AN INVESTIGATION OF THE EFFECT OF THE REDUCTION OF THE CONCRETE AREA ON THE PERFORMANCE OF A REINFORCED BEAM

by

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CONTENTS

INTRODUCTION

In the construction of steel beams the section used is I shaped. The reason for this is the greater efficiency of that steel area which is furtherest from the neutral axis. Therefore, we can say that the more of the cross-sectional area of a beam that you can get further away from the axis, the more economical will be your use of that material. In concrete design the usual shape of the cross-section is rectangular, and rightly so, for concrete is weak in tension and the diagonal tension set up is liable to crack it in the web, if there is not sufficient area to resist it. However, in any beam of average length and lightly loaded, as many are, particularly in small structures; or those of very long length the shear is small compared with the bending moment so that all of the width of the web is not required to resist the shear. In such cases it might be advisable to reduce the thickness of the web to some safe width still capable of resisting the diagonal tension. It was to determine the effect of this reduction on the reaction of a concrete beam to load, that this series of tests were made. In this test two types of beams were used; one with a solid rectangular cross-section, and the other with the same limits of enclosure but with two rectangles removed from each side so that the cross-section resembled a block I. Both were subjected to the same tests, and the results compared.

REVIEW OF LITERATURE

The literature on the testing of reinforced concrete beams is quite complete, dating back to about 1900. The most valuable source of material for these tests came from the bulletins of the various experimental stations of the large universities, especially those of the University of Illinois. These tests conducted in the years from 1904 to 1908 give a pretty complete idea of the results to be expected from the tests of beams of rectangular cross-section. It is to be noted from these tests the descrepency between the calculated and observed values of stress both in the steel and in the concrete at the smaller load values. It was felt that by removing some of the cross-section that these values would more closely resemble the calculated values, for then the effect of some of the tensile area would be removed.

The deflection of the beam of reduced section would of course be expected to be greater, due to the reduced moment of inertia of the cross-section, but a rough approximation based on the gross area shows that it should only be in the neighborhood of three or four per cent.

Mr. G. A. Maney (5) proposed this formula for the deflection of concrete beams, and it will be interesting to note if it is substantiated by this series of tests. The formula is:

$$D = c \frac{l^2}{d} (e_c + e_s)$$

The symbols are the usual ones found in concrete design. The strains are those at the extreme fibers of the concrete and the steel.

THEORY

The formulae used for the calculations of the stresses will be the usual ones based on the straight line variation of concrete stress and strain, although it might perhaps be more nearly correct to use a parabolic distribution of concrete stress. However, as general design practice now universally accepts the former it will be used. In the calculation of the observed depth of the neutral axis, the assumption that plane sections before bending remain plane will be used.

FORMULAE

$$f_s = \frac{M}{a_s\, j\, d}$$

$$f_c = \frac{2M}{j\, k\, b\, d^2}$$

$$k = \sqrt{2np + (np)^2} - np$$

$$jd = d - \frac{kd}{3}$$

$$kd = \frac{e_c\, d}{e_c + e_s}$$

e_c and e_s are the observed strains in the steel and concrete at the extreme fiber.

THE MATERIALS

The materials used for the concrete were crushed limestone, dolomitic sand a manufactured limestone, sand available locally, and a single brand of cement; all furnished by the Research Foundation. The crushed limestone used for the course aggregate was graded from 3/4 to 1/4 inch. The limestone sand, which is not to be confused with crusher screenings, finds a good market locally due to the lack of a commercial of natural sand of good quality. The grading of these aggregates passes the specifications set up by the A.S.T.M.. The cement used was certified to pass the A.S.T.M. requirements, and was not analyzed.

The reinforcing steel rods were $\frac{1}{4}$ inch stock with no deformations. They were not slick smooth but were sufficiently rough to provide an adequate bond between the concrete and the steel. The steel had a modulus of elasticity of 30,000,000 $\#/in^2$ and a yield point of 54,000$\#in^2$.

Sieve Analysis of Fine Aggregate

Sieve	Grams Retained	Percent Retained
10	28.1	5.6 %
20	139.7	27.3
30	141.0	28.2
40	45.1	9.0
60	78.4	15.7
80	20.4	4.1
100	15.0	3.0
Pan	35.3	7.1
	500.0	100.0

Sieve Analysis of Course Aggregate

Sieve	Pounds Retained	Percent Retained
3/4	5# 13-3/4 oz.	23.5 %
5/8	4# 15 oz.	19.8
1/2	6# 3 oz.	24.7
3/8	4# 1-1/4 oz.	16.3
1/4	2# 7 oz.	9.7
Pan	1# 8 oz.	6.0
	25# 0 oz.	100.0

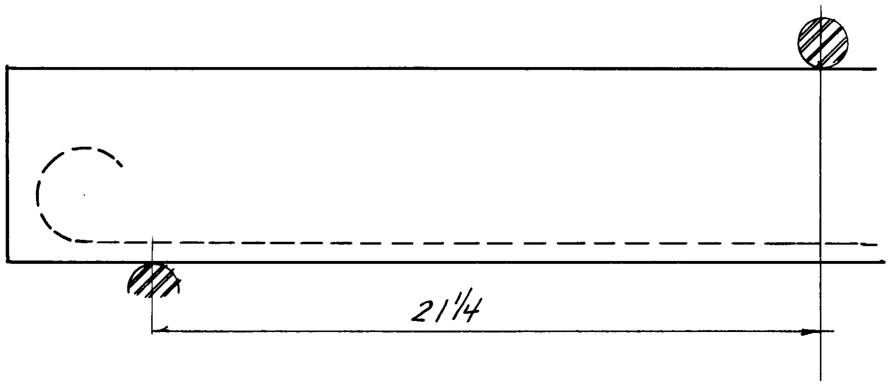
THE BEAMS

The beams were of a rectangular section 6" x 4" and had an overall length of 8' 6". They were reinforced with four of the 1/4 inch round rods with 3000 $\#/in^2$ concrete, n = 10 and an allowable stress of 18000$\#/in^2$ in the steel. Such a beam should be capable of carrying a load of 800 pounds at each of its quarter points. The rods were placed .45 inches from the bottom of the forms, d = 5.55 inches, and were spaced one inch apart. The outside bars therefore were one half inch from the outside of the beam. The ends of the rods were hooked with a diameter of about four inches. By so hooking the ends of the 9' 6" lengths of steel we were just able to fit them in the forms.

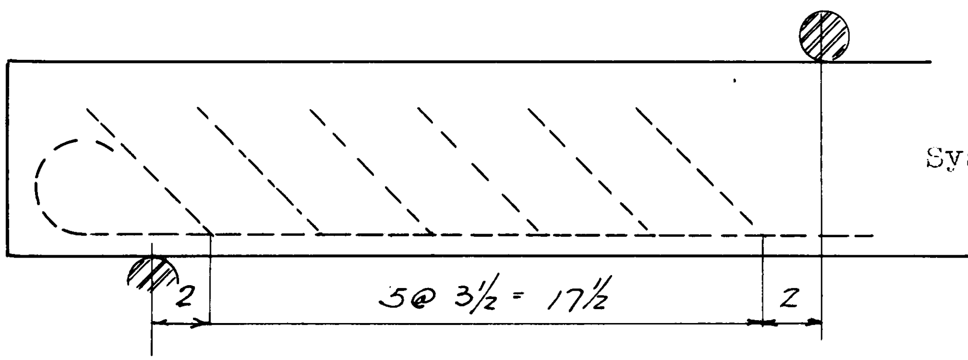
The reduced section beams had the same overall dimensions, but had removed from the cross-section two rectangles 2-3/4" x 1" at a section below the neutral axis which lies at about two inches from the top of the beam. This kept intact the compression area of the beams, but reduced the tension area by about 35%. The cross-section of the reduced beams then consisted of a top flange 4" x 2-1/4", a lower flange 4" x 1" reinforced with four rods, and a web 2" x 2-3/4".

THE STEEL REINFORCING

The longitudinal reinforcing consisted of four 1/4" round rods placed .45" from the bottom of the beam, In the solid beams -- Beams 1, 10, 11, 12, these ran the whole length of the beam and were hooked; no other bars were used. In Beams 2 and 3 this same reinforcing was used. These beams were of reduced section, and contained no web steel. To avoid fail by diagonal tension, and further extend the range of the reduced beams it was decided to reinforce the webs using the same 1/4" rods as stirrups. In Beams 4 and 5 the web reinforcing was six stirrups equally spaced from the end of the beam to the first concentration. The stirrups were $5\frac{1}{2}$" lond and were hooked at each end. One hook was placed around one of the longitudinal bars and the other end allowed to extend into the compression area at an angle of 45 degrees. In Beams 6 and 8 only two stirrups were used at each end, but two of the rods were also bent up to provide web steel. Beams 7 and 8 used no loose stirrups for web steel, but had at each end one of the longitudinal bars bent in a zig-zag fashion. This last system was the most practical one for it was the simplest to place, and had fewer loose parts. These systems are illustrated on the next page and numbered in order of their presentation.



System 1



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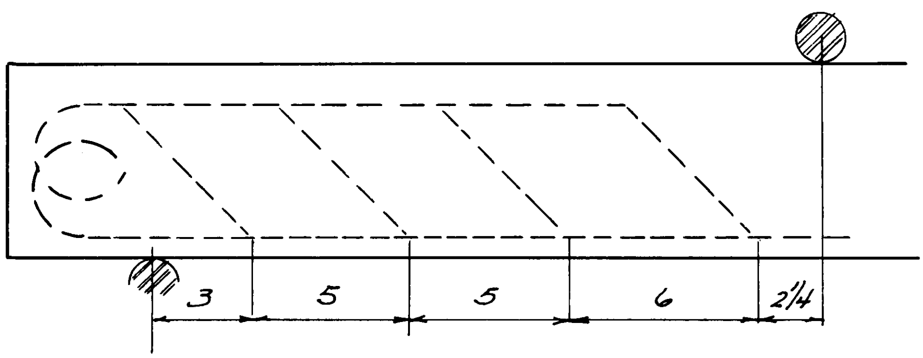

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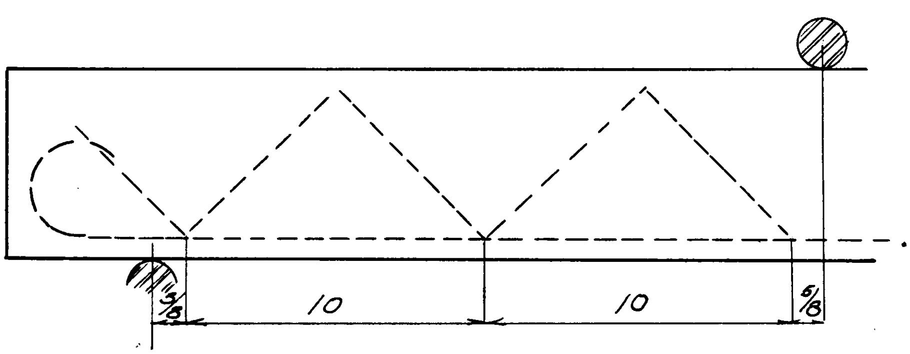


