AERODYNAMIC STABILITY OF SUSPENSION BRIDGES

by

LOUIS ANTHONY BECKER

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APPROVED:

Director of Graduate Studies

Dean of Engineering

APPROVED:

Head of Department

Major Professor

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TABLE OF CONTENTS

I. LIST OF FIGURES

II. SYMBOLS

A	Area of the Deck (Center to Center of Girders)
C_D	Coefficient of Drag
C_L	Coefficient of Lift
C_M	Coefficient of Moment
D	Drag Force
F_1, F_4	Phase Difference Correction Factors, Found by Integrating the Effect of Pressure Distribution Across the Section
G_2	Phase Difference Correction Factors, Found in the Same Way as F_1 and F_4
L	Lift Force
N, n	Frequency of Vibration
R	Resultant of Lift and Drag Forces
S_1	Slope of the Static Lift Curve for the Straight Model
S_2	Slope of the Static Torque Curve for the Straight Model
S_4	Slope of the Static Torque Curve for the Curve Model
V, v	Velocities
X, X', X"	Amplitudes
a_1	Four Times the Radius of Gyration Squared Divided by the Square of the Chord Width
b	Chord Width
d	Depth of the Section
e	Eccentricity

m	Mass
C.P.	Center of Pressure
α	Angle of Attack
δ	Logarithmic Decrement
$\frac{\rho b^2}{m}$	Mass Density Ratio
ρ	Density of the Air

III. INTRODUCTION

For many years engineers have been faced with the problem of aerodynamic effects on structures. The suspension bridge, in particular, throughout its history has been subjected to the destructive effects of wind. Although this is a problem of long standing, little progress was made toward a solution until after the Tacoma disaster in 1940. At the present time, although much work has been done and there are some very promising theories in the literature, it is still important to check and modify all proposed designs by conducting wind tunnel tests on section models of the chosen sections.

This investigation was concerned with a part of the overall problem. It was the purpose of the investigation to determine the stability response of suspension bridge structures to wind velocities actually encountered. The program that was followed treated only the basic motions associated with suspension bridge decks, namely those of vertical motion and torsional motion. No attempt was made to treat coupled motion (combination of vertical motion and torsional motion) although it is known that such motion may also exist in the actual structure.

Most sections encountered had already been subjected to static tests and the coefficients of lift, drag and moment for these sections had previously been determined. However, some sections were encountered that had not previously been subjected to static tests and in such cases the results of these static tests were incorporated in this thesis.

The ultimate goal of this type of investigation is to determine the stability characteristics of basic suspension bridge roadway sections. Such information would prove invaluable to the designer by providing a starting point. It would still be advisable to conduct wind tunnel tests on detailed models to determine the characteristics of the actual design.

IV. REVIEW OF LITERATURE

A. HISTORY

From the early foot bridge to the massive structure of today, wind has been a constant enemy of the suspension bridge. Until the failure of the Tay Bridge(16) in 1879, no consideration was given to the effects of wind in design and hence it is small wonder that there were a large number of failures due to wind before this date. These early failures occurred both in Europe and in the United States. Five of these disasters occurred in the British Isles within a twenty-one year period.(7) There are eye-witness accounts of several of these failures in the literature and it is interesting to note the marked similarity between these early failures and the failure of the Tacoma Narrows Bridge.

The Tay Disaster(16) brought an awareness of the need to include wind effects in bridge design. Following this disaster all bridge designs included a consideration of the horisontal component of wind. For a time, it was felt that the problem was solved, for with the exception of the failure of the Niagara-Clifton Bridge(17) (the cause of which was not completely known since it failed at night) no more failures were experienced until the late nineteen thirties. Several bridges(1),(2),(4),(7) built in the late thirties developed oscillations when subject to certain wind conditions. None of these were of a serious nature and by the use of stays and other devices they were stopped without damage to the bridge.

The idea that the problem of wind effects had been solved was dispelled with the failure of the Tacoma Narrows Bridge.(7) It was noted soon after completion that the bridge was subjected to violent oscillations. On the morning of November 7, 1940 the bridge was destroyed by this wind action. Unlike previous failures this one did not go unnoticed. All during its short life a complete record of its behavior was noted and there are even motion pictures of the failure. Immediately after the failure a flood of literature brought all the old failures before the eyes of the public and, more important, made the engineering world conscious of the glaring need for research into this field.

A more complete history can be found in earlier reports on this subject. An excellent one appears in the thesis on "Aerodynamic Stability of Bridges" by Daniel Frederick and Edward Estes, Jr. (12)

B. TERMINOLOGY

A body immersed in a stream of moving fluid will experience a resultant force, exerted by the fluid, which is dependent upon the relative velocity between the fluid and the body. The usual practice in fluid mechanics is to resolve this resultant into two components, one along the line of velocity called drag and one perpendicular to the line of velocity called lift.(5),(13) Since in the general case, the resultant does not act through the center of gravity of the section a moment is also produced.(13)

From experimentation or dimensional analysis the following equations arise:

$$\text{Lift} = C_L \frac{\rho}{2} V^2 A$$

$$\text{Drag} = C_D \frac{\rho}{2} V^2 A$$

$$\text{Torque} = C_M \frac{\rho}{2} V^2 A b$$

where,

V	is the velocity of the air
A	is the area of the deck (center to center of girders)
b	is the chord or width of the section
ρ	is the density of the air
$\frac{\rho V^2}{2}$	is the dynamic pressure
C_D, C_L, C_M	are dimensional coefficients depending upon the shape of the cross section(5), (13)

In addition to the magnitude and direction of the resultant it is important to know the point of application of the resultant on the body. This point of application of the resultant force is known as the center of pressure. It is customary to express the distance from the center of pressure to the center of the section as a percentage of the chord. This percentage is known as the eccentricity (e) and is determined by dividing the slope of the torque curve by the slope of the lift curve.

$$C_m = \frac{\text{moment}}{\frac{\rho V^2}{2} Ab} \quad \& \quad C_L = \frac{\text{lift}}{\frac{\rho V^2}{2} A} \quad \text{so} \quad \frac{C_m}{C_L} = \frac{\text{moment}}{b \times \text{lift}}$$

$$\frac{C_m}{C_L} = \frac{C.P.}{b} = e$$

where C.P. is the center of pressure.(13),(21)

It can further be shown that the resultant is dependent upon the angle of attack (α). This angle is the angle formed by the chord of the section and the direction of the wind. A plus angle indicates that the wind is blowing upward on the section.

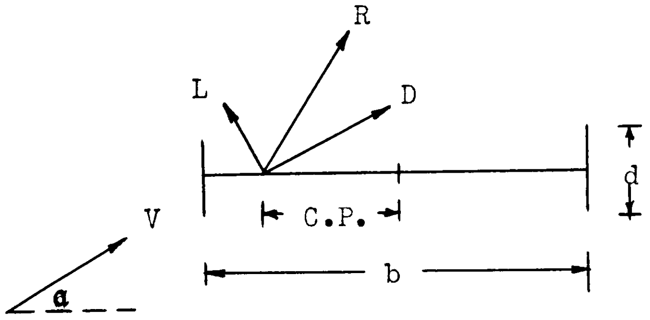


Figure 1 ..... Forces on Basic Bridge Section

The ultimate stability or instability of a section depends upon the amount of damping in the system. The most practical way to measure this damping is by means of the logarithmic decrement "δ". The log decrement is defined as the natural logarithm of the ratio of amplitudes of two successive cycles.(11),(14)

$$\delta = \ln X/X'$$

where X is the amplitude of vibration in the first cycle and X' is the amplitude in the next successive cycle. Practically speaking it is difficult to obtain an accurate decrement working with two successive cycles and therefore a slight modification was made in this investigation. Instead of working with the first two cycles, some set amplitude is chosen and the number of cycles necessary to reach an amplitude of one half the initial value is determined. The decrement can then be computed as follows:

$$\delta = \frac{1}{n} \ln X/X''$$

where X/X" has the value of two and n is the number of cycles.

To transfer results from models to the full size structure several scale ratios are required. First the section ratio(12) of the model must be the same as that of the prototype. This is the ratio of the depth of the girder to the width of the section, center to center of girders (d/b fig. 1).

To transfer the values of the decrement from one system to another, it is necessary to consider the mass density ratio.(14) This ratio is in the form of $\frac{\rho b^2}{m}$ being the density of the air in slugs per cubic foot, b the chord width and m the mass per foot of the structure. This factor represents a ratio of the mass of air displaced to the mass of the displacing object. Variation in the mass density ratio causes a linear change in the value of the decrement.

The final ratio important in the transferring of data from one system to another is the aspect ratio.(13) This is a ratio of the span length to the width of the section. In these experiments an aspect ratio of infinity was maintained by the use of end plates on all models which in effect limits the flow to two dimensions.

C. AEROSTATIC STABILITY

Little trouble has been encountered in handling this part of the overall problem. A bridge that is safe from sliding, uplifting and overturning is aerostatically stable.

The Tay disaster manifested the need to consider the static horizontal pressures caused by the wind, in design. However, the vertical component was virtually overlooked. Even at small angles of attack this component can be quite large and in most cases it exceeds the horizontal component in magnitude. In 1944 the Chester (Illinois) Bridge was destroyed because of the overturning effect of this force.(16)

D. AERODYNAMIC STABILITY

1. Introduction.(6),(15),(18) There are certain sections which, when free to oscillate and subjected to a wind, will vibrate with ever increasing amplitudes. These transverse oscillations are experienced by such sections as a semicircular section with its flat side toward the wind, a rectangular section with its broad side towards the wind and a "T" section with its head to the wind. If these sections are mounted as pinwheels they will rotate in a direction opposite to the direction of the fan causing the wind. These sections are aerodynamically unstable. Other sections such as the semicircular section with its flat side away from the wind, a flat plate with its thin side to the wind and a "T" section with its stem facing the wind will not be subject to this ever increasing amplitude and if mounted as pinwheels they will rotate slowly in the direction of rotation of the fan. Such sections are aerodynamically stable.

2. Early Theories. Several early theories were proposed. These were all based on the autorotation idea and were essentially based on the use of the results of static tests only. Den Hartog's(6)

theory considered the total vertical force on a section and found the variation of this force with the angle of attack. Assuming the angle of attack to be small the ultimate stability reduces to finding the slope of the lift and moment curves for the section in question. Dr. Steinman(18) modified this idea by using the velocity ratio to account for the time required for the wind to traverse the section but the principal criterion for stability was still the same, namely that a positive slope of the lift curve corrected for drag indicates vertical stability and a negative slope indicates vertical instability and that a positive slope to the torque curve indicates torsional stability and a negative slope indicates torsional instability with the degree of stability or instability being determined by the steepness of the slope. A section corrected for drag is stable if $\frac{dL}{d\alpha} + D > 0$.(6)

3. Later Theories. The previous theories are valid for predicting the ultimate stability or instability of a section. However, they are not adequate for predicting the behavior of a section over an entire range of wind velocities. Several more complete theories have been proposed.

Professor Farquharson(7) in connection with his work on the failure of the Tacoma Narrows Bridge, concluded that the formation of vorticies played an important part in the stability of certain sections. Several men have formulated theories concerning stability, from a consideration of vortex formation, but these are difficult to apply because of the complex mathematics arising when the average bridge section is considered.

The most general theory in this field was proposed by Dr. Steinman(22) and the complete treatment can be found in the 1950 Transactions of The American Society of Civil Engineers. The essential idea back of the theory is that all that is necessary for a comprehensive solution to any aerodynamic problem is to write the general aerodynamic equations for the forces acting on the section together with the general dynamic equations for these same forces.

The general aerodynamic equations (equations 1a & 1b in the paper) are a summation of all the lift and moment components acting on the section. Because of the complexity of most bridge sections it is necessary to obtain these components experimentally. This can be done by conducting static tests on straight and curved models. The slopes of the static lift and torque graphs for the straight models represent the contribution of vertical velocity. The slopes of the static lift and torque graphs for the curved models represent the contribution of angular velocity.

In the general vibration system there is a force acting on the body and a displacement and these are usually out of phase. Consider a body in harmonic motion with a displacement x, acted

CRITICAL RANGES FOR STABLE & UNSTABLE SECTIONS

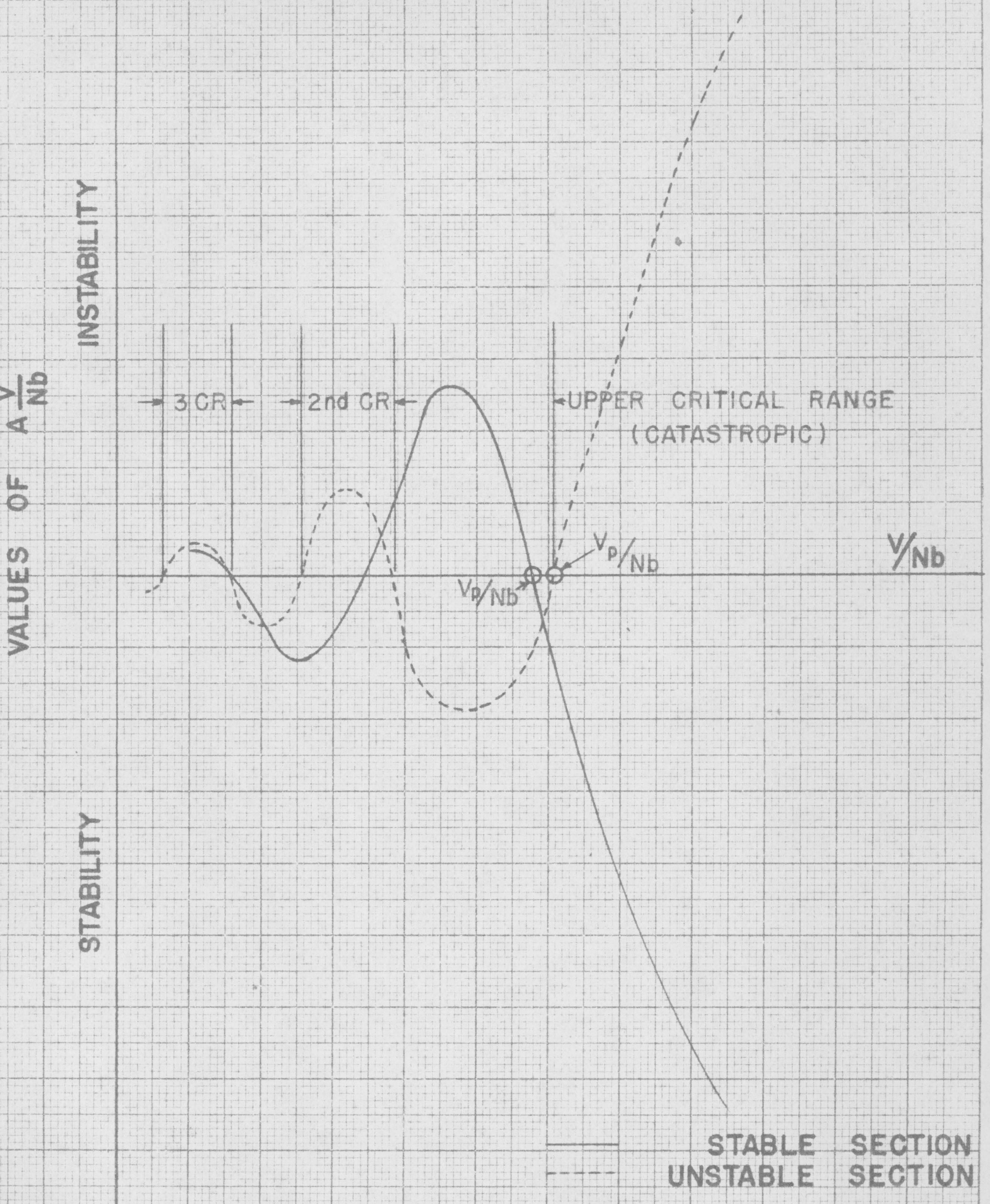


Fig. 2

upon by a force P. In terms of the initial force and displacement, $x = X_0 \sin wt$ and $P = P_0 \sin (wt + \phi)$ where ϕ is the phase angle and w is the angular velocity. The total work done on the section is $W = \pi P_0 X_0 \sin \phi$ (see Fig. 3). This phase angle is a function of the velocity ratio, V/Nb. This dependency arises from the fact that as a particle traverses a section, it meets the section at different positions depending on the above three variables and the magnitude of the force depends on the position and velocity of the section when it meets the particle.

The work done on the section can be either positive or negative depending on the phase angle. If this work is positive to a degree exceeding the structural damping of the section, the amplitude of vibration will tend to increase and depending on the relative increases of all the forces concerned, the section may destroy itself. If the work is negative the amplitude will decrease and the section is then considered stable.

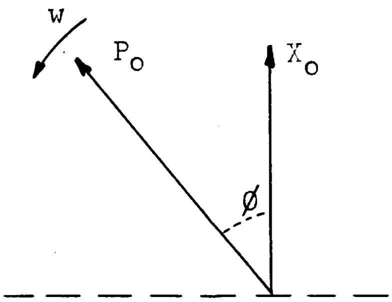
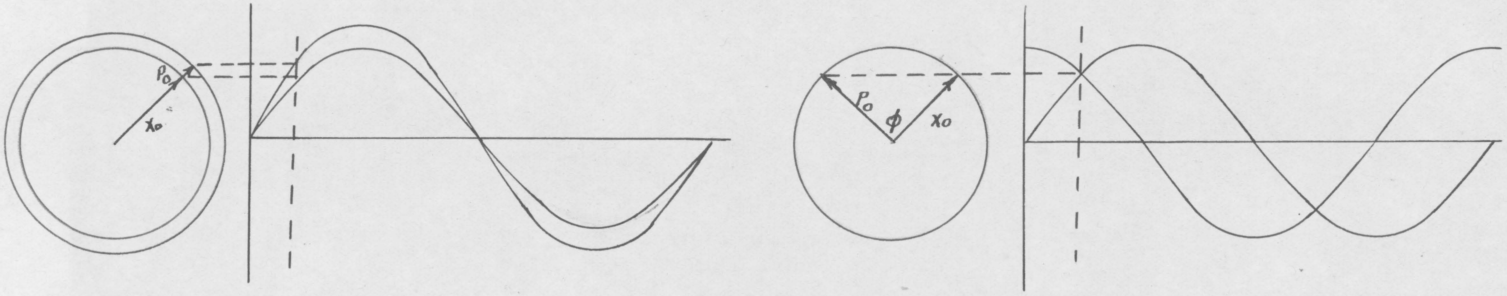
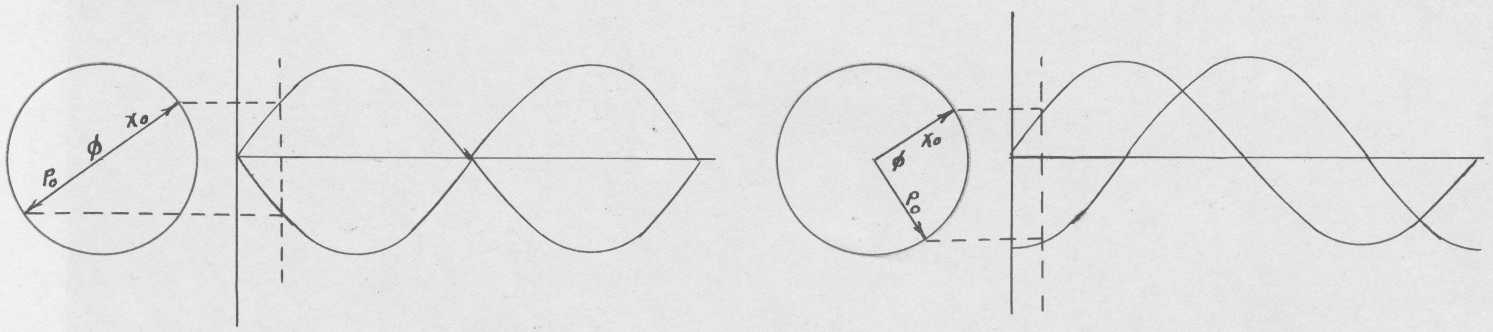


Fig. 3a.. A Force and a Motion of the Same Frequency



No Work Done

Positive Work Done



No Work Done

Negative Work Done

Fig. 3b ... Vector Representation of a Force and a Motion of the Same Frequency

By using the slopes of the static lift and torque graphs for straight models to obtain the contribution of vertical velocity and the slopes of the static lift and torque graphs for curved models to find the contribution of angular velocity, Dr. Steinman(22) was able to determine the general aerodynamic equations for any particular section. Applying correction factors for phase difference, found from the pressure distribution curves (eqs. 5 paper)(22), to these general aerodynamic equations and equating the corrected results to the general dynamic equations for the section, he arrived at the following equations, as end products, for the damping of a section:

for pure vertical motion

$$\delta = -\frac{\mu}{4}(F_1S_1)\frac{V}{Nb} \text{ ....eq. 13 (paper)}$$

and for pure torsional motion

$$\delta = \frac{-\mu}{2a_1}\left(\frac{G_2S_2}{k} + F_4S_4\right)\frac{V}{Nb} \text{ ....eq. 16}$$

where

δ is the logarithmic decrement

μ is the mass density ratio

$\frac{V}{Nb}$ is the velocity ratio

S_1 is the slope of the static lift curve for the straight model

S_2 is the slope of the static torque curve for the straight model

S_4 is the slope of the static torque curve for the curved model

a_1 is four times the radius of gyration squared divided by the square of the chord width

F_1, F_4 and G_2 are correction factors, found by integrating the effect of pressure distribution across the section, to account for the phase difference. (eqs. 5 paper)

This method will also give the modified frequency of the aerodynamic vertical or torsional oscillations in terms of the natural frequency of vibration if this is desired.

