entire deck is analyzed as if it were one deckboard whose width is equal to the sum of the widths of the individual boards. In a typical analysis both decks are checked to determine the maximum stress.

ASSUMPTIONS AND LIMITATIONS: A two dimensional view of the general geometry for RAD pallets is shown in Figure 34 on page 120. The depicted pallet is a three stringer pallet supported at its ends. Two and four stringers may also be analyzed. To maintain structural stability, double winged pallets can not be analyzed if supported under the bottom wing. For these winged pallets the support must be located in the span between outer stringers. For two and three stringer pallets the supports can be located anywhere under the bottom deck. For four stringer pallets the

RACKED ACROSS DECKBOARDS (RAD)
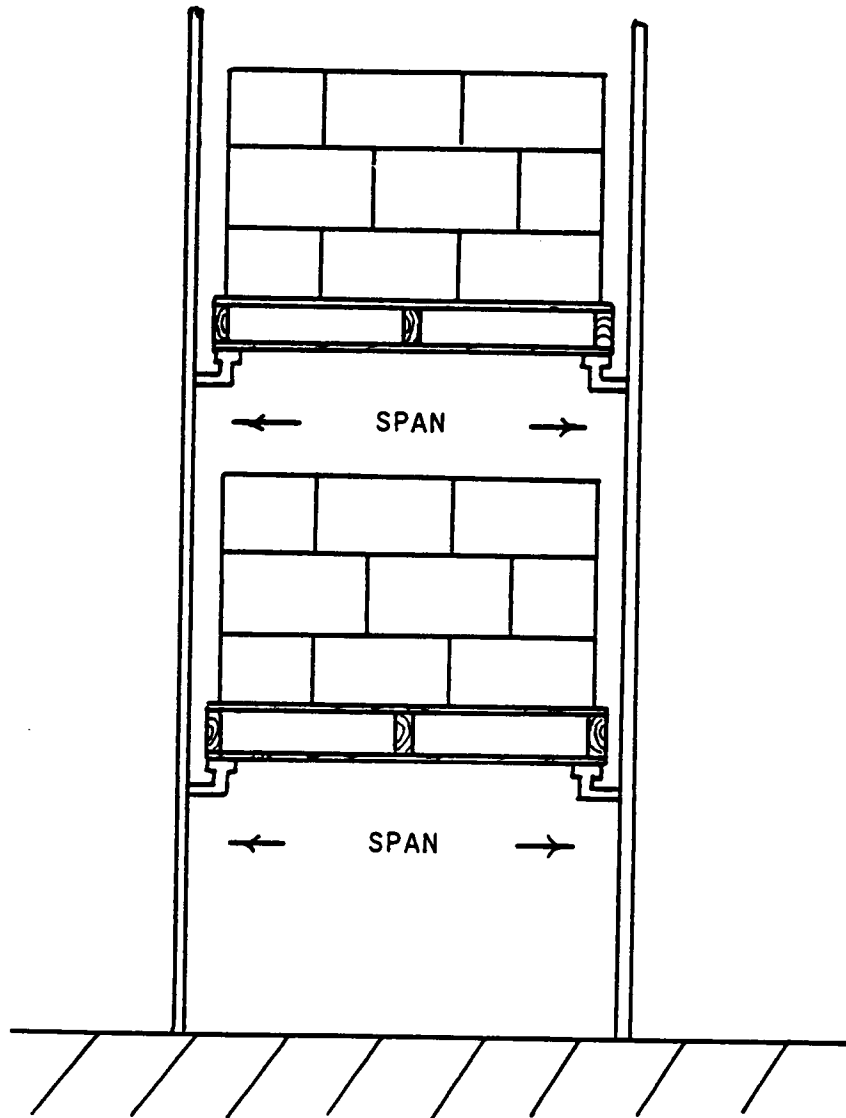
Figure 34. Pallets supported in racked across deckboards (RAD) mode.

support location is restricted to the region between the outside edge of the outer stringer and the outside edge of an inner stringer. However for stability, the supports should generally be located near the outer stringers.

The inner stringers of three-and four-stringer pallets are assumed to cause the top and bottom decks to deflect equally under load. Therefore, the centerline deflection will always be the maximum global pallet deflection. For racked, two stringer pallets the maximum deflection occurs in the top deck because the bottom deck is not loaded.

Two techniques were used to analyze pallets in the RAD mode. A matrix displacement solution is used for all pallets except for single-faced pallets supported under the top deck. For those exceptions a strength of materials solution is used since the structure behaves as a simple beam whose width is equal to the sum of the deckboard widths. Because the structure is inherently unstable a flush pallet with no bottom deck cannot be analyzed RAD. (However this pallet configuration may be analyzed in the stacked mode described in Chapter 6).

5.1 MATRIX STRUCTURAL ANALYSIS SOLUTION

This section describes the technique used to compute the member stress and deflection of RAD pallets by the matrix method. First, a general description of the model and its applications is presented, followed by a detailed derivation of the process used to analyze pallets supported

under the bottom deck. Secondly, the method used to analyze sling sup-
ported winged pallets is described. Last, some experimental verification
of the procedure is presented.

THE ANALOG MODEL: Development of a matrix structural analysis solution
to RAD problem requires selection of an analog model which has flexibility
and the capacity to mimic the action of pallets subject to service con-
ditions. One of the most significant challenges in modeling RAD pallets
dealt with describing the action of the deckboard-stringer joint. Ob-
servations of the behavior of the joints in both full pallets and pallet
sections were used to develop the model for RAD pallets. The pallet
sections were composed of one top and one bottom deckboard nailed to the
stringers and loaded by a center point load. The observations revealed
that the top deckboard always pivots around the inside edge of the outer
stringers. However, the pivot point of the bottom deckboard depends upon
the location of the support relative to the outer stringer-deckboard joint
(Figure 35 on page 123) and the flexural stiffness of the bottom deck and
the unit load magnitude. For example, if the support is located directly
under the stringer, the bottom joint tends to open and the pivot point
is located at the outer edge of the stringer. If the support is located
in the span between the outer and inner stringers, the bottom joint re-
mains closed and appears to become stiffer with increased load [11]. Ad-
ditionally, if the support is extremely wide the portion of the load over

---

[11] This phenomenon was also observed for full sized pallets by Fagan,
and resulted in his recommendation for an improved RAD model.
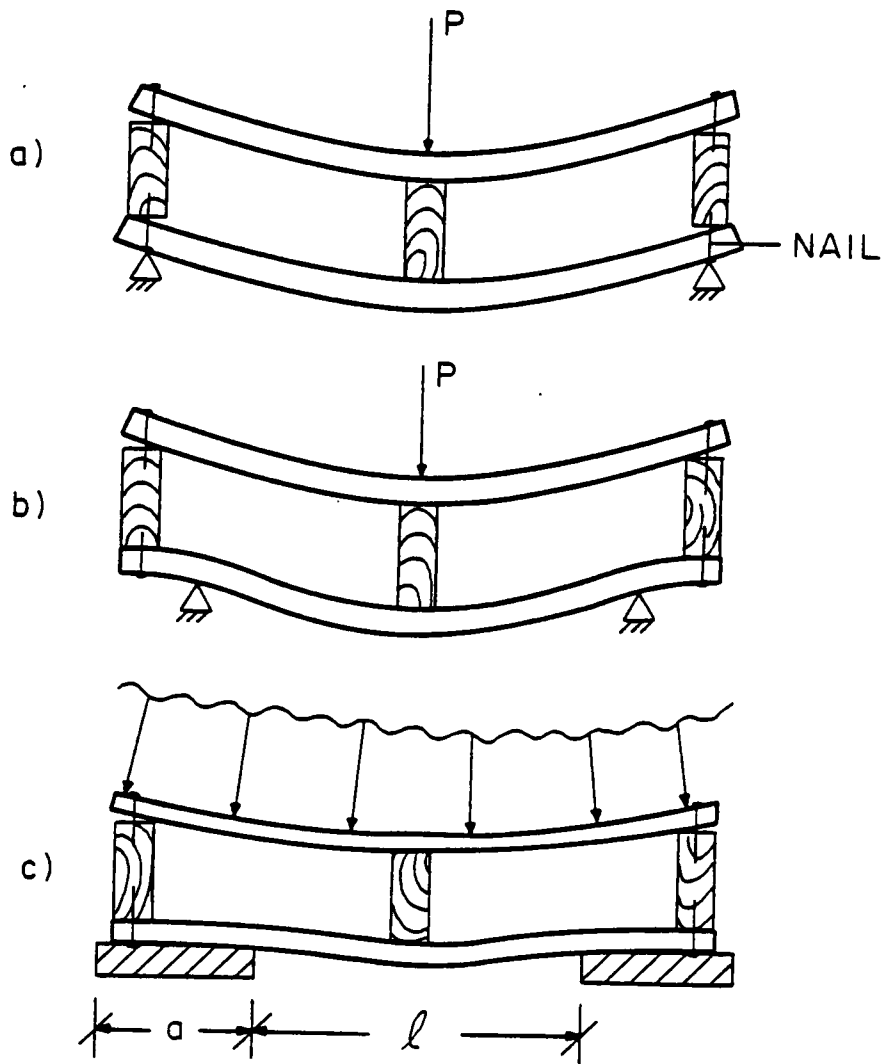
Figure 35. Pivot point for RAD support mode.

the support can cause the bottom deck to remain in contact with the support, thus simulating a fixed end condition instead of the assumed pinned support.

To accurately predict the response of a pallet to applied loads, the analog model must account for these joint actions. Therefore, to simulate the observed joint behavior, zero length spring elements are incorporated into the RAD model. The spring elements are assigned stiffness values corresponding to the lateral, and rotational stiffness of representative test joints. (Details are described elsewhere in this section). Additionally, observations of full size RAD pallets showed that the inner stringers tend to act as rigid bars and cause nearly equal deflection across the width of the pallet. In other words, bending in two directions is not significant in the RAD mode. Based on this observation the action of RAD pallets were simulated using a two, rather than a three dimensional model. The two dimensional model reduces the required computer memory and computation time since the number of degrees of freedom per joint are reduced by half as compared to the three dimensional model.

The RAD model of the left half of a symmetric pallet is shown in Figure 36 on page 125 and is used to represent a wide variety of structures. By selectively assigning material properties to the members the actions of two, three, or four stringer pallets can be simulated. By inverting the model and correctly placing the support, the action of a sling supported winged pallet can also be simulated. For example, to model a two stringer pallet, members 12 and 16 are given material prop-

a) Cross-section of real pallet
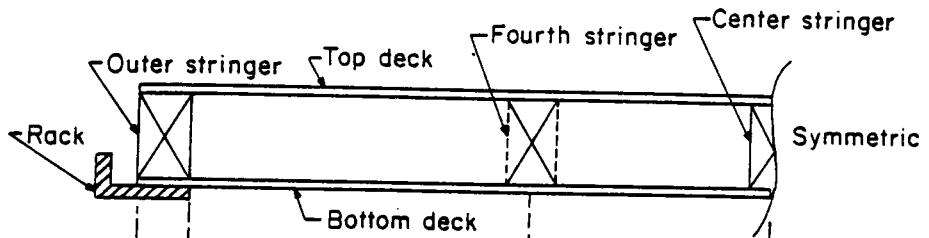
b) RAD model
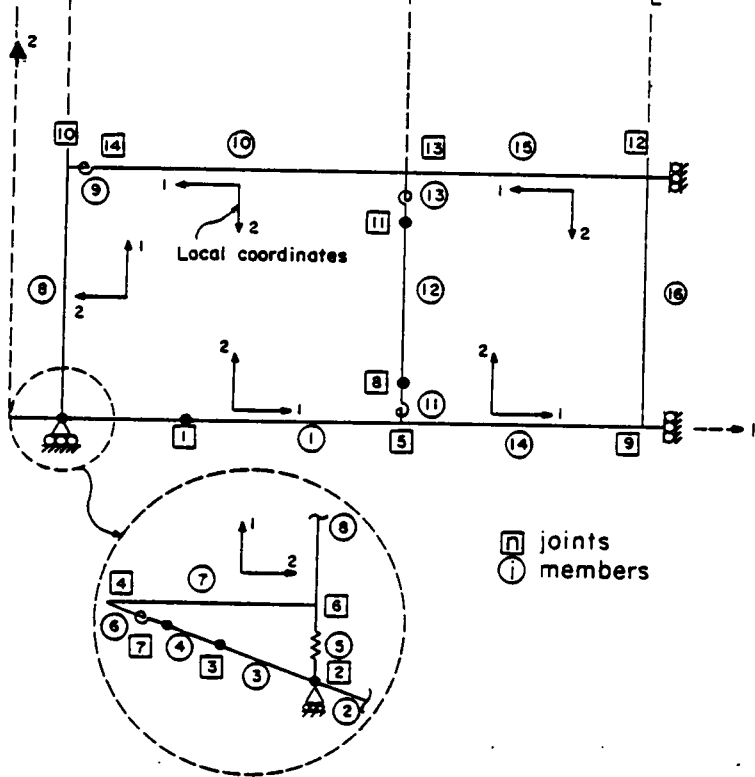
Figure 36. Analog model of RAD pallet

erties equal to zero. For a three stringer pallet the properties of member 12 are set equal to zero and for a four stringer pallet the properties of member 16 are set equal to zero. All the other members are given properties equal to those of the elements they represent in the real pallet. For three stringer pallets the width of the center stringer element (member 16) is set equal to half the width of the real stringer. This is done to conform to the rules of symmetry. The structural action of the symmetric model is identical to that of a full model but has only half the number of joints (or degrees of freedom). Hence, the time required to compute a solution is greatly reduced. Because member 16 is located on the pallet centerline and since symmetry is required, joints 9, and 12, may not experience rotation or lateral slip. Therefore, spring elements are not needed to model the connection between member 16 the top or bottom decks. (This eliminates two elements from the model thus reducing the number of system degrees of freedom).

The model is also versatile regarding the rack support location: For a given condition one of three possible joints (either joint 1, 2, or 3) will represent the rack support. Nodes 1 and 3 are mobile and can be placed anywhere along the bottom deckboard elements simply by adjusting the lengths of elements 1 and 2 (for support at joint 1), or elements 3 and 4 (for support at joint 3). The only limit for the support placement is that the resulting length of any member must be greater than 0.01 inches. (This restriction is imposed to reduce the risk of division by zero errors in the computerized solution). Joint 2 is not mobile and is always located under the inner edge of the stringer. The rack support

is always represented as a pinned support. The joints located on the center line (joints 9 and 12) are represented by shear releases (as in the RAS model), and can transmit moment but are free to deflect vertically. This imitates the action of a real, continuous beam.

Elements 6, 9, 11, and 13 are zero length spring elements used to simulate the action of semi-rigid nail joints. The rotational stiffness of a spring is equal to an estimated joint rotation modulus. Computation of the rotation modulus is described in this chapter. For a racked pallet, members 9 and 13 represent the accumulated stiffness of the nail joints in the top deck and members 6 and 11 correspond to those of the bottom deck.

Member 5 represents a zero length axial spring whose stiffness is usually equal to zero. However, after a matrix solution is obtained the coordinates of the displaced joints are checked for physical compatibility. Specifically, a check is made to determine if joint 2 moved vertically past joint 6--a physical impossibility in a real pallet. If this condition is detected, member 5 is given a high stiffness, the original system stiffness matrix is adjusted accordingly, and a new matrix solution is obtained. Because of this check the model can accurately simulate either the closing action of the bottom joints when the support is located between the inner and outer stringers, or the opening action when the support is located under the stringers.
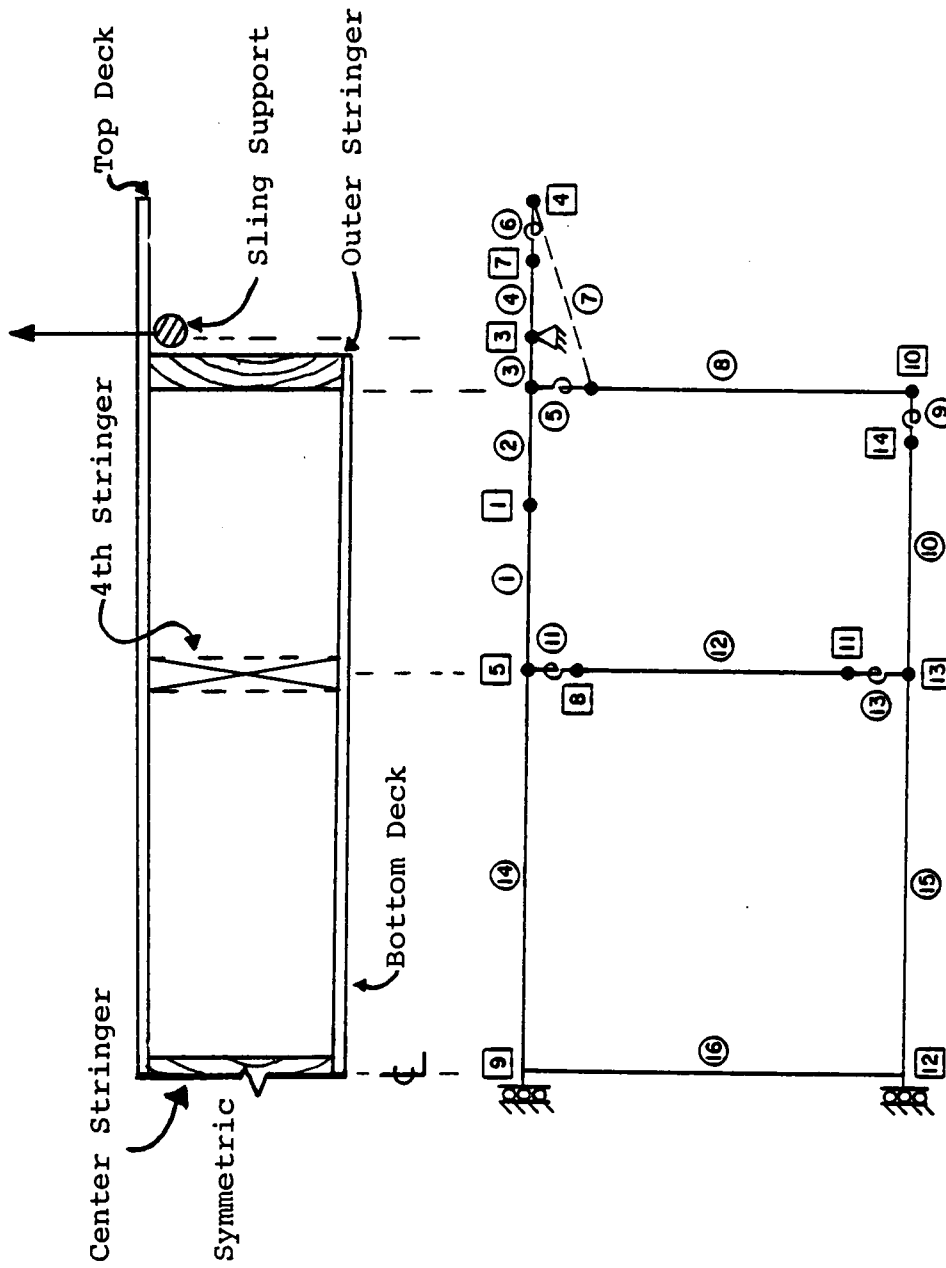
Figure 37. Model for sling supported winged pallet.

WINGED PALLETS--SLING SUPPORTED: A pallet supported under the wing is analyzed by using the same RAD model except that: 1) the model is inverted, 2) the support is located at joint 3, and 3) the stiffness of spring members 5 and 6 are modified. Member 6 is given zero stiffness in each direction to represent a free connection between the deckboard wing and stringer edge (Figure 37 on page 128). The wing can then deflect independently of the stringer edge (element 7). (Note that in this formulation member 7 does not carry load and therefore has no structural function). Member 5 is given the stiffness values of a rotational-lateral spring (and high stiffness in withdrawal) and is used to represent the stiffness of the top deckboard-stringer nail joints. Other members are assigned properties and geometries which correspond to those of the real elements in the pallet.

5.1.1 ASSEMBLY OF SYSTEM STIFFNESS MATRIX FOR THE BASIC MODEL

The RAS analog model was designed to contain a variable number of elements and uses an automatic assembly technique to compute the stiffness matrix. By contrast, the RAD model uses a fixed number of elements to represent pallets of variable geometry. As a result, the system stiffness matrix for the basic RAD model was precomputed and computer encoded to save computation time. Simple modifications to the basic matrix allows for placement of the support at various joints. The "basic" RAD model (Figure 36 on page 125) has joint 2 as the pin supported joint representing the inner edge of the rack. If the support is located at either joint 1 or 3 only ten cells in the basic stiffness matrix need to be

modified. The algorithm of the assembly and solution technique presented for RAS was also used for RAD. The steps to derive the "basic" stiffness matrix are outlined as follows:

1. Define the arrays which contain the unknown joint displacements. First write the JCODE array to define the degrees of freedom associated with each joint as shown in Figure 38 on page 131 . From the JCODE assemble the MCODE array to define, for each element, the identification numbers of the unknown end displacements. This is done by transferring the degree of freedom numbers of the joint located at the "a" end of the element into the first three cells of MCODE. Similarly, the 2nd three cells of MCODE are filled and represent the degree of freedom numbers for the joint connected to the "b" end of the element.

2. Define the local element stiffness matrices. Three different element types are used in the pallet model to represent the various members in the real pallet. Therefore, different element stiffness matrices are needed to define:

   a. Zero length rotational springs

   b. Zero length axial springs

   c. Real elements which can have finite length, stiffness, and cross sectional geometry.

   These matrices are shown in Figure 39 on page 132.

3. Assemble the system stiffness matrix: MCODE is used to identify the cells in the system stiffness matrix which are influenced by the actions of each member. The matrix is assembled, element by element

Design for Racked across the deckboards support mode (RAD)   130
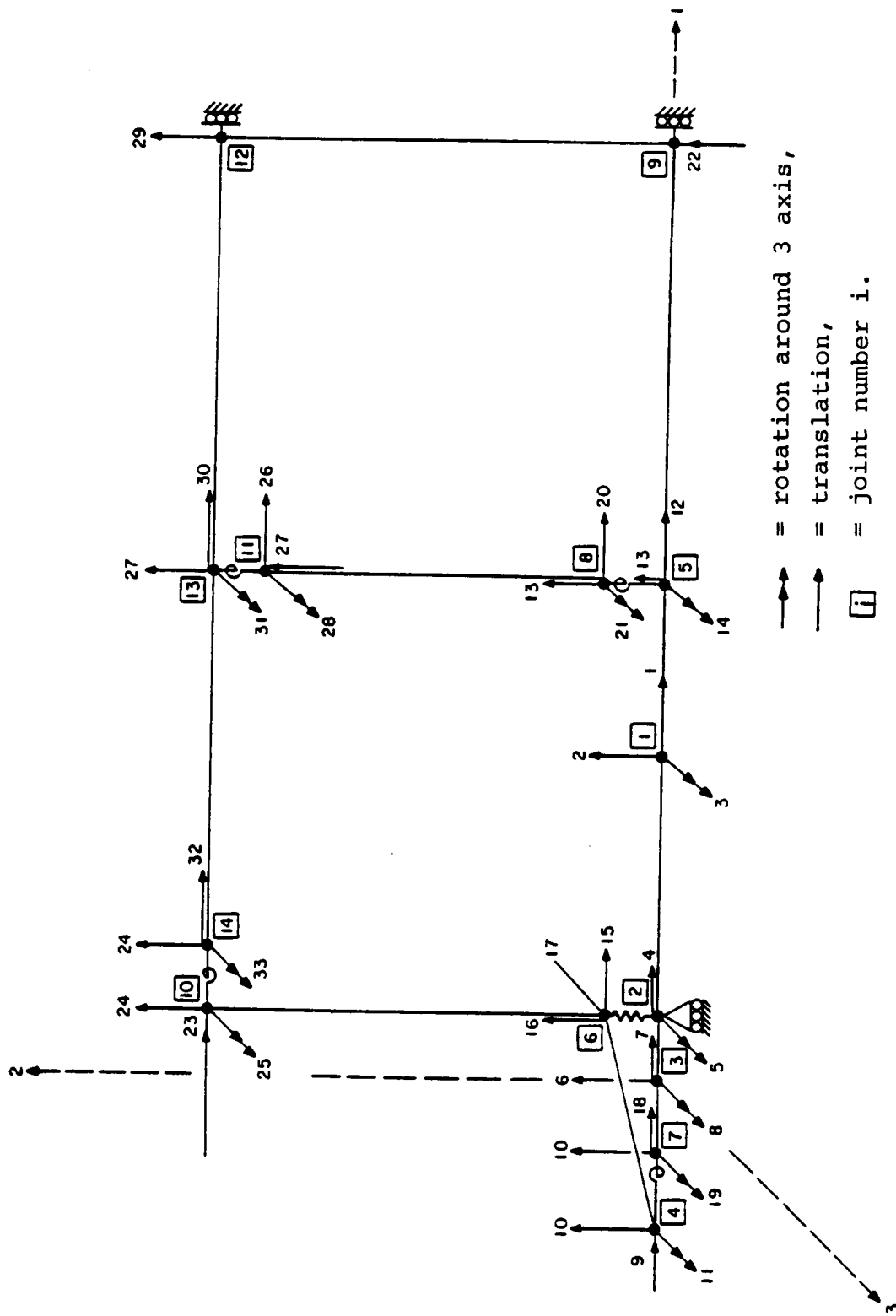
Figure 38. Possible joint motions used to develop JCODE for RAD support at joint 2.

⟶ = rotation around 3 axis,
→ = translation,
[i] = joint number i.

Index:

$$
\begin{bmatrix}
G_1 & G_2 & G_4 & -G_1 & -G_2 & G_4 \\
 & G_3 & G_5 & -G_2 & -G_3 & G_5 \\
 & & G_6 & -G_4 & -G_5 & G_7 \\
 & \text{Symmetric} & & G_1 & G_2 & -G_4 \\
 & & & & G_3 & -G_5 \\
 & & & & & G_6
\end{bmatrix}
$$

Where:

Real Element	Rotational Spring	Axial Spring
$G_1 = \alpha(\beta C_1^2 + 12C_2^2)$	$G_1 = \gamma_1$	$G_1 = 0$
$G_2 = \alpha C_1 C_2 (\beta - 12)$	$G_2 = 0$	$G_2 = 0$
$G_3 = \alpha(\beta C_2^2 + 12C_1^2)$	$G_3 = \gamma_2$	$G_3 = \gamma_4$
$G_4 = -\alpha 6LC_2$	$G_4 = 0.$	$G_4 = 0$
$G_5 = \alpha 6LC_1$	$G_5 = 0$	$G_5 = 0$
$G_6 = \alpha 4L^2$	$G_6 = \gamma_3$	$G_6 = 0$
$G_7 = \alpha 2L^2$	$G_7 = -\gamma_3$	$G_7 = 0$

$\alpha = \dfrac{EI}{L^3}$; $\beta = \dfrac{AL^2}{I}$

C_1, C_2 = direction cosines,

γ_1 = lateral slip stiffness,

γ_2 = withdrawal stiffness,

γ_3 = rotational stiffness,

γ_4 = axial spring stiffness.

Figure 39. Element stiffness matrices for real elements, zero length rotational springs, and zero length axial springs.

until all member actions have been processed. The resulting system matrix is in a general form and is expressed in terms of element length, elastic modulus, and cross sectional area.

4. Compute the equivalent joint load vector, {F}: As shown in Chapter 4, the applied member loads are transformed into equivalent joint loads. Two basic load types are allowed, uniform loads, or line loads. Each load type is discussed separately:

   a. Uniform loads--Uniform loads can act over either all or part of the deck surface. In either case, the load must be continuous over member 15. This places a constraint on the minimum load coverage for pallets RAD as follows: For two and three stringer pallets the length of member 15 is always defined as 1 inch. Because of symmetry, the minimum length of a partial coverage uniform load in the direction of the deckboard length is two inches. For four stringer pallets, the length of member 15 is dependent on the spacing of the inner two stringers: The minimum load distance is therefore equal to the centerline distance between the inner stringers.

   The equations to compute the equivalent joint loads from uniform loads on members 10 and 15 are shown in Figure 40 on page 134 .

   b. Line Loads--For correspondence between a real pallet and the two dimensional model, line loads are represented by point loads acting on the elements representing the top deckboards. The equations to compute the equivalent joint load vector for line loads is shown in Figure 41 on page 135 . Only element 10 is allowed to carry a line load. This places similar constraints
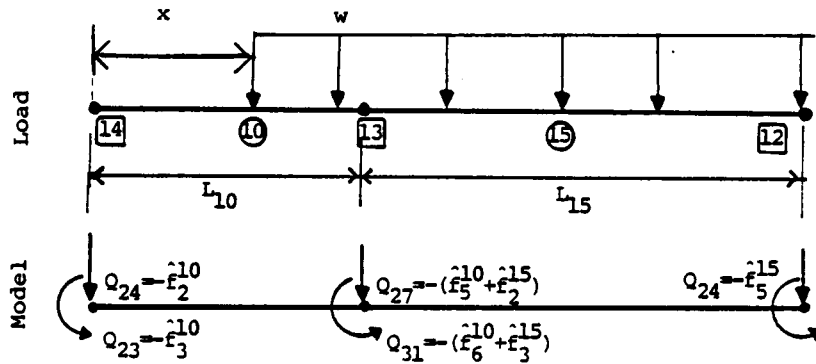
$$\hat{f}_2^{10} = \frac{w}{2L_{10}^3}[2L_{10}^3(L_{10}-x)-2L_{10}(L_{10}^3-x^3)+(L_{10}^4-x^4)]$$

$$\hat{f}_3^{10} = \frac{w}{12L_{10}^2}[6L_{10}^2(L_{10}^2-x^2)-8L_{10}(L_{10}^3-x^3)+3(L_{10}^4-x^4)]$$

$$\hat{f}_5^{10} = w(L_{10}-x)-\hat{f}_2^{10}$$

$$\hat{f}_6^{10} = \frac{-w}{12L_{10}^2}[4L_{10}(L_{10}^3-x^3)-3L_{10}^4(L_{10}^4-x^4)]$$

$$\hat{f}_2^{15} = \hat{f}_5^{15} = \frac{wL_{15}}{2} \; ; \; \hat{f}_3^{15} = \frac{wL_{15}^2}{12} \; ; \; \hat{f}_6^{15} = -\hat{f}_3^{15}$$

Note: $\bar{f}_j^i = f_j^i + \hat{f}_j^i$

Where:

$\bar{f}_j^i$ = actual force vector element i in j direction,

f_j^i = f=kd, internal forces element i in j direction,

$\hat{f}_j^i$ = fixed end forces element i in j direction.

Figure 40 . Computation of equivalent joint loads
caused by the applied uniform member
loads for RAD model.
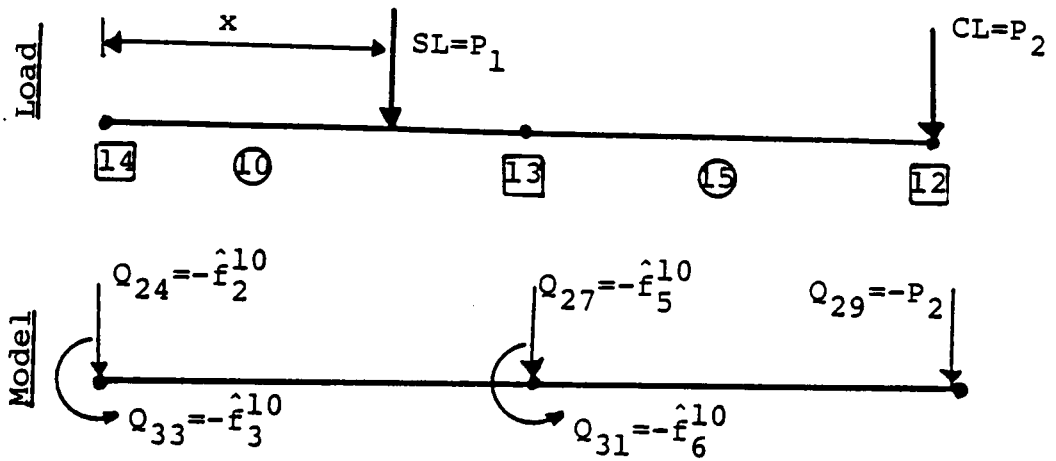
$$\hat{f}_2 = \frac{P_1}{L_{10}^3}[L_{10}-x]^2[L_{10}+2x]$$

$$\hat{f}_3 = P_1 x \left\{ \frac{L_{10}-x}{L_{10}} \right\}^2$$

$$\hat{f}_5 = P_1 - \hat{f}_2$$

$$\hat{f}_6 = -P_1(L_{10}-x)\left(\frac{x}{L_{10}}\right)^2$$

Figure 41. Computation of equivalent joint loads for line loads.

on the load placement as described for uniform loads. However, joint loads can be applied to joints 12, 13, or 14. To conform to the rules of symmetry one half of the centerline point load is always applied to joint 12.

5. Solution of the basic matrix equation: At this point the system stiffness matrix has been computed, and the equivalent joint load vector has been defined. The solution to the matrix equation: $\{F\}$ = $[K]\{D\}$ is obtained by using the Gauss elimination method.

6. Compute member stress: Using the joint displacements found in the solution step, the force, or elastic stress, for each element in the model can be computed. The maximum stress in the model is the load effect used to determine if the design is adequate (DESIGN option) or alternately the maximum allowable pallet load (ANALYSIS option). The sequence for stress computation is as follows:

   a. Member 1--check the stress only if the support is located at either joint 1 or joint 2. Use the equations shown in Figure 42 on page 137 .

   b. Member 14--always check the stress at both ends of the member using the equations shown in Figure 43 on page 138

   c. Member 10--check the stress in this element only if a member load is present. If element 10 is not loaded it is assumed that the maximum stress in the top deck will occur at the centerline, (i.e., member 15, and not in member 10). When member 10 is loaded use the equation shown in Figure 44 on page 139 to compute the stress.

a) Support at Joint 1
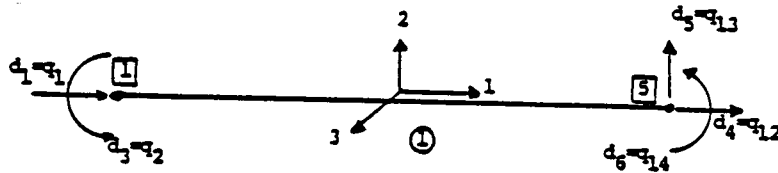
$$f_1 = \frac{EA}{L}(q_1 - q_{12})$$

$$f_3 = \frac{6EI}{L^2}(-q_{13}) + \frac{2EI}{L}(2q_2 + q_{14})$$

$$f_6 = -f_3$$

b) Support at Joint 2 (Basic Model)
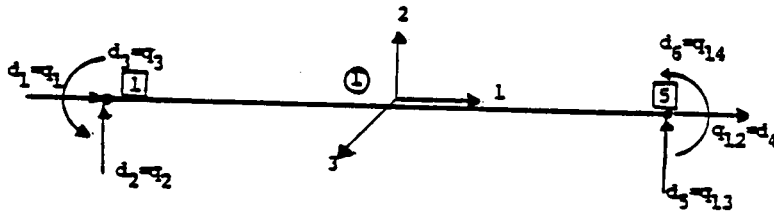
$$f_1 = \frac{EA}{L}(q_1 - q_{12})$$

$$f_2 = \frac{12EI}{L^3}(q_2 - q_{13}) + \frac{6EI}{L^2}(q_3 + q_{14})$$

$$f_3 = \frac{6EI}{L^2}(q_2 - q_{13}) + \frac{2EI}{L}(2q_3 + q_{14})$$

$$f_6 = f_2 L - f_3$$

$$\sigma_1 = \left|\frac{f_1}{A}\right| + \left|\frac{f_3}{S}\right| \qquad \sigma_5 = \left|\frac{f_1}{A}\right| + \left|\frac{f_6}{S}\right|$$

where:

A = x-sectional area of bottom deck,

S = section modulus of bottom deck,

q_i = global degree of freedom i,

d_i = element degree of freedom i.

Figure 42. Computation of stress in RAD member 1: (computed only if either joints 1 or 2 are supported.)
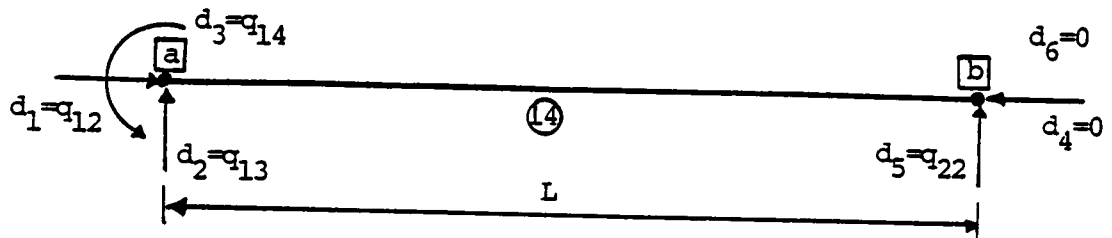
$$f_1 = \frac{EA}{L}(q_{12})$$

$$f_2 = \frac{12EI}{L^3}(q_{13}-q_{22})+\frac{6EI}{L^2}(q_{14})$$

$$f_3 = \frac{6EI}{L^2}(q_{13}-q_{22})+\frac{4EI}{L}\,q_{14}$$

$$f_6 = f_2L-f_3$$

$$\sigma_a = \left|\frac{f_1}{A}\right| + \left|\frac{f_3}{S}\right|$$

$$\sigma_b = \left|\frac{f_1}{A}\right| + \left|\frac{f_6}{S}\right|$$

Where:

A = cross-section area (in.^{2}),
S = section modulus (in.^{3}),
E = elastic modulus (in.^{4}),
L = length (in.),
q_i = joint displacement in i direction.

Figure 43. Computation of stress in RAD member 14.
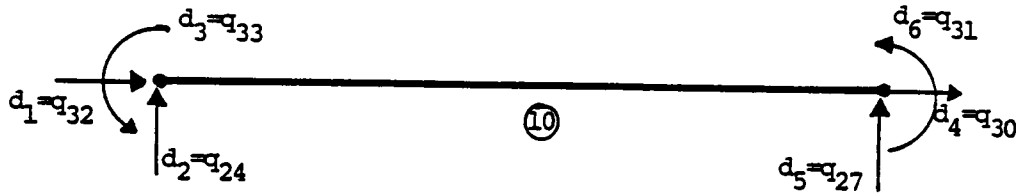
a)   Generalized displacements (local and global)

$$f_1 = \frac{EA}{L}(q_{32} - q_{30})$$

$$f_2 = \frac{12EI}{L^3}(q_{24} - q_{27}) + \frac{6EI}{L^2}(q_{33} + q_{31}) + \hat{f}_2$$

$$f_3 = \frac{6EI}{L^2}(q_{24} - q_{27}) + \frac{2EI}{L}(2q_{33} - q_{31}) + \hat{f}_3$$

b)   Point load on member

$$M_{max} = -f_3 + f_2 x$$

$$\sigma = \left|\frac{f_1}{A}\right| + \left|\frac{-f_3 + f_2 x}{S}\right|$$

c)   Uniform load

$$x_{max} = x + \left|\frac{f_2}{w}\right|$$

$$M_{max} = -f_3 + f_2 \left(\frac{x_{max} + x}{2}\right)$$
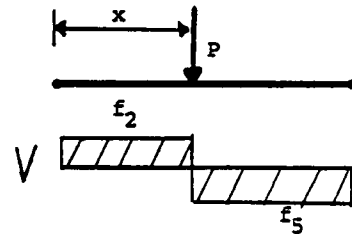
$$\sigma = \left|\frac{f_1}{A}\right| + \left|\frac{M_{max}}{S}\right|$$
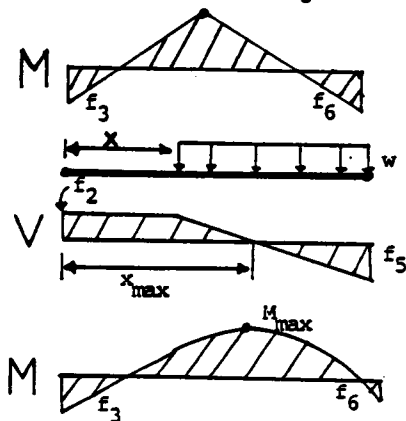
Figure 44.   Computation of stress in RAD
             member 10.
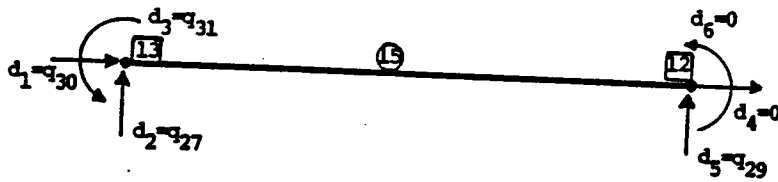
$$f_1 = \frac{EA}{L}(q_{30})$$

$$f_2 = \frac{12EI}{L^3}(q_{27}-q_{29}) + \frac{6EI}{L^2}(q_{31})$$

$$f_3 = \frac{6EI}{L^2}(q_{27}-q_{29}) + \frac{4EI}{L}(q_{31})$$

$$f_6 = f_2 L - f_3$$

$$\bar{f}_2 = f_2 + \hat{f}_2 \quad ; \quad \bar{f}_3 = f_3 + \hat{f}_3^{15}$$

a) Compute stress at each end:

$$\sigma_a = \left|\frac{\bar{f}_1}{A}\right| + \left|\frac{\bar{f}_3}{S}\right| \qquad \sigma_b = \left|\frac{\bar{f}_1}{A}\right| + \left|\frac{\bar{f}_6}{S}\right|$$

b) Compute stress at internal location if member is loaded with uniform load

$$x_{max} = \left|\frac{\bar{f}_2}{w}\right|$$

if: $x_{max} \geq L$ then σ_{max} occurs at member ends as in (a),

if: $x_{max} < L$

$$M_{max} = -\bar{f}_3 + \bar{f}_2 \frac{x_{max}}{2}$$

$$\sigma = \left|\frac{\bar{f}_1}{A}\right| + \left|\frac{M_{max}}{S}\right| \qquad M$$
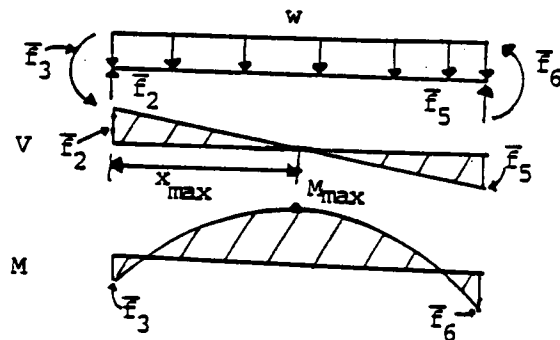
Figure 45. Computation of stress in RAD member 15.

d. Member 15--always check the stress at the ends of member 15 and check the internal stress if an element load is present by using the equations shown in Figure 45 on page 140 .

7. Determine the element with the highest stress, and whether this critical element represents the top or bottom deck. In the DESIGN option, a recommendation regarding the change (either increase or decrease) in the dimensions of the critical deck element is made. This allows the user to optimize the structure in terms of economy and safety. After computing the stress, the next step is to compute the maximum allowable load (ANALYSIS) or to check if the design constraints are met or exceeded (DESIGN). This is done using the first order second moment methods described in chapter 7.

8. Find the maximum deflection: The vertical deflection of every joint in the model is checked and the maximum deflection is saved and reported to the user.

5.1.2 SYSTEM STIFFNESS MATRIX MODIFICATIONS FOR SUPPORT PLACEMENT

The above procedure describes the technique used to analyze a RAD pallet when the support is located directly under the inside edge of the outer stringer (at joint 2). However, if the geometry of the situation requires that the support be placed either in the span between the inner and outer stringer (joint 1), or under the stringer toward the outer edge of the pallet (joint 3), then selected cells of the system stiffness matrix must be changed. The cells are identified by using new JCODE and new MCODE

Figure 46 presents three MCODE matrices. Circled entries denote the modified values.

Basic Model

	1	2	3	4	5	6
1	1	2	3	12	13	14
2	1	2	3	4	0	5
3	4	0	5	4	7	8
4	6	7	8	18	10	19
5	4	0	5	15	16	17
6	9	10	11	18	10	19
7	9	10	11	15	16	17
8	15	16	17	23	24	25
9	23	24	25	32	24	33
10	30	27	31	32	24	33
11	12	13	14	20	13	21
12	20	13	21	26	27	28
13	26	27	28	30	27	31
14	12	13	14	0	22	0
15	0	29	0	30	27	31
16	0	22	0	0	29	0

Support at [1] (circled = modified)

	1	2	3	4	5	6
1	1	(0)	(2)	12	13	14
2	1	(0)	(2)	(3)	(4)	5
3	(3)	(4)	5	6	7	8
4	6	7	8	18	10	19
5	(3)	(4)	5	15	16	17
6	9	10	11	18	10	19
7	9	10	11	15	16	17
8	15	16	17	23	24	25
9	23	24	25	32	24	33
10	30	27	31	32	24	33
11	12	13	14	20	13	21
12	20	13	21	26	27	28
13	26	27	28	30	27	31
14	12	13	14	0	22	0
15	0	29	0	30	27	31
16	0	22	0	0	29	0

Support at [3] (circled = modified)

	1	2	3	4	5	6
1	1	2	3	12	13	14
2	1	2	3	4	(5)	(6)
3	4	(5)	(6)	(7)	(0)	8
4	(7)	(0)	8	18	10	19
5	4	(5)	(6)	15	16	17
6	9	10	11	18	10	19
7	9	10	11	15	16	17
8	15	16	17	23	24	25
9	23	24	25	32	24	33
10	30	27	31	32	24	33
11	12	13	14	20	13	21
12	20	13	21	26	27	28
13	26	27	28	30	27	31
14	12	13	14	0	22	0
15	0	29	0	30	27	31
16	0	22	0	0	29	0

Figure 46. Changes to the MCODE matrix of the basic model to account for support placement at joints 1 or 3.

