ON RATIONAL EXPECTATIONS AND DYNAMIC GAMES

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(ABSTRACT)

We consider the problems of uniting dynamic game theory and the rational expectations hypothesis. In doing so we examine the current trend in macroeconomic literature towards the use of dominant player games and offer an alternative game solution that seems more compatible with the rational expectations hypothesis. Our analysis is undertaken in the context of a simple deterministic macroeconomy. Wage setters are the agents in the economy and are playing a non-cooperative game with the Fed. The game is played with the wage setters selecting a nominal wage based on their expectation of the money supply, and the Fed selects the money supply based on its expectation of the nominal wage.

We find it is incorrect to use the rational expectations hypothesis in conjunction with the assumption that wage setters take the Fed's choices as an exogenous uncontrollable forcing process. We then postulate the use of a Nash equilibrium in which players have rational expectations. This results in an equilibrium that has Stackleberg properties. The nature of the solution is driven by the fact that the wage setter's reaction function is a level maximal set that covers all possible choices of the Fed.

One of the largest problems we encountered in applying rational expectations to a dynamic game is the interdependency of the players' expectations. This problem raises two interesting but as yet unresolved questions regarding the expectations structures of agents: whether an endogenous expectations structure will yield rational expectations; and can endogenous expectations be completely modelled.

In addition to the questions mentioned above we also show that the time inconsistency problem comes from either misspecifying the constraints on the policy maker or an inconsistency in interpreting those constraints. We also show that the Lucas critique holds in a game setting and how the critique relates to the reaction functions of players.

"I will tell you a thing," the Rabbi said, "This 'crucial intersection of living awareness,' as they call it, that is nothing unless you know how your own decisions go out from you like threads into the lives of others."

"To see our own actions in the reactions of others, yes, that is how the Sisters view it."

Frank Herbert, Chapterhouse: Dune

I dedicate this dissertation to my parents in token of my love and respect for them.

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CHAPTER I

INTRODUCTION AND REVIEW OF THE LITERATURE

In 1961 John Muth defined rational expectations as agents having subjective probability distributions of variables of interest that were equal to the objective probability distributions of those variables. In the past 15 years a large body of macroeconomic literature has been developed using this hypothesis. We will be concentrating on this branch of the macroeconomic literature.

One of the first, and certainly more prominent, economists to use John Muth's rational expectations hypothesis in macroeconomics was Robert Lucas. Although, as Lucas puts it, the original impetus for this work was "the attempt to discover a useful theoretical explanation of business cycles,"[1] it has produced results, principally the policy ineffectiveness result, that support Milton Friedman's policy recommendations.[2]

The policy ineffectiveness result led to considerable debate regarding the appropriateness of the rational expectations hypothesis. Reexamination of the ineffectiveness result led to a softening of the hypothesis to mean simply the optimal use of information. However, much of the literature has maintained the Muth hypothesis as an asymptote to which agants' rationally formed expectations converge. Thus, throughout this dissertation we will use the term rational expectations to refer to the Muth hypothesis. Expectations which simply use information optimally will be called rationally formed expectations or referred to as expectational rationality.

A second result of the policy ineffectiveness debate was the rediscovery of the time inconsistency problem of Strotz (1955). This rediscovery of time inconsistency had two effects. First, it added a new dimension to the policy rules versus policy discretion debate. Second, it called into question the use of optimal control theory solutions and led to the use of dynamic dominant player game solutions in rational expectations models.

The main purpose of this dissertation is to address and attempt to resolve certain theoretical difficulties that result from the existing attempts to wed game theory and rational expectations macroeconomic theory. In the process we will examine the impact of the existing difficulties and the resolution of these difficulties on the policy ineffectiveness result and the question of time consistency.

Policy ineffectiveness and time inconsistency are by no means the entire range of questions addressed by the rational expectations literature. They do, however, represent two questions of great practical and theoretical importance. The policy ineffectiveness question is related to the neutrality of money. Time inconsistency depends, at least in part, on policy ineffectiveness and is critical to the policy rules versus policy discretion debate.

In this chapter we will trace the development of part of the rational expectations literature from the policy ineffectiveness result to the use of dominant player games. We will then posit our objections to the rational expectations-dominant player game paradigm and outline the approach of our analysis.

Section 1: The Policy Ineffectiveness Result[3]

The foundation of the policy ineffectiveness result can be found in some of the earliest work using the rational expectations hypothesis. Lucas and Rapping (1969) derived an expectational Phillips curve that, while exhibiting short-run inflation-unemployment trade-offs, exhibited long-run inflation inelasticity of unemployment. This type of an expectational Phillips relationship is indeed the cornerstone of the ineffectiveness result.

The expectational Phillips curve did not become the full blown policy ineffectiveness result until Lucas (1972), Sargent (1973) and Sargent and Wallace (1975) considered the relationship of monetary policy to unemployment, real interest rates and output. These theoretical experiments indicated that the real variables under consideration were "independent of the systematic part of the money supply."[4]

The reason for this result is rather starkly expostulated in Sargent and Wallace (1976). In this analysis the monetary authority is choosing the parameters g_0 and g_1 in the monetary policy feedback rule: $m_t = g_0 + g_1 y_{t-1}$, where y_{t-1} is an endogenous real variable, e.g., deviation of real GNP from "potential" GNP. If it is not true that "the 'structure' of these lags [in y_t] is constant over time and does not depend on how the monetary authority is behaving,"[5] then g_0 and g_1 will be indeterminate and one feedback rule will be as good as another.

There are two conditions that must hold in order to obtain the indeterminancy of g_0 and g_1 and therefore the ineffectiveness result.

The first condition is that the economy exhibit the expectational Phillips curve mentioned above. This in itself is not sufficient to generate the ineffectiveness result since it only posits that if expected and actual outcomes are equal the natural rate of unemployment will obtain. This leaves room for the policy authority to exploit a possible systematic bias in agents expectations structure.[6] The second condition that is necessary is that agents expectations be rational in the sense that they allow no systematic bias. Given these two conditions no systematic policy choice will have an impact on the real endogenous variable y_t.

The implications of the elimination of systematic bias under rational expectations are important to our later analysis, the Lucas (1976) critique of policy evaluation, and the time inconsistency question. In order to eliminate systematic bias in their expectations agents have to know, or be able to learn, the probability distributions of all variables not under their control. This implies that they know not only the structure of the random shocks that effect the economy but also the behavior of the policy authority. This means, as shown in Sargent and Wallace (1976) and discussed in Lucas (1976), that the parameters of the policy rules will be present in the reduced form equations for the endogenous variables. Thus, as Lucas argues, it is incorrect to assume that agents behavior will remain invariant under a change in policy regimes.[7]

Section 2: Time Inconsistency and Rules vs. Discretion

Time inconsistency due to expectational errors[8] was first discussed in Kydland and Prescott (1977), and later expanded on in Barro and Gordon (1983a and 1983b), Buchanan and Lee (1982), Calvo (1978), Fischer (1980), Kydland and Prescott (1980), Lucas and Stokey (1983), Miller and Salmon (1983), Stutzer (1984), and Whiteman (1984). In all of this work the time inconsistency of the optimal policies takes the same form and has the same causes as that in a simple example in Kydland and Prescott (1977).

In the 1977 Kydland and Prescott example the montary authority is faced with an expectational Phillips curve and is trying to minimize unemployment while also minimizing the variance of inflation about some socially optimal rate. The short run Phillips curve has the form:

$$u_t = \lambda(x_t^e - x_t) + u^*;$$

where $\lambda > 0$; u_t is unemployment in time t; x_t is the inflation rate; x_t^e is the expected inflation rate; u^* is the natural rate of unemployment; and u_t is the time t rate of unemployment. If expectations are rational then the monetary authority faces the additional constraint of the long run Phillips curve $u_t = u^*$. Given this long run constraint the optimal policy is to set $x_t = x^*$, where x^* is the optimal inflation rate, and let $u_t = u^*$ in all periods. This policy will not be time consistent since in any period the monetary authority will have an incentive to deviate from $x_t = x^*$ and lower u_t through a "surprise"[9] inflation.

The existence of this inconsistency in the optimal plan has two implications. First, due to the inconsistency the optimal plan will not be obtained if the policy authority is allowed the discretion to replan. Therefore in order to obtain the optimal plan we must somehow constrain the policy maker to the optimal policy rule.

The second implication of the time inconsistency of the optimal plan is that Bellman's principle of optimality no longer holds. Bellman's principle states:

> An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.[10]

Obviously, any policy that adheres to Bellman's principle will be time consistent. The fact that optimal policies in rational expectations models fail to adhere to Bellman's principle implies that it is not always correct to use standard techniques of optimal control theory to solve for the optimal policy.

Section 3: Dynamic Dominant Player Games

The inapplicability of optimal control theory, combined with trends outlined in Sargent (1982), led to the introduction, either implicitly or explicitly, of game theory in rational expectations literature. This use of game theory has most often taken the form of a dominant player game (see e.g., Miller and Solmon, (1983), Roberds (1984), Sargent (1984), and Whiteman (1984)). There are, however, some theoretical problems with the wedding of rational expectations and dominant player games.

First, we would like to emphasize that the objections and arguments below do not apply to "open loop" dominant player games. These games, which do not allow replanning, are essentially complicated static Stacklberg games in which first mover advantage and knowledge of the other player's reaction functions is sufficient to generate dominance, and no assumption needs to be made regarding the information sets of the non-dominant players. We would also like to emphasize that we are dealing with Muth rational expectations and not weaker forms of expectational rationality.

The conditions which generate dominance in open loop games do not hold when we consider dynamic dominant player games with replanning. It is no longer sufficent to assume first mover advantage, since this may be overcome by use of punishment strategies by the other players. In a dynamic replanning setting the dominant player must actually dominate the game. This is typically achieved by assuming that the non-dominant players do not recognize the effect their behavior has on the behavior of the dominant player, while the dominant player recognizes the effect that his behavior has on the behavior of other players. This implies that the information set of the dominant player consists of the history of the game and the reaction or decision functions of all players, while the information sets of the non-dominant players consists only of the history of the game.

Under the rational expectations hypothesis these information sets do not obtain. Rational expectations models typically assume that subjective expectations are equivalent to the mathematical expectations of the model. In a carefully constructed stochastic model the decisions of the dominant player will be well defined random variables whose conditional probability distributions will depend on his decision functions and the statistics of the stochastic elements of the model. Since the rational expectations assumption implies the non-dominant players know the conditional probability distribution of the choice variables of the dominant player, they must also have some information regarding the decision function of the dominant player. Thus the rational expectations hypothesis implies a larger information set for agents than the dynamic dominant player game.

In Chapter II we will consider a policy game, adapted from Canzoneri (1986), in a deterministic setting in which the information structure is compatible with dominant player games. We will enlarge on the above discussion of the informational incompatibility of rational expectations and dynamic dominant player games and show that under the information structure considered, expectations will exhibit a systematic bias. We will also argue that the monetary authority can exploit this bias to affect real output through repeated inflation "surprises" and therefore the game may have no steady state equilibrium.

In Chapter III we will consider an analogous game with information structures compatible with the rational expectations hypothesis. In this game there will be no systematic bias in expectations and monetary policy will no longer affect real variables. Furthermore, this game will have at least one steady state and a countable infinity of asymptotic steady states to which the economy will converge.

In Chapter IV we will discuss the implications of our analyses in Chapters II and III for the policy ineffectiveness result and the time inconsistency problem. We will compare the solutions obtained in Chapters II and III and present arguments defending the game theoretic setting of Chapter III.

In Chapter V we will present our conclusions and indicate areas of further research.

FOOTNOTES

[1]Lucas (1981), p. 2.

[2]See Milton Friedman, 1968.

[3]For an indepth analysis and review of this literature and an extensive bibliography see McCallum (1979, 1980).

[4]Sargent (1973), p. 463.

[5]Sargent and Wallace (1976), p. 171.

[6]This is essentially what we show can happen if the monetary authority is truly a dominant player. See Chapter II.

[7]This implies as we shall argue in Chapter III, that a correct specification and solution of a rational expectations game relies heavily on the reaction functions of the players.

[8]See Stutzer (1984) for a discussion of the various causes of time inconsistency.

[9]There is some question of the legitimacy of policy suprises in a rational expectations world that will be discussed in Chapter IV.