System 4

Reinforcement

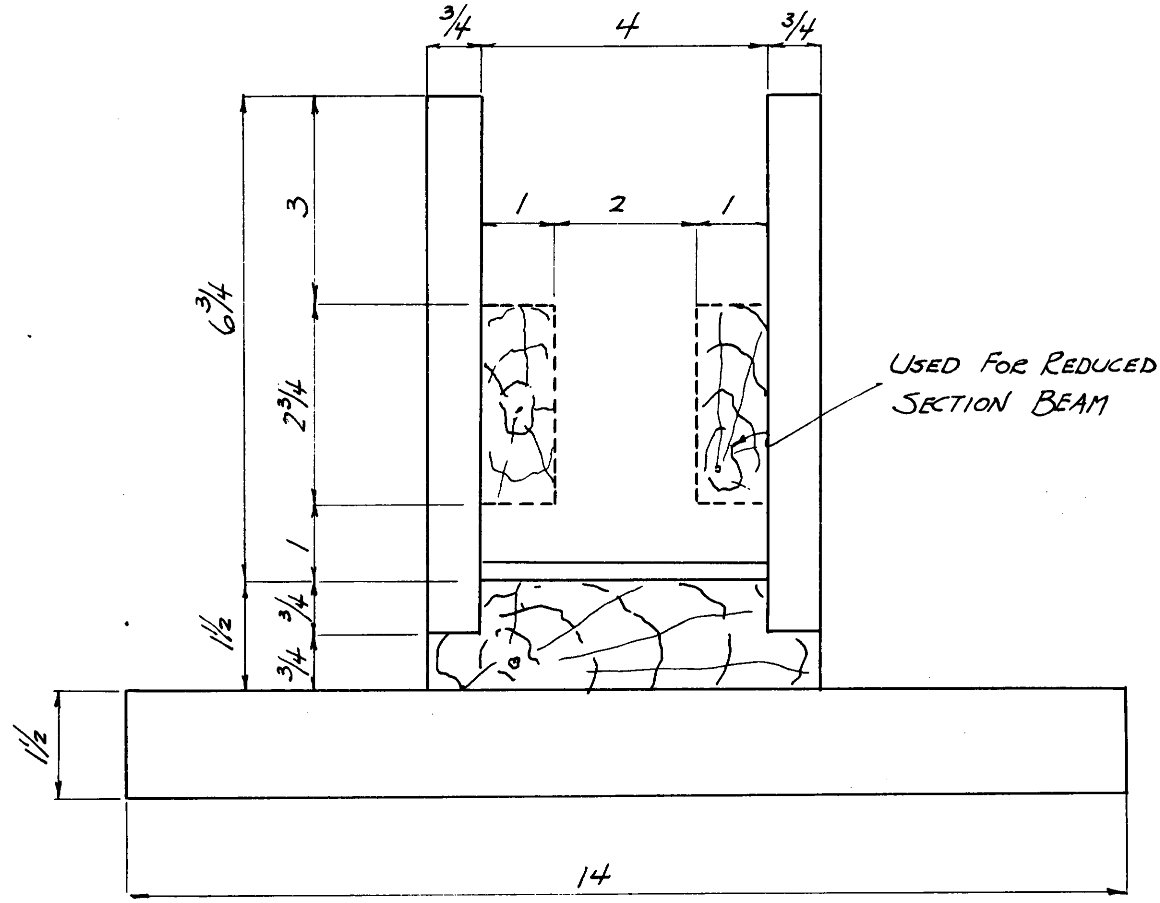
Stirrup
System
Three

Stirrup
System
Four

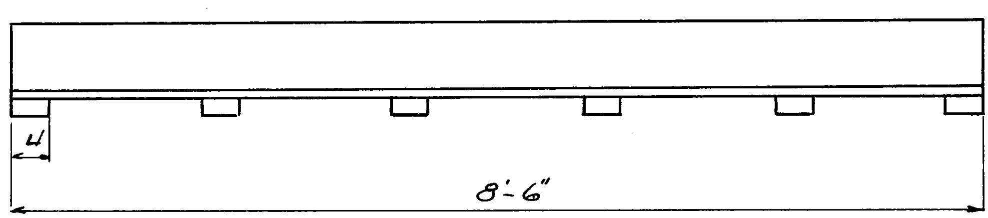
THE FORMS

The forms were constructed of wood and were braced with iron bands. They consisted of a plank $5\frac{1}{2}$" x $1\frac{1}{2}$" x 8' 6" notch along the upper edges to receive the side planks which were $7\frac{1}{2}$" x 3/4" x 8' 6". The lower portion was supported from the floor by five pieced of wood 4" x $1\frac{1}{2}$" x 1' 2" spaced evenly along the length of the form. It was to these pieced that each of the ten angle - shaped steel bands were bolted. In the bottom of the form short lengths of wood were placed sidewise to hold the reinforcement at the proper elevation. One short length of wood was placed at the center to deep bare the under side of one of the bars so that the Berry strain guage could be attached to it. The bars when in their proper position were nailed lightly to the form with small brads. Once the steel had been placed and secured the sides and ends were assembled, and the form checked for dimesions. Slight descrepencies in size could be corrected for by wedges placed between the steel angles and the sides of the form.

When used for the reduced section beams, there was nailed to the inside of the sides of the form two lengths of wood 2 3/4" x 1" x 8' 6". The dimensions of the form are given in the drawings on the following page.



Cross-section of Form



Side View of Form

FABRICATION

The mix used was a 1:2:3 with 7½ gallons of water per sack of cement. This is the same mix that has been used in most of the concrete work at the Virginia Polytechnic Institute, and there was little reason to change it. The first operation in the mixing was to determine the weights of a dry rodded cubic foot of the aggregates. Once this had been found it was no longer necessary to measure the stone and sand by volume, but rather by weight which is a much simpler procedure. These were:

Weight of cubic foot of stone	102#	11 oz.
Weight of cubic foot of sand	105#	11 oz.
Weight of cubic foot of cement	94#	0 oz.

Therefore the quantities used in a single batch of concrete were:

Stone	154#	0 oz.
Sand	105#	0 oz.
Cement	47#	0 oz.
Water	15 qts.	

The sand and stone were weighed and placed in the hopper. The cement and water were measured and set beside the mixer. The mixer is a two cubic foot affair driven by a variable speed direct current motor. The mixer was started and the sand and stone were slid in and allowed to dry mix. After mixing for a few seconds the cement was carefully added. This too, was allowed to thoroughly mix before the water was added. The mixer was then allowed to run until the concrete seemed to be well mixed. This took about two or three minutes, after which the concrete was poured out into a large tub. A slump test was made of each mix according to A.S.T.M. Standards.

Two beams were made in each operation and they required more concrete than could be made in one batch due to the small size of the mixer.

Therefore it was necessary to mix another batch immediately using one fourth of the quantities of the first. The same procedure was used in making this second batch.

The concrete was put into the forms in layers. The first layer being carefully worked in around the reinforcement to prevent any honey-combing which might reduce the bond strength. The subsequent layers were then placed and rodded, and the top struck off at a depth of 6" from the bottom of the form. Two metal bars were then embedded in the top surface at a distance twelve inches of the center line. These were center punched to receive the prongs of the twenty four inch Berry Strain Guage. With each two sets of beams there were made two 6" x 12" cylinders to be used to determine the stress strain relationship of the mix. These were made according to A.S.T.M. specifications; that is in three layers each layer rodded twenty five times.

After pouring the beams and cylinders they were covered with bur-lap and allowed to set for two or three days in the forms. They were then removed and marked. The cylinders were placed in the curing trough, and the beams which were too long for that were covered with wet burlap. The burlap was wetted every few days to keep it from drying out. Each set of beams was cured for twenty eight days before testing. On the twenty seventh day they, the beams and cylinders, were allowed to dry for the test on the following day.

THE TEST APPARATUS

The testing machine used was the 100,000# Riehle machine. This is the only one equiped with wings to support the ends of the beams. This was also the machine that was used to test the cylinders. To determine the compressive strain in the concrete a twenty four inch Berry Strain Guage was used. The prongs set in steel bars embedded in the concrete. The steel strain was measured by an eight inch Berry Strain Guage clamped to one of the reinforcing rods. The deflection of the beam was measured by a deflectometer with an arm ratio of ten to one making it possible to read to .001 of an inch. All these observations were made at the center of the beams.

The load was applied at the quarter points by means of a six inch I beam and two inch and three quarter steel rollers. The supports were the knife edges supplied with the Riehle machine. The beam length was eighty five inches.

The check cylinders were capped with plaster of paris in order to more evenly distribute the load. The strain was measured by means of two Ames Dials set on each side of the cylinder, and at a strain length of ten inches. The true strain reading was the average of the two.

THE TEST PROCEDURE

On the twenty seventh day the cylinders and beams were allowed to dry preparatory to testing the following day. On the twenty eighth day the beams were placed in the machine and the various strain guages were affixed to it. The machine was then balanced and the zero readings of each instrument noted. The load was then applied slowly. The speed of loading was the slowest available on the machine, and the average time from start of load to failure was in the neighborhood of three quarters of an hour. At even two hundred pound intervals simultaneous readings of the instruments were taken. It was found best not to stop the application of the load once it had been started due to the jarring of the instruments when the clutch was engaged and disengaged.

In testing the cylinders the knife edge was removed and the swivel head set in place. After the two strain guages had been attached the cylinder was placed in the machine and the load applied. Readings were taken at even five thousand pound intervals except in some instances where a two thousand five hundred interval was used in order to obtain more significant points.

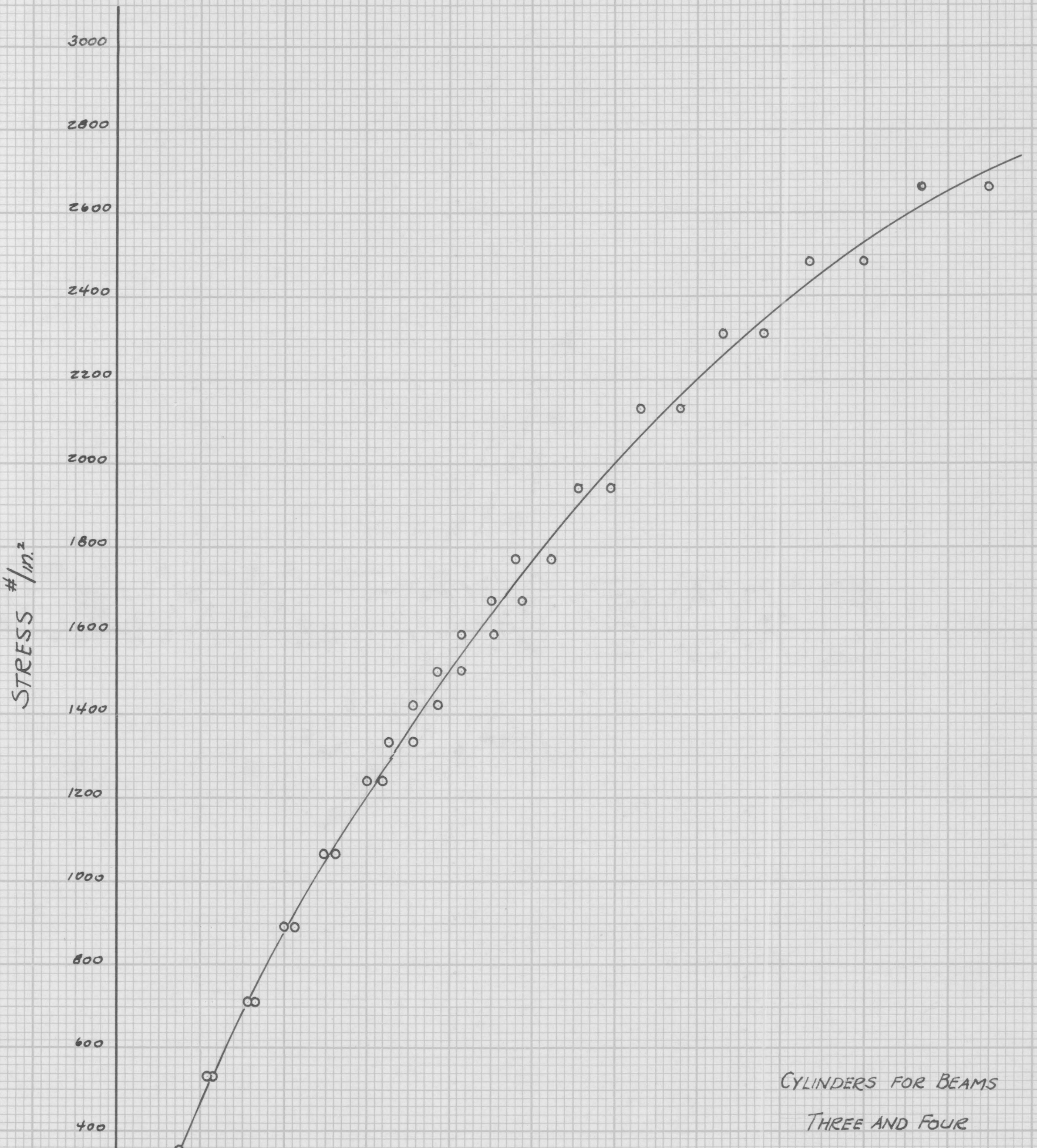
STRESS-STRAIN

RELATIONSHIPS

STRESS-STRAIN RELATIONSHIP FOR BEAMS

Three and Four

Stress	Strain Cylinder 1	Strain Cylinder 2
177	.000035	.000048
354	.000078	.000075
531	.000123	.000110
708	.000165	.000158
885	.000212	.000200
1062	.000263	.000248
1239	.000321	.000300
1328	.000357	.000327
1416	.000383	.000357
1505	.000418	.000385
1594	.000452	.000415
1682	.000488	.000450
1771	.000525	.000480
1948	.000598	.000553
2125	.000682	.000633
2302	.000783	.000727
2479	.000900	.000835
2656	.001048	.000963
2833	.001195	.001123
3260	Ultimate	