There are some other theories but this one seems to be the most complete and it has the advantage of being able to handle any shape that may be encountered.

There are several disadvantages to the theory, however. First of all it requires a large number of tests. There must be four pressure distribution tests in addition to six static tests. Upon completion of the tests the mathematics necessary to reach a conclusion are time consuming. Finally it is difficult to decide how to curve complicated sections to get the correct results. When these disadvantages are weighed against the work necessary to conduct dynamic tests, which through the improvement of equipment and methods in recent years has become much more reliable and easier to conduct, it appears that dynamic tests would be more desirable in most cases.

E. SMALL SCALE MODEL TESTS

Most of the papers written in the field of bridge testing seem to be in agreement on the fact that models will yield sufficiently accurate results for most work. Professor Farquharson(8) in his tests for the redesign of the Tacoma Bridge revealed that all his tests were conducted on small scale models and the most promising designs were tested on larger models. The results obtained from the large models checked the values of the smaller models in all cases. Dr. Steinman made the following two statements in connection with small scale model testing:

> "For an accurate calculation of the overturning moments, the aerodynamic constants of the section are required. These can be easily and quickly obtained in a simple wind tunnel test on a section model, to determine the coefficients of drag, lift and torque."(16)
>
> "The aerodynamic stability or instability of bridge sections is entirely predictable, both in direction and intensity, for all wind velocities and for any angle of attack, without elaborate, costly and time consuming tests on large-scale models of complete structures. All that is needed is a simple test, requiring only a few minutes of time, on an elementary, small scale model of the cross-section.
>
> "For the most accurate determination of the aerodynamic characteristics, a simple pressure-distribution measurement is made. The section model may be only a few inches in dimension, and the test can be made in a miniature wind tunnel......
>
> "In the absence of such pressure-distribution curves for a given section, almost identical results can be obtained from the lift and torque graphs of the elementary section model held stationary at varying angles of incidence in a wind tunnel."(20)

Another statement to verify the use of small models was made by H. A. Thomas:(24)

> "Consideration of the aerodynamic principles involved in the Tacoma Bridge disaster requires knowledge of the value of lift and torque coefficients, which can be obtained by models."

A further verification in the use of small models appears in the report concerning the aerodynamic stability of the Severn Bridge:(10)

> "In the early stages of the investigation it was uncertain whether the stability of a complex bridge could be predicted satisfactorily from experiments on a sectional model alone... To provide a practical programme it was therefore decided to depend on tests of sectional models to a linear scale of 1/100...Whilst the preceding tests were in progress, the design and construction of a full model was also put in hand, with the view to tests of the correlation between the two different experimental methods...The results obtained with the two types of models led to the conclusion that the tests of sectional models were sufficient for reliable full-scale prediction."

F. PREVIOUS TEST RESULTS

Wind tunnel tests have been conducted on bridge sections by Dunn, Farquharson, Rouse, Maher and Fraser.

In connection with the work done on the Tacoma Narrows bridge, the following results were obtained at GALCIT (Guggenheim Aeronautical Laboratory, California Institute of Technology) and at the University of Washington, and reported by Dr. Dunn.(23) The first tests were run on models of two deep truss type girders. The first of these was 53 feet wide and 20 feet deep for a section ratio of 0.38 and the second was 39 feet wide and 24 feet deep for a section ratio of 0.62. Both of these sections were vertically stable. Models of the bridge were then subjected to both static and dynamic tests and the following results obtained:

1. As constructed the models showed vertical stability and torsional instability between the angles of attack of +10° and -10°.

2. When streamlined, the model was found to be both vertically and torsionally stable over the entire range tested.

3. When the depth of the girder was increased the model showed vertical instability between angles of attack of +5° and -5° and torsional instability between the angles of +10° and -10°.

A good correlation was obtained between the work done at GALCIT and at the University of Washington for the streamline section. A discrepancy existed between the curves for the original section. The University of Washington showed a slight bit of vertical instability at low angles of attack while GALCIT showed the section stable over the entire range tested. The difference in the two curves may be explained by the fact that different aspect ratios were used in the two tests.

Additional tests at the University of Washington under the direction of Professor Farquharson(8) are responsible for the following results:

1. Girder sections indicate unstable characteristics regardless of their section ratios.

2. Trussed models have a critical velocity of 100 miles per hour at $\alpha = 0^{o}$.

3. Within the limit of the usual configuration slotted sections experience little motion and this motion is definitely noncatastrophic in nature.

4. Torsional motion in girders is always catastrophic and practically speaking damping is of little help in reducing this motion.

Professor Rouse(19) at the University of Iowa ran pressure distribution tests on "H" sections with the following results: a section with d/b equal to 0.16 is vertically stable and torsionally unstable and a section with d/b equal to 0.33 is unstable in both types of motion.

A more extensive study was made at Virginia Polytechnic Institute under the direction of Professor Maher(12). The following results, based on static tests, were reported:

1. "H" sections with $d/b > 0.075$ are both vertically and torsionally stable. Above this value of the section ratio instability develops.

2. The only deck girder section tested that was completely stable was the one with a section ratio of 0.05.

3. Both deck and through truss sections show complete stability over the range of angles from -6° to +6°. No neutral section, that is a section with zero slope to its lift and torque graphs at the origin, was found and therefore some modifications were tried resulting in the following conclusions:

1. Double "H" sections were tried and although some sections were found that were vertically and torsionally stable no neutral section was found.

2. A proper combination of a center slot and lateral fins on an "H" section almost completely eliminates the instability of the section.

3. The combination of lateral slots and fins on the "H" section improves or eliminates both the vertical and torsional instability.

Dr. Fraser(9), in connection with his work on the proposed Severn Bridge, lists the following conclusions:

1. Bridges stiffened by plate girders are liable to both torsional and vertical oscillations.

2. Bridges with truss stiffening are immune from vertical oscillation and their tendency to be unstable torsionally can be corrected by suitable design of the suspended structure.

These conclusions are based on the results of both static and dynamic tests.

V. EXPERIMENTAL INVESTIGATION

A. OBJECT OF INVESTIGATION

The principal object of this investigation was to determine the stability of basic suspension bridge sections when these sections are subjected to either vertical motion or torsional motion. The stability was determined, for either mode of motion, over a range of wind velocities scaled to represent actual velocities that are likely to be encountered in practice. A secondary object of the investigation was to verify the predictions made concerning the stability of different sections as deduced from the results of static tests.

B. PLAN OF INVESTIGATION

The plan of investigation was divided into the following two parts: (1) to determine the lift, drag and torque graphs, from static tests, for all sections being considered that have not previously been tested; (2) to determine the stability response graphs, over a practical range of wind velocities, for sections subjected to either vertical motion or torsional motion at various angles of attack.

The following sections were considered:

1. A flat plate.

2. An "H" section with a section ratio (d/b) of 0.20.

3. An "H" section with lateral slots and fins.

4. A flat plate modified with details to represent a proposed design for the Tancarville (France) suspension bridge.

C. APPARATUS AND MATERIAL

1. The models used in this investigation were constructed entirely of aluminum except for the Tancarville model which was a combination of aluminum and wood. All of the models were 18" long and approximately 8" wide. End plates were employed in all tests to maintain two dimensional flow, i.e., to give an aspect ratio of infinity.

2. All tests were conducted in the Virginia Polytechnic Institute wind tunnel. This tunnel is a horizontal single return type with a three foot circular open test section. Its 35 horsepower D.C. motor is capable of producing wind velocities up to 150 miles per hour.

3. In the static portion of the investigation the models were held in the wind tunnel by means of the aluminum frame shown in Figure 4. All the struts were streamlined to reduce turbulence in the air stream. A top and bottom floor system was employed to prevent upwash or downwash from striking the frame.

Figure 5 shows the wind tunnel throat, the recording equipment and the wooden frame used to support the models in the dynamic tests. The methods of supporting the models in the frame for the different types of tests are shown in figures 6 and 7. Figure 6 shows the setup for vertical motion tests. The model and end plates are rigidly connected to the steel frame. This frame, in turn, is suspended from the four bar springs as shown in the figure. The other end of each bar spring is attached through mounting blocks to the wooden frame. This arrangement in addition to the guy wires as shown allows the model to oscillate vertically while preventing lateral motion. Figure 7 shows the setup for torsional motion tests. The model and end plates are connected to one end of a shaft. This shaft passes through a bearing held by the vertical struts and is connected to a bar spring. It was necessary to streamline these bar springs and thus they do not show in the photograph. They are underneath the circular streamlining vanes seen in the picture. These springs fasten to the wooden frame and the complete arrangement is such as to let the model oscillate torsionally but not vertically or horizontally.

4. Different methods were employed for changing the angle of attack. In the static tests a protractor was mounted to the model, with a reference mark on the frame. The model could be rotated to any desired angle relative to the frame and that angle read directly from the protractor. In the vertical motion tests

the mounting blocks used to hold the springs to the frame were milled to the desired angles and the angle could be changed by substituting different mounting blocks. For torsional motion tests the method used was somewhat similar to the static tests. The clamps that held the springs to the wooden frame were made in two pieces such that one piece could be rotated relative to the other. Changes in the angle of attack were made by changing the relative orientation of these two pieces.

5. The velocity of the air stream was measured in two ways. For velocities below 1200 feet per minute an Alnor Thermo-Anemometer was used and for velocities above 800 feet per minute a micromanometer connected to two static tubes inside the tunnel, was employed.

6. In the static portion of the investigation the values of lift, drag and moment were obtained by using the four component balance system that is an integral part of the V.P.I. wind tunnel. With this system it is possible to read the values of lift, drag and moment to 1/100 of a pound.

In the dynamic portion of the test the value of the logarithmic decrement was obtained from a time displacement curve as follows. The amplitude of the first cycle was measured. Next the cycle with an amplitude of half the amplitude of the first cycle was located. The number of cycles necessary for the vibration to reach half its initial amplitude could then be determined. From this information the value of the logarithmic decrement was found using the formula

