DESIGN OF A ROBOTIC MANIPULATOR USING VARIABLE GEOMETRY TRUSSES AS JOINTS

by

Babu Padmanabhan

Thesis submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

Master of Science

in

Mechanical Engineering

APPROVED:

Charles F. Reinholtz, Chairman

Arvid Myklebust

Harry H. Robertshaw

December 1988

Blacksburg, Virginia

DESIGN OF A ROBOTIC MANIPULATOR USING VARIABLE GEOMETRY TRUSSES AS JOINTS

by

Babu Padmanabhan

Charles F. Reinholtz, Chairman

Mechanical Engineering

(ABSTRACT)

Parallel robotic manipulators are generally believed to be stiffer under load and more precise than conventional serial manipulators. This is because of their closed loop construction which allows forces to be shared through multiple paths to the ground. Unfortunately, most proposed parallel manipulator designs have severe workspace restrictions. The introduction of Variable Geometry Trusses (VGT's) represents an opportunity to overcome this limitation.

The lack of stiffness in many serial manipulators is primarily due to compliance at the joints. The disadvantage of the series connected device include limitations in lifting capacity and vibration problems. Difficulties of this sort result from the cantilever structure of the device. These factors often limit the degrees of freedom that can be provided in the serial configuration. By replacing the revolute joints with the 'VGT joints', it may be possible to add considerable rigidity at the joints and hence design a highly dextrous manipulator.

The objective of this thesis is to study the feasibility of a design of manipulators using Variable Geometry Trusses. A modeling scheme capable of solving the inverse problem in closed form and finding the range of all possible solutions for a planar VGT has been presented. Another aspect that has been dealt with is in utilizing the extra degree of freedom that becomes available in the proposed manipulator. Enhancing the performance of the manipulator by optimizing relevant parameters has been carried out for a demonstrative case involving a planar truss.

Acknowledgements

I am deeply grateful to Dr. Charles F. Reinholtz, the chairman of my advisory committee, for his invaluable guidance and wholehearted support and encouragement throughout my graduate study. I sincerely appreciate the time he spent reviewing my work, and suggesting new ideas and ways of tackling my problem.

I appreciate all the advice and support given by Dr. Harry H. Robertshaw and Dr. Arvid Myklebust, members of my advisory committee.

My special thanks are due to my friends, , who helped me in my work by means of their helpful criticisms and suggestions.

Finally, I would like to acknowledge the debt I owe my parents and my brother, whose moral support and encouragement has enabled me to accomplish what I have.

Table of Contents

List of Illustrations

Nomenclature

X	-	Coordinate Value in X-axis
Y	-	Coordinate Value in Y-axis
F	-	Fixed Member Lengths in the First bay
V	-	Variable Member Lengths in the First bay
G	-	Ground Member Length in the First bay
l	-	All Variable Link Lengths
U	-	Direction Cosines
r	-	Link Lengths in the Four-bar Linkage
F( )	-	Objective Function
L	-	Mean Value of the Variable Link Lengths
A	-	Upper Limit to Extension of Variable Length member
B	-	Lower Limit to Extension of Variable Length member
QNF	-	Quasi-Newton Factor

Greek Letters

θ	-	Angles Measured in the Planar Truss
ϕ	-	Angles Measured in the Four-bar Linkage

Subscripts

1	-	First Case
2	-	Second Case
3	-	Third Case
4	-	Fourth Case
X	-	Acting in the X-direction
Y	-	Acting in the Y-direction
B	-	Applying to Point 'B'
C	-	Applying to Point 'C'
D	-	Applying to Point 'D'
end	-	At the End Point
ABD	-	Angle Enclosed in the triangle ABD at B
BDC	-	Angle Enclosed in the triangle BDC at D

Superscripts

′	-	Prime - First Derivative
″	-	Double Prime - Second Derivative

Chapter 1

Introduction

1.1 The Evolution of Parallel Robotic Manipulators

Present day robotic manipulators are required to perform a variety of complex tasks. Many designs with similar kinematic structure have evolved during recent years. They are often formed by a sequence of links connected in 'series' at six revolute or prismatic joints, allowing the full six degrees of freedom of end effector motion. These designs have a natural similarity to the human arm. Such anthropomorphic systems have many advantages including flexibility of motion within large workspaces. However, they often suffer from compliance at the joints, and from a lack of rigidity by forming a cantilever structure.

Parallel robotic manipulators were introduced in an effort to increase the rigidity of the devices. The concept of a parallel manipulator is to connect manipulator links in a closed loop so that forces are shared through multiple paths to the ground. Sometimes, the manipulator is designed like a truss to avoid bending stresses and ensure tensile and compressive stresses only. Researchers have devised several planar and spatial configurations for parallel manipulators. However, they all suffer from an acutely restricted workspace.

Studies to date [4, 5] have not yielded parallel manipulators with a workspace nearly as large as serial robots. The introduction of Variable Geometry Trusses in manipulators has presented an opportunity for designing new kinds of parallel manipulators. A great improvement in the workspace of parallel manipulators can be made practical while retaining most of the rigidity. This thesis explores the possibility of using Variable Geometry Trusses as an alternative to existing parallel manipulators, to combine the advantages of both serial and parallel manipulators.

1.2 Literature Review

1.2.1 The Concept of Parallel Manipulators

Hunt [1] initiated the idea of using the Stewart platform, illustrated in Fig.1, created by Stewart [2] as an aircraft simulator, as the mechanism of a robot arm. The Stewart platform is a device that has a top platform which is free and a bottom platform which is fixed. Six Spheric-Prismatic-Spheric (S-P-S) jointed links connect the two platforms together. By varying the length of the S-P-S leg, the position and orientation of the top platform can be controlled. Fichter and McDowell [3] proposed a new design for a manipulator arm based on the Stewart platform and called it the Stewart Platform based Manipulator Arm (SPMA). They claim that SPMA is simple to fabricate and a very stiff manipulator.

GENERAL STEWART PLATFORM

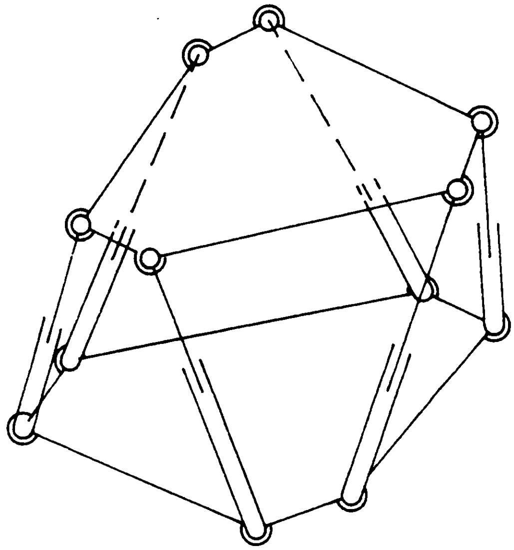


FIG. I A

OCTAHEDRAL PLATFORM

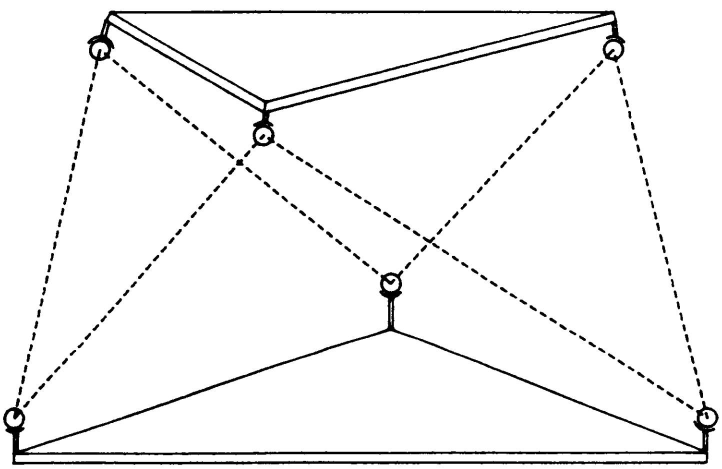


FIG. I B

Figure 1. STEWART PLATFORM - General and Octahedral Cofiguration

Following up his original work, Hunt [4] carried out a systematic study of in-parallel-actuated robot arms. He has suggested various possible actuation methods for the Stewart platform. He also considered different leg configurations for the Stewart platform, one of which is to connect the legs in an octahedral manner. Yang and Lee [5] undertook a feasibility study of the platform type of manipulators. Their study was limited to the Stewart platforms only. They were unfamiliar with Variable Geometry Trusses which includes some forms of Stewart platforms. They have concluded on the basis of actuation at the legs alone that, the six S-P-S configuration of the Stewart platform appears to be the only mechanism of its type that can be adopted as a general maneuverable device. Once the concept of Variable Geometry Trusses is introduced, the possibility of many other devices can be seen.

1.2.2 The Concept of Variable Geometry Trusses

A Variable Geometry Truss or VGT is a truss that varies its geometry by a change in length of any of its members that has a freedom to expand or contract. The VGT is a statically determinate truss structure when all the variable link members are locked up. VGT's can be constructed to be either planar or spatial. They can take shapes of antennas and habitat modules. Typically, VGT's are symmetric, constructed of repeating identical cells, which are termed unit cells, and have exceptional stiffness to weight ratios. The high stiffness can be attributed to the parallel structure and to the presence of only two force members in the truss. Some VGT's can even be folded down to a very compact size. These are known as deployable VGT's. Deployable VGT's were studied first because of their suitability in many space related applications.

The unit cells that repeat to form VGT's could be Tetrahedrons (4 faces), Octahedrons (8 faces), Decahedrons (10 faces), Do-decahedrons (12 faces) of which a Cube truss is a special case, and others. Figure 2 shows some of these unit cells. The tetrahedron is the simplest