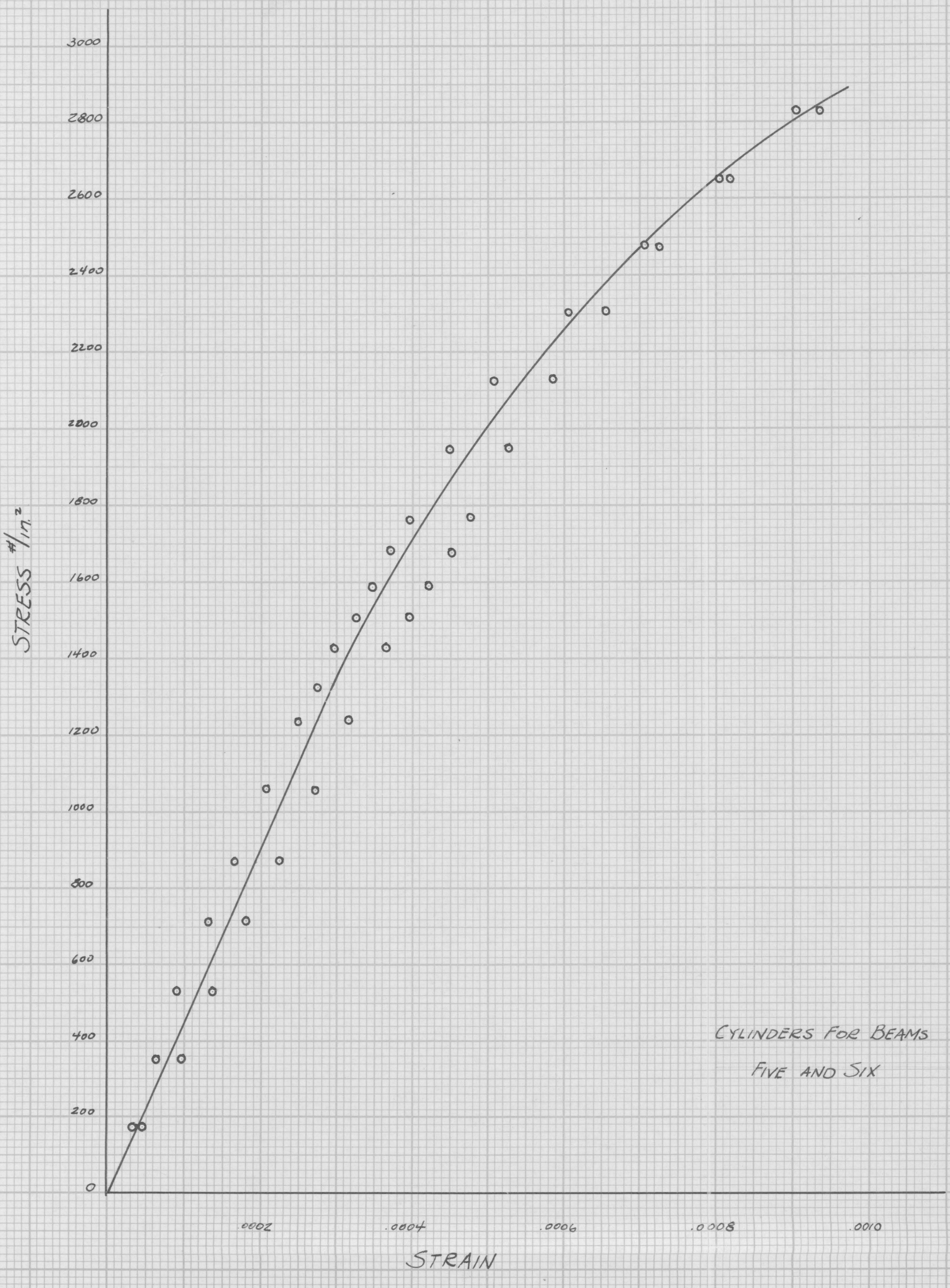


3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
STRESS #/in.²
.0002
.0004
.0006
.0008
.0010
STRAIN
CYLINDERS FOR BEAMS
THREE AND FOUR

STRESS-STRAIN RELATIONSHIP FOR BEAMS

Five and Six

Stress #/in^2	Strain Cylinder 1	Strain Cylinder 2
177	.000030	.000044
354	.000060	.000095
531	.000090	.000134
708	.000127	.000180
885	.000164	.000222
1062	.000201	.000270
1239	.000246	.000318
1328	.000271	---
1416	.000296	.000367
1505	.000327	.000398
1594	.000345	.000423
1682	.000370	.000450
1771	.000396	.000476
1948	.000449	.000527
2125	.000510	.000589
2302	.000608	.000653
2479	.000725	.000708
2656	.000820	.000808
2833	.000936	.000905
3010		.001017
3187		.001165
3364		.001375



STRESS #/in.²
3000
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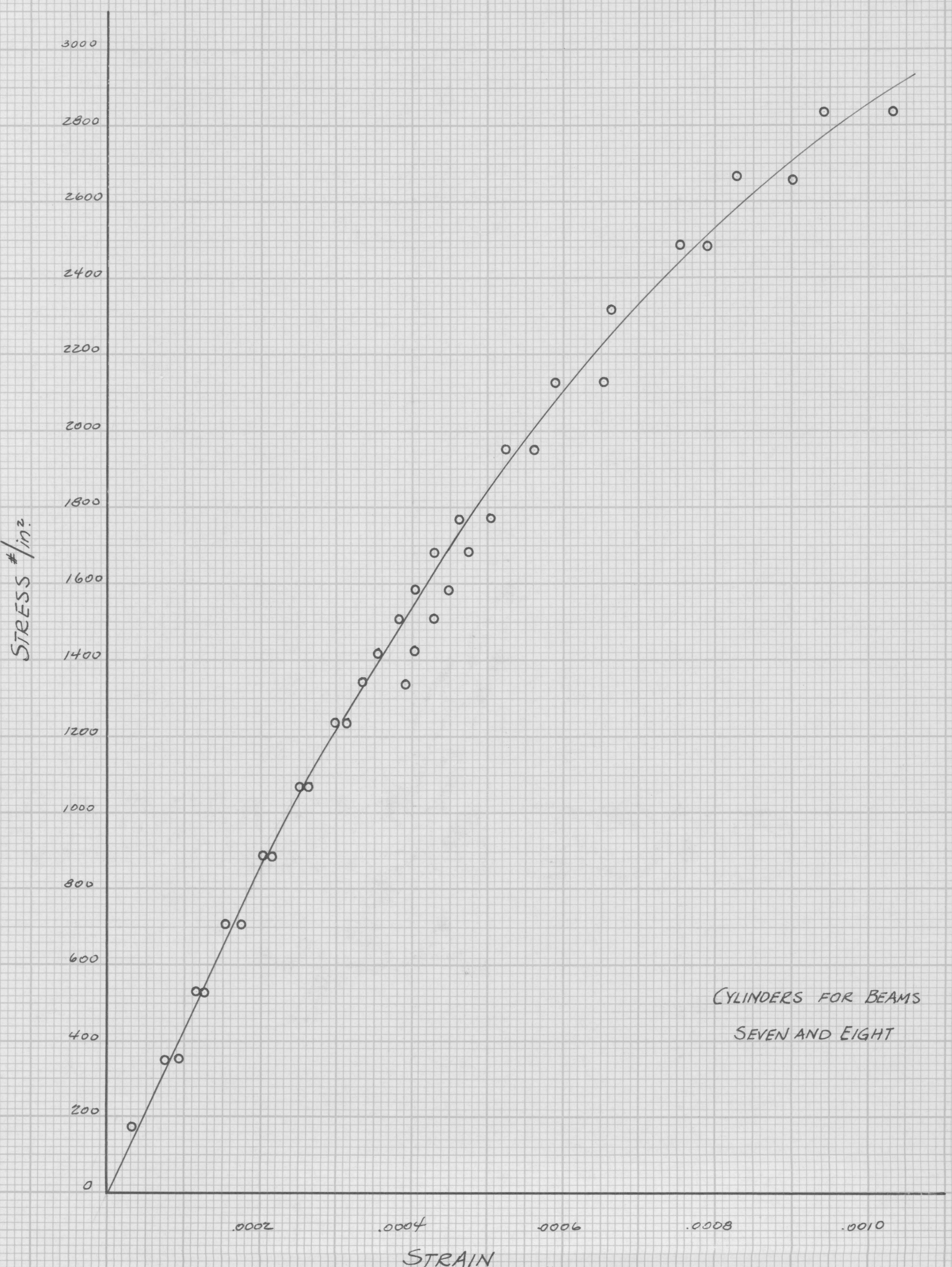
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
.0002
.0004
.0006
.0008
.0010
STRAIN
CYLINDERS FOR BEAMS
FIVE AND SIX

STRESS-STRAIN RELATIONSHIP FOR BEAMS

Seven and Eight

Stress	Strain Cylinder 1	Strain Cylinder 2
177	.000038	.000036
354	.000098	.000073
531	.000129	.000113
708	.000173	.000154
885	.000213	.000202
1062	.000267	.000250
1239	.000318	.000300
1328	.000398	.000338
1416	.000407	.000356
1505	.000432	.000385
1594	.000450	.000410
1682	.000477	.000438
1771	.000506	.000468
1948	.000563	.000527
2125	.000630	.000593
2302	---	.000669
2479	.000791	.000757
2656	.000907	.000832
2833	.001040	.000943
3010	.001227	.001057


3000
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2600
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1200
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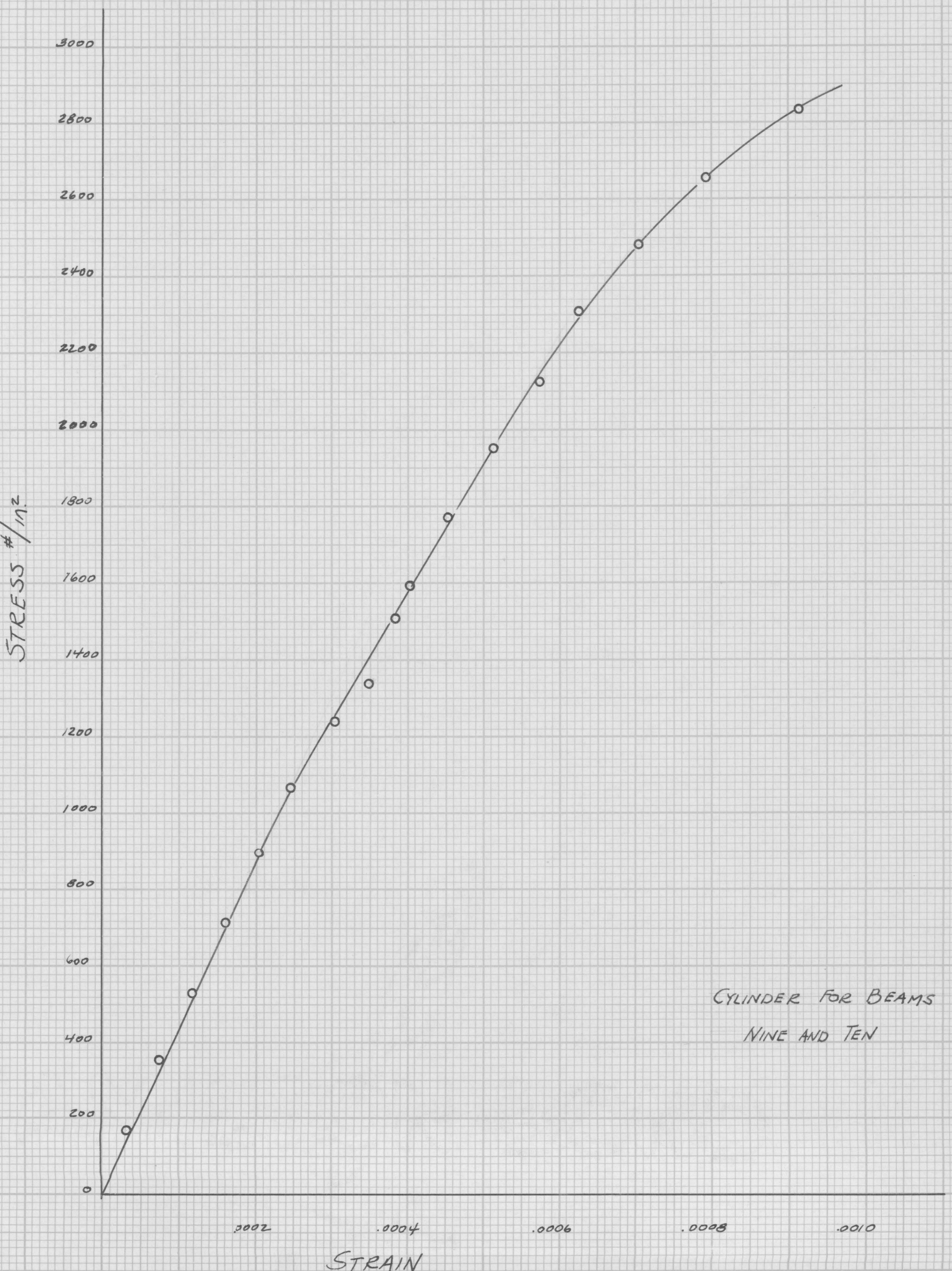
0
STRESS #/in.²
.0002
.0004
.0006
.0008
.0010
STRAIN
CYLINDERS FOR BEAMS
SEVEN AND EIGHT

STRESS-STRAIN RELATIONSHIP FOR BEAMS

Nine and Ten

Stress	Strain Cylinder 1	
177	.000035	No
354	.000078	Other
531	.000120	Cylinder
708	.0001625	
885	.000205	
1062	.000249	
1239	.000308	
1328	.000350	
1416	---	
1505	.000383	
1594	.000406	
1771	.000455	
1948	.000514	
2125	.000574	
2302	.000630	
2479	.000708	
2656	.000798	
2833	.000915	