[10]See Richard Bellman, Dynamic Programming, p. 83.

CHAPTER II

THE DYNAMIC DOMINANT PLAYER GAME

During the 1970's the rational expectations hypothesis rose to prominence in macoreconomics as a reponse to the monetarist-Keynesian debate. One of the early results of this literature was that monetary policy would be ineffective thus vindicating Friedman's x-percent money growth rate rule.

However, a by-product of the policy-ineffectiveness result was the 1976 Lucas critique of policy analysis. This critique pointed out that under rational expectations it is inappropriate to assume that agents behavior will not change when the policy regime changes.

The Lucas critique led almost immediately to an examination of the time-consistency of optimal plans. Kydland and Prescott (1977), Calvo (1978), Fischer (1980) and (most clearly and forcefully) Stutzer (1984) argue that the classical techniques of optimal control developed by Bellman (1957) do not apply in a rational expectations environment. This left macroeconomists with a powerful paradigm but no way to solve the models engendered by it.

As Sargent (1984) points out, the Lucas critique rests on the strategic interdependence of agents and the poilicy authority, therefore the proper theoretical environment for rational expectations is a dynamic game. In most of the rational expectations literature this is implemented by "a dynamic game...in which the government is dominant."[1,2]

Although an appealing alternative to the optimal control solution, in that it overcomes mathematical problems inherent in the design of optimal policy under the rational expectations assumption, the dominant player solution is also flawed. The most glaring fault we find with dynamic dominant player rational expectations models is that the agents' expectational and behavioral information sets are incompatable. Under rational expectations agents know the true underlying model of the economy, including the behavior of the policy authority which, if the policy authority is attempting to pursue an activist policy, will be affected by the actions of the agents. Yet in dynamic dominant player games agents behavior fails to take into account this effect; i.e., they behave as if they didn't know the behavior of the policy authority.

As we will show in this chapter, resolution of the informational inconsistency mentioned above in favor of the dominant player game information set will lead to a systematic expectational bias on the part of the agents. This bias can then be exploited by the monetary authority in an effort to affect real output with monetary policy. Furthermore, given the information structure of the monetary authority, monetary policy will be consistent but the agents' behavior will be inconsistent. Resolution of the informational inconsistency in favor of the rational expectations hypothesis is deferred to the next chapter.

For illustrative purposes we will consider the problem of monetary policy choice in a rather simple discrete time deterministic dynamic macroeconomy adapted from Canzoneri (1983). This economy is populated by only one type of purposeful agent and the monetary authority.

The chapter proceeds as follows. In Section 1 we present some notation and definitions for the game we will be considering. In Section 2 we present the model underlying the game. In Section 3 we define and solve the game. In Section 4 we discuss expectations of the agents and the monetary authority. In Section 5 we discuss the results.

Section 1: Definitions and Notation

In the following discussion each player is assumed to have one choice variable, ν, at his command. This choice variable will be called a control. The value of a control in a particular time period will be called a move and is denoted ν_t. A vector of moves will be called a policy. A policy will be denoted $\{\nu_t\}^{t-i}$, where the superscript t-i denotes the time in which the policy is formed and the subscript t indicates the period in which the policy is implemented. If the superscript t-i is missing, the policy is formed and implemented in the same period. Thus $\{\nu_t\}$ is the policy formed and implemented at time t. A * superscript will denote the optimal move, ν_t^* or policy $\{\nu_t^*\}$.

A policy is said to be time consistent if $\{\nu_{t+j}\}^{t+i} = \{\nu_{t+j}\}^{t+k}$ for $0 \leq i < k \leq j$; i.e., if the future policy remains unchanged under replanning. This means that the given state of the economy in time t+1, as a result of move ν_t, the policy $\{\nu_{t+1}\}^t$ is still optimal. In short, a time consistent optimal policy satisfies Bellman's principle of optimality.

In this chapter much of the discussion will center on expected moves and policies. An e superscript will denote an expected move, ν_t^e, or policy $\{\nu_t^e\}$. Furthermore a superscript t-i will indicate the time in which the expectation is formed; e.g., $\{\nu_t^e\}^{t-i}$ would be player one's time t-i expectation of player two's policy $\{\nu_t\}$. No confusion will arise as to which player is forming the expectations.

In the model under consideration we define the variables as follows:

n_t = level of employment;

p_t = the price level;

y_t = the level of real output;

m_t = the nominal money supply;

w_t = the nominal wage rate; and

c_t = consumption.

All variables are in log form so that we may specify a linear model.

Section 2: The Model

In this section we consider a simple deterministic macro economy with two types of players: a single monetary authority (Fed) and z identical wage setters. The money supply $\{m_t\}$ and wage rate $\{w_t\}$ are the control policies of the Fed and wage setters, respectively. Each is choosing a policy in an attempt to maximize his own utility function subject to the structure of the economy and his belief about the policy choice of the other player. This policy choice occurs at the beginning of each period; however, only the current moves m_t and w_t are implemented at time t and the policy makers are in no way bound to the future policies $\{m_{t+1}\}^t$ and $\{w_{t+1}\}^t$.

In addition to the two players in the economy there exist several single owner firms producing a homogeneous product. The firms optimization problem is assumed to be consistent with a series of single period profit maximizations. Since the economy is structurally static over time, it is possible to solve this problem and derive time independent functional expressions for the firms' labor demand and output supply in terms of those variables exogenous to the firm, viz. w_t, m_t, p_t and c_t. The firms' decisions regarding quantities of labor demanded and output supplied are made after the moves m_t and w_t are known. We also assume that the profits of these firms are the only income of the owners and that owners and wage setters are completely averse to consuming the output of the firm with which they are associated. This last assumption, attributable to Howitt (1981), simply stimulates the use of money in the economy, whereas the assumptions concerning the firms' optimization and the timing of the firms' decisions reduce firms to the role of a non-player in this economy.[3]

In this economy wage setters contract for a nominal wage at the beginning of each period and agree to supply all the labor demanded by firms during that period. Thus, the level of employment in the economy is determined by firms' demand which is a decreasing function of the real wage and is given by:[4]

$$n_t = \phi(p_t - w_t), \qquad \text{(II.1)}$$

where $\phi > 0$ is a parameter.

There is no capital in this economy so output is a function only of labor. Assuming that output is a log-linear function of labor, the aggregate supply of output is:

$$y_t = \theta(p_t - w_t), \qquad \text{(II.2)}$$

where $\theta > z(\phi-1)$ is a parameter.

Following Lucas (1973), we will assume that real output is determined entirely by aggregate supply equation (II.2), and nominal output is determined by aggregate demand. We also assume a simple transactions sequence in each period, viz. wage setters are paid and then wage setters and firm owners purchase output for consumption purposes. This implies that the velocity of money in this economy is one. These assumptions coupled with the no capital assumption above allows us to use the simple quantity theory equation to specify the price level p_t in terms of the money supply m_t and real output y_t:

$$p_t = m_t - y_t. \qquad \text{(II.3)}$$

Wage setters are assumed to be infinitely lived individuals that derive utility from consumption and leisure. In addition, they are assumed to suffer a cost of adjusting their labor supply from one period to the next, reflecting a certain amount of risk aversion. Further, we assume that their utility function is separable, quadratic in both labor supply and labor supply adjustment, quasi-concave and discounted over time. Specifically, we assume their utility function has the form:

$$U^w(\cdot) = \sum_{j=0}^{\infty} b^j\{\delta_1 c_{t+j} - \frac{\delta_2}{2} n_{t+j}^2 - \frac{\delta_3}{2}(n_{t+j} - n_{t+j-1})^2\}, \qquad \text{(II.4)}$$

where $b \in (0,1)$ is the discount factor, $\delta_1, \delta_2, \delta_3 > 0$ are weights and $\delta_1 < \delta_2 + \delta_3$.

We assume that the wage-setters/labor-suppliers have, at least locally, some amount of market power so that their wage, though constrained by the market, will not be set by the market.[5] Therefore they may attempt to maximize (II.4) at time t by choice of $\{w_t\}$, given n_{t-1}, $\{m_t^e\}$ subject to equations (II.1)-(II.3) and the log of their budget constraint

$$p_{t+j} + c_{t+j} = n_{t+j} + w_{t+j}, \qquad j = 0, 1, \ldots \qquad \text{(II.5)}$$

This budget constraint indicates that each period's nominal consumption must equal each period's nominal income; i.e., there is no saving.[6]

Equations (II.1)-(II.3) are constraints on the wage setter's maximization since taken together with w_t and m_t they determine his level of employment, his real wage and the price level. Thus, without any one of these three equations, one or more of the arguments of the utility function will fail to be fully specified. Note also that since firm owners are assumed to consume part of the output, the sum of all wage setters' consumption, $\sum_z c_t$, will be less than total output, y_t.

We also assume that each wage setter has the same n_{t-1} and the same expectations. Since each wage setter is choosing w_t to achieve a particular quantity of labor supplied n_t, and each wage setter has the same utility function, then each wage-setter will select the same wage rate in each period. Thus, in what follows, we will discuss the case of a single representative wage setter. Although it does not seem natural to consider a case in which the Fed responds to the wage demand of an individual wage setter, it would be perfectly natural to assume that it responses to some wage index defined over all wage setters. Since the wage setters are identical, each individual wage will be equal to any reasonably constructed wage index. Thus, we lose no generality by considering an individual wage setter rather than considering several identical wage setters.

The Fed is assumed to be infinitely lived. It is assumed to suffer a quadratic loss around some target level of output, $\bar{y}$, and to suffer a quadratic loss in adjustment of the price level. Further, we assume that the Fed's utility function is separable and discounted over time. We specifically assume a utility function of the form:

$$U^f(\cdot) = \sum_{j=0}^{\infty} \beta^j \{-\frac{\gamma_1}{2}(y_{t+j} - \bar{y})^2 - \frac{\gamma_2}{2}(p_{t+j} - p_{t+j-1})^2\}, \qquad \text{(II.6)}$$

where $\beta \in (0,1)$ is the discount factor and γ_1, $\gamma_2 > 0$ are weights. The Fed desires to maximize (II.6) by choice of $\{m_t\}$, given p_{t-1}, w_{t-1}, $\{w_t^e\}$ subject to equations (II.2) and (II.3). Note that equation (II.1) is not a constraint on the Fed's maximization problem since the Fed is unconcerned with the level of employment and equation (II.1) is not used to determine p_t or y_t.

These maximization problems, (II.4) and (II.6), subject to the condition that m_t and w_t be real numbers, comprise the basic elements of a game. The maximization problems describe the payoffs of the players as functions of their policies; these policies consist of sequences of choices for m_t and w_t. The condition that m_t and w_t be real numbers defines the strategy sets from which policies may be drawn. What we need now are the rules by which the game will be played, and in particular, the order in which moves are made and the manner in which payoffs are determined.

Section 3: The Fed as a Dominant Player

There are many ways in which the game we have set up may be solved. As Sargent (1984) points out, most games in the rational expectations literature are solved as dominant player games with the policy authority (Fed) as the dominant player.

In this section we consider a game between the Fed and the wage setter in which the Fed is a dominant player. In playing the game the Fed and the wage setter move simultaneously and their payoffs in each period will be determined by the realizations of m_t and w_t.

Nash equilibrium consists of expectations and decision functions for the Fed and the wage setter. The decision function of the wage setter will maximize his utility subject to (II.1)-(II.3), (II.5) and his expectations function. The decision function of the Fed will maximize its objective function subject to (II.2), (II.3), its expectations functions, and the wage setters' decision and expectations functions. In other words, the Fed will behave as a Stackleberg leader, in the sense that it has an accurate model of the wage setters behavior and expectations, and the wage setters will behave as Stackleberg followers. This implicitly assumes a belief structure for the wage setter which, as will be discussed in Section 5, is not necessarily rational.

In deriving the dominant player solution we assume that the Fed knows and takes into consideration the wage setter's reaction function. The representative wage setter, however, takes the Fed's policy as an exogenous forcing process and believes that the Fed's policy is not a function of w_t. That is, the wage setter takes $\frac{\partial m^e_{t+j}}{\partial w_{t+i}} = 0$ for all i and j in his Euler equation (II.12) below. As we shall see the money supply expectations of the wage setter, $\{m^e_t\}$, will be the driving force in this solution. We will defer discussion of the processes that drive $\{m^e_t\}$ and $\{w^e_t\}$ until Section 4. For our current purposes we simply assert that m^e_t will be a distributed lag of m_{t-1} and w_{t-1} and that w^e_t will be equal to w_t.

