Analytical and Experimental Investigation of a 1/5 Scale Rail Vehicle Simulator

by

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(ABSTRACT)

Rail vehicle dynamicists frequently employ roller rigs to study the complex dynamic response of vehicles and trucks. This work presents the analytical and experimental investigation of a one-fifth scale model of a typical North American three-piece freight truck running on a roller rig.

The roller rig was built as part of this research for the Department of Mechanical Engineering at Virginia Polytechnic Institute and State University. Linear and nonlinear mathematical models were also developed to simulate the lateral dynamics of the truck running on the roller rig. The nonlinear model incorporates a lookup table for the wheel/rail geometric constraints, a heuristic creep force model to take into account creep force saturation, and it includes the effects of Coulomb friction in the truck bearings.

The linear model predicts the truck's natural frequency versus speed within 5% of the experimental values. It also predicts the damping ratio decrease that occurs with increasing truck speed. The nonlinear model simulates the lateral instability known as hunting. The nonlinear model and the roller rig also exhibit small amplitude hunting, in which flange contact does not occur. The nonlinear model simulates the experimentally observed increase in oscillation frequency that accompanies the onset of flange contact.

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Finally, I would like to thank my parents for their endless love and support. Thanks for always being there and for letting me make my own decisions.

For the passengers and crew of PAN AM Flight 103 who lost their lives because of a senseless act of terrorism.

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Nomenclature

a - ½ rail gauge (ft, m)

d - ½ sideframe separation (ft, m)

D_W - warp damping (lb-ft-s/rad, N-m-s/rad)

D_θ - yaw damping (lb-ft-s/rad, N-m-s/rad)

f_{11} - lateral creep coefficient (lb, N)

f_{12} - spin/lateral creep coefficient (lb-ft, N-m)

f_{22} - spin creep coefficient (lb-ft^2, N-m^2)

f_{33} - longitudinal creep coefficient (lb, N)

F - force at wheel/rail contact (lb)

I_b - bolster moment of inertia about vertical axis (slugs-ft^2, kg-m^2)

I_f - sideframe moment of inertia about vertical axis (slugs-ft^2, kg-m^2)

I_w - wheelset moment of inertia about vertical axis (slugs-ft^2, kg-m^2)

K_W - warp stiffness (lb-ft/rad, N-m/rad)

K_θ - yaw stiffness (lb-ft/rad, N-m/rad)

l_1 - ½ truck wheel base (ft, m)

M - moment at wheel/rail contact (lb-ft, N-m)

M_{bear} - bearing moment (lb-ft, N-m)

M_{break} - bearing break away moment (lb-ft, N-m)

M_b - mass of bolster (slugs, kg)

M_f - mass of one sideframe (slugs, kg)

M_r - mass of restraining bar (slugs, kg)

M_w - mass of one wheelset (slugs, kg)

N - load per wheel (lb, N)

r - instantaneous wheel radius (ft, m)

R - instantaneous roller radius (ft, m)

R_c - rail crown radius (ft, m)

R_o - nominal rolling radius of wheel (ft, m)

R_r - nominal rolling radius of roller (ft, m)

V - nominal speed (ft/s, m/s)

W - load per wheelset (lb, N)

x_T - lateral displacement (ft, m)

$\dot{x}_T$ - lateral velocity (ft/s, m/s)

$\ddot{x}_T$ - lateral acceleration (ft/s^2, m/s^2)

δ - angle between contact plane and axle centerline (rad)

δ_o - wheel rail contact angle, wheelset centered (rad)

γ - creepage

λ - wheel conicity

μ - coefficient of friction

ϕ - wheelset roll angle with respect to horizontal plane (rad)

θ_T - truck yaw angle (rad)

$\dot{\theta}_T$ - truck yaw anglular velocity (rad/s)

$\ddot{\theta}_T$ - truck yaw anglular acceleration (rad/s^2)

θ_W - truck warp angle (rad)

$\dot{\theta}_W$ - truck warp anglular velocity (rad/s)

$\ddot{\theta}_W$ - truck warp anglular acceleration (rad/s^2)

Subscripts

1 - wheel one

2 - wheel two

3 - wheel three

4 - wheel four

12 - wheelset containing wheels one and two

34 - wheelset containing wheels three and four

la - lateral

lo - longitudinal

sp - spin

x - acting in the lateral direction

y - acting in the longitudinal direction

Chapter 1

Introduction and Literature Review

1.1 Introduction

Rail vehicle dynamics has been studied since the beginning of railroad operations. An 1829 study of the first five years of railroad operations stated that "continual rattling during the motion ... is principally produced by the fact that it is scarcely possible to retain the four points of the rails, on which the wheels of the locomotive rest, continually in one place..." (Garg and Dukkipati, 1984). This was one of the first observations leading to the vast amount of research on the complex vehicle dynamics resulting from the simple design of using flanged steel wheels on steel rails.

The purpose of this research was to study the lateral dynamics of a one-fifth scale three-piece freight truck running on a roller rig. Many roller rigs have been built over the past few years. Roller rigs obtain their name from the rollers that replace the railroad tracks. Roller rigs can simulate everything from a single wheel to a whole rail vehicle, such as a freight car, riding on rails. Scaled models and actual full size vehicles have been tested on roller rigs. A roller rig and the scale truck were built as part of this research for the Department of Mechanical Engineering at Virginia Polytechnic Institute and State University.

Initial condition response of the scale model and a phenomenon known as hunting were experimentally and analytically investigated. Hunting is the dynamic instability in which the rail vehicle naturally oscillates between the rails at a certain speed. A linear mathematical model was derived for comparison of the initial condition response with the experimental data. The linear model predicts the truck's natural frequency versus speed within 5% of the experimental values. It also predicts the damping ratio decrease that occurs with increasing truck speed. A nonlinear mathematical model was developed to simulate hunting on the roller rig. The nonlinear model incorporates a lookup table for the wheel/rail geometric constraints and a heuristic creep force model to take into account creep force saturation. The nonlinear model also includes the effects of Coulomb friction in the truck bearings. The nonlinear model simulates hunting and also exhibits small amplitude hunting observed on the roller rig.

1.2 Literature Review

1.2.1 Rail Vehicle Dynamics

Rail vehicle dynamics research has been a world-wide effort. Most of this research has taken place over the past thirty years and has resulted in a massive amount of literature. Law and Cooperrider (1974) presented a comprehensive survey of the analytical research of rail vehicles completed prior to 1974. Garg and Dukkipati's (1984) textbook brings together the major methods used to develop mathematical models of wheelsets, trucks, and whole vehicles.

Research sponsored by the Federal Railroad Administration and the Association of American Railroads resulted in further insight to rail vehicle dynamics. An analytical method for determination of the nonlinear wheel/rail geometric constraint relationships was developed by Cooperrider et al. (1975). Their work provided the basis for the wheel and rail geometry used in this thesis research. General analytical models were developed by Law et al. (1977) for the study of freight car hunting. This work includes the derivation of the equations of motion for a wheelset, an eleven-degree-of-freedom model, and a complete rail vehicle. Work done by Hedrick et al. (1978) provides a good description of rail vehicle nonlinearities. It also provides the quasi-linearization techniques to "combine the computational efficiency of linear analysis with the accuracy of numerical simulation."

Much of the research of rail vehicles has focused on lateral dynamics and the instability known as hunting. Wickens (1969) provided a good overview of the basic concepts of lateral dynamics. Even though Langer and Shamberger's (1935) paper was written over 50 years ago, it provides information on the fundamental concepts of hunting. Clark and Law (1972) gave insight on hunting in their research of hunting and high-speed train design. Matsudaira et al. (1969) studied the problems of hunting on a roller rig and the use of scale models.

Roller rigs provide one method to experimentally study rail vehicle dynamics. The following section discusses the literature describing a wide variety of roller rigs. Most of this literature also contains analytical studies on many aspects of rail vehicle dynamics.

1.2.2 Roller Rigs

Brickle (1973) used a single-roller rig to study the wheel/rail contact forces during flange contact. The rig had a straight conical profile steel wheel riding on a steel roller. The roller was driven by a 3/4 HP motor. Illingworth (1973) studied the effect of track irregularities by using a one-fifth scale roller rig that allowed lateral displacement of the rollers. The rig consisted of a single wheelset (two wheels and an axle) with aluminum wheels riding on aluminum rollers driven by a d.c. motor. Illingworth designed a new roller rig to more accurately simulate travel on real track. This design allowed the rollers to rotate about the vertical axis as well as being laterally displaced.

The German Federal Railways uses a roller rig to simulate trucks running on rails. It consists of a roller for each of the four wheels on a truck. It was used in 1982 and 1983 by Messerschmitt-Bolkow-Blohm to study a truck with a fiber composite frame and creep-controlled wheelsets (Geuenich, 1983). Henderson (1976) designed a roller rig to study a one-fifth scale truck. The rig had four rollers powered by a variable speed electric motor. Henderson's design closely resembles the roller rig used in this research.

Roller rigs do not always incorporate an individual roller for each wheel. The New Technologies Department of the Transport Research Institute in Arcueil-Cedex, France built a roller rig with a single 13-m (42.65 ft) diameter roller (Heliot, 1985). The rig was designed to test a one-quarter scale model of a Y25 freight car truck. Scaled curved rails with a track gauge of 0.36 m (1.18 ft) were attached to the roller. The rig was used to study similitude relations between full size and scaled trucks.

Both passenger and freight car dynamics have been studied on roller rigs. A one-fifth scale model of the Tokaido line vehicles was tested on a roller rig built by the Railway Technical Research Institute of the Japanese National Railways (Matsudaira, 1969). The roller rig had twelve rollers and could be used to test vehicles with two or three trucks. The German Railways repair workshop in Munich tests full size wheelsets, trucks, and rail vehicles (Hahn, 1986) on its roller rig. The roller rig has eight adjustable rollers, one for each wheel. The Association of American Railroads operates a roller rig called the Roll Dynamics Unit located in Pueblo, Colorado. It is used to test full size rail vehicles, including locomotives. A single rollerset is used for each wheelset on the vehicle (Haque, 1983).

The use of roller rigs has provided substantial insight into rail vehicle dynamics over the years. Roller rigs offer the capability to test vehicles in controlled laboratory conditions as an alternative to testing in the field.

1.3 Organization of the Thesis

Chapter 2 contains a description of the roller rig. The development and solution methods of the equations of motion are presented in Chapter 3. Chapter 4 documents the experimental investigation. The analytical and experimental results are compared in Chapter 5. Chapter 6 concludes the report.

Chapter 2

Roller-Rig Description

2.1 Introduction

The roller rig is a rail vehicle simulator. It is composed of two one-fifth scale models, the truck and the rollers. A truck is the running gear found under each end of a freight car. The railroad tracks or rails are simulated by the rollers. The roller rig simulates a truck travelling on the rails. It can be used to study rail vehicle dynamics and wheel/rail contact wear.

Design of the roller rig began as a senior Mechanical Engineering design project at Virginia Polytechnic Institute and State University (Elliot et al., 1987). Modifications were made to the roller rig design as part of this thesis work, and it was built in the summer of 1988. Figure 1 shows the roller rig.

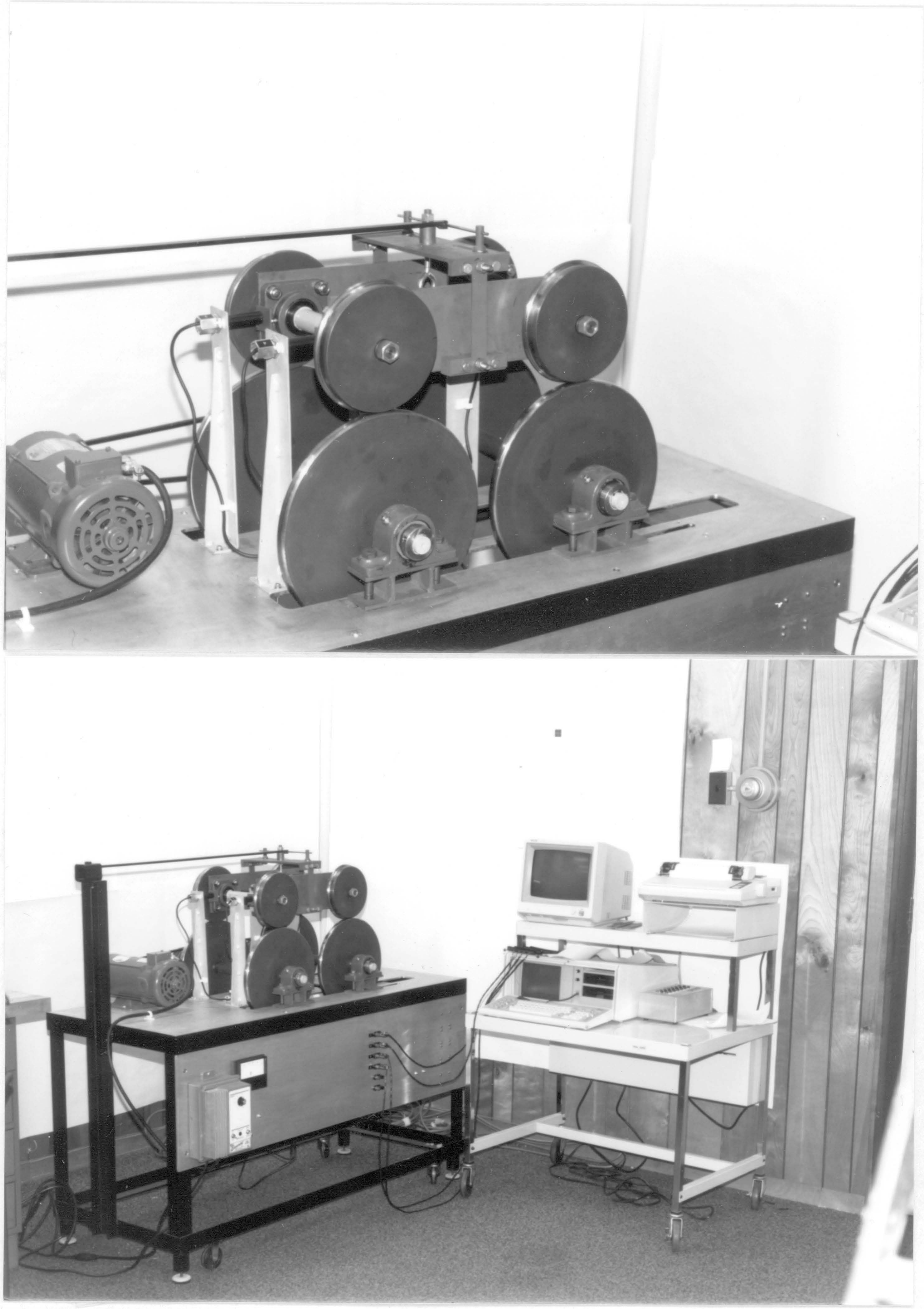


Figure 1. The Roller Rig

2.2 Description

The truck is a one-fifth scale model of a typical North American three-piece freight truck. The truck consists of two wheelsets (two wheels and an axle), two sideframes, a bolster, and mechanisms to vary warp and yaw stiffnesses. Figure 2 shows the truck with its individual parts labeled. A restraining bar connected to the bolster restricts the truck's longitudinal movement. The wheels have a 7.2-in. (182.88-mm) nominal diameter and an AAR Wide Flange Contour (Bethlehem Contour No. G-241) profile. The truck has a 14-in. (355.6-mm) wheelbase. Torsion bars connected to the sideframes and bolster provide warp stiffness. A cantilevered 3/16-in. (4.76-mm) diameter steel rod connected to the restraining bar and the bolster provides yaw stiffness. These stiffnesses are discussed further in the Roller Rig Setup section of Chapter 4. A loaded truck may be simulated by hanging weights from the hook attached to the bottom of the bolster.

The rollers model the rails. The roller profile matches the profile of a 132 RE rail on a 1:40 canted tie plate. Tapered roller bearings support the shafts of each rollerset. The 13-in. (330.2-mm) diameter rollers are set 12 in. (304.8 mm) apart on the shafts to simulate the gage of the track. A 3/4 horsepower d.c. motor drives the rollers through a system of sheaves and belts. The bearings and motor are mounted to a steel frame table. Casters provide portability for the table. Levelers allow the table to be raised off the casters and leveled before experiments are conducted.

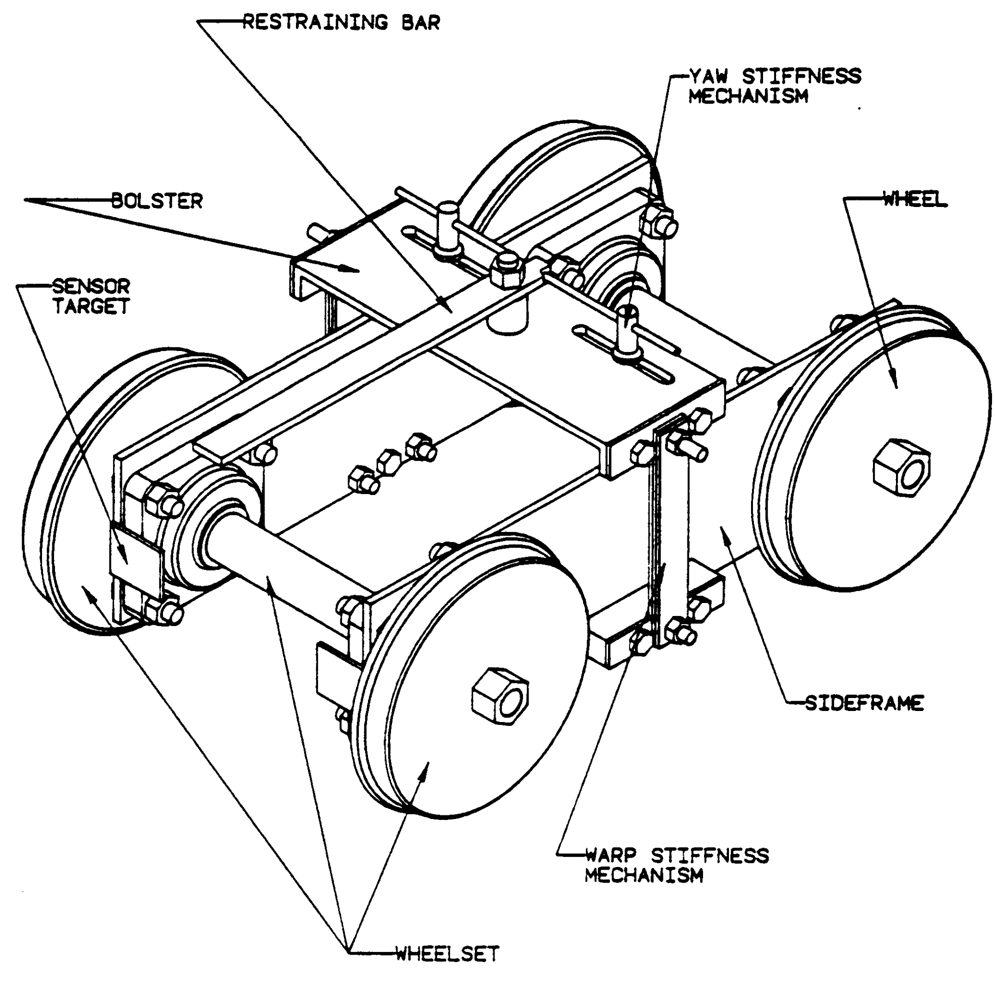


Figure 2. The One-fifth Scale Three Piece Truck

2.3 Instrumentation

Experimental analysis of the roller rig required the measurement of lateral displacement, yaw angle, warp angle, and truck speed. Lateral displacement and yaw and warp angles were calculated from the displacements measured at four points on the truck. These displacements were measured by Electro Corporation's Electro-Mike displacement transducers (model no. PA12D03). These sensors produce a radio frequency electromagnetic field between the sensor and the metal target. The change in the energy of these fields is proportional to the displacement of the target. The targets in this case were four points on the truck's sideframes. The lateral displacement of two points equal distance from the truck's center on a sideframe and the longitudinal displacement of each sideframe were measured.

Rotational speed of the rollers was measured by using a 60-tooth gear attached to one rollerset and an Electro Corporation Di-Mag magnetic pickup (model no. 58423). The pickup produces a signal when a gear tooth passes by. These signals were converted to the rotational speed of the rollers.

2.4 Solid Modeling

Design and analysis of the roller rig was facilitaled by the use of computer solid modeling. The solid modeler used was the CADAM Interactive Solids Design (CADAM Inc., 1987). Parts were modeled and assembled on the computer to guarantee a proper fit before the parts were fabricated. The masses and moments of inertia of the truck and its individual components were calculated by the solid modeler. These values compared favorably with hand calculated and measured values. Table 1 shows these values. The dynamic analysis required values of the inertial parameters, and these values are typically time-consuming to obtain. Use of the solid modeling methods proved to be very beneficial in obtaining the required numerical values.

Table 1. Truck Mass and Moment-of-Inertia Properties

Variable	Numerical Value
I_b	0.0096 slug-ft^2 (0.013 kg-m^2)
I_f	0.122 slug-ft^2 (0.165 kg-m^2)
I_w	0.228 slug-ft^2 (0.309 kg-m^2)
M_b	0.111 slugs (1.62 kg)
M_r	0.103 slugs (1.50 kg)
M_f	0.458 slugs (6.68 kg)
M_w	0.958 slugs (13.98 kg)
Total Truck Mass	3.22 slugs (46.98 kg)

Chapter 3

Dynamic Analysis

3.1 Introduction

This chapter covers the derivation and solution methods of the three-degree-of-freedom nonlinear and linear models of the three-piece truck used on the roller rig. The three degrees of freedom, lateral displacement (x_T), yaw angle (θ_T), and warp angle (θ_W) are shown in Figure 3. Rail vehicle dynamics is governed by the wheel and rail geometries, contact forces, and the interaction of the vehicle components. The first section of this chapter discusses the wheel and roller constraints. This is followed by a discussion of the creep forces. This chapter presents the equations of motion for both the linear and nonlinear roller rig models. Runge-Kutta numerical integration methods are used to solve these equations. In addition, eigenvalue/eigenvector analysis of the linear model is performed.

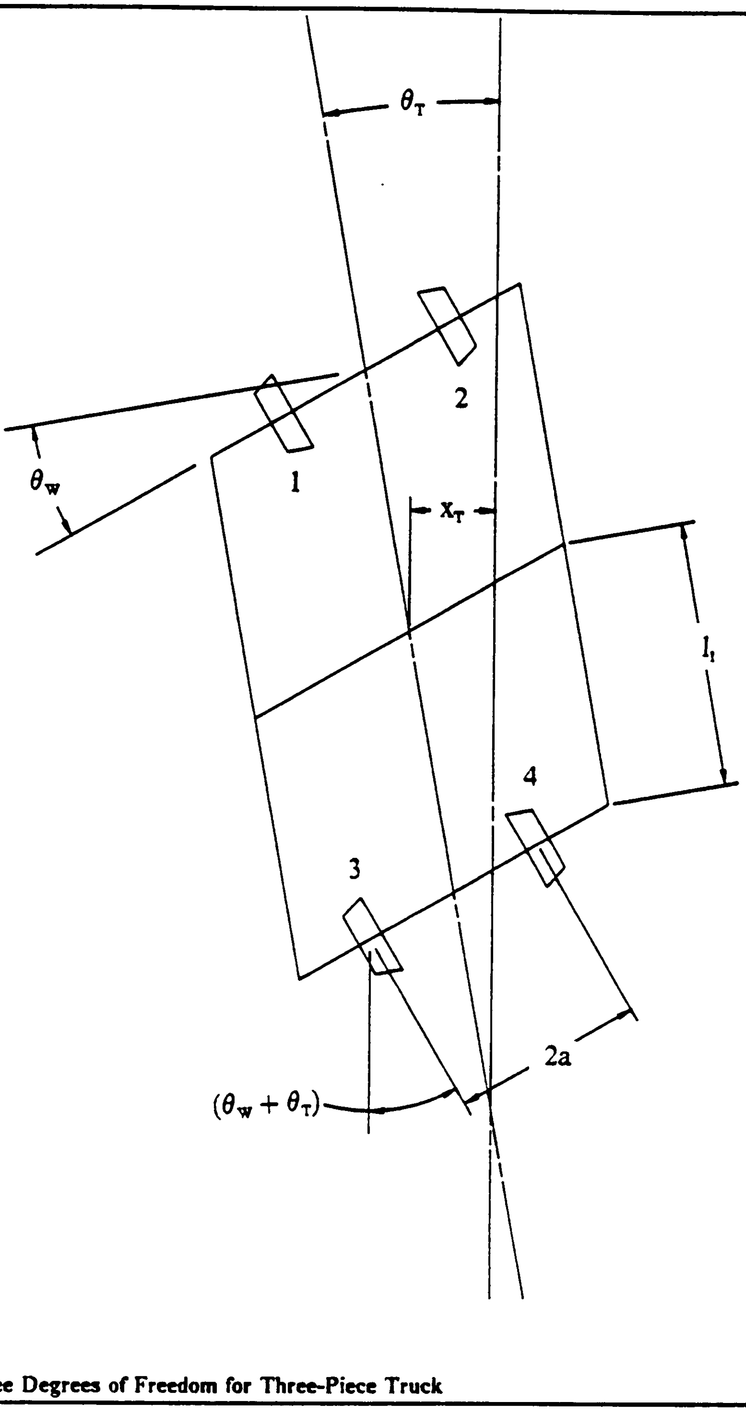


Figure 3. Three Degrees of Freedom for Three-Piece Truck

3.2 Wheel/Roller Constraint Characterization

In rail vehicle dynamic studies, it is necessary to know the radius of each wheel and roller at the contact point with the rail. The wheel/roller contact point moves from the tread region to a position where the roller contacts both the tread and flange, and then just the flange as shown in Figure 4. The profiles used for wheels and rails result in nonlinear functions for the geometric constraint relationships.

For the nolinear model, a method to calculate the wheel and roller radii for each wheelset at different lateral positions was needed. This was accomplished by the use of a FORTRAN progran called WHRAILA1 written by Cooperrider et al. (1975). WHRAILA1 was used to develop a lookup table for a full-size wheelset on rails. This table is included in Appendix C. Two dimensional data about the wheel and rail profiles, and data for the wheel and rail gauges was used by the program to calculate the left and right wheel radii, rail heights, contact angles and wheelset roll angle within a specified range of wheelset lateral displacement. A graphical example of WHRAILA1 output is shown in Figure 5.