3000
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2600
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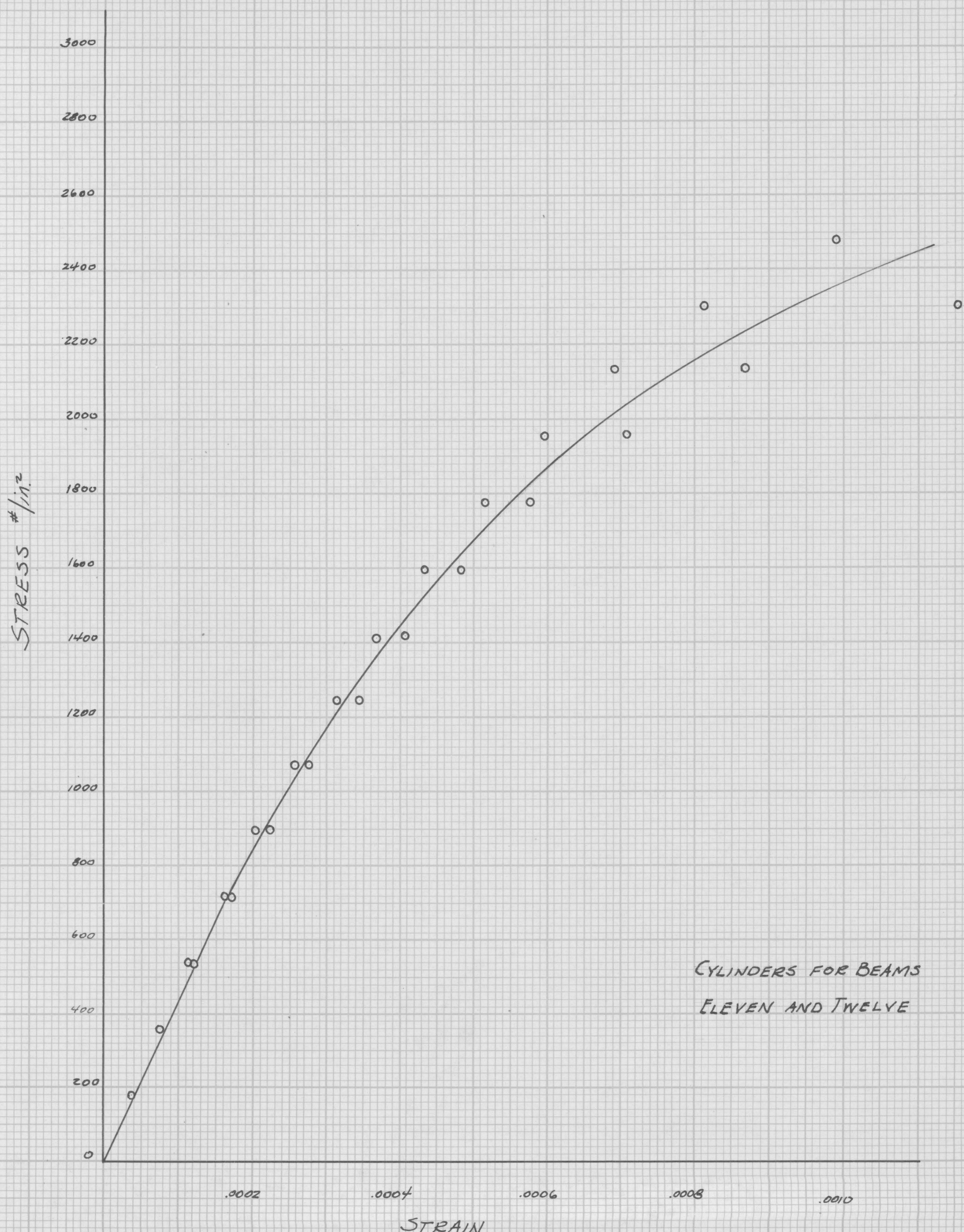
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
STRESS #/IN.²
.0002
.0004
.0006
.0008
.0010
STRAIN
CYLINDER FOR BEAMS
NINE AND TEN

STRESS-STRAIN RELATIONSHIP FOR BEAMS

Eleven and Twelve

Stress	Strain Cylinder 1	Strain Cylinder 2
177	.000038	.000037
354	.000076	.000078
531	.000117	.000125
708	.000165	.000175
885	.000206	.000225
1062	.000258	.000279
1239	.000315	.000343
1416	.000370	.000408
1594	.000435	.000485
1771	.000513	.000580
1948	.000595	.000703
2125	.000693	.000868
2302	.000815	.001155
2479	.000990	.002025
2656	.001270	---


3000
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2600
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1400
1200
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400
200
0
STRESS #/in.²
.0002
.0004
.0006
.0008
.0010
STRAIN
CYLINDERS FOR BEAMS
ELEVEN AND TWELVE

BEAM CALCULATIONS

CALCULATIONS FOR BEAM ONE

No.	Load Pounds	Moment in. lbs.	e_c	e_s	$e_c + e_s$	kd. in.	jd. in.
1	100	3453	.0000250	.0000225	.0000475	2.92	4.61
2	200	4515	.0000319	.0000250	.0000569	3.12	4.51
3	400	6640	.0000361	.0000275	.0000636	3.15	4.50
4	600	8765	.0000583	.0000450	.0001033	3.13	4.51
5	800	10890	.0000875	.0000700	.0001575	3.08	4.52
6	1000	13015	.0001140	.0001000	.0002140	2.96	4.56
7	1200	15140	.0001512	.0001500	.0003012	2.79	4.62
8	1400	17625	.0001945	.0002270	.0004215	2.56	4.70
9	1600	19390	.0002500	.0003400	.0005900	2.35	4.77
10	1800	21515	.0002985	.0004380	.0007365	2.25	4.80
11	2000	23640	.0003480	.0005450	.0008930	2.16	4.83
12	2200	25765	.0003970	.0006380	.0010350	2.13	4.84
13	2400	27890	.0004440	.0007320	.0011760	2.10	4.85
14	2600	30015	.0004880	.0008290	.0013170	2.05	4.87
15	2800	32140	.0005040	.0009340	.0014340	1.95	4.90
16	3000	34265	.0005460	.0010240	.0015700	1.93	4.91
17	3200	36390	.0005880	.0011000	.0016880	1.93	4.91
18	3400	38515	.0006370	.0012050	.0018420	1.92	4.91
19	3600	40640	.0006870	.0013190	.0020060	1.90	4.92
20	3800	42765	.0007300	.0014020	.0021320	1.90	4.92
21	4000	44890	.0007750	.0014980	.0022730	1.89	4.92
22	4100	45953	.0007970	.0015460	.0023430	1.89	4.92
23	4200	47015	.0008190	.0015790	.0023980	1.89	4.92
24	4300	48074	.0008430	.0016380	.0024810	1.89	4.92
25	4400	49140	.0008690	.0016850	.0025540	1.88	4.92
26	4500	50203	.0008950	.0017290	.0026240	1.88	4.92
27	4600	51265	.0009190	.0017750	.0026940	1.89	4.92
28	4700	52328	.0009400	.0018150	.0027550	1.89	4.92
29	4800	53390	.0009600	.0018600	.0028200	1.89	4.92
30	4900	54453	.0098100	.0019600	.0029410	1.85	4.93

Weight 225#
Reinforcement System One

CALCULATIONS FOR BEAM ONE (cont'd)

No.	D in.	Sc obs	Sc cal. (1)	Sc cal. (2)	Ss obs	Ss cal. (1)	Ss cal. (2)
1	.013	100	177	39.5	3600	675	1180
2	.016	140	232	75.5	4720	750	2400
3	.017	160	341	149	6930	825	4800
4	.024	240	450	229	9150	1350	7210
5	.032	360	559	306	11310	2100	9600
6	.039	480	667	394	13600	3000	11900
7	.050	630	778	472	15800	4500	14100
8	.064	800	906	552	18400	6820	16100
9	.084	1010	995	759	20300	10200	18200
10	.104	1190	1110	885	22500	13100	20400
11	.127	1360	1210	1015	24700	16400	22400
12	.148	1510	1320	1150	26900	19100	24600
13	.173	1650	1430	1250	29100	22000	26800
14	.193	1780	1540	1390	31300	24800	28900
15	.217	1820	1650	1555	33500	28000	31000
16	.238	1930	1760	1670	35800	30700	33100
17	.251	2030	1870	1790	38000	33300	35400
18	.271	2140	1980	1910	40200	36100	37500
19	.296	2250	2000	2045	42500	39500	39700
20	.316	2350	2220	2160	44700	42100	41900
21	.338	2440	2300	2280	46900	45000	44600
22	.349	2480	2360	2340	48000	46400	45200
23	.360	2520	2420	2400	49100	47500	46300
24	.371	2580	2470	2460	50300	49100	47400
25	.383	2620	2520	2520	51300	50500	48500
26	.394	2670	2580	2570	52400	51900	49500
27	.405	2720	2630	2630	53500	53200	50600
28	.415	2750	2680	2680	54600	54500	51700
29	.425	2790	2740	2740	55700	55700	52900
30	.436	2820	2790	2800	56600	58700	54000

CALCULATIONS FOR BEAM TWO

No.	Load Pounds	Moment in. lbs.	e_c	e_s	$e_c + e_s$	kd. in.	jd. in.
1	100	2953	.0000834	.0000123	.00002084	2.22	4.81
2	200	4015	.0000416	.0000375	.0000791	2.92	4.58
3	400	6132	.000075	.001000	.0001750	2.38	4.76
4	600	8268	.0000777	.0001025	.0001802	2.39	4.75
5	800	10394	.0001080	.0001525	.0002605	2.30	4.78
6	1000	12515	.0001525	.000250	.0004025	2.10	4.85
7	1200	14640	.0002000	.000337	.0005370	2.06	4.86
8	1400	16745	.000244	.000450	.000694	1.95	4.90
9	1600	18890	.000286	.000535	.000821	1.93	4.91
10	1800	21015	.000324	.000620	.000944	1.91	4.91
11	2000	23340	.000360	.000700	.001060	1.89	4.92
12	2200	25265	.000402	.000787	.001189	1.89	4.92
13	2400	27390	.000438	.000874	.001312	1.85	4.93
14	2600	29515	.000475	.000950	.001425	1.85	4.93
15	2800	31640	.000513	.001040	.001553	1.83	4.94
16	3000	33765	.000534	.001118	.001652	1.79	4.95
17	3200	35890	.000577	.001218	.001795	1.78	4.95
18	3400	38015	.000617	.001302	.001919	1.79	4.95
19	3600	40140	.000657	.001382	.002039	1.79	4.95
20	4000	42265	.000703	.001480	.002183	1.79	4.95
21	4200	44390	.000717	.001493	.002210	1.80	4.95

Weight 179#
Reinforcement System One

CALCULATIONS FOR BEAM TWO (cont'd)

No.	D in.	Sc obs	Sc cal. (1)	Sc cal. (2)	Ss obs	Ss cal. (1)	Ss cal. (2)
1	.007	30	138	158	375	3120	3080
2	.014	170	150	215	1125	4470	4190
3	.025	310	270	329	3000	6560	6390
4	.025	330	364	442	3075	8900	8630
5	.037	440	473	556	4575	11100	10820
6	.056	630	615	670	7500	13200	13080
7	.078	820	731	785	10110	15300	15300
8	.099	1000	877	897	13500	17500	17500
9	.123	1150	996	1010	16050	19700	19700
10	.148	1280	1120	1125	18600	21900	21900
11	.168	1400	1260	1240	21000	24200	24300
12	.192	1530	1365	1360	23610	26200	26400
13	.213	1630	1500	1460	26220	28300	28600
14	.235	1740	1620	1580	28500	29500	30800
15	.258	1840	1750	1690	31200	32700	33000
16	.285	1900	1860	1810	33540	34800	35200
17	.314	2000	2040	1920	36540	37000	37400
18	.340	2090	2140	2040	39060	39200	39600
19	.369	2190	2260	2150	41460	41400	42000
20	.408	2280	2390	2260	44400	43500	44500
21	.445	2320	2490	2330	44790	45500	46200

CALCULATIONS FOR BEAM THREE

No.	Load Pounds	Moment in. lbs.	e_c	e_s	$e_c + e_s$	kd. in.	jd. in.
1	1000	12425	.000194	.000250	.000444	2.40	4.75
2	1100	13488	.000236	.000308	.000544	2.39	4.75
3	1200	14550	.000264	.000380	.000644	2.26	4.80
4	1400	16675	.000316	.000438	.000754	2.31	4.78
5	1600	18800	.000401	.000575	.000976	2.26	4.80
6	1800	20925	.000433	.000615	.001048	2.27	4.79
7	2000	23050	.000477	.000675	.001152	2.28	4.79
8	2200	25175	.000521	.000763	.001284	2.23	4.81
9	2450	27300	.000575	.000847	.001322	2.22	4.81
10	2600	29425	.000630	.000930	.001560	2.22	4.81

Weight 168#
Reinforcement System One

CALCULATIONS FOR BEAM THREE (cont'd)

No.	D in.	Sc obs	Sc cal. (1)	Sc cal. (2)	Ss obs	Ss cal. (1)	Ss cal. (2)
1	.069	840	665	465	7500	12900	11400
2	.084	990	718	594	9240	14000	14500
3	.099	1050	778	672	11400	15100	15500
4	.129	1240	881	755	13500	17300	17800
5	.160	1500	1004	865	17550	19500	20000
6	.180	1600	1118	963	18450	21700	22300
7	.200	1710	1230	1053	20250	23900	24500
8	.231	1830	1342	1171	22990	25000	26600
9	.264	1960	1458	1280	25410	26300	29000
10	.311	2080	1571	1378	27900	30400	31200

