Response to "Comment on 'Linear inviscid wave propagation in a waveguide having a single boundary discontinuity: Part II: Application [J. Acoust. Soc. Am. 75, 356–362 (1984)]' "

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(Received 10 May 1986; accepted for publication 22 June 1986)

This paper addresses the comments of Dr. Bruggeman and Dr. van de Wetering. The source of the error present in the paper entitled "Linear inviscid wave propagation in a waveguide having a single boundary discontinuity: Part II: Application" is addressed. A rational fraction approximation for the transmission coefficient is presented.

PACS numbers: 43.20.Mv

The purpose of a letter to the editor is to alert the general readership to errors and provide clarification where needed. The comments of Dr. Bruggeman and Dr. van de Wetering serve both of these objectives well. They are correct in pointing out the error in the paper by Thompson,[1] which appeared in this *Journal* in 1984. The source of the error stems from an incorrect branch selection when evaluating asymptotic behavior of the velocity potential. A correct branch selection can only be assured if the resulting solution satisfies reciprocity.

This can be seen directly from the asymptotic expansion of the inner solution. The asymptotic behavior of the velocity potential is

$$\lim_{\hat{y}\to\infty}\hat{\phi}=\hat{\phi}^{+}=\frac{1}{\pi}\left[-\ln\left(\frac{4}{1+\gamma^2}\right)-\gamma\pi+\gamma\cos^{-1}\left(\frac{\gamma^2-1}{\gamma^2+1}\right)\right]+\frac{\hat{y}}{h^{+}},$$

$$\lim_{\hat{x}\to-\infty}=\hat{\phi}^{-}=\frac{1}{\pi}\left[\ln\left(\frac{4\gamma^2}{1+\gamma^2}\right)+\frac{\cos^{-1}}{\gamma}\left(\frac{\gamma^2-1}{\gamma^2+1}\right)\right]+\frac{\hat{x}}{h^{-}}.$$

If h^{+} and h^{-} are exchanged, reciprocity dictates that $\hat{\phi}^{-}$ and $\hat{\phi}^{+}$ become $-\hat{\phi}^{+}$ and $-\hat{\phi}^{-}$, respectively.

Following the development by Thompson,[1,2] the first-order outer solution of the acoustic pressure must satisfy the relation

$$\lim_{y^{+}\to 0}p_1^{+}=iq_0\left(\hat{\phi}^{+}-\frac{(\hat{y}-h^{-})}{h^{+}}\right)e^{-it}=\frac{iq_0}{\pi}\left[-\ln\left(\frac{4}{1+\gamma^2}\right)+\gamma\cos^{-1}\left(\frac{\gamma^2-1}{\gamma^2+1}\right)\right]e^{-it},$$

$$\lim_{x^{-}\to 0}p_1^{-}=iq_0\left(\hat{\phi}^{-}-\frac{(\hat{x}+h^{+})}{h^{-}}\right)e^{-it}=\frac{iq_0}{\pi}\left[-\gamma\pi-\ln\left(\frac{4\gamma^2}{1+\gamma^2}\right)+\frac{\cos^{-1}}{\gamma}\left(\frac{\gamma^2-1}{\gamma^2+1}\right)\right]e^{-it}.$$

The correct velocity potential drop is

$$\Delta\hat{\phi}=\gamma+\frac{1}{\pi}\left[2\ln\left(\frac{1+\gamma^2}{4\gamma^2}\right)+\left(\frac{1}{\gamma}-\gamma\right)\cos^{-1}\left(\frac{\gamma^2-1}{\gamma^2+1}\right)\right].$$

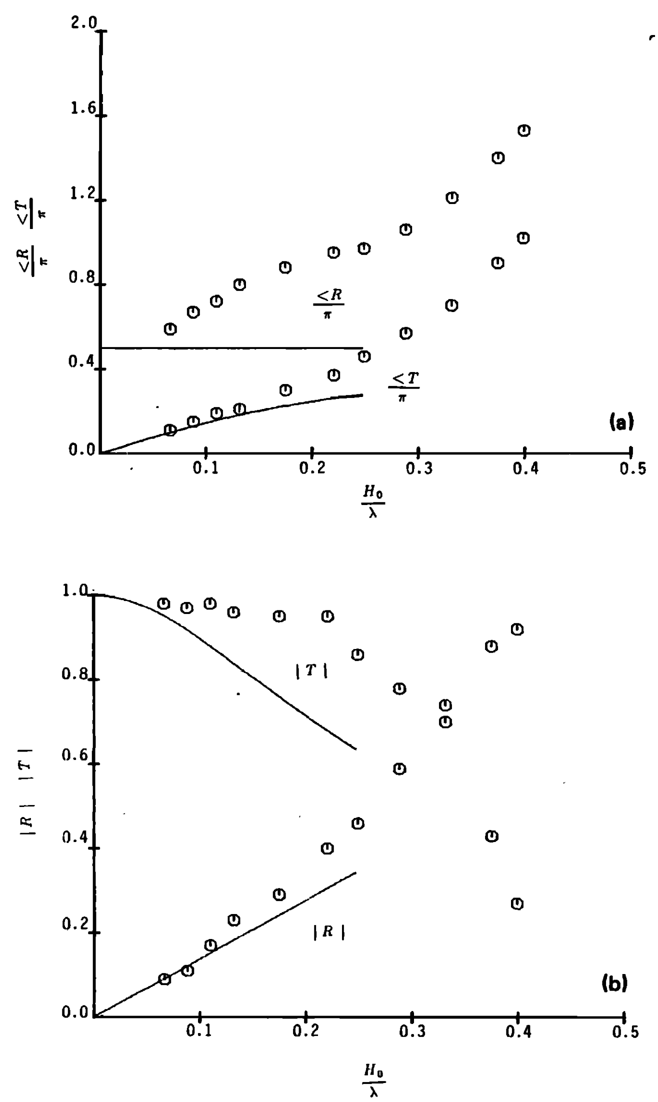

FIG. 1. Reflection and transmission coefficients plotted versus nondimensional wavenumber H_0/λ.

Despite the error, Bruggeman and van de Wetering seem to have accepted the remainder of the theory. It is gratifying to see that close an agreement with my corrected theory. However, I do wish to point out that Lippert's[3] results do not conclusively validate the theory. This is because of the symmetry of the duct height present in his experiment.

I wish to direct attention to one troublesome point. If the transmission coefficient is taken to be of the form $T = T_0 + \epsilon T_1$, its magnitude increases monotonically with frequency. In such a case, the magnitude of T could become greater than one. Bruggeman and van de Wetering overcome this difficulty by truncating the expansion after the $O(1)$ term. A further refinement in the approximation of the transmission coefficient can be made by representing T by a rational fraction in ϵ. In such a case,

$$T = 2\gamma/[\gamma + 1 - ik\epsilon\gamma(\Delta\hat{\phi} + 1)] + O(\epsilon^2).$$

This result satisfies the same asymptotic conditions as presented in the original papers.[1,2] A comparison between Lippert's experimental results and the rational fraction approximation of the transmission coefficient is given in Fig. 1.

[1]C. Thompson, J. Acoust. Soc. Am. **75**, 356–362 (1984).
[2]C. Thompson, J. Acoust. Soc. Am. **75**, 346–355 (1984).
[3]W. K. R. Lippert, Acustica **4**(2), 313–319 (1954).