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Modeling of resonant magneto-electric effect in a magnetostrictive and piezoelectric laminate composite structure coupled by a bonding material

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The harmonic magneto-electro-elastic vibration of a thin laminated composite was considered. A theoretical model, including shear lag and vibration effects was developed for predicting the magneto-electric (ME) effect in a laminate composite consisting of magnetostrictive and piezoelectric layers. To avoid bending, we assumed that the composite was geometrically symmetric. For finite length symmetrically fabricated laminates, we derived the dynamic strain-stress field and ME coefficients, including shear lag and vibration effects for several boundary conditions. Parametric studies are presented to evaluate the influences of material properties and geometries on the strain distribution and the ME coefficient. Analytical expressions indicate that the shear lag and the vibration frequency strongly influence the strain distribution in the laminates and these effects strongly influence the ME coefficients. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.4752271]

I. INTRODUCTION

Multiferroics are a special class of materials that have attracted much attention because of their potential for enhanced functionality in sensors and devices.1,2 For the past fifty years, magnetoelectric (ME) materials have evolved from single phase compounds, to particulate composites, and finally to laminate composites.1–10 The remarkably higher ME effects observed in laminate composites are produced by mechanically coupling continuous magnetostrictive and piezoelectric layers. For example, a ME voltage coefficient of 22 V/cm Oe under a low $H_{bias}$ of 2 Oe was reported by authors.3,11 While several models for laminate ME composites exist, these typically over predict the experimental results significantly. This paper provides an explanation for this discrepancy and a corresponding analytical model validated with experimental results.

Authors provided an analytical foundation for static ME laminate composites. However, this theoretical approach gave a huge disagreement with experiments, because of an assumption that all field functions are homogeneously distributed throughout the composite. To eliminate disagreement between analysis and experiments, authors proposed a model that includes interface coupling parameter $k$ to account for sliding boundary conditions at the ME laminate interfaces. While this provided an approach to better correlate theoretical analysis with the test data, it is unlikely that interface slip occurs at well-bonded continuous interfaces. The magnitude of the ME effect significantly increases in the region of the electromechanical resonance.8,14 Theory of this phenomenon was developed and experimentally verified using samples in the form of disks and plates. A theoretical model that predicts very strong ME interactions at magneto acoustic resonance in single-crystal ferrite-piezoelectric bilayer is discussed in Ref. 15.

Another possible influence on ME laminates that has not been well considered is the shear lag effect. Authors presented a modified shear lag approach to predict load transfer between piezoelectric actuators and an elastic substructure, i.e., electromechanical coupling. The corresponding stress and strain distribution in the piezoelectric laminate composite were studied and confirmed by experiments. Modeling of static shear lag and demagnetization effects in ME laminate composites was proposed by authors.17 However, a dynamic shear lag yet has not applied to electro-magneto-mechanically coupled ME composites.

In this paper, an analytical model is proposed to predict the dynamic response of a laminate ME composite. Shear lag analysis, along with geometrical and vibration characteristics, is incorporated to provide spatial solutions for strain, magnetic field variations, as well as effective ME voltage coefficient.

II. MODEL AND CONSTITUTIVE EQUATIONS FOR A THREE LAYER ME LAMINATE COMPOSITE CONSIDERING AN INTERLAYER BONDING MATERIAL

We consider a tri-layer laminated structure in the shape of a bar with a length $2L$ and a total thickness of $h = 2h_b + h_p + 2h_m (L \gg h)$. The specimen is polarized along the longitudinal ($L$) direction to the planes of the contacts (i.e., the $x_3$-axis). Static (magnetic bias) and alternating magnetic fields were applied along the $L$ direction and across the planes of the contacts ($H_{applied}$). Magnetostrictive and piezoelectric layers are bonded together with a bonding material-layer of a finite thickness $h_b$ and of finite elastic properties $G_b$. If the thickness of the bonding layer $h_b \rightarrow 0$ or shear modulus $G_b \rightarrow \infty$, we can assume that the magnetostrictive and piezoelectric layers are perfectly bonded together. A schematic view of the considered problem is shown in Figure 1. Due to magnetostriction, an alternating magnetic field induces vibrations in the magnetostrictive layers, which propagate both across and along the specimen. Our further
Shear lag analysis then assumes a pure shear in the bonding layer and a pure extension in both the piezoelectric and magnetostrictive layers. The 1D strain-displacement relationships are thus