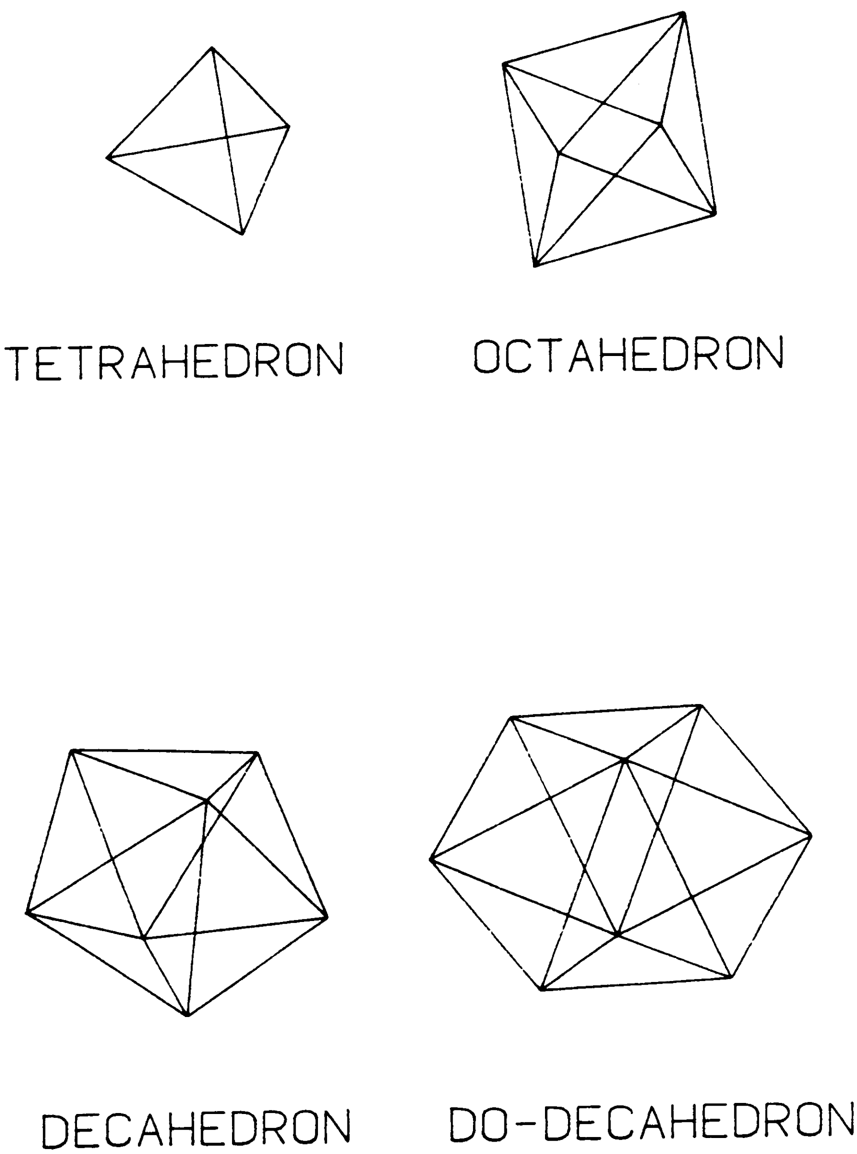


Figure 2. UNIT CELLS - Four Basic types

unit cell. It is made up of just four triangular faces. In general the unit cells are formed by taking an n-sided figure ($n > 3$) and joining each vertex to a node on the top and bottom. Every link must be part of a triangular face and the structure must satisfy the important requirement of being statically determinate when all the link lengths are fixed. This rule results in all possible unit cells excluding the tetrahedron. Joshi [6] has attempted to simplify the process of determining the mobility of the numerous possible configurations.

The concept of Variable Geometry Trusses was conceived by Miura [7] while working on possible deployable structures for applications in space. He and his co-worker Furuya discovered the deployable nature of a truss built with octahedron unit cell. They soon realized that this truss could be used as an adaptive device. They named this truss the Variable Geometry Truss or the VG-Truss. We now know that this is but one of several possible types of Variable Geometry Trusses.

The discovery of an adaptive and deployable structure resulted in much enthusiasm in this field for space related applications. Sincarsin and Hughes [7] studied four truss structures, with the tetrahedron, octahedron and cube as the basic building block. Their study focused on the static characteristics of these trusses for possible use as remote manipulator arms (truss arms). Rhodes and Mikulas [8] analyzed the deployability of the Octahedral VGT. They have named this the deployable controllable geometry truss and suggest applications such as space cranes, remote manipulator arms, masts to position and support feed horns for large antennas, and cantilever beams to deploy and tension blankets of a solar cell power system.

The same Octahedral truss can be found in Fig. 3. Miura, Furuya and Suzuki [9] later experimented on the truss to study its adaptive nature and solved its kinematics. Reinholtz and Gokhale [10] have developed systematic solution methods for both the forward and the inverse kinematic problem of this truss.

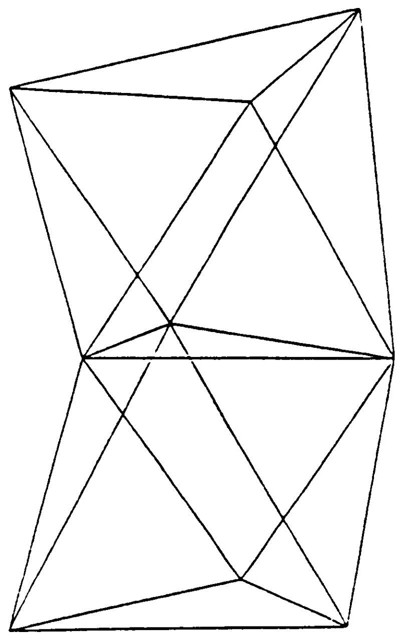


Figure 3. OCTAHEDRAL TRUSS

1.3 Objectives of this work

Some of the work carried out by previous researchers that is pertinent to the research in this thesis was discussed in the previous sections. Based on the preliminary work, a need was perceived to study the feasibility of a kinematic design of manipulators using Variable Geometry Trusses as joints. No such work has been published to date. In this sense, all the work in this thesis dealing with VGT's as manipulator joints represents a new contribution. The kinematic design involves establishing solution techniques for the forward and inverse kinematic problem. It also involves the determination of the workspace of the manipulators. This objective is generally the concern of all manipulator designers.

Apart from this primary objective, there is another aspect that has been addressed in this work. For a manipulator to operate in 3-dimensional space and be able to reach some prescribed position and orientation, a minimum of 6 degrees of freedom are needed. For a manipulator to operate in a 2-dimensional plane, only 3 degrees of freedom are needed. Generally, because of the added compliance, serial manipulators do not provide any more than the minimum number of degrees of freedom that are needed to solve the inverse problem. Since parallel manipulators have inherently smaller workspaces than serial manipulators, it is important to maximize the workspace of the parallel devices. One way of doing this is to provide more degrees of freedom than the minimum number necessary. Throughout this thesis, these are referred to as 'extra' degrees of freedom. This is possible because, unlike serial devices, parallel devices are very stiff. The extra degrees of freedom also provide dexterity and obstacle avoidance capability that are advantageous in some applications. A special objective of this work is to examine methods for treating the extra freedoms.

The work carried out in this thesis has been divided into two distinct sections. The first section is a complete work on a 4-DOF planar VGT manipulator. All the previously stated objectives are dealt with for this VGT manipulator. The forward and the inverse problem have both been solved in closed form. A sample workspace has been determined and plotted. An optimization method has been developed and discussed in Chapter 3. The optimization technique is used to enhance the performance characteristics of the manipulator by using an extra degree of freedom. The second section of this thesis is a preliminary work dealing with the forward analysis of a spatial VGT manipulator. The final chapter of this thesis makes recommendations for future work on the spatial VGT manipulator.

1.4 The Concept of VGT's as a Joint in Manipulators

One of the disadvantages of using serial manipulators is the compliance inherent in the revolute joints. These joints must take the full moment from the entire manipulator and the load beyond the joint. However, the serial configuration provides a relatively large workspace compared to conventional parallel manipulators such as the Stewart Platform. By using VGT's as manipulator joints it is possible to retain the high stiffness of parallel manipulators and at the same time provide a considerable workspace. The introduction of VGT's may transform the field of parallel manipulators.

The VGT's that are often considered for manipulator applications are formed by chaining together identical triangular frames of several unit cells. If each unit cell contains some actuators, a highly dextrous and stiff manipulator can be formed. These manipulators are called "snake-like VGT's". This type of manipulator is extremely useful in tasks requiring obstacle avoidance or in a tortuous workspace. For most industrial application, such a highly dextrous, high degree of freedom device is not needed. An interesting possibility of separating the unit cells with a static member exists. This is, in effect, using VGT's as joints in serial manipulators.

Several types of VGT's can serve as joints in serial manipulators. They can be planar, or any one of the different spatial configurations. This thesis considers one planar and one spatial configuration to demonstrate this new concept. Since well established solution techniques are already available for the octahedral VGT, it has been picked to serve as the joint for the spatial manipulator. A simple planar VGT has been designed to serve as the joint in the planar manipulator.

1.5 Planar Variable Geometry Trusses

The simplest structure in the plane is a pin jointed triangular frame. A rectangular frame with a diagonal forms two such triangles and is a statically determinate truss structure. If two of the fixed links are made variable in length in such a truss it becomes a planar VGT. Figure 4 shows the planar truss which has been analyzed as a planar four degree of freedom manipulator. This VGT is considered for analysis because of its apparent simplicity. This type of planar truss has been studied with respect to vibration control problem and is mentioned only rarely in the literature.

There are certain similarities between the specific planar and spatial manipulator considered for analysis. The inverse kinematics in both cases often results in an under-specified problem. An infinite number of solutions are possible in such cases. Because of its simplicity in analysis, the planar manipulator helps to better understand the under-specified problem.

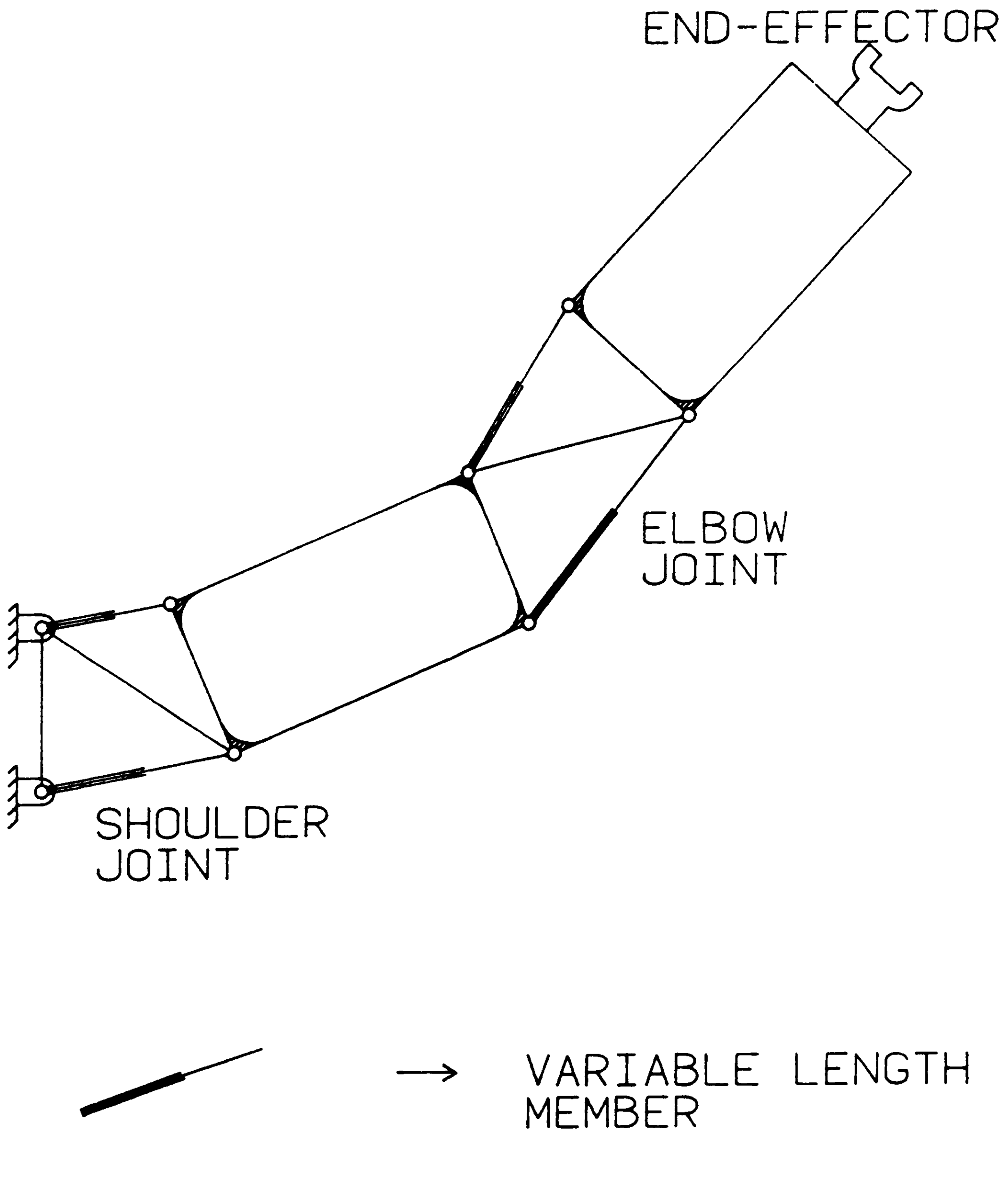


Figure 4. PLANAR VARIABLE GEOMETRY TRUSS - Description

Chapter 2

Analysis of a 4 DOF Planar VGT Manipulator

2.1 Forward Kinematic Analysis

Forward kinematic analysis is determining the position and orientation of the end-effector given the complete set of input parameters, such as link lengths in VGT manipulators, or the rotary or linear displacements in serial manipulators.