In order to find a solution to this game we must derive the Euler equations for each player. These equations govern the selection of each player's time t policy.

The first step in deriving the Euler equations is to write the model in reduced form. Equations (II.1)-(II.3) can be solved for y_t, p_t, and n_t in terms of m_t and w_t. This yields:

$$n_t = \frac{\phi}{1+\theta} m_t - \frac{\phi}{1+\theta} w_t, \tag{II.7}$$

$$y_t = \frac{\theta}{1+\theta} m_t - \frac{\theta}{1+\theta} w_t, \tag{II.8}$$

$$p_t = \frac{1}{1+\theta} m_t + \frac{\theta}{1+\theta} w_t. \tag{II.9}$$

Equations (II.5), (II.7) and (II.9) can now be used to solve for c_t in terms of m_t and w_t. This yields:

$$c_t = \frac{1-\phi}{1+\theta}(w_t - m_t). \tag{II.10}$$

Note that the restriction $\theta > z(\phi-1)$ ensures that $\sum_z c_t < y_t$.

Notice that equations (II.7)-(II.10) describe the endogenous variables, n_t, p_t, y_t, and c_t in terms of the two (exogenous) controls, w_t and m_t. This implies that w_{t-1} and m_{t-1} determine the current state of the economy which forms a constraint on the players in their choice of their optimal time t moves. In order to determine the optimal policy for each player we must follow the evolution of the economy from period to period to find the optimal move in each period given the past policy of the two players.

Equations (II.7), (II.9) and (II.10), w_{t-1}, m_{t-1} and $\{\overset{e}{m}_t\}$ are now the constraints in maximizing equation (II.4). By substituting (II.7), (II.9) and (II.10) into (II.4) the wage setter's utility maximization problem may be written as:

$$\max_{\{w_t\}} U_w(\{w_t\},\{\overset{e}{m}_t\}) = \sum_{j=0}^{\infty} b^j\{\delta_1\frac{1-\phi}{1-\theta}(w_{t+j} - \overset{e}{m}_{t+j}) - \frac{\delta_2}{2}[\frac{\phi}{1+\theta}(\overset{e}{m}_{t+j} - w_{t+j})]^2$$

$$- \frac{\delta_3}{2}[\frac{\phi}{1+\theta}(\overset{e}{m}_{t+j} - w_{t+j} - \overset{e}{m}_{t+j-1} + w_{t+j-1})]^2\}, \tag{II.11}$$

subject to w_{t-1}, m_{t-1}, and $\{\overset{e}{m}_t\}$ given.

The Euler equations associated with (II.11) are

$$b^j\{\delta_1\frac{1-\phi}{1+\theta}(1 - \frac{\partial m^e_{t+j}}{\partial w_{t+j}}) - \delta_2[\frac{\phi}{1+\theta}]^2(m^e_{t+j} - w_{t+j})(\frac{\partial m^e_{t+j}}{\partial w_{t+j}} - 1)$$

$$- [\delta_3[\frac{\phi}{1+\theta}]^2(m^e_{t+j} - w_{t+j} - m^e_{t+j-1} + w_{t+j-1})]$$

$$\cdot [(\frac{\partial m^e_{t+j}}{\partial w_{t+j}} - 1 - \frac{\partial m^e_{t+j-1}}{\partial w_{t+j}})]\}$$

$$+ b^{j+1}\{-[\delta_3[\frac{\phi}{1+\theta}]^2(m^e_{t+j+1} - w_{t+j+1} - m^e_{t+j} + w_{t+j})]$$

$$\cdot [(\frac{\partial m^e_{t+j+1}}{\partial w_{t+j}} - \frac{\partial m^e_{t+j}}{\partial w_{t+j}} + 1)]\} = 0, \quad j = 0,1,2... \qquad \text{(II.12)}$$

The transversality condition associated with (II.11) is:

$$\lim_{T\to\infty} b^T\{\delta_1\frac{1-\phi}{1+\theta}(1 - \frac{\partial m^e_{t+T}}{\partial w_{t+T}}) - \delta_2[\frac{\phi}{1+\theta}]^2(m^e_{t+T} - w_{t+T})(\frac{\partial m^e_{t+T}}{\partial w_{t+T}} - 1)$$

$$- [\delta_3[\frac{\phi}{1+\theta}]^2(m^e_{t+T} - w_{t+T} - m^e_{t+T-1} + w_{t+T-1})]$$

$$\cdot [\frac{\partial m^e_{t+T}}{\partial w_{t+T}} - 1 - \frac{\partial m^e_{t+T-1}}{\partial w_{t+T}})]\} = 0. \qquad \text{(II.13)}$$

The system of equations (II.12) is a second order difference equation system. The constraint that w_{t-1} is given and transversality condition (II.13) are sufficient to ensure a solution. In order for the transversality condition (II.13) to hold it is sufficient to have the processes $\{w_t\}$ and $\{m_t\}$ be of exponential order less than 1/b. If this is the case then $U_w(\{w_t\},\{m_t^e\})$ will be finite for all t.

By the same methodology the maximization problem for the Fed may be written as:

$$\max_{\{m_t\}} U_f(\{m_t\},\{w_t^e\}) = \sum_{j=0}^{\infty} \beta^j\{-\frac{\gamma_1}{2}[\frac{\theta}{1+\theta}(m_{t+j} - w_{t+j}^e) - \bar{y}]^2$$

$$- \frac{1}{2}[\frac{\gamma_2}{1+\theta}(m_{t+j} + \theta w_{t+j}^e - m_{t+j-1} - \theta w_{t+j-1}^e)]^2\},$$

(II.14)

subject to w_{t-1}, m_{t-1}, and $\{w_t^e\}$ given.

The Euler conditions associated with (II.14) are:

$$\beta^j\{-[\gamma_1[\frac{\theta}{1+\theta}](m_{t+j} - w_{t+j}) - \bar{y}][\frac{\theta}{1+\theta}][(1 - \frac{\partial w_{t+j}}{\partial m_{t+j}})]$$

$$- [\gamma_2[\frac{1}{1+\theta}]^2(m_{t+j} + \theta w_{t+j} - m_{t+j-1} - \theta w_{t+j-1})]$$

$$\cdot [(1 + \theta\frac{\partial w_{t+j}}{\partial m_{t+j}} - \theta\frac{\partial w_{t+j-1}}{\partial m_{t+j}})]\}$$

$$+ \beta^{j+1}\{-[\gamma_2[\frac{1}{1+\theta}]^2(m_{t+j+1} + \theta w_{t+j+1} - m_{t+j} - \theta w_{t+j})]$$

$$\cdot [(\theta\frac{\partial w_{t+j+1}}{\partial m_{t+j}} - 1 - \theta\frac{\partial w_{t+j}}{\partial m_{t+j}})]\} = 0, \ j = 0,1,2...$$

(II.15)

The transversality condition associated with (II.14) is:

$$\lim_{T\to\infty}\beta^T\{\gamma_1[\frac{\theta}{1+\theta}](m_{t+T} - w_{t+T}) - \bar{y}][\frac{\theta}{1+\theta}][(1 - \frac{\partial w_{t+T}}{\partial m_{t+T}})]$$

$$- [\gamma_2[\frac{1}{1+\theta}]^2(m_{t+T} + \theta w_{t+T} - m_{t+T-1} - \theta w_{t+T-1})]$$

$$\cdot [(1 + \theta\frac{\partial w_{t+T}}{\partial m_{t+T}} - \theta\frac{\partial w_{t+T}}{\partial m_{t+T}})]\} = 0 \qquad \text{(II.16)}$$

Once again the Euler equations (II.15) form a second order difference equation system, and m_{t-1} and the transversality condition (II.16) ensure that a solution exists. Again, it is sufficient for $\{w_t\}$ and $\{m_t\}$ to be of exponential order less that $1/b$ to have the transversity condition (II.16) hold. We assume, without loss of generality, that $1/b < 1/\beta$.

Solving the Euler equations (II.12) for w_{t+j} when $\frac{\partial m_{t+j}}{\partial w_{t+i}} = 0$ yields the policy rule in feedback form for the wage setter. This policy rule then becomes a constraint on the Fed when maximizing (II.14).

Equations (II.12) can be written in the form:

$$(1 - \lambda_1 L)(1 - \lambda_2 L)w_{t+j+1} = A + B(L)m^e_{t+j+1} \qquad \text{(II.12')}$$

where $\lambda_1\lambda_2 = \frac{1}{b}$, $\qquad \lambda_1 + \lambda_2 = \frac{\delta_2+\delta_3+b\delta_3}{b\delta_3}$ (II.17)

$$A = \delta_1\frac{(\phi-1)(1+\theta)}{b\phi^2},$$

$$B(L) = 1 - (\lambda_1 + \lambda_2)L + \lambda_1\lambda_2 L^2$$

and L is the lag operator. Equations (II.17) and the quadratic formula imply that $\lambda_1 < 1$ and that $\lambda_2 > 1/b$. Following Sargent[7] and solving the unstable root λ_2 forward, we can rewrite (II.12') as:

$$w_{t+j+1} = \lambda_1 w_{t+j} + \frac{A}{1-\lambda_2} + m^e_{t+j+1} - \lambda_1 m^e_{t+j} + c\lambda_2^j; \qquad j = -1,0,1,\ldots \tag{II.18}$$

where $c\lambda_2^j$ is a transient term that must equal zero to satisfy the transversality condition. Taken together equations (II.18) define the optimal policy at time t, given the wage setter's time t expectation of the policy of the Fed. Taking this expression and its partial derivative with respect to m_{t+j} and substituting into the Fed's Euler equations (II.15) we can solve for the Fed's optimal policy given $\{w^e_t\}$;

$$m_{t+j} = \frac{a}{d} w^e_{t+j} + \frac{\theta g}{d} w^e_{t+j-1} - \frac{\beta\theta h}{d} w^e_{t+j+1} + \frac{g}{d} m_{t+j-1}$$

$$- \frac{\beta h}{d} m_{t+j+1} + \gamma_1\theta(1+\theta)\bar{y}\left(1 - \frac{\partial w_{t+j}}{\partial m_{t+j}}\right). \tag{II.19}$$

All coefficients, a, d, g, and h are functions of θ, $\frac{\partial w_{t+j-1}}{\partial m_{t+j}}$, $\frac{\partial w_{t+j}}{\partial m_{t+j}}$ and $\frac{\partial w_{t+j+1}}{\partial m_{t+j}}$. The actual expressions for the coefficients are presented in the Appendix for the interested reader.

Section 4: <u>Information and Expectations</u>

As was stated at the beginning of Section III agents take the behavior of the money supply as an exogenous forcing process. When forming their expectations of the money supply, they will effectively be modelling the behavior of the Fed. This model of the Fed will be conditioned by their beliefs about the Fed's policy function, (i.e., $\partial m_t/\partial w_t = 0$) and the information about the history of the game. Similarly, the Fed will formulate a model of the wage setter in order to form its expectations of the nominal wage.

The wage setters information set, $\{\Omega_t\}$, available at time t, is the set of all past values of variables, wage rates, and money supplies and equations (II.7)-(II.10). Given (II.7)-(II.10), $\{\Omega_t\} = (\{m_{t-j}\}_{j=1}^{\infty}, \{w_{t-j}\}_{j=1}^{\infty})$.

The wage setter is assumed to form his expectation by making a model of the Fed's policy function and using their information to estimate the parameters of that model by a least squares projection of m_t on $\{\Omega_t\}$. Given that the wage setter believes that $\{m_t\}$ is an exogenous forcing process the model should take the form:

$$m_t^e = C(L)m_{t-1} + D(L)w_{t-1} + F; \tag{II.20}$$

where C(L) and D(L) are polynomials in the lag operators and F is a constant. The wage setter's expectation of m_{t+j} will be formed by applying C(L) to the vector $\{m_{t+j-1}^e,\ldots,m_t^e,m_{t-1},\ldots\}$ and adding D(L) applied to $\{w_{t+j-1}^e,\ldots,w_{t+1}^e,w_t,w_{t-1},\ldots\}$ where w_{t+j-1}^e is the planned move of the wage setter in period t+j-1. Thus, the wage setter formulates expectations of future money supplies by projecting the time path of his information sets, $\{\Omega_{t+1}^e\}$, $\{\Omega_{t+2}^e\},\ldots$.