\[ S_{3m} = \frac{\partial u_{3m}}{\partial x_3}, \quad S_{3p} = \frac{\partial u_{3p}}{\partial x_3}, \quad \gamma_b = \frac{u_{3p} - u_{3m}}{h_b}, \]

where \( \gamma_b \) denotes the shear strain in the bonding layer, which is related to shear stress by an isotropic stress-strain relationship \( \tau = G_b \gamma_b \) (see Refs. 16 and 17). For the pure extension assumption, as illustrated in the free body diagram of Fig. 1, the force equilibrium equations for the representative elements are given by

\[ \rho_m \frac{\partial^2 u_{3m}}{\partial t^2} = \frac{\mu_{33} \frac{\partial^2 u_{3m}}{\partial x_3^2}}{s_{33} T_{3m}} + \frac{G_b}{h_m h_b} [u_{3p} - u_{3m}], \]
\[ \rho_p \frac{\partial^2 u_{3p}}{\partial t^2} = \frac{\varepsilon_{33} \frac{\partial^2 u_{3p}}{\partial x_3^2}}{s_{33} \varepsilon_{33} T_{3p}} - \frac{2G_b}{h_p h_b} [u_{3p} - u_{3m}]. \]

Equations (1)–(3) are written under the assumption that all field components do not vary through the thickness and width directions of the laminate composites.

### III. GENERAL AND SPECIFIC SOLUTIONS FOR MODEL

First, we must set up the equations. Using the constitutive equations of Eq. (1), the bonding stress-strain relations of \( \tau = G_b \gamma_b \), the strain-displacement equations of Eq. (2), the equations of motions of Eq. (3), and Maxwell’s equations of Eq. (4), two coupled partial differential equations can be derived in terms of the displacements \( u_{3m}(x_3, t) \) and \( u_{3p}(x_3, t) \)

\[ \frac{\partial^2 u_{3m}}{\partial t^2} = \frac{\mu_{33} \frac{\partial^2 u_{3m}}{\partial x_3^2}}{s_{33} T_{3m}} + \frac{G_b}{h_m h_b} [u_{3p} - u_{3m}], \]
\[ \frac{\partial^2 u_{3p}}{\partial t^2} = \frac{\varepsilon_{33} \frac{\partial^2 u_{3p}}{\partial x_3^2}}{s_{33} \varepsilon_{33} T_{3p}} - \frac{2G_b}{h_p h_b} [u_{3p} - u_{3m}]. \]

Assuming

\[ u_{3m}(x_3, t) = u_m(x_3) e^{i\omega t}, \quad u_{3p}(x_3, t) = u_p(x_3) e^{i\omega t}, \]

\[ H_3(x_3, t) = H_3 e^{i\omega t} \] and \( E_3(x_3, t) = E_3 e^{i\omega t} \)

from Eqs. (5) and (6), we can derive

\[ \frac{\omega^2}{\Omega_m^2} u_m(z) \sim \frac{d^2 u_{3m}}{dx_3^2} + \beta_m [u_{3p} - u_{3m}], \]

where \( \beta_m \) is the bonding shear stiffness. The above two equations can then be solved by the Laplace transform, for both small and large amplitudes of oscillation.
In Eqs. (7) and (8), we introduced following nondimensional parameters:

\[ \Omega_m^2 = \frac{1}{\rho_m s_{33}^2 L}, \quad \Omega_p^2 = \frac{1}{\rho_p s_{33}^2 L}, \quad \alpha_m = \frac{\mu_{33} s_{33}^2}{s_{33}^4 \mu_{33} - q_{33}^2}, \]

\[ \alpha_p = \frac{\mu_{33} s_{33}^2}{s_{33}^4 \mu_{33} - d_{33}^2}, \quad \beta_m = \frac{G_{33} s_{33}^2}{t_m b}, \]

\[ \beta_p = \frac{G_{33} s_{33}^2}{t_p b}, \quad \gamma = 2, \quad z = x_3/L, \quad t_p = h_p/L, \]

\[ t_b = h_b/L, \quad t_m = h_m/L. \]