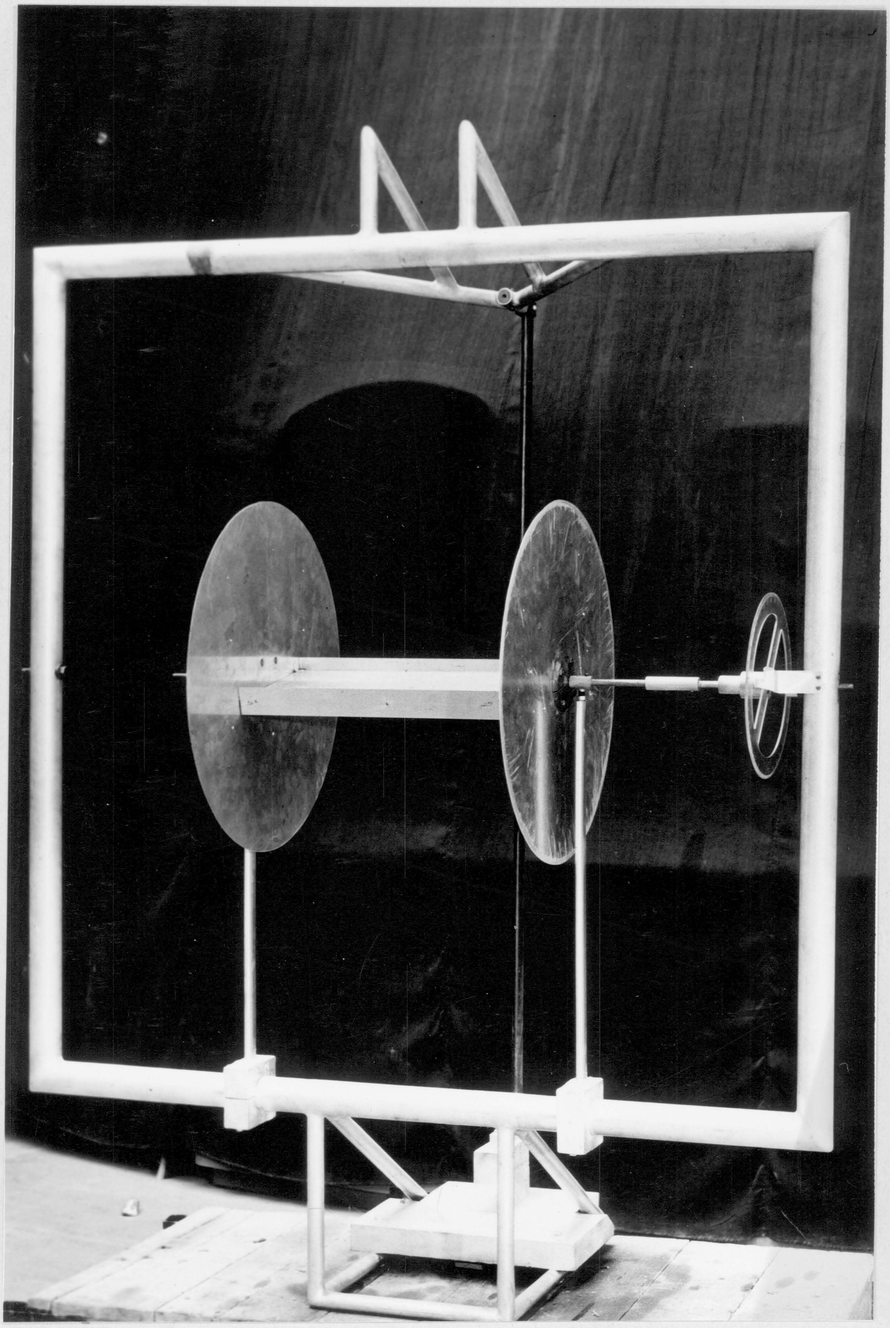


Fig. 4 "H" Section in Static Frame

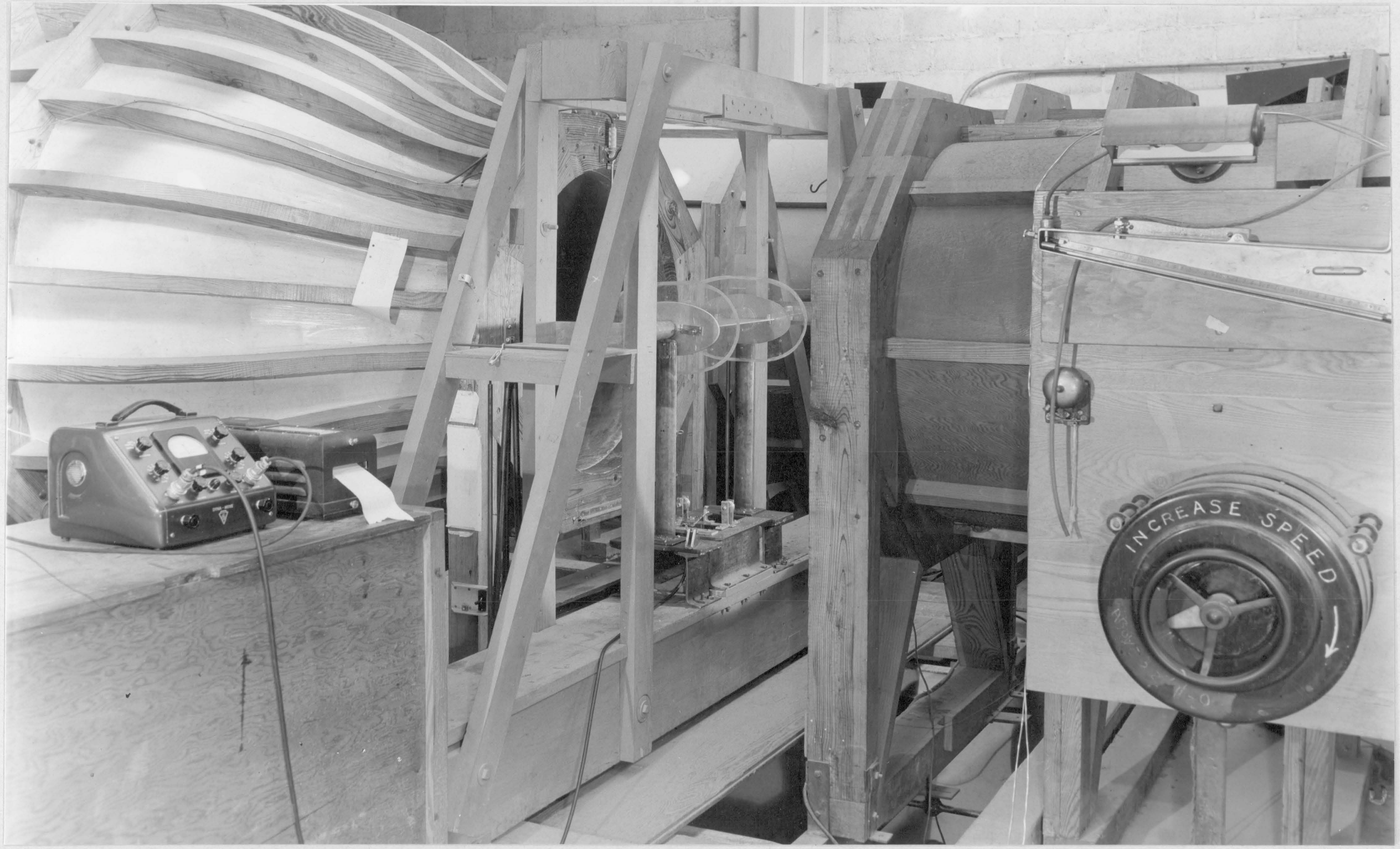


Fig. 5 Wind Tunnel with Model Torsionally Mounted, Frame and Recording Equipment

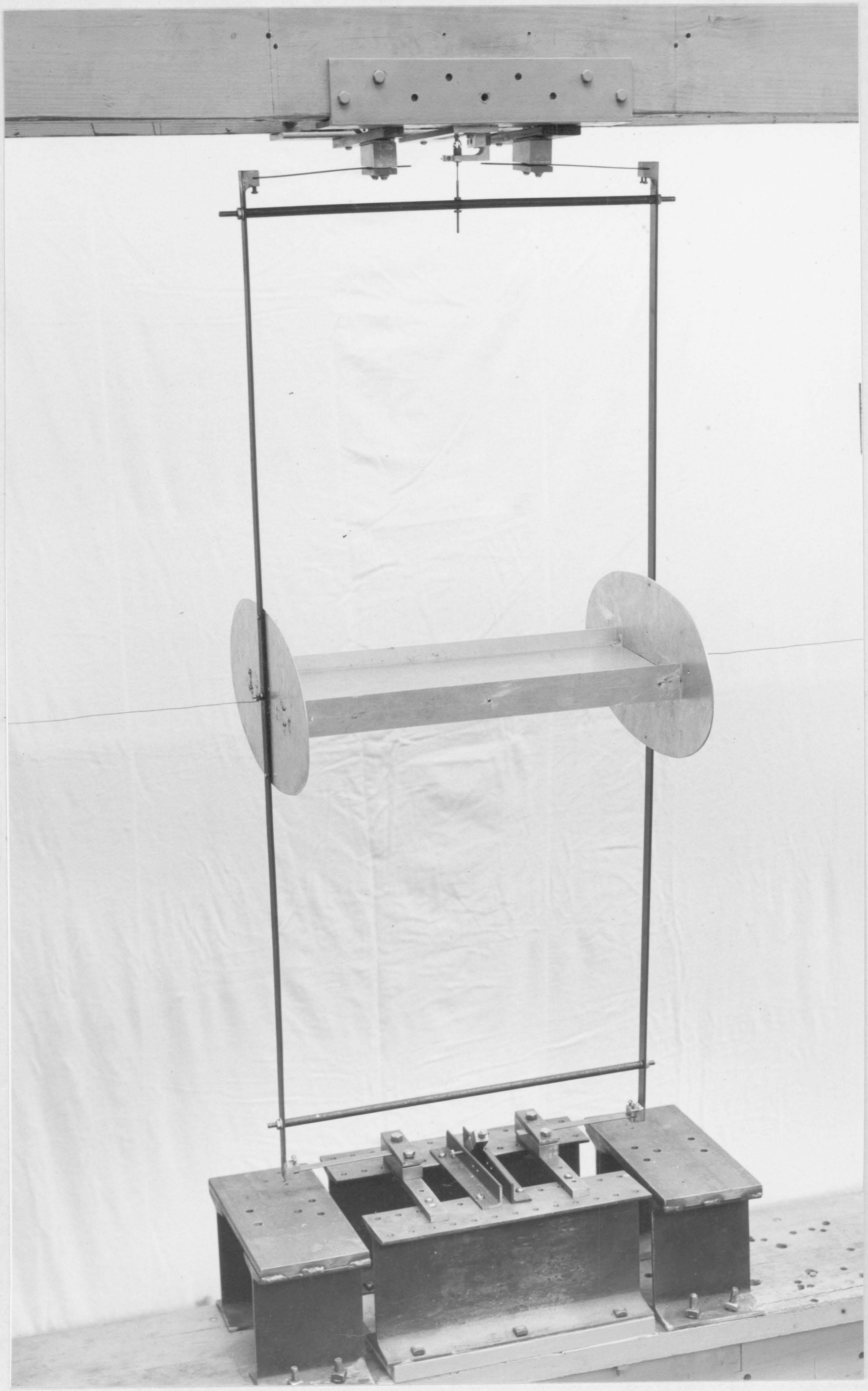


Fig. 6 "H" Section in Vertical Motion Setup

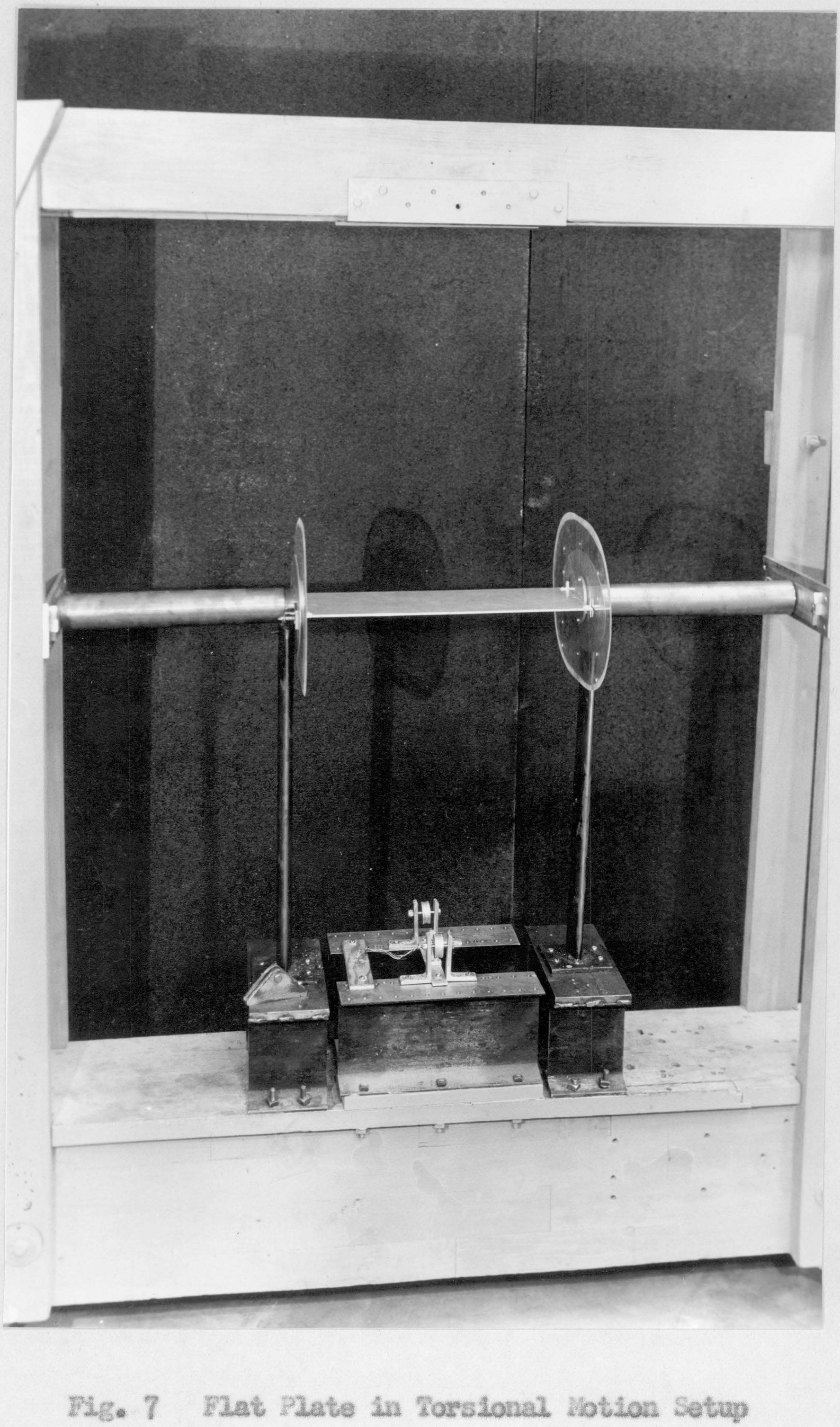


Fig. 7 Flat Plate in Torsional Motion Setup

$\delta = \frac{1}{n} \ln X/X''$ where δ is the log decrement, n the number of cycles necessary to reduce the amplitude of vibration to half the initial value and X/X'' is 2.

This curve was obtained from a Brush Oscillograph used in conjunction with a linear variable differential transformer (LVDT). The is a transformer containing one primary and two secondary windings, wound in a single coil. Centered in the coil is a movable iron core. A relative movement between the coil and the core changes the inductance of the circuit. This change of inductance is converted to strain, or displacement, by electronic circuits of the recording equipment, which are an integral part of the LVDT. There are two secondary windings, wound 180° out of phase, to enable the recording equipment to record the direction of the displacement as well as the magnitude.

In these tests the coil was mounted on the wooden frame and the core was fixed to the model and thus the relative displacement was determined. A permanent record of the displacement was made on a moving tape by means of the Brush oscillograph. Since the speed of the tape was known the frequency of vibration was easily determined from this record.

D. METHOD OF PROCEDURE

1. Static Tests

a. Determination of Tare. In order to determine the effects of the frame, struts, etc., the model was removed from the frame and the values of lift, drag and moment of the frame were taken. This gave a flow pattern similar to that experienced by the frame during the actual tests.

b. Dynamic Pressure. An actual dynamic pressure ($\rho V^2/2$) of 2.67" of water was maintained throughout all tests. This constant pressure eliminated the need to correct for differences in temperature and air density.

c. Test Procedure. The model was held stationary in the air stream by means of the frame shown in figure 4. The values of the lift, drag and moment were read from the scales in the tunnel pit. The first set of readings were taken with the model oriented at -12° to the air stream and subsequent readings were taken every two degrees thereafter until an inclination of $+12^\circ$ was reached.

2. Dynamic Tests.

a. Determination of Tare. In order to determine the effects of the frame, struts, etc., a rectangular strip of steel was substituted for the model and tare readings were taken in still air. This steel strip was the same weight as the model so that the same frequency would result. The strip was placed so as to offer as little area as possible in the direction of motion.

b. Test Procedure. The models were held in the air stream by means of the frames shown in figures 6 and 7. Two methods were employed depending on the amount of damping present in the system. If the section was stable or the amount of instability was less than the structural damping of the entire system the following procedure was used: the model was displaced a known amount from its equilibrium position and held there by a trigger mechanism. When the air stream had reached the desired velocity, the model was released and allowed to oscillate freely in either the vertical or torsional mode of motion. A time displacement record of this decaying motion was recorded by the oscillograph and the log decrement computed from this record.

In the case where the instability exceeded the structural damping, the model was held in its equilibrium position and then released. As its motion increased in amplitude a record of the displacement was made by the oscilligraph. The decrement was computed as before but the values were recorded as negative. The decrement found from these records is for the frame plus the model. To obtain the results for the model alone the tare readings were subtracted from the combined readings.

The first set of readings was taken in still air and all subsequent readings taken as the velocity ratio (V/Nb) was increased in small increments. The final readings were taken at velocity ratios of 5 or better, with the final reading depending on the frequency of vibration as well as on the maximum velocity that could be conveniently reached. This procedure was repeated for each model oriented at 0°, $+1.5^\circ$ and $+5^\circ$ to the wind, for each type of motion.

E. DATA AND RESULTS

1. Static Tests. After the lift, drag and moment for each section had been found, the corresponding coefficients were calculated from the formulas previously given. Because of the slight inclination of the air stream (approximately -1.5°) the curves did not pass through the origin and therefore it was necessary to apply a correction to the results. The correction, which was made graphically, was as follows:

(1) For Lift and Moment

- a. The curve and axis as determined from the actual tests was traced.
- b. This trace was rotated 180° and matched with the actual curve.
- c. The actual curve was then traced with its axis.

d. The mean of these two curves and axes was taken as correct and plotted in this report.

(2) For Drag

A similar procedure was followed for the drag correction except that in step b the curve was turned over instead of rotated.

2. Dynamic Tests. The values of the decrement for each model were calculated from the vibration records by the formula previously stated. These values were plotted against the velocity ratio (V/Nb). The correction made for the inclination of the air stream was to add 1.5° algebraically to the measured values of the angle of attack to get the true values.

3. Results. The results of this investigation are given in the following curves. The static curves for the flat plate and the "H" section with d/b = 0.20 were taken from the thesis of Frederick and Estes.(12)

a. Static Tests. The first three curves are the results of the static tests conducted on the flat plate. They indicate that this section is vertically stable to the same degree when the angle of attack is between -5° and +5°. They also indicate that the section is torsionally stable to the same degree over the entire range tested.

For the remainder of this thesis the results of static tests will only be interpreted between the values of -5° and +5° for the dynamic tests used to verify these results were only conducted between these values of the angle of attack. However the range of evaluation can be extended using the criterion stated on page 18 concerning stability and the slopes of the lift and torque graphs (corrected for drag).

b. Dynamic Tests. The method used in computing the logarithmic decrement involved a large number of cycles and therefore it was necessary to work with different amplitudes. Since a wide range of amplitudes was being used it was decided to check the effect of amplitude on the final results. Figure 9 shows the effect of different amplitudes on the stability response of the flat plate. As can be seen from the curves the numerical values of the decrement are slightly increased as the initial amplitude increased. However this increase is small and the general shape of the curve is unaffected. In the usual case the shape of the curve rather than the actual numerical value is sought and this method offers results within the range of accuracy required.

During the investigation a question arose concerning the effects of frequency on the final results. According to theory the frequency should have no effect on the final results. During the investigation evidence was found that seemed to contradict this idea. Three tests were conducted to determine this effect and the results are recorded in Figures 10, 11, and 12. As the frequency is increased the degree of stability seemed to increase and the final shapes of the curves also changed to some degree. As far as basic stability