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arrays to define the degrees of freedom for the new model. The cells which require changes are shown in Figure 46 on page 142 .

5.1.3 WINGED PALLETS --SLING SUPPORT

Using the "basic" model with minor changes, the sling-supported, double-faced winged pallet can be analyzed (see Figure 37 on page 128). The modifications to the basic model are:

1. Adjust all member lengths and properties to reflect those of the corresponding real elements. Assign element 7 zero stiffness to allow the wing to deflect independently of the stringer edge.

2. Modify the "basic" stiffness matrix to the joint 3 support condition.

3. Change nine cells in the stiffness matrix which are related to the spring element 5. The new equations and cell coordinates are identified in Figure 47 on page 144. These changes transform element 5 from an axial spring to a rotational-lateral spring allowing both lateral slip and rotation. This change together with the stiffness change for element 7 allow the model to behave as a sling supported winged pallet.

4. Compute the equivalent joint load vector. The same techniques and constraints are used to compute the load vector as previously described except for the following modifications:

    a. Uniform or partial uniform loads--Only member 14 must be fully loaded, however, members 1, 2 or 3 can be partially loaded by removing the load from the right end of the member (i.e. the

$$K_{(4,4)} = \gamma_1 + \frac{E_2 A_2}{L_2} + \frac{E_3 A_3}{L_3}$$

$$K_{(4,15)} = -\gamma_1$$

$$K_{(5,5)} = \gamma_2 + \frac{12 E_2 I_2}{L_2^3} + \frac{12 E_3 I_3}{L_3^3}$$

$$K_{(5,10)} = -\gamma_2$$

$$K_{(6,6)} = \gamma_3 + \frac{4 E_2 I_2}{L_2} + \frac{4 E_3 I_3}{L_3}$$

$$K_{(6,17)} = -\gamma_3$$

$$K_{(15,15)} = 12 \left(\frac{EI}{L^3} \right)_{st} + \gamma_1$$

$$K_{(16,16)} = \left(\frac{EA}{L} \right)_{st} + \gamma_2$$

$$K_{(17,17)} = 4 \left(\frac{EI}{L} \right)_{st} + \gamma_3$$

where:

   γ_1 = lateral stiffness of nail joints (lbs/in.),

   γ_2 = withdrawal stiffness of nail joints (lbs/in.),

   γ_3 = rotational stiffness of nail joints (in-lbs/radian),

   K_{ij} = value in stiffness matrix, K, in location row i, column j,

   L_{st} = length of stringer element (in.),

   L_i = length of element i (in.),

   E_i = MOE of element i (psi),

   I_i = moment of inertia of element i (in.^{4}),

   A_i = cross section area of element i (in.^{2}).

Figure 47.   Changes to the system stiffness matrix for joint 3 support condition to model winged pallets.

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A)

$Q_{22} = -\hat{f}_2^{14}$ $Q_{13} = -(\hat{f}_2^{14} + \hat{f}_5^1)$ $Q_2 = -(\hat{f}_2^1 + \hat{f}_2^2)$ $Q_5 = -(\hat{f}_5^2 + \hat{f}_2^3)$

$Q_8 = -(\hat{f}_6^3)$

⑭   5   ①   1   ②   2   ③   3

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$Q_{14} = -(\hat{f}_3^{14} + \hat{f}_6^1)$ $Q_3 = -(\hat{f}_3^1 + \hat{f}_3^2)$ $Q_6 = -(\hat{f}_6^2 + \hat{f}_3^3)$

B)

w   X1

⑭   ①   ②   ③

C)

w   X2

⑭   ①   ②   ③

D)

w   X3

⑭   ①   ②   ③
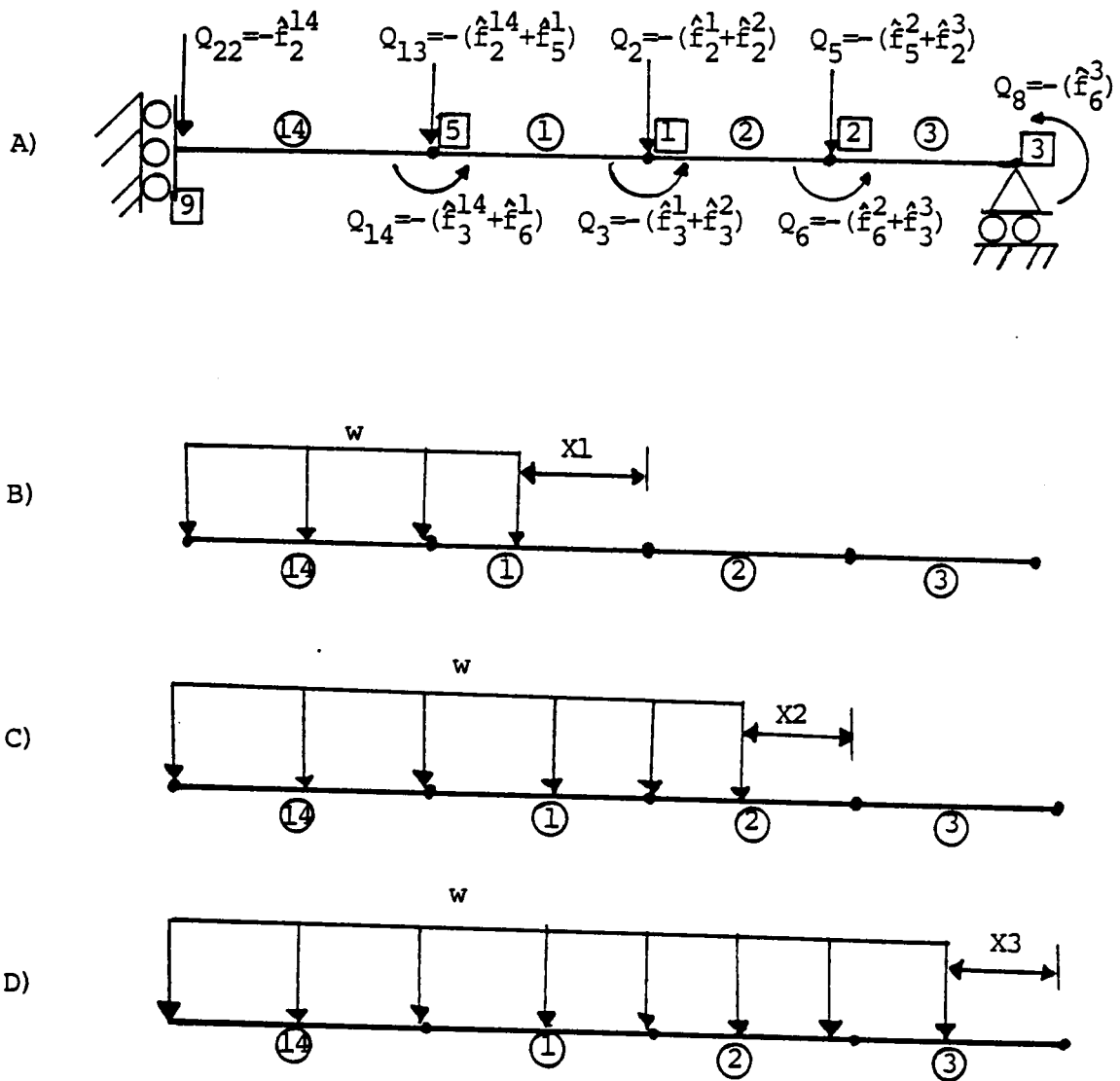
Figure 48. Load models for uniform and partial uniform
loads for sling supported winged pallets.
(a) equivalent fixed end forces (Q_i), corre-
sponding to degree of freedom for structure
(see Table 5.1 for computation); (b) partial
uniform load over member 1; (c) partial uni-
form load over member 2; (d) partial uniform
load over member 3.

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Table 5.1. Equations to compute equivalent fixed end forces--sling supported winged pallets--uniform loads.

Member Number	Equivalent Fixed End Forces
1	$\hat{f}_2^1 = \dfrac{-w}{2L^3}[2L^3(L-X1)-2L(L^3-X1^3)+(L^4-X1^4)]$ $\hat{f}_3^1 = \dfrac{-w}{12L^2}[6L^2(L^2-X1^2)-8L(L^3-X1^3)+3(L^4-X1^4)]$ $\hat{f}_5^1 = -w(L-X1)-\hat{f}_2^1$ $\hat{f}_6^1 = \dfrac{+w}{12L^2}[4L(L^3-X1^3)-3(L^4-X1^4)]$
14	$\hat{f}_2^{14} = \hat{f}_5^{14} = \dfrac{-wL}{2}$ $\hat{f}_6^{14} = -\hat{f}_3^{14} = \dfrac{+wL^2}{12}$
2	$\hat{f}_5^2 = \dfrac{-w}{2L^2}[2L^3(L-X2)-2L(L^3-X2^3)+(L^4-X2^4)]$ $\hat{f}_6^2 = \dfrac{w}{12L^2}[6L^2(L^2-X2^2)-8L(L^3-X2^3)+3(L^4-X2^4)]$ $\hat{f}_2^2 = -w(L-X2)-\hat{f}_5^2$ $\hat{f}_3^2 = \dfrac{-w}{12L^2}[4L(L^3-X2^3)-3(L^4-X2^4)]$
3	$\hat{f}_5^3 = \dfrac{-w}{2L^2}[2L^3(L-X3)-2L(L^3-X3^3)+(L^4-X3^4)]$ $\hat{f}_6^3 = \dfrac{+w}{12L^2}[6L^2(L^2-X3^2)-8L(L^3-X3^2)+3(L^4-X3^4)]$ $\hat{f}_2^3 = -w(L-X3)-\hat{f}_5^3$ $\hat{f}_3^3 = \dfrac{-w}{12L^2}[4L(2^3-X3^3)-3(L^4-X3^4)]$

outboard edges of the pallet). The distance from the pallet edge to the load is input by the user. The magnitude of the applied uniform load is found by computing the pressure; the total load divided by the loaded deckboard surface area. The equivalent fixed end forces are found by using the equations shown in Figure 48 on page 145 and Table 5.1.

b. Line loads--A point load can be placed anywhere between the pallet edge and the beginning of member 14. This represents one of the two symmetrically placed line loads on the real pallet. A center line load is applied to joint 9 and to conform to symmetry rules is represented by half the magnitude of the real center line load. The equations used to compute the equivalent joint load vector for line loads of sling supported pallets is shown in Figure 48 on page 145 .

5. Solution: After the load vector and stiffness matrix are computed the joint displacements are found using the same algorithm as previously described.

6. Compute the member stresses from the joint displacements. The equations for the general case are shown in Figure 49 on page 148. The specific equations are in the program listing in Appendix. Note that the total element force is the sum of the fixed end forces and the force caused by joint displacement. The member end stresses are then computed from the end forces. The internal stress is computed for elements which carry a member load, in a manner similar to that shown in figures 41 and 42.
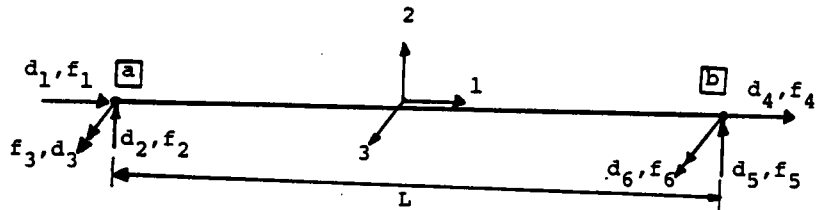
$$f_1 = \gamma(d_1 - d_4)$$

$$f_2 = 12\alpha(d_2 - d_5) + 6L\alpha(d_3 + d_6)$$

$$f_3 = 6L\alpha(d_2 - d_5) + 2L^2\alpha(2d_3 + d_6)$$

$$f_4 = -f_1$$

$$f_5 = -f_2$$

$$f_6 = f_2 L - f_3$$

If member is loaded: $\bar{f} = f + \hat{f}$

Stress:

$$\sigma_a = \left|\frac{\bar{f}_1}{A}\right| + \left|\frac{\bar{f}_3}{S}\right|$$

$$\sigma_b = \left|\frac{\bar{f}_1}{A}\right| + \left|\frac{\bar{f}_6}{S}\right|$$

Where:

$$\gamma = \frac{EA}{L}$$

$$\alpha = \frac{EA}{L^3}$$

$\bar{f}$ = actual force vector,

f = forces caused by displacements,

$\hat{f}$ = fixed end force vector,

A = cross section area,

S = section modulus.

Figure 49. General equations to compute member stresses for sling supported winged pallets.

7. The maximum vertical joint displacement (found in solution step) is
    reported to the user.

5.2 CLOSED FORM SOLUTION FOR SINGLE FACED PALLETS

The steps used to compute the stress and deflection by matrix methods for
RAD pallets have been detailed in the previous section. However, sling
supported pallets with no bottom deck are analyzed using simple beam
theory instead of the matrix method. The justification for the simpler
solution is that this pallet structure behaves as a simply supported beam
rather than a composite beam. The analog model is shown in Figure 50 on
page 150. This section details the steps required to analyze these sling
supported winged pallets.

1. Compute moments: The bending moment is computed at two locations;
    at the sling support, and at the center of the span between the
    stringers. The larger moment is used in the next step to compute the
    load effect.
    The equations used to compute the moments are shown in Figure 51 on
    page 151.

2. Compute the maximum stress: The maximum stress or load effect is
    found by dividing the maximum moment by the section modulus. The
    section modulus is computed using the accumulated width of the top
    deck. The resulting load effect is used in the FOSM equation to de-

a) REAL PALLET
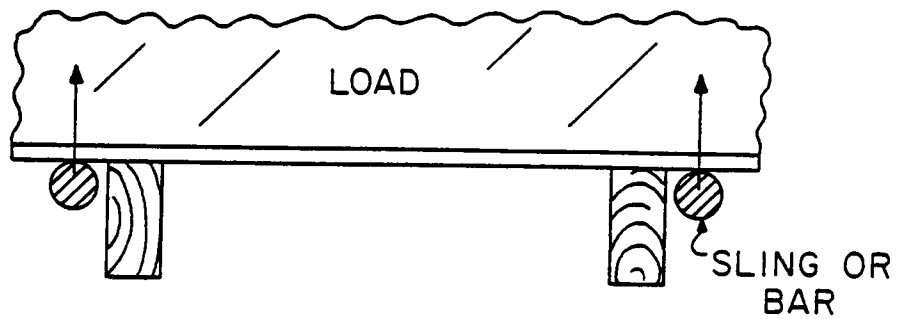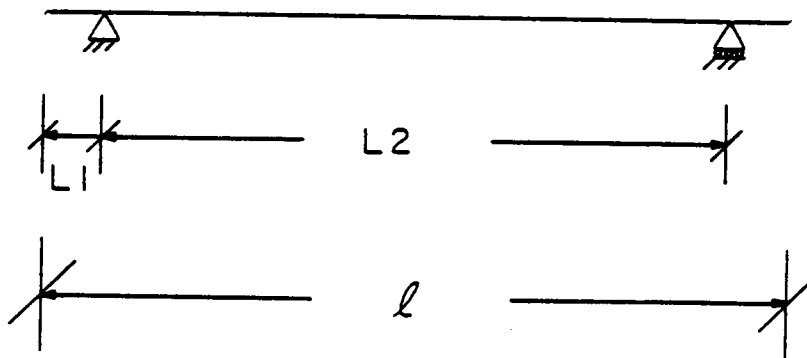
b) ANALOG MODEL

Figure 50. Analog model of single-faced winged pallet.

General Geometry
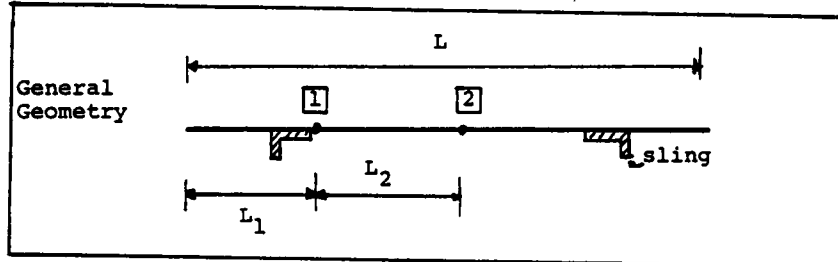
a) Partial and Full Uniform Loads
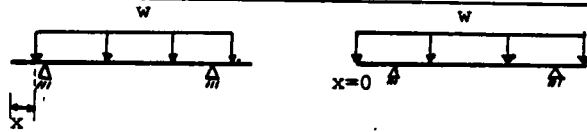
b) Line Loads
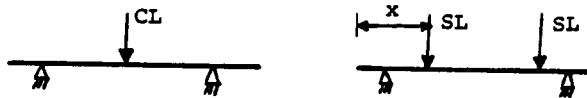
NOTE: Maximum moment occurs at either points ①or ②. Compute each and select the largest to find stress.

Load Type	M1	M2
Full Uniform	$\dfrac{wL1^2}{2}$	$\dfrac{w(\frac{L}{2}-x)^2}{2} + \dfrac{w(\frac{L}{2}-x)L2}{2}$
Partial Uniform	If: x > L1; M1=0 else; $M1=\dfrac{w(L1-x)^2}{2}$	
Line Loads	If: x > L1; M1=0 else; $M1=\dfrac{SL(L1-x)}{1}$	$\dfrac{(SL + \frac{CL}{2}) * L2}{2} - SL(\frac{L}{2}-x)$

$$\text{Stress} = \frac{M_{max}}{S}$$

Where:

M_{max} = greater of M1 or M2,

S = section modulus

Figure 51. Equations to compute member stresses from the joint displacements: for sling supported winged pallets with no bottom decks.

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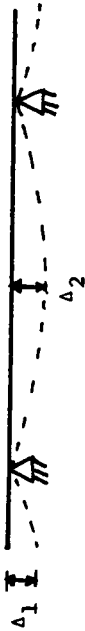
any load type

Load Type	Deflection	
	Δ_1	Δ_2
Full Uniform (x=0) Partial Load	$[8(\frac{wC}{2})(\frac{L2^3}{8} - \frac{L2^3}{2}) + w(\frac{C}{2})(L2^3 - 2BC^2 + C^3 + 2L2C^2) - 2w(\frac{L2}{2} - x + L1)^4]/(48EI)$	$\frac{5wL2^4}{384EI} + \frac{w(L1-x)^2 L2^2}{16EI}$
Center Line Load	$\Delta_2 = \Delta_{CL} = \frac{CL * L2^3}{48EI}$	
2 Side Line Load	$\Delta_2 = \Delta_{SL} = \frac{SL(x-L1)(3L2^2 - 4(x-L1)^2)}{24EI}$	
3 Line Loads	$\Delta_2 = \Delta_{CL} + \Delta_{SL}$	

Where:

$C = L-2X$,
$B = L2-x+4$,
Δ_{SL} = center deflection due to side loads,
Δ_{CL} = center deflection due to center load,

All else defined in Figure 49.

Figure 52. Equations to compute deflection of sling supported winged pallets with no bottom deck.

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termine: 1) if the design is adequate, or, 2) the maximum allowable load based on the corrected-for-safety MOR.

3. Compute the deflection: The maximum deflection at the center of the span is found using the simplified deflection equations shown in Figure 52 on page 152. The equations are based on simple strength of materials and they account for the effect of the load located over the wing and the span between the supports.

5.3 DECKBOARD-STRINGER JOINT CHARACTERIZATION

The spring elements in the RAD model are used to simulate the action of the semi-rigid nail or staple joints under load. (These elements are also used in SPACEPAL.) For design we are interested in the stiffness of the joints in different directions. The deckboard-stringer joint has six possible degrees of freedom or modes of action as shown in Figure 53 on page 154. These stiffnesses are lateral stiffness both parallel and perpendicular to the deckboard grain, in-plane and out-of-plane rotational stiffness, twisting stiffness and withdrawal stiffness (Figure 53 on page 154).

Sensitivity studies using SPACEPAL revealed that "of the six possible components of nail joint displacement only two significantly affect the action of pallets in the RAD mode; lateral slip parallel to the longitudinal axis of the deckboard, and, out of plane rotation of the deckboard relative to the stringer (rotation modulus)" (Mulheren). Accordingly, the spring elements are assigned stiffness values for the lateral slip

1) Lateral parallel
to deckboard

4) In-plane rotation

2) Withdrawal

5) Twisting

3) Lateral perpendicular
to deckboard grain

6) Out of plane rotation
(rotation modulus.)
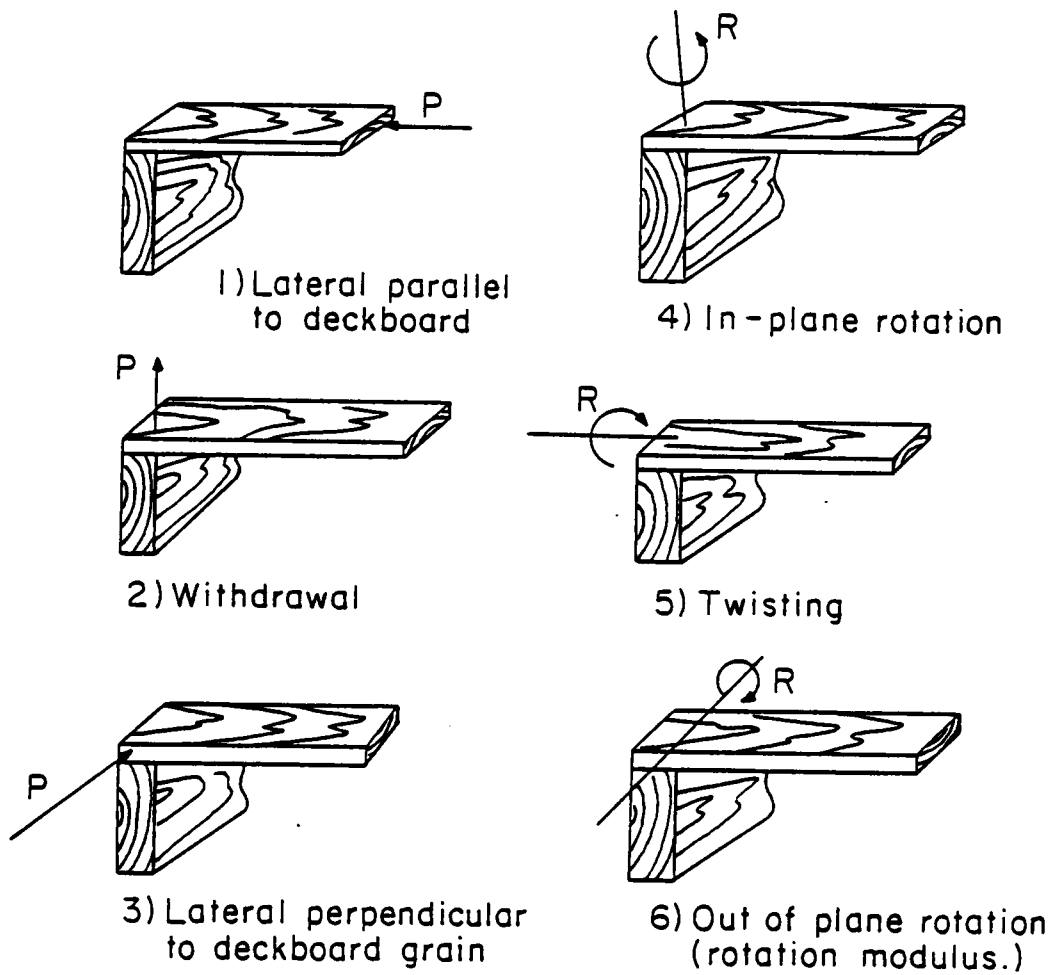
Figure 53.   Six possible stiffness components of joints.

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and the rotation-modulus which reflect those of representative test joints. Because the RAD model is 2-dimensional, only one additional component of spring element displacement is possible; this represents the withdrawal stiffness of the joint. The stiffness in this direction is assumed to be rigid for the purpose of RAD analysis. (Any actual withdrawal tendency is lumped into the rotational stiffness.) This rigid condition is simulated by assigning the same degree of freedom number for the vertical direction in JCODE to both the "a" and the "b" end of the spring element.

The value for the accumulated lateral stiffness of the top or bottom nail joints is found by multiplying the stiffness of a joint containing a single nail by the number of nails in the deck. A deterministic value for the lateral slip of a single nail is used. This value is 30,000 in-pounds and is an estimate of the initial stiffness of laterally loaded joints obtained from limited tests of pallet nails. A sensitivity study showed that a 400% increase in slip stiffness produced only a 2% increase in predicted maximum allowable pallet load thus justifying the use of a deterministic value.

The response of a RAD pallet was found to be more sensitive to the value of the rotation modulus: a 100% increase in the rotation modulus produced a 5% increase in the predicted maximum pallet load and stiffness. Therefore, a regression function was derived to estimate the rotation modulus based on the joint characteristics. It was assumed that rotational stiffness of a nailed joint is influenced by the following; the

Design for Racked across the deckboards support mode (RAD)   155
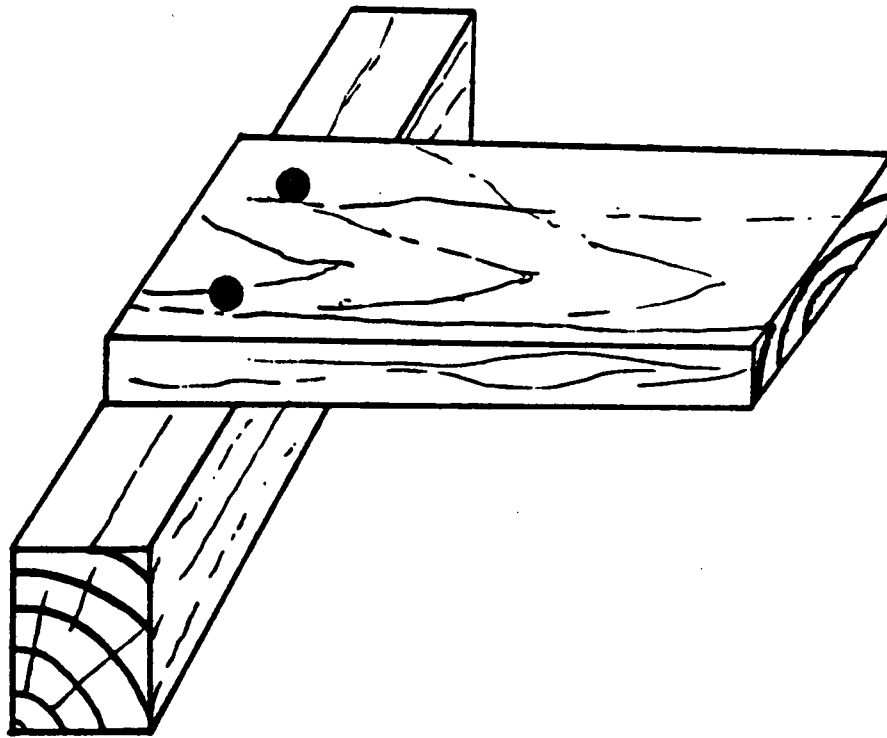
a. Actual System
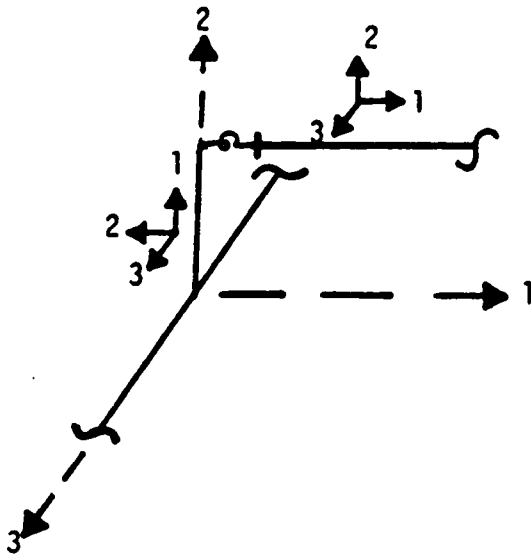
b. Structural Idealization

Figure 54. Analog model of a deckboard-stringer joint.

nail head-pull-through resistance of the deckboard surface, the nail shank withdrawal resistance from the stringer, and the stiffness of the nail itself. (See Figure 54 on page 156). The head pull through resistance was assumed to be a function of specific gravity of the deckboard. The withdrawal resistance was assumed to be a function of the specific gravity of the stringer, the depth of penetration of the nail in the stringer, and nail characteristics such as thread crest and wire diameter, and the thread angle.

Therefore, to predict estimates of the rotation modulus for specific pallet joints, a multivariate regression model was developed using SAS. A description of the variables used in the model follow:

1. Withdrawal strength: Wallin previously derived an empirically based equation to predict the withdrawal strength based upon the characteristics of the joint:

$$FWT = 8.88 * (FQI)(G^{2.25})(P) \tag{5.1}$$

where:

   FWT = Fastener withdrawal load in pounds,

   G = specific gravity of the stringer,

   P = penetration of nail shank into stringer, (inch)

   FQI = fastener quality index

   = 221.24 (WD)[1+(27.15(TD-WD)(H))]

WD = diameter of round wire or equivalent for rectangular or square wire (inches)

TD = thread-crest diameter (inches)

H = number of helices per inch of thread.

2. Specific gravity of the deckboard: Wallin's equation for head-pull-through predicts the load at which a cylinder equal to the thickness of the wood under the head is sheared. However, the parameter needed to predict the rotation modulus should only reflect the load required to indent the nail head into the deckboard surface as opposed to shearing the cylinder under the head. Consequently the specific gravity of the deckboard was used as an additional variable in regression instead of Wallin's head pull through load. It was assumed that the specific gravity was a measure of the hardness of the deckboard surface.

3. Nail stiffness: One additional parameter namely the stiffness of the nail was used as an independent variable in the regression. The nail stiffness can be measured using the MIBANT test--a common test used by both pallet and nail manufacturers (ASTM standard F680, Testing Nails). The test is conducted by dropping a weight from a standard height onto a clamped nail. The resulting angle of the bent nail is a measure of the nails' stiffness.

The rotation modulus was the dependent variable used in the regression model and was computed using equations obtained from Wilkinson (1984) which describe the entire moment-rotation curve. Wilkinson tested joints

constructed with either one, two, or three nails, or a staple.   The
variables he investigated were as follows:

1. nailing patterns--two were used for the one and the three nail joints
    and one pattern was used for two nail joints

2. five combinations of species were used for the deckboards and
    stringers.   These combinations were:

    a.   oak deck, oak stringer

    b.   yellow-poplar deck, yellow-poplar stringer

    c.   Douglas-fir deck, Douglas-fir stringer

    d.   oak deck, yellow-poplar stringer;

    e.   yellow-poplar deck, oak stringer

3. Four fasteners were tested: three types of nails and one type of
    staple:

    a.   2-1/4 inch long by 0.112 inch diameter hardened steel nail

    b.   3 inch long by 0.12 inch diameter hardened steel nail

    c.   2-1/2 inch long by 0.12 inch diameter stiff stock nail

    d.   2-1/2 inch long staple

The details of the fastener characteristics are shown in Table 5.2.

The form of the moment-rotation relationship is curvilinear (Figure 55
on page 160).   Since the RAD analysis assumes joint linearity, the value
of rotational stiffness to be assigned to the springs in the RAD model
were obtained as a secant modulus.   Therefore, the value of rotation at
which to compute the stiffness of the joint must be determined.   Because
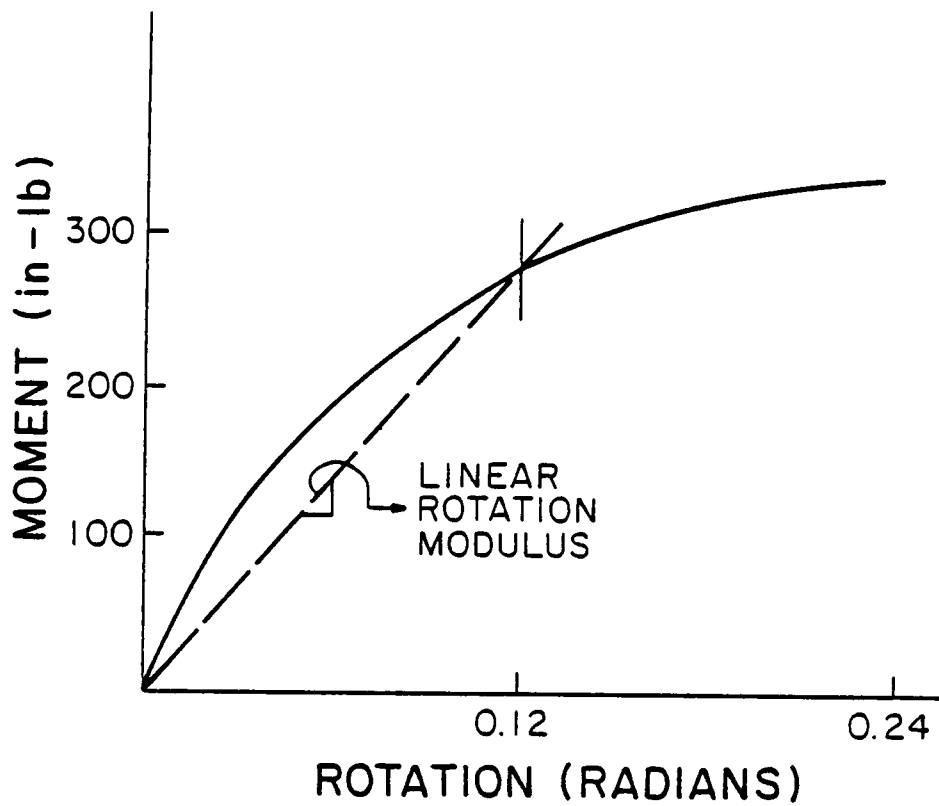
Figure 55.   Typical moment-rotation curve. Estimated
             rotation modulus for rotation of 0.12 radians.

Table 5.2. Physical properties of nails: used to
develop equations to predict rotation
modulus (Wilkinson 1984)

Average Property	3-inch hardened nail	2-1/4 inch hardened nail	2-1/2-inch stiffstock nail
Length, in.	2-7/8	2-1/8	2-7/16
Wire diameter, in.	0.120	0.113	0.122
Thread-crest diameter, in.[1]	0.138	0.127	0.135
Number of flutes[2]	4	4	5
Thread length, in.	2.0	1.50	1.69
No. helix, in.	5	5.33	4.74
MIBANT bend angle, deg.[3]	15	21	48
Head diameter, in.	0.288	0.282	0.259

[1]The diameter measured on the thread crest.

[2]Number of continuous symmetrical depressions along the
nail shank.

[3]A measure of the nail stiffness as obtained following
ASTM standard F680, Testing Nails.

RAD pallets can deflect as much as 5% of the span without exceeding the MOR of the deckboards, the rotation exhibited by the joints of simulated racked pallets, analyzed using SPACEPAL, were used to estimate the value of rotation, at which to evaluate the moment from Wilkinson's equation. This moment divided by the rotation is the rotation modulus. It was determined that a rotation of 0.12 radians was reasonable for this purpose and was used to compute the moment for all test joints included in the regression analysis (Figure 55 on page 160). This rotation corresponds to a deflection of 0.5 inches in a test joint, or a deflection of 1.8 inches for a pallet in a 36 inch span.

Only one nail pattern was selected for use in predicting the rotation modulus, namely, Wilkinson's pattern III, the two nail joint. This was done since it is unrealistic to require the user to input a nail pattern. Pattern III data were selected because most deckboard-stringer joints are assembled with two rather than one or three fasteners. The joint stiffness on a per nail basis was estimated by dividing the rotation modulus of the two nail joint by two. The accumulated rotational stiffness for the deck is estimated by multiplying the rotation modulus of the single-nail joint by the total number of nails in one stringer.

Wilkinson's empirical equation is:

$$M = A \tanh(\phi B)$$

where:

    M = moment (inch-pound)

    ϕ = rotation (radians)

    A and B = parameters shown in Table 5.3 for two nail joints.

Based on the variables described above, a multivariate regression model was developed, using SAS, to predict the rotation modulus. The model selected for use in PDS has 2 independent variables, and a correlation coefficient (r^2) of 0.80, and, a CP statistic of 3.0. The r^2 value indicates that 80% of the variation in the rotation modulus was explained by the independent variables and the low CP statistic indicates that the model had little bias. Better functions may exist, however, since pallet response is not extremely sensitive to the rotation modulus, the following model was used in PDS:

$$RM = -913.34 + 5860 \ (SG) + 4.63 \ (Fwt)$$

where:

    RM = rotation modulus (inch-pound per radian)

    SG = specific gravity of the deckboards (oven-dry basis)

    Fwt = fastener withdrawal strength (defined in equation 5.1).

The MIBANT angle was included in the initial regression model, but it provided only a marginal increase in the correlation coefficient (r^2 =0.801) and a greatly increased CP statistic (CP = 13.25). Therefore, the MIBANT angle was dropped from the final model.

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Table 5.3. Wilkinson's parameters for describing the moment-rotation curve for two-nail joints.

Species		Fastener	A	B	Rotation Modulus (in-lb/radian) Evaluated at 0.12 Radians
Stringers	Boards				
Oak	Oak	2-1/4 HS	563	22.0	4644
		3 HS	595	23.0	4918
		2-1/2 SS	490	25.6	4065
		2-1/2 Staple	463	30.7	3853
Oak	Yellow Poplar	2-1/4 HS	277	27.6	2302
		3 HS	298	29.9	2479
		2-1/2 SS	302	29.6	2512
		2-1/2 Staple	234	44.7	1949
Yellow Poplar	Oak	2-1/4 HS	292	28.6	2428
		3 HS	375	24.3	3106
		2-1/2 SS	329	26.6	2732
		2-1/2 Staple	343	25.4	2845
Yellow Poplar	Yellow Poplar	2-1/4 HS	251	25.1	2081
		3 HS	286	25.0	2371
		2-1/2 SS	235	27.6	1953
		2-1/2 Staple	255	30.9	2122
Douglas-Fir	Douglas-Fir	2-1/4 HS	228	18.2	1852
		3 HS	283	16.4	2267
		2-1/2 SS	256	17.1	2064
		2-1/2 Staple	247	23.3	2043

Where: HS = Hardened steel nail,
SS = Stiff stock nail.

An additional variable was investigated as a possible predictor of the rotation modulus namely the thickness of the deckboards. However, the thickness variable was dropped from the model due to a lack of data and a preliminary investigation that showed the rotation modulus to be into deckboard thickness. One effect of deck thickness is to influence penetration depth which is included in FWT.

5.4 VERIFICATION

The techniques developed to predict the load effects of RAD pallets were evaluated by comparison of predicted and measured pallet stiffness from experimental data collected by Collie (1984). His experimental methods are briefly described in the verification section of Chapter 4.

The pallet stiffness (in the elastic region) is used as the main parameter for verification because RAD pallets constructed of green oak rarely fail in a brittle manner. Instead, tests of such pallets reveal that the testing machine deflection measuring capacity (2 inch maximum) is often exceeded due to excessive pallet deflection. Therefore, the stiffness is used, rather than maximum load, to evaluate the RAD predictions.

The stiffness predictions were generated from PDS by setting the safety index equal to zero, and making three separate analyses of each design using various estimates of the material properties: a)the mean MOR and MOE, b)one standard deviation below the mean MOR and MOE, and c)one standard deviation above the mean MOR and MOE. As described in the RAS

verification section, the resulting predictions represent a region in which the measured pallet stiffness is expected to occur if the model is adequate.

Collie's study provided data on eight pallet designs (five replications of each design). Additional data was obtained from the W.H. Sardo Pallet Laboratory for two pallet designs, representing thirty specimens each. All pallets were tested racked across the deckboards with a uniformly distributed load described previously.

Table 5.4 shows the measured and predicted stiffness values of the test pallets as well as the predicted maximum loads based on deckboard strength. The criteria used for RAS verification were also used for RAD verification. The average absolute error in predicting stiffness using the mean MOR and MOE is 15.5%. The table shows that the upper and lower predicted stiffness bound the measured stiffness for most test pallets. The model does not consistently under- or over-predict the measured stiffness.

There are several reasons for the difference between predicted and measured stiffness. a) The error associated with predicting the exact properties (MOE, and MOR) of a test pallet constructed from highly variable material, as described in the RAS section, also exists for RAD predictions. Also, evaluation of the deckboard MOR contributes to the problem of material property estimation in a manner not exhibited by RAS pallets; Bending tests of individual oak deckboards, in the green condition, show

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Table 5.4. Percent error and actual error in predicted stiffness for RAD tests. (Collie, 1984)

Design No.	Measured Stiffness (lbs/in.)	Percent Error In Predicted Stiffness			Actual Error In Stiffness (lbs/in.)		
		$\bar{X}-\sigma$	$\bar{X}$	$\bar{X}+\sigma$	$\bar{X}-\sigma$	$\bar{X}$	$\bar{X}+\sigma$
1	635	-1.5	-22.0	-39.0	10	-146	-249
2	5390	23.0	7.7	-5.1	1279	420	-279
3	9117	11.4	13.6	-1.0	1048	1245	-92
5	2827	26.7	13.0	-2.7	757	382	-78
6	5090	41.0	31.0	18.8	2128	1592	962
8	4376	35.0	19.5	4.2	1558	854	187
11	3486	28.6	11.0	-4.7	998	396	-166
12	3234	25.0	12.3	-0.5	801	398	-18
VPI#1	6500	--	3.4	--	--	222	--
VPI#2	6400	--	3.5	--	--	223	--
Numerical Average	--	23.6	9.3	-3.7	1072	558.5	-37.9
Absolute Average	--	24.0	13.7	9.5	1072	587.8	249.3

Note: Negative errors indicate over predicting stiffness,
$\bar{X}$ = mean value material properties,
σ = standard deviation of material properties.

that deflections greater than 5 inches are not uncommon. During testing, the limit of the deflection measuring equipment is often exceeded before specimen failure. The load associated with the the maximum deflection is used to compute the "MOR" of a tested, but, unbroken deckboard. The resulting parameter is not a true MOR but rather an artifact of the testing procedure. b) Predicted stiffness is sensitive to another highly variable parameter, namely, the rotation modulus. Errors in the predicted rotation modulus contribute to the overall error of the stiffness prediction. c) Analysis techniques used in PDS do not recognize the nonlinear behavior exhibited by test pallets. Examination of a typical load-deflection plot, obtained from a pallet test, shows linear behavior up to the proportional limit. Beyond the proportional limit the pallet behaves in a nonlinear or plastic manner. Since PDS assumes linear behavior to failure, some inaccuracy in the predicted failure load is expected.

Considering the variability of the important material properties used in pallet construction PDS does an adequate job of predicting RAD pallet stiffness.

6.0 STACKED SUPPORT CONDITION

6.1 GENERAL:

Perhaps the most commonly used support mode for pallets is the stack mode. At the retail level, and in warehouses which have no rack systems, loaded pallets are often stacked in layers for storage. Even those warehouses which have racks utilize the stack mode to ship goods or to temporarily store loaded pallets. The objective of this chapter is to describe the analysis methods developed for computing the load effects of stacked pallets [12]. First the basic assumptions used in the analysis are described. Then the analysis methods used to compute the load effects are discussed. Last, the verification of the analysis method is presented by comparing predicted to experimental response for test pallets.

BASIC ASSUMPTIONS: As in the racked analysis, the analysis of stacked pallets is aimed at computing the load effects, in terms of stress and deflection, of critical members. The critical structural elements in the stacked support mode are the top deckboards of the bottom pallet and the bottom deckboards of the second pallet in the stack. During stacking, some load is transferred to the support directly through the stringers (i.e. the stringers act as columns), as shown in Figure 57 on page 171.

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[12] The techniques and computer code for analyzing stacked pallets were developed by Dr. T. E. McLain.
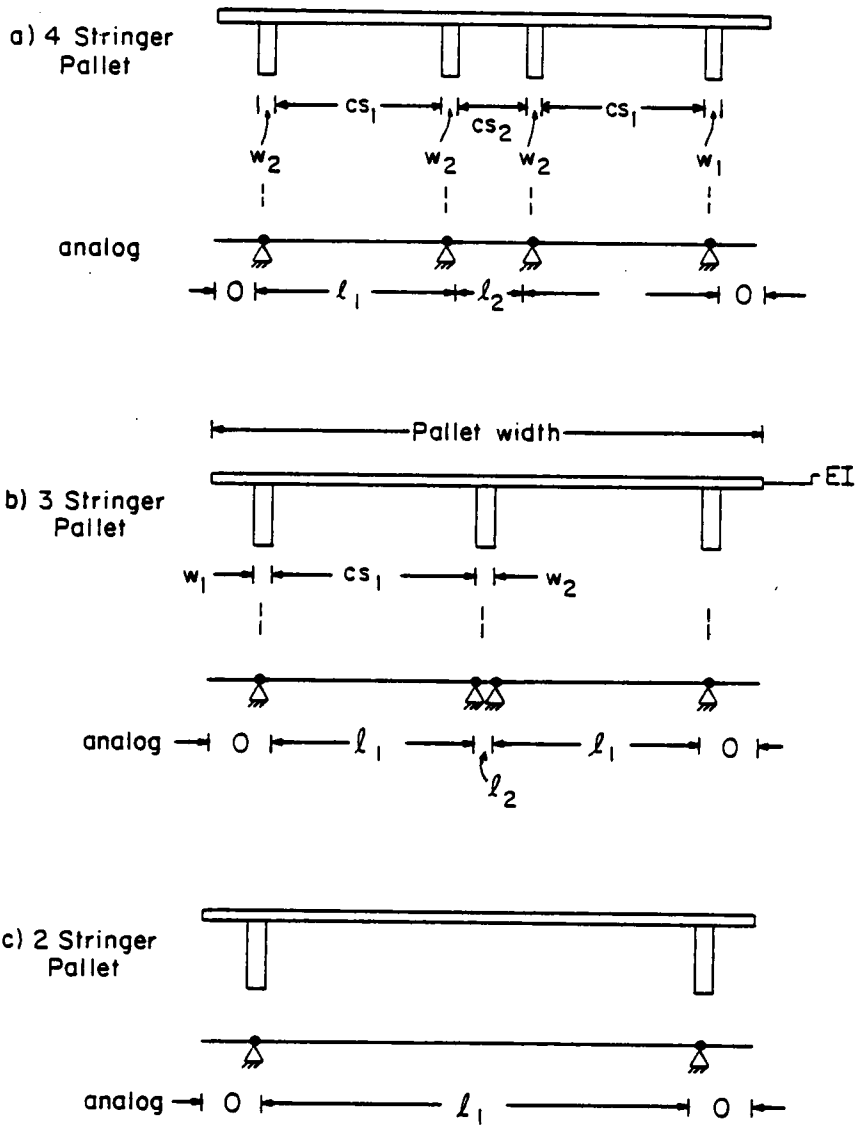
Figure 56. Analog models for rigid support condition.
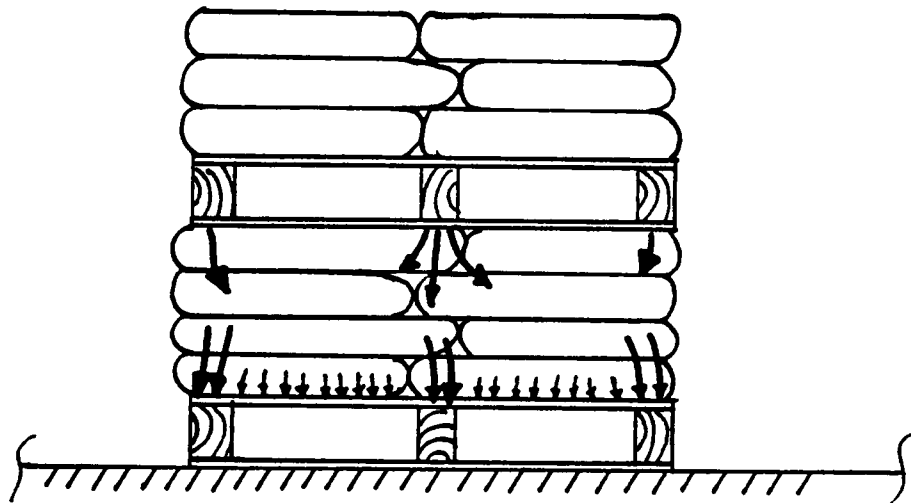
Figure 57 . Schematic diagram showing the assumed load transfer in stacked mode.