A cone wheel and a roller represented by a cylinder with a crown radius, as shown in Figure 6, was used for the linear model. Using these types of wheels and rollers linearize the geometric constraint relationships. The resulting equations for the wheel and roller radii are:

$$r_1 = R_o + \lambda(x_T + l_1\theta_T) \tag{3.2.1}$$

$$r_2 = R_o - \lambda(x_T + l_1\theta_T) \tag{3.2.2}$$

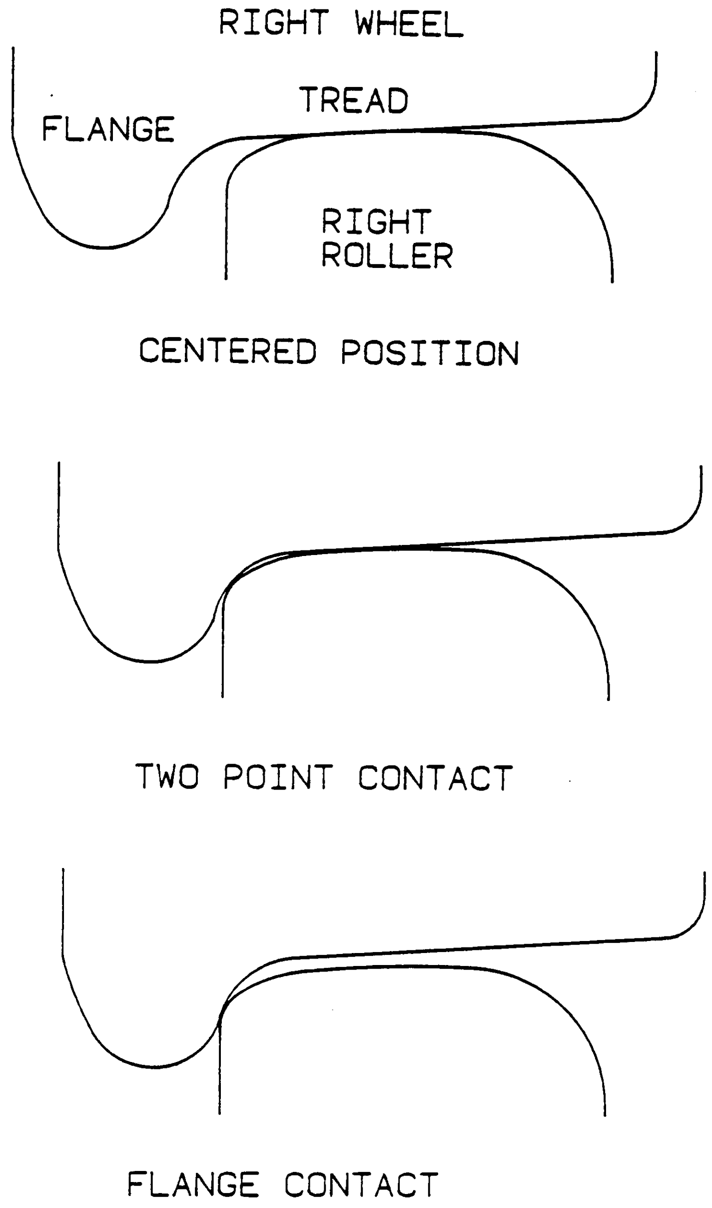


Figure 4. Wheel and Roller Contact Points

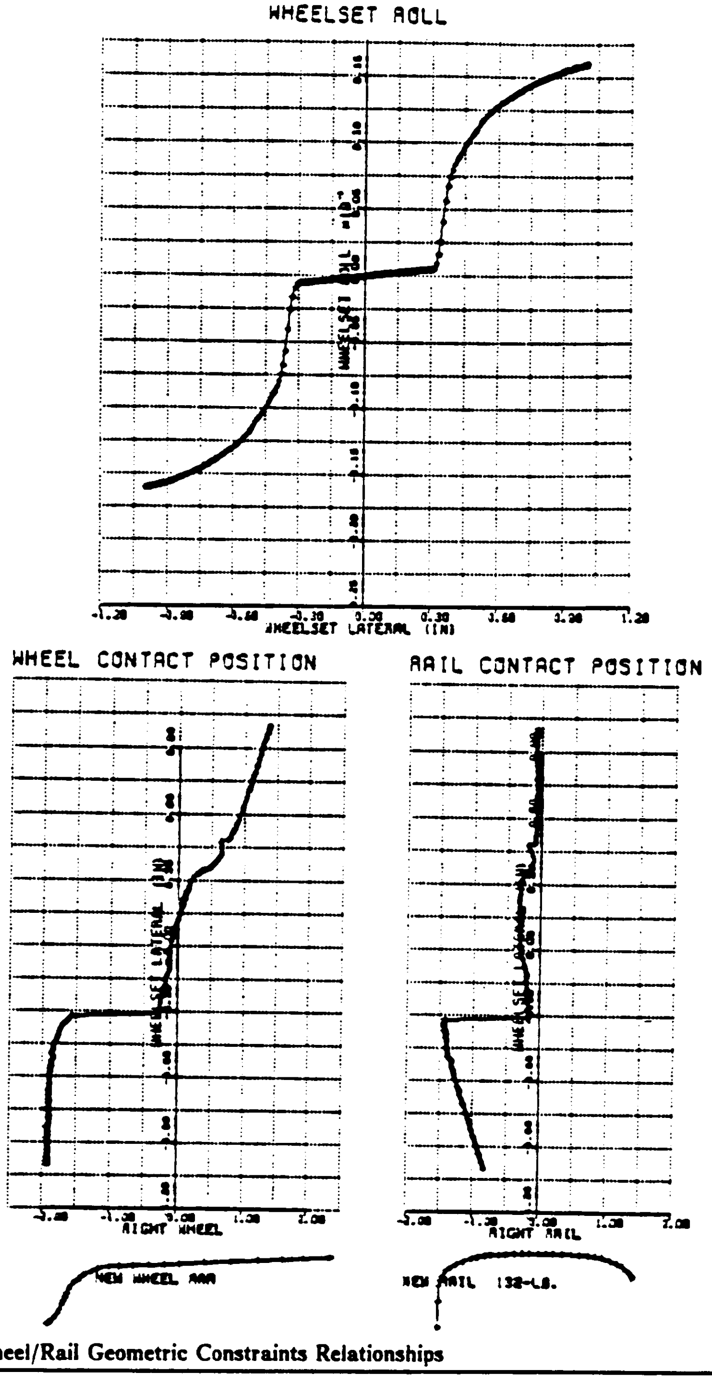


Figure 5. Wheel/Rail Geometric Constraints Relationships

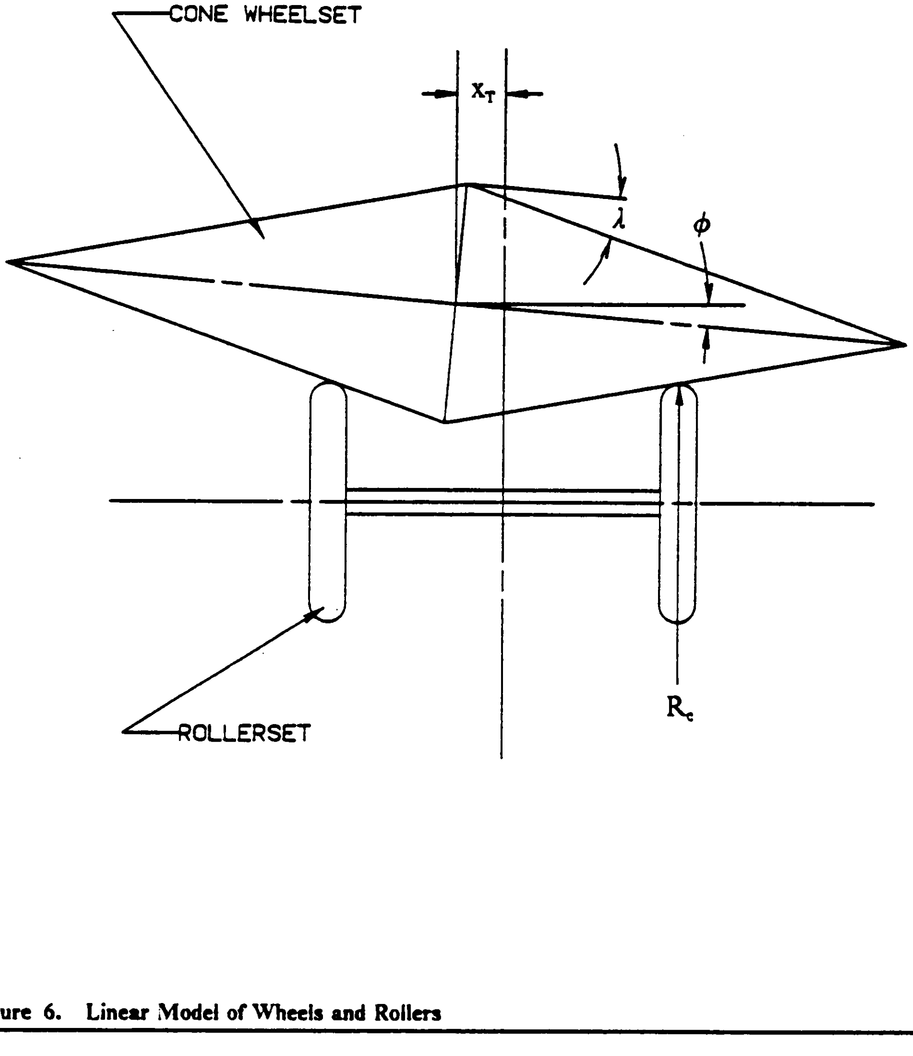


Figure 6. Linear Model of Wheels and Rollers

$$r_3 = R_o + \lambda(x_T - l_1\theta_T) \tag{3.2.3}$$

$$r_4 = R_o - \lambda(x_T - l_1\theta_T) \tag{3.2.4}$$

$$R_1 = R_r + R_c\frac{\lambda^2}{a}(x_T + l_1\theta_T) \tag{3.2.5}$$

$$R_2 = R_r - R_c\frac{\lambda^2}{a}(x_T + l_1\theta_T) \tag{3.2.6}$$

$$R_3 = R_r + R_c\frac{\lambda^2}{a}(x_T - l_1\theta_T) \tag{3.2.7}$$

$$R_4 = R_r - R_c\frac{\lambda^2}{a}(x_T - l_1\theta_T) \tag{3.2.8}$$

These equations describe the wheel and roller radii for use in the linear model's equations of motion.

3.3 Creep Force Determination

The literature is rich with the work done on the rolling contact of two bodies and the determination of creep forces. Garg and Dukkipati (1984) gave a comprehensive overview of the rolling contact theories and the calculation of creep forces. This thesis research was guided by the creep force analysis done by Davila (1986) for a vehicle on rails. This thesis research extends this work to consider a vehicle on a roller rig.

When a wheelset is laterally displaced, slip, or creepage, results between the wheel and rail (roller). Creepage is defined as the rate of sliding between the wheel and rail divided by the forward velocity of the wheel. Figure 7 (Garg and Dukkipati, 1984) shows the creepages and the resulting creep forces in the wheel/rail contact patch. By

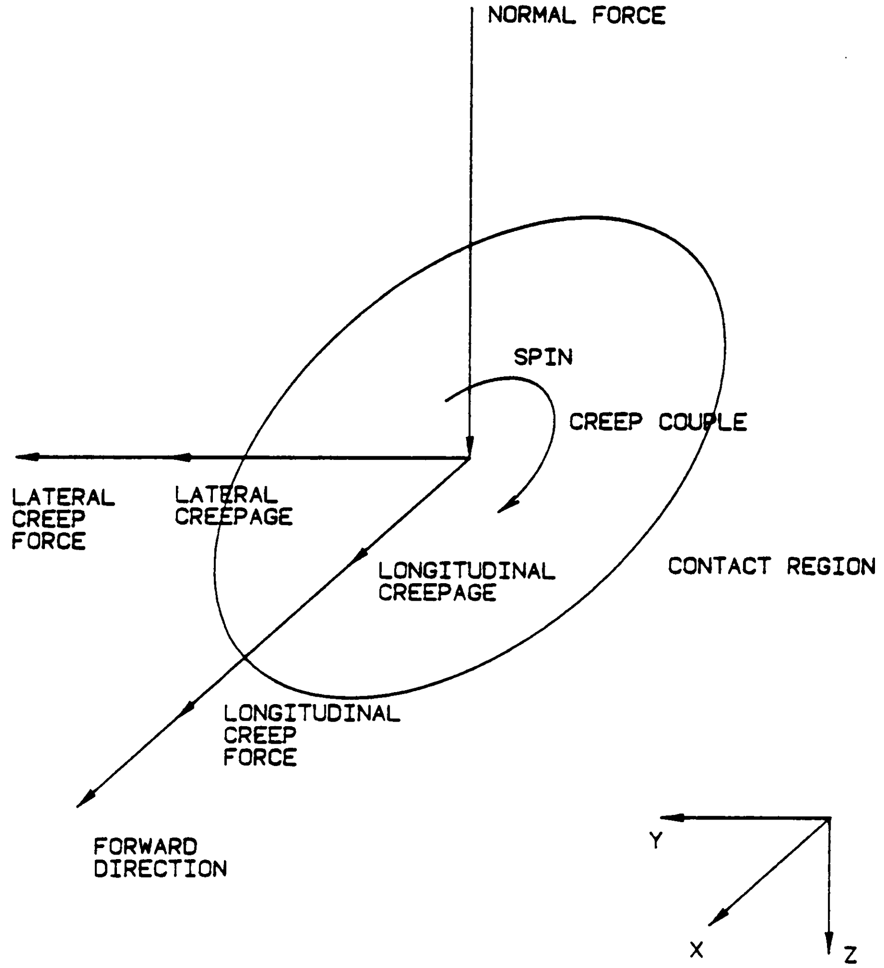


Figure 7. Creep Forces and Creepages

using the definition of creepage, the nonlinear equations for the creepages of each wheel were derived. The longitudinal creepage equations are:

$$\gamma_{lo1} = \frac{-R_1}{R_r} + \frac{r_1}{R_o} + \frac{a}{V}(\dot{\theta}_W + \dot{\theta}_T) \tag{3.3.1}$$

$$\gamma_{lo2} = \frac{-R_2}{R_r} + \frac{r_2}{R_o} - \frac{a}{V}(\dot{\theta}_W + \dot{\theta}_T) \tag{3.3.2}$$

$$\gamma_{lo3} = \frac{-R_3}{R_r} + \frac{r_3}{R_o} + \frac{a}{V}(\dot{\theta}_W + \dot{\theta}_T) \tag{3.3.3}$$

$$\gamma_{lo4} = \frac{-R_4}{R_r} + \frac{r_4}{R_o} - \frac{a}{V}(\dot{\theta}_W + \dot{\theta}_T) \tag{3.3.4}$$

The lateral creepage equations are:

$$\gamma_{la1} = \gamma_{la2} = \frac{\dot{x}_T}{V} - \theta_W - \theta_T + \frac{l_1}{V}\dot{\theta}_T \tag{3.3.5}$$

$$\gamma_{la3} = \gamma_{la4} = \frac{\dot{x}_T}{V} - \theta_W - \theta_T - \frac{l_1}{V}\dot{\theta}_T \tag{3.3.6}$$

The spin creepage equations are:

$$\gamma_{sp1} = -\sin\frac{(\delta_1 + \phi_{12})}{R_r} + \frac{(\dot{\theta}_W + \dot{\theta}_T)}{V} - \sin\frac{(\delta_1)}{R_o} \tag{3.3.7}$$

$$\gamma_{sp2} = \sin\frac{(\delta_2 - \phi_{12})}{R_r} + \frac{(\dot{\theta}_W + \dot{\theta}_T)}{V} + \sin\frac{(\delta_2)}{R_o} \tag{3.3.8}$$

$$\gamma_{sp3} = -\sin\frac{(\delta_3 + \phi_{34})}{R_r} + \frac{(\dot{\theta}_W + \dot{\theta}_T)}{V} - \sin\frac{(\delta_3)}{R_o} \tag{3.3.9}$$

$$\gamma_{sp4} = \sin\frac{(\delta_4 - \phi_{34})}{R_r} + \frac{(\dot{\theta}_W + \dot{\theta}_T)}{V} + \sin\frac{(\delta_4)}{R_o} \tag{3.3.10}$$

The creepage equations for the linear model are:

$$\gamma_{lo1} = -\gamma_{lo2} = \left(\frac{\lambda}{R_o} - \frac{R_c \lambda^2}{R_r a} \right)(x_T + l_1 \theta_T) + \frac{a}{V}(\dot{\theta}_W + \dot{\theta}_T) \quad (3.3.11)$$

$$\gamma_{lo3} = -\gamma_{lo4} = \left(\frac{\lambda}{R_o} - \frac{R_c \lambda^2}{R_r a} \right)(x_T - l_1 \theta_T) + \frac{a}{V}(\dot{\theta}_W + \dot{\theta}_T) \quad (3.3.12)$$

$$\gamma_{la1} = \gamma_{la2} = \frac{\dot{x}_T}{V} - \theta_W - \theta_T + \frac{l_1}{V} \dot{\theta}_T \quad (3.3.13)$$

$$\gamma_{la3} = \gamma_{la4} = \frac{\dot{x}_T}{V} - \theta_W - \theta_T - \frac{l_1}{V} \dot{\theta}_T \quad (3.3.14)$$

$$\gamma_{sp1} = \frac{\lambda}{R_r} \left(-1 - \frac{x_T}{a} - \frac{l_1}{a} \theta_T \right) + \frac{(\dot{\theta}_W + \dot{\theta}_T)}{V} - \frac{\lambda}{R_o} \quad (3.3.15)$$

$$\gamma_{sp2} = \frac{\lambda}{R_r} \left(1 - \frac{x_T}{a} - \frac{l_1}{a} \theta_T \right) + \frac{(\dot{\theta}_W + \dot{\theta}_T)}{V} + \frac{\lambda}{R_o} \quad (3.3.16)$$

$$\gamma_{sp3} = \frac{\lambda}{R_r} \left(-1 - \frac{x_T}{a} + \frac{l_1}{a} \theta_T \right) + \frac{(\dot{\theta}_W + \dot{\theta}_T)}{V} - \frac{\lambda}{R_o} \quad (3.3.17)$$

$$\gamma_{sp4} = \frac{\lambda}{R_r} \left(1 - \frac{x_T}{a} + \frac{l_1}{a} \theta_T \right) + \frac{(\dot{\theta}_W + \dot{\theta}_T)}{V} + \frac{\lambda}{R_o} \quad (3.3.18)$$

Kalker's linear creep theory (Garg and Dukkipati, 1984) shows the creep forces and moments to be:

$$F_{la} = -f_{11} \gamma_{la} - f_{12} \gamma_{sp} \quad (3.3.19)$$

$$F_{lo} = -f_{33} \gamma_{lo} \quad (3.3.20)$$

$$M_{sp} = f_{12} \gamma_{la} - f_{22} \gamma_{sp} \quad (3.3.21)$$

where f_{11}, f_{12}, f_{22}, and f_{33} are the creep coefficients. The calculation of the creep coefficients may be found in Garg and Dukkipati's (1984) book. For the nonlinear model, a heuristic nonlinear creep force model using a saturation coefficient, ρ, was used. This resulted in using the same creep forces and moment equations stated above, but the creep coefficients were multiplied by ρ. A discussion of the heuristic model and the calculation of ρ may be found in the work of Davila (1986) or Shen et al. (1983). The creep coefficients and ρ were calculated by parts of the FORTRAN program FAST1 (Fries, 1987). The linear model used FAST1 to calculate the creep coefficients once and then they were assumed to remain constant.

3.4 Equations of Motion

Derivation of the differential equations of motion was accomplished through the use of Newtonian and Lagrangian methods. The truck's components were assumed to be rigid. Warp and yaw suspension components were represented by torsion springs.

Figure 8 shows the contact forces and moments on a single wheelset. The resultant lateral force exerted by the rollers on the wheels is:

$$\begin{aligned} F_x = {} & N[\tan(\delta_2 - \phi_{12}) - \tan(\delta_1 + \phi_{12}) + \tan(\delta_4 - \phi_{34}) - \tan(\delta_3 + \phi_{34})] \\ & + F_{la1}\cos(\delta_1 + \phi_{12}) + F_{la2}\cos(\delta_2 - \phi_{12}) + F_{la3}\cos(\delta_3 + \phi_{34}) \\ & + F_{la4}\cos(\delta_4 - \phi_{34}) + (F_{lo1} + F_{lo2})\sin\phi_{12} + (F_{lo3} + F_{lo4})\sin\phi_{34} \end{aligned} \tag{3.4.1}$$

The resultant moment on the wheelsets is:

$$\begin{aligned} M = {} & a \bullet \sin(\theta_W + \theta_T)\beta + Na^2\sin(\theta_W + \theta_T)\eta + M_{sp1}\cos(\delta_1 + \phi_{12}) \\ & + M_{sp2}\cos(\delta_2 - \phi_{12}) + M_{sp3}\cos(\delta_3 + \phi_{34}) + M_{sp4}\cos(\delta_4 - \phi_{34}) \\ & + a(F_{lo2} + F_{lo4} - F_{lo1} - F_{lo3}) \end{aligned} \tag{3.4.2}$$

where

$$\begin{aligned} \beta = {} & F_{la2}\cos(\delta_2 - \phi_{12}) - F_{la1}\cos(\delta_1 + \phi_{12}) + F_{la4}\cos(\delta_4 - \phi_{34}) - F_{la3}\cos(\delta_3 + \phi_{34}) \\ & + N[\tan(\delta_1 + \phi_{12}) + \tan(\delta_2 - \phi_{12}) + \tan(\delta_3 + \phi_{34}) + \tan(\delta_4 - \phi_{34})] \end{aligned} \tag{3.4.3}$$

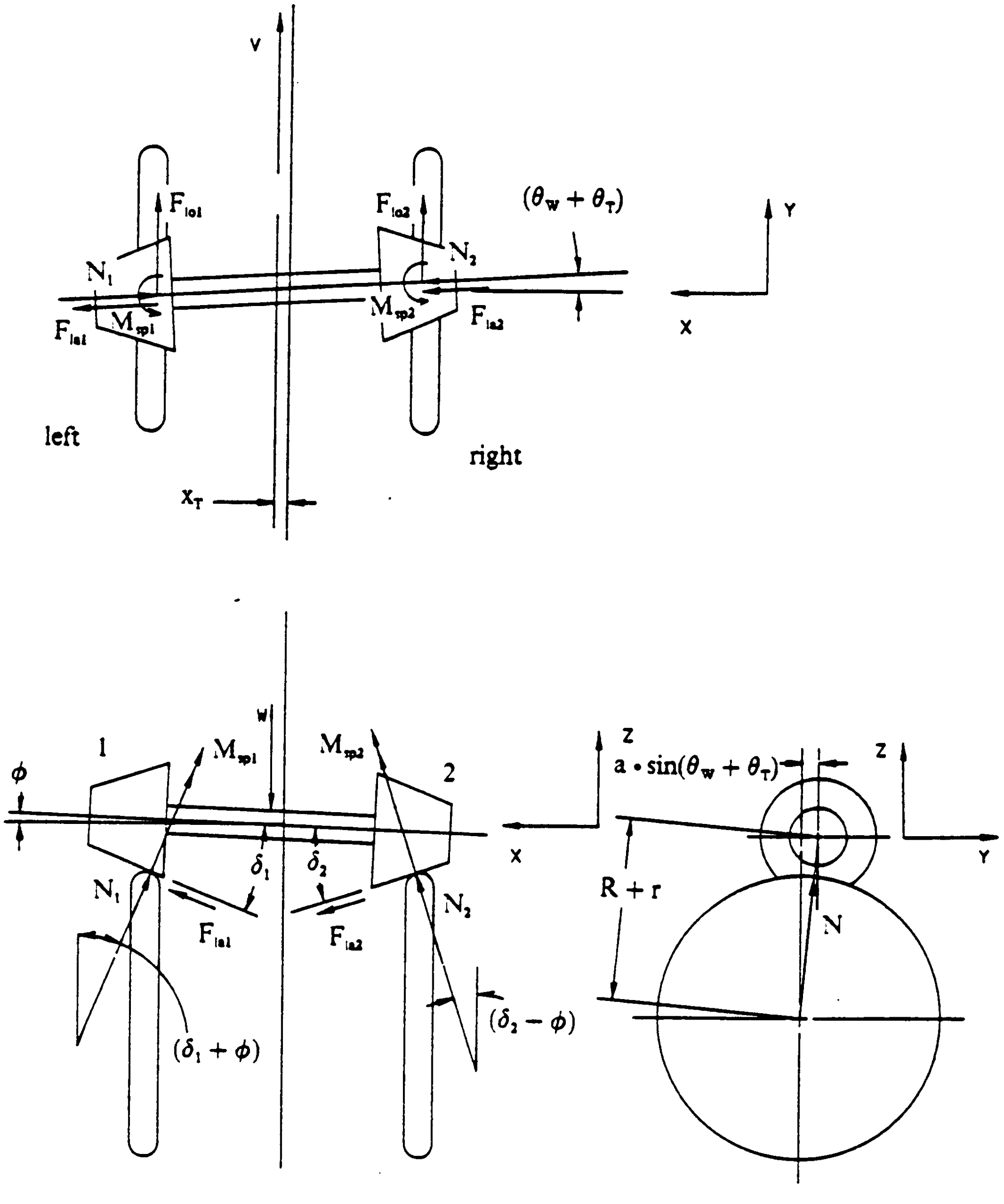


Figure 8. Forces and Moments on a Wheelset

$$\eta = \frac{1}{r_1 + R_1} + \frac{1}{r_2 + R_2} + \frac{1}{r_3 + R_3} + \frac{1}{r_4 + R_4} \qquad (3.4.4)$$

The dynamics of the truck are also affected by the Coulomb friction in the spherical bearings. This friction creates a moment in each bearing that resists the warping of the truck. The model used to describe the Coulomb friction is the same one used by Davila (1986) to model the friction between the car body and truck bolster in his five-degree-of-freedom model of a freight car. The Coulomb friction is represented by the slider model shown in Figure 9. The characteristic curve of Coulomb friction and the curve representing the slider model are shown in Figure 10 (Wang, 1988). The algorithm used to calculate the bearing moment is (Davila, 1986):

1. Calculate $M_{bear} = | K_{bear}(\alpha_i - \theta_{Wi+1}) |$
2. If $M_{bear} < M_{break}$ then $\alpha_{i+1} = \alpha_i$

   If $M_{bear} \geq M_{break}$ then $\alpha_{i+1} = \theta_{Wi+1} - \text{sign}(\dot{\theta}_{Wi}) \dfrac{M_{break}}{K_{bear}}$
3. $M_{bear} = K_{bear}(\alpha_{i+1} - \theta_{Wi+1})$

where α is the position of the rotational spring with stiffness K_{bear}. Heller et al. (1977) recommend K_{bear} be 100 times the value of the stiffness coefficient in the warp equation of motion.