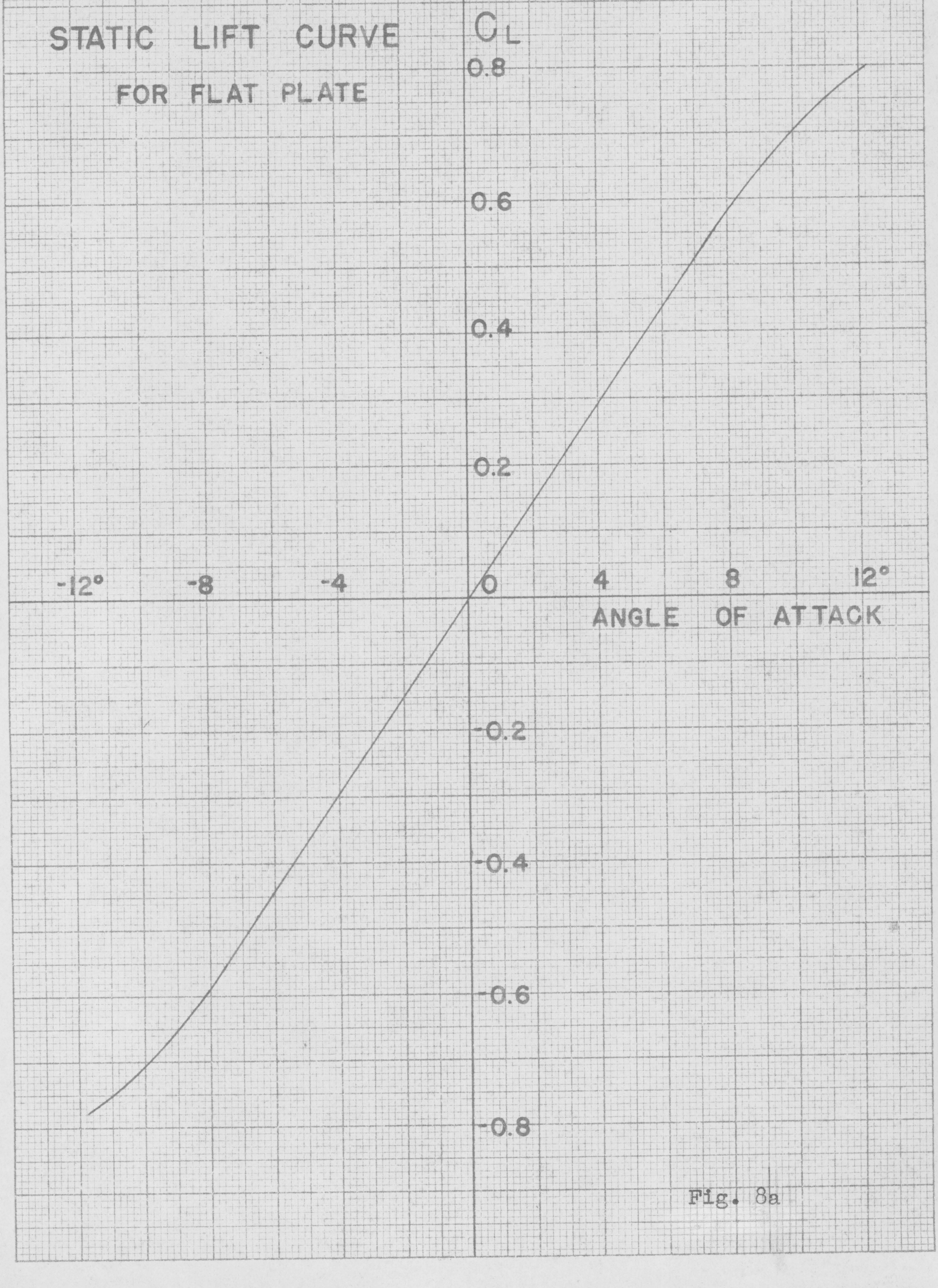

STATIC LIFT CURVE
FOR FLAT PLATE
C_L
0.8
0.6
0.4
0.2
-0.2
-0.4
-0.6
-0.8
-12°
-8
-4
0
4
8
12°
ANGLE OF ATTACK

Fig. 8a

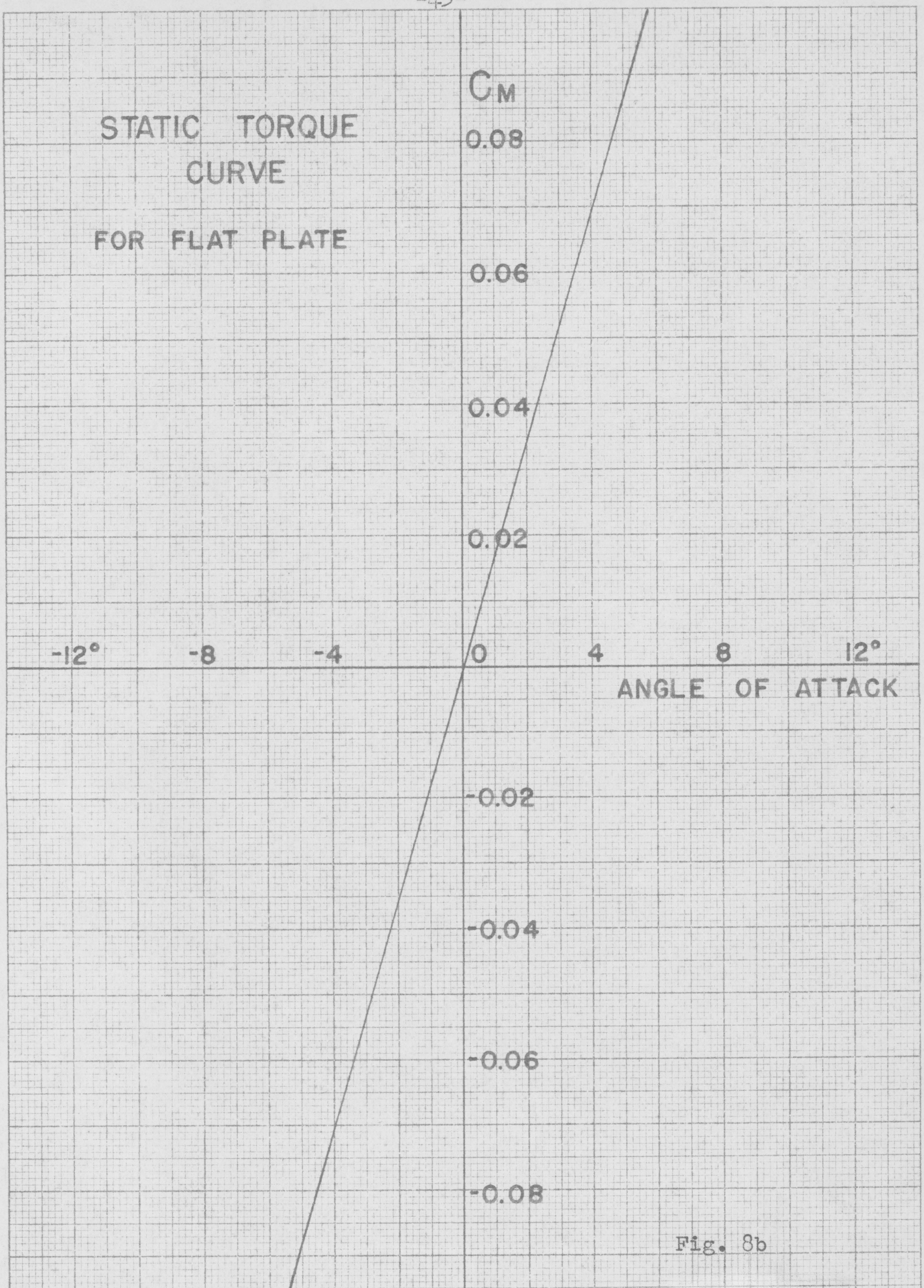

STATIC TORQUE CURVE
FOR FLAT PLATE
C_M
0.08
0.06
0.04
0.02
-0.02
-0.04
-0.06
-0.08
-12°
-8
-4
0
4
8
12°
ANGLE OF ATTACK

Fig. 8b

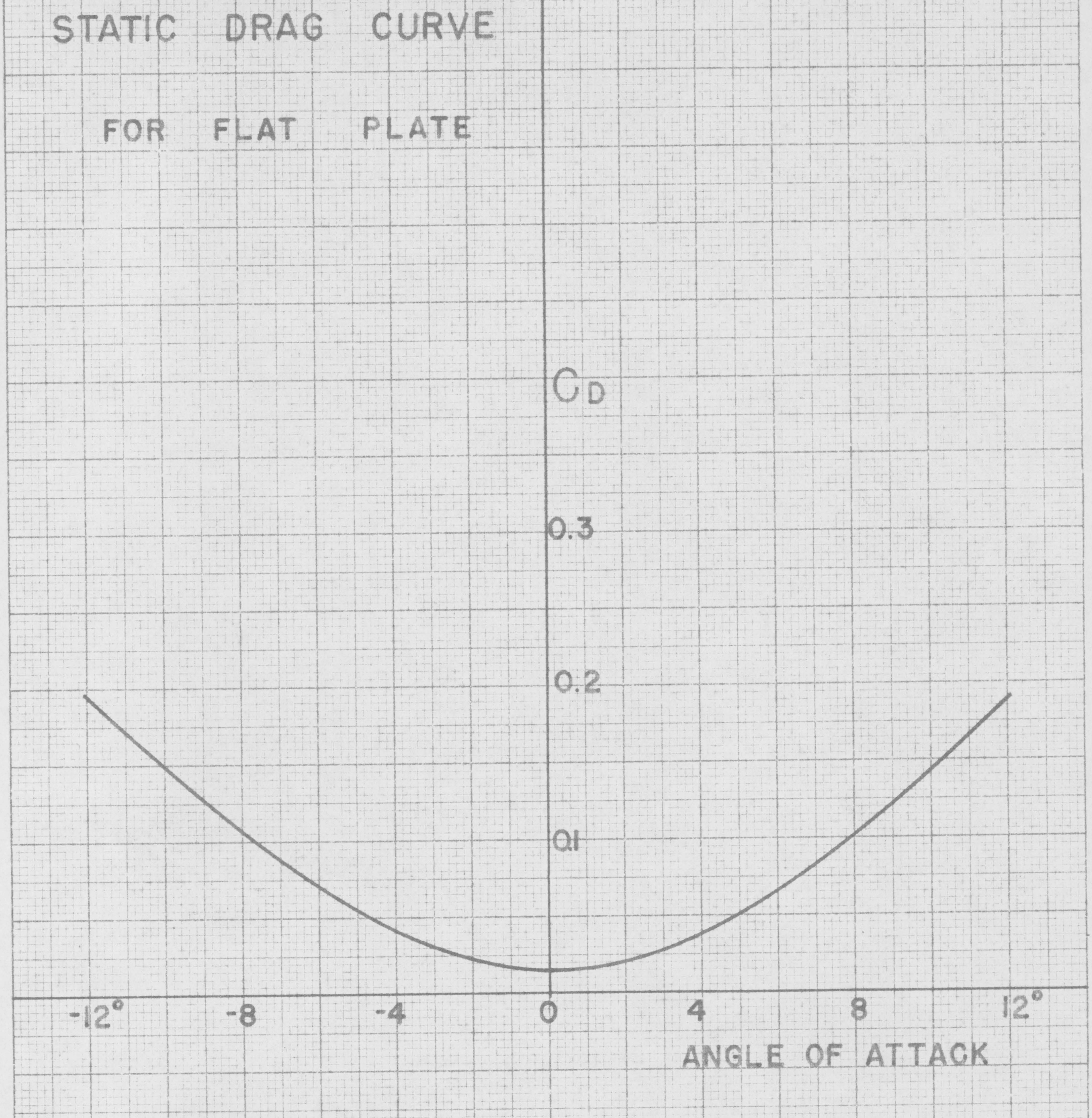

STATIC DRAG CURVE
FOR FLAT PLATE
C_D
0.3
0.2
0.1
-12°
-8
-4
0
4
8
12°
ANGLE OF ATTACK

Fig. 8c

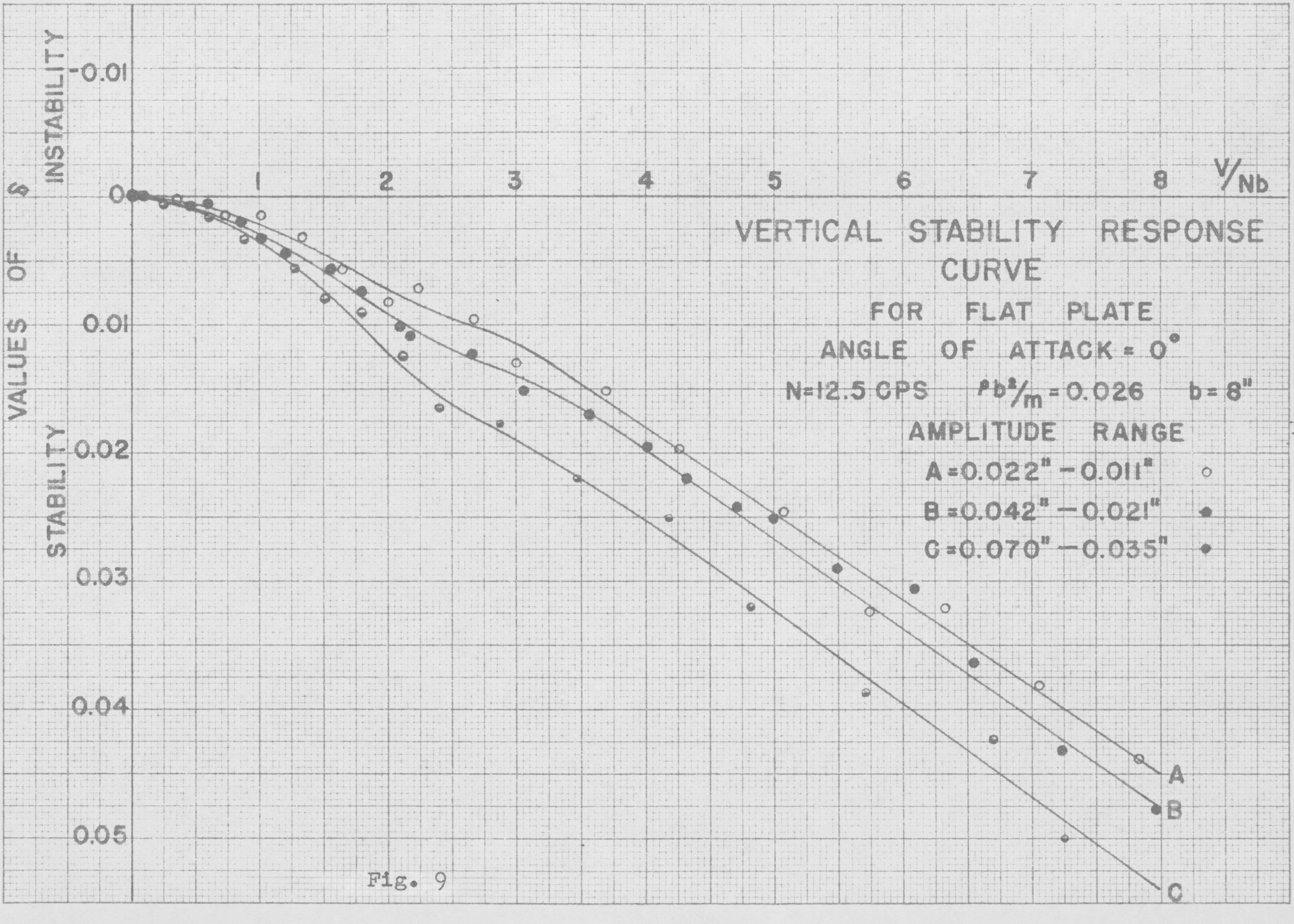
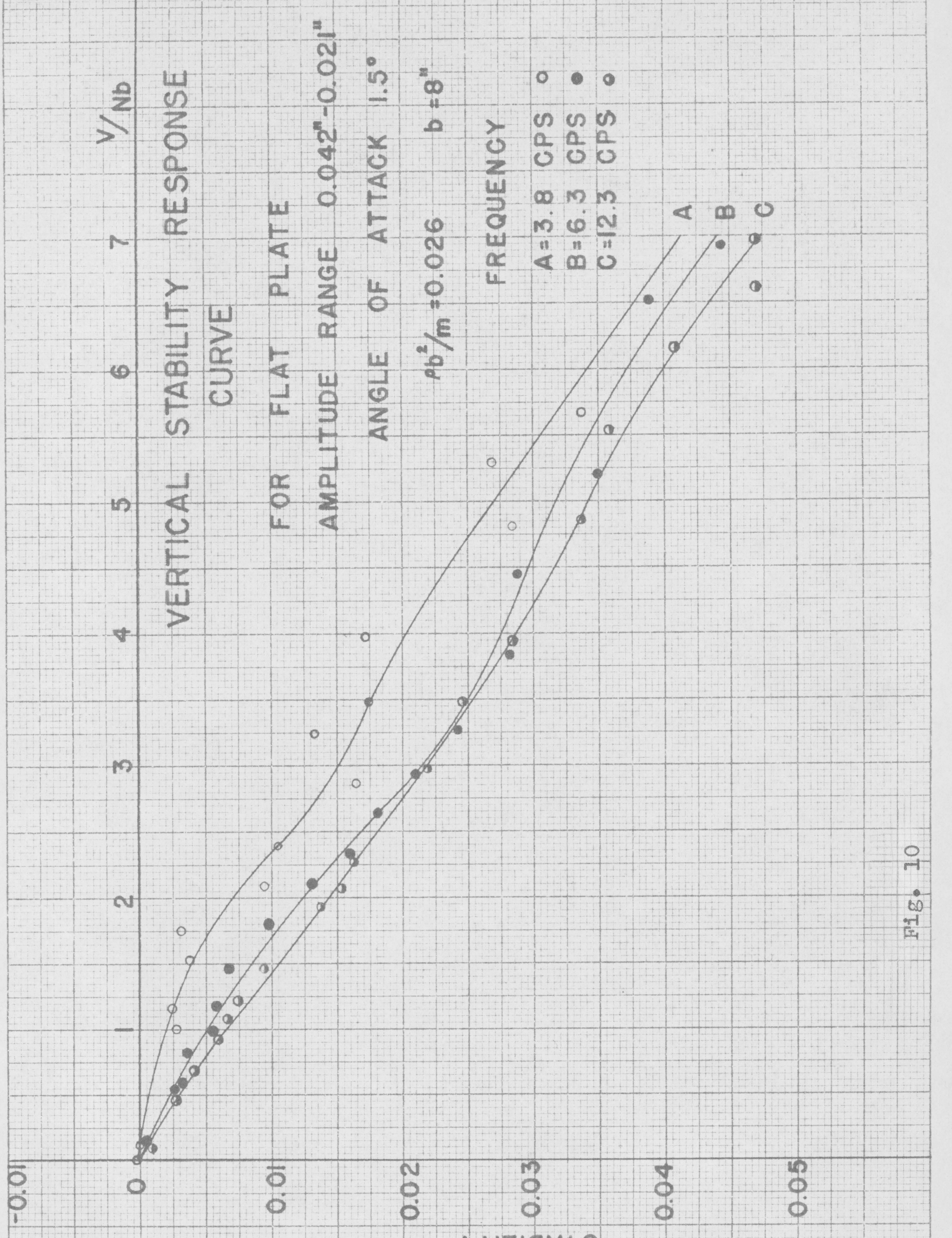

VERTICAL STABILITY RESPONSE CURVE
FOR FLAT PLATE
ANGLE OF ATTACK = 0°
N=12.5 CPS $\rho b^2/m$ = 0.026 b = 8"
AMPLITUDE RANGE
A = 0.022" − 0.011" ○
B = 0.042" − 0.021" ●
C = 0.070" − 0.035" ●
VALUES OF δ
INSTABILITY
STABILITY
-0.01
0
0.01
0.02
0.03
0.04
0.05
1
2
3
4
5
6
7
8
V/Nb
A
B
C

Fig. 9

is concerned the discrepancy is not serious, as the overall characteristics of the section remain unchanged. The tests reported in figures 10 and 11 were all run at the same initial amplitude. Figure 12 is the result of varying both the frequency and the initial amplitude so that the ratio, V/Nx, remained constant. In this ratio V and N have the same meaning as before and x is the initial amplitude. The form of this ratio was determined from a consideration of dimensional analysis, assuming amplitude is a factor in the final result. Considering these results it seems as though the frequency has no effect on the final result provided the initial amplitude is adjusted so that the amplitude ratio remains constant throughout all tests. Naturally no conclusion can be drawn from one test but constant value of Nx seems to be the key to testing when it is necessary to use different frequencies.

Figures 13 and 14 show the effects of different angles of attack on the stability response of the flat plate. When the section was oscillated vertically, it showed complete stability over the entire range of velocity ratios considered. The angle of attack had a very small effect on the final results, although the degree of stability did increase slightly as the angle increased. This increase was small and it did not affect the final shape of the curves appreciably.



VERTICAL STABILITY RESPONSE CURVE

FOR FLAT PLATE

AMPLITUDE RANGE 0.042"-0.021"

ANGLE OF ATTACK 1.5°

$\rho b^2/m = 0.026$ $b = 8"$

FREQUENCY

A = 3.8 CPS ○

B = 6.3 CPS ●

C = 12.3 CPS ◉

Fig. 10

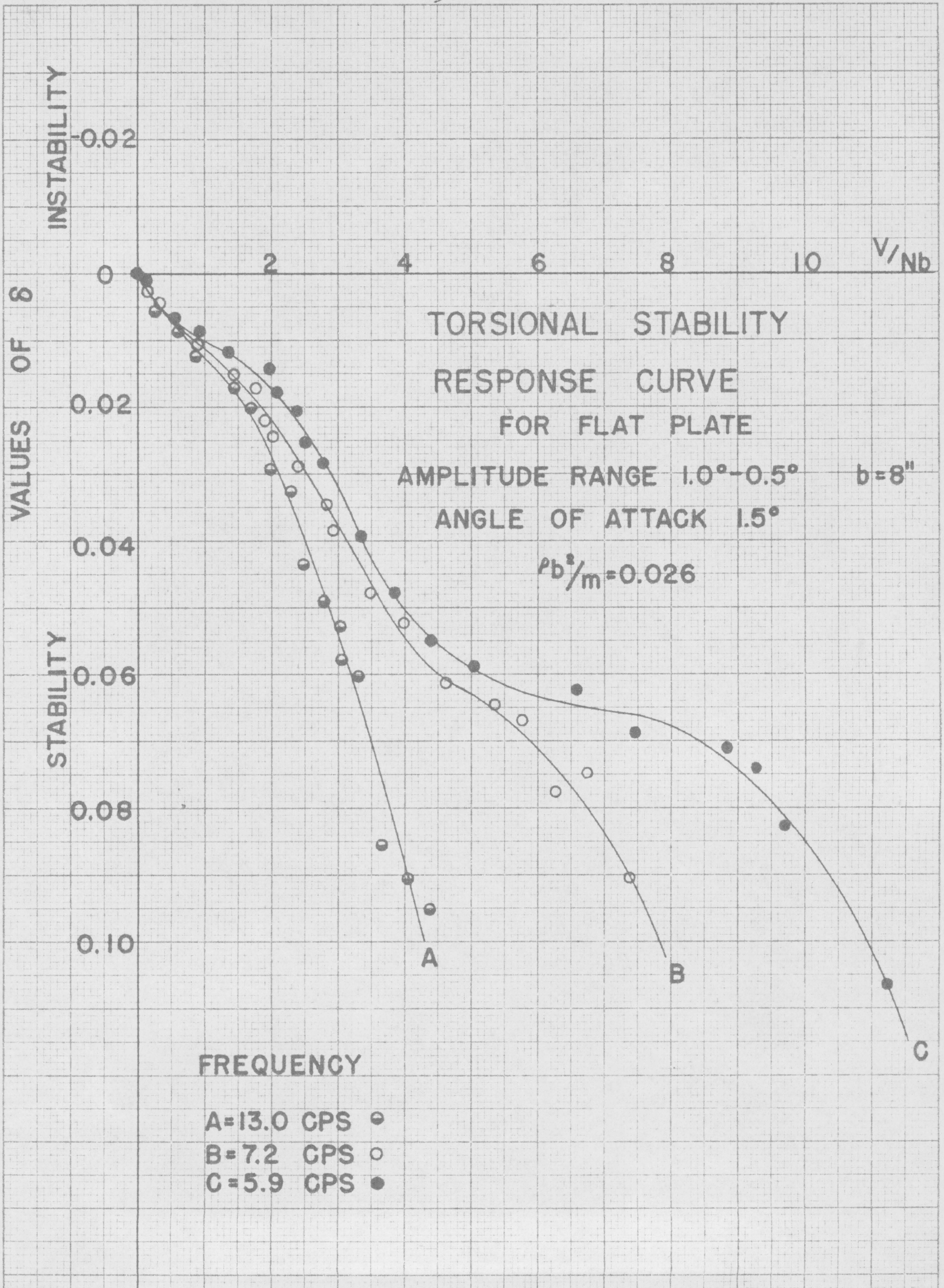

INSTABILITY
-0.02
0
VALUES OF δ
0.02
0.04
0.06
0.08
0.10
STABILITY
2
4
6
8
10
V/Nb
TORSIONAL STABILITY
RESPONSE CURVE
FOR FLAT PLATE
AMPLITUDE RANGE 1.0°-0.5° b=8"
ANGLE OF ATTACK 1.5°
ρb²/m=0.026
A
B
C
FREQUENCY
A=13.0 CPS ◒
B=7.2 CPS ○
C=5.9 CPS ●

Fig. 11

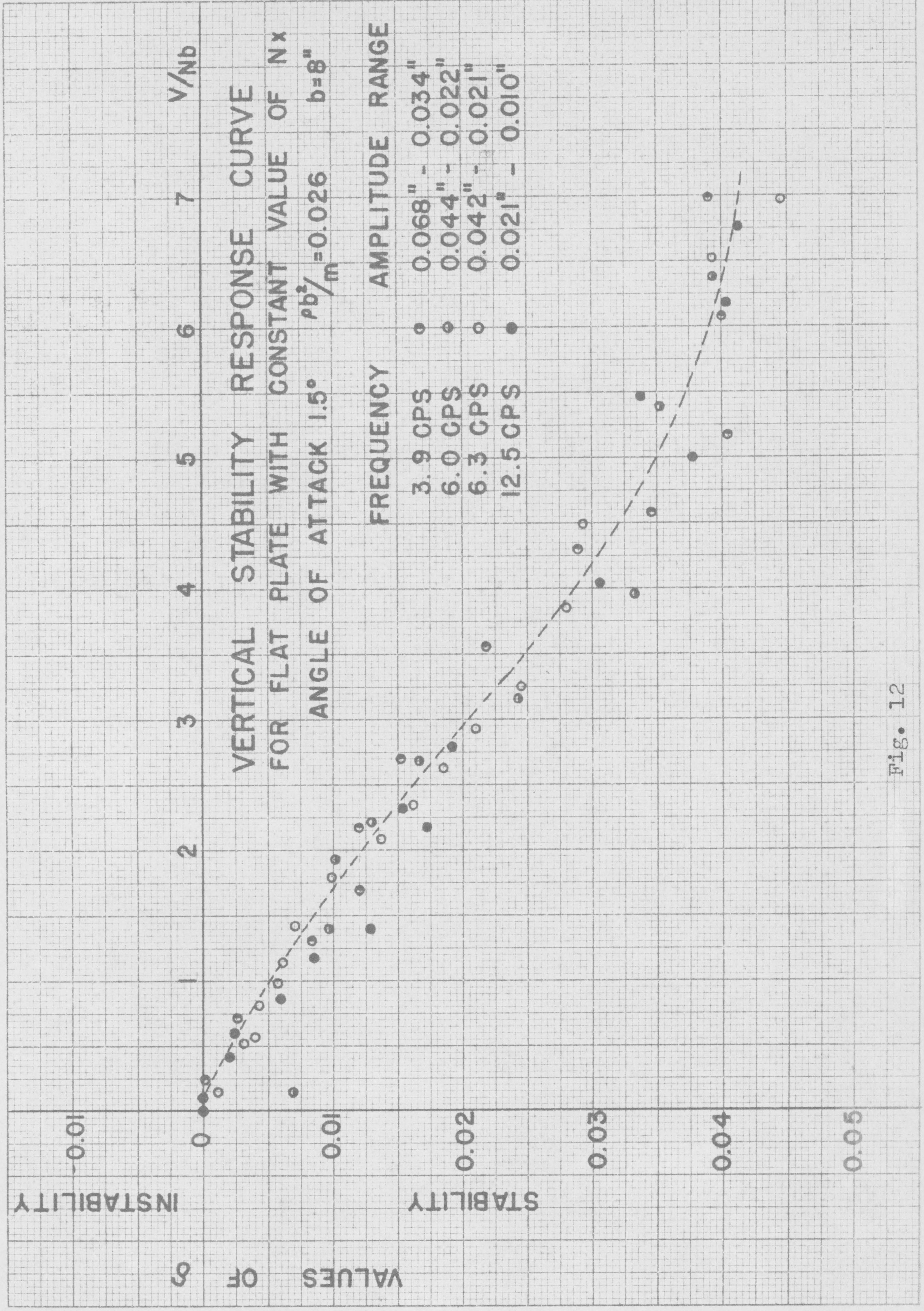

VERTICAL STABILITY RESPONSE CURVE
FOR FLAT PLATE WITH CONSTANT VALUE OF Nx
ANGLE OF ATTACK 1.5° $\rho b^2/m = 0.026$ b=8"
FREQUENCY   AMPLITUDE RANGE
3.9 CPS ⊙ 0.068" - 0.034"
6.0 CPS ⊙ 0.044" - 0.022"
6.3 CPS ○ 0.042" - 0.021"
12.5 CPS ● 0.021" - 0.010"
V/Nb
0
1
2
3
4
5
6
7
-0.01
0
0.01
0.02
0.03
0.04
0.05
INSTABILITY
STABILITY
VALUES OF ζ