In order for the transversality conditions (II.13) and (II.16) to be fulfilled it is sufficient that the roots of the characteristic polynomials $C(R) = 0$, $D(R) = 0$ be greater than $b > \beta$ in modulus and the sequences $\{m_{t-j}\}_{j=0}^{\infty}$ and $\{w_{t-j}\}_{j=0}^{\infty}$ be of exponential order less than $1/b < 1/\beta$.[8] Notice this assumption and (II.20) imply that $\{m^e_{t+j}\}$ will be of exponential order less than 1/b, therefore, by (II.18), $\{w_t\}$ will be of exponential order less than 1/b and, by (II.19) $\{m_t\}$ will be of exponential order less than 1/b. This means that the restrictions placed on the terms of (II.20) are consistent with the model.

The Fed has the same information as the wage setter and, in addition, has a correct model of the wage setter's policy function (II.18) and the wage setter's expectation function (II.19). Given this information it is easy to see that the Fed's expectation of the wage is formed by:

$$w^e_t = w_t = \lambda_1 w_{t-1} + \frac{A}{1-\lambda_2} [C(L)m_{t-1} + D(L)w_{t-1}] - \lambda_1 m_{t-1} \; . \tag{II.21}$$

Like the wage setters the Fed formulates expectations of future wages by updating the information set $\{\Omega_t\}$. But unlike the wage setter the Fed can accurately update the information set and can therefore formulate a time consistent optimal policy at any time t.

Section 5: Discussion

The solution to the dominant player game considered in this chapter consists of the players' policy or decision functions (II.18) and (II.19) and the players' expectations functions (II.20) and (II.21). In this section we consider the implications of this solution for rational expectations, time inconsistency, policy effectiveness and the existence of a steady state.

Notice that equation (II.18) will be sensitive to errors in expectations from period to period. This is because if $m_t^e \neq m_t$ then $\{\Omega_{t+1}^e\} = \{\Omega_t\} \cup (w_t, m_t^e) \neq \{\Omega_t\} \cup (w_t, m_t) = \{\Omega_{t+1}\}$ and $\{m_{t+1}^e\}^t \neq \{m_{t+1}^e\}^{t+1}$. If this occurs then $\{w_{t+1}\}^t \neq \{w_{t+1}\}^{t+1}$. Thus, the policy $\{w_t\}$, described by (II.18) will be optimal only if the wage setter has rational expectations; i.e., $m_{t+j}^e = m_{t+j}$ for all $j = 0,1,2,\ldots$. If expectations are not rational then the time t policy described by (II.18) will be sub-optimal and time inconsistent.

To see that expectations will not be rational, consider equation (II.20). This says that m_t^e is a distributed lag of m_{t-1} and w_{t-1}. In order for $m_t^e = m_t$, equation (II.19) implies a and h must equal zero. This will not be the case even if the Fed ignores the impact of its policy on the nominal wage. Therefore, $m_t^e \neq m_t$ and the wage setter's policy $\{w_t\}$ will not be optimal or time consistent.

This will not be the case for the Fed. Although the above discussion regarding the relationship of the Muth rationality of expectations and the time consistency of policy holds for the Fed, the Fed's policy will be time consistent. This is because (II.21) implies $w_t^e = w_t$, therefore the Fed's expectation of the information set, $\{\Omega_{t+1}^e\} = \{\Omega_t\} \cup (m_t, w_t^e)$ will be correct. Thus, at any time t the Fed can correctly predict the state of the economy at time t+1. By applying successive predictions the Fed can form its optimal time consistent policy $\{m_t^*\}$ at time t.

Essentially the difference between the wage setter and the Fed centers on the accuracy of their models. The wage setter's model fails to take into account the fact that the Fed is modeling his behavior.

However, the Fed's model of the wage setter, as shown in (II.21), includes the wage setter's model of the Fed. Thus, as Blanchard (1977) and Benjamin Friedman (1979) point out the wage setter's expectations fail to be rational because they are trying to estimate the wrong model of the Fed's behavior, and the reason they have the wrong model is that they take the money supply as an exogenous forcing process.

The irrationality of the wage setter's expectations has implications other than the time inconsistency of the wage setters' policy. Since $m_t^e \neq m_t$, there may be a steady state in the real variables c_t, y_t and n_t, but there will be no steady state in the nominal variables m_t, p_t and w_t. This is because the Fed is exploiting the systematic bias in the wage setter's expectations implied by (II.19) and (II.20) to move y_t towards $\bar{y}$, pursuing either an inflation or a deflation in every period. The last statement implies that the Fed's policy can be effective in setting the real variables. This can easily be seen by considering the feasible policy:

$$m_t = \frac{\theta+1}{\theta}\bar{y} + w_t. \qquad \text{(II.22)}$$

The previous discussion regarding time consistency still holds, as does the discussion regarding the fulfillment of the transversality conditions. Using (II.22) will set $y_t = \bar{y}$ in every period. Furthermore, (II.22) will move y_t from its "natural" level[9] unless $\bar{y} = \theta(\phi-1)/\phi^2$, which is the "natural" level of y_t. In fact, the only thing that prevents the Fed from using policy (II.22) is that it does not account for the inflation cost of the policy.

As the reader has probably realized the solution to the game discussed in this chapter begs a few questions. The most important, from the standpoint of the macroeconomist is: "How can the Fed continually trick the wage setter, even though the wage setter has the entire history of the game available to him?" The answer is: it shouldn't be able to. If the wage setter has consistently low expectations of m_t and is rational, then he should assume there is something wrong with his model and adjust it. If he strikes upon the right model he will realize that his assumption that the money supply is an uncontrollable forcing process is wrong. A partial solution to this type of game will be presented in Chapter III.

FOOTNOTES

[1]See Sargent (1984), p. 410.

[2]See also Kydland (1977), Sargent (1984), Stutzer (1984), and Whiteman (1984).

[3]Actually these assumptions reduce the firm to the role of a game playing machine that takes the Fed's and wage setter's moves as inputs and produces pay-offs to the two players as outputs while moving the economy from one period to the next.

[4]This is the log form of an exponential demand function of the form $(w_t/p_t)^{-\phi}$. The same interpretation can be applied to the aggregate supply equation (2).

[5]Alternatively the wage setters may be thought of as trade unions that allocate the labor they contract to supply among its members so that (II.4) represents the group utility.

[6]Wage setters could have savings in the form money, this however would complicate the dynamics of the economy without adding anything to the results we are trying to obtain. For our purposes it will be sufficient to assume that the firm owners hold all money from one period to the next.

[7]See Sargent (1979), pp. 197-199.

[8]This is due to Rudin (1976) Theorem 3.29 (p. 69). See also Hansen and Sargent (1980), Footnote 5.

[9]Here we take the natural level to be that which is consistent with the wage setters' optimum.

CHAPTER III

THE RATIONAL EXPECTATIONS VERSION OF THE GAME

In the previous chapter we considered a dominant player game and treated players' information and expectations in a manner that was dictated by their assumed behavior. This resulted in agents formulating an incorrect model of the Fed and therefore not having rational expectations. This gave policy effectiveness results that are contrary to existing rational expectations results.

In this chapter we will impose information sets consistent with the rational expectations hypothesis and consider behavior that is consistent with this assumption. This will result in possible symmetric treatment of the players and will bring a sharper focus to the Lucas (1976) critique.

The use of the rational expectations hypothesis implies that every player knows (or can learn) the underlying structure of the model. In particular, this means each player knows the optimal reaction functions of the other players and these functions become constraints on his behavior. This creates some rather large simultaneity and existence problems that did not exist in the dominant player version of the game.

The main problem with introducing rational expectations into a game setting is describing the expectations structure of the players. In a two player setting, player 1 must have a complete and accurate model of player 2 in order to have rational expectations. The same is true for player 2. This means player 1's complete model of player 2 must include player 2's model of player 1, and player 2's complete model of player 1 must include player 1's model of player 2. But this means that player 1's complete model of player 2 must include player 2's model of player 1's model of player 2. This will define a new element or level in player 2's model, which in turn will define a new element in player 1's model and so on. In order to describe either player's complete model we must follow this process to the end.

In light of the problem discussed above, we will simply assume that the players have rational expectations. In a dynamic game setting this should be interpreted as assuming that each player has a complete model of all other players and that for each possible value of his choice variable his subjective probability distribution of uncontrollable variables is equal to the final objective probability distribution after accounting for all interactions between the choice variable and the uncontrollable variables.

Given the deterministic nature of the model, this means that $m_t^e = m_t$ and $w_t^e = w_t$ for all t. Granted, this is a deus ex machina expectations structure and some of the weaknesses of this assumption have been discussed in Blanchard (1977), Buiter (1980), B. Friedman (1979) and Howitt (1981). It is, nevertheless, a standard assumption in the rational expectations literature, and to our knowledge, has never been used in a dynamic macroeconomic policy game outside the context of a dominant player game.

Given the assumption of rational expectations neither player can be fooled in any period. Therefore they cannot be moved off their reaction functions and one player's reaction function becomes a constraint for the other player. This means the solution to the game will be a Nash equilibrium--specifically a Nash policy equilibrium. This equilibrium allows a player to consider not only a change in other player's moves but also a change in other player's decision functions as a result of a change in his own move or decision function. Unfortunately, such equilibria are difficult to construct unless they are stationary. Therefore we will first show that a stationary equilibrium exists among the class of policy equilibria and then derive a stationary policy equilibrium for the game.

The chapter is organized as follows: In Part I we use the Federgruen (1978) result to establish the existence of a stationary equilibrium. In Part II we construct the stationary equilibirium and discuss the implications of such an equilibrium.

PART I: The Existence of a Stationary Policy Equilibrium Point

In this part of the chapter we will apply, to the model presented in Chapter II, an existence result for stochastic games derived by Federgruen. In doing so we make certain additional assumptions and it will be convenient to adopt some of the terminology of the stochastic game literature. These assumptions will be detailed in Section 1. Policy equilibria and stationary policies will be defined and a sketch of the Federgruen existence proof will be presented in Section 2.

Section 1: Preliminaries

The non-coopertative game under consideration can be specified by the set of players--the Fed and the wage setters--and by four other elements: the state space S; an action space A^i for each player; a transition probability distribution q; and a bounded real valued one period payoff or return function r^i for each player. Each of these elements must meet certain conditions in order to apply the existence result. As we shall see in Part II of this chapter, some of these conditions are only sufficient to generate an equilibrium, though all are necessary for the use of the Federgruen existence theorem. The parts of the model from Chapter II that correspond to each element and the specifications for each element are detailed below.

The state space, $S = \{s:s = (n_t,p_t)$ for all n_t and $p_t\}$. Note that the state space for the game does not correspond to the typical macroeconomic state space in that it leaves out the endogenous variables y_t and c_t. The state space of the game consists of all endogenous variables that effect the evolution of the game or the future behavior of the players. Since y_t and c_t do not appear in lagged form in the state equations of the model (II.1)-(II.3) and (II.5) or the objective functions (II.4) and (II.6) they are not part of the state space of the game. The Federgruen result requires that S be countable.

The action spaces, $A^w(s)$ for the wage setter and $A^f(s)$ for the Fed, are the sets of possible values for the control variables of two players. The action spaces need not be the same for all states of a game. However, without loss of generality, we will assume that they are in order to simplify notation. This is not a strong assumption since $A^w(s)$ and $A^f(s)$ are subsets of the real line for all $s \epsilon S$ and we assume that $A^w(s)$ and $A^f(s)$ are compact metric spaces for any $s \varepsilon S$. Thus for the compactness assumption to be satisfied there must exist a least upper and a greatest lower bound for $A^w(s)$ and $A^f(s)$ for each $s \varepsilon S$. The non-parameterized action spaces, A^f and A^w, can then be bounded above by the greatest of the parameterized least upper bounds and below by the smallest of the parameterized greatest lower bounds. The idea is that the set of possible actions need not be restricted to the set of probable actions.