CALCULATIONS FOR BEAM FOUR

No.	Load Pounds	Moment in. lbs.	e_c	e_s	$e_c + e_s$	kd. in.	jd. in.
1	400	6190	.0000278	---	----	--	--
2	600	8320	.0000694	.0000175	.0000444	1.57	--
3	800	10440	.0000111	.0000375	.0000486	1.27	--
4	1000	12570	.0000389	.0000675	.0001064	2.03	4.87
5	1200	14690	.0000694	.0001227	.0001921	2.00	4.88
6	1400	16820	.0001122	.0002125	.000325	1.92	4.91
7	1600	18940	.0001580	.000292	.000450	1.95	4.90
8	1800	21070	.000201	.000384	.000585	1.91	4.91
9	2000	23190	.000261	.000480	.000741	1.95	4.90
10	2200	25320	.000290	.000570	.000860	1.87	4.93
11	2400	27440	.000330	.000657	.000987	1.86	4.93
12	2600	29570	.000372	.000740	.001112	1.86	4.93
13	2800	31690	.000410	.000817	.001227	1.86	4.93
14	3000	33820	.000451	.000902	.001353	1.85	4.93
15	3200	35940	.000493	.000992	.001485	1.84	4.94
16	3400	38070	.000544	.001082	.001620	1.86	4.92
17	3600	40290	.000565	.001142	.001707	1.84	4.94
18	3800	42320	.000620	.001250	.001870	1.84	4.93
19	4000	44440	.000666	.001335	.002001	1.85	4.92
20	4200	46570	.000714	.001421	.002135	1.86	4.92
21	4400	48690	.000765	.001512	.002277	1.86	4.92
22	4600	50820	.000810	.001600	.002410	1.87	4.92
23	4800	52940	.000865	.001690	.002555	1.88	4.92
24	5000	55070	.000915	.001782	.002697	1.88	4.92
25	5200	57200	.000960	.001860	.002820	1.83	4.94

Weight 168#
Reinforcement System Two

CALCULATIONS FOR BEAM FOUR (cont'd)

No.	D in.	Sc obs	Sc cal. (1)	Sc cal. (2)	Ss obs	Ss cal. (1)	Ss cal. (2)
1	.003	20	314	--	--	6470	--
2	.010	45	429	--	525	8700	--
3	.017	70	537	--	1120	10900	--
4	.025	180	646	634	2020	13120	13200
5	.035	320	755	754	3680	15330	15300
6	.050	510	865	890	6370	17600	17500
7	.068	700	975	990	8750	19800	19700
8	.087	900	1085	1125	11500	22100	20900
9	.110	1150	1195	1210	14400	24200	24200
10	.130	1260	1305	1370	17100	26400	26200
11	.152	1400	1413	1490	19700	28700	28300
12	.173	1540	1520	1620	22200	30900	30600
13	.192	1640	1630	1730	24500	33200	32800
14	.213	1760	1740	1860	27000	35400	35000
15	.234	1890	1850	1980	29700	37600	37100
16	.255	2020	1960	2080	32500	39800	39500
17	.270	2070	2080	2220	34200	42100	41500
18	.296	2200	2160	2340	37400	44300	43700
19	.318	2310	2290	2440	40000	46400	46000
20	.332	2400	2400	2540	42700	48600	48300
21	.366	2490	2510	2660	45400	50900	50400
22	.391	2560	2620	2750	48000	53100	52700
23	.418	2650	2720	2860	50600	55300	54800
24	.453	2730	2840	2980	53400	57500	57100
25	.488	2800	2950	3160	55700	59800	59000

CALCULATIONS FOR BEAM FIVE

No.	Load Pounds	Moment in. lbs.	e_c	e_s	$e_c \neq e_s$	kd. in.	jd. in.
1	400	6050	.0000471	.0000725	.0001196	2.17	4.83
2	600	8180	.0001013	.0001250	.0002263	2.43	4.74
3	800	10300	.0001304	.0002080	.0003384	2.12	4.84
4	1000	12430	.0001805	.000298	.0004785	2.08	4.85
5	1200	14550	.0002600	.000380	.0006400	2.24	4.82
6	1400	16680	.0002790	.000475	.000754	2.04	4.87
7	1600	18800	.0003370	.000570	.000907	2.04	4.87
8	1800	20930	.0004050	.000673	.001078	2.06	4.86
9	2000	23050	.0004620	.000780	.001242	2.04	4.87
10	2200	25180	.0004870	.000850	.001337	2.01	4.88
11	2400	27300	.0005500	.000938	.001488	2.03	4.87
12	2600	29430	.0006200	.001025	.001645	2.07	4.86
13	2800	31550	.0006950	.001115	.001810	2.11	4.85
14	3000	33680	.0007710	.001208	.001978	2.14	4.84
15	3200	35800	.000855	.001300	.002155	2.18	4.82
16	3400	37930	.000946	.001393	.002339	2.22	4.81
17	3600	40050	.000987	.001488	.002475	2.19	4.82
18	3800	42180	.001058	.001583	.002641	2.20	4.82
19	4000	44300	.001900	.001685	.002875	2.28	4.79
20	4200	46430	.001372	.001788	.003160	2.39	4.75

Weight 183#
Reinforcement System Two

CALCULATIONS FOR BEAM FIVE (cont'd)

No.	D in.	Sc obs	Sc cal. (1)	Sc cal. (2)	Ss obs	Ss cal. (1)	Ss cal. (2)
1	.023	220	323	289	2175	6270	6400
2	.037	470	436	359	3750	8480	8820
3	.053	590	550	501	6240	10700	10850
4	.072	790	665	628	8940	12900	13550
5	.095	1060	778	677	11400	15100	15500
6	.120	1130	881	838	13250	17300	17450
7	.147	1310	1004	945	17100	19500	19700
8	.178	1510	1118	1045	20190	21700	21900
9	.203	1670	1230	1160	23400	23900	25000
10	.230	1740	1342	1285	25500	26300	26400
11	.256	1900	1458	1380	28140	26300	28600
12	.285	2060	1571	1460	30150	30400	30800
13	.314	2210	1681	1560	33450	32700	33200
14	.344	2350	1798	1645	36240	35000	35600
15	.375	2480	1910	1700	39000	37100	39200
16	.405	2590	2020	1780	41790	40300	40300
17	.441	2640	2138	1900	44640	41500	42400
18	.478	2710	2250	1990	47490	43700	44700
19	.525	2830	2360	2025	50550	45900	47200
20	.583	2900	2478	2040	53640	48100	49700

CALCULATIONS FOR BEAM SIX

No.	Load Pounds	Moment in. lbs.	e_c	e_s	$e_c + e_s$	kd. in.	jd. in.
1	400	6180	.0000167	.000050	.0000217	4.27	4.13
2	600	8310	.0000361	.000020	.0000561	3.56	4.36
3	800	10430	.0000625	.0000425	.0001050	3.30	4.45
4	1000	12560	.0000861	.0000625	.0001486	3.22	4.48
5	1200	14680	.0001151	.0000875	.0002056	3.16	4.50
6	1400	16810	.000154	.0001375	.0002915	2.94	4.57
7	1600	18930	.0001942	.000200	.0003942	2.74	4.64
8	1800	21060	.000233	.000275	.000508	2.64	4.67
9	2000	23180	.000278	.000373	.000651	2.37	4.76
10	2200	25310	.000322	.000475	.000797	2.22	4.80
11	2400	27430	.000361	.000568	.000929	2.16	4.83
12	2600	29560	.000414	.000650	.001064	2.15	4.83
13	2800	31680	.000443	.000750	.001193	2.05	4.87
14	3000	33870	.000480	.000828	.001308	2.03	4.87
15	3200	35900	.000520	.000925	.001445	2.00	4.88
16	3400	38060	.000566.	.001015	.001581	1.97	4.89
17	3600	40280	.000615	.001095	.001710	2.00	4.88
18	3800	42310	.000660	.001170	.001830	2.00	4.88
19	4000	44430	.000706	.001260	.001960	1.99	4.89
20	4200	46560	.000745	.001337	.002082	1.98	4.89
21	4400	48680	.000799	.001435	.002234	1.98	4.89
22	4600	50810	.000843	.001525	.002368	1.97	4.89
23	4800	52930	.000887	.001605	.002492	1.98	4.89
24	5000	55060	.000940	.001690	.002630	1.98	4.89

Weight 181#
Reinforcement System Three

CALCULATIONS FOR BEAM SIX (cont'd)

No.	D in.	S_c obs	S_c cal. (1)	S_c cal. (2)	S_s obs	S_s cal. (1)	S_s cal. (2)
1	.004	70	318	174	150	6460	7640
2	.009	80	429	268	600	8700	9700
3	. 017	280	537	355	1275	10900	11950
4	.025	380	646	436	1875	13120	14300
5	.031	520	755	516	2020	15330	16600
6	.042	690	865	628	4130	17000	18800
7	.056	870	975	700	6000	19800	20900
8	.071	1040	1085	855	8250	22100	23000
9	.095	1220	1195	1035	11200	24000	24800
10	.114	1410	1305	1170	14300	26400	26900
11	.135	1560	1413	1320	17000	28700	29000
12	.156	1740	1520	1420	19500	30900	31200
13	.178	1830	1630	1580	22500	33200	33200
14	.197	1940	1740	1700	24800	35400	35400
15	.218	2040	1850	1840	27800	37600	37400
16	.241	2160	1960	1980	30450	39800	39600
17	.263	2280	2080	2060	32900	42100	42100
18	.284	2390	2160	2160	35200	44300	44200
19	.307	2480	2290	2280	37800	46400	46100
20	.328	2550	2400	2400	40100	48600	48600
21	.356	2660	2510	2510	43100	50900	50800
22	.381	2730	2620	2620	45800	53100	52900
23	.404	2790	2720	2720	48200	55300	55100
24	.432	2860	2840	2840	50800	57500	57400

CALCULATIONS FOR BEAM SEVEN

No.	Load Pounds	Moment in. lbs.	e_c	e_s	$e_c + e_s$	kd. in.	jd. in.
1	400	6020	.0000138	.0000250	.0000388	1.98	4.89
2	600	8150	.0000194	.0000125	.0000319	3.38	4.33
3	800	10270	.0000485	.0000350	.0000835	3.22	4.48
4	1000	12400	.0000819	.0000700	.0001519	2.99	4.55
5	1200	14520	.0001193	.0001100	.0002293	2.90	4.58
6	1400	16650	.0001610	.0001750	.000336	2.89	4.58
7	1600	18770	.000214	.000300	.000514	2.32	4.78
8	1800	20900	.000268	.000432	.000700	2.12	4.84
9	2000	23020	.000312	.000550	.000862	2.01	4.88
10	2200	25150	.000360	.000645	.001005	1.99	4.89
11	2400	27270	.000405	.000750	.001155	1.95	4.90
12	2600	29400	.000447	.000838	.001285	1.93	4.91
13	2800	31520	.000500	.000948	.001448	1.92	4.91
14	3000	33650	.000550	.001025	.001575	1.93	4.91
15	3200	35770	.000597	.001102	.001699	1.95	4.90
16	3400	37920	.000647	.001180	.001827	1.96	4.90
17	3600	42020	.000694	.001248	.001942	1.98	4.89
18	3800	42150	.000746	.001338	.002084	1.99	4.89
19	4000	44270	.000804	.001425	.002209	2.02	4.88
20	4200	46400	.000861	.001515	.002376	2.02	4.88
21	4400	48520	.000926	.001612	.002538	2.03	4.87

Weight 168#
Reinforcement System Four

CALCULATIONS FOR BEAM SEVEN (cont'd)

No.	D in.	Sc obs	Sc cal. (1)	Sc cal. (2)	Ss obs	Ss cal. (1)	Ss cal. (2)
1	.005	20	310	--	75	6300	6280
2	.010	80	420	278	375	8520	9600
3	.019	210	529	355	1050	10750	11680
4	.031	360	637	455	2100	12980	13900
5	.047	510	747	547	3300	15180	16200
6	.068	690	857	630	5250	17400	18500
7	.096	910	965	850	9000	19600	20000
8	.123	1090	1075	1020	12960	21920	22000
9	.149	1250	1185	1175	16500	24100	24100
10	.176	1400	1295	1290	19350	26300	26200
11	.200	1540	1405	1430	22500	28500	28400
12	.223	1670	1515	1550	25100	30800	30600
13	.250	1840	1620	1660	28400	33000	32800
14	.276	1980	1735	1780	30750	35200	35200
15	.299	2110	1840	1870	33000	37400	37200
16	.323	2230	1950	1980	35400	39700	39500
17	.345	2330	2060	2060	37400	41800	41800
18	.377	2430	2170	2160	40000	44100	44000
19	.410	2540	2280	2250	42700	46300	46300
20	.443	2630	2390	2350	45500	48500	48500
21	.483	2730	2500	2400	48400	50800	50800