Second, we must find the general solutions of the equations. The general solutions of the system of Eqs. (7) and (8) can be expressed in the following two forms under different conditions:

1. If \( \omega < \sqrt{\beta_p \Omega_p^2 + \beta_m \Omega_m^2} = \omega_0 \), then

\[
\left( \begin{array}{c}
\tilde{u}_m(z) \\
\tilde{u}_p(z)
\end{array} \right) = \left( \begin{array}{cc}
\frac{1}{\bar{\lambda}_1} & A_1 \sin \gamma_1 z + \frac{1}{\bar{\lambda}_1} A_2 \cos \gamma_1 z \\
\frac{1}{\bar{\lambda}_2} & A_3 \sin \gamma_2 z + \frac{1}{\bar{\lambda}_2} A_4 \cos \gamma_2 z
\end{array} \right), \tag{9}
\]

2. If \( \omega > \sqrt{\beta_p \Omega_p^2 + \beta_m \Omega_m^2} = \omega_0 \), then

\[
\left( \begin{array}{c}
\tilde{u}_m(z) \\
\tilde{u}_p(z)
\end{array} \right) = \left( \begin{array}{cc}
\frac{1}{\bar{\lambda}_1} & A_1 \sin \gamma_1 z + \frac{1}{\bar{\lambda}_1} A_2 \cos \gamma_1 z \\
\frac{1}{\bar{\lambda}_2} & A_3 \sin \gamma_2 z + \frac{1}{\bar{\lambda}_2} A_4 \cos \gamma_2 z
\end{array} \right). \tag{10}
\]

In Eqs. (9) and (10), we introduced the following notations:

\[ \tilde{\lambda}_k = \frac{1}{\beta_m} \left( \frac{\omega^2}{\Omega_m^2} - \alpha_m \tilde{\lambda}_k + \beta_m \right), \quad \gamma_k = \sqrt{|\tilde{\lambda}_k|}. \tag{11} \]

From Eq. (12), we can show that, \( \tilde{\lambda}_1 < 0 \) for \( 0 \leq \omega < \infty \), \( \tilde{\lambda}_2 > 0 \) for \( 0 \leq \omega \leq \omega_0 \), and \( \tilde{\lambda}_2 < 0 \) for \( \omega_0 < \omega < \infty \).

Third, we must apply boundary conditions to get specific solutions. Four boundary conditions can now be applied to determine the four unknown constants of \( A_i(i = 1, 2, 3, 4). \) It should be noted that the magnetostrictive and piezoelectric strains \( q_{33} H_3 \) and \( d_{33} E_3 \) do not appear explicitly neither in the motion equations of Eqs. (5) and (6) nor in the solutions of Eqs. (9) and (10), but they enter into the solutions through the boundary conditions. We next have to examine the following three particular boundary conditions applied to the edges of the laminated composite: case (I) both ends are traction free; case (II) one end is clamped and the other end is traction free; case (III) both ends are traction free of the magnetostrictive layers and zero strains on both ends of the piezoelectric layer.

Using the constitutive equations of Eq. (1) and the strain-displacement relations of Eq. (2), the boundary conditions for the above mentioned three cases can be expressed in terms of their displacements as follows:

Case (I)

For \( z = \pm 1 \)

\[ \frac{d\tilde{u}_p(z)}{dz} = d_{33} E_3, \quad \text{and} \quad \frac{d\tilde{u}_m(z)}{dz} = \beta q_{33} H_3. \tag{13} \]

Case (II)

For \( z = -1 \)

\[ \tilde{u}_p(z) = 0 \quad \text{and} \quad \tilde{u}_m(z) = 0. \tag{14} \]