In order to satisfy the compactness assumption it is sufficient that A^w and A^m be closed and bounded. In order to bound the action space we make the following assumptions. First notice that although suppressed in (II.2), y_t is a function of n_t. By assuming a maximum labor supply we effectively assume a maximum output. This, combined with the implicit assumption that money is only demanded to reduce transactions costs,[1] implies there is some maximum value of the money supply, M^+, beyond which money transactions are no more efficient than barter transactions and money no longer affects the economy. We also assume that there is a minimum quantity of money, $M^- > 0$, below which money is too scarce to be efficiently used for transactions. Furthermore we assume there is a maximum real wage that firms are willing to pay in any given period. Since m_t and y_t are bounded (II.3) implies p_t is bounded and therefore there exists an upper bound W^+, on the nominal wage in all periods. In addition, we assume there is a minimum real wage which the wage-setters are willing to accept. Equation (II.3), a maximum of y_t and the existence of M^-imply that there is a minimum price level and therefore a minimum nominal wage $W^- > 0$.[2] Thus the action spaces are $A^f = \{m_t : M^- < e^{m_t} < M^+\}$ and $A^w = \{w_t : W^- < e^{w_t} < W^+\}$.

These assumptions are sufficient to satisfy the action space requirements of the Federgruen theorem since A^f and A^w are closed and bounded subsets of the real line. However, we will also assume that A^f and A^w are countable, in particular that they be expressed in some fraction of a dollar with a finite decimal representation. This, given the nature of the state equations, (II.7) and (II.9), implies the countability of S.

We recognize that the assumptions placed on the action spaces are not terribly palatable, particularly to the macroeconomist. We stress, however, that these are only sufficient and not necessary conditions for the existence of an equilibrium. The assumptions of the compactness of A^f and A^w and the countability of S may be relaxed only at the expense of considerable technical difficulties[3] in proving the existence of an equilibrium for the general case. As we shall see these assumptions can be relaxed more easily in the specific case we are dealing with in this chapter.

The <u>transition probability distribution</u> $q_{ss'}(\vec{a}_t) : S \times A^w \times A^f \to [0,1]$ gives the probability of going to $s' \varepsilon S$ given that we are at $s \varepsilon S$ and action $\vec{a}_t \ \varepsilon\ A = A^w \times A^f$ is taken. The function $q_{ss'}(\vec{a}_t)$ is required to be continuous on A. In our formulation $q_{ss'}(\vec{a}_t)$ is dependent on equations (II.7) and (II.9), i.e.,

$$n_t = \frac{\phi}{1+\theta} m_t - \frac{\phi}{1+\theta} w_t = n(\vec{a}_t) \qquad \text{(III.1)}$$

$$p_t = \frac{1}{1+\theta} m_t + \frac{\theta}{1+\theta} w_t = p(\vec{a}_t). \qquad \text{(III.2)}$$

The transition probability is conditioned by (III.1), (III.2) and the selection of $\vec{a}_t$ and is given by:

$$q_{ss'}(\vec{a}_t) = \begin{cases} 1 & s' = (n(\vec{a}_t'), p(\vec{a}_t')) \\ 0 & \text{otherwise} \end{cases}$$

Since $n(\vec{a}_t)$ and $p(\vec{a}_t)$ are continuous on A, $q_{ss'}(\vec{a}_t)$ will be continuous on A.

Note that $q_{ss'}(\vec{a}_t)$ does not imply that the unconditional probability of going from s to some s' is 1. This is because mixed strategies are available to the players so that at a given state they may choose an action $\vec{a}_t$, with probability less than one. However, given the nature of $q_{ss'}(\vec{a}_t)$ this is essentially a degenerate stochastic game in that the stochastic elements of the transition function has a zero variance, meaning that there is only strategic uncertainty in the game.

The last elements of the game are the one period <u>pay-off functions</u>, r^f for the Fed, and r^w for the wage-setter. The functions r^f and r^w are derived from the objective functions specified in Chapter II by ignoring the intertemporal additivity, substituting (II.7)-(II.10) for the appropriate time t variables, and treating the lagged variables as constants. In our formulation r^f and r^w are defined by:

$$r^f(m_t;w_t,s_t) = -\frac{\gamma 1}{2}\left[\frac{\theta}{1+\theta}(m_t - w_t) - \bar{y}\right]^2$$

$$-\frac{\gamma 2}{2}\left[\frac{1}{1+\theta}(m_t + \theta w_t) - p_{t-1}\right]^2; \qquad \text{(III.3)}$$

$$r^w(w_t;m_t,s_t) = \delta_1(1-\phi)(w_t - m_t)\ \frac{\delta 2}{2}\ -\left[\frac{\phi}{1+\theta}(m_t - w_t)\right]^2$$

$$-\frac{\delta 3}{2}\left[\frac{\phi}{1+\theta}(m_t - w_t) - n_{t-1}\right]^2. \qquad \text{(III.4)}$$

It is obvious that r^f and r^w are continuous on A, real-valued and bounded, due to the boundedness of A and S.[4]

Given these elements, S, A, q, r^f and r^w that satisfy the assumptions:

(A1) S is countable;

(A2) A is a compact metric space;

(A3) q is a countinuous probability distribution function on A;

(A4) r^f and r^w are bounded and continuous real valued functions on A;

and the set of players; we can specify a non-cooperative game and assert the existence of a staionary policy equilibrium.

Section 2: <u>Definition of the Game and Definition of Equilibrium</u>

In the game we consider in this chapter each player chooses a policy in an attempt to maximize his discounted payoff over the life of the game subject to the expected policy of the other players and the transition probability function. Although this is similar to the game played in Chapter II, there is one crucial distinction. We assume that each player knows (or can learn) the policies of the other players,[5] i.e., they have rational expectations. This expectational assumption does not affect the existence proof. It will come into play in deriving the equilibrium poilicies.

The policies under consideration here are slightly different than those considered in Chapter II. In Chapter II we considered a vector of moves. We now turn our consideration to the decision function that generated those moves. A decision function for player i is a mapping $\delta_t^i : S \to A^i(s)$. A policy for player i is a sequence of decision rules $\pi^i = (\delta_1^i, \delta_2^i, \ldots)$. So in state s at time t player i plays $\delta_t^i(s)$.[6] Thus a player's policy may involve using different decision rules in different periods.[7] If a player's policy uses the same decision rule in every period, it is said to be <u>stationary</u>.

In our game $\mu : S \to A^f$ will denote the Fed's decision rule and $\omega : S \to A^w$ will denote the wage setter's decision rule. The policies for the Fed and wage setter will be denoted $\pi^f = \{\mu_1, \mu_2, \ldots\}$ and $\pi^w = \{\omega_1, \omega_2, \ldots\}$ respectively and their stationary policies will be denoted $\{\mu\}$ and $\{\omega\}$. We will denote a policy pair (π^f, π^w) by $\vec{\pi}$.

Now that policies have been defined we need only the discounted payoff and the definition of equilibrium to complete the definition of the game.

The discounted expected payoff or value functions are defined by:

$$V^f(\vec{\pi};s) = E_{\vec{\pi}}\{\sum_{t=0}^{\infty} \beta^t r^f(\vec{a}_t;s_t)|s_0 = s\}; \tag{III.5}$$

for $s \varepsilon S$ and $0 < \beta < 1$;

and

$$V^w(\vec{\pi};s) = E_{\vec{\pi}}\{\sum_{t=0}^{\infty} b^t r^w(\vec{a}_t;s_t)|s_0 = s\}; \tag{III.6}$$

for $s \varepsilon S$ and $0 < b < 1$.

Given (III.3) and (III.4) equations (III.5) and (III.6) are just alternative expressions for the objective functions (II.6) and (II.4) of the Fed and the wage setter respectively. Note that $E_{\vec{\pi}}$ gives the expected future actions, $\vec{a}_t$ and states, s_t. The actual payoffs are given by the realization of the future states.

A <u>policy equilibrium</u> for this game is a policy pair $\vec{\pi}^*$ that, simultaneously for every $s \varepsilon S$, satisfies:

$$V^f(\vec{\pi}^*;s) > V^f(\pi^f,\pi^{w*};s) \text{ for all } \pi^f \varepsilon \Pi^f;$$

$$V^w(\vec{\pi}^*;s) > V^w(\pi^w,\pi^{f*};s) \text{ for all } \pi^w \varepsilon \Pi^w;$$

where Π^f and Π^w are the spaces of all possible policies for the Fed and wage setter. This definition is just an extension of the Nash equilibrium concept to policies and implies that neither player can improve his payoff by unilaterally deviating from $\vec{\pi}^*$. An equilibrium is said to be a <u>stationary policy equilibrium</u> if $\vec{\pi}^* = (\{\mu\},\{\omega\})$.

The game can be formulated as the binary optimization:

$$\max_{\pi^f \in \Pi^f} V^f(\vec{\pi};s) \quad \text{subject to } \pi^w; \tag{III.7}$$

$$\max_{\pi^w \in \Pi^w} V^w(\vec{\pi};s) \quad \text{subject to } \pi^f; \tag{III.8}$$

for the Fed and wage setter. An equilibrium is a solution $\vec{\pi}^*$ which achieves (III.7) and (III.8) simultaneously for every $s \in S$.

We now state the Federgruen existence theorem:

<u>Theorem 1</u>: In a game that satisfies (A1)-(A4), there exists a stationary policy equilibrium.

Like many equilibrium existence proofs the proof of the Federgruen existence theorem consists of constructing a mapping from the strategy set[8] into itself and then showing that the mapping has a fixed point that satisfies the conditions of equilibrium.

The strategy sets, Δ^f and Δ^w, consist of all possible stationary policies for the Fed and the wage setter. The proof of the theorem consists of constructing the reaction functions for the two players. The reaction functions are then used as the components of a vector functional, Φ that maps from $\Delta = \Delta^f \times \Delta^w$ into the closed non-empty subsets of Δ. The functional Φ is then shown to be upper semi-continuous and Δ is shown to be a convex compact subset of a linear Hausdorff locally convex topological space. The Glicksberg (1952) fixed point theorem asserts the existence of a fixed point for Φ, and an extension of Blackwell's (1968) theorem 6-f ensures that the fixed point satisfies the conditions for an equilibrium.

The proof of Theorem 1 implies that the reaction mappings of the players, which are the components of Φ, are upper semi-continuous and have closed graphs. Therefore these reaction mappings intersect due to the compactness and convexity of Δ^f and Δ^w. Thus the geometry of this proof is roughly analogous to the geometry of the well known static Nash game.

PART II: A Solution to the Game

In this part of the chapter we will derive a solution to the rational expectations game. Given Theorem 1 we will be seeking a stationary policy equilibrium. Since players expectations are assumed to be formed according to $m_t^e = m_t$ and $w_t^e = w_t$, or $\mu_t^e = \mu_t$ and $\omega_t^e = \omega_t$, we will effectively be searching for a Nash stationary policy equilibrum.

The reason that the Nash equilibrium is also the rational expectations equilibrium is that if both players have rational expectations then neither player can trick the other. This means that both players will be able to stay on their reaction functions because their beliefs and expectations will be realized. This is exactly what is specified in a Nash equilibrium.

As it turns out we will find a Stackleberg variant of the Nash equilibrium with the Fed as the leader. This is due to the nature of the wage setter's reaction function. As we shall see the wage setter's payoff is the same at all points on his reaction function so he is content to allow the Fed to act as a leader.

In deriving the solution we will first find the reaction function of the wage setter and show it has the above mentioned property. We will then use the wage setter's reaction function as a constraint in deriving the Fed's optimal policy.

In Section 3 we will derive a solution and in Sections 4 and 5 we will discuss the implications of the solution.

Section 3: Derivation of a Solution

In this section we find decision functions μ^*: $S \rightarrow \Delta^f$ and ω^*: $S \rightarrow \Delta^w$ that constitute an equilibrium, where Δ^f and Δ^w are the sets of possible stationary policies for the Fed and the wage-setter.