The remaining load is carried by the deckboards and is transferred to the stringers as bending reactions. It is assumed that deckboard bending and excessive deflection are the critical failure modes and that the compression perpendicular strength of the stringers is not a significant design criterion.

In the proposed pallet design procedure, stacked pallets are modeled as two-dimensional structures. A top or bottom deck is represented by a continuous beam and the stringers are represented by pinned supports as shown in Figure 56 on page 170. In the model, the width of a deck element is equal to the accumulated width of all boards in a deck. In other words, the entire deck is analyzed as if it were a single wide deckboard. The total length of the deck is equal to the width of the pallet and the lengths of the individual members in the model are computed using the equations shown below. A separate analysis is conducted for the top and bottom decks using the same models and assumptions. For some pallets, the assumption of pinned supports may be overly simple and it's use may result in conservative estimates of the load effects. A support width factor is used to modify the deckboard free span to account for the finite support width (i.e. stringer width) and its influence on the deckboard response. This is discussed in detail elsewhere in this chapter.

Linear elasticity is assumed in the analysis of stacked pallets. Three and four stringer pallets are modeled as 3-span continuous beams with overhangs as shown in Figure 56 on page 170. The figure also shows that two stringer pallets are modeled as a simple beam with overhangs. To

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represent a real pallet, the length of the members in the model are adjusted to equal the length of the corresponding members in the real pallet.

DECKBOARD OVERHANG: A limit for the wing length of an overhanging (or winged) deckboard is imposed in PDS. The wing length is limited to the following ratio:

$$\frac{O}{L} < 0.4 \qquad\qquad (6.1)$$

where:

O = length of deckboard over-hang (inch),

L = length of span adjacent to wing (inch),

This limitation was imposed to restrict the possible locations of maximum moment and deflection to the spans L1 and L2, thus reducing computational complexity. Since this ratio is rarely if ever used in practice this restriction is not thought to be significant.

6.2 ANALYSIS METHODS FOR LOAD EFFECTS

This section discusses the techniques developed to compute the load effects in stacked pallets. First, details of the analog models for stacked pallets are presented.   Next the methods for computing the deckboard

bending stress are discussed and last, the technique used to compute the deckboard deflection is detailed.

6.2.1 ANALOG MODELS:

The analog models of stacked pallets define the geometry, spans, and support locations and are used to compute the load effects. This section describes establishment of the effective geometry for use in the calculation of stress and deflection.

MODEL OF THREE AND FOUR STRINGER PALLETS: The geometric correspondence between real three or four stringer pallets and the analog models is shown in Figure 56 on page 170. For continuity the models must be compatible. For example, the analysis of a four stringer pallet having zero spacing between the center stringers should produce the same computed load effects as a three stringer pallet whose center stringer width is equal to the sum of the widths of the corresponding inner stringers (of the four stringer pallet). To maintain this compatibility three effective spans are used in the load effects calculation for both three and four stringer pallets. (Due to symmetry the outer spans are equal). The spans are based upon the clear spacing between inner and outer stringers (CS1), overall pallet width, overhang length, stringer widths, and support width factors. (Support width factors are discussed in the next section). The effective spans are computed as follows:

$$L1=CS1 + SWF_o(Sw) + SWF_c(Sw) \tag{6.2}$$

and:

$$L2=DL- 2(O) - 2(L1) \tag{6.3}$$

where:

L1=effective outboard span 1 (inch),

L2=effective inboard span 2 (inch),

CS1=clear span between faces of inner and outer stringer (inch),

SWF_o, SWF_c=support width factor for outer and inner supports respectively,

Sw=stringer width (inch),

DL=deckboard length (inch),

O=length of wing from end of deckboard to outside face of outboard stringer (inch).

MODEL OF TWO STRINGER PALLETS: The two stringer pallet and corresponding analog model and its' correspondence to a real two stringer pallet are shown in Figure 56 on page 170. Because of limitations imposed on the overhang length the maximum load effects are assumed to occur in the center of the span (L1). Therefore, only one effective span is considered in the analysis. This span is computed as:

$$L1=CS1+SWF_o(Sw) \tag{6.4}$$

where:

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L1=effective between outboards stringers(inch),

CS1=clear span between faces of stringers (inch),

SWF_o =support width factor

Sw=stringer width (inch),

SUPPORT WIDTH FACTOR: To compute stress and deflection of a beam requires an estimate of the effective span. Traditionally, the centerline-to-centerline (C-C) distance between the supports was used to estimate this effective span, regardless of the actual support width. This span is acceptable if the beam is relatively stiff, and the support is narrow. However, for thin, flexible beams, such as deckboards, the support width significantly influences the effective span, and neglecting this influence results in conservative estimates of deflection and stress (Tissel, 1971), (especially if the beam is continuous over multiple spans as in the stack support mode). To account for this influence, support width factors are used in PDS. These factors reduce the traditional estimate of the effective span (i.e. centerline-to-centerline) by a fraction of the support width as described by the American Plywood Association (Tissel, 1971).

Two support width factors are used in PDS: one for the inboard supports. (i.e. center stringers in three and four stringer pallets), and one for the outboard supports (i.e outer stringers). The center SWF is applied to three and four stringer pallets and is assumed to vary linearly with the ratio of clear deckboard spans as:

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$$SWF_c = 0.1 + 0.5\left(\frac{CS2}{CS1}\right) \tag{6.6}$$

where:

SWF_c = center support width factor,

CS1 = clear span between outer and inner stringer (inch),

CS2 = clear span between inner stringers (inch)

(Note that the SWF is equal to 0.5 for a C-C span.) This relationship requires that CS1 must be greater than CS2. In practice CS1 is generally much greater than CS2. Therefore, this restriction is not felt to be significant.

For simplicity the value of the outer SWF was fixed at 0.33. The actual value was found to vary between 0.25 and 0.5 and was related to the ratio of the overhang length to the clear span.

6.2.2 THE STRENGTH OF STACKED PALLETS

The deckboard bending stress is assumed to govern the maximum load capacity of stacked pallets. Because deckboards are relatively thin, horizontal shear is an unlikely failure mechanism. Because of the geometric restrictions imposed, the maximum moment will occur at one of four possible locations: a) at the inner support, b) at the outer support, c) in the span between the outer and inner stringers, or d) in the center

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span of the pallet (only for four stringer pallets loaded with one or three line loads). Therefore, the moment is computed at each of these locations and the maximum is selected for computing the pallet load capacity.

For versatility the equations developed to compute the load effects of deckboards in stacked pallets are based on the theorem of Three Moments for analysis of continuous (two or more supports), statically indeterminate beams (Laursen 1978). The method was obtained from classical structural analysis. The three-moment-method relies on the fact that the slope of the deflection curve must be continuous over the supports. Therefore, "the moments in the beam at three consecutive support points can be related to the load on the intermediate spans" (Laursen, 1978). The development of the three moment equation results in "a set of simultaneous equations in which the moments at the supports are the unknowns" (Laursen, 1978). From the moments in the beam the deflection at any point can be found. The maximum moment (or stress) and deflection can then be used to compute the maximum pallet load (ANALYSIS option) or alternately the optimum deckboard thickness (DESIGN option). Because symmetry about the center line is assumed only half of the beam needs to be considered and the complexity of the analysis is reduced.

COMPUTATION OF MOMENT AT SUPPORTS: A free body diagram of a uniformly loaded three span continuous beam is shown in Figure 58 on page 179. This figure represents the general case for stacked analysis and corresponds to a four stringer pallet. The analysis of two and three stringer pallets

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a)   Analog Loading
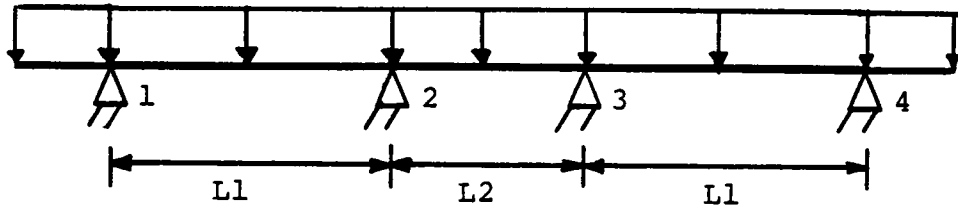
b) Unknowns
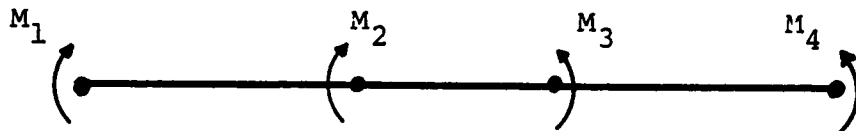
A_1=area

centroid A_2= area

c) Moment Diagrams

$\bar{a}_1$ $\bar{b}_1$ $\bar{a}_2$ $\bar{b}_2$

d)
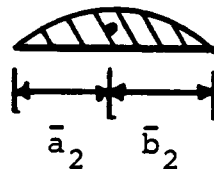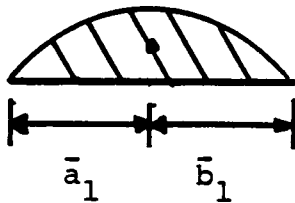
$$M_1 L1 + 2M_2(L1+L2) + M_3 L3 = -\frac{6A_1\bar{a}_1}{L1} - \frac{6A_2\bar{b}_2}{L2}$$

$$M_2 L_2 + 2M_3(L2+L1) + M_4 L1 = -\frac{6A_2\bar{a}_2}{L2} - \frac{6A_1\bar{b}_1}{L1}$$

NOTE: $M_1 = M_4$

$M_2 = M_3$

Figure 58.   Example of three moment method for
three span beam.

Loading

Moment Diagram

a) full uniform load

area = A

$\overline{a}$ $\overline{b}$

b) partial uniform load

area

$\overline{a}$ $\overline{b}$
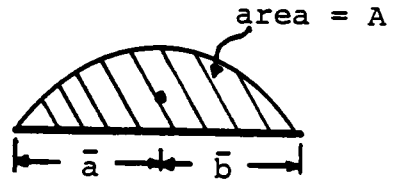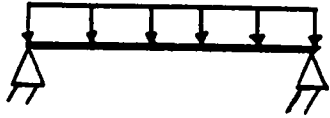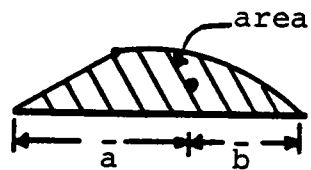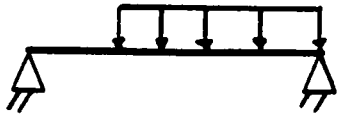
c) off-center line load

centroid

x   P   z

$\overline{a}$ $\overline{b}$

d) center line load

$\overline{a}$ $\overline{b}$
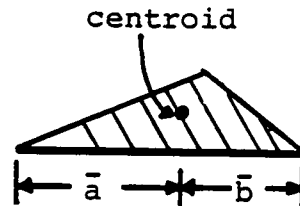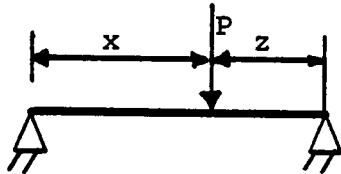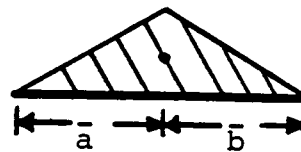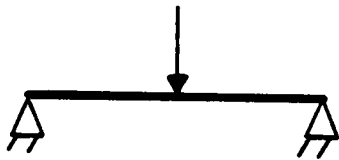
Figure 59. Beam diagrams used to compute moments in stacked pallets.

is similar and differs only in the values assigned to the member lengths (i.e L1, L2). The first step is to construct the moment diagrams for each span as though each section (between supports) was a simply supported beam. The centroid locations and areas under each moment diagram are defined as shown in Figure 59 on page 180. Next, the equations to compute the angle of the beam's rotation on the right and left side of the center support (support 2) are written. Since the beam is continuous these rotation angles must be equal and therefore can be combined into a single equation. The resulting equation is:

$$M_1(L1)+2(M_2)(L1+L2)+M_3(L2)= -\frac{6A_1(\bar{a}_1)}{L1} - \frac{6A_2(\bar{b}_2)}{L2} \quad (6.6)$$

where:

M1,M2,M3=moments at supports 1, 2, 3 respectively,

A1, A2=area under moment diagram for span 1 and 2 respectively,

$\bar{a1}$, $\bar{b2}$=location of centroid of moment diagram (defined in Figure 58 on page 179.),

L1, L2=length of span 1 and 2 respectively.

Equation (6.6) has three unknowns (M1, M2, M3) and therefore must be solved simultaneously with equations developed for each of the other supports. However, because the beam and loading are symmetric, M2 equals M3. Therefore, the number of unknowns in equation (6.6) is reduced to

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two. Further, because the overhang is a cantilever beam and the support is assumed pinned, the moment at support 1 can be computed directly for each load case. (If the pallet lacks an overhang, M1 is assumed equal to zero). Therefore, equation (6.6) has only one unknown term and thus can be solved for the moment at support two. The equations for computing M1 for each load type are shown in Table 6.1. (This table also shows the equations for determining the centroid distance, and the areas under the moment diagram for computing M2). After solving for the value of M1 using Table 6.1, M2 is computed from:

$$M_2 = \frac{\dfrac{6A_1(\bar{a}_1)}{L1} - \dfrac{6A_2(\bar{b}_2)}{L2} - M_1(L1)}{(2L1+3L2)} \tag{6.7}$$

MOMENT BETWEEN SUPPORTS: Because the loads and spans may vary, it is possible to have the maximum moment occur in a span rather than at a support. Therefore the moment in the span (Ms) must also be computed. Since L2 is required to be less than or equal to L1, for most load types the maximum moment in span 1 is greater than that in span 2. (An exception is the case of a four stringer pallet loaded with a center line load.) Therefore, to find the maximum moment in the deckboard span, the moment in the span L1 is computed for all load types except center line loaded four stringer pallets. (Note that three stringer pallets with center line loads are not analyzed because the load is located directly over the center stringer, hence the deckboards are not stressed). For center-line

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Table 6.1. Input for three moment equations for computing load effects in stacked pallets.

Load Case	M_1	A_1	A_2	$\bar{a}_1$	$\bar{b}_2$
Uniform	$\dfrac{w(0)^2}{2}$	$\dfrac{wL1^3}{12}$	$\dfrac{wL2^3}{12}$	$L1/2$	$L2/2$
Partial Uniform					
x > 0	$\emptyset$	$\dfrac{wz^2}{12}(3L1-2z)$	$\dfrac{wL2^3}{12}$	$L1-\dfrac{2L1^2-z^2}{6L1-4z}$	$L2/2$
x < 0	$\dfrac{w(0-x)^2}{2}$	$\dfrac{wL1^3}{12}$	$\dfrac{wL2^3}{12}$	$L1/2$	$L2/2$
Line Loads					
One, <u>Center</u>	$\emptyset$	$\emptyset$	$\dfrac{PL2^2}{8}$	$\emptyset$	$L2/2$
<u>Two:</u> x>0	$\emptyset$	$\dfrac{Pz}{2}(L1-z)$	$\emptyset$	$L1-\dfrac{1}{3}[\dfrac{L1^2-z^2}{L1-z}]$	$\emptyset$
x<0	- - - n o t	- - - a l l o	w e d - - -	- - - - - -	- -
<u>Three:</u> x>0	$\emptyset$	$\dfrac{Pz}{2}(L1-z)$	$\dfrac{CL2^2}{8}$	$L1-\dfrac{1}{3}[\dfrac{L1^2-z^2}{L1-z}]$	$L2/2$
x<0	- - - n o t	- - - a l l o	w e d - - -	- - - - - -	- -

$$z = L1-x$$

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loaded two and four stringer pallets the moment in the center span (Mc) is also computed. The maximum moment in the pallet is found by comparing M1, M2, Ms, and Mc.

The moment in span L1 for uniformly distributed loads is computed using the equations presented in Table 6.2. These equations use beam shear, and reaction to compute maximum moment. (Note that for some pallet geometries loaded with full or partial uniform loads the location of maximum moment is not restricted to the center of span L1. The equations in Table 6.2 account for the location of the maximum moment.)

The moment in span L1 caused by the line-loads is also computed using equations shown in Table 6.2. For the two line load condition the maximum moment occurs under the line load. For the center line load condition in two and four stringer pallets the moment at center-line is also computed. Figure 60 on page 185 shows a flow chart of the general scheme used to analyze stacked pallets.

COMPUTATION OF MAXIMUM STRESS: The maximum stress is computed from:

$$\sigma = \frac{Mmax}{S} \tag{6.8}$$

where:

σ=maximum stress (psi),
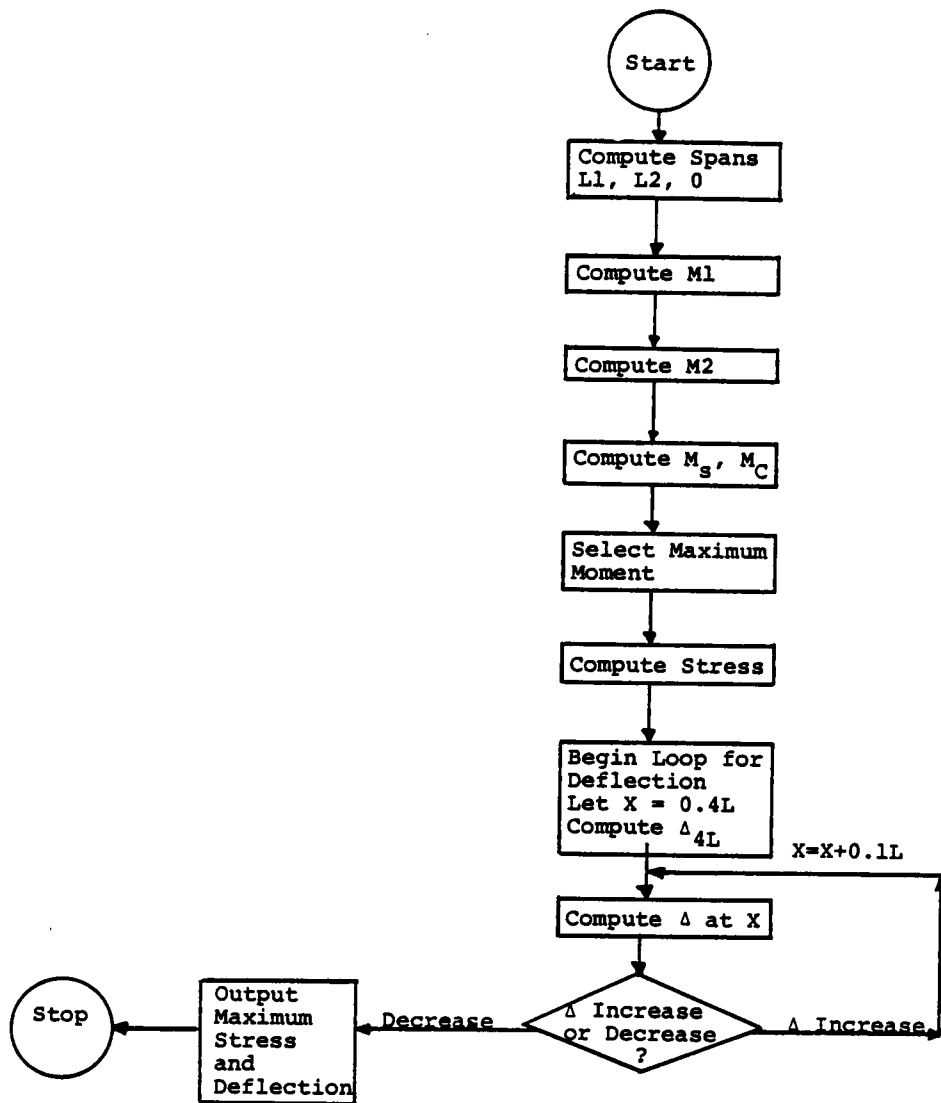
Figure 60. Flow chart of steps to analyze stacked pallets.

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Table 6.2. Equations to compute the moment in the spans for stacked pallets.

Load Case	M_s	V_2	R_1	M_C
Full and Partial Uniform Loads	$M_2 - V_2 x - \dfrac{wx^2}{2}$	$R_1 - w(0) - w(L1-x)$	$\dfrac{M_2 - M_1 + \dfrac{wL1^2}{2}}{L1}$	--
Line Loads: One Center (only two & four stringer pallets)	--	--	--	$M_2 + \dfrac{CL}{2}\left(\dfrac{L^*}{2}\right)$
Two Loads	$M_2 - V_2(L1-x)$	$R_1 - SL$	$\dfrac{M_2 + SL(L1-x)}{L1}$	--
Three Loads	$M_2 - V_2(L1-x)$	$R_1 - SL$	$\dfrac{M_2 + SL(L1-x)}{L1}$	$M_2 + \dfrac{CL}{2}\left(\dfrac{L^*}{2}\right)$

Where:

L^* = L1 if two stringer pallet; L2 if four stringer pallet,

M_s = maximum in span L1,

V_2 = shear at support 2,

R_1 = reaction at support 1,

M_C = maximum moment at center line.

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Mmax=maximum moment in deck (in-lbs),

S= section modulus of deck (in³).

In the DESIGN option the maximum stress is used in the FOSM equation to compute the required mean resistance. The required resistance is then compared to the mean MOR of the deckboards by the criteria described in Chapter 7.

In the ANALYSIS option the allowable mean load effects are computed from the mean MOR using the FOSM equation. The maximum allowable pallet load is then computed from the ratio of the arbitrary input load to the computed stress multiplied by the allowable mean load effect.

6.2.3 DEFLECTION OF STACKED PALLETS

If the decks of a stacked pallet deflect excessively the handling equipment may be unable to enter the pallet. The pallet design procedure provides a method to estimate this deflection. This estimate allows the user to rationally adjust the dimensions of the decks to produce a structure that satisfies the deflection criteria. Due to restrictions in the geometry of the pallet and the load conditions, the maximum deflection is assumed to occur in the span between inner and outer stringers (i.e end span) for full and partial uniform loads, and two line loads. For four stringer pallets loaded with one or three line loads the maximum

deflection may occur in either the end span or the center span, therefore, the deflections at each location are checked.

The maximum deflection can be determined using the principal of superposition, and elementary reference text formulas for basic load and support conditions. For example, the deflection caused by a uniform load is broken into two components: the deflection caused by the load on the span and the deflection caused by the moment applied to the end of the beam (from the adjacent span). The summation of these independent deflections computed at a given point in the span produces the total deflection at that point. Figure 61 on page 189 schematically shows the the principal of superposition.

Since the load and pallet geometry are variable and the resulting equations complex, the exact location of the maximum deflection in the span is not easily known. Therefore, an incremental process is used to compute the deflection at several points along the beam, thus identifying the maximum deflection in the span. The scheme is shown in Figure 60 on page 185). To start the process, the deflection is first computed at a node point located at 0.4L from the support which represents the inner stringer. For this calculation the value of x' in Table 6.3 is equal to 0.4L. The deflection is then computed at the next node point located at 0.5L from the support. The deflection for this node is then compared to the deflection which was computed at the previous node. If the deflection at the current node is less than that of the previous node it is assumed that the maximum deflection for the span occurred at the previous node.

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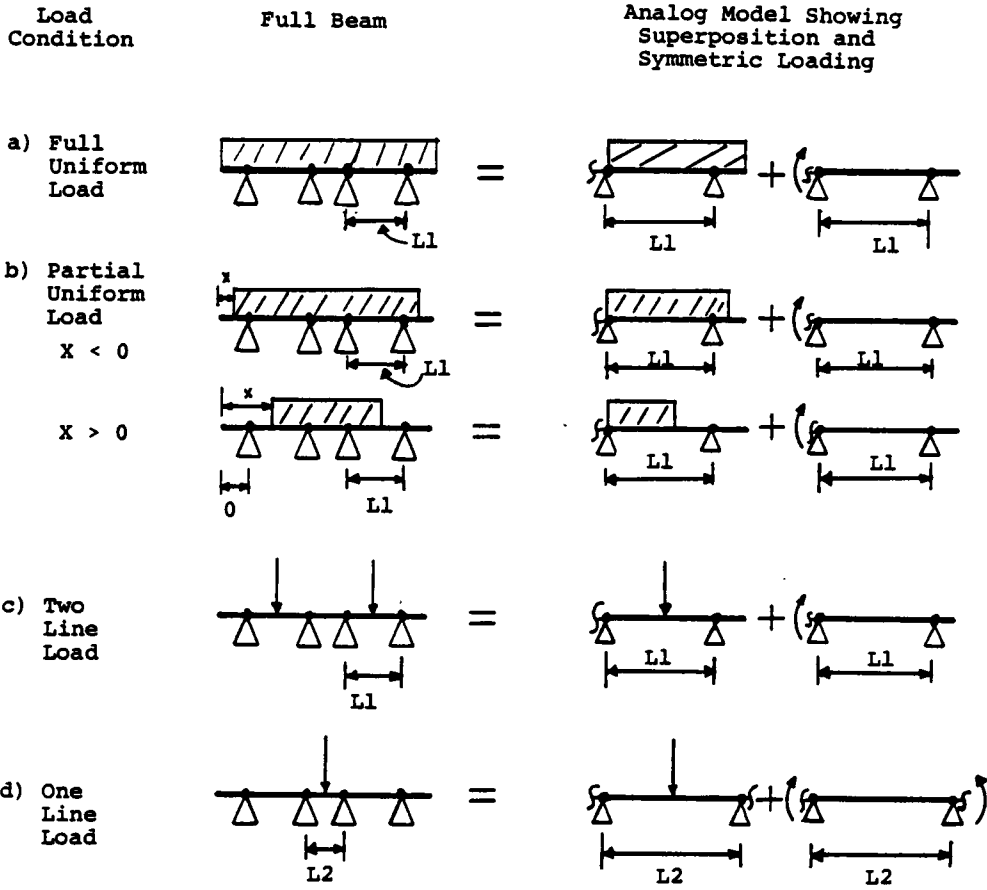
Figure 61. Stacked deflection analog models using symmetry and superposition.

Table 6.3. Equations to compute deflection in decks of stacked pallets.

Load Type	Y_A	Y_B
Full Uniform a = 0 Partial Uniform Load, When: x < 0 (a = 0-x)	$\dfrac{-wX}{2EILl}[Ll^4 - 2Ll^2X^2 + LlX^3 - 2aLl^2 + 2a^2X^2]$	$\dfrac{M_2}{6EI}\left[3X^2 - \dfrac{X^3}{Ll} - 2LlX\right]$
Partial Uniform Load A) x < z	$\dfrac{-wX}{24EILl}(z^2(2Ll-z)^2 - 2zX^2(2Ll-z) + LlX^3)$	
B) x > z	$\dfrac{-wz^2(Ll-X)}{24EILl}(4XLl - 2X^2 - z^2)$	
Two and Three Line Loads A) x < z	$\dfrac{-SLbX}{6EILl}[2Ll(Ll-X) - b^2 - (Ll-X)^2]$	
B) x > z	$\dfrac{-SLz(Ll-X)}{6EILl}[2Llb - b^2 - (Ll-X)^2]$	
One and Three Line Loads	$\dfrac{-CL(Ll)^3}{48EI}$	$\dfrac{M_2X}{2EI}[X-Ll]$

If the deflection at the current node is greater than that at the previous node the variable x' is again incremented by 0.1L. The deflection at the third nodal point is computed and compared to that of the 2nd nodal point. The process is continued until the deflection for the current node is less than the deflection of the previous node: The deflection of the previous nodel point is assumed to be maximum deflection for the span.

For the ANALYSIS option the computed deflection is used in a ratio to determine the deflection at the maximum load:

$$D_{max} = \frac{D_{in}}{P_{in}} (Pmax) \qquad\qquad (6.9)$$

where:

Dmax   = deflection at maximum load Pmax,

Pmax = maximum load capacity,

Din   = deflection computed for arbitrary input load Pin,

Pin = arbitrary input load for analysis option.

For the design option, the computed deflection in reported to the user.

6.3 UNIFORM-LOAD DISTRIBUTION ADJUSTMENTS

The methods described for computing the load effects for stacked pallets assume that the load is transferred primarily through the deckboards to the supports and that the uniformly distributed load is flexible. However, tests of stacked pallets (conducted by Collie (1984)) indicate that the load transfer mechanism is more complex. The experimental results show that some of the total load is actually transferred directly through the stringers to the floor rather than through the deckboards. The exact mechanism of load transfer is unknown and variable, but was found to be related to the number of pallets in the stack and the stiffness of the load.

Collie (1984) suggested that load distribution factors be used to compensate for this effect. These factors may be used if the load is stiff and covers the entire surface of the pallet, such as rigid boxes or bagged goods. The load distribution adjustment factors for pallets in stacks of 1, 2, or 3 or more, are 1.0, 0.8, and 0.65 respectively. Because these factors are not applied in all cases they are not built into the computerized Pallet Design Procedure. Instead, the user must decide if the load distribution factor is necessary and justified for his particular pallet use conditions. The factor is applied as described below.

For the ANALYSIS option the estimated allowable pallet load and deflection are divided by the load distribution factor. Therefore the maximum es-

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timated pallet load capacity is increased compared to the unadjusted estimate.

For the DESIGN option the input load is multiplied by the adjustment factor, thus reducing the design load. (Care must taken when applying this option because in PDS the reduced load would also be applied to the racked modes thus producing erroneous output for those modes.)

6.4 EXPERIMENTAL VERIFICATION

Collie (1984) tested five pallet designs in the rigid (stack) support mode with five replications of each design. The pallets were tested in the testing machine developed by Fagan (1983) and the load was applied by an air-bag at a constant deflection rate corresponding to approximately 0.1 inches per minute. The pallets were placed on a support constructed from four sheets of 3/4" plywood glued face to face. The stiffness of this support was assumed sufficient to provide a rigid foundation similar to a floor. The plywood support was located over four BLH load cells thus allowing accurate measurement of the total applied load. The deckboard deflection was measured with dial gauges located in the center of the spans between stringers. These measurements were taken from the second deckboard from the ends of the pallet. Loads and deflections were recorded at 1000 pound intervals. Testing was stopped at failure or when the machine capacity (6 psi air-bag pressure, or 2 inches of deflection) was reached.

Table 6.4. Percent error and actual error in predicted pallet stiffness for the rigid support tests. (Collie, 1984).

Design No.	Percent Error in Stiffness			Actual Stiffness (lbs/in.)	Actual Error in Stiffness (lbs/in.)		
	$\bar{X}-1s$	$\bar{X}$	$\bar{X}+1s$		$\bar{X}-1s$	$\bar{X}$	$\bar{X}+1s$
1	46.3	34.8	17.3	6012	2689	2026	1006
2	-9.0	-24.2	-41.4	48554	-3529	-9461	-16148
3	25.7	-4.0	-15.6	805912	49451	-7692	-29914
4	6.7	-2.4	-23.5	54851	2857	-1045	-10084
12	-8.7	-15.4	-32.5	150759	-5217	-9231	-28889
Numerical Average	12.2	-2.2	-19.1	--	9250	-5081	-16806
Absolute Average	19.3	16.2	26.1	--	12749	5891	17208

Note: Negative errors indicate over predicting stiffness. Percent error is based on the actual stiffness determined during testing.

Where:

$\bar{X}$ = mean properties (MOR & MOE),

s = standard deviation of properties (MOR & MOE).

The verification of the stacked pallet design procedure was based on comparisons of actual stiffness determined from the tests to those values predicted by PDS. (Because none of the test pallets failed verification was limited to stiffness comparisons.) Due to the inherent variability of pallet material, techniques similar to those described in the verification section of Chapters 4 and 5, were used to verify the analysis techniques developed for the stacked mode. Three separate analyses of each design were made using various estimates of the material properties. These estimates were based on the mean MOR and MOE, one standard deviation above the mean MOR and MOE, and one standard deviation below the mean MOR and MOE.

Table 6.4 shows the percent error in predicting stiffness, and the actual error for each of the designs tested by Collie (1984). The Table shows that the mean error of stiffness predictions based on the mean material properties was 16.2% and the actual error was 5891 lbs/in.

(There are several ^reasons why these error$ occurred.) ? First, the exact material properties were not measured for each deckboard in the pallet. Since the deflection measurements were conducted on individual boards the actual measured stiffness is very sensitive to the properties of those boards. Also, estimates of the dimensions of the pallet parts were used as input to PDS rather than the exact dimensions. These estimates were obtained from a limited subsample of the pallet shook used to construct the test pallets. Differences are likely to exist "between the actual properties of the shook used in the test pallets and that destructively tested."

(Collie, 1984). These differences are extremely important when comparing the response of individual members to the predicted response of the pallet.

Another reason for the differences in predicted and actual stiffness may be due to the assumed joint characteristics. The deckboard-stringer joints in the stack mode are assumed pinned. However, these joints are really semi-rigid connections and therefore transfer some moment into the supports. The analysis procedure does not account for this response. Another possible explanation for the differences between predicted and actual stiffness is that PDS assumes linear response. However, load deflection plots of the deckboards in the test pallets show initial nonlinearity. Presumably, this nonlinear behavior is due to settlement of the deckboard and pallet on the supports.

Based on these observations and considering the variability of material properties of pallet shook, it appears that the proposed design procedure adequately predicts the stiffness of pallets in the stacked mode.

7.0 THE MATERIAL RESISTANCE

Previous chapters described the techniques developed to analyze loaded pallets and determine load effects in the form of member stresses. The other necessary input to the design process is an estimate of the member resistance to the applied stress. Estimation of this resistance, a highly variable property, can come from several sources, such as physical testing, nondestructive evaluation procedures, prediction using parameter correlation, or lumber industry design specifications.

The materials used for pallet construction are variable. For example, some manufacturers construct pallets only from a single species groupings, such as oaks, or pines, while other manufacturers utilize a wide variety of locally available species. Also, the restrictions regarding the allowable mix of lumber quality used to construct pallets varies with customer requirements. Consequently, the material property estimation technique must include a rational method for deriving design values for pallet shook of any species and grade combinations. This task is made more formidable by a lack of information on hardwood properties and the high variability of wood properties in general.

This chapter describes the techniques which form the basis for estimating the material properties that are used in PDS [13].

SPECIES CLASSES: Pallets may be made of almost any species or combination of species found in North America. To simplify matters, major species that are used in pallet construction were segregated into species classes. Species within a class are assumed to have similar strength, stiffness, and specific gravity values. The grouping of some species was also based on marketing practices and regional availability, for user convenience.

Eight classes were defined for hardwood species and four classes were defined for softwood species as shown in Table 7.1 and Table 7.2. The clear wood MOR and MOE assigned to a class are volume-weighted averages based upon clear wood properties and standing timber volume estimates for each species contained in the class. The clear wood properties and timber volume estimates for most species were obtained from ASTM D2555 (1984).

The weighted average technique produces average property estimates that reflect the probability of obtaining material of a given species in a random sample of shook from a given species class. It is the best available information for average properties of the species in a class. However, it may not represent the properties of material in a specific geographic location or manufacturing facility.

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[13] The material property estimation techniques were developed by McLeod (1985). For more specific details see his thesis.

Table 7.1. Hardwood species class for use in PDS.

PDS HARDWOOD SPECIES CLASSES

→ INCREASING STRENGTH ←

1	2	3	4	5	6	7	8
HICKORIES YELLOW BIRCH SWEET BIRCH SUGAR MAPLE BLACK MAPLE RED MAPLE GREEN ASH WHITE ASH BEECH ROCK ELM SLIPPERY ELM BLACK LOCUST BLACK CHERRY EASTERN OAKS DOGWOOD PERSIMMON TANOAK EUCALYPTUS	BIGLEAF MAPLE OREGON ASH	SWEET GUM BLACK TUPELO WATER TUPELO CUCUMBERTREE SOUTHERN MAGNOLIA PAPER BIRCH	OREGON WHITE OAK CA. BLACK OAK CASCARA CHINKAPIN MYRTLE MADRONE	BLACK ASH PUMPKIN ASH HACKBERRY SYCAMORE SILVER MAPLE STRIPED MAPLE BOX ELDER SASSAFRAS SUGARBERRY	RED ALDER	YELLOW-POPLAR EASTERN COTTONWOOD BIGTOOTH ASPEN QUAKING ASPEN CATALPA BUCKEYE BUTTERNUT	BLACK COTTONWOOD BALSAM POPLAR

21	29
VPI EASTERN OAK FILE	VPI YELLOW-POPLAR FILE

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Table 7.2. Softwood species class for use in PDS.

PDS SOFTWOOD SPECIES CLASSES

INCREASING STRENGTH

11	12	13	14
DOUGLAS-FIR	WESTERN HEMLOCK	WHITE SPRUCE	ALASKA YELLOW CEDAR
WESTERN LARCH	MOUNTAIN HEMLOCK	BLACK SPRUCE	INCENSE-CEDAR
LOBLOLLY PINE	CALIFORNIA RED FIR	RED SPRUCE	PORT-ORFORD-CEDAR
LONGLEAF PINE	GRAND FIR	ENGELMANN SPRUCE	ATLANTIC WHITE CEDAR
SHORTLEAF PINE	NOBLE FIR	SITKA SPRUCE	NORTHERN WHITE CEDAR
SLASH PINE	PACIFIC SILVER FIR	SUGAR PINE	EASTERN RED CEDAR
	WHITE FIR	WESTERN WHITE PINE	
		PONDEROSA PINE	
		MONTEREY PINE	
		JACK PINE	
		RED PINE	
		EASTERN WHITE PINE	
		PITCH PINE	
		POND PINE	
		SPRUCE PINE	
		VIRGINIA PINE	
		SUBALPINE FIR	
		BALSAM FIR	
		BALDCYPRESS	
		EASTERN HEMLOCK	
		WESTERN RED CEDAR	

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GRADES: Several schemes for classifying pallet material into grades, based on visual criteria such as knot size or slope of grain, have been produced by the pallet industry. Such grades restrict the lower and upper level of shook quality, and therefore exhibit narrower property distrib- (i.e. MOE and MOR) than ungraded shook. Specifying a minimum allowable grade for use in pallet construction improves communication between a manufacturer and customer. Additionally, a lower quality-threshold is useful (but not essential) for establishing design values (McLeod 1985). Although no grading specification is universally accepted in the United States, sufficient similarities exist between several historic schemes to allow comparison from a common perspective. McLeod provides such a comparison and concludes that "there is reasonable uniformity in the criteria for segregating shook on a visual basis".

For simplicity, McLeod used the grading scheme presented by Sardo and Wallin as a benchmark to compare data sets of pallet shook graded by other schemes. This scheme has four single grades (2-and-better, 3, 4, and cull) and three composite grades (3-and-better, 4-and-better, and all shook). Techniques to develop design values for these single and composite grades are described elsewhere in this chapter.

MATERIAL PROPERTIES: For design purposes two material properties are of primary interest; the modulus of rupture (MOR) and the modulus of elasticity (MOE). The MOR is the computed extreme-fiber stress that causes failure in the material in bending. The MOE is an indication of the stiffness of the material in the elastic range.

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Because PDS utilizes the FOSM method to provide safety and reliability in the resulting pallet designs, the mean and the coefficient of variation (standard deviation divided by the mean) are needed. To obtain estimates of these parameters two techniques were used: In-grade testing, and modification of clear wood properties. These techniques are discussed in the following sections.

7.1 IN-GRADE TESTING

In-grade testing is the most accurate method for obtaining the mean and standard deviation of the properties of full-size pallet shook at any point in time. However, the technique involves a large commitment of time, effort, and money since a large sample of pallet material must be collected from representative locations throughout the principle growth range of the species under investigation. The collection scheme may be based on random sampling of the material contained in a pallet manufacturers inventory, or on serial sampling of pieces that would be used together on a pallet.

After collection, the material is visually evaluated using a scheme such as that outlined above. The grades are based on the maximum allowable defects such as knots or slope of grain. The material is then tested to failure in bending, and the MOR and MOE for each piece is determined and tallied. The statistics (mean and standard deviation, or other distribution parameters) of the population of grade or grade mixes are then

computed. This results in the best available information on the strength and stiffness of pallet shook of that particular population.

To date, only a limited number of species have been extensively tested in this manner: eastern oaks and yellow-poplar. These data sets are available to the PDS user by selection of species classes 21 and 29. Other pallet species have also been tested and the results have been reported in literature. However, the results are difficult to apply directly in PDS because of the limited sampling plans involved.

The data from the in-grade testing of eastern oak were used in the simulations for the development of equations to predict load sharing in RAS pallets (Chapter 4). The same data set was also used in the simulations to calibrate the safety index, Beta (Chapter 8).

Additional details concerning the in-grade testing of pallet material can be found in the theses of H. Spurlock (1982) and J. Holland (1980).

7.2 MODIFICATIONS OF CLEAR-WOOD VALUES

For those many species that have not been tested as shook, another method of estimating properties was used. This approach is similar to the traditional method of establishing design values for structural lumber, with some modifications specific to pallet shook and requirements of PDS. The traditional technique is described in ASTM D2555 (1984), "Standard Methods for Establishing Clear Wood Strength Value", and ASTM D245, "Standard

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Methods for Establishing Structural Grades and Related Allowable Proper-
ties for Visually Graded Lumber."

✳ The ASTM standard methods begin by determining the strength and stiffness
of small clear specimens at a green moisture content. The clear wood
strength values are based upon the lower 5th percentile of the MOR dis-
tribution, while the MOE is based upon the mean value of the distribution.
To establish design values for lumber, the clear property values are
modified by a series of adjustment factors to account for strength re-
ducing characteristics that are present in full-size pieces, moisture
content, size effect, etc. An important factor called the strength ratio
is defined as the ratio of the strength of a piece containing defects to
the strength of a similar piece containing no defects (i.e clear wood).
Traditionally, the minimum strength ratio for a group (or grade) of lum-
ber, categorized by a grading scheme based upon threshold defect size,
is specified for safety and conservatism.

Some modifications to the ASTM method were required for use in PDS. The
mean value of MOR and MOE are required for the FOSM method as well as an
estimate of the standard deviation of both distributions. Therefore,
strength ratios based on the average, rather than minimum, strength ratio
for a grade were applied to pallet shook. Traditionally, grade mixes are
assigned design values equal to the values for the lowest grade in the
group. In other words, the traditional methods intentionally do not
equitably account for the presence of higher quality material in a grade
mix. This results in conservative design values that may be unacceptably

low for use in pallet design. Therefore, a weighted average technique was developed for use in assigning pallet shook design values to grade mixes.

7.2.1 ADJUSTMENT FACTORS TO CLEAR-WOOD PROPERTIES

To translate the properties associated with clear wood into properties that represent full-sized pallet shook, adjustment factors are applied to the clear wood values. These factors account for the grade or quality of the shook, shear, depth, and load duration effects. Each adjustment factor is discussed in the following sections.