When \( z = +1 \)

\[ \frac{d\tilde{u}_p(z)}{dz} = d_{33} E_3, \quad \text{and} \quad \frac{d\tilde{u}_m(z)}{dz} = \beta q_{33} H_3. \tag{15} \]

Case (III)

For \( z = \pm 1 \)

\[ \frac{d\tilde{u}_p(z)}{dz} = 0 \quad \text{and} \quad \frac{d\tilde{u}_m(z)}{dz} = \beta q_{33} H_3. \tag{16} \]

In Eqs. (13), (15), and (16), we introduced a demagnetization coefficient of \( \beta = 1 - N_d (\mu_{33} - 1) \), where \( N_d \) is a demagnetization factor which is a function of sample geometry. The parameter \( \beta \) belongs to the interval of \( 0 < \beta < 1 \) (see Refs. 17 and 20). If, for ferromagnetic plates, \( t_m < 10^{-1} \), then \( \beta \approx 1 \).

Using the general solutions of Eq. (9) or (10) and the boundary conditions for these cases mentioned above, we can uniquely determine the unknown coefficients \( A_i(i = 1, 2, 3, 4). \) Without going into details, the specific solutions for Eq. (9) or (10) can be expressed as follows:

Case (I)

If \( \omega < \sqrt{\beta_p \Omega_p^2 + \beta_m \Omega_m^2} = \omega_0 \), then

\[
\tilde{u}_m(z) = \frac{\tilde{\gamma}_2 \beta q_{33} H_3 - d_{33} E_3 \sin \gamma_1 z}{\tilde{\lambda}_2 - \tilde{\lambda}_1} \sin \gamma_1 z - \frac{\gamma_1 \cos \gamma_1}{\tilde{\lambda}_2 - \tilde{\lambda}_1} \frac{d_{33} E_3 - \tilde{\gamma}_1 \beta q_{33} H_3 \sin \gamma_2 z}{\tilde{\gamma}_2 \cos \gamma_2}, \tag{17}
\]

\[
\tilde{u}_p(z) \quad \text{and} \quad \tilde{u}_m(z) \quad \text{for} \quad z = \pm 1.
\]
\[ u_p(z) = \frac{\gamma_1 \frac{\lambda_2 \beta q_{33} H_3 - d_{33} E_3 \sin \gamma_1 z}{\lambda_2 - \lambda_1}}{\gamma_1 \cos \gamma_1} + \frac{\gamma_2 d_{33} E_3 - \frac{\lambda_1 \beta q_{33} H_3 \sin \gamma_2 z}{\lambda_2 - \lambda_1} \sin \gamma_2 z}{\gamma_2 \cos \gamma_2}. \]

If \( \omega > \sqrt{\frac{\beta_p \Omega_p^2}{\beta_m \Omega_m^2}} = \omega_0 \), then

\[ u_m(z) = \frac{\gamma_1 \frac{\lambda_2 \beta q_{33} H_3 - d_{33} E_3 \sin \gamma_1 z}{\lambda_2 - \lambda_1}}{\gamma_1 \cos \gamma_1} + \frac{\gamma_2 d_{33} E_3 - \frac{\lambda_1 \beta q_{33} H_3 \sin \gamma_2 z}{\lambda_2 - \lambda_1} \sin \gamma_2 z}{\gamma_2 \cos \gamma_2}. \]

\[ u_p(z) = \frac{\gamma_1 \frac{\lambda_2 \beta q_{33} H_3 - d_{33} E_3 \sin \gamma_1 z}{\lambda_2 - \lambda_1}}{\gamma_1 \cos \gamma_1} + \frac{\gamma_2 d_{33} E_3 - \frac{\lambda_1 \beta q_{33} H_3 \sin \gamma_2 z}{\lambda_2 - \lambda_1} \sin \gamma_2 z}{\gamma_2 \cos \gamma_2}. \]

\[ \text{IV. CALCULATION OF ME COEFFICIENT} \]

The magneto-electric coupling coefficient \( z_{ME} \) can be determined under the open circuit condition of

\[ I = \int \frac{d^2 \gamma_s}{dS} = 0, \]

where the integral is evaluated over the surface \( S \) of electrodes.