By proposition 2 of Chapter 6 of Bertsekis (1976) there exist value functions $V_\omega^f(s)$ and $V_\mu^w(s)$ that satisfy:

$$V_\omega^f(s) = \max_{\mu\varepsilon\Delta^f} \{r^f(\mu,\omega,s) + \beta V_\omega^f(q_{ss'}(\mu,\omega,s))\}, \text{ given } \omega;$$

$$\text{(III.9)}$$

$$V_\mu^w(s) = \max_{\omega\varepsilon\Delta^w} \{r^w(\omega,\mu,s) + bV_\mu^w(q_{ss'}(\omega,\mu,s))\}, \text{ given } \mu;$$

for all $s\varepsilon S$. The function $V_\omega^f(s)$ gives the maximum value of the game to the Fed if the game starts at s and the wage setter uses ω. Similarly, $V_\mu^w(s)$ gives the maximum value to the wage setter of a game starting at s when the Fed uses μ.

Notice that the value functions, $V_\omega^f(\cdot)$ and $V_\mu^w(\cdot)$ are real valued functions defined on $\Delta \times S$. Notice also that given r^f and r^w the functions $V_\omega^f(\cdot)$ and $V_\mu^w(\cdot)$ will be continuous on Δ. This is readily apparent, though it may be difficult to prove if S is uncountable and therefore Uryschn's theorem may not be applied to make Δ. is not metrizable. Thus we may now dispense with the countability assumptions on A^f, A^w and

S made in Section 1 above, though we still assume A^f and A^w are compact. Note too, that A^f and A^w are now convex, so the players will use pure strategies.

We can use the right hand sides of (III.9) to define reflexive mappings T_ω^f and T_μ^w which have functions V_ω^f and V_μ^w, as their range and domain. The mappings T_ω^f and T_μ^w are contraction mappings. This implies that for any ω there is a unique $V_\omega^f(s)$, and for any μ there is a unique $V_\mu^w(s)$ that satisfy:

$$V_\omega^f(s) = T_\omega^f(V_\omega^f)(s), \text{ given } \omega; \qquad \text{(III.10a)}$$

$$V_\mu^w(s) = T_\mu^w(V_\mu^f)(s), \text{ given } \mu. \qquad \text{(III.10b)}$$

The Fed's decision function μ which satisfies (III.10a) is the optimal policy response to the wage setter's decision function ω which generated (III.10a). Similarly the ω which satisfies (III.10b) is the optimal reponse to the μ which generated (III.10b). Thus we can obtain the Fed's and the wage setter's reaction functions by solving (III.10a) for μ as a function of ω and by solving (III.10b) for ω as a function of μ. A stationary policy equilibrium pair can then be found by solving these functions simultaneously for μ and ω.

To find the reaction functions we first notice that if μ satisfies (III.10a) and if ω satisfies (III.10b) then (III.10a) and (III.10b) can be restated as:

$$V_\omega^f(s_t) - \beta V_\omega^f(s_{t+1}) = r^f(\mu,\omega,s_t); \qquad \text{(III.11)}$$

$$V^{w}_{\mu}(s_t) - bV^{w}_{\mu}(s_{t+1}) = r^{w}(\mu,\omega,s_t); \qquad \text{(III.12)}$$

for all $s \epsilon S$, and for all t.

Thus finding the μ which will give the Fed the largest payoff given an ω is equivalent to maximizing r^f by choice of μ subject to ω. Similarly to find the optimal ω for a μ we maximize r^w by choice of ω, given μ.

Here we encounter the problem of maximizing a functional by choice of a function. This can be overcome by observing that r^f and r^w are quadratic functions. This implies that m_t and w_t are determined by linear functions of the form:

$$m_t = \hat{\mu}(w_t,s_t); \; w_t = \hat{\omega}(m_t,s_t).$$

But $\hat{\mu}$ and $\hat{\omega}$ can be solved for m_t and w_t as linear functions of only s_t. Thus the functions μ and ω are linear functions of the form:

$$\begin{aligned} m_t &= \mu(s_t) = x_1 n_{t-1} + x_2 p_{t-1} + x_3 \bar{y} + x_4; \\ w_t &= \omega(s_t) = z_1 n_{t-1} + z_2 p_{t-1} + z_3 \bar{y} + z_4. \end{aligned} \qquad \text{(III.13)}$$

Thus we are now faced with the problem of choosing $X = (x_1,x_2,x_3,x_4)$ and $Z = (z_1,z_2,z_3,z_4)$ to maximize r^f and r^w. Again these simultaneous maximizations will yield reaction functions $X = f(Z)$ and $Z = g(X)$ whose simultaneous solutions $X^* = f(g(X^*))$ and $Z^* = g(f(Z^*))$ will, when substituted into (III.13) give a solution to the game.

By substituting (III.13) into (III.4) and maximizing by choice of Z we obtain the first order condition:

$$\frac{\partial r^W}{\partial z_i} = (\delta_2 + \delta_3)n_t - \delta_3 n_{t-1} + \delta_1 \frac{1-\phi}{\phi} = 0, \quad i = 1, 2, 3, 4; \qquad \text{(III.14)}$$

where $n_t = \frac{\phi}{1+\theta}\{(x_1 - z_1)n_{t-1} + (x_2 - z_2)p_{t-1} + (x_3 - z_3)\overline{y} + (x_4 - z_4)\}$. Solving (III.14) for z_1, z_2, z_3, and z_4 we obtain:

$$z_1 = x_1 - \frac{1 + \theta}{\phi} \cdot \frac{\delta_3}{(\delta_2 + \delta_3)}; \qquad \text{(III.15a)}$$

$$z_2 = x_2; \qquad \text{(III.15b)}$$

$$z_3 = x_3; \qquad \text{(III.15c)}$$

$$z_4 = x_4 + \frac{(1+\theta)(1-\phi)}{\phi^2} \cdot \frac{\delta_1}{(\delta_2 + \delta_3)} \qquad \text{(III.15d)}$$

Equations (III.15-a-d) define the component functions of the wage setters reaction function g (i.e., (III.15a-d) defines $Z = g(X)$).

From the above discussion it is obvious that if equations (III.15a-d) are satisfied then:

$$n_t = \frac{\delta_3}{\delta_2 + \delta_3} n_{t-1} + \frac{\delta_1(\phi - 1)}{(\delta_2 + \delta_3)\phi}$$

regardless of the value of X. Therefore, since equation (II.10) implies that $c_t = [(\phi - 1)/\phi]n_t$, as long as the wage setter can enforce the relationship between X and Z implied by (III.15a-d) he will be indifferent to the value of X chosen by the Fed.

Given this the Fed will choose the X which will maximize its payoff subject to the constraint of the wage setter's reaction function. This means the Fed will act as a Stackleberg leader, and the wage setters will act as a Stackleberg follower. Thus the Fed will choose X to maximize r^f subject to (III.15a-d). The first order conditions are:

$$\frac{\partial r^f}{\partial x_1} = -\gamma_2(p_t - p_t^{-1})\frac{1}{1+\theta} = 0 \qquad \text{(III.16)}$$

where $p_t = \frac{1}{1+\theta}\{(x_1 + \theta z_1)n_{t-1} + (x_2 + \theta z_2)p_{t-1} + (x_3 + \theta z_3)\bar{y} + x_4 + \theta z_4)$.
Solving (III.16) for x_1, x_2, x_3 and x_4 yields:

$$x_1 = -\theta z_1 \qquad \text{(III,17a)}$$

$$x_2 = 1+\theta - \theta z_2 \qquad \text{(III.17b)}$$

$$x_3 = -\theta z_3 \qquad \text{(III.17c)}$$

$$x_4 = -\theta z_4 \qquad \text{(III.17d)}$$

Equations (III.17a-d) define the component functions of the Fed's reaction function. Solving (III.15a-d) and (III.17a-d) simultaneously yields:

$$x_1 = \frac{\delta_3\theta}{(\delta_2+\delta_3)\phi};\ z_1 = \frac{-\delta_3}{(\delta_2+\delta_3)\phi}$$

$$x_2 = z_2 = 1;$$

$$x_3 = z_3 = 0;$$

$$x_4 = \frac{\theta(\phi-1)\delta_1}{\phi^2(\delta_2+\delta_3)};\quad z_4 = \frac{1-\phi)\delta_1}{\phi^2(\delta_2+\delta_3)};$$

substituting into (III.13) gives the optimal policies:

$$m_t = \mu(s_t) = \frac{\theta}{\phi}\,\frac{\delta_3}{(\delta_2+\delta_3)}\,n_{t-1} + p_{t-1} + \frac{\theta(\phi-1)}{\phi^2}\,\frac{\delta_1}{(\delta_2+\delta_3)} \qquad \text{(III.18)}$$

$$w_t = \omega(s_t) = \frac{-1}{\phi}\,\frac{\delta_3}{\delta_2+\delta_3}\,n_{t-1} + p_{t-1} - \frac{(\phi-1)}{2\phi^2}\,\frac{\delta_1}{(\delta_2+\delta_3)} \qquad \text{(III.19)}$$

Notice that in deriving the reaction functions and finding their intersection we did not appeal to the compactness of A^f and A^w. This means that for this game there is a mapping $\Phi:\Delta\to\Delta$ that has a fixed point regardless of whether Δ is compact. Thus, we may now relax the compactness assumptions on A^f and A^w and allow m_t and w_t to take any value on the extended real line.

Using these policies and any $(n_o,p_o)\varepsilon S$ we can generate a sequence $\{n_t,p_t\}$ that gives the time path of the state variables by the following:

$$n_t = \left(\frac{\delta_2}{\delta_2+\delta_3}\right)^t n_o + \frac{\phi-1}{\phi}\,\frac{\delta_1}{\delta_2+\delta_3}\sum_{i=1}^{t-1}\left(\frac{\delta_3}{\delta_2+\delta_3}\right)^i \qquad \text{(III.20)}$$

$$p_t = p_o \qquad \text{(III.21)}$$

Thus, each initial state (n_o,p_o) implies an asymptotic steady state to which the economy will converge.

Section 4: <u>Implications of the Solution</u>

The optimality of the Stackleberg solution rests on two conditions: First the wage setter's reaction function (III.15a-d) describes a level maximal set which covers all possible values of X. Second the wage setter has perfect foresight and cannot be tricked into leaving his reaction function. If either of these conditions fails to obtain the Stackleberg solution will not be optimal.

The fact that we solved for the Fed's policy in the same way that we would solve for a Stackleberg leader's policy should not be construed to imply that the Fed is a dominant player. If we restrict the Fed to the reaction function (III.17a-d) and use this as a contraint in the wage setter's maximization we would obtain the exact same policies with the wage setter as the leader. The problem with the latter approach is that equations (III.17a-d) do not describe a natural unconstrained reaction function for the Fed.

The unconstrained reaction function of the Fed is given by:

$$x_1 = \frac{\gamma_1\theta^2\gamma_2\theta}{\gamma_2+\gamma_1\theta^2} z_1; \qquad \text{(III.22a)}$$

$$x_2 = \frac{\gamma_1\theta^2-\gamma_2\theta}{\gamma_2+\gamma_1\theta^2} z_2 + \frac{\gamma_2+\gamma_2\theta}{\gamma_2+\gamma_2\theta^2}; \qquad \text{(III.22b)}$$

$$x_3 = \frac{\gamma_1\theta^2-\gamma_2\theta}{\gamma_2+\gamma_1\theta^2} z_3 + \frac{\gamma_1\theta^2+\gamma_1\theta}{\gamma_2+\gamma_1\theta^2}; \qquad \text{(III.22c)}$$

$$x_4 = \frac{\gamma_1\theta^2-\gamma_2\theta}{\gamma_2+\gamma_1\theta 2} z_4. \qquad \text{(III.22d)}$$

Using (III.22a-d) as a constraint in the wage setter's maximization yields a Nash solution of the form:

$$m_t = \mu(s_t) = \frac{(\gamma_2\theta-\gamma_1\theta^2)\delta_2}{\gamma_2\phi(\delta_2+\delta_3)} n_{t-1} + p_{t-1} + \frac{\gamma_1\theta}{\gamma_2}\bar{y} \tag{III.23}$$

$$w_t = \omega(s_t) = -\frac{(\gamma_1\theta^2+\gamma_2)\delta_2}{\gamma_2\phi(\delta_2+\delta_3)} n_{t-1} + p_{t-1} + \frac{\gamma_1\theta}{\gamma_2}\bar{y} + \frac{(\gamma_1\theta^2+\gamma_2)(1-\phi)\delta_1}{\gamma_2\phi^2(\delta_2+\delta_3)} \tag{III.24}$$

Using these policies and any $(n_o,p_o)\varepsilon S$ we can again generate a sequence $\{n_t,p_t\}$ that gives the time path of the state variables by the following:

$$n_t = \left(\frac{\delta_2}{\delta_2+\delta_3}\right)^t n_o + \frac{(\phi-1)}{\phi}\frac{\delta_1}{(\delta_2+\delta_3)}\sum_{i=1}^{t-1}\left(\frac{\delta_3}{\delta_2+\delta_3}\right)^i \tag{III.25}$$

$$p_t = -\frac{\gamma_1\theta^2\delta_2}{\gamma_2\phi(\delta_2+\delta_3)}\left[\sum_{i=1}^{t}\left(\frac{\delta_2}{\delta_2+\delta_3}\right)^{t-1}n_o + \frac{\phi-1}{\phi}\left(\frac{\delta_1}{\delta_2+\delta_3}\right)\sum_{i=2}^{t}\sum_{j=0}^{t-i}\left(\frac{\delta_2}{\delta_2+\delta_3}\right)^j\right] + p_o + t\left[\frac{\gamma_1\theta}{\gamma_2}\bar{y} + \frac{\gamma_1\theta^2\delta_1}{\gamma_2\phi^2(\delta_2+\delta_3)}\right] \tag{III.26}$$

Note that if $\bar{y} = \theta\delta_1/[\phi^2(\delta_2+\delta_3)]$ then the sequences generated by (III.27) and (III.26) are convergent, and again each initial state (n_o,p_o) implies an asymptotic steady state to which the economy will converge.