The equations of motion were formulated by combining the results of the kinetic and potential energy, wheel/roller contact forces, and the bearing friction analyses. The differential equations of motion for the nonlinear model are:

Nonlinear Lateral Equation of Motion

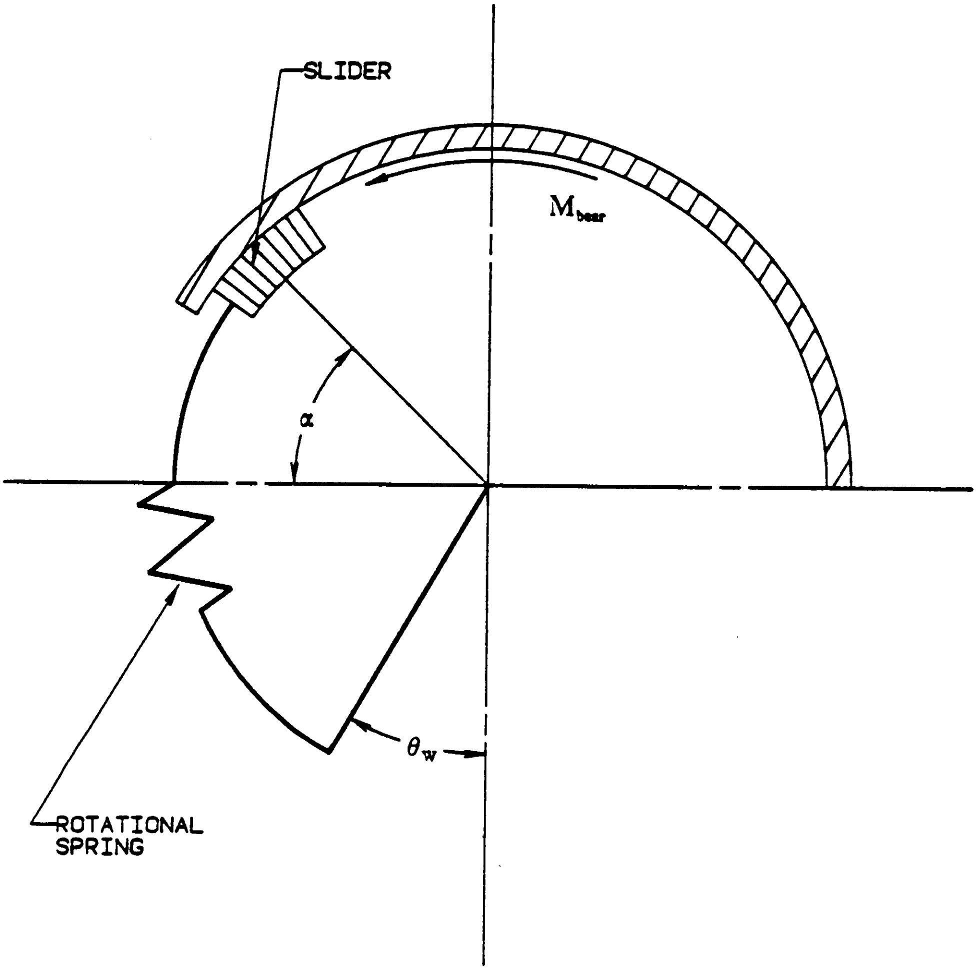


Figure 9. Slider Model of Coulomb Friction

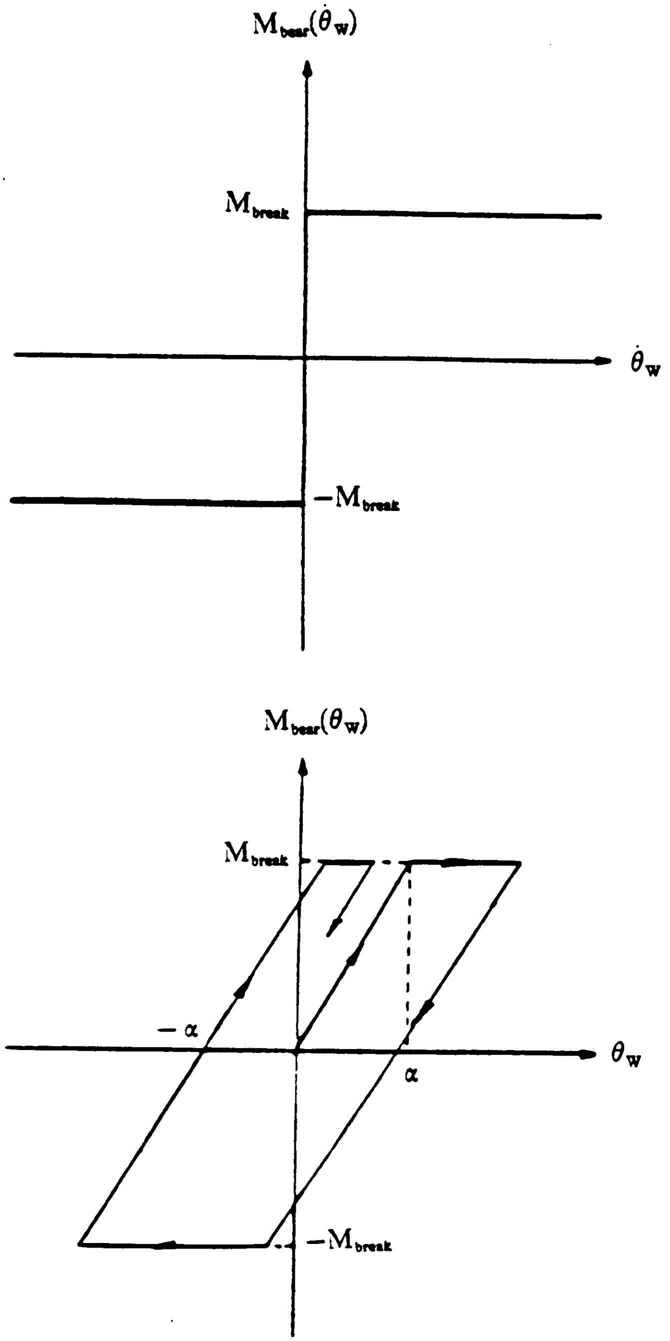


Figure 10. Ideal and Slider Model Characteristic Curves for Coulomb Friction

$$\begin{aligned}
&(2M_w + M_b + \frac{1}{3}M_r + 2M_f)\ddot{x}_T = \\
&\quad F_{la1}\cos(\delta_1 + \phi_{12}) + F_{la2}\cos(\delta_2 - \phi_{12}) + F_{la3}\cos(\delta_3 + \phi_{34}) + \\
&\quad F_{la4}\cos(\delta_4 - \phi_{34}) + (F_{lo1} + F_{lo2})\sin\phi_{12} + (F_{lo3} + F_{lo4})\sin\phi_{34} \\
&\quad + N\{\tan(\delta_2 - \phi_{12}) - \tan(\delta_1 + \phi_{12}) + \tan(\delta_4 - \phi_{34}) - \tan(\delta_3 + \phi_{34})\}
\end{aligned} \tag{3.4.5}$$

Nonlinear Yaw Equation of Motion

$$\begin{aligned}
&(2I_w + 2I_f + 2M_w l_1^2 + 2M_f d^2 + I_b)\ddot{\theta}_T + D_\theta\dot{\theta}_T + K_\theta\theta_T \\
&\quad + (2I_w + 2M_f d^2 + I_b)\ddot{\theta}_W + D_\theta\dot{\theta}_W + K_\theta\theta_W = \\
&M_{sp1}\cos(\delta_1 + \phi_{12}) + M_{sp2}\cos(\delta_2 - \phi_{12}) + M_{sp3}\cos(\delta_3 + \phi_{34}) + M_{sp4}\cos(\delta_4 - \phi_{34}) \\
&\quad + a \bullet \sin(\theta_W + \theta_T)\beta + Na^2\sin(\theta_W + \theta_T)\eta - 4M_{bear} + \\
&l_1 N[\tan(\delta_2 - \phi_{12}) - \tan(\delta_1 + \phi_{12}) - \tan(\delta_4 - \phi_{34}) + \tan(\delta_3 + \phi_{34})] + \\
&l_1[F_{la1}\cos(\delta_1 + \phi_{12}) + F_{la2}\cos(\delta_2 - \phi_{12}) - F_{la3}\cos(\delta_3 + \phi_{34}) - F_{la4}\cos(\delta_4 - \phi_{34})] \\
&\quad + l_1[(F_{lo1} + F_{lo2})\sin\phi_{12} - (F_{lo3} + F_{lo4})\sin\phi_{34}] + \\
&a(F_{lo2} + F_{lo4} - F_{lo1} - F_{lo3})
\end{aligned} \tag{3.4.6}$$

Nonlinear Warp Equation of Motion

$$\begin{aligned}
&(2I_w + 2M_f d^2 + I_b)\ddot{\theta}_W + (D_\theta + D_W)\dot{\theta}_W + (K_\theta + K_W)\theta_W + \\
&\quad (2I_w + 2M_f d^2 + I_b)\ddot{\theta}_T + D_\theta\dot{\theta}_T + K_\theta\theta_T = \\
&M_{sp1}\cos(\delta_1 + \phi_{12}) + M_{sp2}\cos(\delta_2 - \phi_{12}) + M_{sp3}\cos(\delta_3 + \phi_{34}) + \\
&M_{sp4}\cos(\delta_4 - \phi_{34}) + a \bullet \sin(\theta_W + \theta_T)\beta + Na^2\sin(\theta_W + \theta_T)\eta - 4M_{bear} + \\
&a(F_{lo2} + F_{lo4} - F_{lo1} - F_{lo3})
\end{aligned} \tag{3.4.7}$$

The differential equations of motion for the linear model are formulated in the same way as those for the nonlinear model with the addition of a few linearizing assumptions. The assumptions are:

1. The contact angle between each wheel and roller remains constant.

2. The spherical bearings have no Coulomb friction.

3. All angular displacements are small.

4. There is no creep force saturation.

The following are the differential equations of motion for the linear model.

Linear Lateral Equation of Motion

$$\begin{aligned}&(2M_w + M_b + \frac{1}{3}M_r + 2M_f)\ddot{x}_T + \left(\frac{4f_{11}}{V}\right)\dot{x}_T + \left(\frac{4N\lambda}{a} - \frac{4\lambda f_{12}}{R_r a}\right)x_T \\ &+ \left(\frac{4f_{12}}{V}\right)\dot{\theta}_T - (4f_{11})\theta_T + \left(\frac{4f_{12}}{V}\right)\dot{\theta}_W - (4f_{11})\theta_W = 0\end{aligned} \qquad (3.4.8)$$

Linear Yaw Equation of Motion

$$\begin{aligned}&(2I_w + 2I_f + 2M_w l_1^2 + 2M_f d^2 + I_b)\ddot{\theta}_T + \left(D_\theta + \frac{4l_1^2 f_{11}}{V} + \frac{4a^2 f_{33}}{V} + \frac{4f_{22}}{V}\right)\dot{\theta}_T \\ &+ \left(K_\theta - \frac{4\lambda l_1^2 f_{12}}{R_r a} + 4f_{12} - 4a\lambda N - \frac{4Na^2}{R_o + R_r} + \frac{4l_1^2 N\lambda}{a} - \frac{4\lambda l_1^2 f_{12}}{R_o a}\right)\theta_T \\ &+ (2I_w + 2M_f d^2 + I_b)\ddot{\theta}_W + \left(D_\theta + \frac{4f_{22}}{V} + \frac{4a^2 f_{33}}{V}\right)\dot{\theta}_W \\ &+ \left(K_\theta + 4f_{12} - 4a\lambda N - \frac{4Na^2}{R_o + R_r}\right)\theta_W - \left(\frac{4f_{12}}{V}\right)\dot{x}_T \\ &+ \left(\frac{4\lambda a f_{33}}{R_o} - \frac{4\lambda^2 R_c f_{33}}{R_r} - \frac{4\lambda f_{22}}{R_r a}\right)x_T = 0\end{aligned} \qquad (3.4.9)$$

Linear Warp Equation of Motion

$$\begin{aligned}
&(2I_w + 2M_fd^2 + I_b)\ddot{\theta}_W + \left(D_W + D_\theta + \frac{4f_{22}}{V} + \frac{4a^2f_{33}}{V}\right)\dot{\theta}_W \\
&\quad + \left(K_W + K_\theta + 4f_{12} - 4a\lambda N - \frac{4Na^2}{R_o + R_r}\right)\theta_W \\
&\quad + (2I_w + 2M_fd^2 + I_b)\ddot{\theta}_T + \left(D_\theta + \frac{4f_{22}}{V} + \frac{4a^2f_{33}}{V}\right)\dot{\theta}_T \\
&\quad + \left(K_\theta + 4f_{12} - 4a\lambda N - \frac{4Na^2}{R_o + R_r}\right)\theta_T - \left(\frac{4f_{12}}{V}\right)\dot{x}_T \\
&\quad + \left(\frac{4\lambda af_{33}}{R_o} - \frac{4\lambda^2R_cf_{33}}{R_r} - \frac{4\lambda f_{22}}{R_ra}\right)x_T = 0
\end{aligned} \tag{3.4.10}$$

3.5 Linear and Nonlinear Solution Methods

The first step in the solution method for both the linear and nonlinear models was to convert the equations of motion into six first-order differential equations. This results in the matrix equation

$$[\dot{Q}] = [A][Q] + [B] \tag{3.5.1}$$

where

$$q(1) = x_T \tag{3.5.2}$$

$$q(2) = \dot{x}_T \tag{3.5.3}$$

$$q(3) = \theta_T \tag{3.5.4}$$

$$q(4) = \dot{\theta}_T \tag{3.5.5}$$

$$q(5) = \theta_W \tag{3.5.6}$$

$$q(6) = \dot{\theta}_W \tag{3.5.7}$$

and [A] contains mass, damping, and stiffness terms and [B] contains right side of the equation terms. Two FORTRAN programs, LINMOD and NLINMOD, solve the differential equations of the linear and nonlinear models, respectively. Each program uses a fourth-order Runge-Kutta method (Chapra et al., 1985) to solve the differential equations.

LINMOD first calculates the A matrix given in the above equation (B matrix is zero for the linear model). An IMSL subroutine, which uses the IMSL subroutines EVCRG, UMACH, and WRCRN (IMSL, 1987), calculates the eigenvalues and eigenvectors for the A matrix. The differential equations are than solved by the Runge-Kutta method. Lateral displacement, yaw angle, and warp angle versus time output is filed in three seperate files. LINMOD may be found in Appendix A.

NLINMOD begins by setting up an integration loop for the Runge-Kutta method. At each time step, a lookup table is called to find the values for the wheel/roller geometric constraints. Next, the creep forces and moments at each wheel are computed with the heuristic creep force model by using parts of the FORTRAN program FAST1 (Fries, 1987). Also, the moment created by the Coulomb friction in the bearings is found. Eigenvalues and eigenvectors are not calculated by NLINMOD. The output is in the same form as LINMOD's output. NLINMOD may be found in Appendix B.

3.6 Summary

This chapter presented equations of motion and solution methods of the equations of motion for the linear and nonlinear models of the roller rig. The geometric constraints of the wheels and rollers, creep forces, and a slider model simulating the Coulomb friction in the bearings were discussed as part of the derivation. Two programs, LINMOD and NLINMOD, which used the fourth-order Runge-Kutta method to solve the differential equations of the linear and nonlinear models were described. Analytical results are discussed and compared in Chapter 5 with the experimental results presented in Chapter 4.

Chapter 4

Experimental Investigation

4.1 Introduction

The roller rig was built to study the lateral dynamics of a three-piece truck and the phenomenon of hunting. This chapter covers the experimental investigation of the roller rig. Preparation of the roller rig and the methods used during the experimental investigation are presented next. This is followed by a discussion on the characterization of parameters such as the break away moment in the bearings due to Coulomb friction. Finally, the experimental results are presented.

4.2 Roller-Rig Setup

Preparation of the roller rig involves many individual steps that should be performed carefully to ensure experimental accuracy and safety. The roller-rig test stand was raised off the casters by the four machine mounts. A level was used to initially level the stand. Fine adjustments were made by placing a small metal ball on the top of the stand and adjusting the four machine mounts according to the direction the ball rolled. The rollersets were adjusted for the 14-in. (355.6-mm) wheelbase of the truck and made parallel. All the bolts and nuts were checked for tightness. Drive belt tension and alignment were also checked.

After a test run of the roller rig to ensure proper alignment of the components, the displacement transducers were mounted to the test stand. The truck was manually displaced to its maximum lateral and warp displacements and the displacement transducers were adjusted to avoid being contacted by the truck. The displacement transducers were then attached to a data acquisition card in an IBM personal computer. Next, the RPM gauge was calibrated with a stroboscope.

The rollers and wheels required cleaning before experimental data was taken. Illingworth (1973) studied the effects of different cleaning methods of steel wheels. He recommended cleaning the wheels with acetone and abrading with grade 006 emery cloth. Brickle (1973) used carbon tetrachloride and 600 wet and dry paper to clean the wheel and roller surfaces in his experiments. For this research, the roller rig's wheel and roller contact surfaces were degreased with acetone. Contact surfaces were then abraded with 600 wet and dry paper and then wiped with acetone. Finally, the contact surfaces were wiped with a clean dry cloth. The surfaces were periodically wiped with a clean cloth during the experiments.

4.3 Experimental Method

Hunting speed and the lateral displacement data resulting from a lateral step input were the desired data to be collected during the experiments. The first step in obtaining this data was to set the warp and yaw stiffness mechanisms. The warp stiffness is increased by bolting on more stiffness bars. Each side of the truck should have an equal number of stiffness bars. Changing the effective length of the yaw stiffness bar by moving the adjustable pins changes the yaw stiffness. The adjustable pins should be set equal distance from the center of the bolster. Figure 11 shows the components of the stiffness mechanisms. Stiffness values will be discussed in the next section.

The hunting speed was found next. This was accomplished by gradually increasing the truck speed until the truck continuously oscillated laterally and flange contact occurred. Two observations should be mentioned here. First, hunting was not restricted to a specific speed, but occurred within a range of speed. Continuous lateral oscillations would begin at a certain speed but did not involve flange contact. As the speed was increased, the amplitude of the oscillations increased until flange contact occurred. For this thesis research, the hunting speed was recorded when there was flange contact. The other observation was that the speed could be increased past the hunting speed with no oscillations occuring. The truck would appear stable and centered on the rollers. Any type of input applied to the truck would cause it to violently begin oscillating and the

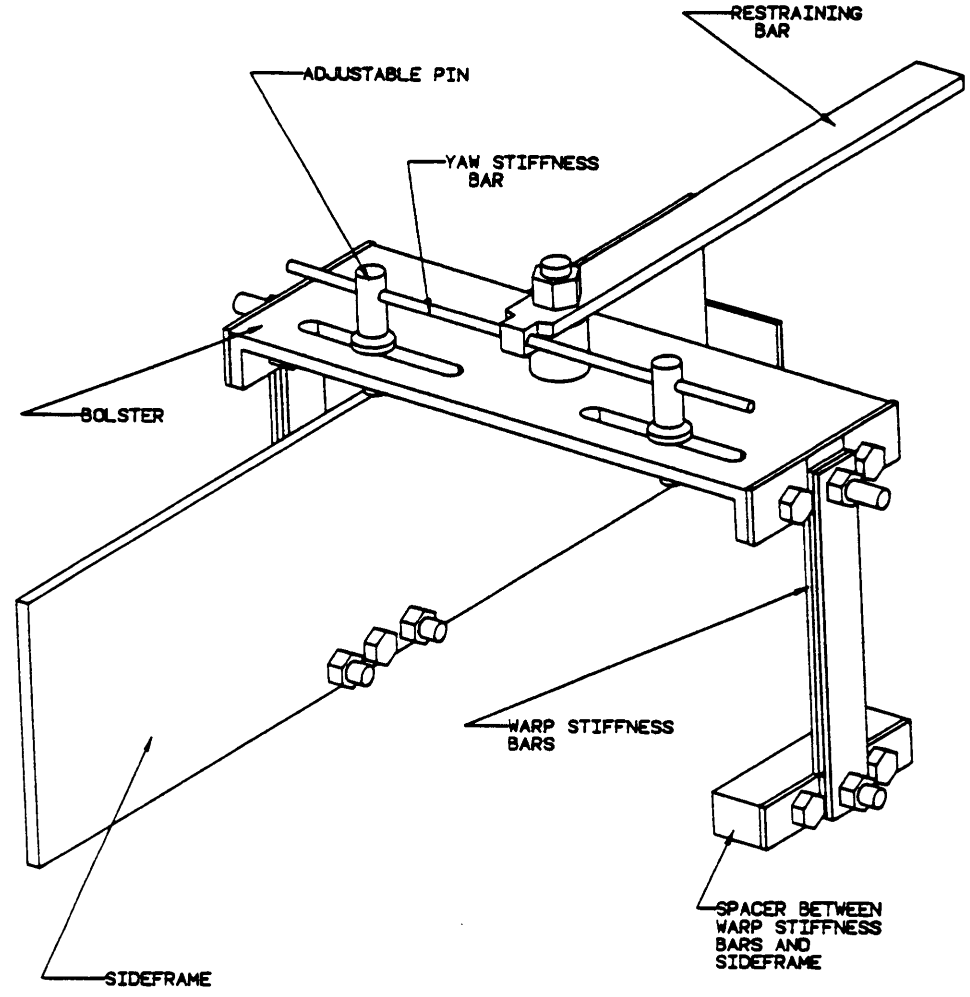


Figure 11. Stiffness Mechanisms

truck would ride up on the flanges. To prevent this, small lateral displacement inputs were made periodically as the speed was increased. This behavior is typical of nonlinear systems in which the stability is dependent upon the system state.

After the hunting speed was found, the speed was reduced and the truck allowed to center itself. A lateral displacement was applied and released. Signals from the displacement transducers were recorded by the computer. This procedure was repeated at different speeds for each experiment. This procedure was also repeated for different settings of the warp and yaw stiffnesses.

4.4 Parameter Characterization

In order to compare the experimental and analytical results, methods to compute the yaw and warp stiffnesses and the break away moment of the bearings were needed. Each warp stiffness bar can be modeled as a rectangular bar subjected to torsional loading. Beer et al. (1981) give a formula for the angle of twist of a rectangular bar as

$$\phi = \frac{TL}{c_2 a b^3 G} \tag{4.4.1}$$

Using this formula, the warp stiffness of one bar was found to be

$$K_W = \frac{T}{\phi} = \frac{c_2 a b^3 G}{12L} \tag{4.4.2}$$

where

T = torsional load (ft-lb)

ϕ = angle of twist (rad)

a = bar width (in)

b = bar thickness (in)

c_2 = function of a/b ratio

L = effective length of bar (in)

G = modulus of rigidity (psi)

For this thesis research, 1/8 × 1 × 5-in. (3.18 × 25.4 × 127.0-mm) steel bars were used. By using the above formula, the warp stiffness for each bar was found to be 113.8 ft-lb/rad (154.3 N-m/rad).