CALCULATIONS FOR BEAM EIGHT

No.	Load Pounds	Moment in. lbs.	e_c	e_s	$e_c + e_s$	kd. in.	jd. in.
1	400	6110	.0000208	.0000125	.0000333	3.46	4.40
2	600	8240	.0000458	.0000275	.0000733	3.46	4.40
3	800	10360	.0000792	.0000750	.0001542	2.85	4.60
4	1000	12490	.0001221	.0001750	.000297	2.28	4.79
5	1200	14610	.0001585	.000290	.0004485	1.96	4.90
6	1400	16740	.0001989	.000425	.0006239	1.77	4.96
7	1600	18860	.000243	.000503	.000746	1.81	4.95
8	1800	20990	.000288	.000580	.000868	1.84	4.94
9	2000	23110	.000324	.000640	.000964	1.87	4.93
10	2200	25240	.000367	.000708	.001075	1.89	4.92
11	2400	27360	.000417	.000803	.001220	1.89	4.92
12	2600	29490	.000461	.000889	.001350	1.89	4.92
13	2800	31610	.000519	.000975	.001494	1.93	4.91
14	3000	33740	.000570	.001050	.001620	1.95	4.90
15	3200	35860	.000606	.001125	.001731	1.94	4.90
16	3400	38010	.000655	.001212	.001867	1.95	4.90
17	3600	40110	.000720	.001296	.002016	1.98	4.89
18	3800	42240	.000765	.001375	.002140	1.98	4.89
19	4000	44360	.000825	.001469	.002294	1.99	4.89
20	4200	46490	.000885	.001550	.002435	2.02	4.88
21	4400	48610	.000950	.001643	.002593	2.03	4.88
22	4600	50740	.001020	.001730	.002750	2.04	4.87
23	4800	52860	.001088	.001820	.002908	2.07	4.86

Weight 175#
System Three

CALCULATIONS FOR BEAM EIGHT (cont'd)

No.	D in.	Sc obs	Sc cal. (1)	Sc cal. (2)	Ss obs	Ss cal. (1)	Ss cal. (2)
1	.003	100	315	201	375	6440	7120
2	.009	210	424	270	825	8620	9550
3	.023	340	533	395	2250	10830	11500
4	.044	520	642	572	5260	13080	13300
5	.068	675	753	761	8720	15300	15200
6	.099	840	862	952	12750	17520	17200
7	.117	1020	970	1050	15090	19720	19400
8	.140	1160	1080	1150	17400	21950	21650
9	.155	1280	1190	1250	19200	24200	23900
10	.175	1410	1300	1350	21250	26450	26200
11	.202	1570	1395	1460	24090	28600	28300
12	.227	1730	1520	1590	26600	30900	30600
13	.253	1890	1630	1670	29200	33100	32800
14	.276	2040	1735	1760	31500	35300	35200
15	.296	2130	1845	1890	33750	37500	37300
16	.321	2250	1960	1990	36360	39800	39600
17	.346	2380	2060	2070	38900	41900	41700
18	.374	2460	2170	2180	41200	44200	44000
19	.400	2570	2280	2280	44000	46400	46100
20	.427	2670	2390	2360	46500	48600	48600
21	.451	2780	2510	2450	48300	50900	50700
22	.484	2880	2610	2550	51900	53100	53000
23	.515	2980	2720	2620	54500	52400	55300

CALCULATIONS FOR BEAM NINE

No.	Load Pounds	Moment in. lbs.	e_c	e_s	$e_c + e_s$	kd. in.	jd. in.
1	400	6150	.0000111	.0000100	.0000211	2.92	4.57
2	600	8280	.0000333	.0000250	.0000583	3.16	4.50
3	800	10400	.0000555	.0000475	.0001030	3.00	4.55
4	1000	12530	.0000782	.0000700	.0001482	2.93	4.57
5	1200	14650	.0001070	.0000975	.0002045	2.90	4.58
6	1400	16780	.0001622	.0001250	.0002872	3.14	4.51
7	1600	18900	.0001830	.0001700	.0003530	2.88	4.59
8	1800	21030	.000208	.000225	.000433	2.67	4.66
9	2000	23150	.000269	.000415	.000684	2.19	4.82
10	2200	25280	.000312	.000502	.000814	2.13	4.84
11	2400	27400	.000354	.000600	.000954	2.06	4.87
12	2600	29530	.000391	.000688	.001079	2.01	4.88
13	2800	31650	.000430	.000775	.001205	1.98	4.89
14	3000	33780	.000480	.000875	.001355	1.97	4.89
15	3200	35900	.000520	.000955	.001475	1.96	4.90
16	3400	38030	.000563	.001070	.001633	1.92	4.91
17	3600	40150	.000611	.001133	.001744	1.95	4.90
18	3800	42280	.000659	.001228	.001887	1.95	4.90
19	4000	44400	.000708	.001325	.002033	1.93	4.91
20	4200	46530	.000755	.001415	.002170	1.93	4.91
21	4400	48650	.000797	.001490	.002287	1.93	4.91
22	4600	50780	.000841	.001575	.002416	1.93	4.91
23	4800	52900	.000897	.001665	.002562	1.94	4.90
24	5000	55030	.000955	.001810	.002765	1.92	4.91

Weight 179#
Reinforcement System Four

CALCULATIONS FOR BEAM NINE (cont'd)

No.	D in.	Sc obs	Sc cal. (1)	Sc cal. (2)	Ss obs	Ss cal. (1)	Ss cal. (2)
1	.004	60	317	230	300	6390	6850
2	.010	150	426	291	750	8600	9380
3	.018	260	535	381	1425	10800	11650
4	.026	360	646	467	2100	13000	14000
5	.033	490	755	552	2930	15200	16300
6	.042	730	874	592	3750	17400	19000
7	.056	820	975	715	5100	19600	21000
8	.072	920	1085	846	6750	21800	23000
9	.102	1140	1193	1100	12450	24050	24500
10	.130	1300	1303	1220	15060	26200	26600
11	.153	1440	1410	1365	18000	28400	28700
12	.178	1560	1520	1505	20640	30700	30900
13	.204	1670	1630	1635	23250	32800	33000
14	.231	1830	1740	1750	26250	35100	35200
15	.254	1940	1850	1860	28750	37300	37400
16	.276	2080	1960	2020	30210	39500	39600
17	.304	2220	2070	2100	33990	41600	41700
18	.331	2360	2205	2210	36840	43900	44000
19	.362	2480	2290	2340	39750	46100	46200
20	.389	2570	2400	2460	42450	48300	48300
21	.399	2650	2500	2560	44700	50500	50600
22	.439	2720	2615	2680	47250	52700	52700
23	.471	2810	2730	2780	49950	54900	55000
24	.	2870	2840	2910	54300	57200	57200

CALCULATIONS FOR BEAM TEN

No.	Load Pounds	Moment in. lbs.	e_s	e_c	$e_c + e_s$	kd. in.	jd. in.
1	600	8660	.0000319	.0000200	.0000519	3.42	4.41
2	800	10780	.0000569	.0000275	.0000844	3.75	4.30
3	1000	12910	.0000847	.0000500	.0001347	3.48	4.39
4	1200	15030	.0001180	.0000775	.0001955	3.36	4.43
5	1400	17160	.0001500	.0001100	.0002600	3.20	4.48
6	1600	19280	.0001835	.0001500	.0003335	3.04	4.54
7	1800	21330	.000236	.0002225	.0004585	2.86	4.60
8	2000	23530	.000295	.0003200	.0006150	2.66	4.66
9	2200	25660	.000350	.000420	.000770	2.52	4.71
10	2400	27780	.000395	.000528	.000923	2.38	4.76
11	2600	29910	.000471	.000640	.001111	2.35	4.77
12	2800	32030	.000534	.000740	.001274	2.33	4.77
13	3000	34160	.000597	.000853	.001450	2.28	4.79
14	3200	36280	.000660	.000945	.001605	2.28	4.79
15	3400	38430	.000720	.001090	.001810	2.21	4.78
16	3600	40530	.000781	.001165	.001946	2.23	4.81
17	3800	42660	.000845	.001278	.002123	2.21	4.81
18	4000	44780	.000904	.001370	.002274	2.20	4.81
19	4200	46910	.000972	.001453	.002425	2.22	4.81
20	4400	49030	.001027	.001575	.002602	2.19	4.82
21	4600	51160	.001098	.001675	.002773	2.20	4.82
22	4800	53280	.001153	.001800	.002953	2.17	4.83

Weight 214#
Reinforcement System One

CALCULATIONS FOR BEAM TEN (con't)

No.	D in.	Sc obs	Sc cal. (1)	Sc cal. (2)	Ss obs	Ss cal. (1)	Ss cal. (2)
1	.008	160	445	298	600	9030	10000
2	.013	270	555	338	825	11240	12800
3	.020	390	666	423	1500	13450	15000
4	.027	540	775	505	2325	15650	17300
5	.034	680	885	598	3300	17900	19500
6	.045	820	994	698	4500	20100	21600
7	.058	1020	1095	810	6675	22200	23600
8	.075	1240	1210	950	9600	24500	25700
9	.095	1430	1320	1080	12600	26800	27800
10	.117	1570	1430	1220	15840	29200	29700
11	.142	1800	1540	1330	19200	31200	32000
12	.165	1990	1650	1440	22200	33400	34200
13	.189	2170	1755	1560	25600	35600	35400
14	.212	2360	1870	1660	28400	37800	38600
15	.236	2500	1870	1820	32700	40000	41000
16	.258	2620	2090	1890	35000	42300	43000
17	.282	2730	2200	2010	38300	44500	45300
18	.306	2810	2300	2110	41100	46700	47500
19	.332	2890	2410	2200	43600	48900	49800
20	.356	2930	2530	2320	47200	51100	51900
21	.381	2970	2640	2420	50300	53300	54300
22	.408	--	2740	2540	54000	55600	56200

CALCULATIONS FOR BEAM ELEVEN

No.	Load Pounds	Moment in. lbs.	e_c	e_s	$e_c + e_s$	kd. in.	jd. in.
1	600	8650	.0000445	.0000225	.0000670	3.69	4.32
2	800	10770	.0000722	.0000425	.0001147	3.49	4.39
3	1000	12900	.0001040	.0000650	.0001690	3.41	4.41
4	1200	15020	.0001331	.0000950	.0002281	3.24	4.47
5	1400	17150	.0001582	.0001250	.0002832	3.10	4.52
6	1600	19270	.0001972	.0001675	.0003647	3.00	4.55
7	1800	21320	.000236	.000235	.000471	2.78	4.62
8	2000	23520	.000287	.000320	.000607	2.63	4.67
9	2200	25650	.000347	.000448	.000795	2.42	4.74
10	2400	27770	.000402	.000540	.000942	2.37	4.76
11	2600	29900	.000468	.000643	.001111	2.01	4.88
12	2800	32020	.000499	.000725	.001224	2.26	4.76
13	3000	34150	.000545	.000820	.001365	2.21	4.78
14	3200	36270	.000598	.000923	.001521	2.18	4.82
15	3400	38420	.000648	.001005	.001653	2.18	4.82
16	3600	40520	.000694	.001100	.001794	2.14	4.84
17	3800	42650	.000743	.001200	.001943	2.12	4.84
18	4000	44770	.000792	.001292	.002084	2.11	4.84
19	4200	46950	.000839	.001375	.002214	2.08	4.86
20	4400	49020	.000891	.001462	.002353	2.10	4.86
21	4600	51150	.000943	.001542	.002485	2.10	4.86
22	4800	53270	.001010	.001628	.002638	2.12	4.84
23	5000	55400	.001043	.001700	.002743	2.11	4.84
24	5200	57530	.001102	.001780	.002892	2.12	4.84

Weight 214#
Reinforcement System One

CALCULATIONS FOR BEAM ELEVEN (cont'd)

No.	D in.	Sc obs	Sc cal. (1)	Sc cal. (2)	Ss obs	Ss cal. (1)	Ss cal. (2)
1	.009	190	445	271	675	9030	10250
2	.016	300	555	351	1275	11240	12500
3	.024	450	666	430	1950	13450	14900
4	.033	570	775	518	2850	15650	17100
5	.042	660	885	614	3750	17900	19400
6	.052	810	994	705	5025	20100	21600
7	.064	950	1095	832	7050	22200	23500
8	.083	1110	1210	1000	9600	24500	25700
9	.105	1290	1320	1120	13440	26800	27700
10	.127	1440	1430	1230	16200	29200	29700
11	.151	1590	1540	1520	19290	31200	31200
12	.170	1660	1650	1490	21750	33400	34400
13	.190	1760	1755	1610	24600	35600	35400
14	.215	1860	1870	1720	27690	37800	38400
15	.247	1960	1970	1830	30150	40000	40600
16	.262	2030	2090	1960	35000	42300	42700
17	.282	2100	2200	2080	36000	44500	45000
18	.305	2160	2300	2190	38760	46700	47100
19	.325	2220	2410	2320	41250	48900	49100
20	.349	2270	2530	2400	43860	51000	51500
21	.373	2320	2640	2500	46260	53300	53600
22	.397	2380	2740	2600	48840	55600	56500
23	.415	2400	2850	2720	51000	57900	58400
24	.442	2450	2980	2800	53400	60100	60600