Fig. 12

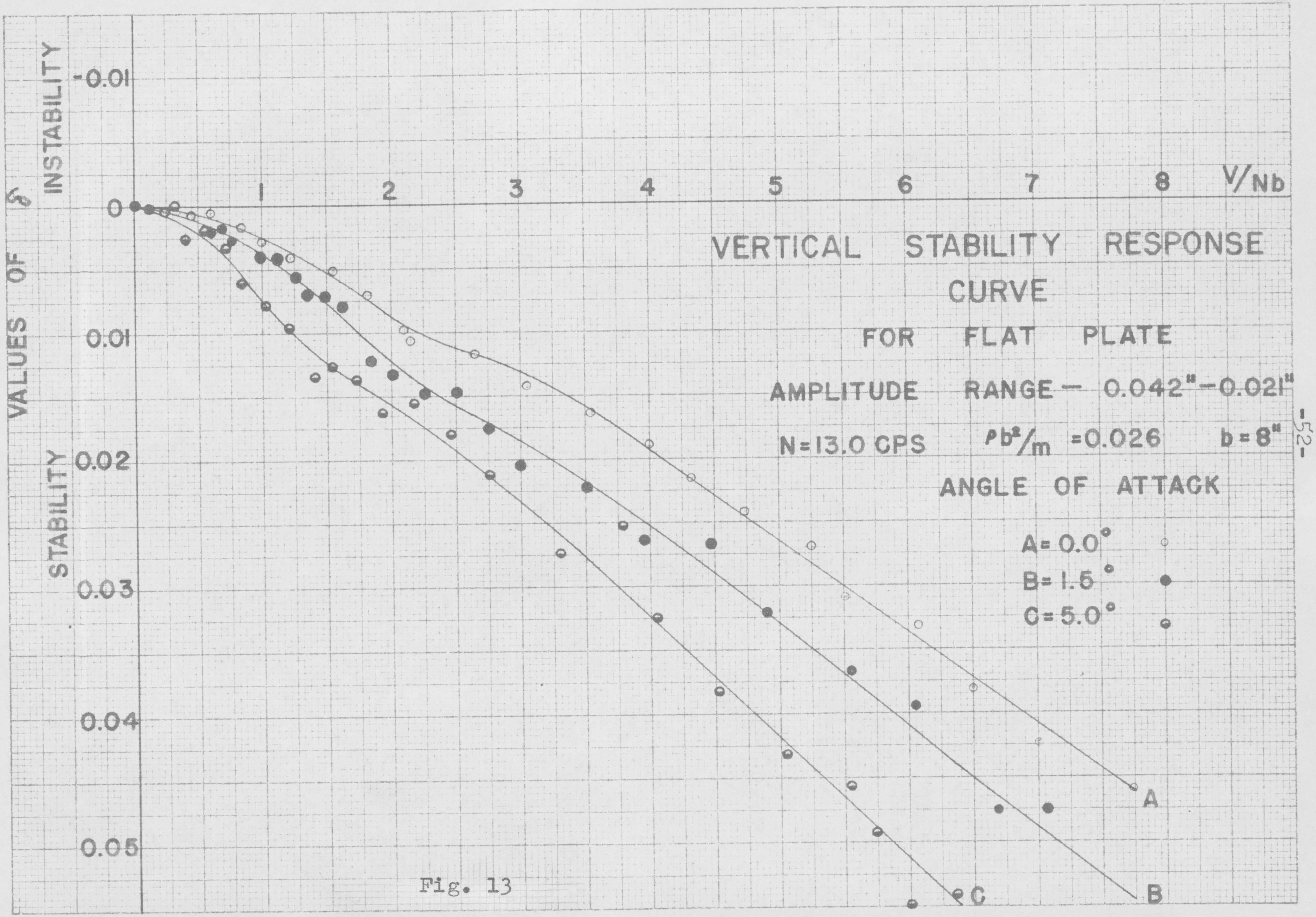

VERTICAL STABILITY RESPONSE
CURVE
FOR FLAT PLATE
AMPLITUDE RANGE — 0.042"–0.021"
N=13.0 CPS
$\rho b^2/m$ =0.026
b = 8"
ANGLE OF ATTACK
A= 0.0° ○
B= 1.5° ●
C= 5.0° ◐
VALUES OF δ
INSTABILITY
STABILITY
-0.01
0
0.01
0.02
0.03
0.04
0.05
1
2
3
4
5
6
7
8
V/Nb
A
B
C

Fig. 13

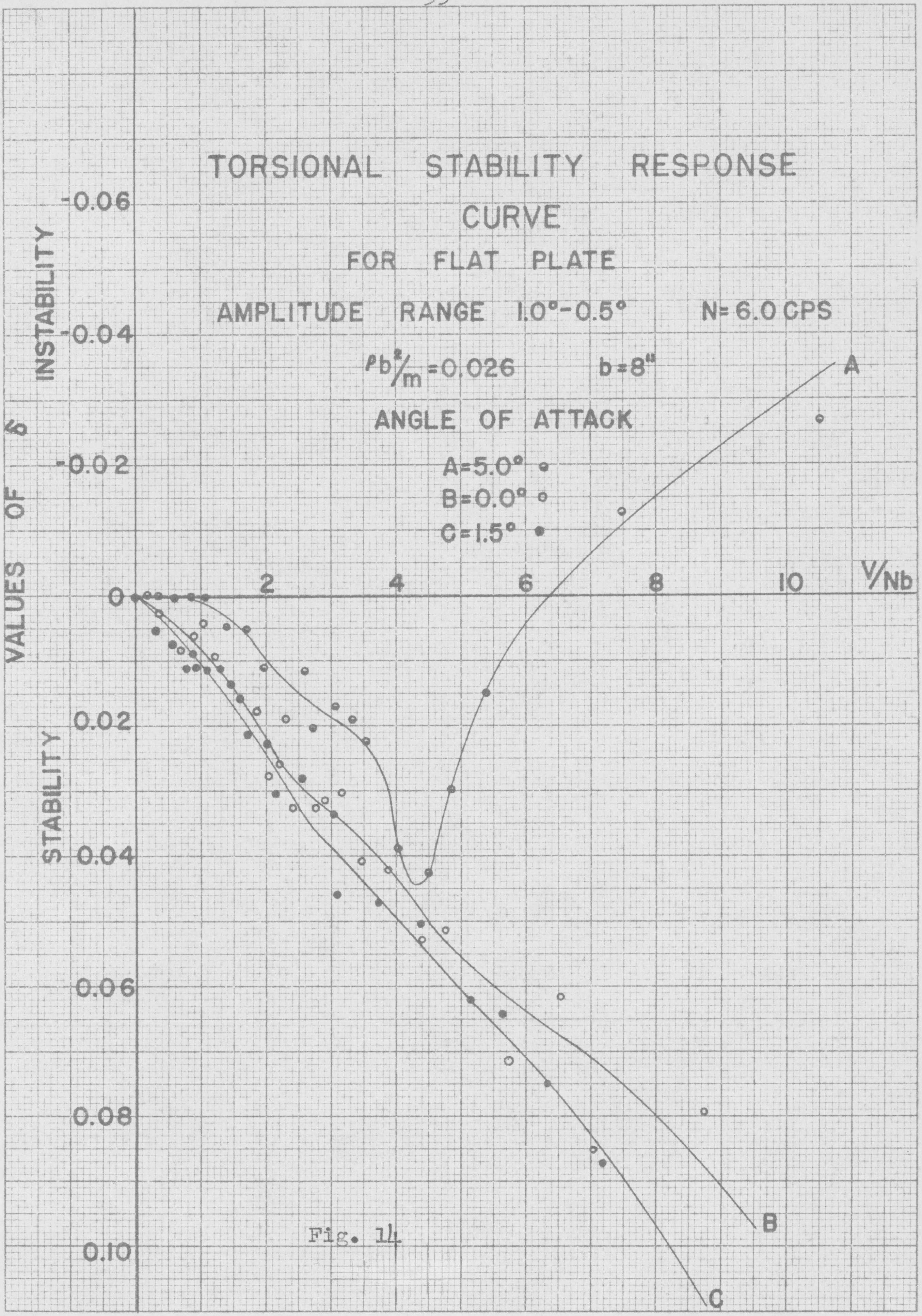

TORSIONAL STABILITY RESPONSE
CURVE
FOR FLAT PLATE
AMPLITUDE RANGE 1.0°-0.5°
N=6.0 CPS
$\rho b^2/m = 0.026$
b=8"
ANGLE OF ATTACK
A=5.0° ◉
B=0.0° ○
C=1.5° ●
VALUES OF δ
INSTABILITY
STABILITY
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.10
2
4
6
8
10
V/Nb
A
B
C

Fig. 14

When torsional motion was considered the angle of attack had considerable effect on the final results. At low angles of attack, (0° & +1.5°), the section was completely stable over the entire range of velocity ratios considered. The degree of stability was almost the same for these two angles of attack although the section was slightly more stable at 1.5° than at 0°. When the angle of attack was increased to +5° the section showed stable characteristics when the velocity ratio was below 6.4 and unstable characteristics when the velocity ratio exceeded this value.

The curves in figure 15 show the results of the static tests on the "H" section with a section ratio, (d/b), of 0.20. When vertical motion is considered the section shows stable characteristics over the entire range in question. The section has its greatest degree of stability when the angle of attack is 0° and the degree of stability decreases as the angle of attack increases.

Torsionally the section is unstable over the entire range considered. The highest degree of instability occurs at 0° and this instability decreases as the angle of attack increases.

Figure 16 shows the results of tests on the "H" section when it is subject to vertical motion. The section is basically stable although there are some regions of instability. The principal region of instability occurs between a velocity ratio of 1.5 and 2.1 but the instability is noncatastrophic in nature. There is another region between 0.5 and 1.0 but because of the difficulty

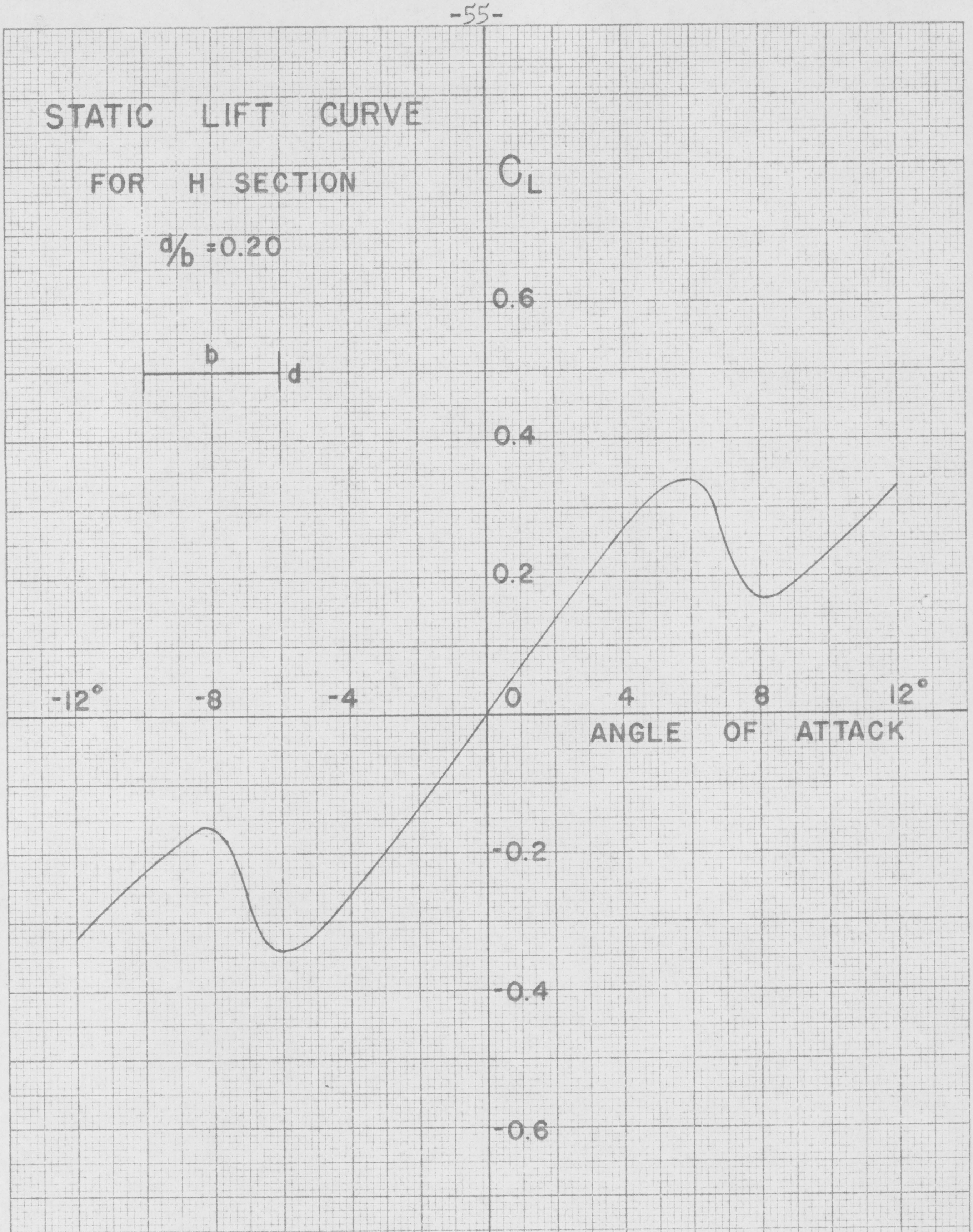

STATIC LIFT CURVE
FOR H SECTION
d/b = 0.20
b
d
C_L
0.6
0.4
0.2
-0.2
-0.4
-0.6
-12°
-8
-4
0
4
8
12°
ANGLE OF ATTACK

Fig. 15a

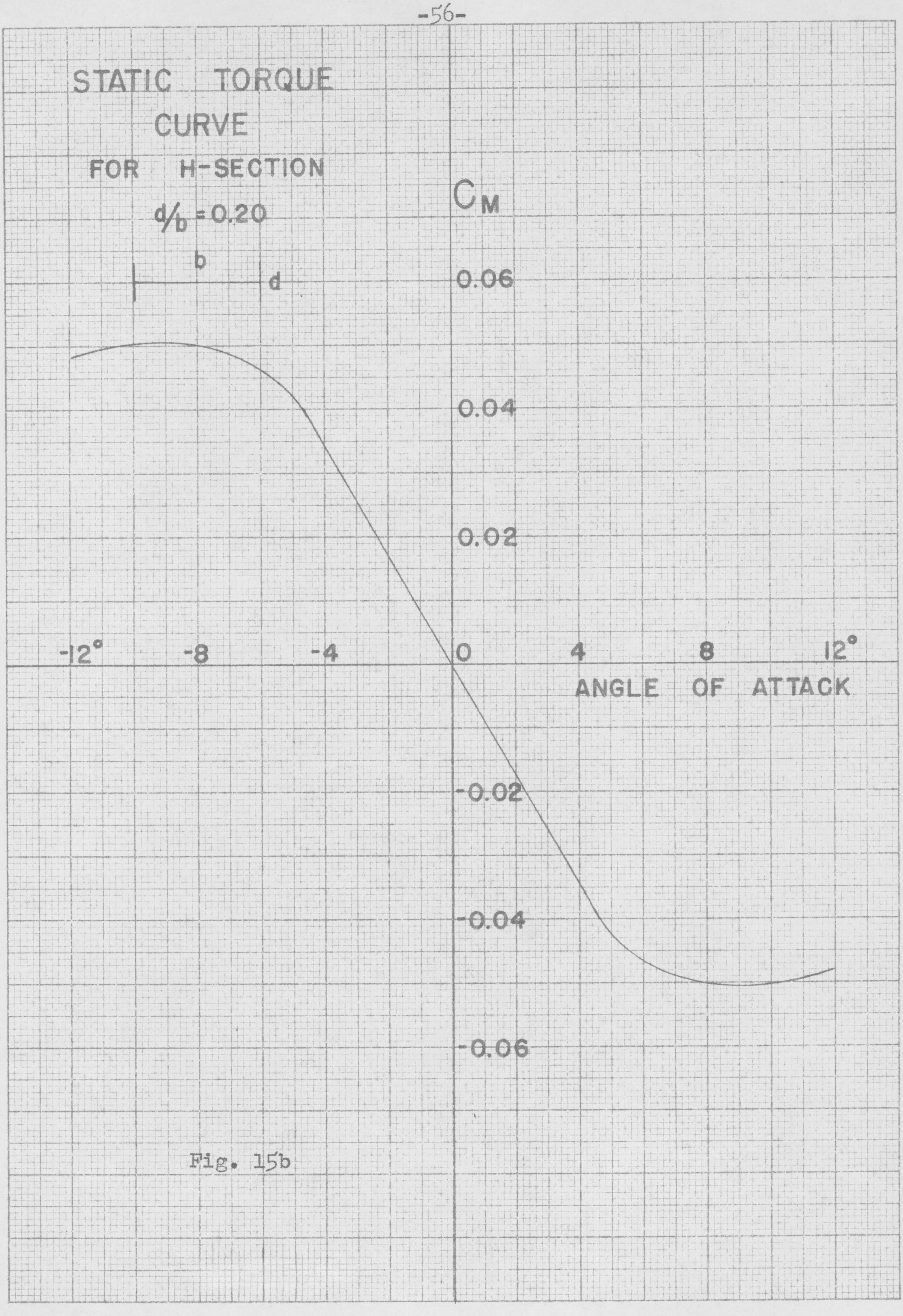

STATIC TORQUE CURVE FOR H-SECTION
d/b = 0.20
b
d
C_M
0.06
0.04
0.02
-0.02
-0.04
-0.06
-12°
-8
-4
0
4
8
12°
ANGLE OF ATTACK