The policies described by (III.23) and (III.24) could be an equilibrium if the Fed believed that it could not move the wage setter off his reaction function and at the same time ignored the implication of (III.15a-d) that monetary policy will be ineffective with respect to the real variables. In fact it is the policy ineffectiveness result that drives the Stackleberg solution (III.18) and (III.19).

One last comment before addressing the Lucas critique. Notice that the Stackleberg solution is both optimal and time consistent. This is because in the Stackleberg solution the Fed recognizes the constraints, implied by the wage setter's reaction function and expectations structure, that its policy will be ineffective and that policy surprises are not possible. Given this the Fed uses the policy, dictated by its objective function and these constraints, that minimizes the inflation cost and ignores the unavoidable output cost.

Section 5: The Lucas Critique

As was mentioned in the introduction to this chapter, the analysis presented above brings a sharp focus to the Lucas critique. As Lucas indicated it is incorrect to assume that agents' decisions or behavior will remain unchanged when the policy authority's decision rule changes. This can be seen quite clearly by examining the wage setter's reaction function.

The equilibrium derived in this chapter rests on the reaction functions of the Fed and wage setter. If the Fed changes its policy rule, this is the same as choosing a new vector of parameters X. When a new X is chosen by the Fed, equations (III.15a-d) imply that a new Z will be chosen by the wage setter. Thus, as Lucas said, the choice of a new monetary policy implies the choice of a new wage policy, and any evaluation of monetary policies that does not take this into account is incorrect.

But the analysis of this chapter reveals even more. The wage setter cannot expect monetary policy to remain unchanged if he changes his wage policy. That is, if the wage setter selects a new Z then the Fed's reaction function (III.17a-d) or (III.22a-d) indicate the Fed will select a new X.

Another thing brought out in this analysis is the generating process for the reaction and decision functions. Reaction and decision functions are products of the objective functions, the structure of the economy and beliefs of the players. Recall that the only difference in the analyses that generated the Stackleberg and Nash solutions was an assumption about the beliefs of the Fed. In the former solution the Fed believed that it could not affect real output, in the latter solution it believed it could affect real output. This change in beliefs on the part of the Fed had very large implications for the nominal variables. The price level went from being stable in (III.21) to being possibly explosive in (III.26).

The importance of beliefs highlights the difficulties in applying the idea of rational expectations to a dynamic game. As will be discussed in Chapter IV the beliefs of one player can affect the beliefs of other players, but the Nash equilibrium concept can only be applied when beliefs are static. Thus given the interaction of beliefs, the application of rational expectations and the Nash equlibrium becomes tenuous and strained. It is only by making large assumptions (i.e., perfect foresight and no surprises) that they may be applied. Once these assumptions break down one is plunged into the morass of interactive beliefs from which there may or may not be an escape.

FOOTNOTES

[1]Notice that we motivated the existence of money through a demand for transactions although we did not exclude barter.

[2]The assumption of the minimum values, M^- and W^-, effectively bounds the level values away from zero. Although it would seem natural to allow zero to be the minimum level value of m and w, to do so would allow the action spaces A^m and A^w, which consists of log values, to be unbounded.

[3]The assumptions on A^f, A^w, and S are used to obtain the metrizability of the space of decision functions, Δ. It is then shown that the payoff or value functions, V are continuous on Δ. Without the metrizability of Δ the standard ε-δ definition of continuity does not hold and some other, less tractable, convergence concept must be used.

[4]The boundedness of S comes from the linearity of q and the boundedness of A.

[5]If the underlying model were a non-deterministic one this knowledge would be within a stochastic error.

[6]In Chapter II the policies under consideration were sequences $\{\delta_1^i(s), \delta_2^i(s), \ldots\}$.

[7]This goes to the heart of the Lucas critique. If one player changes his decision function it is reasonable to assume that other players will also change their decision functions in response.

[8]The strategy set may be a set of moves, decision functions or policies.

CHAPTER IV

DISCUSSION

In this chapter we compare the solutions derived in Chapters II and III. We find the solution of Chapter II has implications for the behavior of the economy and the wage setter that are unplatable from the macroeconomic viewpoint. However, we find that the solution in Chapter III behaves rather well with respect to the movement of the economy and the behavior of the wage setters and the Fed.

We will also consider the impact of the policy ineffectiveness result on the policy in Chapter III and the time inconsistency problem. In light of this, we will consider the relationship of the policy ineffectiveness result, the time inconsistency problem and some criticisms of the rational expectations hypothesis and how this may imply restrictions on the use of rational expectations.

Section 1: Comparison of Results

In Chapter II we derived a solution to a dynamic dominant player game which was characterized by the Fed creating and exploiting a bias in the expectations of the wage setters. This resulted in the possibility of there being no steady state in a structurally static deterministic economy. This result is antithetical to most existing macroeconomic results and makes us uneasy about using this game paradigm to solve macroeconomic models.

The fundamental question that must be addressed to understand why we get such peculiar results is: "Why is it that the wage setters do not or cannot learn the policy function of the Fed even though they can observe the infinite past history of the game and are supposedly rational?" Ideally, the wage setters can and do learn the policy function of the Fed. But, if this occurs the dominance of the Fed would be destroyed.

Recall that the dominance of the Fed was predicated on its informational advantage. If the wage setter learns the Fed's policy function then that informational advantage is destroyed and the Fed is no longer a dominant player. Without the information asymmetry, the Fed and the wage setters are equal players in the game.

This leads to a second question: "Is it necessary that the Fed have superior information in order to be a dominant player or is it sufficient that 'individual [wage setters] do not perceive the effects their decisions have on the form of the policy function'?"[1] To our mind it is necessary to give the Fed an informational advantage, because, while it may be true that the individual wage setters do not perceive the effects of their decisions on the policy function, it is also true that the Fed does not know the effect that its decisions have on the policies of the individual wage setters. Essentially what is happening is that both the wage setters and the Fed are responding to their perceptions of the movement of the aggregate state variables. This type of state contingent response is exactly the solution found in Chapter III.

The solution in Chapter III is fairly well behaved. Although there may not be a classical steady state in which the rate of change of variables is constant, the time path of the real variables will form a well ordered convergent sequence whose rate of convergence will depend only on the parameters of the wage setters objective function. Furthermore, as indicated in Chapter III, Section 3, if the Fed abandons output targets, then the price level will be constant over time.

The lack of a steady state in the Chapter III solution can be traced to the labor supply adjustment cost of the wage setter. Looking at (III.14) it is readily apparent that if the wage setter suffers no loss from adjusting his labor supply; i.e., $\delta_3 = 0$, then $n_t = [\delta_1(\phi-1)]/\delta_2\phi$ for all t. Since $y_t = (\theta/\phi)n_t$ and $c_t = [(\phi-1)/\phi]n_t$, this implies that there will be a steady state for the real variables.

The lack of a steady state in the rational expectations solution is much more palatable than the lack of a steady state in the dominant player solution for two reasons. First, we would like to see the Fed behave in a benign manner, in that it does not undermine the utility of the wage setter. Since a steady state fails to exist in the rational expectations solution only because of utility maximizing behavior on the part of the wage setter, the Fed is acting in a benign manner. This is not true in the dominant player solution. In the dominant player solution a steady state fails to exist because the Fed is attempting to move the wage setter from his optimal employment level, which will obviously reduce the wage setter's utility.

The second reason that the non-existence of a steady state solution in the rational expectations solution is more palatable than the non-existence of a steady state in the dominant player result is that the former result is compatable with the policy ineffectiveness result that one expects in a rational expectations model, while the latter result is not. In the rational expectations solution the fact that the equilibrium may or may not exhibit a steady state in the real variables depends purely on the preferences of the wage setters implies that money has no influence on the real variables, and so is neutral. The fact that monetary policy is the cause of the non-existence of a steady state in the real variables in the dominant player solution implies that money can influence the real variables and, therefore, is not neutral.

The fact that the rational expectations solution exhibits many classical rational expectations macroeconomic equilibrium characteristics (e.g., policy ineffectiveness, tendency towards the "natural" output level, nominal sector stability dependent on the Fed's behavior) and yet admits the possibility of the non-existence of a steady state is interesting. This result seems to indicate that it might be possible to build a frictionless macroeconomic model with an endogenous business cycle that is dependent solely on the utility maximizing behavior of the individual agents. In fact, this model will yield a damped oscillation in the real variables if one wishes to make the heroic assumption that the wage setters enjoy adjusting their labor supply; i.e., $\delta_3 < 0$ in (III.14).

Section 2: Policy Ineffectiveness and Time Inconsistency

The observant reader will have noted that the existence of policy ineffectiveness would seem to imply that the Fed's Nash policy described by (III.23) is sub-optimal. This is because the Fed will suffer price level adjustment costs in every period without affecting real output.

In fact, the policy described by (III.23) is analogous to the consistent policy of Kydland and Prescott's (1977) example. In their example the Fed's consistent policy set the inflation rate at above the social optimum and still did not reduce unemployment. In our model the prices many be explosive if the Fed does not choose the correct output target and yet there will be no effect on output.

Does this mean that the consistent policy of Kydland and Prescott's example and our Nash policy are sub-optimal and that the optimal policies are time inconsistent? The answer is no. It can be claimed that the Nash policy is optimal given the constraints on the players. When we derived the Nash policy we did not constrain the Fed away from the delusion that it could effect real output. If we enforce this constraint; i.e., if y_t is constant with respect to m_t, then the Fed's optimal policy is described by the Stackleberg solution.

Similarly, the consistent "sub-optimal" policy in the Kydland and Prescott example can only be arrived at by totally differentiating the monetary authorities objective function, $V(u_t,x_t)$, while ignoring the expectations constraint $x_t^e = x_t$, which yields:

$$0 = dV = \frac{\partial V}{\partial u_t} du_t + \frac{\partial V}{\partial x_t} dx_t,$$

or

$$\frac{dx_t}{du_t} = \frac{\dfrac{\partial V}{\partial u_t}}{\dfrac{\partial V}{\partial x_t}} = MRS_{xu},$$

where u_t is the unemployment rate and x_t is its inflation rate. By using the short run Phillips curve $u_t = \lambda(x_t^e - x_t) + u^*$ we get:

$$\frac{dx_t}{du_t} = -\frac{1}{\lambda};$$

where $\lambda > 0$, u^* is the natural unemployment rate and x_t^e is the expected inflation rate. Thus we have the familiar result that the marginal rate of substitution is equal to the slope of the constraint. As an after thought, the condition that $x_t^e = x_t$ is appended, implying that $u_t = u^*$. Combining this with the above result we get the consistent policy by finding the x_t that solves:

$$MRS_{xu}\Big|_{u_t = u^*} = \frac{1}{\lambda}.$$

Unfortunately this result cannot be obtained from a constrained maximization. Kydland and Pescott assumed that the monetary authority suffered a loss in unemployment and a quadratic loss in inflation around some optimal level x^*. Thus, assuming $x^* = 0$, the constrained optimization of the monetary authority can be written as:

$$\max_{x_t} V(u_t,x_t) = -\xi_1 u_t - \frac{1}{2}\xi_2 x_t^2$$

subject to $u_t = \lambda(x_t^e - x_t) + u^*$. This will yield $MRS_{xu} = 1/\lambda$, but it is not a well defined problem. Only the choice variable, x_t and one of the constraint variables, u_t are defined at all, and u_t is not well defined because it depends on the undefined variable x_t^e. In order to close the problem we must define x_t^e.