CALCULATIONS FOR BEAM TWELVE

No.	Load Pounds	Moment in. lbs.	e_c	e_s	$e_c + e_s$	kd. in.	jd. in.
1	600	8680	.0000347	.0000150	.0000497	3.87	4.26
2	800	10800	.0000610	.0000275	.0000885	3.82	4.28
3	1000	12930	.0000902	.0000500	.0001402	3.56	4.36
4	1200	15050	.0001205	.0000700	.0001905	3.51	4.38
5	1400	17180	.0001471	.0000950	.0002421	3.38	4.42
6	1600	19300	.0001760	.0001125	.0002885	3.39	4.42
7	1800	21350	.000202	.0001325	.0003345	3.35	4.43
8	2000	23550	.000229	.0001550	.000384	3.31	4.45
9	2200	25680	.000257	.0001825	.0004395	3.24	4.47
10	2400	27800	.000319	.000318	.000637	2.78	4.62
11	2600	29930	.000373	.000490	.000863	2.40	4.75
12	2800	32050	.000431	.000693	.001124	2.13	4.84
13	3000	34180	.000485	.000842	.001327	2.03	4.87
14	3200	36300	.000524	.000950	.001474	1.97	4.89
15	3400	38450	.000570	.001065	.001635	1.94	4.90
16	3600	40550	.000613	.001163	.001776	1.92	4.91
17	3800	42680	.000655	.001252	.001907	1.91	4.91
18	4000	44800	.000699	.001340	.002039	1.90	4.92
19	4200	46930	.000741	.001412	.002153	1.91	4.91
20	4400	49050	.000781	.001477	.002258	1.92	4.92
21	4600	51180	.000829	.001562	.002391	1.92	4.92
22	4800	53300	.000875	.001633	.002508	1.94	4.90
23	5000	55430	.000920	.001700	.002620	1.95	4.90

Weight 217#
Reinforcement System One

CALCULATIONS FOR BEAM TWELVE (cont'd)

No.	D in.	Sc obs	Sc cal. (1)	Sc cal. (2)	Ss obs	Ss cal. (1)	Ss cal. (2)
1	.000	160	447	263	450	9030	10400
2	.003	250	556	331	825	11250	12850
3	.007	380	667	417	1500	13500	15120
4	.015	520	777	490	2100	15700	17500
5	.023	630	885	575	2850	17900	19800
6	.031	730	995	645	3375	20100	22300
7	.039	830	1100	719	3975	22200	24600
8	.048	930	1215	799	4650	24500	27000
9	.058	1020	1325	890	5475	26800	29400
10	.082	1200	1433	1085	9540	29200	30700
11	.114	1640	1540	1310	14700	31200	32200
12	.141	1500	1650	1555	20800	33400	33600
13	.166	1630	1755	1725	25200	35600	35800
14	.190	1710	1870	1890	28500	37800	37800
15	.214	1810	2090	2020	32000	40000	40000
16	.236	1890	2200	2150	34900	42300	42200
17	.259	1970	2300	2280	37600	44500	44300
18	.280	2040	2410	2400	40200	46700	46500
19	.298	2100	2530	2500	42300	48900	48700
20	.315	2150	2640	2600	44300	51000	50800
21	.334	2210	2740	2720	46900	53300	53100
22	.353	2250	2850	2800	49000	55600	55500
23	.374	2300	2980	2900	51000	57900	57600

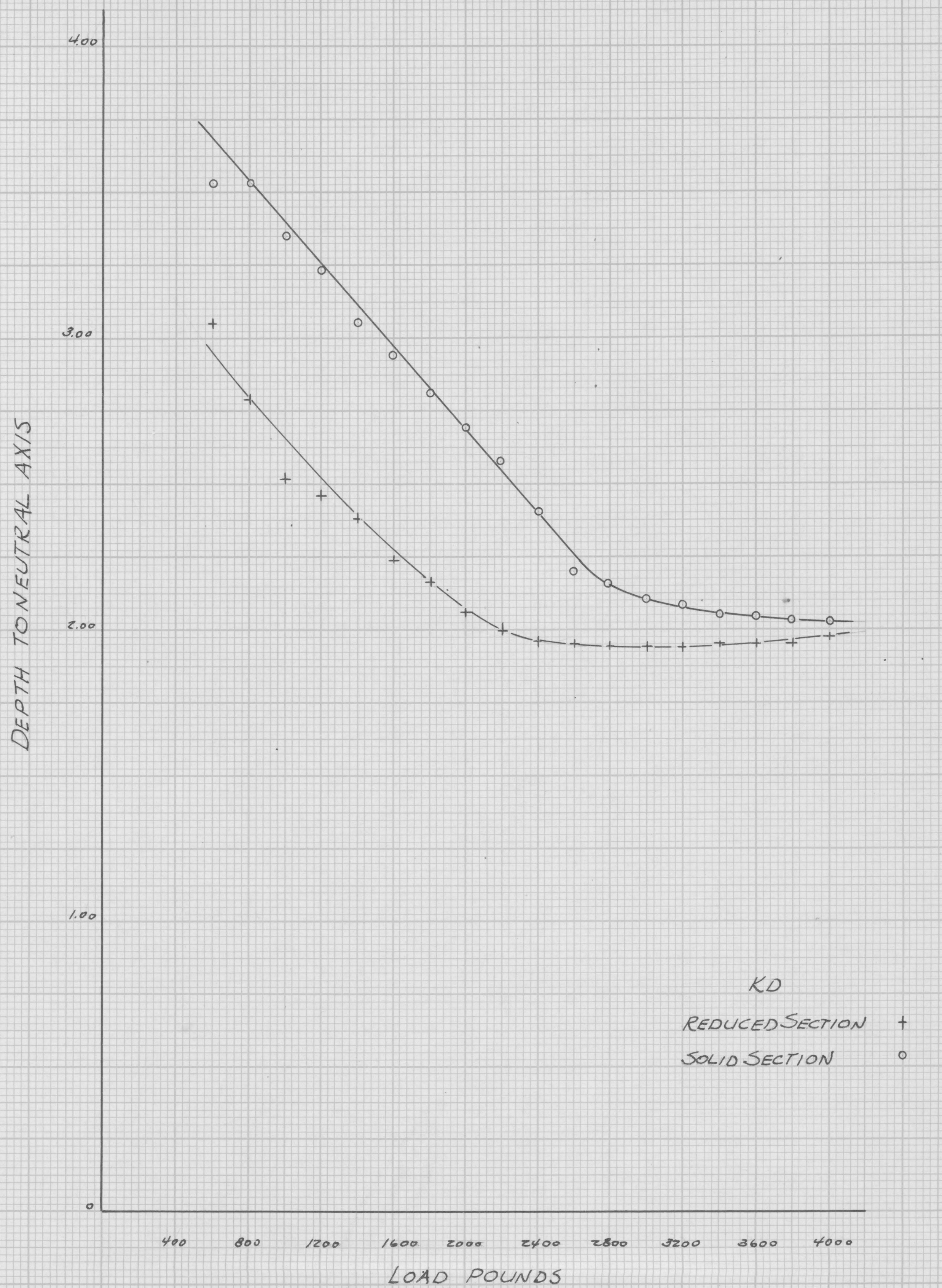
SUMMARY OF CALCULATIONS FOR SOLID BEAMS

Load Pounds	kd Inches	Deflection Inches	Concrete Stress #/in^2	Steel Stress #/in^2
600	3.53	.010	188	768
800	3.53	.016	295	1260
1000	3.35	.023	425	1990
1200	3.23	.031	565	2940
1400	3.06	.041	693	4190
1600	2.94	.053	843	5770
1800	2.81	.065	973	7720
2000	2.69	.081	1160	10060
2200	2.58	.102	1313	12650
2400	2.41	.125	1465	15900
2600	2.20	.150	1703	19500
2800	2.17	.173	1743	23300
3000	2.11	.196	1873	26600
3200	2.09	.217	1990	29400
3400	2.06	.242	2103	32800
3600	2.05	.263	2198	35600
3800	2.04	.285	2288	38500
4000	2.03	.307	2363	41300

SUMMARY OF CALCULATIONS FOR REDUCED BEAMS

Load Pounds	kd Inches	Deflection Inches	Concrete Stress #/in^2	Steel Stress #/in^2
600	3.06	.016	209	1430
800	2.79	.026	313	2572
1000	2.52	.040	460	4255
1200	2.46	.055	628	6190
1400	2.38	.073	770	8430
1600	2.24	.095	954	11010
1800	2.17	.117	1130	13670
2000	2.05	.140	1310	16880
2200	2.00	.164	1450	18020
2400	1.97	.187	1580	22230
2600	1.96	.211	1720	24760
2800	1.95	.236	1846	27500
3000	1.95	.260	1971	30000
3200	1.94	.284	2084	32700
3400	1.96	.309	2203	35110
3600	1.96	.334	2301	37640
3800	1.96	.364	2447	40360
4000	1.98	.395	2504	42880

Does not include Beam Three.


4.00
3.00
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DEPTH TO NEUTRAL AXIS
400
800
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LOAD POUNDS
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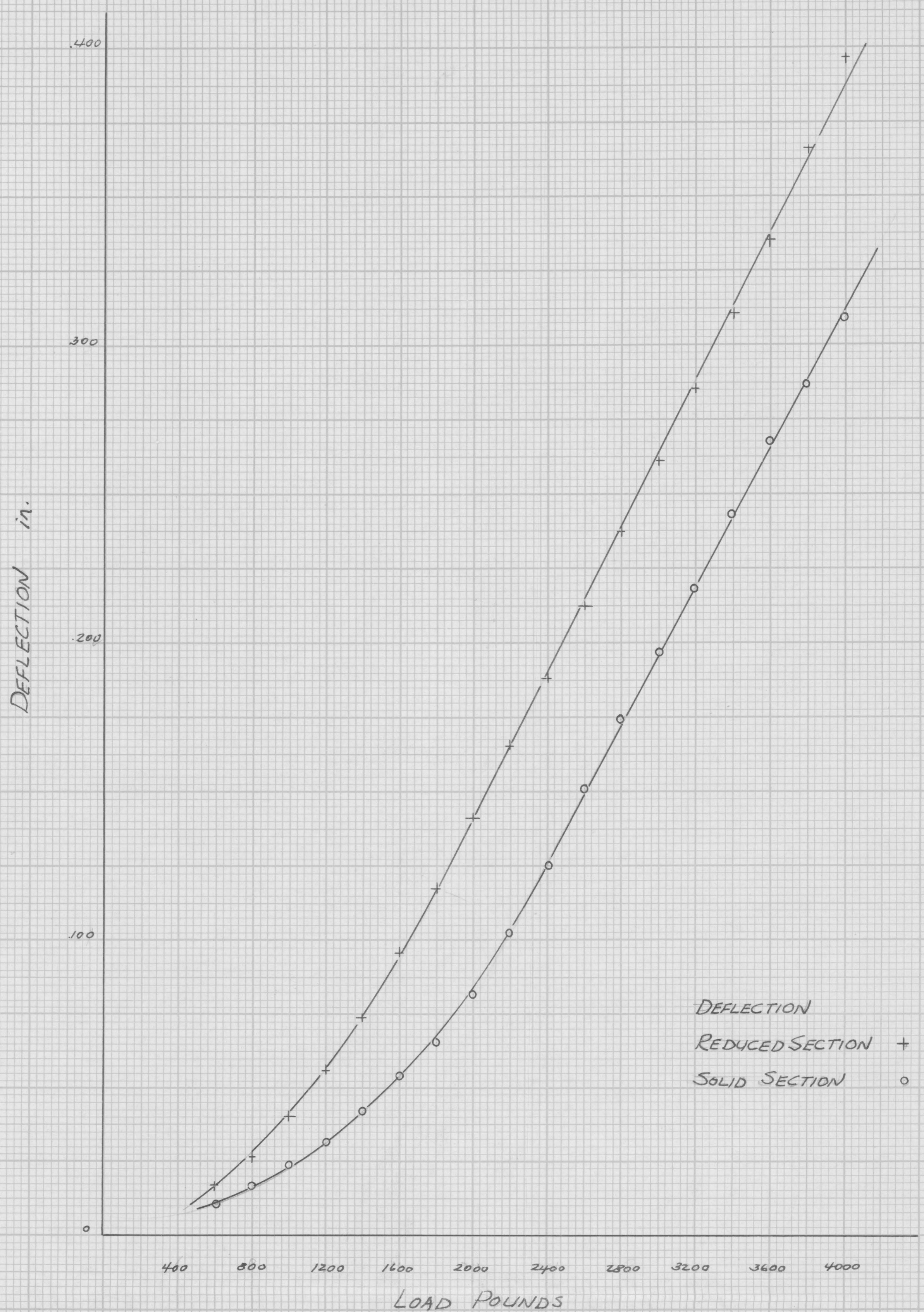
REDUCED SECTION +
SOLID SECTION o