GRADE FACTORS AND QUALITY FACTORS: Grade factors and quality factors are applied to the clear wood values to account for the mixture of quality within a population of shook.   The grade factor modifies the clear MOR and the quality factor modifies the clear wood MOE. The grade factor is based on the ASTM bending strength ratio and the quality factor is experimentally determined from the oak data set collected by Spurlock (1982). Quality and grade factors were developed for single and composite grades.   The derivation of the these factors follow a parallel course, as described in detail in McLeod (1985), and briefly in the following section:

1.   Determine the adjustment factors for each single grade: Two factors are required: one for MOR and one for MOE.

a. The factor that modifies the clear wood MOR for an individual grade is called the grade factor, and is equal to the average ASTM bending strength ratio for a grade. The average strength ratio is the arithmetic mean of the strength ratios for the largest and smallest defects that are allowed in the grade. Each of the four grades is assigned a grade factor.

b. Quality factors are used to modify the clearwood MOE and were determined experimentally from the oak data set. Specifically, the quality factor is found by determining the average reduction in MOE for full-sized material in each grade as compared to the MOE for grade 2-and-better. Each single grade is assigned a quality factor as follows:

$$QFS = \frac{GMOE}{MOE_{2\&B}} \qquad (7.1)$$

where:

QFS=quality factor of single grade

GMOE = MOE of single grade 2-and-better, 3, 4, or cull

$MOE_{2\&B}$ =MOE of grade 2-and-better

2. Determine the adjustment factors for composite grades: A composite grade is used to designate only a minimum level of shook quality that is permissible within a pallet constructed from a mixture of shook grades. (The maximum level may include defect free shook). Three composite grades are commonly used by the industry to market pallets and shook:

• Grade 3 and Better

• Grade 4 and Better

* all shook

As the names imply, material of higher or better quality than the limiting grade is also included within a composite-grade population. Therefore, to rationally assign a grade or quality factor to a composite grade, the contribution of higher quality material to the property distribution (i.e. mean and standard deviation) of the group must be recognized. In PDS, the composite-grade factor is found by weighting the single-grade factors by the expected percentages of the individual grades that are contained in the composite grade. All known existing data sets for graded pallet shook were used to determine the expected percentages of material in each of the four single grades. The species represented were: eastern oak, yellow-poplar, ash, maple, cottonwood, red alder, hemlock, Douglas-fir, and southern pine. The resulting percentages are shown in Table 7.3. The weighted average for an individual grade in a group is found by dividing the expected percentage of the total population of the individual grade, by the sum of all the expected individual grade percentages contained in the group. For example, the weighted average percentage of grade 3 that is contained in a composite grade of "grade 3 and better" is computed as follows:

$$\% \text{ Grade } 3 = 31/(48 + 31) \qquad\qquad (7.2)$$

where:

48 = percent of grade 2-and-better in the total population,

(48 + 31) = sum of individual percentages for all grades that make up "Grade 3 and better" (i.e. Grade 2-and-better + Grade 3)

Other weighted percentages are computed in a similar manner. The factor for the composite grade is then computed by:

$$F_{comp} = [F1(\%1)] + F2[\%2] + F3[\%3] + F4[\%4] \qquad (7.3)$$

where:

F_{comp} = grade or quality factor for the composite grade

F1,F2,F3,F4 = Factor for individual grades 2-and-better, 3, 4, and cull respectively,

%1,%2,%3,%4 = weighted decimal equivalent percentage of shook in the composite grade for grades 2-and better, 3, 4, and cull respectively.

The grade or quality factor for a group mix is used in the same manner as the factors for single grades.

The grade and quality factors for all single and group grades are shown in Table 7.4.

DEPTH: The clearwood MOR is obtained from tests of small clear specimens whose depth is equal to 2 inches. Research has shown that the strength of wood (MOR) decreases as the member size increases. This effect is attributed to the increased probability of strength reducing flaws in larger volumes of material. To reflect this increased probability, ASTM D245 dictates that the MOR must be corrected for sizes larger than the standard specimens. Therefore, a depth correction factor (Fd) is applied only to stringers and is equal to 0.94. Deckboard MOR is not corrected

Table 7.3. Hardwood and Southern Pine Pallet Shook Grades

PDS Input	NWPCA Hardwood Pallet Standards[1]	NWPCA Southern Pine Pallet Specifications[2]	Grades From PEP Study Report[3]	Grade Mix[4]
1	Precision & Better	SP-1 & Better	2 & Better	100, 0, 0, 0
2	Premium/AA & Better	SP-2 & Better	3 & Better	58, 42, 0, 0
3	A & Better	SP-3 & Better	4 & Better	51, 36, 13, 0
4	All Lumber	All Lumber	All Lumber	46, 33, 12, 9
5	Only Premium/AA	Only SP-2	Only 3	0, 100, 0, 0
6	Only A	Only SP-3	Only 4	0, 0, 100, 0
7	Cull	Cull	Cull	0, 0, 0, 100

[1] NWPCA Logo-Mark Hardwood Pallet Standards, March 1, 1982.

[2] NWPCA Specifications for Warehouse, Permanent and Returnable Pallets of Southern Pine, NWPCA, 1962.

[3] Sardo, W. H. and W. B. Wallin, ed. "The Performance of Wooden Pallets in Exchange Programs," 1974.

[4] This is the grade mix of 2 & Better, 3, 4, and Cull lumber used to develop the properties for these classifications. If your grade mix is significantly different from this then use the procedure shown in the example in Appendix C to create your own mix.

Table 7.4. Grade and quality factors for single and group grades. (McLeod, 1985)

| Grade | Grade Factor F | | Quality Factor |
	Deckboards	Stringers	
2 & Better	0.81	0.81	1.00
3	0.57	0.56	0.90
4	0.47	0.38	0.85
Cull	0.20	0.13	0.80
3 & Better	0.71	0.70	0.96
4 & Better	0.67	0.66	0.94
All Shook	0.63	0.61	0.93

for depth because the available data indicates that depth influence in thin wide boards is different as compared to deep narrow beams (Mcleod 1985). In other words, the influence of common defects is less severe in thin, wide planks than in deeper beams.

SHEAR: "The small clear MOE values given in ASTM D 2555 are unadjusted for the effect of shear deflection during the testing procedure " (McLeod 1985). Therefore, to obtain the true MOE values these "apparent" values must be adjusted to correct for the effect of shear. An adjustment factor (Fs) of 1.10 is applied to the small clear MOE values for each species class. (For details see McLeod 1985).

LOAD DURATION: The strength of wood is affected by time and the clear MOR values are adjusted to reflect the difference in strength between a short term test, and the expected accumulated load duration of the full size members. For the case of pallets, which are commonly stored for short durations in racks, a two month cumulative load duration is assumed. Therefore, as per ASTM D245, a load duration factor (Fld) of 0.72 is used to modify the clear MOR of the deckboards and stringers.

7.3 DESIGN PROPERTIES

The average estimated MOR of full size shook is computed by applying the factors as follows:

$$\overline{MOR} = (\overline{MOR}_c)(GF)(Fld)(Fd) \qquad (7.4)$$

where:

$\overline{MOR}$ = mean MOR of full size shook (psi),

$\overline{MOR}_c$ = mean clear wood MOR for either stringers or deckboards,

GF = grade factor,

Fld = load duration factor,

Fd = depth factor.

The average MOE is computed as follows:

$$\overline{MOE} = \overline{MOE}_c (QF)(Fs) \qquad (7.5)$$

where:

$\overline{MOE}$ = mean MOE of full size shook (psi),

$\overline{MOE}_c$ = clear wood mean MOE for either deckboards or stringers,

QF = quality factor,

Fs = shear factor.

(Note that property and factor values for either deckboards or stringers are used.)

The Material Resistance

7.4 ESTIMATING THE VARIANCE OF THE PROPERTIES

The FOSM method requires estimates of the coefficient of variation (standard deviation divided by mean) for the MOR and MOE of the shook. The mean values of the parameters were computed in the previous step using modified ASTM standards. Unfortunately, the same standards do not dictate a procedure that can be used to compute estimates of the variance of the distribution of either the MOR or MOE. (Note the variance is a measure of the dispersion or "spread" of the data about the mean of the distribution. The square root of the variance is defined as the standard deviation and is used to compute the coefficient of variation.)

Since there is no standard method to estimate the property variance of any pallet species, the best available data set, namely the eastern oak data, is used as a reference. It was assumed that the property distributions of other species are similar to the property distributions for the oak data set. Consequently, until better data are available, the average variance of the oak data set is extrapolated to any grade or species in PDS. The resulting values for the coefficient of variation for MOR and MOE are 25% and 28% respectively.

8.0 RELIABILITY BASED DESIGN

This chapter presents details of the reliability-based procedures used to insure an adequate level of safety in wood pallet designs. For safety, the resistance must be greater than the load effects by an amount sufficient to maintain an acceptable probability of failure. However, if the probability of failure is too low the resulting design may be uneconomical. Alternately, if probability of failure is too high the design may be unsafe. Therefore, to produce acceptable designs, the techniques used to compare the load effects to the resistance must balance safety and economy.

ASSUMPTIONS: The exact formulation for comparing lognormall distributed variables was selected for use in the PDS system. At the core of this method is the definition of a safety index as follows:

$$\beta = \frac{\ln\left[\frac{\bar{R}}{\bar{S}}\right]\sqrt{(1+V_S^2)/(1+V_R^2)}}{\sqrt{\ln\{(1+V_S^2)(1+V_R^2)\}}} \qquad (8.1)$$

where:

$\bar{R}$ = mean resistance,

$\bar{S}$ = mean load effects (same units as R),

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β=safety index,

V_S =coefficient of variation of S,

V_R =coefficient of variation of R,

The derivation of the safety index is shown in Chapter 2. The basic equation (8.1) can be rearranged to compute either the required mean resistance as in the DESIGN option, or, the allowable mean load effects as in the ANALYSIS option. For either option two limits states are considered: 1) ultimate and 2) serviceability.

The ultimate limit state defines the load carrying capacity of the critical members. Failure in this limit state indicates that a member has broken and can not carry load. For design purposes this condition is defined as failure of the entire pallet. However, in a real pallet the integrity of the remaining structural members may be sufficient to carry the load. For example, in a four stringer pallet if one of the inner stringers fails the remaining three stringers may have sufficient strength to carry the load. Similar analogies can be made for individual deckboards that have failed in the RAD or Stack modes. Therefore, the assumption that failure of the first member causes total structural failure may lead to conservative estimates of the load capacity for some structures. However, for other structures the initial member failure frequently initiates complete pallet failure, or load instability. Consequently, the proposed pallet design procedure assumes that failure in a critical member is the principle design criteria, regardless of the remaining structural configuration.

The serviceability limit is related to the intended use of the pallet. For example, if the decks of a stacked pallet deflect excessively a fork truck may be unable to insert its tines into the remaining opening without inflicting damage to the load or the pallet. For design purposes, this condition is considered failure in the serviceability limit state. Likewise, in the racked modes, excessive deflection may cause load instability, or may cause the pallet to become inaccessible to automatic pallet handling equipment.

LOAD VARIABILITY: The variability of the load distribution influences the probability of failure of a structure. If the load distribution is narrow (i.e. low COV) the probability of an extreme load, of sufficient magnitude to cause structural failure, is lower than if the load distribution is wide (i.e. high COV). Consequently, equation (8.1) requires the COV of the load distribution.

To allow the user increased flexibility and simplicity in the application of PDS, three discrete levels of load variability are provided for general use: low, medium, and high variability. (An additional feature was included in PDS to allow users to input any load COV. However, this feature is not available to casual users). The levels were chosen to reflect the expected load variability in various warehouse situations. The low-load variability warehouse primarily caters to a single type of load or product and has a relatively tight distribution with an assumed COV of 10%. The medium variability warehouse has a larger amount of load variation and its assumed COV is 25%. The high variability warehouse carries a wide

range of loads and products (for example, a grocery warehouse) and the assumed COV is 45%.

The COV corresponding to the user selected load variability level is used in equation (8.1), for either the ANALYSIS or DESIGN option. The resulting load effect or the required mean MOR reflects the level of load variability. For example, the mean allowable load will be higher for a design situation involving a low load variability than for a high load variability.

The use of the three load variability levels greatly reduces the users' required understanding of statistical distributions and probability based design procedures.

MEAN VS. MAXIMUM LOADS: The output from this method is the mean load capacity, or mean allowable load effects. For some applications the means cannot be used directly by the designer. For example in a warehouse where many types of loads are stored on pallets only a rough estimate of the mean load can be obtained by the pallet designer. Under such circumstances an estimate of the maximum allowable pallet load would be desirable since it is a fairly simple matter to measure the maximum pallet load in a large warehouse. Additionally, use of the mean allowable load can result in misinterpretation of the analysis results by the PDS user, therefore, an option was included in PDS to allow the user to select the type of load which is output by the system, either mean or maximum.

The following equation was used to compute the maximum allowable load from the mean allowable load:

$$P_{max} = P_{mean} (1+1.28(V_S)) \qquad\qquad (8.2)$$

where:

Pmax = maximum allowable load,

Pmean = mean allowable load,

Vs = COV of load effects.

The equation represents the distance between the mean and the 90 percentile value on the load distribution curve. In other words, the maximum load lies 1.28 standard deviations above the mean. This equation assumes that the loads are approximately normally distributed.

Although the estimated maximum load is easier for causal PDS users to understand than the estimated mean load, a curious phenomenon occurs when comparing maximum to mean loads for the various load variabilities. Comparison of PDS output shows that the estimated mean loads increase as the COV of the load distribution decreases. In other words, the probability of failure decreases with decreased load variability. However, the estimated maximum load increases as the load distribution COV increases. A first glance this seems contrary to the logical assumption of increased probability of failure with increased load variability. However, this phenomenon occurs because the maximum load is assumed to
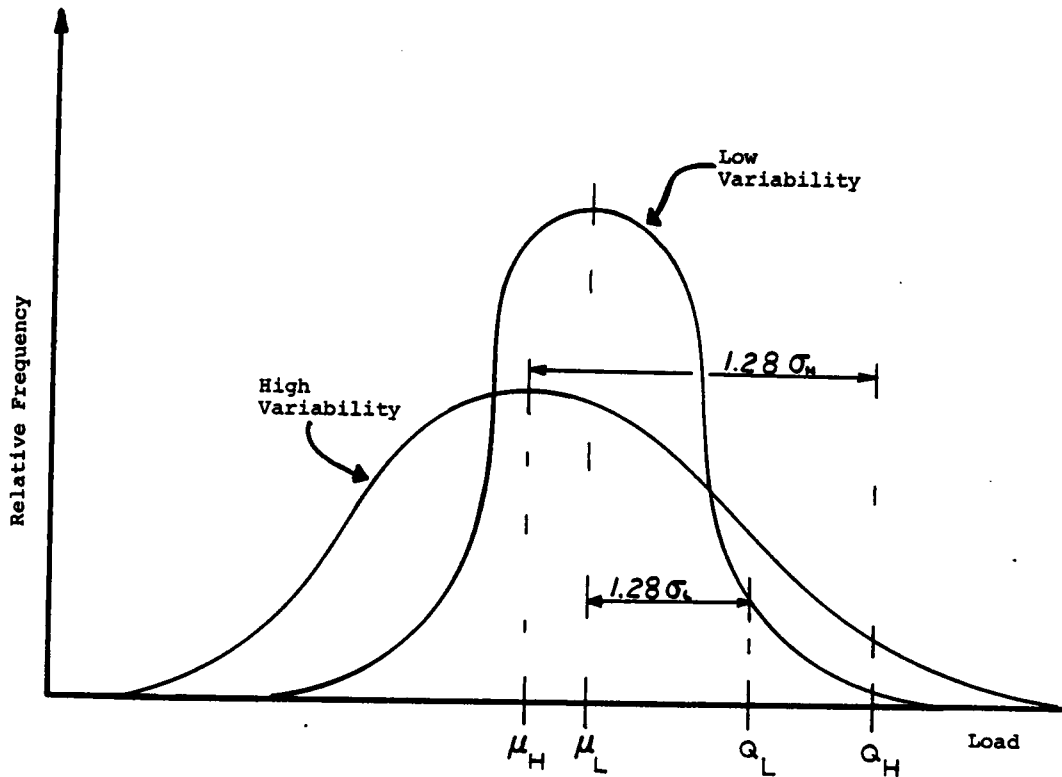
NOTE: $\mu_H < \mu_L$

$Q_H > Q_L$

Where:

μ_H, μ_L = mean loads for High and Low Variability, respectively,

Q_H, Q_L = "maximum" loads for High and Low Variability, respectively,

σ_H, σ_L = standard deviation for High and Low Variability, respectively.

Figure 62. Load variability for mean versus maximum loads.

S BAR VERSUS VS

$\beta = 1.5$

$\beta = 2.0$

$\beta = 2.5$

VR= .25   K= 1.28   R= 5000

V S

S BAR (PSI)

4000.

3000.

2000.

0.00   .05   .10   .15   .20   .25   .30   .35   .40   .45   .5
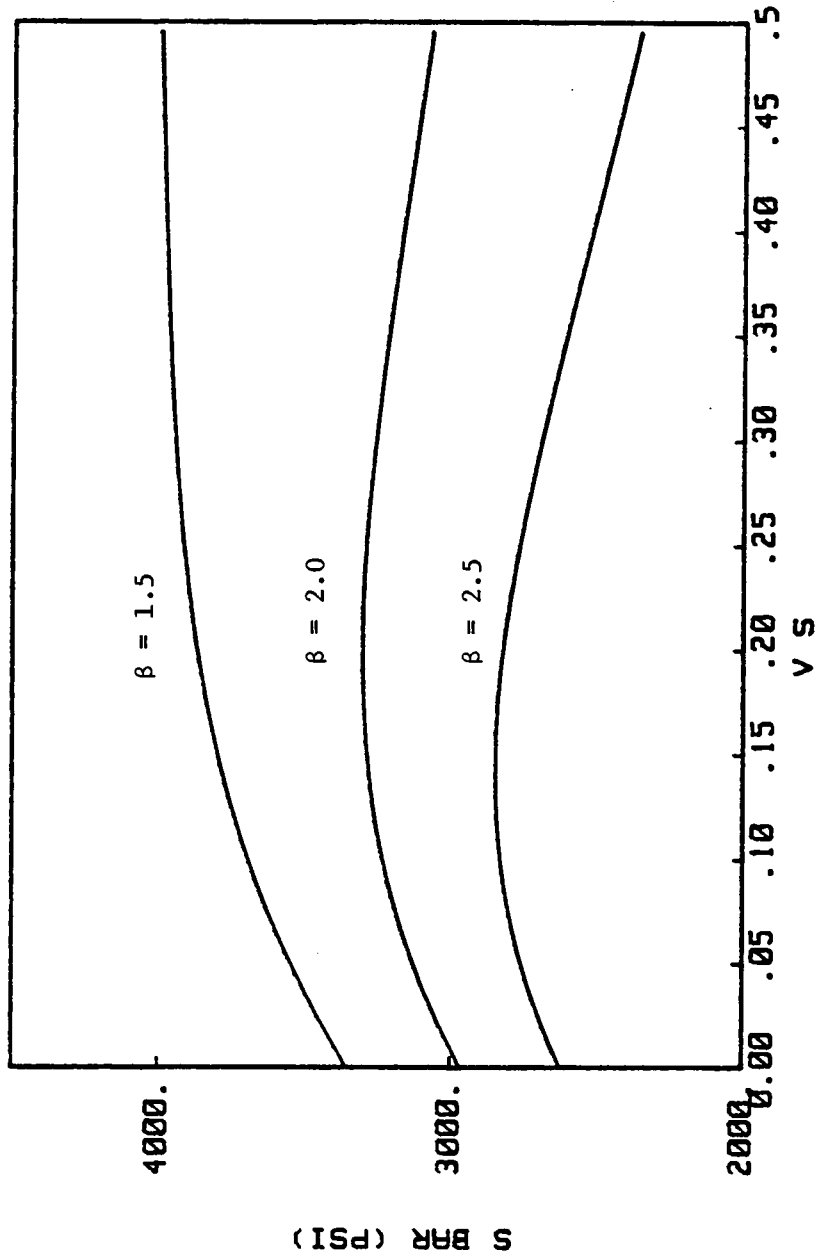
Figure 63. Estimated maximum load effect vs. load COV and Beta.
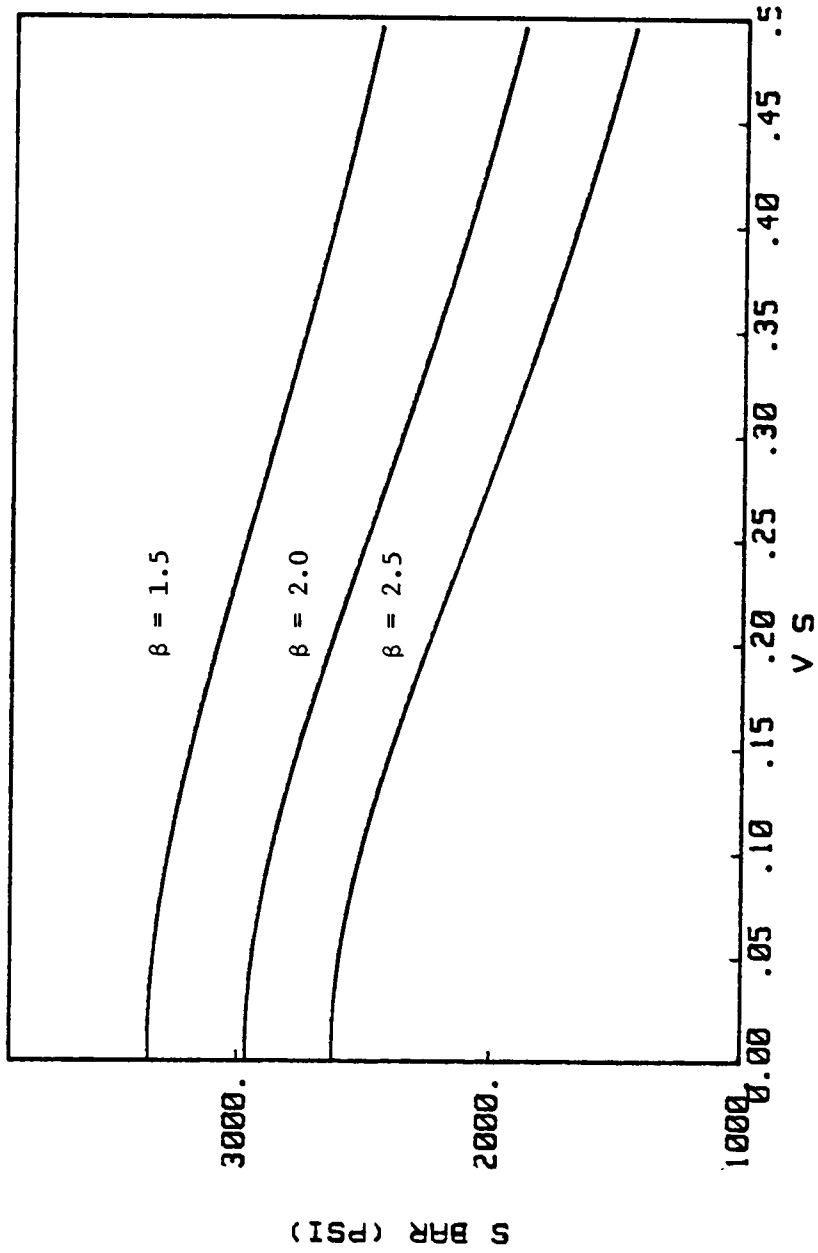
Figure 64. Estimated mean load effect vs. load COV and Beta.

occur at a fixed number of standard deviations above the mean, regardless of the load variability level. Consequently for high load variability, although the mean estimated load is lower than that of the low load variability distribution, the point located 1.28 standard deviations above the mean is actually greater for the high variability distribution than for the low variability distribution. This concept is shown graphically in Figure 62 on page 219.

The problem of establishing the maximum load is compounded when comparing estimates of the allowable load effects computed at different Beta values. Figure 63 on page 220 and Figure 64 on page 221 show this effect for estimated maximum and mean load effects respectively. (The figures represent the solution of equation (8.8) evaluated at three Beta levels. The mean resistance and COV were held constant at 5000 psi and 25% respectively). Figure 63 on page 220 shows that the maximum allowable load effect tends to increase with COV at low Beta values (i.e. Beta=1.5). However, for higher Beta values the maximum allowable load effect decreases with decreased COV. By contrast, Figure 64 on page 221 shows that the estimated mean load effect decreases with increased COV regardless of the level of Beta.

8.1 SAFETY IN THE DESIGN OPTION

ULTIMATE LIMIT STATE: The ultimate limit state of a pallet is exceeded when a critical element breaks or ruptures. The critical elements are the stringers (RAS) or the deckboards (RAD, or stacked).: The basic

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equation for the ultimate limit state in the DESIGN option is found by rearranging equation (8.1) and solving for R. (remember that in the DESIGN option the applied load is known and we wish to check a specific given geometry.) The resulting equation is:

$$\bar{R} = \frac{\bar{S}\left[\exp\left\{(B_b)\sqrt{\ln(1+V_S^2)+\ln(1+V_R^2)}\right\}\right]}{\sqrt{(1+V_S^2)/(1+V_R^2)}} \tag{8.2}$$

where:

B_b =safety index for ultimate limit state

$\bar{S}$ =mean load effects (psi),

$\bar{R}$ =mean resistance (MOR), (psi),

Vs, Vr=Coefficient of variation of load effects and resistance respectively.

To determine design adequacy the mean stress or load effect, S, is first computed from the known pallet load and geometry as described in Chapters 4, 5, and 6. Equation (8.2) is then solved for the required mean resistance, R. The role of the safety index in this equation is to provide the required separation between the mean resistance and the mean load effect, thus insuring safety and maintaining economy in terms of minimum required member dimensions. The required mean resistance can then be compared directly to the mean MOR of the material, either the stringers

or the deckboards.   The decision rule for acceptance or failure of a
particular design is:

Failure: $\bar{R} > \overline{MOR} + 10$ (8.3)

Accept: $\bar{R} < \overline{MOR} + 10$

Note that a 10 psi tolerance level is included in the MOR.   This was done
to reduce conservatism in the PDS system because the values for MOR may
only be accurate to 10 psi, and to reduce the influence of numerical
round-off resulting the methods used in the analysis.

The Pallet Design System also computes an estimated thickness of the
critical element for design optimization, based on the ratio of the re-
quired mean load effects to the MOR.   The optimum thickness is computed
as follows:

$$Tnew = (Tin) \frac{\bar{R}}{MOR} \qquad\qquad\qquad (8.4)$$

where:

    Tnew = estimated optimum thickness,

    Tin = initial thickness input by the user.

Use of the optimum member thickness will result in a structure whose elements are fully stressed to the maximum safe level.

SERVICEABILITY LIMIT STATE: The serviceability limit for pallets is exceeded if the user-defined deflection limit (i.e the mean resistance analogous to the MOR in the ultimate limit state) is less than the mean deflection (corrected for safety) caused by the design load. The mean deflection is found by rearranging the equation (8.1) as follows:

$$\bar{\Delta}_{req} = \frac{\Delta \left[\exp\{(B)(\sqrt{\ln(1+V_s^2)}}\right]}{\sqrt{(1+V_s^2)}} \qquad (8.5)$$

where:

$\bar{\Delta}_{req}$ = mean deflection

Δ = computed deflection at design load,

B = safety index for deflection,

V_s = COV of load effects.

In this formulation, the mean resistance is assumed to equal the mean deflection of a population of pallets subjected to the design load distribution. The mean load effect is the deflection of a pallet loaded with the mean design load (i.e. the deflection associated with the user defined load).

Note that the serviceability limit state equation is essentially the same as that for the ultimate limit state except that the COV of the resistance is assumed to be equal to zero. This assumption was used because the deflection limit is a deterministic value input by the user. The computed deflection at the design load is used as the mean load effect for computing the mean deflection. The decision rule for accepting or rejecting a particular design is:

$$\text{Accept if: } \Delta_{limit} > \bar{\Delta}_{req} \qquad (8.6)$$

$$\text{Reject if: } \Delta_{limit} < \bar{\Delta}_{req}$$

where:

Δ_{Limit} = deflection limit input by user

$\bar{\Delta}_{req}$ = required mean deflection found from equation (8.5).

PDS also computes an estimate of the member thickness required to optimize the design to satisfy a deflection limit. The required thickness to meet a deflection limit is found as follows:

$$T_{req} = T_{in} \left[\frac{\bar{\Delta}_{req}}{\Delta_{Limit}} \right]^{1/3} \qquad (8.7)$$

where:

T_{req} = required thickness of the element,

T_{in} = initial thickness input by the user.

Note that the serviceability and ultimate limit states produce different estimates of the required member thickness. This is because two fundamentally different phenomenon are being investigated. The ultimate limit state is based on the strength of the members, while the serviceability limit state is based upon the stiffness of the members.

8.2 SAFETY IN THE ANALYSIS OPTION

The ANALYSIS option produces an estimate of the maximum safe pallet load. As described in Chapters 4, 5, and 6, the maximum load is computed by transforming the mean allowable load effect (i.e. stress or deflection) into a load. This section describes how safety is provided in this transformation for the ultimate and serviceability limit states.

ULTIMATE LIMIT STATE: The mean allowable load effect is found by rearranging equation (8.1). The resulting equation is:

$$\bar{S} = \frac{\bar{R} \sqrt{\dfrac{1+v_S^2}{1+v_R^2}}}{\exp[\beta_B \sqrt{\ln(1+v_S^2)+Ln(1+v_R^2)}\,]} \qquad (8.8)$$

where: All terms are defined in equation (8.1).

After computing the mean allowable load effect, S, the mean safe design load can be computed by:

$$\bar{P}_{max} = \frac{P_{in}}{\sigma_{in}}(\bar{S})$$

(8.9)

where:

$\bar{P}_{max}$ =mean allowable pallet load (lbs),

P_{in} =arbitrary input load (lbs),

σ_{in}=stress computed at the input load (psi),

$\bar{S}$= mean allowable load effects (psi).

Equation (8.9) compares the ratio of the input load and stress to that of the unknown maximum mean load and the mean allowable load effects.

SERVICEABILITY LIMIT STATE: The ANALYSIS option will also produce an estimate of the maximum load which can be applied to a pallet based on a user defined deflection limit. The mean load effect is computed from the following equation:

$$\bar{\Delta} = \frac{\Delta_{Limit}\sqrt{\dfrac{1+v_S^2}{1}}}{\exp\left[(\beta)\ \sqrt{\ln\ (1+v_S^2)}\right]}$$

(8.10)

where: all terms are defined in equation (8.5).

The load required to cause the mean deflection is computed from:

$$\bar{P}_{max} = \frac{P_{\Delta_{max}}(\bar{S})}{\Delta} \qquad (8.11)$$

where:

$\bar{P}max$ = mean maximum load computed for ultimate limit state,

Δ = deflection at the maximum load, Pmax ,

$\bar{S}$ = mean load effect for the deflection limit,

$P_{\Delta_{max}}$ = maximum load for the deflection limit.

The safety implied by the use of the serviceability limit state is shown graphically in Figure 65 on page 230. The figure shows that the mean of the deflection distribution for the critical member is separated from the deflection limit. The magnitude of the separation is defined by the safety index.

8.3 CALIBRATION:

To produce an acceptable level of safety, the safety index, beta, must be assigned a proper value. A process called calibration was used to determine Beta. As mentioned previously, the safety index defines the distance, in the number of standard deviations, between the origin and the mean of the combined distribution of R and S. Specifically, the combined distribution is defined as the resistance divided by the load effect. The underlying concept of calibration is to analyze existing pallet designs which are known to exhibit satisfactory strength and

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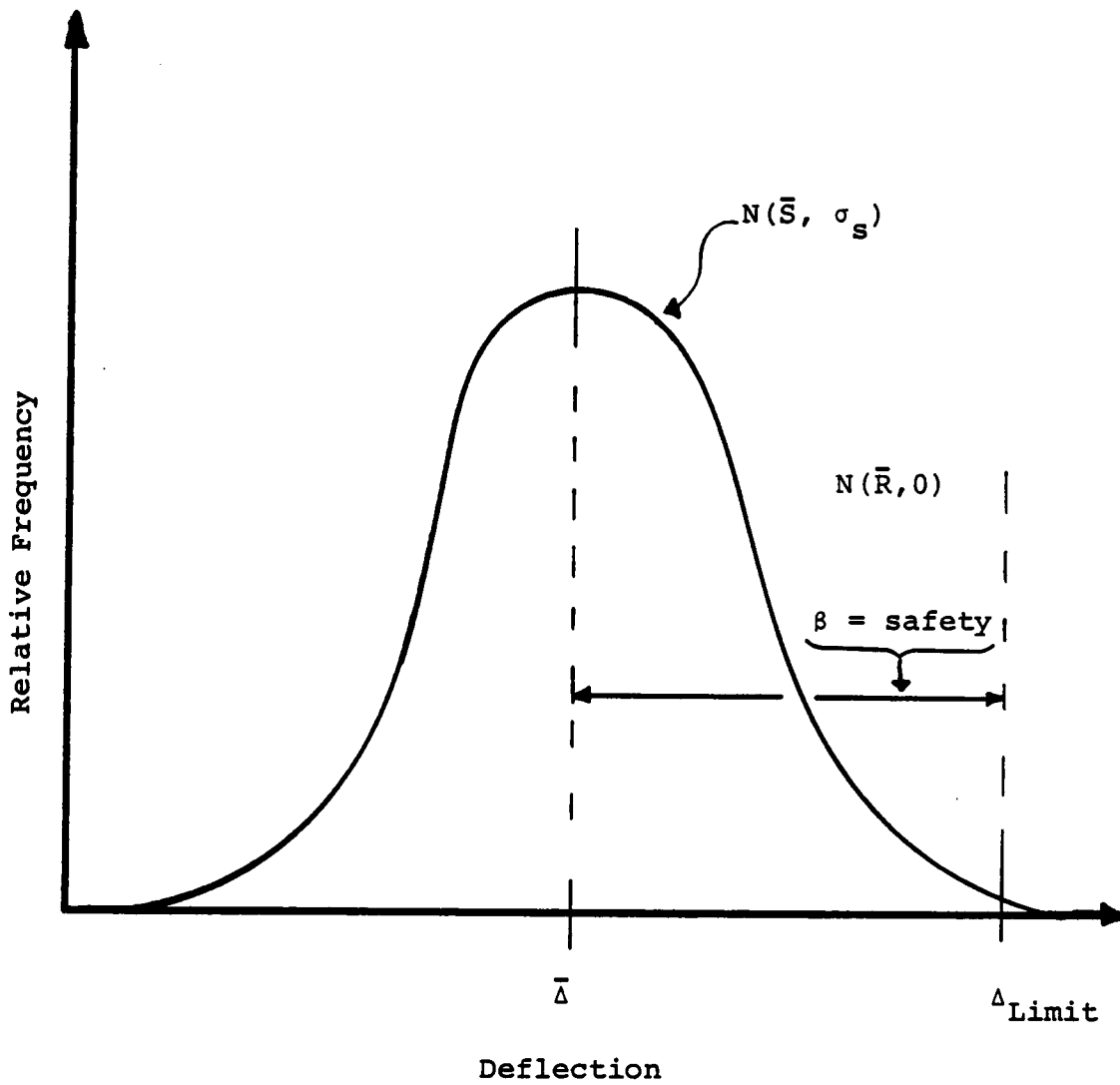
Figure 65. Illustration of serviceability limit
state: analysis option.

stiffness in actual field use. The result of the pallet analysis namely, the load effect, and the input quantity namely, the resistance, are then used in equation (8.1) to compute the safety index. The key to an accurate calibration of Beta is that a large number of structures of a given design, each with material properties that are representative of the population of shook, must be generated and analyzed with known applied loads. This requires detailed knowledge of the structures, material properties, and load distribution. A computer technique called Monte Carlo Simulation was used to generate the structures required for calibration.

WAREHOUSE DATA: Data describing the loads, pallet geometry, part geometry and the support conditions were obtained from a study conducted by Goehring and Wallin (1982). They surveyed 88 materials handling plants or warehouses and obtained pallet specifications and actual pallet loads. However, not all of the obtained data was in a form which was usable for the calibration study. Consequently, only the data from twenty warehouses was used for the calibration of the racked support cases and data from eleven warehouses was used to calibrate the stacked support case.

Two forms of load data were used in the calibration: 1) A random sample of loads obtained from warehouses which store a wide range of products such as a grocery warehouse, and 2) a deterministic load obtained from warehouses which handle a single product resulting in little variation in the load distribution. Three grocery warehouses, and three general merchandise warehouses had randomly sampled load data. The sample size ranged from 51 to 100 individual loads and the COV of the loads ranged

Table 8.1. Weibull distribution parameters. Oak
stringers and deckboards.

Weibull Parameter	Deckboards		Stringers	
	MOR$(\times 10^{-3})$	MOE$(\times 10^{-5})$	MOR$(\times 10^{-3})$	MOE$(\times 10^{-5})$
Shape	4.5752	3.5191	3.767	4.340
Scale	7.8410	12.6701	7.951	12.344
Location	0.0048	1.7831	0.0	0.5447

from 24% to 50%. The remaining warehouses each contained a single load magnitude. However, for the purposes of calibration it was assumed that the actual load distribution had a coefficient of variation of 10%.

RANDOMIZATION: To accurately simulate the response of a population of pallets to applied loads the material properties and part dimensions should be randomized. The randomization of the material properties should reflect the actual probability density functions of the deckboard and stringer MOE distributions. The randomization of the part dimensions should reflect the shook manufacturers tolerance between the actual dimension and the target dimension described in the pallet specification sheet.

Therefore, probability density functions were fit to the best available pallet shook data set, namely the elastic modulus for eastern oak data collected by Spurlock (1983) as part of the Cooperative Pallet Research program. Three distribution forms were investigated for deckboards and stringers: normal, log-normal and Weibull. Based on the results of Chi-square tests, Kolmogorov-Smirnov tests, and visual examination of plots of the probability distribution function superimposed onto a histogram of the actual data, the three parameter Weibull distribution was selected for use in the simulations of both the stringers and deckboards. A program developed by Debonis (1978) was used to estimate the distribution parameters. The parameters for the Weibull distribution are shown in Table 8.1. The probability density function for the three parameter weibull distribution is given by:

$$P(x) = \left(\frac{shape}{scale}\right) \left[\frac{x-Loc}{scale}\right]^{(shape\ -1)} \exp\left[-\left(\frac{x-Loc}{scale}\right)\right]^{shape} \qquad (8.12)$$

where:

shape= shape parameter

loc= location parameter

scale=scale parameter

The oak data set was used to estimate the actual variation of member dimensions. The coefficient of variation of the thickness and widths for both stringers and deckboards were determined for each of the thirty-one mills from which the material was sampled. It was assumed that the variation obtained from these mills was representative of pallet shook in general. The mean COV of the thickness data was found to be 3% and this value was used to randomize the thickness and widths of the pallet parts in the simulations. It was further assumed that the variation of the part dimensions followed a normal distribution.

SIMULATIONS: To execute the simulations used to calibrate the safety index a large amount of computer memory is required. Therefore, the PDS code originally written in the Basic language was translated into the Fortran language and transferred from the mini-computer to an IBM mainframe computer. The program was modified to allow for the simulations of pallets loaded with either the randomly selected loads or the deterministic loads
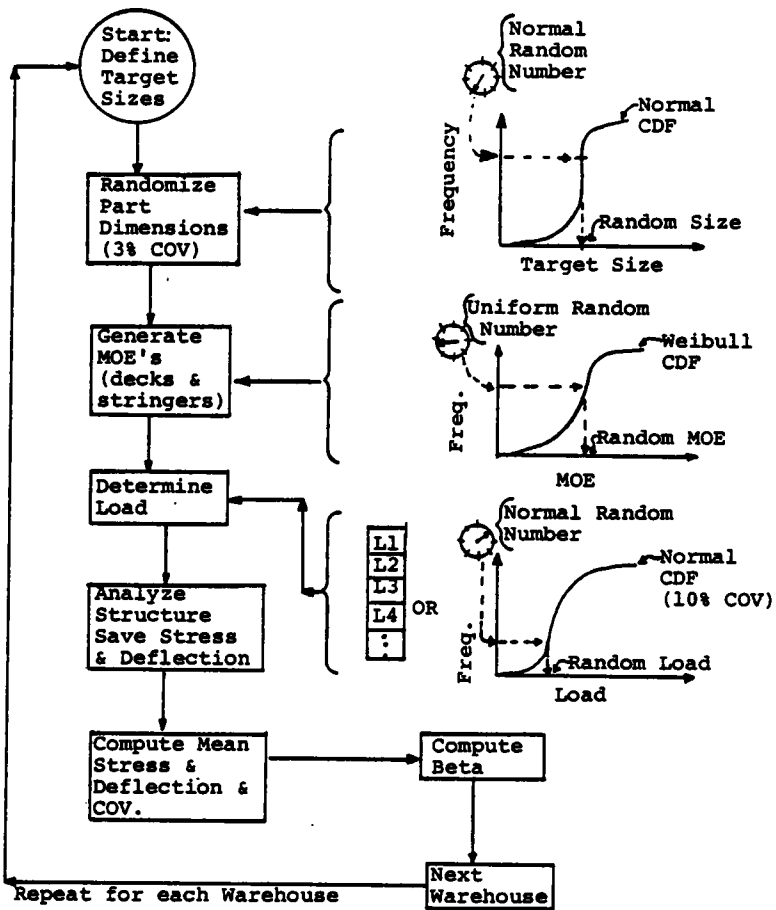
Figure 66. Flow chart of simulations to calibrate Beta.

as shown schematically in Figure 66 on page 235. The specific modifications to the program are described as follows:

1. The target sizes of all parts were defined based upon the description contained in the pallet specifications obtained from Goehring (1982).

2. A counter loop was added to the program and all simulations were executed within this loop. The loop was terminated when all the load data for a given warehouse had been processed. Within this loop the target sizes of the deckboards and stringers were randomized by obtaining a normally distributed random deviate from the International Mathematical and Statistics Library (IMSL) subroutine GGNML. The random deviate was multiplied by the standard deviation of the target size. The resulting value was then added to the target size to obtain the randomized dimension. This equation follows:

$$Rs=Rd(Ts)(COV)+Ts \qquad (8.13)$$

where:

   Ts = target size,

   COV = coefficient of variation of parts dimensions (3%),

   Rs = random size,

   Rd= random normal deviate.

3. The next task conducted within the loop was to generate and deal out the modulus of elasticity values to each member in the pallet. For RAD and stacked analysis the average MOE values for the top and bottom

decks were computed from the values dealt out to each member and assigned to the appropriate elements in the models. To generate the MOE values from the Weibull distribution parameters a standard uniform deviate was generated from the IMSL subroutine called GGUBS. The simulated MOE was then computed from:

$$Smoe = Loc + scale(-\ln(Ru))^{1/shape}$$ (8.14)

where:

Smoe = simulated MOE value,

Loc = Weibull location parameter,

scale = Weibull scale parameter,

shape = Weibull shape parameter,

Ru = uniform random deviate.

The first trial structure was completely defined upon completion of the MOE generation phase. In other words, the simulated pallet had been assigned part dimensions and material properties representative of those found in a real warehouse pallet.

4. Assign a load to the simulated pallet and analyze that pallet in each support condition. Two types of load data were used:

a. The randomly sampled loads were contained in a separate data file. The first load in that file was assigned to the first simulated pallet, and the next load was assigned to the next simulated pallet. This sequence was continued until each load had been used. After the analysis of each simulated pallet the stress and

deflection of the critical members for each support case were saved for later use.

b. The deterministic loads were randomized assuming a 10% COV in a manner which was identical to that of the randomization of the target sizes. For each warehouse which contained a deterministic load, 1000 pallets were simulated. The stress and deflection were saved for computation of the safety index.

The counter loop started in step 2 was terminated after all loads for a warehouse were sampled and analyzed [14].

5. Compute the value of the safety index for each limit state in the three support modes. This was done separately for each warehouse data set by computing the mean and the COV of the stress and the deflection which were obtained from the simulations in the previous step.

a. For the the ultimate limit state the resistance parameters, namely, the mean and COV of the MOR, were obtained directly from the oak data set. The oak deckboard data was used for the RAD and stacked support modes. The mean deckboard MOR was 5000 psi and the COV was 21.2%. The oak stringer data was used for the RAS support mode: The mean unnotched stringer MOR was 5083 psi and the COV was 24.2% and the mean notched stringer MOR was 2695 psi and the COV was 24.2%. (These values were obtained from a limited study of notched oak stringers conducted at VPI). The

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[14] The program was rerun with new seeds at least four additional times for each randomly sampled load data set. This was done to increase the sample size of the simulated pallets in order to obtain a more consistent estimate of the safety index.
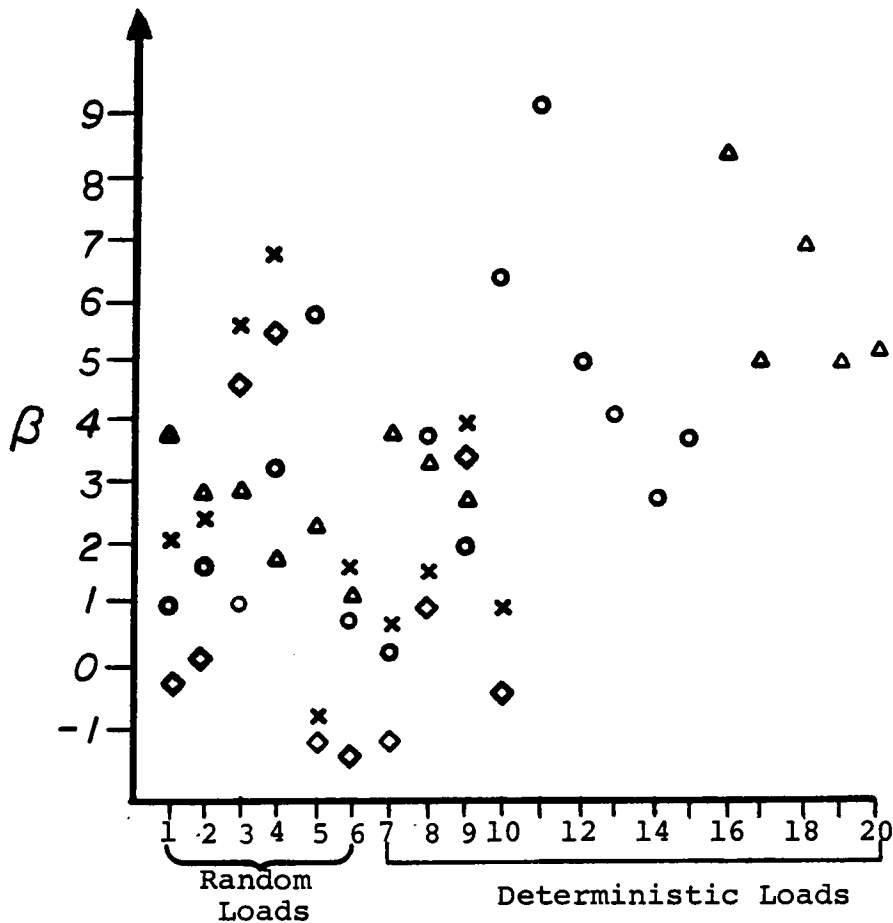
Figure 67. Plot of computed safety index for ultimate limit state.

parameters required to compute the safety index for the ultimate limit state can now be applied. For each warehouse the mean stress, MOR and the COV of each are substituted into equation (8.1) and the safety index was computed. The computed value of the safety index for each support mode was plotted versus the warehouse number as shown in Figure 67 on page 239. The figure shows a wide scatter in the computed value of the safety index. This scatter represents the distribution in the level of safety which is currently accepted by the users of pallets. The goal of the new design procedure is not only to reflect a similar minimum level of safety in the new designs but also to provide a uniform or consistent reliability in the new designs. The values of the safety index for each support mode for use in PDS were selected based upon the mean beta value for the warehouses that were analyzed. The figure shows that negative beta values were computed for the bottom deck of several types of stacked pallets. This indicates that more than 50% of the pallets failed the strength criteria. Also the figure shows that high beta values were computed for the RAS and RAD modes for most warehouses with deterministic loads. This reflects the low probability of failure associated with narrow load distributions.

b. For the serviceability limit state of racked pallets the mean resistance or deflection limit was selected to be equal to 1% of the span. (This limit was selected based upon past recommendations of Wallin, Stern, and Johnson (1976).) For the stack support mode the deflection limit of the deck of the bottom pallet

was assumed to be governed by the height of a handjack which is approximately 3.25 inches.[15] The minimum acceptable gap between the decks resulting from the combined deflection of the top and bottom decks of the second pallet in a stack as assumed to be equal to 2 inches. For all support modes the COV of the deflection limit (resistance) was set equal to zero. The safety index for the deflection limit state was then computed for each warehouse using the mean deflection and its COV (computed from the simulations), and the associated deflection limit.