Using the constitutive equation of Eq. (1), condition (17) can be rewritten as

\[ \left[ \frac{d^2 \gamma_s}{dS} \right] E_3 = - \frac{1}{2} \frac{d^2 \epsilon_{33}}{dS} \left[ u_p(-1) - u_p(+1) \right]. \]

Defining \( z_{ME} = \frac{d^2 \gamma_s}{dS} \), from Eq. (18), we can then derive the following ME coupling coefficients for the identified three cases

**Case (I)**

If \( \omega > \sqrt{\frac{\beta_p \Omega_p^2}{\beta_m \Omega_m^2}} = \omega_0 \), then

\[ z'_{ME} = \frac{\beta q_{33} d_{33}}{s_{33}} \left( \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \right) \left\{ \frac{\tan \gamma_1}{\gamma_1} + \frac{\tan \gamma_2}{\gamma_2} \right\}. \]

**Case (II)**

If \( \omega < \sqrt{\frac{\beta_p \Omega_p^2}{\beta_m \Omega_m^2}} = \omega_0 \), then

\[ z'_{ME} = \frac{\beta q_{33} d_{33}}{s_{33}} \left( \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \right) \left\{ \frac{\tan \gamma_1}{\gamma_1} + \frac{\tan \gamma_2}{\gamma_2} \right\}. \]

**Case (III)**

If \( \omega < \sqrt{\frac{\beta_p \Omega_p^2}{\beta_m \Omega_m^2}} = \omega_0 \), then

\[ z'_{ME} = \frac{\beta q_{33} d_{33}}{s_{33}} \left( \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \right) \left\{ \frac{\tan \gamma_1}{\gamma_1} + \frac{\tan \gamma_2}{\gamma_2} \right\}. \]

**Case (IV)**

If \( \omega > \sqrt{\frac{\beta_p \Omega_p^2}{\beta_m \Omega_m^2}} = \omega_0 \), then

\[ z'_{ME} = \frac{\beta q_{33} d_{33}}{s_{33}} \left( \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \right) \left\{ \frac{\tan \gamma_1}{\gamma_1} + \frac{\tan \gamma_2}{\gamma_2} \right\}. \]
where $\Delta_{H} = 1 - \frac{d_{33}^{2}}{s_{33}^{2}d_{33}} \left\{ 1 - \frac{\tilde{\lambda}_{1}}{\tilde{\lambda}_{1} - \tilde{\lambda}_{2}} \frac{tg2\gamma_{1}}{2\gamma_{1}} + \frac{\tilde{\lambda}_{2}}{\tilde{\lambda}_{1} - \tilde{\lambda}_{2}} \frac{tg2\gamma_{2}}{2\gamma_{2}} \right\}$.

Case (III)

If $\omega < \sqrt{\frac{\beta_{p}\Omega_{p}^{2}}{\beta_{m}\Omega_{m}^{2}}} = \omega_{0}$, then

$$\alpha_{ME}^{(III)} = \frac{\beta_{q}q_{d}d_{33}}{d_{33}^{2} \kappa_{m} \kappa_{m}} \frac{\tilde{\lambda}_{1} \tilde{\lambda}_{2}}{\tilde{\lambda}_{1} - \tilde{\lambda}_{2}} \Delta_{H} \left\{ \frac{tg\gamma_{1}}{\gamma_{1}} - \frac{tg\gamma_{2}}{\gamma_{2}} \right\}, \quad (23)$$

where $\Delta_{H} = 1 - \frac{d_{33}^{2}}{s_{33}^{2}d_{33}}$.

If $\omega > \sqrt{\frac{\beta_{p}\Omega_{p}^{2}}{\beta_{m}\Omega_{m}^{2}}} = \omega_{0}$, then

$$\alpha_{ME}^{(III)} = \frac{\beta_{q}q_{d}d_{33}}{s_{33}^{2}d_{33}} \frac{\tilde{\lambda}_{1} \tilde{\lambda}_{2}}{\tilde{\lambda}_{1} - \tilde{\lambda}_{2}} \Delta_{H} \left\{ \frac{tg\gamma_{1}}{\gamma_{1}} - \frac{tg\gamma_{2}}{\gamma_{2}} \right\}, \quad (24)$$

where $\Delta_{H} = 1 - \frac{d_{33}^{2}}{s_{33}^{2}d_{33}}$.