The only information that Kydland and Prescott give on expectations is that they are rational and that this implies $x_t^e = x_t$. Adding this to the constrained optimization will obviously yield $x_t = 0$, $u_t = u^*$ for all t, which is their optimal policy. Therefore, as with our model, the failure to obtain the globally optimal policy when deriving the time consistent policy results from failing to properly specify the constraints.

Given the simplistic nature of the error which seems to have generated the time inconsistency result we are forced to believe that there is some implicit assumption driving the time inconsistency result. Indeed, most rational expectations models that exhibit time inconsistency seem to assume that policy surprises are possible, and it is these policy surprises that are driving the time inconsistency in these models.

If by rational expectations one means simply that expectations are formed by optimally using available information, then policy surprises are possible, in fact Chapter II provides ample proof of this. However, if by rational expectations one means the Muth (1961) hypothesis, then, as Stutzer (1984) points out, policy surprises represent a logical paradox. If, as Muth suggests, agents know the underlying structure of the model then they can predict "surprises." On the other hand if surprises can occur, agents expectations cannot be Muth rational. Since it is intellectually (due to the apparent short-run inflation-unemployment trade-off) and emotionally appealing to allow the policy authority to enact surprises, the existence of this paradox raises some questions as to the appropriate use of the Muth hypothesis in dynamic macroeconomic games.

Section 3: The Use of Rational Expectations

As indicated in the previous section time inconsistency from policy suprises[2] and rational expectations are mutually exclusive properties of a model. Since policy surprises would seem to be possible in the real world, this seems to imply that the rational expectations model is not a good model of the expectations structure of agents. In actuality this only implies a restriction on the interpretation of rational expectations.

As Friedman (1979) and Taylor (1975) have shown, rational expectations can be derived, from a model with endogenous learning, as the long run limit of a short run expectations structure. In their models agents gather information in each period on variables they are interested in, and use the additional information to update estimates of the parameters of the model generating those variables. Thus, the rational expectations version of Friedman's and Taylor's models contains the limit point of a sequence of paramater estimates in the expectations formulae, assuming the sequence of parameter estimates converges.

There are two problems that may arise in this analysis, both at least partly drive the results of Chapter II. The first problem was raised by Blanchard (1977) and acknowledged by Friedman (1979). Agents may not have the correct model of the generating process of the variables they are interested in, that is, they are estimating the wrong parameters. This may keep the sequence of parameter estimates from converging. If the sequence does converge, its limit point, used in the incorrect model, will fail to give rational expectations. This is exactly what happened to wage setters in Chapter II, they believed that the current money supply was independent of the current wage. Since this was incorrect, their expectations of the money supply was consistently wrong.

The problem of agents having an incorrect model of the economy can be overcome by borrowing an idea from Kreps and Wilson (1982). If an agent has the wrong belief about the structure of the economy then his expectations will fail to coverge towards accuracy, given this the agent will change his beliefs about the economy and hence change the model he is estimating. Thus, in order to overcome this first problem, we can simply add a learning behavior in beliefs to the analyses of Friedman and Taylor. If this belief learning coverges to true beliefs then Friedman's and Taylor's analyses will generate rational expectations as a limit point of the dual process of belief and parameter estimate updating.

The above discussion implicitly assumes away the second problem alluded to above: that of the strategic interdependence of an agent's choice on the variables he has expectations about. To understand what is meant by strategic interdependence let us consider what is meant by strategic independence. A choice variable x and an endogenous variable y are strategically independent if the choice of x does not affect the value of the parameters that determine y. Note that this does not mean that x and y are not or cannot be correlated. It simply means that the choice of x does not affect the structure of the process determining y. The variables x and y are strategically interdependent if the choice of x affects the parameters determining y.

An example of strategic interdependence is provided in Chapter II. When the Fed selects an $m_t \neq m_t^e$ the wage setter will (most likely) change the value of the coefficients in the lag polynomials, C(L) and D(L) in (II.20), in the next period. These coefficients affect the parameters of the wage setter's policy equation (II.18). Therefore, m_{t-1} and w_t are strategically interdependent. Notice too, that equations (II.19) and (II.20) indicate that m_t is strategically interdependent with itself. This highlights the difficulty in overcoming the problem of strategic interdependence and the difficulty in interpreting the traditional rational expectations hypothesis in a game setting.

In the specific example of Chapters II and III the strategic interdependence problem arises in the following manner: the Fed's choice of m_t affects the wage setter's beliefs, and therefore the choice of w_t; the wage setter's choice of w_t affects the Fed's beliefs, and therefore the choice of m_t. Thus, the problem takes the form of a cycle of beliefs. To overcome the problem the cycle must either be broken or shown to converge.

In Chapter II we broke the cycle of beliefs by assuming the wage-setter believed that w_t was strategically independent of m_t. Unfortunately this belief, if it is not true, leads to non-rational expectations and this seems to be the only way to break the cycle if the variables are strategically interdependent.

In Chapter III we assumed the cycle of beliefs converged to the perfect foresight result, $m_t^e = m_t$ and $w_t^e = w_t$. There are, of course, two possible problems with this assumption: first the cycle may not converge; second the cycle may converge but not to the rational expectations perfect foresight result. These are problems that have yet to be resolved. Intuitively, the first problem is much more severe than the second. There does not seen to be any reason to expect the cycle will converge, but if it does, it seems reasonable to expect that it will converge to rational expectations.

Until the two problems regarding the convergence of the belief cycle are resolved it is inappropriate to interpret rational expectations as the long run of or a limit point of an expectations structure in a game setting. It is only appropriate to interpret rational expectations as an exogenous result imposed on a game theoretic model. This, of course, does not mean that the results are any less useful or interesting.

FOOTNOTES

[1]See Whitehead (1984), p. 14.

[2]As Stutzer (1984) points out time inconsistency may result from the structure of the discount term or a change in the objective function, as well as inaccurate expectations.

CHAPTER V

CONCLUSIONS AND QUESTIONS FOR FURTHER RESEARCH

In this chapter we present the conclusions we draw from the analysis in this dissertation. We also present some questions that are raised but not answered by this dissertation.

Section 1: Conclusions

1. The Muth definition of rational expectations is not immediately applicable to dynamic games. This is due to the strategic interdependence of choice variables in a game setting. When player 1 forms a belief about player 2's choice variable, that affects player 2's belief about players 1's beliefs and the value of player 2's choice variable. It is only by acknolwedging this process and assuming that it converges that the rational expectations hypothesis can be applied in a game setting. Thus, in a dynamic game, rational expectations implies more than a knowledge of the underlying model, it also implies intimate knowledge of the beliefs of other players in the game.

2. When choice variables are strategically interdependent it is inappropriate to assume agents have rational expectations and to generate dominance of the policy authority by assuming agents take the authority's choice variables as an exogenous forcing process. Given the latter assumption agents will treat their choice variable as if it were strategically independent of the choice variable of the policy authority, which it is not. This means they will have the wrong model

of, or the wrong beliefs about the policy authority's behavior. Therefore, their expectations will fail to be rational.

3. Under certain conditions the policy authority may act as a Stackleberg leader because agents are content to act as Stackleberg followers. If agents have a level maximal set that covers the action space of the policy authority then they will be content to predict the actions of the policy authority and respond in their maximal level set. This is because agents can not make themselves better off by attempting to move the policy authority to a different action. If, in addition, agents have rational expectations, in the sense described in the first conclusion, then they will be able to perfectly predict (within a white noise error in a stochatic environment) the action of the policy authority. This means that the best the policy authority can do is select the element of the agent's maximal set that gives the authority its highest payoff. Thus, the agents act as Stackleberg followers and the policy authority acts as a Stackleberg leader, but this is purely as a result of the structure of the game. Note that the conditions on the agent's maximal set and expectations imply the policy ineffectiveness result, so this conclusion may be applicable to a large category of games.

4. The time inconsistency problem results from either an inconsistent interpretation or improper specification of constraints that exist in a model. As was mentioned above the rational expectations assumption effectively constrains the policy authority away from policy surprises which are the driving force behind the time inconsistency problem.

5. There may exist a steady state in decision functions which gives rise to a non-steady state in the endogenous variables. This is possible because it may be optimal, from the point of view of agents or the policy authority, to approach a steady state in variables gradually rather than all at once. This leaves open the possibility of generating an endogenous business cycle based on the preferences of agents or the policy authority.

6. In any attempt to estimate or test hypotheses about objective functions from observed actions one must remember that the objective function of a player is only one element in determining his decision function and hence his action. Decision functions will also be effected by the structure of the economy and the structure of the players' beliefs. Thus, any empirical research on objective functions must also take into account the structure of the economy and the structure of beliefs.

Section 2: Suggestions for Further Research

There are two different directions in which this research can go: one is an extension of the game theoretic aspects of the analysis; the other is the extension of the macroeconomic aspects of the analysis.

From a game theoretic standpoint two interesting questions are raised by this dissertation: does the cycle of beliefs converge to the rational expectations hypothesis?; and can a complete model with an endogenous belief structure exist? Paradoxically it may be that the answer to the former question is yes but the answer to the latter is no. What this would mean is that, while agents' expectations converged to a point where they were correct within a white noise error, no model with endogenously defined expectations and belief structures would exhibit rational expectations.

From a macroeconomic standpoint the questions divide into two main groups: econometric analysis, and theoretic analysis.

The econometric questions involve the estimation of structural equations for the economy; objective functions for players; and the belief structure of the game. The estimation of the structural equations is essentially a variant of existing macroeconometric techniques and presents no new problems to the macroeconometrician. The estimation of objective functions necessarily involves the use of microeconometric techniques and will probably require the inventive use of instrumental variables to obtain useful results. The estimation of the belief structure presents a significant challenge both in the operationalizing and testing of any hypothesis of the belief structure.

The macroeconomic theory questions involve expanding the game either to include additional variables, such as capital, savings and interest rates, or additional players, such as firms, different types of wage setters or banks. Any of these expansions may provide better results and better explanations of economic behavior than existing theory holds.

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APPENDIX

COEFFICIENTS OF THE FED'S DECISION RULE FOR THE DOMINANT PLAYER GAME

$$g = -\gamma_2\left(1 + \theta \frac{\partial w_{t+j}}{\partial m_{t+j}} - \theta \frac{\partial w_{t+j-1}}{\partial m_{t+j}}\right)$$

$$h = -\gamma_2\left(\frac{\partial w_{t+j+1}}{\partial m_{t+j}} - 1 - \theta \frac{\partial w_{t+j-1}}{\partial m_{t+j}}\right)$$

$$a = -\gamma_1\theta^2\left(1 - \frac{\partial w_{t+j}}{\partial m_{t+j}}\right) - \theta g + \beta\theta h$$

$$d = -\gamma_1\theta^2\left(1 - \frac{\partial w_{t+j}}{\partial m_{t+j}}\right) + g + \beta h$$

Given (II.18) and (II.20) $\frac{\partial w_{t+j+1}}{\partial m_{t+j}} = 0$ for k = 0,1,..., and $\frac{\partial w_{t+j+1}}{\partial m_{t+j}} = C(1) - \lambda$, therefore:

$$g = -\gamma_2;$$

$$h = -\gamma_2(C(1) - \lambda_1 - 1);$$

$$a = -\gamma_1\theta^2 + \gamma_2\theta - \beta\theta\gamma_2(C(1) - \lambda_1 - 1); \text{ and}$$

$$d = -\gamma_1\theta^2 - \gamma_2\theta - \beta\gamma_2(C(1) - \lambda_1 - 1).$$

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