The resulting beta values for deflection versus the warehouse number are plotted in Figure 68 on page 242. Based on this figure it appears that the assumed stacked deflection limits were conservative because the beta values are extremely high. However, the mean beta values were recommended for use in PDS.

8.4 SUMMARY

A mean value reliability based design method was used to provide safety in the designs resulting from use of PDS. The safety index maintains an adequate separation between the mean load effects and the mean resistance. The safety index values applied in each support mode and limit state were established through the process of calibration, and therefore reflect the level of safety in currently accepted pallet designs.

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[15] Personal conversation with M.S. White and W. Baldwin, September, 1984.
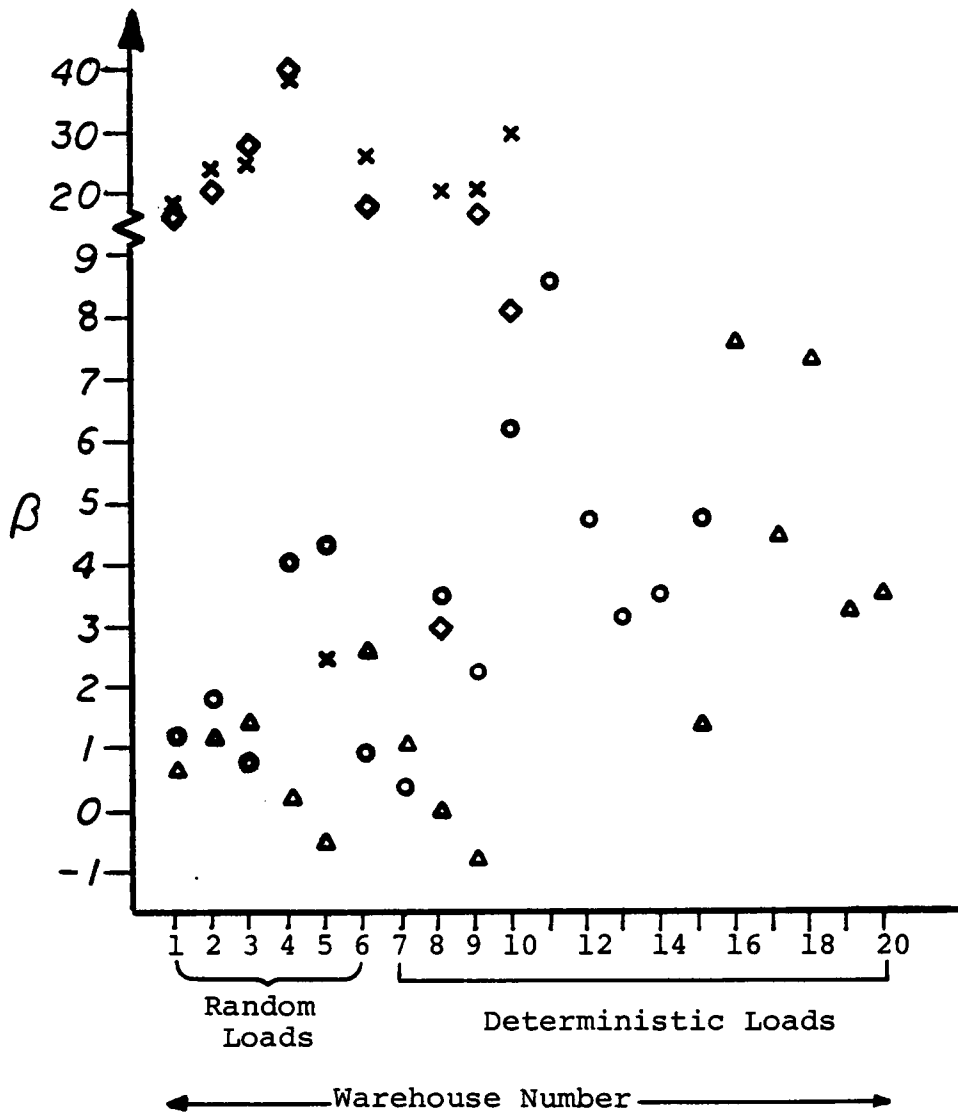
Figure 68 . Plot of computed safety index for
          serviceability limit state.

9.0 PREDICTION OF DURABILITY AND PALLET LIFE-EXPECTANCY

9.1 GENERAL

The preceding chapters have dealt with predicting the preformance of
pallets in specific load and support conditions based on static strength
and stiffness requirements. However, in use, pallets are subjected to
many types of dynamic forces and environmental conditions that affect
performance in terms useful life (i.e. durability). Therefore, the se-
lection of the optimum pallet design for use in a particular environment
should be based on durability as well as static strength and stiffness.
PDS provides techniques for computing estimates of durability and life
expectancy in terms of number of trips or uses, and cost-per-use. These
procedures were developed by Wallin and Whitenack and are described in
this chapter.

Wallin and Whitenack (1974) collected data over a four-year period related
to the performance of 22 different pallet designs; this study was called
the Pallet Exchange Program (PEP). The purpose of the PEP study was to
develop a method to insure uniform in-service pallet performance irre-
spective of the materials used for pallet construction. To evaluate the
influence of factors such as species, defects, or environmental condi-
tions on performance, 2,075 pallets were released into commercial ship-
ping operations and collected data on each use of individual pallets.
The recorded data included the amount of use, number of pallet damages

by part, severity of the pallet damage, and damage to the palletized product. For simplicity of analysis, damage was measured in terms of costs of replacement or repair of either the pallet or the palletized product. Pallet damage was related by economic analysis and regression techniques to both the number of uses and the design characteristics of the pallets. The economic life and the minimum average cost of use were calculated for each of the various designs, species, shook qualities, shook-grade-placements, and nail types.

A computer model based on these results was developed and can be used to compute estimates of the life expectancy, cost-per-use, durability, strength, and stiffness of a pallet design. The program is used in PDS and is the basis of the pallet durability and cost-per-use analysis. This chapter describes the techniques developed by Wallin and Whitenack to predict the durability of pallets.

9.2 OPTIMUM PALLET LIFE AND COST PER TRIP

The average cost-per-use is a measure of the total costs associated with using a pallet during its life. This cost includes the initial purchase price, repair costs, and depreciation costs. The cost per one way trip and the optimum economic life are computed from the average cost-per-use by determining the number of one way trips that produces the minimum average cost-per-use. This point, shown graphically in Figure 69 on page 245, is found by setting the first derivative of an average cost-per-use function equal to zero and solving for the number of uses (U). The number
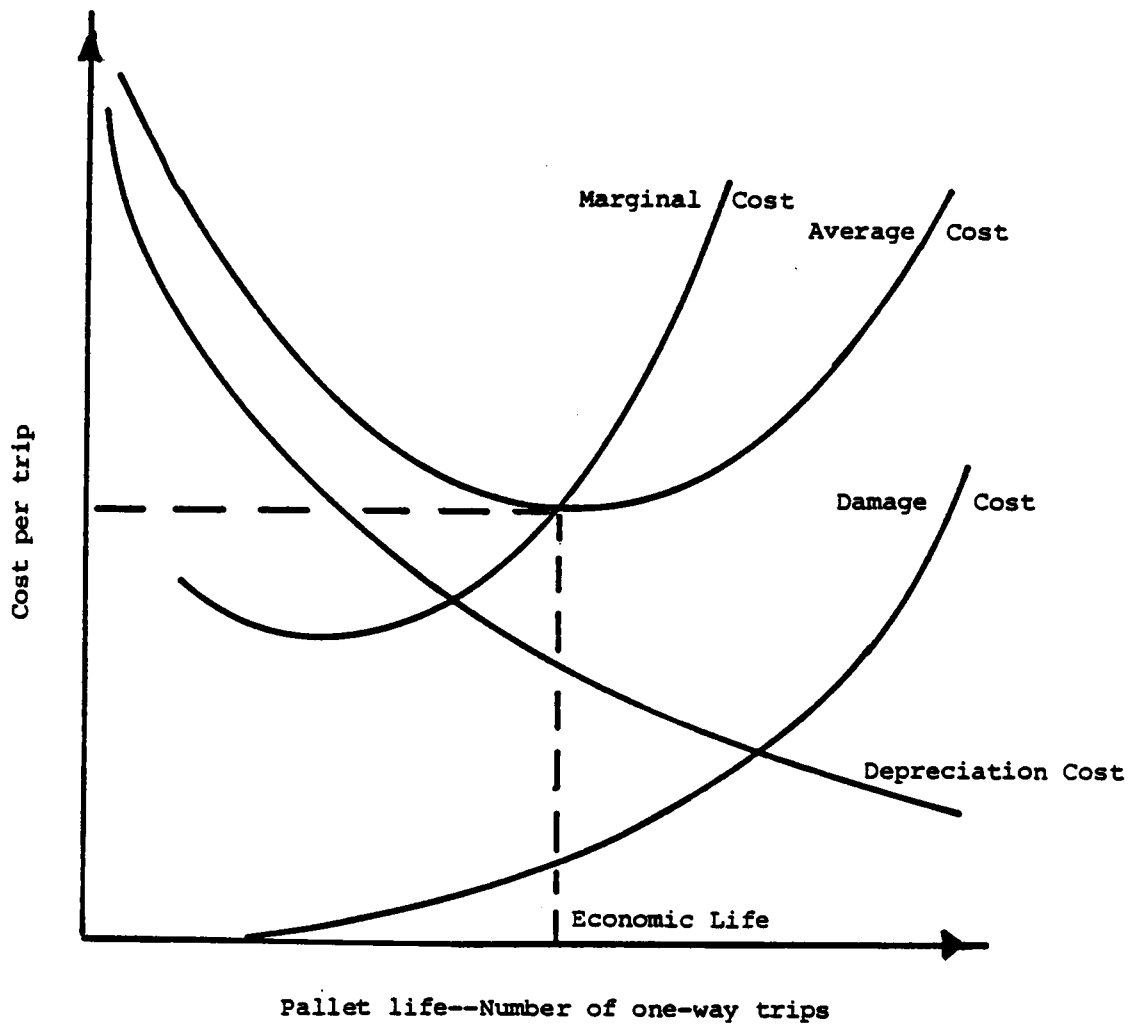
Figure 69. Pallet costs-per-use functions

of uses represents the economic life of the pallet. The cost per trip associated with the economic life (U) is then computed and reported to the user.

The average cost per use is found from:

$$A = (\frac{P+D}{U}) = [\frac{P + C\ (a^U-1)}{U}]$$
(9.1)

where:

A = average cost-per-use (dollars),

P = Price of pallet (dollars),

D = total damage cost = C(F) = C(a^U-1) (dollars)

U = number of one way trips (4 to 6 handlings per trip),

C = Cost per damage = cb^S , (dollars)

F = number of damages = (a^U -1),

a = damage rate factor = (r+1),

r = damage rate,

c = economic coefficient based on costs of repair,

b = technical coefficient based on design of pallet,

s = scale factor developed to measure damage severity.

The optimum pallet life is the number of uses associated with the minimum average cost per use and is found from:

$$\frac{dA}{dU} = C - P + C\ (U)\ [Ln(a)]\ a^U - Ca^U = 0$$
(9.2)

In PDS, U is found using Newton's method of successive approximation. The resulting value of U is then inserted into equation 9.1 and the average cost per trip is computed and reported to the user.

9.2.1 DAMAGE RATE AND SEVERITY

Equations 9.1 and 9.2 contain parameters which are computed from empirical functions that relate the damage rate (r) and severity (s) to quantitative and qualitative measures of the pallet's design and construction features. The damage rate and severity are used to compute the number of damages (F) and the cost per damage (C) as shown in equations 9.1 and 9.2. This section describes the factors that were developed to compute the damage rate and severity.

The damage rate, r, is computed as the product of nine factors (including "F" and "R" factors):

$$r=[1+F(1)][1+F(2)]....[1+F(5)][1+R(1)].....[1+R(4)](0.01) \qquad (9.3)$$

The damage severity is computed as the product of the five "F" factors only:

$$s = [1 + F(1)][1 + F(2)]... [1 + F(5)](2.0) \qquad (9.4)$$
where:

   F(1)...F(5): R(1) to R(4) = factors described in the following subsections.

The factors are computed on a relative basis and relate the performance of the pallet in question to that of a "base pallet." For example, a calculated F(2) of 0.15 means that the damage susceptibility of the example pallet is 15% higher than that of the base pallet. (Note: a negative factor indicates that the given pallet is superior to the base pallet.) The "base pallet" is constructed as follows: 48 inches long by 40 inches wide, class C hardwoods (species group 1) at green moisture content (MC=25%), and oven-dry specific gravity of 0.60. The pallet has fifteen-13/16 inch thick deckboards and three 1-7/8" by 3-7/8" notched stringers. The base pallet is fastened with 114 helically threaded hardened steel nails (108 couples) having the following characteristics: 0.113 inch wire diameter, 0.133-inch thread crest diameter, 2.25-inch length, 1.5 inch threaded length, 60 degree thread angle, 20 degree MIBANT angle, and 5.525 helices.

F(1)--JOINT SEPARATION RESISTANCE FACTOR: The joint separation resistance factor provides a relative rating of joint withdrawal resistance (either head-pull-through or shank withdrawal) for pallets constructed with any type of fastener, compared to the performance of the base pallet. The withdrawal resistance is first computed on a per nail basis and then translated into the F(1) factor by multiplying by the number of fasteners in the pallet. The separation resistance of the joint with one fastener is equal to the lesser of either the head pull-through (HP) or the shank withdrawal resistance (FWT).

The head pull through resistance is the force required to shear a cylinder equal to head diameter and, for a joint with one fastener is computed from the empirical equation:

$$HP = \frac{1.25 \times 10^{6} [HD^{2} - WD^{2}] [T] [G^{2.25}]}{(MC-3)} \qquad (9.5)$$

where:

HD = head diameter (inch),

WD = wire diameter (inch),

T = thickness of deckboard (inch) (0.75 inch maximum),

G = specific gravity of deckboard oven-dry basis,

MC = percent moisture content.

A modified equation is used for a stapled joint:

$$HP = \frac{1.591e6(CL)(WW)(T)(G^{2.25})}{(MC-3)} \qquad (9.6)$$

where:

CL = distance between legs of staple (inch),

WW = width of crown (inch).

The allowable shank withdrawal resistance of a joint (also described in Chapter 4) is computed as follows:

$$FWT = \frac{222.2 \, (FQI) \, (G^{2.25})(P)}{(MC-3)} \qquad (9.7)$$

where:

FWT = fastener withdrawal resistance (pound),

FQI = fastener quality index (defined below),

P = inches of penetration in main member,

G = specific gravity oven-dry basis,

MC = percent moisture content at assembly [16].

Note that the FWT predicts an allowable withdrawal resistance rather than the ultimate.

The fastener quality index (FQI) is used to rate the withdrawal performance of a given fastener relative to the "base nail", independent of the wood material. The FQI for the "base nail" is 100%. The FQI of other nails is found from:

$$FQI = 221.24(WD)[27.15(TD-WD)(H/TL) +1] \qquad (9.8)$$

where:

WD = wire diameter (inch),

TD = thread diameter (inch),

H = number of helices along thread length,

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[16] To simulate actual pallet use joints are assembled green and allowed to dry to approximately 12% MC for testing.

TL = thread length (inch).

The F(1) factor for a pallet measures the increase or decrease in the damage rate and the damage severity for a pallet relative to the base pallet. The factor is computed by determining the total joint separation resistance of a given pallet (the smaller of either the head pull-through or shank withdrawal resistance multiplied by the number of fasteners in the pallet) and comparing it to that of the base pallet:

$$F(1) = \left\{ \frac{\text{withdrawal strength of base pallet}}{\text{withdrawal strength of given pallet}} - 1 \right\} / 20. \qquad (9.9)$$

where:

allowable withdrawal strength of base pallet =54720 pounds (480 pounds per base nail joint).

F(2)--JOINT SHEAR RESISTANCE FACTOR: The shear resistance is a measure of the resistance of the joint to forces that are applied perpendicular to the fastener's longitudinal axis, such as forces that result from fork truck contact. These forces act as lateral or in-plane torsion forces on the deckboard-stringer interfaces. "The efficiency with which the force is distributed among the joints determines the amount of resulting damage." (Wallin 1984). The factors that were found to influence the shear resistance of joints were:

1. The compression strength or specific gravity of the wood,

2. The thickness and width of the deckboards,

3. The stiffness characteristics of the fasteners,

4. The number of fasteners (or couples) per joint,

5. The number of joints per pallet.

The empirically derived equation to compute the total shear resistance for a pallet or a single joint is:

$$FST = \frac{163,025 \ (WD^{1.5}) \ [1/(0.4+0.03M)](G)(T)(C)}{(MC-3)} \qquad (9.10)$$

on where:

FST = shear resistance (pounds),

WD = wire diameter (inch),

M = MIBANT bend angle (degrees),

G = specific gravity oven-dry basis,

T = thickness of deckboards (inch),

C = number of fastener couples per joint [17]

MC = percent moisture content at assembly (assumed to dry in use)

The base joint is constructed with two nails (1 couple), and 13/16 inch thick deckboards and has FST equal to 137.2 pounds. The FST of the base pallet is 14,820 pounds (137.2 X 108 couples). Equivalent wire diameters are used for fasteners other than round-wire nails as follows:

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[17] The number of couples per joint with 2,3,4,5, or 6 nails is 1,3,4,5,6 respectively (Wallin 1984).

1. Square wire nails WD = 1.128 x (width)

2. Rectangular-wire staples, WD = 1.6 x (average wire diameter of one leg)

3. Round-wire staple WD =(2) x (wire diameter of one leg).

The FST for a pallet is found by multiplying the FST, computed on a per couple basis, by the total number of couples in the pallet. The F(2) factor is found by comparing the FST for the base pallet to that of any given pallet by:

$$F(2)=\left\{\frac{[FST\ base\ pallet]}{[FST\ given\ pallet]}\right\}^{0.5} - 1.0 \qquad (9.11)$$

where:

FST base pallet = 14820 pounds

F(3)--JOINT SPLITTING RESISTANCE: The splitting resistance in a joint was found to be a function of the tension perpendicular to the grain strength of the deckboards. Typically, perpendicular to-grain stress forms during drying of pallets which were assembled at high moisture contents. During drying, the nails restrain the shrinkage across the wide face of the deckboards, thereby forming forces perpendicular to the deckboard grain. The resistance to these forces was found to depend upon the following:

1. the distance of the nail from the end of the board,

2. the distance of the nail to edge of the board,

3. the thickness of the board

4. the diameter of the fastener, and

5. the strength of the wood in tension perpendicular to the grain.

The regression equation to compute the splitting resistance is as follows:

$$FSR = [74.174(G^2)(ES)(T)]/WD \qquad (9.12)$$

where:

FSR = splitting resistance (pounds),

G = specific gravity oven-dry basis,

ES = width of edge stringer (inch),

T = thickness of deckboard (inch),

WD = wire diameter (inch).

The splitting resistance factor, F(3), is measured relative to the base pallet and is computed as:

$$F(3) = \left\{ \left[\frac{FSR \text{ base pallet}}{FSR \text{ of given pallet}} \right]^{0.5} - 1 \right\}/10. \qquad (9.13)$$

where:

FSR of the base pallet = 675 pounds.

F(4)--SHOOK QUALITY FACTOR: The damage rate and damage severity were found to vary directly with the minimum shook grade in the pallet. Table 7.3

Table 9.1 Hardwood and Southern Pine Pallet Shook Grades

PDS Input	NWPCA Hardwood Pallet Standards[1]	NWPCA Southern Pine Pallet Specifications[2]	Grades From PEP Study Report[3]	Grade Mix[4]
1	Precision & Better	SP-1 & Better	2 & Better	100, 0, 0, 0
2	Premium/AA & Better	SP-2 & Better	3 & Better	58, 42, 0, 0
3	A & Better	SP-3 & Better	4 & Better	51, 36, 13, 0
4	All Lumber	All Lumber	All Lumber	46, 33, 12, 9
5	Only Premium/AA	Only SP-2	Only 3	0, 100, 0, 0
6	Only A	Only SP-3	Only 4	0, 0, 100, 0
7	Cull	Cull	Cull	0, 0, 0, 100

[1]NWPCA Logo-Mark Hardwood Pallet Standards, March 1, 1982.

[2]NWPCA Specifications for Warehouse, Permanent and Returnable Pallets of Southern Pine, NWPCA, 1962.

[3]Sardo, W. H. and W. B. Wallin, ed. "The Performance of Wooden Pallets in Exchange Programs," 1974.

[4]This is the grade mix of 2 & Better, 3, 4, and Cull lumber used to develop the properties for these classifications. If your grade mix is significantly different from this then use the procedure shown in the example in Appendix C to create your own mix.

shows the correspondence between grades for various grading schemes and the PDS input. The grading scheme used for the F(4) calculation has 4 single grades (2 & better, 3, 4, and all lumber) and was developed by Sardo and Wallin (1974). A regression equation was developed to compute the F(4) factor for pallets constructed from a mix of grades:

$$F(4) = 0.8 \text{ (min. grade)}^{0.75} \qquad (9.14)$$

For pallets constructed from specific grade combinations:

$$F(4) = 0.12 \text{ (WA)} \qquad (9.15)$$

where:

WA = weighted average of the permissible grade mix

F(5)--SELECTIVE PLACEMENT: It has been shown that placing higher quality shook in critical locations reduces the damage rate and severity as compared to random shook placement. The F(5) factor was found to be dependent upon selective shook placement as follows:

F(5) = 0.0 for selective placement,

F(5) = 0.10 for random shook placement.

R(1)--STRINGER STRENGTH: The durability of a pallet was found to be affected by the strength of the stringers (RAS) as compared to the strength of the base pallet. The pallet strength RAS that is used in the dura-

bility section in PDS is based upon the simplified approach developed by Wallin, Stern and Johnson (i.e. not the methods described in Chapter 4). The strength of a pallet is found by:

$$S = 4 \, Y \, b \, h^2 \, F \, g \, L/3(\text{span})$$

where:

S = pallet stringer strength (pounds)

Y = notch reduction factor

$$= [(1-h'/h)(L)]^2 / [4c(L-c)]$$

b = total cumulative width of stringers,

h = height of stringer in inches,

h' = depth of notch,

a = length of notch,

1 = location of notch from stringer end,

c = a + 1,

F = working stress in bending = 0.3 (average MOR)

g = grade factor = 1 - 0.08 (MG)**2

L = length of stringer

Span = load span = L -2h

MG = minimum grade allowed in the mix

The expected damage increase or decrease measured relative to the base pallet is computed as:

[RAS strength of base pallet]
    R(1)= ---------------------------------- - 1.0   (9.17)
                    [RAS strength of given pallet]

where:

    strength of base pallet = 3008 lbs.

R(2)--PALLET STRENGTH RAD: The durability of pallets was found to also

be related to the strength of the pallet racked across the deckboards.

The strength RAD for use in the durability section of PDS is computed from

simplified equations developed by Wallin, Stern and Johnson (1976). The

strength is computed from:

$$\text{pallet strength} = \frac{K\,Y\,b\,h^2\,F\,L\,g}{S^2} \qquad (9.18)$$

where:

    Y = stiffness ratio of top to bottom deck (EIt)/(EIb),

    EIt=elastic modulus multiplied by the moment of inertia of top deck,

    EIb=elastic modulus multiplied by the moment of inertia of bottom

    deck,

    K = moment factor dependent on the number of stringers:

        =1.333 for two stringers,

        =2.133 for three stringers,

=2.667 for four stringers,

b = width of bottom deck (inch),

h = thickness of bottom deck (inch),

F = allowable bending stress of deckboards (psi),

L = length of deck (inch),

g = grade factor (same as in R(1)),

S = L-2h.

The R(2) factor is computed relative to the base pallet as:

$$R(2) = \left\{\frac{\text{strength of base pallet}}{\text{strength of given pallet}}\right\}^{0.5} - 1.0 \qquad (9.19)$$

R(3)--DECK CONSTRUCTION FACTOR: The R(3) factor represents the influence of the construction features that were used to build the pallet. The variables that influence the R(3) factor are:

1. butted endboards

2. use of higher quality material in the end boards and edge stringers

3. use of reinforcement to prevent joint damage, such as straps

4. use of high quality fasteners.

The R(3) factor is computed from damage factors. The damage factors are empirically derived parameters that relate the construction features in the given pallet to those of the base pallet. These parameters are described as follows:

1. Butting of Endboards:

$$DF1 = 1.6 - W(S)/180 \qquad (9.20)$$

where:

W = cumulative width of endboards plus butted boards (inch),

S = number of stringers

2. Density of endboards:

$$DF2 = \frac{(G\ W)_{eb} + (G\ W)_{cb}}{G_{eb} + WW} \qquad (9.21)$$

where:

G = oven-dry specific gravity,

W = cumulative width (inch),

eb = parameter associated with endboards,

cb = parameter assocaited with centerboards.

WW=sum of cumulative widths of end and center boards.

The R(3) factor is computed as follows:

$$R(3)=[(DF1)\ (DF2)] - 1.0 \qquad (9.22)$$

R(4)--MATERIALS HANDLING ENVIRONMENT: The condition of the handling environment in which pallets are used was found to affect the damage rate and severity. The rating of the environment is based upon 17 criteria and includes factors such as proper use of handling equipment, conditions of the loading dock, width of the aisles, speed of the fork truck entry into pallets, etc. The overall condition of the handling environment was

rated into 7 categories. The R(4) factor for each category is 0.1 for excellent, 0.2 for very good, 0.3 for good, 0.35 for average, 0.4 for fair, 0.5 for poor, and 0.6 for very poor. (The exact descriptions of the handling environments are beyond the scope of this dissertation. The interested reader is referred to Wallin and Whitenack (1984) for more specific details).

9.3 INVENTORY ATTRITION ADJUSTMENTS

The average cost per trip for a given pallet design is computed from equation 9.1 and the optimum pallet life is computed from equation 9.2. These calculations assume that no pallets are lost from the inventory. PDS also allows one to estimate the cost-per-use if pallets are frequently lost. "The cost due to loss is the difference between the cost of loss and the expected normal cost. The adjusted total cost including loss therefore may be computed as an adjustment to the expected useful life of the pallet, and its influence on the average cost of use of the inventory of pallets. Pallet loss does not affect either damage severity or damage rate. It only affects the life in terms of the number of uses which may be obtained from the pallet." (Wallin 1984). A mathematical model was developed by Wallin to measure the influence of pallet loss on the life expectancy in terms of number of uses which may be expected from the pallets. "The loss is expressed as a percent of inventory lost per year; and this is translated into a rate of loss per use within the model." (Wallin 1984). The equation to compute adjusted level of uses remaining in a pallet inventory after loss is:

$$N = U(1-L)^V \qquad (9.23)$$

where

N=number of uses remaining,

U=expected life of pallet assuming no loss in terms of number of uses,

L=percent of pallets lost per year (user input),

V=number of uses pallets receive per year (user input).

"In this formulation N represents the life remaining in the inventory after loss has occurred. This may then be employed in the modified cost-per-use formula to obtain the adjusted cost-per-use including pallet loss" (Wallin 1984):

$$A = [P + Ca^N - C]/N \qquad (9.24)$$

where:

A= adjusted cost per use (dollars),

N=expected life after loss (number of uses),

C=cost per damage (dollars),

a=damage rate factor =(1+r)

9.4 SUMMARY

Using the results of the PEP study and economic analysis, Wallin and Whitenack developed techniques to estimate the cost per use, expected number of trips, and inventory attrition costs for pallets. Their techniques were incorporated in PDS. The preceding pages presented these techniques in simplified form. The interested reader is referred to Wallin and Whitenack (1984) for more specific details.

10.0 SUMMARY AND RECOMMENDATIONS

The manufacture of wooden pallets annually consumes a tremendous volume of timber. Approximately 20% of all lumber produced in the United States in 1984 was used to manufacture pallets. Pallets are widely used in warehouses to efficiently store and handle goods and often are subjected to bending and impact loads. Traditionally, pallets were designed emperically with a "trial and error" process, which sometimes resulted in inefficient structures (i.e. member dimensions much greater than required), or unsafe structures (i.e. member dimensions less than required to resist loads). The pallet industry recognized a need for a rational design methodology, based upon engineering principles, to ensure consistent safety and economy in pallets of any geometry. To satisfy this need a cooperative research project between Virginia Polytechnic Institute and State University, the U.S. Forest Service, and the National Wooden Pallet and Container Association was established. The objective of the project was to establish standard methods to design pallets for strength, stiffness, and durability. The results of this project were presented in the preceding pages and are briefly summarized here.

PALLET DESIGN SYSTEM: For simplicity, the developed techniques were computerized for several commonly available minicomputers. The computer program, called the Pallet Design System (PDS), is highly versatile and produces estimates of the maximum load capacity, pallet deflection, optimum member dimensions (to resist specific loads), life expectancy in

specific environments, and estimated cost-per-use. The program is intended to allow pallet manufacturers to design efficient structures to meet customer requirements.

SUPPORT CONDITIONS: PDS analyzes pallets in four main support modes: racked across the stringers, racked across the deckboards, stacked mode, and sling support. (Additionally, resistance to lateral collapse can be analyzed, but these techniques were not presented in this thesis). Two techniques were developed to analyze pallets in the RAS, RAD, and sling modes. 1) Matrix structural analysis methods were applied to pallets whose structural action is too complex for analysis by classical methods. For example, the matrix method was used to analyze unequal sized stringer pallets because of the the difficulty in predicting load sharing among stringers of different stiffness. Also, the matrix method was applied to the RAD and sling modes (for pallets with bottom decks) because it can rationally account for the action of semi-rigid joints. The joints were modeled as zero length spring elements. The stiffness of these elements is variable and is usually equal to the stiffness of representative deckboard-stringer joints. 2) Classical mechanics, based on principles of statics and strength of materials, were used to analyze stacked pallets, and some simpler configurations of RAS, RAD, and sling supported pallets. For example, analysis based on classical mechanics were used for single faced RAD and sling supported pallets, and pallets with equal sized stringers supported in the RAS mode.

LOAD CONDITIONS: Five load types may analyzed: full and partial uniform loads, and single, double, and triple line loads. Chapter 3 describes the specific assumptions and limitations regarding the analysis of these load types.

OPTIONS: Pallets may be ANALYZED or DESIGNED. The ANALYSIS option produces estimates of the load capacity, and deflection in each support mode. The DESIGN option produces estimates of the minimum member dimensions required to safely carry the user defined loads. For either option, the design criteria can be based upon either ultimate or serviceability limit states. In other words, the design criteria can be based upon a strength limit or a user defined deflection limit. The deflection limit state is selected when the user must limit the amount of deflection due to requirements of the handling equipment.

SAFETY: A reliability based design method provides safety in the designs resulting from PDS. The technique accounts for the variability of both the load and resistance distributions. At the core of this method is the safety-index "Beta". The value of this parameter was established through a process called calibration. This was accomplished by analyzing actual pallet designs associated warehouse load data. Monte Carlo simulation techniques were used in the calibration to generate random material properties and load values form the corresponding distributions.

RESISTANCE: A necessary input to the design process is an estimate of the member resistance to the applied load. The important material properties

that provide resistance to loads on pallets are the Modulus of Ruture, and the Modulus of Elasticity. The required input parameters to the reliability based design method are the mean and variance of the properties. The techniques used in PDS to estimate these parameters are based upon either in-grade testing, or modifications of clear-wood properties as described in ASTM D-2555 and ASTM D-245. In-grade testing involves testing large samples of actual pallet material and evaluating the material properties. The modification of clear wood properties involves applying a series of adjustment factors, to the properties of small clear specimens, to account for the effect of strength reducing characteristics that are present in full size lumber. Until better data is available, the variance of the properties for all species in PDS are based on the in-grade testing of oak pallet shook.

DURABILITY: The procedures to predict pallet durability and cost-per-use were developed by Wallin and Whitenack (1984), and are based upon studies of field data. The techniques account for the design characteristics of the pallet (such as butted end-boards) and fasteners, shook quality, and service environment and produce estimates of the "number of uses to first repair", cost-per-use, and economic life.

10.1 RECOMMENDATIONS FOR FURTHER RESEARCH

The methods developed for use in PDS represent a "First Generation" pallet design methodology. A major advantage of this design procedure is that new information, and the results of on-going research can be easily in-

corporated into PDS. This feature allows updating of the methodology to reflect the enhancements in our knowledge of pallet use and behavior. Based upon the results of this study the following areas were identified as being void of sufficient data or techniques and may warrant futher research:

MATERIAL PROPERTIES: 1) The data on the properties of many species are lacking. A continuing effort, based on in-grade testing, is recommended to obtain these properties. This data would enhance the accuracy of PDS for specific species which are currently represented by conservative property estimates.

2) Data on the critical crack extension stress for notched stringers is scanty for all species. A continuing testing program to develop this data for important pallet species is recommended. (A fracture mechanics approach to establish the inherent crack length associated with notched stringers of various species may prove useful for predicting the critical crack extension stress). This effort may be accomplished in conjunction with recommendation number 1.

3) Techniques to predict the rotational stiffness (rotation modulus) for joints constructed from any nail type were developed as described in Chapter 4. However, these estimates may be conservative because of the assumption regarding the deformation of the joint (i.e 0.12 radians), and the limited amount of data used to establish the procedure. Additional research should be aimed at expanding the data base to other species and

fasteners, and investigating other models for predicting the rotation modulus based upon the characteristics of the wood and the fastener. This data will enhance the analysis accuracy for RAD and sling support modes.

MEAN VERSUS MAXIMUM LOADS: As described in Chapter 8, the FOSM method produces estimates of the mean load capacity of a pallet. Because the mean load estimate is of limited use to the pallet industry, provisions were made in PDS translate this estimate into an estimated maximum load, located 1.28 standard deviations above the mean (i.e 90th percentile). Comparison of estimated maximum loads for various levels of load variability shows that the maximum load increases with increased load variability. At first glance, this is contrary to the presumed effect of load variability on probability of failure. However, the phenomenon is mathematically correct as described in Chapter 8, but may lead to confusion for PDS users. Therefore, it is recommended that alternative methods for translating the predicted mean load into a maximum load should be investigated. (Such methods may be based on locating the maximum load at various percentile levels above the corresponding mean load).

CALIBRATION: The value of the safety index is based on calibration. It is recommended that calibration studies should be conducted on a continuing basis. This requires obtaining additional warehouse data for loads and pallet designs. Accurately characterizing the safety index should reduce conservatism in PDS.

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SOFTWARE MODIFICATIONS: The PDS code was written to make allowances for equipment requirements of the users. As computer hardware sophistication increases (and price decreases) the source code of PDS may warrant revision to allow efficient opperation. The user may benefit form such an enhancement through reduced computational time and possibly increased accuracy.

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The following structures were simulated using SPACEPAL to develop estimates of PLOAD and PERROR as described in Chapter 4. Some structures shown in the Table were analyzed with multiple deckboard placement patterns, and therefore occupy multiple lines in the table. Deckboard thicknesses were varied between 3/8" and 1" to produce the stiffness values shown.

| % Coverage | | Length (in.) | | Stiffness (lbs/in.) | | | |
Top Deck	Bottom Deck	Deckboard	Stringer	Top Deck	Bottom Deck	Stringer	Stiffness Ratio
29	29	19.0	60	161.4	96.8	44.6	0.17
72	65	19.0	46	322.9	226.0	99.1	0.18
72	51	19.0	46	322.9	161.4	99.1	0.20
29	17	19.0	60	161.4	32.2	44.6	0.23
72	36	19.0	46	322.9	96.8	91.1	0.24
72	36	19.0	46	322.8	96.8	99.1	0.24
29	17	19.0	60	161.4	32.2	48.6	0.25
29	0	19.0	60	161.4	0	44.6	0.28
72	21	19.0	46	322.9	32.2	99.1	0.28
72	0	19.0	46	322.9	0	99.1	0.31
72	36	19.0	46	322.5	96.8	148.6	0.35
29	17	19.0	60	161.4	32.2	74.1	0.38
36	36	19.0	46	161.4	96.8	99.1	0.38
72	0	19.0	46	322.5	0	148.0	0.46
72	36	19.0	46	322.8	96.8	198.1	0.47
29	17	19.0	60	161.4	32.2	98.3	0.51
36	22	19.0	46	161.4	32.2	99.1	0.51
36	0	19.0	46	161.4	0	99.1	0.61
72	0	19.0	46	322.5	0	198.2	0.61
72	36	19.0	46	322.5	96.8	297.3	0.71
29	17	19.0	60	161.4	32.2	148.3	0.77
72	0	19.0	46	322.5	0	297.3	0.29
93	93	38.5	46	49.9	49.9	99.1	0.99
93	93	38.5	46	49.9	49.9	99.1	0.99
58	0	38.5	60	38.8	0	44.6	1.15
29	29	38.5	60	19.4	11.6	44.6	1.44
72	65	38.5	46	38.8	27.1	99.1	1.50
29	17	38.5	60	19.4	11.6	48.4	1.56
29	17	38.5	60	19.4	11.6	48.4	1.56
72	36	38.5	46	38.8	19.4	99.1	1.70
72	36	38.5	46	19.1	38.8	19.4	1.70
72	51	38.5	46	38.8	19.4	19.1	1.70

% Coverage		Length (in.)		Stiffness (lbs/in.)			
Top Deck	Bottom Deck	Deckboard	Stringer	Top Deck	Bottom Deck	Stringer	Stiffness Ratio
29	17	38.5	60	19.4	3.8	44.6	1.92
72	36	38.5	46	38.8	11.6	99.1	1.96
93	93	38.5	46	49.9	49.9	198.2	1.99
93	93	38.5	46	49.9	49.9	198.2	1.99
29	17	38.5	60	19.4	3.8	49.4	2.13
29	17	38.5	60	19.4	3.8	49.4	2.13
29	0	38.5	60	19.4	0	44.6	2.30
72	21	38.5	46	38.8	3.8	99.1	2.32
72	0	38.5	46	38.8	0	99.1	2.55
72	36	38.5	46	38.8	11.6	148.6	2.95
29	17	38.5	60	19.4	11.6	98.8	3.19
29	17	38.5	60	19.4	11.6	98.8	3.19
29	17	38.5	60	19.4	3.8	74.1	3.19
36	36	38.5	46	19.4	11.6	99.1	3.20
72	36	38.5	46	38.8	19.4	198.2	3.40
72	36	38.5	46	38.8	19.4	198.2	3.40
36	21	38.5	46	19.4	3.8	99.1	4.20
36	0	38.5	46	19.4	0	99.1	5.11
72	36	38.5	46	38.8	11.6	297.3	5.90
29	17	38.5	60	19.4	3.8	148.4	6.39
58	0	75.5	60	5.1	0	44.6	8.68
29	29	75.5	60	2.6	1.5	44.6	10.85
72	65	75.5	46	5.1	3.6	99.1	11.30
72	51	75.5	46	5.1	2.6	99.1	12.85
29	17	75.5	60	2.6	0.5	44.6	14.40
72	36	75.5	46	5.1	1.5	99.1	14.80
72	36	75.5	46	5.1	1.5	99.1	14.80
72	36	75.5	46	5.1	1.5	99.1	14.80
29	17	38.5	60	1.9	1.2	48.4	15.30
29	17	38.5	60	1.9	1.2	48.8	15.49
29	17	75.5	60	2.6	0.5	49.4	16.00
29	17	75.5	60	2.6	0.5	49.4	16.00
29	0	75.5	60	2.6	0	44.6	17.30
72	21	75.5	46	5.1	0.5	99.1	17.50
72	0	75.5	46	5.1	0	99.1	19.20
72	36	75.5	46	5.1	1.5	148.6	22.20
29	17	75.5	60	2.5	0.5	74.1	24.00
36	36	75.5	46	2.6	1.5	99.1	24.10
72	36	75.5	46	5.1	1.5	198.2	29.60
29	17	38.5	60	1.9	1.1	98.8	31.30
29	17	38.5	60	1.9	1.1	98.8	31.30

% Coverage		Length (in.)		Stiffness (lbs/in.)			Stiffness
Top Deck	Bottom Deck	Deckboard	Stringer	Top Deck	Bottom Deck	Stringer	Ratio
29	17	75.5	60	2.6	0.5	98.8	32.00
36	21	75.5	46	2.6	0.5	99.1	32.10
36	0	75.5	46	2.5	0	99.1	38.50
72	36	75.5	46	5.1	1.5	297.3	44.50
29	17	75.5	60	2.5	0.5	148.3	48.00

APPENDIX B. ABSTRACTS OF RELATED THESES

TITLE: LOAD-SUPPORT CONDITIONS AND COMPUTERIZED TEST

APPARATUS FOR WOOD PALLETS

by

G. Brent Fagan

(ABSTRACT)

The in-service loads on wood pallets are often uniformly distributed in nature. Laboratory test methods to evaluate pallets typically use point or line loads because of the difficulty of simulating the actions of a uniform load. In support of research directed at developing design procedures for pallets, a uniform load test machine was developed. This device loads through an air bag and records load-deflection measurements automatically in a micro computer. Test results using the machine were compared to those obtained from tests using bagged goods to simulate a uniform load. The machine is superior in efficiency, speed of testing, and accuracy when compared to bagged goods loading.

The assumption of a uniformly distributed load is very convenient in the design of pallets. However it may be significantly inaccurate for simulation of intrinsically stiff unit loads. The stiffness may cause the load to bridge or redistribute itself into a series of discreet loads. Pallets of varying stiffness were tested with

several bridging and non-bridging type loads to determine the potential error from ignoring load bridging. The results indicated that for stiff pallets where deflection is likely the primary design criterion, the effects of bridging are relatively negligible. However, for flexible pallets where the only restriction on deflection is likely to be load stability, bridging can result in an error greater than 50 percent.

The deflections of pallets stored in a racked mode, whether across the deckboards (RAD) or across the stringers (RAS), depend on the effective span between supports. A study was conducted to determine if the effective span is equal to the clear span and what advantage is gained by increasing rack support width. Tests of pallet sections supported RAD indicated that a pallet functions as a frame with semi-rigid joints whose function differs with the width of the support. A 53 percent reduction in centerline deflection was observed if support width was increased from 1 to 4 inches.

LABORATORY VERIFICATION OF PALLET

DESIGN PROCEDURES

by

Stephen T. Collie

(ABSTRACT)

Three separate investigations were conducted to provide information concerning the development and verification of a computer program, PALLET DESIGN SYSTEM(PDS). The first investigation characterized the distribution of load to pallets used in the stack support condition. Results showed that the load distributed to the top deck span of the bottom pallet varied disproportionately with the cumulative stack load. Load distribution factors were developed which enable the PDS user to account for the fractional load transfer.

The second study determined the effects of load-bridging on pallet design. Theoretical bridging models were developed and empirical tests performed. Results indicated that bridging was dependent on load rigidity and pallet stiffness. Recommendations were made concerning how and when to adjust for this load-support interaction.

The third study experimentally verified the capability of PDS to reasonably predict actual pallet strength and stiffness. A total of twelve pallet designs were tested in up to three support conditions and the results compared to PDS predictions. The PDS procedure was judged reliable in predicting the strength and stiffness of the twelve designs. Differences were primarily due to inadequate estimates of some input parameters. Two potential problems involving maximum strength of deckboards in the RAD mode and modeling of thin deckboards in the stack mode were identified.

DEVELOPMENT OF FLEXURAL DESIGN

VALUES FOR PALLET SHOOK

by

John A. McLeod III

(ABSTRACT)

Rational design of wood pallets requires estimates of average flexural properties of pallet lumber of many species and visual grades.   The objective of this study was to develop procedures for estimating these design values for use in a first-order second-moment design format.

Preliminary studies were performed to assess the effects of increased loading rates on in-grade flexural data, size effects between deckboard and stringer properties, and the effectiveness of the ASTM strength ratio concept as applied to pallet shook.   An increased load rate (ten times the ASTM rate) resulted in an 8.0% increase in average MOR and a 4.7% increase in average MOE. No definite conclusions could be reached concerning the relative strength of deckboards vs. stringers.   Several factors, other than a statistical size effect, may influence their

relative strength. Estimated strength ratios (ESR) generally underpredicted the experimentally determined actual strength ratios (ASR). As knot size increased, the ESR increasingly underpredicted the ASR.