From Eqs. (19)–(24) for the ME coefficient, it follows at the frequencies where $\Delta_{i} = 0$ $(i = I, II, III)$ that there is resonant increase in $\alpha_{ME}(i = I, II, III)$. Next, we need to consider some particular cases for these solutions.

(a) Particular case of low frequency ME coefficients with an account of shear-lag effect (i.e., static shear-lag).

The low frequency ME coefficient can be derived assuming $\omega \rightarrow 0$ in Eqs. (19)–(24). In this case, it is easy to see that $\tilde{\lambda}_{1} = 1$, $\tilde{\lambda}_{2} = 1 - \frac{\omega_{0}}{\omega_{0}} \tilde{\lambda}_{2}(0)$, and $\tilde{\lambda}_{2}(0) = \frac{\omega_{0}}{\omega_{0}} + \frac{\beta_{m}}{\beta_{m}} = \frac{G_{i}\kappa_{i}}{\kappa_{m}} \left( \frac{2}{\gamma_{m}} \gamma_{m}^{2} + \frac{\omega_{0}}{\omega_{0}} \right) = \kappa^{2} \gamma_{0}^{2}$, where $\kappa = \sqrt{\frac{G_{i}\kappa_{i}}{\kappa_{m}}}$, $\gamma_{0} = \sqrt{\frac{\omega_{0}}{\omega_{0}} \gamma_{m}^{2} + \frac{\omega_{0}}{\omega_{0}}}$, $\lambda_{1}(0) = 0$, $\gamma_{1} = 0$, and $\gamma_{2} = \lambda_{2}(0) = \gamma_{20}$. For example, the ME coefficient of case (II) can then be simplified to

$$\alpha_{ME}^{(II)} = \frac{\beta_{q}q_{d}d_{33}}{s_{33}^{2}d_{33}} \frac{\tilde{\lambda}_{1} \tilde{\lambda}_{2}}{\tilde{\lambda}_{1} - \tilde{\lambda}_{2}} \Delta_{H} \left\{ 1 - \frac{tg\gamma_{1}}{\gamma_{1}} - \frac{tg\gamma_{2}}{\gamma_{2}} \right\}, \quad (25)$$

where $\Delta_{H} = 1 - \frac{d_{33}^{2}}{s_{33}^{2}d_{33}} \left\{ 1 - \frac{\omega_{0}}{\omega_{0}} \omega_{0} + \frac{\omega_{0}}{\omega_{0}} + \frac{\omega_{0}}{\omega_{0}} \right\}$.

This case is consistent with the results of authors.\(^9\) In addition to $\omega \rightarrow 0$, if we also assume perfect bonding between layers (i.e., $\kappa = \sqrt{\frac{G_{i}\kappa_{i}}{\kappa_{m}}}$), then from Eq. (25), we can obtain the following expression:

$$\alpha_{ME}^{(II)} = \frac{\beta_{q}q_{d}d_{33}}{s_{33}^{2}d_{33}} \frac{\tilde{\lambda}_{1} \tilde{\lambda}_{2}}{\tilde{\lambda}_{1} - \tilde{\lambda}_{2}} \Delta_{H} \left\{ 1 - \frac{\omega_{0}}{\omega_{0}} \omega_{0} + \frac{\omega_{0}}{\omega_{0}} \right\}, \quad (26)$$

where $\eta = \frac{G_{i}\kappa_{i}}{s_{33}^{2}d_{33}}$.  