Figure 4 shows the planar manipulator considered for analysis. It has two planar trusses as part of the manipulator arm forming the shoulder and elbow joints. Each truss consists of two variable links and they provide four degrees of freedom in the arm. It is a structure when the length of the variable links are fixed. Hence, specifying lengths for the variable link members should result in a unique solution for a particular starting assembly.

The dimensions of all the fixed links are known. To start the analysis some arbitrary lengths are to be specified for the variable length members. Once this is done, all the link dimensions are known. The points 'A' and 'B' in Fig. 5 are on the ground link. The links 'AD' and 'BD' are pinned to these two points respectively. Given two points in a plane and the distance a third point is from each of them, determines the location of the third point. Two solutions are geometrically possible. However, Only one of these can physically be reached from the given starting configuration. To reach the other would require disassembly of the manipulator. The algorithm used for this analysis eliminates the unwanted solution automatically, resulting in the desired solution. Using the law of cosines the angle θ_{ABD} formed between the sides 'AB' and 'BD' is given by the Eq. (2.1.1). Fig. 5 describes all the notations used in the following equations.

$$\theta_{ABD} = \cos^{-1}\left[\frac{F_1^2 + G^2 - V_1^2}{2F_1G}\right] \tag{2.1.1}$$

This expression, when implemented on a computer, provides only the positive angle which eliminates the unwanted solution resulting from the other angle. The coordinates of the point 'D' can now be obtained by rotating the link 'BD' from the link 'AB' about the point 'B'. Thus,

$$X_D = X_B + F_1 \sin\theta_{ABD}$$

$$Y_D = Y_B - F_1 \cos\theta_{ABD} \tag{2.1.2}$$

The points 'D' and 'B' are now known. The links 'DC' and 'BC' are pinned to these two points respectively. The angle θ_{BDC} formed between the links 'DB' and 'DC' is given by Eq. (2.1.3)

$$\theta_{BDC} = \cos^{-1}\left[\frac{F_1^2 + F_2^2 - V_2^2}{2F_1F_2}\right] \tag{2.1.3}$$

The coordinates of point 'D' can now be obtained by rotating link 'DC' from 'DB' about 'D'.

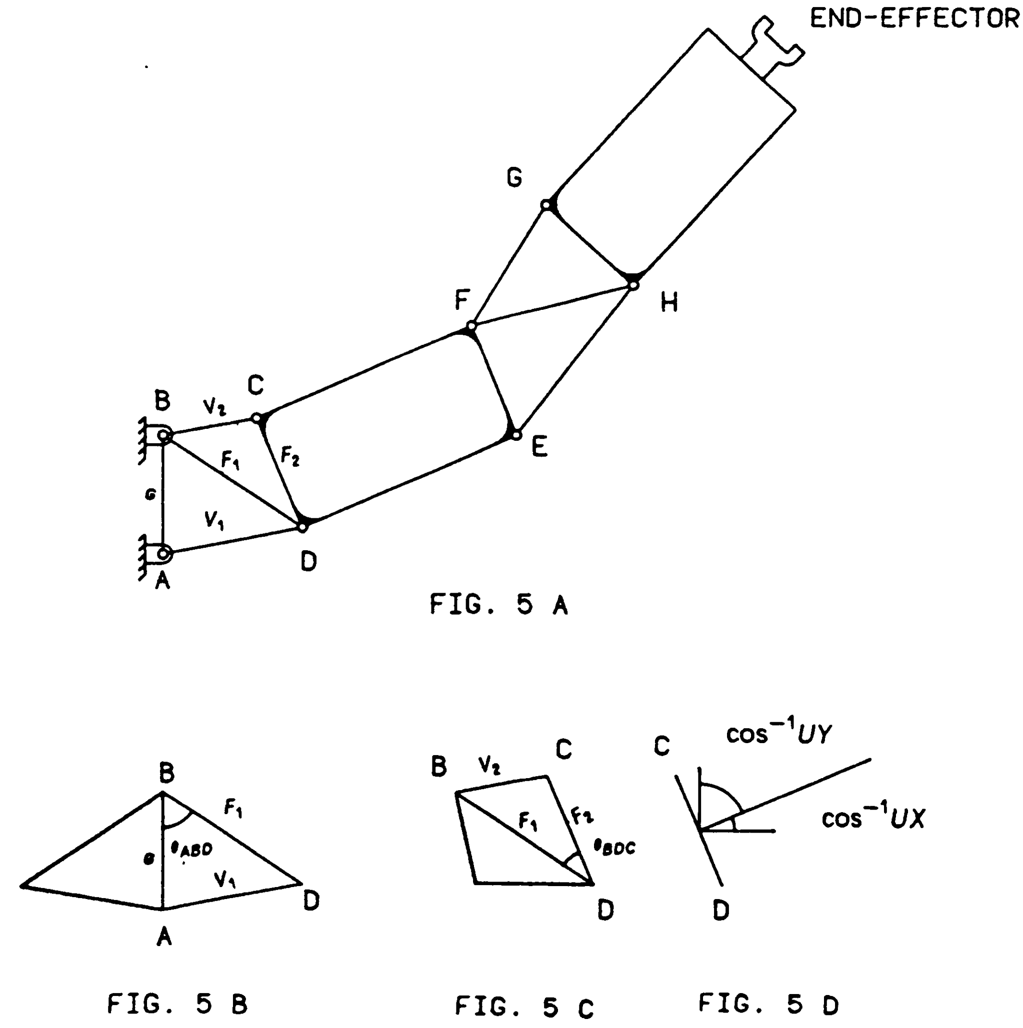


Figure 5. PLANAR VARIABLE GEOMETRY TRUSS - Forward Analysis of the Planar VGT

The direction cosines of the arm (the components of the unit vector normal to link CD) are given by Eq (2.1.4).

$$U_X = \frac{(Y_C - Y_D)}{\sqrt{(X_D - X_c)^2 + (Y_C - Y_D)^2}} \tag{2.1.4a}$$

$$U_Y = \frac{(X_D - X_C)}{\sqrt{(X_D - X_C)^2 + (Y_C - Y_D)^2}} \tag{2.1.4b}$$

The length of the arm is known. The coordinates of the starting points of the second truss are found by translating the points 'C' and 'D' along the unit vector by the length of the arm. Once these points are obtained the process is repeated to solve the forward kinematics of the next module and the position and orientation of the end point is determined.

2.2 *Exact Analysis of the Under-Specified Problem*

2.2.1 Objective of the Analysis

The manipulator being analyzed has four degrees of freedom. Only three degrees of freedom, namely, the positions 'X_{end}', 'Y_{end}' and orientation 'θ_{end}' of the end-effector are specified for a planar manipulator. Hence, there are infinite number of solutions to the inverse problem in the general case. The objective is to find a method to solve the inverse problem in closed-form, to find the range of all possible solutions, and to determine the best solution.

The model that has been created isolates a single degree of freedom. If desired, all possible solutions can be obtained by varying the parameter corresponding to this degree of freedom within a valid range. This model has been named the four-bar linkage model and is described in the next section.

2.2.2 Four-bar Linkage Model

Figure 6 shows the four-bar linkage model as a part of the manipulator. It can be clearly seen that fixing the position and orientation of the end-effector fixes point 'H' in Fig. 5, or 'O_1' in Fig. 6 in the four-bar linkage model. The various possible solutions to the inverse problem are given by the full range of motion of the four-bar linkage. If the four-bar linkage does not assemble for a particular input then the inverse problem has no solution.

The orientation of the ground link, ϕ_1, is fixed by the given position and orientation of the end effector. The input angle ϕ_2 prescribes all possible solutions. The range of the input angle can be determined from the four-bar linkage. Once the angle range is obtained, a possible solution can be obtained by selecting an input angle within this range and solving the four-bar linkage. There are two possible solutions resulting from the two branches, and both are to be considered. Once the input and output angles are known, all the coordinates and the lengths of the extensible manipulator links can be obtained. The link lengths are then verified if they lie within the desired range.