Two approaches were used to derive pallet shook design values. The best is full-size in-grade testing of commercial material. However, only yellow-poplar and eastern oak species have currently been evaluated in this manner. For all other species, a modified procedure based largely on the methods of ASTM D 2555 and D 245 was recommended. This procedure yields conservative estimates of strength for grades allowing large knots.

These equations were developed by Gerhardt (1984) and are presented here in a condensed form. (For details the interested reader is refered to Gerhardt).

<u>Stress:</u>

$$\sigma_{max} = (6M/th^2)(f_1(\phi)) + (6V/th)(f_2(\phi))$$

where:

M = bending moment at notch,
V = shear at notch,
t = thickness,
h = beam depth,

$$f_1(\phi) = \frac{1}{(-1.26\phi + 1)}$$

$$f_2(\phi) = (1.13\phi + 0.3)$$

ϕ = notch depth/beam depth.

<u>Deflection:</u>

$$\delta_P = \frac{Ps^3}{48EI} \psi_{1P} \qquad \text{CP load}$$

$$\delta_Q = \frac{Pa(3s^2 - 4a^2)}{48EI} \psi_{1Q} \qquad \text{CP load}$$

$$\delta_P = \frac{Qa(3s^2 - 4a^2)}{48EI} \psi_{2P} \qquad \text{TP load}$$

$$\delta_Q = \frac{4Qa^2(3s - 4a)}{48EI} \psi_{2Q} \qquad \text{TP load}$$

$$\delta_P = \frac{5ws^4}{384EI} \psi_{3P} \qquad \text{U load}$$

$$\delta_Q = \frac{wa(s^3 - 2sa^2 + a^3)}{24EI} \psi_{3Q} \qquad \text{U load}$$

Table C1.

Dimensionless factors ψ_i for double-notched stringers (fig. 2) are defined as follows:

$$\psi_{1p} = \frac{8(n_o^3 - m_o^3)}{(1-\phi)^3} + \frac{12\alpha h_o(\beta_o^2 + \gamma_o^2)}{(1-\phi)^2} - \frac{48\alpha^2 h_o^2(\beta_o + \gamma_o)}{(1-\phi)} - 12\alpha h_o C_1 + 48\alpha^2 h_o^2 C_2 + 12\alpha^3 h_o^3 C_3 + C_4$$

$$\psi_{1q} = \frac{1}{a_o(3-4a_o^2)}\left[\frac{12n_o^2 a_o - 8m_o^3 - 4a_o^3}{(1-\phi)^3} + \frac{12\alpha h_o(\beta_o^2 - \gamma_o a_o)}{(1-\phi)^2} + \frac{24\alpha^2 h_o^2(a_o - 2\beta_o)}{(1-\phi)} + 12a_o C_5 - 24\alpha^2 h_o^2 C_6 + 12\alpha^3 h_o^3 C_7 + C_8\right]$$

$$\psi_{2p} = \psi_{1q}$$

$$\psi_{2q} = \frac{1}{a_o^2(3-4a_o)}\left[\frac{6n_o a_o^2 - 2m_o^3 - 4a_o^3}{(1-\phi)^3} + \frac{3\alpha h_o(\beta_o^2 + a_o^2)}{(1-\phi)^2} - \frac{12\alpha^2 h_o^2\beta_o}{(1-\phi)} - 3\alpha h_o C_{13} + 12\alpha^2 h_o^2 C_{14} + 3\alpha^3 h_o^3 C_{15} + C_{16}\right]$$

$$\psi_{3p} = 0.2\left[\frac{16[n_o(4-3n_o) + m_o^3(-4+3m_o)]}{(1-\phi)^3} + \frac{96\alpha h_o[\beta_o^2(1-\beta_o) + \gamma_o^2(1+\gamma_o)]}{(1-\phi)^2} + \frac{192\alpha^2 h_o^2[\beta_o(-2+3\beta_o) - \gamma_o(2+3\gamma_o)]}{(1-\phi)} \right.$$
$$\left. + 96\alpha h_o C_9 + 192\alpha^2 h_o^2 C_{10} + 96\alpha^3 h_o^3 C_{11} + C_{12}\right]$$

$$\psi_{3q} = \frac{1}{a_o(1-2a_o^2 + a_o^3)}\left[\frac{[a_o^2(-2 + a_o) + m_o^3(-4+3m_o) + 2n_o^2 a_o(3 - 2n_o)]}{(1-\phi)^3} + \frac{6\alpha h_o[\beta_o^2(1-\beta_o) - \gamma_o a_o(1+\gamma_o)]}{(1-\phi)^2}\right.$$
$$\left. + \frac{12\alpha^2 h_o^2[\beta_o(-2 + 3\beta_o) + a_o(1 + 2\gamma_o)]}{(1-\phi)} + 6\alpha h_o C_{17} + 12\alpha^2 h_o^2 C_{18} + 6\alpha^3 h_o^3 C_{19} + C_{20}\right]$$

The dimensionless constants are defined by (see fig. 2): $\phi = D/h$, $n_o = (m + W)/s$, $a_o = a/s$, $m_o = m/s$, $\beta_o = m_o + \alpha h_o(1 - \phi)$, and $\gamma_o = -n_o + \alpha h_o(1 - \phi)$. The constants C_i ($i = 1,20$) are defined in table B1.

Table C2. Relationships for constants C_i ($i = 1, 20$)

	Appropriate notch depth interval		
	I_1	I_2	I_3
$^cC_1$	$\beta_o^2 + \gamma_o^2$	γ_o^2	$\gamma_o^2\beta_o^2$
C_2	$\beta_o + \gamma_o$	γ_o	$\gamma_o\beta_o$
$^aC_3$	$-4\ell n(1-\phi)$	$3 - 2\ell n(1-\phi) - 2\ell n(\theta_o)$	$3 - 2\ell n(\theta_o) - 2\ell n(\theta_o)$
$^cC_4$	$1 + 8(x_b^2 - x_t^2)$	$1 - 8x_b^2$	0
C_5	$\gamma_o a_o - \beta_o^2$	a_o	$\gamma_o a_o\beta_o$
C_6	$-2\beta_o + a_o$		$a_o\beta_o$
C_7	$-2\ell n(1-\phi)$	$3 - 2\ell n(\theta_o)$	$3 - 2\ell n(\theta_o)$
C_8	$3a_o + 8x_b^2 - 12x_t^2 a_o$	$3a_o - 12x_b^2 a_o$	0
C_9	$\beta_o^2(-1 + \beta_o) - \gamma_o^2(1 + \gamma_o)$	$-\gamma_o^2(1 + \gamma_o)$	$-\gamma_o^2(1 + \gamma_o)\beta_o^2$
C_{10}	$\beta_o(2 - 3\beta_o) + \gamma_o(2 + 3\gamma_o)$	$\gamma_o(2 + 3\gamma_o)$	$\gamma_o(2 + 3\gamma_o)\beta_o$
C_{11}	$[-4 + 6(\beta_o - \gamma_o)]\ell n(1-\phi)$	$3 + 2m_o - 5\beta_o - 2a_o\phi h_o$ $-2(1 + 3\gamma_o)\ell n(1-\phi) - 2(1 - 3\beta_o)\ell n(\theta_o)$	$2 + 2m_o + 2n_o - 5\beta_o - 2(1 + 3\gamma_o)\ell n(\theta_o)$ $-2(1 - 3\beta_o)\ell n(\theta_o)$
C_{12}	$5 + 16x_t^2(-4 + 3x_t) + 16x_b^2(4 - 3x_b)$	$5 + 16x_b^2(-4 + 3x_b)$	0
C_{13}	$\beta_o^2 + a_o^2$	a_o^2	$a_o^2\beta_o^2$
C_{14}	β_o	0	0
C_{15}	$-2\ell n(1-\phi)$	$3 - 2\ell n(\theta_o)$	$3 - 2\ell n(\theta_o)$
C_{16}	$3a_o^2 + 2x_t^2 - 6x_t a_o^2$	$3a_o^2 - 6x_b a_o^2$	0
C_{17}	$\beta_o^2(-1 + \beta_o) + \gamma_o a_o(1 + \gamma_o)$	$\gamma_o a_o(1 + \gamma_o)$	$\gamma_o a_o(1 + \gamma_o)\beta_o^2$
C_{18}	$\beta_o(2 - 3\beta_o) - a_o(1 + 2\gamma_o)$	$-a_o(1 + 2\gamma_o)$	$-a_o(1 + 2\gamma_o)\beta_o$
C_{19}	$2(-1 + \beta_o + 3\beta_o)\ell n(1-\phi) + 2m_o - 2x_t$	$2a_o\ell n(1-\phi) + 2(-1 + 3\beta_o)\ell n(\theta_o) + 3$ $+ 2m_o - 5\beta_o$	$2a_o\ell n(\theta_o) + 2(-1 + 3\beta_o)\ell n(\theta_o) + 3 + 2m_o$ $- 5\beta_o$
C_{20}	$a_o + x_t^2(4 - 3x_t) + 2x_b^2 a_o(-3 + 2x_b)$	$a_o + 2x_b^2 a_o(-3 + 2x_b)$	0

$^aI_1$: $D \leq m/a$ and $D \leq q/a$ (see fig. 2)

$^aI_2$: $D > m/a$ and $D \leq q/a$

$^aI_3$: $D > m/a$ and $D > q/a$

$^a\theta_o = 2a_o h_o/(1 + 2\gamma_o)$

$^b\theta_o = a h_o(1 - \phi)/\beta_o$; $\theta_o = (1 - \phi)h_o$,

$^cx_t = \beta_o - a h_o$; $x_b = -\gamma_o + a h_o$

Appendix C. Equations to compute stress and deflection of notched stringers

FigureC1. Double-notched stringer geometry.