---

FIG. 2. (a) Locations of first resonant frequencies for three boundary cases. Case (I): blue line, case (II): purple line, and case (III): green line. Graphs introduced for following parameters: $t_{m} = 0.02$, $t_{p} = 0.05$, $2L = 0.04$ m, and $\kappa = 100$. (b) Locations of first resonant frequencies for three boundary cases. Case (I): blue line, case (II): purple line, and case (III): brown line. Graphs introduced for following parameters: $t_{m} = 0.002$, $t_{p} = 0.005$, $2L = 0.04$ m, and $\kappa = 0.01$. (c) Locations of first resonant frequencies for three boundary cases. Case (I): blue line, case (II): purple line, and case (III): brown line. Graphs introduced for following parameters: $t_{m} = 0.02$, $t_{p} = 0.005$, $2L = 0.04$ m, and $\kappa = 100$. (d) Locations of first resonant frequencies for three boundary cases. Case (I): blue line, case (II): purple line, and case (III): brown line. Graphs introduced for following parameters: $t_{m} = 0.002$, $t_{p} = 0.05$, $2L = 0.04$ m, and $\kappa = 100$.  

---
(b) Case of ME coefficients for perfect bonding between layers.

If we assume perfect bonding between layers (i.e., \( G_m \rightarrow \infty \)), Eqs. (19)–(24) can be simplified. In this case

\[
\lambda_1 \rightarrow -\xi^2 \omega^2, \quad \lambda_2 \rightarrow \infty, \quad \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \rightarrow 1 + 2\eta \xi_m / \xi_p,
\]

where \( \xi^2 = \Omega_p^{-2} \frac{1}{\xi_p + \eta \xi_m} (1 + 2\eta \Omega_m^2 \Omega_p^{-2}) \).

The ME coefficient in case (I) is then reduced to

\[
\chi'_{ME} = -\frac{\beta q_{33} d_{33}}{s_{33}^M} \frac{1}{\Delta_1} \frac{2\eta \Omega_m}{\Delta_2} \left( \frac{f_{\chi x} \xi_2}{\xi_p} \right),
\]

(27)

where \( \Delta_1 = 1 - \frac{d_{33}^2}{s_{33}^M} \left( 1 - \frac{\xi_p}{\xi_p + 2\eta \xi_m} \frac{f_{\chi x} \xi_2}{\xi_p} \right) \).

Equation (27) is consistent to that developed in Ref. 21. For frequencies of \( \omega = \frac{1}{\xi} (\xi + \pi k), k = 0, 1, \ldots \), Eq. (27) then simplifies to

\[
\chi'_{ME} = -\frac{\beta q_{33}}{d_{33}},
\]

(28)

From Eq. (28), we can see for a composite made of Metglas and piezoelectric lead zirconium titanate (PZT) layers \( (q_{33} = 50 \cdot 10^{-9} \text{ m/A} \) and \( d_{33} = 400 \cdot 10^{-12} \text{ m/V} \), the ME coefficient becomes \( \chi'_{ME} \approx 300 \text{ V/cmOe} \), and for a composite made of Permandur and PZT layers, \( (q_{33} = 3 \cdot 10^{-9} \text{ m/A} \) and \( d_{33} = 1.7 \cdot 10^{-10} \text{ m/V} \) the ME coefficients are equal to \( \chi'_{ME} \approx 16 \text{ V/cmOe} \).

V. NUMERICAL DISCUSSIONS

For calculations, we will make use of the following parameters for Metglas-PZT-Metglas three layer composite:

\[
\begin{align*}
\varepsilon_{33}^M &= 10 \cdot 10^{-12} \text{ m}^2/\text{N}, & q_{33} &= 50 \cdot 10^{-9} \text{ m/A}, \\
\mu_{33} &= \mu_0 \cdot 4.5 \cdot 10^4, & \mu_0 &= 4\pi \cdot 10^{-7} \text{ N} \cdot \text{A}^{-2}, \\
\rho_m &= 7180 \text{ kg/m}^3, & \rho_p &= 7600 \text{ kg/m}^3, \\
\rho_{23} &= 400 \cdot 10^{-12} \text{ m/V}, & \varepsilon_{33} &= 15.3 \cdot 10^{-12} \text{ m}^2/\text{N}, \\
\rho_{23} &= 400 \cdot 10^{-12} \text{ m/V}, & \varepsilon_{33} &= 1750 \cdot \varepsilon_0, \\
\end{align*}
\]