Since the yaw stiffness bar is not connected to the restraining bar at the center of rotation, a method to compute the yaw stiffness was difficult to formulate. The yaw stiffness bar was assumed to act like two cantilever beams rigidly connected to the adjustable pins. If a force is applied to the end of the beam, the deflection is given by (Beer et al., 1981)

$$\Delta = \frac{PL^3}{3EI} \cong L\theta \tag{4.4.3}$$

where

Δ = beam deflection

E = modulus of elasticity (lb/ in^2)

I = area moment of inertia (in^4)

L = effective length of one side of the rod (in)

P = force (lb)

θ = angle of deflection (rad)

Thus, the formula for the total yaw stiffness is

$$K_\theta = 2\left(\frac{PL}{\theta}\right) = \frac{6EI}{12L} = \frac{EI}{2L} \tag{4.4.4}$$

If a moment, M, is applied to the end of the cantilever beam, the deflection is (Beer et al., 1981)

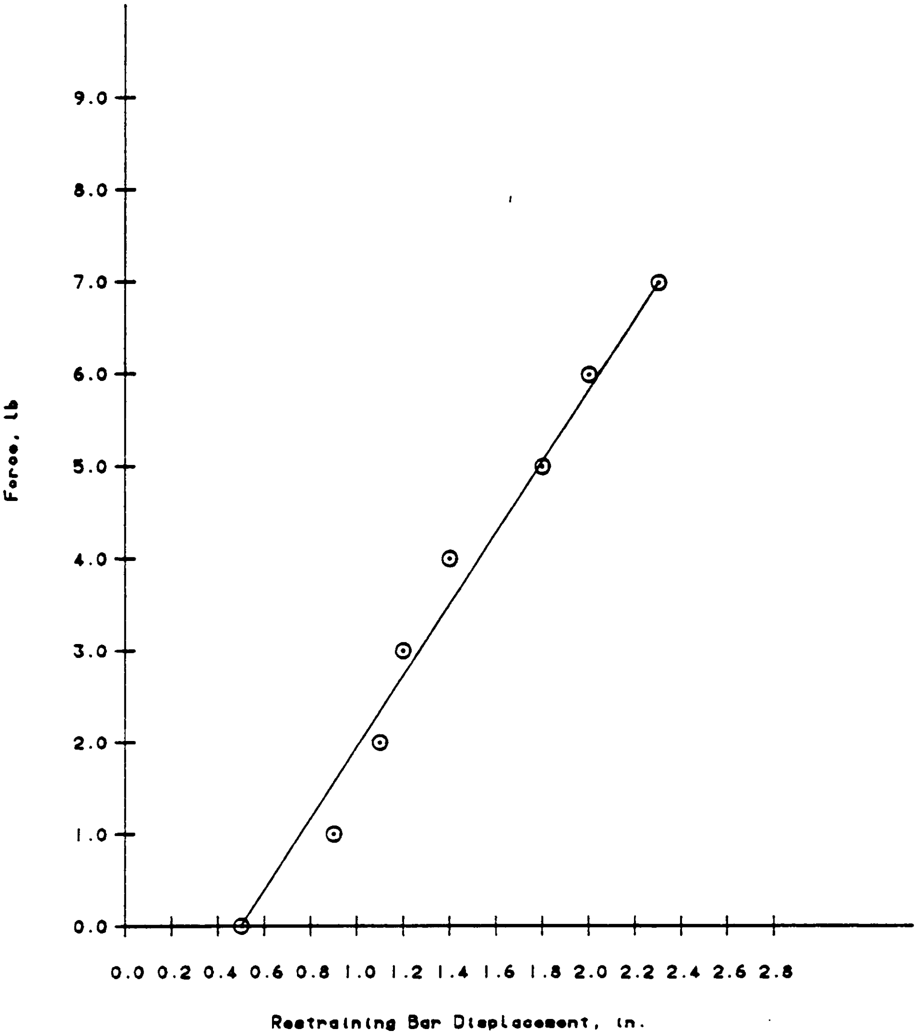
$$\Delta = \frac{ML^2}{2EI} \cong L\theta \tag{4.4.5}$$

The resulting yaw stiffness formula is

$$K_\theta = \frac{2M}{\theta} = \frac{4EI}{12L} = \frac{EI}{3L} \tag{4.4.6}$$

In order to determine which formula should be used, an experiment was set up to measure the yaw stiffness. While the truck was held fixed, a force was applied to the restraining bar (after it was disconnected from the test stand) and the resulting deflection of the restraining bar was measured. The data collected is graphically displayed in Figure 12. This results in a stiffness of 3.9 lb/in. (682.9 N/m). A formula to convert the rotational stiffness into a linear stiffness which can be compared to the above result was found to be

$$K_{\text{linear}} = \frac{K_{\text{rotational}}}{l_B^2} \tag{4.4.7}$$



(1 lb = 4.448 N)
(1 in. = 0.0254 m)

Figure 12. Experimental Data for Yaw Stiffness

where l_B is the distance between the yaw stiffness bar and the point on the restraining bar where the force was applied. This distance was 29 in. (0.737 m). Using a 3/16-in. (4.76-mm) diameter steel rod for the yaw stiffness bar and L = 3.45 in. (0.0876 m), stiffness equation 4.4.3 gives a linear stiffness of 3.85 lb/in. (674.2 N/m). Yaw stiffness equation 4.4.6 gives a linear stiffness of 2.51 lb/in. (439.5 N/m). Based on these results, equation 4.4.3, derived by modeling the yaw stiffness as two cantilevered beams with a force applied to the end of each beam, was used in the analytical studies.

The break-away moment due to Coulomb friction in the bearings was also found experimentally. The truck was lifted off the rollers by the use of a hydraulic jack. A force in a longitudinal direction was applied at opposite corners of the truck as shown in Figure 13. This force was gradually increased until the truck warped. This force was found to be 4 lb (17.8 N). The break-away moment was then calculated to be 0.36 ft-lb (6.49 N-m) per bearing. Only fifty percent of the value for the break-away moment was used in the analytical investigation. This was done to take into account the truck vibrations which keep the bearings moving, thus reducing the effects of the Coulomb friction. This is a characteristic of real trucks.

4.5 Experimental Results

Experimental data was collected from the roller rig for different combinations of yaw and warp stiffness. One, two, and three bars per side were used for the warp stiffness, which resulted in total warp stiffnesses of 227.6 (308.6), 455.2 (617.1), and 682.8 ft-lb/rad (925.7 N-m/rad), respectively. Yaw stiffness was varied by removing the stiffness bar,

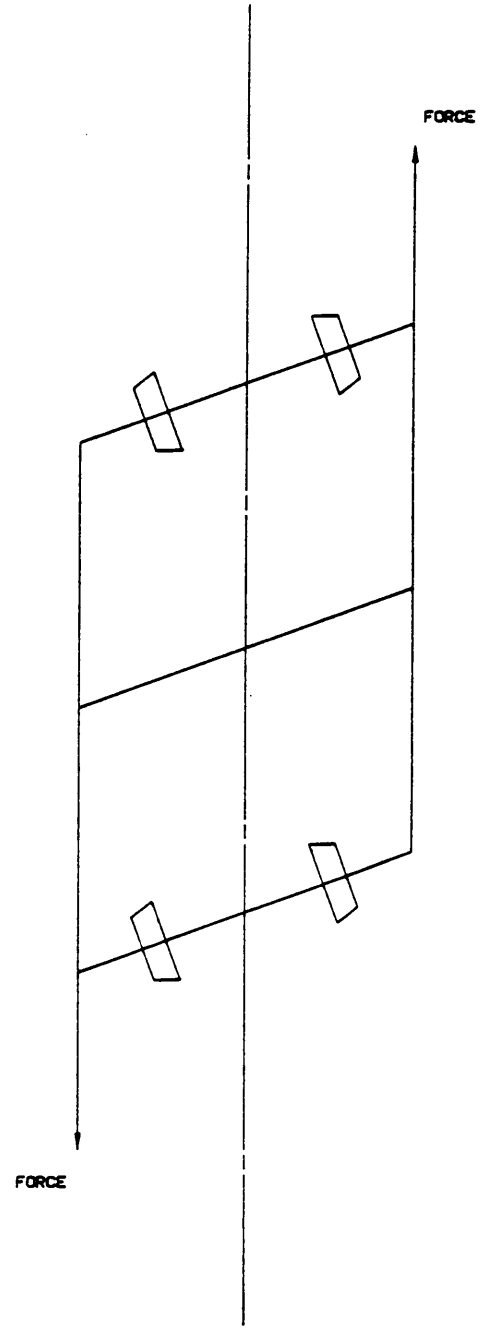
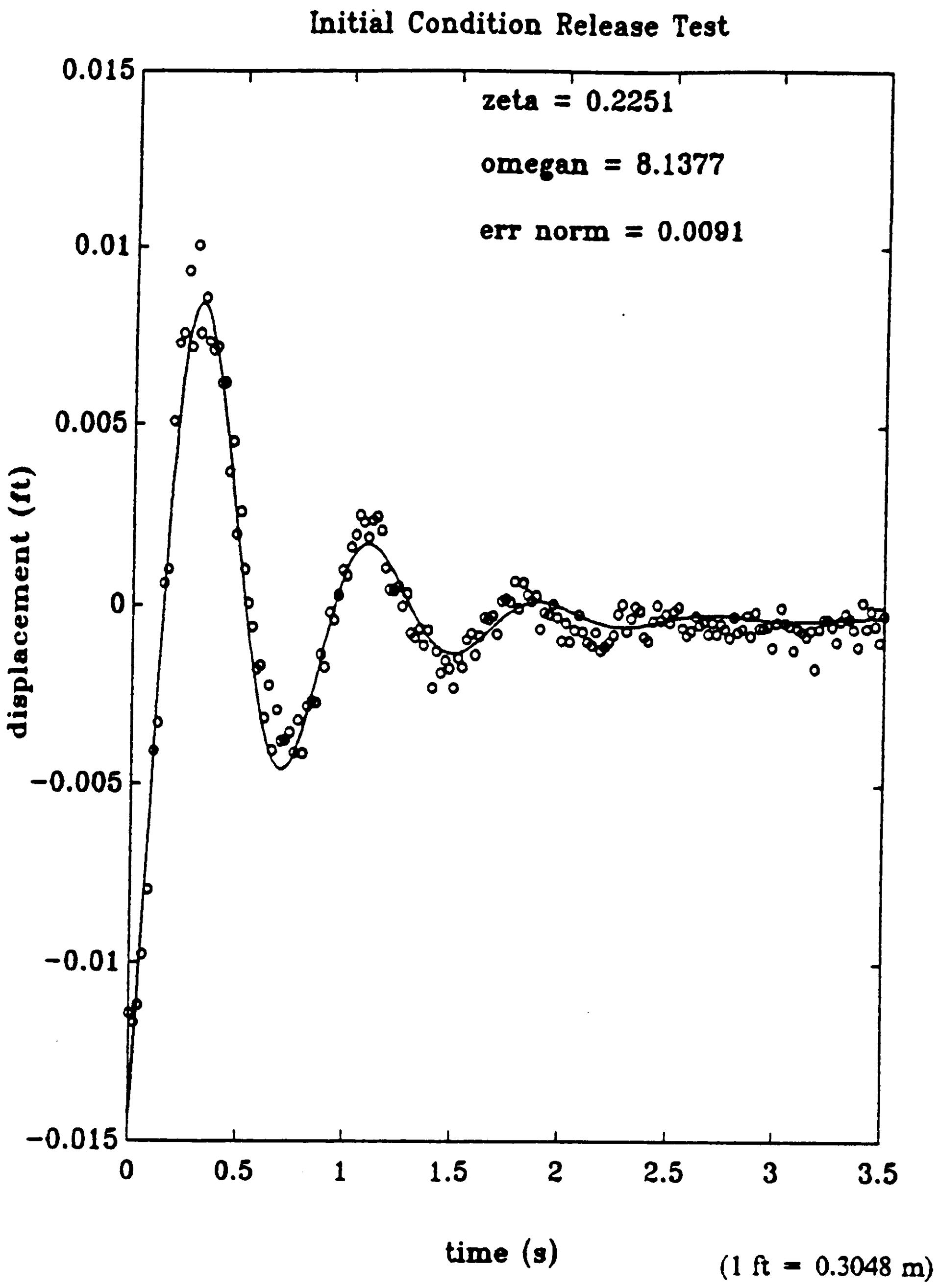


Figure 13. Force Applied to Calculate Break Away Moment

and by making the distance between the restraining bar and the adjustable pins equal to 2.60 (0.066 m) and 3.45 in. (0.088 m). These setups resulted in total yaw stiffnesses of 0.0 (0.0), 350.0 (474.5), and 263.8 ft-lb/rad (357.6 N-m/rad), respectively.

The lateral displacement versus time data collected at different speeds for each setup was used to estimate the damping ratio and natural frequency for each setup. Damping ratio and natural frequency were found by fitting an exponentially decaying sine wave to the experimental data. The damping ratio and natural frequency are parameters of the exponentially decaying sine wave. The curve fit was a least-square method implemented using Matlab (The MathWorks, Inc., 1987). An example of the output from Matlab is shown in Figure 14. The results of this analysis are tabulated in Table 2. Table 4 shows how the hunting speed was affected by the different stiffness setups. These results will be discussed further and compared to the analytical results in the next chapter.



(1 ft = 0.3048 m)

Figure 14. Example of Matlab Output

Table 2. Damping Ratio and Natural Frequency Data

Truck Speed (ft/sec)	Warp Stiffness (ft-lb/rad)	Yaw Stiffness (ft-lb/rad)	Damping Ratio	Natural Frequency (rad/sec)
14.18	227.6	350.0	0.17	8.3
17.02	227.6	350.0	0.13	10.2
19.85	227.6	350.0	0.12	12.0
22.69	227.6	350.0	0.09	14.0
14.18	455.2	350.0	0.21	8.0
17.02	455.2	350.0	0.19	9.5
19.85	455.2	350.0	0.16	10.9
22.69	455.2	350.0	0.14	13.2
25.53	455.2	350.0	0.12	14.9
17.02	682.8	350.0	0.26	8.8
19.85	682.8	350.0	0.20	11.3
22.69	682.8	350.0	0.18	13.3
28.36	682.8	350.0	0.13	14.6
8.51	227.6	0.0	0.027	5.5
11.34	227.6	0.0	0.018	7.3
14.18	227.6	0.0	0.017	8.5
11.34	227.6	263.8	0.05	6.6
14.18	227.6	263.8	0.04	8.3
17.02	227.6	263.8	0.03	10.2
11.34	455.2	263.8	0.07	6.7
17.02	455.2	263.8	0.04	9.4
22.69	455.2	263.8	0.01	13.1

Table 3. Damping Ratio and Natural Frequency Data (SI Units)

Truck Speed (m/sec)	Warp Stiffness (N-m/rad)	Yaw Stiffness (N-m/rad)	Damping Ratio	Natural Frequency (rad/sec)
4.32	308.6	474.5	0.17	8.3
5.19	308.6	474.5	0.13	10.2
6.05	308.6	474.5	0.12	12.0
6.92	308.6	474.5	0.09	14.0
4.32	617.1	474.5	0.21	8.0
5.19	617.1	474.5	0.19	9.5
6.05	617.1	474.5	0.16	10.9
6.92	617.1	474.5	0.14	13.2
7.78	617.1	474.5	0.12	14.9
5.19	925.7	474.5	0.26	8.8
6 05	925.7	474.5	0.20	11.3
6.92	925.7	474.5	0.18	13.3
8.64	925.7	474.5	0.13	14.6
8.51	308.6	0.0	0.027	5.5
3.46	308.6	0.0	0.018	7.3
4.32	308.6	0.0	0.017	8.5
3.46	308.6	357.6	0.05	6.6
4.32	308.6	357.6	0.04	8.3
5.19	308.6	357.6	0.03	10.2
3.46	617.1	357.6	0.07	6.7
5.19	617.1	357.6	0.04	9.4
6.92	617.1	357.6	0.01	13.1

Table 4. Hunting Speed Data

Yaw Stiffness (ft-lb/rad)	Warp Stiffness (ft-lb/rad)	Hunting Speed (ft/sec)
0.0	227.6	18
0.0	455.2	20
263.8	227.6	23
263.8	455.2	24
350.0	227.6	33
350.0	455.2	36
350.0	682.8	41

Yaw Stiffness (N-m/rad)	Warp Stiffness (N-m/rad)	Hunting Speed (m/sec)
0.0	308.6	5.5
0.0	617.1	6.1
357.6	308.6	7.0
357.6	617.1	7.3
474.5	308.6	10.1
474.5	617.1	10.9
474.5	925.7	12.5

Chapter 5

Comparison of Analytical and Experimental Results

5.1 Introduction

Chapter 3 contains the linear and nonlinear mathematical models of the one-fifth scale three-piece freight truck. Analytical results from the simulation programs, LINMOD and NLINMOD, are presented in this chapter and compared with the experimental results. The linear model is used to compare the effects of a lateral displacement initial condition to the truck with the experimental data. This data is used to compare the damping ratio and natural frequency at different operating speeds. The nonlinear model was used in the study of hunting. Values for the variables used in all of the analytical work are presented in Table 5.

Table 5. Values for Varibles Used in Analytical Investigation

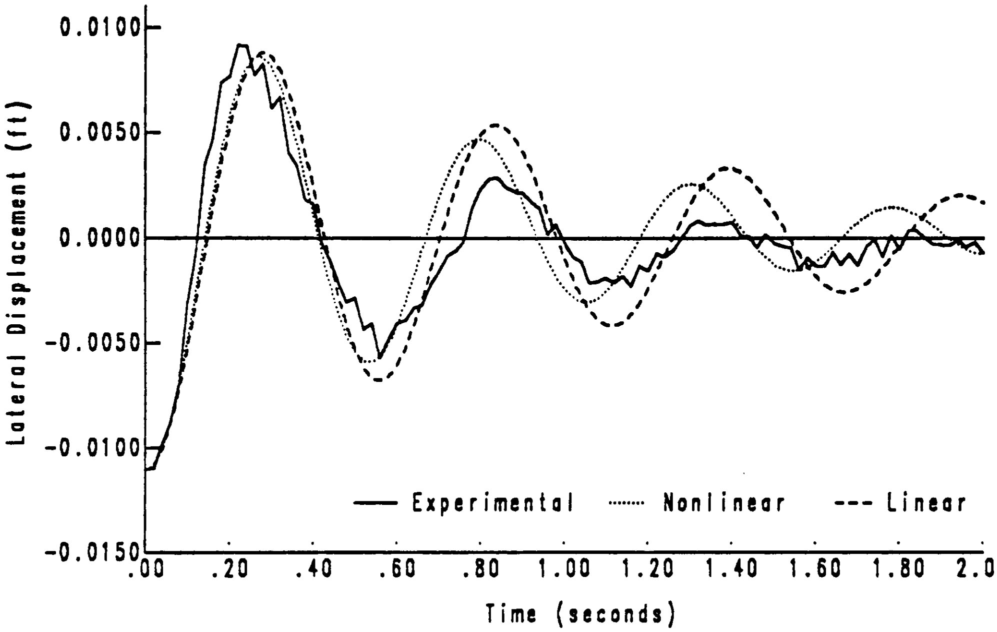
Variable	Numerical Value
a	0.5 ft (0.15 m)
d	0.364 ft (0.11 m)
D_θ	0.0 lb-ft-s/rad (0.0 N-m-s/rad)
D_W	0.0 lb-ft-s/rad (0.0 N-m-s/rad)
f_{11} (100%)	8656.3 lb (38503.2 N)
f_{12} (100%)	4.14 lb-ft (5.61 N-m)
f_{22} (100%)	0.00338 lb-ft^2 (0.0014 N-m^2)
f_{33} (100%)	9547.5 lb (42467.3 N)
I_b	0.0096 slugs-ft^2 (0.013 kg-m^2)
I_f	0.122 slugs-ft^2 (0.165 kg-m^2)
I_w	0.228 slugs-ft^2 (0.309 kg-m^2)
l_1	0.583 ft (0.18 m)
M_{break}	0.18 lb-ft (0.24 N-m)
M_b	0.111 slugs (1.62 kg)
M_f	0.458 slugs (6.68 kg)
M_r	0.103 slugs (1.50 kg)
M_w	0.958 slugs (13.98 kg)
N	25.9 lb (115.2 N)
R_0	0.3 ft (0.09 m)
R_r	0.542 ft (0.17 m)
R_c	0.167 ft (0.05 m)
μ	0.65

5.2 Initial Condition Response

In the experimental analysis of the roller rig, the truck was set up with different warp and yaw stiffnesses and then subjected to a lateral displacement initial condition at different operating speeds. The lateral displacement used was approximately 0.011 ft (0.00335 m). This same initial condition was used in the simulation programs. An example of the initial condition response from both simulation programs and the roller rig is shown in Figure 15. Warp and yaw stiffnesses of 455.2 (617.1) and 350.0 ft-lb/rad (474.5 N-m/rad), respectively, and a truck speed of 19.85 ft/sec (6.05 m/sec) were used in this example. As can be seen in the figure, the truck stabilizes itself and moves back to the center position between the rollers. In order to simulate real track conditions, researchers often use a percentage of the creep coefficients. For this thesis research, 70% of the creep coefficients was used. This percentage was found by varying the percentage and comparing the analytical and experimental step responses and selecting a percentage that gave a good match. If the rollers had not been cleaned, a smaller percentage would have been used. The linear simulation program, LINMOD, used the values of stiffnesses and truck speed from the experimental investigation to calculate the damping ratios and natural frequencies presented in the next section.

5.3 Damping Ratio and Natural Frequency vs. Speed

Damping ratio and natural frequency versus speed data for the experimental investigation of the roller rig was presented at the end of the previous chapter. The



(1 ft = 0.3048 m)

Figure 15. Example of Initial Condition Response

linear model resulted in the damping ratio and natural frequency data shown in Table 6. Figure 16 shows damping ratio versus speed for the experimental and linear model investigations for warp and yaw stiffnesses of 455.2 (617.1) and 350.0 ft-lb/rad (474.5 N-m/rad), respectively. Figure 17 shows the natural frequency versus speed for the same case.

As shown in these tables and figures, the damping ratio decreases and the natural frequency increases as the truck speed is increased. The linear model predicts the truck's natural frequency versus speed within 5% of the experimental values. Damping ratios from the linear model were lower than the experimental values at low speeds and higher than the experimental values at high speeds. One result is that the linear model predicts a higher hunting speed than was found experimentally as discussed in the next section.

5.4 Hunting or Unstable Response

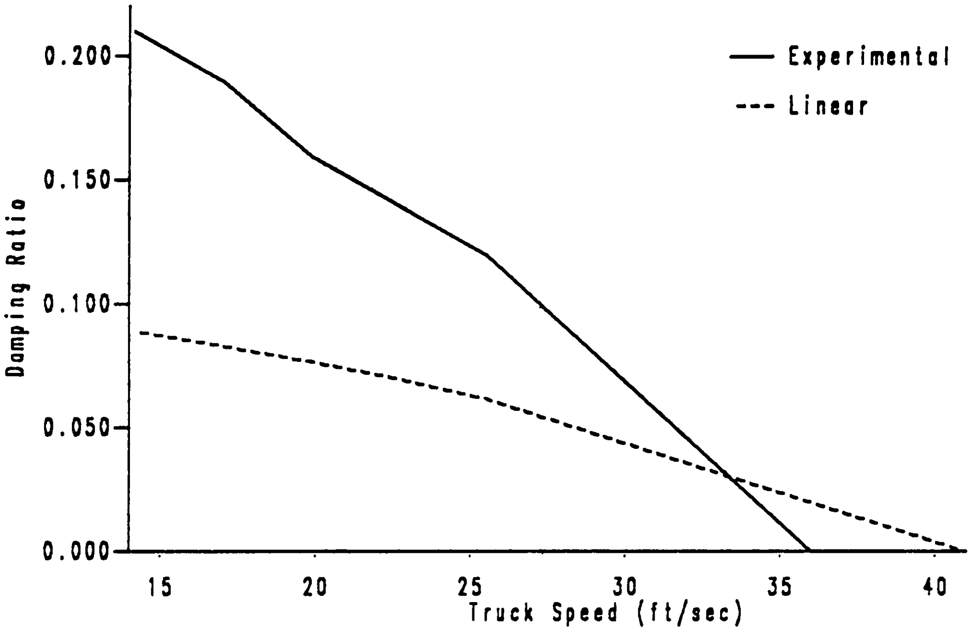
Since hunting typically involves flange contact, the nonlinearities of the wheel and rail geometries needed to be included in the mathematical model. When LINMOD is used at hunting speed, the lateral oscillations continuously increase in amplitude as shown in Figure 18. Figure 19 illustrates hunting on the roller rig. The truck continuously oscillates but the amplitude tends to remain constant. This figure also shows that the roller rig can simulate small amplitude hunting in which flange contact does not occur. This is a common occurance on real rail vehicles. NLINMOD can simulate the hunting of the truck as shown in Figure 20. The nonlinear model simulates

Table 6. Damping Ratio and Natural Frequency Data From LINMOD

Truck Speed (ft/sec)	Warp Stiffness (ft-lb/rad)	Yaw Stiffness (ft-lb/rad)	Damping Ratio	Natural Frequency (rad/sec)
14.18	227.6	350.0	0.060	8.15
17.02	227.6	350.0	0.054	9.78
19.85	227.6	350.0	0.047	11.40
22.69	227.6	350.0	0.040	13.03
14.18	455.2	350.0	0.089	8.13
17.02	455.2	350.0	0.083	9.76
19.85	455.2	350.0	0.077	11.38
22.69	455.2	350.0	0.070	13.01
25.53	455.2	350.0	0.062	14.64
41.00	455.2	350.0	0.000	23.26
17.02	682.8	350.0	0.113	9.73
19.85	682.8	350.0	0.107	11.34
22.69	682.8	350.0	0.099	12.97
28.36	682.8	350.0	0.082	16.21
8.51	227.6	0.0	0.020	4.89
11.34	227.6	0.0	0.017	6.51
14.18	227.6	0.0	0.013	8.14
11.34	227.6	263.8	0.052	6.51
14.18	227.6	263.8	0.048	8.14
17.02	227.6	263.8	0.042	9.78
11.34	455.2	263.8	0.082	6.50
17.02	455.2	263.8	0.072	9.76
22.69	455.2	263.8	0.058	13.00

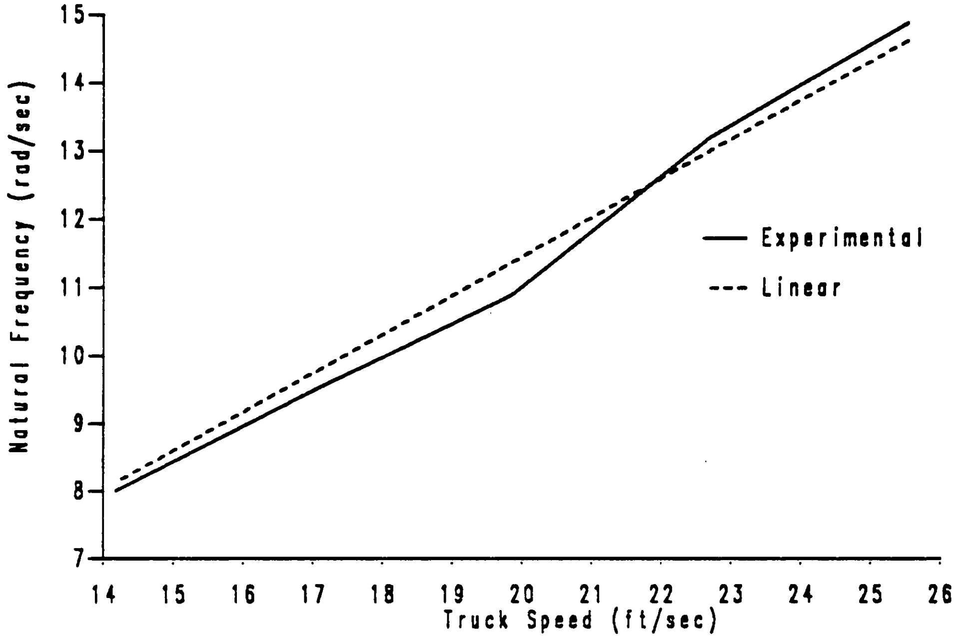
Table 7. Damping Ratio and Natural Frequency Data From LINMOD (SI Units)