Appendix C. Equations to compute stress and deflection of notched stringers

APPENDIX D. PROGRAM LISTING

The following programs are PDS subroutines to compute the load effects for racked pallets.

```
1448 DEFINT I,J,K,N
1449 REM    $include: 'dim1450'
1452 COMMON SUPLN$,SUPLA$,SUPLA1$,SUPLA2$,SUPLA3$
1454 COMMON COSTN$,COSTA$,COSTA1$,COSTA2$,COSTA3$
1456 COMMON ID$,LO,WO,IC2,IE2,Q1,LTYPE,NLINE,PLC,PLC1,XDECK,XTGR,LPP,SPAN,SPAN1,
ILOAD,TLOAD,ULOAD,CLOAD,SLOAD,X,LIM,DLIM(),SY1,SY2
1458 COMMON H,L,O,J,I9,IJ ,YLT(),XLOC(),XL1(),XL2(),FA()
1460 COMMON U(),U1(),U2(),P(),P1(),P2()
1462 COMMON MO,G,ISPAC,TTOP,TSAC,NEB,NCT,NT4,NT3,NT2,ILEA
1464 COMMON H1,L1,O1,K,JK
1466 COMMON MO1,G1,ISPAC1,TBOT,BSAC,NBT,NB4,NB3,NB2
1468 COMMON H2,L2,J1,IJ1,FT
1470 COMMON MO2,G2,ISPAC2,TSTI,SSAC,H7,L7,LO,R7,STR4,LSTART,ZLOC,HV
1472 COMMON IW1,IW2,IEO,PR
1474 COMMON W.TOT(),W.DEF(),Y.W(),TINC.(),TDINC.(),IFAIL(),JFAIL()
1476 COMMON CR(),EL(),CPT(),LTFR(),IL,NTRP(),PCL()
1478 COMMON DMOR,DMOE,BMOE,BMOR,SMOR,SMOE,BVCR,BVCE,DVCR,DVCE,SVCR,SVCE
1479 COMMON V.S(),V.SD(),BTR(),BTE(),CRITK,G1T,G1B,G3T,G3B,MOE.,IG,ISOW$,ILVAR,I
REP,IDRW,IDFLAG,GSTART,JEFLAG,MAXAVG$,LOADF
1481 DIM JN(19),LC(9),XL(67),XI(67),E(67),MINC(67,2),JCODE(30,3),INDEX(6,6),XG(8
),SS(67,11),P3(9),DIAG(75),XJ(67),MCODE(67,6),Q(75),MOMENT(2)
1482 ON ERROR GOTO 8000
1483 FATAL=0: IF TTOP<TBOT THEN FATAL=1: GOTO 2041 :'PRINT "***warning---too few
top deck boards for the model****"
1485 '***************************************************************************
1486 'Pallet Design System...P D S...                        Version 0.97
1487 '
1488 'SUB URAS..Solution of Racked across stringers -- grid model
1489 '
1490 '
1491 'J.R.Loferski, VIRGINIA TECH,Blacksburg,Va
1492 '***************************************************************************
1500 'GOSUB 10000:COLOR 7,1: 'set up all variables for debuging ----delete this
line later
1501 J2=J1:G1TT=G1T:G3TT=G3T:G1BB=G1B:G3BB=G3B:XTE9=X : UL9=ULOAD : TL9=TLOAD :
CL9=CLOAD : SL9=SLOAD:FATAL=0:IF ILOAD=2 THEN ON  LTYPE GOTO 1502,1502,1503,1504
,1504 ELSE GOTO 1506
1502 W.TOT(1)=TLOAD :GOTO 1505
1503 W.TOT(1)=CLOAD :GOTO 1505
1504 W.TOT(1)=SLOAD*2+CLOAD
1505 W.TOT(2)=W.TOT(1)
1506 DATA 3,6,6,9,9,12,12,15,15,18,18,21,21,24,24,27,27,30,30:' joint number arr
ay
1510 FOR I=1 TO 19: READ JN(I): NEXT I: REM read in number of elements
1525 SUPLOC=SPAN/2!: NJ=JN(TTOP)+3: NE=(NJ*2) +((NJ/3)-3)
1527 IF H7> O THEN GOSUB 20000:' sub notch
1530 ' compute joint coordinates of dkbds on stringers. ie global 1 direction
1531 LC(1)=0:FLAG=0:LC(NJ/3)=LO/2 -YLT(1,1): IF TTOP =1 THEN SUPJNT=6:LC(NJ/3)=S
UPLOC: GOTO 1570
1532 IF SUPLOC<=LC(NJ/3) THEN GOTO 1540
1533 SUPJNT=NJ:LC(NJ/3)=SUPLOC: NNN=NJ/3: FOR I=1 TO NNN:'support to right of en
d board
1534 IF NNN-I < 2 THEN GOTO 1536
1535 LC(NNN-I) = LO/2-YLT(I,1):NEXT I
1536 GOTO 1570:'begin support between deckboards section
1540 NNN=NJ/3: FOR I=1 TO NNN: IF FLAG=1 THEN GOTO 1555
```

```
1545 LC(NJ/3-I)=L0/2-YLT(I+1,1): IF SUPLOC < LC(NJ/3-I) THEN GOTO 1565: 'nexti
1550 SUPJNT=(NJ/3-I)*3 : LC(SUPJNT/3) =SUPLOC: FLAG=1
1555 IF NJ/3-I-1 < 2 THEN GOTO 1570
1560 LC(NJ/3-I-1)=L0/2-YLT(I+1,1)
1565 NEXT I
1570 ' continue--all global 1 coordinates are defined: compute member lengths
1573 'PRINT "nj=";NJ;"ne=";NE;"supjnt=";SUPJNT:BEEP:YN$=INPUT$(1)
1575 FOR I=1 TO NJ/3-1: JJJ=I*7 : XL(JJJ)=LC(I+1)-LC(I): IF XL(JJJ) < .1.THEN XL
(JJJ)=.1
1580 XL(JJJ-1)=XL(JJJ): XL(JJJ-2)=XL(JJJ): NEXT I
1581 'COMPUTE DECK ELEMENT LENGTHS--IE GLOBAL 3
1585 DUM1=WO/2-U2(1,2)/2: DUM3= 0!: IF TSTI =2 OR TSTI = 3 THEN DUM2=DUM1/2 ELSE
DUM2=DUM1-STR4
1590 XL(1) =DUM1-DUM2: XL(3)=DUM2-DUM3
1595 FOR I=1 TO NJ/3: JJJ=(I-1)*7 +1: XL(JJJ)=XL(1): XL(JJJ+1)=XL(1): XL(JJJ+2)=
XL(3): XL(JJJ+3)=XL(3): NEXT I: 'all dkbd lengths are defined
1597 SME=SMOE:IF H7> 0 THEN SME=SMOE/XNOTCH
1599 ' define member properties--stringers first- zero out unneeded ones later
1600 FOR I=1 TO NJ/3-1: JJJ=(I*7): XI(JJJ)=U2(J1,2)*(H2^3)/24: XI(JJJ-1)=U2(J1,2
)*(H2^3)/12: XI(JJJ-2)=U2(1,2)*(H2^3)/12
1601 XJ(JJJ)=XI(JJJ)+ H2*(U2(J1,2)^3)/24: XJ(JJJ-1)=XI(JJJ-1)+ H2*(U2(J1,2)^3)/1
2: XJ(JJJ-2)=XI(JJJ-2)+ H2*(U2(1,2)^3)/12
1605 E(JJJ)=SME: E(JJJ-1)=SME: E(JJJ-2)= SME: GAMMA=30000!
1610 IF TSTI = 2 OR TSTI= 3 THEN XI(JJJ-1)=.1: E(JJJ-1)=1: XJ(JJJ-1)=1!
1615 IF TSTI=2 OR TSTI=4 THEN XI(JJJ)=.1: E(JJJ)=1:XJ(JJJ)=1!
1620 NEXT I: ' stringer elements have been defined-- start deckbds
1625 IF TTOP MOD 2 = 0 THEN GOTO 1635 :' center line dkbd properties
1630 XI(1)=YLT(INT(TTOP/2)+1,2)*(H^3)/24: XI(3)=XI(1):E(1)=DMOE: E(3)=E(1):XJ(1)
=XI(1)+H*(YLT(INT(TTOP/2)+1,2)^3)/24:XJ(3)=XJ(1):GOTO 1640
1635 XI(1)=.1: XI(3)=.1: E(1)=1!: E(3)=E(1):XJ(1)=1:XJ(3)=1
1640 IF TBOT MOD 2 = 0 THEN GOTO 1655
1645 IF Q1>0 THEN W=XL1(INT(TBOT/2)+1,2)-XL1(INT(TBOT/2)+1,1) :XI(2)=W*(H1^3)/24
:XI(4)=XI(2): E(2)=BMOE:
1650 E(4)=BMOE:XJ(2)=XI(2)+H1*(W^3)/24:XJ(4)=XJ(2): GOTO 1660
1655  XI(2)=.1: XI(4)=.1:E(2)=1!: E(4)=1!:XJ(2)=1:XJ(4)=1
1660 ' remaining dkbds-- first dummy out those near the supports
1665 JJJ=((SUPJNT/3-1)*7)+1: FOR KK=JJJ TO JJJ+3
1670 E(KK)=1!: XI(KK)= .1: XJ(KK)=1: NEXT KK: ' deal out properties to top deck
elements with higher numbers than the supported boards
1675 IF SUPJNT = NJ GOTO 1695
1680 FOR I=SUPJNT/3 TO NJ/3-1: KKK=NJ/3-I: JJ=I*7+1: JJJ=JJ+2
1685 XI(JJ)=(H^3)*YLT(KKK,2)/12: E(JJ)=DMOE:XI(JJJ)=XI(JJ): E(JJJ)=E(JJ):XJ(JJ)=
XI(JJ)+H*(YLT(KKK,2)^3)/12: XJ(JJJ)=XJ(JJ)
1690 NEXT I
1695 'deal out properties to top deck elements with lower numbers than supported
boards
1700 IF SUPJNT = 6 GOTO 1715
1705 FOR I=1 TO SUPJNT/3-2: JJ=I*7+1: KK=NJ/3-I-1: XI(JJ)=(H^3)*YLT(KK,2)/12
1710 E(JJ)=DMOE : XI(JJ+2)=XI(JJ): E(JJ+2)=E(JJ):XJ(JJ)=XI(JJ)+H*(YLT(KK,2)^3)/1
2:XJ(JJ+2)=XJ(JJ): NEXT .I
1715 ' all top deckbd properties have been dealt out-- bottom deck next
1720 FOR I=2 TO NJ/3: IF I=SUPJNT/3 THEN GOTO 1730:'zero out all elements first
1725 JJ=(I-1)*7+2 : XI(JJ)=.1: E(JJ)=1! :XI(JJ+2)=.1: E(JJ+2)=E(JJ): XJ(JJ)=1: X
J(JJ+2)=1
1730 NEXT I
1735 IF Q1=0 GOTO 1770: ' no bottom deck-- jump
1740 I=INT(TBOT/2): FOR JJ=2 TO NJ/3:
1744 IF JJ = SUPJNT/3 THEN GOTO 1765
1745 CL=L0/2-((XL1(I,2)-XL1(I,1))/2 + XL1(I,1)): JJJ=(JJ-1)*7+2
1750 IF JJ=SUPJNT/3-1 THEN GOTO 1753 ELSE GOTO 1755 :'check for support location
relative to current joint number
1753 IF NJ=SUPJNT THEN DISTB=L0/2: GOTO 1758:'support at end of model-hope that-
i=1
1754 DISTB=(LC(JJ+2)-LC(JJ))/2+LC(JJ): GOTO 1758:'more joints to right of supp
1755 IF JJ=NJ/3 THEN DISTB=L0/2 ELSE DISTB=(LC(JJ+1)-LC(JJ))/2+LC(JJ):'check if
more joints are to right of current joint
```

```
1758 IF CL > DISTB THEN GOTO 1765
1760 XI(JJJ)=(H1^3)*(XL1(I,2)-XL1(I,1))/12: E(JJJ)=BMOE:XI(JJJ+2)=XI(JJJ): E(JJJ
+2)=E(JJJ):XJ(JJJ)=XJ(JJJ)+H1*((XL1(I,2)-XL1(I,1))^3)/12:XJ(JJJ+2)=XJ(JJJ):I=I-1
: IF I=0 THEN GOTO 1770:'jump out side loop--all boards placed
1765 NEXT JJ: IF I>= 1 THEN FATAL=1:'PRINT "*****warning not all bottom boards w
ere placed---number of top boards must be >= number of bottom boards"
1770 ' bottom deck properties are now defined--read in minc and jcode- then
compute mcode array
1775 DATA 1,2,1,2,2,3,2,3,1,4,2,5,3,6,4,5,4,5,5,6,5,6,4,7,5,8,6,9,7,8,7,8,8,9,8,
9,7,10,8,11,9,12,10,11,10,11,11,12,11,12,10,13,11,14,12,15,13,14,13,14,14,15,14,
15,13,16,14,17,15,18
1776 DATA 16,17,16,17,17,18,17,18,16,19,17,20,18,21,19,20,19,20,20,21,20,21,19,2
2,20,23,21,24,22,23,22,23,23,24,23,24,22,25,23,26,24,27,25,26,25,26,26,27,26,27,
25,28,26,29,27,30,28,29,29,30,29,30:' minc(i,j) i=1,53: j=1,2
1780 FOR I=1 TO 67: FOR JJ=1 TO 2: READ MINC(I,JJ): NEXT JJ:NEXT I
1785 MAXID=0:' read in jcode
1786 DATA 1,1,0,1,1,0,0,1,0,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,0,1,
1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,0,1,1,1,1,1,
1,1,1,0,1,1,1,1,1,1,0,1,1:'              jcode(24,3)
1787 FOR I=1 TO 30: FOR JJ=1 TO 3: READ JCODE(I,JJ): NEXT JJ : NEXT I: ' modify
jcode to account for support location
1790 JCODE(SUPJNT,2)=0: JCODE(SUPJNT-1,2)=0: JCODE(SUPJNT-2,2)=0:' assign number
s in sequence to jcode
1795 KK=1:FOR I=1 TO NJ: FOR JJ=1 TO 3: IF JCODE(I,JJ)= 0 THEN GOTO 1805
1800 JCODE(I,JJ)=KK: KK=KK+1:
1805 NEXT JJ:NEXT I: ' next generate mcode
1810 NDOF=0 : FOR I=1 TO NE: JJ1=MINC(I,1): JJ2=MINC(I,2): FOR JJ=1 TO 3: MCODE(
I,JJ)=JCODE(JJ1,JJ): IF MCODE(I,JJ) > NDOF THEN NDOF=MCODE(I,JJ)
1815 KK=JJ+3: MCODE(I,KK)=JCODE(JJ2,JJ): IF MCODE(I,KK) > NDOF THEN NDOF=MCODE(I
,KK)
1820 NEXT JJ:KKK=1:'find half band width
1821 ISS=MCODE(I,KKK)
1822 IF ISS=0 THEN KKK=KKK+1: GOTO 1821 :'smallest gd
1823 KKK=6
1824 ILL=MCODE(I,KKK):IF ILL=0 THEN KKK=KKK-1:GOTO 1824
1825 IDD=ILL-ISS:IF IDD> MAXID THEN MAXID=IDD
1826 NEXT I: HBW=MAXID+1 :'set half band width= max dif in dof for an element
1829 ' begin to assemble stiffness matrix--first define index array
1830   DATA 1,2,4,8,-2,4,2,3,5,2,-3,5,4,5,6,-4,-5,7,8,2,-4,1,-2,-4,-2,-3,-5,-2,3,
-5,4,5,7,-4,-5,6: 'index array
1831 FOR I=1 TO 6: FOR JJ=1 TO 6: READ INDEX(I,JJ): NEXT JJ: NEXT I
1835 FOR I=1 TO NE: ALPHA=E(I)*XI(I)/(XL(I)^3): GAMMA=30000*XJ(I)/XL(I)
1836 IF ALPHA<1 THEN ALPHA =1: GAMMA=1
1838 'PRINT I;"alpha=";ALPHA,"gamma=";GAMMA
1839 IF I MOD 7 =0 THEN GOTO 1860
1840 IF (I+1) MOD 7 = 0 THEN GOTO 1860
1845 IF (I+2) MOD 7 = 0 THEN GOTO 1860
1849 ' begin deck element stiffnes matrix
1850 XG(1)=4*(XL(I)^2)*ALPHA: XG(2)=6*XL(I)*ALPHA: XG(3)=12*ALPHA: XG(4)=0: XG(5
)=0:XG(6)=GAMMA:XG(7)=-GAMMA:XG(8)=2*(XL(I)^2)*ALPHA:' deckbd stiffness matrix
1855 GOTO 1870 :' start stringer element stiffness matrix
1860 XG(1)=GAMMA:XG(2)=0: XG(3)=12*ALPHA: XG(4)=0: XG(5)=6*XL(I)*ALPHA: XG(6)=4*
(XL(I)^2)*ALPHA: XG(7)=2*(XL(I)^2)*ALPHA: XG(8)=-GAMMA: 'end string element stif
fness matrix
1870 '_transfer_ element matrix into system stiffnes matrix
1875 FOR JM=1 TO 6: JJ=MCODE(I,JM): IF JJ=0 THEN GOTO 1905
1880    FOR KM=JM TO 6: KK=MCODE(I,KM): IF KK=0 THEN GOTO 1900
1885       KB=KK-JJ+1: LL=INDEX(JM,KM): IF LL > 0 THEN GOTO 1895
1890       LL=-LL: SS(JJ,KB)=SS(JJ,KB)-XG(LL): GOTO 1900
1895       SS(JJ,KB)=SS(JJ,KB)+XG(LL)
1900    NEXT KM:
1905 NEXT JM
1910 NEXT I: ' system stiffness matrix is finished
1915 ' compute equivilent joint loads from member loads on deckboards
1916 ' full or partial uniform loads only-- compute pressure ww=#/in^2
1920 TW=0;IFLAG=0 :P3(SUPJNT/3)=0:IF LTYPE=1 THEN XTGR=0
```

```
1924 ' find the loaded width of top deckbds
1925 FOR I=1 TO NJ/3: DUM=NJ/3+1-I: IF DUM=SUPJNT/3 THEN IFLAG=1: GOTO 1965
1930 JJ=I: IF IFLAG>=1 THEN JJ=JJ-1
1935 P3(DUM)=0: IF I=NJ/3 AND TTOP MOD 2 =0 THEN GOTO 1965
1940 IF I = NJ/3 AND TTOP MOD 2 <> 0 THEN IFLAG = 2
1945 IF XTGR <= XLOC(JJ,2) THEN P3(DUM)=XLOC(JJ,2)-XTGR
1950 IF P3(DUM) > YLT(JJ,2) THEN P3(DUM)=YLT(JJ,2):' fix up loaded width if too
large
1955 IF IFLAG =2 THEN P3(DUM)=P3(DUM)/2
1960 TW=TW+P3(DUM)
1965 NEXT I: ' total width (tw) has been found---next compute pressure
1970 TW=TW*2:IF ILOAD=1 THEN TLOAD=2000: 'SBAR=SMOR*EXP(-BTR(1)*SQR(V.S(1)^2+SVC
R^2))
1971 IF ILOAD=1 OR H7>0 THEN AAA=SMOR*SQR((1+V.S(1)^2)/(1+SVCR^2)):BBB=EXP(-BTR(
1)*SQR(LOG(1+V.S(1)^2)+LOG(1+SVCR^2))): SBAR=AAA*BBB
1975 PPERIN=TLOAD/(TW*WO):' compute fixed end forces--initialize load vector
1980 FOR I=1 TO NDOF: Q(I)=0: NEXT I: 'process element actions
1985 FOR I=1 TO NJ/3: KK=(I-1)*7+1: KK2=KK+2
1990     FOR JJ= KK TO KK2 STEP 2: ACT=P3(I)*PPERIN: F2=ACT*XL(JJ)/2:
1995        F3=ACT*(XL(JJ)^2)/12: F5=F2:F6=-F3:' transfer local forces into global

2000          FOR LL=1 TO 6: KKK=MCODE(JJ,LL):IF KKK = 0 THEN GOTO 2020
2010            ON LL GOTO 2011,2012,2013,2014,2015,2013
2011            Q(KKK)= Q(KKK)-F3: GOTO 2020
2012            Q(KKK)=Q(KKK)-F2: GOTO 2020
2013            Q(KKK)= 0: GOTO 2020
2014            Q(KKK)=Q(KKK)-F6: GOTO 2020
2015            Q(KKK)=Q(KKK)-F5
2020        NEXT LL
2025      NEXT JJ
2030  NEXT I: ' load vector is finished
2040 GOSUB 3785:'GOTO SOLVE SUBROUTINE
2041 IF FATAL=1 THEN W.TOT(1)=1:IFAIL(1)=5: Y.W(1)=0:W.DEF(1)=0: GOTO 3510: 'fal
tal error   was detected----jump to next subroutine
2050 SIGMAX=0: IMEMB=0:' find max stress from member displacements--only look
stringer elements 5,6,7 and at notch location if present
2055 FOR I=5 TO 7: ALPHA= E(I)*XI(I)/(XL(I)^3): IF E(I)< 10 THEN GOTO 2085
2060 D2=MCODE(I,2): D5=MCODE(I,5): D3=MCODE(I,3): D6=MCODE(I,6)
2065 F3=6*XL(I)*ALPHA*(Q(D2)-Q(D5))+2*(XL(I)^2)*ALPHA*(Q(D6))
2070 IF I=5 THEN SM=U2(1,2)*H2^2/6
2074 IF I =7 THEN SM=U2(J2,2)*(H2^2)/12
2075 IF I =6 THEN SM=U2(J2,2)*(H2^2)/6
2080 STRESS=ABS(F3/SM): IF STRESS > SIGMAX THEN SIGMAX=STRESS:IMEMB=I
2081 'PRINT I;SM;F3;"d2=";D2;"d6";D6;"alpha=";ALPHA;"stress=";STRESS
2085 NEXT I
2086 IF TSTI=3 THEN Y.W(1)=ABS(Q(5)):' max deflection
2087 IF TSTI=4 THEN Y.W(1)=ABS(Q(4))
2088 'PRINT "q(5)=";Q(5);"q(4)=";Q(4):BEEP:YN$=INPUT$(1)
2090 IF H7=0 THEN GOTO 3010:'JUMP IF UNNOTCHED
2092 XNUMER1=SBAR*U2(1,2)*H2^2/6:XNUMER2=SBAR*U2(J2,2)*H2^2/6: IF TSTI=3 THEN XN
UMER2=XNUMER2/2 :' find max allowable moment at notch-numer1 is outside stringe
r--numer2 is inner stringer
2093 DEN=((1/(1-1.26*PHI))+VOM*H2*(1.13*PHI+.295)): XMMAX1=XNUMER1/DEN:XMMAX2=XN
UMER2/DEN:' max allow moment at notch
2094 GOSUB 5000:' sub find--compute moment at notch
2095 IF ILOAD=1 THEN DUM=TLOAD*XMMAX1/MOMENT(1):DUM2=TLOAD*XMMAX2/MOMENT(2):IF D
UM< DUM2 THEN W.TOT(1)=DUM ELSE IF DUM2<DUM THEN W.TOT(1)=DUM2:'max load notched
 analysis
2096 IF ILOAD=1 THEN GOTO 3010
2097 'design option for notches
3005 IF MOMENT(1)> XMMAX1 OR MOMENT(2)> XMMAX2 THEN IFAIL(1)=1 ELSE IFAIL(1)=0
3006 IF DLIM(1)< .0001 THEN GOTO 3510: 'no defl limit--finished notch design
3007 DEFLEC=Y.W(1): RDEF=DEFLEC*EXP(BTE(1)*SQR(LOG(1+V.SD(1)^2)))/SQR(1+V.SD(1)^
2)
3008 IF RDEF <= DLIM(1) THEN JFAIL(1)=0 ELSE JFAIL(1)=1:GOTO 3510:'finished notc
h design option with defl limit
```

```
3010 IF ILOAD=2 THEN DEFLIM=DLIM(1):THK=H2:XMOR=SMOR:IOUT=1:RCOV=SVCR:DEFLEC=Y.W
(1): RESS=SIGMAX: GOSUB 4205: GOTO 3510: 'gosub design--finished design option
3011 IF ILOAD=1 AND H7=0 THEN W.TOT(1)=TLOAD*SBAR/SIGMAX:' max load for analysis
 option --unnotched stringer
3012 IF ILOAD=1 THEN Y.W(1)=Y.W(1)*W.TOT(1)/TLOAD:' compute deflection at max lo
ad
3013 IF ILOAD=1 AND DLIM(1)>0 THEN DELTS=DLIM(1)*SQR(1+V.SD(1)^2)/EXP(BTE(1)*SQR
(LOG(1+V.SD(1)^2)))
3015 IF ILOAD=1 AND DLIM(1) > 0   THEN   DEFL=ABS(Y.W(1)):W.DEF(1)=DELTS*W.TOT(1)/
DEFL: 'max load for deflection limit--both notch or unnotched --finished analysi
s option
3510 CHAIN "a:sub1500"
3785 COLOR 15,1,1:CLS : LOCATE 12,20 :  PRINT "WAIT , I'M THINKING"
3795 RMIN = 1 : IDECAY = 0
3805 FOR I = 1 TO NDOF : DIAG(I) = SS(I,1) : NEXT I
3815 FOR N9 = 1 TO NDOF
3825 FOR L9 = 2 TO HBW
3835 IF SS(N9,L9) = 0 GOTO 3905
3845 I = N9 + L9 - 1 : J9 = 0 : C = SS(N9,L9) / SS(N9,1)
3855 FOR K9 = L9 TO HBW
3865 J9 = J9 + 1
3875 SS(I,J9) = SS(I,J9) - C * SS(N9,K9)
3885 NEXT K9
3895 SS(N9,L9) = C
3905 NEXT L9
3915 NEXT N9
3925 FOR I = 1 TO NDOF
3935 DECAY = SS(I,1) / DIAG(I)
3945 IF ABS(DECAY) >= ABS(RMIN) GOTO 3965
3955 RMIN = DECAY
3965 IF DECAY < 0 THEN FATAL=1: 'PRINT "*** ERROR in subroutine SOLVE in diagona
l of row ";I;" *** structure may be unstable"
3975 IF DECAY < 0 THEN PRINT "*** caution *** structure may be unstable in this
stringer support mode"
3985 NEXT I
3995 FOR N9 = 1 TO NDOF
4005 FOR L9 = 2 TO HBW
4015 IF SS(N9,L9) = 0 GOTO 4045
4025 I = N9 + L9 - 1
4035 Q(I) = Q(I) - SS(N9,L9) * Q(N9)
4045 NEXT L9
4055 IF SS(N9,1) = 0 THEN FATAL=1 :'PRINT "***ERROR in subroutine SOLVE ";N9;" e
lement of banded system stiffness matrix ";SS(N9,1);"  check input"  : GOTO 3125

4065 IF SS(N9,1)=0 THEN GOTO 4085
4075 Q(N9) = Q(N9) / SS(N9,1)
4085 NEXT N9
4095 FOR I = 2 TO NDOF
4105 N9 = NDOF+1 - I
4115 FOR L9 = 2 TO HBW
4125 IF SS(N9,L9) = 0 GOTO 4155
4135 K9 = N9 + L9 - 1
4145 Q(N9) = Q(N9) - SS(N9,L9) * Q(K9)
4155 NEXT L9
4165 NEXT I
4175 'PRINT " Minimum decay ratio = ";RMIN
4185 IF ABS(RMIN) < 9.999999E-06 THEN FATAL=1:'PRINT "***ERROR in SOLVE **** ill
-conditioning detected"
4195 RETURN
4205 '**********************************************************************
4215 ' sub design for finding mim mor and defl limit
4225 '**********************************************************************
4235 BBT=BTR(1):SCOV=V.S(1):DCOV=V.SD(1) :DBT=BTE(1)
4245 RREQ=RESS*EXP(BBT*SQR(LOG(1+SCOV*SCOV)+LOG(1+RCOV*RCOV)))/SQR((1+SCOV^2)/(1
+RCOV^2))
4255 XMOR=XMOR+50:IF RREQ<XMOR THEN GOTO 4285
```

```
4265 REM PRINT " this design failed try increasing dimensions and rerun"
4275 IFAIL(IOUT)=1 : GOTO 4295
4285 IFAIL(IOUT)=0:REM fix up dimensions for rerun
4295 TINC.(IOUT)=ABS(THK*(SQR(RREQ/XMOR)-1))
4305 IF DEFLIM=0 THEN RETURN
4315 RDEF=DEFLEC*EXP(DBT*SQR(LOG(1+DCOV^2)))/SQR(1+DCOV^2)
4325 IF RDEF<DEFLIM THEN JFAIL(IOUT)=Q : GOTO 4345
4335 JFAIL(IOUT)=1 : REM PRINT "THIS DESIGN FAILED DEFLECTION LIMITS"
4345 REM fixup dimensions for rerun for defl limit
4355 TREQ=(RDEF*(THK^3)/DEFLIM)^.333333: REM PRINT "5140 treq=";TREQ
4365 TDINC.(IOUT)=ABS(TREQ-THK)
4375 RETURN
5000 '*********************************************************************
5005 ' sub find-compute moment at notch
5006 '*********************************************************************
5010 DIST=L0/2-(L7+LO): SUMLEN=0: 'find member which contains notch and dist in
mem
5015 FOR I=1 TO NJ/3-1: JJJ=I*7: SUMLEN=SUMLEN+XL(JJJ):IF SUMLEN>=DIST THEN NMEM
=JJJ: DISTX=XL(JJJ)-(SUMLEN-DIST) :GOTO 5021
5020 NEXT I: 'FIND MOMENT AT NOTCH
5021 NOTCHM=0:NMEM=NMEM-2:II=0: FOR I=JJ TO NMEM
5022 IF E(I)< 10 THEN GOTO 5040
5025 ALPHA=E(I)*XI(I)/(XL(I)^3): D2=MCODE(I,2): D3=MCODE(I,3): D5=MCODE(I,5): D6
=MCODE(I,6)
5030 F3=6*XL(I)*ALPHA*(Q(D2)-Q(D5))+2*XL(I)^2*ALPHA*(2*Q(D3)+Q(D6))
5031 F2=12*ALPHA*(Q(D2)-Q(D5))+6*XL(I)*ALPHA*(Q(D3)+Q(D6)): F6=F2*XL(I)-F3:II=II
+1:'IF F3>0 AND F6>0 OR F3<0 AND F6<0 THEN PRINT " ********warning---both momen
ts are same sign**********"
5032 F3=ABS(F3):F6=ABS(F6):IF F3 > F6 THEN MOMENT(II)=(F3-F6)*(XL(I)-DISTX)/XL(I
)+F6
5033 IF F6>F3 THEN  MOMENT(II)=(F6-F3)*DISTX/XL(I)+F3
5034 IF F6=F3 THEN MOMENT(II)=F3
5035 MOMENT(II)=ABS(MOMENT(II)): IF MOMENT(II)> MNOTCH THEN MNOTCH=MOMENT(II):II
MEMB=II
5036 'PRINT "ii=";II,"moment(ii)=";MOMENT(II),"phi=";PHI,"vom="VOM
5037 'PRINT "distx=";DISTX,"f3=";F3,"f5=";F5;"f6=";F6
5040 NEXT I
5045 RETURN
8000 COLOR 14,12:CLS:LOCATE 12,20:PRINT "FATAL ERROR DETECTED (uras)....Check in
put and retry":LOCATE 14,20:PRINT "Error No. :";ERR,"Error Line :";ERL:LSTART=18
4:GSTART=0:RESUME 8010
8010 CHAIN "A:main1s"
10000 BMOE=1800000!: DMOE=1800000!: SMOE=1800000!: TTOP=5: TBOT=5:LTYPE=1:TSTI=3
: SPAN=44:L0=48: WO=40: Q1=1: J=1: K=1: J1=1:H=.75:H1=H: H2=3.5:L=40: L1=40: L2=
48:J2=1
10010 U2(1,2)=1.5: U2(2,2)=1.5: YLT(1,1)=2.75: YLT(2,1)=13.375: YLT(3,1)=24!:YLT
(1,2)=5.5: YLT(2,2)=5.5: YLT(3,2)=5.5:XL1(1,1)=0!: XL1(1,2)=5.5
10015 XL1(2,1)=10.625: XL1(2,2)=16.125: XL1(3,1)=21.25: XL1(3,2)=26.75: XL1(4,1)
=31.875: XL1(4,2)=37.375: XL1(5,1)=42.5: XL1(5,2)=48!
10016 XLOC(1,2)=5.5: XLOC(2,2)=16.125: XLOC(3,2)=26.75: XLOC(4,2)=37.375: XLOC(5
,2)=48
10017 ILOAD=1: SMOR=8000:CRITK=1729:H7=1.5:L7=9:LO=6: R7=.5:V.S=.25:BTR(1)=3.2:
SVCR=.25
10020 RETURN
20000 '*********************************************************************
20010 'sub notchdef...computes adjustment factor to defln of notched stringer
20020 '*********************************************************************
20030 PHI=H7/H2 :H.=H2/SPAN :M.E=LO-(L2-SPAN)/2:M.=M.E/SPAN: XN.=(M.E+L7)/SPAN
20040 A=3:B.=M. + A * H.*(1-PHI) : G.=-XN. + A*H.*(1-PHI) : Q.=SPAN/2 -L7-M.E
20050 X.L=B.-A*H. : X.R=-G.+A*H. :T1.=2*A*H./(1+2*G.) : T2.=A*H.*(1-PHI)/B. :
20060 Z.=LO+L7-R7 : OH=(L2-SPAN)/2 :
20070 T3.=(1-PHI)*T1.:A.= (XTGR-(L2-SPAN)/2)/SPAN:IF A.<0 AND LTYPE >=4 THEN PRI
NT "BAD X VALUE":END
20080 IF H7<=M.E/A AND H7<=Q./A THEN GOSUB 20450:GOTO 20120
20090 IF H7> M.E/A AND H7<=Q./A THEN GOSUB 20490:GOTO 20120
20100 IF H7> M.E/A AND H7> Q./A THEN GOSUB 20520:GOTO 20120 ELSE PRINT "Geometry
```

```
out of program bounds!!!!"
20110 PRINT "input error!!!! wrong notch depth interval":END
20120 IF LTYPE>=3 THEN NLINE =LTYPE-2 ELSE NLINE =0
20130 ON LTYPE GOTO 20140,20210,20250,20250,20250
20140 VOM=(2*Z.-L2)/(Z.^2-L2*(Z.-OH))
20150 P1=16*(XN.^3*(4-3*XN.) + M.^3*(-4+3*M.))/(1-PHI)^3
20160 P2=96*A*H.*(B.^2*(1-B.)+G.^2*(1+G.))/(1-PHI)^2
20170 P3=192*A^2*H.^2*(B.*(-2+3*B.)-G.*(2+3*G.))/(1-PHI)
20180 P4=96*A*H.*C9 + 192*A^2*H.^2*C10 + 96*A^3*H.^3*C11 +C12
20190 XNOTCH=.2*(P1+P2+P3+P4) : '...psi3p
20200 GOTO 20540
20210 'partial uniform load....use psi3p
20220 R.1=(L2-2*XTGR)/2 : VOM=(Z.-XTGR-R.1)/((Z.-XTGR)^2/2-R.1*(Z.-OH)):
20230 GOTO 20150
20240 END
20250 ON NLINE GOTO 20260,20310,20380
20260 VOM=1/(Z.-OH):'center line load ....psi1p
20270 P1=8*(XN.^3-M.^3)/(1-PHI)^3 : P2=12*A*H.*(B.^2 + G.^2)/(1-PHI)^2
20280 P3=48*A^2*H.^2*(B.+G.)/(1-PHI)
20290 P4=-12*A*H.*C1 +48*A^2*H.^2*C2 + 12*A^3*H.^3*C3 + C4
20300 XNOTCH=P1+P2-P3-P4 :IF NLINE=3 THEN GOTO 20390 ELSE GOTO 20540:'...psi1p
20310 'two line loads....use psi2p
20320 IF XTGR>=Z. THEN VOM=1/(Z.-OH) ELSE VOM=0
20330 P1=(12*XN.^2*A.-8*M.^3-4*A.^3)/(1-PHI)^3
20340 P2=12*A*H.*(B.^2-G.*A.)/(1-PHI)^2
20350 P3=24*A^2*H.^2*(A.-2*B.)/(1-PHI)
20360 P4=12*A*H.*C5 -24*A^2*H.^2*C6 + 12*A^3*H.^3*C7 +C8
20370 XNOTCH=(P1+P2+P3+P4)/(A.*(3-4*A.^2)) :IF NLINE=3 THEN GOTO 20400 ELSE GOTO
 20540 :'...psi2p
20380 RAT.D=2*(SLOAD/CLOAD)*A.*(3*SPAN^2-4*A.^2)/(SPAN^3) : GOTO 20270
20390 PSI1P=XNOTCH :GOTO 20310
20400 XNOTCH=(XNOTCH*RAT.D + PSI1P)/(RAT.D+1) : 'psi1 and 2 comb
20410 IF XTGR> Z. THEN VOM=1/(Z.-OH):GOTO 20540
20420 R.1=(CLOAD + 2*SLOAD)/2 : VOM=(SLOAD-R.1)/(R.1*(Z.-OH)-SLOAD*(Z.-XSTGR))
20430 GOTO 20540
20440 END
20450 C1=B.^2+G.^2 : C2=B.+G. : C3=-4*LOG(1-PHI) :C4=1+8*(X.L^3-X.R^3) :C5=G.*A.
-B.^2 : C6=-2*B.+A.
20460 C7=-2*LOG(1-PHI): C8=3*A.+8*X.L^3-12*X.R^2*A. :C9=B.^2*(-1+B.)-G.^2*(1+G.)

20470 C10=B.*(2-3*B.)+G.*(2+3*G.) : C11=(-4+6*(B.-G.))*LOG(1-PHI) : C12=5+16*X.R
^3*(-4+3*X.R)+16*X.L^3*(4-3*X.L) : C13=B.^2+A.^2 : C14=B. : C15=-2*LOG(1-PHI)
20480 C16=3*A.^2 +2*X.L^3 -6*X.R*A.^2 :C17=B.^2*(-1+B.) + G.*A.*(1+G.) : C18=B.*
 (2-3*B.)-A.*(1+2*G.) : C19=2*(-1+A.+3*B.)*LOG(1-PHI) +2*M. -2*X.L : C20=A.+X.L^
3*(4-3*X.L)+2*X.R^2*A.*(-3+2*X.R):RETURN
20490 C1=G.^2 : C2=G. : C3=3-2*LOG(1-PHI)-2*LOG(T2.) :C4=1-8*X.R^3 : C5=G.*A.
20500 C6=A. : C8=3*A.-12*X.R^2*A. : C7= 3-2*LOG(T2.) : C9=-G.^2*(1+G.) :C10=G.*(
2+3*G.):C11=3+2*M. -5*B. -2*A*PHI*H. -2*(1+3*G.)*LOG(1-PHI)-2*(1-3*B.)*LOG(T2.)
20510 C12=5+16*X.R^3*(3*X.R-4) : C13=A.^2:C14=0:C15=3-2*LOG(T2.) : C16=3*A.^2-6
*X.R*A.^2 : C17=G.*A.*(1+2*G.) : C19=2*A.*LOG(1-PHI) + 2*(-1+3*B.)*LOG(T2.) + 3
+2*M. -5*B. :C20=A.+2*X.R^2*A.*(2*X.R-3):RETURN
20520 C1=G.^2*T1.^2 : C2=G.*T1. : C3=3-2*LOG(T2.)-2*LOG(T3.) : C4=0 : C5=G.*A.*T
1.^2 : C6=A.*T1. : C7=3-2*LOG(T2.) : C8=0 : C9= -(G.^2)*(1+G.)*T1.^2 : C10=G.*(2
+3*G.)*T1. : C11=2+2*M. -5*B. -2*(1+3*G.)*LOG(T3.) - 2*(1-3*B.)*LOG(T2.)
20530 C12=0 : C13= A.^2*T1.^2 : C14=0 : C15=3-2*LOG(T2.) : C16=0 : C17=G.*A.*(1+
G.)*T1.^2 : C18=-A.*(1+2*G.)*T1. : C19= 2*A.*LOG(T3.) + 2*(3*B.-1)*LOG(T2.) + 3
+2*M. - 5*B. : C20=0 : RETURN
20540 RETURN
```

```
1520 BINERT = (H1 ^ 3) * U1(1,2)1448 DEFINT I,J,K,N
1449 REM    $include: 'dim1450'
1452 COMMON SUPLN$,SUPLA$,SUPLA1$,SUPLA2$,SUPLA3$
1454 COMMON COSTN$,COSTA$,COSTA1$,COSTA2$,COSTA3$
1456 COMMON ID$,LO,WO,IC2,IE2,Q1,LTYPE,NLINE,PLC,PLC1,XDECK,XTGR,LPP,SPAN,SPAN1,
ILOAD,TLOAD,ULOAD,CLOAD,SLOAD,X,LIM,DLIM(),SY1,SY2
1458 COMMON H,L,O,J,I9,IJ ,YLT(),XLOC(),XL1(),XL2(),FA()
1460 COMMON U(),U1(),U2(),P(),P1(),P2()
1462 COMMON MO,G,ISPAC,TTOP,TSAC,NEB,NCT,NT4,NT3,NT2,ILEA
1464 COMMON H1,L1,O1,K,JK
1466 COMMON MO1,G1,ISPAC1,TBOT,BSAC,NBT,NB4,NB3,NB2
1468 COMMON H2,L2,J1,IJ1,FT
1470 COMMON MO2,G2,ISPAC2,TSTI,SSAC,H7,L7,LO,R7,STR4,LSTART,ZLOC,HV
1472 COMMON IW1,IW2,IEO,PR
1474 COMMON W.TOT(),W.DEF(),Y.W(),TINC.(),TDINC.(),IFAIL(),JFAIL()
1476 COMMON CR(),EL(),CPT(),LTFR(),IL,NTRP(),PCL()
1478 COMMON DMOR,DMOE,BMOE,BMOR,SMOR,SMOE,BVCR,BVCE,DVCR,DVCE,SVCR,SVCE
1479 COMMON V.S(),V.SD(),BTR(),BTE(),CRITK,G1T,G1B,G3T,G3B,MOE.,IG,ISOW$,ILVAR,I
REP,IDRW,IDFLAG,GSTART,JEFLAG,MAXAVG$,LOADF
1481 DIM XQ(33),ZQ(33),ZK(33,16),SS(33,16),DIAG(33),P3(10),P4(10)
1482 ON ERROR GOTO 8000
1485 '*******************************************************************************
1486 'Pallet Design System...P D S...                          Version 1.0
1487 '
1488 'SUB1500..Solution of Racked Support Conditions--equal sized stringers
1489 '        ..Sets Material Properties
1490 '
1491 'J.R.Loferski,T.E.McLain, and H.R.Glasser,VIRGINIA TECH,Blacksburg,Va
1492 '*******************************************************************************
1501 G1TT=G1T:G3TT=G3T:G1BB=G1B:G3BB=G3B:XTE9=X : UL9=ULOAD : TL9=TLOAD : CL9=CL
OAD : SL9=SLOAD:
1502 IF J1>1 AND LTYPE<3 AND U2(1,2)<>U2(2,2) THEN GOTO 1850:'check if ras is al
ready done
1505 '*************** Begin RAS analysis for equal sized stringers ***********
1506 IF ILOAD=2 THEN ON  LTYPE GOTO 1507,1507,1508,1509,1509 ELSE GOTO 1510
1507 W.TOT(1)=TLOAD :GOTO 1510
1508 W.TOT(1)=CLOAD :GOTO 1510
1509 W.TOT(1)=SLOAD*2+CLOAD
1510 W.TOT(2)=W.TOT(1)
1511 DKSPAN = WO -2*O-U2(1,2) : BSTIFF = 0 : IF  Q1 = 0 GOTO 1526
1515 IF  K > 1 GOTO 1525
1520 BINERT = (H1 ^ 3) * U1(1,2) * (TBOT - 2) / 12 : BSTIFF = BMOE * BINERT /(DK
SPAN ^ 3) : GOTO 1526
1525 BINERT = (H1 ^ 3) * (BSAC - 2 * U1(1,2)) / 12 : BSTIFF = BMOE * BINERT / (D
KSPAN ^ 3):TW=0
1526 IF SPAN < .1 GOTO 1850
1529 IX=YLT(0,0): IX1=YLT(0,0) :IF TTOP MOD 2 <> 0 THEN IX1=IX1-1
1530 SECMOD = (H2 * H2) * U2(1,2) / 6
1531 TW=0:IF LTYPE > 2 THEN GOTO 1539
1532 PFLAG=IX1
1533 FOR I=1 TO IX1: P3(I)=0: IF XTGR <= XLOC(I,2) THEN P3(I)=XLOC(I,2)-XTGR:IF
P3(I)>YLT(I,2) THEN P3(I)=YLT(I,2)
1534 IF LTYPE =2 AND H7>0 AND XLOC(I,2) <=LO+L7 THEN PFLAG=I
1535 IF IX1< YLT(0,0) THEN P3(IX)=2*(LO/2-XTGR): IF P3(IX) > YLT(IX,2) THEN  P3(
IX)=YLT(IX,2):
1536 TW=TW+P3(I): NEXT I:TW=TW*2
1537 IF IX1< YLT(0,0) THEN P3(IX)=2*(LO/2-XTGR): IF P3(IX) > YLT(IX,2) THEN  P3(
IX)=YLT(IX,2):
1538 IF IX1< YLT(0,0) THEN TW=TW+P3(IX)
1539 STINER = (H2 ^ 3) * U2(1,2) / 12 : STSTIF = SMOE * STINER / (SPAN ^ 3) : TI
NERT = (H ^ 3) * TSAC / 12 : TSTIFF = DMOE * TINERT / (DKSPAN ^ 3)
1540 REM PRINT"stiner=":STINER . "biner=":BINERT."tinert=":TINERT
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```
1545 REM PRINT "ststiff=";STSTIF,"tstiff=";TSTIFF,"bstiff=";BSTIFF
1547 IF H7>0 THEN GOSUB 4385: STSTIF=STSTIF/XNOTCH
1548 IF LTYPE > 2 THEN GOTO 1561
1550 R = STSTIF / (TSTIFF + BSTIFF) : FLR = LOG(R) / 2.302585 : PLOAD = 53.193 +
    6.09 * FLR - .147 * R
1551 IF R > 27 THEN R=27: 'keep the r variable in range of regression
1552 IF R< .1 THEN R=.1
1553 FLR = LOG(R) / 2.302585 : PLOAD = 53.193 + 6.09 * FLR - .147 * R
1555 XK = 15.57 + R * .239 - 14.875 * FLR : REM PRINT "smoe=";SMOE, "r=";R,"xk="
    ;XK,"dkspan=";DKSPAN,"str4=";STR4
1556 IF TSTI=4 THEN R=R*(STR4^3)/(DKSPAN^3):SP=(DKSPAN-2*STR4-U2(1,2))/DKSPAN:PL
    OAD=(.3087-.0079*R+.1399*SP)*100:XK=-.123-1.531*R+.16*(1/R):XD=1.5989-4.518*R+.4
    258*(1/R)
1557 IF TSTI=4 AND XK>13.3 THEN XK=13.3
1558 IF TSTI=4 AND XD>30 THEN XD=30
1559 IF TSTI=2 THEN PLOAD=50:XK=0:XD=0: ' two stringer adjustment
1560 FXK=1-XK/100:REM PRINT "r ratio=";R, " xk=" ;XK, " xd=";XD, " pload=";PLOAD

1561 IF ILOAD = 1 OR H7>0 THEN SV2=V.S(1)*V.S(1) :VSTR2=SVCR*SVCR: SBAR=SMOR*SQR
    ((1+SV2)/(1+VSTR2))/EXP(BTR(1)*SQR(LOG(1+SV2)+LOG(1+VSTR2)))
1565 IF H7=0 GOTO 1580
1570 XXX=LO+L7-(L2/2-SPAN/2): ZZ=LO+L7: XNUMER=SBAR*U2(1,2)*(H2*H2)/6
1575 DEN=((1/(1-1.26*PHI))+VOM*H2*(1.13*PHI+.295)): XMMAX=XNUMER/DEN:REM end com
    mon notch stuff
1580 IF ILOAD = 1 THEN XM=(SBAR*(H2*H2)*U2(1,2))/6: ' max allowable moment
1581 IF LTYPE > 2 GOTO 1846: 'jump for line loads
1582 IF ILOAD=1 GOTO 1600: 'goto analysis for uniform loads
1585 Q = PLOAD / 100 * TLOAD
1590 REM PRINT "q=";Q,"tload=";TLOAD
1595 IF LTYPE<>2 AND H7=0 THEN GOTO 1660
1600 REM analysis option
1601 BBB=LO/2: IF LTYPE=2 AND H7>0 THEN BBB=LO+L7
1605 SUMWD = 0 : FOR I = 1 TO PFLAG : IF P3(I)<> YLT(I,2) THEN XA= BBB-XLOC(I,2)
    +P3(I)/2 ELSE XA = BBB - YLT(I,1)
1606 SUMWD = SUMWD + XA * P3(I) * FXK : NEXT I
1607 IF ILOAD=2 THEN GOTO 1660
1610 REM PRINT "sumwd=";SUMWD
1625 DENOM = (SPAN / 4) * FXK - (1/ TW) * SUMWD: XM=(SBAR*(H2*H2)*U2(1,2))/6
1630 REM PRINT "denom=";DENOM, "unotched moment=xm=";XM
1635 IF H7=0 GOTO 1650 : REM notch moment
1640 XM=XMMAX*((SPAN^2)/(4*XXX*(SPAN-XXX)))
1641 IF LTYPE = 2 THEN Q=ABS(XMMAX/((SUMWD/TW)-(XXX*FXK/2))): GOTO 1651
1645 REM -notched moment=xm=";XM,xmmax=max allowalbe moment at notch
1650 Q = XM / DENOM
1651 TOTLD=Q/(PLOAD/100)
1655 REM PRINT "1731--analysis output---q=";Q,"totload=";TOTLD
1660 PPERIN = Q / TW : LOCATE 12,1
1665 FOR I = 1 TO IX : P4(I) = PPERIN * P3(I) * FXK : NEXT I
1670 IF ILOAD = 1 GOTO 1740
1675 QMOMEN = (Q / 2) * (SPAN / 2) * FXK : PMOMEN = 0
1680 FOR I = 1 TO IX1: IF P3(I)<> YLT(I,2) THEN XARM=LO/2-XLOC(I,2)+P3(I)/2 ELSE
    XARM = LO / 2 - YLT(I,1)
1685 PMOMEN = PMOMEN + P4(I) * XARM : NEXT I
1690 XMOMEN=QMOMEN-PMOMEN
1695 REM PRINT "1695-design option--xmomen=";XMOMEN,"qmomen=";QMOMEN,"pmomen=";P
    MOMEN
1700 IF H7=0 GOTO 1730 :REM notch design option
1705 REM PRINT "max allow moment at notch (xmmax)=";XMMAX
1710 XMNOT=(4*XXX*(SPAN-XXX)/(SPAN*SPAN))*XMOMEN : IF   LTYPE =2      THEN XMNOT=
    ABS(Q*((SUMWD/TW)-(XXX*FXK/2)))
1715 XMMAX=XMMAX+25: IF ABS(XMNOT) >ABS(XMMAX) THEN IFAIL(1)=1: REM PRINT "faile
    d at notch XMNOT= ";XMNOT,"xmmax=";XMMAX
1720 IF ABS(XMNOT) <=ABS(XMMAX) THEN IFAIL(1)=0: REM PRINT "strength at notch ok
    -"
1725 GOTO 1740 :REM end notch design
1730 STRESS = XMOMEN / SECMOD
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1735 REM PRINT "1735 ---no notch--design option- secmod=";SECMOD,"stress=";STRES
S
1740 DEFL = O : RECDST = (LO - SPAN) / 2
1750 FOR I = 1 TO IX1 : IF P3(I)<> YLT(I,2) THEN A=XLOC(I,2)-RECDST-P3(I)/2 ELSE
A = YLT(I,1) - RECDST
1751 DIST=3*(SPAN ^ 2)-4*(A^2):IF TSTI=4 THEN P4(I)=(P4(I)/FXK)*(1-XD/100)
1752 DEFL=DEFL+A*P4(I)*DIST/(24*SMOE*STINER)
1755 NEXT I
1756 IF IX1 < YLT(0,0) AND TSTI=4 THEN P4(IX1+1)=(P4(IX1+1)/FXK)*(1-XD/100)
1760 IF IX1 < YLT(0,0) THEN DEFL = DEFL + P4(IX1+1) * (SPAN ^ 3) / (48 * SMOE *
STINER)
1761 REM PRINT "deflection=";DEFL
1765 IF H7>O THEN DEFL=ABS(DEFL*XNOTCH) :RDEF=DEFL*EXP(BTE(1)*SQR(LOG(1+V.SD(1)^
2)))/SQR(1+V.SD(1)^2): IF RDEF > DLIM(1) THEN JFAIL(1)=1 ELSE JFAIL(1)=0 :REM co
rrected notch defl
1770 IF ILOAD=1 THEN W.TOT(1)=ABS(TOTLD)
1775 Y.W(1)=ABS(DEFL)
1780 IF ILOAD=2 AND H7>O THEN GOTO 1850 : REM design for notch finished
1785 DEFLIM=DLIM(1):THK=H2:XMOR=SMOR:IOUT=1:RCOV=SVCR:DEFLEC=DEFL: RESS=STRESS
1790 IF ILOAD =2 THEN GOSUB 4215
1795 IF ILOAD=2 THEN GOTO 1850 : REM finished design option
1800 IF DLIM(1) < .0001 GOTO 1850 : REM find max load for defl limit
1805 TLOAD=2000 : Q=PLOAD/100*TLOAD : PPERIN = Q / TW
1806 IF TSTI=4 THEN FXK=1-XD/100
1810 FOR I = 1 TO IX : P4(I) = PPERIN * P3(I) * FXK : NEXT I
1815 DEFL = O : RECDST = (LO - SPAN) / 2
1820 FOR I = 1 TO IX1 : IF P3(I)<>YLT(I,2) THEN A=XLOC(I,2)-RECDST-P3(I)/2 ELSE
A = YLT(I,1) - RECDST :
1825 DIST = 3 * (SPAN * SPAN) - 4 * (A * A) : DEFL = DEFL + A * P4(I) * DIST / (
24 * SMOE * STINER)
1830 NEXT I
1835 IF IX1 < YLT(0,0) THEN DEFL = DEFL + P4(IX1+1) * (SPAN ^ 3) / (48 * SMOE *
STINER) : REM chain here for notch defl limit stuff*********
1839 DELTS=DLIM(1)*SQR(1+V.SD(1)^2)*EXP(-BTE(1)*SQR(LOG(1+V.SD(1)^2))) : IF H7>O
THEN DEFL=DEFL*XNOTCH
1840 REM-find load to meet defl criteria
1845 DEFL=ABS(DEFL) : QQQ=DELTS*Q/DEFL : TOTLD=QQQ/(PLOAD/100): W.DEF(1)=ABS(TOT
LD):GOTO 1850
1846 IF LTYPE > 2 THEN GOSUB 6100
1847 '***************** Begin RAD Analysis ********************************
1850 IF SPAN1 < .1 GOTO 3755
1851 IF Q1=0 AND IG=0 THEN GOTO 3755
1852 IF IG=1 AND Q1=0 THEN GOSUB 6800: GOTO 3755
1855 IF LTYPE>=3 THEN NLINE =LTYPE-2 ELSE NLINE =0
1860 F210 = 0 : F310 = 0 : F610 = 0 : F315 = 0 : F515 = 0 : F615 = 0
: F215 = 0:F314=0:F514=0:F614=0:F214=0: F21=0: F31=0: F51=0: F61=0: F22=0:F32=0:
F52=0: F62=0: F23=0: F33=0: F53=0: F63=0
1865 MLOAD = 0 : SIGA = 0 : SIGB = 0 : SIGMAX = 0 : SIGI = 0 : RADLTH = WO - 2 *
U2(1,2)-2*O: SUPLOC = RADLTH - SPAN1
1870 IFLAG = 2 : IF SUPLOC > .2 THEN IFLAG = 1
1875 IF SUPLOC < -.2 THEN IFLAG = 3
1880 IF IFLAG <> 1 GOTO 1910
1885 YL2 = SUPLOC / 2 : YL3 = U2(1,2) / 2 : YL4 = YL3 : YL10 = RADLTH / 2 - 1
1890 IF TSTI = 4 THEN YL10 = STR4 - U2(1,2) / 2
1895 YL1 = YL10 -YL2 : IF YL1 < .2 THEN PRINT "Support is placed incorrectly"
: YN$ = INPUT$(1)
1900 YL14 = RADLTH / 2 - YL10  : YL15 = YL14 : IF YL14 < .2 THEN PRINT "Members
14 and 15 are too short"  : YN$ = INPUT$(1)
1905 GOTO 1965
1910 IF IFLAG <> 2 GOTO  1935
1915 YL3 = U2(1,2) / 2 : YL4 = YL3 : YL10 = RADLTH / 2 - 1
1920 IF TSTI = 4 THEN YL10 = STR4 - U2(1,2) / 2
1925 YL1 = YL10 / 2 : YL2 = YL1 : YL14 = RADLTH / 2 - YL10 : YL15 = YL14 : IF YL
14 < .2 THEN PRINT "Members 14 and 15 are too short"
1930 GOTO 1965
1935 IF IFLAG <> 3 THEN PRINT "Something is wrong with flag"
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1940 YL3 = ABS(SUPLOC) / 2 : YL4 = U2(1,2) - YL3 : IF YL4 > .2 GOTO 1950
1945 YL4 = .2 : YL3 = U2(1,2) - YL4
1950 IF YL3 < .2 THEN PRINT "Something is wrong with Stringer or Span"
1951 IF IG=1 THEN YL3=(SPAN1-RADLTH)/2: YL4=(WO-2*O-SPAN1)/2:IF YL4 <.2 THEN YL4
=.2
1955 YL10 = RADLTH / 2 - 1 : IF TSTI = 4 THEN YL10 = STR4 - U2(1,2) / 2
1960 YL1 = YL10 / 2 : YL2 = YL1 : YL14 = RADLTH / 2 - YL10 : YL15 = YL14
1965 YL7 = U2(1,2) : XLST = H2 + H / 2 + H1 / 2
1970 BINERT = (H1 ^ 3) * BSAC / 12 : TINERT = ( H ^ 3) * TSAC / 12 : VINERT = 83
3.3
1975 BAREA = H1 * BSAC : TAREA = H * TSAC : VAREA = 100
1980 BSEC = BSAC * H1 * H1 / 6 : TSEC = TSAC * H * H / 6 : VE = 10000000#
1985 FOR I = 1 TO 33 : XQ(I) = 0 : FOR I9 = 1 TO 16 : ZK(I,I9) = 0 : NEXT I9 : N
EXT I
1990 EAB = DMOE * BAREA : EAT = DMOE * TAREA : EIB = DMOE * BINERT : EIT = DMOE
* TINERT : EIST = VE * VINERT : EAST = VE * VAREA: EAST7=EAST:EIST7=EIST
1991 IF IG=1 THEN EE=EAB:EAB=EAT: EAT=EE: EE=EIB: EIB=EIT: EIT=EE: EAST7=0: EIST
7=0
1995 EIB12 = EIB * 12 : EIT12 = EIT * 12 : EIB6 = EIB * 6 : EIT6 = EIT * 6 : EIB
4 = EIB * 4 : EIT4 = EIT * 4 : EIB2 = EIB * 2 : EIT2 = EIT * 2
2000 EIST6 = EIST * 6 / (XLST ^ 2) : EIST12 = EIST * 12 / (XLST ^ 3) : EIST4 = E
IST * 4 / XLST : EIST2 = EIST * 2 / XLST
2005 G4 = 0: IF IG=1 THEN GG=G1T: G1T=G1B: G1B=GG: GG=G3T: G3T=G3B: G3B=GG
2010 ZK(1,1) = EAB / YL1 + EAB / YL2 : ZK(1,4) = -EAB / YL2 : ZK(1,12) = -EAB /Y
L1
2015 ZK(2,1) = EIB12 / (YL1 ^ 3) + EIB12 / (YL2 ^ 3) : ZK(2,2) = EIB6 / (YL1 ^ 2
) - EIB6 / (YL2 ^ 2) : ZK(2,4) = -EIB6 / (YL2 ^ 2) : ZK(2,12) = -EIB12 / ( YL1 ^
3) : ZK(2,13) = EIB6 / ( YL1 ^ 2)
2020 ZK(3,1) = EIB4 * (1 / YL1 + 1 / YL2) : ZK(3,3) = EIB2 / YL2 : ZK(3,11) = -
EIB6 / (YL1 ^ 2) : ZK(3,12) = EIB2 / YL1
2025 ZK(4,1) = EAB * (1 / YL2 + 1 / YL3) : ZK(4,3) = -EAB / YL3
2030 ZK(5,1) = EIB4 * (1 / YL2 + 1 / YL3) : ZK(5,3) = EIB6 / (YL3 ^ 2) : ZK(5,4)
= EIB2 / YL3
2035 ZK(6,1) = EAB * (1 / YL3 + 1 / YL4) : ZK(6,13) = -EAB / YL4
2040 ZK(7,1) = EIB12 * (1 / (YL3 ^ 3) + 1 / (YL4 ^ 3)) : ZK(7,2) = EIB6 * ( 1 /
(YL3 ^ 2) - 1 /(YL4 ^ 2)) : ZK(7,4) = -EIB12 / (YL4 ^ 3) : ZK(7,13) = -EIB6 / (Y
L4 ^ 2)
2045 ZK(8,1) = EIB4 * (1 / YL3 + 1 / YL4) : ZK(8,3) = EIB6 / (YL4 ^ 2) : ZK(8,12
) = EIB2 / YL4
2050 ZK(9,1) = G1B + EAST7 / YL7 : ZK(9,7) = -EAST7 / YL7 : ZK(9,10) = -G1B
2055 ZK(10,1) = EIB12 / (YL4 ^ 3) + 12 * EIST7 / (YL7 ^ 3) : ZK(10,2) = 6 * EIST
7/ (YL7 ^ 2) : ZK(10,7) = -12 * EIST7/ (YL7 ^ 3) : ZK(10,8) = 6 * EIST7/ (YL7 ^
2) : ZK(10,10) = EIB6 / (YL4 ^ 2)
2060 ZK(11,1) = G3B + 4 * EIST7/ YL7 : ZK(11,6) = -6 * EIST7/ (YL7 ^ 2) : ZK(11,
7) = 2 * EIST7/ YL7 : ZK(11,9) = -G3B
2065 ZK(12,1) = EAB * (1 / YL1 + 1 / YL14) + G1B : ZK(12,9) = -G1B
2070 ZK(13,1) = EIB12 * (1 / (YL1 ^ 3) + 1 / (YL14 ^ 3)) + EAST / XLST : ZK(13,2
) = EIB6 * (1 / (YL14 ^ 2) - 1 / (YL1 ^ 2)) : ZK(13,10) = -EIB12 / (YL14 ^ 3) :
ZK(13,15) = -EAST / XLST
2075 ZK(14,1) = EIB4 * (1 / YL1 + 1 / YL14) + G3B : ZK(14,8) = -G3B : ZK(14,9) =
-EIB6 / (YL14 ^ 2)
2080 ZK(15,1) = EAST7/ YL7 + EIST12 : ZK(15,3) = -EIST6 : ZK(15,9) = -EIST12 : Z
K(15,11) = -EIST6