We will also assume a laminate length \( L = 40 \text{ mm} \) and the demagnetization coefficient \( \beta = 1 \). The behavior of the ME coefficients \( \chi'_{ME} (i = I, II, III) \) was then calculated in terms of the following parameters: a nondimensional frequency \( \Omega = \omega / \Omega_p \), the relative thicknesses of magnetostrictive and piezoelectric layers \( t_m \) and \( t_p \), and the rigidity coefficient \( \kappa = \frac{G_{023}^M}{w} \). Figures 2–5 show the predicted dependence of \( \chi'_{ME} (i = I, II, III) \) calculated using Eqs. (19)–(24) for different values of the parameters \( \Omega = \omega / \Omega_p, t_m, t_p, \) and \( \kappa = \frac{G_{023}^M}{w} \). From these figures, we can see that

• Boundary conditions have significant influence on resonant frequency of the ME coefficient. For case (I), the resonant frequency was about 80 kHz, for case (II), it was about 40 kHz, and for case (III), it was about frequency 57 kHz [see Figs. 2(a)–2(d)].

• Comparisons of Figs. 2(a) and 2(b) show that the rigidity parameter \( \kappa \) not only changes the magnitude of the ME coefficient but also the location of the resonant frequency.

• The thickness parameters \( t_m \) and \( t_p \) strongly influence the location of the ME resonance frequency [see Figs. 2(c) and 2(d)] and also the 3D representation of the ME coefficient [see Figs. 3(a) and 3(b)].

• The strain distribution is strongly inhomogeneous in the layers. There are strong end effects [see Figs. 4(a)–4(d)]. The strain distribution is strongly influenced by frequency, thickness of the layers, and the rigidity parameter.

• Figures 4(c) and 4(d) show for certain values of the frequency and other parameters that the strain becomes large in magnitude and has an oscillatory type of distribution. This means that a linear theory is not completely suitable to applications describing ME interactions.

• For an ac magnetic field of frequency \( \omega_{ac} = \sqrt{2} \beta_p \Omega_p^2 + \beta_m \Omega_m^2 \), the ME coefficient is not continuous

FIG. 3. (a) 3D dependence of ME coefficient \( \chi_{ME} \) on frequency and thickness of magnetic layer \( t_m \). The other parameters chosen as \( \kappa = 100, t_p = 0.005 \), \( 2L = 0.04 \text{ m} \) case (I). (b) 3D dependence of ME coefficient \( \chi_{ME} \) on frequency and thickness of magnetic layer \( t_m \). The other parameters chosen as \( \kappa = 100, t_p = 0.01 \), \( 2L = 0.04 \text{ m} \) case (I).
(i.e., the ME coefficient jumps from one value to another [see Fig. 2(b) near 45 kHz]).

• The dependence of the ME coefficient on thickness of the magnetic layer at low frequencies is shown on Figs. 5(a)–5(c). From these figures, we can see that for \( \kappa = \frac{G_b t_m}{33} \), highest values of the low frequency ME coefficient occur for case (II) (i.e., cantilever beam-layers).

**VI. CONCLUSIONS**

An analytical model, including dynamic shear-lag effect has been proposed for magneto-electric laminate composites. The theory is applied to Metglas-PZT-Metglas tri-layer composite structures. The frequency dependence of the ME coefficient predicts that the resonance depends strongly on the physical and geometrical parameters of the laminates.
Nondimensional shear lag parameter $\kappa$, frequency $\Omega = \omega / \Omega_p$, and relative thicknesses of magnetostrictive and piezoelectric layers $t_m$ and $t_p$ were used to study the influences caused by material properties and sample geometries on the ME coefficient. The results indicate that shear lag causes substantial strain inhomogeneity near free ends. For certain values of frequency, the distribution of strain becomes large and results in an oscillatory behavior, which makes questionable a linear theory for ME interactions. We then show, when the value of the ac magnetic field frequency equals $\omega_{0m} = \sqrt{\beta_\mu^2 \Omega_p^2 + \beta_m^2 \Omega_m^2}$, the ME coefficient is not continuous but rather jumps from one value to another.

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