2.2.3 Optimization

The extra degree of freedom allows the possibility of looking for the best solution among the many solutions that exist. The objective function that forms the basis of such a search is determined first. The objective function can be formed based on the dynamic, static, or kinematic characteristics of the manipulator among other things. These functions can either

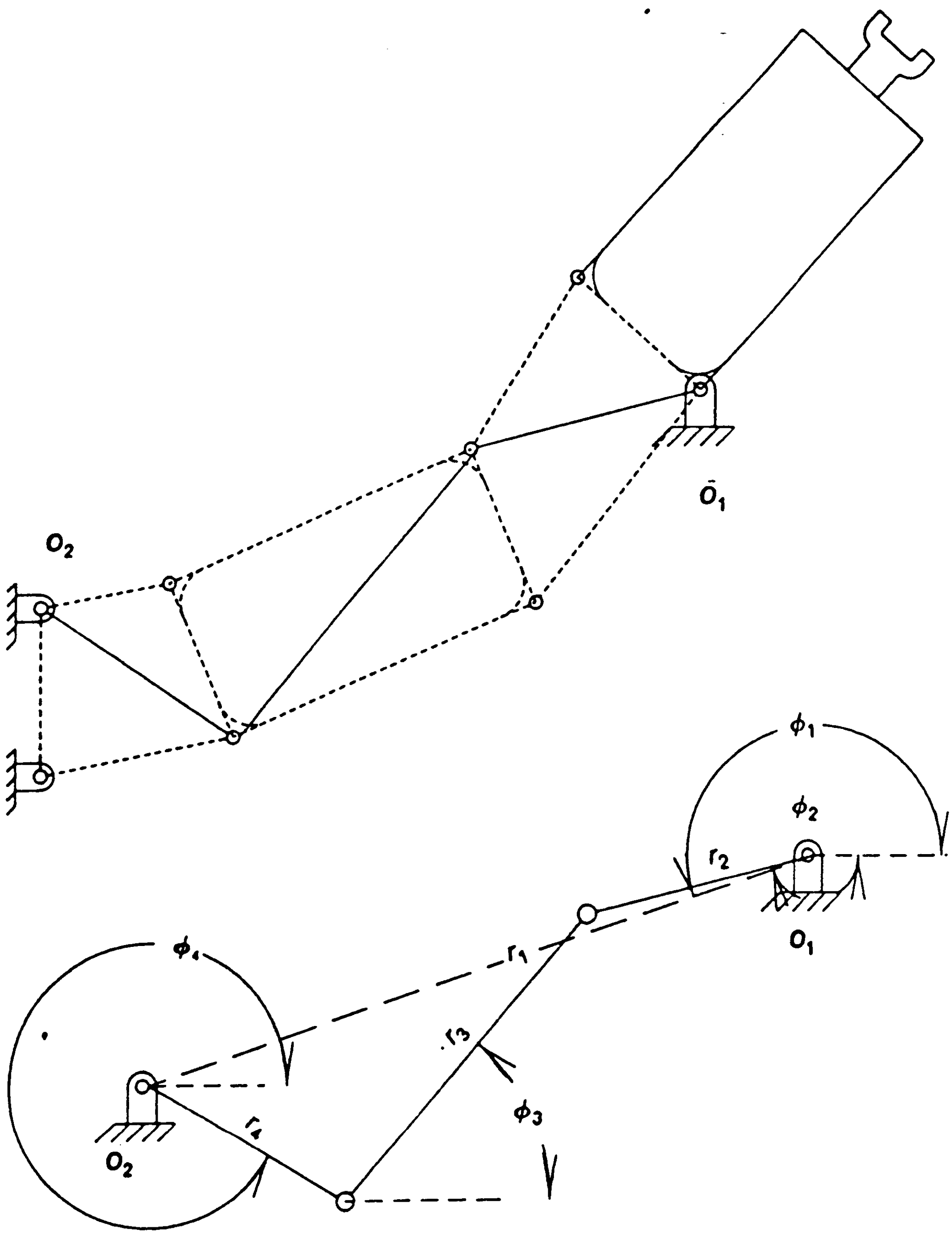


Figure 6. PLANAR VARIABLE GEOMETRY TRUSS - Four-Bar Linkage Model

be optimized individually or together with each carrying some weight. By introducing a set of constraints the need to check separately whether the link lengths are within the desired range is eliminated.

The solution method proposed here is based on numerical techniques, since it was not possible to write a closed-form expression for the link lengths in terms of the extra degree of freedom, the input angle of the four-bar linkage. The next chapter describes how such an optimization could be carried out.

2.3 Inverse Kinematic Analysis

The Inverse kinematic analysis is determining the set of input parameters (such as link lengths in VGT manipulators) which will satisfy a given position and orientation of the end-effector.

The first step in solving the inverse problem is to solve the position problem for the four-bar linkage model. This problem entails determining an output angle for a given input angle. The solution procedure is described in appendix 1 found at the end of this thesis. When this procedure is carried out, two solutions are obtained resulting from the two branches of the four bar linkage. Both solutions may be found to be valid during the analysis of the manipulator. From each solution of the four-bar linkage model a set of manipulators link lengths which satisfy the given input parameters can be immediately computed.

The optimization search is carried out in both the branches within the range of the input angle. The ensuing section will discuss the basis for picking a valid input angle before the four-bar linkage can be solved.

2.3.1 Range of the Input angle

The range of movement of the four-bar linkage can be found by finding the maximum and minimum angle the input angle can take. Fig. 7 displays the two extreme cases for a particular position. This range of motion provides all the solutions to the inverse problem. Based on the notation in that figure, the maximum and minimum input angle can be calculated as in Eqs. (2.3.1) and (2.3.2).

$$\min \phi_2 = \phi_1 - \cos^{-1}\left[\frac{r_2^2 + r_1^2 - (r_3 + r_4)^2}{2r_2r_1}\right] \tag{2.3.1}$$

$$\max \phi_2 = \phi_1 + \cos^{-1}\left[\frac{r_2^2 + r_1^2 - (r_3 + r_4)^2}{2r_2r_1}\right] \tag{2.3.2}$$

2.3.2 Determination of Link Lengths

The determination of the VGT link lengths can be carried out once the input and output angles of the four-bar linkage are determined, since the points 'D' and 'F' shown in Fig. 5 becomes known. This fixes the length of two of the variable members. The diagonal of the first arm is to be positioned on the line joining the two points. Once this is carried out the length of the other two variable members can be calculated.

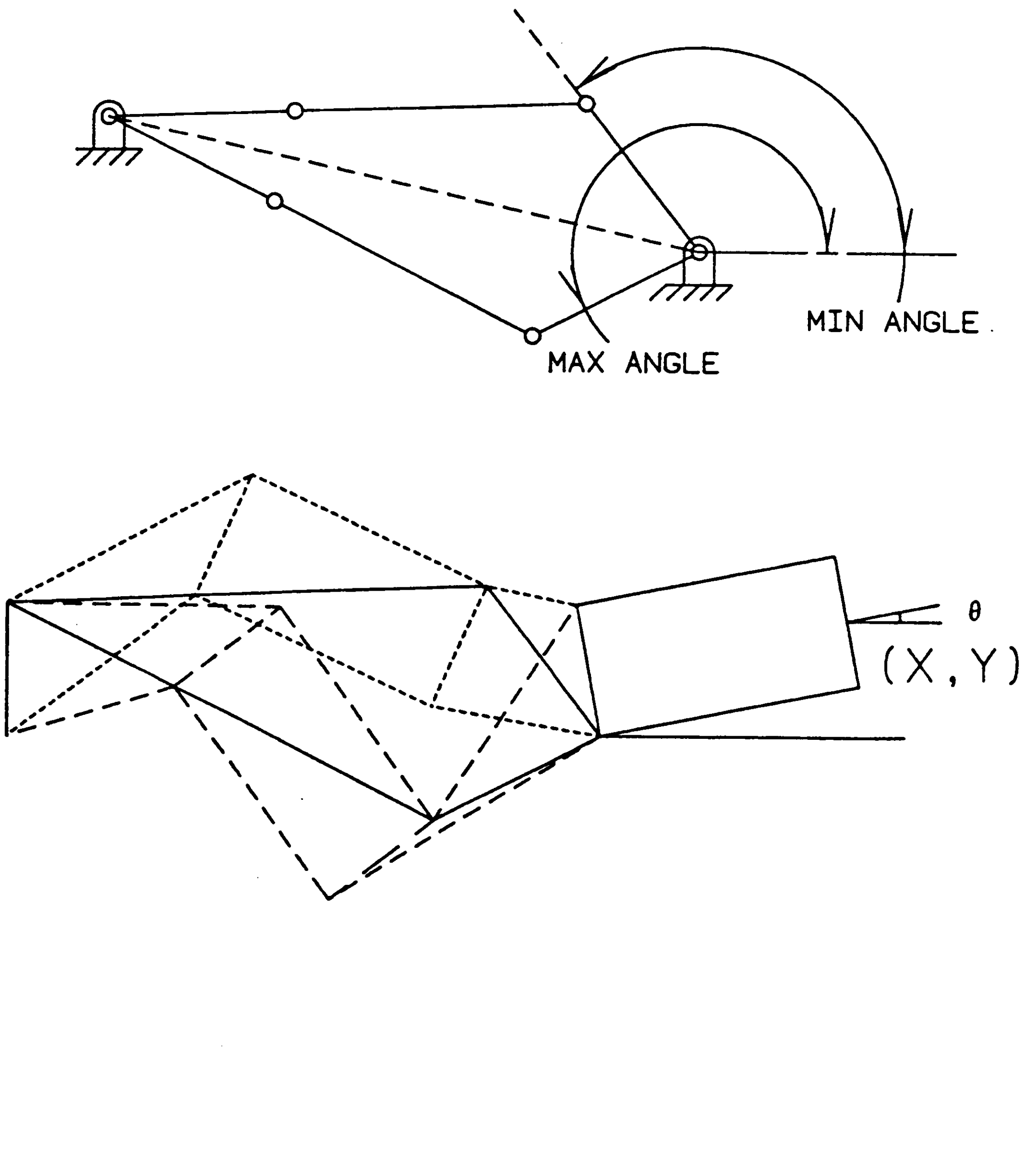


Figure 7. PLANAR VARIABLE GEOMETRY TRUSS - Range of the Input Angle

2.4 Workspace

The workspace of a robotic manipulator can be defined as the volume or area that the end effector of the manipulator can reach. Workspace can also be interpreted as the space in which a kinematic solution exists for the robotic manipulator. A manipulator's workspace can be further divided into dextrous workspace and reachable workspace . Dextrous workspace describes the volume or area the manipulator can reach with all possible orientations of the end effector, while reachable workspace is the volume or area which the end effector can reach with some orientation.

Figure 8 depicts the reachable workspace of the planar manipulator. Due to the availability of the extra degree of freedom, the manipulator can reach a point inside the boundary of the reachable workspace in many different orientations. The workspace shown is for a case where the fixed link lengths, other than the diagonal links are each 5 units. The two diagonal links are each 7.071 units. The boundary of the work area has been created with the variable links varying from 3 units to 10 units.

The workspace of the planar manipulator is asymmetric due to the asymmetry in the manipulator itself. Since the diagonal element is not symmetric about any coordinate system, the resulting workspace is also asymmetric.