Truck Speed (m/sec)	Warp Stiffness (N-m/rad)	Yaw Stiffness (N-m/rad)	Damping Ratio	Natural Frequency (rad/sec)
4.32	308.6	474.5	0.060	8.15
5.19	308.6	474.5	0.054	9.78
6.05	308.6	474.5	0.047	11.40
6.92	308.6	474.5	0.040	13.03
4.32	617.1	474.5	0.089	8.13
5.19	617.1	474.5	0.083	9.76
6.05	617.1	474.5	0.077	11.38
6.92	617.1	474.5	0.070	13.01
7.78	617.1	474.5	0.062	14.64
12.50	617.1	474.5	0.000	23.26
5.19	925.7	474.5	0.113	9.73
6.05	925.7	474.5	0.107	11.34
6.92	925.7	474.5	0.099	12.97
8.64	925.7	474.5	0.082	16.21
2.59	308.6	0.0	0.020	4.89
3.46	308.6	0.0	0.017	6.51
4.32	308.6	0.0	0.013	8.14
3.46	308.6	357.6	0.052	6.51
4.32	308.6	357.6	0.048	8.14
5.19	308.6	357.6	0.042	9.78
3.46	617.1	357.6	0.082	6.50
5.19	617.1	357.6	0.072	9.76
6.92	617.1	357.6	0.058	13.00



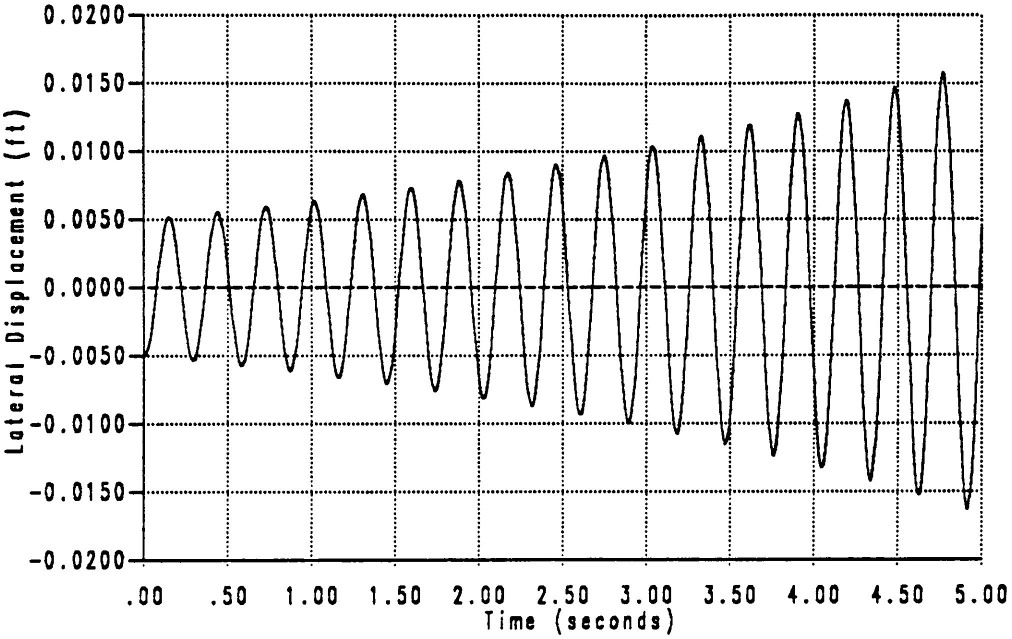
(1 ft = 0.3048 m)

Figure 16. Comparison of Damping Ratio vs. Truck Speed



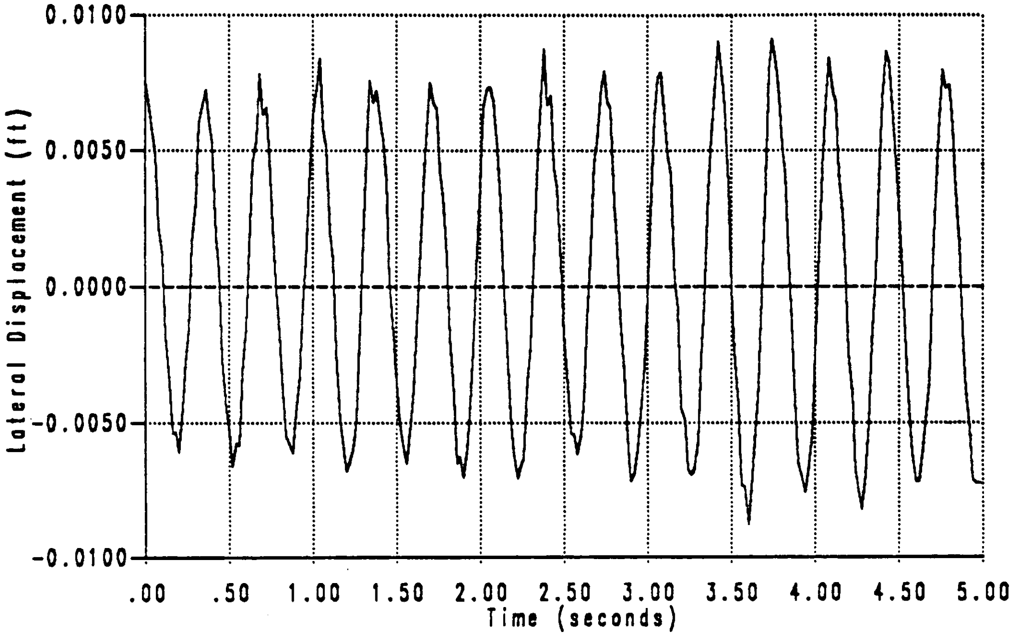
(1 ft = 0.3048 m)

Figure 17. Comparison of Natural Frequency vs. Truck Speed



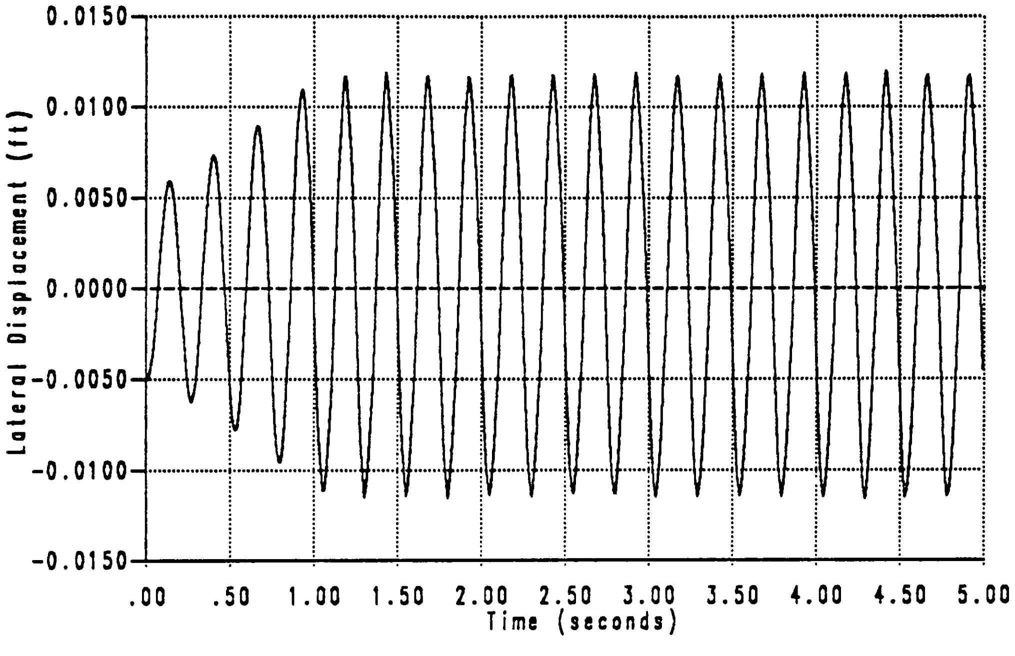
(1 ft = 0.3048 m)

Figure 18. Linear Simulation of Truck Hunting



(1 ft = 0.3048 m)

Figure 19. Truck Hunting on Roller Rig



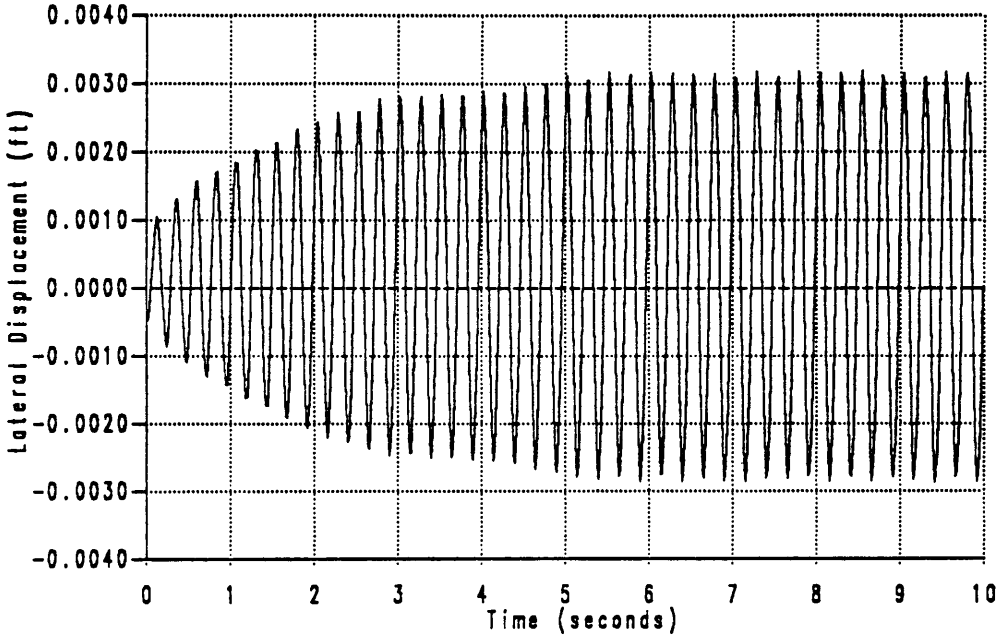
(1 ft = 0.3048 m)

Figure 20. Nonlinear Simulation of Truck Hunting

the experimentally observed increase in oscillation frequency that accompanies the onset of flange contact.

By using the lookup table for the wheel and rail geometric constraints, the nonlinear model can simulate the increase in the wheel/roller contact angles which cause an increase of the lateral forces on the wheels. NLINMOD simulates small amplitude hunting as shown in Figure 21. A truck speed of 40 ft/sec (12.19 m/sec) and yaw and warp stiffnesses of 350.0 (474.5) and 455.2 ft-lb/rad (617.1 N-m/rad), respectively, were used for this case.

The hunting speed data for the experimental investigation was presented in the previous chapter. Table 8 contains the hunting speeds found for different stiffness setups using both LINMOD and NLINMOD. Data from both the analytical and experimental investigations show that the hunting speed increases as the stiffnesses are increased. Hunting speed calculated using LINMOD and NLINMOD tended to be 5 to 15% higher than those found experimentally. NLINMOD offers a safe means to study the effects of different wheel/roller geometries on the hunting speed.



(1 ft = 0.3048 m)

Figure 21. Small Amplitude Hunting Simulated by NLINMOD

Table 8. Hunting Speed Data From LINMOD and NLINMOD

Yaw Stiffness (ft-lb/rad)	Warp Stiffness (ft-lb/rad)	Hunting Speed (ft/sec)		
		LINMOD	NLINMOD	Experimental
0.0	227.6	21	22	18
0.0	455.2	31	32	20
263.8	227.6	32	32	23
263.8	455.2	39	40	24
350.0	227.6	34	35	33
350.0	455.2	41	42	36
350.0	682.8	47	47	41

Yaw Stiffness (N-m/rad)	Warp Stiffness (N-m/rad)	Hunting Speed (m/sec)		
		LINMOD	NLINMOD	Experimental
0.0	308.6	6.4	6.7	5.5
0.0	617.1	9.4	9.8	6.1
357.6	308.6	9.8	9.8	7.0
357.6	617.1	11.9	12.2	7.3
474.5	308.6	10.4	10.7	10.1
474.5	617.1	12.5	12.8	10.9
474.5	925.7	14.3	14.3	12.5

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

This thesis contains the results of experimental and analytical investigations of the lateral dynamics of a one-fifth scale model of a typical North American freight truck running on a roller rig. Linear and nonlinear mathematical models were developed to simulate the lateral dynamics of the truck on the roller rig. The following conclusions can be drawn from this research:

1. The roller rig built for this research can be used to simulate a real truck running on rails. It simulates the lateral instability known as hunting and would be a useful tool for future studies.

2. The linear model predicts the truck's natural frequency versus speed within 5% of the experimental values.

3. The linear model predicts the characteristic of decreasing damping ratio as the truck speed increases that was found in the experimental investigation. Damping ratios from the linear model were lower than the experimental values at low speed and higher than the experimental values at higher speeds. One result is that the linear model predicts a higher hunting speed than was found experimentally. The agreement may be improved by following the recommendations in the next section.

4. The nonlinear model simulates the characteristics of hunting, including small amplitude hunting. By using a lookup table for the wheel/rail geometries and the heuristic method to calculate creep force saturation, flange contact is simulated. Hunting speeds found by the nonlinear model were about 5 to 15% higher than those found experimentally.

5. The nonlinear model simulates the experimentally observed increase in oscillation frequency that accompanies the onset of flange contact.

6.2 Recommendations

In order to obtain better results from the linear model for the damping ratio analysis, the following should be considered:

1. Use nonzero values for the warp and yaw damping.

2. Identify the parameters.

    a. Do static tests to find the warp and yaw stiffnesses.

    b. Do dynamic tests for the warp and yaw spring and bearing friction characteristics.

3. Use a curve fitting method to find the damping ratios from the nonlinear model results and compare with the linear model and experimental damping ratios.

Three modifications to the roller rig should be made. They are:

1. Reinforce the mounting brackets for the displacement transducers to reduce their vibrations.

2. Redesign the yaw stiffness mechanism so that the yaw stiffness bar is connected to the restraining bar over the center of the truck.

3. Install larger machine mounts for levelling and supporting the test stand to reduce the lateral rocking of the test stand during hunting.

A number of other investigations remain to be done on the roller rig. The roller rig has the capability to investigate:

1. effects of different wheel profiles and wheel wear

2. the influence of weight added to the truck by hanging weights from the hook under the bolster

3. the dynamics of novel truck designs

4. the effect of different roller radii.

The mathematical models also provide insight in the above investigations.

Appendix A

Program LINMOD

```
**********************************************************************
*
*         LINEAR SIMULATION PROGRAM
*
*     THIS PROGRAM USES THE FORTH ORDER RUNGA KUTTA
*     METHOD TO SOLVE 6 FIRST ORDER DIFFERENTIAL
*     EQUATIONS DESCRIBING THE THREE-DEGREES-OF-
*     FREEDOM-1/5-SCALE FREIGHT CAR TRUCK.
*     IT ALSO FINDS THE EIGENVALUES.
*
**********************************************************************
      REAL Q(6),K1(6),K2(6),K3(6),K4(6),H,A(6,6),V,AMAT(6,6)
*
* DATA FOR INITIAL VALUES
*
      DATA Q/-0.011,0.0,0.0,0.0,0.0,0.0/
*
* END OF DATA
*
*
* DEFINE THE SIX FIRST ORDER DIFFERENTIAL EQUATIONS
*
      DQ1(Q1,Q2,Q3,Q4,Q5,Q6) = A(1,1)*Q1 + A(1,2)*Q2 + A(1,3)*Q3 +
     >                         A(1,4)*Q4 + A(1,5)*Q5 + A(1,6)*Q6
      DQ2(Q1,Q2,Q3,Q4,Q5,Q6) = A(2,1)*Q1 + A(2,2)*Q2 + A(2,3)*Q3 +
     >                         A(2,4)*Q4 + A(2,5)*Q5 + A(2,6)*Q6
      DQ3(Q1,Q2,Q3,Q4,Q5,Q6) = A(3,1)*Q1 + A(3,2)*Q2 + A(3,3)*Q3 +
     >                         A(3,4)*Q4 + A(3,5)*Q5 + A(3,6)*Q6
      DQ4(Q1,Q2,Q3,Q4,Q5,Q6) = A(4,1)*Q1 + A(4,2)*Q2 + A(4,3)*Q3 +
     >                         A(4,4)*Q4 + A(4,5)*Q5 + A(4,6)*Q6
      DQ5(Q1,Q2,Q3,Q4,Q5,Q6) = A(5,1)*Q1 + A(5,2)*Q2 + A(5,3)*Q3 +
     >                         A(5,4)*Q4 + A(5,5)*Q5 + A(5,6)*Q6
      DQ6(Q1,Q2,Q3,Q4,Q5,Q6) = A(6,1)*Q1 + A(6,2)*Q2 + A(6,3)*Q3 +
     >                         A(6,4)*Q4 + A(6,5)*Q5 + A(6,6)*Q6
*
* OPEN OUTPUT FILES
*
```

```
      OPEN(10, FILE = 'LAT', STATUS = 'NEW')
      OPEN(11, FILE = 'YAW', STATUS = 'NEW')
      OPEN(12, FILE = 'WARP', STATUS = 'NEW')
*
* ASK USER FOR STEP SIZE
*
      WRITE(6,*)'ENTER STEP SIZE (H IN SECONDS)'
      READ(5,*) H
*
* ASK FOR ENDING TIME
*
      WRITE(6,*)'ENTER ENDING TIME (SECONDS)'
      READ(5,*) TIME
*
* ASK FOR VELOCITY
*
      WRITE(6,*)'ENTER VELOCITY (FT/SEC)'
      READ(5,*) V
*
* ASK FOR STIFFNESSES AND DAMPING VALUES
*
      WRITE(6,*)'ENTER THE WARP AND YAW STIFFNESSES (FT-LB/RAD)'
      READ(5,*) WARPK, YAWK
      WRITE(6,*)'ENTER THE WARP AND YAW DAMPING (FT-LB-SEC)'
      READ(5,*) WARPD, YAWD
*
* CALL SUBROUTINE CALC TO GET VALUE OF MATRIX A
*
      CALL CALC(V,WARPK,YAWK,WARPD,YAWD,A)
*
* CALL SUBROUTINE EIGEN TO GET THE EIGENVALUES
*
      DO 10 I = 1,6
         DO 5 J = 1,6
              AMAT(I,J) = A(I,J)
    5    CONTINUE
   10 CONTINUE
      CALL EIGEN(AMAT)

*
*
      LIMIT = TIME/H
      LFILE = LIMIT/490

*
* START INTEGRATION LOOP
*
      T = 0.0
      N = 0
      DO 100 I = 1,LIMIT
         T = T + H
         N = N + 1
*
*        CALCULATION OF K1'S
*
         K1(1) = DQ1(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
         K1(2) = DQ2(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
         K1(3) = DQ3(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
         K1(4) = DQ4(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
         K1(5) = DQ5(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
         K1(6) = DQ6(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
*
*        CALCULATION OF K2'S
*
         K2(1) = DQ1(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
     >               Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
         K2(2) = DQ2(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
```

```
     >                Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
        K2(3) = DQ3(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
     >                Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
        K2(4) = DQ4(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
     >                Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
        K2(5) = DQ5(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
     >                Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
        K2(6) = DQ6(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
     >                Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
*
*       CALCULATION OF K3'S
*
        K3(1) = DQ1(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >                Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
        K3(2) = DQ2(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >                Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
        K3(3) = DQ3(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >                Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
        K3(4) = DQ4(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >                Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
        K3(5) = DQ5(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >                Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
        K3(6) = DQ6(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >                Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
*
*       CALCULATION OF K4'S
*
        K4(1) = DQ1(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >                Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
        K4(2) = DQ2(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >                Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
        K4(3) = DQ3(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >                Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
        K4(4) = DQ4(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >                Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
        K4(5) = DQ5(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >                Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
        K4(6) = DQ6(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >                Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
*
*       FIND VALUES OF Q FOR NEXT TIME STEP.
*
        DO 50 J = 1,6
             Q(J) = Q(J) + H*(K1(J) + 2.*K2(J) + 2.*K3(J) + K4(J))/6.
   50   CONTINUE
*
*       OUTPUT TO FILES
*
        IF (N .EQ. LFILE) THEN
        WRITE(10,200) T, Q(1)
        WRITE(11,200) T, Q(3)
        WRITE(12,200) T, Q(5)
        N = 0
        ENDIF
  100 CONTINUE
  200 FORMAT(4X,F10.7,4X,F15.7)
      END
*******************************************************************
*
*         SUBROUTINE CALC
*
*     THIS SUBROUTINE CALCULATES THE ELEMENTS OF
*     THE A MATRIX WHICH CONTAINS THE COEFFICIENTS
*     THIS IS ALSO WHERE THE VALUES FOR THE DIFFERENT
*     VARIABLES ARE ENTERED.
*
*******************************************************************
```

```
*  LIST OF VARIABLES
*
*  AGAGE = HALF RAIL GAGE
*  D = HALF SIDEFRAME SEPARATION
*  FSUB11 = LATERAL CREEP COEFFICIENT
*  FSUB12 = SPIN/LATERAL CREEP COEFFICIENT
*  FSUB22 = SPIN CREEP COEFFICIENT
*  FSUB33 = LONGITUDINAL CREEP COEFFICIENT
*  ISUBB = BOLSTER INERTIAL MOMENT
*  ISUBF = SIDEFRAME INERTIAL MOMENT
*  ISUBW = WHEELSET INERTIAL MOMENT
*  LAMDA = WHEELSET CONICITY
*  L1 = HALF WHEELBASE
*  MSUBB = BOLSTER MASS
*  MSUBF = SIDEFRAME MASS
*  MSUBW = WHEELSET MASS
*  MSUBRR = RESTRAINING ROD MASS
*  NLOAD = LOAD PER WHEEL
*  RSUBC = RAIL CROWN RADIUS
*  RSUBO = NOMINAL ROLLING RADIUS
*  RSUBR = RAIL ROLLER RADIUS
*  VEL = WHEEL/RAIL RELATIVE VELOCITY
*  Q(1) = LATERAL DISPLACEMENT
*  Q(2) = LATERAL VELOCITY
*  Q(3) = YAW ANGLE
*  Q(4) = YAW ANGULAR VELOCITY
*  Q(5) = WARP ANGLE
*  Q(6) = WARP ANGULAR VELOCITY
*  WARPK = WARP STIFFNESS
*  WARPD = WARP DAMPING
*  YAWK = YAW STIFFNESS
*  YAWD = YAW DAMPING
*
*
      SUBROUTINE CALC(VEL,WARPK,YAWK,WARPD,YAWD,A)
      REAL K(3,3),M(3,3),A(6,6),C(3,3),VEL
      REAL MSUBW,MSUBB,MSUBRR,MSUBF,ISUBW,ISUBF,ISUBB,NLOAD,LAMDA,L1
*
* DATA FOR MASSES
*
      DATA MSUBW/0.958/, MSUBB/0.111/,
     >     MSUBRR/0.103/,MSUBF/0.458/
*
* DATA FOR INERTIAS
*
      DATA ISUBW/0.228/,ISUBF/0.122/,ISUBB/0.0096/
*
* DATA FOR CREEP COEFFICIENTS
*
      DATA FSUB11/8656.3/,FSUB12/4.1406/,
     >     FSUB22/0.0033763/,FSUB33/9547.5/
*
* DATA FOR OTHER VARIABLES
*
      DATA AGAGE/0.5/, NLOAD/25.9/, LAMDA/.05/,
     >     RSUBO/0.3/, RSUBR/0.5417/,
     >     D/0.3646/, L1/0.5833/,
     >     RSUBC/0.167/
*
* END OF DATA
*
*
* CALCULATION OF COEFFICIENTS WHERE M, K, AND C REPRESENT
* MASS, STIFFNESS, AND DAMPING TERMS, RESPECTIVELY.
*
      M(1,1) = 2.*MSUBW + MSUBB + MSUBRR/3. + 2.*MSUBF
      M(2,1) = 2.*ISUBW + 2.*ISUBF + 2.*MSUBW*(L1**2.) + 2.*MSUBF*
```

```
     >(D**2.) + ISUBB
      M(2,2) = 2.*ISUBW + 2.*MSUBF*(D**2.) + ISUBB
      M(3,1) = M(2,2)
      M(3,2) = M(2,2)
      M(3,3) = M(2,2)
*
      K(1,1) = (4.*LAMDA/AGAGE)*(NLOAD - (FSUB12/RSUBR))
      K(1,2) = 4.*FSUB11
      K(1,3) = K(1,2)
      K(2,1) = YAWK - 4.*LAMDA*FSUB12*(L1**2.)/(RSUBR*AGAGE) +
     >         4.*FSUB12 - 4.*AGAGE*LAMDA*NLOAD -
     >         4.*NLOAD*(AGAGE**2.)/(RSUBO + RSUBR) +
     >         4.*NLOAD*LAMDA*(L1**2.)/AGAGE -
     >         4.*LAMDA*FSUB12*(L1**2.)/(RSUBO*AGAGE)
      K(2,2) = YAWK + 4.*FSUB12 - 4.*AGAGE*LAMDA*NLOAD -
     >         4.*NLOAD*(AGAGE**2.)/(RSUBO + RSUBR)
      K(2,3) = 4.*LAMDA*(FSUB33*(AGAGE/RSUBO - LAMDA*RSUBC/RSUBR) -
     >         FSUB22/(RSUBR*AGAGE))
      K(3,1) = WARPK + K(2,2)
      K(3,2) = K(2,2)
      K(3,3) = K(2,3)
      C(1,1) = 4.*FSUB11/VEL
      C(1,2) = 4.*FSUB12/VEL
      C(1,3) = C(1,2)
      C(2,1) = YAWD + (4./VEL)*(FSUB11*(L1**2.) +FSUB33*(AGAGE**2.) +
     >         FSUB22)
      C(2,2) = YAWD + (4./VEL)*(FSUB22 + FSUB33*(AGAGE**2.))
      C(2,3) = C(1,2)
      C(3,1) = WARPD + C(2,2)
      C(3,2) = C(2,2)
      C(3,3) = C(1,2)
*
* CALCULATION OF A MATRIX ELEMENTS
*
      A(1,1) = 0.0
      A(1,2) = 1.0
      A(1,3) = 0.0
      A(1,4) = 0.0
      A(1,5) = 0.0
      A(1,6) = 0.0
      A(2,1) = -K(1,1)/M(1,1)
      A(2,2) = -C(1,1)/M(1,1)
      A(2,3) = K(1,2)/M(1,1)
      A(2,4) = -C(1,2)/M(1,1)
      A(2,5) = K(1,3)/M(1,1)
      A(2,6) = -C(1,3)/M(1,1)
      A(3,1) = 0.0
      A(3,2) = 0.0
      A(3,3) = 0.0
      A(3,4) = 1.0
      A(3,5) = 0.0
      A(3,6) = 0.0
      A(4,1) = 0.0
      A(4,2) = 0.0
      DEN1 = M(2,1) - M(3,2)
      A(4,3) = (K(3,2)-K(2,1))/DEN1
      A(4,4) = (C(3,2)-C(2,1))/DEN1
      A(4,5) = (K(3,1)-K(2,2))/DEN1
      A(4,6) = (C(3,1)-C(2,2))/DEN1
      A(5,1) = 0.0
      A(5,2) = 0.0
      A(5,3) = 0.0
      A(5,4) = 0.0
      A(5,5) = 0.0
      A(5,6) = 1.0
      DEN2 = M(2,1)/(M(2,1) - M(2,2))
      A(6,1) = (K(2,3)/M(2,1) - K(3,3)/M(3,1))*DEN2
```

```
      A(6,2) = (C(3,3)/M(3,1) - C(2,3)/M(2,1))*DEN2
      A(6,3) = (K(2,1)/M(2,1) - K(3,2)/M(3,1))*DEN2
      A(6,4) = (C(2,1)/M(2,1) - C(3,2)/M(3,1))*DEN2
      A(6,5) = (K(2,2)/M(2,1) - K(3,1)/M(3,1))*DEN2
      A(6,6) = (C(2,2)/M(2,1) - C(3,1)/M(3,1))*DEN2
*
* RETURN TO MAIN PROGRAM
*
      RETURN
      END
***********************************************************************
*
*       SUBROUTINE EIGEN
*
*     THIS SUBROUTINE WAS CREATED FROM THE PROGRAM
*     TO CALCULATE EIGENVALUES AND EIGENVECTORS FROM
*     THE FOLLOWING REFERENCE
*
*     IMSL USER'S MANUAL, 1987, VOL. 1,
*     INTERNATIONAL MATHEMATICAL AND STATISTICAL
*     LIBRARIES, INC., HOUSTON, TEXAS.
*
***********************************************************************
      SUBROUTINE EIGEN(AA)
      REAL AA(6,6)
      INTEGER N,NOUT,LDA,LDEVEC
      COMPLEX EVAL(6), EVEC(6,6)
      N = 6
      LDA = 6
      LDEVEC = 6

C  FIND EIGENVALUES AND VECTORS OF AA

      CALL EVCRG(N,AA,LDA,EVAL,EVEC,LDEVEC)

C  PRINT RESULTS

      CALL UMACH(2,NOUT)
      CALL WRCRN('EVAL',1,N,EVAL,1,0)
      CALL WRCRN('EVEC',N,N,EVEC,LDEVEC,0)
      RETURN
      END
```