.400
.300
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0
DEFLECTION in.
400
800
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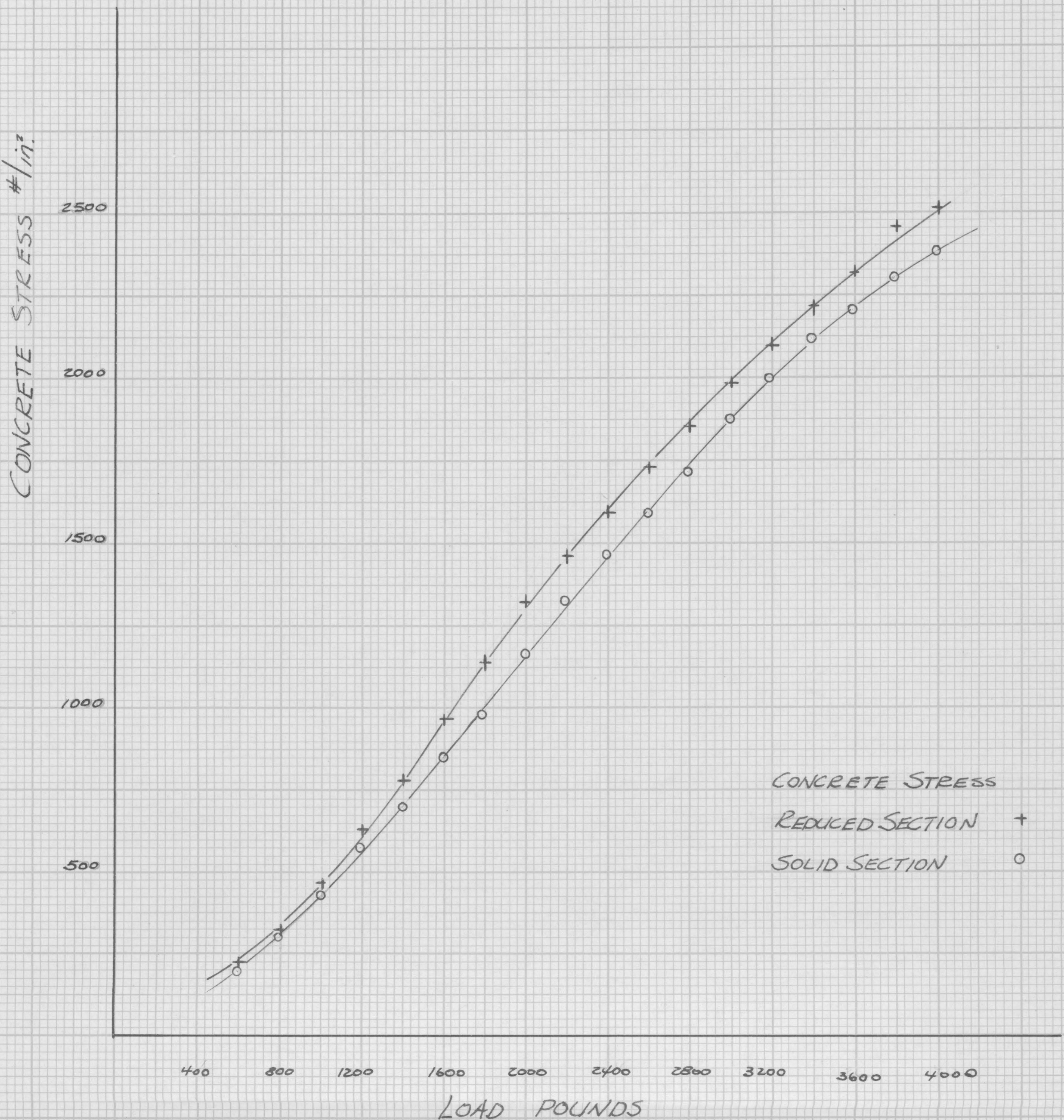
LOAD POUNDS
DEFLECTION
REDUCED SECTION +
SOLID SECTION o


CONCRETE STRESS #/in.²
2500
2000
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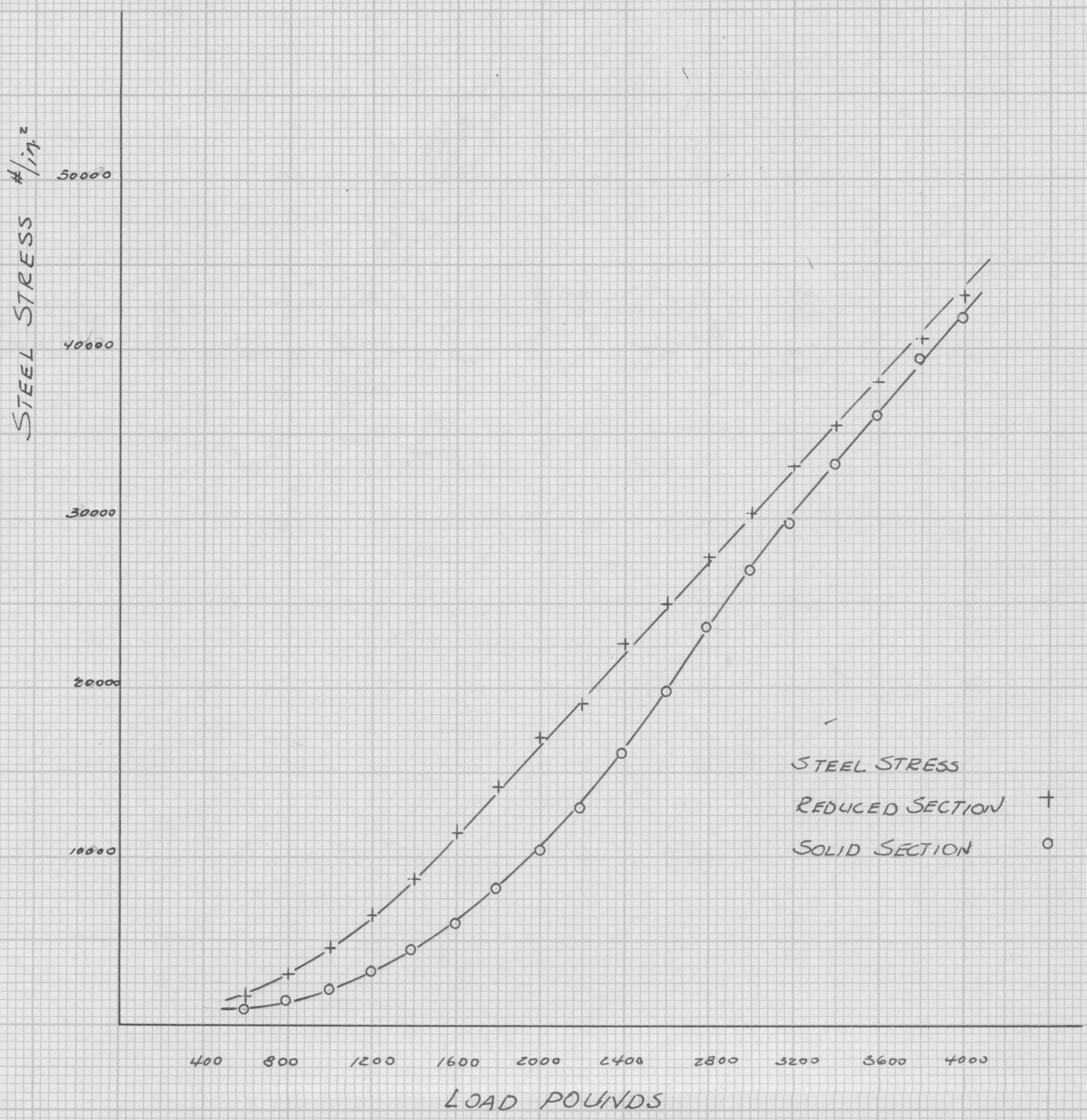
LOAD POUNDS
CONCRETE STRESS
REDUCED SECTION +
SOLID SECTION o


STEEL STRESS #/in.²
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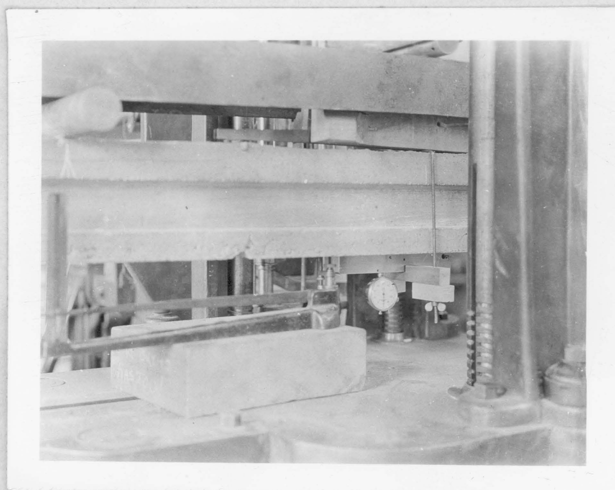
40000
30000
20000
10000
400
800
1200
1600
2000
2400
2800
3200
3600
4000
LOAD POUNDS
STEEL STRESS
REDUCED SECTION +
SOLID SECTION o

Failure of

Beam Seven

Set up of

Guages



THE RESULTS OF THE TESTS

The method of failure of almost all of the beams was due to the yielding of the steel. Such a failure will be typical of practice for beams of economical section will reach the yield point of the steel before the concrete stresses reach a maximum. This was borne out here for when the steel stresses were at 5500 $\#/in^2$ the concrete stresses averaged 2700 $\#/in^2$ where as the concrete should be expected to carry about 3600 $\#/in^2$.

Cracking of the concrete became apparent at about a load of 2500 pounds, but it is quite evident that they must have existed before this time. They were confined to the central portion of the beam, and were most pronounced under the concentrations, undoubtedly due to the local stress concentrations. From an examination of the performance curves of the beams we might say that the cracks became apparent when they had ceased to rise any longer. From this load upward the stresses and the deflection varied directly with the load indicating that no more assistance to resisting the load could be received from the tensile concrete.

METHOD OF FAILURE

All the solid beams and all the reduced beams that had web reinforcement, except beam seven, failed by yielding of the steel. The two reduced beams with no web steel failed by diagonal tension as did beam seven. The yielding failures of the beams are similar in action to the yielding of steel in tension. That is, they will continue to resist at the maximum load but there is a continuation of the deflection without an increase in the load. As the deflection of the beam continues the load carrying ability drops off slightly. If the loading is continued past this point, the concrete will crush and the failure will be more or less complete.

The diagonal tension failues are not so slow, and from a practical point of view are more dangerous. The beam will act quite normal up to failure and then with little warning will split and collapse.

THE POSITION OF THE NEUTRAL AXIS

If we assume no cracking of the beam and "n" equals ten, we will find that the neutral axis should lie at a depth of 3.19 inches from the top of the beam. If we assume a maximum of cracking then it should lie at a depth of 1.89 inches. An analysis of the results of these tests reveals that at the early stages of loading when cracking is not severe the value of "kd" is close to 3.19 and as the load progresses gradually drops down to nearly 1.89. It remains nearly constant at this level until failure. A comparision of the action of the two types of beams shows that they are both approaching this lower value as a limit. The solid beam more gradually, and the reduced section a good deal more rapidly. However, near failure the position of the neutral axis in both types is practically the same.

The method of calculating the depth of the neutral axis should be explained for it is based on the assumption that plane sections remain plane after bending, and that there is no slipping of the steel. The unit strains at the top fiber and in the steel being observed a simple relationship can be obtained which will indicate the depth of the neutral axis. This relationship is among the formulae previously set forth.

THE TENSILE STRESS IN THE STEEL

The stresses set up by the bending of the beam were calculated from the concrete design formulae using a constant value of "n" equal to ten. The observed values were obtained from the strain guage readings using a value of E equal to 30,000,000. A comparision of the observed and calculated values for both beams shows that the steel stresses at load within the safe design load can be expected to be smaller than the calculated values. At the higher loads the stresses more closely approach the calculated ones.

The stresses in the steel of the reduced beam were constantly higher than those of the solid beam, but still within the calculated stress. Such an action is indicative of a higher efficiency in the use of the steel. This difference can be credited to the lack of tensile concrete area.

COMPRESSIVE STRESSES IN THE CONCRETE

The stresses in the concrete were also at the lower loads consistently smaller than the calculated values, but just above the design load for the reduced beam and at eight hundred pounds above this for the solid section beam the concrete stresses rose above the calculated ones. Just before failure the observed stresses again dropped below the calculated ones.

The method of obtaining the observed stresses was by use of a stress strain curve drawn for each batch. These had been obtained from a compression test of each one. It was assumed that the concrete in the beams would act similarly.

DEFLECTION OF THE BEAMS

The computation of the deflection of a concrete beam is complicated by the inability to determine the EI factor. Both of these are changing so that the deflection of the beam does not vary directly with the load. This was evident in this series of tests. At the early loads there is not a straight line variation but after about 2500 pounds had been applied the deflection curves became straight lines. The slope of both of the deflection curves was the same indicating a similiar EI variation. The reduced section deflected more than the solid section until about 2500 pounds when it remained about .070. more than the solid beam. This seems to be an indication that the deflection of a concrete beam depends entirely upon how much of the beam remains uncracked.

The formula proposed by Maney was found to be accurate within a few per cent for the solid beams, and about eight per cent low for the reduced section. The constant which he sets at .1065 for the beam loaded at the third point and which should be about the same for quarter point loading was very closely approximated by the solid beams. In the reduced beams a value of .115 would have more closely approached the true value. However, this formula is of little practical use, as are most of the concrete deflection formulas, for we cannot determine e_c and e_s with any accuracy. Cross says (9), "the deflection of the simplest beam is difficult to predict ... we may as will guess at the deflection in the first place", and his observation apparently is quite correct.

CONCLUSIONS

From this series of tests we may conclude that if the area below the neutral axis is reduced by about thirty five per cent:

1. The stress in the concrete will be slightly higher than the stress in a beam of solid section, but still with in the design stresses. At the higher loads these stresses will approximate each other.

2. The steel stresses will undergo a similar reaction.

3. The deflection of the reduced section beam will be higher than the solid beam by about 80% with in the design load, but as they are only small deformations they are not at all excessive.

4. The saving of 20% of the weight of the concrete in the beam can be accomplished with no practical disadvantage of the reduced section beam over the solid one.

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