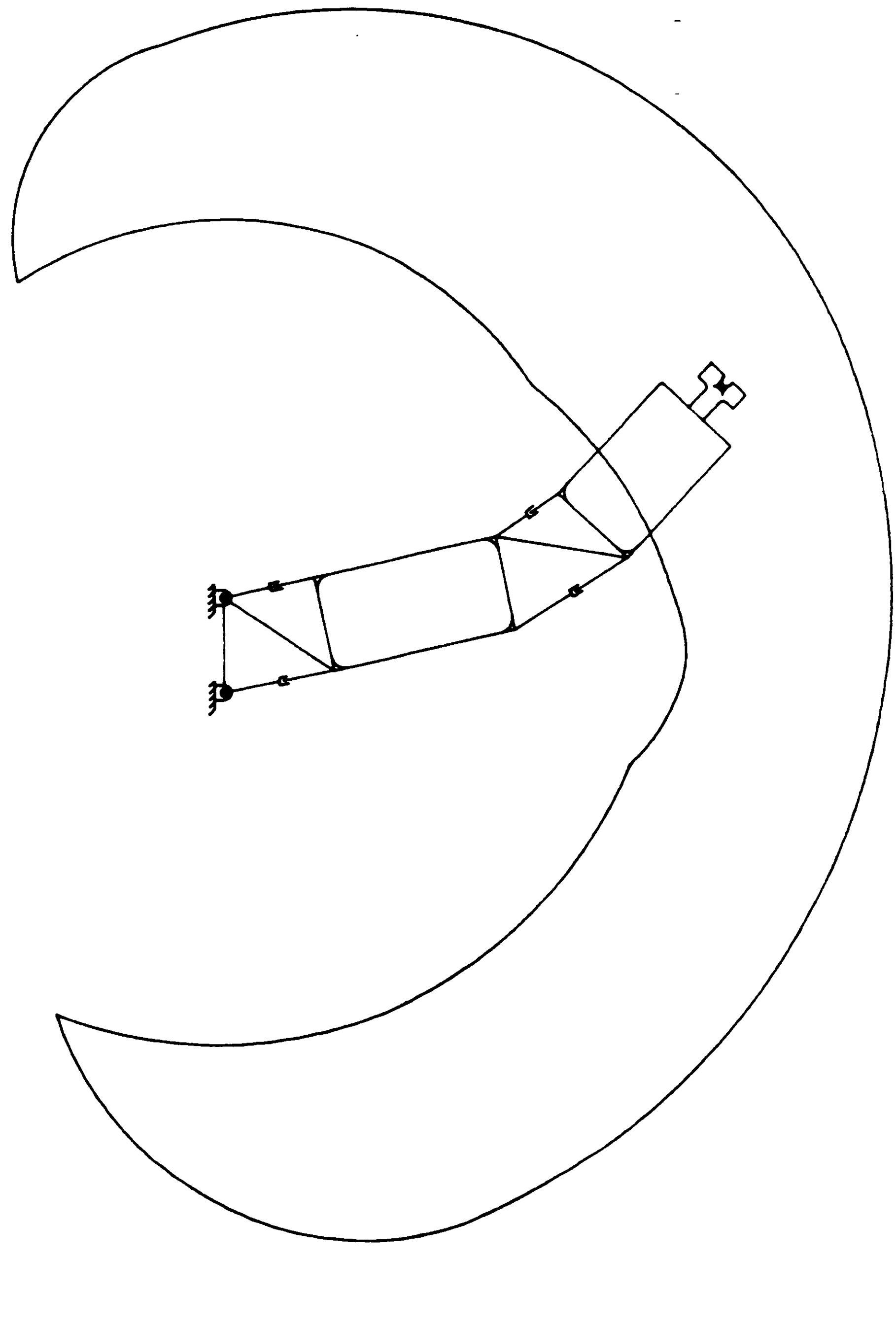


Figure 8. PLANAR VARIABLE GEOMETRY TRUSS - Reachable Workspace

Chapter 3

Optimization

3.1 Optimization Problem

The four-degree of freedom planar manipulator has an extra degree of freedom. Only three degrees of freedom, namely, the position 'X_{end}', 'Y_{end}' and the orientation 'θ_{end}' of the end effector can be specified. This results in an infinite number of solutions to the inverse problem in the general case. This is sometimes referred to as the "under-specified problem". This extra degree of freedom allows dexterity in the manipulators and presents a possibility for optimization. The previous chapter discussed the solution to the inverse problem. The modeling approach made it possible to determine the entire range of solutions. An optimization procedure can be carried out to obtain the best possible solution to the inverse problem based on an objective function and a set of constraints.

Some of the factors on which the objective function can be formed are,

1. Dynamic Characteristics
   a. Velocity Response
   b. Acceleration Response
2. Static Characteristics
   a. Load Carrying Capacity (Stress Analysis)
   b. Force Analysis
3. Kinematic Characteristics
   a. Link Length Ratio
   b. Link Variation from Mean Position

The objective functions so formed can either be optimized individually or in a group with each carrying a certain weight.

3.2 Objective Function

The various objective functions that can be formed based on the factors mentioned in the previous section are described here.

1) Dynamic Characteristics

This includes velocity and inertial response. Any of the three norms can be used for obtaining the objective function. They are described by Johnson and Riess [15].

a) Velocity response

$$F(\phi_2) = F(l_1, l_2, l_3, l_4) = \text{norm}\begin{bmatrix} \frac{\partial x}{\partial l_1} & \frac{\partial x}{\partial l_2} & \frac{\partial x}{\partial l_3} & \frac{\partial x}{\partial l_4} \\ \frac{\partial y}{\partial l_1} & \frac{\partial y}{\partial l_2} & \frac{\partial y}{\partial l_3} & \frac{\partial y}{\partial l_4} \\ \frac{\partial \theta}{\partial l_1} & \frac{\partial \theta}{\partial l_2} & \frac{\partial \theta}{\partial l_3} & \frac{\partial \theta}{\partial l_4} \end{bmatrix} \qquad (3.3.1)$$

When this function is minimized, higher mechanical advantage and hence lower motor torque is obtained. On the other hand, when this is maximized the response at the end-effector is faster.

b) Acceleration response

$$F(\phi_2) = F(l_1, l_2, l_3, l_4) = \text{norm}\begin{bmatrix} \frac{\partial^2 x}{\partial l_1^2} & \frac{\partial^2 x}{\partial l_2^2} & \frac{\partial^2 x}{\partial l_3^2} & \frac{\partial^2 x}{\partial l_4^2} \\ \frac{\partial^2 y}{\partial l_1^2} & \frac{\partial^2 y}{\partial l_2^2} & \frac{\partial^2 y}{\partial l_3^2} & \frac{\partial^2 y}{\partial l_4^2} \\ \frac{\partial^2 \theta}{\partial l_1^2} & \frac{\partial^2 \theta}{\partial l_2^2} & \frac{\partial^2 \theta}{\partial l_3^2} & \frac{\partial^2 \theta}{\partial l_4^2} \end{bmatrix} \qquad (3.3.2)$$

This helps in controllling the starting and stalling accelerations during a point to point move.

2) Static Characteristics

The stresses induced in the links are different at various configurations of the truss. The maximum stress in the links can be obtained by providing the information about the geometry of the truss and the forces to a finite element routine. The value obtained from the stress analysis can be used as an objective function. The force analysis involves computing motor stalling and starting torque ratings based on the forces on the truss and the gear ratios. The truss can be maintained in certain configurations where the torque requirement is low.

3) Kinematic Characteristics

The range of movement in the variable links is predetermined during the design of the manipulator. When the manipulator is performing a point to point move, it is expected to react as quickly as possible. Looking at the geometry of the manipulator intuitively, the best possible configurations appear to be when the link lengths stay close to the mean position, or the link length ratio tends to be unity. These cases result in simple objective functions that are easy to handle. They have yielded satisfactory results for the cases tested. The two objective functions give rise to totally different solutions in some cases, as shown in Fig. 10.

a) Link Length ratios

$$F(\phi_2) = F(l_1, l_2, l_3, l_4) \; = \; \max\left[\frac{l_1}{l_2},\ \frac{l_2}{l_1},\ \frac{l_3}{l_4},\ \frac{l_4}{l_3} \right] \qquad (3.3.3)$$

b) Minimum variation from mean position

$$F(\phi_2) = F(l_1, l_2, l_3, l_4) \; = \; |l_1 - L| + |l_2 - L| + |l_3 - L| + |l_4 - L| \qquad (3.3.4)$$

where L is a certain mean position.

Figures 9 and 10 illustrate the two objective functions obtained based on kinematic characteristics. The X-axis carries the total range of the input angle. Due to added constraints described in the next section, solutions exist only in a smaller range. The objective functions clearly have a single minimum value with in this range. The optimization algorithm has been developed to find the optimum values which might lie on the boundary, which occurs in Fig. 10. The function values were determined by varying the input angle between the minimum and maximum allowable limit and calculating all the link lengths. In some cases, no closure is possible due to restriction in link length movements or link interferences.

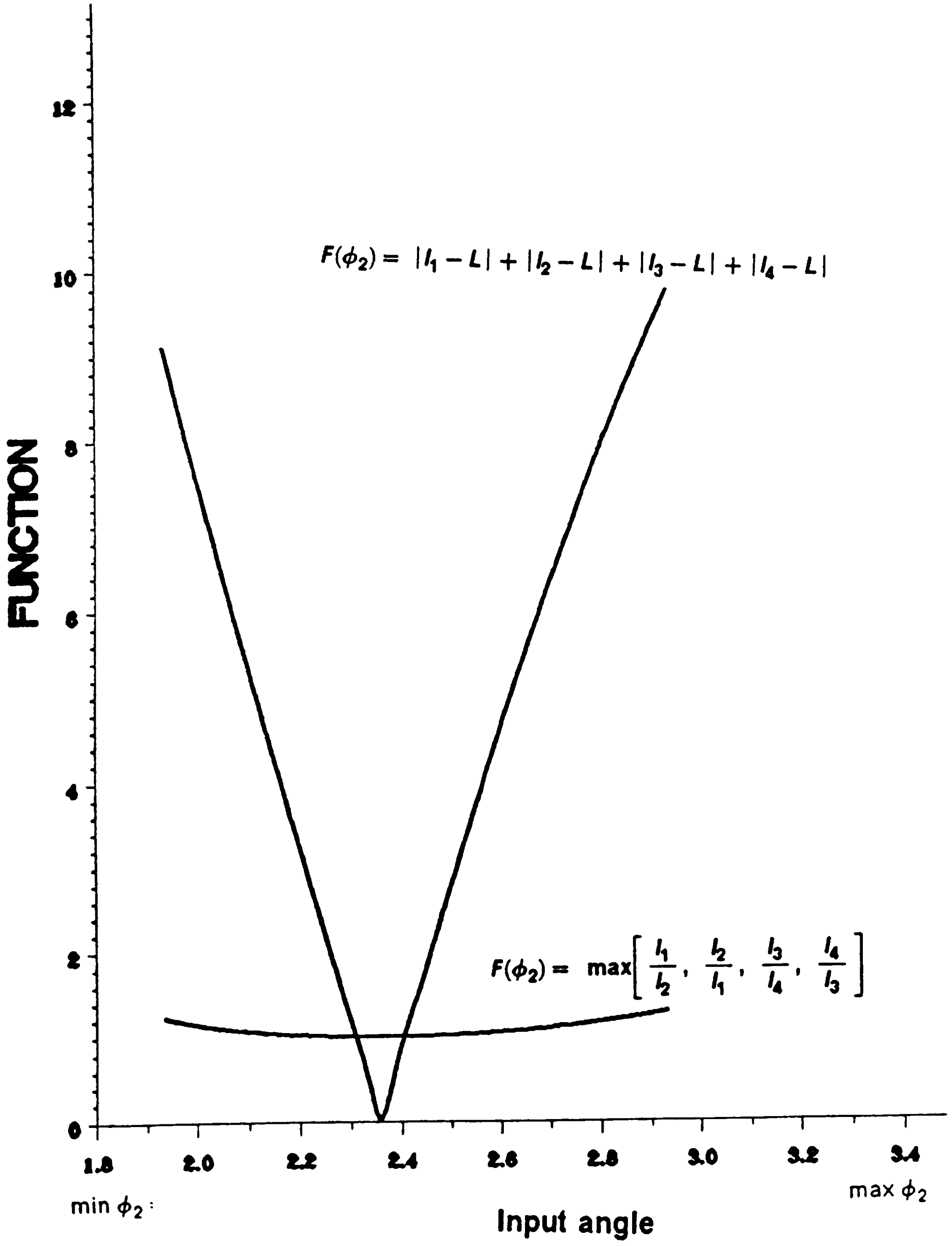


Figure 9. OBJECTIVE FUNCTIONS - First Case Showing Range of Input angle and Effect of Constraints