Appendix B

Program NLINMOD

```
**********************************************************************
*
*         NONLINEAR SIMULATION PROGRAM
*
*     THIS PROGRAM USES THE FORTH ORDER RUNGA KUTTA
*     METHOD TO SOLVE 6 FIRST ORDER DIFFERENTIAL
*     EQUATIONS DESCRIBING THE THREE DEGREES OF
*     FREEDOM 1/5 SCALE FREIGHT CAR TRUCK.
*
**********************************************************************
      REAL*8 Q(6),K1(6),K2(6),K3(6),K4(6),H,A(6,6),V,B(6),TABLE(241,8)
      REAL*8 Q1,Q2,Q3,Q4,Q5,Q6,ALPHA
      REAL*8 DQ1,DQ2,DQ3,DQ4,DQ5,DQ6,TIME,TT,WARPD,WARPK,YAWD,YAWK,XLIM
      INTEGER LIMIT,LFILE
      COMMON TABLE
*
* DATA FOR INITIAL VALUES
*
      DATA Q/-0.011,0.0,0.0,0.0,0.0,0.0/
      DATA ALPHA/0.0/
*
* END OF DATA
*
*
* DEFINE THE SIX FIRST ORDER DIFFERENTIAL EQUATIONS
*
      DQ1(Q1,Q2,Q3,Q4,Q5,Q6) = A(1,1)*Q1 + A(1,2)*Q2 + A(1,3)*Q3 +
     >                         A(1,4)*Q4 + A(1,5)*Q5 + A(1,6)*Q6 + B(1)
      DQ2(Q1,Q2,Q3,Q4,Q5,Q6) = A(2,1)*Q1 + A(2,2)*Q2 + A(2,3)*Q3 +
     >                         A(2,4)*Q4 + A(2,5)*Q5 + A(2,6)*Q6 + B(2)
      DQ3(Q1,Q2,Q3,Q4,Q5,Q6) = A(3,1)*Q1 + A(3,2)*Q2 + A(3,3)*Q3 +
     >                         A(3,4)*Q4 + A(3,5)*Q5 + A(3,6)*Q6 + B(3)
      DQ4(Q1,Q2,Q3,Q4,Q5,Q6) = A(4,1)*Q1 + A(4,2)*Q2 + A(4,3)*Q3 +
     >                         A(4,4)*Q4 + A(4,5)*Q5 + A(4,6)*Q6 + B(4)
      DQ5(Q1,Q2,Q3,Q4,Q5,Q6) = A(5,1)*Q1 + A(5,2)*Q2 + A(5,3)*Q3 +
     >                         A(5,4)*Q4 + A(5,5)*Q5 + A(5,6)*Q6 + B(5)
      DQ6(Q1,Q2,Q3,Q4,Q5,Q6) = A(6,1)*Q1 + A(6,2)*Q2 + A(6,3)*Q3 +
```

```
     >                            A(6,4)*Q4 + A(6,5)*Q5 + A(6,6)*Q6 + B(6)
*
* OPEN OUTPUT FILES
*
      OPEN(10, FILE = 'LAT', STATUS = 'NEW')
      OPEN(11, FILE = 'YAW', STATUS = 'NEW')
      OPEN(12, FILE = 'WARP', STATUS = 'NEW')
*
*  OPEN INPUT FILE
*
      OPEN(13, FILE = 'LKUP',ACCESS = 'SEQUENTIAL',FORM = 'FORMATTED',
     >     STATUS = 'OLD')
*
*  INITIALIZE T
*
      DO 15 I = 1,241
         DO 10 J = 1,8
              TABLE(I,J) = 0.D0
   10    CONTINUE
   15 CONTINUE
*
*  READ LOOKUP TABLE VALUES INTO MATRIX
*
      DO 25 JJ = 1,3
         READ(13,*)
   25 CONTINUE
      DO 30 I = 1,241
         READ(13,1000) TABLE(I,1),TABLE(I,2),
     >    TABLE(I,3),TABLE(I,4),TABLE(I,5)
     >   ,TABLE(I,6),TABLE(I,7),TABLE(I,8)
   30 CONTINUE
 1000 FORMAT(5X,D6.3,D9.5,D8.5,D9.5,D8.5,D8.5,D8.5,D10.6)

*
* ASK USER FOR STEP SIZE
*
      WRITE(6,*)'ENTER STEP SIZE (H IN SECONDS)'
      READ(5,*) H
*
* ASK FOR ENDING TIME
*
      WRITE(6,*)'ENTER ENDING TIME (SECONDS)'
      READ(5,*) TIME
*
* ASK FOR VELOCITY
*
      WRITE(6,*)'ENTER VELOCITY (FT/SEC)'
      READ(5,*) V
*
* ASK FOR STIFFNESSES AND DAMPING VALUES
*
      WRITE(6,*)'ENTER THE WARP AND YAW STIFFNESSES (FT-LB/RAD)'
      READ(5,*) WARPK, YAWK
      WRITE(6,*)'ENTER THE WARP AND YAW DAMPING (FT-LB-SEC)'
      READ(5,*) WARPD, YAWD
      LIMIT = TIME/H
      LFILE = LIMIT/490
      XLIM = 1./(5.*12.)
*
* START INTEGRATION LOOP
*
      TT = 0.0
      N = 0
      DO 100 I = 1,LIMIT
         TT = TT + H
         N = N + 1
*
```

```
* CALL SUBROUTINE CALCNL TO GET VALUE OF MATRIX A AND MATRIX B
*
      CALL CALCNL(V,WARPK,YAWK,WARPD,YAWD,A,B,Q,ALPHA)
*
*        CALCULATION OF K1'S
*
         K1(1) = DQ1(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
         K1(2) = DQ2(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
         K1(3) = DQ3(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
         K1(4) = DQ4(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
         K1(5) = DQ5(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
         K1(6) = DQ6(Q(1),Q(2),Q(3),Q(4),Q(5),Q(6))
*
*        CALCULATION OF K2'S
*
         K2(1) = DQ1(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
     >               Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
         K2(2) = DQ2(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
     >               Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
         K2(3) = DQ3(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
     >               Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
         K2(4) = DQ4(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
     >               Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
         K2(5) = DQ5(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
     >               Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
         K2(6) = DQ6(Q(1)+.5*H*K1(1),Q(2)+.5*H*K1(2),Q(3)+.5*H*K1(3),
     >               Q(4)+.5*H*K1(4),Q(5)+.5*H*K1(5),Q(6)+.5*H*K1(6))
*
*        CALCULATION OF K3'S
*
         K3(1) = DQ1(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >               Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
         K3(2) = DQ2(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >               Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
         K3(3) = DQ3(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >               Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
         K3(4) = DQ4(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >               Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
         K3(5) = DQ5(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >               Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
         K3(6) = DQ6(Q(1)+.5*H*K2(1),Q(2)+.5*H*K2(2),Q(3)+.5*H*K2(3),
     >               Q(4)+.5*H*K2(4),Q(5)+.5*H*K2(5),Q(6)+.5*H*K2(6))
*
*        CALCULATION OF K4'S
*
         K4(1) = DQ1(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >               Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
         K4(2) = DQ2(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >               Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
         K4(3) = DQ3(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >               Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
         K4(4) = DQ4(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >               Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
         K4(5) = DQ5(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >               Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
         K4(6) = DQ6(Q(1) + H*K3(1),Q(2) + H*K3(2),Q(3) + H*K3(3),
     >               Q(4) + H*K3(4),Q(5) + H*K3(5),Q(6) + H*K3(6))
*
*        FIND VALUES OF Q FOR NEXT TIME STEP.
*
         DO 50 J = 1,6
              Q(J) = Q(J) + H*(K1(J) + 2.D0*K2(J) +
     >               2.D0*K3(J) + K4(J))/6.D0
   50    CONTINUE
*
*        OUTPUT TO FILES
*
```

```
         IF (N .EQ. LFILE) THEN
         WRITE(10,200) TT, Q(1)
         WRITE(11,200) TT, Q(3)
         WRITE(12,200) TT, Q(5)
         N = 0
         ENDIF
  100 CONTINUE
  200 FORMAT(4X,F10.7,4X,F15.7)
      CLOSE(UNIT = 13)
      STOP
      END
**********************************************************************
*
*         SUBROUTINE CALCNL
*
*     THIS SUBROUTINE CALCULATES THE ELEMENTS OF
*     THE A AND B MATRICES WHICH CONTAINS THE
*     COEFFICIENTS FOR THE SIX FIRST ORDER DIFFERENTIAL
*     EQUATIONS.  THIS IS ALSO WHERE THE VALUES FOR
*     THE DIFFERENT VARIABLES ARE ENTERED.
*
**********************************************************************
*  LIST OF VARIABLES
*
*  AGAGE = HALF RAIL GAGE
*  D = HALF SIDEFRAME SEPARATION
*  ISUBB = BOLSTER INERTIAL MOMENT
*  ISUBF = SIDEFRAME INERTIAL MOMENT
*  ISUBW = WHEELSET INERTIAL MOMENT
*  L1 = HALF WHEELBASE
*  MSUBB = BOLSTER MASS
*  MSUBF = SIDEFRAME MASS
*  MSUBW = WHEELSET MASS
*  MSUBRR = RESTRAINING ROD MASS
*  NLOAD = LOAD PER WHEEL
*  RSUBC = RAIL CROWN RADIUS
*  RSUBO = NOMINAL ROLLING RADIUS
*  RSUBR = RAIL ROLLER RADIUS
*  VEL = WHEEL/RAIL RELATIVE VELOCITY
*  Q(1) = LATERAL DISPLACEMENT
*  Q(2) = LATERAL VELOCITY
*  Q(3) = YAW ANGLE
*  Q(4) = YAW ANGULAR VELOCITY
*  Q(5) = WARP ANGLE
*  Q(6) = WARP ANGULAR VELOCITY
*  WARPK = WARP STIFFNESS
*  WARPD = WARP DAMPING
*  YAWK = YAW STIFFNESS
*  YAWD = YAW DAMPING
*
*
      SUBROUTINE CALCNL(VEL,WARPK,YAWK,WARPD,YAWD,A,B,Q,ALPHA)
      REAL*8 K(3,3),M(3,3),A(6,6),C(3,3),VEL,B(6),ALPHA
      REAL*8 MSUBW,MSUBB,MSUBRR,MSUBF,ISUBW,ISUBF,ISUBB,NLOAD,L1,MBEAR
      REAL*8 DELTA(4),PHIW12,PHIW34,FX(4),FY(4),R(4),RR(4),MOM(4)
      REAL*8 Q(6),RIGHT1,RIGHT2,RIGHT3,T1, T2, T3, T4, C1, C2, C3, C4
      REAL*8 BETA, S12, S34, ETA, TABLE(241,8)
      REAL*8 AGAGE,D,DEN1,RSUBC,RSUBO,RSUBR,WARPD,WARPK,YAWD,YAWK
      REAL*8 DELTAT(4),PHIW12T,PHIW34T,RT(4),RRT(4)
      COMMON TABLE
*
* DATA FOR MASSES
*
      DATA MSUBW/0.958/, MSUBB/0.111/,
     >     MSUBRR/0.103/,MSUBF/0.458/
*
```

```
* DATA FOR INERTIAS
*
      DATA ISUBW/0.228/,ISUBF/0.122/,ISUBB/0.0096/
*
* DATA FOR OTHER VARIABLES
*
      DATA AGAGE/0.5/, NLOAD/25.9/,
     >     RSUBO/0.3/, RSUBR/0.5417/,
     >     D/0.3646/, L1/0.5833/,
     >     RSUBC/0.167/
*
* END OF DATA
*
* INITIALIZE
*
      PHIW12 = 0.D0
      PHIW34 = 0.D0
      DO 10 I = 1,4
         R(I) = 0.D0
         RR(I) = 0.D0
         MOM(I) = 0.D0
         FX(I) = 0.D0
         FY(I) = 0.D0
         DELTA(I) = 0.D0
   10 CONTINUE
      DO 20 I = 1,3
         DO 15 J = 1,3
              K(I,J) = 0.D0
              C(I,J) = 0.D0
              M(I,J) = 0.D0
   15    CONTINUE
   20 CONTINUE
      DO 200 II = 1,6
         DO 150 JJ = 1,6
              A(II,JJ) = 0.D0
  150    CONTINUE
         B(II) = 0.D0
  200 CONTINUE
*
*  CALL SUBROUTINE LOOKUP
*
      CALL LOOKUP(Q,R,RR,DELTA,PHIW12,PHIW34,L1)
*
*  CALL SUBROUTINE CREEP
*
      CALL CREEP(RR,R,RSUBR,RSUBO,AGAGE,VEL,Q,L1,DELTA,FX,FY,
     >           MOM,NLOAD,PHIW12,PHIW34)
*
* CALCULATION OF TAN AND COS AND SIN TERMS
*
      T1 = DTAN(DELTA(1) + PHIW12)
      T2 = DTAN(DELTA(2) - PHIW12)
      T3 = DTAN(DELTA(3) + PHIW34)
      T4 = DTAN(DELTA(4) - PHIW34)
      C1 = DCOS(DELTA(1) + PHIW12)
      C2 = DCOS(DELTA(2) - PHIW12)
      C3 = DCOS(DELTA(3) + PHIW34)
      C4 = DCOS(DELTA(4) - PHIW34)
      S12 = DSIN(PHIW12)
      S34 = DSIN(PHIW34)
      BETA = FX(2)*C2 - FX(1)*C1 - FX(3)*C3 + FX(4)*C4 +
     >       NLOAD*(T1+T2+T3+T4)
      ETA = 1./(R(1) + RR(1)) + 1./(R(2) + RR(2)) +
     >      1./(R(3) + RR(3)) + 1./(R(4) + RR(4))
*
* CALCULATION OF COEFFICIENTS WHERE M, K, AND C REPRESENT
```

```
* MASS, STIFFNESS, AND DAMPING TERMS, RESPECTIVELY.
*
      M(1,1) = 2.*MSUBW + MSUBB + MSUBRR/3. + 2.*MSUBF
      M(2,1) = 2.*ISUBW + 2.*ISUBF + 2.*MSUBW*(L1**2.) + 2.*MSUBF*
     >(D**2.) + ISUBB
      M(2,2) = 2.*ISUBW + 2.*MSUBF*(D**2.) + ISUBB
      M(3,1) = M(2,2)
      M(3,2) = M(2,2)
*
      K(1,1) = 0.0
      K(1,2) = 0.0
      K(2,1) = YAWK
      K(2,2) = K(2,1)
      K(3,1) = WARPK + YAWK
      K(3,2) = K(2,1)
      C(2,1) = YAWD
      C(2,2) = YAWD
      C(3,1) = WARPD + YAWD
      C(3,2) = C(2,2)
*
* CALCULATION OF A MATRIX ELEMENTS
*
      A(1,1) = 0.0
      A(1,2) = 1.0
      A(1,3) = 0.0
      A(1,4) = 0.0
      A(1,5) = 0.0
      A(1,6) = 0.0
      A(2,1) = 0.0
      A(2,2) = 0.0
      A(2,3) = 0.0
      A(2,4) = 0.0
      A(2,5) = 0.0
      A(2,6) = 0.0
      A(3,1) = 0.0
      A(3,2) = 0.0
      A(3,3) = 0.0
      A(3,4) = 1.0
      A(3,5) = 0.0
      A(3,6) = 0.0
      A(4,1) = 0.0
      A(4,2) = 0.0
      DEN1 = M(2,1) - M(2,2)
      A(4,3) = (K(3,2) - K(2,1))/DEN1
      A(4,4) = 0.0
      A(4,5) = (K(3,1)-K(2,2))/DEN1
      A(4,6) = (C(3,1)-C(2,2))/DEN1
      A(5,1) = 0.0
      A(5,2) = 0.0
      A(5,3) = 0.0
      A(5,4) = 0.0
      A(5,5) = 0.0
      A(5,6) = 1.0
      A(6,1) = 0.0
      A(6,2) = 0.0
      A(6,3) = (K(2,1) - M(2,1)*K(3,2)/M(3,1))/DEN1
      A(6,4) = (C(2,1) - M(2,1)*C(3,2)/M(3,1))/DEN1
      A(6,5) = (K(2,2) - M(2,1)*K(3,1)/M(3,1))/DEN1
      A(6,6) = (C(2,2) - M(2,1)*C(3,1)/M(3,1))/DEN1
*
*  CALL SUBROUTINE BEAR
*
      CALL BEAR(Q(5),Q(6),K(3,1),MBEAR,ALPHA)
*
* CALCULATION OF RIGHT1, RIGHT2, AND RIGHT3
*
      RIGHT1 = NLOAD*(T2-T1+T4-T3) + FX(1)*C1 + FX(2)*C2
```

```
     >         + FX(3)*C3 + FX(4)*C4 + (FY(1) + FY(2))*S12
     >         + (FY(3) + FY(4))*S34
      RIGHT2 = L1*(NLOAD*(T2-T1-T4+T3) + FX(1)*C1 + FX(2)*C2
     >         - FX(3)*C3 - FX(4)*C4 + (FY(1) + FY(2))*S12
     >         - (FY(3) + FY(4))*S34) + MOM(1)*C1 + MOM(2)*C2
     >         + MOM(3)*C3 + MOM(4)*C4 + AGAGE*(FY(2) + FY(4)
     >         - FY(1) -FY(3)) - 4.*MBEAR
     >         + AGAGE*DSIN(Q(5)+Q(3))*BETA + NLOAD*(AGAGE**2.)*
     >         DSIN(Q(5)+Q(3))*ETA
      RIGHT3 = MOM(1)*C1 + MOM(2)*C2 + MOM(3)*C3 + MOM(4)*C4 +
     >         AGAGE*(FY(2) + FY(4) - FY(1) -FY(3)) - 4.*MBEAR
     >         + AGAGE*DSIN(Q(5)+Q(3))*BETA + NLOAD*(AGAGE**2.)*
     >         DSIN(Q(5)+Q(3))*ETA


*
* CALCULATION OF B MATRIX ELEMENTS
*
      B(1) = 0.0
      B(2) = RIGHT1/M(1,1)
      B(3) = 0.0
      B(4) = (RIGHT2 - RIGHT3)/DEN1
      B(5) = 0.0
      B(6) = (-RIGHT2 + M(2,1)*RIGHT3/M(3,1))/DEN1
*
* RETURN TO CALLING PROGRAM
*
      RETURN
      END
************************************************************************
*
*        SUBROUTINE LOOKUP
*
*     THIS SUBROUTINE FINDS THE WHEEL AND RAIL GEOMETRIC
*     VARIABLES FROM A LOOKUP TABLE.
*
************************************************************************
*
      SUBROUTINE LOOKUP(Q,R,RR,DELTA,PHIW12,PHIW34,L1)
      REAL*8 Q(6),R(4),RR(4),DELTA(4),PHIW12,PHIW34,MR0,ML0,MR1,ML1
      REAL*8 L1, TABLE(241,8)
      REAL*8 XW0,XW1,FRONTX,REARX,WR0,WR1, RRMID, YRMID, RLMID
      REAL*8 YLMID, RR0, RR1, YR0, YR1, RL0, RL1, YL0, YL1
      REAL*8 XW2, RR2,YR2, RL2, YL2,WR2,MR2,ML2
      COMMON TABLE
*
*  CALCULATE FRONT AND REAR LATERAL DISPLACEMENTS
*
*  INIT
      FRONTX = 0.D0
      REARX = 0.D0
         XW0 = 0.D0
         RR0 = 0.D0
         YR0 = 0.D0
         RL0 = 0.D0
         YL0 = 0.D0
         MR0 = 0.D0
         ML0 = 0.D0
         WR0 = 0.D0
*
         XW1 = 0.D0
         RR1 = 0.D0
         YR1 = 0.D0
         RL1 = 0.D0
         YL1 = 0.D0
         MR1 = 0.D0
         ML1 = 0.D0
```

```
         WR1 = 0.D0
         XW2 = 0.D0
         RR2 = 0.D0
         YR2 = 0.D0
         RL2 = 0.D0
         YL2 = 0.D0
         MR2 = 0.D0
         ML2 = 0.D0
         WR2 = 0.D0
         PHIW12 = 0.D0
         PHIW34 = 0.D0
         DELTA(1) = 0.D0
         DELTA(2) = 0.D0
         DELTA(3) = 0.D0
         DELTA(4) = 0.D0
*
*
      FRONTX = Q(1) + L1*DSIN(Q(3))
      REARX = Q(1) - L1*DSIN(Q(3))
*
*  CONVERT TO FULL SIZE FOR LOOKUP TABLE AND CONVERT TO INCHES
*
      FRONTX = 5.*12.*FRONTX
      REARX = 5.*12.*REARX
*
*
*
*  FIND LOW VALUE IN TABLE FIRST FOR FRONTX
*
      LOW = 0
      LOW = FRONTX * 100. + 121.
      IF(TABLE(LOW,1).EQ.FRONTX) THEN
         RRMID = TABLE(LOW,2)
         YRMID = TABLE(LOW,3)
         RLMID = TABLE(LOW,4)
         YLMID = TABLE(LOW,5)
         DELTA(1) = TABLE(LOW,7)
         DELTA(2) = TABLE(LOW,6)
         PHIW12 = TABLE(LOW,8)
         GOTO 150
      ENDIF
         XW0 = TABLE(LOW,1)
         RR0 = TABLE(LOW,2)
         YR0 = TABLE(LOW,3)
         RL0 = TABLE(LOW,4)
         YL0 = TABLE(LOW,5)
         MR0 = TABLE(LOW,6)
         ML0 = TABLE(LOW,7)
         WR0 = TABLE(LOW,8)
*
         XW1 = TABLE(LOW+1,1)
         RR1 = TABLE(LOW+1,2)
         YR1 = TABLE(LOW+1,3)
         RL1 = TABLE(LOW+1,4)
         YL1 = TABLE(LOW+1,5)
         MR1 = TABLE(LOW+1,6)
         ML1 = TABLE(LOW+1,7)
         WR1 = TABLE(LOW+1,8)
*
         XW2 = TABLE(LOW+2,1)
         RR2 = TABLE(LOW+2,2)
         YR2 = TABLE(LOW+2,3)
         RL2 = TABLE(LOW+2,4)
         YL2 = TABLE(LOW+2,5)
         MR2 = TABLE(LOW+2,6)
         ML2 = TABLE(LOW+2,7)
```

```
         WR2 = TABLE(LOW+2,8)
*
*  CALL INTERP TO GET INTERPOLATED VALUES
*
      CALL INTERP(FRONTX,XW0,XW1,XW2,RRMID,RR0,RR1,RR2)
      CALL INTERP(FRONTX,XW0,XW1,XW2,YRMID,YR0,YR1,YR2)
      CALL INTERP(FRONTX,XW0,XW1,XW2,RLMID,RL0,RL1,RL2)
      CALL INTERP(FRONTX,XW0,XW1,XW2,YLMID,YL0,YL1,YL2)
      CALL INTERP(FRONTX,XW0,XW1,XW2,DELTA(1),ML0,ML1,ML2)
      CALL INTERP(FRONTX,XW0,XW1,XW2,DELTA(2),MR0,MR1,MR2)
      CALL INTERP(FRONTX,XW0,XW1,XW2,PHIW12,WR0,WR1,WR2)
*
*  CONVERT DISTANCE VALUES TO SCALED MODEL VALUES AND CONVERT
*  TO FEET ALSO.
*
  150 R(1) = RLMID/60.D0 + 3033.D-4/12.D0
      R(2) = RRMID/60.D0 + 3033.D-4/12.D0
      RR(1) = YLMID*(6.475/7.125)*1.00363/12.D0
      RR(2) = YRMID*(6.475/7.125)*1.00363/12.D0
*
*  REPEAT FOR REARX
*
*  INIT
         XW0 = 0.D0
         RR0 = 0.D0
         YR0 = 0.D0
         RL0 = 0.D0
         YL0 = 0.D0
         MR0 = 0.D0
         ML0 = 0.D0
         WR0 = 0.D0
         XW1 = 0.D0
         RR1 = 0.D0
         YR1 = 0.D0
         RL1 = 0.D0
         YL1 = 0.D0
         MR1 = 0.D0
         ML1 = 0.D0
         WR1 = 0.D0
         XW2 = 0.D0
         RR2 = 0.D0
         YR2 = 0.D0
         RL2 = 0.D0
         YL2 = 0.D0
         MR2 = 0.D0
         ML2 = 0.D0
         WR2 = 0.D0
*
*  FIND LOW VALUE IN TABLE FIRST FOR REARX
*
      LOW = 0
      LOW = REARX * 100. + 121.
      IF(TABLE(LOW,1).EQ.REARX) THEN
         RRMID = TABLE(LOW,2)
         YRMID = TABLE(LOW,3)
         RLMID = TABLE(LOW,4)
         YLMID = TABLE(LOW,5)
         DELTA(3) = TABLE(LOW,7)
         DELTA(4) = TABLE(LOW,6)
         PHIW34 = TABLE(LOW,8)
         GOTO 350
      ENDIF
         XW0 = TABLE(LOW,1)
         RR0 = TABLE(LOW,2)