Fig. 15b

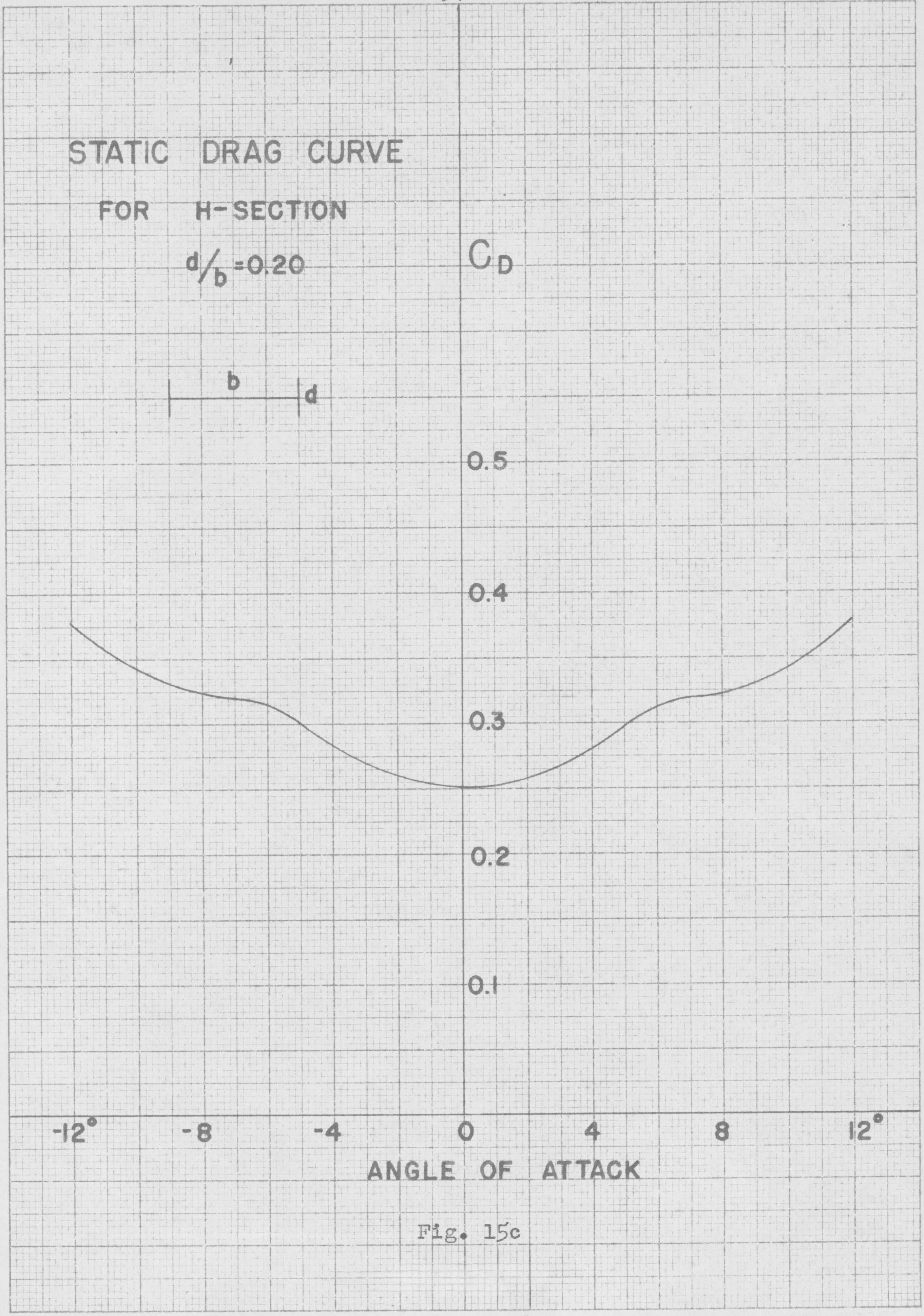

STATIC DRAG CURVE
FOR H-SECTION
d/b = 0.20
b
d
C_D
0.5
0.4
0.3
0.2
0.1
-12°
-8
-4
0
4
8
12°
ANGLE OF ATTACK

Fig. 15c

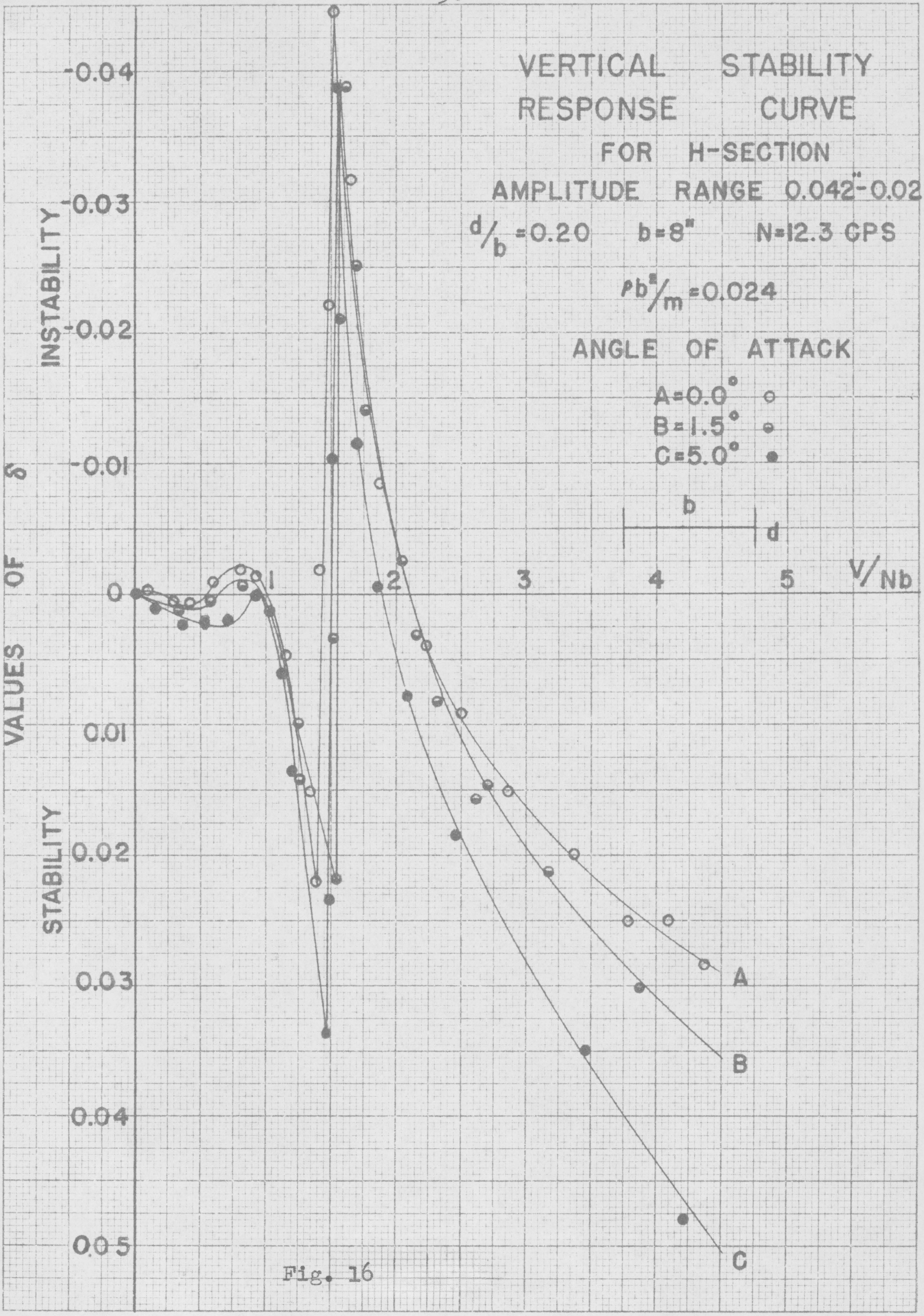

VERTICAL STABILITY RESPONSE CURVE
FOR H-SECTION
AMPLITUDE RANGE 0.042"-0.021"
d/b = 0.20 b = 8" N = 12.3 CPS
ρb²/m = 0.024
ANGLE OF ATTACK
A = 0.0° ○
B = 1.5° ⊖
C = 5.0° ●
b
d
V/Nb
0
1
2
3
4
5
-0.04
-0.03
-0.02
-0.01
0.01
0.02
0.03
0.04
0.05
INSTABILITY
STABILITY
VALUES OF δ
A
B
C

Fig. 16

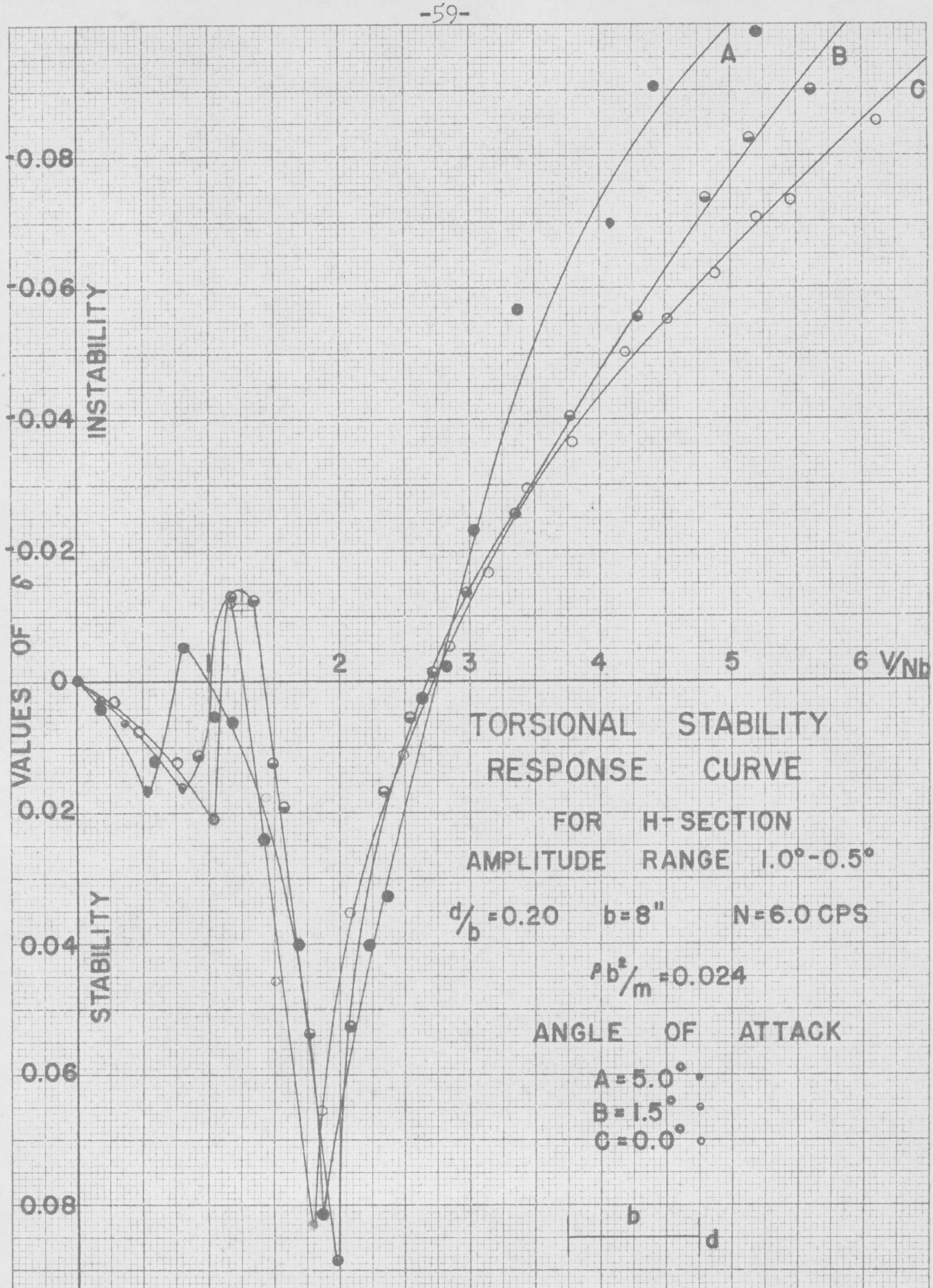

INSTABILITY
STABILITY
VALUES OF δ
0.08
0.06
0.04
0.02
0
0.02
0.04
0.06
0.08
1
2
3
4
5
6 V/Nb
A
B
C
TORSIONAL STABILITY
RESPONSE CURVE
FOR H-SECTION
AMPLITUDE RANGE 1.0°-0.5°
d/b = 0.20 b = 8" N = 6.0 CPS
ρb²/m = 0.024
ANGLE OF ATTACK
A = 5.0° ●
B = 1.5° ◐
C = 0.0° ○
b
d

Fig. 17

of obtaining reliable data at these low velocities this region is not well defined. The angle of attack has little effect on this section until the velocity ratio reaches 2. Above this value the section appears to be stable to a greater degree as the angle of attack increases.

When this section is oscillated torsionally it is basically unstable. There is a stable region between the velocity ratios of approximately 1 and 2.7, with the exact values depending on the angle of attack. There is another region of stability somewhat below 1 but this region is poorly defined due to the difficulty of obtaining data at low velocities. Above the velocity ratio of 2.7 the section becomes unstable with the degree of instability increasing as the angle of attack increases. This instability is of a catastrophic nature.

The results of the static tests conducted on the "H" section with lateral slots and fins are shown in figure 18. When vertical motion is considered, the section exhibits a very slight degree of instability at an angle of attack of $+1.5^{\circ}$, and a slightly higher degree of instability at 0°. When the angle of attack is increased to $+5^{\circ}$ the section becomes stable.

When torsional motion is considered the section exhibits exactly opposite characteristics. It is stable to a very slight degree when the angle of attack is $+1.5^{\circ}$. This degree of stability

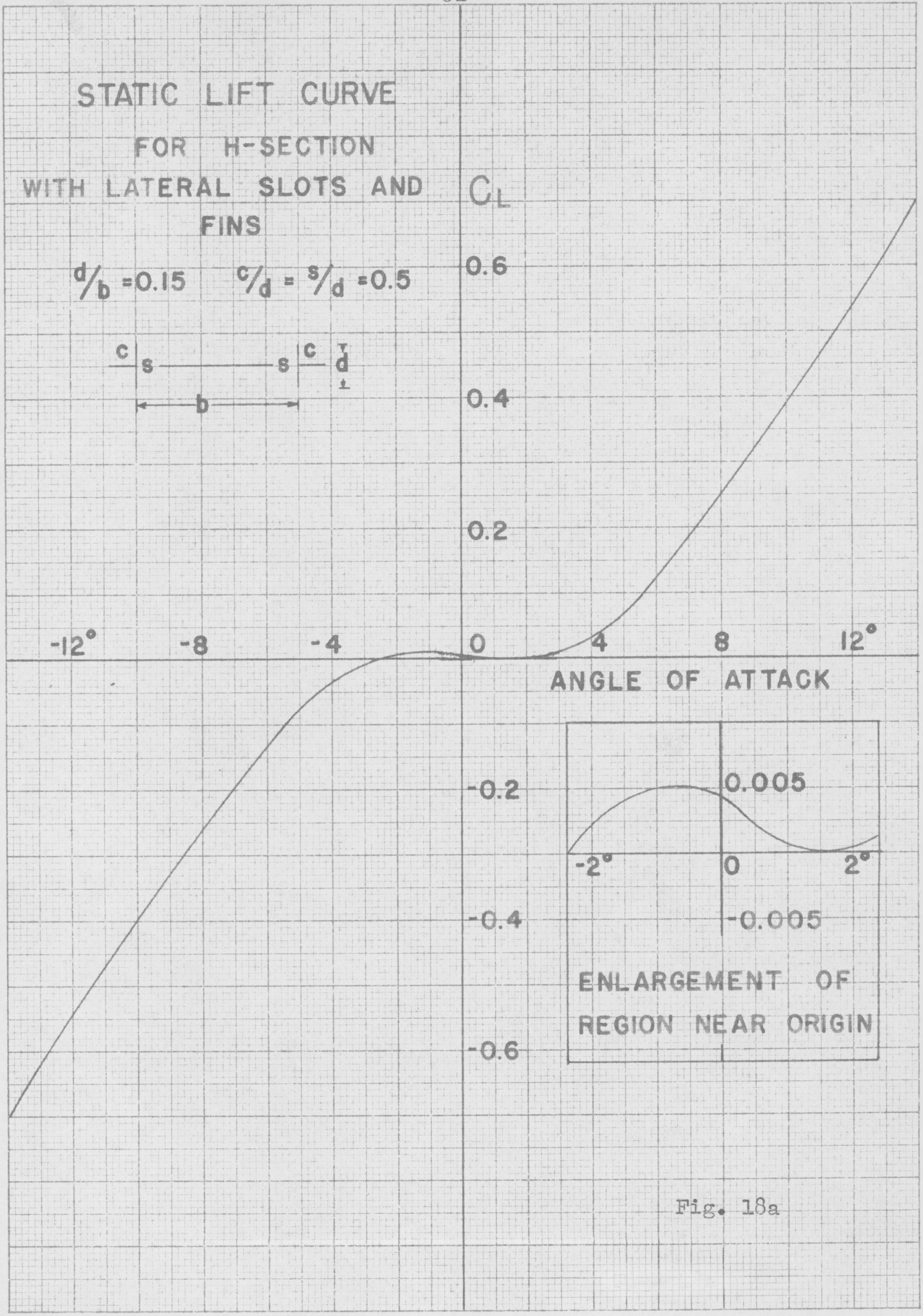

STATIC LIFT CURVE
FOR H-SECTION
WITH LATERAL SLOTS AND FINS
d/b = 0.15 c/d = s/d = 0.5
c
s
s
c
d
b
C_L
0.6
0.4
0.2
-0.2
-0.4
-0.6
-12°
-8
-4
0
4
8
12°
ANGLE OF ATTACK
0.005
-2°
0
2°
-0.005
ENLARGEMENT OF REGION NEAR ORIGIN

Fig. 18a

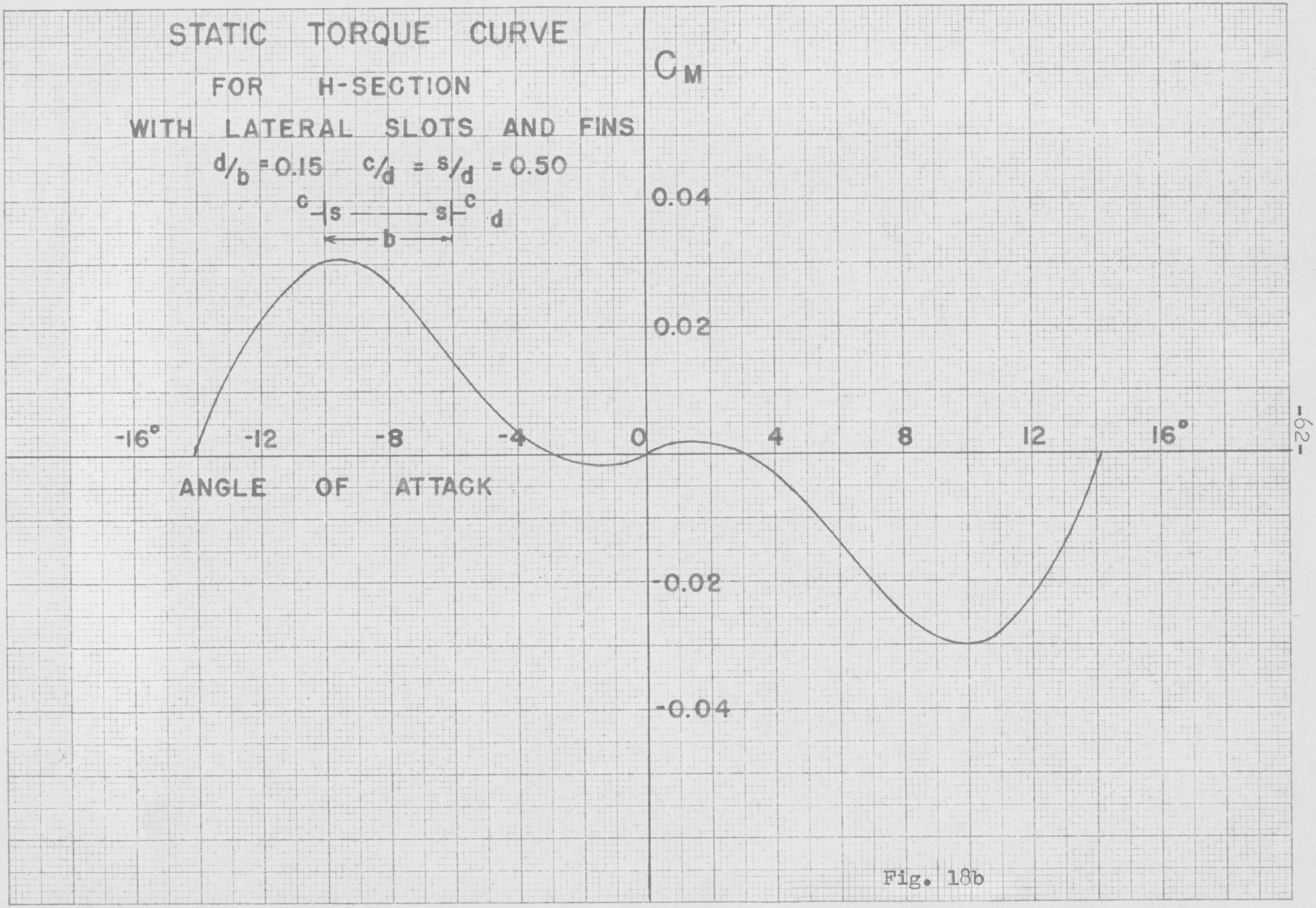
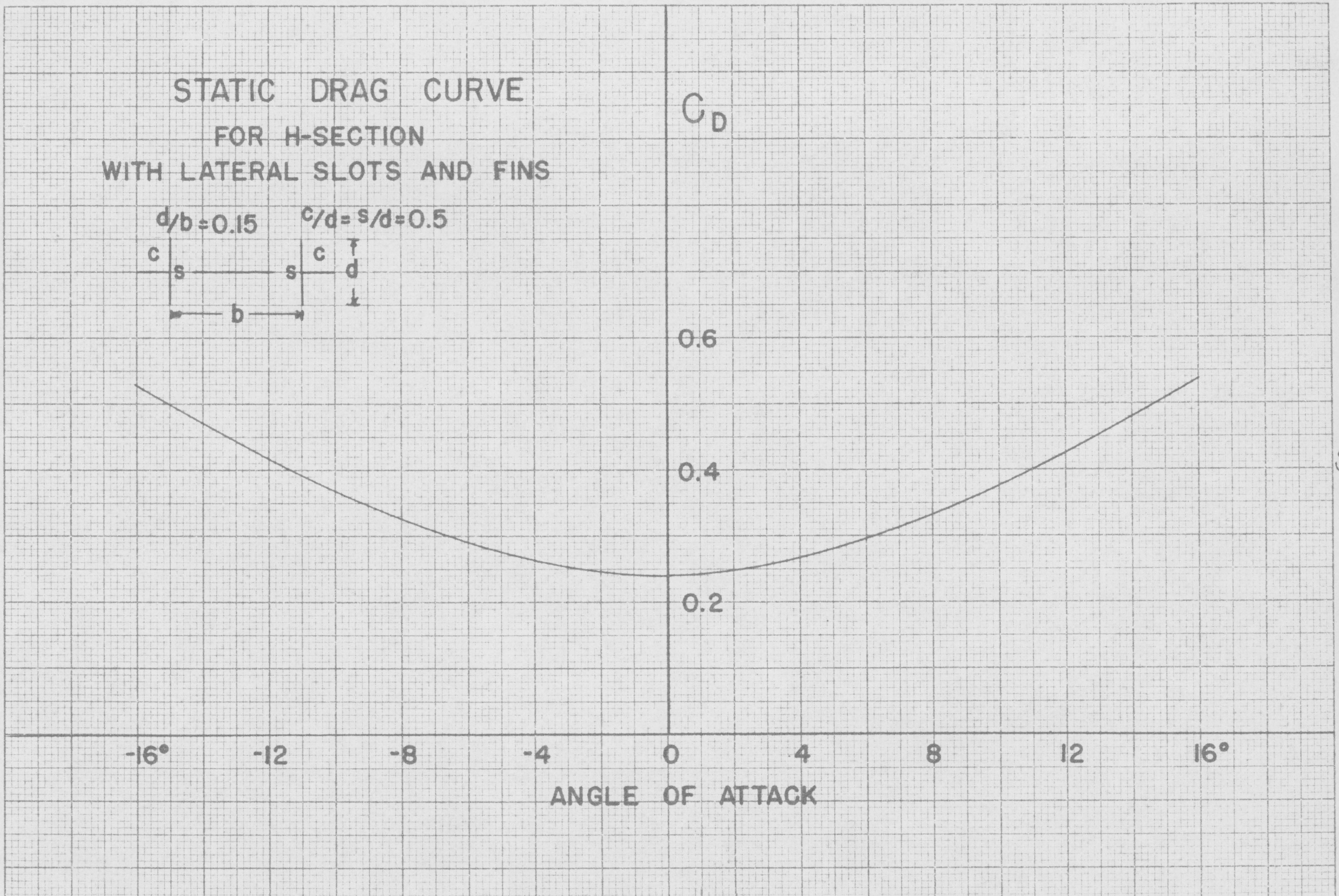

STATIC TORQUE CURVE
FOR H-SECTION
WITH LATERAL SLOTS AND FINS
$d/b = 0.15$ $c/d = s/d = 0.50$
c
s
s
c
d
b
C_M
0.04
0.02
-0.02
-0.04
-16°
-12
-8
-4
0
4
8
12
16°
ANGLE OF ATTACK

Fig. 18b



STATIC DRAG CURVE
FOR H-SECTION
WITH LATERAL SLOTS AND FINS

Fig. 18c

is somewhat higher for an angle of attack of 0°. When the angle of attack is increased to $+5^\circ$ the section shows unstable characteristics.