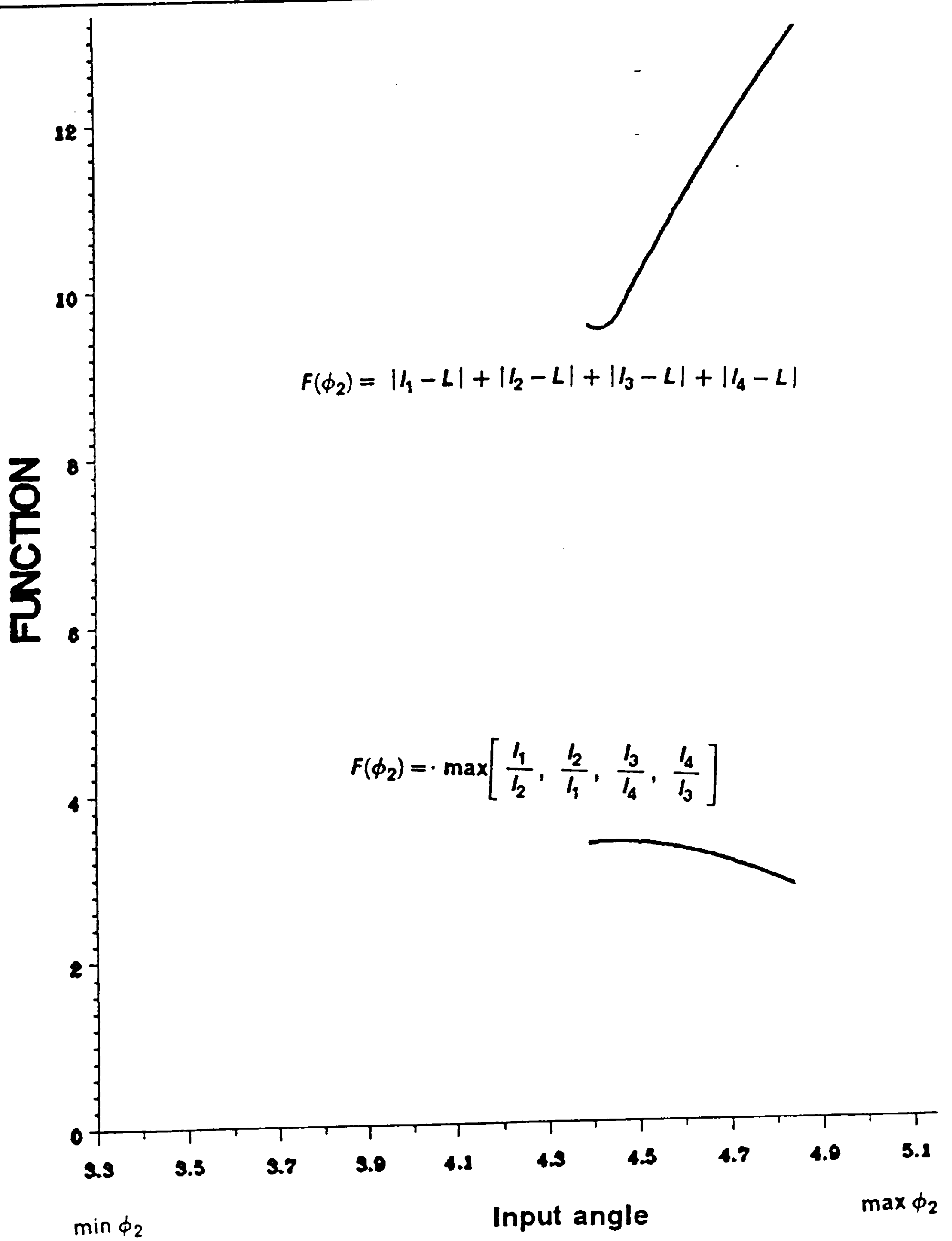


Figure 10. OBJECTIVE FUNCTIONS - Second Case Showing Nature of the Function

3.3 Constraints

There are physical limitations to the extent each variable link can shrink or expand. These conditions are built into the optimization procedure itself, avoiding any additional check. These constraints are:

$$A_1 \leq l_1 \leq B_1$$

$$A_2 \leq l_2 \leq B_2$$

$$A_3 \leq l_3 \leq B_3$$

$$A_4 \leq l_4 \leq B_4$$

Where A_i and B_i are the respective minimum and maximum link lengths that each l_i can attain. Other geometric constraints to avoid link interferences are also considered. These include testing the region in which the input and output angles are located.

3.4 Optimization method

The method to find the bounding values for the objective function has already been discussed. The mid-value is picked and the inverse solution carried out. If the mid-value is not a solution, a search is carried out alternatively toward the minimum and maximum input angle ϕ_2 till a solution is obtained. Once a starting value is obtained, the optimization can be carried out.

3.4.1 Quasi-Newton Method

The optimization method used in this research is a variation of the Newton-Raphson method discussed by Beighter, et al., [12]. The initial guess for the input angle ϕ_2 is chosen. The search direction in which the function optimizes is obtained by the Eq. (3.5.1).

$$d\phi_2 = \frac{-F'(\phi_2)}{F''(\phi_2)} \tag{3.5.1}$$

Optimization is performed by iterating on the following equation.

$$\phi_2^{i+1} = \phi_2^i - \frac{F'(\phi_2^i)}{F''(\phi_2^i)} \times QNF \tag{3.5.2}$$

The values of $F'(\phi_2)$ and $F''(\phi_2)$ are obtained numerically. By introducing the Q-N factor 'QNF', a check is carried out which prevents over-shooting, allows boundary values as solutions and provides faster convergence.

Chapter 4

Design and Analysis of a VGT Jointed Spatial Manipulator

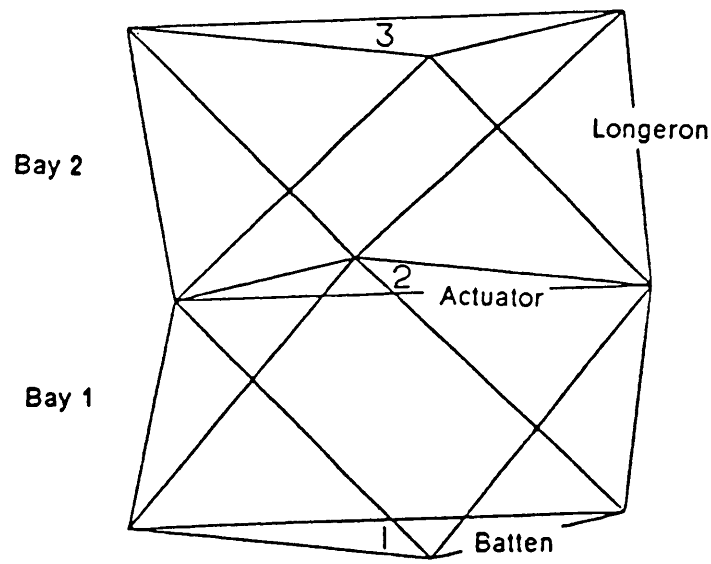
The concept of unit cells was introduced while discussing variable geometry trusses in Chapter 1. There are numerous ways such unit cells can assemble to form VGT's. The unit cells can be stacked on one another with a triangular frame in common, resulting in a long chain. The octahedral truss, formed by stacking two octahedral unit cells, is one of the VGT's, on which much attention has been focused.

4.1 Octahedral VGT

Figure 11 shows one module of an octahedral VGT. There are two bays in each module. Each module is made up of twenty one members. They are identified in Fig. 11 as battens, longerons and actuators. In the ensuing discussion it will be assumed that, the battens are of equal length and form an equilateral triangle called the batten frame, and that the twelve longerons are also of equal length; they connect the batten frames to the actuators. The batten frames form the ends of a typical module. The actuators are extensible members and they are also connected together to form a triangular frame called the actuator frame. The truss is symmetric about the actuator frame and capable of collapsing due to these assumptions. The components of the truss are two force members, thereby giving it excellent strength and stiffness.

4.2 Model Construction

A model was constructed to evaluate the concept of using a VGT as an actuated 3 degree of freedom joint in a "serial" manipulator. Figure 12 shows a photograph of the model. The model of the serial manipulator incorporates two VGT joints. Each VGT joint has one module, with a batten length of 8 inches. All members are fabricated from 1/4 inch diameter wooden dowels with the exception of the actuators which are brass tubes. Each VGT extends for a total of six inches from the fully collapsed position. The actuated links vary in length from 4.6 to 9.2 inches. A photograph of the joint is shown in Fig. 12 The links of the manipulator are made of strips of pine wood.



PLANES 1 & 3 - BATTEN PLANES

PLANE 2 - ACTUATOR PLANE

Figure 11. SPATIAL VARIABLE GEOMETRY TRUSS - Octahedral VGT Description

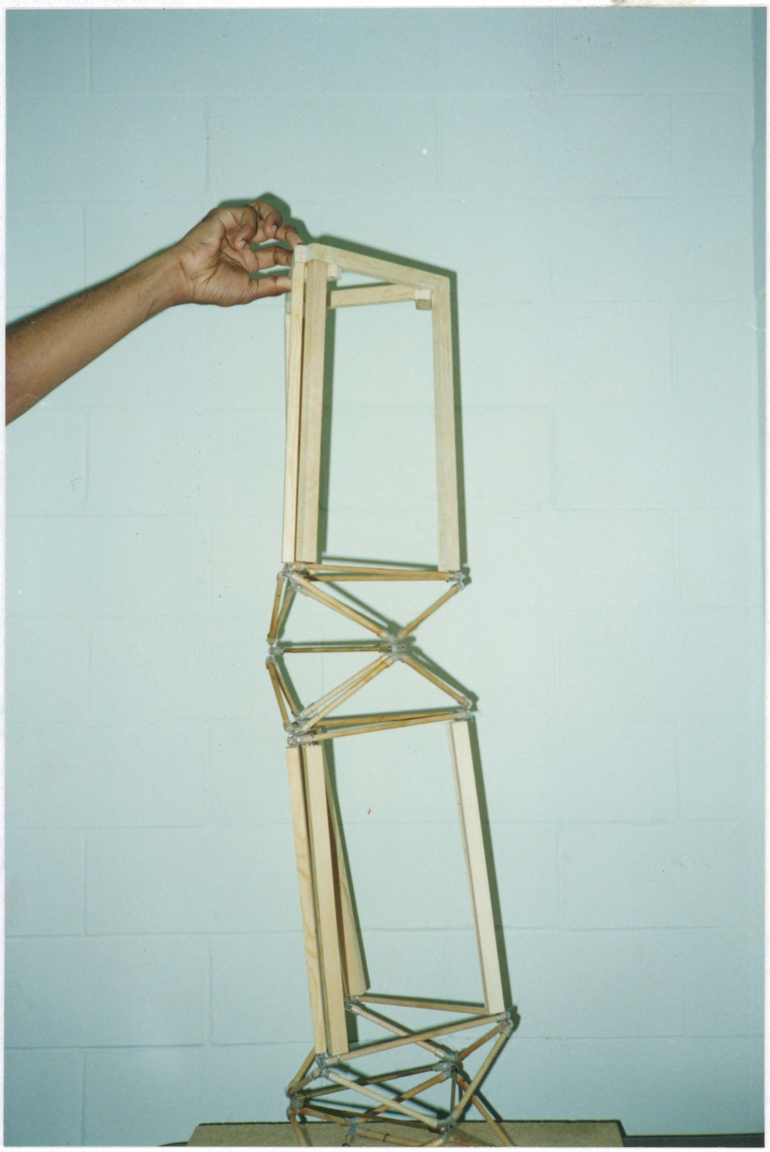


Figure 12. SPATIAL VARIABLE GEOMETRY TRUSS - Photograph of Manipulator Model

4.3 Kinematic Analysis

In order to control a robotic manipulator and predict its behavior for a set of input parameters, it is essential to have prior knowledge of the forward and inverse kinematic solutions and of the manipulator's workspace.