         YR0 = TABLE(LOW,3)
         RL0 = TABLE(LOW,4)
         YL0 = TABLE(LOW,5)
```

```
         MR0 = TABLE(LOW,6)
         ML0 = TABLE(LOW,7)
         WR0 = TABLE(LOW,8)
*
         XW1 = TABLE(LOW+1,1)
         RR1 = TABLE(LOW+1,2)
         YR1 = TABLE(LOW+1,3)
         RL1 = TABLE(LOW+1,4)
         YL1 = TABLE(LOW+1,5)
         MR1 = TABLE(LOW+1,6)
         ML1 = TABLE(LOW+1,7)
         WR1 = TABLE(LOW+1,8)
*
         XW2 = TABLE(LOW+2,1)
         RR2 = TABLE(LOW+2,2)
         YR2 = TABLE(LOW+2,3)
         RL2 = TABLE(LOW+2,4)
         YL2 = TABLE(LOW+2,5)
         MR2 = TABLE(LOW+2,6)
         ML2 = TABLE(LOW+2,7)
         WR2 = TABLE(LOW+2,8)
*
*  CALL INTERP TO GET INTERPOLATED VALUES
*
      CALL INTERP(REARX,XW0,XW1,XW2,RRMID,RR0,RR1,RR2)
      CALL INTERP(REARX,XW0,XW1,XW2,YRMID,YR0,YR1,YR2)
      CALL INTERP(REARX,XW0,XW1,XW2,RLMID,RL0,RL1,RL2)
      CALL INTERP(REARX,XW0,XW1,XW2,YLMID,YL0,YL1,YL2)
      CALL INTERP(REARX,XW0,XW1,XW2,DELTA(3),ML0,ML1,ML2)
      CALL INTERP(REARX,XW0,XW1,XW2,DELTA(4),MR0,MR1,MR2)
      CALL INTERP(REARX,XW0,XW1,XW2,PHIW34,WR0,WR1,WR2)
*
*  CONVERT DISTANCE VALUES TO SCALED MODEL VALUES AND CONVERT
*  TO FEET ALSO.
*
  350 R(3) = RLMID/60.D0 + 3033.D-4/12.D0
      R(4) = RRMID/60.D0 + 3033.D-4/12.D0
      RR(3) = YLMID*(6.475/7.125)*1.00363/12.D0
      RR(4) = YRMID*(6.475/7.125)*1.00363/12.D0
*
*
*  RETURN TO CALLING PROGRAM
*
      RETURN
      END
**********************************************************************
*
*               SUBROUTINE INTERP
*
*     THIS SUBROUTINE USES QUADRATIC INTERPOLATION TO
*     FIND THE INTERMEDIATE VALUE BETWEEN TWO DATA POINTS.
*
**********************************************************************
*
      SUBROUTINE INTERP(Z,Z0,Z1,Z2,F,F0,F1,F2)
      REAL*8 A0,A1,A2,B0,B1,B2
      REAL*8 F, F0, F1, Z, Z0, Z1,Z2,F2
*
*  INITIALIZE F
*
      F = 0.D0
*
*  CALCULATE F
*
      B0 = F0
      B1 = (F1 - F0)/(-1.D-2)
```

```
      B2 = ((F2 - F1)/(-1.D-2) - (F1 - F0)/(-1.D-2))/(-2.D-2)
*
      A0 = B0 - B1*Z0 + B2*Z0*Z1
      A1 = B1 - B2*Z0 - B2*Z1
      A2 = B2
*
      F = A0 + A1*Z + A2*Z**2
*
*  RETURN TO CALLING PROGRAM
*
      RETURN
      END
*****************************************************************
*
*              SUBROUTINE CREEP
*
*         THIS SUBROUTINE CALCULATES THE CREEPAGES AND
*         THE CREEP FORCES AND MOMENTS AT EACH WHEEL
*
*****************************************************************
*
*  RR = INSTANEOUS ROLLER RADIUS
*  R = INSTANEOUS WHEEL RADIUS
*  DELTA = CONTACT ANGLE
*  FX = WHEEL FORCE IN THE LATERAL DIRECTION
*  FY = WHEEL FORCE IN THE LONGITUDINAL DIRECTION
*  MOM = WHEEL MOMENT IN Z DIRECTION
*  F11 = LATERAL CREEP COEFFICIENT
*  F12 = SPIN/LATERAL CREEP COEFFICIENT
*  F22 = SPIN CREEP COEFFICIENT
*  F33 = LONGITUDINAL CREEP COEFFICIENT
*  MU = COEFFICIENT OF FRICTION
*
*
      SUBROUTINE CREEP(RR,R,RSUBR,RSUBO,AGAGE,VEL,Q,L1,DELTA,FX,FY,MOM,
     >NLOAD,PHIW12,PHIW34)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 PI,G1,G2,E1,E2,XNU1,XNU2,SMRI,G,XNU,XK1,XK2,APB,BMA,COST
      REAL*8 THETA,AI,XM,XN,TERM,P,ALONG,BTRANS,AREA,AVGBST,Q0
      REAL*8 TAUMAX, AOVERB,C11,C22,C23,C33,C,SMR,CTANGL
      REAL*8 FLAT,FLON,FRP,FMAX,FRAT,GLON,GLAT,GSP
      REAL*8 RR(4),R(4),Q(6),L1,DELTA(4),FX(4),FY(4),MOM(4),GAMMA(3,4)
      REAL*8 NLOAD,F11,F12,F22,F33,MU,LIMIT
      REAL*8 MSP,NUX,NUY,FR
      REAL*8 RRAD,RINV,RPINV,ROLRI,PHIW12,PHIW34
      REAL*8 RSUBO,RSUBR,VAR1,VEL,AGAGE
      REAL*8 ARG(23),VALM(23),VALN(23)
C     ***********************************************************
C     ARGUMENT RANGE FOR M,N HAS BEEN EXTENDED FROM 0.5 TO 30 DEG., SEE
C     KORNHAUSER, J. APPL. MECH., SEPTEMBER, 1951
C     ***********************************************************
C     LIST VALUES OF VALM(I), VALN(I) AFTER THIS CARD
C     ***********************************************************
      ARG(1) = .5
      ARG(2) = 1.
      ARG(3) = 1.5
      ARG(4) = 2.
      ARG(5) = 3.
      ARG(6) = 4.
      ARG(7) = 6.
      ARG(8) = 8.
      ARG(9) = 10.
      ARG(10) = 20.
      ARG(11) = 30.
      ARG(12) = 35.
      ARG(13) = 40.
      ARG(14) = 45.
```

```
      ARG(15) = 50.
      ARG(16) = 55.
      ARG(17) = 60.
      ARG(18) = 65.
      ARG(19) = 70.
      ARG(20) = 75.
      ARG(21) = 80.
      ARG(22) = 85.
      ARG(23) = 90.
      VALM(1) = 61.4
      VALM(2) = 36.89
      VALM(3) = 27.48
      VALM(4) = 22.26
      VALM(5) = 16.5
      VALM(6) = 13.31
      VALM(7) = 9.79
      VALM(8) = 7.86
      VALM(9) = 6.604
      VALM(10) = 3.813
      VALM(11) = 2.731
      VALM(12) = 2.397
      VALM(13) = 2.136
      VALM(14) = 1.926
      VALM(15) = 1.754
      VALM(16) = 1.611
      VALM(17) = 1.486
      VALM(18) = 1.378
      VALM(19) = 1.284
      VALM(20) = 1.202
      VALM(21) = 1.128
      VALM(22) = 1.061
      VALM(23) = 1.
      VALN(1) = .1018
      VALN(2) = .1314
      VALN(3) = .1522
      VALN(4) = .1691
      VALN(5) = .1964
      VALN(6) = .2188
      VALN(7) = .2552
      VALN(8) = .285
      VALN(9) = .3112
      VALN(10) = .4123
      VALN(11) = .493
      VALN(12) = .53
      VALN(13) = .567
      VALN(14) = .604
      VALN(15) = .641
      VALN(16) = .678
      VALN(17) = .717
      VALN(18) = .759
      VALN(19) = .802
      VALN(20) = .846
      VALN(21) = .893
      VALN(22) = .944
      VALN(23) = 1.
*
*  INITIALIZE GAMMA
*
      DO 10 I = 1,3
         DO 5 J = 1,4
              GAMMA(I,J) = 0.D0
    5    CONTINUE
   10 CONTINUE
*
*  CALCULATION OF LONGITUDINAL CREEPAGES
*
      GAMMA(1,1) = -RR(1)/RSUBR + R(1)/RSUBO + AGAGE*(Q(6)+Q(4))/VEL
```

```
      GAMMA(1,2) = -RR(2)/RSUBR + R(2)/RSUBO - AGAGE*(Q(6)+Q(4))/VEL
      GAMMA(1,3) = -RR(3)/RSUBR + R(3)/RSUBO + AGAGE*(Q(6)+Q(4))/VEL
      GAMMA(1,4) = -RR(4)/RSUBR + R(4)/RSUBO - AGAGE*(Q(6)+Q(4))/VEL
*
*  CALCULATION OF LATERAL CREEPAGES
*
      GAMMA(2,1) = Q(2)/VEL - Q(5) - Q(3) + L1*Q(4)/VEL
      GAMMA(2,2) = GAMMA(2,1)
      GAMMA(2,3) = Q(2)/VEL - Q(5) - Q(3) - L1*Q(4)/VEL
      GAMMA(2,4) = GAMMA(2,3)
*
*  CALCULATION OF SPIN CREEPAGES
*
      VAR1 = (Q(6) + Q(4))/VEL

      GAMMA(3,1) = -DSIN(DELTA(1))/RSUBO -
     >             DSIN(DELTA(1) + PHIW12)/RSUBR + VAR1
      GAMMA(3,2) = DSIN(DELTA(2))/RSUBO +
     >             DSIN(DELTA(2) - PHIW12)/RSUBR + VAR1
      GAMMA(3,3) = -DSIN(DELTA(3))/RSUBO -
     >             DSIN(DELTA(3) + PHIW34)/RSUBR + VAR1
      GAMMA(3,4) = DSIN(DELTA(4))/RSUBO +
     >             DSIN(DELTA(4) - PHIW34)/RSUBR + VAR1
*
*  CALCULATE FORCES AND MOMENTS AT EACH WHEEL
*
      DO 1010 III = 1,4
*
* FOLLOWING SECTION IS FROM A PROGRAM CALLED FAST1
*
*
C
C     REFERENCES:
C
C         CONTACT AREA,  TIMOSHENKO + GOODIER,'THEORY OF ELASTICITY',
C                        MCGRAW-HILL, 2ND ED., 1951
C
C         CREEP,  KALKER,J.J.,"THE TANGENTIAL FORCE TRANSMITTED BY TWO
C                 ELASTIC BODIES ROLLING OVER EACH OTHER WITH PURE
C                 CREEPAGE", WEAR II,1968, PP.421-430
C
C
C         HEURISTIC MODEL,  Shen, Z. Y., et al. "A Comparison of
C                           Alternative Creep Force Models for Rail
C                           Vehicle Dynamic Analysis," 8th IAVSD
C                           Symposium Preceedings, August 1983,
C                           pp 591-605.
C
C     VARIABLES:
C
C                   RINV = 1/WHEEL PROFILE RADIUS,  1/FT
C                   SMR    = WHEEL ROLLING RADIUS, FT
C                   RPINV = 1/RAIL HEAD PROFILE RADIUS, 1/FT
C                     IF RINV,RPINV < 0 THEN THE CENTER OF CURVATURE
C                     OF THE PROFILE IS OUTSIDE THE BODY
C                   ROLRI  = 1/ROLLER RADIUS, 1/FT, ROLRI = 0 FOR RAIL
C
C                   P      = NORMAL LOAD ACROSS CONTACT PATCH, LB
C                   CTANGL = CONTACT ANGLE ON WHEEL
C                   MU     = COEFFICIENT OF FRICTION
C
C                   E1     = YOUNG'S MODULUS RAIL, PSI
C                   XNU1   = POISSON'S RATIO RAIL
C
C                   E2     = YOUNG'S MODULUS WHEEL, PSI
C                   XNU2   = POISSON'S RATIO WHEEL
C
```

```
C                     GLON   = REAL LONGITUDINAL CREEPAGE
C                     GLAT   = REAL LATERAL CREEPAGE
C                     GSP    = REAL SPIN CREEPAGE (1/FT)
C
C
C                     FLAT   = LATERAL CREEP FORCE (LB)
C                     FLON   = LINGITUDINAL CREEP FORCE (LB)
C                     MSP    = SPIN MOMENT (FT-LB)
C
C     OUTPUT VARIABLES:
C
C                     ALONG  = CONTACT ELLIPSE LONGITUDINAL SEMIAXIS (IN)
C                     BTRANS = CONTACT ELLIPSE TRANSVERSE SEMIAXIS (IN)
C                     AREA   = CONTACT ELLIPSE AREA (IN**2)
C
C                     CIJ    = KALKER COEFFICIENTS IN KALKER'S NOTATION
C                     FIJ    = CREEP COEFFICIENTS IN COOPERRIDER AND
C                              LAW NOTATION (DIFFERS FROM KALKER'S)
C
C
C
C
C     PROGRAM VARIABLES:
C
C
C                     G1     = SHEAR MODULUS RAIL, PSI
C                     G2     = SHEAR MODULUS WHEEL,PSI
C
C        Kalker's coordinate system has X in the longitudinal
C        direstionand Y in the lateral direction.
C
C     Written by R. H. Fries 6/87
C
C
C
C     INPUT SECTION
C
      SMR = R(III)
      RRAD= RR(III)
      P = NLOAD
      MU = 0.65D0
      CTANGL = DELTA(III)

      RINV = 0.D0
      RPINV = 6.D0
      ROLRI = 1.D0/RRAD
      E1 = 30.D6
      XNU1 = .25D0
      E2 = 30.D6
      XNU2 = .25D0

C
      GLAT = GAMMA(2,III)
      GLON = GAMMA(1,III)
      GSP = GAMMA(3,III)


C
      PI = DACOS(-1.D0)
      G1 = E1/(2.*(1.+XNU1))
      G2 = E2/(2.*(1.+XNU2))
 3000 KK=0
      SMRI=DCOS(CTANGL)/SMR
      G = 2./((1./G1)+(1./G2))
      XNU =(G/2.)*((XNU1/G1)+(XNU2/G2))
      XK1 =(1.-XNU1*XNU1)/(PI*E1)
```

```
      XK2 =(1.-XNU2*XNU2)/(PI*E2)
C
C     REDEFINE WHEEL ROLLING RADIUS AS PRINCIPAL RADIUS
C
      APB = 0.5*(SMRI + RINV + RPINV + ROLRI)
      BMA = 0.5*DABS(SMRI - RINV - RPINV + ROLRI)
      COST = BMA/APB
      THETA = DACOS(COST) * 57.3
      DO 15 I=1,13
      AI=I
      K=I+10
   15 ARG(K) = 25.+(5.*AI)
      CALL SLI(THETA,ARG,VALM,XM,23,IERM)
      CALL SLI(THETA,ARG,VALN,XN,23,IERN)
 1000 CONTINUE
      TERM = 0.75*PI*P*(XK1+XK2)/(APB/12.)
C
C     ALONG AND BTRANS ARE IN INCHES, AREA IN INCHES**2
C
      IF((SMRI + ROLRI) - (RPINV + RINV)) 4000,4000,4005
C     NOTE  IF SMRI + ROLRI > RPINV + RINV, THEN BTRANS>ALONG
C           IF SMRI + ROLRI < RPINV + RINV, THEN BTRANS < ALONG
 4000 ALONG = XM*(TERM**(1./3.))
      BTRANS = XN*(TERM**(1./3.))
      GO TO 4010
 4005 ALONG = XN*(TERM**(1./3.))
      BTRANS = XM*(TERM**(1./3.))
 4010 CONTINUE
      AREA = PI*ALONG*BTRANS
      AVGBST = P/AREA
C     MAX CONTACT PRESSURE = Q0, MAX SHEAR STRESS = 0.31*Q0
C
      Q0 = 1.5*P/(PI*ALONG*BTRANS)
      TAUMAX = 0.31*Q0
      AOVERB = ALONG/BTRANS
      IF(KK-0)2025,2020,2025
 2020 CALL CONST (ALONG,BTRANS,XNU,C11,C22,C23,C33)
      KK=1
 2025 CONTINUE
      C = DSQRT(ALONG*BTRANS)
C     F11 IN LB/WHEEL, LATERAL
C     F12 IN LBFT/WHEEL, LAT-SPIN
C     F22 IN LBFT2/WHEEL, SPIN
C     F33 IN LB/WHEEL, LONG.
      PERCENT = 0.30
      F11 = PERCENT*C*C*G*C22
      F12 = PERCENT*C*C*C*G*C23/12.
      F22 = PERCENT*C*C*C*C*G*C33/144.
      F33 = PERCENT*C*C*G*C11
 5000 CONTINUE
C
C        This section computes the linear creep forces and moment
C
      FLAT = -F11*GLAT - F12*GSP
      FLON = F33*GLON
      MSP  = F12*GLAT - F22*GSP
C
C        This section computes the forces from the heuristic model
C
      FRP  = DSQRT(FLAT*FLAT + FLON*FLON)
      FMAX = MU*P
      FRAT = FRP/FMAX
      IF (FRP.GE.(3.*FMAX)) THEN
         FLAT = -FMAX*GLAT/DSQRT(GLAT*GLAT+GLON*GLON)
         FLON =  FMAX*GLON/DSQRT(GLAT*GLAT+GLON*GLON)
         MSP  = MSP/FRAT
      ELSE
```

```
         FR = FMAX*(FRAT-FRAT**2./3.+FRAT**3./27.)
         FRAT = FR/FRP
         FLAT = FLAT*FRAT
         FLON = FLON*FRAT
         MSP  = MSP*FRAT
      ENDIF
*******************************************************
         FY(III) = FLON
         FX(III) = FLAT
         MOM(III) = MSP
 1010 CONTINUE
*
*
*  RETURN TO MAIN PROGRAM
*
      RETURN
      END
C   ******************************************************************
      SUBROUTINE CONST(A1,B1,XNU,C11,C22,C23,C33)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 PI,RG,SG,GAM,C11,C22,C23,C33
      DIMENSION AL(4),BE(4),D(12),E(12,20),AR(20),CNT(4),D1(12,9),
     1    D2(12,9),D3(12),D4(12)
C   ******************************************************************
C   SUBROUTINE CONST CALCULATES THE KALKER LINEAR CREEP COEFFICIENTS BY
C
C        (A) LINEAR AND QUADRATIC INTERPOLATION FOR 0 < XNU <0.5
C
C            AND 0.1 < A/B < 11.0
C
C        (B)  CALCULATION FROM ASYMPTOTIC EXPANSIONS FOR A/B > 11.0
C
C        A = SEMI AXIS OF ELLIPSE IN ROLLING DIRECTION
C
C        B = SEMI AXIS OF ELLIPSE IN TRANSVERSE DIRECTION
C
C   ******************************************************************
      REAL NU
      DATA D1/
     $  2.51,3.31,4.85,2.51,2.52,2.53,.334,.473,.731,6.42,8.28,11.7,
     $  2.59,3.37,4.81,2.59,2.63,2.66,.483,.603,.809,3.46,4.27,5.66,
     $  2.68,3.44,4.80,2.68,2.75,2.81,.607,.715,.889,2.49,2.96,3.72,
     $  2.78,3.53,4.82,2.78,2.88,2.98,.720,.823,.977,2.02,2.32,2.77,
     $  2.88,3.62,4.83,2.88,3.01,3.14,.827,.929,1.07,1.74,1.93,2.22,
     $  2.98,3.72,4.91,2.98,3.14,3.31,.930,1.03,1.18,1.56,1.68,1.86,
     $  3.09,3.81,4.97,3.09,3.28,3.48,1.03,1.14,1.29,1.43,1.50,1.60,
     $  3.19,3.91,5.05,3.19,3.41,3.65,1.13,1.25,1.40,1.34,1.37,1.42,
     $  3.29,4.01,5.12,3.29,3.54,3.82,1.23,1.36,1.51,1.27,1.27,1.27/
      DATA D2/
     $ 3.40,4.12,5.20,3.40,3.67,3.98,1.33,1.47,1.63,1.21,1.19,1.16,
     $ 3.51,4.22,5.30,3.51,3.81,4.16,1.44,1.59,1.77,1.16,1.11,1.06,
     $ 3.65,4.36,5.42,3.65,3.99,4.39,1.58,1.75,1.94,1.10,1.04,.954,
     $ 3.82,4.54,5.58,3.82,4.21,4.67,1.76,1.95,2.18,1.05,.965,.852,
     $ 4.06,4.78,5.80,4.06,4.50,5.04,2.01,2.23,2.50,1.01,.892,.751,
     $ 4.37,5.10,6.11,4.37,4.90,5.56,2.35,2.62,2.96,.958,.819,.650,
     $ 4.84,5.57,6.57,4.84,5.48,6.31,2.88,3.24,3.70,.912,.747,.549,
     $ 5.57,6.34,7.34,5.57,6.40,7.51,3.79,4.32,5.01,.868,.674,.446,
     $ 6.96,7.78,8.82,6.96,8.14,9.79,5.72,6.63,7.89,.828,.601,.341/
      DATA D3/
     $ 10.7, 11.7, 12.9, 10.7, 12.8, 16.0, 12.2, 14.6, 18.0, 0.795,
     $ 0.562, 0.228/
      DATA D4/
     $ 11.08, 12.01, 13.10, 11.08, 13.38, 16.90, 13.72, 16.34, 20.20,
     $ 0.785, 0.552, 0.208/
      DATA AR / 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0,1.111111,
     $1.25,1.428571,1.666667,2.0,2.5,3.333333,5.0,10.0,11.0/
      NU=XNU
```

```
      DO 6 I=1,12
      DO 5 J=1,9
      E(I,J)=D1(I,J)
  5   E(I,J+9)=D2(I,J)
      E(I,19)=D3(I)
  6   E(I,20)=D4(I)
      PI=DACOS(-1.D0)
      RG=A1/B1
      IF(RG.GT.11.0) GO TO 14
      GO TO 15
C    ******************************************************************
C
C     CALCULATIONS FOR A/B.GT.11.0
C
C    ******************************************************************
  14  SG=B1/A1
      GAM=DLOG(16.0/(SG*SG))
      C11=2.0*PI/(SG*(GAM-2.0*NU))*(1.0+( 1.613706    )/(GAM-2.0*NU))
      C22=(( 1.613706 )*(1.0-NU))/(2.0*NU+GAM*(1.0-NU))
      C22=2.0*PI*(1.0+C22)/(SG*(2.0*NU+GAM*(1.0-NU)))
      C23=2.0*PI/((DSQRT(SG)*SG*3.0)*((1.0-NU)*GAM-2.0+4.0*NU))
      C33=PI/4.0*(GAM*(1.0-2.0*NU)-2.0+6.0*NU)/(GAM*(1.0-NU)-2.0+4.0*NU)
      GO TO 80
C    ******************************************************************
C
C     INTERPOLATION FROM TABLES FOR A/B.LE.11.0 AND A/B.GE.0.1
C
C    ******************************************************************
  15  DO 20 I=2,20
      IF(RG.LE.AR(I)) GO TO 25
  20  CONTINUE
  25  J=I
      DO 30 I=1,12
  30  D(I)=E(I,J-1)+(E(I,J)-E(I,J-1))*(RG-AR(J-1))/(AR(J)-AR(J-1))
      DO 40 I=1,4
      AL(I)=8.0*(D(3*I)-2.0*D(3*I-1)+D(3*I-2))
      BE(I)=2.0*(-D(3*I)+4.0*D(3*I-1)-3.0*D(3*I-2))
  40  CNT(I)=AL(I)*NU**2+BE(I)*NU+D(3*I-2)
      C11=CNT(1)
      C22=CNT(2)
      C23=CNT(3)
       C33=CNT(4)
  80  CONTINUE
      RETURN
      END
C
      SUBROUTINE SLI(X,ARG,VAL,Y,NDIM,IER)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ARG(23),VAL(23)
      IER = 0
      J = 0
      IF ((X.LT.ARG(1)).OR.(X.GT.ARG(NDIM))) GO TO 40
      DO 10 I = 1,NDIM
      IF (X-ARG(I)) 20,20,10
   20 J = I
      GO TO 30
   10 CONTINUE
   30 IF (X.EQ.ARG(J)) GO TO 25
      Y = ((VAL(J)-VAL(J-1))/(ARG(J)-ARG(J-1)))*(X-ARG(J-1))+VAL(J-1)
      GO TO 35
   25 Y = VAL(J)
      GO TO 35
   40 IER = 1
      WRITE(*,100)X
 100  FORMAT(/' **** INPUT X VAL =',E15.7,' IS OUT OF RNG FOR SLI ROUTI
     1NE ****')
   35 RETURN
```

```
      END
*******************************************************************
*
*      SUBROUTINE BEAR
*
*   THIS SUBROUTINE CALCULATES THE BEARING MOMENT
*
*******************************************************************
*
      SUBROUTINE BEAR(THETAW,THETAD,KK,MBEAR,ALPHA)
      REAL*8 KK,MBEAR,MBREAK,ALPHA
      REAL*8 THETAW, THETAD,STIFF
*
*  DATA FOR BREAK AWAY MOMENT
*
      DATA MBREAK/0.18/
      STIFF = 100. * KK
      MBEAR = DABS(STIFF*(ALPHA-THETAW))
      IF(MBEAR.GE.MBREAK) THEN
         ALPHA = THETAW - DSIGN(1.D0,THETAD)*MBREAK/STIFF
      ENDIF
      MBEAR = STIFF * (ALPHA - THETAW)
*
      RETURN
      END
```