The results of the dynamic tests on the "H" section with lateral slots and fins are given in figures 19 and 20. For this particular section the angle of attack is extremely critical. In vertical motion the section was unstable at low angles of attack (0 and +1.5). The degree of instability was higher at 0 than at 1.5 degrees. The critical velocity ratio for the section at 0° was 4.35 and for the section at 1.5° it was 4.0. Above these values the motion was of the catastrophic type. Another unstable region occurred for each angle of attack between the velocity ratios of 1.1 and 1.7 but this range of instability was noncatastrophic in nature. When the angle of attack was increased to $+5^\circ$ the section showed stable characteristics. It did, however, have an unstable range, noncatastrophic in nature, between the values of V/Nb of 1.0 and 1.5.

Torsionally the section showed stable characteristics for angles of attack of 0° and 1.5°. There was an unstable range at each angle between the values of V/Nb of 2 and approximately 7 but these were not dangerous. There were also some unstable ranges at low velocity ratios but again the difficulty of getting reliable data in this region prevents the region from being well

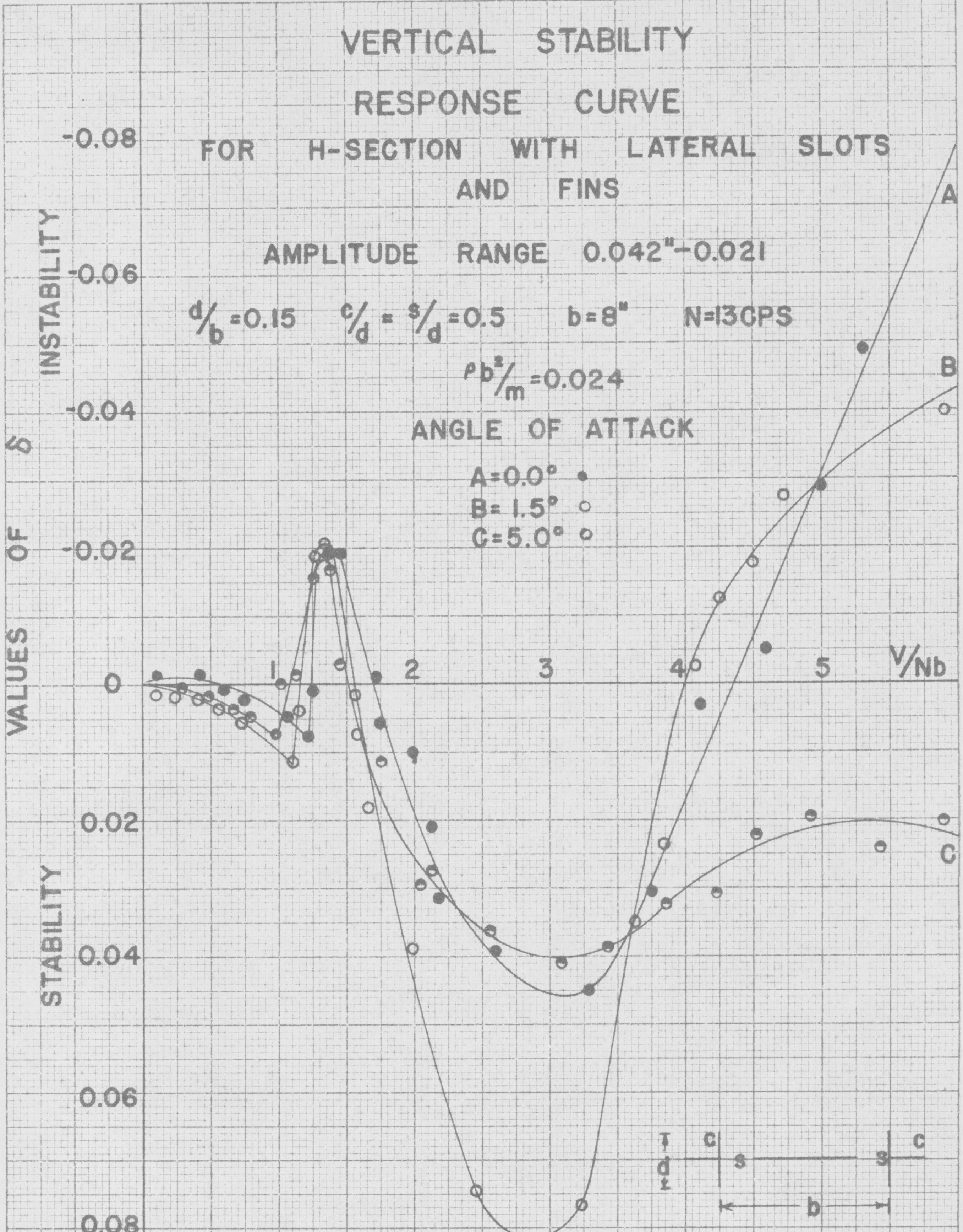

VERTICAL STABILITY
RESPONSE CURVE
FOR H-SECTION WITH LATERAL SLOTS AND FINS
AMPLITUDE RANGE 0.042"-0.021
d/b=0.15 c/d = s/d=0.5 b=8" N=13CPS
ρb²/m=0.024
ANGLE OF ATTACK
A=0.0° ●
B= 1.5° ○
C=5.0° ○
VALUES OF δ
INSTABILITY
STABILITY
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
1
2
3
4
5
V/Nb
A
B
C
d
c
s
s
c
b

Fig. 19

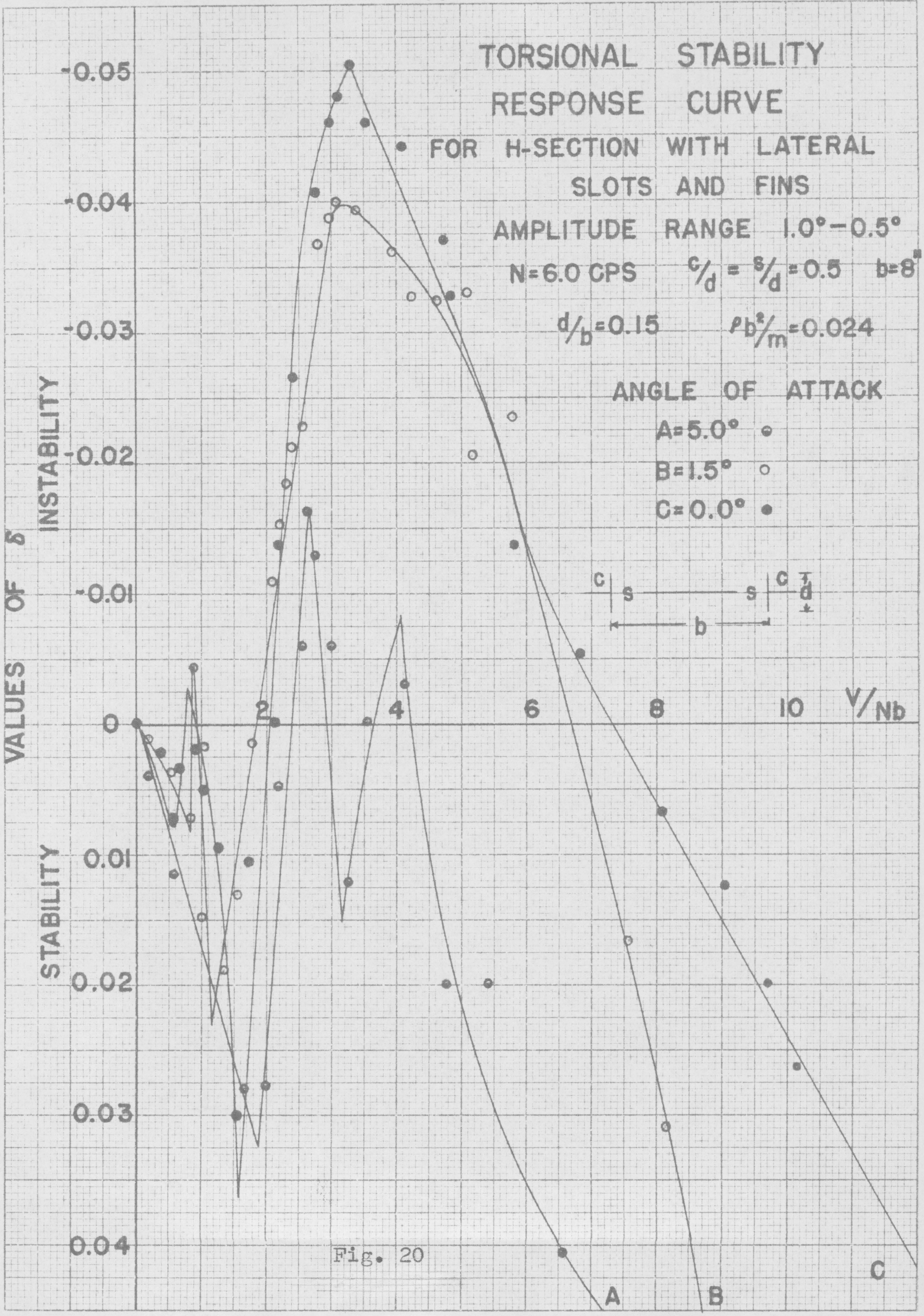

TORSIONAL STABILITY
RESPONSE CURVE
● FOR H-SECTION WITH LATERAL
SLOTS AND FINS
AMPLITUDE RANGE 1.0°-0.5°
N=6.0 CPS c/d = s/d = 0.5 b=8"
d/b=0.15 ρb²/m=0.024
ANGLE OF ATTACK
A=5.0° ⊙
B=1.5° ○
C=0.0° ●
c
s
s
c
d
b
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
VALUES OF δ
INSTABILITY
STABILITY
2
4
6
8
10
V/Nb
A
B
C

Fig. 20

defined. At the highest angle of attack (5°) there are several unstable regions but the final points show the section to be stable.

Figure 22 shows the results of the static tests on the section model of the proposed Tancarville* Bridge. These tests show the section to be vertically stable over the entire range considered. The degree of stability is greatest when the angle of attack equals 0° and this degree of stability decreases as the angle of attack is increased.

When torsional motion is considered the section is stable to the same degree over the entire range considered. In this mode of motion the angle of attack seems to have no effect on the degree of stability.

The results of the dynamic tests are given in figures 23 and 24. Vertically the section is stable over the entire range considered. It has its greatest degree of stability when the angle of attack is 5° and its least degree of stability when the angle of attack is 1.5°.

Torsionally the section is also stable over the entire range considered. The section has its greatest degree of stability for an angle of attack of 5° and its least degree of stability for an angle of 1.5°

---

* This section is one of the designs considered for the proposed Tancarville (France) Suspension Bridge.

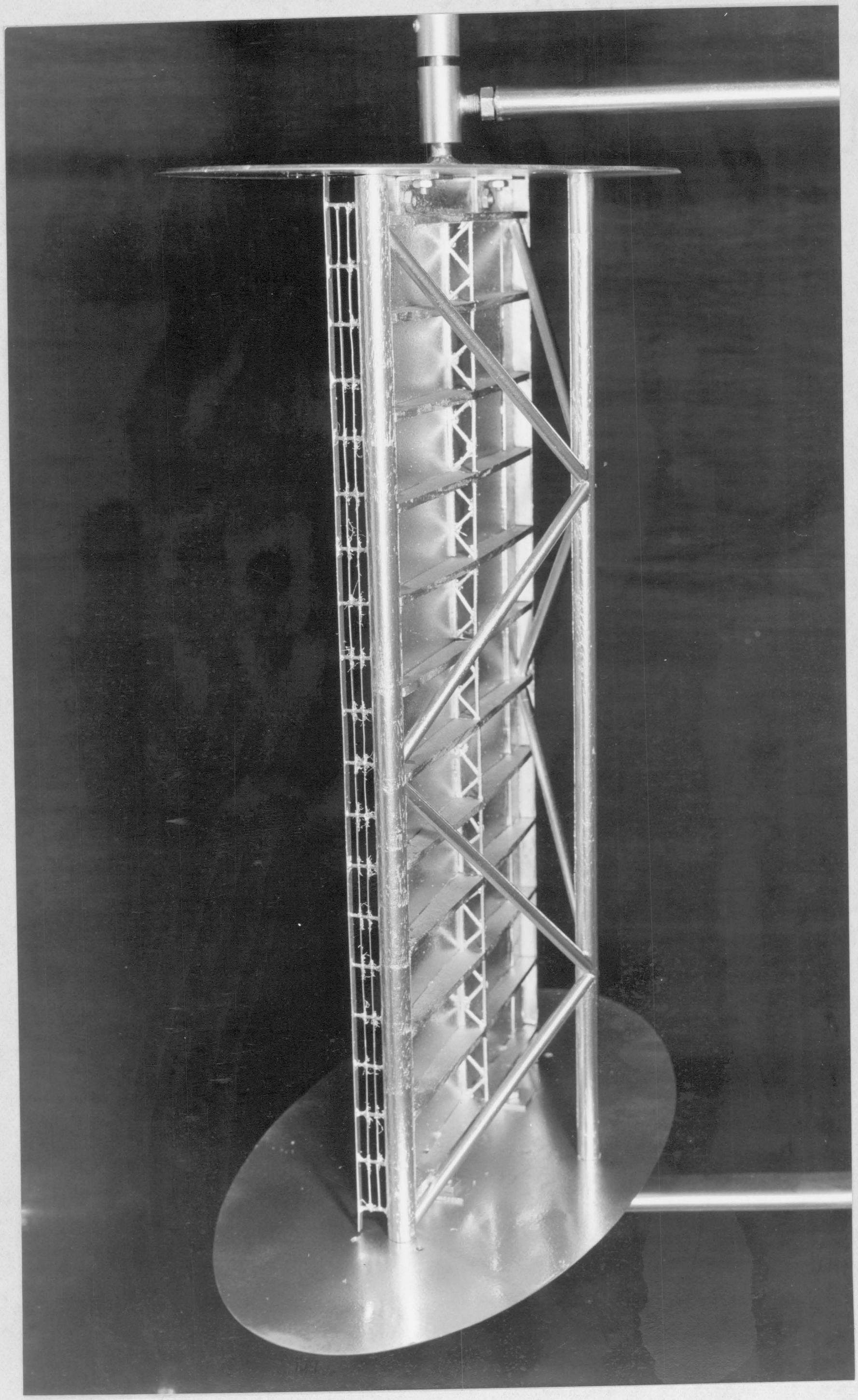


Fig. 21 ... Section Model of the Proposed Tancarville Bridge Section

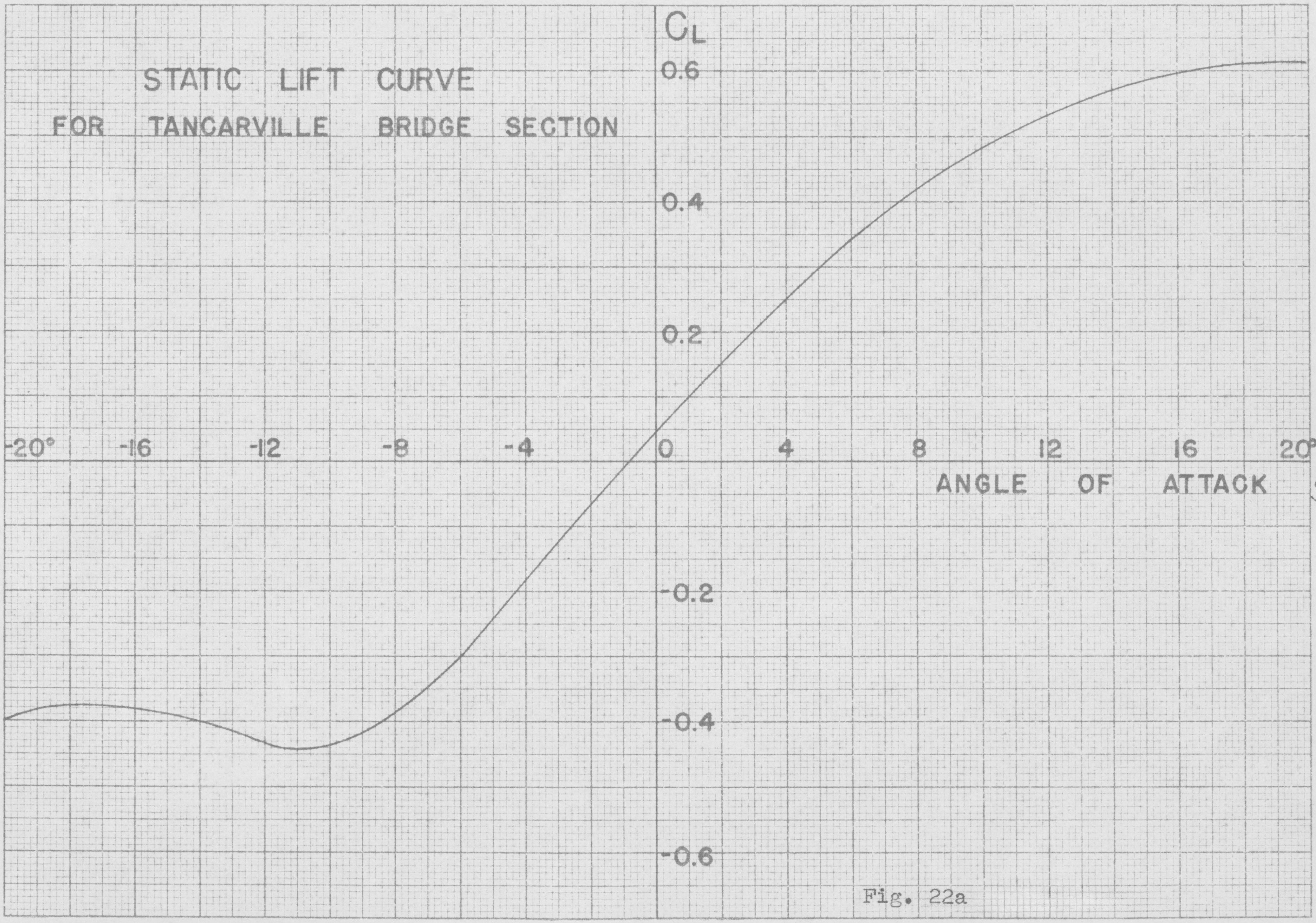

STATIC LIFT CURVE
FOR TANCARVILLE BRIDGE SECTION

Fig. 22a

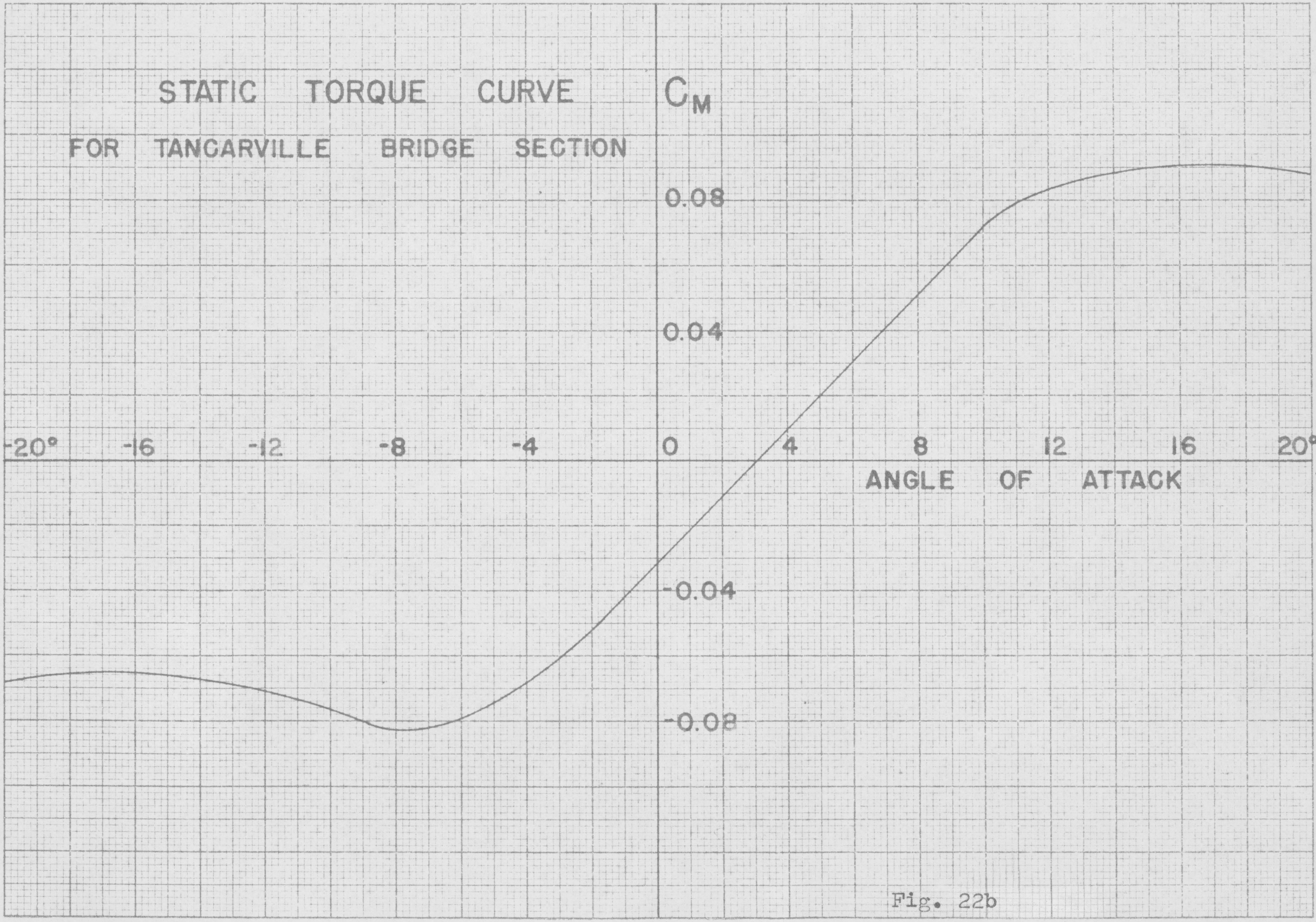

STATIC TORQUE CURVE
FOR TANCARVILLE BRIDGE SECTION
C_M
0.08
0.04
0
-0.04
-0.08
-20°
-16
-12
-8
-4
4
8
12
16
20°
ANGLE OF ATTACK