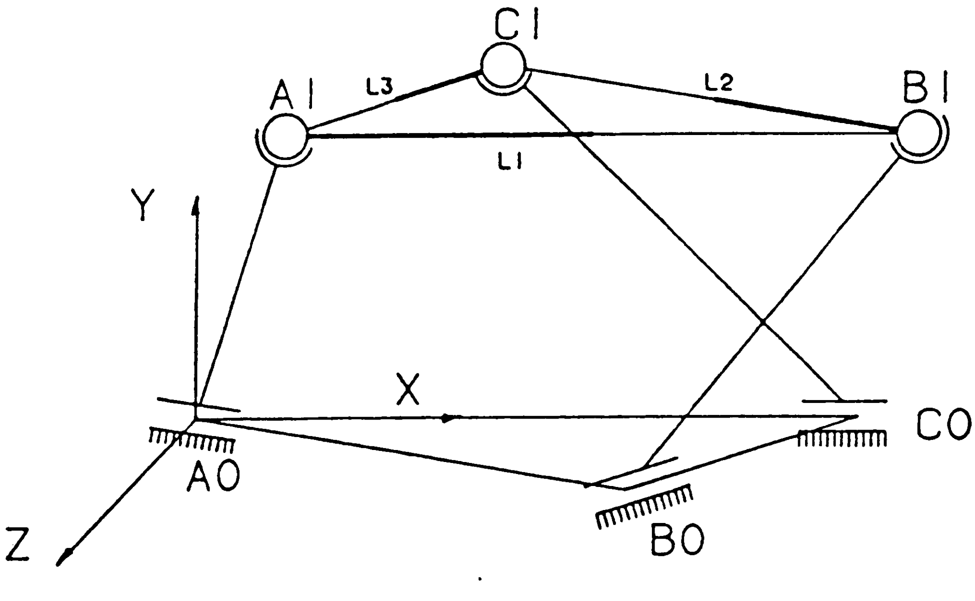
FORWARD KINEMATIC SOLUTION

The forward kinematic problem for a robotic manipulator has been defined in Chapter 2. In the above case, this translates into finding the coordinates of the end effector for a given set of values for the six actuated variable links.

The approach used in solving the forward kinematics problem is based in part on the work done by Gokhale [10]. Figure 14 shows one module of an octahedral VGT. It shows the lower half of the module modeled as a ten link mechanism consisting of three revolute joints, three prismatic joints, and six spheric joints. This mechanism has six degrees of freedom as calculated using the Kutzbach equation [11].

Of these six degrees of freedom, three are idle rotations of the two-link chains A1B1, B1C1, and C1A1 between the spheric joints, hence effectively the mechanism has only three degrees of freedom. This mechanism can be visualized as being made up of three interconnected RSSR mechanisms A0-A1-B1-B0, B0-B1-C1-C0, and C0-C1-A1-A0, each having one degree of freedom. This mechanism is a kinematic equivalence of the VGT and easier to analyze.

For the three RSSR mechanisms with a given set of lengths L_1, L_2, L_3 between the S-S joints, the unknowns are the three angles θ_1, θ_2, θ_3 the links A0-A1, B0-B1, and C0-C1 make with the ground plane (plane A0B0C0). The constraint used in solving for the unknowns is that all three RSSR mechanisms should assemble simultaneously.



THE THREE RSSR MECHANISMS ARE :
A0-A1-B1-B0
B0-B1-C1-C0
C0-C1-A1-A0

Figure 13. SPATIAL VARIABLE GEOMETRY TRUSS - Three RSSR Model of One Octahedral VGT bay

The procedure for kinematic analysis of a RSSR mechanism is given in the text by Suh and Radcliffe [13]. Applying the procedure to the three RSSR approximation of the VGT results in three non-linear transcendental equations in three unknowns. No closed form solution exists, hence an iterative root finding technique is used. Because of the existence of multiple branches, the iteration must proceed cautiously, or else a solution in a different branch will result. Therefore, the Quasi-Newton algorithm, which performs a line search on the iterated values, is used to solve these equations.

The face angles $\theta_1, \theta_2, \theta_3$ are then used to calculate the coordinates of the joints A1, B1, and C1. The geometry and the inherent symmetry of the truss are used to calculate the positions of the joints A2, B2, C2. (Refer to figure [14] for notation). The position of the base plane of the second VGT joint is determined by calculating the unit normal to the top plane of the first joint and advancing in that direction by the length of the arm (12 inches in this case). The coordinates of the second VGT joint are calculated in a similar manner, using the new link lengths, and the joint is then translated to its new location above the arm. The position of the end effector is calculated by using the unit normal from the top plane of the second VGT joint.

The above procedure was incorporated in a computer program. Graphic output was obtained using graPHIGS, the IBM version of the ISO 3D graphics standard. Figure 15 shows photographs of the graphic output.

4.4 Workspace

The program developed for the forward kinematic analysis was used to calculate the reachable workspace. The range of travel of the actuated links is from 4.6 to 9.2 inches. For gener-

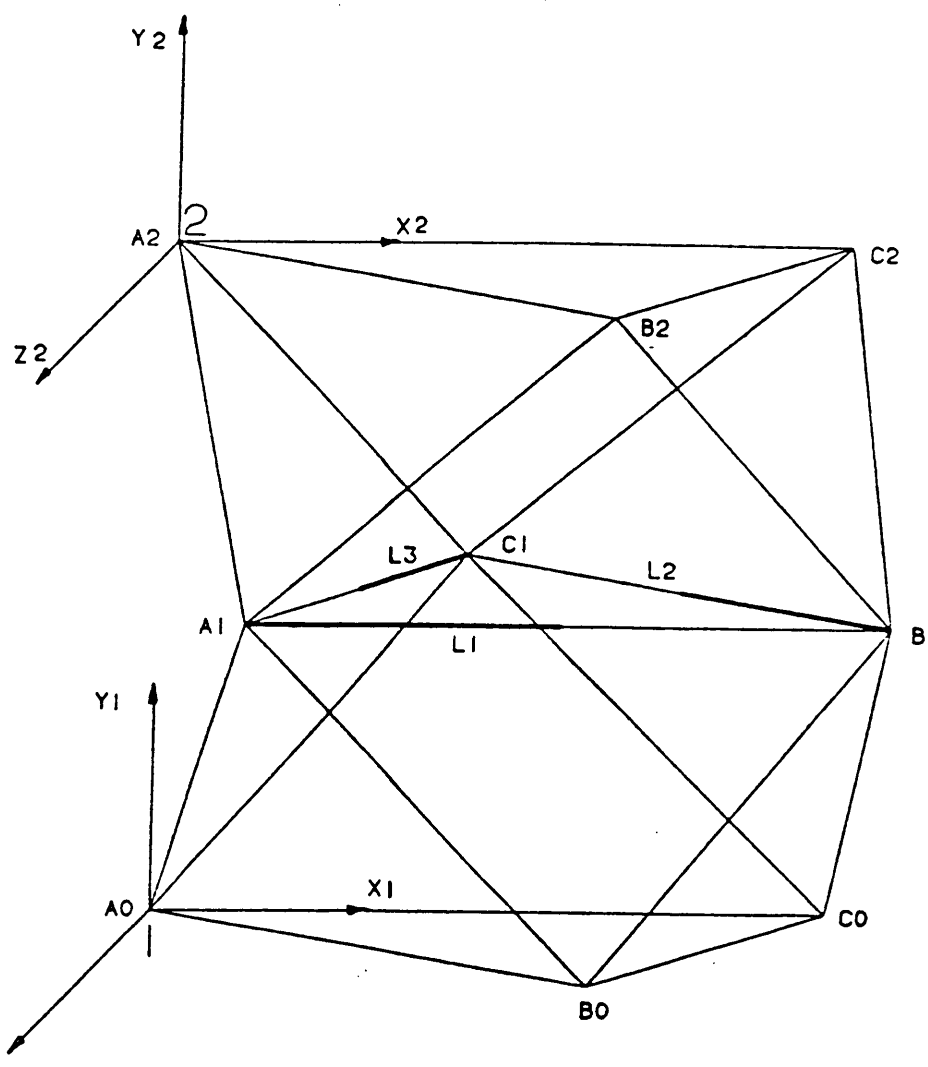


Figure 14. SPATIAL VARIABLE GEOMETRY TRUSS - One Module of Octahedral VGT With Associated Notation

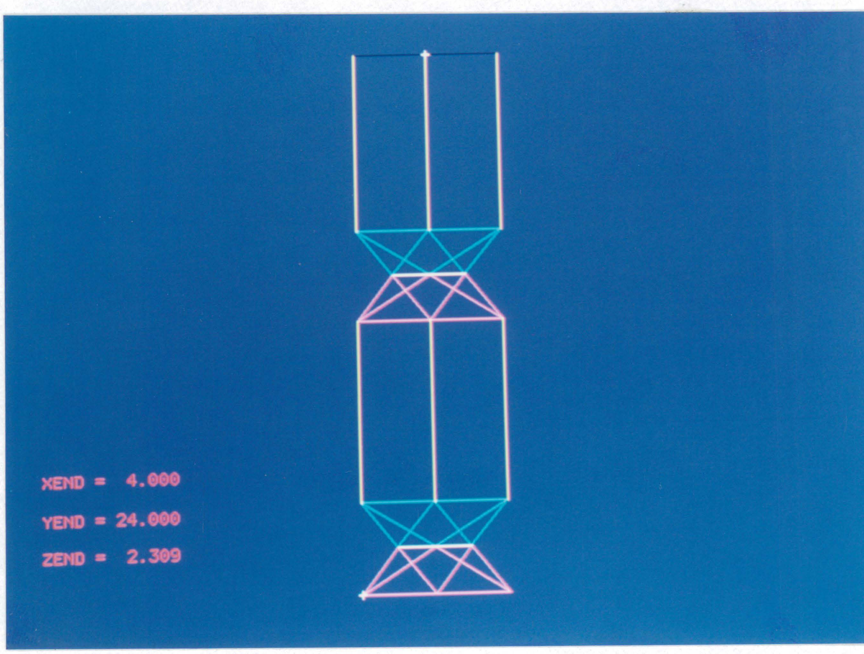


Figure 15. SPATIAL VARIABLE GEOMETRY TRUSS - Output from graPHIGS

ating the workspace, the six links were varied from one extreme to another in all possible combinations. Different views of the workspace are presented in Fig. 16. The dimensions shown are in inches. Each point represents a reachable position. As can be observed from the plots, the distance that can be reached in all three directions (X, Y, and Z) is quite appreciable compared to the total extension and the total length varied. The boundaries are shown approximately using sections of cubic splines.