Appendix C

Lookup Table

I	XW (IN)	RR (IN)	YR (IN)	RL (IN)	YL (IN)	MR (RAD)	ML (RAD)	WR (RAD)
1	-1.200	17.50340	7.01631	16.40080	7.12491	0.39225	0.04701	-0.016434
2	-1.190	17.49918	7.01544	16.40125	7.12492	0.40880	0.04325	-0.016338
3	-1.180	17.49011	7.01933	16.40125	7.12496	0.44128	0.04325	-0.016243
4	-1.170	17.49011	7.01377	16.40125	7.12499	0.44128	0.04325	-0.016138
5	-1.160	17.48529	7.01284	16.40125	7.12500	0.45719	0.04325	-0.016032
6	-1.150	17.48027	7.01187	16.40167	7.12500	0.47286	0.04016	-0.015921
7	-1.140	17.48027	7.00585	16.40167	7.12500	0.47286	0.04016	-0.015809
8	-1.130	17.48027	6.99974	16.40167	7.12498	0.47286	0.04016	-0.015695
9	-1.120	17.48529	6.98881	16.40205	7.12497	0.45719	0.03769	-0.015583
10	-1.110	17.50745	6.96257	16.40205	7.12494	0.37553	0.03769	-0.015497
11	-1.100	17.50745	6.95825	16.40244	7.12493	0.37553	0.03580	-0.015414
12	-1.090	17.50745	6.95388	16.40244	7.12489	0.37553	0.03580	-0.015331
13	-1.080	17.50340	6.95328	16.40279	7.12488	0.39225	0.03446	-0.015245
14	-1.070	17.50340	6.94873	16.40279	7.12482	0.39225	0.03446	-0.015159
15	-1.060	17.49918	6.94805	16.40312	7.12481	0.40880	0.03361	-0.015068
16	-1.050	17.49474	6.94733	16.41171	7.12420	0.42514	0.05274	-0.014973
17	-1.040	17.49474	6.94202	16.41277	7.12409	0.42514	0.05310	-0.014869
18	-1.030	17.49011	6.94111	16.41331	7.12411	0.44128	0.05315	-0.014764
19	-1.020	17.48529	6.94015	16.41383	7.12414	0.45719	0.05311	-0.014654
20	-1.010	17.48027	6.93912	16.41437	7.12416	0.47286	0.05299	-0.014539
21	-1.000	17.47473	6.93796	16.41544	7.12406	0.47685	0.05258	-0.014414
22	-0.990	17.46947	6.93682	16.41595	7.12409	0.49102	0.05230	-0.014293
23	-0.980	17.46404	6.93561	16.41647	7.12411	0.50660	0.05198	-0.014168
24	-0.970	17.45837	6.93430	16.41699	7.12414	0.52364	0.05164	-0.014038
25	-0.960	17.45247	6.93289	16.41751	7.12417	0.54218	0.05128	-0.013902
26	-0.950	17.44630	6.93136	16.41801	7.12420	0.56224	0.05090	-0.013759
27	-0.940	17.43987	6.92970	16.41852	7.12423	0.58382	0.05053	-0.013609
28	-0.930	17.43309	6.92786	16.41852	7.12438	0.60686	0.05053	-0.013451
29	-0.920	17.42596	6.92585	16.41949	7.12430	0.63132	0.04996	-0.013284
30	-0.910	17.41846	6.92359	16.42000	7.12434	0.65709	0.04995	-0.013105
31	-0.900	17.41052	6.92106	16.42101	7.12426	0.68408	0.04993	-0.012914
32	-0.890	17.40213	6.91834	16.42149	7.12430	0.71212	0.04992	-0.012713
33	-0.880	17.39323	6.91510	16.42250	7.12422	0.74106	0.04992	-0.012493
34	-0.870	17.38382	6.91137	16.42349	7.12414	0.77072	0.04991	-0.012255

I	XW (IN)	RR (IN)	YR (IN)	RL (IN)	YL (IN)	MR (RAD)	ML (RAD)	WR (RAD)
35	-0.860	17.37381	6.90702	16.42400	7.12420	0.80091	0.04991	-0.011997
36	-0.850	17.36317	6.90188	16.42499	7.12413	0.83142	0.04991	-0.011713
37	-0.840	17.35185	6.89592	16.42599	7.12406	0.86204	0.04992	-0.011404
38	-0.830	17.33983	6.88845	16.42699	7.12401	0.89259	0.04993	-0.011055
39	-0.820	17.32703	6.87909	16.42799	7.12396	0.92288	0.04995	-0.010660
40	-0.810	17.29887	6.88227	16.42899	7.12392	0.98197	0.04997	-0.010219
41	-0.800	17.28342	6.86919	16.43050	7.12375	1.01050	0.05005	-0.009713
42	-0.790	17.26659	6.85077	16.43150	7.12375	1.05674	0.05004	-0.009090
43	-0.780	17.22945	6.84459	16.43300	7.12363	1.09454	0.05002	-0.008328
44	-0.770	17.18929	6.83384	16.43449	7.12354	1.12124	0.04997	-0.007434
45	-0.760	17.12489	6.83645	16.43649	7.12333	1.14516	0.04992	-0.006353
46	-0.750	17.08031	6.81299	16.44148	7.12211	1.15197	0.04992	-0.005156
47	-0.740	16.99077	6.83391	16.44299	7.12211	1.14447	0.04997	-0.003957
48	-0.730	16.92673	6.83879	16.44449	7.12202	1.11693	0.04999	-0.002921
49	-0.720	16.85155	6.86770	16.44550	7.12213	1.03275	0.05000	-0.002114
50	-0.710	16.79913	6.89982	16.44649	7.12201	0.83646	0.05000	-0.001763
51	-0.700	16.76891	6.91509	16.44699	7.12213	0.74525	0.05000	-0.001500
52	-0.690	16.75128	6.92021	16.44798	7.12194	0.70072	0.05000	-0.001277
53	-0.680	16.51849	7.12196	16.44849	7.12199	0.04990	0.05000	-0.001176
54	-0.670	16.51799	7.12195	16.44899	7.12200	0.04990	0.04999	-0.001158
55	-0.660	16.51749	7.12194	16.44949	7.12201	0.04991	0.04999	-0.001141
56	-0.650	16.51698	7.12193	16.45000	7.12202	0.04991	0.04998	-0.001124
57	-0.640	16.51649	7.12192	16.45050	7.12203	0.04992	0.04998	-0.001106
58	-0.630	16.51601	7.12191	16.45100	7.12204	0.04993	0.04997	-0.001089
59	-0.620	16.51550	7.12190	16.45151	7.12204	0.04993	0.04996	-0.001072
60	-0.610	16.51450	7.12219	16.45200	7.12205	0.04995	0.04995	-0.001055
61	-0.600	16.51401	7.12218	16.45250	7.12206	0.04996	0.04994	-0.001038
62	-0.590	16.51350	7.12217	16.45300	7.12207	0.04997	0.04991	-0.001020
63	-0.580	16.51300	7.12217	16.45351	7.12208	0.04997	0.04990	-0.001003
64	-0.570	16.51250	7.12216	16.45399	7.12209	0.04998	0.04989	-0.000986
65	-0.560	16.51199	7.12215	16.45448	7.12209	0.04999	0.04988	-0.000968
66	-0.550	16.51149	7.12214	16.45499	7.12210	0.04999	0.04988	-0.000951
67	-0.540	16.51099	7.12213	16.45549	7.12211	0.05000	0.04988	-0.000934
68	-0.530	16.51048	7.12212	16.45599	7.12212	0.05000	0.04989	-0.000916
69	-0.520	16.50999	7.12212	16.45650	7.12213	0.05000	0.04989	-0.000899
70	-0.510	16.50949	7.12211	16.45699	7.12214	0.05000	0.04990	-0.000882
71	-0.500	16.50899	7.12210	16.45749	7.12214	0.05000	0.04991	-0.000864
72	-0.490	16.50848	7.12209	16.45799	7.12215	0.05001	0.04993	-0.000847
73	-0.480	16.50800	7.12208	16.45850	7.12216	0.05000	0.04994	-0.000830
74	-0.470	16.50749	7.12207	16.45898	7.12217	0.04999	0.04995	-0.000812
75	-0.460	16.50699	7.12207	16.45949	7.12218	0.04999	0.04997	-0.000795
76	-0.450	16.50648	7.12206	16.45999	7.12219	0.04998	0.04998	-0.000778
77	-0.440	16.50600	7.12205	16.46049	7.12219	0.04997	0.04999	-0.000760
78	-0.430	16.50549	7.12204	16.46149	7.12191	0.04996	0.05002	-0.000744
79	-0.420	16.50499	7.12203	16.46199	7.12191	0.04994	0.05002	-0.000726
80	-0.410	16.50449	7.12202	16.46249	7.12192	0.04993	0.05003	-0.000709
81	-0.400	16.50398	7.12202	16.46300	7.12193	0.04992	0.05003	-0.000692
82	-0.390	16.50349	7.12201	16.46349	7.12194	0.04991	0.05003	-0.000674
83	-0.380	16.50299	7.12200	16.46400	7.12195	0.04990	0.05001	-0.000657
84	-0.370	16.50249	7.12199	16.46449	7.12196	0.04989	0.04999	-0.000640
85	-0.360	16.50198	7.12198	16.46500	7.12197	0.04988	0.04997	-0.000622
86	-0.350	16.50150	7.12197	16.46550	7.12197	0.04988	0.04996	-0.000605
87	-0.340	16.50101	7.12196	16.46600	7.12198	0.04988	0.04994	-0.000588
88	-0.330	16.50050	7.12196	16.46651	7.12199	0.04987	0.04993	-0.000570
89	-0.320	16.50000	7.12195	16.46700	7.12200	0.04988	0.04992	-0.000553
90	-0.310	16.49950	7.12194	16.46750	7.12201	0.04988	0.04992	-0.000536
91	-0.300	16.49899	7.12193	16.46799	7.12202	0.04989	0.04991	-0.000519
92	-0.290	16.49850	7.12192	16.46849	7.12203	0.04990	0.04991	-0.000501
93	-0.280	16.49800	7.12191	16.46899	7.12203	0.04992	0.04990	-0.000484
94	-0.270	16.49750	7.12190	16.46948	7.12204	0.05001	0.04990	-0.000467
95	-0.260	16.49651	7.12219	16.46999	7.12205	0.05003	0.04990	-0.000450
96	-0.250	16.49600	7.12218	16.47049	7.12206	0.05003	0.04991	-0.000433
97	-0.240	16.49550	7.12217	16.47099	7.12207	0.05004	0.04991	-0.000415
98	-0.230	16.49500	7.12217	16.47150	7.12208	0.05003	0.04991	-0.000398

I	XW (IN)	RR (IN)	YR (IN)	RL (IN)	YL (IN)	MR (RAD)	ML (RAD)	WR (RAD)
99	-0.220	16.49451	7.12216	16.47198	7.12208	0.05003	0.04992	-0.000381
100	-0.210	16.49399	7.12215	16.47249	7.12209	0.05003	0.04993	-0.000363
101	-0.200	16.49348	7.12214	16.47299	7.12210	0.05002	0.04994	-0.000346
102	-0.190	16.49300	7.12213	16.47350	7.12211	0.05001	0.04995	-0.000329
103	-0.180	16.49249	7.12213	16.47398	7.12212	0.05000	0.04996	-0.000311
104	-0.170	16.49199	7.12212	16.47449	7.12213	0.04999	0.04997	-0.000294
105	-0.160	16.49149	7.12211	16.47499	7.12213	0.04998	0.04998	-0.000277
106	-0.150	16.49100	7.12210	16.47549	7.12214	0.04997	0.05001	-0.000259
107	-0.140	16.49049	7.12209	16.47598	7.12215	0.04995	0.05001	-0.000242
108	-0.130	16.48999	7.12208	16.47649	7.12216	0.04994	0.05002	-0.000225
109	-0.120	16.48949	7.12207	16.47699	7.12217	0.04993	0.05002	-0.000207
110	-0.110	16.48898	7.12207	16.47749	7.12218	0.04992	0.05002	-0.000190
111	-0.100	16.48849	7.12206	16.47800	7.12218	0.04991	0.05002	-0.000173
112	-0.090	16.48799	7.12205	16.47849	7.12219	0.04991	0.05002	-0.000155
113	-0.080	16.48749	7.12204	16.47951	7.12190	0.04990	0.05001	-0.000139
114	-0.070	16.48698	7.12203	16.48000	7.12191	0.04990	0.05001	-0.000121
115	-0.060	16.48650	7.12202	16.48050	7.12192	0.04990	0.05000	-0.000104
116	-0.050	16.48599	7.12202	16.48100	7.12193	0.04991	0.04999	-0.000087
117	-0.040	16.48550	7.12201	16.48151	7.12194	0.04992	0.04999	-0.000069
118	-0.030	16.48500	7.12200	16.48199	7.12195	0.04993	0.04998	-0.000052
119	-0.020	16.48450	7.12199	16.48250	7.12196	0.04994	0.04997	-0.000035
120	-0.010	16.48399	7.12198	16.48300	7.12197	0.04994	0.04996	-0.000017
121	0.000	16.48351	7.12197	16.48351	7.12197	0.04995	0.04995	0.000000
122	0.010	16.48300	7.12197	16.48399	7.12198	0.04996	0.04994	0.000017
123	0.020	16.48250	7.12196	16.48450	7.12199	0.04997	0.04994	0.000035
124	0.030	16.48199	7.12195	16.48500	7.12200	0.04998	0.04993	0.000052
125	0.040	16.48151	7.12194	16.48550	7.12201	0.04999	0.04992	0.000069
126	0.050	16.48100	7.12193	16.48599	7.12202	0.04999	0.04991	0.000087
127	0.060	16.48050	7.12192	16.48650	7.12202	0.05000	0.04990	0.000104
128	0.070	16.48000	7.12191	16.48698	7.12203	0.05001	0.04990	0.000121
129	0.080	16.47951	7.12190	16.48749	7.12204	0.05001	0.04990	0.000139
130	0.090	16.47849	7.12219	16.48799	7.12205	0.05002	0.04991	0.000155
131	0.100	16.47800	7.12218	16.48849	7.12206	0.05002	0.04991	0.000173
132	0.110	16.47749	7.12218	16.48898	7.12207	0.05002	0.04992	0.000190
133	0.120	16.47699	7.12217	16.48949	7.12207	0.05002	0.04993	0.000207
134	0.130	16.47649	7.12216	16.48999	7.12208	0.05002	0.04994	0.000225
135	0.140	16.47598	7.12215	16.49049	7.12209	0.05001	0.04995	0.000242
136	0.150	16.47549	7.12214	16.49100	7.12210	0.05001	0.04997	0.000259
137	0.160	16.47499	7.12213	16.49149	7.12211	0.04998	0.04998	0.000277
138	0.170	16.47449	7.12213	16.49199	7.12212	0.04997	0.04999	0.000294
139	0.180	16.47398	7.12212	16.49249	7.12213	0.04996	0.05000	0.000311
140	0.190	16.47350	7.12211	16.49300	7.12213	0.04995	0.05001	0.000329
141	0.200	16.47299	7.12210	16.49348	7.12214	0.04994	0.05002	0.000346
142	0.210	16.47249	7.12209	16.49399	7.12215	0.04993	0.05003	0.000363
143	0.220	16.47198	7.12208	16.49451	7.12216	0.04992	0.05003	0.000381
144	0.230	16.47150	7.12208	16.49500	7.12217	0.04991	0.05003	0.000398
145	0.240	16.47099	7.12207	16.49550	7.12217	0.04991	0.05004	0.000415
146	0.250	16.47049	7.12206	16.49600	7.12218	0.04991	0.05003	0.000433
147	0.260	16.46999	7.12205	16.49651	7.12219	0.04990	0.05003	0.000450
148	0.270	16.46948	7.12204	16.49750	7.12190	0.04990	0.05001	0.000467
149	0.280	16.46899	7.12203	16.49800	7.12191	0.04990	0.04992	0.000484
150	0.290	16.46849	7.12203	16.49850	7.12192	0.04991	0.04990	0.000501
151	0.300	16.46799	7.12202	16.49899	7.12193	0.04991	0.04989	0.000519
152	0.310	16.46750	7.12201	16.49950	7.12194	0.04992	0.04988	0.000536
153	0.320	16.46700	7.12200	16.50000	7.12195	0.04992	0.04988	0.000553
154	0.330	16.46651	7.12199	16.50050	7.12196	0.04993	0.04987	0.000570
155	0.340	16.46600	7.12198	16.50101	7.12196	0.04994	0.04988	0.000588
156	0.350	16.46550	7.12197	16.50150	7.12197	0.04996	0.04988	0.000605
157	0.360	16.46500	7.12197	16.50198	7.12198	0.04997	0.04988	0.000622
158	0.370	16.46449	7.12196	16.50249	7.12199	0.04999	0.04989	0.000640
159	0.380	16.46400	7.12195	16.50299	7.12200	0.05001	0.04990	0.000657
160	0.390	16.46349	7.12194	16.50349	7.12201	0.05003	0.04991	0.000674
161	0.400	16.46300	7.12193	16.50398	7.12202	0.05003	0.04992	0.000692
162	0.410	16.46249	7.12192	16.50449	7.12202	0.05003	0.04993	0.000709

I	XW (IN)	RR (IN)	YR (IN)	RL (IN)	YL (IN)	MR (RAD)	ML (RAD)	WR (RAD)
163	0.420	16.46199	7.12191	16.50499	7.12203	0.05002	0.04994	0.000726
164	0.430	16.46149	7.12191	16.50549	7.12204	0.05002	0.04996	0.000744
165	0.440	16.46049	7.12219	16.50600	7.12205	0.04999	0.04997	0.000760
166	0.450	16.45999	7.12219	16.50648	7.12206	0.04998	0.04998	0.000778
167	0.460	16.45949	7.12218	16.50699	7.12207	0.04997	0.04999	0.000795
168	0.470	16.45898	7.12217	16.50749	7.12207	0.04995	0.04999	0.000812
169	0.480	16.45850	7.12216	16.50800	7.12208	0.04994	0.05000	0.000830
170	0.490	16.45799	7.12215	16.50848	7.12209	0.04993	0.05001	0.000847
171	0.500	16.45749	7.12214	16.50899	7.12210	0.04991	0.05000	0.000864
172	0.510	16.45699	7.12214	16.50949	7.12211	0.04990	0.05000	0.000882
173	0.520	16.45650	7.12213	16.50999	7.12212	0.04989	0.05000	0.000899
174	0.530	16.45599	7.12212	16.51048	7.12212	0.04989	0.05000	0.000916
175	0.540	16.45549	7.12211	16.51099	7.12213	0.04988	0.05000	0.000934
176	0.550	16.45499	7.12210	16.51149	7.12214	0.04988	0.04999	0.000951
177	0.560	16.45448	7.12209	16.51199	7.12215	0.04988	0.04999	0.000968
178	0.570	16.45399	7.12209	16.51250	7.12216	0.04989	0.04998	0.000986
179	0.580	16.45351	7.12208	16.51300	7.12217	0.04990	0.04997	0.001003
180	0.590	16.45300	7.12207	16.51350	7.12217	0.04991	0.04997	0.001020
181	0.600	16.45250	7.12206	16.51401	7.12218	0.04994	0.04996	0.001038
182	0.610	16.45200	7.12205	16.51450	7.12219	0.04995	0.04995	0.001055
183	0.620	16.45151	7.12204	16.51550	7.12190	0.04996	0.04993	0.001072
184	0.630	16.45100	7.12204	16.51601	7.12191	0.04997	0.04993	0.001089
185	0.640	16.45050	7.12203	16.51649	7.12192	0.04998	0.04992	0.001106
186	0.650	16.45000	7.12202	16.51698	7.12193	0.04998	0.04991	0.001124
187	0.660	16.44949	7.12201	16.51749	7.12194	0.04999	0.04991	0.001141
188	0.670	16.44899	7.12200	16.51799	7.12195	0.04999	0.04990	0.001158
189	0.680	16.44849	7.12199	16.51849	7.12196	0.05000	0.04990	0.001176
190	0.690	16.44798	7.12194	16.75128	6.92021	0.05000	0.70072	0.001277
191	0.700	16.44699	7.12213	16.76891	6.91509	0.05000	0.74525	0.001500
192	0.710	16.44649	7.12201	16.79913	6.89982	0.05000	0.83646	0.001763
193	0.720	16.44550	7.12213	16.85155	6.86770	0.05000	1.03275	0.002114
194	0.730	16.44449	7.12202	16.92673	6.83879	0.04999	1.11693	0.002921
195	0.740	16.44299	7.12211	16.99077	6.83391	0.04997	1.14447	0.003957
196	0.750	16.44148	7.12211	17.08031	6.81299	0.04992	1.15197	0.005156
197	0.760	16.43649	7.12333	17.12489	6.83645	0.04992	1.14516	0.006353
198	0.770	16.43449	7.12354	17.18929	6.83384	0.04997	1.12124	0.007434
199	0.780	16.43300	7.12363	17.22945	6.84459	0.05002	1.09454	0.008328
200	0.790	16.43150	7.12375	17.26659	6.85077	0.05004	1.05674	0.009090
201	0.800	16.43050	7.12375	17.28342	6.86919	0.05005	1.01050	0.009713
202	0.810	16.42899	7.12392	17.29887	6.88227	0.04997	0.98197	0.010219
203	0.820	16.42799	7.12396	17.32703	6.87909	0.04995	0.92288	0.010660
204	0.830	16.42699	7.12401	17.33983	6.88845	0.04993	0.89259	0.011055
205	0.840	16.42599	7.12406	17.35185	6.89592	0.04992	0.86204	0.011404
206	0.850	16.42499	7.12413	17.36317	6.90188	0.04991	0.83142	0.011713
207	0.860	16.42400	7.12420	17.37381	6.90702	0.04991	0.80091	0.011997
208	0.870	16.42349	7.12414	17.38382	6.91137	0.04991	0.77072	0.012255
209	0.880	16.42250	7.12422	17.39323	6.91510	0.04992	0.74106	0.012493
210	0.890	16.42149	7.12430	17.40213	6.91834	0.04992	0.71212	0.012713
211	0.900	16.42101	7.12426	17.41052	6.92106	0.04993	0.68408	0.012914
212	0.910	16.42000	7.12434	17.41846	6.92359	0.04995	0.65709	0.013105
213	0.920	16.41949	7.12430	17.42596	6.92585	0.04996	0.63132	0.013284
214	0.930	16.41852	7.12438	17.43309	6.92786	0.05053	0.60686	0.013451
215	0.940	16.41852	7.12423	17.43987	6.92970	0.05053	0.58382	0.013609
216	0.950	16.41801	7.12420	17.44630	6.93136	0.05090	0.56224	0.013759
217	0.960	16.41751	7.12417	17.45247	6.93289	0.05128	0.54218	0.013902
218	0.970	16.41699	7.12414	17.45837	6.93430	0.05164	0.52364	0.014038
219	0.980	16.41647	7.12411	17.46404	6.93561	0.05198	0.50660	0.014168
220	0.990	16.41595	7.12409	17.46947	6.93682	0.05230	0.49102	0.014293
221	1.000	16.41544	7.12406	17.47473	6.93796	0.05258	0.47685	0.014414
222	1.010	16.41437	7.12416	17.48027	6.93912	0.05299	0.47286	0.014539
223	1.020	16.41383	7.12414	17.48529	6.94015	0.05311	0.45719	0.014654
224	1.030	16.41331	7.12411	17.49011	6.94111	0.05315	0.44128	0.014764
225	1.040	16.41277	7.12409	17.49474	6.94202	0.05310	0.42514	0.014869
226	1.050	16.41171	7.12420	17.49474	6.94733	0.05274	0.42514	0.014973

I	XW (IN)	RR (IN)	YR (IN)	RL (IN)	YL (IN)	MR (RAD)	ML (RAD)	WR (RAD)
227	1.060	16.40312	7.12481	17.49918	6.94805	0.03361	0.40880	0.015068
228	1.070	16.40279	7.12482	17.50340	6.94873	0.03446	0.39225	0.015159
229	1.080	16.40279	7.12488	17.50340	6.95328	0.03446	0.39225	0.015245
230	1.090	16.40244	7.12489	17.50745	6.95388	0.03580	0.37553	0.015331
231	1.100	16.40244	7.12493	17.50745	6.95825	0.03580	0.37553	0.015414
232	1.110	16.40205	7.12494	17.50745	6.96257	0.03769	0.37553	0.015497
233	1.120	16.40205	7.12497	17.48529	6.98881	0.03769	0.45719	0.015583
234	1.130	16.40167	7.12498	17.48027	6.99974	0.04016	0.47286	0.015695
235	1.140	16.40167	7.12500	17.48027	7.00585	0.04016	0.47286	0.015809
236	1.150	16.40167	7.12500	17.48027	7.01187	0.04016	0.47286	0.015921
237	1.160	16.40125	7.12500	17.48529	7.01284	0.04325	0.45719	0.016032
238	1.170	16.40125	7.12499	17.49011	7.01377	0.04325	0.44128	0.016138
239	1.180	16.40125	7.12496	17.49011	7.01933	0.04325	0.44128	0.016243
240	1.190	16.40125	7.12492	17.49918	7.01544	0.04325	0.40880	0.016338
241	1.200	16.40080	7.12491	17.50340	7.01631	0.04701	0.39225	0.016434

XW = lateral displacement
RR = radius of right wheel
RL = radius of left wheel
YR = height of right rail
YL = height of left rail
MR = contact angle between right wheel and right rail
ML = contact angle between left wheel and left rail
WR = roll angle of wheelset

(1 in. = 25.4 mm)

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