Fig. 22b

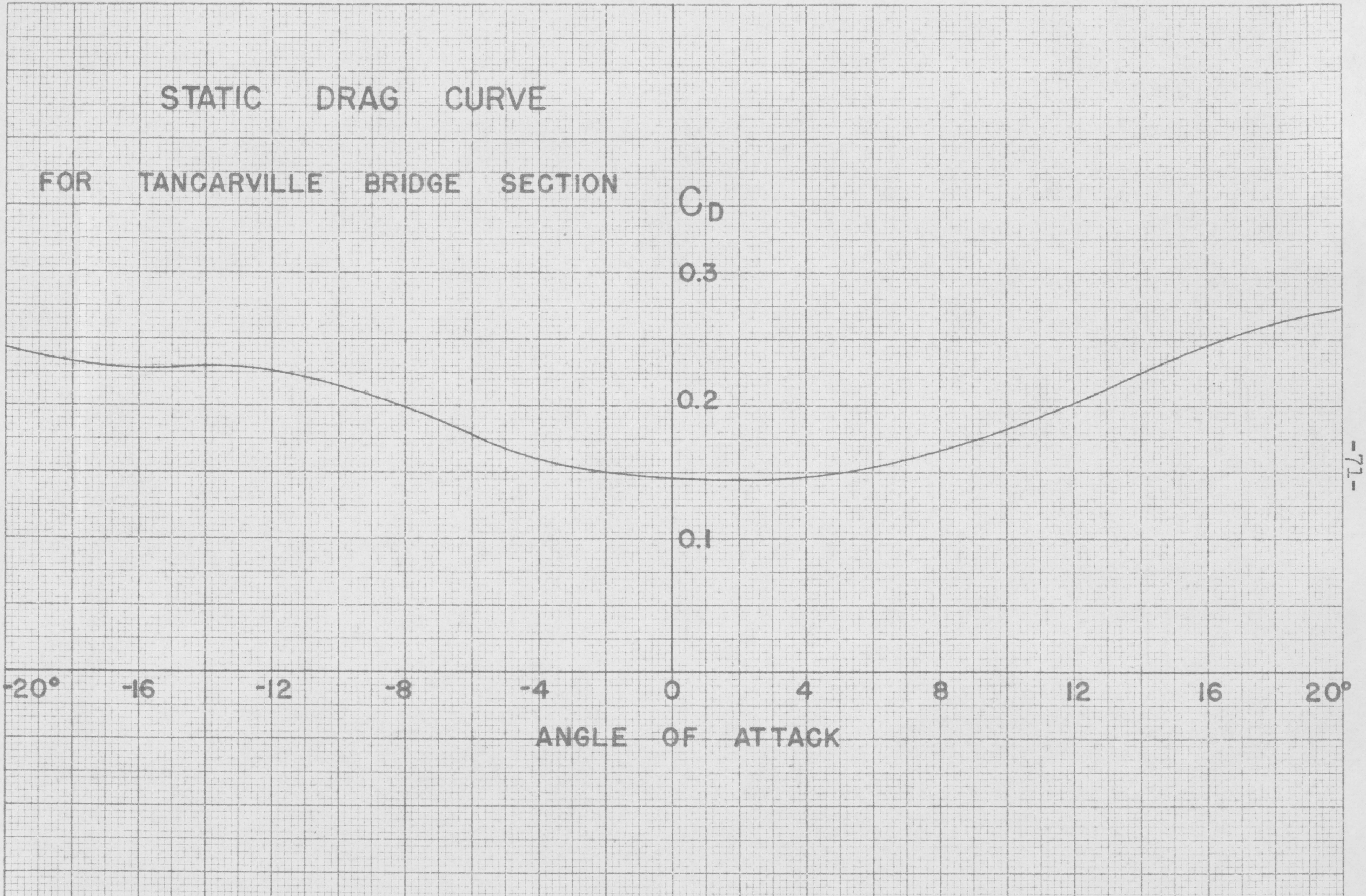

STATIC DRAG CURVE
FOR TANCARVILLE BRIDGE SECTION
C_D
0.3
0.2
0.1
-20°
-16
-12
-8
-4
0
4
8
12
16
20°
ANGLE OF ATTACK

Fig. 22c

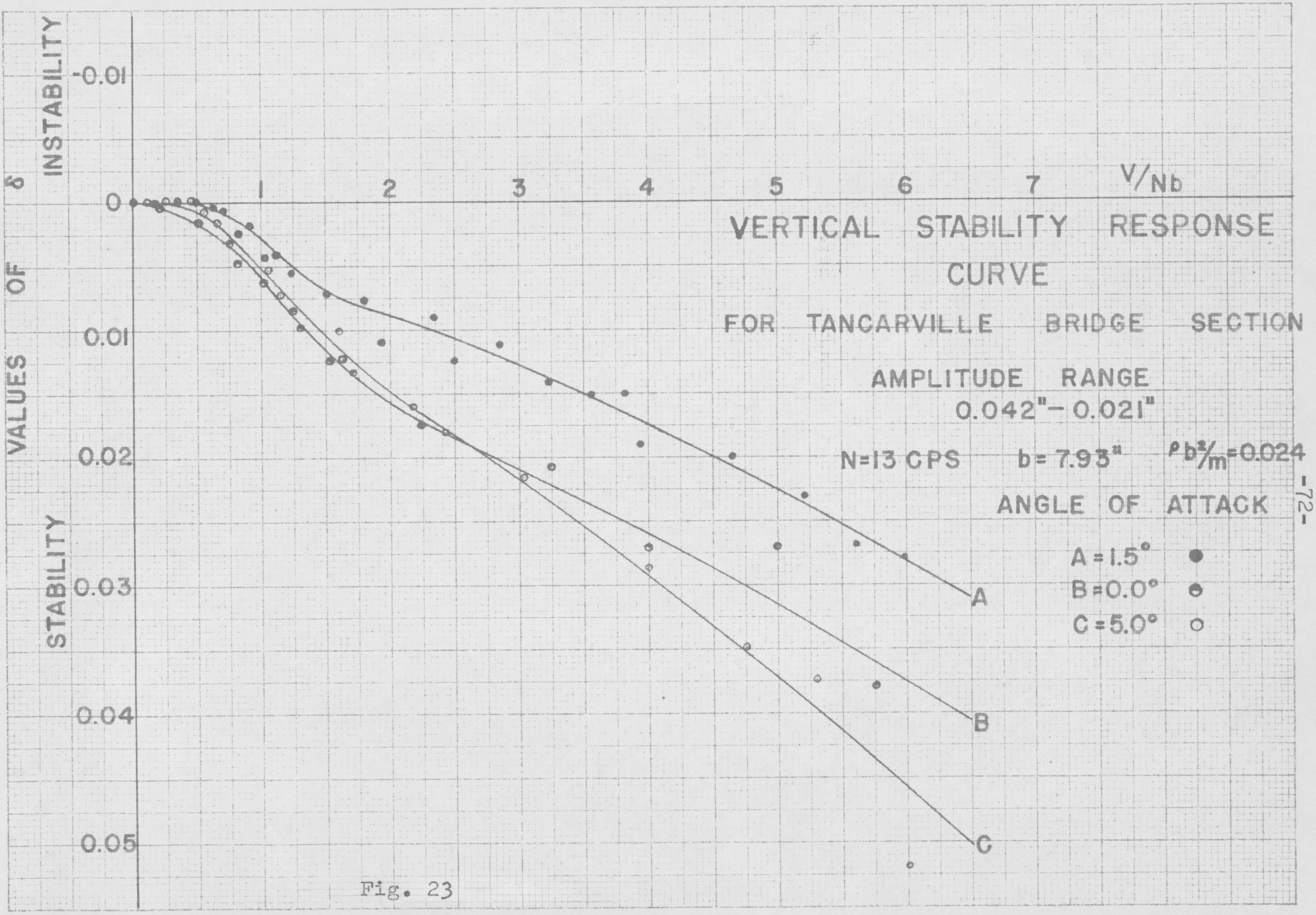
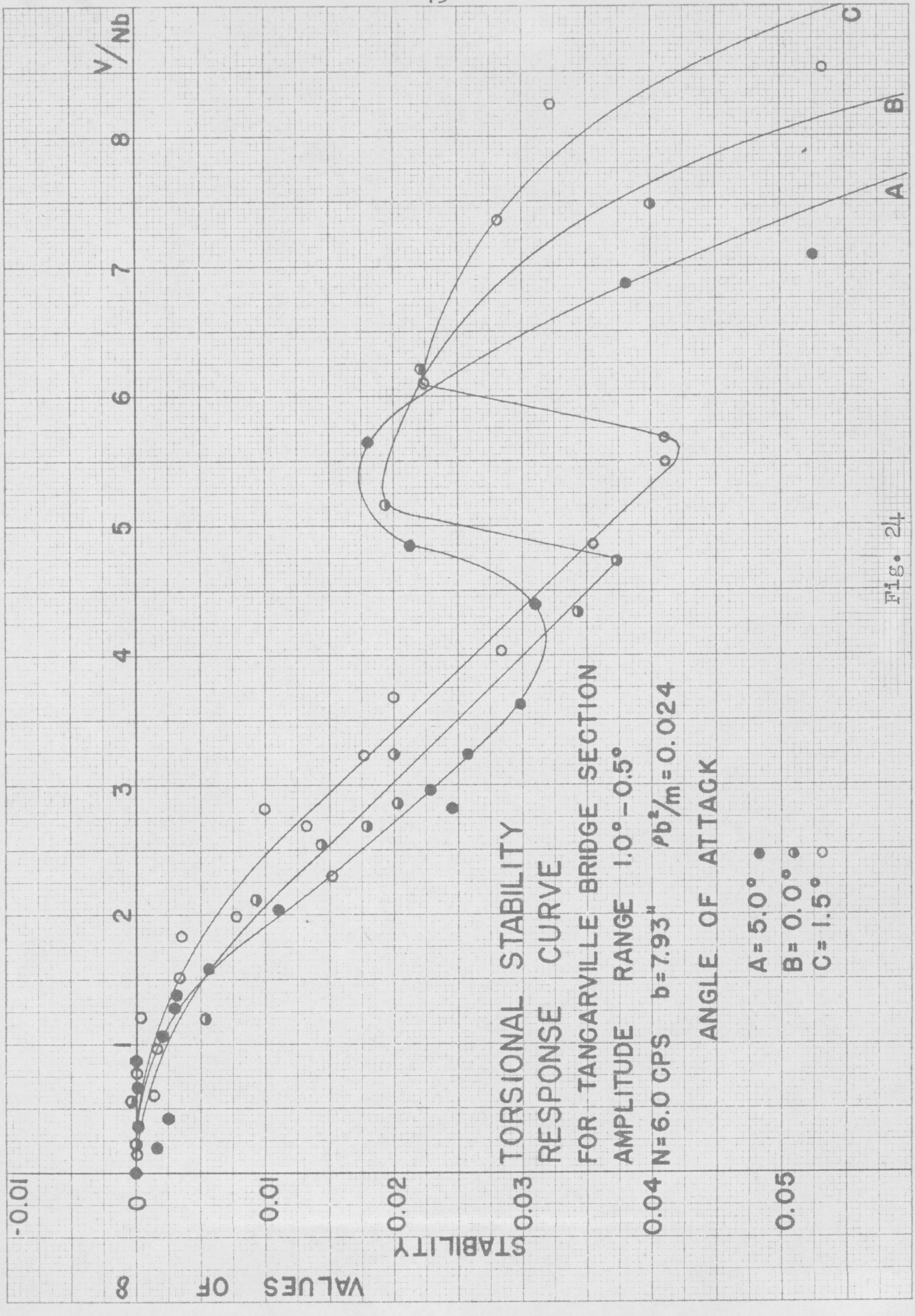

VERTICAL STABILITY RESPONSE CURVE
FOR TANCARVILLE BRIDGE SECTION
AMPLITUDE RANGE
0.042" - 0.021"
N=13 CPS
b = 7.93"
$\rho b^2/m = 0.024$
ANGLE OF ATTACK
A = 1.5° ●
B = 0.0° ◉
C = 5.0° ○
V/Nb
0
1
2
3
4
5
6
7
-0.01
0
0.01
0.02
0.03
0.04
0.05
VALUES OF δ
INSTABILITY
STABILITY
A
B
C

Fig. 23



TORSIONAL STABILITY RESPONSE CURVE
FOR TANGARVILLE BRIDGE SECTION
AMPLITUDE RANGE 1.0° – 0.5°
N = 6.0 CPS b = 7.93" $\rho b^2/m = 0.024$

ANGLE OF ATTACK

A = 5.0° ●
B = 0.0° ◉
C = 1.5° ○

Fig. 24

VI. DISCUSSION OF RESULTS

A. Test Results. In almost all cases the results of the dynamic tests checked the predictions made concerning the stability of the various sections, based on the results of the static tests. There were a few exceptions but there are several possible reasons for these exceptions.

From the results of the static tests the flat plate was predicted to be stable to the same degree over the entire range tested. This was true for both torsional motion and vertical motion. The dynamic tests showed the section to be stable over the entire range tested when vertical motion was considered. There was a slight change in the degree of stability with a change in the angle of attack but this was small and did not offer a serious disagreement with the predictions made from the static tests.

Torsionally the section was stable to the same degree over the entire range tested when the angle of attack was low (0° or 1.5°). When the angle of attack was increased to 45° a region of torsional instability developed as the velocity ratio increased above 6.4. This region of torsional instability is the only place where the results of the dynamic tests differ from the predictions made from the results of the static tests. Further tests are needed to determine the reason for this discrepancy.

The predictions made from the static tests concerning the stability of the "H" section were checked by the results of the dynamic tests. The degree of stability was slightly different than predicted but these differences are small and offer no real difficulty.

A discrepancy arises between the prediction made from the results of the static tests and the results of the dynamic tests, when torsional motion is considered in the "H" section with lateral slots and fins. The static tests show this section to be unstable at an angle of attack of 45°. The dynamic tests show the section to be stable at this angle of attack. However, the final points show the curve reducing in slope and it is the belief of the author that this curve will become unstable and that this is just a stable region in a basically unstable section. Further tests, at a higher velocity ratio, are needed to verify this belief. The section at the other angles of attack in torsional motion and at all angles of attack in vertical motion behaved exactly as predicted from the results of the static tests. Former tests on this section showed it to be neutral, that is having a zero slope to the lift and torque graphs at the origin. The results of these tests seemed to indicate that such sections are not as good as they first appeared since small details can shift the curves either way and may prove troublesome. Also scale effects may cause a shift in the curves when predictions made from model tests are applied to the prototype.

The final section tested behaved just as the static tests predicted it would except for the degree of stability. The difference was again small and proposed no serious question to the theory that dynamic behavior can be predicted from static tests. The main object behind the testing of this section was to determine the effects of detail on the final results. As shown in Figure 21 this section is essentially a flat plate modified by details. Comparing the results of these curves with those of the flat plate the following conclusions can be drawn: vertically, the curves are almost identical, with the details reducing the degree of stability only slightly; torsionally the reduction in the degree of stability is considerable but the same conclusions concerning stability may be reached. It therefore would appear that initial tests for the determination of the most desirable sections could be made on models void of most details and the more promising sections checked on more complete models.

E. Recommendations. During the course of this investigation several points were noted which need further study before a definite conclusion can be reached. The effect of frequency requires an investigation of its own. According to theory there should be no effect on the final results from varying frequencies. In this investigation two tests were run which showed an effect, due to frequency changes, on the final results. A third test was run which offers a possible explanation for this difference in results.

However no valid conclusion can be drawn from these few tests and therefore the author suggests that more work be done along this line to determine the complete answer.

It is also recommended that further tests be run on the flat plate at high angles of attack in torsional motion to determine the cause of the instability found there by this investigation. For the same reason it is recommended that additional tests be conducted on the "H" section with lateral slots and fins, subjected to torsional motion at an angle of attack of 45°.

Although a very good start has been made toward the final solution of the problem of aerodynamic stability of suspension bridges, there are still many sidelines that must be explored before the problem can be considered completely solved.

VII. CONCLUSIONS

From the results of the static tests the following conclusions are evident:

1. The flat plate is stable over the entire range tested in both vertical motion and torsional motion.
2. The "H" section, $(d/b = 0.20)$, is vertically stable and torsionally unstable.
3. The "H" section with lateral slots and fins is vertically unstable to a slight degree when the wind is nearly

horizontal but as the angle of attack increases the stability also increases. Torsionally the section is stable near zero angle of attack but becomes unstable as the angle of attack increases.

4. The proposed section for the Tancarville Bridge is stable in both modes of motion over the entire range tested.

The results of the dynamic tests reveal the same basic conclusions except that the flat plate is torsionally unstable at 45° when high velocity ratios are considered and the "H" section with lateral slots and fins remains stable at the highest angle of attack considered in torsional motion. The dynamic tests show up several unstable regions, noncatastrophic in nature, for the "H" section in vertical motion and several stable regions for the same section when torsional motion is considered. The dynamic tests show both stable and unstable regions, in both modes of motion, for the "H" section with lateral slots and fins.

When amplitude is changed the numerical values of the stability response are affected but the general shape of the curves are unaffected. Increasing the initial amplitude increases the stability of the section slightly.

When only the frequency was changed the stability of the section was changed with the stability increasing as the frequency was increased. However when the value of the initial amplitude was increased so that the product of the frequency times the amplitude remained constant the final stability response curves were unaffected.

The effects of different angles of attack were small except for torsional motion in the flat plate and vertical motion in the "H" section with lateral slots and fins. In the general case the differences caused by the angle of attack occurred only at high velocity ratios and then it only changed the degree of stability slightly. The angle of attack did change the stability characteristics in the above mentioned cases. It made the flat plate torsionally unstable at +5° whereas it had been stable at the other angles and it made the "H" section with lateral slots and fins vertically stable at +5° whereas it had been unstable at the lower values of the angle of attack.

These tests indicate that the best way to design the final section is first to conduct static tests on all proposed sections and then conduct dynamic tests on the most desirable sections. The static tests give the basic stability characteristics of the section but do not show the regions of instability that may be troublesome. From the dynamic tests, the magnitude of these regions can be determined. Such unstable regions should be minimized by modifying final designs and it would be desirable to approach a neutral section. However it is not wise to design a neutral section for the reasons already mentioned.

It further appears that the initial tests can be conducted on models void of most details. However the final sections should be checked on models complete with details.

Modification toward final design should take the form of reducing the positive slope of lift and torque graphs without actually obtaining a neutral section. The neutral section as explained previously may prove dangerous if details of the actual bridge cause a shift to negative slopes. Small positive slopes then present some factor of safety with respect to catastrophic instability.

In general, it might be concluded that the modified flat plate offers the best solution, exclusive of truss sections, to the problem of aerodynamic instability. If adequate structural stiffness and strength can be provided, no aerodynamic problem occurs at least for uncoupled motion.

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