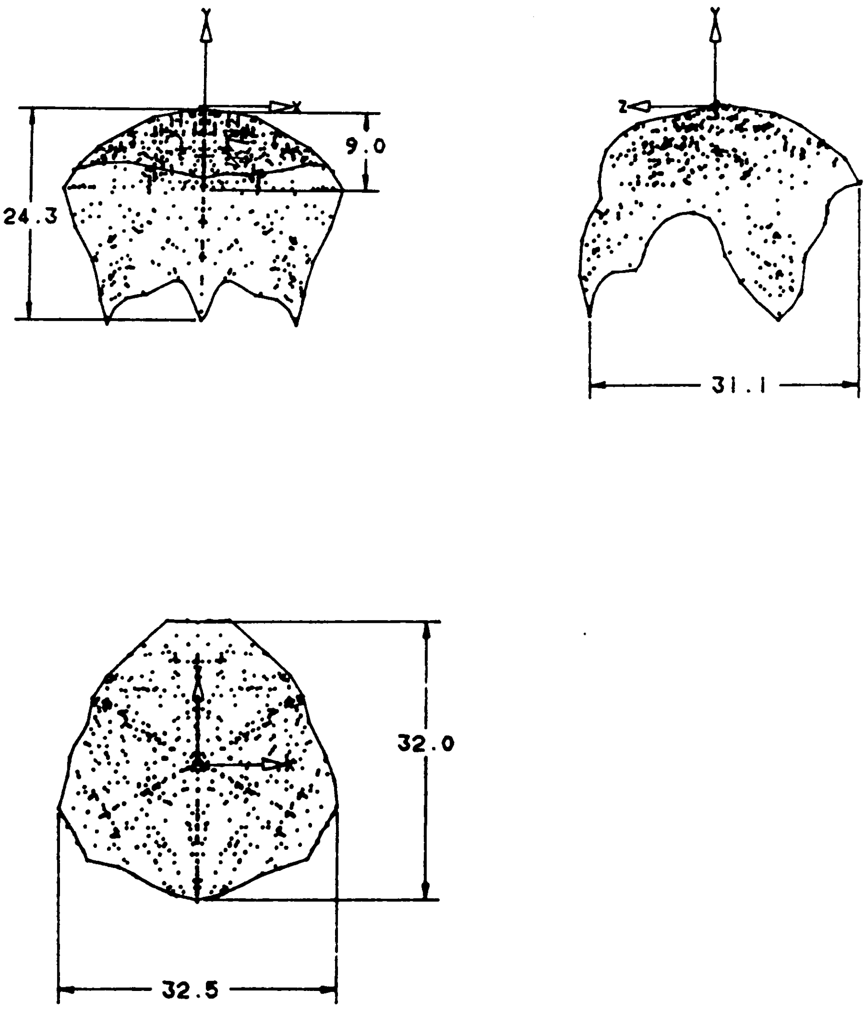


Figure 16. SPATIAL VARIABLE GEOMETRY TRUSS - Workspace in XY, XZ, and YZ Plane

Chapter 5

Conclusions and Recommendations

Even though serial manipulators suffer from a lack of rigidity, they have large workspaces which are critical in the design of manipulators. On the other hand conventional parallel manipulator designs are able to provide better rigidity. However, they have a highly restricted workspace. By using the Variable Geometry Trusses as joints in serial manipulator arms, it is possible to increase the rigidity of manipulators and at the same time provide workspaces that are comparable to the serial manipulators. The feasibility of achieving a larger workspace while retaining the rigidity is a significant step forward from the design of parallel manipulators that are based on single platform type devices. The possibility of achieving large workspaces should generate much interest in this concept.

Serial manipulators normally have six or fewer degrees of freedom. Sometimes a higher degree of freedom device is desired for tasks requiring obstacle avoidance or in a tortuous workspace. The lack of stiffness in serial manipulators has so far prevented the creation of high degree of freedom systems. Serial manipulators that are constructed with numerous VGT joints have excellent stiffness throughout the range of motion. Such a manipulator still retains a great amount of the rigidity inherent in the joints. This kind of manipulator may be extremely useful as remote manipulator arm in space. Present remote manipulator arms are long and slender and have vibration problems [16]. In these manipulators only the vibrations that occur in the plane containing the actuators can be controlled. However vibrations can occur in any plane. By using the VGT jointed manipulator control of most modes of vibration is possible [17] .

The concept of optimization allows variations in dynamic characteristics while the manipulator is in motion. Properties such as velocity response, mechanical advantage, and load carrying capacity are controllable in manipulators with high degree of freedom. This thesis introduced the new concept of VGT jointed manipulators and has dealt with specific problems such as the under-specified problem and the optimization problem.

One area yet to be investigated is the feasibility of using other variable geometry trusses as joints. The octahedral truss has no problems with link interferences. When using other trusses such problems may arise. The range of motion that can be produced with other trusses is an important area yet to be studied. This is important because small improvements in workspace are highly critical in the design of VGT jointed manipulators.

References

1. Hunt, K. H., **Kinematic Geometry of Mechanisms** , Oxford University Press, London, 1978.
2. Stewart, D., " A Platform with Six Degrees of Freedom ", **Proc. I. Mech. E. (London)** , Vol. 180, Pt. 1, 1965-6, pp. 371-386
3. Fichter, E. F.; and McDowell, E. D., " A Novel Design for a Robot Arm", **Advances in Computer Technology** , an ASME Publication, pp. 250-256.
4. Hunt, K. H., "Structural Kinematics of In-parallel Actuated Robot Arm", ASME Paper 82-DET-105, 1982.
5. Yang, D. C. H.; and Lee, T. W., "Feasibility Study of a Platform Type of Robotic Manipulators from a Kinematic Viewpoint", ASME paper 83-DET-35, 1983.
6. Joshi, N., "Mobility Analysis of Variable Geometry Trusses ", MS Thesis, Department of Mechanical Engineering, VPI&SU, Blacksburg, VA, Feb. 1988.
7. Miura, K.; and Furuya, H., "an Adaptive Structure Concept for Future Space Applications", IAF-85-211, Proceedings of the 36th Congress of the International Astronautical Federation, Pergamon Press, 1985.
8. Rhodes, M. D.; and Mikulas Jr., M. M., "Deployable Controllable Geometry Truss Beam", **NASA Technical Memorandum 86366** , June 1985.

9. Miura, K.; Furuya, H.; and Suzuki, K., "Variable Geometry Truss and its Application to Deployable Truss and Space Crane Arm" **Acta Astronautica** , 1985, IAF-84-394, Vol. 12, No. 7-8.

10. Reinholtz, C. F.; and Gokhale, D., "Design and Analysis of Variable Geometry Truss Robots", Proceedings of the 10th Applied Mechanisms Conference, Dec 1987.

11. Mabie, H. H.; and Reinholtz, C. F., **Mechanisms and Dynamics of Machinery** , 4th Ed., John Wiley and Sons, NY, 1987.

12. Beighter, S. C.; Phillips, T. D.; and Wilde, J. D., **Foundation of Optimization** , 2nd Ed., Prentice-hall, Chapter 2.

13. Suh, C. H.; and Radcliffe, C. W., **Kinematics and Mechanisms Design** , John Wiley and Sons, NY, 1978.

14. Gokhale, D., "Kinematic Analysis and Animation of a Variable Geometry Truss Robot", MS Thesis, Department of Mechanical Engineering, VPI&SU, Blacksburg, VA, Dec 1987.

15. Johnson, L. W.; and Riess D. R., **Numerical Analysis** , 2nd Ed., Addison Wesley.

16. Karl-Heinz Doetsch, Jr., "The Remote Manipulator System for the Space Shuttle Orbiter", Z. Flugwiss. Weitraumforch. 2 (1978), Heft 4, PP. 261-269.

17. Patten, W. N., H. H. Robertshaw, D. Pierpont, and R. H. Wynn, "Active Vibration Mitigation of Distributed Parameter, Smart-Type Structures Using Psuedo Feedback Optimal Control", presented at Computational Aspects in the Control of Flexible Structures Workshop, NASA Langley Research center, July 12-14, 1988.

18. Padmanabhan, B.; and Arun, V., "VGT's - A New Concept for Manipulator Joints", ASME Student Mechanisms Design Contest, Graduate Student Division, Orlando, Oct 1988.

Appendix A

Solution to the Four-bar Linkage Model

The solution to the four-bar linkage model has been carried out as in Mabie and Reinholtz [11]. Equation (1) is the loop loop closure equation for the linkage shown in Fig. 6 .

$$r_1 e^{i\phi_1} + r_4 e^{i\phi_4} + r_3 e^{i\phi_3} - r_2 e^{i\phi_2} = 0 \tag{1}$$

This equation contains two parts. It can be expanded in terms of the sines and cosines as in Eqs. (2) and (3).

$$r_1 \cos\phi_1 + r_4 \cos\phi_4 + r_3 \cos\phi_3 - r_2 \cos\phi_2 = 0 \tag{2}$$

$$r_1 \sin\phi_1 + r_4 \sin\phi_4 + r_3 \sin\phi_3 - r_2 \sin\phi_2 = 0 \tag{3}$$

These two equations can be rearranged, squared and added to eliminate the unknown angle ϕ_3 to result in Eq. (4).

$$r_3^2 - r_2^2 - r_1^2 - r_4^2 + 2r_1r_2\cos\phi_1\cos\phi_2 - 2r_1r_4\cos\phi_1\cos\phi_4 + 2r_2r_4\cos\phi_2\cos\phi_4 + 2r_1r_2\sin\phi_1\sin\phi_2 - 2r_1r_4\sin\phi_1\sin\phi_4 + 2r_2r_4\sin\phi_2\sin\phi_4 = 0 \tag{4}$$

The value of the output angle ϕ_4 is given by

$$\phi_4 = 2\tan^{-1}\left[\frac{C \pm \sqrt{C^2 - A^2 + B^2}}{A + B}\right] \tag{5}$$

Where:

$$A = r_3^2 - r_2^2 - r_1^2 - r_4^2 + 2r_1r_2\cos(\phi_1)\cos(\phi_2) + 2r_1r_2\sin(\phi_1)\sin(\phi_2) \tag{6}$$

$$B = 2r_1r_4\cos(\phi_1) - 2r_2r_4\cos(\phi_2) \tag{7}$$

$$C = 2r_1r_4\sin(\phi_1) - 2r_2r_4\sin(\phi_2) \tag{8}$$

This completes the solution of the four-bar linkage model.

The vita has been removed from the scanned document