2085 ZK(16,1) = G4 + 12 * EIST7/ (YL7 ^ 3) + EAST / XLST : ZK(16,2) = -6 * EIST7
/ (YL7 ^ 2) : ZK(16,9) = - EAST / XLST
2090 ZK(17,1) = 4 * EIST7/ YL7 + EIST4 : ZK(17,7) = EIST6 : ZK(17,9) = EIST2
2095 ZK(18,1) = EAB / YL4 + G1B
2100 ZK(19,1) = EIB4 / YL4 + G3B
2105 ZK(20,1) = EIST12 + G1B : ZK(20,2) = -EIST6 : ZK(20,7) = -EIST12 : ZK(20,9)
= -EIST6
2110 ZK(21,1) = G3B + EIST4 : ZK(21,6) = EIST6 : ZK(21,8) = EIST2
2115 ZK(22,1) = EIB12 / (YL14 ^ 3) + EAST / XLST : ZK(22,8) = -EAST / XLST
2120 ZK(23,1) = EIST12 + G1T : ZK(23,3) = EIST6 : ZK(23,10) = -G1T
2125 ZK(24,1) = EAST / XLST + EIT12 / (YL10 ^ 3) : ZK(24,4) = -EIT12 / (YL10 ^ 3
) : ZK(24,8) = EIT6 / (YL10 ^ 2) : ZK(24,10) = ZK(24,8)
2130 ZK(25,1) = EIST4 + G3T : ZK(25,9) = -G3T
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2135 ZK(26,1) = EIST12 + G1T : ZK(26,3) = EIST6 : ZK(26,5) = -G1T
2140 ZK(27,1) = EIT12 * (1 / (YL10 ^ 3) + 1 / (YL15 ^ 3)) + EAST / XLST : ZK(27,
3) = -EIT12 / (YL15 ^ 3) : ZK(27,5) = EIT6 * ((1 / YL15 ^ 2) - 1 / (YL10 ^ 2)) :
 ZK(27,7) = -EIT6 / (YL10 ^ 2)
2145 ZK(28,1) = EIST4 + G3T : ZK(28,4) = -G3T
2150 ZK(29,1) = EIT12 / (YL15 ^ 3) + EAST / XLST : ZK(29,3) = -EIT6 / (YL15 ^ 2)

2155 ZK(30,1) = EAT * (1 / YL10 + 1 / YL15 ) + G1T : ZK(30,3) = -EAT / YL10
2160 ZK(31,1) = EIT4 * (1 / YL10 + 1 / YL15) + G3T : ZK(31,3) = EIT2 / YL10
2165 ZK(32,1) = EAT / YL10 + G1T : ZK(33,1) = EIT4 / YL10 + G3T
2170 IF IFLAG <> 1 GOTO 2190
2175 ZK(1,3) = -EAB / YL2 : ZK(1,4) = 0 : ZK(2,1) = EIB4 * (1 / YL1 + 1 / YL2) :
 ZK(2,2) = 0 : ZK(2,3) = EIB6 / (YL2 ^ 2) : ZK(2,4) = EIB2 / YL2 : ZK(2,12) = -E
IB6 / (YL1 ^ 2) : ZK(2,13) = EIB2 / YL1
2180 ZK(3,1) = EAB * (1 / YL2 + 1 / YL3) : ZK(3,3) = 0 : ZK(3,4) = -EAB / YL3 :
ZK(3,11) = 0 : ZK(3,12) = 0
2185 ZK(4,1) = EIB12 * (1 / (YL2 ^ 3) + 1 / (YL3 ^ 3)) + G4 : ZK(4,2) = EIB6 * (
1 / (YL2 ^ 2) - 1 / (YL3 ^ 2)) : ZK(4,3) = 0 : ZK(4,4) = -EIB12 / (YL3 ^ 3) : ZK
(4,5) = -EIB6 / (YL3 ^ 2) : ZK(4,13) = -G4
2190 IF IFLAG <> 3 GOTO 2210
2195 ZK(2,4) = -EIB12 / (YL2 ^ 3) : ZK(2,5) = -EIB6 / (YL2 ^ 2) : ZK(3,3) = EIB6
 / (YL2 ^ 2) : ZK(3,4) = EIB2 / YL2 : ZK(4,3) = 0 : ZK(4,4) = -EAB / YL3
2200 ZK(5,1) = EIB12 * (1 / (YL2 ^ 3) + 1 / (YL3 ^ 3)) + G4 : ZK(5,2) = EIB6 * (
1 / (YL2 ^ 2) - 1 / (YL3 ^ 2)) : ZK(5,3) = 0 : ZK(5,4) = -EIB6 / (YL3 ^ 2) : ZK(
5,12) = -G4
2205 ZK(6,1) = EIB4 * (1 / YL2 + 1 / YL3) : ZK(6,3) = EIB2 / YL3 : ZK(6,13) = 0
: ZK(7,1) = EAB * (1 / YL3 + 1 / YL4) : ZK(7,2) = 0 : ZK(7,4) = 0 : ZK(7,12) = -
EAB / YL4 : ZK(7,13) = 0
2206 IF IG=1 THEN G2B=1E+08: ZK(4,1)=EAB/YL2+EAB/YL3+G1B: ZK(4,12)=-G1B:ZK(5,1)=
EIB12/(YL2^3)+EIB12/(YL3^3)+G2B:ZK(5,12)=-G2B: ZK(6,1)=EIB4/YL2+EIB4/YL3+G3B: ZK
(6,12)=-G3B:ZK(15,1)=EIST12+G1B:ZK(16,1)=EAST/XLST+G2B:ZK(17,1)=EIST4+G3B
2210 IF TSTI = 4 GOTO 2220 ELSE ZK(27,1) = (EIT12/(YL10 ^ 3)) + (EIT12/(YL15 ^ 3
)) : ZK(28,1) = G3T
2215 ZK(13,1) = (EIB12/(YL1 ^ 3)) + (EIB12/(YL14 ^ 3)) : ZK(13,15) = 0 : ZK(20,1
) = G1B : ZK(20,2) = 0 : ZK(20,7) = 0 : ZK(20,9) = 0 : ZK(21,1) = G3B : ZK(21,6)
 = 0 : ZK(21,8) = 0 : ZK(26,1) = G1T : ZK(26,3) = 0
2220 IF TSTI = 3 GOTO 2230
2225 ZK(22,1) = EIB12 / (YL14 ^ 3) : ZK(22,8) = 0 : ZK(29,1) = EIT12 / (YL15 ^ 3
)
2230 REM FOR I = 1 TO 33 : FOR I9 = 1 TO 16
2235 REM PRINT "i=";I;ZK(I,I9); : NEXT I9 : NEXT I
2240 ON LTYPE GOTO 2245,2250,2365,2365,2365 :REM uniform load first
2245 ULOAD = ULOAD /RADLTH : IF IG=1 THEN ULOAD=RADLTH*ULOAD/SPAN1
2246 GOTO 2257
2250 REM ltype=2--partial uniform load
2255 IF IG=0 THEN X = X - U2(1,2)
2256 ULOAD=ULOAD/(RADLTH-2*X): IF IG=1 THEN ULOAD=ULOAD*(RADLTH-2*X)/(SPAN1-2*X)

2257 IF IG=0 GOTO 2305: REM winged pallet fixed end forces
2258 F214=-ULOAD*YL14/2:F514=F214: F614=ULOAD*(YL14^2)/12: F314=-F614: XX1=0: XX
2=0: XX3=0: IF X <= YL3 AND X > 0 THEN XX3=X: MLOAD=3
2259 SUML = YL3 + YL2: IF X > YL3 AND X <= SUML THEN XX2=X-YL3: MLOAD = 2: XX3=
YL3
2260 SUML2 = YL3 + YL2 + YL1: IF X > SUML AND X <= SUML2 THEN XX1 = X-SUML: XX3=
YL3: XX2 = YL2: MLOAD = 1
2261 F21=(-ULOAD/(2*YL1^3))*((2*YL1^3)*(YL1-XX1)-(2*YL1)*(YL1^3-XX1^3)+(YL1^4-XX
1^4)): F31=(-ULOAD/(12*YL1*YL1))*((6*YL1^2)*(YL1^2-XX1^2)-(8*YL1)*(YL1^3-XX1^3)+
3*(YL1^4-XX1^4))
2262 F51=-ULOAD*(YL1-XX1)-F21: F61=(ULOAD/(12*YL1*YL1))*((4*YL1)*(YL1^3-XX1^3)-3
*(YL1^4-XX1^4)): F52=(-ULOAD/(2*YL2^3))*((2*YL2^3)*(YL2-XX2)-(2*YL2)*(YL2^3-XX2^
3)+(YL2^4-XX2^4))
2263 F62=(+ULOAD/(12*YL2*YL2))*((6*YL2^2)*(YL2^2-XX2^2)-(8*YL2)*(YL2^3-XX2^3)+3*
(YL2^4-XX2^4)): F22=-ULOAD*(YL2-XX2)-F52: F32=(-ULOAD/(12*YL2^2))*((4*YL2)*(YL2^
3-XX2^3)-3*(YL2^4-XX2^4))
2264 F53=(-ULOAD/(2*YL3^3))*((2*YL3^3)*(YL3-XX3)-(2*YL3)*(YL3^3-XX3^3)+(YL3^4-XX
3^4)): F63=(+ULOAD/(12*YL3^2))*((6*YL3^2)*(YL3^2-XX3^2)-(8*YL3)*(YL3^3-XX3^3)+3*
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(YL3^4-XX3^4))
2265 F23=-ULOAD*(YL3-XX3)-F53: F33=(-ULOAD/(12*YL3^2))*((4*YL3)*(YL3^3-XX3^3)-3*
(YL3^4-XX3^4))
2266 XQ(22)=-F214: XQ(14)=-(F314+F61): XQ(13)=-(F214+F51): XQ(3)=-(F31-F32): XQ(
2)=-(F21+F22): XQ(5)=-(F52+F23): XQ(6)=+(F62+F33): XQ(8)=+F63
2267 REM finished wing load vectors
2295 GOTO 2475
2305 IF X < YL10 THEN MLOAD = 10
2315 X4 = (YL10 ^ 4) - (X ^ 4) : X3 = (YL10 ^ 3) - (X ^ 3) : X2 = (YL10 ^ 2) - (
X ^ 2) : X1 = YL10 - X
2325 F210 = (ULOAD / ((YL10 ^ 3) * 2)) * ((2 * (YL10 ^ 3) * X1) - (2 * YL10 * X3
) + X4) : W12 = ULOAD / ((YL10 ^ 2) * 12)
2335 F310 = W12 * ((6 * YL10 * YL10 * X2) - (8 * YL10 * X3) + (3 * X4)) : F510 =
 ULOAD * X1 - F210 : F610 = -W12 * ((4 * YL10 * X3) - (3 * X4))
2345 F215 = ULOAD * YL15 / 2 : F515 = F215 : F315 = ULOAD * YL15 * YL15 / 12 : F
615 = -F315
2355 XQ(24) = -F210 : XQ(27) = - (F510 + F215) : XQ(29) = -F515 : XQ(31) = - (F6
10 + F315) : XQ(33) = -F310 : GOTO 2475
2365 IF NLINE = 1 GOTO 2415 : REM line loads
2375 IF NLINE <> 2 GOTO 2395
2385 X = X - U2(1,2) : GOTO 2415
2395 IF NLINE <> 3 THEN PRINT " ERROR in number of line-loads--- retry ---" : ST
OP
2405 X = X - U2(1,2)
2415 CLOAD = CLOAD / 2
2425 IF X > YL10 THEN PRINT "** ERROR** - line-load is not placed correctly--che
ck input and rerun": YN$ = INPUT$(1)
2435 IF X < YL10 THEN MLOAD = 10
2445 F210 = (SLOAD / (YL10 ^ 3)) * (YL10 + 2 * X) * ((YL10 - X) ^ 2) : F310 = (S
LOAD * X) * (((YL10 - X) / YL10) ^ 2) : F510 = SLOAD - F210
2455 F610 = -SLOAD * (YL10 - X) * ((X / YL10) ^ 2)
2465 XQ(24) = -F210 : XQ(27) = -F510 : XQ(29) = -CLOAD : XQ(31) = -F610 : XQ(33)
 = -F310
2475 FOR I = 1 TO 33 : ZQ(I) = XQ(I) : REM IF XQ(I) <> 0 THEN PRINT I,XQ(I)
2485 NEXT I
2495 FOR I = 1 TO 33 : FOR I9 = 1 TO 16 : SS(I,I9) = ZK(I,I9) : NEXT I9 : NEXT I

2505 GOSUB 3785
2515 REM CLS : FOR I = 1 TO 33 : IF XQ(I) <> 0 THEN PRINT I,XQ(I)
2525 REM NEXT I
2535 IF IFLAG <> 1 GOTO 2595
2545 IF XQ(4) <= XQ(16) GOTO 2595
2555 G4 = 10000000#
2565 FOR I = 1 TO 33 : XQ(I) = ZQ(I) : FOR I9 = 1 TO 16 : ZK(I,I9) = SS(I,I9) :
NEXT I9 : NEXT I
2575 ZK(4,1) = EIB12 * (1 / (YL2 ^ 3) + 1 / (YL3 ^ 3)) + G4 : ZK( 4,13) = - G4 :
 GOSUB 3785
2585 GOTO 2646
2595 IF IFLAG <> 2 GOTO 2646
2605 IF XQ(16) >= 0 GOTO 2646
2615 G4 = 10000000#
2625 FOR I = 1 TO 33 : XQ(I) = ZQ(I) : FOR I9 = 1 TO 16 : ZK(I,I9) = SS(I,I9) :
NEXT I9 : NEXT I
2635 ZK(16,1) = G4 + 12 * EIST / (YL7 ^ 3) + EAST / XLST
2645 GOSUB 3785:REM finished pallet analysis- find stresses from deflections
2646 IF IG=0 THEN GOTO 2685: REM start winged pallet stresses
2650 F1=(EAB/YL14)*XQ(12): F2=(EIB12/(YL14^3))*(XQ(13)-XQ(22))+(EIB6/(YL14^2))*X
Q(14): F3=(EIB6/(YL14^2))*(XQ(13)-XQ(22))+(EIB4/YL14)*XQ(14): F6=F2*YL14-F3
2651 F2=F2+F214: F3=F3+F314: F6=F6+F614: SIGI=ABS(F1/TAREA)+ABS(F3/TSEC): SIG2=A
BS(F1/TAREA)+ABS(F6/TSEC): SIGMAX=SIGI:IMEMB=14: IF SIGMAX < SIG2 THEN SIGMAX =
SIG2
2652 IF LTYPE <> 1 OR LTYPE <> 2 GOTO 2655
2653 XMAX=ABS(F2/ULOAD): SIGI=0: IF XMAX >= YL14 GOTO 2655
2654 XMMAX=(F2*XMAX/2)-F3: SIGI=ABS(F1/TAREA)+ABS(XMMAX/TSEC): IF SIGI > SIGMAX
THEN SIGMAX = SIGI
2655 F1=(EAB/YL1)*(XQ(1)-XQ(12)): F2=(EIB12/(YL1^3))*(XQ(2)-XQ(13))+(EIB6/(YL1^2
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))*(XQ(14)+XQ(3)): F3=(EIB6/(YL1^2))*(XQ(2)-XQ(13))+(EIB2/YL1)*(2*XQ(3)+XQ(14)):
 F6=F2*YL1-F3
2656 F2=F2+F21: F3=F3+F31: F6=F6+F61: SIG1=ABS(F1/TAREA)+ABS(F3/TSEC): SIG2=ABS(
F1/TAREA)+ABS(F6/TSEC): DUM=YL1-XX1: SIGI=0: IF ABS(DUM) < .01 GOTO 2658
2657 XMAX=XX1+ABS(F2/ULOAD): XMMAX=F3-F2*(XMAX+XX1)/2: SIGI=ABS(F1/TAREA)+ABS(XM
MAX/TSEC): IF XMAX > YL1 THEN SIGI=0
2658 IF SIGI > SIGMAX OR SIG1 > SIGMAX OR SIG2 > SIGMAX THEN IMEMB = 1
2659 IF SIGI > SIGMAX THEN SIGMAX = SIGI
2660 IF SIG1 > SIGMAX THEN SIGMAX = SIG1
2661 IF SIG2 > SIGMAX THEN SIGMAX = SIG2
2662 F1=(-EAB/YL2)*(XQ(1)-XQ(4)): F2=(EIB12/(YL2^3))*(XQ(2)-XQ(5))-(EIB6/(YL2^2)
)*(XQ(6)+XQ(3)): F3=(EIB6/(YL2^2))*(XQ(2)-XQ(5))-(EIB2/YL2)*(2*XQ(3)+XQ(6)): F6=
F2*YL2-F3: F2=F2+F22: F3=F3+F32: F6=F6+F62
2663 SIG1=ABS(F1/TAREA)+ABS(F3/TSEC): SIG2=ABS(F1/TAREA)+ABS(F6/TSEC): DUM=YL2-X
X2: SIGI=0: IF ABS(DUM) < .01 THEN GOTO 2665
2664 XMAX=ABS(F2/ULOAD): XMMAX=-F3-F2*(XMAX)/2: SIGI=ABS(F1/TAREA)+ABS(XMMAX/TSE
C): IF XMAX > YL2 THEN SIGI=0
2665 IF SIGI > SIGMAX OR SIG1 > SIGMAX OR SIG2 > SIGMAX THEN IMEMB = 2
2666 IF SIGI > SIGMAX THEN SIGMAX = SIGI
2667 IF SIG1 > SIGMAX THEN SIGMAX = SIG1
2668 IF SIG2 > SIGMAX THEN SIGMAX = SIG2
2669 F1=(-EAB/YL3)*(XQ(4)-XQ(7)): F2=(EIB12/(YL3^3))*(XQ(5))-(EIB6/(YL3^2))*(XQ(
6)+XQ(8)): F3=(EIB6/(YL3^2))*(XQ(5))-(EIB2/YL3)*(2*XQ(6)+XQ(8)): F6=F2*YL3-F3: F
2=F2+F23: F3=F3+F33: F6=F6+F63
2670 SIG1=ABS(F1/TAREA)+ABS(F3/TSEC): SIG2=ABS(F1/TAREA)+ABS(F6/TSEC): DUM=YL3-X
X3: SIGI=0: IF ABS(DUM) < .01 THEN GOTO 2672
2671 XMAX =ABS(F2/ULOAD): XMMAX=-F3-F2*XMAX/2: SIGI=ABS(F1/TAREA)+ABS(XMMAX/TSEC
): IF XMAX > YL3 THEN SIGI=0
2672 IF SIGI > SIGMAX OR SIG1 > SIGMAX OR SIG2 > SIGMAX THEN IMEMB = 3
2673 IF SIGI > SIGMAX THEN SIGMAX = SIGI
2674 IF SIG1 > SIGMAX THEN SIGMAX = SIG1
2675 IF SIG2 > SIGMAX THEN SIGMAX = SIG2
2684 GOTO 2955 : REM finished winged stresses except member 15
2685 IF IFLAG <> 1 GOTO 2735
2695 F1 =(EAB / YL1) * (XQ(1) - XQ(12))
2705 F3 = (EIB6 / (YL1 ^ 2)) * (-XQ(13)) + (EIB2 / YL1) * (2 * XQ(2) + XQ(14)) :
F2 = (EIB12/(YL1^3)) * (-XQ(13)) + (EIB6/(YL1*YL1))*(XQ(2)+XQ(14)) : F6 = F2 * Y
L1 - F3
2715 SIG1 = ABS(F1 / BAREA) + ABS(F3 / BSEC) : SIG2 = ABS(F1 / BAREA) + ABS(F6 /
 BSEC) : IF SIG2 > SIG1 THEN SIG1 = SIG2
2725 GOTO 2775
2735 F1 = (EAB / YL1) * (XQ(1) - XQ(12))
2745 F2 = (EIB12 / (YL1 ^ 3)) * (XQ(2) - XQ(13)) + (EIB6 / (YL1 * YL1)) * (XQ(3)
 + XQ(14))
2755 F3 = (EIB6 / (YL1 * YL1)) * (XQ(2) - XQ(13)) + (EIB2 / YL1) * (2 * XQ(3) +
XQ(14))
2765 F6 = F2 * YL1 - F3 : SIG1 = ABS(F1 / BAREA) + ABS(F6 / BSEC) : SIG2 = ABS(F
1 / BAREA) + ABS(F3/BSEC) : IF SIG2 > SIG1 THEN SIG1 = SIG2
2775 SIGMAX = SIG1 : IMEMB = 1 : REM PRINT IMEMB,SIG1
2785 F1 = (EAB /YL14) * XQ(12)
2795 F2 = (EIB12 / (YL14 ^ 3)) * (XQ(13) - XQ(22)) + (EIB6 / (YL14 ^ 2)) * XQ(14
)
2805 F3 = (EIB6 / (YL14 * YL14)) * (XQ(13) - XQ(22)) + (EIB4 / YL14) * XQ(14)
2815 F6 = F2 * YL14 - F3 : SIGA = ABS(F1 / BAREA) + ABS(F3 / BSEC) : SIGB = ABS(
F1 / BAREA) + ABS(F6 / BSEC)
2825 IF SIGA > SIGMAX OR SIGB > SIGMAX THEN IMEMB = 14
2835 IF SIGA > SIGMAX THEN SIGMAX = SIGA
2845 IF SIGB > SIGMAX THEN SIGMAX = SIGB
2855 IF MLOAD <> 10 GOTO 2955
2865 F1 = (EAT / YL10) * (XQ(32) - XQ(30))
2875 F2 = (EIT12 / (YL10 ^ 3)) * (XQ(24) - XQ(27)) + (EIT6 / (YL10 * YL10)) * (X
Q(33) + XQ(31)) + F210
2885 F3 = (EIT6 / (YL10 * YL10)) * (XQ(24) - XQ(27)) + (EIT2 / YL10) * (2 * XQ(3
3) + XQ(31)) + F310
2895 ON LTYPE GOTO 2905,2905,2935,2935,2935
2905 XMAX = X + ABS(F2 / ULOAD)
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2915 IF XMAX >= YL10 GOTO 2955
2925 XMMAX = -F3 + F2 * ((XMAX + X) / 2) : SIGI = ABS(F1 / TAREA) + ABS(XMMAX /
TSEC) : GOTO 2945
2935 SIGI = ABS(F1 / TAREA) + ABS((F2 * X - F3)/TSEC)
2945 IF SIGI > SIGMAX THEN IMEMB = 10 : SIGMAX = SIGI
2955 F1 = (EAT / YL15) * XQ(30)
2965 F2 = (EIT12 / (YL15 ^ 3)) * (XQ(27) - XQ(29)) + (EIT6 / (YL15 * YL15)) * XQ
(31)
2975 F3 = (EIT6 / (YL15 * YL15)) * (XQ(27) - XQ(29)) + (EIT4 / YL15) * XQ(31)
2985 F6 = F2 * YL15 - F3 : F2 = F2 + F215 : F3 = F3 + F315 : F6 = F6 + F615
2995 SIGA = ABS(F1 / TAREA) + ABS(F6 / TSEC) : SIGB = ABS(F1 / TAREA) + ABS(F3 /
 TSEC) : SIGI = 0
3005 IF LTYPE <> 1 OR LTYPE <> 2 OR IG=1 GOTO 3025
3015 XMAX = ABS(F2 / ULOAD) : XMMAX = -F3 + F2 * (XMAX / 2) : SIGI = ABS(F1 / TA
REA) + ABS(XMMAX / TSEC)
3025 IF SIGA > SIGMAX OR SIGB > SIGMAX OR SIGI > SIGMAX THEN IMEMB = 15
3035 IF SIGA > SIGMAX THEN SIGMAX = SIGA
3045 IF SIGB > SIGMAX THEN SIGMAX = SIGB
3055 IF SIGI > SIGMAX THEN SIGMAX = SIGI
3065 DEFMAX = 0 : FOR I = 1 TO 33
3075 IF ABS(XQ(I)) > ABS(DEFMAX) THEN IDEF = I : DEFMAX = ABS(XQ(I))
3085 NEXT I
3095 REM all analysis has been done compute results
3105 DEFMAX=ABS(DEFMAX):IOUT=2
3115 REM both option output check for max stress in top or bot deck
3125 IF IMEMB = 15 AND IG=0 OR IMEMB=10 AND IG=0 THEN RCOV=DVCR:XMOR=DMOR: THK=H
:  IDFLAG=1 : ELSE RCOV=BVCR:XMOR=BMOR:THK=H1: IDFLAG=0
3135 IF ILOAD<> 2 GOTO 3175
3145 DEFLIM=DLIM(2):DEFLEC=DEFMAX: RESS=SIGMAX: Y.W(2)=DEFMAX
3155 GOSUB 4215
3165 GOTO 3755 : REM *********finished design option ************
3175 REM analysis option******
3185 IF IMEMB = 15 AND IG=0 OR IMEMB = 10 AND IG=0 THEN XMOR=DMOR:RCOV=DVCR:IDFL
AG=1: ELSE XMOR=BMOR :RCOV=BVCR :IDFLAG=0: REM find req. mean load effects
3195 SBAR=XMOR*SQR((1+V.S(2)^2)/(1+RCOV^2))*EXP(-BTR(2)*SQR(LOG(1+RCOV^2)+LOG(1+
V.S(2)^2))): IF DLIM(2) > 0 THEN DELTS=DLIM(2)*SQR(1+V.SD(2)^2)*EXP(-BTE(2)*SQR(
LOG(1+V.SD(2)^2)))
3205 ON LTYPE GOTO 3215,3235,3275,3305,3305
3215 IF IG=0 THEN ULOAD=ULOAD*RADLTH ELSE ULOAD=ULOAD*SPAN1-(X*2)
3225 GOTO 3245
3235 IF IG=0 THEN ULOAD=ULOAD*(RADLTH-X*2) ELSE ULOAD=ULOAD*SPAN1-(X*2)
3245 PMAX=SBAR*ULOAD/SIGMAX:DEFP=PMAX*DEFMAX/ULOAD: W.TOT(2)=PMAX: Y.W(2)=DEFP
3255 IF DLIM(2) > 0 THEN  W.DEF(2)=DELTS*ULOAD/DEFMAX: REM
 finished uniform load analysis output
3265 GOTO 3755 :REM start output for 1 line load
3275 CLOAD=CLOAD*2: PMAX=SBAR*CLOAD/SIGMAX:W.TOT(2)=PMAX: Y.W(2)=PMAX*DEFMAX/CLO
AD
3285 IF DLIM(2)> 0 THEN W.DEF(2)=DELTS*CLOAD/DEFMAX: 'FINISHED 1 LINE LOAD
3295 GOTO 3755 : REM start output for 2 and 3 line loads.
3305 PMAX=SBAR*SLOAD/SIGMAX: Y.W(2)=PMAX*DEFMAX/SLOAD
3315 IF DLIM(2) > 0 THEN W.DEF(2)=DELTS*SLOAD/DEFMAX : 'FINISHED 2&3 LINE LOADS
3325 IF LTYPE=4 THEN W.TOT(2)=2*PMAX:W.DEF(2)=W.DEF(2)*2: ELSE W.TOT(2)=3*PMAX:
W.DEF(2)=W.DEF(2)*3
3335 GOTO 3755
3345 '****** Finished all Analysis Begin SCREEN OUTPUT ************************

3355 GOSUB 9000:SCREEN 0,1 : WIDTH 80 :COLOR 14,1,0 : CLS : LOCATE 2,20 : COLOR
15,4 : PRINT "RACKING STRENGTH and STIFFNESS ANALYSIS" : COLOR 14,1
3365 LOCATE 4,9 : IF ILOAD = 1 THEN PRINT "ANALYSIS OPTION..." ELSE PRINT "DESIG
N OPTION...": LOCATE 5,14 : PRINT "KNOWN PROPERTIES... Total ";MAXAVG$;" load =
";W.TOT(1)*LOADF;" lbs"
3366 TYP1$="HIGH":LZF=2.047:IF ILVAR=1 THEN TYP1$="MED.":LZF=1.582
3367 IF ILVAR=2 THEN TYP1$="LOW":LZF=1.233
3368 IF ILVAR=4 THEN TYP1$=STR$(V.S(1)):LZF=1+2.326*V.S(1)
3370 IF ILOAD=1 AND W.TOT(1)<=2 AND SPAN <>0 THEN GOSUB 8500
3371 IF ILOAD=1 AND W.TOT(2)<=2 AND SPAN1 <> 0 THEN GOSUB 8510
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3372 IF ILOAD=2 AND IFAIL(1)=5 THEN GOSUB 8500:IFAIL(1)=1
3373 IF ILOAD=2 AND IFAIL(2)=5 THEN GOSUB 8510:IFAIL(2)=1
3375 LOCATE 4,27 : ON LTYPE GOTO 3385,3395,3405,3415,3425
3385 PRINT "FULL UNIFORM LOAD" : GOTO 3435
3395 PRINT "PART. UNIFORM LOAD" : GOTO 3435
3405 PRINT "CENTER LINE LOAD" : GOTO 3435
3415 PRINT "TWO-LINE LOADS" : GOTO 3435
3425 PRINT "THREE-LINE LOADS"
3435 LOCATE 4,46:PRINT " ...";TYP1$;" Load Variability"
3436 LOCATE 7,15 :IF SPAN=0 THEN PRINT "This Pallet is not racked across Stringe
rs" : GOTO 3585 ELSE COLOR 15,4 : LOCATE 7,25 : PRINT "Rack across Stringers" :
COLOR 14,1
3445 IF XTGR>0 THEN LOCATE 8,2 : PRINT "Load ";XTGR;" in. from end"
3455 LOCATE 9,2 : PRINT CHR$(95);CHR$(95); : FOR I = 1 TO 5 : PRINT TAB(I*4)CHR$
(95);CHR$(95); : NEXT I : PRINT CHR$(95);CHR$(95) : PRINT TAB(2);: FOR I = 1 TO
22 : PRINT CHR$(219); : NEXT I
3456  IF IE2 <> 2 THEN LOCATE 10,7:PRINT CHR$(223);CHR$(223);CHR$(223): LOCATE 1
0,16:PRINT CHR$(223);CHR$(223);CHR$(223)
3485 PRINT TAB(2)CHR$(169);TAB(23)CHR$(170)
3486 PRINT TAB(4)CHR$(17);CHR$(196);CHR$(196);TAB(10)SPAN;TAB(19)CHR$(196);CHR$(
196);CHR$(16)
3494 ZW.=U2(1,2):IF J1=2 THEN ZW.=U2(2,2)
3495 IF ILOAD = 2 GOTO 3525
3505 LOCATE 9,26 : PRINT MAXAVG$;" Pallet load = ";:PRINT USING "######.";W.TOT(
1)*LOADF;:PRINT " lbs" : LOCATE 10,26 : PRINT "Deflection @ ";MAXAVG$;" load = "
;:PRINT USING "#.##";Y.W(1)*LOADF; : PRINT " in."
3515 IF ABS(DLIM(1))<=.0001 THEN GOTO 3585 ELSE W.DEF(1)=W.DEF(1)*(1+2.326*V.SD(
1)):' apply dfactor
3516 IF W.DEF(1)>W.TOT(1)*LOADF AND MAXAVG$="MAXIMUM" THEN GOTO 3518 ELSE IF MAX
AVG$="MAXIMUM" THEN GOTO 3519
3517 IF MAXAVG$="AVERAGE" AND W.DEF(1) < W.TOT(1)*LZF THEN GOTO 3519
3518 LOCATE 12,26:PRINT "Strength, not deflection, governs design":GOTO 3585
3519 LOCATE 12,26 :PRINT "MAXIMUM load for ";: PRINT USING "#.##";DLIM(1);:PRINT
 " in. deflection limit ";:LOCATE 13,55:PRINT "=";: PRINT USING "######.";W.DEF(
1);: PRINT " lbs": GOTO 3585
3525 LOCATE 9,26 : PRINT "Strength :"; :IF IFAIL(1)=1 THEN PRINT "..FAILS ": LOC
ATE 10,26: PRINT "(New properties or change dimensions)" ELSE GOTO 3535
3526 IF IE2 =4 THEN LOCATE 11,26:PRINT "(Could also change notch geometry)":GOTO
 3585
3527 LOCATE 11,26:PRINT "    (Increase Height ";:PRINT USING "#.##";TINC.(1);:PRI
NT " *OR*   Width ";:PRINT USING "#.##";ABS(U2(1,2)*(1-((H2-TINC.(1))/H2)^2)):G
OTO 3585
3535 IF IE2=4 THEN PRINT "..OK..(Could change properties,dimensions,":LOCATE 10,
45:PRINT " or Notch geometry if needed)":GOTO 3537
3536 PRINT "..OK..(could reduce Height ";: PRINT USING "#.##";TINC.(1);:PRINT "
in. ": LOCATE 10,49: PRINT " *OR*   Width  ";:PRINT USING "#.##";ABS(ZW.*(1-((H2-
TINC.(1))/H2)^2));: PRINT " in.)"
3537 LOCATE 11,26: PRINT MAXAVG$;" Deflection = ";:PRINT USING "##.##";Y.W(1)*LO
ADF;: PRINT " in."
3545 IF ABS(DLIM(1))<=.0001 GOTO 3585
3555 LOCATE 12,26 : PRINT "Deflection Criterion:";: IF JFAIL(1)=0 THEN PRINT "..
OK  for limit of ";: PRINT USING "##.##";DLIM(1);: PRINT " in." : GOTO 3585
3565 PRINT "..FAILS  for limit of ";: PRINT USING "#.##";DLIM(1); : PRINT " in."

3575 LOCATE 13,30: PRINT "(New properties or Increase  Height ";:PRINT USING "#
.##";TDINC.(1);:PRINT " in.";
3576 LOCATE 14,54:PRINT "*OR*  Width   ";:PRINT USING "#.##";ABS(ZW.*(1-((H2-TDIN
C.(1))/H2)^3));:PRINT " in.)"
3585 IF TSTI=2 THEN LOCATE 25,10:COLOR 15,4:PRINT "WARNING!..Deckboard response
may govern..Check R A D and STACK results ";:COLOR 14,1
3588 LOCATE 16,15 : IF SPAN1 = 0 OR Q1=0 AND IG=0 THEN  PRINT "This Pallet is no
t racked across Deckboards" : COLOR 14,1 : GOTO 3735
3589 LOCATE 16,25 : COLOR 15,4 :IF IG=0 AND Q1=1 THEN PRINT "Rack across Deckboa
rds" : COLOR 14,1 ELSE IF IG=1 THEN PRINT "Supported Under Top Deck" :COLOR 14,1

3595 IF XDECK>0 THEN LOCATE 17,26 : PRINT "Load starts ";XDECK;" in. from end"
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3605 ILINE=19 :GOSUB 20000
3635 PRINT TAB(3)CHR$(169);TAB(5)CHR$(17);CHR$(196);CHR$(196);TAB(10)SPAN1;TAB(1
7)CHR$(196);CHR$(196);CHR$(16);TAB(21)CHR$(170)
3645 IF ILOAD = 2 GOTO 3675
3655 LOCATE 18,26 : PRINT MAXAVG$;" Pallet load = ";:PRINT USING "######.";W.TOT
(2)*LOADF;:PRINT " lbs" : LOCATE 19,26 : PRINT "Deflection @ ";MAXAVG$;" load =
";:PRINT USING "##.##";Y.W(2)*LOADF; : PRINT " in."
3665 IF ABS(DLIM(2))<=.0001 THEN GOTO 3735 ELSE W.DEF(2)=W.DEF(2)*(1+2.326*V.SD(
2)):' apply dfactor
3666 IF W.DEF(2)>W.TOT(2)*LOADF AND MAXAVG$="MAXIMUM" THEN GOTO 3668 ELSE IF MAX
AVG$="MAXIMUM" THEN GOTO 3669
3667 IF MAXAVG$="AVERAGE" AND W.DEF(2) < W.TOT(2)*LZF THEN GOTO 3669
3668 LOCATE 21,26:PRINT "Strength, not deflection, governs design":GOTO 3735
3669 LOCATE 21,26 :  PRINT "MAXIMUM load for ";: PRINT USING "#.##";DLIM(2);:PRI
NT " in. deflection limit ";:LOCATE 22,55:PRINT "=";: PRINT USING "######.";W.DE
F(2);: PRINT " lbs": GOTO 3735
3675 LOCATE 18,26:PRINT "Strength :";:IF IFAIL(2)=1 THEN PRINT "..FAILS";:LOCATE
 19,26:PRINT "(New properties or increase thickness ";:PRINT USING "##.###";TINC
.(2);:PRINT " in.)":GOTO 3735
3685 PRINT "..OK (could reduce Thickness "; : PRINT USING "##.###";TINC.(2);:PRI
NT " in.)": LOCATE 19,26: PRINT MAXAVG$;" Deflection = ";:PRINT USING "##.##";Y.
W(2)*LOADF; : PRINT " in."
3695 IF ABS(DLIM(2))<=.0001 GOTO 3735
3705 LOCATE 21,26 : PRINT "Deflection Criterion:";: IF JFAIL(2)=0 THEN PRINT "..
OK  for limit of ";: PRINT USING "##.##";DLIM(2);: PRINT " in." : GOTO 3735
3715 PRINT "..FAILS  for limit of ";: PRINT USING "##.##";DLIM(2); : PRINT " in.
"
3725 LOCATE 22,26 : PRINT "(New properties or increase deck thickness"; : PRINT
USING "##.##";TDINC.(2); : PRINT " in.)"
3735 IF SPAN1=0 OR IG=0 AND Q1=0 THEN 3740
3736 COLOR 15,0:IF IDFLAG=1 THEN LOCATE 18,4: PRINT "Top Deck Critical" ELSE LOC
ATE 23,2: PRINT "Bottom Deck Critical"
3740 BEEP : YN$=INPUT$(1)
3745 RETURN
3755 GOSUB 3355
3765 ERASE XQ,ZQ,ZK,SS,DIAG,P3 : X=XTE9 : ULOAD=UL9 : TLOAD=TL9 : SLOAD =SL9 : C
LOAD=CL9 : G1T=G1TT : G3T=G3TT : G1B=G1BB : G3B=G3BB
3775 CHAIN "A:STACK" : ' *** STACKF ***
3785 COLOR 15,1,1:CLS : LOCATE 12,20 :   PRINT "WAIT , I'M THINKING"
3795 RMIN = 1 : IDECAY = 0
3805 FOR I = 1 TO 33 : DIAG(I) = ZK(I,1) : NEXT I
3815 FOR N9 = 1 TO 33
3825 FOR L9 = 2 TO 16
3835 IF ZK(N9,L9) = 0 GOTO 3905
3845 I = N9 + L9 - 1 : J9 = 0 : C = ZK(N9,L9) / ZK(N9,1)
3855 FOR K9 = L9 TO 16
3865 J9 = J9 + 1
3875 ZK(I,J9) = ZK(I,J9) - C * ZK(N9,K9)
3885 NEXT K9
3895 ZK(N9,L9) = C
3905 NEXT L9
3915 NEXT N9
3925 FOR I = 1 TO 33
3935 DECAY = ZK(I,1) / DIAG(I)
3945 IF ABS(DECAY) >= ABS(RMIN) GOTO 3965
3955 RMIN = DECAY
3965 'IF DECAY < 0 THEN W.TOT(2)=1:IFAIL=5:W.DEF(2)=0:Y.W(2)=0:GOTO 3755:'PRINT
"*** ERROR in subroutine SOLVE in diagonal of row ";I;" *** structure may be uns
table"
3975 REM IF DECAY < 0 THEN PRINT "*** caution *** structure may be unstable in t
his deckboard support mode"
3985 NEXT I
3995 FOR N9 = 1 TO 33
4005 FOR L9 = 2 TO 16
4015 IF ZK(N9,L9) = 0 GOTO 4045
4025 I = N9 + L9 - 1
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```
4035 XQ(I) = XQ(I) - ZK(N9,L9) * XQ(N9)
4045 NEXT L9
4055 REM IF ZK(N9,1) = 0 THEN PRINT "***ERROR in subroutine SOLVE ";N9;" element
     of banded system stiffness matrix ";ZK(N9,1);"  check input"  : GOTO 3125
4065 IF ZK(N9,1)=0 THEN GOTO 4085
4075 XQ(N9) = XQ(N9) / ZK(N9,1)
4085 NEXT N9
4095 FOR I = 2 TO 33
4105 N9 = 34 - I
4115 FOR L9 = 2 TO 16
4125 IF ZK(N9,L9) = 0 GOTO 4155
4135 K9 = N9 + L9 - 1
4145 XQ(N9) = XQ(N9) - ZK(N9,L9) * XQ(K9)
4155 NEXT L9
4165 NEXT I
4175 REM PRINT " Minimum decay ratio = ";RMIN
4185 REM IF ABS(RMIN) < 9.999999E-06 THEN PRINT "***ERROR in SOLVE **** ill-cond
itioning detected"
4195 RETURN
4205 '***************************************************************************
4215 ' sub design for finding mim mor and defl limit
4225 '***************************************************************************
4235 IF IOUT=1 THEN BBT=BTR(1):SCOV=V.S(1):DCOV=V.SD(1) :DBT=BTE(1) ELSE BBT=BTR
(2):SCOV=V.S(2):DCOV=V.SD(2):DBT=BTE(2)
4245 RREQ=RESS*EXP(BBT*SQR(LOG(1+SCOV^2)+LOG(1+RCOV^2)))/SQR((1+SCOV^2)/(1+RCOV^
2))
4255 XMOR=XMOR+50:IF RREQ<XMOR THEN GOTO 4285
4265 REM PRINT " this design failed try increasing dimensions and rerun"
4275 IFAIL(IOUT)=1 : GOTO 4295
4285 IFAIL(IOUT)=0:REM fix up dimensions for rerun
4295 TINC.(IOUT)=ABS(THK*(SQR(RREQ/XMOR)-1))
4305 IF DEFLIM=0 THEN RETURN
4315 RDEF=DEFLEC*EXP(DBT*SQR(LOG(1+DCOV^2)))/SQR(1+DCOV^2)
4325 IF RDEF<DEFLIM THEN JFAIL(IOUT)=0 : GOTO 4345
4335 JFAIL(IOUT)=1 : REM PRINT "THIS DESIGN FAILED DEFLECTION LIMITS"
4345 REM fixup dimensions for rerun for defl limit
4355 TREQ=(RDEF*(THK^3)/DEFLIM)^.333333: REM PRINT "5140 treq=";TREQ
4365 TDINC.(IOUT)=ABS(TREQ-THK)
4375 RETURN
4385 '***************************************************************************
4395 'sub notchdef...computes adjustment factor to defln of notched stringer
4405 '***************************************************************************
4415 PHI=H7/H2 :H.=H2/SPAN :M.E=LO-(L2-SPAN)/2:M.=M.E/SPAN:XN.=(M.E+L7)/SPAN
4425 A=3:B.=M. + A * H.*(1-PHI) : G.=-XN. + A*H.*(1-PHI) : Q.=SPAN/2 -L7-M.E
4435 X.L=B.-A*H.  : X.R=-G.+A*H.  :T1.=2*A*H./(1+2*G.) : T2.=A*H.*(1-PHI)/B.  :
4445 Z.=LO+L7-R7 : OH=(L2-SPAN)/2 :
4455 T3.=(1-PHI)*T1.:A.= (XTGR-(L2-SPAN)/2)/SPAN:IF A.<0 AND LTYPE >=4 THEN PRIN
T "BAD X VALUE":END
4465 IF H7<=M.E/A AND H7<=Q./A THEN GOSUB 4835:GOTO 4505
4475 IF H7> M.E/A AND H7<=Q./A THEN GOSUB 4875:GOTO 4505
4485 IF H7> M.E/A AND H7> Q./A THEN GOSUB 4896:GOTO 4505 ELSE PRINT "Geometry ou
t of program bounds!!!!"
4495 PRINT "input error!!!! wrong notch depth interval":END
4505 IF LTYPE>=3 THEN NLINE =LTYPE-2 ELSE NLINE =0
4515 ON LTYPE GOTO 4525,4595,4635,4635,4635
4525 VOM=(2*Z.-L2)/(Z.^2-L2*(Z.-OH))
4535 P1=16*(XN.^3*(4-3*XN.) + M.^3*(-4+3*M.))/(1-PHI)^3
4545 P2=96*A*H.*(B.^2*(1-B.)+G.^2*(1+G.))/(1-PHI)^2
4555 P3=192*A^2*H.^2*(B.*(-2+3*B.)-G.*(2+3*G.))/(1-PHI)
4565 P4=96*A*H.*C9 + 192*A^2*H.^2*C10 + 96*A^3*H.^3*C11 +C12
4575 XNOTCH=.2*(P1+P2+P3+P4) : '...psi3p
4585 GOTO 4905
4595 'partial uniform load....use psi3p
4605 R.1=(L2-2*XTGR)/2 : VOM=(Z.-XTGR-R.1)/((Z.-XTGR)^2/2-R.1*(Z.-OH)):
4615 GOTO 4535
4625 END
```

```
4635 ON NLINE GOTO 4645,4695,4765
4645 VOM=1/(Z.-OH):'center line load ....psi1p
4655 P1=8*(XN.^3-M.^3)/(1-PHI)^3 : P2=12*A*H.*(B.^2 + G.^2)/(1-PHI)^2
4665 P3=48*A^2*H.^2*(B.+G.)/(1-PHI)
4675 P4=-12*A*H.*C1 +48*A^2*H.^2*C2 + 12*A^3*H.^3*C3 + C4
4685 XNOTCH=P1+P2-P3+P4 :IF NLINE=3 THEN GOTO 4775 ELSE GOTO 4905:'...psi1p
4695 'two line loads....use psi2p
4705 IF XTGR>=Z. THEN VOM=1/(Z.-OH) ELSE VOM=0
4715 P1=(12*XN.^2*A.-8*M.^3-4*A.^3)/(1-PHI)^3
4725 P2=12*A*H.*(B.^2-G.*A.)/(1-PHI)^2
4735 P3=24*A^2*H.^2*(A.-2*B.)/(1-PHI)
4745 P4=12*A*H.*C5 -24*A^2*H.^2*C6 + 12*A^3*H.^3*C7 +C8
4755 XNOTCH=(P1+P2+P3+P4)/(A.*(3-4*A.^2)) :IF NLINE=3 THEN GOTO 4785 ELSE GOTO 4
905 :'...psi2p
4765 RAT.D=2*(SLOAD/CLOAD)*A.*(3*SPAN^2-4*A.^2)/(SPAN^3) : GOTO 4655
4775 PSI1P=XNOTCH :GOTO 4695
4785 XNOTCH=(XNOTCH*RAT.D + PSI1P)/(RAT.D+1) : 'psi1 and 2 comb
4795 IF XTGR> Z. THEN VOM=1/(Z.-OH):GOTO 4905
4805 R.1=(CLOAD + 2*SLOAD)/2 : VOM=(SLOAD-R.1)/(R.1*(Z.-OH)-SLOAD*(Z.-XTGR))
4815 GOTO 4905
4825 END
4835 C1=B.^2+G.^2 : C2=B.+G. : C3=-4*LOG(1-PHI) :C4=1+8*(X.L^3-X.R^3) :C5=G.*A.-
B.^2 : C6=-2*B.+A.
4845 C7=-2*LOG(1-PHI): C8=3*A.+8*X.L^3-12*X.R^2*A. :C9=B.^2*(-1+B.)-G.^2*(1+G.)
4855 C10=B.*(2-3*B.)+G.*(2+3*G.) : C11=(-4+6*(B.-G.))*LOG(1-PHI) : C12=5+16*X.R^
3*(-4+3*X.R)+16*X.L^3*(4-3*X.L) : C13=B.^2+A.^2 : C14=B. : C15=-2*LOG(1-PHI)
4865 C16=3*A.^2 +2*X.L^3 -6*X.R*A.^2 :C17=B.^2*(-1+B.) + G.*A.*(1+G.) : C18=B.*
(2-3*B.)-A.*(1+2*G.) : C19=2*(-1+A.+3*B.)*LOG(1-PHI) +2*M. -2*X.L : C20=A.+X.L^3
*(4-3*X.L)+2*X.R^2*A.*(-3+2*X.R):RETURN
4875 C1=G.^2 : C2=G. : C3=3-2*LOG(1-PHI)-2*LOG(T2.) :C4=1-8*X.R^3 : C5=G.*A.
4885 C6=A. : C8=3*A.-12*X.R^2*A. : C7= 3-2*LOG(T2.) : C9=-G.^2*(1+G.) :C10=G.*(2
+3*G.):C11=3+2*M. -5*B. -2*A*PHI*H. -2*(1+3*G.)*LOG(1-PHI)-2*(1-3*B.)*LOG(T2.)
4895 C12=5+16*X.R^3*(3*X.R-4) : C13=A.^2 :C14=0:C15=3-2*LOG(T2.) : C16=3*A.^2-6*
X.R*A.^2 : C17=G.*A.*(1+2*G.) : C19=2*A.*LOG(1-PHI) + 2*(-1+3*B.)*LOG(T2.) + 3 +
2*M. -5*B. : C20=A.+2*X.R^2*A.*(2*X.R-3):RETURN
4896 C1=G.^2*T1.^2 : C2=G.*T1. : C3=3-2*LOG(T2.)-2*LOG(T3.) : C4=0 : C5=G.*A.*T1
.^2 : C6=A.*T1. : C7=3-2*LOG(T2.) : C8=0 : C9= -(G.^2)*(1+G.)*T1.^2 : C10=G.*(2+
3*G.)*T1. : C11=2+2*M.+2*XN. - 5*B. -2*(1+3*G.)*LOG(T3.) - 2*(1-3*B.)*LOG(T2.)
4897 C12=0 : C13= A.^2*T1.^2 : C14=0 : C15=3-2*LOG(T2.) : C16=0 : C17=G.*A.*(1+G
.)*T1.^2 : C18=-A.*(1+2*G.)*T1. : C19= 2*A.*LOG(T3.) + 2*(3*B.-1)*LOG(T2.) + 3 +
2*M. - 5*B. : C20=0 : RETURN
4905 RETURN
6100 '****************************************************************
6101 ' sub ras line loads
6102 '****************************************************************
6110 IF ILOAD <> 2 GOTO 6500
6113 CL=CLOAD/TSTI: SL=SLOAD/TSTI
6115 ON LTYPE GOTO 6120,6120,6125,6140,6150
6120 RETURN: '(ltype=1 or 2 )
6125 'ltype=3--ie center line load
6130 XMOMEN=CL*SPAN/4: DEFL=CL*(SPAN^3)/(48*SMOE*STINER)
6135 IF H7=0 THEN GOTO 6240 ELSE XMNOT=CL*XXX/2: GOTO 6300
6140 'ltype=4--ie two line loads
6141 A=XTGR-(L0/2-SPAN/2):XMOMEN=SL*A: DEFL= SL*A*(3*(SPAN^2)-4*(A^2))/(24*SMOE*
STINER)
6145 IF H7=0 THEN GOTO 6240 ELSE XMNOT=SL*XXX: GOTO 6300
6150 'ltype=5-- ie three line loads
6151 A=XTGR-(L0/2-SPAN/2): XMOMEN=SL*A+CL*SPAN/4: DEFL=(SL*A*(3*(SPAN^2)-4*(A^2)
)/(24*SMOE*STINER))+CL*(SPAN^3)/(48*SMOE*STINER)
6155 IF H7=0 THEN GOTO 6240
6156 XMNOT=(CL/2+SL)*A+(CL/2)*(XXX-A): GOTO 6300
6240 STRESS=XMOMEN/SECMOD: Y.W(1)=DEFL
6245 DEFLIM=DLIM(1):THK=H2:XMOR=SMOR:IOUT=1:RCOV=SVCR:DEFLEC=DEFL: RESS=STRESS
6250 GOSUB 4215: RETURN
6300 IF ABS(XMNOT) >ABS(XMMAX) THEN IFAIL(1)=1: REM PRINT "failed at notch XMNOT
= ":XMNOT."xmmax=":XMMAX
```

```
6310 IF ABS(XMNOT) <=ABS(XMMAX) THEN IFAIL(1)=0: REM PRINT "strength at notch ok
-"
6315 DEFL=DEFL*XNOTCH:Y.W(1)=DEFL:
6320 IF DEFL > DLIM(1) THEN JFAIL(1)=1 ELSE JFAIL(1)=0 :REM corrected notch defl

6325 RETURN : 'finished line load design option
6500 'analysis option--line loads
6505 ON LTYPE GOTO 6515,6515,6520,6540,6560
6515 RETURN: ' ltype 1 or 2
6520 'ltype=3--ie center line load
6525 DENOM=SPAN/4: AA=SPAN^3/(48*SMOE*STINER)
6530 IF H7=0 THEN GOTO 6600 ELSE P=XMMAX*2/XXX: TOTLD=P*TSTI: GOTO 6700
6540 'ltype=4 --ie 2 line loads
6545 A=XTGR-(LO/2-SPAN/2): DENOM=A: AA=A*(3*(SPAN^2)-4*(A^2))/(24*SMOE*STINER)
6550 IF H7=0 THEN GOTO 6600 ELSE P=XMMAX/XXX: TOTLD=2*P*TSTI: GOTO 6700
6560 'ltype=5 ie three line loads
6565 A=XTGR-(LO/2-SPAN/2): DENOM=A+SPAN/4: AA=(SPAN^3)/(48*SMOE*STINER)+A*(3*(SP
AN^2)-4*(A^2))/(24*SMOE*STINER)
6570 IF H7=0 THEN GOTO 6600 ELSE P=XMMAX/((3/2*A)+(XXX-A)/2): TOTLD=3*P*TSTI:   G
OTO 6700
6600 P=XM/DENOM: TOTLD=P*TSTI*(LTYPE-2): DEFL=P*AA: Y.W(1)=ABS(DEFL): W.TOT(1)=
ABS(TOTLD)
6610 IF DLIM(1) < .001 THEN RETURN
6611 GOTO 6720 ' jump for max load for a deflection limit
6700 'max line loads for notchs
6705 W.TOT(1)=ABS(TOTLD): DEFL=P*AA*XNOTCH: Y.W(1)=ABS(DEFL)
6710 IF DLIM(1) < .001 THEN RETURN
6720 ' max load for a deflection limit
6725 DELTS=DLIM(1)*SQR(1+V.SD(1)^2)*EXP(-BTE(1)*SQR(LOG(1+V.SD(1)^2)))
6730 P=2000: DEFL=ABS(P*AA): QQQ=DELTS*P/DEFL: TOTLD=QQQ*TSTI*(LTYPE-2)
6735 W.DEF(1)=ABS(TOTLD): RETURN
6800 '**********************************************************************
6801 ' rad support under top deck w/ no bottom deck
6802 '**********************************************************************
6803 IDFLAG=1
6805 L.1=0+SY2: L.2=SPAN1:
6810 ON LTYPE GOTO 6815,6830,6850,6865,6875
6815 'ltype=1 ----full uniform load
6820 XDECK=0: IF ILOAD=1 THEN UL=2000 ELSE UL=TLOAD
6825 W=UL/L: M.1=ABS(W*(L.1^2)/2): GOTO 6845
6830 'ltype=2 partial uniform load
6835 IF ILOAD=1 THEN UL=2000 ELSE UL=TLOAD
6840 W=UL/(2*(L/2-XDECK)): IF XDECK >= L.1 THEN M.1=0 ELSE M.1=ABS(W*((L.1-XDECK
)^2)/2)
6845 M.2=ABS(-W*((L/2-XDECK)^2)/2+W*(L/2-XDECK)*L.2/2): GOTO 6890
6850 ' ltype=3 --center line load
6855 SL=0: IF ILOAD=1 THEN CL=2000 ELSE CL=CLOAD
6860 GOTO 6880
6865 ' ltype=4 ----2 line loads
6870 CL=0: IF ILOAD=1 THEN SL=2000: ELSE SL=SLOAD
6873 GOTO 6880
6875 ' ltype=5---three line loads
6877 IF ILOAD=1 THEN SL=1000: CL=SL: ELSE SL=SLOAD: CL=CLOAD
6880 IF XDECK >= L.1 THEN M.1=0 ELSE M.1=ABS(SL*(L.1-XDECK))
6885 M.2=ABS((SL+CL/2)*L.2/2-SL*(L/2-XDECK))
6890 IF M.1> M.2 THEN M.MAX=M.1 ELSE M.MAX=M.2:' max. moment
6895 IF ILOAD = 2 THEN GOTO 6950
6900 ' analysis option--------------- find max load
6905 'SBAR=DMOR*EXP(-BTR(2)*SQR(DVCR*DVCR+V.S(2)*V.S(2))): IF DLIM(2) > 0 THEN D
ELTS=DLIM(2)*EXP(-BTE(2)*V.SD(2))
6906 SBAR=DMOR*SQR((1+V.S(2)^2)/(1+DVCR^2))*EXP(-BTR(2)*SQR(LOG(1+DVCR^2)+LOG(1+
V.S(2)^2))): IF DLIM(2) > 0 THEN DELTS=DLIM(2)*SQR(1+V.SD(2)^2)*EXP(-BTE(2)*SQR(
LOG(1+V.SD(2)^2)))
6910 XM=SBAR*(H^2)*TSAC/6
6915 ON LTYPE GOTO 6920,6920,6925,6930,6935
6920 PMAX=XM*UL/M.MAX: W.TOT(2)=ABS(PMAX)
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6921 IF LTYPE=1 THEN W=PMAX/L ELSE W=PMAX/(2*(L/2-XDECK))
6922 GOTO 6950
6925 PMAX=XM*CL/M.MAX: CL=PMAX:W.TOT(2)=ABS(PMAX): GOTO 6950
6930 PMAX=XM*SL/M.MAX: W.TOT(2)=ABS(2*PMAX): SL=PMAX: GOTO 6950
6935 PMAX=XM*SL/M.MAX: W.TOT(2)=ABS(3*PMAX): SL=PMAX: CL=PMAX
6950 ' find deflections***** both options ********************************
6955 DINERT=TSAC*(H^3)/12: ON LTYPE GOTO 6960,6965,6985,6990,6990
6960 XDECK=0
6965 IF XDECK > L.1 THEN GOTO 6975
6970 DMAX=ABS((-5*W*L.2^4)/(384*DMOE*DINERT)+W*((L.1-XDECK)^2)*(L.2^2)/(16*DMOE*
DINERT)): GOTO 7000
6975 C.=L-2*XDECK: B.=L.2-XDECK+L.1
6980 DMAX=ABS((8*(W*C./2)*((L.2^3)/8-(L.2^3)/2)+W*(C./2)*(L.2^3-2*B.*C.^2+C.^3+2
*L.2*C.^2)-2*W*(L.2/2-XDECK+L.1)^4)/(48*DMOE*DINERT)): GOTO 7000
6985 DMAX=CL*L.2^3/(48*DMOE*DINERT): GOTO 7000
6990 DMAX=SL*(XDECK-L.1)*(3*L.2^2-4*(XDECK-L.1)^2)/(24*DMOE*DINERT)
6995 IF LTYPE= 5 THEN DMAX=DMAX+ CL*L.2^3/(48*DMOE*DINERT)
7000 Y.W(2)=ABS(DMAX)
7005 IF ILOAD = 1 GOTO 7020: 'jump for analysis
7010 RCOV=DVCR:XMOR=DMOR: THK=H: IOUT=2: RESS=ABS(M.MAX/(TSAC*H^2/6)):DEFLIM=DLI
M(2): DEFLEC=DMAX
7015 GOSUB 4215 : RETURN
7020 ' analysis option--------------
7025 IF DLIM(2)=0 THEN RETURN
7030 ON LTYPE GOTO 7035,7035,7040,7045,7050
7035 W.DEF(2)=DELTS*UL/DMAX: RETURN
7040 W.DEF(2)=DELTS*CL/DMAX: RETURN
7045 W.DEF(2)=2*SL*DELTS/DMAX: RETURN
7050 W.DEF(2)=3*SL*DELTS/DMAX: RETURN
7056 '*********** finished top deck support case****************************
8000 COLOR 14,12:CLS:LOCATE 12,20:PRINT "FATAL ERROR DETECTED(Sub1500)....Check
input and retry":LOCATE 14,20:PRINT "Error No. :";ERR,"Error Line :";ERL:LSTART=
184:GSTART=0:RESUME 8010
8010 CHAIN "A:main1s"
8500 COLOR 14,1:LOCATE 25,1:PRINT SPACE$(76);:LOCATE 25,5:COLOR 14,12:BEEP:PRINT
 "Racked Across Stringers CANNOT be analyzed...See Users Guide";:COLOR 14,1 :RET
URN
8510 COLOR 14,1:LOCATE 25,1:PRINT SPACE$(76);:LOCATE 25,5:COLOR 14,12:BEEP:PRINT
 "Racked Across Deckboards CANNOT be analyzed...See Users Guide";:COLOR 14,1: RE
TURN
9000 'Warning and Disclaimer Message
9005 CLS:COLOR 14,12:LOCATE 4,35 :PRINT "N O T I C E !":LOCATE 8,5
9010 PRINT "   THE RECOMMENDATIONS AND ESTIMATED LOADS THAT FOLLOW ARE BASED ON
A CONTINUING PROGRAM OF LABORATORY AND FIELD RESEARCH.   THEY  REPRESENT THE BES
T AVAILABLE  ENGINEERING INFORMATION AND CONSENSUS JUDGEMENT TO-DATE."
9011 PRINT
9015 PRINT "   HOWEVER, VIRGINIA TECH, THE U.S. FOREST SERVICE AND THE NWPCA HAV
E NO CONTROL";
9016 PRINT " OVER THE MANUFACTURE AND USE OF PALLETS OR THE CORRECTNESS AND APPL
ICABILITY OF THE INPUT DATA.  HENCE, THEY CANNOT ASSUME ANY RESPONSIBILITY FOR A
CTUAL PALLET";
9020 PRINT " PERFORMANCE OR THE CONSEQUENCES THEREOF."
9021 PRINT
9025 LOCATE 23,25:COLOR 11,1:PRINT "Press any key to continue":BEEP:YN$=INPUT$(1
):RETURN
20000 'sub to create RAD screen figures...iline indicates where fig is placed
20001 IF TSTI=4 THEN NW2=194:NW3=196:NW4=179:NW5=0 ELSE NW2=196:NW3=194:NW4=0:NW
5=179
20005 IF TSTI=2 THEN NW2=196:NW3=196:NW4=0:NW5=0
20010 IF IC2=1 THEN NW1=0:NW1A=218:NW1B=191 ELSE NW1=196:NW1A=194:NW1B=194
20020 LOCATE ILINE,2:PRINT CHR$(NW1);CHR$(NW1A);CHR$(194);CHR$(196);CHR$(196);CH
R$(196);CHR$(196);CHR$(NW2);CHR$(NW2);CHR$(196);CHR$(NW3);CHR$(NW3);CHR$(196);CH
R$(NW2);CHR$(196);CHR$(196);CHR$(196);CHR$(196);CHR$(194);CHR$(NW1B);C
HR$(NW1)
20030 'end top deck.begin bottom
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20040 IF TSTI=4 THEN NW2=193:NW3=196 ELSE NW2=196:NW3=193
20045 IF TSTI=2 THEN NW2=196:NW3=196
20050 IF IC2<=2 THEN NW1=0:NW1A=192:NW1B=217 ELSE NW1=196:NW1A=193:NW1B=193
20055 IF Q1=0 THEN 20068
20060 LOCATE ,2:PRINT CHR$(NW1);CHR$(NW1A);CHR$(193);CHR$(196);CHR$(196);CHR$(19
6);CHR$(196);CHR$(NW2);CHR$(NW2);CHR$(196);CHR$(NW3);CHR$(NW3);CHR$(196);CHR$(NW
2);CHR$(NW2);CHR$(196);CHR$(196);CHR$(196);CHR$(196);CHR$(193);CHR$(NW1B);CHR$(N
W1)
20065 GOTO 20075
20068 IF TSTI=4 THEN NW2=192:NW2A=217:NW3=0 :NW3A=0 ELSE NW2=0:NW2A=0:NW3=192:NW
3A=217
20069 IF TSTI=2 THEN NW2=0:NW3=0:NW2A=0:NW3A=0
20070 LOCATE ,2:PRINT CHR$(NW1);CHR$(NW1A);CHR$(217);CHR$(0);CHR$(0);CHR$(0);CHR
$(0);CHR$(NW2);CHR$(NW2A);CHR$(0);CHR$(NW3);CHR$(NW3A);CHR$(0);CHR$(NW2);CHR$(NW
2A);CHR$(0);CHR$(0);CHR$(0);CHR$(0);CHR$(192);CHR$(NW1B);CHR$(NW1)
20075 RETURN
```